Investor Protection and Asset Prices*

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Abstract

Empirical evidence suggests that investor protection has significant effects on ownership concentration and asset prices. We develop a dynamic asset pricing model to address the empirical regularities and uncover some of the underlying mechanisms at play. Our model features a controlling shareholder who endogenously accumulates control over a firm, and diverts a fraction of its output. Better investor protection decreases stock holdings of controlling shareholders, increases stock mean-returns, and increases stock return volatilities when ownership concentration is sufficiently high, consistent with the related empirical evidence. The model also predicts that better protection increases interest rates and decreases the controlling shareholder's leverage.

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1. Introduction

In many countries corporations are dominated by controlling shareholders who can divert resources for their private benefit. Consequently, the protection of minority shareholders against expropriation by controlling shareholders is of much importance for understanding ownership concentration and asset prices. In line with this observation, the empirical literature documents significant effects of investor protection on asset prices and their dynamics. However, an overarching theory that sheds light simultaneously on the effects of investor protection on the firm ownership concentration, stock returns, interest rates, personal leverage, risk sharing, and extent of expropriation is missing in the literature. In this paper, we develop such a theory in a parsimonious general equilibrium setting and use it to address some of the empirical evidence on asset holdings of controlling shareholders and stock returns, and to provide new predictions.

The dynamic accumulation of control and the ability of shareholders to trade and share risks are key features of our analysis which play a leading role in determining the effects of investor protection on asset holdings and returns. Due to these key features, our analysis generates rich dynamics of asset holdings and returns, and reveals that the excess ownership concentration (relative to the full investor protection benchmark) is the main channel through which investor protection influences stock mean-returns, volatilities, interest rates, and the controlling shareholder's leverage. In particular, we find that the stock holdings of controlling shareholders change in response to economic shocks and decrease with better investor protection, stock mean-returns increase with better protection, and stock return volatilities also increase with better protection when ownership concentration is sufficiently high. We relate these results to empirical findings and formulate additional predictions below.

We consider a dynamic general equilibrium economy with a representative competitive firm that produces an exogenous stream of output. The firm's stock is owned by two types of shareholders with heterogeneous constant relative risk aversion (CRRA) preferences, a minority shareholder and a controlling shareholder who can divert a fraction of the firm's output for himself. The diverted fraction satisfies an investor protection constraint, which limits the scope of available diversion strategies. This constraint becomes tighter with better protection and looser with higher stock holdings that increase the controlling shareholder's power over the firm. The diversion of output is further tempered by pecuniary costs of stealing. We provide tractable expressions for the equilibrium processes that admit intuitive comparative statics and are explicit up to the controlling shareholder's stock holding, which

is found numerically.

We find that the controlling shareholder's stock holding is larger in economies with imperfect investor protection than in economies with full protection. Intuitively, poor protection expands the set of diversion strategies and increases the potential gains from higher control power over the firm, which induces buying more shares. However, the relationship between investor protection and optimal stock holdings is non-monotonic and depends on whether the investor protection constraint binds or not. This is because investor protection has two opposing effects on the controlling shareholder's optimal portfolio decision. An increase in investor protection reduces the marginal benefit of control and hence reduces the incentive to acquire more shares, while on the other hand, makes the investor protection constraint relevant for a wider range of stock holdings thereby providing an incentive to acquire more shares to relax the constraint. The relative importance of these effects depends on the consumption share of the minority shareholder, which acts as an endogenous state variable in the model. We also find that the shareholders' stock holdings change in response to economic shocks. In particular, the controlling shareholder is more exposed to stock market fluctuations because he invests a larger fraction of wealth in stocks than the minority shareholder. Consequently, the controlling shareholder sells stocks to finance consumption in bad times, when the firm output is hit by negative shocks, and buys them back in good times.

The acquisition of the controlling shareholder's shares is financed by personal leverage, and hence this leverage is higher in economies with imperfect protection. We establish a tight link between the personal leverage and stock holdings of the controlling shareholder. In particular, the leverage-stock price ratio in our model is simply given by the controlling shareholder's excess ownership concentration, defined as the stock holding over and above his holding in the full protection economy. We also show that the fraction of diverted output is a hump-shaped function of the shares held by the controlling shareholder. In our model, a higher stock holding relaxes the investor protection constraint by increasing the control power over the firm, and the controlling shareholder diverts more as his holding increases. On the other hand, the higher stock holding decreases his incentive to divert due to pecuniary costs of stealing. After some point, the investor protection constraint no longer binds and the equilibrium amount of diverted output decreases with the stock holding.

We demonstrate that the stock mean-return decreases with poor investor protection in equilibrium. The intuition is that, in contrast to the minority shareholder, the controlling shareholder is compensated for holding risky assets not only by the risk premium but also by the fraction of the diverted output. Therefore, the controlling shareholder hoards shares even if the realized risk premium is low, which drives down the stock mean-return in equilibrium.

Following this intuition, we show that the asset holdings of the controlling shareholder are determined by a previously unexplored quantity, which we refer to as the effective risk premium. We decompose the effective risk premium into a conventional risk premium implied by stock price dynamics and an additional term capturing the diverted output per share. In our decomposition the diverted output per share can be interpreted as an adjustment to the dividend that is received by the controlling shareholder.

We also show that the equilibrium stock return volatility is higher with imperfect protection than with full protection and exceeds the volatility of the aggregate output. Another novel prediction of our model is that across economies with varying imperfect protection, the relation between volatility and investor protection is non-monotone, and when the stock holding of the controlling shareholder is sufficiently high the volatility is higher in economies with better protection. We find that the excess volatility relative to the full protection benchmark economy is proportional to the excess personal leverage-stock price ratio relative to the benchmark, and to the best of our knowledge, such a simple characterization of volatility in terms of personal leverage is new to the literature. Intuitively, personal leverage finances the acquisition of shares by the controlling shareholder when protection is low, and hence increases the sensitivity of the controlling shareholder's wealth to economic shocks, which translates into higher stock return volatility via the state variable that tracks wealth inequality. The non-monotonicity of volatilities with respect to investor protection is explained by the non-monotonicity of the stock holdings, which determine personal leverage, as discussed earlier.

Furthermore, we find that the risk-free interest rates decrease with lower protection due to the following two effects in equilibrium. First, because of low risk premium and high volatility, the minority shareholder turns to bond markets and is more willing to provide cheap credit. Second, the acquisition of shares by the controlling shareholder is partially covered by the diverted output, which moderates his demand for credit. These two effects are partially offset by the surge of the controlling shareholder's leverage under poor protection, which increases the demand for borrowing, but the net effect on the equilibrium interest rate is negative.

The effects of investor protection on stock holdings of controlling shareholders, stock mean-returns and volatilities in our model go in the same direction as in the related empirical literature. In particular, this literature finds that the controlling shareholder's stock holdings are larger in economies with lower investor protection (La Porta, Lopez-de-Silanes, and Shleifer, 1999) and change in response to economic shocks (e.g., Denis and Sarin, 1999; Holderness, Kroszner, and Sheehan, 1999). Moreover, it shows that the expected stock

returns are negatively related to investor protection as measured by the degree of corporate governance, entrenchment and managerial perks (e.g., Gompers, Ishii and Metrick, 2003; Yermack, 2006; Bebchuk, Cohen and Ferrell, 2009; Daines, Gow and Larcker, 2010, among others), although there is an ongoing discussion of the robustness of this relation (Core, Guay and Rusticus, 2006; Giroud and Mueller, 2011; Bebchuk, Cohen and Wang, 2013). We contribute to this discussion by providing a theoretical argument in favor of the positive relation between the stock mean-return and investor protection. The empirical literature also finds that the stock return volatility is higher in economies with better protection (e.g., Morck, Yeung and Yu, 2000; Jin and Myers, 2006; Bartram, Brown and Stulz, 2012), similar to our model when ownership concentration is sufficiently high.

We note that the above empirical literature does not explore the link between the dynamics of ownership concentration and the observed effects of investor protection on the stock mean-returns and volatilities, as predicted by our model. Therefore, further empirical work would be needed to better match the empirical evidence on investor protection to this prediction. Additional new testable predictions of our model include a higher personal leverage of controlling shareholders and lower risk-free interest rates when investor protection is low, the non-monotonicity of stock return volatility with respect to investor protection, and the link between excess volatility and excess personal leverage in the economy with imperfect protection.

Finally, we study the equilibrium effects of the pecuniary costs of stealing such as bribes, fines payable if controlling shareholders are caught stealing, payments to lawyers and other expenses for arranging diversion schemes. We find that a higher pecuniary cost of stealing is associated with a higher stock gross return and interest rate, and a lower volatility and stock holding of the controlling shareholder. We also consider an extension of our analysis to incorporate cross-firm differences in investor protection, albeit in a simple one-period economy. We show that our earlier results on stock holdings, mean-returns, and interest rates remain valid. We additionally find that cross-firm differences in investor protection give rise to a positive investor protection premium, defined as the spread between mean-returns of stocks with higher and lower protection. This result is particularly valuable because much of the empirical evidence on the positive relation between stock returns and investor protection (discussed above) is for cross-sections of firms with different levels of investor protection whereas our main analysis features a representative firm. We also consider other extensions of our analysis that feature alternative stealing technologies and non-pecuniary costs of diverting output. We find that all our results on the effects of investor protection remain qualitatively the same.

Our paper also makes a methodological contribution by integrating a model with corporate frictions such as investor protection and acquisition of control into a general equilibrium asset pricing framework, albeit in a pure-exchange setting.[1] Solving models with frictions such as constraints on certain choice variables is a daunting task. We achieve tractability by allowing investors to optimize two-period CRRA preferences repeated over time, similar to models with myopic investors and overlapping generations. This approach allows us to focus on the effects of investor protection and abstract away from hedging demands for stocks, which are more relevant for the portfolio choice literature. However, we demonstrate that the equilibrium processes in the full protection benchmark economy share some features of dynamic Lucas-type (1978) economies with heterogeneous investors. In particular, the Sharpe ratio in the latter economy and our model have the same structure. A notable feature of our model is that it is stationary in the sense that both shareholders survive in the long run and the distributions of their consumption shares are non-degenerate. The stationarity is achieved by endowing shareholders with non-financial labor incomes, since future incomes help investors gradually accumulate wealth and our preference specification keeps investors' wealth positive, restricting financial losses.

1.1. Related Literature

The most related to our paper are the works that study asset pricing implications of investor protection against expropriation by controlling shareholders. Shleifer and Wolfenzon (2002) introduce a static model that explains why firms are larger and more valuable with better investor protection. Albuquerque and Wang (2008) develop a dynamic production economy with buy-and-hold controlling shareholders who extract more benefits in larger firms. They show that weaker investor protection implies over-investment which leads to higher stock risk premia, volatility, and interest rates. The stock mean-returns, volatility, and interest rates are constant due to the absence of trading between shareholders. Giannetti and Koskinen (2010) study a static model of two countries with different levels of investor protection, where investors make portfolio decisions at the initial date and do not rebalance their portfolios. They find that stock returns decrease with weaker protection, similar to our paper. We complement these works by focusing on different aspects of investor protection arising due to its effects on asset demands and accumulation of control. In particular, the controlling shareholder in our model extracts private benefits by dynamically increasing his stock holding

[1] Focusing on a pure-exchange setting allows us to incorporate trading between different groups of shareholders in a tractable way. As discussed below, a general equilibrium production economy with investor protection and a buy-and-hold controlling shareholder has been studied by Albuquerque and Wang (2008).

in the firm whereas in these works the stock holding is fixed. Consequently, we uncover new economic forces that decrease stock mean-returns and interest rates when investor protection is low, leading also to excess stock return volatility, personal leverage, time-variation of all equilibrium processes and wealth transfers between different categories of shareholders.

Our paper is also related to the literature where the stock holdings of controlling shareholders are endogenous and influence managerial incentives and firm output in a dynamic setting. DeMarzo and Urosevic (2006) and Gorton, He, and Huang (2014) present models with stock trading and risk sharing between a controlling shareholder and small investors. The controlling shareholder in these models trades dynamically and accounts for the relationship between her stock holding on one hand and effort level, firm output, and stock price on the other. In particular, the latter paper shows that stock mean-returns and volatility increase or decrease depending on investors' risk aversions. Haddad (2015) studies a dynamic model with endogenous ownership concentration where active shareholders increase the output mean-growth rate and are compensated by the firm for being under-diversified. He shows that active capital amplifies stock-return volatility. Jung, Subramanian, and Zeng (2016) study a model where a large shareholder faces a tradeoff between diversification and incentive provision to firm managers determining the value of the firm. Our paper differs from these works in that we focus on the economic effects of investor protection in a setting where the controlling shareholder's power over the firm and the ability to divert output depend on his stock holdings. As a result, we uncover several new economic effects of investor protection on excess ownership concentration, mean-returns, volatility, interest rates, and personal leverage.

Also related to our paper are various works that share some of the key features of our analysis such as investor protection, expropriation of minority shareholders, and endogenous ownership concentration. Doidge, Karolyi, and Stulz (2004) consider a static model where controlling shareholders of firms with growth opportunities reduce their cost of capital by cross-listing firms in the U.S., which commits them to better investor protection. Doidge, Karolyi, and Stulz (2007) show, both theoretically and empirically, the importance of country characteristics for the quality of investor protection. Dow, Gorton, and Krishnamurthy (2005) study a model with the separation of ownership and control where firm managers expropriate shareholders by diverting cash flows to inefficient investments. Aslan and Kumar (2012) present a three-period model in which the endogenous choice of ownership concentration affects the cost of borrowing, probability of default, and the post-default restructuring. Li and Li (2018) show, both theoretically and empirically, that better corporate governance leads to higher stock returns during market upturns when firms have good growth options,

but low stock returns in the downturns. In contrast to this literature, our paper studies the joint effects and interactions of investor protection, expropriation, and ownership concentration, which give rise to distinct economic predictions.

2. The Economy with Investor Protection

We consider a pure-exchange continuous-time infinite-horizon economy with a representative firm that produces one consumption good and is owned by two types of shareholders with heterogeneous control power over the firm. In this Section, we discuss the firm, the financial markets, and shareholder optimization problems.

2.1. Firms and Financial Markets

The uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a Brownian motion w. The stochastic processes are adapted to the filtration $\{\mathcal{F}_t, t \in [0, \infty]\}$, generated by w. There is one representative firm in the economy which stands for a large number of identical firms. The firm produces an exogenous stream of output $\widehat{D}_t$, which follows a geometric Brownian motion (GBM) process:

$$d\widehat{D}_t = \widehat{D}_t\big[\mu_D dt + \sigma_D dw_t\big], \tag{1}$$

where the output mean-growth rate μ_D and volatility σ_D are constants.

There are two types of shareholders, a controlling C and a representative minority M shareholders. The representative minority shareholder stands for a group of identical shareholders. Fractions l_C and l_M of the aggregate output are paid to the controlling and minority shareholders as their labor incomes, respectively. The shareholders trade continuously in two securities, a riskless bond in zero net supply with an instantaneous risk-free interest rate r_t and a stock in positive net supply, normalized to one unit. The stock is a claim to the stream of dividends, which are paid each date t out of the net output after paying labor incomes, given by $D_t = (1 - l_C - l_M)\widehat{D}_t$. The dividend payout is determined by the controlling shareholder, as discussed below. We focus on Markovian equilibria in which bond price, B_t, and stock price, S_t, follow processes

$$dB_t = B_t r_t dt, \tag{2}$$

$$dS_t = S_t\big[\mu_t dt + \sigma_t dw_t\big], \tag{3}$$

where the risk-free interest rate r_t, stock mean-return μ_t, and volatility σ_t are endogenously determined in equilibrium, and the bond price at time 0 is normalized so that $B_0 = 1$.

2.2. Investor Protection and Shareholder Objectives

The minority shareholder does not have control power, and cannot influence the dividend payout policy. The controlling shareholder can divert a fraction x_t of the firm's output for himself. The remaining non-diverted output $(1 - x_t)D_t$ is paid as a time-t dividend. The diverted fraction x_t is constrained by investor protection in the economy, so that $x_t \leq (1 - p)q(n_t)$, where $p \in [0, 1]$ is interpreted as the *protection of minority shareholders*, with higher p indicating better investor protection, and $q(n) \in [0, 1]$ indicating the controlling shareholder's power over the firm. Consequently, the above *investor protection constraint* on fraction x_t is determined jointly by investor protection p and the controlling shareholder's power $q(n)$ over the firm. To simplify the analysis, we assume that $q(n) = n$, so that the power over the firm is linearly increasing in the the number of shares, and the controlling shareholder has the same control and cash flow rights in the firm. In Section 5 and the Internet Appendix IA.2, we consider alternative stealing technologies that temper the incentives to divert output when the controlling shareholder's stock holding is small.

The investor protection constraint captures the fact that better protection restricts the set of available expropriation strategies. Controlling shareholders may divert output by employing a wide range of complex strategies rather than outright theft. For example, cash flows can be tunneled through intra-group activities which can be economically large (Bertrand, Mehta, and Mullainathan, 2002; Cheung, Rau, and Stouraitis, 2006; Khanna and Yafeh, 2007; Jiang, Lee and Yue, 2010). Companies can give each other, or to controlling shareholders directly, high (or low) interest loans (Bertrand, Mehta, and Mullainathan, 2002), pay special dividends to controlling shareholders (DeAngelo and DeAngelo, 2000), engage in abnormal sales (Jian and Wong, 2010; Lo, Wong, and Firth, 2010), sell assets below or above their market values (Cheung, Rau, and Stouraitis, 2006) and provide loan guarantees (Berkman, Cole and Fu, 2009). One could also include jobs given to relatives, large bonuses, perquisites etc. that are enjoyed by controlling shareholders as a form of wealth transfer from minority shareholders.

The investor protection constraint also demonstrates that more power $q(n)$ over the firm makes it easier to orchestrate wealth transfers for private benefits by expanding the set of available diversion strategies through various channels. First such channel is that a higher stake in the firm gives the controlling shareholder better control over the board and its decisions which allows implementing diversion strategies that would otherwise not be possible. For instance, Anderson and Reeb (2004) analyze board composition of family controlled firms and find that families often seek to minimize the presence of independent

directors. The literature provides both empirical and anecdotal evidence of how controlling shareholders divert firm assets to their private use without substantive interference from the board (e.g., DeAngelo and DeAngelo, 2000; Anderson and Reeb, 2004; Enriques and Volpin, 2007). Second channel is that a higher stake thwarts possible takeover attempts by third parties, widely considered as a market disciplining mechanism (Grossman and Hart, 1980) in which poorly performing firms become targets for acquisition by third parties (perhaps as a result of extraction of private benefits). Consistent with this argument Stulz (1988) shows that an increase in the fraction of voting rights controlled by management decreases the probability of a successful tender offer, mitigating the disciplining role played by outside takeover threats. Anti-takeover clauses are often used as part of governance indices (Gompers, Ishii and Metrick, 2003). Third possible channel is that a larger stake of the controlling shareholder deters a formation of large blocks by other shareholders who might oppose the extraction of private benefits through better monitoring.[2]

We further assume that the controlling shareholder incurs a pecuniary cost $f(x, D)$ from diverting output because stealing is by nature inefficient, in line with the related literature (e.g., Shleifer and Wolfenson, 2002; Albuquerque and Wang, 2008). The cost function $f(x, D)$ is an increasing function of the diverted fraction x_t and net output D_t. Throughout the paper, we assume for tractability that the cost function $f(x_t, D_t)$ is quadratic in x_t and given by

$$f(x_t, D_t) = \frac{k\, x_t^2 D_t}{2}, \tag{4}$$

where the parameter k captures the magnitude of the cost. The pecuniary costs of diverting output may include bribes, fines payable if controlling shareholders are caught stealing, payments to lawyers and other expenses for arranging diversion schemes. The pecuniary cost does not disappear from the economy but is paid to the minority shareholder either via a government transfer or by the controlling shareholder.

In reality, there could also be non-pecuniary costs of diverting output such as disutility from stealing due to social norms that promote fairness, honesty and morality (Kahneman, Knetsch and Thaler, 1986), and loss of reputation. Such non-pecuniary costs reduce the controlling shareholder's utility without affecting the budget constraint. In Section 5 and the Internet Appendix IA.3, we discuss an alternative formulation with non-pecuniary costs and find that all our main economic mechanisms at play and results remain equally valid for this alternative.

The investor protection constraint $x_t \leq (1-p)n_t$ and the cost function $f(x_t, D_t)$ capture

[2]We take the investor protection constraint as given. However, it would be of interest to microfound this constraint as arising due to some underlying agency conflict between different groups of shareholders.

different barriers to expropriation in the economy, and hence, lead to distinct economic implications in equilibrium, as highlighted in Section 4.3 below. The former constraint proxies for legal protection of minority shareholders that limits wealth transfer strategies. In contrast, cost function $f(x_t, D_t)$ quantifies pecuniary costs of stealing.

All shareholders have standard constant relative risk aversion (CRRA) preferences

$$u_i(c) = \frac{c^{1-\gamma_i} - 1}{1 - \gamma_i}, \quad i = C, M, \tag{5}$$

with the risk aversion parameters $\gamma_M \geq \gamma_C > 0$. The controlling shareholder being less risk averse is natural in our setting since a typical endogenous occupation choice model would predict that less risk averse people self-select into entrepreneurial activities and become controlling shareholders (Kihlstrom and Laffont, 1979).[3] For tractability, we assume that investors are guided by myopic preferences over current consumption c and wealth W, given by:

$$V_i(c_t, W_t, W_{t+dt}) = \rho u_i(c_t)dt + (1 - \rho dt)\mathbb{E}_t\Big[u_i(W_{t+dt})\Big], \tag{6}$$

where utility function $u_i(\cdot)$ is given by (5), $i = \{C, M\}$, and $\rho > 0$ is a time-preference parameter. Myopic preferences admit considerable tractability yielding closed-form solution for the optimal portfolios (Section 3.1), and are widely employed in various contexts (e.g., DeLong, Shleifer, Summers and Waldmann, 1990; Pastor, 2000; Acharya and Pedersen, 2004). They also help us focus on the effects of investor protection and abstract away from hedging demands, which are more relevant for the portfolio choice literature.[4]

The preferences (6) have the additional benefit that, along with labor income, they make our economy stationary in that both shareholders continue to have significant economic impact in the long run (Section 3.2). The reason is that these preferences ensure that the shareholders hold positive next-period financial wealth W_{t+dt} at all times. Our economy then becomes stationary because keeping the wealth W_{t+dt} positive restricts financial losses while future non-financial labor incomes help investors gradually accumulate financial wealth even when their current wealth is close to zero.[5]

[3]We thank an anonymous referee for pointing this out. Consistently, Hvide and Panos (2014) provide empirical evidence that risk tolerant individuals are more likely to become entrepreneurs.

[4]Such preferences may also naturally arise in an OLG-type framework. Furthermore, in many models logarithmic preferences give rise to investor myopia, similar to that in the objective function (6) (e.g., Detemple and Murthy, 1997; Basak and Croitoru, 2000). In particular, it can be shown that the value function of a dynamic infinite-horizon consumption choice problem with logarithmic preferences has the following structure: $J(W_t, z_t) = (1/\rho)\ln(W_t) + \tilde{J}(z_t, t)$, where z_t is a certain state variable. Then, from the Hamilton-Jacobi-Bellman equation for the dynamic problem it is immediate to observe that solving the dynamic problem is equivalent to solving a myopic problem with an objective function $\rho\ln(c_t)dt + (1 - \rho dt)\mathbb{E}_t[\ln(W_{t+dt})]$. The objective function (6) retains the latter structure of problems with logarithmic preferences but has the additional benefit of accounting for risk aversion γ_i.

[5]We note that our myopic preferences do not account for future labor incomes, and hence are not a

2.3. Shareholders' budget constraints and optimizations

Each shareholder chooses consumption c_{it}, the number of shares n_{it}, and the controlling shareholder additionally chooses the fraction x_t of diverted output for private consumption. Each shareholder's wealth at time t is given by $W_{it} \equiv b_{it}B_t + n_{it}S_t$, where b_{it} is the number of units of bonds in the shareholder's portfolio. The shareholders' dynamic self-financing budget constraints are as follows:

$$\begin{aligned} dW_{it} &= \Big(W_{it}r_t + n_{it}(S_t(\mu_t - r_t) + (1-x_t)D_t) - c_{it} + l_i\widehat{D}_t\Big)\,dt \\ &\quad + \big(1_{\{i=C\}}(x_tD_t - f(x_t, D_t)) + 1_{\{i=M\}}f(x_t, D_t)\big)\,dt + n_{it}S_t\sigma_t dw_t, \end{aligned} \tag{7}$$

where $S_t(\mu_t - r_t) + (1-x_t)D_t$ is the gross dollar excess return on the stock in absolute terms, $(1-x_t)D_t$ is the dividend per share, $l_i\widehat{D}_t$ is shareholder i's labor income, x_tD_t is the diverted output, $f(x_t, D_t)$ is the pecuniary cost of diverting output, and $1_{\{i\}}$ denotes the indicator function.

The minority shareholder maximizes the following objective function over current consumption c and next period's wealth W:

$$\max_{n_t, c_t} V_M(c_t, W_t, W_{t+dt}), \tag{8}$$

where the function $V_M(\cdot)$ is given by equation (6) for $i = M$, subject to self-financing budget constraint (7). The controlling shareholder maximizes his objective function

$$\max_{x_t, n_t, c_t} V_C(c_t, W_t, W_{t+dt}), \tag{9}$$

where the function $V_C(\cdot)$ is as given in (6) for $i = C$, subject to the budget constraint (7), the investor protection constraint $x_t \leq (1-p)n_t$ and the maximum share constraint $n_t \leq 1$. An important feature of our model is that we allow the controlling shareholder to dynamically rebalance the portfolio of assets, consistent with empirical evidence on the dynamics of firm

reduced form of any intertemporal preferences (such as CRRA or Epstein-Zin). In settings with intertemporal preferences, investors' decisions are guided by their total wealth $\widehat{W}_t = W_t + L_t$, where L_t is the present value of future labor income and W_t is financial wealth which can be negative. Nevertheless, the non-negativity of wealth and the reduced role of future labor income in our model are in the spirit of models with intertemporal preferences and non-pledgeability of labor income, which could also arise in our setting due to difficulties of borrowing against stolen output and fully expropriating labor income in the event of investor's default. In these models, the constraint $W_{t+dt} \geq 0$ is imposed to prevent investors with limited liabilities from defaulting on risky positions (e.g., Detemple and Serrat, 2003; Chien and Lustig, 2010; Chabakauri and Han, 2018). Moreover, due to the non-pledgeability of labor income its value is significantly reduced (e.g., Chabakauri and Han, 2018), and the risk-taking decisions are driven by financial W_{t+dt} rather than total wealth $\widehat{W}_{t+dt}$. Another common feature that these models share with ours is the stationarity of equilibrium, which arises for the same reason that wealth W_t is positive at all times.

ownership by controlling shareholders such as managers and board members (e.g., Denis and Sarin, 1999; Holderness, Kroszner, and Sheehan, 1999). Finally, we note that the controlling shareholders in our setting act as price takers and do not manipulate their firm's stock price.[6]

3. Equilibrium with Investor Protection

In this Section, we first solve for investors' optimal strategies in a partial equilibrium setting, in which asset price dynamics are taken as given. Then, by substituting the optimal strategies into the market clearing conditions, we obtain the dynamics of asset prices in equilibrium.

3.1. Shareholders' Optimal Strategies

We now solve for the optimal consumptions and stock holdings of controlling and minority shareholders. We first note that the maximization of objective functions (8) and (9) turns out to be equivalent to separate optimization problems for consumption c_t and stock holding n_t.[7] In particular, investor i's optimal consumption maximizes the following objective function:

$$\max_{c_t} \rho \frac{c_t^{1-\gamma_i} - 1}{1-\gamma_i} - W_{it}^{-\gamma_i} c_t, \tag{10}$$

whereas the optimal stock holding n_{Ct}^* of the controlling shareholder and diverted fraction x_t^* solve a quadratic optimization problem

$$\max_{n_t,\ x_t} J_C(n_t; x_t) = \frac{n_t S_t}{W_{Ct}}\Big((\mu_t - r_t) + (1-x_t)\frac{D_t}{S_t}\Big) + x_t \frac{D_t}{W_{Ct}} - \frac{k\, x_t^2}{2}\frac{D_t}{W_{Ct}} - \frac{\gamma_C}{2}\Big(\frac{n_t S_t}{W_{Ct}}\sigma_t\Big)^2, \tag{11}$$

[6]This is due to the following reasons. First, their trading, consumption and stealing decisions do not affect Sharpe ratios of their own stocks because all firms are small and have identical outputs driven by the same Brownian motion w_t. In such an economy, any deviation of a firm's Sharpe ratio from that of the market portfolio leads to an arbitrage opportunity (e.g., Basak and Croitoru, 2000), which can be easily eliminated by minority shareholders who do not face any trading frictions. Second, we posit that the minority shareholders cannot observe the stock demands of controlling shareholders and believe that firms are identical and the controlling shareholders do not manipulate prices. Consequently, they are indifferent to which stocks to hold. If they observe that the stock price of a particular firm k deviates from the prices of other firms they assume that too many minority shareholders bought or sold stocks of firm k and trade in the direction of eliminating the mispricing. As a result, the controlling shareholders do not attempt to alter stock prices knowing that the mispricing will be eliminated, which confirms the beliefs of the minority shareholders that all firms are identical and the controlling shareholders do not manipulate prices.

[7]To demonstrate this, we rewrite the second term in equation (6) for investor preferences as $\mathbb{E}_t[u_i(W_{i\,t+dt})] = u_i(W_{it}) + \mathbb{E}_t[du_i(W_{it})]$, apply Itô's Lemma to $u_i(W_{it})$, where $u_i(\cdot)$ is given by (5), and then, after some algebra, we find that the optimal consumption and the number of shares solve two separate optimization problems.

subject to constraints $x_t \leq (1-p)n_t$ and $n_t \leq 1$, and the optimal stock holding n^*_{Mt} of the minority shareholder solves the optimization problem

$$\max_{n_t} J_M(n_t) = \frac{n_t S_t}{W_{Mt}}\Big((\mu_t - r_t) + (1-x_t)\frac{D_t}{S_t}\Big) - \frac{\gamma_M}{2}\Big(\frac{n_t S_t}{W_{Mt}}\sigma_t\Big)^2. \tag{12}$$

Solving for shareholders' optimal consumptions, we find that consumptions c^*_i are given by:

$$c^*_{it} = \rho^{1/\gamma_i} W_{it}. \tag{13}$$

The shareholders' consumption-wealth ratios are constant, similar to frictionless models with logarithmic preferences.

Solving the portfolio choice problem of the controlling shareholder is complicated by the presence of constraints on stock holding n and diverted fraction of output x, which renders the value function (11) non-concave function of stock holding n. We proceed in two steps. We first maximize the objective function $J_C(n;x)$ in equation (11) with respect to x, taking the stock holding n as given, and find the optimal fraction of diverted output $x^*(n)$. Then, we substitute the fraction $x^*(n)$ back into equation (11) and find the optimal stock holding n^*_C by maximizing the objective function $J_C(n;x^*(n))$. Proposition 1 below summarizes our results in partial equilibrium.

Proposition 1 (Partial equilibrium). *The fraction of diverted output x^*_t and optimal stock holdings n^*_{it} are given by:*

$$x^*(n_t) = \min\Big((1-n_t)/k; (1-p)n_t\Big); \tag{14}$$

$$n^*_{Ct} = \begin{cases} n^*_{Ct,1} = \dfrac{\mu_t - r_t + (2-p)\frac{D_t}{S_t}}{\gamma_C \sigma_t^2 \frac{S_t}{W_{Ct}} + \Big(2(1-p) + (1-p)^2 k\Big)\frac{D_t}{S_t}}, & \text{if } J^*_C = J_C(n^*_{Ct,1}; x^*_t) \text{ (region 1)}, \\ n^*_{Ct,2} = \dfrac{\mu_t - r_t + \frac{k-1}{k}\frac{D_t}{S_t}}{\gamma_C \sigma_t^2 \frac{S_t}{W_{Ct}} - \frac{1}{k}\frac{D_t}{S_t}}, & \text{if } J^*_C = J_C(n^*_{Ct,2}; x^*_t) \text{ (region 2)}, \\ n^*_{Ct,3} = \dfrac{1}{1+(1-p)k}, & \text{if } J^*_C = J_C(n^*_{Ct,3}; x^*_t) \text{ (region 3)}, \\ n^*_{Ct,4} = 1, & \text{if } J^*_C = J_C(n^*_{Ct,4}; x^*_t) \text{ (region 4)}; \end{cases} \tag{15}$$

$$n^*_{Mt} = \frac{\mu_t - r_t + (1-x_t)\frac{D_t}{S_t}}{\gamma_M \sigma_t^2 \frac{S_t}{W_{Mt}}}, \tag{16}$$

where $J^*_C = \max\Big(J_C(n^*_{Ct,1}; x^*_t), J_C(n^*_{Ct,2}; x^*_t), J_C(n^*_{Ct,3}; x^*_t), J_C(n^*_{Ct,4}; x^*_t)\Big)$ *and* $J_C(n;x)$ *is as in* (11). *Moreover, the shareholders' optimal consumptions are given by Equation* (13).

The optimal fraction of diverted output $x^*(n_t)$ is a hump-shaped function of the number of shares n_t, and is depicted on Figure 1. On one hand, the endogenous accumulation of control power by the controlling shareholder relaxes the investor protection constraint and allows him to divert more as his stock holdings increase. On the other hand, marginal benefit of stealing decreases due to larger cash flow rights. Initially, the equilibrium level of stealing increases as the controlling shareholders' stock holdings increase. However, after the kink in Figure 1 where the stock holding is sufficiently large, the amount of diverted output decreases because the payout through dividends becomes a larger share of the total output, and hence the incentive to divert output is weaker given the natural inefficiencies of stealing.

The hump-shaped relation between the diverted output and the number of shares is a notable feature of our model. Previous literature focuses only on cases where the fraction of diverted output is exogenous or is decreasing due to larger cash flow rights as in our region 2 (Shleifer and Wolfenzon, 2002; Albuquerque and Wang, 2008). Region 1 where the diverted output increases with the stock holding n is in line with the evidence that firms without controlling shareholders have higher valuations than firms with controlling shareholders (Laeven and Levine, 2008). Therefore, when a firm transitions from being widely held (when $n \approx 0$) to being controlled by a large shareholder its value would go down. On the other hand, Claessens, Djankov, Fan and Lang (2002) show that the firm value increases with increases in the stock holding of the controlling shareholder, consistent with a decreasing function $x^*(n)$. However, we note that the latter work uses a cutoff level of $n = 10\%$ to identify a firm as being effectively controlled by a shareholder, and hence truncates the region where $x^*(n)$ might be an increasing function of the stock holding n.

The controlling shareholder's optimal stock holding n^*_{Ct} in Proposition 1 captures the tradeoff between the benefits and costs of diverting the output. The expression for stock holding n^*_{Ct} in Equation (15) differs across four regions in the space of the state variables. Region 1 is such that the fraction of the diverted output is given by $x^*(n^*_{Ct}) = (1-p)n^*_{Ct}$, that is, the investor chooses to divert the maximum possible fraction of output. In this region, laws and regulations that protect minority shareholder rights is the binding constraint on stealing. This is because marginal benefit from stealing is high with low ownership rights, the constraint on stealing is tight because controlling shareholder's power is low and the pecuniary cost of stealing is low at low levels of stealing.

Region 2 is such that $x^*(n^*_{Ct}) = (1 - n^*_{Ct})/k$, that is, the cost of diverting output kicks in. In this region, the controlling shareholder has higher power over the firm which makes the constraint imposed by investor protection relatively relaxed. On the other hand, high stake in the firm reduces incentive to expropriate. Consequently, in this region, the cost

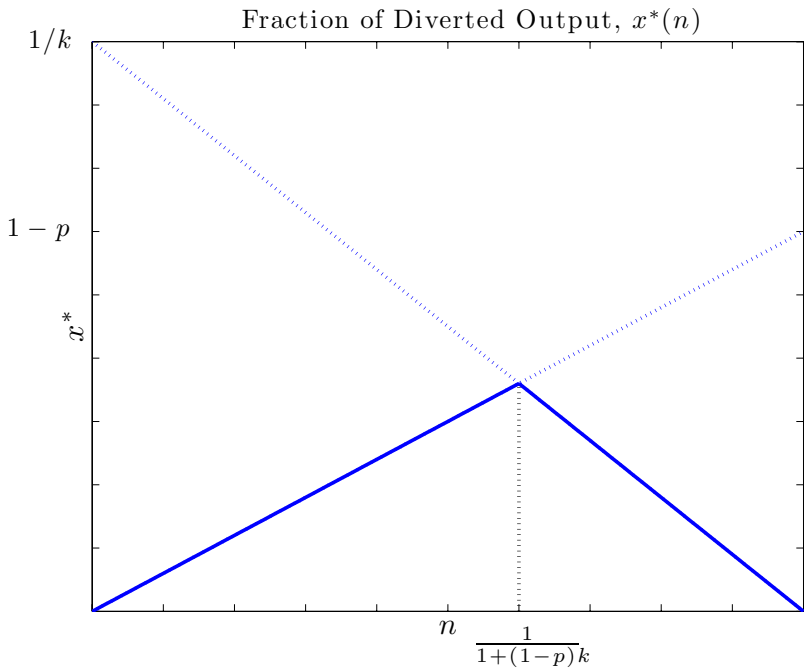

Figure 1
Optimal Fraction of Diverted Output $x^*(n)$
This Figure shows the tent-shaped optimal fraction of diverted output x^* as a function of the controlling shareholder's stake n in the firm.

of stealing rather than investor protection determines the optimal amount of stealing. We observe that, after simple algebra, the stock holding n_C^* in region 2 can be rewritten in the following equivalent way:

$$n_{Ct}^* = \frac{\mu_t - r_t + (1 - x_t^*)\frac{D_t}{S_t}}{\gamma_C \sigma_t^2 \frac{S_t}{W_{Ct}}}, \tag{17}$$

where $x^*(n_{Ct}^*) = (1 - n_{Ct}^*)/k$. Therefore, both types of shareholders hold a simple mean-variance portfolio when the economy is in region 2, where the investor protection constraint is not binding.[8]

Finally, we discuss regions 3 and 4. Region 3 is a point where $(1-p)n_{Ct}^* = (1 - n_{Ct}^*)/k$, and hence the two forces of diverting the maximum and the cost of stealing equate. Region 4 is where $n_{Ct}^* = 1$, and hence the controlling shareholder has full cash flow rights and no incentives to steal.

[8]This result follows from the envelope condition and is due to the fact that the derivative of objective function $J_C(n; x_t)$ in (11) with respect to x_t is zero in region 2 because fraction of diverted output x_t is chosen precisely to satisfy the first order condition with respect to x_t, that is, $\partial J_C(n; x_t)/\partial x_t = 0$. Therefore, the first order condition with respect to stock holding n_C is given by $\partial J_C(n; x_t)/\partial n_t = 0$, and hence, the optimal stock holdings of controlling and minority shareholders are similar.

3.2. Asset Price Dynamics in Equilibrium

In this subsection, we derive the equilibrium mean-return μ_t, volatility σ_t, risk-free rate r_t and shareholder stock holdings n^*_{it}. The definition of equilibrium in our pure-exchange economy is standard: the *equilibrium* is a set of processes r_t, μ_t and σ_t, optimal stock and bond holdings, n^*_{it} and b^*_{it}, and consumptions c^*_{it} that satisfy the market clearing conditions

$$n^*_{Ct} + n^*_{Mt} = 1, \tag{18}$$

$$b^*_{Ct} + b^*_{Mt} = 0, \tag{19}$$

$$c^*_{Ct} + c^*_{Mt} = \widehat{D}_t. \tag{20}$$

All equilibrium processes are derived as functions of minority shareholder's *share in the aggregate consumption*, defined as $y_t = c^*_{Mt}/\widehat{D}_t$. Similarly to the literature on equilibrium with heterogeneous investors (e.g., Chabakauri, 2013) the consumption share y_t of one of the investors emerges as a crucial state variable that determines the dynamics of asset prices in the economy. Following the literature, we conjecture and then verify that the consumption share follows a Markovian process

$$dy_t = \mu_{yt}dt + \sigma_{yt}dw_t, \tag{21}$$

where drift μ_{yt} and volatility σ_{yt} are determined in equilibrium as functions of y_t.

To facilitate the intuition, we provide first the equilibrium in the benchmark economy with full investment protection $p = 1$, and then compare it with the equilibrium with imperfect protection. In the benchmark economy, the difference in risk aversions γ_C and γ_M is the only source of heterogeneity between investors C and M.

Proposition 2 (Benchmark equilibrium with full protection $p = 1$). *In the economy with full investor protection $p = 1$ the shareholders' optimal consumptions are given by* (13). *The stock mean-return, Sharpe ratio, volatility, and the risk-free interest rate are given by:*

$$\mu^{bmk}_t = r^{bmk}_t + \kappa^{bmk}_t \sigma^{bmk}_t - \frac{1 - l_M - l_C}{y_t\rho^{-1/\gamma_M} + (1 - y_t)\rho^{-1/\gamma_C}}, \tag{22}$$

$$\kappa^{bmk}_t = \Gamma_t \sigma_D, \tag{23}$$

$$\sigma^{bmk}_t = \sigma_D + \frac{y_t(1 - y_t)\left(\rho^{-1/\gamma_M} - \rho^{-1/\gamma_C}\right)\left(\frac{1}{\gamma_M} - \frac{1}{\gamma_C}\right)\Gamma_t\sigma_D}{y_t\rho^{-1/\gamma_M} + (1 - y_t)\rho^{-1/\gamma_C}}, \tag{24}$$

$$r^{bmk}_t = y_t\rho^{1/\gamma_M} + (1 - y_t)\rho^{1/\gamma_C} + \mu_D - \Gamma_t\sigma^2_D - l_M\rho^{1/\gamma_M} - l_C\rho^{1/\gamma_C}, \tag{25}$$

respectively, where Γ_t is the risk aversion of the representative investor, given by:

$$\Gamma_t = \frac{1}{y_t/\gamma_M + (1-y_t)/\gamma_C}. \tag{26}$$

The minority shareholder's consumption share volatility and drift are

$$\sigma_{yt}^{bmk} = y_t(1-y_t)\Gamma_t\sigma_D\Big(\frac{1}{\gamma_M} - \frac{1}{\gamma_C}\Big), \tag{27}$$

$$\begin{aligned}\mu_{yt}^{bmk} &= y_t(1-y_t)\Big(\rho^{1/\gamma_C} - \rho^{1/\gamma_M} + \Big(\frac{1}{\gamma_M} - \frac{1}{\gamma_C}\Big)\Gamma_t(\Gamma_t - 1)\sigma_D^2\Big) \\ &\quad +(1-y_t)l_M\rho^{1/\gamma_M} - y_t l_C\rho^{1/\gamma_C}.\end{aligned} \tag{28}$$

The shareholders' optimal stock holdings are given by:

$$n_{Ct}^{bmk} = \frac{\frac{1-y_t}{\gamma_C\rho^{1/\gamma_C}}}{\frac{y_t}{\gamma_M\rho^{1/\gamma_M}} + \frac{1-y_t}{\gamma_C\rho^{1/\gamma_C}}}, \quad n_{Mt}^{bmk} = \frac{\frac{y_t}{\gamma_M\rho^{1/\gamma_M}}}{\frac{y_t}{\gamma_M\rho^{1/\gamma_M}} + \frac{1-y_t}{\gamma_C\rho^{1/\gamma_C}}}. \tag{29}$$

The distribution of consumption share y is stationary and its probability density function (pdf) is given in closed form by:

$$\psi(y) = \exp\Big\{a_0 + a_1 y - \frac{a_C}{y} - \frac{a_M}{1-y}\Big\}\frac{y^{b_C}(1-y)^{-b_M}}{\Gamma^2}, \tag{30}$$

where the constants a_0, a_1, $a_i > 0$, and b_i, $i = M, C$, are provided in the Appendix.

Proposition 2 provides in closed-form all the relevant equilibrium processes in the benchmark economy with full investor protection. We note several similarities of these processes with those in comparable models with non-myopic investors. In particular, the equilibrium processes depend on the risk aversion Γ of the representative investor, with the Sharpe ratio and the consumption share volatility being given by the same expressions as in the comparable economy with non-myopic investors. Moreover, the stock return volatility exceeds the output volatility, $\sigma_t^{bmk} > \sigma_D$. We also note that the gross stock-mean return is given by $r_t^{bmk} + \kappa_t^{bmk}\sigma_t^{bmk}$, as in the economy with non-myopic investors. Equation (22) reports this gross mean-return net of the dividend yield D/S, given by the last term (22), to highlight the effect of stealing entering through the dividend yield in the ensuing analysis. The risk-free interest rate r_t in (25) is, however, different from that in models with non-myopic investors. Specifically, it does not have a prudence parameter of the representative investor, and features additional terms that are proportional to shareholders' income shares.

The equilibrium in our benchmark economy is stationary, in contrast to models with non-myopic investors. The stationarity follows from the structure of the volatility σ_y and

drift μ_y of the state variable process in equations (27) and (28). From these equations, we observe that the volatility σ_y is zero at the boundaries $y = 0$ and $y = 1$, while the drift is $\mu_y = l_M \rho^{1/\gamma_M} > 0$ at $y = 0$ and $\mu_y = -l_C \rho^{1/\gamma_C} < 0$ at $y = 1$. Therefore, the boundaries $y = 0$ and $y = 1$ are repulsive. When the consumption share y approaches the boundary, its volatility decreases and the drift pushes it back into the internal region. This behavior at the boundaries indicates that none of the investors disappear in the long run. The probability density function (pdf) of the state variable y is obtained in closed form given by (30). As discussed in Section 2, the stationarity arises because the preferences require the shareholders to maintain positive financial wealth and non-financial income allows them to accumulate wealth over time.

Proposition 3 (Equilibrium with imperfect protection). *In the equilibrium with imperfect protection $p < 1$ the shareholders' optimal consumptions are given by* (13), *and the stock mean-return, Sharpe ratio, volatility, and the risk-free interest rate are given by:*

$$\mu_t = r_t + \sigma_t \kappa_t - \Big(1 - x_t^*\Big) \frac{1 - l_M - l_C}{y_t \rho^{-1/\gamma_M} + (1 - y_t)\rho^{-1/\gamma_C}}, \tag{31}$$

$$\kappa_t = \kappa_t^{bmk} - \frac{(n_{Ct}^* - n_{Ct}^{bmk})\rho^{1/\gamma_C}\Gamma_t \sigma_D}{(\rho^{1/\gamma_M}(1 - n_{Ct}^*) + \rho^{1/\gamma_C} n_{Ct}^*) n_{Mt}^{bmk}}, \tag{32}$$

$$\sigma_t = \sigma_t^{bmk} + \sigma_t^{bmk} \frac{(n_{Ct}^* - n_{Ct}^{bmk})(\rho^{1/\gamma_M} - \rho^{1/\gamma_C})}{\rho^{1/\gamma_M}(1 - n_{Ct}^*) + \rho^{1/\gamma_C} n_{Ct}^*}, \tag{33}$$

$$\begin{aligned} r_t = {} & r_t^{bmk} - (1 - l_M - l_C) x_t^* \rho^{1/\gamma_C} - (\rho^{1/\gamma_M} - \rho^{1/\gamma_C})(1 - l_M - l_C) \frac{k (x_t^*)^2}{2} \\ & + \frac{(n_{Ct}^* - n_{Ct}^{bmk})\Gamma_t \sigma_D}{(\rho^{1/\gamma_M}(1 - n_{Ct}^*) + \rho^{1/\gamma_C} n_{Ct}^*) n_{Mt}^{bmk}}, \end{aligned} \tag{34}$$

respectively, and the minority shareholder's consumption share y volatility and drift are

$$\sigma_{yt} = \sigma_{yt}^{bmk} - \frac{(n_{Ct}^* - n_{Ct}^{bmk}) y_t \Gamma_t \sigma_D}{(\rho^{1/\gamma_M}(1 - n_{Ct}^*) + \rho^{1/\gamma_C} n_{Ct}^*)\gamma_M n_{Mt}^{bmk}}, \tag{35}$$

$$\mu_{yt} = y_t \Big(r_t + \frac{\kappa_t^2}{\gamma_M} - \rho^{1/\gamma_M} - \mu_D \Big) - \sigma_{yt}\sigma_D + l_M \rho^{1/\gamma_M} + (1 - l_M - l_C) \frac{k (x_t^*)^2}{2} \rho^{1/\gamma_M}, \tag{36}$$

where κ^{bmk}, σ^{bmk}, r^{bmk}, σ_y^{bmk} and n_C^{bmk} are the corresponding equilibrium processes in the full protection economy given by (23)–(27), *and* (29), *respectively. The controlling shareholder's optimal stock holding n_{Ct}^* solves the fixed-point equation*

$$\begin{aligned} n_{Ct}^* = {} & \underset{n_t,\, n_t \le 1}{\operatorname{argmax}} \Big\{ \frac{n_t S_t}{W_{Ct}} \Big(\mu_t - r_t + (1 - x^*(n_t)) \frac{D_t}{S_t} \Big) \\ & + x^*(n_t) \frac{D_t}{W_{Ct}} - \frac{k\, x^*(n_t)^2}{2} \frac{D_t}{W_{Ct}} - \frac{\gamma_C}{2} \Big(\frac{n_t S_t}{W_{Ct}} \sigma_t \Big)^2 \Big\}, \end{aligned} \tag{37}$$

where $x^*(n)$ is given by equation (14), *μ_t, r_t and σ_t are given by equations* (31)–(33) *and themselves depend on n^*_{Ct} in equilibrium, and ratios D_t/W_{Ct} and S_t/W_{Ct} are given by:*

$$\frac{D_t}{S_t} = \frac{1 - l_M - l_C}{y_t \rho^{-1/\gamma_M} + (1 - y_t)\rho^{-1/\gamma_C}}, \quad \frac{D_t}{W_{Ct}} = \frac{\rho^{1/\gamma_C}(1 - l_M - l_C)}{1 - y_t}, \quad \frac{S_t}{W_{Ct}} = \frac{y_t \rho^{-1/\gamma_M} + (1 - y_t)\rho^{-1/\gamma_C}}{(1 - y_t)\rho^{-1/\gamma_C}}. \tag{38}$$

Proposition 2 derives the equilibrium processes (31)–(34) as functions of minority shareholder's consumption share y_t and demonstrates that these processes are given by their counterparts (22)–(25) in the benchmark economy, plus additional adjustment terms arising due to imperfect protection. These adjustment terms depend on the fraction of diverted output x_t and the *excess ownership concentration* relative to the full protection benchmark, given by $n^*_{Ct} - n^{bmk}_{Ct}$. In contrast to the full protection benchmark economy, simple closed-form expressions for stock holdings n^*_{it} are no longer available. Imperfect investor protection gives rise to complex dynamics of equilibrium processes via the effects of protection on the stock holding of the controlling shareholder n^*_{Ct}. In particular, stock holding n^*_{Ct} now solves a fixed-point problem in equation (37) in which the equilibrium processes on the right-hand side of this equation are functions of n^*_{Ct} itself.

The structure of the equilibrium processes in Proposition 3 is sufficiently tractable admitting several comparative statics. In particular, from equations (32) and (33) we observe that the effects of investor protection on the stock Sharpe ratio κ and volatility σ depend on the excess ownership concentration $n^*_{Ct} - n^{bmk}_{Ct}$. Intuitively, one would expect that controlling shareholders hold more shares under weak investor protection, and hence, $n^*_{Ct} - n^{bmk}_{Ct} \geq 0$, as we confirm in Section 4 after computing the equilibrium. Therefore, the expressions for the volatility (33) and the Sharpe ratio (32) suggest that weaker investor protection increases volatility and decreases the Sharpe ratio relative to the full protection benchmark.

From equation (35) we also observe that the state variable volatility σ_y is negative because σ^{bmk}_y in (27) is negative and $n^*_{Ct} - n^{bmk}_{Ct} \geq 0$. Hence, the state variable y is negatively correlated with shocks to output dw. As a result, this variable is countercyclical in the sense that it tends to be higher (lower) during periods of negative (positive) output shocks dw which we label as bad (good) times. The countercyclicality of y arises because the controlling shareholder is under-diversified and less risk averse than the minority shareholder. Therefore, negative output shocks hurt the controlling shareholder and benefit the minority shareholder so that the minority shareholder's consumption share y increases in bad times.

The economy with weak investor protection remains stationary because at the boundaries $y = 0$ and $y = 1$ the equilibrium processes coincide with those in the full protection

benchmark, and hence the boundaries are repulsive as in the benchmark. This is due to the fact that at both boundaries there is no stealing in equilibrium because at $y = 0$ the economy is dominated by minority shareholders and at $y = 1$ it is dominated by controlling shareholders who do not steal from themselves when they control the entire economy, as discussed in Section 2. However, the pdf of the consumption share y is no longer possible in closed form, and we compute it by Monte-Carlo simulation in Section 4. We also observe that the consumption share y_t is not a martingale because its drift μ_y is non-zero and is reverted back into the interior of $(0, 1)$ interval when y_t is close to the boundary. Consequently, the consumption share y_t and all equilibrium processes which are functions of y_t are predictable over short time intervals.

We note that the endogenous accumulation of control and the participation of the controlling shareholder in asset markets are key ingredients for explaining certain empirical regularities. In particular, endogenizing the stock holdings allows us to explain the higher ownership concentration in countries with low protection and shed light on the portfolio choice between the risky asset, which provides higher control rights, and the riskless asset. Furthermore, dynamic asset holdings generate endogenous wealth transfers between controlling and minority shareholders which give rise to the stochastic time-variation in asset returns and excess volatility. Finally, these asset holdings also give rise to the controlling shareholder's leverage and help us shed light on the role of leverage in the accumulation of control and its effect on the stock return volatility.

4. Economic Implications of Investor Protection

In this Section, we present our results with plots as depicted in Figures 2-6 for a plausible set of baseline parameters as functions of the consumption share y of the minority shareholders in the economy.[9] The state variable y is countercyclical, as discussed in Section 3.2, and hence the dependence on this variable provides information on how the equilibrium processes change over good and bad times in the economy.

Figure 2 presents the controlling shareholder's equilibrium stock holding n_C^*, the fraction

[9]We set $\mu_D = 1.5\%$ and $\sigma_D = 13\%$, matching the mean-growth rate and volatility of dividends (e.g., Brennan and Xia, 2001; Dumas, Kurshev, and Uppal, 2009), and set $\gamma_C = 3$, $\gamma_M = 3.5$, $\rho = 0.01$, and $k = 3$. We set the labor income shares to $l_C = 0.1$ and $l_M = 0.5$. The labor income shares capture the fact that although individual controlling shareholders may have large incomes they are fewer than minority shareholders in reality. Hence, their combined income is still lower than the income of minority shareholders. We recall that the shareholders in our model are representative, that is, each shareholder stands for a group of identical shareholders.

of diverted output x^*, and the controlling shareholder's leverage-stock price ratio for different values of investor protection p in the economy, holding the stealing cost parameter k fixed. Then, we use the results in Figure 2 for the analysis of equilibrium expected gross returns, interest rates, and volatilities as depicted in Figure 3 for the same protection p and stealing cost k parameters. Figure 4 presents the stationary distributions of the consumption share y for the same sets of parameter values as in previous figures. Figure 5 explores the effects of the stealing cost by varying parameter k and holding investor protection p fixed. Finally, Figure 6 shows the expected stock gross returns and interest rates in an economy without the investor protection constraint for a range of cost parameters k. The numerical approach for deriving the equilibrium processes is explained in the Appendix.

4.1. Stock Holdings and Diverted Output

We start our analysis with Figure 2 presenting the controlling shareholder's stock holding, fraction of diverted output and leverage. Panel (a) of Figure 2 demonstrates that lower protection tends to increase the controlling shareholder's stock holding n_C^* relative to the full protection benchmark, i.e., $n_{Ct}^* \geq n_{Ct}^{bmk}$, in the region where the investor protection constraint binds. This is because when investor protection is imperfect the controlling shareholder can divert a larger fraction of output when he owns more shares. This gives the controlling shareholder an incentive to acquire more shares in equilibrium when his consumption share is low (i.e., y is high). However, when the controlling shareholder's consumption share is high (i.e., y is low), the stock holding is the same as in the benchmark economy, i.e., $n_{Ct}^* = n_{Ct}^{bmk}$. In this case, the controlling shareholder holds a large number of shares, which reduces the benefits of diverting the output. Therefore, the economy is in region 2 (equation (15) of Proposition 1), where the investor protection constraint does not bind and the diversion of output is tempered only by the cost of stealing. The cost parameter k, however, does not depend on the stock holding. Consequently, buying shares beyond the benchmark level n_{Ct}^{bmk} does not help divert more output and harms the controlling shareholder by reducing portfolio diversification benefits. Hence, $n_{Ct}^* = n_{Ct}^{bmk}$ in region 2.

We note that the controlling shareholder's stock holding n_{Ct}^* is non-monotone in protection p. In particular, in panel (a) we observe that whether stock holding n_{Ct}^* is higher in the economy with $p = 0.9$ or $p = 0.6$ critically depends not only on the protection p but also on the consumption share y. This is because an increase in p has two opposing effects on stock holding n_C^*. On one hand, stock holding n_C^* decreases because the benefit of control is lower when protection is high. On the other hand, the investor protection constraint becomes

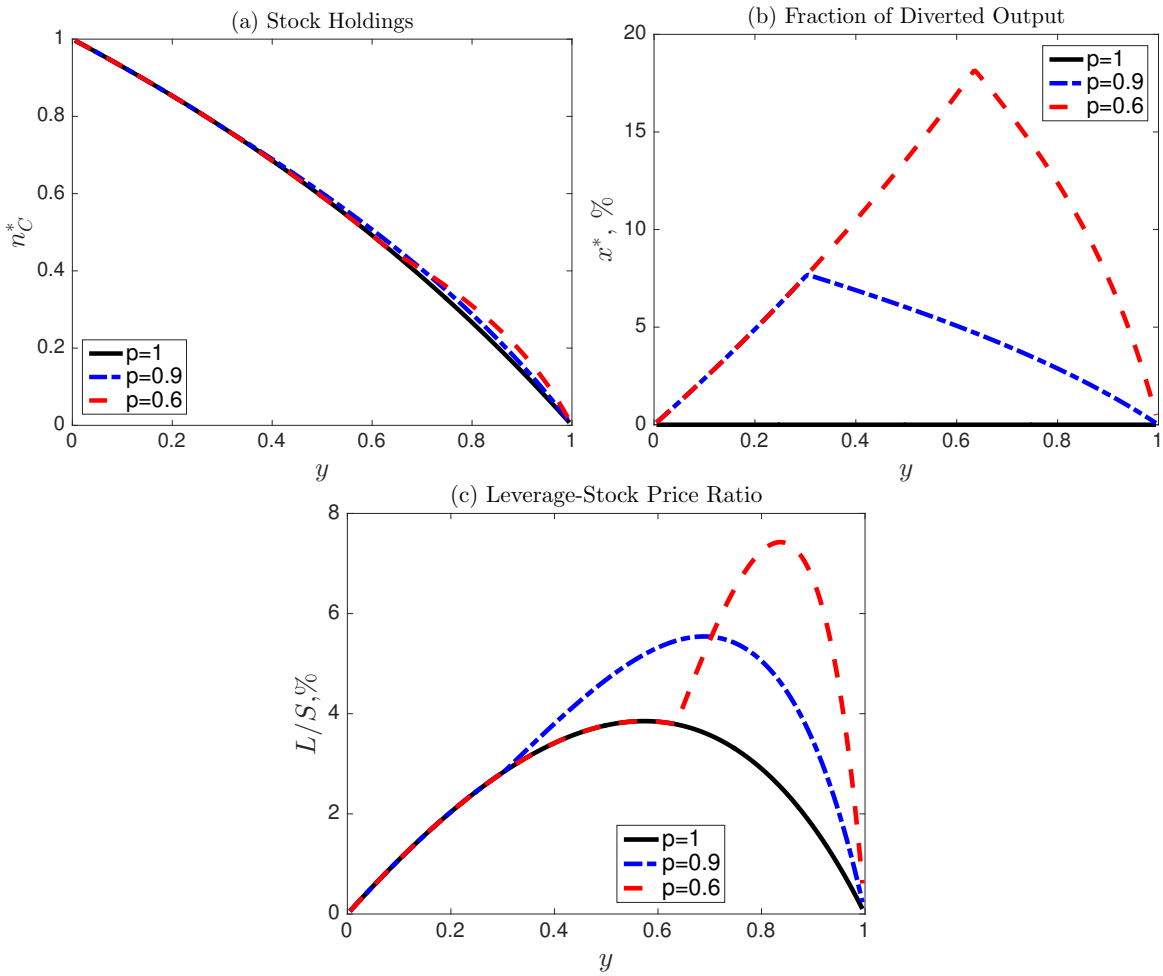

Figure 2
The effect of investor protection on controlling shareholder's stock holding, fraction of diverted output and leverage
This Figure shows the controlling shareholder's equilibrium stock holdings n_C^*, fraction of diverted output x^* and leverage-stock price ratio L/S as functions of consumption share y for different levels of investor protection p and the baseline parameter values.

tighter thereby providing incentives to acquire more shares to relax it. Moreover, because of a tighter investor protection constraint, region 1 in equation (15) for stock holding n_C^* where this constraint binds becomes larger.

Panel (b) of Figure 2 shows the fraction of diverted output x^* and how it is affected by investor protection. As would be expected, the fraction of diverted output is considerably reduced in economies with better protection. The diverted output $x^*(n_{Ct}^*)$ has a kink at the separation point of regions 1 and 2 in equation (15) for the stock holding n_{Ct}^*, which correspond to situations when the investor protection constraint $x_t \leq (1-p)n_t$ is binding

or not, respectively. The latter constraint is loose when n is sufficiently large, so that the controlling shareholder does not want to steal from himself. Therefore, because the stock holding n^*_{Ct} is a decreasing function of consumption share y (panel (c) of Figure 2), the economy is in region 1 (region 2) when y lies to the right-hand (left-hand) side of the kink. We remark that all the equilibrium processes have kinks, which arise via the dependence of the latter processes on the fraction of diverted output $x^*(n^*_{Ct})$.

The economy transits from region 1 to region 2 following a sequence of positive shocks $dw > 0$ because the controlling shareholder accumulates more wealth and stock. The economy moves from region 2 to region 1 if it is hit by bad shocks $dw < 0$ in which case the controlling shareholder decreases his stock holding to finance consumption in bad times. These transitions between the regions and the time-variation of the state variable y make the diverted output $x^*(n^*_{Ct})$ and all other equilibrium processes time-varying in our model.

Panel (c) of Figure 2 shows the controlling shareholder's leverage-stock price ratio. The leverage is given by the wealth invested in stocks in excess of total wealth. The personal leverage in the full protection and imperfect protection economies is given by $L^{bmk}_t = n^{bmk}_{Ct} S^{bmk}_t - W^{bmk}_{Ct}$ and $L_t = n^*_{Ct} S_t - W_{Ct}$, respectively. Then, taking into account that the ratios S^{bmk}_t / W^{bmk}_{Ct} and S_t / W_{Ct} are the same, conditional on the consumption share y, and given in equation (38), we obtain the personal leverage as:

$$\frac{L_t}{S_t} = \frac{L^{bmk}_t}{S^{bmk}_t} + n^*_{Ct} - n^{bmk}_{Ct}. \tag{39}$$

Equation (39) reveals that the excess ownership concentration $n^*_{Ct} - n^{bmk}_{Ct}$ is exactly equal to the change in leverage $L_t/S_t - L^{bmk}_t/S^{bmk}_t$. Therefore, the acquisition of additional shares in economies with poor protection is financed by personal leverage because, as shown below, borrowing is cheap in such economies. Hence, the leverage increases, as shown on panel (c) of Figure 2. We emphasize that the leverage here represents the personal leverage of the controlling shareholder while the firm is unlevered.

4.2. Stock Return, Volatility, and Interest Rate

Panel (a) of Figure 3 shows that a higher investor protection p leads to a higher gross stock return $\mu + (1 - x^*)D/S$, where μ is the mean capital gain and $(1 - x^*)D/S$ is dividend yield. This finding is consistent with the empirical literature documenting a positive relationship between corporate governance and realized returns. Future returns are positively correlated with a governance index of shareholder rights (Gompers, Ishii and Metrick, 2003), lower managerial perks (Yermack, 2006), a lower entrenchment index (Bebchuk, Cohen and Ferrell,

2009), and a governance index (AGR) from Audit Integrity (Daines, Gow and Larcker, 2010). We note that these empirical findings are for cross-sections of firms with different investor protections whereas our main analysis features a representative firm. However, our main insights and intuition remain valid for a cross-section of firms, as we demonstrate in Section 5.3 below, albeit in a simple one-period setting.

Similar relations are documented in other countries and cross-country studies. Firms with higher governance scores in Germany (Drobetz, Schillhofer, and Zimmermann, 2004), firms that do not engage in tunneling using inter-corporate loans in China (Jiang, Lee and Yue, 2010) and countries with better legal institutions (Lombardo and Pagano, 2006) have higher returns. Despite the supporting empirical evidence, it is not immediately clear why such a relationship between expected returns and investor protection exists in equilibrium. For example, taking away a constant fraction of dividends reduces the value of the firm but does not affect the expected return in equilibrium. There is also an ongoing discussion about whether the empirical relationship is robust (Core, Guay and Rusticus, 2006; Giroud and Mueller, 2011; Bebchuk, Cohen and Wang, 2013). Therefore, further guidance from theory, as a contribution to this debate, would be helpful.

To understand the intuition, we first consider the benchmark economy with full protection $p = 1$. In this economy, gross stock returns are determined by investors' risk aversions and are sufficiently high to compensate investors for risk taking. Lower investor protection $p < 1$ opens up an opportunity to divert firm cash flows to benefit the controlling shareholders. Therefore, the controlling shareholder is compensated for excessive risk taking not only via risk premia but also via stealing. Consequently, the compensation for risk of the controlling shareholder is determined by a new previously unexplored quantity which we refer to as the *effective* risk premium, and which appears to be higher than the risk premium implied by the stock price dynamics. More formally, from the budget constraint (7) of the controlling shareholder C it is immediate to observe that his effective risk premium for holding stocks is given by $\mu - r + (1-x^*)D/S + x^* D/(S\, n_C^*)$, and hence, is higher than the risk premium $\mu - r + (1-x^*)D/S$ for the minority shareholder by the *diverted output yield per share* $x^* D/(S\, n_C^*)$. Therefore, a low risk premium $\mu - r + (1-x^*)D/S$ implied by asset prices is indeed consistent with our equilibrium because the stock market clears due to high demand for stocks by the controlling shareholders. Consistent with our intuition, it has been documented that the demand by controlling shareholders for voting shares increases with poor investor protection, which then affects prices.[10]

[10]This is consistent with the evidence that the value of control is negatively correlated with variation in investor protection across countries (Nenova, 2003; Dyck and Zingales, 2004) and over time (Albuquerque

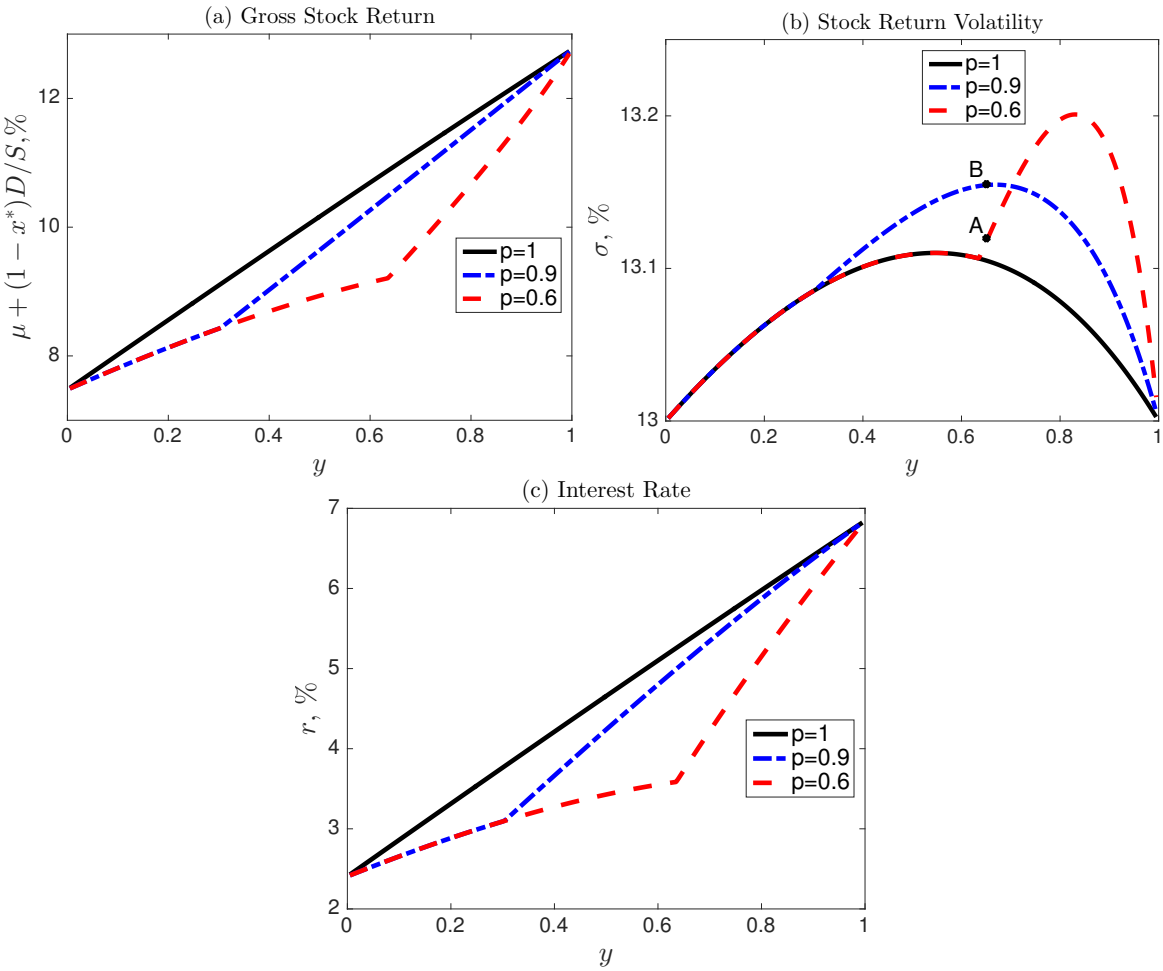

Figure 3
The effect of investor protection on stock gross returns, volatilities and interest rates
This Figure shows the equilibrium gross stock returns $\mu + (1 - x^*)D/S$, stock return volatilities σ and risk-free interest rates r as functions of consumption share y for different levels of investor protection p and the baseline parameter values.

Panel (b) depicts the stock return volatility σ and demonstrates that in our calibration of the model it is higher than in the benchmark economy with full protection, i.e., $\sigma \geq \sigma^{bmk}$, and hence, the stock price is more volatile than output. This result is consistent with the comparative statics in Section 3.2. The volatility increases because with imperfect protection $p < 1$ the controlling shareholder holds more shares than in the full protection benchmark, and hence is under-diversified. Therefore, his wealth and consumption are more volatile, which then translates into the stock market volatility via the market clearing conditions.

and Schroth, 2010).

We note an important link between the volatility and personal leverage. In particular, combining equations (33) and (39) for the volatility and leverage, after simple algebra, we obtain the following expression for the percentage change in volatility in terms of change in leverage:

$$\frac{\sigma_t - \sigma_t^{bmk}}{\sigma_t^{bmk}} = \left(\frac{L_t}{S_t} - \frac{L_t^{bmk}}{S_t^{bmk}} \right) \frac{\rho^{1/\gamma_M} - \rho^{1/\gamma_C}}{\rho^{1/\gamma_M}(1 - n_{Ct}^*) + \rho^{1/\gamma_C} n_{Ct}^*}. \tag{40}$$

Equation (40) explains the close resemblance of volatility σ and personal leverage-stock price ratio L/S. Moreover, in our calibration, the latter equation implies a positive relationship between the change in volatility and the change in personal leverage. When the investor protection constraint does not bind, that is, the economy is in region 2, the equilibrium stock holding n_{Ct}^* is the same as in the benchmark economy, as elaborated in Section 4.1. Consequently, equation (39) implies that there is no excess leverage in the economy in region 2, and hence equation (40) implies that the volatility is the same as in the full protection benchmark.

Our analysis further uncovers previously unknown effects of investor protection on stock return volatilities. First, in the region where the investor protection constraint is binding, stock return volatility σ is a concave function of minority shareholder's consumption share y, consistent with the empirical evidence provided by Gul, Kim and Qui (2010). Second, similar to the controlling shareholder's stock holding n_C^*, we observe that volatility σ is non-monotone in protection p. This is because the interaction between volatility σ and protection p depends on the minority shareholder's consumption share y. Clearly, the income inequality or distribution of wealth could have a direct effect on portfolio holdings of controlling versus minority shareholders and our model implies that this would interact with the effect of investor protection on equilibrium volatility.

Empirical evidence indicates that the idiosyncratic and total volatilities are higher in more developed countries such as U.S. (Morck, Yeung and Yu, 2000; Bartram, Brown and Stulz, 2012), where investor protection is also relatively higher. Indeed, minority investor protection, property rights protection and opaqueness are suggested as likely culprits (Morck, Yeung and Yu, 2000; Jin and Myers, 2006; Bartram, Brown and Stulz, 2012). Our model can potentially shed light on this empirical relation between investor protection and volatility. For example, in economies where the stock holding of the controlling shareholder is sufficiently high, and hence the corresponding consumption share y is sufficiently low, stock price volatility can be higher in an economy with high level of investor protection (e.g., point B in panel (b) of Figure 3) than in an economy with low level of protection (e.g., point A in panel (b) of Figure 3).

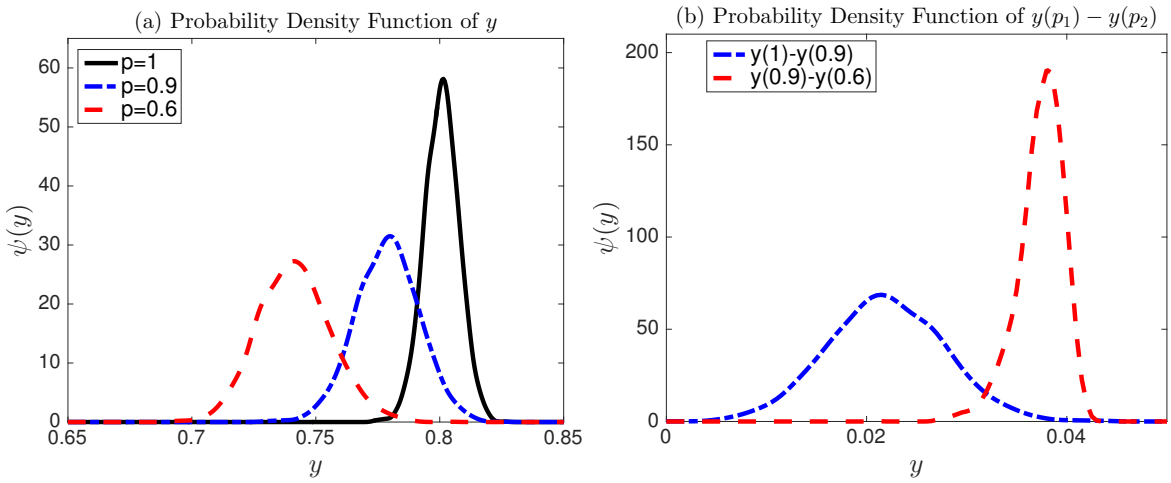

Figure 4
Stationary probability density functions of consumption share y
Panel (a) shows the probability density function of the minority shareholder's consumption share y for different levels of investor protection and the baseline parameter values. Panel (b) depicts for the same parameter values the simulated distribution of the consumption share difference $y(p_1) - y(p_2)$ between two economies with differing protections p_1 and p_2 but otherwise identical in all other respects, including the output shocks dw.

Panel (c) of Figure 3 shows that the risk-free interest rate r is lower in the economy with poor protection. Because the risk premium faced by the minority shareholders is low, investment in stocks is less attractive for them than in the economy with full protection. Therefore, the minority shareholders run to the bond market, and hence, are willing to provide cheap credit, which decreases the interest rates. Furthermore, the purchases of stocks can be partially covered by the diverted output, which contributes to the decreases in interest rates. The negative effect of stealing on interest rates is captured by the second and third terms equation (34) for the interest rate r. We also observe that the gross stock return and the interest rate have similar shapes, and their comparison shows that the interest rates account for a significant fraction of the variation in gross returns. In particular, the ratio $r/(\mu + (1 - x^*)D/S)$ ranges from 0.32 (when $y \approx 0$) to 0.54 (when $y \approx 1$).

From the results on Figures 2 and 3, we observe that the effects of investor protection are more conspicuous when both investors have significant stock holdings, i.e, the economy is away from the boundaries $y = 0$ and $y = 1$. Intuitively, when $y \approx 1$ the controlling shareholder accounts for a tiny fraction of aggregate wealth and consumption, and hence, the effect of stealing is small. Furthermore, when $y \approx 0$ the economy is dominated by the controlling shareholder. As a result, the controlling shareholder holds almost all shares, i.e., $n^*_{Ct} \approx 1$, and hence, the diverted fraction $x^*_t = \min((1 - n^*_{Ct})/k; (1 - p)n^*_{Ct})$ is small because,

	$p = 1$	$p = 0.9$	$p = 0.6$
$\mathbb{E}[y]$	0.80	0.78	0.74
$\mathbb{E}[\frac{dS+Ddt}{S}]$	11.74%	11.38%	10.14%
$\mathbb{E}[r]$	5.98%	5.76%	4.61%
$\mathbb{E}[\sigma]$	13.08%	13.14%	13.17%

Table 1
Unconditional means of consumption shares, stock returns, interest rates, and volatilities
This Table reports the unconditional means of consumption shares, stock returns, interest rates, and volatilities for different levels of investor protection $p = 1, 0.9, 0.6$, and the baseline parameter values.

given the cost of diverting the output, the controlling shareholder finds it sub-optimal to divert the output of a firm in which he is entitled to almost 100% of cash flows. Therefore, there is no diversion of output when $y = 0$ and the effect of investor protection vanishes.

Figure 4a shows the stationary probability density functions for the minority shareholder's consumption share y. The distribution of the variable y is stationary due to the presence of labor incomes and the shareholders' wealth being positive at all times, as elaborated in Section 3. The distributions are not available in closed form, and so we compute them using Monte-Carlo simulations of the consumption share y dynamics given in equation (21). These distributions are shifted towards the boundary $y = 1$ because the minority shareholders as a group receive higher labor income than the controlling shareholder. The distributions have supports concentrated over a small interval $[0.7, 0.85]$.

We use these stationary distributions to evaluate the unconditional means of consumption shares, stock returns, interest rates, and volatilities, and report them in Table 1 for different levels of investor protection. These unconditional means capture the following two effects of investor protection. The first is the direct effect of lower protection holding the consumption share y fixed, which is presented in our main analysis above. The second is the indirect effect of investor protection through y and arises because lower investor protection shifts the distribution of consumption towards the less risk averse controlling shareholder. We observe that lower protection levels lead to lower unconditional means of consumption shares, returns, and interest rates, and increases the volatilities. Consequently, accounting for the indirect effect does not change our main results.

We explore the indirect effect of investor protection further. Figure 4a and Table 1 indeed demonstrate that lower investor protection shifts consumption towards the less risk averse controlling shareholder, so that the minority shareholder's consumption share y decreases on average. Moreover, the pdfs of Figure 4a have little overlap, and hence, the consumption share y randomly sampled from the distribution for $p = 1$ exceeds with a high probability those sampled from the distributions for $p = 0.9$ and $p = 0.6$. Furthermore, Figure 4b plots the simulated pdfs of the consumption share differences $y(p_1) - y(p_2)$ between two economies that differ in investor protections p_1 and p_2, but are otherwise identical in all other respects, including the output shocks dw_t (so that the effects are not confounded by the differences in exogenous variables and shocks). We observe that $y(p_1) - y(p_2) > 0$ for $p_1 > p_2$ with probability 1, and hence the consumption share of minority shareholders is lower in economies with low protection.

From Figures 3a and 3c we observe that the stock mean-returns and interest rates are increasing functions of both p, as discussed above, and of y, because the controlling shareholder is less risk averse. As a result, when investor protection decreases, the stock mean-returns and interest rates decrease because of lower p (direct effect) and lower y (indirect effect). Consequently, the indirect effect reinforces the direct effect of investor protection. Similarly, from Figure 3b we observe that in economies with high protection ($p = 1, 0.9$) the stock volatility σ is higher for $p = 0.9$ than for $p = 1$, and is a decreasing function of y over the interval $[0.7, 0.85]$ where all consumption shares lie with probability 1, and hence, the indirect effect reinforces the direct effect. In an economy with low protection $p = 0.6$ the volatility σ exceeds the volatility for $p = 1$, and is an increasing function of y over the interval $[0.7, 0.85]$, and hence, the indirect effect partially offsets the direct effect, but the latter effect dominates for our baseline parameter values and the volatility increases on average (Table 1).

We also note that consumption share y takes a broader range of values before its distribution converges to the stationary one. Therefore, the comparison of the effects for different levels of protection p conditional on the variable y should be performed for the entire interval $(0, 1)$. This conditional comparison has two economic interpretations. First, it reveals the effects of an unanticipated change in investor protection p when the equilibrium processes change in response to p but variable y has not yet adjusted to the new level. Second, such a comparison reveals the effects of investor protection across economies that have different levels of protection p but similar distribution of consumption.

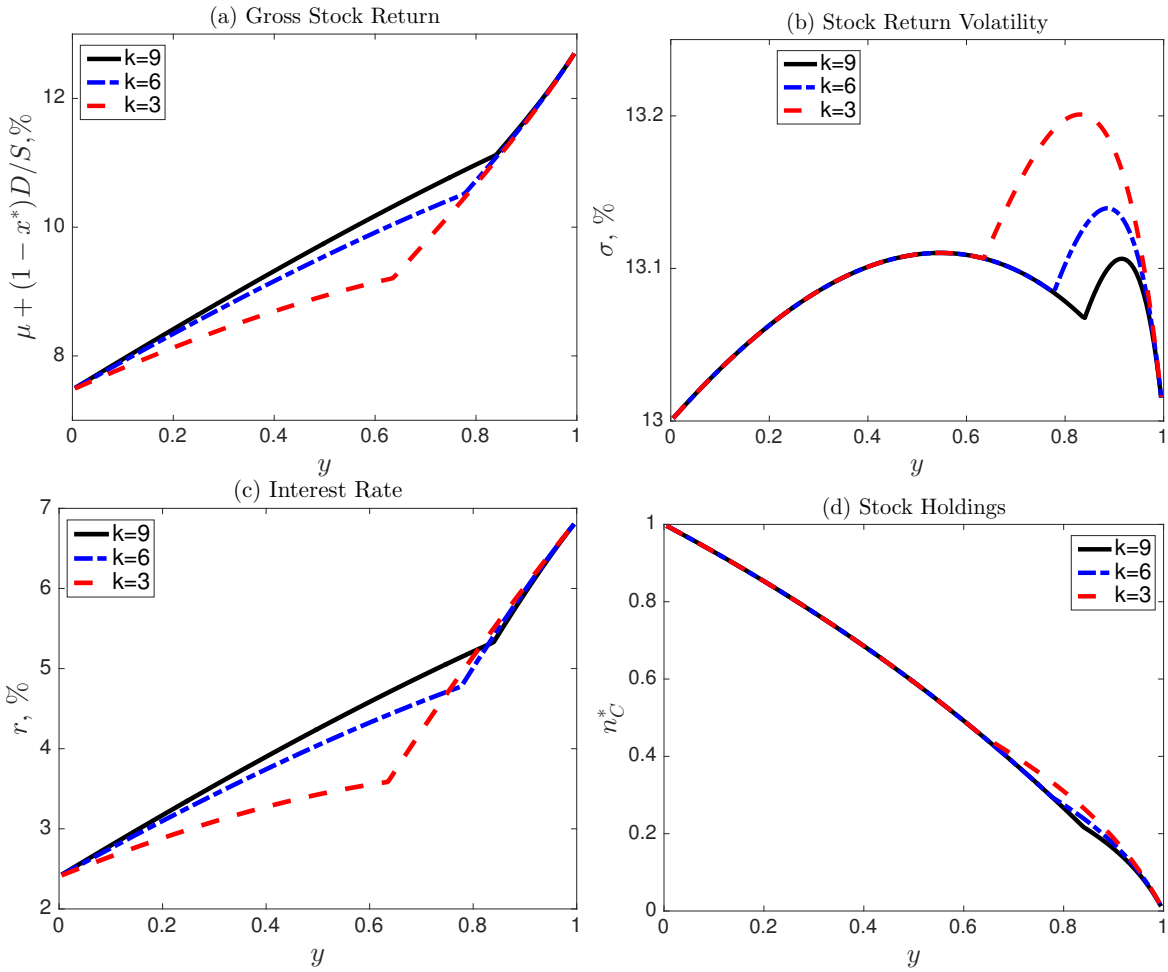

Figure 5
The effect of stealing costs on stock gross returns, volatilities, interest rates, and stock holdings
This Figure shows the equilibrium gross returns $\mu + (1 - x^*)D/S$, stock return volatilities σ, risk-free interest rates r and stock holdings n_C^* as functions of consumption share y for fixed investor protection $p = 0.6$ and the baseline parameter values.

4.3. Effect of Stealing Costs

Figure 5 shows the effect of a change in the cost function parameter k on equilibrium processes when the level of investor protection is fixed. It demonstrates that higher cost of stealing (i.e., high cost parameter k) is associated with higher stock gross return and interest rate, and lower volatility and stock holding of the controlling shareholder. Overall, the effect of higher cost of stealing on equilibrium is consistent with the effect of better protection p.

A comparison of the results in Figures 3 and 5 reveals an important difference between the economic effects of the protection p and cost k parameters on stock gross returns and

interest rates. In particular, the change in protection p has stronger effects when consumption share y is high (i.e., the economy is in region 1), while the change in stealing costs k has stronger effects when consumption share y is relatively low (i.e., the economy is in region 2). Intuitively, as can be formally seen from expression (14) for diverted fraction of output x^*, this is due to the fact that lowering the cost of stealing k increases the diverted output x^* only when the investor protection constraint does not bind, that is, when the economy is in region 2 where the variable y is low.

The effect of the cost parameter k on the stock volatility and holdings is present only when consumption share y is large, in contrast to the effect of k on gross stock return and interest rate. The reason for such an asymmetry is as follows. When consumption share y is low, the economy is dominated by the controlling shareholder, and hence, the stock holding of the controlling shareholder n_C^* is high. Therefore, the investor protection constraint $x_t \leq (1-p)n_t$ is not binding because the controlling shareholder does not have an incentive to steal from himself. Hence, the economy is in region 2 where the diversion of output is only tempered by the cost of stealing and the stock holding is the same as in the benchmark economy. Consequently, the stock volatility equation (33) implies that the volatility is the same as in the benchmark economy.

Furthermore, the cost parameter k has a significant effect on stock return μ_t and interest rate r_t in region 2 via the terms that depend on the fraction of diverted output $x_t^* = (1-n_{Ct}^*)/k$ in equations (31) and (34) when consumption share y is low. The intuition is that despite the fact that $n_{Ct}^* = n_t^{bmk}$ in region 2, the wealth and consumption of the controlling shareholder continue to increase at a higher rate than the wealth and consumption of the minority shareholder due to the diverted cash flow $x_t^* D_t$ in the budget constraint (7) of the controlling shareholder. Therefore, the controlling shareholder extracts higher effective returns from holding stocks, which affects the valuation of stocks and their returns.

Finally, to shed further light on the distinction between the effects of investor protection constraint and the cost function we consider an economy without the investor protection constraint so that stealing is only moderated by the pecuniary cost function. Figure 6 presents the gross stock returns and interest rates in such an economy for different values of the cost parameter k. This figure shows that lowering the cost of stealing decreases asset returns, consistent with the results in Figure 5 and the effects of weaker investor protection in Section 4.1.

With the investor protection constraint absent, the volatility of stock returns and the stockholdings of the controlling shareholder are driven only by the indirect effect of investor

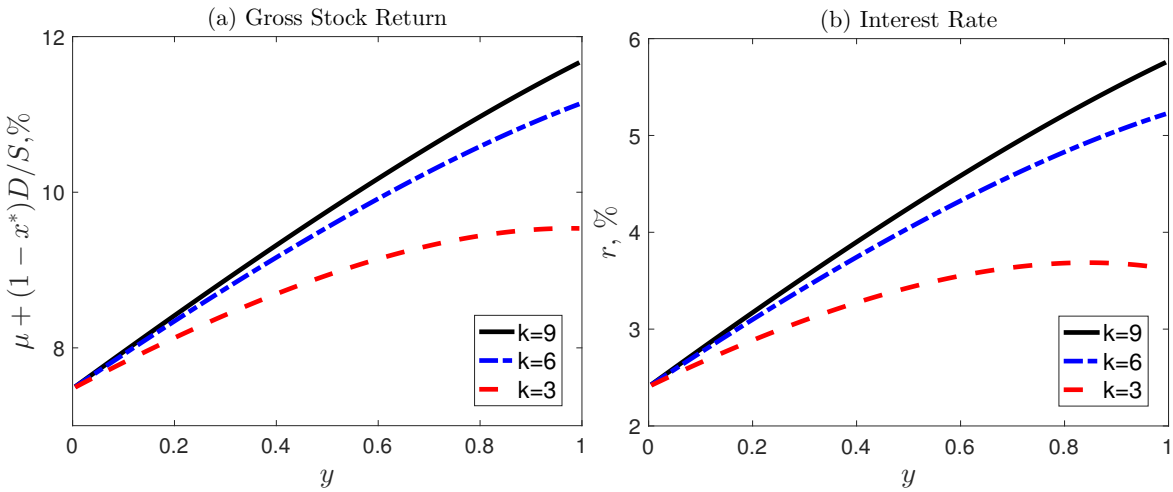

Figure 6
The effect of stealing costs on stock gross returns and interest rates when investor protection constraint is absent
This Figure shows the equilibrium gross returns $\mu + (1 - x^*)D/S$ and risk-free interest rates r for different levels of cost parameter k in the economy where the investor protection constraint is absent, for the baseline parameter values.

protection (discussed in Section 4.2), and hence are not shown for brevity. This is because region 2 is now the entire interval $(0, 1)$ and the controlling shareholder's portfolio in that region is given by the same expression in terms of consumption share y as in the benchmark economy, as discussed in Section 4.1. The cost parameter k affects the volatility and stock holdings only via the distribution of the minority shareholder's consumption share y. This consumption share will be smaller in economies with low stealing costs where the controlling shareholder has more consumption and wealth. As a result, the stockholding n_C^* will be larger in such economies. The effect of the cost parameter k on volatility σ is ambiguous because σ is a non-monotone function of the variable y (panel (b) of Figure 3).

5. Extensions

In this Section, we discuss extensions of our main analysis that incorporate cross-firm differences in investor protection, alternative stealing technologies, and non-pecuniary costs of diverting output. The details of these extended formulations are provided in the Internet Appendix.

Investor protection and cross-section of asset returns. For tractability, we adopt a simpler, one-period economy, which allows us to study stock holdings and mean-returns

but not stock return volatilities because they are equal to exogenous dividend volatilities. This economy has two firms with exogenous outputs at the final date, given by $\widetilde{D}_j = D(1 + \mu_D + \sigma_D \Delta w_j)$, $j = 1, 2$, where Δw_j are shocks. Both firms have the same value in the full protection benchmark because their outputs are ex-ante the same. Firm 1 has imperfect investor protection, as in our main analysis, and firm 2 has full protection. The controlling and minority shareholders trade both stocks and a riskless bond, and have CRRA preferences as in the main analysis.

We show in the Internet Appendix IA.1 that our earlier results on stock holdings, mean-returns, and interest rates remain valid. We additionally find that the firm with high investor protection has higher return than the firm with low protection, consistent with the cross-sectional empirical findings at Gompers, Ishii and Metrick (2003), Yermack (2006), Bebchuk, Cohen, and Ferrell (2009), Daines, Gow, and Larcker (2010). The intuition is essentially the same as in the main analysis and is based on the fact that the controlling shareholder is compensated not only by stock 1 mean-return μ_1 but also by the diverted output $x\widetilde{D}_1$. Consequently, this shareholder invests more in firm 1 stocks relative to the benchmark, and hence drives down its mean-return relative to the mean-return of firm 2.

Alternative stealing technologies. We explore the economic effects of alternative plausible stealing technologies that temper the incentives to divert output when the controlling shareholder's stock holding n_{Ct} is small. First, we consider a formulation with a more elaborate control power $q(n) = \delta \max(n - A, 0) + B$, which incorporates specifications when atomistic investors with small stock holdings are excluded from gaining control in the firm ($B = 0$, $A > 0$, $\delta > 0$). For comparison and generality, it also incorporates the possibility of stealing by managers and other corporate insiders with no or little stake in the firm ($B > 0$), and nests the linear control $q(n) = n$ of our main analysis as a special case. Then, we consider a formulation where the net stolen output is proportional to the stock holding n_{Ct} of the controlling shareholder, which precludes atomistic shareholders from diverting output. In this latter specification, the incentives to divert output near $n_{Ct} = 0$ are weaker than in our main analysis, and are not dictated by the specification of the control power $q(n)$.

We show in the Internet Appendix IA.2 that the main economic insights of our analysis in Section 4 remain valid with these alternative specifications. In particular, as in our main analysis, lower investor protection gives rise to higher ownership concentration and stock return volatilities, and lower stock mean-returns and interest rates. The reason is that with these specifications the controlling shareholder still has incentives to hold more stocks than in the benchmark economy and the effects of the investor protection on equilibrium are again determined by the excess ownership concentration $n^*_{Ct} - n^{bmk}_t > 0$ (Proposition IA.1 in the

Internet Appendix), as in our main analysis. Consequently, the results of Section 4 remain valid with the alternative stealing technologies.

Non-pecuniary costs of diverting output. Our main analysis incorporates pecuniary costs from diverting output. In the Internet Appendix IA.3, we provide an alternative formulation with non-pecuniary costs of diverting output. Such intangible costs could arise because of disutility from stealing due to social norms or loss of reputation. We take the non-pecuniary cost function $f(x, D)$ to be the same as the pecuniary cost given by (4), where this cost is now interpreted as the certainty equivalent of utility loss due to stealing and directly affects the controlling shareholder's preferences:

$$V_i(c_t, W_t, W_{t+dt}) = \rho u_i(c_t)dt + (1 - \rho dt)\mathbb{E}_t\Big[u_i(W_{t+dt} - f(x_t, D_t)dt)\Big],$$

where $u_C(c)$ is the utility function in (5). The controlling shareholder's optimal portfolio turns out to be the same as for the main analysis with pecuniary costs (15), because non-pecuniary costs are subtracted from wealth W_{t+dt} similarly to that of pecuniary costs reducing wealth through the budget constraint. Moreover, all the effects of investor protection on the equilibrium processes remain qualitatively the same as in Section 4. This is because the pecuniary costs directly affect only the interest rate r and drift μ_y and have small effects on these processes.

6. Conclusion

We develop a dynamic asset pricing model where a controlling shareholder can divert a firm's output but is constrained by investor protection and pecuniary costs of stealing. By incorporating endogenous accumulation of control, we show that controlling shareholder's excess ownership concentration interacts with investor protection to determine equilibrium level of stock mean-return, volatility and interest rates. We demonstrate that in equilibrium the controlling shareholder's asset concentration in the firm is larger with imperfect investor protection. We also find that better investor protection increases stock mean-returns, and increases stock return volatilities when ownership concentration is sufficiently high. Our findings provide support for some of the empirical evidence on asset prices. However, it would also be of interest to empirically investigate the link between the effects of investor protection and the endogenous accumulation of control, as predicted by our model.

Our main analysis employs a preference specification that exhibits myopia. It would be of interest to extend our analysis to the familiar case of dynamic preferences of Epstein and Zin (1989) with heterogeneous risk aversions and unit intertemporal elasticities of substitution,

in line with the literature on economies with heterogeneous investors (e.g., Gârleanu and Panageas, 2015). However, such an analysis appears rather difficult because solving the model is further complicated by additional hedging demands in shareholder portfolios, non-linear differential equations for the value functions, and the non-convexity of the controlling shareholder's portfolio optimization.

Appendix: Proofs

Proof of Proposition 1. We observe that the controlling shareholder's objective function (11) is a quadratic function of the share of diverted output x_t. Maximizing this function with respect to x_t subject to the constraint $x_t \leq (1-p)n_t$, we obtain the optimal fraction of diverted output as $x^*(n_t) = \min\left((1-n_t)/k; (1-p)n_t\right)$. Substituting $x^*(n_t)$ back into the objective function (11), we find that the objective function $J_C(n_t)$ is given by:

$$J_C(n_t) = \begin{cases} J_{C1}(n_t), \; n_t & \leq \; \frac{1}{1+(1-p)k}, \\ J_{C2}(n_t), \; n_t & > \; \frac{1}{1+(1-p)k}, \end{cases} \tag{A.1}$$

where $J_{C1}(n_t)$ and $J_{C2}(n_t)$ are quadratic functions of n_t defined as follows:

$$\begin{aligned} J_{C1}(n_t) &= \frac{n_t S_t}{W_{Ct}}\Big((\mu_t - r_t) + \Big(1-(1-p)n_t\Big)\frac{D_t}{S_t}\Big) + (1-p)n_t\frac{D_t}{W_{Ct}} \\ &\quad -\frac{k\,(1-p)^2 n_t^2}{2}\frac{D_t}{W_{Ct}} - \frac{\gamma_C}{2}\Big(\frac{n_t S_t}{W_{Ct}}\sigma_t\Big)^2, \end{aligned} \tag{A.2}$$

$$\begin{aligned} J_{C2}(n_t) &= \frac{n_t S_t}{W_{Ct}}\left((\mu_t - r_t) + \left(1 - \frac{1-n_t}{k}\right)\frac{D_t}{S_t}\right) + \frac{1-n_t}{k}\frac{D_t}{W_{Ct}} \\ &\quad -\frac{(1-n_t)^2}{2k}\frac{D_t}{W_{Ct}} - \frac{\gamma_C}{2}\Big(\frac{n_t S_t}{W_{Ct}}\sigma_t\Big)^2. \end{aligned} \tag{A.3}$$

It is immediate to observe that the function $J_{C1}(n_t)$ is a concave function of n_t and achieves a unique global maximum given by $n^*_{Ct,1}$ in equation (15). In contrast, the function $J_{C2}(n_t)$ can be either convex or concave, depending on the cost parameter k. It achieves a global maximum or minimum (depending on k) at point $n^*_{Ct,2}$ in equation (15). Two other potential points of global maximum are $n^*_{Ct,3} = 1/(1+(1-p)k)$, where $J_{C1}(n_t) = J_{C2}(n_t)$, and $n^*_{Ct,4} = 1$ at which the constraint $n_t \leq 1$ becomes binding. We then determine the global maximum by direct search over points $n^*_{Ct,1}$, $n^*_{Ct,2}$, $n^*_{Ct,3}$, $n^*_{Ct,4}$ to find the point at which the function achieves global maximum, which gives rise to equation (15). The minority shareholder's optimal portfolio (16) is easily obtained by maximizing the quadratic concave objective function (12). ■

Proof of Proposition 2. We consider the benchmark economy with full protection. Because there is no stealing in this economy, and hence $x_t = 0$, both investors have the same objective function (12) and their portfolios are given by

$$n^{bmk}_{Ct} = \frac{\mu_t - r_t + \frac{D_t}{S_t}}{\gamma_C \sigma_t^2 \frac{S_t}{W_{Ct}}}, \quad n^{bmk}_{Mt} = \frac{\mu_t - r_t + \frac{D_t}{S_t}}{\gamma_M \sigma_t^2 \frac{S_t}{W_{Mt}}}. \tag{A.4}$$

In what follows we omit the superscript "*bmk*" over the benchmark equilibrium processes for convenience. Substituting the portfolios (A.4) and optimal consumptions $c_{it}^* = \rho^{1/\gamma_i} W_{it}$ into the shareholders' self-financing budget constraints (7) with $x_t = 0$, we obtain that their wealths under the optimal strategies follow the dynamics:

$$dW_{it} = W_{it}\left(r_t + \frac{\kappa_t^2}{\gamma_i} - \rho^{1/\gamma_i}\right)dt + l_i \widehat{D}_t dt + \frac{W_{it}\kappa_t}{\gamma_i} dw_t, \tag{A.5}$$

where $\kappa_t \equiv (\mu_t - r_t + D_t/S_t)/\sigma_t$ is the Sharpe ratio. Substituting the shareholders' optimal consumptions $c_{it}^* = \rho^{1/\gamma_i} W_{it}$ into the consumption clearing condition (20), we obtain the equation $\rho^{1/\gamma_C} W_{Ct} + \rho^{1/\gamma_M} W_{Mt} = \widehat{D}_t$. Applying Itô's Lemma to both sides of this equation, matching the dt and dw terms and then dividing both sides of the resulting equations by the output $\widehat{D}_t$, we obtain the following system of equations for r and κ:

$$(1-y_t)\left(r_t + \frac{\kappa_t^2}{\gamma_C} - \rho^{1/\gamma_C}\right) + y_t\left(r_t + \frac{\kappa_t^2}{\gamma_M} - \rho^{1/\gamma_M}\right) = \mu_D - \rho^{1/\gamma_M} l_M - \rho^{1/\gamma_C} l_C, \tag{A.6}$$

$$(1-y_t)\frac{\kappa_t}{\gamma_C} + y_t \frac{\kappa_t}{\gamma_M} = \sigma_D. \tag{A.7}$$

Solving equations (A.6) and (A.7), we obtain the interest rate (25) and Sharpe ratio (23).

Next, we obtain the drift μ_{yt} and volatility σ_{yt} of the consumption share $y_t = c_{Mt}/\widehat{D}_t$. Because $c_{Mt} = \rho^{1/\gamma_M} W_{Mt}$, the consumption share can be rewritten as $y_t = \rho^{1/\gamma_M} W_{Mt}/\widehat{D}_t$, where W_{Mt} follows the process (A.5). Applying Itô's Lemma to both sides of this equation for the consumption share y_t, and matching dt and dw terms, we obtain equations (27) and (28) for σ_y and μ_y, respectively.

Towards obtaining the volatility σ_t and the trading strategies, we first derive the ratios $\widehat{D}_t/S_t$, W_{Ct}/S_t, and W_{Mt}/S_t. The ratio D_t/S_t is found using the market clearing condition $W_{Ct} + W_{Mt} = S_t$ and optimal consumptions $c_{it}^* = \rho^{1/\gamma_i} W_{it}$ as follows:

$$\begin{aligned} \frac{\widehat{D}_t}{S_t} &= \frac{\widehat{D}_t}{W_{Ct} + W_{Mt}} = \frac{\widehat{D}_t}{c_{Ct}\rho^{-1/\gamma_C} + c_{Mt}\rho^{-1/\gamma_M}} \\ &= \frac{1}{(1-y_t)\rho^{-1/\gamma_C} + y_t \rho^{-1/\gamma_M}}. \end{aligned} \tag{A.8}$$

The ratios W_{Ct}/S_t and W_{Mt}/S_t are determined analogously to D_t/S_t as follows:

$$\begin{aligned} \frac{W_{Ct}}{S_t} &= \frac{W_{Ct}}{W_{Ct} + W_{Mt}} = \frac{c_{Ct}\rho^{-1/\gamma_C}}{c_{Ct}\rho^{-1/\gamma_C} + c_{Mt}\rho^{-1/\gamma_M}} \\ &= \frac{(1-y_t)\rho^{-1/\gamma_C}}{(1-y_t)\rho^{-1/\gamma_C} + y_t \rho^{-1/\gamma_M}}, \end{aligned} \tag{A.9}$$

$$\frac{W_{Mt}}{S_t} = 1 - \frac{W_{Ct}}{S_t} = \frac{y_t \rho^{-1/\gamma_M}}{(1-y_t)\rho^{-1/\gamma_C} + y_t \rho^{-1/\gamma_M}}. \tag{A.10}$$

The market clearing in the stock and bond markets implies that the aggregate wealth equals the value of the stock market, that is, $W_{Ct} + W_{Mt} = S_t$. From the latter equation and the expressions for the optimal consumption $c^*_{it} = \rho^{1/\gamma_i} W_{it}$ we obtain the stock price:

$$\begin{aligned} S_t &= c^*_{Ct}\rho^{-1/\gamma_C} + c^*_{Mt}\rho^{-1/\gamma_M} \\ &= ((1-y_t)\rho^{-1/\gamma_C} + y_t\rho^{-1/\gamma_M})\widehat{D}_t. \end{aligned} \tag{A.11}$$

Applying Itô's Lemma to equation (A.11) we find that $\sigma_t = \sigma_D + (\rho^{-1/\gamma_M} - \rho^{-1/\gamma_C})\sigma_{yt}\widehat{D}_t/S_t$. Substituting σ_y from (27) and $\widehat{D}_t/S_t$ from (A.8) into the latter equation for σ_t we obtain the volatility σ_t in equation (24). The stock return is then given by $\mu_t = \kappa_t\sigma_t + r_t - (1 - l_M - l_C)\widehat{D}_t/S_t$, which yields equation (22).

Finally, we obtain the optimal portfolios (29) by substituting the equilibrium processes into equations (A.4). The processes μ_t, r_t, and σ_t are substituted from (22)–(24), the ratio $D_t/S_t \equiv (1 - l_M - l_C)\widehat{D}_t/S_t$ is substituted from (A.8), and W_{it}/S_t from (A.9) and (A.10).

The stationary pdf is given by (Karlin and Taylor, 1981, p. 221):

$$\psi(y) = \frac{\exp\left(a_0 + \int_{0.5}^{y} \frac{2\mu_y(z)}{\sigma_y(z)^2} dz\right)}{\sigma_y(y)^2}, \tag{A.12}$$

where a_0 is a normalizing constant that makes the pdf integrate to 1. After some tedious algebra, we find that:

$$\frac{2\mu_y}{\sigma_y^2} = a_1 + \frac{a_C}{y^2} - \frac{a_M}{(1-y)^2} + \frac{b_C}{y} + \frac{b_M}{1-y}, \tag{A.13}$$

where the constants are given by:

$$\begin{aligned} a_1 &= -2\frac{\rho^{1/\gamma_C} - \rho^{1/\gamma_M}}{\sigma_D^2}, \\ b_i &= \frac{2}{\sigma_D^2(1/\gamma_M - 1/\gamma_C)^2}\left(\frac{\rho^{1/\gamma_C} - \rho^{1/\gamma_M}}{\gamma_i^2} + \sigma_D^2\left(\frac{1}{\gamma_M} - \frac{1}{\gamma_C}\right)\frac{\gamma_i - 1}{\gamma_i} + \frac{\rho^{1/\gamma_M} l_M - \rho^{1/\gamma_C} l_C}{\gamma_i^2}\right. \\ &\quad \left. + \frac{2\rho^{1/\gamma_i} l_i}{\gamma_i}\left(\frac{1}{\gamma_M} - \frac{1}{\gamma_C}\right)\right), \\ a_C &= \frac{\rho^{1/\gamma_M} l_M}{\gamma_C^2}, \quad a_M = \frac{\rho^{1/\gamma_C} l_C}{\gamma_M^2}. \end{aligned} \tag{A.14}$$

Substituting (A.13) into (A.12) and integrating, we obtain the pdf (30). Because $a_M > 0$ and $a_C > 0$, the pdf (30) converges to 0 exponentially fast at the boundaries. Consequently, $\lim_{y\to 0}\psi(y) = \lim_{y\to 1}\psi(y) = 0$, and $\int_0^1 \psi(y) < +\infty$, and hence, the pdf is well-defined. ■

Proof of Proposition 2. Equation (31) for the mean-stock return μ_t follows readily from the definition of the Sharpe ratio $\kappa_t = (\mu_t - r_t + (1-x_t^*)D_t/S_t)/\sigma_t$ and the expression (A.8) for the ratio $\widehat{D}_t/S_t$. Next, we derive the interest rate r_t. The wealths of the controlling and minority shareholders satisfy the budget constraints (7). Applying Itô's Lemma to both sides of the consumption clearing condition $\rho^{1/\gamma_C}W_{Ct} + \rho^{1/\gamma_M}W_{Mt} = \widehat{D}_t$, matching dt and dw terms, and dividing both sides by $\widehat{D}_t$, we obtain the equations:

$$(1-y_t)\left(r_t - \rho^{1/\gamma_C} + \frac{n_{Ct}^* S_t\sigma_t\kappa_t}{W_{Ct}}\right) + (1-l_M-l_C)\left(x_t^* - 0.5k(x_t^*)^2\right)\rho^{1/\gamma_C} + l_C\rho^{1/\gamma_C}$$
$$+y_t\left(r_t - \rho^{1/\gamma_M} + \frac{n_{Mt}^* S_t\sigma_t\kappa_t}{W_{Mt}}\right) + 0.5k(x_t^*)^2\rho^{1/\gamma_M} + l_M\rho^{1/\gamma_M} = \mu_D, \quad \text{(A.15)}$$

$$(1-y_t)\frac{n_{Ct}^* S_t\sigma_t}{W_{Ct}} + y_t\frac{n_{Mt}^* S_t\sigma_t}{W_{Mt}} = \sigma_D. \quad \text{(A.16)}$$

From equations (A.15)–(A.16) and equations (A.9)–(A.10) for the ratios W_C/S and W_M/S (which remain the same for the model with imperfect protection), we obtain:

$$\begin{aligned} r_t &= \mu_D + y_t\rho^{1/\gamma_M} + (1-y_t)\rho^{1/\gamma_C} - \kappa_t\sigma_D - l_M\rho^{1/\gamma_M} - l_C\rho^{1/\gamma_C} \\ &\quad -(1-l_M-l_C)x_t^*\rho^{1/\gamma_C} - (\rho^{1/\gamma_M} - \rho^{1/\gamma_C})(1-l_M-l_C)\frac{k(x_t^*)^2}{2}, \qquad \text{(A.17)} \\ \sigma_t &= \frac{\sigma_D}{(\rho^{1/\gamma_M}(1-n_{Ct}^*) + \rho^{1/\gamma_C}n_{Ct}^*)(y_t\rho^{-1/\gamma_M} + (1-y_t)\rho^{-1/\gamma_C})}. \qquad \text{(A.18)} \end{aligned}$$

From equation (16) for n_M^* and the fact that $n_M^* = 1 - n_C^*$ we find that $\kappa_t = \gamma_M\sigma_t(1 - n_{Ct}^*)/(W_{Mt}/S_t)$. Substituting the ratio W_M/S from (A.10) and σ_t from (A.18) into the latter equation, we obtain:

$$\kappa_t = \frac{\gamma_M\sigma_D(1-n_{Ct}^*)\rho^{1/\gamma_M}}{\rho^{1/\gamma_M}(1-n_{Ct}^*) + \rho^{1/\gamma_C}n_{Ct}^*}\frac{1}{y_t}. \quad \text{(A.19)}$$

Using exactly the same steps as above, we obtain the following expressions for σ^{bmk} and κ^{bmk} in the benchmark economy:

$$\sigma_t^{bmk} = \frac{\sigma_D}{(\rho^{1/\gamma_M}(1-n_{Ct}^{bmk}) + \rho^{1/\gamma_C}n_{Ct}^{bmk})(y_t\rho^{-1/\gamma_M} + (1-y_t)\rho^{-1/\gamma_C})}. \quad \text{(A.20)}$$

$$\kappa_t^{bmk} = \frac{\gamma_M\sigma_D(1-n_{Ct}^{bmk})\rho^{1/\gamma_M}}{\rho^{1/\gamma_M}(1-n_{Ct}^{bmk}) + \rho^{1/\gamma_C}n_{Ct}^{bmk}}\frac{1}{y_t}. \quad \text{(A.21)}$$

We next rewrite κ_t as $\kappa_t = \kappa_t^{bmk} + (\kappa_t - \kappa_t^{bmk})$, where κ_t^{bmk} is given by equation (A.21). Substituting (A.19) into the right-hand side of the latter equation, after some algebra, we obtain equation (32) for the Sharpe ratio. Similarly, we rewrite the volatility as $\sigma_t = \sigma_t^{bmk} + (\sigma_t - \sigma_t^{bmk})$, where σ_t^{bmk} is given by equation (A.20). Substituting (A.18) into the right-hand side of the latter equation, after some algebra, we obtain equation (33) for σ_t. Substituting κ_t from (32) into equation (A.17) for r_t, after some algebra, we obtain equation (34) for r_t.

Next, we find the volatility σ_y and drift μ_y of the consumption share $y_t = c_{Mt}^*/\widehat{D}_t$. Noting that $c_{Mt}^* = \rho^{1/\gamma_M} W_{Mt}$, we apply Itô's Lemma to $y_t = \rho^{1/\gamma_M} W_{Mt}/\widehat{D}_t$ and obtain σ_y and μ_y by matching the dw and dt terms on both sides of the equation, as in the benchmark economy.

The controlling shareholder's optimization problem (11) implies the fixed-point equation (37) for the stock holding n_C^*, in which the equilibrium processes depend on n_C^* itself. The ratios D_t/S_t and S_t/W_{Ct} in (38) are derived in the same way as in equations (A.8)–(A.10). The ratio D_t/W_{Ct} in (38) is found by multiplying ratios D_t/S_t and S_t/W_{Ct}. ■

Lemma A.1 (Equilibrium stock holding). *The optimal stock holding $n_{C,t}^*$ of the controlling shareholder in equilibrium has the following representation:*

$$n_{Ct}^* = \begin{cases} n_{Ct,1}^* = n_{Ct,2}^* + n_{Ct,2}^* \left(1 - \dfrac{n_{Ct,1}^*}{n_{Ct,3}^*}\right) \left(\rho^{1/\gamma_C} n_{Ct,1}^* + \rho^{1/\gamma_M}(1 - n_{Ct,1}^*)\right)^2 \\ \qquad\qquad \times \dfrac{(1-p)(1-l_M-l_C) y_t \rho^{-1/\gamma_M}}{\gamma_M \sigma_D^2}, & \text{(region 1)}, \\ n_{Ct,2}^* = \dfrac{\frac{1-y_t}{\rho^{1/\gamma_C}\gamma_C}}{\frac{1-y_t}{\rho^{1/\gamma_C}\gamma_C} + \frac{y_t}{\rho^{1/\gamma_M}\gamma_M}}, & \text{(region 2)}, \\ n_{Ct,3}^* = \dfrac{1}{1+(1-p)k}, & \text{(region 3)}, \\ n_{Ct,4}^* = 1. & \text{(region 4)}, \end{cases} \tag{A.22}$$

Furthermore, for all consumption shares $y \in [0,1]$ there always exists $n_{Ct,1}^* \in [0,1]$, which solves a third degree polynomial equation.

Proof of Lemma A.1. First, we find the stock holding $n_{Ct,2}^*$ in region 2. The constraint $x_t \leq (1-p)q(n_t)$ is not binding by the definition of region 2. Hence, $n_{Ct,2}^*$ satisfies the

following first order condition:

$$\begin{aligned}&\frac{S_t}{W_{Ct}}\left(\mu_t - r_t + (1-x_t^*)\frac{D_t}{S_t}\right) - \gamma_C n_t \left(\frac{S_t\sigma_t}{W_{Ct}}\right)^2 \\ &+ \left(\frac{S_t}{W_{Ct}}\left(\mu_t - r_t + (1-x_t^*)\frac{D_t}{S_t}\right) + \left(x_t^* - \frac{k(x_t^*)^2}{2}\right)\frac{D_t}{W_{Ct}}\right)_x' (x^*(n_t))' = 0.\end{aligned} \tag{A.23}$$

The second line in (A.23) is zero from the first order condition for x_t^*. Hence, we obtain:

$$n_{Ct}^* = \frac{\mu_t - r_t + (1-x_t^*)\frac{D_t}{S_t}}{\gamma_C \sigma_t^2 \frac{S_t}{W_{Ct}}}. \tag{A.24}$$

Substituting κ_t from (A.19), σ_t from (A.18), D_t/W_{Ct} and S_t/W_{Ct} from (38) into the expression for $n_{Ct,2}^*$ in (A.24), after some algebra, we obtain:

$$n_{Ct,2}^* = \frac{\frac{1-y_t}{\rho^{1/\gamma_C}\gamma_C}}{\frac{1-y_t}{\rho^{1/\gamma_C}\gamma_C} + \frac{y_t}{\rho^{1/\gamma_M}\gamma_M}}. \tag{A.25}$$

We note that $n_{Ct,3}^*$ and $n_{Ct,4}^*$ remain the same as in (15).

To obtain the stock holding $n_{Ct,1}^*$ in region 1, we observe that by the definition of region 1 $n_{Ct,1}^*$ satisfies the following first order condition for the objective function $J_{C1}(n_t)$ in (A.2):

$$n_{Ct,1}^* = \frac{\kappa_t}{\gamma_C(S_t/W_{Ct}\sigma_t)} + \frac{(1-p)D_t/W_{Ct}(1-n_{Ct,1}^* - k(1-p)n_{Ct,1}^*)}{\gamma_C\left(S_t/W_{Ct}\sigma_t\right)^2}. \tag{A.26}$$

Substituting κ_t from (A.19) and σ_t from (A.18), D_t/W_{Ct} and S_t/W_{Ct} from (38) into equation (A.26), after straightforward algebra, we obtain the first line of (A.22). It can be easily verified that the left-hand side of the equation in the first line of (A.22) is lower than its right-hand side for $n_{Ct,1}^* = 0$, and vise versa for $n_{Ct,1}^* = 1$. Therefore, by the intermediate value theorem, there exists $n_{Ct,1}^* \in [0,1]$. ■

Numerical method for the main analysis in Section 4. First, we derive the optimal stock holding n_{Ct}^* as a function of the consumption share y in each of the regions 1, 2, 3, and 4 in equation (15). Lemma A.1 above demonstrates the existence of the stock holding n_{Ct}^* and derives it as an implicit function of the consumption share y_t, which can easily be computed by solving a third-degree polynomial equation.[11] Then, for each value of the

[11] As demonstrated in Lemma A.1, this polynomial always has a solution in the interval $[0,1]$. Although in general this solution may not be unique, we only obtain unique solutions in our calibration of the model. Furthermore, because the objective function (A.1) may have regions of non-concavity, the optimal portfolio holding n_{Ct}^* may appear to be a discontinuous function of y. However, such a discontinuity requires extreme values of exogenous model parameters and does not occur in our calibration of the model.

consumption share y, we substitute $n^*_{Ct,1}$, $n^*_{Ct,2}$, $n^*_{Ct,3}$, and $n^*_{Ct,4}$ in turn into the objective function on the right-hand side of (37) and find the element that maximizes the objective function. This way, we obtain the optimal stock holding n^*_{Ct}. Substituting n^*_{Ct} into the expressions for equilibrium processes in Proposition 2, we obtain all equilibrium processes as functions of the consumption share y.

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Internet Appendix for "Investor Protection and Asset Prices"

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In this Internet Appendix, we consider extensions of our analysis that feature cross-firm differences in investor protection, alternative stealing technologies, and non-pecuniary costs of diverting output. We find that the underlying mechanisms at play and our results on the effects of investor protection remain qualitatively the same as in Section 4, and also uncover additional insights due to these extended formulations. The proofs and the details of the numerical methods employed are provided in Section IA.4.

IA.1 Investor Protection and Cross-Section of Asset Returns

The empirical literature on the effects of investor protection on firm stock returns is largely cross-sectional. Hence, it would be valuable to extend our main analysis to feature an additional firm with a different level of investor protection. However, such an extension turns out to be intractable in a dynamic continuous-time setting due to cumbersome expressions for equilibrium processes, an extra state variable, complex differential equations and fixed-point problems for optimal stock holdings.[1] Therefore, we adopt a simpler, one-period economy. This economy lends considerable tractability to the analysis of stock holdings and mean-returns but does not allow us to study stock return volatilities because they are equal to exogenous dividend volatilities. For further tractability, we exclude pecuniary costs of stealing to avoid non-convexity of the controlling shareholder's optimization (discussed in Section 3.1).[2]

We consider an economy with two dates, 0 and 1, and three equally likely states ω_1, ω_2, and ω_3 at date $t = 1$. There are two representative firms that produce equal outputs D at the initial date and state-dependent outputs $\widetilde{D}_1$ and $\widetilde{D}_2$ at the final date $t = 1$, given by

$$\widetilde{D}_j = D[1 + \mu_D + \sigma_D \Delta w_j], \quad j = 1, 2, \tag{IA.1}$$

[1] The analysis of stock prices and volatilities is complicated even in frictionless economies with heterogeneous investors, where these processes are given by complex expressions in terms of special functions and integrals (e.g., Chabakauri, 2013). Moreover, a dynamic two-firm setting requires an additional state variable capturing the dynamics of the output share of one of the firms, which further complicates the analysis.

[2] We justify removing pecuniary costs as follows. First, these costs are introduced in our main analysis for realism, and the economic effects are primarily driven by the investor protection constraint. Second, they have similar effects on asset returns as investor protection constraint (Section 4.3). Third, firms are more likely to differ in investor protection implied by corporate culture rather than in pecuniary costs such as bribes, fines, and similar expenses that are likely to be determined at an economy level.

where the output mean-growth rate μ_D and volatility σ_D are constants, and Δw_j are economic shocks. The shocks take values $\Delta w_1 \in \{\sqrt{3/2}, 0, -\sqrt{3/2}\}$ and $\Delta w_2 \in \{\sqrt{3/2}, -\sqrt{3/2}, 0\}$ in states ω_1, ω_2, ω_3, respectively. Their mean and variance are given by $\mathbb{E}[\Delta w_j] = 0$ and $\text{var}[\Delta w_j] = 1$, respectively, and the correlation is 0.5. The outputs (IA.1) are deliberately chosen to be ex-ante identical to exclude confounding sources of firm heterogeneity.

The economy is populated by representative controlling and minority shareholders. The shareholders trade in three securities, a riskless bond in zero net supply with interest rate r and two stocks with prices S_1 and S_2, which represent claims to dividends paid by firms 1 and 2 at the final date, respectively. Hence, the underlying financial market is complete.

Firm 1 has imperfect investor protection. The controlling shareholder C of firm 1 can divert a fraction x of this firm's output, subject to the investor protection constraint $x \leq (1-p)n_{C1}$, similar to our main analysis, where n_{C1} is this shareholder's holding of stock 1. Firm 2 has full protection. This firm may also have a controlling shareholder who manages it. However, this shareholder cannot divert output, and hence faces the same portfolio and consumption choice as a minority shareholder. Therefore, we subsume this shareholder and all minority shareholders within the representative minority shareholder M of firm 1. In an economy with full protection for both firms the stock prices are the same, $S_1 = S_2$, because the outputs (IA.1) are equivalent from the viewpoint of date 0 before shocks Δw_j are realized. Hence, the difference in investor protections is the only source of cross-sectional variation in stock prices and returns in this setting.

The shareholders have preferences over initial and final consumptions c and $\tilde{c}$, given by:

$$u_i(c) + \beta\, \mathbb{E}[u_i(\widetilde{c})], \quad i = C, M, \tag{IA.2}$$

where $u_i(\cdot)$ are CRRA utilities given by (5) and $\beta > 0$ is a time-preference parameter. The shareholders' self-financing budget constraints are given by

$$\widetilde{c}_i = (W_i - c_i)(1+r) + n_{i1}((1-x)\widetilde{D}_1 - S_1) + n_{i2}(\widetilde{D}_2 - S_2) + 1_{\{i=C\}} x \widetilde{D}_1, \quad i = C, M, \tag{IA.3}$$

where W_i denotes initial wealth.

The minority shareholder M maximizes the objective function (IA.2) with respect to consumption c and stock holdings n_1 and n_2 subject to the budget constraint (IA.3). The controlling shareholder C maximizes the same objective with respect to c, n_1, n_2, and diverted output x subject to the budget constraint (IA.3), the investor protection constraint $x \leq (1-p)n_{C1}$, and a non-negative date $t = 1$ financial wealth constraint

$$\widetilde{W}_C = (W_C - c_C)(1+r) + n_{C1}((1-x)\widetilde{D}_1 - S_1) + n_{C2}(\widetilde{D}_2 - S_2) \geq 0. \tag{IA.4}$$

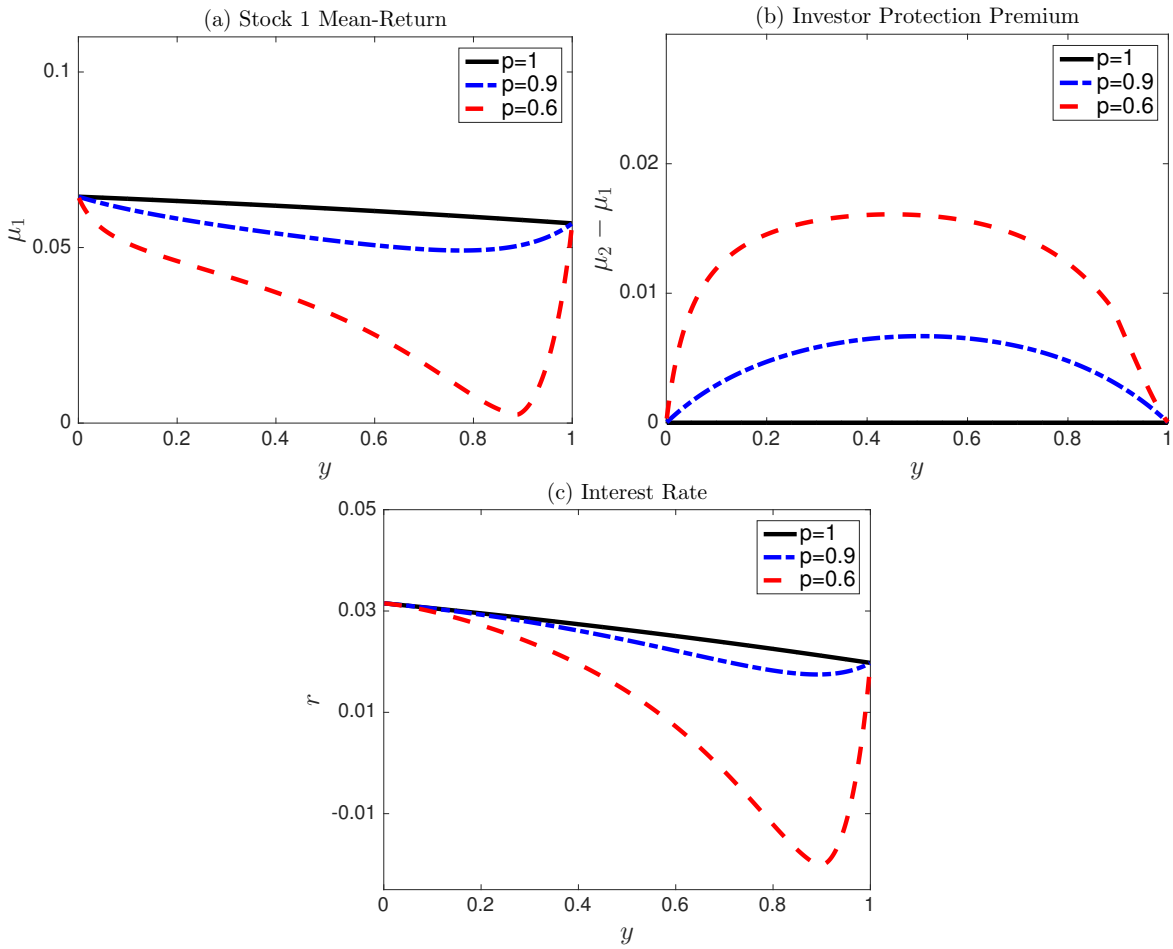

Figure IA.1
The effect of firm 1 investor protection on stock 1 mean-returns, investor protection premiums, and interest rates
This Figure shows the equilibrium stock 1 mean-return μ_1, investor protection premium $\mu_2 - \mu_1$ and risk-free interest rates r as functions of the consumption share y for different levels of firm 1 investor protection p and the baseline parameter values.

The latter constraint rules out unrealistic risky positions backed by future stolen output, consistent with our main analysis where a similar constraint emerges due to shareholder preferences over next-period financial wealth (Section 2.2).

We derive the equilibrium processes as functions of the consumption share y of the minority shareholder at date $t = 0$. Finding equilibrium reduces to solving a system of four non-linear equations for the controlling shareholder's consumptions in sates ω_1, ω_2, ω_3, and fraction x. These equations are significantly more complex than in the full protection economy where the system breaks down into separate one-dimensional equations for consumptions in states ω. We solve the model numerically, as detailed in Section IA.4.

The effects of investor protection on stock returns, interest rates, and stock holdings are in line with our main analysis. For brevity, we here only present the stock mean-returns, defined as $\mu_1 = \mathbb{E}[(1-x)\widetilde{D}_1]/S_1 - 1$ and $\mu_2 = \mathbb{E}[\widetilde{D}_2]/S_2 - 1$, which are the focus of the related empirical literature, and also the interest rate r for comparison. Figure IA.1 reports the mean-return μ_1, spread $\mu_2 - \mu_1$, and interest rate r for different values of firm 1 investor protection p and baseline parameter values.[3] Panels (a) and (c) demonstrate that mean-return μ_1 and rate r are lower with weaker investor protection, as in our main analysis.

Panel (b) depicts the *investor protection premium*, defined as the spread $\mu_2 - \mu_1$ between the mean-returns of stocks with full and imperfect protection, respectively. This premium is zero when both firms have full protection because the firm outputs are ex-ante identical. Our key result is that this premium is positive when firm 1 has imperfect protection $p < 1$, in line with the empirical evidence discussed in Section 4.2. The intuition is that if the premium remains zero for $p < 1$ (i.e., $\mu_2 = \mu_1$) the controlling shareholder would be willing to invest more in stock 1 to receive extra compensation in the form of diverted output $x\widetilde{D}_1$ thereby reducing the mean-return μ_1. Hence the positive premium in equilibrium.

IA.2 Alternative Stealing Technologies

The controlling shareholder in our main analysis is able to divert output even with a small stock holding n_{Ct}, and has strong incentives to do so around $n_{Ct} = 0$. Our main objective here is to explore the economic effects of alternative plausible stealing technologies that temper the incentives to divert when the stock holding n_{Ct} is small, and to demonstrate that our results are not driven by the linearity of the control power $q(n)$. First, we consider a specification with a more general control power $q(n)$, and then a formulation where the net stolen output is proportional to the stock holding, which reduces the controlling shareholder's incentives to divert around $n_{Ct} = 0$ irrespective of the control power $q(n)$ specification. We show that the main economic insights of our analysis with these alternative technologies remain the same as in Section 4.

We start with the analysis of a more general control function $q(n) = \delta \max(n - A, 0) + B$, which incorporates the situation of a sufficiently large stock holding n being required to gain control in the firm ($B = 0$), and hence, there being no stealing around $n_{Ct} = 0$. For comparison and generality, it also incorporates the possibility of stealing by hired managers and other firm insiders with no stake in the firm ($B > 0$), and nests the linear control specification $q(n) = n$ of our main analysis as a special case. The equilibrium processes with

[3] We set $\mu_D = 1.5\%$, $\sigma_D = 13\%$, $\gamma_C = 3$, $\gamma_M = 3.5$, as in our main analysis, and $\beta = 0.95$.

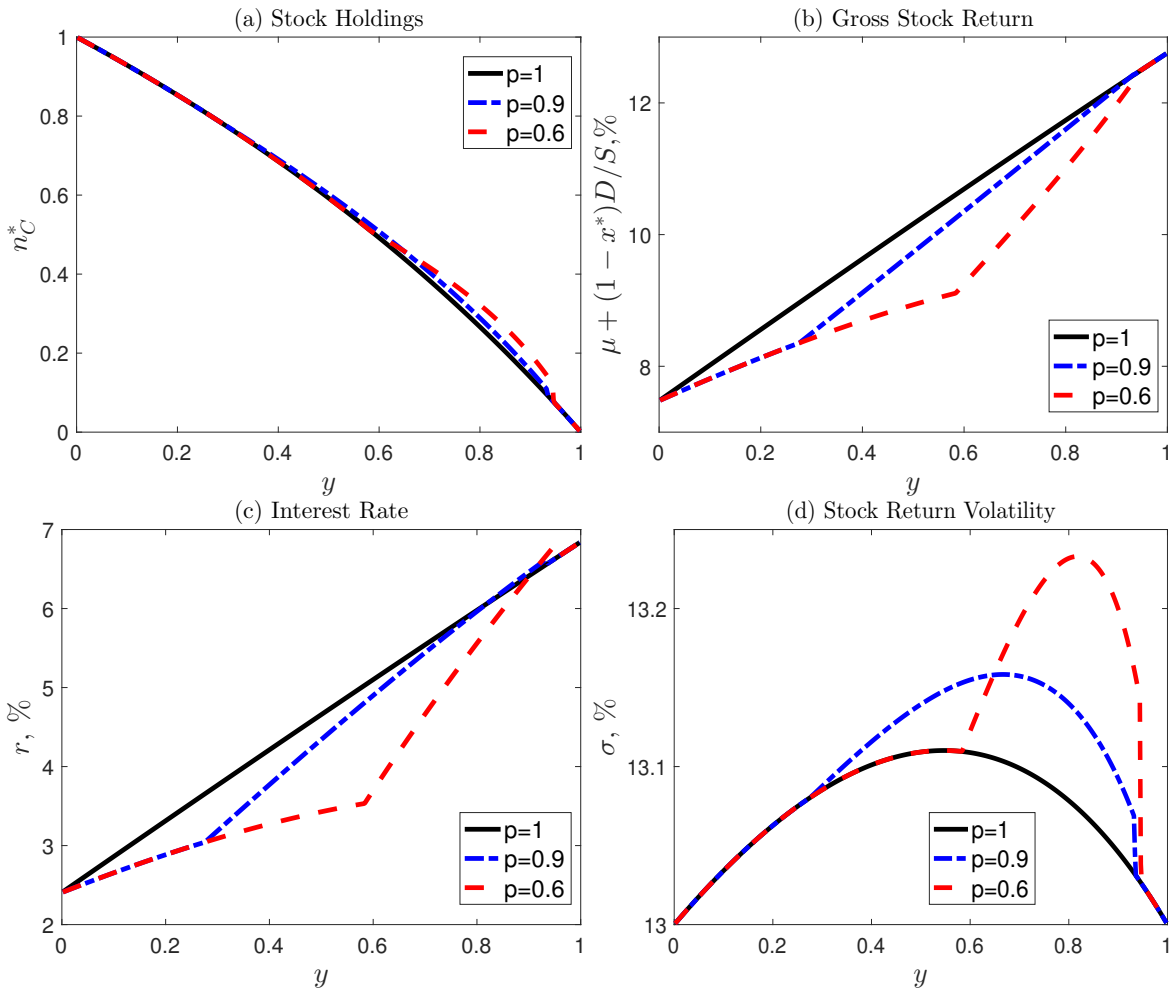

Figure IA.2
Equilibrium processes when control power is given by $q(n) = \max(n - A, 0)$
This Figure shows the equilibrium stock holdings, gross stock returns, risk-free interest rates and stock return volatilities for different levels of investor protection p and the baseline parameter values when the control power is given by $q(n) = \max(A - n, 0)$, and $A = 0.1$.

this more general control power are given by the same equations (31)–(37) as in Proposition 3, but with a new diverted output $x^*(n) = \min\Big((1-n)/k; (1-p)(\delta \max(n - A, 0) + B)\Big)$. It follows from the latter equations for the equilibrium processes that the effects of investor protection are determined by the excess ownership concentration $n_C^* - n^{bmk}$, as in Section 4. The excess concentration $n_C^* - n^{bmk}$ remains positive with a non-linear control $q(n)$ because the controlling shareholder still has incentives to increase his stock holding relative to the benchmark economy to gain access to stealing opportunities. As a result, the qualitative effects of investor protection are similar to those in Section 4.

Figure IA.2 presents the controlling shareholder's stock holding, gross stock mean-return,

interest rate, and the stock return volatility when the control power is given by $q(n) = \max(n - A, 0)$ for the baseline parameter values and $A = 0.1$. In this economy, stealing is not feasible for atomistic shareholders with small stock holdings. We observe that all economic insights of our main analysis remain valid. In particular, low investor protection increases the stock holding of the controlling shareholder and the stock return volatility, and decreases the expected stock return and interest rate, relative to the full protection benchmark. The main difference with Section 4 is that the equilibrium processes in the economies with weak protection are the same as in the economy with full protection when the consumption share of minority shareholders y is close to 1. This is because the stock holding of the controlling shareholder is small when $y \approx 1$, and hence stealing is not feasible.

For comparison, we next consider an economy where the control power is given by $q(n) = B + n$, and hence the company managers can divert output even with zero stock holdings. Figure IA.3 reports the equilibrium stock holding n_C^*, gross mean-return, interest rate, and volatility in this economy. We observe that the results are qualitatively the same as in our main analysis but the effects of investor protection have smaller magnitudes. In particular, the stock holding n_C^* is lower than that in our main analysis because the controlling shareholder has less incentives to increase stock holdings. The main difference from Section 4 is that now the stock gross return (panel (b)) and interest rate (panel (c)) under weaker protection are lower than those in the economy with full protection even when the stock holding n_{Ct}^* is close to zero around $y \approx 1$.

Finally, we study an economy where the net stolen output is proportional to the stock holding of the controlling shareholder and is given by $(x_t^* - k(x_t^*)^2/2)D_t n_{Ct}^*$. This specification of net diverted output reduces the incentives to divert output when the stock holding n_C^* is low irrespective of the specification of the control power $q(n)$. The equilibrium interest rate and the optimization problem of the controlling shareholder in this new economy differ from those in our main analysis due to the additional non-linearities of the net diverted output. Proposition IA.1 reports the equilibrium.

Proposition IA.1. *In the equilibrium with imperfect protection $p < 1$ and net diverted output $(x_t^* - k(x^*)^2/2)D_t n_{Ct}^*$, the shareholders' optimal consumptions are given by* (13), *the fraction of diverted output is given by equation* (14), *the equilibrium stock mean-return, Sharpe ratio, and volatility are given by equations* (31)–(33) *in Proposition 3, and the risk-free in-*

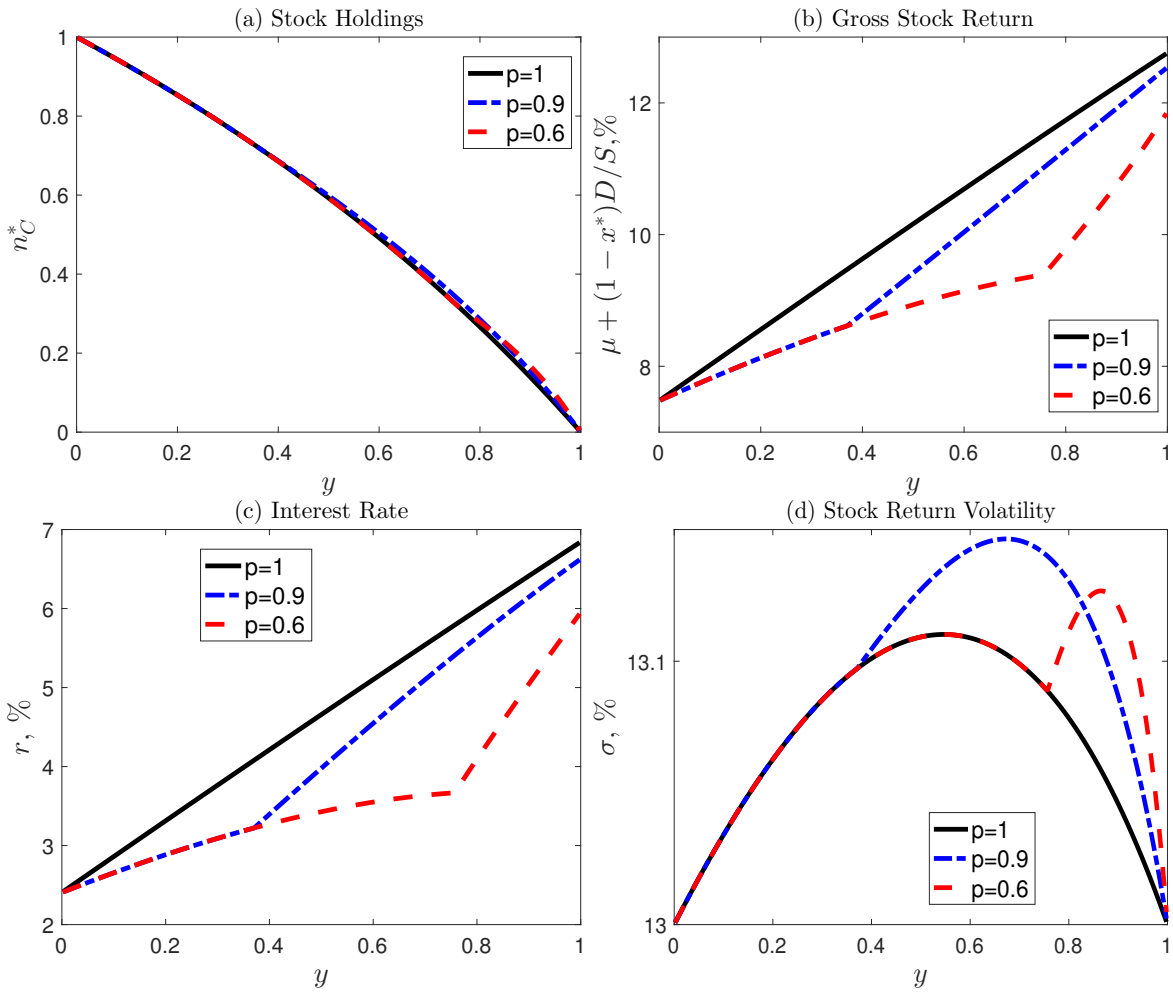

Figure IA.3
Equilibrium processes when control power is given by $q(n) = n + B$
This Figure shows the equilibrium stock holdings, gross stock returns, risk-free interest rates and stock return volatilities for different levels of investor protection p and the baseline parameter values when the control power is given by $q(n) = B + n$, and $A = 0.1$.

terest rate is given by:

$$r_t = r_t^{bmk} - (1 - l_M - l_C)x_t^* n_{Ct}^* \rho^{1/\gamma_C} - (\rho^{1/\gamma_M} - \rho^{1/\gamma_C})(1 - l_M - l_C)\frac{k(x_t^*)^2 n_{Ct}^*}{2} + \frac{(n_{Ct}^* - n_{Ct}^{bmk})\Gamma_t \sigma_D}{(\rho^{1/\gamma_M}(1 - n_{Ct}^*) + \rho^{1/\gamma_C} n_{Ct}^*) n_{Mt}^{bmk}}, \quad \text{(IA.5)}$$

where r^{bmk} is the equilibrium interest rate in the full protection economy given by (25), *and*

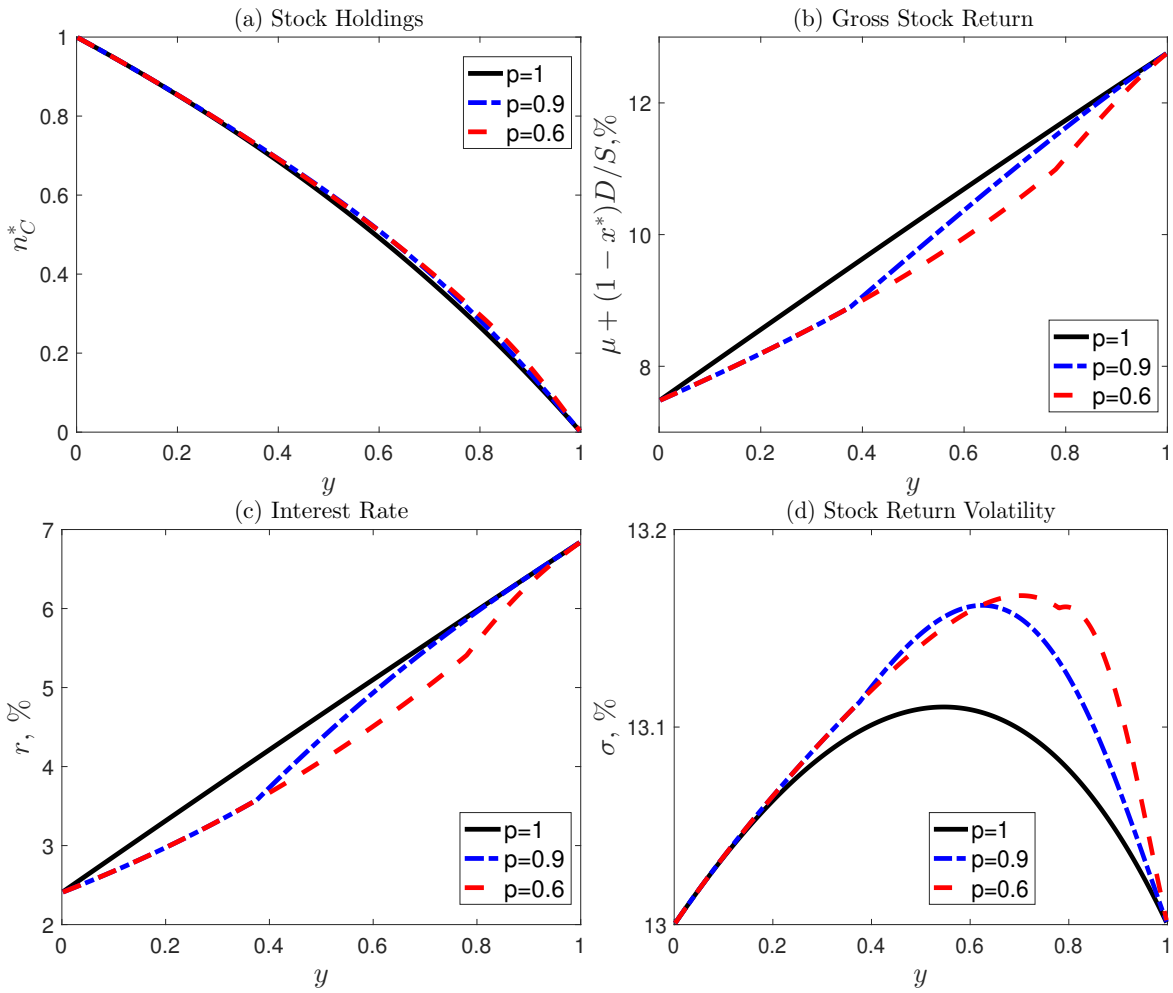

Figure IA.4
Equilibrium processes with weaker incentives to divert output around $n_{Ct}^* = 0$
This Figure shows the equilibrium stock holdings, gross stock returns, risk-free interest rates and stock return volatilities for different levels of investor protection p and the baseline parameter values when the net diverted output is given by $(x_t^* - k(x^*)^2/2)D_t n_{Ct}^*$.

the controlling shareholder's optimal stock holding n_{Ct}^* solves the fixed-point equation

$$\begin{aligned} n_{Ct}^* &= \underset{n_t,\, n_t \le 1}{\operatorname{argmax}} \Big\{ \frac{n_t S_t}{W_{Ct}} \Big(\mu_t - r_t + (1 - x^*(n_t) n_t) \frac{D_t}{S_t} \Big) \\ &\quad + x^*(n_t) n_t \frac{D_t}{W_{Ct}} - \frac{k x^*(n_t)^2 n_t}{2} \frac{D_t}{W_{Ct}} - \frac{\gamma_C}{2} \Big(\frac{n_t S_t}{W_{Ct}} \sigma_t \Big)^2 \Big\}, \end{aligned} \tag{IA.6}$$

where μ_t, r_t and σ_t are equilibrium processes and themselves depend on n_{Ct}^*, and the ratios D_t/W_{Ct} and S_t/W_{Ct} are given by equations (38).

Proposition IA.1 demonstrates that the equilibrium processes have similar structures to those in our main analysis in Section 4. In particular, the processes are driven by the excess

ownership concentration $n_C^* - n^{bmk}$, which is positive in this economy because the controlling shareholder still has incentives to increase stock holdings relative to the benchmark, albeit these incentives are now weaker when n_C^* is small. As a result, the effects of investor protection on equilibrium remain qualitatively the same as in Section 4. However, the magnitudes of the effects are smaller, especially when the stock holding n_{Ct}^* is close to zero, due to reduced incentives to divert output around $n_C^* \approx 0$. Figure IA.4 presents the equilibrium stock holding n_C^*, gross mean-return, risk-free interest rate, and volatility in this economy and demonstrates the robustness of our results.

IA.3 Non-pecuniary Costs of Diverting Output

We here extend our analysis to feature non-pecuniary costs of diverting output, instead of pecuniary costs in our main analysis. The non-pecuniary costs capture the possibility that misuse of control power is more costly in terms of the required effort and reputation, or agents feel disutility from stealing due to social norms that promote fairness, honesty and morality. In particular, Kahneman, Knetsch and Thaler (1986) argue that people have a preference for fairness and provide examples when economic agents commonly allocate resources fairly to others even when they are free to do otherwise. Consequently, the controlling shareholder may not only be limited by laws and regulations that protect minority shareholders but also by social norms that promote fairness. These norms could be self enforced by personal emotions or by others through disapproval, ridicule or ostracism (Posner, 1997). We capture the effect of norms in the controlling shareholder's decision to steal simply by incorporating a disutility from stealing.

The non-pecuniary cost function $f(x, D)$ is the same as the pecuniary cost given by (4). However, this cost is now interpreted as the certainty equivalent of the disutility of stealing and directly affects the controlling shareholder's preferences:

$$V_C(c_t, W_t, W_{t+dt}) = \rho u_C(c_t)dt + (1 - \rho dt)\mathbb{E}_t\Big[u_C(W_{t+dt} - f(x_t, D_t)dt)\Big], \tag{IA.7}$$

where $u_C(c)$ is the utility function in (5). The controlling shareholder maximizes the objective function (IA.7) subject to the investor protection constraint $x_t \leq (1-p)q(n_t)$, maximum share constraint $n_t \leq 1$, and the budget constraint. The utility of the minority shareholder is as before, and the shareholders' budget constraints are as follows:

$$dW_{it} = \Big(W_{it}r_t + n_{it}(S_t(\mu_t - r_t) + (1 - x_t)D_t) - c_{it} + l_i\widehat{D}_t\Big)\, dt + 1_{\{i=C\}}x_tD_t + n_{it}S_t\sigma_t dw_t, \tag{IA.8}$$

and no longer include the cost function $f(x, D)$.

The optimal portfolio n_C^* of the controlling shareholder in the partial equilibrium can be obtained following the same steps as in Proposition 1 and turns out to be the same as for the main model with pecuniary costs in equation (15). This is because the non-pecuniary costs are subtracted from wealth W_{t+dt}, and hence their effect is similar to that of pecuniary costs, which reduce the wealth through the budget constraint (7). However, the dynamics of wealths in the economies with pecuniary and non-pecuniary costs, given by (7) and (IA.8), are different.

The equilibrium can be derived along the same lines as in Proposition 3. The equilibrium processes are again given by equations as in (32)–(36) but without the quadratic costs $k(x_t^*)^2/2$. All economic effects of investor protection on the equilibrium processes remain qualitatively the same as in the main analysis of Section 4. This is because the pecuniary costs directly affect only the interest rate r and drift μ_y, and moreover, have small effects on these processes. In particular, the term $(\rho^{1/\gamma_M} - \rho^{1/\gamma_C})(1 - l_M - l_C)k(x_t^*)^2/2$ through which the costs affect the interest rate (34) has a magnitude smaller than 0.1% for our baseline parameter values.

IA.4 Numerical methods and proofs

Numerical method for the model with cross-section of firms in Section IA.1. For tractability, we solve the optimization of shareholder C subject to the financial wealth constraint (IA.4) using the method of penalty functions. In particular, we incorporate the latter constraint into the utility function by adding a penalty term $\widetilde{\beta}\mathbb{E}[u_C(\widetilde{W})]$, where $\widetilde{\beta} = 10^{-7}$ is a small penalty parameter. Economically, this term captures the disutility of violating constraint (IA.4). We then maximize utility function $u_C(c) + \beta\mathbb{E}[u_C(\widetilde{c})] + \widetilde{\beta}\mathbb{E}[u_C(\widetilde{W})]$, subject to the budget constraint (IA.3) and investor protection constraint $x \leq (1-p)n$. The latter constraint always binds because there are no pecuniary costs of stealing, and hence $x = (1-p)n$. We also use the fact that $\widetilde{W} = \widetilde{c} - xD_1$.

The FOCs with respect to c and $\widetilde{c}$ for both shareholders, stacked together, give equations:

$$\frac{1}{1+r} = \beta\mathbb{E}\left[\frac{u_C'(\widetilde{c}_C^*)}{u_C'(c_C^*)}\right] + \widetilde{\beta}\mathbb{E}\left[\frac{u_C'(\widetilde{c}_C^* - x^*D_1)}{u_C'(c_C^*)}\right] = \beta\mathbb{E}\left[\frac{u_M'(\widetilde{c}_M^*)}{u_M'(c_M^*)}\right], \tag{IA.9}$$

$$\begin{aligned} S_1 &= \beta\mathbb{E}\left[(2-p-2x^*)D_1\frac{u_C'(\widetilde{c}_C^*)}{u_C'(c_C^*)}\right] + \widetilde{\beta}\mathbb{E}\left[(1-2x^*)D_1\frac{u_C'(\widetilde{c}_C^* - x^*D_1)}{u_C'(c_C^*)}\right] \quad \text{(IA.10)} \\ &= \beta\mathbb{E}\left[D_1\frac{u_M'(\widetilde{c}_M^*)}{u_M'(c_M^*)}\right], \end{aligned}$$

$$S_2 = \beta\mathbb{E}\left[D_2\frac{u_C'(\widetilde{c}_C^*)}{u_C'(c_C^*)}\right] + \widetilde{\beta}\mathbb{E}\left[D_2\frac{u_C'(\widetilde{c}_C^* - x^*D_1)}{u_C'(c_C^*)}\right] = \beta\mathbb{E}\left[D_2\frac{u_M'(\widetilde{c}_M^*)}{u_M'(c_M^*)}\right]. \tag{IA.11}$$

Multiplying the budget constraint (IA.3) for shareholder C in turn by Δw_1 and then by Δw_2 and taking expectations, we obtain two equations:

$$\mathbb{E}[\widetilde{c}_C^*\Delta w_1] = (n_{1C}^*(1-x^*)+x^*)D\sigma_D + 0.5n_{2C}^*D\sigma_D, \tag{IA.12}$$

$$\mathbb{E}[\widetilde{c}_C^*\Delta w_2] = 0.5(n_{1C}^*(1-x^*)+x^*)D\sigma_D + n_{2C}^*D\sigma_D. \tag{IA.13}$$

Noting that $n_{1C}^* = x^*/(1-p)$ and solving (IA.12)–(IA.13) we obtain:

$$\left(x^* + \frac{x^*(1-x^*)}{1-p}\right)D\sigma_D = (4/3)\mathbb{E}\left[\widetilde{c}_C^*(\Delta w_1 - 0.5\Delta w_2)\right]. \tag{IA.14}$$

Let $c_M^* = 2Dy$, and define date $t=1$ consumption share $\widetilde{y} = \widetilde{c}_M^*/(\widetilde{D}_1+\widetilde{D}_2)$ in state ω. Then, using the market clearing conditions for consumption, we find $c_C^* = 2D(1-y)$, $\widetilde{c}_C^* = (\widetilde{D}_1+\widetilde{D}_2)(1-\widetilde{y})$. Substituting consumptions in terms of the variables y and $\widetilde{y}$ into equations (IA.9)–(IA.11) and (IA.14), and also solving equations (IA.9)–(IA.11) for marginal utility ratios $u_M'(\widetilde{c}_M^*)/u_M'(c_M^*)$, we obtain the following system of equations for $\widetilde{y}$ and x:

$$\left(\frac{\widetilde{y}}{y}\frac{\widetilde{D}_1+\widetilde{D}_2}{2D}\right)^{-\gamma_M} = \left(\frac{1-\widetilde{y}}{1-y}\frac{\widetilde{D}_1+\widetilde{D}_2}{2D}\right)^{-\gamma_C} + \frac{\widetilde{\beta}}{\beta}\left(\frac{1-\widetilde{y}}{1-y}\frac{\widetilde{D}_1+\widetilde{D}_2}{2D} - \frac{x^*}{1-y}\frac{\widetilde{D}_1}{2D}\right)^{-\gamma_C} + \delta\widetilde{v} \tag{IA.15}$$

$$\delta = \frac{(1-p)\mathbb{E}\left[\frac{\widetilde{D}_1}{D}\left(\frac{1-\widetilde{y}}{1-y}\frac{\widetilde{D}_1+\widetilde{D}_2}{2D}\right)^{-\gamma_C}\right] - x^*\mathbb{E}\left[\frac{\widetilde{D}_1}{D}\left(\frac{\widetilde{y}}{y}\frac{\widetilde{D}_1+\widetilde{D}_2}{2D}\right)^{-\gamma_M}\right]}{1-x^*(1+\mathbb{E}[(\widetilde{D}_1/D)\widetilde{v}])}, \tag{IA.16}$$

$$\left(x^* + \frac{x^*(1-x^*)}{1-p}\right)\sigma_D = (4/3)\mathbb{E}\left[\frac{\widetilde{D}_1+\widetilde{D}_2}{D}(1-\widetilde{y})(\Delta w_1 - 0.5\Delta w_2)\right], \tag{IA.17}$$

where variable $\widetilde{v}$ takes values v_k in states ω_k, given by:

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ \widetilde{D}_1(\omega_1)/D & \widetilde{D}_1(\omega_2)/D & \widetilde{D}_1(\omega_3)/D \\ \widetilde{D}_2(\omega_1)/D & \widetilde{D}_2(\omega_2)/D & \widetilde{D}_2(\omega_3)/D \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

We then solve equations (IA.15)–(IA.17) numerically along the following steps:

(i) For each value of y, consider equation (IA.15) for fixed δ. For each state ω, this is an equation for one variable $\widetilde{y}(\omega)$, which we simply denote as $\widetilde{y}$. Solving it by the method of bisections, we find $\widetilde{y}(\delta; x^*)$.

(ii) Substituting $\widetilde{y}(\delta; x^*)$ into (IA.16), we solve the latter equation using the method of bisections and find $\delta(x^*)$.

(iii) Substituting $\widetilde{y}(\delta(x^*); x^*)$ into (IA.17), by the same method we find x^*.

(iv) Hence, we find date $t = 1$ consumption shares $\widetilde{y}$ in all states ω.

(v) Then, we find $c^*_M = 2Dy$ and $\widetilde{c}^*_M = (\widetilde{D}_1 + \widetilde{D}_1)\widetilde{y}$. Substituting these optimal consumptions into (IA.9)–(IA.11), we find the interest rate r, and stock prices S_1 and S_2.

Proof of Proposition IA.1. The fraction of diverted output is given by $x_t n_{Ct}$, the net diverted output is $(x_t D_t - f(x_t, D_t))n_{Ct}$, and pecuniary cost $f(x_t, D_t)n_{Ct}$ of diversion is paid to the minority shareholder either via a government transfer or by the controlling shareholder. The shareholders maximize preferences (6) over the current consumption and next-period wealth subject to the following dynamic budget constraint:

$$\begin{aligned} dW_{it} &= \Big(W_{it} r_t + n_{it}(S_t(\mu_t - r_t) + (1 - x_t n_{Ct})D_t) - c_{it} + l_i \widehat{D}_t\Big)\, dt \\ &\quad + \big(1_{\{i=C\}}(x_t D_t - f(x_t, D_t)) + 1_{\{i=M\}} f(x_t, D_t)\big)\, n_{Ct} dt + n_{it} S_t \sigma_t dw_t, \quad \text{(IA.18)} \end{aligned}$$

Similar to our main analysis, the optimal consumptions are given by $c^*_{it} = \rho^{1/\gamma_i} W_{it}$. Applying Itô's Lemma to the market clearing condition $c^*_{Ct} + c^*_{Mt} = (1 - l_C - l_M)D_t$ and matching dt and dw terms, following the same steps as in the proof of Proposition 3, we obtain that the equilibrium processes for stock returns, Sharpe ratio, volatility, and the fraction of diverted output are the same as in Proposition 3, the interest rate is given by (IA.5), and the optimization problem of the controlling shareholder is given by (IA.6). ■

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