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Online Supplement:
When to Deploy Test Auctions in Sourcing

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Proof of Proposition 1

No-test. The dominant bidding strategy result follows from Krishna (2002). Since the marginal cost of recruiting a supplier is nondecreasing (as $k(m)$ is increasing convex), to show convexity of total cost in m_N it suffices to show that the expected per-unit purchase price is decreasing discrete convex in the number of recruited suppliers. This in turn is implied by regularity; to see this, take the auction's expected per-unit clearing price, $a+\int_a^b(1-F(s))^{n+m_N}ds+\int_a^b(n+m_N)F(s)(1-F(s))^{n+m_N-1}ds$, and integrate the last term by parts and then differentiate with respect to m_N twice.

Test. We first characterize the buyer's "best response" m_T^* under the supposition that suppliers remain in an auction until either winning or reaching their true cost. We then show that given the buyer's strategy the supposed supplier strategy indeed constitutes an equilibrium.

Note that the buyer's recruitment decision depends on whether the test auction reserve price is met or not, and we analyze both cases one by one. If the reserve price is not met, no units are awarded in the test auction and all incumbents are discarded. So the second auction is equivalent to running a no-test strategy to auction off Q units with zero incumbents to start with.

If the reserve price is met, then the outcome of the test auction is $x=\min\{r_1, X_{(2:n)}\}$ (due to our supposition of suppliers' equilibrium bidding strategy). For the second auction, let $\mathbb{E}[\mathrm{MB}_m(x)]$ denote the expected per-unit purchasing price when $m-1$ entrants are recruited minus the same when m entrants are recruited. Let $\hat{\hat{X}}$ represent the random variable following distribution $\hat{\hat{F}}(y)=F(y)/F(x)$ for $y\in[a,x]$. We can write

$$\mathbb{E}[\mathrm{MB}_m(x)]=F(x)\left[\sum_{i=0}^{m-1}\binom{m-1}{i}F(x)^i(1-F(x))^{m-1-i}[\mathbb{E}[\hat{\hat{X}}_{(2:i+1)}]-\mathbb{E}[\hat{\hat{X}}_{(2:i+2)}]]\right],$$

where $\mathbb{E}[\hat{\hat{X}}_{(2:1)}]\triangleq x$. The leading $F(x)$ reflects the fact that the m^{th} entrant has an effect only if its cost lies below x, and the summation over i corresponds to i of the other $m-1$ entrants having a cost below x as well. Similarly, we can write

$$\begin{aligned}\mathbb{E}[\mathrm{MB}_{m+1}(x)]=&(1-F(x))\mathbb{E}[\mathrm{MB}_m(x)]\\&+\big(F(x)\big)^2\left[\sum_{i=0}^{m-1}\binom{m-1}{i}F(x)^i(1-F(x))^{m-1-i}[\mathbb{E}[\hat{\hat{X}}_{(2:i+2)}]-\mathbb{E}[\hat{\hat{X}}_{(2:i+3)}]]\right]<\mathbb{E}[\mathrm{MB}_m(x)]\end{aligned}$$

where the inequality follows since $\mathbb{E}[\hat{\hat{X}}_{(2:i+1)}]-\mathbb{E}[\hat{\hat{X}}_{(2:i+2)}]>\mathbb{E}[\hat{\hat{X}}_{(2:i+2)}]-\mathbb{E}[\hat{\hat{X}}_{(2:i+3)}]$ by regularity of $\hat{\hat{F}}$ (which follows from regularity of F). Since the supplier recruitment cost is convex, the buyer's expected total (recruitment plus second-auction purchase) cost is discrete convex in m_T.

To prove that the buyer's optimal number of entrants to recruit is nondecreasing in the clearing price, we first establish the following lemma.

LEMMA EC. 1. *The expected marginal benefit of adding the m^{th} supplier is nondecreasing in x.*

Proof of Lemma 1 This can be seen using a sample path argument: Suppose there are $m-1$ entrants. Order the cost of the incumbent who won the test auction and the $m-1$ realizations of entrant costs such that $x_1 \le x_2 \le \cdots \le x_m$. Now consider adding an m^{th} entrant with cost realization z. We will show that the buyer's marginal benefit does not decrease in the test auction clearing price x under any sample path. We know that $x_1 \le x$ since the incumbent's cost is at most x. There are two cases to consider.

Case 1. $x_1 \le x \le x_2$ or $m = 1$. The buyer's marginal benefit of recruiting the m^{th} entrant is

$$x - x_1 \quad \text{if} \quad z \le x_1 \le x, \tag{12}$$

$$x - z \quad \text{if} \quad x_1 \le z \le x, \quad \text{and} \tag{13}$$

$$0 \quad \text{if} \quad x_1 \le x \le z. \tag{14}$$

Now consider the case where the test auction clearing price is $x' > x$. Under this sample path, (12) becomes $\min\{x', x_2\} - x_1$, (13) becomes $\min\{x', x_2\} - z$ (where if $m = 1$ we take $x_2 = \infty$), and (14) becomes a nonnegative quantity (as the second auction's clearing price cannot increase when the m^{th} entrant is added). Thus, the buyer's marginal benefit of adding the m^{th} entrant when the clearing price is x' is greater than or equal to her marginal benefit when the clearing price is x under all sample paths where $x_1 \le x \le x_2$.

Case 2. $x_2 \le x$. The buyer's marginal benefit of recruiting the m^{th} entrant is $x_2 - x_1$ if $z \le x_1 \le x_2$; $x_2 - z$ if $x_1 \le z \le x_2$; and 0 if $x_1 \le x_2 \le z$. Hence the marginal benefit is unchanged when clearing price is $x' > x$.

In conclusion, since the stated result holds for any sample path, it must hold in expectation. □

Note that the marginal cost of adding an additional entrant is independent of x. Hence, by Lemma 1, the buyer's expected total (recruitment plus second-auction purchase) cost is submodular in the number of entrants to recruit and the clearing price. Hence, the optimal number of entrants to recruit is nondecreasing in the clearing price.

We now address the supplier bidding strategy. Working backwards, in the second auction, the result holds as it is analogous to a single auction. In the test auction, if supplier i remains in the auction at a price below his true cost x_i, he will lose money on any units he is awarded and can only make negative profit because the second auction opens at the clearing price of the test auction. If the supplier drops out of the test auction at a price above his true cost x_i, if he does not meet the test auction reserve price, he is discarded and earns zero profit. If he does meet the reserve price, he still loses the test auction and cannot make more profit in the second auction than if he bid down to x_i because the number of entrants the buyer will recruit is nondecreasing in the outcome of the test auction. In either case, the supplier would have been better off remaining in the auction until winning or reaching his true cost, whichever happens first.

Note that the above analysis holds for any quantity auctioned off in the test auction. That is, the incumbent cost information gleaned by the buyer in the test auction is the same regardless of the quantity auctioned off in the test auction (as long as that quantity is at least y, the minimum quantity needed to entice the incumbents to participate in the auction). Thus, it is optimal for the buyer to auction the minimum quantity possible in the test auction, y, because when the reserve price is met, the clearing price in the second auction is no higher than the test auction, and when the reserve price is not met, this quantity does not affect the buyer's cost.

Proof of Proposition 2

(i)(a). Although an increase in y does not affect the buyer's cost under the no-test strategy, it weakly increases her expected cost under any given sample path using the test without reserve price strategy. For $y = 0$, the test without reserve price strategy weakly dominates the no-test strategy as the buyer could achieve the same expected cost as the no-test strategy by recruiting m_N^* entrants regardless of the outcome of the test auction.

(i)(b). As discussed in Proposition 1's proof, the expected unit cost under the no-test strategy is decreasing discrete convex in the number of recruited suppliers and approaches a as $n+m \to \infty$. As the marginal cost of recruiting an entrant is nondecreasing, there must exist a n^* such that $m_N^* = 0 \;\forall\; n \geq n^*$. If $m_N^* = 0$, the test without reserve price strategy is a weakly dominant strategy.

(i)(c). We model the new supplier recruitment cost growing proportionately larger by letting $\gamma \cdot k(m)$ denote the recruitment cost, and considering the constant γ. As $k(1) > 0$, there exists a constant, $\overline{\gamma}$, such that when $\gamma = \overline{\gamma}$ the buyer prefers the test without reserve price strategy because $m_N^* = 0$, and the test without reserve price strategy weakly dominates the no-test strategy. Further, the test without reserve price strategy will be preferred for all $\gamma > \overline{\gamma}$ as $m_N^* = 0$ for all such γ.

(i)(d). First we establish the following lemma:

LEMMA EC. 2. *Let the scaled distribution $\tilde{f}$ be such that $\tilde{f}(x) = f(x/\alpha)/\alpha$. As α becomes large (that is, the distribution is scaled up), the buyer prefers the no-test strategy. Similarly, as α becomes small (that is, the distribution is scaled down), the buyer prefers the test without reserve price strategy.*

Proof of Lemma 2. We show that scaling up (down) the distribution by α is equivalent to scaling down (up) the recruitment cost function by α. Then the result follows from part (i)(c). Define scaled distribution $\tilde{F}(x) \triangleq F(x/\alpha)$, and scaled recruitment cost function $\tilde{k}(m) \triangleq \alpha k(m)$. Let $\tilde{X}_{(i:j)}$ represent the i^{th}-lowest order statistic out of j draws from the distribution $\tilde{F}$. Then the buyer's problem given by the parameters $\tilde{F}$, $\tilde{k}(m)$, Q, y, and n can be written as follows. For the no-test strategy,

$$\mathbb{E}[\text{total cost under no-test}|m_N] = \mathbb{E}[\tilde{X}_{(2:n+m_N)}]Q + \tilde{k}(m_N) = \alpha\big(\mathbb{E}[X_{(2:n+m_N)}]Q + k(m_N)\big). \tag{15}$$

The analysis for the test without reserve price strategy is analogous. So, the problem with scaled parameters $\tilde{f}$ and $\tilde{k}(m)$ is equivalent to the original problem with all supplier and recruitment costs scaled by α. □

Now consider the effect of shifting the distribution by Δ. A Δ-shift of the suppliers' cost distribution is represented by the distribution $\tilde{f}$ with $\tilde{f}(x) = f(x+\Delta)$. It is trivial to show that such a shift will not change the buyer's strategy preference — it will simply shift the expected cost of both strategies by $\Delta \cdot Q$.

Combining this result with Lemma 2 completes the proof of (i)(d).

(ii). The test strategy results in an expected cost that is less than or equal to the buyer's expected cost under the test without reserve price strategy as $r_1 = b$ is a feasible reserve price. Thus, if the test without reserve price strategy has an expected cost less than the no-test strategy, so does the test strategy.

Proof of Proposition 3

As the first step in proving the optimal mechanism, we first establish that the revelation principle can be applied to our setting. To do so, consider the following class of multi-stage mechanisms featuring a form of costly communication between the buyer and suppliers. In particular, incumbents are free to initiate communication with, but entrants are costly to initiate communication with. Incumbents and entrants each have a production cost, which is their private information. The buyer's contract award decision and payments to suppliers are based on the information reported by the set of suppliers the buyer communicated with, and not on information from suppliers the buyer did not communicate with. Finally, there are two additional points: (i), the buyer cannot convey information about the mechanism to a supplier without communicating with them; and (ii), communication does not occur between the suppliers themselves (this is meant to disallow mechanisms where the buyer, say, recruits one entrant and then asks that entrant to gather information from the other entrants).

McAfee and McMillan (1988) formally establish that the revelation principle holds for such dynamic mechanisms, which are "principal-initiated (point (i) above) and "principal-centered (point (ii) above). Namely, the revelation principle implication is that for any Bayesian Nash equilibrium of a mechanism described above, there exists a direct, truthful sequential (the buyer communicates with suppliers one at a time) mechanism which produces the same distribution of decisions and suppliers communicated with. In such a mechanism, the buyer seeks out suppliers one-by-one and asks them to report their cost, and the suppliers report truthfully; once the buyer decides to stop, based on the costs reported up to that point a payment and allocation decision is made.

Restricting our search for an optimal mechanism to the class of direct sequential mechanisms greatly simplifies the problem at hand. Moreover, when searching for an optimal direct sequential mechanism, because it is costless to communicate with incumbent suppliers, without loss of optimality we can assume that the buyer communicates with all incumbent suppliers before she ever decides to communicate with an entrant. We will call stage 1 the stage where the buyer costlessly and simultaneously communicates with all the incumbents.

Up to now, we have described a class of multi-stage mechanisms with costly communication for which, without loss of optimality, to find an optimal mechanism we only have to consider mechanisms in which the buyer initially communicates with all the incumbents, and then one-by-one communicates with entrants. However, in our setting, as mentioned in the Introduction, to complete the procurement process in a timely manner the entrants must be recruited simultaneously. We can account for this while still deploying the revelation principle: Namely, we consider a sequential mechanism to be feasible only if the buyer commits to ignoring the information she collects from the entrants until she has communicated with every entrant she will communicate with. Of course, this can be collapsed into one simultaneous recruitment and communication stage for the entrants. We will call this stage 2.

To summarize, we wish to find an optimal multi-stage mechanism in which recruitment of entrants is costly and must be done simultaneously. The above arguments imply that to find such a mechanism, it suffices to restrict our search to the following class of two-stage direct mechanisms: In stage 1 the incumbents report their true costs to the buyer; in stage 2 the buyer recruits entrants who then report their true costs to the buyer; the buyer then makes contract allocation and payment decisions based on the costs of the incumbents, and recruited entrants (if any).

Given that the revelation principle allows us to focus on direct mechanisms, incentive compatibility constraint (7) can be stated as $u_i(x_i) \triangleq \max_{s_i}\{p_i(s_i) - x_i q_i(s_i)\}$. By the envelope theorem, this gives $u_i'(x_i) = -q_i(x_i)$, which implies that $u_i(x_i) = u_i(b) + \int_{x_i}^b q_i(s_i)\mathrm{d}s_i$. Solving for $p_i(x_i)$ gives

$$p_i(x_i) = u_i(b) + \int_{x_i}^b q_i(s_i)\mathrm{d}s_i + x_i q_i(x_i). \tag{16}$$

It is easy to check (e.g., Lemma 2 in Myerson (1981)) that, if (16) determines the payment rule, then q_i nonincreasing implies that incentive compatibility constraint (7) holds, and $u_i(b) \geq 0$ for all i implies that the individually rationality constraint (6) holds. Moreover,

$$\begin{aligned}
&\sum_{l=0}^{\infty}\int_{\{x^I|m_T(x^I)=l\}}\left[\int_{x^{E,l}} P_i(x^I, x^{E,l}) f(x^{E,l})\mathrm{d}x^{E,l}\right] f(x^I)\mathrm{d}x^I = \int_{x_i} p_i(x_i) f(x_i)\mathrm{d}x_i \\
=&u_i(b) + \int_{x_i} \psi(x_i) q_i(x_i) f(x_i)\mathrm{d}x_i \quad \text{by applying (16) and integrating,} \\
=&u_i(b) + \sum_{l=0}^{\infty}\int_{\{x^I|m_T(x^I)=l\}}\left[\int_{x^{E,l}} \psi(x_i) Q_i(x^I, x^{E,l}) f(x^{E,l})\mathrm{d}x^{E,l}\right] f(x^I)\mathrm{d}x^I.
\end{aligned}$$

Given these observations, we can rewrite the buyer's mechanism design problem (5)–(9) as

$$\min_{Q_i(\cdot), m_T(\cdot)} \quad \sum_{l=0}^{\infty}\int_{\{x^I|m_T(x^I)=l\}}\left[k(l) + \sum_{i=1}^{n+l} u_i(b) + \sum_{i=1}^{n+l}\int_{x^{E,l}} \psi(x_i) Q_i(x^I, x^{E,l}) f(x^{E,l})\mathrm{d}x^{E,l}\right] f(x^I)\mathrm{d}x^I \tag{17}$$

$$\text{s.t.} \quad u_i(b) \geq 0 \qquad \forall i \tag{18}$$

$$q_i(\cdot) \text{ nonincreasing} \qquad \forall i \tag{19}$$

$$\text{constraints } (8) - (9).$$

Note that an optimal mechanism will have $u_i(b) = 0$ for all i. For now, relax constraint (19). Since $\psi(\cdot)$ is increasing, for any given number of recruited entrants $m_T(x^I)$, objective function (17) is minimized by

$$Q_i^*(x^I, x^{E, m_T(x^I)}) = \begin{cases} Q & \text{if} \quad x_i = x_{(1:n+m_T(x^I))} \\ 0 & \text{otherwise,} \end{cases} \tag{20}$$

where $x_{(1:n+m_T(x^I))}$ is the lowest cost among the incumbents and recruited entrants.

Given (20), consider the optimal number of entrants to recruit, $m_T^*(x^I)$. Note that $m_T^*(x^I)$ is not affected by the incumbents with costs $x^I_{(2:n)}, x^I_{(3:n)}, \ldots, x^I_{(n:n)}$ because they are "out of the running" and $Q_i^*(x^I, x^{E,m_T(x^I)}) = 0$ for these incumbents. Hence, the optimal number of entrants to recruit only depends on the lowest-cost incumbent's cost, $x^I_{(1:n)}$, and (10) describes the optimal recruitment rule. We establish the following lemma.

Lemma EC. 3. *The optimal number of entrants to recruit, $m_T^*(x^I)$, is nondecreasing in $x^I_{(1:n)}$.*

Proof of Lemma 3 Similar to the proof of Proposition 1, we show (i) the buyer's total expected payment plus recruitment cost is discrete convex in m_T and (ii) the expected marginal benefit of recruiting the m^{th} entrant is nondecreasing in $x^I_{(1:n)}$. Let H be the distribution of $\psi(\cdot)$. Note that $\mathbb{E}[\psi(X_{(1:n+m)})|x^I_{(1:n)}] =$

$\int_0^{\psi(x^I_{(1:n)})}(1-H(x))^m dx$ is decreasing discrete convex in m. Since the supplier recruitment cost function is increasing convex in m, the buyer's expected total (recruitment plus purchase) cost is discrete convex in m. Next, note that the marginal reduction in expected purchase cost from recruiting the m^{th} entrant equals

$$\mathbb{E}[\psi(X_{(1:n+m-1)})|x^I_{(1:n)}] - \mathbb{E}[\psi(X_{(1:n+m)})|x^I_{(1:n)}] = \int_0^{\psi(x^I_{(1:n)})}(1-H(x))^{m-1}H(x)dx,$$

and the derivative of the right hand side with respect to $x^I_{(1:n)}$ equals $(1-H(x^I_{(1:n)}))^{m-1}H(x^I_{(1:n)})$, which is positive. Thus, the buyer's optimal number of entrants to recruit is nondecreasing in $x^I_{(1:n)}$. $\square$

Thus far we have shown that (20) and (10) minimize the objective function (17). Furthermore, it is easy to check that constraints (8)-(9) are satisfied. We next show that q_i is nonincreasing (constraint (19)).

Suppose i is an entrant supplier. Since supplier i's report only affects the buyer's allocation decision, which is to give the contract to the lowest-cost bidder, clearly entrant i can not improve its allocation probability (increase q_i) by reporting a higher cost. Next suppose i is an incumbent supplier. For any number of entrants recruited by the buyer, supplier i's quantity allocation is nonincreasing in his cost report per (20). Moreover, the quantity allocation per (20) is nonincreasing in the number of recruited entrants he competes against. Since this number, m^*_T, is nondecreasing in supplier i's report (by Lemma 3), we must have that supplier i cannot increase his allocation probability by reporting a higher cost. In summary, the purchase quantity rule q_i is nonincreasing in x_i for all i, so the mechanism is incentive compatible.

To summarize, (20) and (10) solve program (17)-(19), (8)-(9). To complete Proposition 3's proof, it remains only to show that payment rule (11) implements (16) and so is an optimal payment rule. Define $x^I_{-i} \triangleq (x_1,\dots,x_{i-1},x_{i+1},\dots,x_n)$, and $x^E_{-i} \triangleq (x_{n+1},\dots,x_{i-1},x_{i+1},\dots,x_{n+|x^E|})$.

If i is an entrant, then (11) is equivalent to $P^*_i(x^I,x^E) = Q^*_i(x^I,x^E)x_i + \int_{s_i=x_i}^b Q^*_i(x^I,(s_i,x^E_{-i}))\mathrm{d}s_i$, hence

$$\begin{aligned}
p^*_i(x_i) =& \sum_{l=0}^{\infty}\int_{\{x^I|m_T(x^I)=l\}}\left[\int_{x^{E,l}_{-i}}\left\{Q^*_i(x^I,x^{E,l})x_i + \int_{s_i=x_i}^b Q^*_i(x^I,(s_i,x^{E,l}_{-i}))\mathrm{d}s_i\right\}f(x^{E,l}_{-i})\mathrm{d}x^{E,l}_{-i}\right]f(x^I)\mathrm{d}x^I\\
=& q^*_i(x_i)x_i + \int_{s_i=x_i}^b\left\{\sum_{l=0}^{\infty}\int_{\{x^I|m_T(x^I)=l\}}\left[\int_{x^{E,l}_{-i}} Q^*_i(x^I,(s_i,x^{E,l}_{-i}))f(x^{E,l}_{-i})\mathrm{d}x^{E,l}_{-i}\right]f(x^I)\mathrm{d}x^I\right\}\mathrm{d}s_i,\\
=& q^*_i(x_i)x_i + \int_{s_i=x_i}^b q^*_i(s_i)\mathrm{d}s_i, \quad \text{which matches (16).}
\end{aligned}$$

Finally, if i is an incumbent, then (11) is equivalent to

$$P^*_i(x^I,x^E) = Q^*_i(x^I,x^E)x_i + \int_{s_i=x_i}^b\left[\int_{\left\{\tilde{x}^E\middle||\tilde{x}^E|=m^*_T(s_i,x^I)-m^*_T(x^I)\right\}} Q^*_i\left((s_i,x^I_{-i}),(x^E,\tilde{x}^E)\right)f(\tilde{x}^E)\mathrm{d}\tilde{x}^E\right]\mathrm{d}s_i,$$

and hence

$$\begin{aligned}
p^*_i(x_i) =& \sum_{l=0}^{\infty}\int_{\{x^I_{-i}|m_T(x_i,x^I_{-i})=l\}}\left[\int_{x^{E,l}} Q^*_i(x^I,x^{E,l})x_i f(x^{E,l})\mathrm{d}x^{E,l}\right]f(x^I_{-i})\mathrm{d}x^I_{-i}\\
&+\sum_{l=0}^{\infty}\int_{\{x^I_{-i}|m_T(x_i,x^I_{-i})=l\}}\Big[\int_{x^{E,l}}\int_{s_i=x_i}^b\Big\{\int_{\left\{\tilde{x}^E\middle||\tilde{x}^E|=m^*_T(s_i,x^I)-m^*_T(x_i,x^I_{-i})\right\}}\\
&Q^*_i\left((s_i,x^I_{-i}),(x^{E,l},\tilde{x}^E)\right)f(\tilde{x}^E)\mathrm{d}\tilde{x}^E\Big\}f(x^{E,l})\mathrm{d}x^{E,l}\Big]f(x^I_{-i})\mathrm{d}x^I_{-i}\\
=& q^*_i(x_i)x_i + \int_{s_i=x_i}^b\Big[\sum_{l=0}^{\infty}\int_{\left\{x^I_{-i}|m^*_T(x_i,x^I_{-i})=l\right\}}\Big\{\int_{x^{E,m^*_T(s_i,x^I_{-i})}}
\end{aligned}$$

$$Q_i^*\left((s_i, x_{-i}^I), x^{E,m_T^*(s_i,x_{-i}^I)}\right) f(x^{E,m_T^*(s_i,x_{-i}^I)})\mathrm{d}x^{E,m_T^*(s_i,x_{-i}^I)}\Big\} f(x_{-i}^I)\mathrm{d}x_{-i}^I\Big]\mathrm{d}s_i,$$

$$= q_i^*(x_i)x_i + \int_{s_i=x_i}^{b}\Big[\sum_{l=0}^{\infty}\int_{\{x_{-i}^I | m_T^*(s_i,x_{-i}^I)=l\}}\Big\{\int_{x^{E,l}} Q_i^*\left((s_i,x_{-i}^I),x^{E,l}\right) f(x^{E,l})\mathrm{d}x^{E,l}\Big\} f(x_{-i}^I)\mathrm{d}x_{-i}^I\Big]\mathrm{d}s_i,$$

$$= q_i^*(x_i)x_i + \int_{s_i=x_i}^{b} q_i^*(s_i)\mathrm{d}s_i, \quad \text{which matches (16) and completes the proof of Proposition 3.}$$

Proof of Proposition 4

(i)(a). Regardless of the suppliers' bidding strategies, for $y=0$ the test without reserve price strategy can do no worse in expectation than the no-test strategy: Under the test without reserve price strategy, a feasible strategy is to always recruit m_N^* entrants and not use the outcome of the test auction as a cap on the second auction, which achieves the expected cost of the no-test strategy.

(i)(b). Under the no-test strategy, the bidders are ex ante symmetric so by the revenue equivalence theorem, the buyer will pay the same expected cost (up to a constant) under the open-bid second-price, sealed-bid first-price, and Dutch auction formats. Thus, as in Proposition 2(i)(b), as the marginal cost of recruiting an entrant is nondecreasing, there must exist a n^* such that $m_N^* = 0\ \forall\ n \geq n^*$ for these auction formats. If $m_N^* = 0$, the test without reserve price strategy is a weakly dominant strategy since the buyer could replicate the outcome of the no-test strategy by announcing that she will always recruit zero entrants after the test auction.

(i)(c). The proof of Proposition 2(i)(c) continues to hold, where as in part (i)(b) above we use the fact that the test without reserve price strategy weakly dominates the no-test strategy when $m_N^* = 0$.

(i)(d). The proof of Proposition 2(i)(d) holds with the following change: To prove the equivalent of Lemma 2, let CP and $\widetilde{\mathsf{CP}}$ represent the clearing price of a sealed-bid first-price or Dutch auction when suppliers have cost distribution F and $\tilde{F}$, respectively. Then replacing $X_{(2:n+m_N)}$ and $\widetilde{X}_{(2:n+m_N)}$ with CP and $\widetilde{\mathsf{CP}}$, respectively, provides the result. Finally, note that if the bidders' costs are shifted by Δ, then one can just make the following change: Suppose the bidders' play exactly the same strategy as before the shift, except that they add Δ to their bid(s). The buyer's best response to this bidding strategy does not change (the buyer's total cost just shifts by $\Delta \cdot Q$), so the bidders earn exactly the same payoffs as before. Thus the shifted bidding strategy and the buyer's original strategy form an equilibrium in which the buyer's total cost shifts by $\Delta \cdot Q$ under both strategies but her preference over the two remains unaffected.

(ii). The proof of Proposition 2(ii) continues to hold.

Proof of Proposition 5

(i). Consider a supplier's bidding strategy under the no-test strategy and in the second auction of the test strategy. It is still a weakly dominant strategy for a supplier to remain in the auction until the price reaches their true cost for the same reasons as in Proposition 1. Now consider the test auction. Due to the restriction that only the incumbent supplier who is awarded units in the test auction can compete in the second auction, incumbent suppliers will not remove themselves from the test auction while the price is greater than their cost because they will lose the test auction and will not be considered in the second auction. Further, incumbent suppliers will not stay in the test auction when the price is less than their cost because they would make negative profit if they win the test auction, and the test auction's clearing price serves as a cap on the unit price of the second auction.

(ii). Suppose the buyer uses a test auction reserve price of r_1 and the test auction clears. Then, following the test auction, the buyer aims to minimize her expected cost by choosing to recruit $m_T^* = \arg\min_{m_T \in \mathbb{N}} \left\{ \mathbb{E}[\hat{X}_{(2:n+m_T)} | r_1, b_1, b_2, \ldots, b_{n-1}](Q-y) + k(m_T) \right\}$ entrants where $\hat{X}$ follows the buyer's updated supplier cost distribution based on the realizations of the drop-out bids $b_1, \ldots, b_{n-1}$ of the $n-1$ incumbent suppliers who do not win the test auction, where the i^{th} drop-out bid $b_i = X_{(n-i+1:n)}$ if $X_{(n-i+1:n)} \le r_1$, or "N/A" otherwise. Thus, m_T^* is a function of the test auction reserve price r_1 and the drop-out bids of the incumbent suppliers that do not win the test auction. An analogous argument holds when the test auction reserve price r_1 is not met.

Proof of Proposition 6

(i)(a)-(i)(c). The proofs of Proposition 2(i)(a)-(i)(c) continue to hold.

(i)(d). The proof of Proposition 2(i)(d) applies to the information updating case with one supplementation to Lemma 2. Define the scaled distribution $\tilde{F}(x)$, scaled recruitment cost function $\tilde{k}(m)$, and $\tilde{X}_{(i:j)}$ as before. Let the $n-1$ incumbent drop-out bids be $b^{\mathsf{inc}} \triangleq [b_1, b_2, \ldots, b_{n-1}]$ (where inc stands for incumbent) and let the scaled version of the $n-1$ incumbent drop-out bids be $\widetilde{b}^{\mathsf{inc}} \triangleq b^{\mathsf{inc}}/\alpha = [b_1/\alpha, b_2/\alpha, \ldots, b_{n-1}/\alpha]$. Then, for any recruitment rule, the test without reserve price strategy's equivalent to (15) is given by

$$\begin{aligned}\mathbb{E}[\text{total cost under test without reserve price}] &= \mathbb{E}[\tilde{X}_{(2:n)}]y + \mathbb{E}_{\widetilde{b}^{\mathsf{inc}}}\left[\mathbb{E}[\tilde{X}_{(2:n+m_T(\widetilde{b}^{\mathsf{inc}}))} | \widetilde{b}^{\mathsf{inc}}](Q-y) + \tilde{k}(m_T(\widetilde{b}^{\mathsf{inc}}))\right] \\ &= \alpha\left(\mathbb{E}[X_{(2:n)}]y + \mathbb{E}_{b^{\mathsf{inc}}}\left[\mathbb{E}[X_{(2:n+m_T(b^{\mathsf{inc}}))} | b^{\mathsf{inc}}](Q-y) + k(m_T(b^{\mathsf{inc}}))\right]\right).\end{aligned}$$

(ii). The proof of Proposition 2(ii) continues to hold.

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