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Optimal Procurement of Flexibility Services within Electricity Distribution Networks

Arnaud Laur[a], Jesus Nieto-Martin[a,*], Derek W. Bunn[a], Alejandro Vicente-Pastor[a]

[a]*Management Science & Operations, London Business School, London NW1 4SA, UK*

Abstract

The increased injection of volatile renewable energy at local levels into the electricity grid is forcing the distribution network operators to seek participation in emerging service markets in order to obtain the flexibility required to balance their systems. However, the distribution companies lack experience in designing new market arrangements. We consider a market framework wherein a proactive distribution company is willing to purchase reserve capacity for overload management, using a two-part tariff. The problem is modelled as a three-stage stochastic market including Day-Ahead, Intra-Day and Real-Time, with uncertainty on both demand and generation. After assessing our formulation, we discuss the impact of risk-aversion at each stage with an objective function based on CVaR. Finally, different Intra-Day clearing horizons and delivery settings are evaluated. The results show that risk-aversion close to Real-Time becomes the main driver for decision makers and that early hedging strategies lead to sub-optimal solutions.

Keywords: Stochastic programming, energy, risk-aversion, intraday trading

1. Introduction

1.1. The need for flexibility in power distribution networks

Supported by government incentives for low-carbon targets, the rapid uptake of Renewable Energy Systems and Distributed Generation (DG) has caused major shifts in the way the grid is operated and the electricity is delivered. According to Ackermann et al. (2001), DG is an electric power source connected directly to the distribution network or on the customer side of the meter. Indeed, the large number of connections of very volatile DGs at medium and low voltage levels has substantially complicated the obligations of Distribution Network Operators (DNOs) to safely and efficiently operate their distribution grids (De Joode et al., 2009). In the short term, DG has exacerbated the already-existing operational issues which include voltage deviations, fault (or current) limits, thermal constraints as well as transmission constraints.

*Corresponding author

Nomenclature

Sets

- t Time periods
- st, stt Stages
- n Nodes
- l Lines
- i Generators - flexibility providers
- j Loads
- $tr(n)$ Nodes connected to transmission level
- ω Scenarios

Parameters

- Cap_l Line capacity.
- $F_{i,t}^{up}$ Up-regulation volume bid by unit i at time t.
- $F_{i,t}^{down}$ Down-regulation volume bid by unit i at time t.
- π_{st}^{tot} Total cost per stage for the DSO
- π^{CO} Equivalent cost of overloaded lines
- $\pi_{i,st}^{res}$ Flexibility reservation price for unit i at stage st.
- $\pi_{i,st}^{act}$ Flexibility activation price for unit i at stage st.
- $\pi_{i,st}^{sys}$ Price charged by SO at the transmission interface for time t during RT.
- $VOLL$ Value Of Lost Load.
- $VOLW$ Value Of Lost Wind.
- $VOLS$ Value Of Lost Sun.
- $d_{j,t}$ Deterministic load profile for each consumer j and time t.
- $Cap_{i,t}^{g}$ Power injection from renewable producer i at time t.
- B_l^g Line susceptance (S).
- $P_{i,t}^{WS}$ Energy commitment from the Wholesale Market by generator i and time t.
- $P_{tr,t}^{TSO,WS}$ Energy from TSO cleared during the Wholesale Market

Variables

- S_{st}^{D} Stochastic coefficient per stage, demand-side.
- S_{st}^{W} Stochastic coefficient per stage, generation-side.
- $f_{l,t,st}$ Power flow through line l at time t and stage st.
- $f_{l,t,st}^{+}$ Absolute value of power flow.
- $P_{i,t}^{RES,Curt}$ Renewable injection curtailment. Superscript W denotes Wind while S is for Solar.
- $P_{tr,t,st}^{TSO}$ Energy input from TSO via transmission interface at time t and stage st.
- $LS_{j,t}$ Load shed applied to consumer j at time t.
- $\Delta_{tr,t}^{SO,+}$ Positive component of the power fluctuation at the TI.
- $\Delta_{tr,t}^{SO,-}$ Negative component of the power fluctuation at the TI.
- $\Delta_{l,t,st}^{f+,up}$ Positive deviation from flow threshold.
- $\Delta_{l,t,st}^{f+,down}$ Negative deviation from flow threshold.
- $\theta_{n,t,st}$ Phase angle at node t, time t and stage st.
- $R_{i,t,st}^{up}$ Up-regulation volume reserved at time t and stage st.
- $R_{i,t,st}^{down}$ Down-regulation volume reserved at time t and stage st.
- $RB_{i,t}^{up}$ Up-regulation volume reserved at time t and stage 1, used for stage 2 power balance.
- $RB_{i,t}^{down}$ Down-regulation volume reserved at time t and stage 1, used for stage 2 power balance.
- $A_{i,t,st}^{up}$ Up-regulation volume activated at time t and stage st.
- $A_{i,t,st}^{down}$ Down-regulation volume activated at time t and stage st.

Voltage drops typically increase in stretched rural networks at lower voltages while thermal issues prevail in dense urban areas. Line sags, provoked by thermal overload, may result in serious electric arcs or in short-circuits if they touch a ground-connected object. Furthermore, if the DNO operates lines close to their thermal limits for extended periods it shortens their lifespan: frequent heating affects their mechanical and electrical properties over time by inducing permanent sag and deteriorates their electrical properties. In the UK for instance, thermal limits alone represent around 44% of the constraints for new connections while voltage drops amount to 4% (Western Power Distribution, 2016). Furthermore, thermal congestion has been increasing with distributed generation since most networks were configured for central dispatch operations. (Bjørndal et al., 2018). In addition to creating delivery charges borne by the wider society, these thermal congestions may eventually result in costly grid failures and cascading outages, as described in Henneaux (2015) and Yang & Jiang (2017). Apart from short-term adaptations, the historical long-term business model of DNOs to ensure reliability has been through asset reinforcement. With DG accelerating, the rate at which additional grid capacity is needed is increasing to a point where it jeopardises this long-established business model. Not only are the reinforcement costs challenging, they are highly risky as network power flows become harder to predict. Therefore it is becoming more efficient for DNOs to delay these upgrades by directly procuring flexibility through service markets. These organisational changes have resulted in DNOs transitioning towards Distributions System Operators (DSOs) as emerging players in the electricity markets (Ruester et al., 2014). As such, they now have to use advanced metering and control techniques, as well as building their own prediction and optionality models to assess the flexibility levels required for secure operations. This raises a number of challenges:

• Traditionally, most services were contracted in (or prior to) the Day-Ahead market and activated in Real-Time operation. However, the inherent volatility of Renewable Energy Systems requires dynamic, close-to-delivery trading and scheduling in order to be managed efficiently. Thus, Intra-Day (ID) markets have been of growing interest for several countries since it allows traders to update their forecasts and requirements more frequently. When addressing uncertainties in power systems, a wide range of methods have been proposed, including stochastic approaches (Ruszczyński & Shapiro, 2003), robust optimisation (Zugno & Conejo, 2015), information gap decision theory (Ben-Haim, 2006) and hybrid techniques (Torbat et al., 2018). In particular, stochastic formulations, which consider a finite set of potential realisations (or scenarios), are widely used in practice and research for their accuracy, see for instance Botterud et al. (2012), Mohan et al. (2015), Abbaspourtorbati et al. (2017a), Zavala et al. (2017) and Morales et al. (2014). Nevertheless, these approaches are usually simplified as appropriate because of their computational requirements. For the particularly demanding case of a stochastically-cleared market, there are very few large-scale practical implementations like that of Switzerland (Abbaspourtorbati & Zima, 2016). This will change with greater DG penetrations, although concerns linger regarding the ability of such formulations to retain all the properties of deterministic clearings.

• DNOs and DSOs are natural local monopolies and, as such, are heavily regulated bodies. Regulators prevent them from being profit-maximisers to protect the interests of end-consumers. While they acknowledge the issues and may reward innovative and efficient

solutions, their top priority still remains the securing of supply delivery and of secure grid operations. Therefore, DSOs behave by nature in a risk-averse fashion, and even more so when facing these new business challenges. Much of their budgets are, as a consequence, allocated to ensure the worst-case scenarios are avoided. From a modelling perspective, the risk aversion complicates the decision analysis away from the traditional Expected Value (EV) analysis, especially when considered in a dynamic trading environment.
• The reconciliation between Renewable Energy Systems' uncertainty and the DSO's risk-aversion is not trivial: the DSO would prefer to reserve enough capacity ahead of time but asset operators have no incentive to trade early in sequential markets. In order to hedge against uncertainty, a pricing mechanism satisfying both constraints must be implemented. One approach is to consider a typical Two-Part Tariff design which consists of both a reservation and an activation payment. This scheme allows in our case the DSO to reduce its upfront costs and the generators to capture reservation payments without always being called upon. To efficiently run a service market, the DSOs must choose an optimal reservation policy to obtain the best trade-off between cost and risk.
• The mathematical and numerical complexity arising from adding all these features into a single model is in itself another challenge when size increases. Moreover, the validity of the proposed stochastic formulation must be evaluated against standard benchmarks in terms of both market and algorithmic design. Market-wise, the benefits of having 3 stages must be evaluated against 2- and 1-stage markets. Algorithm-wise, the efficacies of the stochastic solution must be assessed using the classic metrics, eg as in (Escudero et al., 2007).
Fortunately, as we observed earlier that voltage considerations are relatively minor compared to the thermal limits, linearised direct current network models can be sufficiently accurate, thereby avoiding the more complicated non-linear alternating current formulations.

In conclusion, motivated by the requirements of this new business environment, the technical challenges and the multidimensionality of this problem, we propose a solution framework in which our contribution is three-fold:

- Implementation of a three-stage stochastic algorithm with a two part tariff scheme to solve the DSO's service procurement problem. Development of a case study to assess the performances against 2 market benchmarks, perfect information and a deterministic model.
- Analysis of the impact of the DSO's risk-aversion at each stage on the final costs by means of a novel objective function.
- Compare a set of intra day setups in order to provide guidance on the optimal service market design.

The rest of this paper is organised as follows: section 2 motivates this study by showing that the existing research literature provides insufficient solutions to solve the DSO's problem. Section 3 covers the methodology and mathematical model, along with the stochastic and risk-aversion settings. In section 4, we show and discuss the results of the case study, of the benchmark comparison and on the behavioural analysis. Finally, conclusions are summarised in section 5.

2. Literature limitations: the need for a more extensive approach

2.1. The value of Intraday trading and Multistage programming

Most bulk electricity is usually traded day-ahead in the wholesale market and residual deviations are traded in the balancing market (also known as real-time) by the Transmission System Operator (TSO). Any regional service market must therefore accommodate these pre-established constraints. For proactive DSOs, their activities in managing local congestion can now adversely impact the TSO's business. Intraday energy markets are already being used at local and national level in many countries (Hu et al., 2018) either through discrete auctions or continuous trading. Scharff & Amelin (2016) found in their empirical study that it indeed facilitated Renewable Energy trading but without identifying any clear Pareto-optimal design. There have already been a number of early attempts for DSO market pilots and local models such as in De Joode et al. (2009) and Olivella-Rosell et al. (2018), utilising local assets and demand-side response from clusters of pro-active consumers, although these are mostly concerned with resource scheduling and do not address line overload nor the periodicity of forward trading. Yet, Hu et al. (2018), who write on new market designs, reveal the lack of inter-market coordination and the need for a higher trading periodicity close to real-time. From that perspective, Zavala et al. (2017) have shown the substantial improvements in expected costs and distortions by jointly optimising all markets in a stochastic framework.

In terms of algorithmic features, these additional intraday stages translate into multistage stochastic models with recourse. Tractability considerations often cause them to be replaced by rolling horizon approaches, wherein a sequence of 2-stage models is solved instead (Devine & Bertsch, 2018); or simple 2-stage formulations. This assumption can be sufficient for many applications, but it inherits two noticeable drawbacks. First, it contradicts Hu et al. (2018) and the discussion in section 1, for it disregards the value of the intraday adjustments and reduces market liquidity. Second, it has been demonstrated by Abbaspourtorbati et al. (2017b) that, for similar problems, 3-stage models can achieve savings of up to 12% on expected costs over 2-stage equivalents. Therefore, in order to model accurately the DSO service market, a multistage algorithm looks desirable. However, the upgrade to multistage is not trivial; the effective design of multistage models and in our context, of additional intraday stages still remains unresolved (Scharff & Amelin, 2016). For further reading on multistage stochastic optimisation theory (later used in section 3.3), the reader may refer to Ruszczyński & Shapiro (2003), Kozmik (2014) and Vigerske (2013).

2.2. Risk aversion and the Two Part Tariff

As detailed previously, the DSO is a risk-averse entity who needs tools to constrain the cost volatility and hedge against overload exposures (Garry et al., 2018). Therefore the problem falls into a class defined as Risk-Averse Multistage Stochastic Programs (RAMSP). These models, along with adequate risk measure selection issues, are discussed, by Ruszczyński & Shapiro (2003), Kozmik (2014), Pagnoncelli (2016), Homem-de Mello & Pagnoncelli (2016) and Detlefsen & Scandolo (2005). There are also a few implementations.

For example, Chen et al. (2017) carried out a multi-objective power flow optimisation with EV, variance and skewness, but they do not engage in multistage considerations. Because of the extra complexity, some studies such as Zugno & Conejo (2015) have preferred robust solutions, but at the cost of suboptimality.
With risk considerations, it becomes appropriate to use two-part tariffs, which have option-like characteristics, instead of single-price fixed contracts in which full payment and delivery are always due regardless of actual utilisation. Formulations involving two-part tariffs appear in Benjamin (2013), Antweiler (2017) and Hu et al. (2018) but do not fully capture all the RAMSP features. We argue in particular that the non-refundable reservation price of the tariff together with the renewable output volatility suggest the need for adapting the RAMSP formulation to include variable risk aversion in the forward rounds. Generally, the research literature does not formulate dynamic risk evolution and often targets the terminal stage only. It is this feature in the RAMSP model which we present in the next section that provides a novel risk-based objective function for our analysis.

3. A RAMSP model for the DSO services market

In sections 1 and 2, we discussed the DSO's new business challenges and the lack of an existing method to consider all infrastructural, financial and dynamical aspects of the problem. Thus, we introduce a service market using a RAMSP model that effectively captures the requirements of the DSO. This section introduces the design, architecture and novelties of the algorithm.

3.1. Market Structure

The proposed framework is a local service market, run by the DSO in parallel to the wholesale energy market. The sellers (local asset owners and flexibility providers) are remunerated on a Two-Part Tariff basis. While many countries run continuous intraday trading, Hu et al. (2018) revealed these tend to decrease liquidity and increase transaction costs. Empirical examinations of these also tend to show patterns of trading concentrations at a few distinct periods (eg close to gate closure). As a result, we use an auction-based market structure, as in the successful Spanish market (Chaves-Ávila & Fernandes, 2015). Auctions are also more suitable for stochastic algorithms, as a clearing can readily be defined as a unique stage. We therefore aggregate several intraday rounds into one session to obtain a three-stage model (day-ahead, intraday and real-time), which is depicted in Figure 1.

After the wholesale day-ahead market is cleared on D-1 at noon, the DSO identifies the lines at risk of potential overload. Then, it can reserve up/down capacity from a pool of vendors during the two first stages (day-ahead and intraday). The DA for the service market is cleared before D-Day and covers the whole trading horizon (24 hours). At 11am on D-Day, the intraday is cleared, early enough to cover the afternoon peak while being late enough so that the renewable uncertainty has been reduced from the DA auction. This, however, would be subject to local considerations (see section 3.3). Finally, in real-time (24h), the uncertainty is realised and the DSO can activate the reserved capacity to prevent any physical overload. If the solution is still not feasible or optimal, it can also resort to

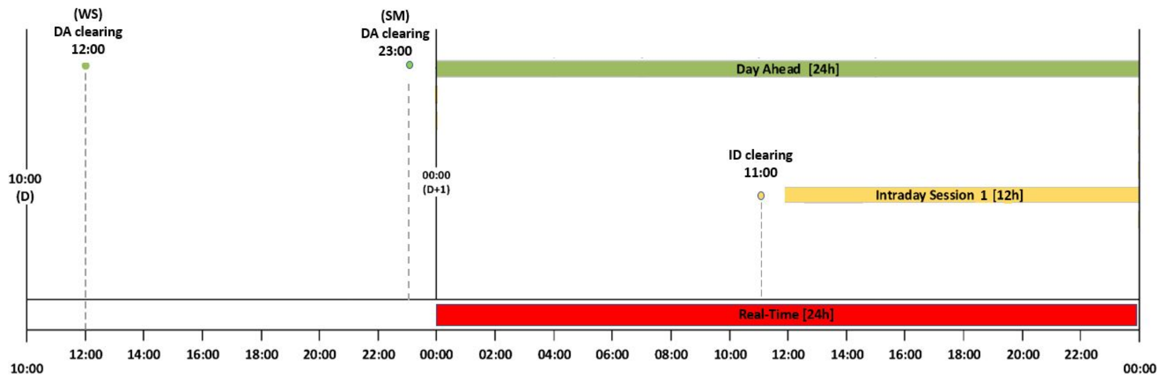

Figure 1: Market Microstructure Dynamics (D: day; WS: wholesale; SM: services market; DA: day-ahead; ID: intraday)

curtailing Renewable Energy and shedding some of the system's loads. Regarding the hourly time steps, while this may undervalue some of the flexibility compared with half-hourly, or 15min periods, it is consistent with the Spanish reference and the modelling has no loss of generality.

The market will inevitably create real-time deviations at the transmission interface from the volumes committed as a result of the wholesale market and TSO balancing. Depending on the market regulations, the arrangements to settle such imbalances can be very different. For our study we assume deviations will force the System Operator to, in turn, take further actions in order to ensure balancing. As a result, we elect a settlement rule in which the System Operator charges any net imbalance (positive or negative) at the transmission interface in real-time with a cost-reflective system balancing price. Finally, we assume the TSO does not procure its services in this region to anticipate these effects.

3.2. Formulation

This section introduces the algorithm used to model the problem.

$$\text{Objective function:} \qquad Min \quad \sum_{st} \left(\mathbb{E}_{\omega} \left[\pi^{tot}_{st} \right] \right) \tag{1}$$

The objective is first defined as the explicit Expected Value (operator $\mathbb{E}[-]$) of the cost constraint (2) over all stages st and scenarios ω. It will, however, be developed further in section 3.3.

Subject to:

$$\pi_{st}^{tot} = \sum_t \left(\sum_i \pi_{i,st}^{res}.(R_{i,t,st}^{up} + R_{i,t,st}^{down}) + \left(\sum_{stt} \sum_i \pi_{i,stt}^{act}.(A_{i,t,stt}^{up} + A_{i,t,stt}^{down}) + \sum_l (\pi^{CO}.\Delta_{l,t,st}^{f+,up}) \right.\right.$$
$$\left.\left. + \sum_{tr} \pi_{tr,t}^{sys}.(\Delta_{tr,t,st}^{SO,+} + \Delta_{tr,t,st}^{SO,-}) + \sum_i (VOLW.P_{i,t}^{W,Curt} + VOLS.P_{i,t}^{S,Curt}) + \sum_j VOLL.LS_{j,t} \right)_{st=3} \right) \tag{2}$$

The cost constraint (2) represents the DSO's expenditures at each stage. In day ahead and intraday, the cash-out is simply the cost of the reserved up and down volumes. In real-time ($st = 3$), the costs include the product of volume activated and its the activation price, the cost of curtailment and load shed, the imbalances charged by the System Operator and the realised overload penalised at a large cost π^{CO} (as detailed in section 4.1). To determine the imbalance and overload volume, we use the following method:

$$f_{l,t,st}^{+} \geq f_{l,t,st} \quad \forall l, t, st \tag{3}$$

$$f_{l,t,st}^{+} \geq -f_{l,t,st} \quad \forall l, t, st \tag{4}$$

$$P_{tr,t,st=3}^{TSO} - P_{tr,t}^{TSO,WS} = \Delta_{tr,t}^{SO,+} - \Delta_{tr,t}^{SO,-} \quad \forall tr, t \tag{5}$$

$$f_{l,t,st=3}^{+} - 0.85Cap_l = \Delta_{l,t}^{f+,up} - \Delta_{l,t}^{f+,down} \ \forall l, t \tag{6}$$

With (3) and (4), we compute the absolute line power flow ($f_{l,t,st}^{+}$) without resorting to non-linear operators. In parallel, (6) decomposes the difference between the absolute flow and the overload threshold into a positive and a negative component. Here we set the threshold at 85% of the lines' rated capacity, but actual values may vary (Capitanescu & Van Cutsem, 2007). Only the positive component $\Delta_{l,t}^{f+,up}$ (overload) is then charged in (1) in real-time. In (5), the same method is applied to determine the deviations between the actual real-time flow at the transmission interface and the wholesale commitments, except both components are here charged in the cost function. This method only works when $f_{l,t,st}^{+}$, $\Delta_{l,t}^{f+,up}$, $\Delta_{l,t}^{f+,down}$, $\Delta_{tr,t}^{SO,+}$ and $\Delta_{tr,t}^{SO,-}$ are defined as positive variables.

$$\sum_{l \in rec\_bus} f_{l,t,st} - \sum_{l \in snd\_bus} f_{l,t,st} + \sum_{i \in gen\_bus} \left(P_{i,t}^{WS} + (S_{st}^{W}.Cap_{i,t}^{g} - P_{i,t,\forall st=3}^{RES,Curt})\right)$$
$$+ \sum_{i,st=1,2} (R_{i,t,st}^{up} + R_{i,t,st}^{down}) + \sum_{i,st=2} (RB_{i,t}^{up} + RB_{i,t}^{down}) + \sum_{i,st=3} \left(\sum_{stt \in [1,2]} (A_{i,t,stt}^{up} + A_{i,t,stt}^{down}) \right)$$
$$+ \sum_{tr} P_{tr,t,st}^{TSO} - \sum_j (S_{st}^{D}.d_{j,t} - LS_{j,t,\forall st=3}) = 0 \quad \forall n, t, st \tag{7}$$

Constraint (7) is a modified power flow balance. It balances the line flows, the generation levels (wholesale commitments from dispatchable generators and renewable injections) and the loads to determine the best reservation/activation volumes. Uncertainty on both Renewable Energy Systems output and load levels is taken into account.

$$R^{up}_{i,t,st} \leq F^{up}_{i,t} \quad \forall i,t,st \tag{8}$$

$$R^{down}_{i,t,st} \leq F^{down}_{i,t} \quad \forall i,t,st \tag{9}$$

$$RB^{up}_{i,t} \leq R^{up}_{i,t,st=1} \quad \forall i,t \tag{10}$$

$$RB^{down}_{i,t} \leq R^{down}_{i,t,st=1} \quad \forall i,t \tag{11}$$

$$R^{up}_{i,t,2} \leq F^{up}_{i,t,1} - R^{up}_{i,t,1} \quad \forall i,t \tag{12}$$

$$R^{down}_{i,t,2} \leq F^{down}_{i,t,1} - R^{up}_{i,t,1} \quad \forall i,t \tag{13}$$

$$A^{up}_{i,t,st} \leq R^{up}_{i,t,st} \quad \forall i,t,st \tag{14}$$

$$A^{down}_{i,t,st} \leq R^{down}_{i,t,st} \quad \forall i,t,st \tag{15}$$

In the day-ahead market, the DSO makes a first set of reservations whose volumes are capped by the providers' offers in (8) and (9). The limits $F^{up}_{i,t,st}$ and $F^{down}_{i,t,st}$ depend on the assets'nominal capacities and the results of the wholesale market. Next, the intraday consists of another reservation round wherein the decision is conditional on the day-ahead results. Indeed, the day-ahead provisions must be discounted from the offered volumes (equations (12) and (13)) but also accounted for in (7). To be exact, these volumes cannot be added as such in the balance at stage 2 because they are not imposed injections/withdrawals but a dispatchable flexibility portfolio the DSO is already in possession of. Therefore, the variables $RB^{up}_{i,t}$ and $RB^{down}_{i,t}$ are defined as fictive intraday activations of the day-ahead reservations with (10) and (11) to ensure only necessary reservations are performed intraday. Finally, in real-time, only the real activations are considered in the flow balance. These cannot be higher than the reserved quantities in forward markets, as per stage-3 equations (14) and (15).

$$R^{up}_{i=6,t,st=1,2} + A^{up}_{i=6,t,st=3} \leq 12 \quad \forall t \tag{16}$$

$$R^{down}_{i=6,t,st=1,2} + A^{down}_{i=6,t,st=3} \leq 12 \quad \forall t \tag{17}$$

$$P^{Curt}_{i,t} \leq S^{W}_{st}.Cap^{g}_{i,t} \quad \forall i,t \tag{18}$$

$$LS_{j,t} \leq S^{D}_{st}.d_{j,t} \quad \forall j,t \tag{19}$$

$$f_{l,t,st} = B_l * (\theta^{rec}_{n,t,st} - \theta^{snd}_{n,t,st}) \quad \forall l,t,st \tag{20}$$

$$\theta_{slack,t,st} = 0 \quad \forall t,st \tag{21}$$

$$f_{l,t,st} \geq -Cap_l \quad \forall l,t,st \tag{22}$$

$$f_{l,t,st} \leq Cap_l \quad \forall l,t,st \tag{23}$$

$$R^{up}_{i,t,st=3} = R^{down}_{i,t,st=3} = 0 \quad \forall i,t \tag{24}$$

$$A^{up}_{i,t,st=3} = A^{down}_{i,t,st=3} = 0 \quad \forall i,t \tag{25}$$

$$R^{up}_{i,t<12,st=2} = R^{down}_{i,t<12,st=2} = 0 \quad \forall i \tag{26}$$

The remaining equations of the model ((16)-(26)) are more generic and only briefly commented. Equations (16) and (17) account for the storage technical constraints (if any). Battery owners can submit up/down offers for the entire day but the service cannot be physically sustained for more than a few hours before the battery gets either fully charged or discharged. Therefore we conjecture that the DSO can only trade during 50% of the time periods, and that idle hours are used by the owner to re-adapt the State-Of-Charge. This formulation prevents the algorithm from having full control over the battery (since the DSO does not own the asset). Next, curtailment and load shed in real-time are capped in (18) and (19) by the realised power injection and load level, respectively.

Constraints (20) to (22) correspond to the standard equations of a security-constrained DC power flow. The linearisation of Kirchhoff's equations has the enormous advantage of keeping the problem linear and solvable by commercial software with fast computation times. While it is not usually applied for low-voltage grids, we noted earlier that the large thermal as distinct from voltage considerations made it suitable in our case. In contrast, it is well-known that the DC approximation performs well at high voltage where voltage magnitudes are more stable, conduction losses drop and resistance/reactance ratios are relatively small. Finally, (24), (25) and (26) prevent impossible reservations and activations from occurring. The EV problem is now fully described and the next section covers the inclusion of risk.

3.3. A versatile objective function for a RAMSP model

The above-detailed algorithm is a 3-stage stochastic problem with recourse. For this general class of problems, the solver implicitly optimises:

$$\min_{x_1} \; f(x_1) + \mathbb{E}\left[\min_{x_2} \; f_{[1]}(x_2) + \mathbb{E}\left[\min_{x_3} \; f_{[2]}(x_3) \right] \right] \tag{27}$$

Where $f(x_{st})$ is an objective function. In our model, uncertainty is considered in stages 2 and 3 with the random variables S^W_{st} and S^D_{st}, accounting for weather (Renewable Energy output) and demand (load level) respectively. As stated in section 2, a finite set of outcomes per stage is created through a scenario tree. Mathematically, the EV operator becomes the probability-weighted sum for the discrete distribution set Ω_t :

$$\mathbb{E}[f(x_{st})] = \mathbb{E}_{\omega_{st}}[f(x_{st})] = \sum_{\omega_{st} \in \Omega_{st}} p(\omega_{st}) * [f(x_{st})] \tag{28}$$

In this setup, the algorithm minimises the cost on average over the entire set of scenarios. However, the DSO's requirements mean that this EV formulation should be replaced by a risk measure ρ in the objective function, in order to account for the hedging strategy. Risk measures are extensively used in many sectors with various metrics (Homem-de Mello & Pagnoncelli, 2016; Kettunen et al., 2010). In our formulation, we seek to reduce cost in the worst scenarios, without penalizing outperformance, and for that purpose, the Conditional Value-at-Risk (CVaR) is appropriate. Various studies such as Sarykalin et al. (2008) have proven its superiority over Value-at-Risk both theoretically and computationally. We note that the right-tail CVaR, unlike left-tail, is a coherent risk-aversion measure because it satisfies the 4 following properties for all random outcomes X, Y of a stochastic process :

$$\begin{aligned}
&(i)\ Convexity: \quad \rho(\alpha X + (1-\alpha)Y) \leq \alpha\rho(X) + (1-\alpha)\rho(Y) \quad \forall\ \alpha \in \mathbb{R}\\
&(ii)\ Monotonicity: \quad X \geq Y \Rightarrow \rho(X) \geq \rho(Y)\\
&(iii)\ Translational\ Equivariance: \quad \rho(X+t) = \rho(X) + t \quad \forall\ t \in \mathbb{R}\\
&(iv)\ Positive\ Homogeneity: \quad \rho(tX) = t\rho(X) \quad \forall\ t \in \mathbb{R}
\end{aligned}$$

However, a purely risk-focused objective can also lead to much larger costs on average due to a somewhat excessive hedging strategy. The benefits of combining several objectives were addressed by Najafi & Mushakhian (2015) and thus, as in many applications of CVaR, our objective is defined as an EV-weighted compromise:

$$\rho_{st}(f(x_{st})) = (1-\lambda_{st})\mathbb{E}_{\omega_{st}}[f(x_{st})] + \lambda_{st}.CVaR_\alpha[f(x_{st})] \quad \forall \lambda_t \in (0,1) \tag{29}$$

Where α is the quantile level for CVaR and the parameter λ is a behavioural feature: λ being close to 1 at stage st indicates a purely risk-averse DSO at this stage, while lower values lead to a less risk-averse, more cost-minimising conduct. Adapting such an objective function to our model is not trivial. Indeed, complications arise with the extension to multistage programs ($st > 3$). For these, there is no obvious way of modelling cumulative risk, as the ρ_{st} can be added, multiplied or even nested to obtain different global risk measures (Kozmik, 2014). There are also cases where risk is considered for the final stage only (Najafi & Mushakhian, 2015).

Thus, we need to consider various desirable properties for ρ_{st}. The most significant is that of time consistency. In brief, it requires the global solution to be also a local optimum, i.e the solution for the whole scenario tree must hold true for any subsection of that tree. It also implies that the optimal decision at a given stage cannot involve infeasibility in any subsequent stages. This prevents the solution from becoming degenerative. With this criterion, two methods are attractive: the multi-period (also called Expected Conditional Risk Measure, ECRM) and nested risk models (Ruszczyński & Shapiro, 2003; Pagnoncelli, 2016). However, we discard the latter for two reasons. Firstly, its recursive definition imposes that information at $t+1$ is used to calculate stage t: the algorithm is therefore not solely based on historical values, and that causes a problem of predictability. Secondly, its complex structure makes the risk measure intuitively opaque. Alternatively, ECRM is a hybrid function based on an additive and a nested formulation. It is defined as:

$$ECRM(X) = X_1 + \rho_1(X_2) + \mathbb{E}_{[2]}\left[\rho_3^{[2]}(X_3)\right] \tag{30}$$

Combined with (29), it is rewritten as:

$$\begin{aligned} ECRM(X) = X_1 + (1-\lambda_2)\mathbb{E}_{\omega_2}(X_2) + \lambda_2.CVaR_{\alpha_2}(X_2) + \\ + \mathbb{E}_{\omega_3}^{[2]}\left[(1-\lambda_3)\mathbb{E}_{\omega_3}(X_3) + \lambda_3.CVaR_{\alpha_3}(X_3)\right] \end{aligned} \tag{31}$$

This versatile formulation is our final objective following (1) and (27), with X_{st} being replaced by π_{st}^{tot}. It combines both EV and Risk Measures, under different levels and weights at each stage. It also provide better forecasts over the nested approach: for any stage $t < t_{max}$, the Expected $t+1$ Risk and Return sent back to each node includes all subsequent scenarios and not only the descendants of that node.

In section 4, our case study evaluates the performances of both the market and the stochastic algorithm using the following set of benchmarks:

• A real-time-only service market. Only physical adjustments are permitted, in order to simulate a passive DNO and benchmark the value of the service market. These adjustments include curtailment, load sheds, drawing more from the transmission interface or trade with the battery. We assume the storage owner would be willing to provide close-to-real-time flexibility in this situation. The new objective is therefore a 1-stage reduction from Eq.(31):

$$Min \quad (1-\lambda_3)\mathbb{E}[\pi_{st=3}^{tot}] + \lambda_3 CVaR_\alpha[\pi_{st=3}^{tot}] \tag{32}$$

• A day ahead & real-time service market in which the DSO cannot trade intradaily. This benchmark demonstrates the value of the intraday round. The market mechanisms and the parameters remain unchanged from the main model, with the exception that reservations in stage 2 are disabled. The objective is reduced to the stage-1 costs and the expected stage-3 EV-CVaR balance:

$$Min \quad \pi^{tot}_{st=1} + \mathbb{E}^{[1]}_{\omega_3}\Big[(1-\lambda_3)\mathbb{E}[\pi^{tot}_{st=3}] + \lambda_3 CVaR_\alpha[\pi^{tot}_{st=3}]\Big] \tag{33}$$

An important feature is that both these benchmarks remain 3-stage stochastic models so that the uncertainty design remains unchanged and the results unbiased.

• Two of the main standard stochastic benchmarks (see Birge & Louveaux (2011)): the perfect information and the deterministic model.

4. Numerical studies

We apply the above formulations to a case-study whose settings are detailed below in 4.1. Next, we comment the results in 4.2 and show the 3-stage RAMSP provides solutions of higher quality when compared to the benchmarks. Afterwards, we carry out a sensitivity analysis on the DSO's risk-aversion and the intraday clearing stage in 4.3, to identify the best dynamics to adopt. Finally, the limitations of the approach are discussed in 4.3.

4.1. Simulation settings

The problem size is low enough to allow for an analytical development of the results. Hence, we use a modified IEEE 5-bus test network, which can be found in Li & Bo (2010). This network is originally a model for transmission but its topology remains applicable for meshed high-voltage distribution grids. To obtain values typical of such grid layers, we scale down all line, demand and generation capacities by a factor 10. We also add the transmission interface (denoted TI) at node 3 and some storage at node 1 where both renewable producers are connected (see Figure 2).

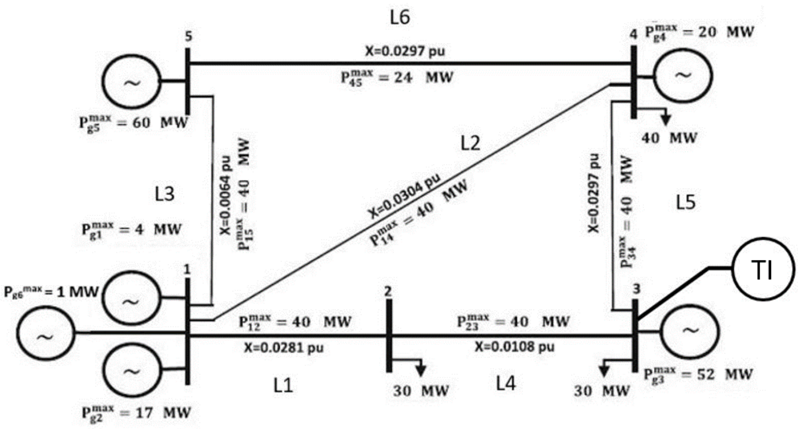

Figure 2: Modified IEEE 5-bus test network

Vendor	Tech.	$\pi^{res}_{st=1}$	$\pi^{res}_{st=2}$	$\pi^{act}_{st=1,2}$
g1	Wind	12	6	72.5
g2	PV	10	8	69
g3	Turbine	5	3	90
g4	CHP	7	5	75
g5	CHP	7	5	75
g6	Stor.	6	4	60

Table 1: Technologies and Two-part tariffs (£/MWh)

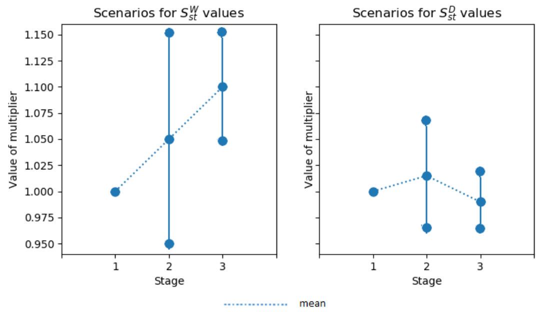

Figure 3: Stochastic Multipliers by Stage

We also arbitrarily define a mix of technologies for the vendors in Table 1, along with their associated two-part tariff prices. Two-part-tariff prices are based on the Short Term Operating Reserve (STOR) data from the British TSO (NationalGrid, 2013), since the nature of this service and its time frames are quite similar to ours. The distinction per technology is made upon considerations of volatility and technological constraints. The principle of time decay is quite intuitive as the reservation price decreases as the delivery period approaches.

The deviation penalty is an average of system price data from the same TSO, over the period 04/01/15 to 16/08/17. As inputs, we use £3,000/MWh for the VOLL, £200/MWh for the VOLW and a VOLS of £150/MWh (assuming the last two include, among others, the lost revenue from a technology-specific Feed-In-Tariff subsidy). Our overload penalty is computed as follows:

$$\pi^{CO} = Mean\ Line\ Reinforcement\ cost \left[\frac{£}{MW.km}\right] \times \\ \times\ Mean\ line\ length\ [km] \times\ degradation\ coefficient\ [-] \quad (34)$$

The mean reinforcement cost is taken from Mohtashami et al. (2016), with an average line length of 5 km and a conservative degradation factor of 1% are assumed, leading to a π^{CO} of 731.53 £/MW.

Next, we generate individual profiles for the loads ($d_{j,t}$) and the Renewable Energy injections ($Cap^g_{i,t}$), which correspond to initial day-ahead forecasts. In section 3, we defined our random variables S^D_{st} and S^W_{st} as being multiplied to these profiles. In stage 1 (day-ahead), both are set to 1 but in later stages, several scenarios for these values are created. That way, S^D_{st} and S^W_{st} can be understood as deviations from the original forecasts. We allocate 3 possible realisations (low, medium and high) to each and for each stage, i.e a "high" S^D_{st} represents the largest upper load level deviation for that stage, while a "low" S^W_{st} designates the worst weather conditions and so the lowest renewable injections. This brings the number of scenarios to $3^{2*2} = 81$. Figure 3 depicts the chosen distribution for the two stochastic variables. The medium scenarios following the dotted line have a probability of 0.6 while high and low are both set at 0.2. While the distance between these 3 realisations must decrease throughout stages (more accurate forecasts as we get closer to RT), we chose an

arbitrary upward trend for S^D_{st} so as to stress the system (higher expected injections). Since weather and load are typically inversely correlated, S^W_{st} follows a downward trend. Finally, for our objective, we take a confidence level of 0.9 for CVaR so that the 10% worst cost scenarios are given more emphasis.
The algorithm was implemented on GAMS using the EMP Library, providing very convenient handling of stochasticity, EV and CVaR functions. The solver first creates a deterministic equivalent (DE) of the stochastic model (all scenarios being weighted by their probability) before calling CPLEX as the subsolver. The EMP Library also automatically adds the non-predictability constraints to the algorithm.

4.2. Simulation results

As a reference, we first use an EV-CVaR weight λ of 0.5 for stages 2 and 3. The Wholesale market is first simulated exogenously (see Appendix), then the main model is solved to optimality under 3s with an Intel i5-7300HQ CPU and 8 GB of RAM. We structure the analysis of the results into a physical section which considers the outcomes of the market, and an algorithmic section which discusses the performances of the RAMSP model.

4.2.1. Results assessment: physical standpoint

The main model reaches an objective (ECRM) of £15,588.03. The breakdown of the ECRM into all its theoretical sub-components will be addressed in the next subsection, but for now we simply study what it represents in terms of costs per stage for the DSO.
For that, we focus on three aspects: the cost of trades, of overload and of deviations at the transmission interface. The cost of forward trades (day-ahead and intraday reservations) is computed as $\pi^{res} * \sum_t \sum_i (R^{up} + R^{down})$, while the real-time trades (activations) are obtained using $\pi^{act} * \sum_t \sum_i (A^{up} + A^{down})$. The costs of overload and of imbalances at the transmission interface are defined as $\sum_{l,t}(\pi^{CO}.\Delta^{f+,up}_{l,t,st})$ and $\sum_{tr,t}(\pi^{sys}.(\Delta^{SO,+}_{tr,t,st} + \Delta^{SO,-}_{tr,t,st}))$ respectively. Table 2 summarises the values obtained through the simulation. When the wholesale market is

	WS	DA	ID	RT	Total savings
Overload	44,335.11	27,610.87	51.94	0.11	+ 44,335.0
Trades	0	509.73	748.58	13,356.11	- 14,614.42
Imbalance	0	1,057.58	2,229.03	482.73	- 482.73
Total	44,335.11	28,668.45	2,280.97	482.84	+30,496.16

Table 2: Cost of trades, overload and imbalances (£)

cleared, the DSO computes a first forecast revealing a substantial expected real-time overload and amounting to an equivalent cost of £44,335.11. If there is no service market in place, then this is an estimation of the damage expenditures the DSO incurs (as the real value is determined by the real-time realisations). As can be seen on the 'Overload' row in Table 2, the service market plays its role well, as the overload is trimmed down to neglectable amounts in real-time. Note that the overload costs displayed for day-ahead and intraday are estimations by the algorithm for that period, but do not represent actual cash-out flows.

This reduction is permitted by the flexibility trades performed along the timeline. Since activation prices far exceed those of reservations (see Table 1), the real-time market incurs most of the cost (91.4%). Here, the costs of trades displayed at each stage all represent direct expenditures. Finally, the market also generates some imbalances. The associated costs increase substantially in the day ahead and real-time because these are not penalised by the model. In real-time however, the TSO will require an imbalance payment and so the DSO makes sure to optimally activate its reserve portfolio. In total, the savings enabled by the service market amount to 68.8% of the initial wholesale cost forecast.
Next, the daily distribution of the trades is depicted in Figure 4.

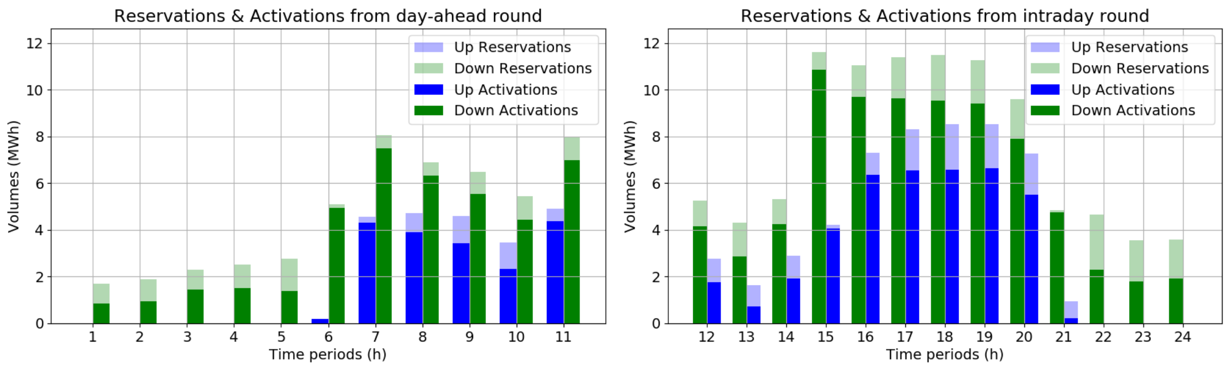

Figure 4: Volumes reserved and activated per stage

In terms of volume, the total down-reservations amount to 147.43 MWh, which is 2.1 times higher than the up-reservations. This is the expected outcome for the market whose main purpose is overload reduction (up reservations only help lowering imbalances). The DSO trades in the day ahead only for hours 1 to 11. From hour 12 onwards, all reservations are done through the intraday, since prices have dropped (see 1). The algorithm performs efficiently by avoiding over-contracting: 82% of the day-ahead trades are activated in real-time, against 79% from the intraday. Normally, one would expect the intraday to have a higher activation rate (being closer to real-time with lower uncertainty), but this is not the case here. The main reason is that our stochastic variables are defined as deviations relative to an initial profile. The afternoon peak features the highest absolute demand and therefore the highest absolute possible deviations (which is empirically verified). Since all reservations for that period are done through the intraday, this round ends up performing slightly worse in that regard.
We now compare the results of the main model with the two market benchmarks, detailed in section 3.3. For this, we use the following set of key metrics: global objective value (Obj), cost in the worst and best scenarios (Min, Max), cost standard deviation (StD) and realised overload. Figure 5 plots the results in terms of variations with the main model (note that the given mean values represent here the arithmetic average, not the scenario-weighted value). For the Real-time-only, the results are quite self-explanatory: without a service

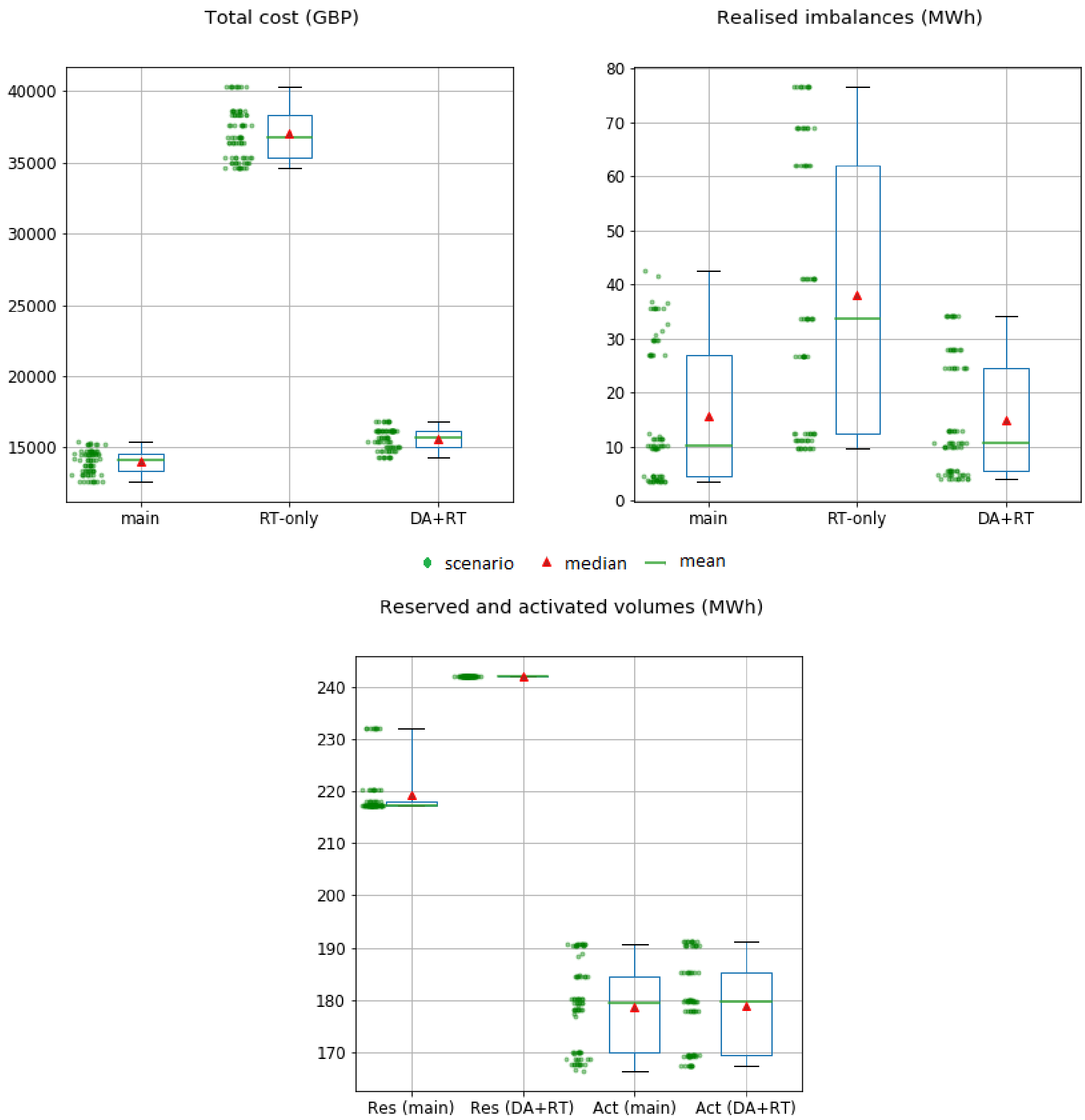

Figure 5: Key performance indicators by simulation (£)

market, a passive DNO with only late corrective actions would be faced with more than double the costs entailed by the market (mean cost and deviations increased by 143.8% and 128% respectively). Because full demand serving is always a hard constraint, the lack of flexible options forces the DSO to concede some expensive thermal overload through some lines (resulting in initial wholesale costs cut down by 31% only).
The outcome of benchmark 2 (Day-ahead+Real-time) is much closer to the reference simulation. While it also manages to completely avoid any overload, its overall performance is worse, with higher mean (+2.6%), maximum (+1.4%) and minimum (+3%) costs. The reserve activation rate is reduced to 74% and the scenario cost spread is also greater, with +1.3% in terms of standard deviation. There are however some cases where it outperforms the main model in terms of minimising the achieved deviations at the interface. Looking at the last boxplot, it is clear that the Day-ahead+Real-time benchmark overbooks day-ahead due to the risk-averse objective function and the higher uncertainty at this stage. An interesting remark is that this benchmark reserves the same volume for every scenario because there are no recourse actions intraday (no market) and therefore no potential costs to hedge against. As a result, the benchmark often has some reserve surplus in real-time, which is used instead of drawing more power from the transmission grid. This is further confirmed by the slight upward shift of the Day-ahead + Real-time activation. The 3-stage model enables the DSO to perform a tighter reservation schedule and to avoid wasting payments for non-activated volumes. While this may lead to higher fluctuations at the transmission interface, the penalty imposed by the System Operator is not enough to counterbalance the achieved benefits. Nonetheless, while it is not included in this research, reliance on the System Operator to cover deviations may create cross-impacts since the latter has, in turn, to procure services and keep the grid balanced. Now that we have discussed the advantages of our 3-stage market setup, we will study in more details the algorithmic implications of our RAMSP model.

4.2.2. Results and assessment: algorithmic standpoint

Recall our ECRM formulation from Eq.(31). In order to evaluate in more detail its behaviour, we have computed its sub-components in terms of EV and CVaR and display them in Table 3.

	DA	ID	RT	Total
ECRM	-	-	-	15,588.03
$\mathbb{E}(\pi^{tot})$	509.79	748.57	13,864.08	15,122.44
$CVaR$	0	748.88	14,794.96	15,543.84

Table 3: ECRM, EVs and CVaRs per stage (£)

A first comment is that the value of the ECRM is higher than both the pure EV problem and the pure risk-averse (CVaR) objectives, despite being their weighted average. The pure EV problem features - as anticipated - the lowest total costs, including the immediate day-ahead decisions $\mathbb{E}(\pi^{tot}_{st=1}) = \pi^{tot}_{st=1}$ and the expectations for the next stages. The pure CVaR problem is more costly since only the 10% worst scenarios are considered. There is, however, no risk-aversion for immediate decisions, which is why $CVaR_{st=1} = 0$. Because the ECRM combines the two formulations, it includes both the day-ahead EV expenditures and the larger costs arising from risk considerations with the weight $\lambda = 0.5$, and its higher value can indeed be verified by backward calculation. Another interesting aspect is the ratio between the EV and CVaR values: in the intraday, the right-tail scenarios represent a mere 0.04% increase from the main scenario pool, against 6.7% in real-time. This suggests that the volatility is higher close to delivery and that this stage should therefore require more attention. We will confirm this assumption in the next section.

Further, the validity of the RAMSP algorithm must be tested by comparing it against the classic stochastic standards of the literature. There exists several of these references with idiosyncratic properties but two of them are widely used: the Wait-and-See (W&S) and the Expected EV solution (EEV) models (Birge & Louveaux, 2011). In essence, a typical stochastic program with recourse like our RAMSP solves the so-called "Here-and-Now" Recourse Problem (RP): decisions must be taken immediately, then nature creates realisations to which the program must re-adapt. On the contrary, the W&S provides the solutions when decisions are taken after the realisations: it represents the hypothetical "perfect information" (or perfect forecast) model. The EEV determines the performances of using the deterministic EV formulation wherein all scenarios are replaced by their expected average. Therefore, for a minimisation problem, the following relation has been mathematically verified:

$$W\&S \ \leq \ RP \ \leq EEV \tag{35}$$

The difference $EVPI = RP - W\&S$ is commonly known as the Expected Value of Perfect Information and determines how much additional savings a DSO would achieve from obtaining perfect forecasts. The difference $VSS = EEV - RP$ characterises the Value of the Stochastic Solution, which indicates the benefits of the RP solution over the deterministic one (how valuable the scenario design is). These measures have been extensively used for 2-stage models, but again the generalisation to multistage programs adds some complexity. Computing the W&S solution is not impacted because we just need to solve the RP model with the optimal decision path for each scenario. The EEV case, however, is more intricate. Indeed, to calculate it, the 2-stage procedure has to first solve the deterministic EV problem (using the single average value scenario) and then solve the RP model with its stage-1 variables fixed at the EV solutions. For multistage models, it is not usually clear which variables and at which stage they must be fixed. We resort to Escudero et al. (2007) whose methodology is to define a stage-wise EEV_{st} (thus VSS_{st}) so that, for instance, EEV_2 represents the impact on stage 2 of fixing all previous stages at the EV values.

We proceed as follows: first, the solution to the deterministic EV problem is computed. In the later, the RP random variables are replaced by their expected realisations (conveniently corresponding to our medium scenario across all variables; see 3). Next, we compute the W&S, EEV_2 and EEV_3 solutions as described above. Lastly, we produce the values for the EVPI, VSS_2 and VSS_3 and summarise our findings in Figure 6.

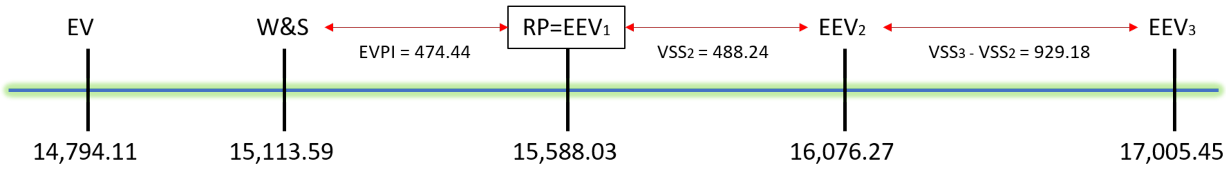

Figure 6: Solutions of the RAMSP and the stochastic benchmarks (£)

First of all, note that $EV < W\&S$. This is a paradox also found in Escudero et al. (2007), and caused by the uncertainty in the recourse matrix. In brief, the deterministic EV removes any volatility to the profit of one average scenario and therefore heavy tail-costs do not arise and the CVaR - which increase the costs as per Table 2 - does not play any role. Next, the W&S value is 3.04% lower than the RP (main RAMSP) so that the DSO would be ready to pay up to an EVPI of £474.44 to access perfect forecasts. EEV_2 and EEV_3 respectively corresponds to 3.1% and 9.1% increases from the main objective function. As a result, our stochastic ID has an expected added value of $VSS_2 = £488.24$ and our stochastic RT one of $VSS_3 = £1417.42$ on the model, over EV stages. In terms of relative distance, our RAMSP outperforms the case study in Escudero et al. (2007) since it is much closer to the W&S than the full expected EV problem EEV_3 which suggests our design is appropriate and accurate. It also surpasses Abbaspourtorbati et al. (2017b) in terms of achieved VSS (their EEV_3 increased by a maximum of 5.1% only). Knowing that the core algorithm is correct, we can now safely focus on our end-goal: highlighting best practice for a DSO. In the next section, we carry out further analyses in terms of optimal market architecture and decision policies.

4.3. DSO behaviour and ID location

So far, we have demonstrated the advantages of our formulation over physical and algorithmic benchmarks. To do so, we defined arbitrarily the intraday auction and the risk aversion of the DSO during all forward trading rounds. We suggested in the Introduction that the benefit of the intraday market is to facilitate trades closer to real-time with less uncertainty. Yet, only the periods following the clearing are re-optimised. We argue that there is a trade-off between an early intraday clearing with high uncertainty, long delivery period and a late intraday with reduced uncertainty and shorter delivery period. We also explore the flexibility of our risk measure function by varying λ_{st} to evaluate different hedging attitudes the DSO. Consequently, we carry out sensitivity analyses on both the risk weight and the intraday temporal location.

4.3.1. Sensitivity analysis: Risk aversion

To consider all possible configurations, we vary both λ_2 (risk aversion for intraday) and λ_3 (risk aversion for real-time) across the whole (0,1) interval. For all the simulations, we use the following set of metrics: Objective function, Standard deviation (StD), Max and Min cost. As a result, we are able to provide a mapping of these metrics that can be used as a decision tool for DSOs to inform the impact of risk. This mapping is provided in Figure 7. The results prove to be coherent since a higher risk-aversion does increase overall costs

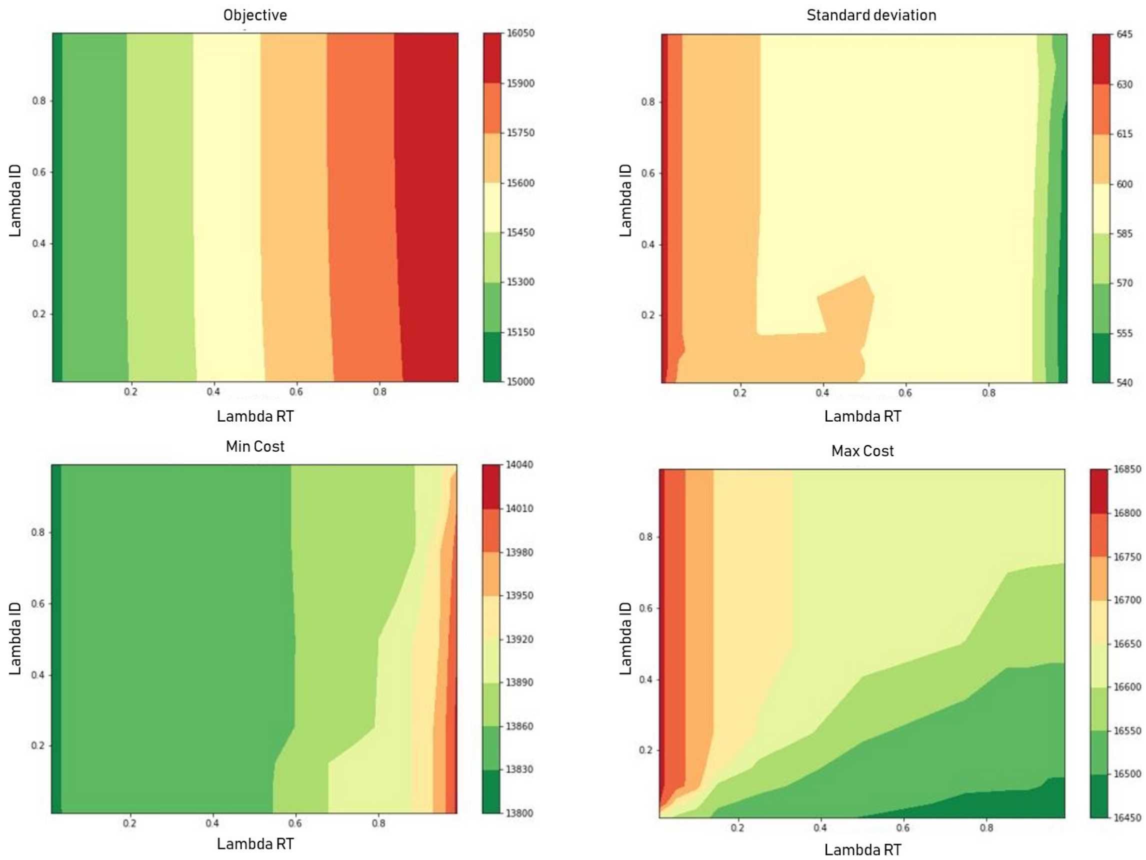

Figure 7: Mapping on the effect of Risk-aversion (values in £)

but reduces StD and Max Cost, due to the CVaR properties. However, several additional comments can be made:

• The Objective and StD's gradients are almost purely unidirectional and a function of λ_3. The impact of risk-aversion in early rounds on these variables is very limited.

• While the Objective is essentially a linear function of λ_3, the StD undergoes strong variations only in extreme cases (close to 0 or 1).

• The impact of λ_2 is more significant for the Min and Max cost values. The Min Cost is mostly driven by λ_3 and actually increases with a risk-averse behaviour. As far as the Max Cost is concerned, the opposite is true. Further, for a given λ_3, a higher λ_2 surprisingly contributes to a higher value of the Max Cost (despite the CVaR properties).

• Hence we conjecture that the lower right-hand side corner may contain solutions offering a good trade-off for typical DSOs. Indeed, the region approximated by $\lambda_2 \simeq 0$ and $\lambda_3 \in [0.7; 0.9]$ provides acceptable StD, Min Cost and Max Cost. This comes at the expense of a higher Objective but this is the price of ensuring safe grid operations.

We conclude that real-time risk-aversion is the dominant factor and that conservative behaviours in earlier stages can lead to sub-optimal solutions. There is no clear benefit of using any $\lambda_2 > 0$ according to our results. This is, however, assuming that there is no uncertainty in the intraday available volumes (which may not be the case in real applications). Thus, we set $\lambda_2 = 0.1$ as a constant for our last analysis.

4.3.2. Sensitivity analysis : Intraday Auction Time

While the Spanish Market that inspired our design is organised around several ID rounds, we considered only a single auction. Beyond numerical considerations, another reason is that the volumes of our market are much smaller. Starting from the postulate of having only one auction, a smart DSO would have to define what is the best clearing time period to make the most out of that round. Thus, we consider (on top of our 12pm reference) three other possible delivery periods starting at 5am, 9am and, 4pm. These correspond to horizons of respectively 20h, 16h and 9h. The last has been chosen to precisely correspond to the beginning of the evening peak. Any clearing further in time would be quite inefficient by missing these critical periods. For each of these new settings, we adapt the level of uncertainty and the two-part tariffs accordingly (early rounds with higher uncertainty and price, late rounds with lower uncertainty and price). We compute the same set of metrics as functions of the clearing period and the value of λ_3.

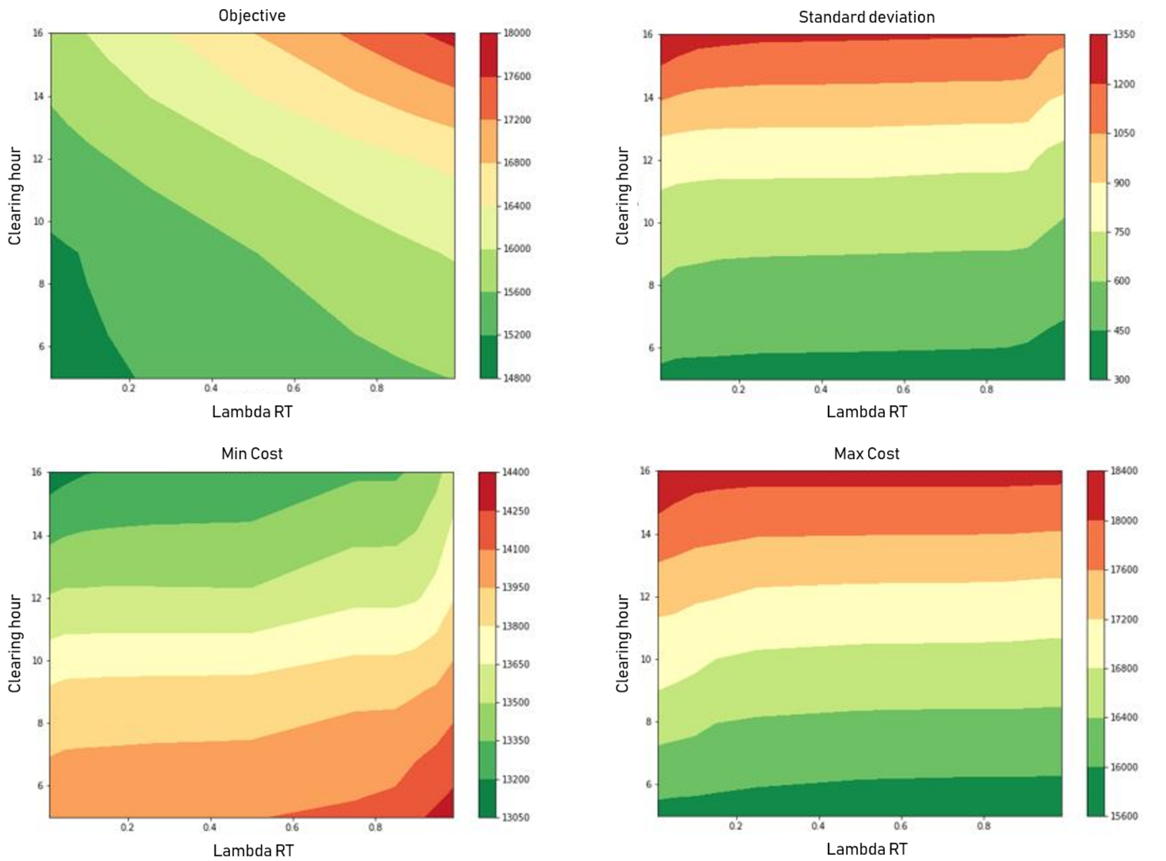

Figure 8: Mapping on the effect of ID location (values in £)

Results are displayed in Figure 8 and the following remarks are provided:
• The risk aversion and the intraday timing both contribute to increasing the value of the Objective. The largest Objective variations, however, are observed along λ for late intraday rounds (when peak time draws closer).
• The StD increases with a late clearing because fewer hours are re-optimised from the day-ahead. As in Figure 7, it is affected by the value of λ_3 only when the latter is extreme.
• The Max and Min cost values are almost mirrored. While the first one becomes higher as the intraday auction is delayed (due to reduced hedging possibilities), the second gets lower because the reduced uncertainty causes some real-time scenarios to be much cheaper.
• Unlike Figure 7, Figure 8 does not seem to feature any region with a reasonable compromise but rather a unidirectional trend: the sooner the round, the better the results.
We observe that this apparently "naive" conclusion might not apply to real applications and actually exposes our model's limitation. The fact is that we have considered the stage-specific uncertainty to be constant for every hour of the day because of numerical constraints. Generally, at each round, the uncertainty on the furthest delivery period should be higher than the nearest. Therefore, an early intraday would normally feature a much greater spread during evening peak time and we would expect enlarged costs. Following on from this restriction, the next section finally details the other limits of our work and some of the remaining issues from both the service market and the algorithm.

4.4. Limitations and transferability

4.4.1. Market-wise

In terms of market setting, our RAMSP is based on several critical assumptions. First of all, we have chosen to concentrate exclusively on thermal loading given its preponderance among the various network constraints, but others and especially voltage would benefit the real-life optimal service procurement process. Next, the 3-stage model features a single intraday clearing while many countries typically use a multi-clearing or a continuous trading approach. While multi-clearing can to some extent be approximated by a single stage (it has been seen in practice that most trades are concentrated in one or two rounds), the current RAMSP cannot accommodate a continuous mechanism whose bidding, allocation and settlement rules must be adequately modelled. On settlement rules, we have used single (bidirectional) prices for reservations and activations but up and down reserves can also be differentiated to account for differences in costs for each technology to provide such services. Yet, this upgrade is very straightforward. Moreover, a 1h settlement period - based on the Spanish model - has been used. Nowadays, many markets are converging towards lower resolutions to promote flexibility. This is yet another direct addition to the model which could provide further savings for DSOs.
Other elements to consider are the technical requirements necessary to run this service market. To function optimally, the latter would indeed require updated forecasts and measurements from one stage to the next. For many years, DSOs have been lacking such information, as the reconciliation with the meter could take up to several months (manual reading). However, with the new Smart Meters (typically half-hourly automated readings)

whether to support Time-of-Use (ToU) tariffs, high-frequency trading or potential innovative peer-to-peer business propositions, most users should be equipped with adequate data monitoring devices, (Sovacool et al., 2017).
One aspect to consider further are the potential cross-impacts with the other stakeholders. Indeed here we assumed the entire flexibility pool to be for the sole use of the DSO, but in reality such services can also be procured by Retailers and TSOs. The way these procurement processes are coordinated may impact the findings by adding more uncertainty on available volumes.

4.4.2. Algorithm-wise

The focus on thermal loading at high distribution voltage allowed us to use a DC network assumption in the algorithm. However, this limits the applicability in lower grid levels for which an AC model would normally be required. While an AC design would readily enable the considerations of voltage issues and network losses, it would also make the algorithm non-linear and critically affect its scalability and convergence properties. Moreover, we chose a 5-bus network to focus on the optimal stochastic design rather than on the model's scalability. While the latter is ensured by the linear nature of the program, the transferability of the results to larger system could be impacted as the numbers of consumers and generators grow. Our case study featured three possible stochastic realisations (low, medium and high) for each random variable. More scenarios could be considered to further improve the forecasts, but the computational trade-off must be carefully considered. The design of the random variables as deviation coefficients (as opposed to absolute injections/withdrawals) increases the robustness of the results, but the issues picked up in section 4.3.2 could be resolved by constructing a time-dependent random vector within each stage instead of a single value for all hours.

5. Conclusions

We have formulated a methodology that addresses the challenges expected by emerging DSOs in the face of the rapid penetration of local renewable energy technologies. After discussing how grid relief needs can be answered by opening a service market where flexibility is traded with local agents, we identified the necessity for this market to be multistage, close to real-time, and equipped with hedging tools. Consequently, a stochastic clearing formulation with an intraday round to absorb part of the uncertainty, a two-part tariff scheme and a customised risk-aversion function were modelled. This informed our research objectives:

- The algorithm has proved viable and the three-stage market also outperforms standard formulations. By strategically contracting with local providers, the DSO is able to bring overloaded lines back to operational safety and spares large expenses. The intraday market provided greater value by fully covering the evening peak with better service provision, while the benchmarks relied more on the System Operator to fill those gaps. We analysed both expected values and also scenario-specific results, showing the benefits of our Expected Conditional Risk Measure (ECRM) function.

• We exploited the ECRM to map out the impact of several trading strategies on the final real-time costs. Bespoke guidelines depend on the actual requirements of the DSO, but our results demonstrate that forward risk-aversion has very little weight and does not provide the best trade-off, whereas risk-aversion in real-time significantly impacts the outcome (the last round concentrating most of the market costs). In other words, we advise DSOs to run an early intraday auction where low risk aversion is considered and a more pragmatic close-to-real-time round where risk should weight more in the decision criterion. Another key insight is that despite the unidirectional restrictions of CVaR, its inclusion in the objective function changes the decision pattern and, in this setting, contributes to increase the cost of the cheapest scenarios.

• Finally, we explored a joint search of real-time risk-aversion impact and the temporal location of the intra-day market. We were able to exhibit reasonable properties for late intraday settings, in which higher end-costs and variability arise because of the shortening of the delivery period (hours not re-optimised from day-ahead). Yet, results for early rounds are biased by the constant stage-wise stochastic multipliers which makes them consistently more efficient.

Our study has also contributed to the understanding of multistage, risk averse stochastic programming. For future work perspectives, a dynamic hourly stochastic evolution could be assigned to the random variables at each stage for more realism. Further analyses on the CVaR confidence level α would also extend the present analysis.

6. Appendix

This Appendix provides details on how the wholesale market data (inputs for the main simulation) are obtained.
First of all, we assume all the dispatchable (non renewable) generators in our DSO's area participate in the day ahead wholesale market. They all offer a tuple of price and energy volume for each delivery period of the next day. After the market clears, we assume the DSO is able to retrieve the cleared volumes for each of these generators. Using this data as inputs, a power flow model is run to obtain the full description of the region according to the wholesale commitments. Because we aim at solving congestion issues, we select arbitrarily cleared volumes $P_{i,t}^{WS}$ such that the wholesale network state features a risk of stress. The system solved is the following:

$$\sum_{l \in rec\_bus} f_{l,t}^{WS} - \sum_{l \in snd\_bus} f_{l,t}^{WS} + \sum_{i \in gen\_bus} \left(P_{i,t}^{WS} + Cap_{i,t}^{g}\right) + \sum_{tr} P_{tr,t}^{TSO,WS} - \sum_{j} d_{j,t} = 0 \quad \forall n, t \tag{36}$$

$$f_{l,t,st}^{WS} = B_l * (\theta_{n,t}^{rec,WS} - \theta_{n,t}^{snd,WS}) \quad \forall l, t \tag{37}$$

$$\theta_{slack,t}^{WS} = 0 \quad \forall t \tag{38}$$

Where the only variables $f_{l,t}^{WS}$, $P_{tr,t}^{TSO,WS}$ and $\theta_{n,t}^{WS}$ are directly computed because all the injections and withdrawals are known. In parallel, from the commitments, we also compute the up and down volumes initially available for trades with the DSO:

$$F_{i,t}^{up} = P_{i,t}^{rated} - \left(P_{i,t}^{WS}\right)_{\forall \text{ dispatchable}} - \left(Cap_{i,t}^{g}\right)_{\forall \text{ non-dispatchable}} \quad \forall i, t \tag{39}$$

$$F_{i,t}^{down} = \left(P_{i,t}^{WS}\right)_{\forall \text{ dispatchable}} + \left(Cap_{i,t}^{g}\right)_{\forall \text{ non-dispatchable}} \quad \forall i, t \tag{40}$$

Finally, the values of $P_{tr,t}^{TSO,WS}$, $P_{i,t}^{WS}$, $F_{i,t}^{up}$ and $F_{i,t}^{down}$ are passed as parameters to the main model described in section 3.

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