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Online Appendix for "Governance Under Common Ownership"

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Appendix E. Governance: Additional Equilibria

This section considers additional equilibria to the most efficient equilibria focused on in Section 2.

E.1 Governance through exit: additional equilibria

While Section 2.1 compared the most efficient equilibrium under common ownership with the benchmark, Proposition 7 considers all equilibria under common ownership.

Proposition 7 (Comparison of equilibria, exit): Suppose $L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$. There is $\beta^{*} \in [0, 1)$ s.t., if $\beta \geq \beta^{*}$, any equilibrium under common ownership is strictly more efficient than any equilibrium under separate ownership.

Proof of Proposition 7. Suppose $L/n \leq \underline{v}\left(1 - F\left(\Delta\right)\right)$. From Lemma 3, the unique working threshold in any equilibrium under separate ownership is strictly smaller than Δ—that is, $c^{*}_{so,exit} < \Delta$. Moreover, since $c^{*}_{so,exit}$ solves $\phi_{exit}\left(F\left(c^{*}\right)\right) = c^{*}$, $\lim_{\beta \to 1} c^{*}_{so,exit} = \Delta\left(1 - \omega\right)$. From Lemma 4, a type (i) equilibrium under common ownership exists in which the working threshold is Δ. From the proof of Lemma 4, the only other possible equilibrium is a type (i) equilibrium in which the investor retains bad firms if $\theta = 0$. If such an equilibrium exists, the working threshold must satisfy $\psi_{exit}\left(F\left(c^{*}\right)\right) = c^{*}$. Note that, as $\beta \to 1$, any solution of $\psi_{exit}\left(F\left(c^{*}\right)\right) = c^{*}$ converges to Δ. Therefore, if β is sufficiently close to 1, any equilibrium under common ownership is strictly more efficient than any equilibrium under separate ownership. ■

Proposition 7 shows that, if β is sufficiently high, *any* equilibrium under common ownership is strictly more efficient than the separate ownership benchmark. If β is sufficiently low, there exist less efficient equilibria—these are the small-shock equilibria where the investor retains some bad firms. In such equilibria, the incentives to work are decreasing in the frequency with which a bad firm is retained. This frequency is greater if β is low, since a bad firm is always retained upon no shock, and if L is low, since a smaller shock allows the investor to retain more bad firms upon a shock. However, under the efficiency criterion, the most efficient equilibrium will be chosen and so governance is always weakly stronger under common ownership.

E.2 Governance through voice: full analysis

Recall that Proposition 2, for the exit model, studied the most efficient equilibrium under common ownership and that Proposition 7 in Online Appendix E.1 required an extra condition on β to guarantee that governance is stronger under all equilibria under common ownership. In contrast, Proposition 3, for the voice model, holds for all equilibria under common ownership, without the need for an extra condition on β. This is because, while multiple equilibria exist, they differ only in terms of the investor's trading strategy, and not her monitoring strategy. For $L/n \leq \underline{v}(1-F(\Delta))$, even if price informativeness is lower under common ownership, per-security monitoring incentives remain higher. This is because the only way in which price informativeness can be lower is if the investor retains bad firms upon a shock, and so being retained is not fully revealing that a firm has been monitored. However, this does not affect the investor's incentives to monitor, since her payoff from a monitored and retained firm is its fundamental value of $\overline{v}$, regardless of the stock price. Thus, the threshold is Δ in *any* equilibrium for which $L/n \leq \underline{v}(1-F(\Delta))$.

While unnecessary for the main result in Section 2.2 (which holds for $L/n \leq \underline{v}(1-F(\Delta))$), for completeness, Lemma 9 gives the most efficient equilibrium under common ownership when $L/n > \underline{v}(1-F(\Delta))$.

Lemma 9 Suppose $L/n > \underline{v}(1-F(\Delta))$. There are $\underline{v}(1-F(\Delta)) < \underline{y} \leq \overline{y} \leq \underline{v}$ such that the monitoring threshold in the most efficient equilibrium is given by

$$c^{**}_{co,voice} = \begin{cases} c^{**}_{ii} \equiv \textit{the largest solution of } c^* = \zeta_{voice}\left(F\left(c^*\right)\right) & \textit{if } \underline{v}\left(1-F\left(\Delta\right)\right) < L/n < \underline{y} \\ \max\{c^{**}_{ii}, c^{**}_{iii}\} & \textit{if } \underline{y} \leq L/n < \overline{y} \\ c^{**}_{iii} \equiv \textit{the largest solution of } c^* = \phi_{voice}\left(F(c^*)\right) & \textit{if } L/n \geq \overline{y}, \end{cases} \tag{32}$$

where

$$\zeta_{voice}(\tau) \equiv \Delta \left[1 - \frac{L/n - \underline{v}(1-\tau)}{\underline{v}(\frac{1-\beta}{\beta}(1-\tau)+\tau) + \Delta\tau} \frac{1-\beta+\beta\tau}{\tau} \right]. \tag{33}$$

Prices and trading strategies are characterized by Lemma 2.

Below we state and prove an auxiliary lemma used in the proof of Lemma 9.

Lemma 10 Suppose $L/n > \underline{v}(1 - F(\Delta))$. Consider an equilibrium under common ownership in which each firm is good w.p. $\tau^* = F(c^*)$ where $L/n > \underline{v}(1-\tau^*)$. Then:

(i) If the equilibrium is type (ii), then

$$\zeta_{voice}(F(c^*)) - \beta(1 - \overline{x}_{co}(F(c^*)))\Delta \leq c^* \leq \zeta_{voice}(F(c^*)), \tag{34}$$

where $\zeta_{voice}(\cdot)$ is given by Equation (33).

(ii) If the equilibrium is type (iii), then

$$c^* = \phi_{voice}(F(c^*)), \tag{35}$$

where $\phi_{voice}(\cdot)$ is given by Equation (15).

Proof of Lemma 10. Let $\Pi(c^*, c)$ be as defined in the proof of Lemma 6. Then, $c^* = F^{-1}(\tau^*)$ is an equilibrium only if $c^* \in \arg\max_{c \geq 0} \Pi(c^*, c)$. Also, let $\overline{x}^* = \overline{x}_{co}(\tau^*)$ and $\overline{p}^* = \overline{p}_{co}(\tau^*)$ if the equilibrium is type (ii) and let $\overline{x}^* = \overline{x}_{so}(\tau^*)/n$ and $\overline{p}^* = \overline{p}_{so}(\tau^*)$ if the equilibrium is type (iii).

If $\theta = 0$, the investor obtains a payoff of $\overline{v}$ from a good firm. She can obtain a payoff of $\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\underline{v}$ from a bad firm by selling $\overline{x}^*$ of each bad firm, which generates the highest payoff. Recall that, from Lemma 2, the investor has no incentives to sell less than $\overline{x}^*$ of a bad firm when $\theta = 0$. Thus, for any $x_i' < \overline{x}^*$,

$$\begin{aligned} \overline{x}^*\overline{p}^* + (1-\overline{x}^*)\underline{v} &> x_i' p(x_i') + (1-x_i')\underline{v} \\ &\Rightarrow \overline{x}^*\overline{p}^* - x_i' p(x_i') > (\overline{x}^* - x_i')\underline{v} > 0. \end{aligned}$$

Therefore, in any equilibrium, the pricing function in the range $[0, \overline{x}^*)$ must satisfy this condition. This condition also implies that the investor cannot raise more revenue from a single deviation to selling $x' < \overline{x}^*$ from one particular bad firm. Without loss of generality and to simplify the exposition, hereafter we assume $x_i^*(\overline{v}, 0) = 0$ and $x' \in (0, \overline{x}^*) \Rightarrow p(x_i') = \overline{p}^*$. These off-equilibrium prices preserve monotonicity, and $\overline{x}^*\overline{p}^* - x_i'p(x_i') > (\overline{x}^* - x_i')\underline{v}$. Moreover, note that if the investor found it optimal to monitor τ^* firms under the general pricing function (i.e., before specializing to $p(x_i') = \overline{p}^*$), deviating to sell $x' < \overline{x}^*$ from a given firm cannot be sufficiently beneficial to induce deviation from monitoring τ^* firms under the pricing rule $x' \in (0, \overline{x}^*) \Rightarrow p(x_i') = \overline{p}^*$. Indeed, since the pricing rule $x' \in (0, \overline{x}^*) \Rightarrow p(x_i') = \overline{p}^*$ is the lowest that satisfies monotonicity, a deviation from monitoring τ^* firms is less beneficial than under the general pricing function, and hence suboptimal as well. Intuitively, if she sold less than $\overline{x}^*$ from bad firms, she would receive the same price as if she sold $\overline{x}^*$, and so her incentives to monitor are no different.

Suppose $\theta = L$, and consider a type (iii) equilibrium. We argue that the investor has no incentives to deviate from selling $\overline{x}^*$ from each firm. We consider two cases.

1. Suppose $L/n \geq \underline{v}$. We first argue $\overline{x}^*\overline{p}^* \geq \underline{v}$. To see why, recall that in this case that $\overline{x}^* = \min\{\frac{L/n}{\overline{p}^*}, 1\}$. Therefore, either $\overline{x}^*\overline{p}^* = L/n \geq \underline{v}$ or $\overline{x}^* = 1$. Since $\overline{p}^* \geq \underline{v}$, $\overline{x}^* = 1 \Rightarrow \overline{x}^*\overline{p}^* \geq \underline{v}$. Second, since $\overline{x}^*\overline{p}^* \geq \underline{v}$, the investor raises more funds when she chooses $x_i = \overline{x}^*$ rather than $x_i = 1$ (when $\overline{x}^* < 1$). Therefore, she will sell $\overline{x}^*$ from each bad firm and $\frac{L/n - \overline{x}^*\overline{p}^*(1-\tau)}{\overline{p}^*\tau}$ from each good firm. If $\overline{x}^*\overline{p}^* = L/n$ then $\frac{L/n - \overline{x}^*\overline{p}^*(1-\tau)}{\overline{p}^*\tau} = \overline{x}^*$, and if $\overline{x} = 1$ then $\overline{p}^* \leq L/n$, which implies that she has to sell the entire portfolio to raise L. Either way, she will sell $\overline{x}^*$ from each firm in her portfolio.

2. Suppose $L/n < \underline{v}$. Note that $\overline{x}^* = \min\{\frac{L/n}{\overline{p}^*}, 1\}$ and $\overline{p}^* > \underline{v} > L/n$ imply $\overline{x}^*\overline{p}^* = L/n$, and so $\overline{x}^*\overline{p}^* < \underline{v}$. If the investor deviates from selling $\overline{x}^*$ from each firm, she would deviate to fully selling $\min\{\frac{L/n}{\underline{v}}, 1-\tau\}$ bad firms, selling a fraction $\overline{x}^*$ of $(1-\tau) - \min\{\frac{L/n}{\underline{v}}, 1-\tau\}$ bad firms, and selling a fraction $\hat{x} = \max\{0, \frac{L/n-(1-\tau)\underline{v}}{\tau\overline{p}^*}\}$ of all good firms. Deviation is

not strictly preferred if and only if

$$\begin{aligned}
& \tau\left[\hat{x}\overline{p}^* + (1-\hat{x})\,\overline{v}\right] + \left[(1-\tau) - \min\left\{\frac{L/n}{\underline{v}}, (1-\tau)\right\}\right]\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\underline{v}\right] + \\
& \min\left\{\frac{L/n}{\underline{v}}, (1-\tau)\right\}\underline{v} \\
\leq\; & \tau\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\overline{v}\right] + (1-\tau)\left[\overline{x}^*\overline{p}^* + (1-\overline{x}^*)\,\underline{v}\right] \Leftrightarrow \\
& \max\left\{0, \frac{L/n - (1-\tau)\,\underline{v}}{\tau\overline{p}^*}\right\} \geq \overline{x}^*\left[1 - \min\left\{\frac{L/n}{\underline{v}}, 1-\tau\right\}\frac{(\overline{p}^* - \underline{v})}{\tau(\overline{v} - \overline{p}^*)}\right] \qquad (36)
\end{aligned}$$

If $\tau \leq 1 - \frac{L/n}{\underline{v}}$, then Condition (36) holds if and only if $\tau \leq \frac{L/n}{\underline{v}}\frac{\beta\tau^*}{1-\tau^*}$, where we used $\overline{p}^* = \overline{p}_{so}(\tau^*)$. Note that

$$1 - \frac{L/n}{\underline{v}} < \frac{L/n}{\underline{v}}\frac{\beta\tau^*}{1-\tau^*} \Leftrightarrow \frac{1-\tau^*}{1-\tau^*+\beta\tau^*}\underline{v} < L/n$$

which must hold if the equilibrium is type (iii). If $\tau > 1 - \frac{L/n}{\underline{v}}$ then Condition (36) holds if and only if $L/n \geq \frac{1-\tau^*}{1-\tau^*+\beta\tau^*}\underline{v}$, which must hold if the equilibrium is type (iii). Therefore, Condition (36) holds, and deviation is suboptimal.

Combining cases 1 and 2 above, the investor has no incentives to deviate from selling $\overline{x}^*$ from each firm, and so her payoff is given by

$$\begin{aligned}
\Pi(c^*, c) &= F(c)\left[\overline{v} - \beta\overline{x}^*(\overline{v} - \overline{p}^*)\right] + (1 - F(c))\left[\underline{v} + \overline{x}^*(\overline{p}^* - \underline{v})\right] - F(c)\,E\left[c_i | c_i < c\right] \qquad (37) \\
&= \underline{v} + \Delta\left[F(c) + \overline{x}^*(\tau^* - F(c))\frac{\beta}{\beta\tau^* + 1 - \tau^*}\right] - F(c)\,E\left[c_i | c_i < c\right],
\end{aligned}$$

and so

$$\frac{\partial\Pi(c^*, c)}{\partial c}\frac{1}{f(c)} = \Delta\left[1 - \overline{x}^*\frac{\beta}{\beta\tau^* + 1 - \tau^*}\right] - c.$$

Substituting $\overline{x}^* = \overline{x}_{so}(\tau^*)$ into the first-order condition $\frac{\partial\Pi(c^*,c)}{\partial c} = 0$ yields $c^* = \phi_{voice}(F(c^*))$, as required. This completes part (iii).

Suppose $\theta = L$, and consider the type (ii) equilibrium. Recall that we must have $L/n < \underline{v}$.

Also recall that, by construction,

$$\tau^* \overline{x}^* \overline{p}^* + (1-\tau^*)\, \underline{v} = L/n.$$

Therefore, the investor can raise L by fully selling $(1-\tau^*)\, n$ bad firms and selling a fraction $\overline{x}^*$ of $\tau^* n$ good firms. Also, since $\overline{x}^* \overline{p}^* < \underline{v}$, selling $\overline{x}^*$ from every firm will not raise enough revenue to satisfy the shock. Note that, regardless of firm value, the investor has strict incentives to sell $\overline{x}^*$ rather 1. Indeed, in the latter case the payoff is $\underline{v}$, the lowest possible. Therefore, regardless of the proportion of good firms (i.e., even if $\tau \neq \tau^*$), she will fully sell exactly $(1-\tau^*)\, n$ firms and a fraction $\overline{x}^*$ of all other firms. The investor will prefer fully selling a bad firm if there are sufficient numbers. Therefore, her expected payoff from choosing cutoff c is:

$$\begin{aligned}
\Pi(c^*, c) &= (1-F(c)) \left[\begin{array}{c} (1-\beta)\left[\overline{x}^* \overline{p}^* + (1-\overline{x}^*)\, \underline{v}\right] \\ +\beta \left(\begin{array}{c} \underline{v} \min\left\{1, \frac{1-\tau^*}{1-F(c)}\right\} \\ +\left[\overline{x}^* \overline{p}^* + (1-\overline{x}^*)\, \underline{v}\right] \left(1 - \min\left\{1, \frac{1-\tau^*}{1-F(c)}\right\}\right) \end{array} \right) \end{array} \right] \\
&\quad + F(c) \left[(1-\beta)\, \overline{v} + \beta \left(\begin{array}{c} \underline{v} \max\left\{0, \frac{F(c)-\tau^*}{F(c)}\right\} \\ +\left[\overline{x}^* \overline{p}^* + (1-\overline{x}^*)\, \overline{v}\right] \left(1 - \max\left\{0, \frac{F(c)-\tau^*}{F(c)}\right\}\right) \end{array} \right) \right] \\
&\quad - F(c)\, E\left[c_i | c_i < c\right].
\end{aligned}$$

Using $\overline{x}^* = \overline{x}_{co}(\tau^*)$ and $\overline{p}^* = \overline{p}_{co}(\tau^*)$, we obtain:

$$\begin{aligned}
\Pi(c^*, c) &= \underline{v} + F(c)\,\Delta + \overline{x}^* (\tau^* - F(c))\, \beta\Delta \frac{1-\beta+\beta\tau^*}{\beta\tau^* + (1-\beta)(1-\tau^*)} - \beta(1-\overline{x}^*)\,\Delta \max\left\{F(c) - \tau^*, 0\right\} \\
&\quad - F(c)\, E\left[c_i | c_i < c\right].
\end{aligned}$$

Note that

$$\frac{\partial \Pi(c^*, c)}{\partial c} \frac{1}{f(c)} = \zeta_{voice}(\tau^*) - c - \begin{cases} 0 & \textit{if } F(c) < \tau^* \\ \beta(1-\overline{x}^*)\,\Delta & \textit{if } F(c) > \tau^*, \end{cases}$$

which is a strictly decreasing function of c, and $\zeta_{voice}(\cdot)$ is given by Equation (33). Since f is strictly positive, $f(0) > 0$ and $\frac{\partial \Pi(c^*,c)}{\partial c}\frac{1}{f(c)}|_{c=0} > 0$. Therefore, $\Pi(c^*, c)$ is maximized at

$$\arg\max_{c\geq 0} \Pi(c^*, c) = \begin{cases} \zeta_{voice}(\tau^*) & \textit{if } F(\zeta_{voice}(\tau^*)) < \tau^* \\ F^{-1}(\tau^*) & \textit{if } F(\zeta_{voice}(\tau^*) - \beta(1-\overline{x})\Delta) \leq \tau^* \leq F(\zeta_{voice}(\tau^*)) \\ \zeta_{voice}(\tau^*) - \beta(1-\overline{x})\Delta & \textit{if } \tau^* < F(\zeta_{voice}(\tau^*) - \beta(1-\overline{x})\Delta). \end{cases}$$

In equilibrium, we require $c^*(\tau^*) = F^{-1}(\tau^*)$. Therefore, τ^* must satisfy

$$\zeta_{voice}(\tau^*) - \beta(1-\overline{x}(\tau^*))\Delta \leq F^{-1}(\tau^*) \leq \zeta_{voice}(\tau^*),$$

as required. ■

The full proof of Lemma 9 now follows.

Proof of Lemma 9. We prove the result in three steps. First, suppose $L/n \geq \underline{v}$. Based on Lemma 2, the equilibrium must be type (iii). Based on part (ii) of Lemma 10, the monitoring threshold must solve $c^* = \phi_{voice}(F(c^*))$. Note that $\phi_{voice}(F(c))$ is continuous, $\phi_{voice}(F(0)) = \Delta(1-\beta)$ and $\phi_{voice}(1) = \Delta(1 - \min\{\frac{L/n}{\underline{v}+\Delta}, 1\})$, and hence, by the intermediate value theorem, a solution always exists. Given a threshold that satisfies $c^* = \phi_{voice}(F(c^*))$, by construction there is a type (iii) equilibrium with this threshold.

Second, we prove that if $\underline{v}(1 - F(\Delta)) < L/n < \underline{v}$, there always exists a type (ii) equilibrium where the monitoring threshold is given by part (i) of Lemma 10—that is, the largest solution of $c^* = \zeta_{voice}(F(c^*))$. In particular, it is sufficient to show that $c^* = \zeta_{voice}(F(c^*))$ has a solution such that $F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) < c^*$ (which is equivalent to $\underline{v}(1-\tau^*) < L/n$). Indeed, when $c^* = F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)$ then $\zeta_{voice}(F(c^*)) = \Delta$. Since $\underline{v}(1 - F(\Delta)) < L/n$, then $c^* = F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right) \Rightarrow \zeta_{voice}(F(c^*)) > F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)$. Furthermore, when $F(c^*) = 1$ then $\zeta_{voice}(F(c^*)) = \Delta\left[1 - \frac{L/n}{\underline{v}+\Delta\tau}\right] < \infty$, since $F(c^*) = \tau^* = 1$. Since $\zeta_{voice}(F(c^*))$ is continuous in c^*, by the intermediate value theorem, a solution strictly greater than $F^{-1}\left(1 - \frac{L/n}{\underline{v}}\right)$ always exists. By construction, there is a type (ii) equilibrium with such a threshold.

Third, suppose $\underline{v}(1 - F(\Delta)) < L/n < \underline{v}$. We compare the efficiency of the sustainable equilibria. First note that any type (i) equilibrium is less efficient than a type (ii) equilibrium.

Indeed, in the former case the equilibrium threshold c_i^* must satisfy $L/n \le \underline{v}(1-F(c_i^*))$, and in the latter case the equilibrium threshold c_{ii}^* must satisfy $L/n > \underline{v}(1-F(c_{ii}^*))$. Therefore, $c_{ii}^* > c_i^*$, as required. Next, consider type (iii) equilibria. When $L/n < \underline{v}$, such equilibria exhibit $\overline{x}^*\overline{p}^* = L/n$, where $\overline{p}^* = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau}$. Therefore, whenever these equilibria exist,

$$\phi_{voice}(\tau) \equiv \Delta\left[1-\beta\frac{L/n}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau}\right].$$

Note that $\zeta_{voice}(\tau) > \phi_{voice}(\tau)$ if and only if

$$\Delta\left[1-\frac{L/n-\underline{v}(1-\tau)}{\underline{v}\left(\frac{1-\beta}{\beta}(1-\tau)+\tau\right)+\Delta\tau}\frac{1-\beta+\beta\tau}{\tau}\right] \;>\; \Delta\left[1-\beta\frac{L/n}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau}\right] \Leftrightarrow$$

$$\left(\frac{1-\beta}{1-\beta+\beta\tau}+\frac{\beta\tau}{1-\beta+\beta\tau}\frac{\underline{v}}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau}\right)L/n \;<\; \underline{v}. \tag{38}$$

Also note that

$$1 \ge \frac{1-\beta}{1-\beta+\beta\tau}+\frac{\beta\tau}{1-\beta+\beta\tau}\frac{\underline{v}}{\underline{v}+(\Delta\beta-\underline{v}(1-\beta))\tau} \Leftrightarrow \beta \ge \frac{\underline{v}}{\underline{v}+\Delta}.$$

Therefore, if $\beta \ge \frac{\underline{v}}{\underline{v}+\Delta}$, then Condition (38) always holds, which implies that the most efficient equilibrium is type (ii). In this case, $\underline{y} = \overline{y} = \underline{v}$. In other words, whenever a type (ii) equilibrium exists (i.e., $\underline{v}(1-F(\Delta)) < L/n < \underline{v}$), it is the most efficient equilibrium.

Suppose $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$. Note that Condition (38) is equivalent to $\Gamma(\tau) < 0$, where

$$\Gamma(\tau) = \tau^2 - \tau\left[\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} - \frac{1-\beta}{\beta}\right]\frac{\underline{v}-L/n}{\underline{v}} - \frac{1-\beta}{\beta}\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\frac{\underline{v}-L/n}{\underline{v}}.$$

Note that $\min\Gamma(\tau) < 0$. Also, recall $\underline{v}(1-\tau_{ii}^{**}) < L/n$. Therefore, it is sufficient to focus on $\underline{v}(1-\tau) < L/n \Leftrightarrow \frac{\underline{v}-L/n}{\underline{v}} < \tau$. It can be verified that $\Gamma\left(\frac{\underline{v}-L/n}{\underline{v}}\right) < 0$. Therefore, there is

$\hat{\tau} > \frac{\underline{v}-L/n}{\underline{v}}$ such that $\Gamma(\tau) \geq 0 \Leftrightarrow \tau \geq \hat{\tau}$ where $\hat{\tau}$ is the largest root of $\Gamma(\tau)$, given by

$$\hat{\tau} = \frac{1}{2}\frac{\underline{v}-L/n}{\underline{v}}\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} - \frac{1-\beta}{\beta}\right) + \frac{1}{2}\frac{\underline{v}-L/n}{\underline{v}}\sqrt{\left(\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} - \frac{1-\beta}{\beta}\right)^2 + 4\frac{1-\beta}{\beta}\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta}\frac{\underline{v}}{\underline{v}-L/n}}. \tag{39}$$

Note that a type (iii) equilibrium requires

$$\underline{v}\frac{1-\tau}{\beta\tau+1-\tau} < L/n \Leftrightarrow \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta} < \tau$$

where $\frac{\underline{v}-L/n}{\underline{v}} < \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}$. Also note that $\tau^* < F(\Delta)$ in both a type (ii) and type (iii) equilibrium. Therefore, the relevant range is $\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta} \leq \tau \leq F(\Delta)$. This interval is non-empty if and only if

$$\frac{\underline{v}}{1+\frac{F(\Delta)}{1-F(\Delta)}\beta} < L/n \Leftrightarrow \frac{\underline{v}-L/n}{L/n}\frac{1-F(\Delta)}{F(\Delta)} < \beta.$$

Note that $\underline{v}(1-F(\Delta)) < \frac{\underline{v}}{1+\frac{F(\Delta)}{1-F(\Delta)}\beta}$ for all β. Since $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$ if $L/n < \frac{\underline{v}}{1+\frac{F(\Delta)}{1-F(\Delta)}\frac{\underline{v}}{\underline{v}+\Delta}}$, the most efficient equilibrium is type (ii). This establishes the existence of $\underline{y}$, the threshold below which a type (ii) equilibrium is most efficient.

Suppose

$$\frac{\underline{v}-L/n}{L/n}\frac{1-F(\Delta)}{F(\Delta)} < \beta < \frac{\underline{v}}{\underline{v}+\Delta}. \tag{40}$$

If $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$, then $\phi_{voice}(\tau)$ is a decreasing function, and so τ^{**}_{iii}, given by the solution of $\tau = F(\phi_{voice}(\tau))$, is unique. Therefore, the equilibrium with τ^{**}_{iii} is most efficient if and only if

$$\max\left\{\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}, \hat{\tau}\right\} < \tau^{**}_{iii}.$$

We now prove that $\hat{\tau} \geq \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}$. To do so, we first prove that

$$\hat{\tau} < x \Leftrightarrow \frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}}-\beta} < x\frac{\frac{1-\beta}{\beta}+x\frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+x}. \tag{41}$$

To see this, note that

$$\hat{\tau} \geq x \Leftrightarrow \sqrt{\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right)^2+4\frac{1-\beta}{\beta}\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}\frac{v}{\underline{v}-L/n}} \geq 2x\frac{v}{\underline{v}-L/n}-\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) \Leftrightarrow$$

$$\begin{aligned}
2x\frac{v}{\underline{v}-L/n}-\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) &< 0 \text{ or} \\
2x\frac{v}{\underline{v}-L/n}-\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) &\geq 0 \text{ and} \\
4\frac{1-\beta}{\beta}\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}\frac{v}{\underline{v}-L/n} &\geq \left[2x\frac{v}{\underline{v}-L/n}\right]^2-2\left[2x\frac{v}{\underline{v}-L/n}\right]\left(\frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta}-\frac{1-\beta}{\beta}\right) \Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
2x\frac{v}{\underline{v}-L/n}+\frac{1-\beta}{\beta} &< \frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta} \text{ or} \\
2x\frac{v}{\underline{v}-L/n}+\frac{1-\beta}{\beta} &\geq \frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta} \text{ and } \frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta} \geq \frac{x\frac{1-\beta}{\beta}+x^2\frac{v}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+x}.
\end{aligned}$$

Note

$$2x\frac{v}{\underline{v}-L/n}+\frac{1-\beta}{\beta} > \frac{x\frac{1-\beta}{\beta}+x^2\frac{v}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+x} \Leftrightarrow 2x\frac{v}{\underline{v}-L/n}\frac{1-\beta}{\beta}+x^2\frac{v}{\underline{v}-L/n}+\left[\frac{1-\beta}{\beta}\right]^2 > 0,$$

which proves Condition (41). Using Condition (41), we have

$$\hat{\tau} > \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta} \Leftrightarrow \frac{\frac{v}{\Delta+\underline{v}}}{\frac{v}{\Delta+\underline{v}}-\beta} > \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}\frac{\frac{1-\beta}{\beta}+\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}\frac{v}{\underline{v}-L/n}}{\frac{1-\beta}{\beta}+\frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}}.$$

This eventually yields

$$\frac{\beta}{\frac{v}{\Delta+\underline{v}}-\beta}\left(1+\frac{L/n}{\underline{v}-L/n}\beta\right) > -\frac{L/n}{\underline{v}-L/n}\frac{\beta\frac{L/n}{\underline{v}-L/n}(1-\beta)}{\frac{1}{\beta}+\frac{L/n}{\underline{v}-L/n}(1-\beta)},$$

which always holds.

Since $\hat{\tau} \geq \frac{1}{1+\frac{L/n}{\underline{v}-L/n}\beta}$, τ_{iii}^{**} is most efficient only if $\hat{\tau} < \tau_{iii}^{**}$ and $\beta < \frac{\underline{v}}{\underline{v}+\Delta}$, that is,

$$\frac{\frac{\underline{v}}{\Delta+\underline{v}}}{\frac{\underline{v}}{\Delta+\underline{v}} - \beta} < \tau_{iii}^{**} \frac{\frac{1-\beta}{\beta} + \tau_{iii}^{**} \frac{\underline{v}}{\underline{v}-L/n}}{\frac{1-\beta}{\beta} + \tau_{iii}^{**}}.$$

Note that $\lim_{L/n \to \underline{v}} \tau_{iii}^{**} > 0 = \lim_{L/n \to \underline{v}} \hat{\tau}$. By continuity, there is $\overline{y} \in [\underline{y}, \underline{v})$ such that, if $L/n > \underline{y}$, the most efficient equilibrium is type (iii). ■

Appendix F. Robustness: Full Analysis

F.1 Two firms

We define $\mathbf{x} \equiv (x_i, x_j)$, $\mathbf{v} \equiv (v_i, v_j)$, $\mathbf{x}^T \equiv (x_j, x_i)$, $\mathbf{v}^T \equiv (v_j, v_i)$, $\overline{\mathbf{v}} \equiv (\overline{v}, \overline{v})$, and $\underline{\mathbf{v}} \equiv (\underline{v}, \underline{v})$. We denote by $\mathbf{e}(\theta, \mathbf{v}) \equiv (e_i(\theta, \mathbf{v}), e_j(\theta, \mathbf{v}))$ the equilibrium strategy of type $(\theta, \mathbf{v})$. By symmetry, $p_i(\mathbf{x}) = p_j(\mathbf{x}^T)$ for all x_i and x_j, and $e_j(\theta, \mathbf{v}) = e_i(\theta, \mathbf{v}^T)$. We therefore omit the subscript whenever there is no risk of confusion. Let $\Pi^*(\theta, \mathbf{v}) \equiv \Pi(\mathbf{e}(\theta, \mathbf{v}), \mathbf{v})$ denote the equilibrium payoff of type $(\theta, \mathbf{v})$, where

$$\Pi(\mathbf{x}, \mathbf{v}) = x_i p_i(\mathbf{x}) + (n/2 - x_i) v_i + x_j p_i(\mathbf{x}^T) + (n/2 - x_j) v_j.$$

We start by analyzing the case of separate market makers and then a single market maker. We focus on the case of small liquidity shocks ($L/n \leq \underline{v}/2$, so that a shock can be met by fully selling one bad firm) since this is where our results are strongest.

F.1.1 Separate market makers

Lemma 11 (Common ownership, two firms, separate market makers): Suppose $L/n \leq \underline{v}/2$. An equilibrium under common ownership always exists and is unique.[31]

[31] If $L/n \leq \underline{v}/2$ and $\tau = \frac{1}{1+\beta}$ then the equilibria in parts (i) and (ii) coexist.

(i) If $\tau \geq \frac{1}{1+\beta}$, then

$$x^*_{co,i}(v_i, v_j, \theta) = \begin{cases} 0 & if\ \ v_i = \overline{v}\ \ and\ \ \theta = 0 \\ \overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)} & else \end{cases} \tag{42}$$

and prices of firm i are

$$p^*_i(x_i) = \begin{cases} \overline{v} & if\ \ x_i = 0 \\ \overline{p}_{co}(\tau) = \underline{v} + \frac{\beta\tau}{\beta\tau+1-\tau}\Delta & if\ \ x_i \in (0, \overline{x}_{co}(\tau)], \\ \underline{v} & if\ \ x_i > \overline{x}_{co}(\tau). \end{cases}$$

(ii) If $\tau < \frac{1}{1+\beta}$, then

$$x^*_{co,i}(\mathbf{v}, \theta) = \begin{cases} 0 & if\ \ \mathbf{v} = (\overline{v}, \underline{v}),\ or\ \mathbf{v} = \overline{\mathbf{v}}\ and\ \theta = 0 \\ \overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)} & if\ \ \mathbf{v} = (\underline{v}, \overline{v})\ \ and\ \ \theta = 0,\ \mathbf{v} = \underline{\mathbf{v}}\ ,\ or\ \mathbf{v} = \overline{\mathbf{v}}\ and\ \theta = L \\ n/2 & if\ \ \mathbf{v} = (\underline{v}, \overline{v})\ \ and\ \ \theta = L, \end{cases} \tag{43}$$

and prices of firm i are

$$p^*_i(x_i) = \begin{cases} \overline{v} & if\ \ x_i = 0 \\ \overline{p}_{co}(\tau) = \underline{v} + \frac{\tau^2\beta}{(1-\beta)(1-\tau)+(\tau^2+(1-\tau)^2)\beta}\Delta & if\ \ x_i \in (0, \overline{x}_{co}(\tau)], \\ \underline{v} & if\ \ x_i > \overline{x}_{co}(\tau). \end{cases} \tag{44}$$

The intuition is as follows. If $\tau \geq \frac{1}{1+\beta}$, then the investor retains a firm only if it is good and she suffers no shock. If the firm is bad, or if she suffers a shock, she sells the firm to the same degree ($\overline{x}_{co}(\tau)$ in this case), just as under separate ownership. In particular, even if firm i is good and firm j is bad, she still sells firm i upon a shock. Even though she could meet her liquidity need by fully selling only firm j, doing so would lead to the lowest possible price of $\underline{v}$. When $\tau \geq \frac{1}{1+\beta}$, the probability of a good firm (τ) and the probability of a liquidity shock

(β) are both high, and so the price of a partially sold firm $\overline{p}_{co}(\tau)$ is also high as there is a high probability that this is a good firm sold due to a shock. Thus, the investor prefers to meet the shock by selling only $\overline{x}_{co}(\tau)$ of the bad firm, even though doing so also requires her to sell $\overline{x}_{co}(\tau)$ of the good firm to meet the shock. Put differently, even though common ownership gives the investor the flexibility to meet the shock by selling only bad firms, she chooses not to take advantage of this flexibility. This issue does not arise with a continuum of firms since it is never the case that all firms are good. Thus, a sold firm cannot be a good firm sold due to a shock, and so it receives the lowest possible price of $\underline{v}$.

If $\tau < \frac{1}{1+\beta}$, then the price of a partially-sold firm is sufficiently low that an investor with exactly one bad firm does take advantage of her flexibility to meet a liquidity shock by selling only the bad firm and retaining the good one.

F.1.2 Single market maker

Here, the market maker can condition p_i on x_j. We focus on equilibria with non-increasing prices in the following sense: If $0 \leq x < x + \varepsilon \leq n/2$ then $p_i(\mathbf{x}) \geq p_i(\mathbf{x} + \varepsilon)$ and $p_j(\mathbf{x}) \geq p_j(\mathbf{x} + \varepsilon)$. In other words, among balanced exit strategies, if an investor sells less of all firms, the prices of all firms must (weakly) increase.

Lemma 12 (Common ownership, two firms, single market maker): Suppose $L/n \leq \underline{v}/2$. An equilibrium under common ownership always exists.[32]

(i) If $\tau > \frac{1}{1+\sqrt{\beta}}$, then the equilibrium is unique where:

$$x^*_{co,i}(v_i, v_j, \theta) = \begin{cases} 0 & \textit{if } \mathbf{v} = (\overline{v}, \overline{v}) \textit{ and } \theta = 0 \\ \overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)} & \textit{else} \end{cases} \tag{45}$$

[32] If $L/n \leq \underline{v}/2$ and $\tau = \frac{1}{1+\sqrt{\beta}}$, then the equilibria in parts (i) and (ii) coexist.

and prices of firm i are

$$p_i^*(x_i,x_j)=\begin{cases}\overline{v} & \textit{if } \; x_i=0\\ \overline{p}_{co}(\tau)=\underline{v}+\tau\frac{\beta+(1-\beta)(1-\tau)}{\beta+(1-\beta)(1-\tau^2)}\Delta & \textit{if } \; 0<x_i\le\min\{\overline{x}_{co}(\tau),x_j\},\\ \underline{v} & \textit{if } \; \min\{\overline{x}_{co}(\tau),x_j\}<x_i\end{cases}$$

(ii) If $\tau<\frac{1}{1+\sqrt{\beta}}$, then in any equilibrium

$$x_{co,i}^*(\mathbf{v},\theta)=\begin{cases}0 & \textit{if } \; \mathbf{v}=\overline{\mathbf{v}} \textit{ and } \theta=0, \textit{ or } \mathbf{v}=(\overline{v},\underline{v}) \; \textit{ and } \theta=L\\ \overline{x}_{co}(\tau)=\frac{L/2}{\overline{p}_{co}(\tau)} & \textit{if } \; \mathbf{v}=\overline{\mathbf{v}} \textit{ and } \theta=L, \textit{ or } \mathbf{v}=\underline{\mathbf{v}}\\ n/2 & \textit{if } \; \mathbf{v}=(\underline{v},\overline{v}) \; \textit{ and } \theta=L\end{cases} \tag{46}$$

and

$$\left(x_{co,i}^*((\underline{v},\overline{v}),0),x_{co,j}^*((\underline{v},\overline{v}),0)\right)\neq(\overline{x}_{co}(\tau),\overline{x}_{co}(\tau)).$$

Moreover,

$$\overline{p}_{co}(\tau)=\underline{v}+\frac{\beta\tau^2}{\beta\left((1-\tau)^2+\tau^2\right)+(1-\beta)(1-\tau)^2}\Delta.$$

The equilibrium with the lowest price informativeness satisfies

$$\left(x_{co,i}^*((\underline{v},\overline{v}),0),x_{co,j}^*((\underline{v},\overline{v}),0)\right)=(0,0)$$

and

$$p_i^*(x_i,x_j)=\begin{cases}p(0)=\underline{v}+\Delta\frac{\tau(1-\tau)+\tau^2}{\tau(1-\tau)+(1-\tau)\tau+\tau^2} & \textit{if } \; x_i=0\\ \overline{p}_{co}(\tau) & \textit{if } \; 0<x_i\le\min\{\overline{x}_{co}(\tau),x_j\}\\ \underline{v} & \textit{if } \; \min\{\overline{x}_{co}(\tau),x_j\}<x_i\end{cases}.$$

The equilibria are similar to the separate market maker case, except for when the investor has exactly one bad firm and does not suffer a shock. With separate market makers, the investor voluntarily sells $\overline{x}_{co}(\tau)$ of the bad firm, to disguise the sale as being of a good firm and due to a shock, and retains the good firm. With a single market maker, such disguise is no longer possible, since the market maker would see that the good firm is fully retained, infer that there has been no shock, and price the partially-sold firm at $\underline{v}$. She may thus choose to sell both the bad and good firm to the same degree, to disguise their sale as being motivated by a shock. The lowest level of price informativeness arises when the investor retains both firms upon no shock, because now being fully retained no longer fully reveals that a firm is good.

F.1.3 Price informativeness

Proposition 8 (Price informativeness, two firms): Suppose $L/n \leq \underline{v}/2$.

(i) Separate market makers: If $\tau < \frac{1}{1+\beta}$, then in any equilibrium under separate and common ownership:

$$P_{co,Separate}\left(\overline{v},\tau\right) > P_{so}\left(\overline{v}\right) \text{ and } P_{co,Separate}\left(\underline{v},\tau\right) < P_{so}\left(\underline{v}\right).$$

(ii) Single market maker: There is $\overline{\beta} \in (0,1)$ such that, if $\tau < \frac{1}{2}$ and $\beta \in (\overline{\beta}, 1]$, then in any equilibrium under separate and common ownership:

$$P_{co,Single}\left(\overline{v},\tau\right) > P_{so}\left(\overline{v}\right) \text{ and } P_{co,Single}\left(\underline{v},\tau\right) < P_{so}\left(\underline{v}\right).$$

The intuition is as follows; we discuss the required parameters for the separate market makers case but the intuition is similar for the single market maker case. Price informativeness can be higher under common ownership only if $\tau < \frac{1}{1+\sqrt{\beta}}$, because only in this case does an investor with one bad firm take advantage of her flexibility to meet a liquidity shock by selling only the bad firm. For price informativeness to be higher under common ownership in *any* equilibrium, the price of a bad firm must be lower than under separate ownership. Under separate ownership, a bad firm's price is increasing in β, since the higher the probability of a shock, the higher the probability that a partial sale is of a good firm due to a shock. Thus, a high β (to increase the price of a bad firm under separate ownership) and a low τ (so that $\tau < \frac{1}{1+\sqrt{\beta}}$) ensure that price informativeness is higher under common ownership in any equilibrium.

F.1.4 Proofs

Proof of Lemma 11. We start by proving several claims.

1. In any equilibrium, there is a unique $\overline{x}_G \in (0, n/2)$ such that $\mathbf{e}(L, \overline{\mathbf{v}}) = \overline{\mathbf{x}}_G$. Proof: A symmetric equilibrium requires the investor to sell the same quantities from both firms if they have the same value. A pure strategy equilibrium requires $\overline{x}_G$ to be unique. Since $\beta > 0$, such $\overline{x}_G$ exists. Since $L > 0$, if $\overline{x}_G = 0$ then the investor has a profitable deviation to selling $n/2$ units from each firm and raising more revenue. If $\overline{x}_G = n/2$, since $p(\overline{x}_G) \geq \underline{v}$, $\overline{x}_G p(\overline{x}_G) \geq n/2\underline{v} \geq L > L/2$, which contradicts $\overline{x}_G p(\overline{x}_G) = L/2$ that we prove in claim 2 below.

2. In any equilibrium, (i) $p_i(\overline{x}_G) \in (\underline{v}, \overline{v})$; (ii) $\overline{x}_G p(\overline{x}_G) = L/2$. Proof of part (i): Since $\beta > 0$, $p(\overline{x}_G) > \underline{v}$. Suppose on the contrary $p(\overline{x}_G) = \overline{v}$. Since prices are non-increasing, there is $\bar{x} \geq \overline{x}_G$ such that $x \leq \bar{x} \Rightarrow p(x) = \overline{v}$ and either $\bar{x} = n/2$ or $x > \bar{x} \Rightarrow p(x) < \overline{v}$. Let $\overline{x}_B = e_i(L, \underline{\mathbf{v}}) = e_j(L, \underline{\mathbf{v}})$. Then, $p(\overline{x}_B) < \overline{v}$. Moreover, since $x \leq \bar{x} \Rightarrow p(x) = \overline{v}$, it must be $\overline{x}_B > \bar{x}$. If $p(\overline{x}_B) = \underline{v}$ then type $\underline{\mathbf{v}}$ has a profitable deviation to $(\bar{x}, \bar{x})$. Therefore, $p(\overline{x}_B) > \underline{v}$, which requires either $\mathbf{e}(L, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_B$ or $\mathbf{e}(0, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_B$. Type $(\overline{v}, \underline{v})$ prefers $\overline{\mathbf{x}}_B$ over $\overline{\mathbf{x}}_G$ only if

$$\begin{aligned} 2\overline{x}_B p(\overline{x}_B) + (n/2 - \overline{x}_B)(\underline{v} + \overline{v}) &\geq 2\overline{x}_G \overline{v} + (n/2 - \overline{x}_G)(\underline{v} + \overline{v}) \Leftrightarrow \\ p(\overline{x}_B) &\geq \frac{\overline{x}_G}{\overline{x}_B} \frac{\overline{v} - \underline{v}}{2} + \frac{\overline{v} + \underline{v}}{2} \end{aligned}$$

which is strictly greater than $\frac{\overline{v}+\underline{v}}{2}$. However, note that

$$p(\overline{x}_B) \leq \underline{v} + \frac{\tau(1-\tau)\Delta}{\tau(1-\tau) + (1-\tau)\tau + \beta(1-\tau^2)}$$

that is, the highest possible value of $p(\overline{x}_B)$ arises when type $(\overline{v}, \underline{v})$ chooses $\overline{\mathbf{x}}_B$ regardless of her liquidity need and type $\underline{\mathbf{v}}$ chooses $\overline{\mathbf{x}}_B$ only when there is a shock. In addition,

$$\underline{v} + \frac{\tau(1-\tau)\Delta}{\tau(1-\tau) + (1-\tau)\tau + \beta(1-\tau^2)} < \frac{\overline{v} + \underline{v}}{2} \Leftrightarrow 0 < \beta\left(1 - \tau^2\right)$$

which always holds. Therefore, type $(\overline{v}, \underline{v})$ never chooses $\overline{\mathbf{x}}_B$, a contradiction.

Proof of part (ii): Suppose on the contrary $\overline{x}_G p(\overline{x}_G) > L/2$. Since prices are non-increasing, there is $\varepsilon > 0$ such that $(\overline{x}_G - \varepsilon) p(\overline{x}_G - \varepsilon) \geq L/2$. Since $p(\overline{x}_G) < \overline{v}$, if $\theta = L$ then type $\underline{\mathbf{v}}$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to $(\overline{x}_G - \varepsilon, \overline{x}_G - \varepsilon)$. Suppose on the contrary $\overline{x}_G p(\overline{x}_G) < L/2$. Since $L/n \leq \underline{v}/2$, the investor can sell $n/2$ from both firms, get a price no lower than $\underline{v}$, and thus raise enough liquidity. This creates a profitable deviation.

3. In any equilibrium, $e_i(0, \overline{\mathbf{v}}) < \overline{x}_G$ and $p(e_i(0, \overline{\mathbf{v}})) = \overline{v}$. Proof: Since the investor can fully retain both firms, $p(e_i(0, \overline{\mathbf{v}})) = \overline{v}$. Suppose on the contrary $e_i(0, \overline{\mathbf{v}}) \geq \overline{x}_G$. Since $p(e_i(0, \overline{\mathbf{v}})) = \overline{v}$ and $p(\overline{x}_G) < \overline{v}$ (from claim 2), it cannot be $e_i(0, \overline{\mathbf{v}}) = \overline{x}_G$. Suppose $e_i(0, \overline{\mathbf{v}}) > \overline{x}_G$. Since $p(e_i(0, \overline{\mathbf{v}})) = \overline{v} > p(\overline{x}_G)$, if $\theta = L$ then type $\overline{\mathbf{v}}$ has a profitable deviation from $\overline{\mathbf{x}}_G$ to $\mathbf{e}(0, \overline{\mathbf{v}})$.

4. In any equilibrium, $e_i(0, (\overline{v}, \underline{v})) < \overline{x}_G$ and $p(e_i(0, (\overline{v}, \underline{v}))) = \overline{v}$. Proof: the same as claim 3.

5. In any equilibrium, if the investor sells x_B from a bad firm, either $x_B = \overline{x}_G$ or $p(x_B) = \underline{v}$. Proof: Suppose on the contrary the investor sells $x_B \neq \overline{x}_G$ from the bad firm w.p. $\gamma > 0$ in equilibrium, and $p(x_B) > \underline{v}$. Therefore, $p(x_B) < \overline{v}$. Based on claims 1-4, $p(x_B) > \underline{v}$ requires the investor to sell x_B from the good firm when $\theta = L$ and the other firm is bad. Let $\hat{x}$ be the quantity sold from the bad firm in this case. That is, $\mathbf{e}(L, (\overline{v}, \underline{v})) = (x_B, \hat{x})$. Note that $\hat{x} \geq x_B$. Otherwise, type $(\overline{v}, \underline{v})$ has a strictly optimal deviation from $(x_B, \hat{x})$ to $(\hat{x}, x_B)$, that is, selling more from the bad firm and still meeting her liquidity need. Moreover, note that $\hat{x} = x_B$. To show this, if $\hat{x} \neq x_B$ then based on claims 1-4, we must have $x_B < \hat{x} = \overline{x}_G$ or $p(\hat{x}) = \underline{v}$.

   (a) Suppose $x_B < \hat{x} = \overline{x}_G$. If type $(\overline{v}, \underline{v})$ prefers $(x_B, \overline{x}_G)$ over $(\overline{x}_G, \overline{x}_G)$ when $\theta = L$, then type $\overline{\mathbf{v}}$ must also prefer $(x_B, \overline{x}_G)$ over $(\overline{x}_G, \overline{x}_G)$ when $\theta = L$. However, since $\mathbf{e}(L, \overline{\mathbf{v}}) = (\overline{x}_G, \overline{x}_G)$, type $\overline{\mathbf{v}}$ must be indifferent between the two. Both strategies must generate the same payoff, but also raise exactly L, otherwise the investor can always sell less from the good firm and still meet her liquidity need. However, this implies $x_B = \overline{x}_G$, a contradiction.

(b) Suppose $p(\hat{x}) = \underline{v}$. Then, type $(\overline{v}, \underline{v})$ has a strictly profitable deviation from $(x_B, \hat{x})$ to $(0, n/2)$. Under both strategies, she raises enough revenue (recall $\underline{v}n/2 \geq L$). However, under the latter strategy her payoff is $n/2(\underline{v} + \overline{v})$, and under the former strategy her payoff is strictly smaller: her payoff is $\underline{v}$ for the bad firm, and since $p(x_B) < \overline{v}$, her payoff for the good firm is strictly smaller than $\overline{v}$.

Therefore, $\mathbf{e}(L, (\overline{v}, \underline{v})) = (x_B, x_B)$. By revealed preference,

$$\Pi\left((x_B, x_B), (\overline{v}, \underline{v})\right) \geq n/2(\underline{v} + \overline{v}) \Leftrightarrow p(x_B) \geq \frac{\underline{v} + \overline{v}}{2}.$$

Suppose type $(\overline{v}, \underline{v})$ chooses (x_B, x_B) w.p. $\sigma > 0$, then

$$p(x_B) = \underline{v} + \Delta \frac{\sigma\tau(1-\tau)}{\sigma\left[\tau(1-\tau) + (1-\tau)\tau\right] + \gamma(1-\tau)} < \frac{\underline{v} + \overline{v}}{2},$$

a contradiction.

6. In any equilibrium, if $\theta = 0$, then the investor sells $\overline{x}_G$ from a bad firm. Proof: Based on claim 5, if the investor sells x_B from a bad firm, either $x_B = \overline{x}_G$ or $p(x_B) = \underline{v}$. Since $p(\overline{x}_G) > \underline{v}$, if the investor does not need liquidity, she will maximize her payoff by choosing $\overline{x}_G$.

7. In any equilibrium, $\mathbf{e}(L, \underline{\mathbf{v}}) = (\overline{x}_G, \overline{x}_G)$. Proof: By symmetry, the investor must sell in equilibrium the same quantity from both firms. Based on claim 5, if $\mathbf{e}(L, \underline{\mathbf{v}}) \neq (\overline{x}_G, \overline{x}_G)$, the investor must receive $\underline{v}$ from the firms, and so her payoff is $\underline{v}$. Since $p(\overline{x}_G) > \underline{v}$ and $\overline{x}_G p(\overline{x}_G) = L/2$, the investor maximizes her payoff by choosing $(\overline{x}_G, \overline{x}_G)$.

8. In any equilibrium, if $p(\overline{x}_G) > \frac{\underline{v}+\overline{v}}{2}$, then $\mathbf{e}(L, (\overline{v}, \underline{v})) = (\overline{x}_G, \overline{x}_G)$, and if $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$, then $e_i(L, (\overline{v}, \underline{v})) < \overline{x}_G < e_j(L, (\overline{v}, \underline{v}))$. Proof: Recall $L/n \leq \underline{v}/2$ implies that selling $(0, n/2)$ yields type $(\overline{v}, \underline{v})$ a payoff of $n/2(\underline{v} + \overline{v})$ and enough revenue to cover her liquidity need. Therefore, she prefers $(\overline{x}_G, \overline{x}_G)$ if and only if

$$\Pi\left((\overline{x}_G, \overline{x}_G), (\overline{v}, \underline{v})\right) \geq n/2(\underline{v} + \overline{v}) \Leftrightarrow p(\overline{x}_G) \geq \frac{\underline{v} + \overline{v}}{2}.$$

Suppose $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$. If the investor chooses balanced exit $(x, x) \notin \{(\overline{x}_G, \overline{x}_G), (0,0)\}$, then by symmetry, $p(x) = \frac{\underline{v}+\overline{v}}{2}$. The revenue raised must be exactly L, else she can sell slightly less from the good firm and make a strictly higher profit. Therefore, it cannot be $x < \overline{x}_G$, else type $\overline{\mathbf{v}}$ will deviate from $(\overline{x}_G, \overline{x}_G)$ to (x, x) when $\theta = L$. Also, it cannot be $x > \overline{x}_G$. Indeed, if $x > \overline{x}_G$ then $\Pi((x,x),(\overline{v},\underline{v})) \geq \Pi((\overline{x}_G,\overline{x}_G),(\overline{v},\underline{v}))$ implies $\Pi((\overline{x}_G,\overline{x}_G),(\underline{v},\underline{v})) > \Pi((x,x),(\underline{v},\underline{v}))$, which means that type $\underline{\mathbf{v}}$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x, x) when $\theta = L$. Therefore, the investor cannot choose balanced exit when $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$.

Next, suppose type $(\overline{v}, \underline{v})$ chooses imbalanced exit (x_G, x_B). Note that the investor always sells more from the bad firm—that is, $x_G < x_B$. Also, since the investor can always meet her liquidity need choosing $(0, n/2)$, she must raise enough liquidity by choosing (x_G, x_B). We wish to prove $e_i(L, (\overline{v}, \underline{v})) < \overline{x}_G < e_j(L, (\overline{v}, \underline{v}))$, which requires us to prove $x_G < \overline{x}_G < x_B$. Suppose on the contrary $x_B \leq \overline{x}_G$. Since prices are non-increasing, $p(x_B) \geq p(x_G) \geq p(\overline{x}_G)$, and type $\overline{\mathbf{v}}$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x_G, x_B) when $\theta = L$, a contradiction. Therefore, $\overline{x}_G < x_B$. Suppose $\overline{x}_G \leq x_G$. If $x_G > \overline{x}_G$ then the based on all the claims above, no other type is choosing x_G, and so $p(x_G) = \overline{v}$. However, since $x_G > \overline{x}_G$ type $\underline{\mathbf{v}}$ has a profitable deviation from $(\overline{x}_G, \overline{x}_G)$ to (x_G, x_G). Suppose $x_G = \overline{x}_G$. Note that according to claim 5, $x_B > \overline{x}_G$ implies $p(x_B) = \underline{v}$. Therefore, the investor's payoff from the bad firm is $\underline{v}$. Since $p(\overline{x}_G) < \overline{v}$, the investor's payoff from the good firm is strictly lower than $\overline{v}$. Therefore, the investor's overall payoff is strictly smaller than $n/2(\underline{v} + \overline{v})$, which means that she has a profitable deviation to $(0, n/2)$. We conclude that $x_G < \overline{x}_G < x_B$, as required.

Based on claims 3 and 4, we can assume without loss of generality that $e_i(0, \overline{\mathbf{v}}) = e_i(0, (\overline{v}, \underline{v})) = 0$. In addition, the above claims imply there are both balanced exit and imbalanced exit equilibria. Consider first the balanced exit equilibrium of part (i). In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (i) of the proposition. Note that, based on claim 8, $p(\overline{x}_G) \geq \frac{\underline{v}+\overline{v}}{2}$, and since $\overline{x}_G = \overline{x}_{co}(\tau)$, given the expression of $\overline{p}_{co}(\tau)$ in part (i), it must be $\tau \geq \frac{1}{1+\beta}$.

We now show that profitable deviations do not exist. There are four cases to consider:

1. If $\theta = 0$, the investor cannot receive more than $\overline{v}$ from a good firm and $\overline{x}_{co}(\tau)\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\underline{v}$ from a bad firm, which she does by following the equilibrium strategies.

2. Suppose $\theta = L$ and both firms are bad. Then, the equilibrium strategy yields the highest possible payoff given prices.

3. Suppose $\theta = L$ and both firms are good. The only possible profitable deviation is selling $\varepsilon \in (0, n/2 - \overline{x}_{co}(\tau)]$ more from one firm and $\delta \in (0, \overline{x}_{co}(\tau)]$ less from the other, while still raising enough revenue. This deviation generates at least L in revenue if and only if

$$(\overline{x}_{co}(\tau) + \varepsilon)\underline{v} + (\overline{x}_{co}(\tau) - \delta)\overline{p}_{co}(\tau) \geq L \Leftrightarrow \delta \leq \frac{(\overline{x}_{co}(\tau) + \varepsilon)\underline{v} - L/2}{\overline{p}_{co}(\tau)}. \tag{47}$$

This deviation increases the investor's payoff if and only if

$$\begin{aligned}
&(\overline{x}_{co}(\tau) + \varepsilon)\underline{v} + (n/2 - (\overline{x}_{co}(\tau) + \varepsilon))\overline{v} + (\overline{x}_{co}(\tau) - \delta)\overline{p}_{co}(\tau) + (n/2 - (\overline{x}_{co}(\tau) - \delta))\overline{v} \\
&> 2\overline{x}_{co}(\tau)\overline{p}_{co}(\tau) + 2(n/2 - \overline{x}_{co}(\tau))\overline{v} \Leftrightarrow \\
\delta &> \frac{\varepsilon\Delta + \overline{x}_{co}(\tau)(\overline{p}_{co}(\tau) - \underline{v})}{\overline{v} - \overline{p}_{co}(\tau)}.
\end{aligned}$$

However, note that

$$\frac{\varepsilon\Delta + \overline{x}_{co}(\tau)(\overline{p}_{co}(\tau) - \underline{v})}{\overline{v} - \overline{p}_{co}(\tau)} > \frac{(\overline{x}_{co}(\tau) + \varepsilon)\underline{v} - L/2}{\overline{p}_{co}(\tau)} \Leftrightarrow \overline{p}_{co}(\tau) > \underline{v},$$

and so a profitable deviation does not exist.

4. Suppose $\theta = L$ and exactly one firm is bad. The only possible profitable deviation is selling $\varepsilon \in (0, n/2 - \overline{x}_{co}(\tau)]$ more from the bad firm and $\delta \in (0, \overline{x}_{co}(\tau)]$ less from the good firm. As in case 3, this deviation generates at least L in revenue if and only if

$\delta \leq \frac{(\overline{x}_{co}(\tau)+\varepsilon)\underline{v}-L/2}{\overline{p}_{co}(\tau)}$. This deviation is profitable if and only if

$$\begin{aligned}
&(\overline{x}_{co}(\tau)+\varepsilon)\underline{v}+(n/2-(\overline{x}_{co}(\tau)+\varepsilon))\underline{v}+(\overline{x}_{co}(\tau)-\delta)\overline{p}_{co}(\tau)+(n/2-(\overline{x}_{co}(\tau)-\delta))\overline{v} \\
&> 2\overline{x}_{co}(\tau)\overline{p}_{co}(\tau)+(n/2-\overline{x}_{co}(\tau))(\underline{v}+\overline{v}) \Leftrightarrow \\
\delta &> \overline{x}_{co}(\tau)\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)}.
\end{aligned}$$

Overall, the deviation is feasible and profitable if and only if

$$\overline{x}_{co}(\tau)\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)} < \delta \leq \min\left\{\overline{x}_{co}(\tau), \frac{(\overline{x}_{co}(\tau)+\varepsilon)\underline{v}-L/2}{\overline{p}_{co}(\tau)}\right\}.$$

Note that a larger δ implies a more profitable deviation, and that the profitability of the deviation is invariant to ε as long as $\varepsilon > 0$. Therefore, it is sufficient to focus on $\varepsilon = n/2 - \overline{x}_{co}(\tau)$. Therefore, a profitable deviation exists if and only if

$$\begin{aligned}
\overline{x}_{co}(\tau)\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)} &< \min\left\{\overline{x}_{co}(\tau), \frac{(\overline{x}_{co}(\tau)+n/2-\overline{x}_G)\underline{v}-L/2}{\overline{p}_{co}(\tau)}\right\} \Leftrightarrow \\
\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)} &< \min\left\{1, \frac{n\underline{v}-L}{L}\right\} \Leftrightarrow \\
\frac{\overline{p}_{co}(\tau)-\underline{v}}{\overline{v}-\overline{p}_{co}(\tau)} &< 1 \Leftrightarrow \\
\tau &< \frac{1}{1+\beta}
\end{aligned}$$

which contradicts $\tau \geq \frac{1}{1+\beta}$.

Next, consider the imbalanced exit equilibrium of part (ii). Given claim 8, we assume without loss of generality that $\mathbf{e}(L,(\overline{v},\underline{v})) = (0, n/2)$. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (ii) of the proposition. Note that, based on claim 8, $p(\overline{x}_G) < \frac{\underline{v}+\overline{v}}{2}$, and since $\overline{x}_G = \overline{x}_{co}(\tau)$, given the expression of $\overline{p}_{co}(\tau)$ in part (ii), it must be $\tau < \frac{1}{1+\beta}$.

We now show that profitable deviations do not exist. As in the balanced exit equilibrium, it is straightforward to see that there is no profitable deviation when $\theta = 0$, when $\theta = L$ and both firms are bad, or when $\theta = L$ and both firms are good. Suppose $\theta = L$ and exactly one firm is bad. First note that selling more of the good firm and less from the bad firm (but still more than $\overline{x}_{co}(\tau)$ units) is suboptimal: the investor's payoff from the bad firm does not change, but since $x > 0 \Rightarrow p(x) < \overline{v}$, her payoff from the good firm decreases. Therefore, a profitable deviation requires selling $\overline{x}_{co}(\tau)$ from the bad firm. However, since by construction $\overline{x}_{co}(\tau)\overline{p}_{co}(\tau) = L/2$, this deviation generates revenue of at least L if and only if the investor also sells $\overline{x}_{co}(\tau)$ from the good firm. Such a deviation is profitable if and only if

$$\begin{aligned}\underline{v}n/2 + \overline{v}n/2 &< \overline{x}_{co}(\tau)\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\underline{v} + \overline{x}_{co}(\tau)\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\overline{v} \Leftrightarrow \\ \frac{\underline{v}+\overline{v}}{2} &< \overline{p}_{co}(\tau) \Leftrightarrow \frac{1}{1+\beta} < \tau,\end{aligned}$$

which contradicts $\tau > \frac{1}{1+\beta}$. ■

Proof of Lemma 12. We start by proving several claims.

1. In any equilibrium, if $\mathbf{x}$ such that $x_i < x_j$ is on the path, then $p_i(\mathbf{x}) = \overline{v}$ and $p_j(\mathbf{x}) = \underline{v}$. Proof: by symmetry, $\mathbf{v} \in \{(\underline{v}, \overline{v}), (\overline{v}, \underline{v})\}$. The investor's payoff is $\Pi(\mathbf{x}, \mathbf{v})$. Note that for $\mathbf{x}$ satisfying $x_i < x_j$

   $$\Pi(\mathbf{x}, \mathbf{v}) > \Pi\left(\mathbf{x}^T, \mathbf{v}\right) \Leftrightarrow v_i > v_j.$$

   Since $\mathbf{x}$ is on the equilibrium path (by symmetry, so is x^T), $p_i(\mathbf{x}) \geq p_j(\mathbf{x})$. Since $x_i < x_j$ implies $v \in \{(\underline{v}, \overline{v}), (\overline{v}, \underline{v})\}$, it must be $\mathbf{v} = (\overline{v}, \underline{v})$, and so $p_i(\mathbf{x}) = \overline{v}$ and $p_j(\mathbf{x}) = \underline{v}$, as required.

2. In any equilibrium, if $v_i > v_j$, then $x_i \leq x_j$. Proof: suppose on the contrary $v_i > v_j$ and $x_i < x_j$. Since $\Pi(\mathbf{x}, \mathbf{v}) > \Pi\left(\mathbf{x}^T, \mathbf{v}\right) \Leftrightarrow v_i > v_j$, the investor has a profitable deviation to sell x_i units from firm v_j and x_j units from v_i.

3. In any equilibrium, there is a unique $\overline{x}_G \in (0, n/2)$ such that $\mathbf{e}(L, \overline{\mathbf{v}}) = \overline{\mathbf{x}}_G$. Proof: the same as claim 1 in the proof of Lemma 11, where the price is $p(x, x)$ instead of $p(x)$.

4. In any equilibrium, (i) $p_i(\overline{\mathbf{x}}_G) \in (\underline{v}, \overline{v})$; (ii) $\overline{x}_G p(\overline{\mathbf{x}}_G) = L/2$. Proof: the same as claim 2 in the proof of Lemma 11, where the price is $p(x, x)$ instead of $p(x)$.

5. In any equilibrium, $e_i(0, \overline{\mathbf{v}}) < \overline{x}_G$ and $p(\mathbf{e}(0, \overline{\mathbf{v}})) = \overline{v}$. Proof: the same as claim 3 in the proof of Lemma 11, where the price is $p(x, x)$ instead of $p(x)$.

6. In any equilibrium, $e_i(0, \underline{\mathbf{v}}) = e_i(L, \underline{\mathbf{v}}) = \overline{x}_G$. Proof: By symmetry, if $v_i = v_j = \underline{v}$ then $e_i(\theta, \underline{\mathbf{v}}) = e_j(\theta, \underline{\mathbf{v}})$. Let $\mathbf{x}_B(\theta) \equiv \mathbf{e}(\theta, \underline{\mathbf{v}})$. Note that if $x < \overline{x}_G$ then $xp(x, x) < L/2$. Otherwise, since $p(\overline{\mathbf{x}}_G) < \overline{v}$, claim 1 implies that type $(\overline{\mathbf{v}}, L)$ has strict incentives to deviate and sell less than $\overline{x}_G$ from each firm. Therefore, $x_B(L) \geq \overline{x}_G$. Suppose that, instead of $x_B(L) = \overline{x}_G$, we have $x_B(L) > \overline{x}_G$. So far we have shown it cannot be $x_B(L) < \overline{x}_G$. Now, we rule out $x_B(L) > \overline{x}_G$. Since $p(\overline{\mathbf{x}}_G) > \underline{v}$, type $\underline{v}$ chooses $\mathbf{x}_B(L)$ only if $p(\mathbf{x}_B(L)) > \underline{v}$. This is possible only if $\mathbf{e}(L, (\overline{v}, \underline{v})) = \mathbf{x}_B(L)$ or $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{x}_B(L)$. Note that type $(\overline{v}, \underline{v})$ can always obtain a payoff of $n/2(\underline{v} + \overline{v})$ and revenue of $n/2\underline{v} \geq L$ by choosing $(0, n/2)$. Therefore, she would choose $\mathbf{x}_B(L)$ only if

$$2x_B(L)\, p(\mathbf{x}_B(L)) + (n/2 - x_B(L))(\underline{v} + \overline{v}) \geq n/2(\underline{v} + \overline{v}) \Leftrightarrow p(\mathbf{x}_B(L)) \geq \frac{\underline{v} + \overline{v}}{2}$$

   Let $\gamma > 0$ be the probability type $\underline{v}$ chooses $\mathbf{x}_B(L)$, then the highest possible value of $p(\mathbf{x}_B(L)))$ arises when type $(\overline{v}, \underline{v})$ chooses $\mathbf{x}_B(L)$ w.p. one. In this case,

$$p(\mathbf{x}_B(L)) = \underline{v} + \Delta \frac{\tau(1-\tau)}{[\tau(1-\tau) + (1-\tau)\tau] + \gamma(1-\tau)^2} < \frac{\underline{v} + \overline{v}}{2},$$

   a contradiction.

7. In any equilibrium, if $p(\overline{\mathbf{x}}_G) > \frac{\underline{v}+\overline{v}}{2}$, then $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{e}(L, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_G$, and if $p(\overline{\mathbf{x}}_G) < \frac{\underline{v}+\overline{v}}{2}$ then $\mathbf{e}(0, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$ and $\mathbf{e}(L, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$. Proof: Note that if type $(\overline{v}, \underline{v})$ chooses $(0, n/2)$ then she obtains a payoff of $n/2(\underline{v} + \overline{v})$, and since $\underline{v}n/2 \geq L$, this strategy raises enough revenue to meet the liquidity need. Therefore, regardless of her liquidity need, type $(\overline{v}, \underline{v})$ chooses $\overline{\mathbf{x}}_G$ if and only if

$$2\overline{x}_G p(\overline{\mathbf{x}}_G) + (n/2 - \overline{x}_G)(\underline{v} + \overline{v}) > n/2(\underline{v} + \overline{v}) \Leftrightarrow p(\overline{\mathbf{x}}_G) > \frac{\underline{v} + \overline{v}}{2}, \tag{48}$$

   as required.

Based on the claims above, there are two types of equilibria: those in which $\mathbf{e}(\theta, (\overline{v}, \underline{v})) = \overline{\mathbf{x}}_G$

(balanced exit), and those in which $\mathbf{e}(\theta, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$ (which can potentially be imbalanced exit). Consider first the balanced exit equilibrium. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (i) of the proposition. In particular $\overline{x}_G = \overline{x}_{co}(\tau)$. Note that Condition (48) and $p(\overline{\mathbf{x}}_G) = \overline{p}_{co}(\tau)$ imply that $\tau \geq \frac{1}{1+\sqrt{\beta}}$ is necessary. We now show profitable deviations do not exist. There are four cases to consider:

1. If $\theta = 0$ and $\mathbf{v} = \overline{\mathbf{v}}$, the investor cannot receive more than $\overline{v}$ on each of her good firms, and so fully retains both.

2. Suppose $\mathbf{v} = \underline{\mathbf{v}}$. The equilibrium payoff is

$$\Pi^*(\underline{\mathbf{v}}) = 2\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) + 2(n/2 - \overline{x}_{co}(\tau))\,\underline{v} > \underline{v}.$$

   Any deviation to sell $x > \overline{x}_{co}(\tau)$ from one of the firms generates a strictly lower payoff: the price of the sold firm falls to $\underline{v}$, and the price of the other firm does not increase unless it is fully retained, in which case it generates a payoff of $\underline{v}$ since the firm is bad. Any deviation to sell $x < \overline{x}_{co}(\tau)$ from one of the firms also generates a strictly lower payoff: the price of either firm does not increase unless it is fully retained in which case its payoff is its fundamental value of $\underline{v}$. Therefore, there is no profitable deviation.

3. Suppose $\mathbf{v} = (\overline{v}, \underline{v})$. Note that by construction $\overline{\mathbf{x}}_G p(\overline{\mathbf{x}}_G) = L/2$, and so the investor can meet her liquidity need by choosing $\overline{\mathbf{x}}_G$. Since Condition (48) holds, by choosing $\overline{\mathbf{x}}_G$ the investor obtains a payoff higher than $n/2(\underline{v} + \overline{v})$. Based on the pricing function, any deviation that involves selling strictly more than $\overline{x}_{co}$ from at least one firm is dominated by fully selling the bad firm and fully retaining the good firm, a strategy that generates a payoff of $n/2(\underline{v} + \overline{v})$. Consider a deviation that involves selling strictly $x < \overline{x}_{co}$ from at least one firm. If it is a balanced exit strategy, it generates a payoff of $2x\overline{p}_{co}(\tau) + 2(n/2 - x)\frac{\overline{v}+\underline{v}}{2}$. However,

$$\begin{aligned} x\overline{p}_{co}(\tau) + (n/2 - x)\frac{\overline{v}+\underline{v}}{2} &< \overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\frac{\overline{v}+\underline{v}}{2} \Leftrightarrow \\ x[\overline{p}_{co}(\tau) - \frac{\overline{v}+\underline{v}}{2}] &< \overline{x}_{co}(\tau)\,[\overline{p}_{co}(\tau) - \frac{\overline{v}+\underline{v}}{2}]. \end{aligned}$$

Since $x < \overline{x}_{co}$ and $\overline{p}_{co}(\tau) - \frac{\overline{v}+\underline{v}}{2} \geq 0$, choosing $x < \overline{x}_{co}$ is suboptimal. If it is an imbalanced exit strategy, the investor can profit from a deviation only if she sells less from the good firm. However, any such deviation lowers the price of the bad firm to $\underline{v}$. Therefore, an optimal deviation must involve the investor fully retaining the good firm. However, this deviation creates a payoff of $n/2\,(\underline{v} + \overline{v})$, which is lower than the equilibrium payoff.

4. Suppose $\theta = L$ and $\mathbf{v} = \overline{\mathbf{v}}$. The equilibrium payoff is

$$\Pi^*(\overline{\mathbf{v}}) = 2\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) + 2\,(n/2 - \overline{x}_{co}(\tau))\,\overline{v},$$

Based on the pricing function, any deviation to selling more than $\overline{x}_{co}(\tau)$ from both firms requires the investor to sell more of the good firm but receive a price strictly lower than $\overline{p}_{co}(\tau)$ and $\overline{v}$, and hence generates a strictly lower payoff. Moreover, since $\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) = L/2$, any deviation to selling less than $\overline{x}_{co}(\tau)$ from both firms does not generate enough revenue to meet the liquidity need, and is thus suboptimal. Consider a deviation to imbalanced exit (x_i, x_j) such that $x_i < \overline{x}_{co}(\tau) < x_j$. The payoff and revenue are, respectively,

$$\hat{\Pi} = x_i\overline{p}_{co}(\tau) + x_j\underline{v} + (n - (x_i + x_j))\,\overline{v},$$

and

$$\hat{R} = x_i\overline{p}_{co}(\tau) + x_j\underline{v}.$$

This deviation is optimal only if $\hat{\Pi} \geq \Pi^*(\overline{\mathbf{v}})$ and $\hat{R} \geq L$. If there is such a deviation, then there is another optimal deviation in which $\hat{R} = L$ (selling less from either firm does not decrease the price). Since $2\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) = L$, $\hat{\Pi} \geq \Pi^*(\overline{\mathbf{v}})$ requires

$$(n - (x_i + x_j))\,\overline{v} \geq 2\,(n/2 - \overline{x}_{co}(\tau))\,\overline{v} \Leftrightarrow (x_i + x_j) \leq 2\overline{x}_{co}(\tau)$$

Using $x_i\overline{p}_{co}(\tau) + x_j\underline{v} = L$ and $\overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)}$, the condition becomes $x_i \geq 2\overline{x}_{co}(\tau)$, which contradicts $x_i < \overline{x}_{co}(\tau)$. Therefore, there is no profitable deviation.

Next, consider the equilibrium with $\mathbf{e}(\theta, (\overline{v}, \underline{v})) \neq \overline{\mathbf{x}}_G$. This implies that $\overline{x}_G = \overline{x}_{co}(\tau)$, and from Bayes' rule, $p(\overline{\mathbf{x}}_G) = \overline{p}_{co}(\tau)$ is given as in part (ii) of the proposition. Since we require

$p(\overline{\mathbf{x}}_G) < \frac{\underline{v}+\overline{v}}{2}$, the expression for $\overline{p}_{co}(\tau)$ in part (ii) implies that $\tau < \frac{1}{1+\sqrt{\beta}}$.

We argue that if $\theta = L$ then type $(\overline{v}, \underline{v})$ must follow imbalanced exit, that is $e_i(L, (\overline{v}, \underline{v})) < e_j(L, (\overline{v}, \underline{v}))$. Suppose on the contrary $\mathbf{e}(L, (\overline{v}, \underline{v})) = \mathbf{x} = (x, x)$ where $x \neq \overline{x}_G$. Based on claims 3, 5, and 6, no other type is choosing $\mathbf{x}$. Therefore, $p(\mathbf{x}) = \frac{\underline{v}+\overline{v}}{2}$. Since $p(\overline{\mathbf{x}}_G) < \frac{\underline{v}+\overline{v}}{2}$, if $x > \overline{x}_G$ then type $\underline{v}$ has a strictly profitable deviation from $\overline{\mathbf{x}}_G$ to $\mathbf{x}$: she can sell more units from each firm for a price higher than $p(\overline{\mathbf{x}}_G)$, and thereby get a strictly larger payoff. Consider $x < \overline{x}_G$. Note that $xp(\mathbf{x}) < L/2$, otherwise, type $\overline{\mathbf{v}}$ would have a profitable deviation from $\overline{\mathbf{x}}_G$ to $\mathbf{x}$ when $\theta = L$. Therefore, it cannot be $x < \overline{x}_G$. Thus, $e_i(L, (\overline{v}, \underline{v})) \neq e_j(L, (\overline{v}, \underline{v}))$, and from claim 2 we have $e_i(L, (\overline{v}, \underline{v})) < e_j(L, (\overline{v}, \underline{v}))$. Given claims 1 and 2, we can assume without loss of generality (and to ease the exposition) that the imbalanced strategy involves $\mathbf{e}(L, (\overline{v}, \underline{v})) = (0, n/2)$. Since $\underline{v}n/2 \geq L$, this strategy raises enough revenue to meet the liquidity need.

The equilibrium therefore hinges on type $(\overline{v}, \underline{v})$'s strategy when $\theta = 0$. The lowest price informativeness arises when type $(\overline{v}, \underline{v})$ follows balanced exit. There are two cases to consider:

1. If $\mathbf{e}(0, (\overline{v}, \underline{v})) \neq \mathbf{e}(0, \overline{\mathbf{v}})$, then

$$\begin{aligned} P'_{co,single}(\underline{v}) &= \tau\left(\beta\underline{v} + (1-\beta)\frac{\overline{v}+\underline{v}}{2}\right) + (1-\tau)\overline{p}_{co}(\tau) \\ P'_{co,single}(\overline{v}) &= \beta\tau\overline{p}_{co}(\tau) + (1-\beta)\tau\overline{v} + \beta(1-\tau)\overline{v} + (1-\beta)(1-\tau)\frac{\overline{v}+\underline{v}}{2}. \end{aligned}$$

2. If $\mathbf{e}(0, (\overline{v}, \underline{v})) = \mathbf{e}(0, \overline{\mathbf{v}})$, then

$$\begin{aligned} P''_{co,single}(\underline{v}) &= \tau(\beta\underline{v} + (1-\beta)p(0)) + (1-\tau)\overline{p}_{co}(\tau) \\ P''_{co,single}(\overline{v}) &= \beta\tau\overline{p}_{co}(\tau) + (1-\beta)\tau p(0) + \beta(1-\tau)\overline{v} + (1-\beta)(1-\tau)p(0). \end{aligned}$$

Note that

$$p(0) \in \left(\frac{\overline{v}+\underline{v}}{2}, \tau\overline{v} + (1-\tau)\frac{\overline{v}+\underline{v}}{2}\right)$$

and

$$\begin{aligned} P''_{co,single}\left(\underline{v}\right) &> P'_{co,single}\left(\underline{v}\right) \Leftrightarrow p\left(0\right) > \frac{\overline{v}+\underline{v}}{2}, \text{ and} \\ P''_{co,single}\left(\overline{v}\right) &< P'_{co,single}\left(\overline{v}\right) \Leftrightarrow p\left(0\right) < \tau\overline{v} + \left(1-\tau\right)\frac{\overline{v}+\underline{v}}{2}. \end{aligned}$$

Therefore, the equilibrium with the lowest price informativeness is obtained when type $(\overline{v}, \underline{v})$ chooses $(0,0)$ when $\theta = 0$.

We now show that an equilibrium with $\mathbf{e}(0,(\overline{v},\underline{v})) = \mathbf{e}(0,\overline{\mathbf{v}})$ exists. In this case, the equilibrium strategies and prices (from Bayes' rule) are described in part (ii) of the proposition. We now show that profitable deviations do not exist. As in the balanced exit equilibrium, it is straightforward to see that there is no profitable deviation when $\mathbf{v} = \overline{\mathbf{v}}$ or $\mathbf{v} = \underline{\mathbf{v}}$ (cases 1, 2, and 4 above). Indeed, the equilibrium strategies of these type do not change. Moreover, the proof only depends on the property $\overline{x}_{co}(\tau) = \frac{L/2}{\overline{p}_{co}(\tau)}$, which holds. One difference from balanced exit is that $p(0,0) < \overline{v}$, but this does not affect the investor's incentives since her payoff does not depend on prices when she fully retains the firms.

There is one remaining case to consider. Suppose $\mathbf{v} = (\overline{v}, \underline{v})$. The equilibrium payoff is $n/2\left(\underline{v}+\overline{v}\right)$ and the revenue is when $\theta = L$ is $\underline{v}n/2 \geq L$ (the investor can meet her liquidity need by choosing $(0, n/2)$). Note that any deviation to imbalanced exit will continue to generate a payoff of $\underline{v}$ from the bad firm and at most $\overline{x}_{co}(\tau)\,\overline{p}_{co}(\tau) + (n/2 - \overline{x}_{co}(\tau))\,\overline{v} < \overline{v}$ from the good firm. Therefore, such a deviation is suboptimal. Any deviation to balanced exit (x,x), where $x > \overline{x}_{co}(\tau)$, will generate a payoff of $\underline{v}$ on the bad firm and a payoff of at most $\overline{x}_{co}(\tau)\,\underline{v} + (n/2 - \overline{x}_{co}(\tau))\,\overline{v} < \overline{v}$ on the good firm, which is suboptimal. Consider a deviation to balanced exit (x,x), where $x \leq \overline{x}_{co}(\tau)$. The investor's payoff is

$$2x\overline{p}_{co}\left(\tau\right) + \left(n/2 - x\right)\left(\overline{v}+\underline{v}\right),$$

which is smaller than $n/2\left(\underline{v}+\overline{v}\right)$ if and only if $\overline{p}_{co}(\tau) \leq \frac{\overline{v}+\underline{v}}{2}$, which always holds if $\tau < \frac{1}{1+\sqrt{\beta}}$. Therefore, the investor has no profitable deviation, as required. Note that that monotonicity is preserved: $p(0) > \overline{p}_{co}(\tau) \Leftrightarrow \frac{(1-\tau)}{\tau^2} > \beta$, which always holds if $\tau < \frac{1}{1+\sqrt{\beta}}$. ■

Proof of Proposition 8. Recall that according to Lemma 1, $P_{so}(\underline{v}) = \overline{p}_{so}(\tau)$ and $P_{so}(\overline{v}) = \beta\overline{p}_{so}(\tau) + (1-\beta)\overline{v}$, where

$$\overline{p}_{so}(\tau) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau + 1 - \tau}.$$

Consider part (i). Based on part (i) of Lemma 11, if $\tau \geq \frac{1}{1+\beta}$ then the equilibrium is the same as under the benchmark, and so price informativeness is the same. If $\tau < \frac{1}{1+\beta}$ then

$$\begin{aligned} P_{co,Separate}(\underline{v},\tau) &= \beta\tau\underline{v} + (1-\beta\tau)\overline{p}_{co}(\tau) \\ P_{co,Separate}(\overline{v},\tau) &= \beta\tau\overline{p}_{co}(\tau) + (1-\beta\tau)\overline{v} \end{aligned}$$

Using

$$\overline{p}_{co}(\tau) = \underline{v} + \frac{\tau^2\beta}{(1-\beta)(1-\tau) + (\tau^2 + (1-\tau)^2)\beta}\Delta.$$

Using simple algebra, it can be verified that

$$P_{co,Separate}(\overline{v},\tau) > P_{so}(\overline{v}) \text{ and } P_{co,Separate}(\underline{v},\tau) < P_{so}(\underline{v}). \tag{49}$$

Consider part (ii). Based on Lemma 12, price informativeness is higher with two firms only if $\tau < \frac{1}{1+\sqrt{\beta}}$. Part (ii) of Lemma 12 also gives the fully uninformative equilibrium. In this equilibrium

$$\begin{aligned} P_{co,Single}(\underline{v},\tau) &= \tau(\beta\underline{v} + (1-\beta)p(0)) + (1-\tau)\overline{p}_{co}(\tau) \\ P_{co,Single}(\overline{v},\tau) &= \beta\tau\overline{p}_{co}(\tau) + \beta(1-\tau)\overline{v} + ((1-\beta)\tau + (1-\beta)(1-\tau))p(0) \end{aligned}$$

where

$$\begin{aligned} p(0) &= \underline{v} + \Delta\frac{\tau(1-\tau)+\tau^2}{\tau(1-\tau) + (1-\tau)\tau + \tau^2} = \underline{v} + \Delta\frac{1}{2-\tau} \\ \overline{p}_{co}(\tau) &= \underline{v} + \frac{\beta\tau^2}{\beta\left((1-\tau)^2 + \tau^2\right) + (1-\beta)(1-\tau)^2}\Delta = \underline{v} + \frac{\beta\tau^2}{\beta\tau^2 + (1-\tau)^2}\Delta \end{aligned}$$

Using simple algebra, it can be verified that

$$\begin{aligned} P_{co,Single}\left(\underline{v},\tau\right) &< P_{so}\left(\underline{v}\right) \Leftrightarrow \\ \frac{1-\beta}{2-\tau}+\frac{\left(1-\tau\right)\beta\tau}{\beta\tau^{2}+\left(1-\tau\right)^{2}} &< \frac{\beta}{\beta\tau+1-\tau} \end{aligned}$$

and

$$\begin{aligned} P_{co,Single}\left(\overline{v},\tau\right) &> P_{so}\left(\overline{v}\right) \Leftrightarrow \\ \frac{\beta^{2}\tau^{3}}{\beta\tau^{2}+\left(1-\tau\right)^{2}}+\left(1-\tau\right)\frac{3\beta-1-\tau\beta}{2-\tau} &> \frac{\beta^{2}\tau}{\beta\tau+1-\tau} \end{aligned}$$

Note that as $\beta \to 1$, the two conditions hold for any τ. Also note that if $\tau < \frac{1}{2}$ then $\tau < \frac{1}{1+\sqrt{\beta}}$ for all $\beta \in [0,1]$. Combined, it concludes the proof. ■

F.2 Single market maker

This section considers the case of a single market maker, who observes the order flows of all firms when setting prices. Since liquidity shocks are i.i.d. across investors under separate ownership, x_j contains no information relevant for the pricing of firm $i \neq j$. Therefore, the equilibria under separate ownership do not change. Below we derive the equilibria under common ownership. We focus on equilibria with monotonic prices, defined as in the two firm, single market maker case.

Proposition 9 (Single market maker): For all L, there exists an equilibrium under common ownership where price informativeness is strictly higher than under separate ownership. There also exists an equilibrium under common ownership where price informativeness is strictly lower than under separate ownership.

Proof of Proposition 9. First, suppose $L/n \leq \underline{v}(1-\tau)$. Since the market maker observes all trades, and due to the law of large numbers, in equilibrium he knows (ex post) the exact measure of bad firms sold across all firms he buys. Thus, the investor will always receive the expected value of her portfolio ($\underline{v} + \tau\Delta$) in all states.[33] We show that there is an equilibrium in which, under common ownership, $x^*(\overline{v}, \theta) = 0$ and $x^*(\underline{v}, \theta) = \overline{x} \; \forall \; \theta \in \{0, L\}$, with $\overline{x} = \frac{L/n}{\underline{v}(1-\tau)} \leq 1$. For this candidate equilibrium, prices can be:

$$p_i(\mathbf{x}) = \overline{v} \text{ if } x_i = 0$$
$$p_i(\mathbf{x}) = \underline{v} \text{ otherwise.}$$

Clearly, there are no profitable deviations, and when $\theta = L$, the investor generates revenue of $\overline{x}(1-\tau)\underline{v}n = L$, which is sufficient to cover her shock. Finally, prices are clearly monotonic. Thus, this is an equilibrium.

Next, suppose $L/n \in (\underline{v}(1-\tau), \underline{v} + \Delta\tau)$. Then, the investor must sell a positive amount of good firms upon a shock. Consider the candidate equilibrium where $x^*(\overline{v}, L) = \overline{x}$ and $x^*(\underline{v}, L) = 1$, with $\overline{x} = \frac{L/n - \underline{v}(1-\tau)}{\overline{v}\tau} < 1$, and $x^*(\overline{v}, 0) = 0$ and $x^*(\underline{v}, 0) = 1$. Prices in this equilibrium can be:

$$p_i(\mathbf{x}) = \overline{v} \text{ if } x_i \leq \overline{x} \text{ and } \int_{j: x_j > \overline{x}} dj = (1-\tau)\, n$$
$$p_i(\mathbf{x}) = \underline{v} \text{ otherwise.}$$

Clearly, there are no profitable deviations, and when $\theta = L$, the investor by construction generates revenue of L, which is sufficient to cover her shock. Finally, prices are monotonic. Thus, this is an equilibrium.

Given these first two cases, if $L/n < \underline{v} + \Delta\tau$, then there exists an equilibrium under common ownership in which $P_{co}(\overline{v}, \tau) = \overline{v}$ and $P_{co}(\underline{v}, \tau) = \underline{v}$—that is, prices are fully informative and thus more informative than under separate ownership.

There is also an equilibrium under common ownership with fully uninformative prices when $L/n < \underline{v} + \Delta\tau$. Consider a candidate equilibrium in which $x^*(v, \theta) = \overline{x} \equiv \frac{L/n}{\underline{v} + \Delta\tau} < 1$ for all v

[33] With separate market makers, a maker maker does not observe the entire vector of trades, and thus may not know the exact measure of bad firms among the purchased assets.

and θ. Let equilibrium prices be

$$p_i(\mathbf{x}) = \underline{v} + \Delta\tau \text{ if } x_i = x_j : \forall i, j$$
$$p_i(\mathbf{x}) = \underline{v} \text{ otherwise.}$$

Under these strategies and prices, the investor raises $L = n\overline{xp}$ and receives a payoff of $\underline{v} + \Delta\tau$. Furthermore, there are no profitable deviations: deviating to another balanced exit strategy will still yield $\underline{v} + \Delta\tau$, while deviating to imbalanced exit will yield at most $\underline{v} + \Delta\tau$. Finally, since the prices for balanced exit are always $\underline{v} + \Delta\tau$, they satisfy monotonicity. Since prices are $\underline{v} + \Delta\tau$ for all firms, they are fully uninformative.

Finally, consider the case when $L/n \geq \underline{v} + \Delta\tau$. Then, in any equilibrium under common ownership, the investor must fully sell all firms when $\theta = L$. This implies that $p_i(\mathbf{x}) = \underline{v} + \Delta\tau$ when $x_i = 1$ for all i. The investor has two potential strategies when $\theta = 0$. The first is balanced exit, where trades and thus prices are fully uninformative. The second is imbalanced exit, where she sells $\underline{x}$ when $v_i = \overline{v}$, and $\overline{x} > \underline{x}$ when $v_i = \underline{v}$. Expected prices are:

$$P_{co}(\overline{v}, \tau) = (1 - \beta)\overline{v} + \beta(\underline{v} + \Delta\tau)$$
$$P_{co}(\underline{v}, \tau) = (1 - \beta)\underline{v} + \beta(\underline{v} + \Delta\tau) = \underline{v} + \Delta\tau\beta.$$

(Prices off-equilibrium can be described in a similar way to $L/n \in (\underline{v}(1 - \tau), \underline{v} + \Delta\tau)$.) Under separate ownership, expected prices are:

$$P_{so}(\overline{v}, \tau) = (1 - \beta)\overline{v} + \beta\left(\underline{v} + \Delta\frac{\beta\tau}{\beta\tau + 1 - \tau}\right)$$
$$P_{so}(\underline{v}, \tau) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau + 1 - \tau}.$$

Since $P_{so}(\underline{v}, \tau) > P_{co}(\underline{v}, \tau)$ and $P_{so}(\overline{v}, \tau) < P_{co}(\overline{v}, \tau)$, prices are strictly more informative under common ownership. ■

F.3 Unobserved initial portfolio and information endowment

This section analyzes the trade-only model when the market makers do not observe whether the investor is concentrated or diversified. It also allows for the investor to be uninformed; her information endowment is also her private information. The market maker of firm i believes the investor is concentrated w.p. $\mu \in (0,1)$ and diversified w.p. $1-\mu$. Moreover, the market maker believes that she is informed with probability $\lambda \in (0,1)$ and uninformed with probability $1-\lambda$. The concentration of the investor's portfolio and her information endowment are independent of each other. For simplicity, we assume $L/n \leq \underline{v}(1-\tau)$ so that we are in the small-shock equilibrium of Lemma 2 where our results are strongest. We also assume that $L \leq \underline{v}$, so that the investor can satisfy her liquidity need by selling one unit of an asset. This assumption guarantees that the concentrated investor does not have incentives to separate herself from the diversified investor by simply selling more than one unit.[34]

Proposition 10 presents the equilibrium. We slightly abuse notation by using $v_i = \underline{v} + \tau\Delta$ to denote the case in which the investor is uninformed.

Proposition 10 (Anonymous Trade): Suppose the investor is concentrated w.p. μ and informed w.p. λ. If $L \leq \underline{v}$ and $1 \leq n(1-\tau)$, then in any equilibrium the concentrated investor's trading strategy is[35]

$$x^*_{so}(v_i, \theta) = \begin{cases} 0 & \text{if } v_i \in \{\overline{v}, \underline{v} + \tau\Delta\} \text{ and } \theta = 0 \\ \overline{x}^* \equiv L/\overline{p} & \text{otherwise,} \end{cases} \tag{50}$$

[34] If such separation is feasible, the anonymity of the investor plays no significant role as the investor's trade can reveal her portfolio. Note that since the investor's information endowment does not affect the set of feasible trades, this assumption does not change the investor's ability or incentives to reveal her information endowment through her trades.

[35] There is another equilibrium which is identical to the one described in the Proposition except that, if $v_i = \underline{v} + \tau\Delta$ and $\theta = 0$, then the investor sells $x_i > 0$ units, where $p(x_i) = \underline{v} + \tau\Delta$ and x_i is sufficiently small such that no other type mimics this strategy. In the proof we show that, in this equilibrium, price informativeness also decreases with μ.

the diversified investor's trading strategy is

$$x^*_{co}(v_i,\theta) = \begin{cases} 0 & \text{if } v_i = \overline{v} \text{ or, } v_i = \underline{v} + \tau\Delta \text{ and } \theta = 0 \\ \overline{x}^* & \text{if } v_i = \underline{v} \text{ or, } v_i = \underline{v} + \tau\Delta \text{ and } \theta = L \end{cases}, \tag{51}$$

and prices are

$$p^*_i(x_i) = \begin{cases} \underline{v} + \Delta\frac{\tau\lambda(1-\beta\mu)+\tau(1-\lambda)(1-\beta)}{\tau\lambda(1-\beta\mu)+(1-\lambda)(1-\beta)} & \text{if } x_i = 0 \\ \overline{p} \equiv \underline{v} + \Delta\frac{\lambda\mu\beta\tau+(1-\lambda)\beta\tau}{\lambda(\mu\beta\tau+1-\tau)+(1-\lambda)\beta} & \text{if } x_i \in (0,\overline{x}] \\ \underline{v} & \text{if } x_i > \overline{x}. \end{cases} \tag{52}$$

Moreover, price informativeness in equilibrium decreases with μ.

Intuitively, since $L/n < \underline{v}(1-\tau)$, if the market maker knows that a sale is from a diversified and informed investor, she knows that the asset must be bad for the same reasons as in the core model. Thus, a diversified investor will pool with a concentrated investor who suffers a liquidity shock. Thus, the equilibrium is similar to that under separate ownership except that the market maker takes into account the fact that a sale may be of a bad asset by a diversified investor, or by an uninformed investor who needs liquidity. This yields the expression for $\overline{p}$, the price upon a sale order of $\overline{x}^*$. Note that $\overline{p}$ increases in μ and that $p^*_i(0)$, which reflects the possibility that the uninformed investor fully retains her assets when unshocked, is decreasing in μ. Overall, price informativeness decreases with the probability that the investor is concentrated. This extends the result of Proposition 1 to the case in which the investor's portfolio is private information, and so the market maker attaches an interior probability to her being concentrated.

Proof of Proposition 10. We first note that $L \leq \underline{v}$ and $1 \leq n(1-\tau)$ imply $L/n < \underline{v}(1-\tau)$. Therefore, the diversified (and informed) investor sells a positive amount of her good assets only if the price is $\overline{v}$. Without loss of generality, we assume that the diversified investor fully retains good assets—that is, $x^*_{co}(\overline{v},\theta) = 0$. Similarly, the concentrated (and informed) investor who does not suffer a liquidity shock sells a positive amount from her good asset only if the price is $\overline{v}$, so without loss of generality $x^*_{so}(\overline{v},0) = 0$ as well.

We first show a set of results regarding the possible equilibrium selling strategies, similar to the proof of Lemma 1.

1. We prove that in any equilibrium, if $x_i > 1$ is on the path, then $p_i(x_i) = \underline{v}$. Let $\hat{x}_i$ be the highest quantity sold in equilibrium such that $p_i(\hat{x}_i) > \underline{v}$. Suppose on the contrary that $\hat{x}_i > 1$. Since $p_i(\hat{x}_i) > \underline{v}$, it must be that the investor sells $\hat{x}_i$ if $v_i \in \{\overline{v}, \underline{v} + \tau\Delta\}$. Since $L \leq \underline{v}$ and $1 \leq n(1-\tau)$, it must be $\hat{x}_i p_i(\hat{x}_i) > L$. Since prices are non-increasing, the investor can raise L in revenue by selling strictly less than $\hat{x}_i$. Therefore, selling $\hat{x}_i$ is weakly optimal only if $p_i(\hat{x}_i) = \overline{v}$ when $v_i = \overline{v}$ and $p_i(\hat{x}_i) \geq \underline{v} + \tau\Delta$ when $v_i = \underline{v} + \tau\Delta$. Suppose $p_i(\hat{x}_i) = \overline{v}$. Then, the investor has strict incentives to sell $\hat{x}_i$ units of firm i when $v_i = \underline{v}$. Indeed, since $\hat{x}_i$ is the highest quantity sold in equilibrium such that $p_i(\hat{x}_i) > \underline{v}$ and $p_i(\hat{x}_i) = \overline{v}$ is the highest price possible, by selling $\hat{x}_i$ the investor maximizes her profit. Therefore, it cannot be $p_i(\hat{x}_i) = \overline{v}$. Suppose $p_i(\hat{x}_i) \in [\underline{v} + \tau\Delta, \overline{v})$. Since $p_i(\hat{x}_i) < \overline{v}$, the investor never chooses $\hat{x}_i$ when $(v_i, \theta) = (\overline{v}, 0)$. Moreover, since $\hat{x}_i p_i(\hat{x}_i) > L$ and prices are non-increasing, there exists $x_i < \hat{x}_i$ such that $x_i p_i(x_i) \geq L$. Since $p_i(\hat{x}_i) < \overline{v}$, the investor has strict incentives to choose $x_i < \hat{x}_i$ when $(v_i, \theta) = (\overline{v}, L)$. Therefore, it must be $p_i(\hat{x}_i) = \underline{v} + \tau\Delta$, which implies that the investor sells $\hat{x}_i$ units when $v_i = \underline{v} + \tau\Delta$, but she never sells $\hat{x}_i$ units when $v_i = \underline{v}$. If the investor never sells $\hat{x}_i$ units when $v_i = \underline{v}$, then there must be $x'_i < \hat{x}_i$ such that $p_i(x'_i) > \underline{v} + \tau\Delta$ (or else, the investor has strict preferences to sell $\hat{x}_i$ units when $v_i = \underline{v}$, for the same reasons as in the case where $p_i(\hat{x}_i) = \overline{v}$). Since $p_i(x'_i) > \underline{v} + \tau\Delta$, the investor must be selling x'_i units when $v_i = \overline{v}$. Since the investor sells x'_i units when $v_i = \underline{v}$, $p_i(x'_i) < \overline{v}$. Since $p_i(x'_i) < \overline{v}$, the investor sells x'_i units when $v_i = \overline{v}$ only if she needs liquidity. Therefore, it must be $x'_i p_i(x'_i) \geq L$, or else the investor has incentives to sell strictly more than x'_i when $(v_i, \theta) = (\overline{v}, L)$ in order to meet her liquidity need. However, note that the investor strictly prefers selling x'_i units over selling $\hat{x}_i$ units when $v_i = \underline{v} + \tau\Delta$, irrespective of her liquidity need. Indeed, by selling $\hat{x}_i$ units the investor's payoff is exactly $\underline{v} + \tau\Delta$. By selling x'_i units the investor raises enough revenue to meet her liquidity need and she is getting $p_i(x'_i) > \underline{v} + \tau\Delta$ for the x'_i units she is selling, which increases her profit. This implies that $\hat{x}_i$ must be off-equilibrium, a contradiction.

2. $x_\chi(\underline{v}, \theta) > 0$ and $x_\chi(\underline{v} + \tau\Delta, L) > 0$, where $\chi \in \{so, co\}$. To see this, note that if $\theta = L$,

the investor will sell a positive amount. If $\theta = 0$ and $v_i = \underline{v}$, suppose that $x_i = 0$. Her payoff in this case is $\underline{v}$. Since $x_\chi(\underline{v} + \tau\Delta, L) > 0$, $p_i(x_\chi(\underline{v} + \tau\Delta, L)) > \underline{v}$, and so type $\underline{v}$ has a profitable deviation to $x_\chi(\underline{v} + \tau\Delta, L)$.

3. $x_\chi(\underline{v}, \theta) \in \{x_{so}(\overline{v}, L), x_{so}(\underline{v} + \Delta\tau, L), x_{co}(\underline{v} + \Delta\tau, L)\}$. Suppose not. Then, it cannot be $x_\chi(\underline{v}, \theta) \notin \{x_{so}(\overline{v}, 0), x_{co}(\overline{v}, \theta), x_{so}(\underline{v} + \Delta\tau, 0), x_{co}(\underline{v} + \Delta\tau, 0)\}$, since then $p\left(x_\chi(\underline{v}, \theta)\right) = \underline{v}$ and the investor's payoff is $\underline{v}$. Indeed, since $x_{co}(\underline{v}+\tau\Delta, L) > 0$ and $p_i\left(x_{co}(\underline{v} + \tau\Delta, L)\right) > \underline{v}$, the investor has a profitable deviation to $x_{co}(\underline{v} + \tau\Delta, L) > 0$. However, since $x_{so}(\overline{v}, 0) = x_{co}(\overline{v}, \theta) = 0$, it must be $x_\chi(\underline{v}, \theta) \in \{x_{so}(\underline{v} + \Delta\tau, 0), x_{co}(\underline{v} + \Delta\tau, 0)\}$. Moreover, if $x_\chi(\underline{v} + \Delta\tau, 0) > 0$, then it must be $x_\chi(\underline{v} + \Delta\tau, 0) \notin \{x_{so}(\overline{v}, 0), x_{co}(\overline{v}, \theta)\}$. Therefore, if $x_\chi(\underline{v}, \theta) = x_{\chi'}(\underline{v} + \Delta\tau, 0) > 0$, then it must be $p_i(x_{\chi'}(\underline{v} + \Delta\tau, 0)) < \underline{v} + \Delta\tau$. However, if $p_i(x_{\chi'}(\underline{v} + \Delta\tau, 0)) < \underline{v} + \Delta\tau$, type $(\underline{v} + \Delta\tau, 0)$ has a profitable deviation to $x = 0$, which implies it must be $x_{\chi'}(\underline{v} + \Delta\tau, 0) = 0$, and therefore, $x_\chi(\underline{v}, \theta) = 0$. However, this contradicts point 2. Therefore, $x_\chi(\underline{v}, \theta) \in \{x_{so}(\overline{v}, L), x_{so}(\underline{v} + \Delta\tau, L), x_{co}(\underline{v} + \Delta\tau, L)\}$ as required.

4. The investor sells strictly less than one unit from firm i. Suppose not. Based on point 1, if $x_i \geq 1$, then the price is $\underline{v}$, which means that the investor never sells more than one unit when $v_i \in \{\overline{v}, \underline{v} + \Delta\tau\}$. Based on point 3, it must be $x_\chi(\underline{v}, \theta) < 1$ as well.

5. $x_{so}(\underline{v}, 0) = x_{so}(\underline{v}, L) = x_{co}(\underline{v}, 0) = x_{co}(\underline{v}, L) = \overline{x} > 0$. Suppose on the contrary this is not the case. Based on point 4, $x_\chi(\underline{v}, \theta) < 1$. Since $n > 1$ and $\underline{v} \geq L$, the investor always meets her liquidity need in equilibrium. Therefore, the liquidity constraint does not bind, and the strategies of the investor when $v_i = \underline{v}$ must generate the same payoff. Suppose there are $(\chi', \theta') \neq (\chi'', \theta'')$ such that $x_{\chi'}(\underline{v}, \theta') = x' \neq x'' = x_{\chi''}(\underline{v}, \theta'')$. Payoffs are the same when $v_i = \underline{v}$ if

$$x' p_i(x') + (1 - x')\underline{v} = x'' p_i(x'') + (1 - x'')\underline{v} \Leftrightarrow$$
$$x' p_i(x') - x'' p_i(x'') = (x' - x'')\underline{v}.$$

Recall $x_\chi(\underline{v}, \theta) \in \{x_{so}(\overline{v}, L), x_{so}(\underline{v} + \Delta\tau, L), x_{co}(\underline{v} + \Delta\tau, L)\}$. There are two cases. First $x' = x_{so}(\overline{v}, L)$ and $x'' = x_\chi(\underline{v} + \Delta\tau, L)$. Note that $x' \neq x''$ implies $x_{so}(\overline{v}, L) \neq x_\chi(\underline{v} +$

$\Delta\tau, L)$, and by revealed preference:

$$\begin{aligned} x'p_i(x') - x''p_i(x'') &\geq (x' - x'')\left(\underline{v} + \Delta\tau\right) \text{ and,} \\ x'p_i(x') - x_L p_i(x'') &\leq (x' - x'')\overline{v} \end{aligned}$$

which holds if and only if

$$(x' - x'')\overline{v} \geq (x' - x'')\underline{v} \geq (x' - x'')\left(\underline{v} + \Delta\tau\right),$$

which can never hold. Second, $x' = x_{co}(\underline{v} + \Delta\tau, L)$ and $x'' = x_{so}(\underline{v} + \Delta\tau, L)$. Note that $x' \neq x''$ implies $x_{co}(\underline{v} + \Delta\tau, L) \neq x_{so}(\underline{v} + \Delta\tau, L)$, and by revealed preference

$$\begin{aligned} x'p_i(x') - x''p_i(x'') &\geq (x' - x'')\left(\underline{v} + \Delta\tau\right) \text{ and,} \\ x'p_i(x') - x_L p_i(x'') &\leq (x' - x'')\left(\underline{v} + \Delta\tau\right) \end{aligned}$$

which holds if and only if

$$(x' - x'')\left(\underline{v} + \Delta\tau\right) \geq (x' - x'')\underline{v} \geq (x' - x'')\left(\underline{v} + \Delta\tau\right),$$

which holds if and only if $x' = x''$, a contradiction.

6. $x_\chi(\underline{v}+\Delta\tau, L) = \overline{x}$. Suppose instead $x_\chi(\underline{v}+\Delta\tau, L) = x'_\chi \neq \overline{x}$. Since $x'_\chi \neq \overline{x}$, point 5 yields $p_i(x'_\chi) \geq \underline{v} + \Delta\tau$. Moreover, since $n > 1$ and $\underline{v} \geq L$, the investor meets her liquidity need by selling x'_χ units. There are two cases:

   (a) Suppose $p_i(\overline{x}) < p_i(x'_\chi)$. If $x'_\chi > \overline{x}$, then type $(\underline{v}, 0)$ has a profitable deviation to x'_χ, since she can sell more shares for a higher price. If $x'_\chi < \overline{x}$, then if $x_{so}(\overline{v}, L) = \overline{x}$, then type $(\overline{v}, L)$ has a profitable deviation to x'_χ (which satisfies her liquidity need). Therefore, it must be $x_{so}(\overline{v}, L) \neq \overline{x}$ and $p_i(\overline{x}) < \underline{v} + \Delta\tau$. This implies that $x_{\chi'}(\underline{v} + \Delta\tau, L) \neq \overline{x}$ as well: indeed, if $x_i = \overline{x}$ and $v_i = \underline{v} + \Delta\tau$, then the investor's payoff is strictly smaller than $\underline{v} + \Delta\tau$, but if $x_i = x'_\chi$ and $v_i = \underline{v} + \Delta\tau$, then the investor's payoff is strictly larger as she sells fewer units at a higher price. Therefore, it must be that $p_i(\overline{x}) = \underline{v}$. However, in this case and type $(\underline{v}, 0)$ has a profitable deviation to x_i', a contradiction.

(b) Suppose $p_i(\overline{x}) \geq p_i(x_i')$. Since $p_i(x_\chi') \geq \underline{v}+\Delta\tau$, type $(\overline{v}, L)$ must play $\overline{x}$ with positive probability. By revealed preference, this means that

$$\overline{x}p_i(\overline{x}) - x_{\chi'}p_i(x_{\chi'}) \geq (\overline{x} - x_{\chi'})\overline{v}.$$

Since type $\underline{v}$ also weakly prefers $\overline{x}$ over $x_{\chi'}$,

$$\overline{x}p_i(\overline{x}) - x_{\chi'}p_i(x_{\chi'}) \geq (\overline{x} - x_{\chi'})\underline{v}.$$

However, type $(\underline{v} + \Delta\tau)$ weakly prefers $x_{\chi'}$ over $\overline{x}$,

$$\overline{x}p_i(\overline{x}) - x_{\chi'}p_i(x_{\chi'}) \leq (\overline{x} - x_{\chi'})\left(\underline{v} + \Delta\tau\right).$$

The combination of the three conditions implies $\overline{x} - x_{\chi'} = 0$, a contradiction.

7. $p_i(\overline{x}) < \underline{v} + \tau\Delta$. Based on points 1–6 and the application of Bayes' rule,

$$p_i(\overline{x}) \leq \max_{\gamma\in[0,1]}\left\{\underline{v} + \Delta\frac{\lambda\mu\beta\tau + (1-\lambda)\left(\beta + (1-\beta)\gamma\right)\tau}{\lambda\left[\mu\left(\beta\tau + 1 - \tau\right) + (1-\mu)\left(1-\tau\right)\right] + (1-\lambda)\left(\beta + (1-\beta)\gamma\right)}\right\}.$$

Indeed, the highest possible value of $p_i(\overline{x})$ arises if $x_{so}(\overline{v}, L) = \overline{x}$ w.p. 1, and $x_\chi(\underline{v} + \Delta\tau, 0) = \overline{x}$ w.p. γ. Note that since $\tau \in (0, 1)$ and $\lambda \in (0, 1)$, for every $\gamma \in [0, 1]$ the RHS is strictly smaller than $\underline{v} + \Delta\tau$.

8. $x_{so}(\overline{v}, L) = \overline{x}$. Suppose instead $x_{so}(\overline{v}, L) = x_i' \neq \overline{x}$. Based on points 6 and 7, $p_i(x_i') > \underline{v} + \Delta\tau > p_i(\overline{x})$. Therefore, type $(\underline{v} + \Delta\tau, L)$ has strict incentives to deviate to x_i' since it leads to a trading profit and also satisfies her liquidity need.

9. $x_\chi(\underline{v} + \Delta\tau, 0) \neq \overline{x}$. Suppose instead that $x_\chi(\underline{v} + \Delta\tau, 0) = \overline{x}$. Based on point 7, $p_i(\overline{x}) < \underline{v}+\Delta\tau$. Therefore, type $(\underline{v}+\Delta\tau, 0)$ has strict incentives to deviate from $x_i = \overline{x}$ to $x_i = 0$.

Given the claims above, Bayes' rule implies $p_i(\overline{x}) = \overline{p}$. Note that $L \leq \underline{v}$ implies $\overline{x}^* = L/\overline{p} \leq$

1. We prove that, in any equilibrium, $\overline{x} = \overline{x}^*$. Suppose on the contrary that $\overline{x} > \overline{x}^*$. Then, it has to be $\overline{xp} > \overline{x}^*\overline{p} = L$. Since the pricing function is non-increasing, there is $\varepsilon > 0$ such that $(\overline{x} - \varepsilon) p (\overline{x} - \varepsilon) \geq L$. This implies that type $(\overline{v}, L)$ of the concentrated investor will strictly prefer deviating to $\overline{x} - \varepsilon$, a contradiction. We conclude that $\overline{x} \leq \overline{x}^*$. Next, suppose on the contrary $\overline{x} < \overline{x}^*$. Then, it has to be $\overline{xp} < \overline{x}^*\overline{p} = L$. Type $(\overline{v}, L)$ of the concentrated investor does not raise revenue of L in equilibrium by selling $\overline{x}$ units of the firm. Consider a deviation to selling one unit. Since $p(1) \geq \underline{v}$, the revenue raised would be at least $\underline{v} \geq L$, and so the deviation is optimal, a contradiction.

Note that $x_\chi(\underline{v} + \Delta\tau, 0) \neq 0$ can be sustained in equilibrium, if it is small enough to prevent mimicking by types who choose $\overline{x}^*$. At the same time, $x_\chi(\underline{v} + \Delta\tau, 0) = 0$ can also be in equilibrium. We consider these two cases separately below:

1. Suppose $x_\chi(\underline{v} + \Delta\tau, 0) = 0$. The pricing function in Proposition 10 is consistent with the trading strategy and it is non-increasing. Note that the trading strategy is incentive-compatible given prices. Consider the concentrated investor. First, the equilibrium payoff of type $(\overline{v}, 0)$ is $\overline{v}$, the highest possible. Second, since $\overline{x}^*\overline{p} = L$ and $p^*(x)$ is flat on $(0, \overline{x}^*]$, deviating to $(0, \overline{x}^*]$ generates revenue strictly lower than L, and so is suboptimal if $\theta = L$. Moreover, since $x > \overline{x}^* \Rightarrow p^*(x) = \underline{v}$, the investor has no optimal deviation to $x > \overline{x}^*$, regardless of firm value. Last, it is easy to see that $x = \overline{x}^*$ is optimal for type $(\underline{v}, 0)$. Consider the diversified investor. Since $\overline{p} \in (\underline{v}, \underline{v} + \Delta\tau)$, $p^*(x)$ is flat on $(0, \overline{x}^*]$, and $x > \overline{x}^* \Rightarrow p^*(x) = \underline{v}$, she has incentives to sell $\overline{x}^*$ from all bad assets. Moreover, since $\overline{x}^*\overline{p} = L$, $L \leq \underline{v}$ and $1 \leq n(1 - \tau)$ imply that by selling $\overline{x}^*$ the diversified investor raises enough revenue to meet her liquidity need.

   Note that the expected price in equilibrium conditional on $\underline{v}$ and $\overline{v}$, respectively is,

$$
\begin{aligned}
P(\underline{v}) &= \lambda\overline{p} + (1 - \lambda)[\beta\overline{p} + (1 - \beta) p^*(0)] \\
P(\overline{v}) &= \lambda(\beta\mu\overline{p} + (1 - \beta\mu) p^*(0)) + (1 - \lambda)[\beta\overline{p} + (1 - \beta) p^*(0)].
\end{aligned}
$$

   It can be verified that $P(\underline{v})$ is increasing in μ. Since $\tau P(\overline{v}) + (1 - \tau) P(\underline{v}) = \underline{v} + \Delta\tau$, it follows that higher μ implies higher $P(\underline{v})$ and a lower $P(\overline{v})$, and therefore, price informativeness decreases in μ.

2. Suppose $x_\chi(\underline{v}+\Delta\tau,0)\neq 0$. The only difference from the previous case is that $p^*(0)=\overline{v}$ and $p^*(x_\chi(\underline{v}+\Delta\tau,0)) = \underline{v}+\Delta\tau$. The proof that the trading strategy is incentive-compatible given prices is very similar to the case 1 above, and for brevity it is omitted. The expected price in equilibrium conditional on $\underline{v}$ and $\overline{v}$, respectively is,

$$\begin{aligned} P(\underline{v}) &= \lambda\overline{p}+(1-\lambda)\left[\beta\overline{p}+(1-\beta)(\underline{v}+\Delta\tau)\right] \\ P(\overline{v}) &= \lambda(\beta\mu\overline{p}+(1-\beta\mu)\overline{v})+(1-\lambda)\left[\beta\overline{p}+(1-\beta)(\underline{v}+\Delta\tau)\right]. \end{aligned}$$

It can be verified that $\overline{p}$ is increasing in μ. Therefore, $P(\underline{v})$ is also increasing in μ, which implies that price informativeness decreases in μ.

■

F.4 Heterogeneous firms

In the core model, all firms have the same valuation distribution Δ. We now analyze the case in which firms have different valuation distributions, and thus differ in their information asymmetry and the price impact of selling. For brevity, we consider the small shock case, since this is where our results are strongest.

Let there be $J\geq 1$ classes of firms. The valuation of firm i in class $j\in\{1,...,J\}$ is $v_{i,j}\in\{\underline{v}_j,\overline{v}_j\}$, where $\Delta_j\equiv\overline{v}_j-\underline{v}_j>0$. We assume $\underline{v}_{j'}<\overline{v}_{j''}$ for all $j'\in J$ and $j''\in J$—that is, the worst good firm is more valuable than the best bad firm. We also index by j the exogenous parameters τ, $\overline{v}$, and $\underline{v}$. As in the core model, each firm has its own market maker, and the class to which firm i belongs is common knowledge. All random variables are independent across firms and classes.

The analysis of separate ownership remain unchanged, with the addition of a subscript j to denote that quantities apply to a firm of class j. Under common ownership, we assume the investor owns a mass of $n_j\geq 0$ firms from class j.

Proposition 11 (Heterogeneous firms): Suppose $L\leq\sum_{j=1}^{J}n_j\underline{v}_j(1-\tau_j)$, and let γ_j denote the probability that $x_{j,i}=0$ when $v_{j,i}=\underline{v}_j$. For firm class j, if $\gamma_j\leq\frac{\beta\tau_j}{\beta\tau_j+(1-\beta)(1-\tau_j)}$, price informativeness is weakly higher under common ownership than separate ownership. Conversely, if $\gamma_j > \frac{\beta\tau_j}{\beta\tau_j+(1-\beta)(1-\tau_j)}$, price informativeness is strictly higher under separate ownership.

Proof of Proposition 11. Since $L \leq \sum_{j=1}^{J} n_j \underline{v}_j(1-\tau_j)$, the investor can meet her liquidity need by selling bad firms alone. Then, in any equilibrium, as in the core model it must be that for firm i in class j, $x_i > 0 \rightarrow p_{j,i}(x_i) = \underline{v}_j$. Then, if $v_{j,i} = \overline{v}_j$, $x^*_{j,div}(v_{j,i},\theta) = 0$. If $v_{j,i} = \underline{v}_j$, similar to the core model, $x^*_{j,div}(v_{j,i},\theta) = \underline{x}_j(\theta) \in \{0,1\}$, with $\underline{x}_j$ such that $\sum_{j=1}^{J} \underline{x}_j(\theta) n_j \underline{v}_j(1-\tau_j) \in \left[\theta, \sum_{j=1}^{J} n_j \underline{v}_j(1-\tau_j)\right]$. Letting $\gamma_j \in \{0,\beta,1-\beta,1\}$ denote the probability that $x_{j,i} = 0$ when $v_{j,i} = \underline{v}_j$, prices on the equilibrium path are then:

$$
\begin{aligned}
p_{j,i}(x_{j,i}) &= \underline{v}_j \;\; if\, x_{j,i} > 0 \\
p_{j,i}(x_{j,i}) &= \underline{v}_j + \Delta_j \left(\frac{\tau_j}{\tau_j + (1-\tau_j)\gamma_j}\right) \;\; if\, x_{j,i} = 0.
\end{aligned}
$$

Then, expected prices under common ownership are:

$$
\begin{aligned}
P_{j,div}(\overline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \left(\frac{\tau_j}{\tau_j + (1-\tau_j)\gamma_j}\right) \\
P_{j,div}(\underline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \gamma_j \left(\frac{\tau_j}{\tau_j + (1-\tau_j)\gamma_j}\right).
\end{aligned}
$$

Under separate ownership, expected prices are:

$$
\begin{aligned}
P_{j,con}(\overline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \left(\frac{\beta\tau_j + (1-\beta)(1-\tau_j)}{\beta\tau_j + (1-\tau_j)}\right) \\
P_{j,con}(\underline{v}_j, \tau_j) &= \underline{v}_j + \Delta_j \left(\frac{\beta\tau_j}{\beta\tau_j + (1-\tau_j)}\right).
\end{aligned}
$$

Then, we have

$$\begin{aligned} P_{j,div}(\underline{v}_j, \tau_j) \leq P_{j,con}(\underline{v}_j, \tau_j) &\iff \\ \frac{\gamma_j \tau_j}{\tau_j + (1-\tau_j)\gamma_j} \leq \frac{\beta \tau_j}{\beta \tau_j + (1-\tau_j)} &\iff \\ \beta \tau_j \gamma_j + (1-\tau_j)\gamma_j \leq \beta \tau_j + \beta(1-\tau_j)\gamma_j &\iff \\ \frac{\beta \tau_j}{\beta \tau_j + (1-\beta)(1-\tau_j)} \geq \gamma_j. \end{aligned}$$

Price informativeness is higher under common ownership if this inequality holds, which is the same as in the core model (with subscript j added). ■

F.5 Discontinuing relationships

In this section, we apply our model to situations in which the investor decides whether to (partly) discontinue a relationship with the firm. We thus distinguish between two concepts—the price $p_i(x_i)$ reflects the impact of (dis)continuation on firm i's reputation, and the payoff upon selling is what the investor receives. In the core model, these were the same, but now the latter is the investor's outside option, $r < \overline{v}$.[36] Importantly, unlike in the core model, this reservation payoff is fixed and independent of the impact of sale on the firm's reputation. However, we nevertheless show that common ownership can still improve price informativeness.

We consider two cases based on the magnitude of r. In the first case, $r < \underline{v}$, and so the investor discontinues only if she needs liquidity. In the second case, $r \in (\underline{v}, \overline{v})$, and so the investor discontinues her relationship with bad firms, but retains it with good firms.

Proposition 12 (Discontinuing relationships, low reservation payoff): Suppose $r < \underline{v}$. Price informativeness under common ownership is always weakly higher than under separate ownership, and strictly higher when $L/n < r$.

Proof of Proposition 12. First, consider separate ownership. The optimal strategies are

[36] If $r \geq \overline{v}$, the analysis is trivial since the investor is weakly better off discontinuing all assets, regardless of their value and her liquidity needs. This behavior will result in identical price informativeness under separate and common ownership and, in particular, expected prices will be the same for both good and bad firms.

$x_i^*(v_i, L) = \overline{x} \equiv \min\{\frac{L/n}{r}, 1\}$ for all v_i, and $x_I^*(v_i, 0) = 0$ for all v_i. Then, prices are fully uninformative.

Next, consider common ownership. Note first that, for all L/n, it is optimal not to discontinue when $\theta = 0$. Now, suppose $L/n \leq r(1-\tau)$. Then, when $\theta = L$, the optimal strategies are $x_i^*(\overline{v}, L) = 0$ and $x_i^*(\underline{v}, L) = \overline{x} \equiv \frac{L/n}{(1-\tau)r}$. In that case $p(x_i) = \underline{v}$ for $x_i = \overline{x}$, and $p(x_i) = \underline{v} + \Delta\frac{\tau}{\tau+(1-\beta)(1-\tau)}$ for $x_i = 0$. If, instead, $L/n \in (r(1-\tau), r)$, optimal strategies when $\theta = L$ are $x_i^*(\underline{v}, L) = 1$ and $x_i^*(\overline{v}, L) = \frac{L/n - r(1-\tau)}{r\tau}$. Thus, again prices are partially informative. Finally, if $L/n \geq r$, we have $x_i^*(v_i, L) = 1$ for all v_i, and so prices are fully uninformative.

Therefore, prices are strictly more informative under common ownership when $L/n < r$, and equally (un)informative when $L/n \geq r$. ■

Proposition 13 (Discontinuing relationships, high reservation payoff): Suppose $r \in (\underline{v}, \overline{v})$. Price informativeness under common ownership is always identical to that under separate ownership.

Proof of Proposition 13. Note that when $r \in (\underline{v}, \overline{v})$, the investor always wants to discontinue bad firms fully. First, consider separate ownership. Then, $x_i^*(\underline{v}, \theta) = 1$ for all θ, $x_i^*(\overline{v}, 0) = 0$, and $x_i^*(\overline{v}, L) = \overline{x} \equiv \min\{\frac{L/n}{r}, 1\}$. Then, if $L/n < r$, prices are $p_i(x_i) = \underline{v}$ if $x_i = 1$, and $p_i(x_i) = \overline{v}$ if $x_i \in \{0, \overline{x}\}$.[37] Otherwise, they are $p_i(x_i) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau}$ if $x_i = 1$ and $p_i(x_i) = \overline{v}$ if $x_i = 0$.

Under common ownership, $x_i^*(\underline{v}, \theta) = 1$ for all θ, and $x_i^*(\overline{v}, 0) = 0$. If $\frac{L}{n} < r$, then $x_i^*(\overline{v}, L) = \overline{x} \equiv \max\{0, \frac{L/n - r(1-\tau)}{\tau r}\}$. In this case, prices are $p_i(x_i) = \overline{v}$ if $x_i \in \{0, \overline{x}\}$, and $p_i(x_i) = \underline{v}$ if $x_i = 1$.

Alternatively, if $\frac{L}{n} \geq r$, then $x_i^*(\overline{v}, L) = 1$. Prices are $p_i(x_i) = \overline{v}$ if $x_i = 0$, and $p_i(x_i) = \underline{v} + \Delta\frac{\beta\tau}{\beta\tau+1-\tau}$ if $x_i = 1$. Thus, if $L/n < r$,

$$P_{so}(\overline{v}, \tau) = \overline{v} = P_{co}(\overline{v}, \tau),$$

and

$$P_{so}(\underline{v}, \tau) = \underline{v} = P_{co}(\underline{v}, \tau).$$

[37] Since off-equilibrium prices are irrelevant for the seller's discontinuation decision, we do not specify them to ease the exposition.

If instead $L/n \geq r$,

$$P_{so}(\overline{v}, \tau) = \underline{v} + \Delta \frac{\beta\tau + (1-\beta)(1-\tau)}{\beta\tau + 1 - \tau} = P_{co}(\overline{v}, \tau)$$
$$P_{so}(\underline{v}, \tau) = \underline{v} + \Delta \frac{\beta\tau}{\beta\tau + 1 - \tau} = P_{co}(\underline{v}, \tau)$$

Thus, price informativeness under separate ownership is identical to that under common ownership. ■

The intuition is as follows. Since the investor is unconcerned with price impact, she sells a bad firm fully and thus receives the lowest possible price of $\underline{v}$ under any ownership structure. Thus, if the shock is not large enough to force the sale of her entire portfolio ($\frac{L}{n} < r$), prices are already fully informative under separate ownership and so no higher under common ownership.