Optimal Bank Regulation and Fiscal Capacity

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Abstract

Financial regulation is harmonized across countries even though countries vary in their ability to bail-out their banking sector in the event of a crisis. This paper addresses the question of whether countries with different fiscal capacity should optimally have different bank regulation, implemented — among other tools — through capital requirements — a question so far ignored by the theoretical banking literature. I show that countries with larger fiscal capacity should have lower ex-ante minimum bank capital requirements, in an environment with endogenously incomplete markets and overinvestment due to "Too-Big-To-Fail" moral hazard and pecuniary externalities. I also show that, in addition to a minimum bank capital requirement, regulators in countries with strong "Too-Big-To-Fail" moral hazard should impose a limit on the liabilities pledged by financial institutions in a crisis state. This implies limits on put options/CDS contracts. Finally, I argue that the type of regulatory instrument used is crucial as to whether larger fiscal capacity implies more or less stringent bank regulation.

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1 Introduction

While financial regulation across countries has become even more harmonized, the bail-out that banks receive in the event of a crisis remains tightly linked to the fiscal capacity of the country.[1] The last crisis provided plenty of evidence that countries vary widely in their abilities to provide a financial sector bail-out. Ireland recapitalized its banks in 2009 which turned out to be prohibitively costly and led to a sovereign debt crisis and an IMF/EU government bail-out. Iceland did not even try to bail-out its failing banking sector as it was "Too-Big-To-Save". In contrast, the US provided a substantial bail-out to its financial sector which had no impact on the governments' ability to borrow at very low rates.

Switzerland, whose financial sector assets in 2007 were 650 percent of the GDP of the country, became more concerned about its ability to provide a bank bail-out.[2] The bail-out of UBS prompted Swiss regulators to more than double the minimum bank capital requirements for systemically important banks.[3]

This paper addresses the question of whether governments with different abilities to bail-out their banking system during a financial crisis (different fiscal capacity) should have more- or less-stringent ex-ante regulation, which limits the amount of risky investment.[4] It also examines what other regulation, such as derivatives regulation, is required, given the fiscal capacity of the country and bank size. The reason why regulation is needed in the first place is due to "Too-Big-To-Fail" moral hazard and pecuniary externalities. I study how these externalities interact with the fiscal capacity of the country and with the optimal amount of regulation. I consider two types of policy instruments, which control the amount of risky investment — a "quantity" instrument such as a minimum bank capital requirement versus a "price" instrument such as a tax on investment. I show

[1] Many countries (and almost all advanced economies) impose the minimum bank capital ratio suggested by the Basel Accords as a minimum regulatory standard (See Figure 1 in the Appendix). Kalemli-Ozcan, Papaioannou, and Peydro (2010) provide evidence of the synchronization of financial sector regulation in the European Union. More recently, the establishment of the Banking Union in the European Union in 2012 has promoted further harmonization of bank regulation. (See "Europe's Radical Banking Union," Veron, p.11). However, an agreement on common deposit insurance and bail-out scheme remains elusive.

[2] In contrast, the same number for the US for 2007 was 330 percent of the US GDP. The data source is the "Global Shadow Banking Monitoring Report 2014," and the financial sector includes the following categories: banks, public financial institutions and other financial intermediaries (OFIs) and financial auxiliaries.

[3] Swiss systemically important banks now face a minimum bank capital ratio of up to 19 percent, which is a significant increase relative to the 8 percent pre-crisis level (See "Regulatory Consistency Assessment Programme (RCAP); Assessment of Basel III regulations – Switzerland, BIS Report, 2013").

[4] The interpretation of the results is not limited to banks. The results derived in this paper apply to any financial institution or firm that, upon fireselling assets, generates significant dead-weight loss to society to warrant a government bail-out.

that the type of regulatory instrument — "quantity" versus "price" instrument — is crucial as to whether larger fiscal capacity implies more or less regulation.

I build a model, in which markets are endogenously incomplete due to friction in the spirit of Hart and Moore (1994). Bankers can borrow using state-contingent debt, but they can run away with the cash flow, which generates endogenous borrowing constraints. In a crisis, bankers are forced to sell part of their capital stock to foreign arbitrageurs, which leads to fire sales. The government provides an optimal bail-out to the bankers by taxing the consumers. However, taxation is costly since the government has access only to distortionary labor taxes. I also assume that there is an additional exogenous deadweight loss from collecting taxes, which proxies the ability of the government to enforce tax collection and the degree of government inefficiency and corruption.

I define fiscal capacity as the marginal cost of an extra dollar of a bail-out, for a given level of an aggregate bail-out. Given this definition, endogenously, a country has a larger fiscal capacity if it has a more productive labor-intensive sector and lower dis-utility of labor, which is a proxy for a large tax base. Alternatively, the fiscal capacity of a country is larger if the government is more efficient and less corrupt. Thus, we can think of fiscal capacity consisting of two components: GDP size and efficiency of tax collection.

First, I consider a model where banks are infinitesimally small. In this model, the presence of fire sales generates inefficient pecuniary externalities in the spirit of Lorenzoni (2008), which lead to ex-ante overinvestment. Bankers do not internalize the fact that the more they invest ex-ante, the larger the fire sale of financial assets is during a future crisis, which tightens the budget constraints of the other bankers. This channel is welfare-reducing because it increases the inefficient transfer of capital from the bankers to the foreign arbitrageurs. When banks are infinitesimally small a single instrument that regulates ex-ante investment is sufficient to eliminate the externalities. Furthermore, conditional on being able to set ex-ante policy optimally, there would be no welfare improvement if the bail-out was determined ex-ante — i.e., if there was a commitment mechanism. Therefore, according to this model, the regulatory focus should be on making sure banks are well regulated before the crisis rather than to tie the hands of governments in the middle of the crisis.

In this framework, one can prove the following result — that countries with smaller fiscal capacity should have higher ex-ante minimum bank capital ratio (regulators should require banks to finance a larger fraction of the risky investment using equity). The intuition is as follows: For a given level of productive capital, countries with smaller fiscal capacity will be less able to support their banks during a crisis, and hence fire sales of banks' assets will be larger. Therefore, the constrained Central Planner in more fiscally constrained countries perceives ex-ante investment as less attractive, and he optimally chooses to invest less, relative to the constrained Central Planner of a country with a larger fiscal capacity. Since the ex-ante investment chosen by the constrained Central Planner and the optimal minimum bank capital ratio are inversely related, smaller fiscal capacity implies a higher ex-ante minimum bank capital ratio. In summary, countries with larger fiscal capacity can prop up asset prices more during a crisis and can alleviate any inefficiencies arising from fire sales. As a result, they can "afford" to have larger investment booms ex-ante.

Second, I augment the model to allow for large banks, which introduces a second source of ex-ante inefficiency, in addition to the inefficient pecuniary externalities, — "Too-Big-To-Fail" moral hazard. When banks are large, and they anticipate a bail-out in the future, they internalize the fact that the more they invest ex-ante, the larger the aggregate fire sale during a crisis is, which leads to a bigger bail-out ex-post. However, unlike the constrained Central Planner, they do not internalize the cost of the bail-out. As a result, large bankers underestimate the social cost of the fire sale which leads to ex-ante overinvestment.

In the large banks' case, the main result derived in the small banks' case still holds. Namely, that conditional on assuming that the policy maker has a sufficient number of instruments to replicate the constrained Central Planner's allocation, countries with smaller fiscal capacity should have a higher ex-ante minimum bank capital ratio. The intuition why that is the case is the same as in the small banks' case.

A second result emerges; in addition to imposing a minimum bank capital requirement, countries with strong "Too-Big-To-Fail" moral hazard should also impose a limit on the liabilities of large banks in a future crisis state, when a government bail-out is anticipated. The intuition is the following. If the banker promises a larger payment in a crisis, his net worth in a crisis is smaller and the fire sale is more severe. Since large banks internalize the fact that larger fire sale implies a bigger bail-out, they might value wealth more in normal times than in a crisis. As a result, large banks might try to shift risk using state-contingent contracts, when they are prevented from increasing their risky investment further.[5] The desire to over-borrow against a crisis state is stronger, the larger the perceived marginal increase of the bail-out is when the fire sale increases; i.e. the stronger the "Too-Big-To-Fail" moral hazard is. Larger fiscal capacity does not always imply a stronger "Too-Big-To-Fail" moral hazard. The "Too-Big-To-Fail" moral hazard is strong if a country has a more productive labor-intensive sector and lower dis-utility of labor — it has a large fiscal capacity due to a large tax base. However, if the fiscal capacity is large because governments are more efficient and less corrupt, then the "Too-Big-To-Fail" moral hazard is weak rather than strong.[6]

According to the results of this paper, the "sufficient" statistic which regulators should target is the net assets of the banking sector in a potential future crisis, when a bail-out will be required. There are various ways for financial institutions to affect their net worth in a crisis, with derivative

[5]In contrast, infinitesimally small banks don't internalize that larger fire sale leads to a larger bail-out. As a result, they always value wealth in a crisis by more than during normal times, similarly to the Central Planner, which implies that limiting the liabilities of small banks against the bad state of nature is not needed.

[6]AIG is a prime example of a large financial institution located in a country with a large tax base (the US), which shifted risk using state contingent contracts. It sold insurance against CDO defaults, where CDOs were the financial instruments at the epicentre of the 2008 Global Financial Crisis. AIG ended up being bailed out by the US government.

instruments being one of the most effective ways. Therefore, in addition to imposing minimum bank capital requirements, the regulators of countries with strong "Too-Big-To-Fail" moral hazard should, among other measures, limit the sale of put option contracts by banks, such as credit default swap (CDS) contracts. Finally, according to the results of this model, regulating derivative contracts is important even if there is no counterparty risk.

One of the key results of the paper is that larger fiscal capacity implies an optimally lower minimum bank capital requirement, which is a "quantity" regulatory instrument. An interesting question to ask is whether fiscal capacity implies more or less regulation if one uses a "price" instrument instead to regulate the banks' period zero investment, such as an ex-ante tax on investment. "Price" regulatory instruments have entered the policy debate on bank regulation more recently (see, for example, Mooij and Nicodeme (2014)). Moreover, comparing the behavior of the two types of instruments will shed light on the seemingly surprising result that countries with stronger "Too-Big-To-Fail" moral hazard due to a larger tax base would require lower (rather than higher) ax-ante minimum bank capital ratio.

The result emerges that if regulators use a "price" instrument, larger fiscal capacity due to a larger tax base would imply a higher optimal tax on period zero investment when banks are large. In contrast, if fiscal capacity is larger due to the government being more efficient at collecting taxes, the optimal tax on period zero investment is lower.

The difference in the comparative statics of the "price" and "quantity" instruments can be attributed to the fact that the "quantity" instrument is a function only of the constrained Central Planner's allocation and, therefore, does not depend on the type and the strength of the externalities in the model. In contrast the "price" instrument, which is a Pigouvian tax, equals the difference in the perceived marginal valuations of ex-ante investment between the banker and the Central Planner. The reason why the two marginal valuations differ is due to the presence of pecuniary externalities and "Too-Big-To-Fail" moral hazard. Therefore, the comparative static of the optimal ex-ante tax on capital with respect to fiscal capacity reflects how the fiscal capacity affects the strength of these two externalities. This is why stronger externalities lead to higher "price" regulatory instruments but not to higher "quantity" regulatory instruments.[7]

The paper is related to a few strands of literature. It features a "Too-Big-To-Fail" moral hazard, where the size of the banks and their strategic behavior play a crucial role. Therefore, it is related to the literature on moral hazard, pioneered by the seminal work of Bagehot (1873). Nosal and Ordonez (2016) also emphasize the potential adverse effects of large banks in a moral hazard framework, albeit due to different channels. In general, the literature that studied how moral hazard relates to bank size is underdeveloped. The first contribution is to define the "Too-Big-To-Fail" moral hazard within a

[7] In a model with pecuniary externalities Bianchi (2011) also finds that the behavior of "price" and "quantity" regulatory instruments differ as one varies the amount of debt.

model. Second, I study how fiscal capacity affects the strength of the "Too-Big-To-Fail" moral hazard and show that it is not always the case that larger fiscal capacity implies stronger "Too-Big-To-Fail" moral hazard. Lastly, I show that this type of moral hazard can generate the need to regulate state contingent contracts such as derivative contracts.

The second main source of inefficiency in the model – inefficient pecuniary externalities – dates back to Hart (1975), Stiglitz (1982) and Geanakoplos and Polemarchakis (1986). More recently, a growing literature on financial sector regulation has emerged, which has brought the role of fire sales and pecuniary externalities to the forefront of the policy debate (some prominent examples include Gromb and Vayanos (2002), Lorenzoni (2008), Stein (2012), Jeanne and Korinek (2017), He and Kondor (2012), Brunnermeier and Sannikov (2014), Bianchi and Mendoza (2018)). I build on a simplified version of the paper by Lorenzoni (2008), who shows how pecuniary externalities can emerge in a micro-founded environment with endogenously incomplete markets.[8] The key difference between this paper and Lorenzoni (2008) is that he does not allow for an ex-post bail-out and, as a result, does not study the link between fiscal capacity and optimal regulation. Also Lorenzoni (2008) does not allow for a concentrated banking sector and optimal policy plays a minor role in his paper.

This paper also relates to the literature on different types of regulatory instruments, pioneered by Weitzman (1974). According to Weitzman (1974), if the policy maker has access to state-contingent policy instruments, she can replicate the constrained Central Planner's allocation using either a "price" or a "quantity" instrument. Relative to the existing literature, which uses the two types of instruments interchangeably, I argue that there are significant differences in the comparative statics of these instruments with respect to key parameters, such as fiscal capacity.

To my knowledge, the only other paper that studies the mix of ex-ante regulation and optimal ex-post bailouts, and features both pecuniary externalities and moral hazard is the one by Jeanne and Korinek (2017). In contrast to this paper, in Jeanne and Korinek (2017), markets are exogenously incomplete, and there is no state-contingent borrowing and, therefore, no need to regulate derivative-like contracts. Their model also does not consider the case of a concentrated banking sector and, hence, does not feature a "Too-Big-To-Fail" type of moral hazard and bail-outs are non-targeted. Most importantly, Jeanne and Korinek (2017) do not focus on the fundamental question raised by this paper: How should the optimal mix of ex-ante and ex-post bank regulation vary with the country's fiscal capacity and why?

A class of papers studies the role for regulating state-contingent borrowing. In Nosal and Ordonez (2016), more risk sharing implies that upon observing a bank failure, the government assumes that it is caused by an aggregate shock with a higher probability, which increases the likelihood of bailing-out

[8] There is a large literature on pecuniary externalities in which the inefficiency comes from binding borrowing constraints, where prices enter the borrowing constraint (for example, Stein (2012), Bianchi (2011)). In this paper, as in Lorenzoni (2008), the source of the pecuniary externality is that bankers do not internalize the fact that their actions are tightening the *budget* constraints (not the *borrowing* constraints) of the other bankers.

the first bank that gets into trouble. This increases ex-ante risk-taking. Simsek (2013) also argues that too much financial innovation might be suboptimal in a model with heterogeneous traders' beliefs since it can increase the traders' portfolio risk.[9] In this paper, I provide a different reason as to why too much state-contingent borrowing might be inefficient. Also, I argue that not all types of state-contingent debt need to be regulated — only the borrowing against states of nature where there will be a bail-out.

The paper is structured as follows. In section 2 I present the model set-up. Section 3 discusses the case where banks are infinitesimally small. In sub-section 3.2, I study the solution to the constrained Central Planner's problem while in sub-section 3.3, I answer the question how the constrained Central Planner's allocation can be decentralized and how the optimal policy instruments should vary with the fiscal capacity of the country when banks are small. Section 4 allows for banks to be large and discusses how the results change and section 5 concludes.

2 Model Set-Up

The model has three periods, $t = 0, 1, 2$, and, without loss of generality, I assume that there is no discounting between the periods. There are two goods — a perishable consumption good and a capital good, where a perfectly competitive capital goods sector produces the capital good. There are four main types of agents in the model — domestic consumers, domestic bankers, modeled as entrepreneurs, foreign arbitrageurs, and a bail-out authority which designs optimal bail-outs/transfers to the financial system. There are also a constrained Central Planner and a policy maker, who is responsible for decentralizing the constrained Central Planner's allocation. All agents are risk neutral, and there is aggregate uncertainty only in $t = 1$. In $t = 1$, the state of nature can be either good or bad with probabilities π_g and π_b, respectively, where $\pi_b + \pi_g = 1$. The state of nature will be bad when the banker's productivity is low and good when the banker's productivity is high. Throughout the paper, I will use the notation $x_{t,s}$ to denote the variable x in period t if state s is realized in $t = 1$, where $s \in \{g, b\}$. To simplify the notation, I assume that $x_{0,s} = x_0$. This paper considers two different banking structures – a continuum of banks and large banks.

2.1 Consumers

The domestic consumers are identical, infinitesimally small and of measure one. They consume and lend to/borrow from banks using state-contingent contracts. Every consumer also operates a labor-

[9] Regulating the amount of safe assets is another way to ensure that financial institutions have sufficient net worth in a crisis, in addition to controlling debt against the crisis state. Some of the papers that find a role for a minimum liquidity regulation are Farhi and Tirole (2012), Acharya, Shin, and Yorulmazer (2011), Repullo (2005), Bengui (2014) and Keister (2016).

intensive production technology and owns an equal share of the equity of the firms producing the capital good.

Since the problem of the consumer is time-consistent, without loss of generality, one can solve for the period zero problem under commitment. In $t = 0$, the representative consumer maximizes

$$\max_{d^c_{t,s}, c^c_{t,s}, l_{t,s}} \left(c^c_0 - \omega l_0\right) + \sum_{t=1}^{2} \sum_{s} \pi_s \left(c^c_{t,s} - \omega l_{t,s}\right)$$

The optimization problem is subject to the consumer's budget constraints in $t = 0, 1, 2$

$$\begin{aligned} c^c_0 + \sum_s p_{1,s} d^c_{1,s} &\leq m + \tilde{\pi}_0 + (1 - \tau_0)\, a l^\alpha_0 + d^c_0 \\ c^c_{1,s} + p_{2,s} d^c_{2,s} &\leq m + \tilde{\pi}_{1,s} + (1 - \tau_{1,s})\, a l^\alpha_{1,s} + d^c_{1,s} \\ c^c_{2,s} &\leq m + \tilde{\pi}_{2,s} + (1 - \tau_{2,s})\, a l^\alpha_{2,s} + d^c_{2,s} \end{aligned}$$

where $c^c_{t,s}$ is consumption in period t, state s. The consumer's production technology is given by $a l^\alpha_{t,s}$ and it has decreasing returns to scale $(0 < \alpha < 1)$, where a is the time-invariant productivity of the labor-intensive sector and $l_{t,s}$ is the labor supplied by the consumer. The amount of state-contingent debt purchased by the consumer at the price $p_{t,s}$ is $d^c_{t,s}$. The dis-utility from labor is ω and $\tau_{t,s}$ is a distortionary labor tax such that $0 \leq \tau_{t,s} \leq 1$. Finally, $\tilde{\pi}_{t,s}$ are the profits from the firms producing the capital good. Every period, the representative consumer receives an exogenous endowment of the consumption good, m, which is assumed to be large enough so that $c^c_{t,s}$ is always positive in equilibrium and the corner equilibrium $(c^c_{t,s} = 0)$ does not exist. The assumptions on m are formally stated in the Appendix under assumptions 3, 5 and 8. If these assumptions are satisfied, the first order conditions with respect to the state-contingent debt imply $p_{1,s} = \pi_s$ and $p_{2,s} = 1$. The first order condition with respect to $l_{t,s}$ implies

$$l^*_{t,s}\left(\tau_{t,s}\right) = \left(\frac{\left(1 - \tau_{t,s}\right) \alpha a}{\omega}\right)^{\frac{1}{1-\alpha}}, \tag{1}$$

where $l^*_{t,s}\left(\tau_{t,s}\right)$ stands for the optimal labor allocation as a function of the labor taxes. The standard trade-off of distortionary labor taxation is apparent; higher taxes lead to lower labor supply and lower output. I define the welfare gain to consumers from operating the labor-intensive production technology as

$$e\left(\tau_{t,s}\right) = \left(1 - \tau_{t,s}\right) a \left(l^*_{t,s}\left(\tau_{t,s}\right)\right)^\alpha - \omega l^*_{t,s}\left(\tau_{t,s}\right). \tag{2}$$

2.2 Perfectly Competitive Capital Goods Sector

Assume that there is a continuum of firms of measure one, which are perfectly competitive, and can produce new capital stock by transforming consumption into capital one-to-one but not the other way round (capital is irreversible). They have a static optimization problem given by $\max_{k^o_{t,s}} q_{t,s}k^o_{t,s} - k^o_{t,s}$ subject to the constraint that the capital produced is positive, $k^o_{t,s} \geq 0$, with a Lagrange multiplier $\mu^o_{t,s}$. The price of capital is $q_{t,s}$ and the first order condition implies $q_{t,s} = 1 - \mu^o_{t,s} \leq 1$. If new capital stock is produced, then $\mu^o_{t,s} = 0$ and the price of capital is $q_{t,s} = 1$. If no new capital stock is produced, then $\mu^o_{t,s} > 0$ and $q_{t,s} < 1$. In equilibrium, profits are zero.

2.3 Foreign Arbitrageurs

Foreign arbitrageurs are infinitesimally small and have a mass of η^f. In period t and state s the representative arbitrageur invests $k^f_{t,s}$ units of capital, which produce $F\left(k^f_{t,s}\right)$ units of the consumption good in the following period. Upon production, capital depreciates one hundred percent. Given that the problem is time consistent, in $t = 0$, the representative arbitrageur maximizes

$$\max_{c^f_{t,s},k^f_{t,s}} c^f_0 + \sum_{t=1}^{2}\sum_{s} \pi_s c^f_{t,s},$$

where $c^f_{t,s}$ is his consumption. The optimization problem is subject to the budget constraint $c^f_{t,s} \leq F\left(k^f_{t-1,s}\right) + m^f - q_{t,s}k^f_{t,s}$ and the non-negative capital constraint $k^f_{t,s} \geq 0$ with a Lagrange multiplier $\mu^f_{t,s}$. The arbitrageur receives an exogenous per period endowment of the consumption good equal to m^f, where I assume that $k^f_0 = 0$ and m^f is such that, in equilibrium, the arbitrageur consumes a positive amount every period. The first order conditions imply $F'\left(k^f_{t,s}\right) + \mu^f_{t,s} = q_{t,s}$. Therefore, if the arbitrageur employs his production technology, then $\mu^f_{t,s} = 0$ and $F'\left(k^f_{t,s}\right) = q_{t,s}$.

One can assume that the arbitrageurs are domestic rather than foreign agents and that the Central Planner assigns a positive weight to the arbitrageur's welfare. All the results would go through since the marginal productivity of the arbitrageurs is assumed to be lower than the one of the bankers, which is sufficient for the presence of inefficient pecuniary externalities as in Lorenzoni (2008).

2.4 Bankers

The domestic banking sector is of measure one, and the banker can be large or infinitesimally small depending on the specification considered. In this sub-section, I introduce the general set-up of the banker's problem. The first order conditions in the case of small and in the case of large banks are relegated to sections 3 and 4, respectively.

2.4.1 No Ex-Ante Regulation

First, consider the banker's problem in the presence of ex-post bail-outs but no ex-ante regulation. The banker's problem is solved via backwards induction. At the end of periods $t = 0, 1, 2$, banker i

maximizes the net present value of his expected future consumption, subject to the budget constraint in period t, where the respective budget constraints for each period are given by

$$\begin{aligned} c_0^i + q_0 k_0^i &\leq n_0 + \sum_s p_{1,s} d_{1,s}^i + T_0^i \text{ in } t = 0 \\ c_{1,s}^i + q_{1,s} k_{1,s}^i + d_{1,s}^i &\leq A_{1,s} k_0^i + (q_{1,s} - \gamma)\, k_0^i + T_{1,s}^i + p_{2,s} d_{2,s}^i \text{ in } t = 1 \\ c_{2,s}^i + d_{2,s}^i &\leq A_{2,s} k_{1,s}^i + T_{2,s}^i \text{ in } t = 2. \end{aligned} \tag{3}$$

Banker i chooses his consumption, $c_{t,s}^i$, state-contingent debt $d_{t+1,s}^i$, and capital stock, $k_{t,s}^i$. The variable $T_{t,s}^i$ stands for bank specific transfers from the bail-out authority to banker i, γ is a refinancing cost, which will be specified later on, and n_0 is the period zero bank endowment, which also stands for bank equity. Each banker has an access to a linear production technology which produces $A_{t+1,s} k_{t,s}^i$ units of the consumption good the following period.

The differences between the budget constraints in $t = 0, 1, 2$ are due to the following assumptions. For simplicity, I assume that there is no pre-existing capital stock and debt in $t = 0$ and every banker starts with the same exogenous endowment and receives no endowment in $t = 1, 2$. Since capital produces with a lag and the world ends in $t = 2$, the value of the collateral in $t = 2$ is $q_{2,s} = 0$ and it is not optimal to refinance the project in $t = 2$. In contrast, given assumption 2 specified below, it will always be optimal for the banker to refinance the capital stock in $t = 1$.

The banker is modelled as an entrepreneur, which is equivalent to assuming that there are no frictions between the banker and the firm. Bank loans tend to be more flexible and more "equity-like" relative to other types of debt financing due to the long term relationship nature of the contract. For example, in addition to loans, banks provide lines of credit and tend to insure their borrowers against certain types of shocks (see Berger and Udell (1995), Kashyap, Rajan, and Stein (2002) and Gatev and Strahan (2006)). Furthermore, bank loan covenants tend to be more flexible than corporate debt covenants (see the literature review in Denis and Mihov (2003)).

In order for the model to have fire sales — a realistic feature of the data — I assume that for capital to remain productive in the next period, it has to be refinanced at the cost of $\gamma k_{t,s}^i$, where $\gamma > 0$. One can justify the assumption with the fact that loans are often accompanied by promises of future lines of credit, which could force banks to fire-sell assets.[10]

Alternatively, one could change the model to allow for a liquidity shock in the form of a wholesale funding freeze. To do that one would have to assume that banks borrow using short-term debt contracts. While realistic and interesting, such a set-up would imply that the contract between the banker and the consumer is no longer endogenous and state-contingent and would introduce bank

[10] Ivashina and Scharfstein (2010) show that immediately after the Lehman Brothers failure, there was a spike in commercial and industrial loans due to firms drawing down their credit lines which would be consistent with the refinancing cost in the model. For empirical evidence on banks fire-selling assets, see Irani and Meisenzahl (2017) and Duarte and Eisenbach (2018), among others.

default. State contingent contracts proxy the growing use of derivative contracts and key results in the model rely on the state-contingent nature of contracts. Furthermore, introducing default will make the model significantly less tractable.

Borrowing constraints are another very important feature in the data, in addition to fire sales, which are also necessary for the model to have an inefficient decentralized allocation and, hence, a role for regulation. In particular, I assume that the banker faces endogenous borrowing constraints due to agency frictions. The banker can run away with the cash flow but the consumer can seize the capital stock and sell it after paying the refinancing cost. I assume that when the amount owed is higher than the resale value of the capital stock minus the refinancing cost, the banker makes a "take-it-or-leave-it" offer to the consumer and has a commitment device that allows him to follow through on his costly threat. He offers to pay only the amount equal to the resale value of the capital stock minus the refinancing cost. Given that the consumer cannot seize any of the cash flow, he accepts the offer. Therefore, the banker can borrow only against the collateral he owns. Such a set-up is a good proxy for the empirical observation that the majority of bank loans are collateralized.[11]

Therefore, endogenously, the banker will face the following borrowing constraints:

$$
\begin{aligned}
d_{1,s}^{i} &\leq \left(q_{1,s}-\gamma\right) k_{0}^{i} \text{ in } t=0 \text{ for } s \in \{g,b\} && (4)\\
d_{2,s}^{i} &\leq 0 \text{ in } t=1. && (5)
\end{aligned}
$$

Finally, the banks' optimization problem is also subject to the non-negative consumption and investment constraints $c_{t,s}^{i} \geq 0$ and $k_{t,s}^{i} \geq 0$ and the future best response functions of all the agents in the economy in periods greater than t.

2.4.2 Ex-Ante Regulation

In this sub-section I specify the set of ex-ante regulatory instruments. The instruments are chosen to allow the policy maker to replicate the constrained Central Planner's allocation.

Every banker faces either a minimum bank capital requirement constraint – a "quantity" instrument – or a tax on capital – a "price" instrument. The minimum bank capital requirement is modeled as banker $i's$ period zero optimization problem being subject to the following constraint $k_0^i \leq \frac{n_0}{\rho^i}$, which implies that at least a fraction ρ^i of bank capital has to be financed using bank equity. I also consider a tax on period zero capital, $\tau_0^{k,i}$, instead of a minimum bank capital requirement. It affects the effective period zero price of capital, which becomes $q_0\left(1+\tau_0^{k,i}\right)$.[12]

[11] Paravisini (2008) documents the importance of borrowing constraints for banks while Berger and Udell (1990) argue that the majority of bank loans are collaterized.

[12] A minimum bank capital requirement is the instrument currently employed by regulators worldwide. However, policy makers and the literature on pecuniary externalities (see Stein (2012), Bianchi (2011) and Jeanne and Korinek (2017)) have recently suggested the use of "price" rather than "quantity" instruments.

The second type of ex-ante regulation the banker faces is a quantity constraint on the amount of period zero state-contingent debt against the bad state given by $d^i_{1,b} \leq \nu^i$. Such a regulation implies that banker i cannot issue debt larger than ν^i against the bad state and can be interpreted as a constraint on the number of derivative instruments sold, such as sovereign or bank credit default swaps.

2.5 Bail-Out Authority

The role of the bail-out authority is to choose the optimal transfers from the consumers to the bankers – the optimal bail-out. The bail-out authority can be thought of as the fiscal authority of the country, which historically has been responsible for bailing out the financial sector. It places an equal weight on consumers and bankers and its ex-ante welfare is given by

$$\left(c^c_0 - \omega l^*_0\left(\tau_0\right) + c_0\right) + \sum_{t=1}^{2}\sum_{s}\pi_s\left(c^c_{t,s} - \omega l^*_{t,s}\left(\tau_{t,s}\right) + c_{t,s}\right). \tag{6}$$

The bail-out authority chooses $\tau_{t,s}$ and $T_{t,s}$ and it internalizes the fact that transfers are costly which is captured by the constraint that the amount transferred to banks has to be funded by costly labor taxes

$$T_{t,s} \leq \chi\tau_{t,s}al^{\alpha}_{t,s}, \text{ where } 0 \leq \chi \leq 1. \tag{7}$$

The parameter χ captures the fact that, in some countries, part of the collected taxes will be dissipated due to corruption or tax evasion. I assume that the diverted and non-paid taxes do not increase welfare. Therefore, $(1-\chi)\,\tau_{t,s}al^{\alpha}_{t,s}$ is an exogenous deadweight loss from taxation, which is in addition to the deadweight loss from distorting the labor supply decision. Without loss of generality, I assume that the bail-out authority can provide transfers only in period one (i.e., I assume that $T_0 = T_{2,s} = \tau_0 = \tau_{2,s} = 0$).

2.6 Constrained Central Planner and Policy Maker

This sub-section specifies the problem of the constrained Central Planner. The solution to the Central Planner's problem, when compared to the decentralized allocation, allows me to analyze the presence of externalities in the model and to solve for the optimal regulation. Given that the paper focuses on externalities in the banking sector, the benevolent Central Planner is allowed to choose the allocation of the bankers and also the choice variables of the bail-out authority. This set-up is equivalent to designing a centralized financial sector regulation and bail-out policy. The allocation of the constrained Central Planner can be interpreted as the highest social welfare attainable if all externalities due to the behavior of the bankers can be eliminated and costly bail-outs are available.

Similarly to the bail-out authority, the constrained Central Planner places equal weights on consumers and bankers. The Central Planner faces the same constraints as the bankers and the bail-out authority.

The constrained Central Planner's ex-ante welfare is also given by equation (6), where the Central Planner's choice variables are $\tau_{t,s}, T_{t,s}, k_{t,s}, c_{t,s}$ and $d_{t+1,s}$. The Central Planner internalizes the banker's budget and borrowing constraints given by equations (3), (4) and (5), the market clearing conditions and the first order conditions of the consumers, arbitrageurs and the firms producing the capital good. Similarly to the bail-out authority, she also internalizes equation (7).

The role of the policy maker in the model is to choose the ex-ante regulatory instruments, specified in sub-section 2.4.2, and the level of labor taxes and bank transfers to decentralize the constrained Central Planner's allocation. One can think of this set-up as the policy maker regulating both the bankers and the bail-out authority. I will prove that a subset or all of the specified policy instruments are sufficient to replicate the constrained Central Planner's allocation. The number and type of instruments required will depend on whether we consider the continuum of banks' case or the large banks' case and also on the fiscal capacity of the country.

2.7 Key Assumptions

In addition to the assumptions made thus far, I make the following assumptions, which simplify the analysis and ensure that the problem is well behaved and that there are fire sales.

Assumption 1 *The assumptions on the banker's production technology are the following*

$$\begin{aligned}\sum_s \pi_s A_{1,s} &> 1; \quad A_{2,s} = A_2 > 1 \\ 0 &\leq A_{1,b} < \gamma; \quad \chi A_2 \leq 1 \\ \gamma &\leq G \text{ where } G = \frac{1 - A_{1,b} + \frac{\gamma}{\pi_b}}{\frac{\pi_g}{\pi_b} A_{1,g} + 1}.\end{aligned}$$

To capture the idea that after the crisis the economy converges to a steady state, regardless of which state was realized in $t = 1$, I assume that period two productivity is not state-contingent $A_{2,s} = A_2$. The fact that $\sum_s \pi_s A_{1,s} > 1$ and $A_2 > 1$ ensure that it is optimal for the banker to always invest a positive amount. The condition $0 \leq A_{1,b} < \gamma$ is necessary but not sufficient for there to be a fire sale in the bad state. Combining $A_{1,b} < \gamma$ and $\sum_s \pi_s A_{1,s} > 1$ implies that $A_{1,g} > 1$, which will be a necessary condition for there to be no fire sale in the good state. All the results in the paper go through without assuming $\chi A_2 \leq 1$ but if imposed, this assumption guarantees that there will be no bail-out in $t = 1$ unless there is a fire sale. Finally, $\gamma \leq G$ is a necessary but not a sufficient condition for the existence of an interior equilibrium.

The production technology of the arbitrageurs is assumed to satisfy the following properties.

Assumption 2 *The assumptions on the arbitrageur's production technology are the following*

$$\begin{aligned} F'\left(k_{t,s}^f\right) &> 0;\quad F''\left(k_{t,s}^f\right) < 0 \\ F'(0) &= 1;\quad \lim_{k_{t,s}^f \to \infty} F'\left(k_{t,s}^f\right) \geq \gamma \\ F'\left(k_{t,s}^f\right) + F''\left(k_{t,s}^f\right) k_{t,s}^f &> 0 \\ 2F''\left(k_{t,s}^f\right) + F'''\left(k_{t,s}^f\right) k_{t,s}^f &< 0 \text{ where } k_{t,s}^f \in [0, \infty). \end{aligned}$$

The concavity assumption implies that the arbitrageurs have a downward sloping demand for capital. The assumption $F'(0) = 1$ simplifies the analytical solution since it implies that the arbitrageurs use their production technology only when no new capital is produced; i.e. when the bankers sell capital to the foreign arbitrageurs, which represents a fire sale in this model. The fact that $\lim_{k^f \to \infty} F'\left(k_{t,s}^f\right) \geq \gamma$ guarantees that it will be always optimal for the banker to refinance the project in $t = 1$. The last two inequalities imply that the total amount spent by arbitrageurs to purchase capital increases with the capital purchased at a decreasing rate, $\frac{\partial\left(q_{t,s}k_{t,s}^f\right)}{\partial k_{t,s}^f} > 0$ and $\frac{\partial^2\left(q_{t,s}k_{t,s}^f\right)}{\partial\left(k_{t,s}^f\right)^2} < 0$, which is a necessary condition to have an unique equilibrium.

In the sections that follow I assume that the assumptions made thus far are satisfied and any additional assumptions made later on will be specified explicitly. The following Lemma links the price of capital to the size of the fire sale. The aggregate capital stock of the banking sector is defined as $k_{t,s}$.

Lemma 1 *Given the market clearing condition for capital, $\eta^f k_{t,s}^f + k_{t,s} = k_{t,s}^o + k_{t-1,s}$, the first order conditions of the producer of capital and of the foreign arbitrageur, it follows that*

$$q_{t,s} = \begin{cases} 1 \text{ iff } k_{t,s}^f = 0 \\ F'\left(k_{t,s}^f\right) < 1 \text{ iff } k_{t,s}^f > 0 \end{cases} \tag{8}$$

$$k_{t,s}^f = \max\left\{\frac{1}{\eta^f}\left(k_{t-1,s} - k_{t,s}\right), 0\right\}. \tag{9}$$

Proof of Lemma 1: See section 6.A.1 in the Appendix. □

Lemma 1 implies that if the banking sector is a net seller of capital, then the arbitrageurs employ their production technology, $k_{t,s}^f > 0$, no new capital is produced, $k_{t,s}^o = 0$, and the price of capital is less than one, $q_{t,s} < 1$.[13]

[13] η^f proxies the ability of the arbitrageurs to absorb the capital sold by the banking sector. The larger η^f is, the higher the price of capital will be if there was a fire sale.

3 Infinitesimally Small Banks

In this section, I solve the problem of the infinitesimally small banker, where I assume that there is a continuum of banks of measure one. Aggregate bank variables are defined as $x_{t,s} = \int_0^1 x_{t,s}^i di$. First, I solve the model with no ex-ante regulation and an optimal ex-post bail-out assuming that the bail-out authority cannot commit. While the no commitment case is the more realistic and, therefore, the focus of this paper, I also briefly discuss how the results change if the bail-out authority can commit. Second, I solve for the constrained Central Planner's allocation and study the externalities in the model. Third, I solve for the optimal regulation that decentralizes the constrained Central Planner's allocation. After defining what fiscal capacity means in this model, the section concludes with a discussion of how the optimal regulation changes as we vary the fiscal capacity of a country. The equilibrium concept is sub-game perfect and I solve for the symmetric equilibrium.

3.1 No Ex-Ante Regulation

3.1.1 Banker's Problem: First Order Conditions

In this sub-section, I solve the banker's problem with no ex-ante regulation via backward induction. At the end of period two, all agents produce and consume. Given that $q_{2,s} = 0$ and $p_{2,s} = 1$, at the end of period one, after $T_{1,s}^i$ is determined, banker i maximizes

$$\begin{aligned}
&\max_{c_{1,s}^i, k_{1,s}^i, d_{2,s}^i} c_{1,s}^i + \left(A_2 k_{1,s}^i - d_{2,s}^i\right) \text{ subject to} \\
&c_{1,s}^i + q_{1,s} k_{1,s}^i + d_{1,s}^i \leq \left(A_{1,s} + q_{1,s} - \gamma\right) k_0^i + T_{1,s}^i + d_{2,s}^i \qquad \left[\hat{\lambda}_{1,s}^i\right] \qquad (10) \\
&c_{1,s}^i \geq 0 \quad \left[\hat{z}_{1,s}^i\right]; \; k_{1,s}^i \geq 0 \quad \left[\hat{\kappa}_{1,s}^i\right]; \; d_{2,s}^i \leq 0 \quad \left[\hat{\mu}_{2,s}^i\right].
\end{aligned}$$

All the Lagrange multipliers are in square brackets. A "hat" indicates that the Lagrange multiplier is associated with the period one optimization problem while the absence of a "hat" implies that it is associated with the period zero optimization problem. Bankers are infinitesimally small and they take prices and aggregate variables as given. The first order conditions imply that the period one marginal value of wealth is $\hat{\lambda}_{1,s}^i = \hat{\mu}_{2,s}^i + 1 = 1 + \hat{z}_{1,s}^i = \frac{A_2 + \hat{\kappa}_{1,s}^i}{q_{1,s}} > 1$, where the inequality follows from assumption 1 and Lemma 1, which imply $A_2 > 1 \geq q_{1,s}$. The fact that the marginal value of wealth is above one in period one, but it is one in period two implies that banker i chooses zero consumption and savings in period one ($c_{1,s}^i = d_{2,s}^i = 0$). Finally, the period one budget constraint determines $k_{1,s}^i$ as a function of prices and transfers and $\hat{\kappa}_{1,s}^i = 0$.

At the end of $t = 0$, banker i solves the following problem

$$\max_{d_{1,s}^i, c_0^i, k_{1,s}^i, k_0^i} c_0^i + \sum_s \pi_s A_2 k_{1,s}^i$$

subject to the borrowing constraints, the budget constraints and the non-negative consumption and capital constraints. I use the fact that $p_{1,s} = \pi_s$ and $q_0 = 1$, where $q_0 = 1$ follows from Lemma 1 and the fact that there is no pre-existing capital in $t = 0$. The constraints are given by

$$
\begin{gathered}
(q_{1,s} - \gamma) k_0^i \geq d_{1,s}^i \qquad \left[\pi_s \mu_{1,s}^i\right] \\
n_0 + \sum_s \pi_s d_{1,s}^i - k_0^i - c_0^i \geq 0 \qquad \left[\lambda_0^i\right] \\
(A_{1,s} - \gamma + q_{1,s}) k_0^i + T_{1,s}^i - q_{1,s} k_{1,s}^i - d_{1,s}^i \geq 0 \qquad \left[\pi_s \lambda_{1,s}^i\right] \\
c_0^i \geq 0 \qquad \left[z_0^i\right]; \quad k_0^i \geq 0 \qquad \left[\kappa_0^i\right]
\end{gathered}
$$

The first order conditions are as follows

$$
\begin{aligned}
MC_{k_0} &= \lambda_0^i = \kappa_0^i + \sum_s \pi_s \left(\lambda_{1,s}^i (A_{1,s} + q_{1,s} - \gamma) + \mu_{1,s}^i (q_{1,s} - \gamma)\right) = 1 + z_0^i = MB_{k_0} > 1 && \text{(11a)} \\
\lambda_{1,s}^i &= \frac{A_2}{q_{1,s}} > 1; && \text{(11b)} \\
\mu_{1,s}^i &= \lambda_0^i - \lambda_{1,s}^i. && \text{(11c)}
\end{aligned}
$$

When deriving the first order conditions, I assumed that the banker takes $T_{1,s}^i$ as given. If the bail-out authority can commit, then the transfers are determined at the beginning of period zero before the banker chooses his allocation. In the no commitment case, which is the focus of this paper, the optimal bail-out will be a function of only aggregate variables, the proof of which is presented in sub-section 3.1.3. Therefore, in that case, the infinitesimally small banker will take $T_{1,s}^i$ as given as well. This will not be the case in the large banks' case, where the banker will no longer take $T_{1,s}^i$ as given.

The first order conditions with respect to capital determine the marginal values of wealth in periods zero and one, λ_0^i and $\lambda_{1,s}^i$, as perceived by the banker. An extra dollar in period one can purchase $\frac{1}{q_{1,s}}$ units of capital which, in turn, will deliver A_2 units of the consumption good in period two. An extra dollar in period zero will purchase one unit of capital, the returns of which will be reinvested in period one. Furthermore, if the borrowing constraint against any state of nature binds, an extra unit of capital invested will relax it. The first order condition with respect to the state-contingent debt, equation (11c), determines the pattern of borrowing by the banks. For example, if $\mu_{1,s}^i > 0$, banker i borrows to the maximum against state s in period one as the marginal value of wealth is higher in period zero relative to state s in period one.

The proof why $\lambda_{1,s}^i > 1$ is the same as to why $\hat{\lambda}_{1,s}^i > 1$. When combined with the fact that $EA_{1,s} \geq 1$ and $q_{1,s} - \gamma \geq 0$, the first order conditions imply that the period zero marginal value of wealth is greater than one, $\lambda_0^i > 1$, and that the banker chooses zero consumption in period zero, $c_0^i = 0$. Using a similar argument, one can prove by contradiction that the period zero investment is always positive, $k_0^i > 0$.[14]

The following Lemma proves that there will be no fire sale in the good state and there will be a fire sale in the bad state if the equilibrium is interior.

Lemma 2 *There is no fire sale in the good state, $q_{1,g} = 1$, and there is a fire sale in the bad state, $q_{1,b} < 1$, if the equilibrium is interior (i.e. if the banker does not borrow to the maximum in $t = 0$).*

Proof of Lemma 2: See section 6.A.2 in the Appendix. □

The solution for the rest of the endogenous variables is summarized in Proposition 1 in sub-section 3.1.3 where the various types of possible equilibria are discussed.

3.1.2 Bail-Out Authority — No Commitment

This subsection presents the solution to the bail-out authority's problem without commitment. The lack of commitment is defined as the bail-out authority choosing $T_{1,s}$ and $\tau_{1,s}$ in the beginning of period one after the uncertainty is realized and after the banker chooses his period zero allocation. Due to the linear production technology of the bankers and the risk neutrality assumption, the bail-out authority's problem becomes equivalent to choosing only aggregate bank variables.[15] At the beginning of period one, the bail-out authority chooses period one aggregate taxes and transfers, $\tau_{1,s}$ and $T_{1,s}$ and it maximizes $\sum_{t=1}^{2} \left(c_{t,s} + c_{t,s}^c - \omega l_{t,s}^* \left(\tau_{t,s}\right)\right)$. The optimization problem is subject to the periods one and two best response functions of all agents, which implies the following Lagrangian:

$$\max_{\tau_{1,s},k_{1,s}} 2m + e\left(\tau_{1,s}\right) + d_{1,s}^c + e\left(0\right) + A_2 k_{1,s} + \hat{\lambda}_{1,s}^B \left(\left(A_{1,s} - \gamma\right) k_0 + T_{1,s}\left(\tau_{1,s}\right) - d_{1,s} - q_{1,s}\left(k_{1,s} - k_0\right)\right) + \hat{\varpi}_{1,s}^B \tau_{1,s},$$

where $\hat{\varpi}_{1,s}^B$ and $\hat{\lambda}_{1,s}^B$ are the Lagrange multipliers on the non-negative tax constraint and the banker's budget constraint respectively and $T_{1,s}\left(\tau_{1,s}\right) = \chi \tau_{1,s} a \left(l_{1,s}^*\right)^\alpha$. The first order condition with respect to $k_{1,s}$ determines the marginal value of wealth of the banking sector, as perceived by the bail-out authority in period one

$$\hat{\lambda}_{1,s}^B = \frac{A_2}{q_{1,s} + \frac{\partial q_{1,s}}{\partial k_{1,s}}\left(k_{1,s} - k_0\right)} \geq A_2 > 1 \tag{12}$$

$$\text{where } q_{1,s} + \frac{\partial q_{1,s}}{\partial k_{1,s}}\left(k_{1,s} - k_0\right) = \begin{cases} F'\left(k_{1,s}^f\right) + F''\left(k_{1,s}^f\right) k_{1,s}^f < 1 \text{ if } k_{1,s}^f > 0 \\ 1 \text{ if } k_{1,s}^f = 0 \end{cases} \tag{13}$$

The only difference between the expressions for $\hat{\lambda}_{1,s}^B$ and $\hat{\lambda}_{1,s}^i$ is that the bail-out authority internalizes the fact that, conditional on k_0, a larger period one investment increases the price of capital if there

[14] Assume that $k_0^i = 0$ and the banker saves his initial endowment instead. This would imply that there is no fire sale and the marginal value of wealth is equated across all states of nature, $\lambda_0^i = \lambda_{1,s}^i$, as the borrowing constraints do not bind. However, from assumption 1, $A_2\left(E_0 A_1 + (1-\gamma)\right) > A_2$, which implies that $\lambda_0^i > \lambda_{1,s}^i$, and that is a contradiction. Therefore, $k_0^i > 0$ and $\kappa_0^i = 0$.

[15] For a formal argument, see the large banks case in Section 6.B of the Appendix.

is a fire sale, thus, making an extra dollar in the hands of the banker even more valuable. The inequalities follow directly from Lemma 1 and the assumptions made.

The first order condition with respect to $\tau_{1,s}$ determines the optimal tax rate by equating the marginal cost (MC_{τ_1}) and the marginal benefit (MB_{τ_1}) of increasing labor taxation

$$MB_{\tau_1} = \hat{\lambda}^B_{1,s} T'_{1,s} \left(\tau_{1,s}\right) \leq -e' \left(\tau_{1,s}\right) = MC_{\tau_1}, \tag{14}$$

where $-e'\left(\tau_{1,s}\right) = a \left(l^*_{1,s}\right)^{\alpha} > 0$ is the marginal cost from increasing the tax on labor due to the decreased welfare of the consumers. The marginal benefit is given by the marginal benefit of an extra dollar transferred to the banker in $t = 1$, $\hat{\lambda}^B_{1,s}$, multiplied by how much more transfers the government can provide by marginally increasing the labor tax rate, $T'_{1,s}\left(\tau_{1,s}\right) = \chi a \left(l^*_{1,s}\right)^{\alpha} \left(1 - \frac{\alpha \tau_{1,s}}{(1-\alpha)(1-\tau_{1,s})}\right)$.

3.1.3 Key Lemmas and Propositions

In this sub-section, I present the key results from the decentralized equilibrium with no ex-ante regulation and a bail-out authority that cannot commit. Lemma 3 formalizes the solution for the optimal tax rate.

Lemma 3 *If the bail-out authority cannot commit, the optimal labor tax rate can be expressed as*

$$\tau_{1,s} = \begin{cases} \frac{\chi \hat{\lambda}^B_{1,s} - 1}{\frac{\chi \hat{\lambda}^B_{1,s}}{(1-\alpha)} - 1} \text{ iff } \hat{\lambda}^B_{1,s}\chi > 1 \\ 0 \text{ iff } \hat{\lambda}^B_{1,s}\chi \leq 1 \end{cases}, \tag{15}$$

where $\hat{\lambda}^B_{1,s}$ is given by equation (12). If there is no fire sale in state s, then $T_{1,s} = 0$, where $T_{1,s}$ is given by equation (7). if there is a fire sale, a larger fire sale leads to a larger marginal value of wealth in the hands of the bankers, as perceived by the bail-out authority, $\hat{\lambda}^{B\prime}_{1,s}\left(k^f_{1,s}\right) > 0$ and to a larger bail-out, $T'_{1,s}\left(k^f_{1,s}\right) > 0$.

Proof of Lemma 3: See section 6.A.3 in the Appendix. □

The expression for the optimal tax rate follows directly from equation (14). Since $\hat{\lambda}^B_{1,s}$ is a function of only the aggregate fire sale, Lemma 3 implies that the optimal bail-out is a function of only aggregate variables. Given that the policy maker is indifferent how to distribute the bail-out across bankers, in order to solve for the symmetric equilibrium, I assume that each banker receives the same bail-out, $T^i_{1,s} = T_{1,s}$. This is why the conjecture that the infinitesimally small banker takes $T^i_{1,s}$ as given was correct, despite the bail-out being targeted.

The optimal tax rate and, as a result, the optimal bail-out can be zero due to the exogenous deadweight loss from collecting taxes, captured by χ, which plays the role of a fixed cost. If the exogenous deadweight loss from taxation was zero ($\chi = 1$), taxes would always be positive since $\hat{\lambda}^B_{1,s} > 1$. However, if $\chi < 1$ it can be the case that $\hat{\lambda}^B_{1,s}\chi \leq 1$ and the optimal tax will be zero. Since

$\hat{\lambda}^B_{1,g} = A_2$, if we assume that $A_2\chi \leq 1$ is satisfied, then there will be no bail-out in the good state.

Lemma 3 also proves that the larger the fire sale is, the more the bail-out authority values an extra dollar in the hands of the banker in period one. Therefore, a larger fire sale implies that it is more likely that the optimal labor tax rate, and, hence, bail-out will be positive. Also, conditional on a positive bail-out, a larger fire sale implies a larger optimal bail-out.[16]

The following Proposition characterizes the different types of equilibria as a function of the pecking order of borrowing.

Proposition 1 *The equilibrium allocation exists and is unique. There are two types of equilibria: Type 1) An interior equilibrium (the banker borrows to the maximum in* $t = 0$ *against the good state);* $\lambda_0 = \lambda_{1,b} > \lambda_{1,g}$ $(\mu_{1,g} > 0$ *and* $\mu_{1,b} = 0)$ *and*

$$q_{1,b} = F'\left(k^f_{1,b}\right) = G < 1; \tag{17}$$

Type 2) A corner equilibrium (the banker borrows to the maximum in $t = 0$ *against both states);* $\lambda_0 > \lambda_{1,s}$ $(\mu_{1,s} > 0)$.

Proof of Proposition 1: See section 6.A.4 in the Appendix.[17] $\square$

The key takeaways from this sub-section are the following. There will be no fire sale and bail-out in the good state. If the equilibrium is interior, there will be a fire sale in the bad state, and there might be a bail-out depending on the size of the exogenous deadweight loss from taxation. Furthermore, for any level of the fire sale, each banker values wealth more in the bad state relative to the good state, $\lambda_{1,b}\left(k^f_{1,b}\right) > \lambda_{1,g}$. Therefore, he will always borrow first to the maximum against the good state and, only then, against the bad state. This pecking order of borrowing will be potentially different in the large banks' case. This difference will be at the core of the proof as to why an instrument that constrains the state-contingent borrowing of large banks might be required.

3.1.4 Bail-Out Authority: Commitment

In this sub-section, I discuss how the optimal bail-out changes if the bail-out authority can commit and there is no ex-ante regulation. Commitment is defined as the bail-out authority choosing state

[16] To see why, one can express the bail-out as the tax rate times the tax base times the fraction of non-wasted taxes. More precisely,

$$T_{1,s} = \chi\tau_{1,s}\left(\chi, k^f_{1,s}\right) a\left(l^*_{1,s}\left(\tau_{1,s}\left(\chi, k^f_{1,s}\right), a, \omega\right)\right)^\alpha, \tag{16}$$

where $l^*_{1,s}$ and $\tau_{1,s}$ are given by equations (1) and (15), respectively. A higher fire sale leads to a higher labor tax rate, as it increases the marginal benefit of an extra dollar in the hands of the banks in a crisis, as perceived by the bail-out authority. Higher distortionary labor tax rate leads to a lower labor supplied, and hence to a smaller tax base. Given that I assume that the labor tax rate chosen is always on the left side of the Laffer curve, the increase in the labor tax rate dominates the fall in the tax base, and the overall effect is that a larger fire sale leads to a higher optimal bail-out.

[17] In an online Appendix, I provide numerical examples which visualize the proofs of a few of the Propositions and Lemmas presented in the paper, including this one.

contingent labor taxes and bank transfers in the beginning of period zero before the banker chooses his period zero allocation. I focus on the interior equilibrium, as defined in Proposition 1. Section 6.A.5 in the Appendix provides the formal derivations and further details. The discussion is purposefully brief as for the rest of the paper I focus only on the no commitment case, which is the more realistic case.

The main difference between the commitment and the no commitment problems in the small banks' case is that if the bail-out authority can commit, it internalizes the fact that a larger ex-post bail-out will just encourage more bank investment ex-ante without decreasing the equilibrium fire sale in the bad state (i.e. it takes into account equation (17)). This result is in contrast to the no commitment case where the bail-out authority chooses the bail-out in the beginning of period one and, therefore, it takes k_0 as given. Conditional on k_0 being pre-determined, larger bail-out decreases the equilibrium fire sale. For these reasons, the bail-out authority that can commit optimally provides a smaller bail-out. As a result, the problem of the bail-out authority is time inconsistent if there is no ex-ante regulation.[18]

Notice that even if the bail-out authority can commit, the optimal bail-out might be still positive because it relaxes the borrowing constraint of the banker.

3.2 Constrained-Efficient Central Planner

In this sub-section I solve the constrained-efficient Central Planner's problem defined in section 2.6. I consider both the no commitment and commitment cases, the formal solutions to which are presented in Sections 6.A.6 and 6.A.7 in the Appendix. The allocations of the two problems coincide, which implies that the Central Planner's problem is time consistent.

Below I present the key first order conditions and results from the Central Planner's problem without commitment. As in the decentralized equilibrium, the Central Planner's allocation is such that the only possible equilibria types are an interior equilibrium $\left(\lambda_0^{CP} = \lambda_{1,b}^{CP} > \lambda_{1,g}^{CP}\right)$ and a corner equilibrium $\left(\lambda_0^{CP} > \lambda_{1,s}^{CP}\right)$. I will focus on parametrization such that the Central Planner's equilibrium is always interior (see assumption 4 in the Appendix for the required condition). Moreover, as in the decentralized allocation, there will be a fire sale in the bad state and no fire sale and no bail-out in the good state.

The first order conditions with respect to capital determine the marginal value of wealth in the

[18] In the case of large banks, there will be another important difference between the commitment and no commitment cases. In the commitment case, large banks will internalize that they can no longer affect the bail-out they receive with their period zero actions, which will eliminate the "Too-Big-To-Fail" moral hazard. Formal derivations of the large banks case with commitment and no ex–ante regulation are not presented as the problem is significantly less tractable.

hands of the banker as perceived by the Central Planner

$$\lambda_{1,s}^{CP} = \frac{A_2}{q_{1,s} + \frac{\partial q_{1,s}}{\partial k_{1,s}} (k_{1,s} - k_0)} \tag{18}$$

$$MC_{k_0}^{CP} = \lambda_0^{CP} = \sum_s \pi_s \left(\mu_{1,s}^{CP} \left(q_{1,s} - \gamma + \frac{\partial q_{1,s}}{\partial k_0} k_0 \right) + \lambda_{1,s}^{CP} \left((A_{1,s} - \gamma + q_{1,s}) + \frac{\partial q_{1,s}}{\partial k_0} (k_0 - k_{1,s}) \right) \right) = MB_{k_0}^{CP}. \tag{19}$$

The first order conditions with respect to the state-contingent debt determine the Lagrange multipliers on the borrowing constraints — $\mu_{1,g}^{CP} = \lambda_0^{CP} - \lambda_{1,g}^{CP} > 0$ and $\mu_{1,b}^{CP} = \lambda_0^{CP} - \lambda_{1,b}^{CP} = 0$. The only difference between the Central Planner's first order conditions and the first order conditions of the infinitesimally small banker is that the Central Planner internalizes the fact that her decision impacts prices. This difference will be at the core of the inefficient pecuniary externalities.

One can solve for the equilibrium fire sale, $k_{1,b}^{f,CP}$, using the fact that the interior equilibrium implies that $\lambda_0^{CP} = \lambda_{1,b}^{CP}$. The fire sale is determined by

$$F' \left(k_{1,b}^{f,CP} \right) + F'' \left(k_{1,b}^{f,CP} \right) k_{1,b}^{f,CP} = G, \tag{20}$$

where I denote the Central Planner's allocation with the superscript "CP".

The first order condition with respect to taxes is the same as inequality (14) since $\lambda_{1,b}^{CP} \left(k_{1,b}^f \right) = \hat{\lambda}_1^B \left(k_{1,b}^f \right)$, and it implies that the optimal tax is given by equation (15). The Central Planner will choose the same ex-post bail-out as the bail-out authority in the middle of the crisis, conditional on being able to choose the ex-ante allocation of the banker. Therefore, when we discuss how the Central Planner's allocation can be decentralized, it will imply that the policy maker will not need to regulate (or replace) the bail-out authority, which cannot commit. To link it to reality, if regulators can design the ex-ante regulation optimally, they do not need to regulate how the fiscal authority sets bail-outs in the middle of a crisis and do not have to introduce commitment mechanisms.

If the equilibrium is interior for both the Central Planner and the banker in the decentralized equilibrium, the banker values his wealth in the bad state by less than the Central Planner does since

$$\lambda_{1,b}^{CP} \left(k_{1,b}^f \right) = \frac{A_2}{F' \left(k_{1,b}^f \right) + F'' \left(k_{1,b}^f \right) k_{1,b}^f} > \lambda_{1,b} \left(k_{1,b}^f \right) = \frac{A_2}{F' \left(k_{1,b}^f \right)}. \tag{21}$$

The fact that inequality (21) is satisfied is crucial for the presence of ex-ante over-investment. The following Lemma makes this point formally, where I denote the allocation from the decentralized problem with no ex-ante regulation and ex-post bail-out with no commitment with a star.

Lemma 4 *The banker in the decentralized equilibrium with no ex-ante regulation and ex-post bail-out without commitment overinvests relative to the constrained Central Planner;* $k_0^{CP} < k_0^*$. *The source of* *the externality is inefficient pecuniary externalities and it is captured by the difference in the perceived marginal benefit to cost ratio of an extra k_0 between the banker and the Central Planner, evaluated at the Central Planner's allocation. This difference is equal to the following "inefficiency" wedge, which captures the strength of the externalities*

$$\Pi\left(k_{1,b}^{f,CP}\right) = RMV_{k_0} - RMV_{k_0}^{CP} = \Phi\left(\frac{1}{\lambda_{1,b}\left(k_{1,b}^{f,CP}\right)} - \frac{1}{\lambda_{1,b}^{CP}\left(k_{1,b}^{f,CP}\right)}\right) = -\frac{\Phi}{A_2}F''\left(k_{1,b}^{f,CP}\right)k_{1,b}^{f,CP} > 0, \quad (22)$$

$$\textit{where } \Phi = \left(\pi_g A_{1,g} + \pi_b\right)A_2 > 0, RMV_{k_0} = \frac{MB_{k_0}\left(k_{1,b}^{f,CP}\right)}{MC_{k_0}\left(k_{1,b}^{f,CP}\right)}, RMV_{k_0}^{CP} = \frac{MB_{k_0}^{CP}\left(k_{1,b}^{f,CP}\right)}{MC_{k_0}^{CP}\left(k_{1,b}^{f,CP}\right)}.$$

Proof of Lemma 4: See section 6.A.8 in the Appendix.□

I sketch the key parts of the proof. To prove the presence of over-investment, it is sufficient to prove that the fire sale in the Central Planner's allocation is smaller than the one in the decentralized equilibrium. Conditional on interior equilibria, from equations (11a), (11b), (18) and (19), the marginal benefit to cost ratios can be expressed as

$$RMV_{k_0}^{CP} = 1 - \gamma + \pi_b\left(A_{1,b} - 1\right) + \Phi\frac{1}{\lambda_{1,b}^{CP}\left(k_{1,b}^{f}\right)} \quad (23)$$

$$RMV_{k_0} = 1 - \gamma + \pi_b\left(A_{1,b} - 1\right) + \Phi\frac{1}{\lambda_{1,b}\left(k_{1,b}^{f}\right)}, \quad (24)$$

where the equilibrium fire sales, $k_{1,b}^{f,*}$ and $k_{1,b}^{f,CP}$, are determined by $RMV_{k_0} = 1$ and $RMV_{k_0}^{CP} = 1$, respectively.

The marginal values of wealth in the bad state always increase with the size of the fire sale and, if inequality (21) is satisfied, it implies that $RMV_{k_0}^{CP} < RMV_{k_0}$. Therefore, $k_{1,b}^{f,CP} < k_{1,b}^{f,*}$.

The wedge between the Central Planner's and the banker's first order conditions, which captures the strength of the externalities, is given by combining equations (23) and (24). The fact that it is positive implies that the banker perceives the marginal benefit to cost ratio of an extra k_0 to be higher than the Central Planner does, which is why we observe the overinvestment. The more the Central Planner values wealth in the hands of the banker in the bad state relative to the banker, the bigger this wedge is and the stronger the externality is. The "inefficiency" wedge will be a key variable in understanding how the "price" regulatory instrument varies with the fiscal capacity of the country.

From equations (11a), (11b), (18) and (19) it is clear that the only reason why the "inefficiency" wedge is positive is because the Central Planner internalizes that the banker's actions affect the price of bank capital while the banker, who is small, does not. Therefore the only inefficiency is due to the pecuniary externalities.[19]

Intuitively, unlike the infinitesimally small banker, the Central Planner internalizes the fact that if banker i invests a lot ex-ante, his wealth in the crisis state will be low and the aggregate fire sale will be large. A large fire sale tightens the budget constraints of the other bankers and leads to lower investment and consumption for all bankers and larger profits for the foreign arbitrageurs. This is why the pecuniary externalities in this model are inefficient. In the case of large banks, where banks also internalize that their actions impact the size of the bail-out, there will be a second externality due to "Too-Big-To-Fail" moral hazard and the "inefficiency" wedge will be a function of it as well.

3.3 Ex-Ante Regulation and Fiscal Capacity

Given the presence of ex-ante over-investment, in this sub-section, I study how one can decentralize the constrained Central Planner's allocation and how the optimal regulation varies with the fiscal capacity of the country. For this section and for the rest of the paper I assume that χ is such that the bail-out in the bad state is positive in both the Central Planner's and the decentralized allocation with no ex-ante regulation.

3.3.1 Fiscal Capacity and Key Comparative Statics

First, I define fiscal capacity within the framework of the model and derive some useful comparative statics of the endogenous variables with respect to fiscal capacity.

Definition 1 *A country has a larger fiscal capacity relative to another country, if it has a lower marginal cost of bank bail-out, $MC_{T_1} = \frac{MC_{\tau_1}}{T'_{1,s}(\tau_{1,s})}$, for a given level of bank bail-out, $T_{1,s}$.*

Lemma 5 *Given definition 1, and holding all else constant, a country has a larger fiscal capacity relative to another country if it has a more productive labor-intensive sector (a is higher), lower dis-utility from labor (ω is lower) and lower exogenous dead-weight loss of taxation (χ is larger).*

Proof of Lemma 5: See section 6.A.9 in the Appendix. □

Countries with a more productive labor-intensive sector and lower dis-utility from labor have a larger tax base, holding all else constant. As a result, these countries can provide the same amount of bail-out by imposing a lower distortionary labor tax, which leads to a lower marginal cost of the bail-out for a given $T_{1,s}$. The marginal cost of the bail-out is also a function of the efficiency of the government, χ. If χ is high, to finance a certain bail-out, the policy maker will impose a lower distortionary labor tax rate, $\tau_{1,s}$, leading to a lower marginal cost of the bail-out.

Lemma 6 examines the question how the fiscal capacity impacts the size of both the bail-out and the marginal increase of the bail-out as the fire sale increases. These derivatives will be at the heart of a lot of the proves that follow.

[19] Unlike this paper, in Bianchi (2016) bank transfers generate moral hazard even when banks are small. The reason why is because the transfers received are exogenously designed as a fraction of each bank's debt.

Lemma 6 *For a given level of the fire sale, $k^f_{1,b}$, larger fiscal capacity implies a larger equilibrium bail-out*

$$\frac{\partial T_{1,b}}{\partial \chi} > 0; \frac{\partial T_{1,b}}{\partial \omega} < 0; \frac{\partial T_{1,b}}{\partial a} > 0.$$

Larger fiscal capacity due to lower dis-utility from labor and higher productivity of the labor-intensive technology leads to a larger marginal increase of the bail-out when the fire sale is large than when it's small. In contrast, larger fiscal capacity due to a lower exogenous distortionary cost of taxation leads to the opposite result

$$\frac{\partial^2 T_{1,b}}{\partial \left(k^f_{1,b}\right) \partial \chi} < 0; \frac{\partial^2 T_{1,b}}{\partial \left(k^f_{1,b}\right) \partial \omega} < 0; \frac{\partial^2 T_{1,b}}{\partial \left(k^f_{1,b}\right) \partial a} > 0.$$

Proof of Lemma 6: See section 6.A.10 in the Appendix.□

The marginal benefit of the bail-out, given by $MB_{T_1}\left(k^f_{1,b}\right) = \lambda^B_{1,b}\left(k^f_{1,b}\right)$, is not a function of the fiscal capacity, while larger fiscal capacity implies lower marginal cost of the bail-out (see Definition 1 and Lemma 5). As a result, larger fiscal capacity is associated with a higher bail-out, for a given fire sale.

The intuition why the cross partial derivatives differ, depending on the source of the fiscal capacity is due to the fact that a and ω affect only the tax base of the country, while χ affects both the tax base and the optimal tax rate (see the expression in footnote (15)). While larger fiscal capacity always implies a larger tax base, higher χ also implies lower marginal increase of the tax rate as the fire sale increases, $\frac{\partial^2 \tau_{1,b}(\chi)}{\partial\left(k^f_{1,b}\right)\partial \chi} < 0$. This latter force pushes $\frac{\partial^2 T_{1,b}}{\partial\left(k^f_{1,b}\right)\partial \chi}$ to be negative and it dominates, generating the difference in the comparative statics of $\frac{\partial T_{1,b}}{\partial\left(k^f_{1,b}\right)}$ with respect to the various fiscal capacity parameters.

The next Lemma derives the comparative statics of the optimal period zero investment and the fire sale with respect to fiscal capacity in the Central Planner's equilibrium. I consider the case where the Central Planner's equilibrium is interior.

Lemma 7 *The equilibrium fire sale in the Central Planner's allocation, $k^{f,CP}_{1,b}$, does not depend on the fiscal capacity of the country; $\frac{\partial k^{f,CP}_{1,b}}{\partial x} = 0$ for $x \in \{a, \omega, \chi\}$. In contrast, larger fiscal capacity implies optimally higher period zero investment chosen by the Central Planner: $\frac{\partial k^{CP}_0}{\partial a} > 0$, $\frac{\partial k^{CP}_0}{\partial \omega} < 0$, $\frac{\partial k^{CP}_0}{\partial \chi} > 0$.*

Proof of Lemma 7 : The fact that $k^{f,CP}_{1,b}$ is not a function of the fiscal capacity of the country follows directly from equation (20) and, importantly, will imply that the pecuniary externality will not be a function of the fiscal capacity. The comparative statics of k^{CP}_0 coincide with the comparative statics of $T_{1,b}\left(k^f_{1,b}\right)$ with respect to the fiscal capacity parameters (see equation (40) in the Appendix). From Lemma 6, larger fiscal capacity implies a larger optimal bail-out and, therefore, a larger k^{CP}_0, which finishes the proof. □

Lemma 7 implies that, in equilibrium, the Central Planner responds to a higher bail-out in the future by increasing his period zero investment in a way that keeps the equilibrium fire sale constant. Moreover, larger fiscal capacity is associated with a higher optimal ex-ante investment chosen by the Central Planner – a result which will be crucial to understand the comparative statics of the ex-ante minimum bank capital requirement with respect to fiscal capacity.

In Lemma 12 in section 6.A.11 of the Appendix I derive similar comparative statics for the fire sale and the ex-ante investment chosen by the banker in the decentralized equilibrium with small banks and no ex-ante regulation. The comparative statics for k_0^* and $k_{1,b}^{f,*}$ with respect to fiscal capacity are identical to the comparative statics of k_0^{CP} and $k_{1,b}^{f,CP}$ with respect to fiscal capacity and the intuition as to why is similar. These results help answer the following question: Do banks overinvest and overborrow by more if fiscal capacity is large rather than small. In other words, is regulation more important for a country with a large versus a country with a small fiscal capacity? The answer is that it depends on the source of the fiscal capacity. If the fiscal capacity is larger because the country has a larger tax base, then overinvestment and overborrowing against the bad state by banks is even bigger (i.e. the equilibrium allocation deviates by more from the constrained Central Planner's allocation). However, the opposite is true if the fiscal capacity is larger because a country is less corrupt and more efficient at collecting taxes. (For formal derivations see Lemma 13 in section 6.A.11 of the Appendix.)

These results make it clear that the strength of the externality, as captured by the "inefficiency" wedge given by equation (22), is not the same concept as the degree of the overinvestment and overborrowing. While the strength of the externality in the small banks' case is not a function of the fiscal capacity, the degree of overinvestment is. The distinction is subtle but important to understand the mechanisms in the model.

3.3.2 Ex-Ante Regulation

Next I solve for the optimal ex-ante regulation. The following Proposition proves formally that one can use either a "price" or a "quantity" instrument, as defined in sub-section 2.4.2, to regulate ex-ante investment in order to decentralize the constrained Central Planner's allocation and emphasizes the link between the two instruments.

Weitzman (1974) argues that if there is full information the two regulatory instruments are interchangeable. The main reason why I consider both a "price" and a "quantity" instrument, even though either one of them will be sufficient to decentralize the constrained Central Planner's allocation, is to point out that the comparative statics of the two types of instruments with respect to the fiscal capacity of the country will vary significantly. More precisely, whether larger fiscal capacity implies more or less regulation will depend on the type of policy instrument used, where I will use the term "more regulation" to refer to a higher ρ or a higher τ_0^k.

Proposition 2 *The constrained Central Planner's allocation can be decentralized using a single ex-ante instrument — either a minimum bank capital requirement constraint or a tax on period zero investment;* $\rho^* = \frac{n_0}{k_0^{CP}}$ *or* $\tau_0^{k,*} = \Pi\left(k_{1,b}^{f,CP}\right) > 0$. *The star denotes the optimal policy and* $\Pi(\cdot)$ *is defined in equation (22). Furthermore,* $\tau_0^{k,*} = \xi(\rho^*)$, *where* $\lambda_0 \xi$ *denotes the Lagrange multiplier on the minimum bank capital requirement constraint.*

Proof of Proposition 2: See section 6.A.12 in the Appendix.□

The reason why in order to decentralize the constrained Central Planner's allocation one needs an instrument that controls ex-ante bank investment is due to the fact that with no ex-ante regulation the banker overinvests relative to the constrained optimal allocation (see Lemma 4). For a given k_0, the rest of the endogenous variables in the decentralized equilibrium with no commitment and no ex-ante regulation and the constrained Central Planner's allocation coincide. The main reason why that is the case is because both the banker and the Central Planner value wealth by more in a crisis than in the good state for any value of the fire sale, which is why they choose the same state contingent borrowing. As a result, a single instrument is sufficient to decentralize the constrained Central Planner's allocation. This will not be the case in the large banks' case where a second instrument might be required.

Next, I discuss how the "quantity" and "price" instruments differ and Proposition 3 explores how these differences translate into different comparative statics of each instrument with respect to the fiscal capacity of the country.

The optimal minimum bank capital requirement is a function only of the Central Planner's first order conditions, which determine k_0^{CP}. In contrast, the Pigouvian tax on period zero bank investment, $\tau_0^{k,*}$, is equal to the "inefficiency" wedge given by equation (22), which is driven by the presence of inefficient pecuniary externalities as discussed in Lemma 4. The larger this wedge is, the higher the optimal tax rate is. In summary, these differences between the two instruments imply that the strength of the externality does not impact ρ^* in any way, but it is the main driver of $\tau_0^{k,*}$.

The following Proposition solves for the comparative statics of the optimal ex-ante regulation with respect to the different fiscal capacity parameters.

Proposition 3 *If the policy maker uses a minimum bank capital requirement to decentralize the constrained Central Planner's allocation, a larger fiscal capacity implies a lower optimal minimum bank capital ratio. More precisely,* $\frac{\partial \rho^*}{\partial \chi} < 0$, $\frac{\partial \rho^*}{\partial a} < 0$ *and* $\frac{\partial \rho^*}{\partial \omega} > 0$. *If a tax on capital is used instead, fiscal capacity does not affect the optimal tax on capital. More precisely,* $\frac{\partial \tau_0^{k,*}}{\partial x} = 0$ *for* $x \in \{a, \omega, \chi\}$.

Proof of Proposition 3: The relationship between fiscal capacity and the optimal regulation depends crucially on whether a "quantity" or a "price" instrument is used. Consider the case of a "quantity" instrument. Given the presence of over-investment, the minimum bank capital constraint binds and $\rho^* = \frac{n_0}{k_0^{CP}}$. Since n_0 is exogenous, to prove that larger fiscal capacity implies lower ρ^*, it is sufficient to prove that large fiscal capacity implies large k_0^{CP}, which was done in Lemma 7. Since $\tau_0^{k,*}$, specified in Proposition 2, is not a function of any of the fiscal capacity variables, all the derivatives of $\tau_0^{k,*}$ with respect to the fiscal capacity parameters are zero.$\square$

The intuition of the proof of Proposition 3 is the following. The size of the optimal ρ^* is a function only of the period zero quantity invested chosen by the Central Planner. For a given level of ex-ante investment and borrowing, the Central Planner of a country with a large fiscal capacity optimally provides a large bail-out in a crisis. This alleviates the fire sale and increases the re-sale price of capital. As a result, from the perspective of the Central Planner, the marginal benefit of period zero investment is higher if the country has a larger fiscal capacity. Therefore, such a Central Planner optimally chooses a higher ex-ante investment, which implies lower ex-ante minimum bank capital ratio.

In contrast, the optimal ex-ante tax on capital is equal to the "inefficiency" wedge. This wedge is driven by the pecuniary externalities and, as such, depends on the effect of the aggregate fire sale on the price of capital, summarized by $F''\left(k_{1,b}^{CP}\right)$. However, the equilibrium fire sale in the Central Planner's allocation, $k_{1,b}^{CP}$, is not a function of the fiscal capacity of the country (see Lemma 7), which implies that the strength of the pecuniary externalities, and, hence, the optimal "price" instrument, $\tau_0^{k,*}$, do no depend on the fiscal capacity of the country. This result will change in the large banks' case where the "inefficiency" wedge will vary with the fiscal capacity of the country due to the "Too-Big-To-Fail" moral hazard.

4 Large Banks

In this section, I consider a set-up with large banks and an equilibrium concept where bankers internalize the effect of their actions on the fire sale price and the bail-out that they receive. I assume that there are N symmetric banks, each of which has a measure $\frac{1}{N}$. Aggregate bank variables are given by $x_{t,s} = \sum_{i=1}^{N} \frac{1}{N} x_{t,s}^{i}$, which guarantees that the allocation is finite. It will be the case that as $N \to \infty$, the large banks' case converges to the continuum of banks' case.

I apply a sub-game perfect Cournot-Nash equilibrium concept. The equilibrium is Cournot-Nash because in period t banker i internalizes the demand function for capital, which can be obtained by combining the period t market clearing conditions and the first order conditions of the foreign arbitrageurs. As a result, he takes into account that his actions in period t affect equilibrium prices in period t. However, banker i takes as given the period t actions of the other bankers. The equilibrium is also sub-game perfect since I solve the problem using backward induction. Given that in period t banker i takes into account the future best response functions of all agents, including the bail-out authority, he internalizes his impact on the bail-out. This channel will be the source of the "Too-Big-To-Fail" moral hazard, which is present only in the large banks' case.

The timing and notation are the same as in the small banks' case. The Lagrange multipliers and the equilibrium variables are differentiated from the small banks' case with the superscript "l".

Finally, in addition to assumptions 1, 2 and 4, in this section, I also introduce assumptions 6, 7 and 9, which are defined in the Appendix. They are necessary for the problem to be well-behaved and interesting, i.e. for there to be a positive bail-out in the bad state and for the equilibrium to exist and to be unique.

4.1 No Ex-Ante Regulation

This sub-section collects the most important first order conditions and discusses the types of equilibria present in the case of large banks, no commitment of the bail-out authority and no ex-ante regulation. Furthermore, I compare the decentralized equilibrium of the large banks' case to the small banks' case. Details of the set-up and the solution are presented in section 6.B of the Appendix.

In the following Proposition, I prove that the solution to the decentralized equilibrium with large banks, no commitment by the bail-out authority and no ex-ante regulation has two types of equilibria, as in the case with a continuum of banks. The results in Proposition 4 correspond to the results in Proposition 1 and in Lemmas 3 and 2.

Proposition 4 *There are two types of equilibria:*
Type 1) An interior equilibrium (the banker borrows to the maximum in $t = 0$ against the good state); $\lambda_0^l = \lambda_{1,b}^l > \lambda_{1,g}^l$ $\left(\mu_{1,g}^l > 0 \text{ and } \mu_{1,b}^l = 0\right)$ *and $k_{1,b}^{f,l,*}$ is determined by*

$$q_{1,b}\left(k_{1,b}^{f,l,*}\right) = F'\left(k_{1,b}^{f,l,*}\right) = \left(1 + \underbrace{\frac{S\left(k_{1,b}^{f,l,*}\right)}{Z\left(k_{1,b}^{f,l,*}\right)}}_{\text{strategic term}}\right) G, \tag{25}$$

$$\text{where } S\left(k_{1,b}^{f}\right) = \left(\overbrace{-\frac{1}{N} F''\left(k_{1,b}^{f}\right) k_{1,b}^{f}}^{\text{"price impact" term}}\right) - \overbrace{\frac{1}{N\eta^f} T'_{1,b}\left(k_{1,b}^{f}\right)}^{\text{"Too-Big-To-Fail" moral hazard term}}, \tag{26}$$

$$Z\left(k_{1,b}^{f}\right) = F'\left(k_{1,b}^{f}\right) + F''\left(k_{1,b}^{f}\right) k_{1,b}^{f} + \frac{1}{\eta^f} T'_{1,b}\left(k_{1,b}^{f}\right), \tag{27}$$

and $T'_{1,b}\left(k_{1,b}^{f}\right) > 0$ is given by equation (37) in the Appendix.
Type 2) A corner equilibrium (the banker borrows to the maximum in $t = 0$ against both states); $\lambda_0^l > \lambda_{1,s}^l$ $\left(\mu_{1,s}^l > 0\right)$.
Lemma 2 holds. Similarly to the small banks' case, the optimal labor tax rate is given by equation (15), where the bail-out authority has the same marginal valuation of wealth in the bad state, i.e. $\hat{\lambda}_1^{l,B}\left(k_{1,b}^{f}\right) = \hat{\lambda}_1^{B}\left(k_{1,b}^{f}\right)$. *Finally, if $N \to \infty$, the allocation coincides with the allocation in the small banks' case.*

Proof of Proposition 4: See section 6.B.1 in the Appendix. □

Lemmas 2 and 3 apply to the large banks' case as well. More specifically, if banks are large, the optimal labor tax rate is given by the same formula as in the small banks' case, and there is always a fire sale in the bad state if the equilibrium is interior and no fire sale and no bail-out in the good state.

Similarly to the case with a continuum of banks, the key endogenous variable is the marginal value of wealth in the bad state as perceived by the banker. If the equilibrium is interior, $\lambda^l_{1,b}$ and $\lambda^l_{1,g}$ are given by the following equations

$$\lambda^l_{1,b} = \left(1 + \underbrace{\frac{S\left(k^f_{1,b}\right)}{Z\left(k^f_{1,b}\right)}}_{\text{strategic term}}\right) \underbrace{\frac{A_2}{F'\left(k^f_{1,b}\right)}}_{\text{non-strategic term}} \tag{28}$$

$$\lambda^l_{1,g} = A_2. \tag{29}$$

The only difference between the marginal values of wealth in the bad state in the large banks' case, $\lambda^l_{1,b}$, and the small banks' case, $\lambda_{1,b}$, is due to the strategic term in equation (28). The larger N is (the smaller the banks are), the smaller the strategic term is in absolute terms and as $N \to \infty$, the large banks' case coincides with the continuum of banks' case.[20] This result is intuitive since the smaller the banks are, the smaller their perceived effect on the price of capital and the bail-out is.

The scaled strategic term, $S\left(k^f_{1,b}\right)$, has two components — a "Too-Big-To-Fail" moral hazard term, $\frac{1}{N\eta^f}T'_{1,b}\left(k^f_{1,b}\right) > 0$, and a "price impact" term, $-\frac{1}{N}F''\left(k^f_{1,b}\right)k^f_{1,b} > 0$. The "Too-Big-To-Fail" moral hazard term captures the fact that the large banker realizes that a larger fire sale increases the bail-out he receives in the bad state. At the same time, the fire sale is large when the banker's wealth in the bad state is small. Therefore, from equation (28), the "Too-Big-To-Fail" moral hazard term makes a larger banker value wealth in the bad state less than a smaller banker does. The "price impact" term captures the fact that, when banks are large, they realize that larger fire sale implies a lower resale price of capital, less investment in the bad state and lower welfare. Therefore, the "price impact" term makes the large banker value wealth in the bad state more than the small banker does.

Whether a large banker values wealth in the bad state more or less relative to a small banker will depend on the relative size of the "Too-Big-To-Fail" moral hazard term and the "price impact" term, which is captured by whether $S\left(k^f_{1,b}\right)$ is positive or a negative. For a given $k^f_{1,b}$, if the "price impact" term is smaller than the "Too-Big-To-Fail" moral hazard term, $S\left(k^f_{1,b}\right) < 0$, a large banker values wealth in the bad state less than an infinitesimally small banker does and the other way round.

[20]The proof is as follows. Since $\lim_{N\to\infty}\lambda^l_{1,b}\left(k^f_{1,b}\right) = \lambda_{1,b}\left(k^f_{1,b}\right)$, then $\lim_{N\to\infty}k^{f,l,*}_{1,b} = k^{f,*}_{1,b}$ and the rest of the endogenous variables are determined by the same system of equations.

The following Lemma builds on this intuition and further compares the allocation in the infinitesimally small banks' case to the one in the large banks' case, assuming the equilibria are interior.

Lemma 8 *If the "Too-Big-To-Fail" moral hazard term is larger than the "price impact" term, evaluated at the equilibrium fire sale, $k_{1,b}^{f,l,*}$, (i.e. $S\left(k_{1,b}^{f,l,*}\right) < 0$), then the large banks' allocation features a larger fire sale, lower fire sale price of capital, larger period zero investment, and larger debt against the bad state than the small banks' allocation. If the "Too-Big-To-Fail" moral hazard term is smaller than the "price impact" term, (i.e. $S\left(k_{1,b}^{f,l,*}\right) > 0$), then the opposite is true.*

Proof of Lemma 8: See section 6.B.2 in the Appendix. □

Lemma 8 points out that the severity of a crisis and the degree of the ex-ante boom and overinvestment, holding the fiscal capacity of a country constant, depends on the size of the banks. Whether larger banks contribute to more or less severe financial crises will depend crucially on the comparison between the size of the "price impact" term and the "Too-Big-To-Fail" moral hazard term. The former is linked to the pecuniary externalities while the latter is linked to the "Too-Big-To-Fail" moral hazard in the large banks' case, which are defined formally in the next sub-section.

4.2 Constrained-Efficient Central Planner

In this sub-section I discuss how the constrained Central Planner's allocation compares to the decentralized allocation with large banks and what are the externalities present when banks are large relative to the case when banks are small.

Notice that the constrained Central Planner's problem is the same in the small and in the large banks' cases. The reason why is because all agents are risk neutral and the banker's production technology is linear. As a result, the Central Planner chooses only aggregate variables and the size of the banks does not affect the Central Planner's optimization problem.

The following Lemma is the large banks' case counterpart to Lemma 4.

Lemma 9 *For any N, the banker in the decentralized equilibrium with no ex-ante regulation and ex-post bail-out without commitment overinvests relative to the constrained Central Planner; $k_0^{CP} < k_0^{l,*}$. The sources of externality are inefficient pecuniary externalities and "Too-Big-To-Fail" moral hazard and they are captured by the difference in the perceived marginal benefit to cost ratio of an extra k_0 between the banker and the Central Planner, evaluated at the Central Planner's allocation. This* *difference is equal to the following "inefficiency" wedge*

$$\Pi^l\left(k_{1,b}^{f,CP}\right) = RMV_{k_0}^l - RMV_{k_0}^{CP} = \Phi\left(\frac{1}{\lambda_{1,b}^l\left(k_{1,b}^{f,CP}\right)} - \frac{1}{\lambda_{1,b}^{CP}\left(k_{1,b}^{f,CP}\right)}\right) \tag{30}$$

$$= \left(-\frac{\underbrace{\frac{S\left(k_{1,b}^f\right)}{Z\left(k_{1,b}^f\right)}}_{\textit{strategic term}}}{1+\underbrace{\frac{S\left(k_{1,b}^f\right)}{Z\left(k_{1,b}^f\right)}}_{\textit{strategic term}}}\right)\Phi\frac{F'\left(k_{1,b}^{f,CP}\right)}{A_2} + \Pi\left(k_{1,b}^{f,CP}\right) > 0,$$

which captures the strength of the externality. For $N = 1$, the "inefficiency" wedge is a function only of the "Too-Big-To-Fail" moral hazard while for $N \to \infty$, it's a function only of the inefficient pecuniary externalities. For $1 < N < \infty$, the wedge is driven both by the "Too-Big-To-Fail" moral hazard and the pecuniary externalities.

Proof of Lemma 9: See section 6.B.3 in the Appendix. □

The proof parallels the proof of Lemma 4, where $RMV_{k_0}^l$ is given by equation (24) with $\lambda_{1,b}$ being replaced by $\lambda_{1,b}^l$. The rest of the proof is identical to the small banks case.

The differences in the sources of the inefficiency between the small banks' case and the large banks' case can be observed by comparing the "inefficiency" wedges $\Pi\left(k_{1,b}^{f,CP}\right)$ and $\Pi^l\left(k_{1,b}^{f,CP}\right)$. They differ only because of the strategic term which is non-zero when $N < \infty$. Let's focus on the scaled strategic component $S\left(k_{1,b}^{f,CP}\right)$ which consists of the "Too-Big-To-Fail" moral hazard and the "price impact" terms. The "price impact" term weakens the inefficient pecuniary externalities relative to the small banks' case. This term pushes $\Pi^l\left(k_{1,b}^{f,CP}\right)$ to be smaller than $\Pi\left(k_{1,b}^{f,CP}\right)$, but does not fully eliminate the pecuniary externalities unless $N = 1$. The intuition why this is the case is the following. Even though the large banker internalizes the fact that he partially affects the resale price of capital, which, in turn, affects his own welfare, he underestimates the social cost of the fire sale. Unlike the Central Planner, he does not internalize the fact that a lower resale price of capital in a crisis lowers the welfare of the other bankers as well.[21] The "Too-Big-To-Fail" moral hazard term captures the presence of a second type of externality due to "Too-Big-To-Fail" moral hazard. This term pushes $\Pi^l\left(k_{1,b}^{f,CP}\right)$ to be larger than $\Pi\left(k_{1,b}^{f,CP}\right)$. The large banker internalizes the fact that a larger fire sale will increase the transfer he receives in a crisis, but does not internalize the cost of this transfer, which the Central Planner does. As a result, he underestimates the social cost of the fire sale.

Whether the "inefficiency" wedge, and, therefore, the strength of the externality, is larger or

[21]For the corresponding derivations see section 6.B in the Appendix.

smaller in the large banks' case versus the small banks' case once again depends on the relative valuation of wealth in the crisis state by large and small banks and on the strategic term. The following Lemma formalizes the result.

Lemma 10 *If the "Too-Big-To-Fail" moral hazard term is larger than the "price impact" term,* $S\left(k_{1,b}^{f,CP}\right) < 0$, *then the "inefficiency" wedge in the large banks' case is larger than the small banks' case,* $\Pi^l\left(k_{1,b}^{f,CP}\right) > \Pi\left(k_{1,b}^{f,CP}\right)$. *If the "Too-Big-To-Fail" moral hazard term is smaller than the "price impact" term,* $S\left(k_{1,b}^{f,CP}\right) > 0$, *then the opposite is true.*

Proof of Lemma 10: The proof follows directly from equation (30). □

As discussed above, the pecuniary externalities are weaker when banks are large. However, large banks also lead to "Too-Big-To-Fail" moral hazard which is not present in the infinitesimally small banks' case. Therefore, whether the "inefficiency" wedge is smaller or larger in the large banks' case relative to the infinitesimally small banks' case will depend on how the strength of the "Too-Big-To-Fail" moral hazard externality compares to the reduction in the strength of the pecuniary externalities when banks are large, which is summarized by the sign of $S\left(k_{1,b}^{f,CP}\right)$.[22]

4.3 Ex-Ante Regulation and Fiscal Capacity

In this sub-section, I study how one can decentralize the constrained Central Planner's allocation when banks are large and how fiscal capacity affects the optimal ex-ante regulation.

4.3.1 Fiscal Capacity and Key Comparative Statics

Before doing that, I briefly discuss how fiscal capacity affects the decentralized allocation with no ex-ante regulation and no commitment by the bail-out authority when banks are large.

Lemma 15 in section 6.B.4 of the Appendix proves that the equilibrium fire sale in the case with large banks is a function of the fiscal capacity of the country, which is in contrast to the results in the small banks' case and the Central Planner's problem. More precisely, it shows that larger fiscal capacity due to a more productive labor intensive sector or lower dis-utility of labor (i.e. larger tax base) leads to a larger equilibrium fire sale and period zero investment. Larger fiscal capacity due to the government being less corrupt or more efficient at collecting taxes implies lower fire sale.

[22] One can ask the question under what conditions the large banker overinvests and overborrows by more than the small banker does. If the "Too-Big-To-Fail" moral hazard term is larger than the "price impact" term, evaluated at the equilibrium fire sale, $k_{1,b}^{f,l,*}$, (i.e. $S\left(k_{1,b}^{f,l,*}\right) < 0$), then the large banker overborrows and overinvests by more than the small banker does; $k_0^{l,*}(N<\infty) - k_0^{CP} > k_0^* - k_0^{CP}$ and $d_{1,b}^{l,*}(N<\infty) - d_{1,b}^{CP} > d_{1,b}^* - d_{1,b}^{CP}$. If the "Too-Big-To-Fail" moral hazard term is smaller than the "price impact" term, (i.e. $S\left(k_{1,b}^{f,l,*}\right) > 0$), then the opposite is true. The proof follows directly from Lemma 8 and from the fact that the Central Planner's allocation does not depend on the size of the banks.

The reason why the equilibrium fire sale in the case of large banks is a function of the fiscal capacity, while in the small banks' case it is not, is because of the "Too-Big-To-Fail" moral hazard, which is present only in the large banks' case. One can show that how the fire sale varies with the fiscal capacity of a country is driven by how the strength of the moral hazard varies with the fiscal capacity; $\frac{\partial k_{1,b}^{f,l,*}}{\partial x} \propto \frac{1}{N\eta^f} \frac{\partial^2 T_{1,b}\left(x; k_{1,b}^f = k_{1,b}^{f,l,*}\right)}{\partial\left(k_{1,b}^f\right)\partial x}$ for $x \in \{a, \omega, \chi\}$.

Lemma 16 in section 6.B.4 of the Appendix derives the counterpart to Lemma 13 in the Appendix and asks the question whether large banks overinvest and overborrow by more when fiscal capacity is large relative to the case when it is small. Despite the differences in the comparative statics of the endogenous variables in the large banks' case versus the small banks' case, the answer to this question remains exactly the same as in the small banks' case.

4.3.2 Ex-Ante Regulation

A natural question to ask is whether a single instrument that controls the period zero investment is sufficient to decentralize the constrained Central Planner's allocation when banks are large, as in the small banks' case. One way to answer this question is to set the minimum bank capital requirement such that $\rho = \frac{n_0}{k_0^{CP}}$ and check if the large banker chooses the rest of his choice variables to be the same as the ones in the Central Planner's allocation. Lemma 17 in the Appendix solves this problem formally and shows that it will not be always the case that one can decentralize the Central Planner's allocation with only a minimum bank capital requirement when banks are large. A second instrument which controls the large bank's borrowing against the bad state might have to be imposed. Whether a limit on the debt against the bad state has to be imposed, will depend on the fiscal capacity of the country.

The following Proposition solves for what fiscal capacity parameter space a second regulatory instrument will be needed and presents the solution for the optimal regulatory instruments.

Proposition 5 *A single instrument, given by $\rho^{l,*} = \frac{n_0}{k_0^{CP}}$ (or $\tau_0^{k,l,*} = \Pi^l\left(k_{1,b}^{f,CP}\right) > 0$), is sufficient to replicate the constrained Central Planner's allocation if*

(i) the productivity of the labor-intensive sector is lower than a particular threshold, $a < \tilde{a}(\omega, \chi)$

(ii) the dis-utility of labor is higher than a particular threshold, $\omega > \tilde{\omega}(a, \chi)$

(iii) government efficiency is higher than a particular threshold, $\chi > \tilde{\chi}(a, \omega)$

Otherwise, two instruments, $\rho^{l,*}$ (or $\tau_0^{k,l,*}$) and $\nu^{l,*} = d_{1,b}^{CP}$, will be needed to replicate the constrained Central Planner's allocation.

Proof of Proposition 5: See section 6.B.6 in the Appendix. □

The intuition why a single instrument is sufficient to decentralize the Central Planner's allocation when banks are small, but it might not be sufficient when banks are large, is the following. Both the small and the large banker do not like selling capital at fire sale prices and, for that reason, perceive the fire sale to be costly. However, unlike the small banker, the large banker realizes that a larger fire sale also implies a larger bail-out. If this latter force dominates, the large banker might not value wealth in the crisis state by more than in normal times when facing a minimum bank capital requirement. As a result, he might end up borrowing against the bad state before he has exhausted his ability to borrow against the good state. More formally, from equations (28) and (29), the large banker will value wealth by more in the bad state relative to the good state if the following condition is satisfied

$$1 - F'\left(k_{1,b}^{f}\left(k_0^{CP}\right)\right) > -\frac{S\left(k_{1,b}^{f}\left(k_0^{CP}\right)\right)}{Z\left(k_{1,b}^{f}\left(k_0^{CP}\right)\right)}, \tag{31}$$

where $k_{1,b}^{f}\left(k_0^{CP}\right)$ stands for the equilibrium fire sale, conditional on $\rho = \frac{n_0}{k_0^{CP}}$. A necessary but not sufficient condition for this inequality to be violated is that $S\left(k_{1,b}^{f}\left(k_0^{CP}\right)\right) < 0$, which implies that the "Too-Big-To-Fail" moral hazard term is larger than the "price impact" term. To answer the question whether a larger or smaller fiscal capacity makes it more likely that a second regulatory instrument will be needed (i.e. inequality (31) is violated), it is sufficient to examine how the fiscal capacity affects the size of the "Too-Big-To-Fail" moral hazard term. Lemma 6 addresses this question. Larger a, smaller ω and smaller χ lead to a larger marginal increase of the bail-out as the fire sale increases, holding all else constant, and, to a stronger "Too-Big-To-Fail"moral hazard, thus, making it more likely that inequality (31) is violated. As a result, the cutoffs in Proposition 5 with respect to the fiscal capacity parameters, which define when it is necessary to employ a second regulatory instrument, are intuitive. A second instrument which regulates bank state-contingent borrowing will be required if the country has a large enough tax base or if it is sufficiently inefficient at collecting taxes. The fact that the result differs depending on how one defines fiscal capacity implies that micro-founding the source of the fiscal capacity is important when studying how optimal bank regulation should vary with the fiscal capacity of a country.

Finally, similarly to the small banks' case, the formulas for $\rho^{l,*}$ and $\tau_0^{k,l,*}$ are the same with the exception that in the case of $\tau_0^{k,l,*}$, $\Pi^l\left(k_{1,b}^{f,CP}\right)$ replaces $\Pi\left(k_{1,b}^{f,CP}\right)$. The proof is identical to the small banks' case.

The following Proposition generalizes Proposition 3 for the large banks' setting.

Proposition 6 *Assume that the policy maker has an access to a sufficient number of instruments to replicate the constrained Central Planner's allocation. If the policy maker uses a minimum bank capital requirement, a larger fiscal capacity implies a lower $\rho^{l,*}$. More precisely, $\frac{\partial \rho^{l,*}}{\partial \chi} < 0$, $\frac{\partial \rho^{l,*}}{\partial a} < 0$ and $\frac{\partial \rho^{l,*}}{\partial \omega} > 0$. If a tax on period zero investment is used instead, then a more productivity labor-intensive sector (larger a) and lower dis-utility from labor (lower ω) imply a higher $\tau_0^{k,l,*}$. More precisely $\frac{\partial \tau_0^{k,l,*}}{\partial a} > 0$, $\frac{\partial \tau_0^{k,l,*}}{\partial \omega} < 0$. In contrast, lower government efficiency (lower χ) implies a higher $\tau_0^{k,l,*}$, $\frac{\partial \tau_0^{k,l,*}}{\partial \chi} < 0$. Finally, if a borrowing constraint against the bad state has to be imposed, a larger fiscal capacity implies higher optimal borrowing against the bad state, $\frac{\partial \nu^{l,*}}{\partial \chi} > 0$, $\frac{\partial \nu^{l,*}}{\partial a} > 0$ and $\frac{\partial \nu^{l,*}}{\partial \omega} < 0$.*

Proof of Proposition 6: The proof how $\rho^{l,*}$ varies with the fiscal capacity of the country is the same as the proof of Proposition 3. The optimal debt in the bad state chosen by the Central Planner is such that $d_{1,b}^{CP} \propto k_0^{CP}$. Therefore, the comparative statics of $\nu^{l,*} = d_{1,b}^{CP}$ with respect to the fiscal capacity parameters are exactly the same as the comparative statics of k_0^{CP} with respect to the fiscal capacity parameters given by Lemma 12. The comparative statics of $\tau_0^{k,*}$ with respect to the fiscal capacity variables differ in the case with large banks relative to the continuum of banks' case. The fact that banks are large implies that they internalize their impact on the bail-out which affects their perceived marginal value of wealth $\lambda_{1,b}^l$ in the crisis state. As before, $k_{1,b}^{CP}$ is still not a function of a, ω or χ. From Proposition 5

$$\frac{\partial \tau_0^{k,l,*}}{\partial x} = \frac{\partial \Pi^l \left(k_{1,b}^{f,CP}\right)}{\partial x} \propto \frac{1}{N\eta^f} \frac{\partial^2 T_{1,b}\left(k_{1,b}^{f,CP}\right)}{\partial \left(k_{1,b}^f\right) \partial x} \text{ for } x \in \{a, \omega, \chi\}\,. \tag{32}$$

From Lemma 6, $\frac{\partial \tau_0^{k,l,*}}{\partial a} > 0$; $\frac{\partial \tau_0^{k,l,*}}{\partial \omega} < 0$ and $\frac{\partial \tau_0^{k,l,*}}{\partial \chi} < 0$. $\square$

The reason why the comparative statics of the "price" and "quantity" instruments behave very differently is due to the fact that the "price" instrument is equal to the "inefficiency" wedge, given by equation (30), which captures the source and the strength of the externalities. In contrast, the "quantity" instrument reflects only the Central Planner's allocation.

Similarly to the small banks' case, the pecuniary externalities are not a function of the fiscal capacity since $k_{1,b}^{f,CP}$ does not depend on the fiscal capacity parameters. Therefore, the comparative statics of the "inefficiency" wedge and, as a result, the optimal tax on capital reflect how the strength of the "Too-Big-To-Fail" moral hazard varies with the fiscal capacity of the country, as captured by the cross partial derivative in equation (32).

Intuitively, when large bankers perceive the marginal effect of the fire sale on the bail-out received to be larger (which will happen when a is large, χ is small and ω is small), they want to have an even larger fire sale in the bad state, which they can achieve by investing more in period zero. In order to discourage them from overinvesting, the policy maker sets the effective cost of period zero capital, $\left(1 + \tau_0^{k,l,*}\right)$ to be even higher when the bankers are more tempted to overinvest which happens when the "Too-Big-To-Fail" moral hazard term is larger. This intuition explains the comparative statics of $\tau_0^{k,l,*}$ with respect to fiscal capacity and why the strength of the externality affects the size of the "price" instrument.

Given that the Central Planner's allocations in the small and large banks' cases coincide, the result that larger fiscal capacity implies lower ex-ante minimum bank capital requirement holds even when banks are large. The result that the limit on borrowing against the bad state, if such a limit is imposed, is increasing with the fiscal capacity of the country is due to the same reason as to why larger fiscal capacity implies a lower minimum bank capital requirement since both instruments are "quantity" instruments. More precisely, both results follow directly from the fact that when the government has deep pockets, banks can optimally afford to borrow and invest more ex-ante as the bail-out authority can provide a large bail-out ex-post and alleviate the inefficiency due to the fire sale.

Finally, I discuss how the results would change if one were to assume a concave banker's production technology. The main result that larger fiscal capacity implies a lower ex-ante minimum bank capital ratio remains as long as during a crisis the marginal benefit of investing is not very small. The intuition is the following. Introducing concavity in the production technology of the banker will imply that the larger the fiscal capacity is, the larger the bail-out is, for a given k_0, and the lower the marginal product of $k_{1,b}$ is (since $k_{1,b}$ is higher).

As a result, for a given k_0, the concavity will push the marginal benefit of period zero investment, as perceived by the Central Planner, to decrease as the fiscal capacity increases. However, as long as the decreasing returns to scale are not too strong in a crisis, this channel will not be sufficient to dominate the fact that larger fiscal capacity implies smaller fire sale and a greater marginal benefit of the period zero investment. If the latter force dominates, k_0^{CP} will be higher for countries with large fiscal capacity, and the optimal minimum bank capital ratio will be lower.

Regarding the comparative statics of $\tau_0^{k,*}$ with respect to fiscal capacity, in the version of the model with a concave banker's production technology, larger fiscal capacity implies smaller equilibrium fire sale, $k_{1,b}^{CP}$, and weaker pecuniary externalities. At the same time, larger fiscal capacity in the form of a larger tax base indicates stronger "Too-Big-To-Fail" moral hazard. Therefore, if the production technology is concave, it will be no longer clear whether larger fiscal capacity implies smaller or larger $\tau_0^{k,*}$.

5 Conclusion

This paper studies the interaction between optimal ex-ante bank regulation and the ability of the government to provide a bank bail-out in the event of a crisis and presents a number of normative results. First, countries with larger fiscal capacity should have lower ex-ante minimum bank capital ratios relative to countries with a smaller fiscal capacity, conditional on the policy maker having sufficient instruments to replicate the constrained Central Planner's allocation. Second, countries with a concentrated banking sector and a large tax base relative to the size of the banking industry or governments that are inefficient at collecting taxes should also impose a limit on the amount of bank borrowing against a future crisis state of nature. This includes regulating derivative contracts, which will leave the financial sector with a high liability during a systemic banking crisis. One can consider a number of interesting extensions to the model, which I discuss below and are left for future work.

In the model presented in this paper, the government can raise revenue only via distortionary taxation. In reality, a bank bail-out can be financed by taxing, borrowing or printing money (if an independent monetary policy is available). The access to and the cost of these policy tools jointly determine a country's fiscal capacity. A country will optimally use all of these instruments up to the point where the marginal costs of each of them are equalized. The marginal cost of each of these instruments, in equilibrium, will be equal to the cost of an extra dollar of the bail-out. Therefore, one can think of the marginal cost of the bail-out as a sufficient statistic. Since the fact that a country has a higher marginal cost of the bail-out is enough to prove that it should impose a higher ex-ante minimum bank capital ratio, the first key result of the paper will not change, even if one were to introduce sovereign borrowing and money in the model. However, given that the strength of the "Too-Big-To-Fail" moral hazard depends crucially on how one models the fiscal capacity of the country, one needs to introduce borrowing and printing money explicitly, to study how they will affect the second key result. Namely, that a second regulatory instrument might be required to decentralize the constrained Central Planner's allocation.

In practice, regulating banks' net assets in a crisis can be challenging due to the ability of banks to evade the regulation via financial innovation and due to the complexity involved in imposing such regulation. If the model is extended by introducing bank size heterogeneity, one can show that by conditioning ex-ante regulation and ex-post bail-outs on bank size, the policy maker can eliminate the need to regulate derivative contracts. More precisely, he can either prevent large banks from risk shifting by making their capital ratios very high or he can distribute the bail-out only to the infinitesimally small banks which will eliminate the "Too-Big-To-Fail" moral hazard. Both of these policies will eliminate the need to impose regulation on state contingent debt.[23]

To know whether one country should have a lower ex-ante minimum bank capital ratio relative to another, a policy maker has to forecast the country's fiscal capacity in future crisis states of nature. She can do this by considering variables such as the size of the banking sector relative to GDP, the availability of independent monetary policy and a forecast of the cost of sovereign borrowing in a crisis. However, in many cases getting a reliable estimate could be challenging given that the cost of sovereign borrowing can be very different during a banking crisis. Furthermore, complicated Value at Risk models and detailed balance sheet data of financial institutions will be required to regulate the banks' net assets in a future crisis. Detailed study of the constraints policy makers might face when implementing the optimal policy is left for future work.

The model also abstracts from a number of complexities in how banks operate by modelling them as entrepreneurs, by having exogenous bank equity and by not modelling a richer and more complex asset structure. Enriching the model will be important when translating its lessons into policy and regulation. Yet, these abstractions are fruitful to highlight the key trade-offs in the debate on why

[23]Following this paper, Davila and Walther (2018) study bank size heterogeneity and optimal bail-outs.

and how we should regulate financial institutions of different size in countries with varying degrees of fiscal capacity.

Finally, this model does not explicitly consider what might happen if one were to introduce heterogeneous regulation and banks were allowed to relocate to different countries. A few papers have emphasized the optimality of harmonized bank regulation using the argument of creating a "level playing field" for banks.[24] To generate the result that there will be a race to the bottom in regulatory standards, as suggested by this literature, a crucial implicit assumption is that banks enter foreign markets via branches, which do not have to abide by the regulatory standards of the country where they operate. As more countries introduce regulatory standards for branches and subsidiaries of foreign banks similar to those for domestic banks, the competition over commercial loans is less likely to be affected by the heterogeneity in cross country regulation.[25] Furthermore, markets are naturally segmented since monitoring costs are lower if the banks are closer to the borrowers. Such natural segmentation will prevent banks from locating in a country with laxer regulatory standards and providing loans to firms in another country with more stringent regulation. Given this natural market segmentation and the ability of governments to impose the same regulation on branches and subsidiaries of foreign banks, governments could have significant leeway regarding having differential regulation.[26]

In summary, the results of this paper suggest that harmonization in bank regulation is sub-optimal and differences in fiscal capacity is an important source of heterogeneity to consider when designing optimal bank regulation.

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[24]The main idea behind this class of papers is that the lack of harmonization in regulation will lead to a "race to the bottom" since countries will lower minimum bank capital ratios in an attempt to gain market share for their banks (see for example Dell'Aricciaa and Marquez (2006), Boot, Dezelan, and Milbourn (2000) and the literature review in Santos (2001)).

[25]For example, the United States regulates branches and subsidiaries of large foreign banks similarly to how it regulates domestic banks.

[26]Another class of papers has emphasized the need for coordinating regulation across countries. This might be the case since different regulatory jurisdictions have an incentive to free ride, to manipulate prices in their favor, to extract resources from foreign-owned banks that operate domestically and not to share information (see for example Sinn (2002/2003), Calzolari and Loranth (2001), Acharya (2003), Bengui (2014) and Dalen and Olsen (2003)). The idea that coordination is welfare improving does not imply that all countries should use the same regulatory standards. Right on the contrary, in the presence of heterogeneity, the optimal policy even with coordination tends to be heterogeneous (see, for example, Acharya (2003)).

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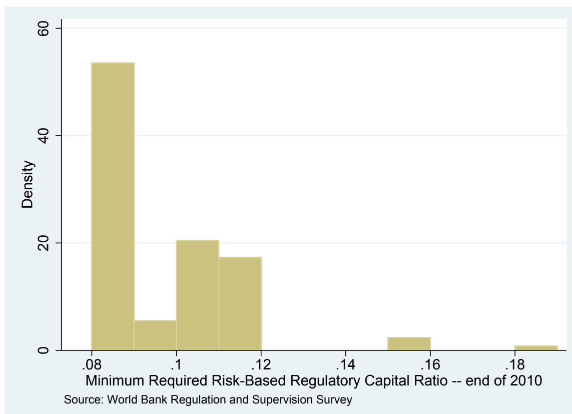
6 Appendix



Appendix 6.A Appendix: Small Banks — Proofs and Derivations

6.A.1 Proof of Lemma 1

The first order conditions with respect to capital of the producer of capital and the foreign arbitrageur are $q_{t,s} = 1 - \mu^o_{t,s}$ and $F'\left(k^f_{t,s}\right) + \mu^f_{t,s} = q_{t,s}$. Since $\mu^f_{t,s} \geq 0$ and $\mu^o_{t,s} \geq 0$, the first order conditions, when combined with assumption 2 imply equation (8). More specifically if $q_{t,s} = 1$, since $F'(0) = 1$ and $F'\left(k^f_{t,s}\right)$ is strictly monotone and decreasing then $k^f_{t,s} = 0$. If $k^f_{t,s} = 0$ then $q_{t,s} = 1$ since $q_{t,s} \leq 1, F'(0) = 1$ and $\mu^f_{t,s} \geq 0$. If $q_{t,s} = F'\left(k^f_{t,s}\right) < 1$ then $k^f_{t,s} > 0$ since $F'(0) = 1$ and $F'\left(k^f_{t,s}\right)$ is strictly monotone and decreasing. If $k^f_{t,s} > 0$ then $\mu^f_{t,s} = 0$ and $F'\left(k^f_{t,s}\right) = q_{t,s} < 1$ where the inequality follows from the previous argument. Equation (8), when combined with the assumption $\lim_{k^f_{t,s} \to \infty} F'\left(k^f_{t,s}\right) \geq \gamma$, implies that the banker refinances all of his capital stock. Therefore, the market clearing condition for capital is $\eta^f k^f_{t,s} + k_{t,s} = k^o_{t,s} + k_{t-1,s}$ and equation (9) follows from the market clearing condition for capital and the non-reversability of capital constraint. $\square$

The following comparative statics will be useful for future reference. From assumption 2 and Lemma 1

$$\begin{aligned} \frac{\partial q_{1,s}}{\partial k^f_{1,s}} &= \begin{cases} F''\left(k^f_{1,s}\right) < 0 \text{ if } k^f_{1,s} > 0 \\ 0 \text{ if } k^f_{1,s} = 0 \end{cases} \\ \frac{\partial k^f_{1,s}}{\partial k_{1,s}} &= -\frac{\partial k^f_{1,s}}{\partial k_0} = -\frac{1}{\eta^f} < 0 \end{aligned} \tag{33}$$

6.A.2 Proof of Lemma 2

The period one budget constraint of the banker, equation (10), combined with the period zero borrowing constraint, equation (4), implies that

$$k^i_{1,s} - k^i_0 = \frac{(A_{1,s} - \gamma)\, k^i_0 + T^i_{1,s} - d^i_{1,s}}{q_{1,s}} \geq \frac{(A_{1,s} - q_{1,s})\, k^i_0 + T^i_{1,s}}{q_{1,s}} \tag{34}$$

There will be a fire sale in state s as long as $k^i_{1,s} - k^i_0 < 0$. First, I consider the good state of nature. From assumption 1 and Lemma 1, it follows that $A_{1,g} > 1 \geq q_{1,g}$. Also the constraint $\tau^i_{1,s} \geq 0$ implies that $T^i_{1,s} \geq 0$. Therefore, since $\hat{\lambda}^i_{1,b} > 1$, in the good state of nature every banker will increase the amount of capital invested relative to period zero and there will be no fire sale. One can prove by contradiction that there is a fire sale in the bad state conditional on the equilibrium not being the corner equilibrium where the banker borrows and invests to the maximum. Assume that there is no fire sale in the bad state. From equations (11a) and (11b) and the assumptions made it follows that the marginal value of wealth in period zero, λ^i_0, is always greater than the marginal value of wealth in period one, $\lambda^i_{1,s} = A_2$, since $A_2 \left(\sum_s \pi_s A_{1,s} + (1-\gamma)\right) > A_2$. This implies that the equilibrium has to be a corner one where the banker borrows to the maximum in period zero. So if an interior equilibrium exists, there has to be a fire sale in the bad state.[27] $\square$

6.A.3 Proof of Lemma 3

I prove the Lemma by proving the following steps:

(i) The optimal tax is such that $0 \leq \tau_{1,s} < 1 - \alpha$.

(ii) $\tau_{1,s} \begin{cases} > 0 \text{ iff } \hat{\lambda}^B_{1,s}\chi > 1 \\ = 0 \text{ iff } \hat{\lambda}^B_{1,s}\chi \leq 1 \end{cases}$.

(iii) The optimal tax rate is given by equation (15) in the text.

(iv) Finally, I prove Lemma 11 which implies that if there is a fire sale, $\hat{\lambda}^{B\prime}_{1,s}\left(k^f_{1,s}\right) > 0$ and if there are positive transfers, then $T'_{1,s}\left(k^f_{1,s}\right) > 0$.

[27]In the online Appendix, I provide examples of parametrizations which ensure that the equilibrium is interior.

The set-up generates a standard Laffer curve, where the same amount of bank transfers can be financed with a low and a high tax rate. I assume that the policy maker will choose the lower and less distortionary tax rate. Since $\tau_{t,s}^{\max} = \arg\max_{\tau_{t,s}} T_{t,s} = \arg\max_{\tau_{t,s}} \chi \tau_{t,s} a \left(l_{t,s}^{*}\left(\tau_{t,s}\right)\right)^{\alpha} = 1-\alpha$, the optimal tax rate will be such that $\tau_{1,s} \leq 1-\alpha$. The first order condition with respect to $\tau_{1,s}$ can be re-written as

$$\hat{\lambda}_{1,s}^{B} \chi \frac{\left(1-\alpha-\tau_{1,s}\right)}{\left(1-\tau_{1,s}\right)(1-\alpha)} + \frac{\hat{\varpi}_{1,s}^{B}}{a\left(l_{1,s}^{*}\left(\tau_{1,s}\right)\right)^{\alpha}} = 1 \tag{35}$$

Consider the case $\hat{\varpi}_{1,s}^{B} = 0$, which implies $\tau_{1,s} \geq 0$. This case will be an equilibrium outcome only if $\chi > 0$. If $\chi > 0$ and given that $\hat{\lambda}_{1,s}^{B} > 1$, it follows that $\infty > \hat{\lambda}_{1,s}^{B} \chi = \frac{(1-\alpha)(1-\tau_{1,s})}{(1-\alpha-\tau_{1,s})} > 0$. This result, when combined with the assumption that $1 > \alpha$ and the fact that $\tau_{1,s} \leq \tau_{1,s}^{\max} = 1-\alpha$, implies that the optimal tax has to be such that $1-\alpha > \tau_{1,s}$. The fact that $\tau_{1,s} \geq 0$ follows directly from the non-zero constraint on taxes, which finishes the proof of part (i). If $\hat{\varpi}_{1,s}^{B} = 0$ and $\tau_{1,s} > 0$, since $\alpha > 0$, it follows that $\frac{(1-\alpha)(1-\tau_{1,s})}{(1-\alpha-\tau_{1,s})} > 1$ and, hence, $\hat{\lambda}_{1,s}^{B} \chi > 1$. The reverse statement that $\hat{\lambda}_{1,s}^{B} \chi > 1$ implies $\tau_{1,s} > 0$ can be proven by contradiction. Assume that if $\hat{\lambda}_{1,s}^{B} \chi > 1$, then $\tau_{1,s} = 0$. If $\hat{\varpi}_{1,s}^{B} \geq 0$ and $\tau_{1,s} = 0$, then from equation (35) $\hat{\lambda}_{1,s}^{B} \chi \leq 1$, which is a contradiction. Therefore, if $\hat{\lambda}_{1,s}^{B} \chi > 1$ it will be the case that $\tau_{1,s} > 0$. If $\hat{\varpi}_{1,s}^{B} > 0$ then $\tau_{1,s} = 0$. Equation (35) implies $\hat{\lambda}_{1,s}^{B} \chi < 1$. If $\hat{\lambda}_{1,s}^{B} \chi < 1$, I prove by contradiction that $\tau_{1,s} = 0$. Assume that $\hat{\lambda}_{1,s}^{B} \chi < 1$ and $\tau_{1,s} > 0$. From equation (35) if $\hat{\varpi}_{1,s}^{B} = 0$ and $\tau_{1,s} > 0$, then $\hat{\lambda}_{1,s}^{B} \chi > 1$, which is a contradiction. This finishes the proof of part (ii). Finally, the optimal tax rate, part (iii) is derived by combining equation (35) with the results in part (ii). Finally I prove part (iv) which is separated as Lemma 11 since I will refer to it later on in the Appendix.

Lemma 11

(i) If $k_{1,s}^{f} > 0$, then $\hat{\lambda}_{1,s}^{B\prime}\left(k_{1,s}^{f}\right) > 0$ and if $k_{1,s}^{f} = 0$, then $\hat{\lambda}_{1,s}^{B\prime}\left(k_{1,s}^{f}\right) = 0$.

(ii) If $k_{1,s}^{f} > 0$ and $\tau_{1,s} > 0$, then $\tau_{1,s}^{\prime}\left(k_{1,s}^{f}\right) > 0$ and $T_{1,s}^{\prime}\left(k_{1,s}^{f}\right) > 0$. If $\tau_{1,s} = 0$ or $k_{1,s}^{f} = 0$, then $T_{1,s}^{\prime}\left(k_{1,s}^{f}\right) = 0$ and $\tau_{1,s}^{\prime}\left(k_{1,s}^{f}\right) = 0$.

Proof of Lemma 11: From equation (12) and from assumption 2, it follows that if $k_{1,s}^{f} = 0$, then $\hat{\lambda}_{1,s}^{B\prime}\left(k_{1,s}^{f}\right) = 0$ and if $k_{1,s}^{f} > 0$, then

$$\hat{\lambda}_{1,s}^{B\prime}\left(k_{1,s}^{f}\right) = -\frac{\left(2F''\left(k_{1,s}^{f}\right) + F'''\left(k_{1,s}^{f}\right) k_{1,s}^{f}\right)}{\left(F'\left(k_{1,s}^{f}\right) + F''\left(k_{1,s}^{f}\right) k_{1,s}^{f}\right)} \hat{\lambda}_{1,s}^{B}\left(k_{1,s}^{f}\right) > 0.$$

From equation (15) in the text and the equation above, one can show that

$$\tau_{1,s}^{\prime}\left(k_{1,s}^{f}\right) = \begin{cases} \frac{\chi \hat{\lambda}_{1,s}^{B\prime}\left(k_{1,s}^{f}\right)(1-\alpha)\alpha}{\left(\chi \hat{\lambda}_{1,s}^{B} - (1-\alpha)\right)^{2}} > 0 \text{ if } \tau_{1,s} > 0 \text{ and } k_{1,s}^{f} > 0 \\ 0 \text{ if } \tau_{1,s} = 0 \text{ or } k_{1,s}^{f} = 0 \end{cases} \tag{36}$$

From the equation above and since $T'_{1,s}(\tau_{1,s}) = \chi a \left(\frac{\alpha a}{\omega}\right)^{\frac{\alpha}{1-\alpha}} (1-\tau_{1,s})^{\frac{\alpha}{1-\alpha}-1} \frac{1-\alpha-\tau_{1,s}}{1-\alpha} > 0$, it follows that

$$T'_{1,s}\left(k^f_{1,s}\right) = \begin{cases} \tau'_{1,s}\left(k^f_{1,s}\right) T'_{1,s}(\tau_{1,s}) = \\ T_{1,s}\left[\frac{1-\frac{\tau_{1,s}}{(1-\alpha)}}{\tau_{1,s}(1-\tau_{1,s})}\right] \tau'_{1,s}\left(k^f_{1,s}\right) > 0 \text{ if } \tau_{1,s} > 0 \text{ and } k^f_{1,s} > 0 \quad \square \\ 0 \text{ if } \tau_{1,s} = 0 \text{ or } k^f_{1,s} = 0 \end{cases} \tag{37}$$

6.A.4 Proof of Proposition 1

Re-writing and combining the first order conditions of the banker given by equations (11) implies

$$\lambda_0 = \frac{\kappa_0 + A_2\left(\pi_g A_{1,g} + \pi_b \frac{1}{q_{1,b}} A_{1,b}\right)}{1 - \pi_b (q_{1,b} - \gamma) - \pi_g (1-\gamma)} \tag{38}$$

$$\lambda_{1,g} = A_2; \; \lambda_{1,b} = \frac{A_2}{q_{1,b}}. \tag{39}$$

To characterize the solution, it is sufficient to consider the various borrowing scenarios captured by equation (11c). There are four potential types of equilibria based on whether $\mu_{1,s}$ binds or not. First, consider the case where none of the period zero borrowing constraints bind — $\mu_{1,s} = 0$ for $s \in \{g, b\}$ — which implies that the marginal value of wealth is equalized across all states of nature, $\lambda_0 = \lambda_{1,g} = \lambda_{1,b}$. Given that Lemma 2 implies that $q_{1,g} = 1$ and $q_{1,b} < 1$, then this case is not an equilibrium as the marginal value of wealth is higher in the bad state relative to the good state, $\lambda_{1,g} < \lambda_{1,b}$. Next consider the case where the banker borrows to the maximum only against the bad state, $\mu_{1,g} = 0$, $\mu_{1,b} > 0$. This case implies $\lambda_0 = \lambda_{1,g} > \lambda_{1,b}$ which, once again, cannot be an equilibrium for the same reason.

There are only two plausible types of equilibria that remain. The type 1 equilibrium implies that the banker borrows to the maximum only against the good state, $\mu_{1,g} > 0$ and $\mu_{1,b} = 0$, and this is the interior equilibrium. In this equilibrium $\lambda_0 = \lambda_{1,b} > \lambda_{1,g}$, which implies that $k^f_{1,b}$ is determined by equation (17). The rest of the endogenous bank variables are determined by the following system of equations

$$k_0 = \frac{T_{1,b}\left(k^f_{1,b}\right) + F'\left(k^f_{1,b}\right) k^f_{1,b}\eta^f + \frac{n_0}{\pi_b}}{\left(1 + \frac{\gamma}{\pi_b} - A_{1,b}\right)} \tag{40}$$

$$d_{1,b} = \left(1 + \frac{\pi_g}{\pi_b}\gamma\right) k_0 - \frac{n_0}{\pi_b}; \; d_{1,g} = (1-\gamma) k_0 \tag{41}$$

$$k_{1,g} = A_{1,g} k_0 + T_{1,g}; \quad k_{1,b} = k_0 - k^f_{1,b}\eta^f.$$

Notice that once we solve for the aggregate fire sale, we can solve for all other endogenous bank variables. The second type of equilibrium is the type 2 corner equilibrium, where the banker borrows to the maximum against both states of nature ($\mu_{1,s} > 0$), which implies that he values wealth in period zero the most.

The system of equations for the type 2 equilibrium is:

$$\begin{aligned} d_{1,s} &= (q_{1s} - \gamma) k_0; \; q_{1,b} = F'\left(k^f_{1,b}\right) \\ k_0 &= \frac{n_0}{1 - \sum_s \pi_s (q_{1,s} - \gamma)} = \frac{n_0}{\pi_b (1 - q_{1,b}) + \gamma} \\ k_{1,s} &= \frac{A_{1,s} k_0 + T_{1,s}}{q_{1,s}}, \end{aligned} \tag{42}$$

where $\mu_{1,s} > 0$ and $\lambda_0 > \lambda_{1,s}$.

Proof of Existence and Uniqueness:

It is sufficient to prove existence and uniqueness for $k^f_{1,b}$. Let $k^{f,\max}_{1,b}$ and $k^{\max}_0$ denote the equilibrium fire sale and period zero investment if the equilibrium is of type 2. Also $k^{f,\max}_{1,b}$ and $k^{\max}_0$ are the upper bounds on the admissible sets for k_0 and $k^f_{1,b}$. First I prove that if the equilibrium is of type 2, it is unique and exists. It is sufficient to prove that $k^{f,\max}_{1,b}$ is unique and exists.

First consider the case where $q^{\max}_{1,b} = 1$. In that case we can solve for the equilibrium allocation in closed form and it is given by

$$\begin{aligned} k^{\max}_0 &= \frac{n_0}{\gamma}; d^{\max}_{1,s} = (1 - \gamma) k^{\max}_0; \; q_{1,b} = 1 \\ k^{\max}_{1,s} &= A_{1,s} k^{\max}_0 + T^{\max}_{1,s}; T^{\max}_{1,s} = 0 \end{aligned} \tag{43}$$

Next consider the case $q^{\max}_{1,b} = F'\left(k^{f,\max}_{1,b}\right) < 1$. It will be the case that $k^{\max}_0 < \frac{n_0}{\gamma}$. Also $k^{f,\max}_{1,b} > 0$ is determined by $\Theta\left(k^{f,\max}_{1,b}\right) = 0$, where

$$\Theta\left(k^f_{1,b}\right) = k^f_{1,b} - \frac{1}{\eta^f} \frac{n_0}{\pi_b \left(1 - F'\left(k^f_{1,b}\right)\right) + \gamma} \left(1 - \frac{A_{1,b}}{F'\left(k^f_{1,b}\right)}\right) + \frac{1}{\eta^f} \frac{T_{1,b}\left(k^f_{1,b}\right)}{F'\left(k^f_{1,b}\right)} \tag{44}$$

From Lemmas 1 and 2, it follows that $F'\left(k^f_{1,b}\right) < 1$, $F''\left(k^f_{1,b}\right) < 0$, $T'_{1,b}\left(k^f_{1,b}\right) > 0$ and $A_{1,b} < \gamma \leq$

$F'\left(k_{1,b}^{f}\right)$, the latter of which implies $1-\frac{A_{1,b}}{F'\left(k_{1,b}^{f}\right)}>0$. Therefore,

$$\Theta'\left(k_{1,b}^{f}\right)=1-\frac{1}{\eta^{f}}\frac{F''\left(k_{1,b}^{f}\right)n_{0}}{\pi_{b}\left(1-F'\left(k_{1,b}^{f}\right)\right)+\gamma}\left[\frac{\pi_{b}\left(1-\frac{A_{1,b}}{F'\left(k_{1,b}^{f}\right)}\right)}{\pi_{b}\left(1-F'\left(k_{1,b}^{f}\right)\right)+\gamma}+\frac{A_{1,b}}{\left(F'\left(k_{1,b}^{f}\right)\right)^{2}}\right]$$
$$+\frac{1}{\eta^{f}}\frac{T_{1,b}'\left(k_{1,b}^{f}\right)}{F'\left(k_{1,b}^{f}\right)}-\frac{1}{\eta^{f}}F''\left(k_{1,b}^{f}\right)\frac{T_{1,b}\left(k_{1,b}^{f}\right)}{\left(F'\left(k_{1,b}^{f}\right)\right)^{2}}>0.$$

From equation (44), $lim_{k_{1,b}^{f}\to\infty}\Theta\left(k_{1,b}^{f}\right)\to\infty$ and

$$lim_{k_{1,b}^{f}\to0^{+}}\Theta\left(k_{1,b}^{f}\right)=\frac{1}{\eta^{f}}\left(lim_{k_{1,b}^{f}\to0^{+}}T_{1,b}\left(k_{1,b}^{f}\right)-\frac{n_{0}}{\gamma}\left(1-A_{1,b}\right)\right)<0$$

as $lim_{k_{1,b}^{f}\to0^{+}}T_{1,b}\left(k_{1,b}^{f}\right)=0$ and $(1-A_{1,b})>0$. Therefore, the type 2 equilibrium will be unique since $\Theta'\left(k_{1,b}^{f}\right)>0$.

Next, consider the interior equilibrium where $k_{1,b}^{f}$ is determined by equation (17). Define $\Xi\left(k_{1,b}^{f}\right)=F'\left(k_{1,b}^{f}\right)-G$, and consider the range $k_{1,b}^{f}\in(0,k_{1,b}^{f,\max}]$. $\Xi\left(k_{1,b}^{f}\right)$ is a continuos function on this range. From assumption 2, it follows that $\Xi'\left(k_{1,b}^{f}\right)<0$ and $lim_{k_{1,b}^{f}\to0^{+}}\Xi\left(k_{1,b}^{f}\right)=1-G>0$. If $\Xi\left(k_{1,b}^{f,\max}\right)\leq0$, then there will be an unique solution to $\Xi\left(k_{1,b}^{f}\right)=0$ and the equilibrium will be of type 1. If $\Xi\left(k_{1,b}^{f,\max}\right)>0$, the equilibrium will be of type 2, where one can also show that $\Xi\left(k_{1,b}^{f,\max}\right)>0$ implies that $\lambda_{0}\left(k_{1,b}^{f,\max}\right)>\lambda_{1,s}\left(k_{1,b}^{f,\max}\right)$. A sufficient and necessary condition for the equilibrium to be of type1 is given by $F'\left(k_{1,b}^{f,\max}\right)\leq G$.

Finally, I derive the sufficient but not necessary conditions on m which guarantee that the consumer's consumption is always strictly positive in the decentralized equilibrium with no ex-ante regulation and small banks.

Assumption 3 *If $m>0$ is such that $((\pi_{b}+\pi_{g}\gamma)+\pi_{g}(1-\gamma))\frac{T_{1,b}\left(k_{1,b}^{f,*}\right)+F'\left(k_{1,b}^{f,*}\right)k_{1,b}^{f,*}\eta^{f}+\frac{n_{0}}{\pi_{b}}}{\left(1+\frac{\gamma}{\pi_{b}}-A_{1,b}\right)}-n_{0}<m$, where $T_{1,b}\left(k_{1,b}^{f,*}\right)=\chi\tau_{1,b}\left(k_{1,b}^{f,*}\right)a\left(\left(\frac{\left(1-\tau_{t,b}\left(k_{1,b}^{f,*}\right)\right)\alpha a}{\omega}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha}$ and $\tau_{1,b}$ and $k_{1,b}^{f,*}$ are given by equations (15) and (17), respectively, then it will be always the case that the consumer has strictly positive consumption in $t=0$ in the decentralized equilibrium with no ex-ante regulation and small banks. If $-\left(1+\frac{\pi_{g}}{\pi_{b}}\gamma\right)\frac{T_{1,b}\left(k_{1,b}^{f,*}\right)+F'\left(k_{1,b}^{f,*}\right)k_{1,b}^{f,*}\eta^{f}+\frac{n_{0}}{\pi_{b}}}{\left(1+\frac{\gamma}{\pi_{b}}-A_{1,b}\right)}+\frac{n_{0}}{\pi_{b}}<m$ and $-(1-\gamma)\frac{T_{1,b}\left(k_{1,b}^{f,*}\right)+F'\left(k_{1,b}^{f,*}\right)k_{1,b}^{f,*}\eta^{f}+\frac{n_{0}}{\pi_{b}}}{\left(1+\frac{\gamma}{\pi_{b}}-A_{1,b}\right)}<m$, then also the consumer's consumption in $t=1$ is strictly positive.*

To see why this assumption is sufficient to eliminate the corner equilibrium in the consumer's problem in $t=0$, notice that if $c_{0}^{c}=0$ then $p_{1,s}<\pi_{s}$ (i.e. the net interest rate that the banker faces will be higher than zero). As a result, if $c_0^c = 0$, the period one state contingent debt level demanded by the banker will be smaller than the equilibrium debt level derived in equation (41) under the assumption of a zero net interest rate. The market clearing condition implies that $d_{1,s}^* = d_{1,s}^{c,*}$, where by assumption $d_0^* = d_0^{c,*} = 0$. Therefore, the sufficient but not necessary condition to eliminate the corner equilibrium in the consumer's problem in $t = 0$ is derived from $\pi_b d_{1,b}^* + \pi_g d_{1,g}^* < m$.

A similar argument applies to period $t = 1$. Since $d_{2,s}^{c,*} = d_{2,s}^* = 0$, the corner equilibrium, $c_{1,s}^c = 0$, is eliminated as long as $-d_{1,s}^* < m$ for $s \in \{g, b\}$. Finally, $c_{2,s}^c$ is always strictly positive. $\square$

6.A.5 Problem of the Bail-Out Authority with Commitment and No Ex-Ante Regulation

Lemma 2 and Proposition 1 apply in this case as well. Since the bail-out authority can commit, it internalizes all future best response functions of all agents in the economy. I focus only on the type 1 interior equilibrium.

The bail-out authority maximizes aggregate welfare, and the period zero optimization problem can be written as

$$
\begin{aligned}
\max_{\tau_{1,s}} \sum_s \pi_s \sum_{t=0}^{2} \left(c_{t,s}^c - \omega l_{t,s}^* \left(\tau_{t,s}\right) + c_{t,s}\right) &= \max_{\tau_{1,s}} 3m + 2e\left(0\right) + \pi_b \left(e\left(\tau_{1,b}\right) + A_2 \left(k_0 - \left(F'\right)^{-1}\left(G\right)\eta^f\right)\right) \\
&+\pi_g \left(e\left(\tau_{1,g}\right) + A_2 \left(A_{1,g} k_0 + T_{1,g}\right)\right) \text{ subject to} \\
&\tau_{1,s} \geq 0 \qquad \left[\pi_s \varpi_{1,s}^{B,C}\right] \quad \text{where} \\
&k_0\left(\tau_{1,b}\right) = \frac{T_{1,b}\left(\tau_{1,b}\right) + G\left(F'\right)^{-1}\left(G\right)\eta^f + \frac{n_0}{\pi_b}}{\left(1 + \frac{\gamma}{\pi_b} - A_{1,b}\right)}.
\end{aligned}
$$

I have substituted out the system of equations of the small banker with no ex-ante regulation given by (40), the first order conditions of all other agents and the market clearing conditions. The first order conditions are:

$$
\begin{aligned}
\tau_{1,b} &: \quad \pi_b \left(e'\left(\tau_{1,b}\right) + A_2 \frac{\partial k_0}{\partial \tau_{1,b}}\right) + \pi_g A_2 A_{1,g} \frac{\partial k_0}{\partial \tau_{1,b}} + \pi_b \varpi_{1,b}^{B,C} = 0 \\
\tau_{1,g} &: \quad e'\left(\tau_{1,g}\right) + A_2 T_{1,g}'\left(\tau_{1,g}\right) + \varpi_{1,g}^{B,C} = 0 \\
\frac{\partial k_0}{\partial \tau_{1,b}} &= \frac{T_{1,b}'\left(\tau_{1,b}\right)}{\left(1 + \frac{\gamma}{\pi_b} - A_{1,b}\right)}.
\end{aligned}
$$

Simplifying the first order conditions above implies

$$
\begin{aligned}
\tau_{1,b} \quad &: \quad e'(\tau_{1,b}) + \frac{A_2}{F'\left(k^f_{1,b}\right)} T'_{1,b}(\tau_{1,b}) + \varpi^{B,C}_{1,b} = 0 \\
\tau_{1,g} \quad &: \quad e'(\tau_{1,g}) + A_2 T'_{1,g}(\tau_{1,g}) + \varpi^{B,C}_{1,g} = 0.
\end{aligned}
$$

One can summarize the first order conditions in a way comparable to the case with no commitment, equations 12 and 14:

$$
\begin{aligned}
\lambda^{B,C}_1 T'_1(\tau_1) &\leq -e'(\tau_1) \text{ where} \\
\lambda^{B,C}_1 &= \frac{A_2}{q_1}
\end{aligned}
$$

Lemma 3 and its proof apply to the commitment case as well with the exception that $\lambda^{B,C}_1$ replaces $\hat{\lambda}^B_1$.

The first order conditions in the commitment case imply the same marginal cost – marginal benefit trade-off from raising taxes as in equation (14). The only difference is that $\lambda^{B,C}_1 = \frac{A_2}{q_1}$ replaces $\hat{\lambda}^B_1$, where $\lambda^{B,C}_1$ is the marginal value of wealth in the hands of the banker in the bad state, as perceived by the bail-out authority that can commit. Lemma 3 applies to the commitment case as well with the exception that $\lambda^{B,C}_1$ replaces $\hat{\lambda}^B_1$. Since the the bail-out authority which can commit internalizes the fact that $k^f_{1,b} = (F')^{-1}(G)$, it internalizes that that a larger ex-post bail-out will just encourage more investment ex-ante without decreasing the fire sale in the bad state. For these reasons, the bail-out authority, that can commit, perceives wealth to be less valuable in the hands of the banker in the bad state, $\lambda^{B,C}_{1,b} < \hat{\lambda}^B_{1,b}$, and optimally provides a smaller bail-out.

6.A.6 Constrained Central Planner (CP) – No Commitment

In $t = 1$, the objective function of the constrained CP is given by

$$
\max_{d_2, k_1, \tau_1, c_1} (c_1 + A_2 k_1 - d_2) + e(0) + 2m + e(\tau_1) + d^c_1 \text{ subject to}
$$

subject to

$$
\begin{gathered}
c_1 + q_1 k_1 + d_1 \leq (A_1 + q_1 - \gamma) k_0 + T_1(\tau_1) + d_2 \qquad \left[\hat{\lambda}^{CP}_1\right] \\
c_1 \geq 0 \quad \left[\hat{z}^{CP}_1\right]; \quad d_2 \leq 0 \quad \left[\hat{\mu}^{CP}_2\right]; \quad k_1 \geq 0 \quad \left[\hat{\kappa}^{CP}_1\right]; \quad \tau_1 \geq 0 \quad \left[\hat{\varpi}^{CP}_1\right]
\end{gathered}
$$

where the CP takes into account the market clearing conditions, the budget constraint of the bail-out authority and the first order conditions of the non-bank agents. The first order conditions are given by

$$\hat{\lambda}_1^{CP} = \frac{\hat{\kappa}_1^{CP} + A_2}{q_1 - \frac{\partial q_1}{\partial k_1}(k_0 - k_1)} = 1 + \hat{z}_1^{CP} = 1 + \hat{\mu}_2^{CP} \tag{45}$$

$$e'(\tau_1) + \hat{\lambda}_1^{CP} T_1'(\tau_1) \leq 0 \tag{46}$$

where if the first order condition, given by (46), holds with an inequality, then $\tau_1 = 0$. From assumption 1 and given that $q_1 + \frac{\partial q_1}{\partial k_1}(k_1 - k_0) \leq 1$, it follows that $\hat{\lambda}_1^{CP} > 1$ and $c_1 = d_2 = 0$. In order for the budget constraint to be satisfied it has to be the case that $k_1 > 0$ and $\hat{\kappa}_1^{CP} = 0$. Similarly to Lemma 3, one can prove that the optimal tax is given by equation (15), where $\hat{\lambda}_1^B\left(k_1^f\right) = \hat{\lambda}_1^{CP}\left(k_1^f\right)$. Lemma 11 also holds. In $t = 0$, the CP maximizes

$$\max_{d_{1,s}, k_{1,s}, k_0, c_0} c_0 + 3m + 2e(0) + \sum_s \pi_s \left(A_2 k_{1,s} + e(\tau_{1,s})\right) \text{ subject to}$$

$$\begin{aligned}
(q_{1,s} - \gamma) k_0 \geq d_{1,s} \qquad & \left[\pi_s \mu_{1,s}^{CP}\right] \\
n_0 + \sum_s \pi_s d_{1,s} - k_0 - c_0 \geq 0 \qquad & \left[\lambda_0^{CP}\right] \\
(A_{1,s} - \gamma + q_{1,s}) k_0 + T_{1,s} - q_{1,s} k_{1,s} - d_{1,s} \geq 0 \qquad & \left[\pi_s \lambda_{1,s}^{CP}\right] \\
c_0 \geq 0 \qquad \left[z_0^{CP}\right]; \quad k_0 \geq 0 \qquad & \left[\kappa_0^{CP}\right]
\end{aligned}$$

where the CP takes into account the best response functions in periods one and two, the market clearing conditions, the budget constraint of the bail-out authority, the banker's budget constraint in period zero and the first order conditions of the non-bank agents. The first order conditions are

$$A_2 + \left(e'(\tau_{1,s}) + \lambda_{1,s}^{CP} \frac{\partial T_{1,s}}{\partial \tau_{1,s}}\right) \frac{\partial \tau_{1,s}}{\partial k_{1,s}} + \mu_{1,s}^{CP} \frac{\partial q_{1,s}}{\partial k_{1,s}} k_0 + \lambda_{1,s}^{CP} \left(\frac{\partial q_{1,s}}{\partial k_{1,s}}(k_0 - k_{1,s}) - q_{1,s}\right) = 0 \tag{47}$$

$$\mu_{1,s}^{CP} = \lambda_0^{CP} - \lambda_{1,s}^{CP}; \qquad 1 + z_0^{CP} = \lambda_0^{CP}$$

$$\lambda_0^{CP} = \kappa_0^{CP} + \sum_s \pi_s \left[\begin{array}{c} \left(e'(\tau_{1,s}) + \lambda_{1,s}^{CP} \frac{\partial T_{1,s}}{\partial \tau_{1,s}}\right) \frac{\partial \tau_{1,s}}{\partial k_0} + \mu_{1,s}^{CP}\left(q_{1,s} - \gamma + \frac{\partial q_{1,s}}{\partial k_0} k_0\right) \\ + \lambda_{1,s}^{CP}\left((A_{1,s} - \gamma + q_{1,s}) + \frac{\partial q_{1,s}}{\partial k_0}(k_0 - k_{1,s})\right) \end{array}\right].$$

The proof that $k_0^{CP} > 0$ and $\kappa_0^{CP} = 0$ is similar to the proof in the decentralized equilibrium with small banks. One can prove that Lemma 2 holds as well. As before, there will be four potential equilibria depending on whether $\mu_{1,s}^{CP}$ binds or not. First I prove that it will be always the case that $\lambda_{1,b}^{CP} > \lambda_{1,g}^{CP}$ which implies that the case $\lambda_{1,s}^{CP} = \lambda_0^{CP}$ for $s \in \{g, b\}$ and $\lambda_{1,g}^{CP} > \lambda_{1,b}^{CP} = \lambda_0^{CP}$ cannot be equilibria. Note that both of those equilibrium are interior and hence from Lemma 2, $q_{1,b} < 1$. Since also $q_{1,g} = 1$, then there is no fire sale and hence no bail-out in the good state and $\lambda_{1,g}^{CP} = A_2$.

$$\lambda_{1,b}^{CP} = \frac{A_2 + \left(e'\left(\tau_{1,b}\right) + \lambda_{1,b}^{CP}\frac{\partial T_{1,b}}{\partial \tau_{1,b}}\right)\frac{\partial \tau_{1,b}}{\partial k_{1,b}} + \mu_{1,b}^{CP}\frac{\partial q_{1,b}}{\partial k_{1,b}}k_0}{q_{1,b} - \frac{\partial q_{1,b}}{\partial k_{1,b}}\left(k_0 - k_{1,b}\right)}$$

First, consider the case where $\tau_{1,b} = 0$. It implies that

$$\lambda_{1,b}^{CP} = \frac{A_2 + \mu_{1,b}^{CP}\frac{\partial q_{1,b}}{\partial k_{1,b}}k_0}{q_{1,b} - \frac{\partial q_{1,b}}{\partial k_{1,b}}\left(k_0 - k_{1,b}\right)} > \lambda_{1,g}^{CP} = A_2$$

which follows from the inequalities (33) and (13).

Next, consider the case $\tau_{1,b} > 0$ and $q_{1,b} < 1$. First, I prove that $\lambda_{1,b}^{CP} > \hat{\lambda}_{1,b}^{CP}$. The proof is by contradiction. Conjecture that $\lambda_{1,b}^{CP} < \hat{\lambda}_{1,b}^{CP}$. From equation (46), which holds with an equality since $\tau_{1,b} > 0$, if $\lambda_{1,b}^{CP} < \hat{\lambda}_{1,b}^{CP}$ then $e'\left(\tau_{1,b}\right) + \lambda_{1,b}^{CP}\frac{\partial T_{1,b}}{\partial \tau_{1,b}} < 0$. However, since $\mu_{1,b}^{CP} \geq 0$ and from inequalities (33) and Lemma 11, $\frac{\partial \tau_{1,b}}{\partial k_{1,b}} < 0$ and $\frac{\partial q_{1,b}}{\partial k_{1,b}} > 0$, it follows that

$$\begin{aligned} \lambda_{1,b}^{CP} &= \frac{A_2 + \left(e'\left(\tau_{1,b}\right) + \lambda_{1,b}^{CP}\frac{\partial T_{1,b}}{\partial \tau_{1,b}}\right)\frac{\partial \tau_{1,b}}{\partial k_{1,b}} + \mu_{1,b}^{CP}\frac{\partial q_{1,b}}{\partial k_{1,b}}k_0}{q_{1,b} - \frac{\partial q_{1,b}}{\partial k_{1,b}}\left(k_0 - k_{1,b}\right)} \\ &> \hat{\lambda}_{1,b}^{CP} = \frac{A_2}{q_{1,b} - \frac{\partial q_{1,b}}{\partial k_{1,b}}\left(k_0 - k_{1,b}\right)} \end{aligned}$$

which is a contradiction and this finishes the proof that $\lambda_{1,b}^{CP} > \hat{\lambda}_{1,b}^{CP}$. Combining this result with equation (13) it follows that

$$\lambda_{1,b}^{CP} > \hat{\lambda}_{1,b}^{CP} > \lambda_{1,g}^{CP} = A_2.$$

There are only two possible types of equilibria remaining.

Type 1 Equilibrium: $\mu_{1,g}^{CP} > 0$, $\mu_{1,b}^{CP} = 0$

One can re-write the first order conditions as

$$\hat{\lambda}_{1,b}^{CP} = \lambda_{1,b}^{CP} = \lambda_0^{CP} = \frac{A_2}{F'\left(k_{1,b}^f\right) + F''\left(k_{1,b}^f\right)k_{1,b}^f} > 1$$

which implies

$$F'\left(k_{1,b}^f\right) + F''\left(k_{1,b}^f\right)k_{1,b}^f = G \tag{48}$$

and $c_0 = 0$. The rest of the endogenous variables are determined by the system of equations specified in (40).

Type 2 Equilibrium: $\mu_{1,s}^{CP} > 0$

The allocation of the type 2 equilibrium is the same as the one given by the system of equations in (42).

Existence and Uniqueness:

It is sufficient to solve for $k^f_{1,b}$. Consider the interior equilibrium where $k^f_{1,b}$ is determined by equation (48). Define $\Xi^{CP}\left(k^f_{1,b}\right) = F'\left(k^f_{1,b}\right) + F''\left(k^f_{1,b}\right)k^f_{1,b} - G$, and consider the range $k^f_{1,b} \in (0, k^{f,\max}_{1,b}]$. Notice that $\Xi^{CP}\left(k^f_{1,b}\right)$ is a continuos function. Given assumption 2, it follows that $\Xi^{CP\prime}\left(k^f_{1,b}\right) < 0$ and $lim^{CP}_{k^f_{1,b}\to 0^+}\Xi\left(k^f_{1,b}\right) = 1 - G > 0$. If $\Xi^{CP}\left(k^{f,\max}_{1,b}\right) = F'\left(k^{f,\max}_{1,b}\right) + F''\left(k^{f,\max}_{1,b}\right)k^{f,\max}_{1,b} - G \leq 0$, then there will be an unique solution to $\Xi^{CP}\left(k^f_{1,b}\right) = 0$ and the equilibrium will be of type1. If $\Xi^{CP}\left(k^{f,\max}_{1,b}\right) = F'\left(k^{f,\max}_{1,b}\right) + F''\left(k^{f,\max}_{1,b}\right)k^{f,\max}_{1,b} - G > 0$, the equilibrium will be of type 2, where one can also show that $\Xi^{CP}\left(k^{f,\max}_{1,b}\right) > 0$ implies that $\lambda^{CP}_0\left(k^{f,\max}_{1,b}\right) > \lambda^{CP}_{1,s}\left(k^{f,\max}_{1,b}\right)$. Therefore, a sufficient and necessary condition for the CP's equilibrium to be of type1 is for the following assumption to be satisfied

Assumption 4 *The condition for an interior equilibrium in the CP's problem is:*

$$F'\left(k^{f,\max}_{1,b}\right) + F''\left(k^{f,\max}_{1,b}\right)k^{f,\max}_{1,b} \leq G,$$

and for a positive optimal bail-out in the CP's problem is:

$$A_2\chi > G.$$

$A_2\chi > G$ guarantees positive optimal bail-out for the following reasons. Given that the CP's allocation is assumed to be interior, in order for the optimal bail-out to be positive in equilibrium, it will have to be the case that $\lambda^{CP}_{1,b}\chi > 1$, which implies $A_2\chi > G$.

Finally, I derive the sufficient but not necessary conditions on m which guarantee that the consumer's consumption is always strictly positive in the Central Planner's allocation.

Assumption 5 *If $m > 0$ is such that $((\pi_b + \pi_g\gamma) + \pi_g(1-\gamma))\frac{T_{1,b}\left(k^{f,CP}_{1,b}\right) + F'\left(k^{f,CP}_{1,b}\right)k^{f,CP}_{1,b}\eta^f + \frac{n_0}{\pi_b}}{\left(1 + \frac{\gamma}{\pi_b} - A_{1,b}\right)} - n_0 < m$, where $T_{1,b}\left(k^{f,CP}_{1,b}\right) = \chi\tau_{1,b}\left(k^{f,CP}_{1,b}\right)a\left(\left(\frac{\left(1-\tau_{t,b}\left(k^{f,CP}_{1,b}\right)\right)\alpha a}{\omega}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha}$ and $\tau_{1,b}$ and $k^{f,CP}_{1,b}$ are given by equations (15) and (48), then it will be always the case that the consumer has positive consumption in $t = 0$ in the Central Planner's allocation. If $-\left(1 + \frac{\pi_g}{\pi_b}\gamma\right)\frac{T_{1,b}\left(k^{f,CP}_{1,b}\right) + F'\left(k^{f,CP}_{1,b}\right)k^{f,CP}_{1,b}\eta^f + \frac{n_0}{\pi_b}}{\left(1 + \frac{\gamma}{\pi_b} - A_{1,b}\right)} + \frac{n_0}{\pi_b} < m$ and $-(1-\gamma)\frac{T_{1,b}\left(k^{f,CP}_{1,b}\right) + F'\left(k^{f,CP}_{1,b}\right)k^{f,CP}_{1,b}\eta^f + \frac{n_0}{\pi_b}}{\left(1 + \frac{\gamma}{\pi_b} - A_{1,b}\right)} < m$, then also the consumer's consumption is positive in $t = 1$.*

The derivations and the intuition are equivalent to the ones under assumption 3.□

6.A.7 Constrained Central Planner – Commitment

In this section of the Appendix, I solve the problem of the constrained CP with commitment and prove that the allocation is equivalent to the CP's problem without commitment. The CP that can commit maximizes

$$\max_{\tau_{1,s},d_{2,s},d_{1,s},k_{1,s},c_{1,s},k_0,c_0} E_0 \sum_{t=0}^{2} \left(c_t^c - \omega l_t^* \left(\tau_t\right) + c_t\right)$$

$$= \max_{\tau_{1,s},d_{2,s},d_{1,s},k_{1,s},c_{1,s},k_0,c_0} 3m + 2e\left(0\right) + c_0 + \sum_s \pi_s \left(c_{1,s} + e\left(\tau_{1,s}\right) + A_2 k_{1,s} - d_{2,s}\right) \text{ subject to}$$

$$d_{1,s} \leq \left(q_{1,s} - \gamma\right) k_0 \quad \left[\pi_s \mu_{1,s}^{CP,C}\right]$$

$$d_{2,s} \leq 0 \quad \left[\pi_s \mu_{2,s}^{CP,C}\right]$$

$$n_0 + \sum_s \pi_s d_{1,s} - k_0 - c_0 \geq 0 \quad \left[\lambda_0^{CP,C}\right]$$

$$c_{1,s} + q_{1,s} k_{1,s} + d_{1,s} \leq \left(A_{1,s} + q_{1,s} - \gamma\right) k_0 + T_{1,s}\left(\tau_{1,s}\right) + d_{2,s} \quad \left[\pi_s \lambda_{1,s}^{CP,C}\right]$$

$$c_0 \geq 0 \quad \left[z_0^{CP,C}\right]; \quad c_{1,s} \geq 0 \quad \left[\pi_s z_{1,s}^{CP,C}\right]$$

$$k_0 \geq 0 \quad \left[\kappa_0^{CP,C}\right]; \quad k_{1,s} \geq 0 \quad \left[\pi_s \kappa_{1,s}^{CP,C}\right]$$

$$\tau_{1,s} \geq 0 \quad \left[\pi_s \varpi_{1,s}^{CP,C}\right]$$

where the CP takes into account the budget constraint of the bail-out authority which determines $T_{1,s}\left(\tau_{1,s}\right)$, the market clearing conditions and the first order conditions of the non-bank agents which determine $q_{1,s}$. The first order conditions are given by

$$\begin{aligned}
c_0 &: 1 + z_0^{CP,C} = \lambda_0^{CP,C}; \; c_{1,s} : 1 + z_{1,s}^{CP,C} = \lambda_{1,s}^{CP,C} \\
d_{2,s} &: \mu_{2,s}^{CP,C} = \lambda_{1,s}^{CP,C} - 1; \; d_{1,s} : \mu_{1,s}^{CP,C} = \lambda_0^{CP,C} - \lambda_{1,s}^{CP,C} \\
k_{1,s} &: A_2 + \mu_{1,s}^{CP,C} \frac{\partial q_{1,s}}{\partial k_{1,s}} k_0 + \lambda_{1,s}^{CP,C} \left(\frac{\partial q_{1,s}}{\partial k_{1,s}} \left(k_0 - k_{1,s}\right) - q_{1,s}\right) + \kappa_{1,s}^{CP,C} = 0 \\
k_0 &: \sum_s \pi_s \left(\mu_{1,s}^{CP,C} \left(\left(q_{1,s} - \gamma\right) + \frac{\partial q_{1,s}}{\partial k_0} k_0\right) + \lambda_{1,s}^{CP,C} \left(\frac{\partial q_{1,s}}{\partial k_0} \left(k_0 - k_{1,s}\right) + \left(A_{1,s} + q_{1,s} - \gamma\right)\right)\right) + \kappa_0^{CP,C} = \lambda_0^{CP,C} \\
\tau_{1,s} &: e'\left(\tau_{1,s}\right) + \lambda_{1,s}^{CP,C} T_1'\left(\tau_{1,s}\right) + \varpi_{1,s}^{CP,C} = 0
\end{aligned}$$

The proof that $k_0 > 0$ and $\kappa_0^{CP,C} = 0$ is similar to the proof in the decentralized equilibrium with small banks. One can solve out for

$$\lambda_{1,s}^{CP,C} = \frac{A_2 + \kappa_{1,s}^{CP,C} + \mu_{1,s}^{CP,C} \frac{\partial q_{1,s}}{\partial k_{1,s}} k_0}{q_{1,s} - \frac{\partial q_{1,s}}{\partial k_{1,s}} \left(k_0 - k_{1,s}\right)} \geq A_2 > 1$$

Since $\lambda_{1,s}^{CP,C} > 1$, $c_{1,s} = 0$ and $z_{1,s}^{CP,C} > 0$. In order for the period one budget constraint to hold,

$k_{1,s} > 0$ and $\kappa_{1,s}^{CP,C} = 0$. Finally since $\mu_{1,s}^{CP,C} \geq 0$, then $\lambda_0^{CP,C} \geq \lambda_{1,s}^{CP,C} > 1$ and $c_0 = 0$ and $z_0^{CP,C} > 0$. 2 holds as well. Similarly to the case where the CP cannot commit, one can prove that the cases $\lambda_{1,s}^{CP,C} = \lambda_0^{CP,C}$ for $s \in \{g, b\}$ and $\lambda_{1,g}^{CP,C} > \lambda_{1,b}^{CP,C} = \lambda_0^{CP,C}$ cannot be equilibria. One can derive the same expression for the optimal tax as in equation (15) in the text with the only exception that $\hat{\lambda}_1^B$ is replaced by $\lambda_1^{CP,C}$. There are only two types of equilibria left, which are exactly the same as in the case where the CP cannot commit. This finishes the proof that there is no time inconsistency with respect to the CP's problem.

6.A.8 Proof of Lemma 4

First, I prove that $k_{1,b}^{f,CP} < k_{1,b}^{f,*}$. After that, I show that $k_0^{CP} < k_0^*$. Define the following functions

$$\psi^{CP}\left(k_{1,b}^f\right) = \lambda_{1,b}^{CP} - \lambda_0^{CP} = \Omega_1 \lambda_{1,b}^{CP} - \Omega_2 \tag{49}$$

$$\psi\left(k_{1,b}^f\right) = \lambda_{1,b} - \lambda_0 = \Omega_1 \lambda_{1,b} - \Omega_2 \tag{50}$$

$$\Omega_1 = \left(\frac{1 + \frac{1}{\pi_b}\gamma - A_{1,b}}{1 + \frac{\pi_g}{\pi_b}\gamma}\right); \; \Omega_2 = A_2 \frac{\frac{\pi_g}{\pi_b} A_{1,g} + 1}{1 + \frac{\pi_g}{\pi_b}\gamma} \tag{51}$$

If the equilibrium is of type1 for both the CP and the banker from the decentralized equilibrium with no ex-ante regulation, then $k_{1,b}^{f,*}$ and $k_{1,b}^{f,CP}$ are determined by the system of equations $\psi\left(k_{1,b}^{f,*}\right) = 0$ and $\psi^{CP}\left(k_{1,b}^{f,CP}\right) = 0$. Given the assumptions made and Lemma 11, it follows that $\lambda_{1,b}^{CP\prime}\left(k_{1,b}^f\right) > 0$ and $\lambda_{1,b}'\left(k_{1,b}^f\right) > 0$, which imply $\psi'\left(k_{1,b}^f\right) > 0$ and $\psi^{CP\prime}\left(k_{1,b}^f\right) > 0$. From inequality (21) in the text, which implies $\lambda_{1,b}^{CP}\left(k_{1,b}^f\right) > \lambda_{1,b}\left(k_{1,b}^f\right)$, it follows that $\psi^{CP}\left(k_{1,b}^f\right) > \psi\left(k_{1,b}^f\right)$ which finishes the proof that $k_{1,b}^{f,CP} < k_{1,b}^{f,*}$. Next, I prove that the larger the equilibrium fire sale is, the larger the period zero investment is $\frac{\partial k_0}{\partial k_{1,b}^f} > 0$, which implies that $k_0^{CP} < k_0^*$.

$$\frac{\partial k_0}{\partial k_{1,b}^f} = \frac{T_{1,b}'\left(k_{1,b}^f\right) + \left(F'\left(k_{1,b}^f\right) + F''\left(k_{1,b}^f\right) k_{1,b}^f\right)\eta^f}{\left(1 + \frac{\gamma}{\pi_b} - A_{1,b}\right)} > 0 \tag{52}$$

and the inequalities follow from Lemma 11. If the equilibrium is of type 2 for the banker in the decentralized equilibrium, then there is overinvestment since the type 2 equilibrium implies investing to the maximum in $t = 0$ while the CP's equilibrium is interior.□

6.A.9 Proof of Lemma 5

Consider the case $\tau_{1,s} > 0$. For a given $T_{1,s}$, I totally differentiate equation, (7):

$T_{1,s} = \chi\tau_{1,s}(T_{1,s})\, a\left(\frac{(1-\tau_{1,s}(T_{1,s}))\alpha a}{\omega}\right)^{\frac{\alpha}{1-\alpha}}$, which determines $\tau_{1,s}$ as a function of $T_{1,s}$. Given that $\alpha < 1$ and from Lemma 3

$$
\begin{aligned}
\frac{\partial\tau_{1,s}(\omega;T_{1,s})}{\partial\omega} &= \frac{\alpha}{1-\alpha}\frac{\tau_{1,s}(1-\alpha)(1-\tau_{1,s})}{\omega(1-\alpha-\tau_{1,s})} > 0 \\
\frac{\partial\tau_{1,s}(\chi;T_{1,s})}{\partial\chi} &= -\frac{(1-\alpha)(1-\tau_{1,s})}{(1-\alpha-\tau_{1,s})}\frac{\tau_{1,s}}{\chi} < 0 \\
\frac{\partial\tau_{1,s}(a;T_{1,s})}{\partial a} &= -\frac{\tau_{1,s}}{a}\frac{(1-\tau_{1,s})}{(1-\alpha-\tau_{1,s})} < 0.
\end{aligned}
$$

The comparative statics above imply that, for a given size of the bail-out, the optimal labor tax is lower the higher the fiscal capacity of a country is.

For a given $T_{1,s}$, one can differentiate $MC_{T_1} = -\frac{e'(\tau_{1,s})}{T'_{1,s}(\tau_{1,s})} = \frac{(1-\alpha)(1-\tau_{1,s})}{\chi(1-\alpha-\tau_{1,s})}$

$$
\begin{aligned}
\frac{\partial MC_{T_1}(\omega;T_{1,s})}{\partial\omega} &= \frac{\partial\tau_{1,s}(\omega;T_{1,s})}{\partial\omega}\frac{(1-\alpha)\alpha}{\chi(1-\alpha-\tau_{1,s})^2} > 0 \\
\frac{\partial MC_{T_1}(\chi;T_{1,s})}{\partial\chi} &= -\frac{(1-\alpha)(1-\tau_{1,s})}{\chi^2(1-\alpha-\tau_{1,s})} + \frac{\partial\tau_{1,s}(\chi;T_{1,s})}{\partial\chi}\frac{(1-\alpha)\alpha}{\chi(1-\alpha-\tau_{1,s})^2} < 0 \\
\frac{\partial MC_{T_1}(a;T_{1,s})}{\partial a} &= \frac{\partial\tau_{1,s}(a;T_{1,s})}{\partial a}\frac{(1-\alpha)\alpha}{\chi(1-\alpha-\tau_{1,s})^2} < 0.
\end{aligned}
$$

This completes the proof that larger fiscal capacity implies lower MC_{T_1} for a given $T_{1,s}$. □

6.A.10 Proof of Lemma 6

First I prove that larger fiscal capacity implies larger $T_{1,s}$ for a given $k^f_{1,s}$. From equations (*7)*, (1), (18) and Lemma 3:

$$\begin{aligned}
T_{1,s} &= \tau_{1,s}\left(\chi;k_{1,s}^{f}\right)\left(1-\tau_{1,s}\left(\chi;k_{1,s}^{f}\right)\right)^{\frac{\alpha}{1-\alpha}}K \qquad (53)\\
\text{where } K &= \left[\frac{\alpha a}{\omega}\right]^{\frac{\alpha}{1-\alpha}}a\chi,\ \tau_{1,s}\left(\chi;k_{1,s}^{f}\right)=\frac{\chi\hat{\lambda}_{1,s}^{B}-1}{\frac{\chi\hat{\lambda}_{1,s}^{B}}{(1-\alpha)}-1}\\
\text{and } \hat{\lambda}_{1,s}^{B} &= \hat{\lambda}_{1,s}^{CP}=\frac{A_2}{F'\left(k_{1,s}^{f}\right)+F''\left(k_{1,s}^{f}\right)k_{1,s}^{f}}\\
\frac{\partial T_{1,s}\left(a;k_{1,s}^{f}\right)}{\partial a} &\propto \frac{\partial K}{\partial a}>0;\frac{\partial T_{1,s}\left(\omega;k_{1,s}^{f}\right)}{\partial \omega}\propto\frac{\partial K}{\partial \omega}<0\\
\frac{\partial T_{1,s}\left(\chi;k_{1,s}^{f}\right)}{\partial \chi} &= \frac{\partial \tau_{1,s}\left(\chi;k_{1,s}^{f}\right)}{\partial \chi}\left(1-\tau_{1,s}\right)^{\frac{\alpha}{1-\alpha}-1}\frac{\left(1-\alpha-\tau_{1,s}\right)}{(1-\alpha)}K \qquad (54)\\
+\tau_{1,s}\left(1-\tau_{1,s}\right)^{\frac{\alpha}{1-\alpha}}\frac{\partial K}{\partial \chi} &> 0 \text{ where } \frac{\partial \tau_{1,s}\left(\chi;k_{1,s}^{f}\right)}{\partial \chi}=\frac{\hat{\lambda}_{1,s}^{B}\alpha}{(1-\alpha)\left(\frac{\chi\hat{\lambda}_{1,s}^{B}}{(1-\alpha)}-1\right)^{2}}>0.
\end{aligned}$$

For a given $k_{1,b}^{f}$,

$$\frac{\partial^2 T_{1,b}}{\partial x\partial k_{1,b}^{f}}=\tau_{1,b}'\left(k_{1,b}^{f}\right)\left(1-\tau_{1,b}\right)^{\frac{\alpha}{1-\alpha}-1}\left(\frac{1-\alpha-\tau_{1,b}}{1-\alpha}\right)\frac{\partial K}{\partial x} \text{ for } x\in\{a,\omega\}.$$

From the proof of Lemma 3 part (i), $1-\alpha-\tau_{1,b}>0$, and also from equation (36), $\tau_{1,b}'\left(k_{1,b}^{f}\right)>0$. Therefore

$$\begin{aligned}
\frac{\partial^2 T_{1,b}}{\partial a\partial k_{1,b}^{f}} &\propto \frac{\partial K}{\partial a}>0\\
\frac{\partial^2 T_{1,b}}{\partial \omega\partial k_{1,b}^{f}} &\propto \frac{\partial K}{\partial \omega}<0.
\end{aligned}$$

One can re-write equation (54) further as

$$\frac{\partial T_{1,b}}{\partial \chi}=\left(\frac{\alpha}{\left(\chi\hat{\lambda}_{1,s}^{B}-1\right)\left(\chi\hat{\lambda}_{1,s}^{B}-(1-\alpha)\right)}+1\right)\tau_{1,b}\left(1-\tau_{1,b}\right)^{\frac{\alpha}{1-\alpha}}\frac{K}{\chi}.$$

Since if the bail-out is positive then $\chi\hat{\lambda}_{1,b}^{B}\geq 1>(1-\alpha)$ and from the proof of Lemma 3, $\frac{\partial\hat{\lambda}_{1,b}^{B}}{\partial k_{1,b}^{f}}>0$, it follows that

$$\frac{\partial^2 T_{1,b}}{\partial\chi\partial k_{1,b}^f} = -\frac{\partial\hat{\lambda}_{1,b}^B}{\partial k_{1,b}^f} K \frac{\alpha(1-\alpha)(1-\tau_{1,b})^{\frac{\alpha}{1-\alpha}}}{\left(\chi\hat{\lambda}_{1,b}^B-(1-\alpha)\right)^3}\left(1+\frac{1}{\chi\hat{\lambda}_{1,b}^B}\right) < 0.$$

Finally, since $\hat{\lambda}_1^{CP\prime}\left(k_{1,b}^f\right) > 0$, then $\frac{\partial\tau_{1,b}}{\partial\chi\partial k_{1,b}^f} = -\hat{\lambda}_1^{CP\prime}\left(k_{1,b}^f\right)\frac{\alpha}{(1-\alpha)\left(\frac{\chi\hat{\lambda}_1^{CP}}{(1-\alpha)}-1\right)^2}\frac{\chi\hat{\lambda}_1^{CP}+(1-\alpha)}{\chi\hat{\lambda}_1^{CP}-(1-\alpha)} < 0$, which is the comparative static discussed in the main text.□

6.A.11 Overinvestment and Overborrowing and Fiscal Capacity: Small Banks' Case

Lemma 12 *The equilibrium fire sales in the decentralized allocation with small bank and no ex-ante regulation, $k_{1,b}^{f,*}$ does not depend on the fiscal capacity of the country; $\frac{\partial k_{1,b}^{f,*}}{\partial x}=0$ for $x\in\{a,\omega,\chi\}$. In contrast, larger fiscal capacity implies optimally higher period zero investment chosen by the small banker: $\frac{\partial k_0^*}{\partial a}>0, \frac{\partial k_0^*}{\partial\omega}<0, \frac{\partial k_0^*}{\partial\chi}>0$.*

Proof of Lemma 12 : The fact that $k_{1,b}^{f,*}$ is not a function of the fiscal capacity of the country follows directly from equations (17). The comparative statics of k_0^* coincide with the comparative statics of $T_{1,b}\left(k_{1,b}^f\right)$ with respect to the fiscal capacity parameters (see equation (40)). From Lemma 6, larger fiscal capacity implies a larger optimal bail-out and, therefore, a larger k_0^*, which finishes the proof. □

Lemma 13 links the degree of overinvestment and overborrowing against the bad state of nature, measured as $k_0^*-k_0^{CP}$ and $d_{1,b}^*-d_{1,b}^{CP}$, respectively, to the fiscal capacity of a country. I consider the case where the banker's and the Central Planner's equilibria are both interior.

Lemma 13 *Larger fiscal capacity due to a more productive labor intensive sector or lower dis-utility of labor (i.e. larger tax base) implies that the banker will overinvest by more relative to the Central Planner; $\frac{\partial\left(k_0^*-k_0^{CP}\right)}{\partial a}>0$ and $\frac{\partial\left(k_0^*-k_0^{CP}\right)}{\partial\omega}<0$. However, larger fiscal capacity due to the government being less corrupt or more efficient at collecting taxes implies lower overinvestment; $\frac{\partial\left(k_0^*-k_0^{CP}\right)}{\partial\chi}<0$. The exact same comparative statics hold with respect to the degree of overborrowing against the bad state of nature.*

Proof of Lemma 13

Equation (40) determines k_0 and

$$k_0^*-k_0^{CP} = \frac{T_{1,b}\left(k_{1,b}^{f,*}\right)-T_{1,b}\left(k_{1,b}^{f,CP}\right)+\left(F'\left(k_{1,b}^{f,*}\right)k_{1,b}^{f,*}-F'\left(k_{1,b}^{f,CP}\right)k_{1,b}^{f,CP}\right)\eta^f}{\left(1+\frac{\gamma}{\pi_b}-A_{1,b}\right)},$$

where $T_{1,b}\left(k_{1,b}^f\right)$ is given by equation (53). Since $k_{1,b}^{f,*}$ and $k_{1,b}^{f,CP}$ are determined by equations (17) and (20) respectively, they are not a function of the fiscal capacity parameters. Therefore, since $A_{1,b}<\gamma$,

$$\frac{\partial k_0^* - k_0^{CP}}{\partial x} \propto \frac{\partial \left(T_{1,b}\left(k_{1,b}^{f,*}\right) - T_{1,b}\left(k_{1,b}^{f,CP}\right)\right)}{\partial x} \text{ for } x \in \{a, \chi, \omega\}.$$

The comparative statics of the overinvestment with respect to fiscal capacity are driven entirely by how much the bail-out increases as the fiscal capacity increases in the decentralized equilibrium relative to the Central Planner's equilibrium. Since $k_{1,b}^* > k_{1,b}^{CP}$, the result follows directly from the comparative statics of the cross partial derivatives, $\frac{\partial^2 T_{1,b}}{\partial\left(k_{1,b}^f\right)\partial x}$ for $x \in \{a, \omega, \chi\}$, presented in Lemma 6.

$$\begin{aligned}
\frac{\partial\left(k_0^* - k_0^{CP}\right)}{\partial a} &> 0 \\
\frac{\partial\left(k_0^* - k_0^{CP}\right)}{\partial \omega} &< 0 \\
\frac{\partial\left(k_0^* - k_0^{CP}\right)}{\partial \chi} &< 0.
\end{aligned}$$

In both the Central Planner's and the banker's allocation, $d_{1,b} = \left(1 + \frac{\pi_g}{\pi_b}\gamma\right) k_0 - \frac{n_0}{\pi_b}$. Therefore, the same comparative statics apply with respect to $d_{1,b}^* - d_{1,b}^{CP}$. $\square$

6.A.12 Proof of Proposition 2

Lemma 4 proves that $k_0^{CP} < k_0^*$. For a given k_0, the rest of the endogenous variables in the decentralized equilibrium with no commitment and no ex-ante regulation and the constrained Central Planner's allocation coincide. The variable $k_{1,s}$ is determined by the same period one budget constraint and $c_0 = c_{1,s} = 0$ in both problems. In $t = 0$, both the constrained Central Planner and the banker always borrow to the maximum against the good state and only then they borrow against the bad state, and there is no borrowing or lending in $t = 1$. Finally, the period one taxes and transfers chosen are the same in both problems for a given k_0. As a result, the constrained Central Planner's allocation can be decentralized with a single instrument that controls the banker's choice of k_0.

"Quantity" Regulatory Instrument

First, I consider the minimum bank capital requirement. At the beginning of period $t = 0$, the policy maker chooses ρ^i. At the end of period zero, banker i chooses his optimal allocation subject to the constraint $k_0^i \leq \frac{n_0}{\rho^i}$, where the Lagrange multiplier on the constraint is $\lambda_0^i \xi^i$. The first order condition of the banker with respect to k_0^i, subject to the regulatory constraint, becomes

$$\lambda_0^i \left(1 + \xi^i\right) = \sum_s \pi_s \left(\lambda_{1,s}^i \left(A_{1,s} - \gamma + q_{1,s}\right) + \mu_{1,s}^i \left(q_{1,s} - \gamma\right)\right). \tag{55}$$

The rest of the first order conditions are the same as in section 3.1.1. Since $k_0^{CP} < k_0^*$, then $\lambda_0^i \xi^i > 0$, and the constrained Central Planner's allocation will be replicated if $\rho^{i,*} = \rho^* = \frac{n_0}{k_0^{CP}}$.

"Price" Regulatory Instrument

Instead of using a minimum bank capital requirement, the policy maker can replicate the Central Planner's allocation with a tax instrument. The optimization problem of the banker is the same, with the only difference being that the period-zero budget constraint becomes $k_0^i \left(1+\tau_0^{k,i}\right) - n_0 \leq \sum_s p_{1,s} d_{1,s}^i + T_0^k$, where $T_0^k = k_0 \tau_0^k$. I assume that the lump sum rebates from capital taxation, T_0^k, are non-targeted and that $\tau_0^{k,i}$ is optimally chosen by the policy maker at the beginning of $t = 0$ before the banker chooses his allocation. The first order condition with respect to k_0^i becomes

$$\lambda_0^i \left(1+\tau_0^{k,i}\right) = \sum_s \pi_s \left(\lambda_{1,s}^i \left(A_{1,s} - \gamma + q_{1,s}\right) + \mu_{1,s}^i \left(q_{1,s} - \gamma\right)\right). \tag{56}$$

Since the equilibrium is symmetric and using the fact that $\lambda_0 = \lambda_{1,b}$, equation (56) can be re-written as

$$\tau_0^k = \frac{A_2}{\lambda_{1,b}} \left(\pi_g A_{1,g} + \pi_b\right) + \pi_b \left(A_{1,b} - 1\right) - \gamma \tag{57}$$

In order to calculate the optimal τ_0^k, equation (57) is combined with the equivalent first order condition from the Central Planner's problem, equation (18), which can be re-written as $0 = \frac{A_2}{\lambda_{1,b}^{CP}} \left(\pi_g A_{1,g} + \pi_b\right) + \pi_b \left(A_{1,b} - 1\right) - \gamma$. Combining the last two equations gives the formula for the optimal tax rate. Finally, from equations (55) and (56), one can prove that the optimal tax on capital is equal to the scaled Lagrange multiplier on the minimum bank capital requirement constraint, where the latter is the shadow price of the "quantity" constraint, $\xi\left(\rho^*\right) = \tau_0^{k,*}$. A similar point regarding the link between "price" and "quantity" instruments has been made by Stein and Kashyap (2012).□

Appendix 6.B Appendix: Large Banks

In this section of the Appendix, I solve the problem with N banks. I solve sequentially the banker's problem and the problem of the bail-out authority with no commitment as each agent takes into account future best response functions. At the end of period one, banker i maximizes $\max_{c_{1,s}^i, k_{1,s}^i, d_{2,s}^i} A_2 k_{1,s}^i + c_{1,s}^i - d_{2,s}^i$ subject to

$$\begin{aligned} c_{1,s}^i + q_{1,s} k_{1,s}^i + d_{1,s}^i \quad &\leq \quad \left(A_{1,s} + q_{1,s} - \gamma\right) k_0^i + T_{1,s}^i + d_{2,s}^i \qquad \left[\hat{\lambda}_{1,s}^{l,i}\right] \\ c_{1,s}^i \quad &\geq \quad 0 \qquad \left[\hat{z}_{1,s}^{l,i}\right];\ d_{2,s}^i \leq 0 \qquad \left[\hat{\mu}_{2,s}^{l,i}\right];\ k_{1,s}^i \geq 0 \qquad \left[\hat{\kappa}_{1,s}^{l,i}\right]. \end{aligned}$$

The first order conditions imply

$$
\begin{aligned}
\hat{\lambda}_{1,s}^{l,i} &= \frac{\hat{\kappa}_{1,s}^{i}+A_2}{q_{1,s}-\frac{\partial q_{1,s}}{\partial k_{1,s}^{i}}\left(k_0^i-k_{1,s}^{i}\right)} = 1+\hat{z}_{1,s}^{l,i} = 1+\hat{\mu}_{2,s}^{l,i} \\
\frac{\partial q_{1,s}}{\partial k_{1,s}^{i}} &= -\frac{\partial q_{1,s}}{\partial k_0^{i}} = \begin{cases} -F''\left(k_{1,s}^{f}\right)\frac{1}{\eta^f N} \text{ if } q_{1,s}<1 \\ 0 \text{ if } q_{1,s}=1 \end{cases},
\end{aligned}
$$

where $k_{1,s}^{f}=\max\left\{\frac{1}{\eta^f N}\sum_{i=1}^{N}\left(k_0^i-k_{1,s}^{i}\right),0\right\}=\max\left\{\frac{1}{\eta^f}\left(k_0-k_{1,s}\right),0\right\}$. Given the Assumptions made and from Lemma 1, $A_2>1\geq q_{1,s}-\frac{\partial q_{1,s}}{\partial k_{1,s}^{i}}\left(k_0-k_{1,s}\right)=F'\left(k_{1,s}^{f}\right)+\frac{1}{N}F''\left(k_{1,s}^{f}\right)k_{1,s}^{f}$. Therefore, $\hat{\lambda}_{1,s}^{l,i}>1$ and $c_{1,s}^{i}=d_{2,s}^{i}=0$. In order for the budget constraint to be satisfied, it has to be the case that $k_{1,s}^{i}>0$. In the beginning of $t=1$, the bail-out authority maximizes $\sum_{t=1}^{2}\left(c_{t,s}^{c}-\omega l_{t,s}^{*}\left(\tau_{t,s}\right)+\sum_{i=1}^{N}\frac{1}{N}c_{t,s}^{i}\right)$, where the Lagrangian is given by

$$
\begin{aligned}
&\max_{\left\{k_{1,s}^{i},T_{1,s}^{i}\right\}_{i=1}^{N},\tau_{1,s}} \quad 2m+e\left(\tau_{1,s}\right)+d_{1,s}+e\left(0\right)+ \\
&\sum_{i=1}^{N}\frac{1}{N}\left(A_2k_{1,s}^{i}+\hat{\lambda}_{1,s}^{l,B,i}\left(\left(A_1-\gamma\right)k_0^i+T_{1,s}^{i}-d_{1,s}^{i}-q_{1,s}\left(k_{1,s}^{i}-k_0^i\right)\right)\right) \\
&+\hat{\iota}_{1,s}^{l,B}\left(\chi\tau_{1,s}a\left(\frac{\left(1-\tau_t\right)\alpha a}{\omega}\right)^{\frac{\alpha}{1-\alpha}}-\sum_{i=1}^{N}\frac{1}{N}T_{1,s}^{i}\right)+\hat{\varpi}_{1,s}^{l,B}\tau_{1,s}
\end{aligned}
$$

and I have substituted for the optimal labor decision given by equation (1). The Lagrange multipliers on the government budget constraint and the non-zero tax rate constraint are $\hat{\iota}_{1,s}^{l,B}$ and $\hat{\varpi}_{1,s}^{l,B}$, respectively. The first order conditions are

$$
-e'\left(\tau_{1,s}\right) \geq \hat{\iota}_{1,s}^{l,B}\chi\left(1-\tau_{1,s}\right)^{\frac{\alpha}{1-\alpha}-1}a\left[\frac{\alpha a}{\omega}\right]^{\frac{\alpha}{1-\alpha}}\frac{\left(1-\alpha-\tau_{1,s}\right)}{\left(1-\alpha\right)} \tag{58}
$$

$$
\hat{\lambda}_{1,s}^{l,B,i} = \hat{\iota}_{1,s}^{l,B}
$$

$$
A_2-\hat{\lambda}_{1,s}^{l,B,i}q_{1,s} = \sum_{j=1}^{N}\hat{\lambda}_{1,s}^{l,B,j}\frac{\partial q_{1,s}}{\partial k_{1,s}^{j}}\left(k_{1,s}^{j}-k_0^j\right). \tag{59}
$$

$\hat{\iota}_{1}^{l,B}=\hat{\lambda}_{1}^{l,B,i}=\hat{\lambda}_{1}^{l,B}$ Since $\hat{\iota}_{1,s}^{l,B}=\hat{\lambda}_{1,s}^{l,B,i}=\hat{\lambda}_{1,s}^{l,B}$, the marginal value of wealth in the hands of each banker from the perspective of the bail-out authority is equated. Equation (59) can be re-written as

$$
\hat{\lambda}_{1,s}^{l,B}=\frac{A_2}{F'\left(k_{1,s}^{f}\right)+F''\left(k_{1,s}^{f}\right)k_{1,s}^{f}}, \tag{60}
$$

where $\hat{\lambda}_{1,s}^{l,B}\left(k_{1,s}^{f}\right)=\hat{\lambda}_{1,s}^{B}\left(k_{1,s}^{f}\right)=\hat{\lambda}_{1,s}^{CP}\left(k_{1,s}^{f}\right)$. Substituting $\hat{\lambda}_{1,s}^{l,B,i}=\hat{\iota}_{1,s}^{l,B}$ and $e'\left(\tau_{1,s}\right)=-a\left(l_{1,s}^{*}\right)^{\alpha}$ into equation (58) and using a proof similar to the proof of Lemma 3 one can show that $\tau_{1,s}$ is given by equation (15).

The marginal value of wealth in the hands of the banker, as perceived by the bail-out authority, is given by equation (60). It is the same for all bankers and a function only of the aggregate fire sale, which implies that the policy maker is indifferent how to distribute the bail-out across bankers during the crisis. To solve the model, I assume symmetric equilibrium, where each banker receives the same amount of bail-out $T_{1,s}=T_{1,s}^{i}$. At the end of $t=0$, banker i solves the following optimization problem

$$\begin{aligned}
& \max_{d_{1,s}^{i},c_{0}^{i},k_{0}^{i},\left\{k_{1,s}^{j}\right\}_{j=1}^{N}} c_{0}^{i}+\sum_{s}\pi_{s}A_{2}k_{1,s}^{i} \\
\left(q_{1,s}-\gamma\right)k_{0}^{i} &\geq d_{1,s}^{i} \qquad \left[\pi_{s}\mu_{1,s}^{l,i}\right] \\
n_{0}+\sum_{s}\pi_{s}d_{1,s}^{i}-k_{0}^{i}-c_{0}^{i} &\geq 0 \qquad \left[\lambda_{0}^{l,i}\right] \\
\left(A_{1,s}-\gamma+q_{1,s}\right)k_{0}^{j}+T_{1,s}^{j}-q_{1,s}k_{1,s}^{j}-d_{1,s}^{j} &\geq 0 \qquad \left[\pi_{s}\lambda_{1,s}^{l,i,j}\right] \\
c_{0}^{i} &\geq 0 \qquad \left[z_{0}^{l,i}\right] \\
k_{0}^{i} &\geq 0 \qquad \left[\kappa_{0}^{l,i}\right]
\end{aligned}$$

where $\lambda_{1,s}^{l,i,j}$ is the marginal benefit of banker i from relaxing the budget constraint of banker j in $t=1$ and $\lambda_{1,s}^{l,i}=\lambda_{1,s}^{l,i,i}$. Banker i internalizes the period one budget constraints of the other bankers as they determine their period one investment and capture future best response functions. These budget constraints were ignored in the case of small banks since the only channel through which banker i could impact them is via his effect on the optimal fire sale, which affects the bail-out and the price of capital. However, small bankers take aggregate variables as given. The problem is also subject to the ex-ante regulatory constraints

$$\begin{aligned}
k_{0}^{i} &\leq \frac{n_{0}}{\rho^{i}} \qquad \left[\lambda_{0}^{l,i}\xi^{l,i}\right] \\
d_{1,b}^{i} &\leq \nu^{i} \qquad \left[\pi_{b}\varphi^{l,i}\right].
\end{aligned}$$

The first order conditions with respect to $d_{1,s}^{i}$ and c_{0}^{i} are given by

$$\begin{aligned}
\mu_{1,g}^{l,i} &= \lambda_{0}^{l,i}-\lambda_{1,g}^{l,i,i} \\
\mu_{1,b}^{l,i}+\varphi^{l,i} &= \lambda_{0}^{l,i}-\lambda_{1,b}^{l,i,i} \\
1+z_{0}^{l,i} &= \lambda_{0}^{l,i}.
\end{aligned}$$

The first order condition with respect to $k_{1,s}^i$ is

$$A_2 + \mu_{1,s}^{l,i} \frac{\partial q_{1,s}}{\partial k_{1,s}^f} \frac{\partial k_{1,s}^f}{\partial k_{1,s}^i} k_0^i + \sum_{j=1}^{N} \lambda_{1,s}^{l,i,j} \frac{\partial k_{1,s}^f}{\partial k_{1,s}^i} \left(\frac{\partial T_{1,s}}{\partial k_{1,s}^f} + \frac{\partial q_{1,s}}{\partial k_{1,s}^f} \left(k_0^j - k_{1,s}^j \right) \right) = \lambda_{1,s}^{l,i,i} q_{1,s}.$$

The first order condition with respect to $k_{1,s}^j$, where $j \neq i$, is

$$\mu_{1,s}^{l,i} \frac{\partial q_{1,s}}{\partial k_{1,s}^f} \frac{\partial k_{1,s}^f}{\partial k_{1,s}^j} k_0^i + \sum_{k=1}^{N} \lambda_{1,s}^{l,i,k} \frac{\partial k_{1,s}^f}{\partial k_{1,s}^j} \left(\frac{\partial T_{1,s}}{\partial k_{1,s}^f} + \frac{\partial q_{1,s}}{\partial k_{1,s}^f} \left(k_0^k - k_{1,s}^k \right) \right) = \lambda_{1,s}^{l,i,j} q_{1,s}.$$

The first order condition with respect to k_0^i is

$$\begin{aligned} MC_{k_0}^l &= \lambda_0^{l,i} \left(1 + \xi^{l,i}\right) = \\ \kappa_0^{l,i} + \sum_s \pi_s \left[\begin{array}{c} \lambda_{1,s}^{l,i} \left(A_{1,s} - \gamma + q_{1,s}\right) + \sum_{j=1}^{N} \lambda_{1,s}^{l,i,j} \left(\frac{\partial T_{1,s}^j}{\partial k_{1,s}^f} + \frac{\partial q_{1,s}}{\partial k_{1,s}^f} \left(k_0^j - k_{1,s}^j\right) \right) \frac{\partial k_{1,s}^f}{\partial k_0^i} \\ + \mu_{1,s}^{l,i} \left(q_{1,s} - \gamma + \frac{\partial q_{1,s}}{\partial k_{1,s}^f} \frac{\partial k_{1,s}^f}{\partial k_0^i} k_0^i \right) \end{array} \right] &= MB_{k_0}^l, \end{aligned}$$

where if there is a fire sale in $t = 1$, $\frac{\partial k_{1,s}^f}{\partial k_{1,s}^j} = -\frac{1}{\eta^f N}$ for every j and $\frac{\partial k_{1,s}^f}{\partial k_0^i} = \frac{1}{\eta^f N}$. Simplifying the first order conditions with respect to $k_{1,s}^i$ and $k_{1,s}^j$, one can show that

$$\lambda_{1,s}^{l,i,j} = \lambda_{1,s}^{l,i} - \frac{A_2}{q_{1,s}} \text{ if } j \neq i \tag{61}$$

Assume a symmetric equilibrium. If there is no fire sale in state s in $t = 1$, $\lambda_{1,s}^{l,i} = \lambda_{1,s}^l = A_2$, while if there is a fire sale in state s in $t = 1$,

$$\lambda_{1,s}^{l,i} = \lambda_{1,s}^l = \frac{A_2}{F'\left(k_{1,s}^f\right)} \left(1 - \frac{\frac{1}{N}\left(\frac{1}{\eta^f} \frac{\partial T_{1,s}}{\partial k_{1,s}^f} + F''\left(k_{1,s}^f\right) k_{1,s}^f \right)}{Z\left(k_{1,s}^f\right)} \right) - \frac{1}{N} \frac{\frac{1}{\eta^f} \mu_{1,s}^l F''\left(k_{1,s}^f\right) k_0}{Z\left(k_{1,s}^f\right)}, \tag{62}$$

where $Z\left(k_{1,s}^f\right)$ is given by equation (27) in the text. Notice that as $N \to \infty, \lambda_{1,s}^l = \frac{A_2}{F'\left(k_{1,s}^f\right)}$ and $\lambda_{1,s}^{l,i,j} = 0$ if $j \neq i$ as in the small banks' case. The first order condition with respect to k_0^i, assuming symmetric equilibrium, can be re-written as

$$\lambda_0^l \left(1 + \xi^l\right) = \kappa_0^l + \sum_s \pi_s \left(\lambda_{1,s}^l \left(A_{1,s} - \gamma\right) + A_2 + \mu_{1,s}^l \left(F'\left(k_{1,s}^f\right) - \gamma \right) \right). \tag{63}$$

Finally, it could be the case that $\lambda_{1,s}^{l,i,j}$ is negative or positive if $j \neq i$ — i.e. banker i can perceive an extra dollar of wealth in the hands of banker j to increase or decrease his own welfare. From equations (61) and (62), if there is a fire sale and if $\mu_{1,s}^{l}=0$,

$$\lambda_{1,s}^{l,i,j}=\frac{A_2}{F'\left(k_{1,s}^{f}\right)}\frac{\frac{1}{N}\left(-\frac{1}{\eta^f}\frac{\partial T_{1,s}}{\partial k_{1,s}^{f}}-F''\left(k_{1,s}^{f}\right)k_{1,s}^{f}\right)}{Z\left(k_{1,s}^{f}\right)}.$$

Then the marginal value of relaxing banker's j period one budget constraint from the perspective of banker i, $\lambda_{1,s}^{l,i,j}$, is positive if $-F''\left(k_{1,s}^{f}\right)k_{1,s}^{f}-\frac{1}{\eta^f}\frac{\partial T_{1,s}}{\partial k_{1,s}^{f}}>0$, negative if the reverse is true and zero if this term is equal to zero or if $N\to\infty$. If $-F''\left(k_{1,s}^{f}\right)k_{1,s}^{f}-\frac{1}{\eta^f}\frac{\partial T_{1,s}}{\partial k_{1,s}^{f}}>0$, then banker i would like to relax hers and the other bankers' budget constraints in $t=1$ to prop up the price of capital. If $-F''\left(k_{1,s}^{f}\right)k_{1,s}^{f}-\frac{1}{\eta^f}\frac{\partial T_{1,s}}{\partial k_{1,s}^{f}}<0$, banker i would like to tighten the period one budget constraints of the other bankers in order to maximize the fire sale and, hence, the bail-out received.

6.B.1 Proof of Proposition 4

One can easily show that Lemma 2 holds in the case of large banks, implying that there is no fire sale in the good state and there will be a fire sale in the bad state if the equilibrium is not the corner one. Also, similarly to the small banks' case, one can prove that $\kappa_0^l=0$ and $k_0>0$. Define

$$\begin{aligned} M\left(k_{1,s}^{f}\right) &= F'\left(k_{1,s}^{f}\right)-1+\frac{1}{N}\left(1-\frac{F'\left(k_{1,s}^{f}\right)}{Z\left(k_{1,s}^{f}\right)}\right) \\ &= F'\left(k_{1,s}^{f}\right)-1+\frac{1}{N}\left(\frac{\frac{1}{\eta^f}\frac{\partial T_{1,s}}{\partial k_{1,s}^{f}}+F''\left(k_{1,s}^{f}\right)k_{1,s}^{f}}{Z\left(k_{1,s}^{f}\right)}\right). \end{aligned} \tag{64}$$

Next, I prove the following Lemma.

Lemma 14 *Conditional on the following assumption*

Assumption 6 *Assume that the following inequalities, which assure that the bail-out increases with respect to the fire sale but at a decreasing rate, are satisfied*

$$\begin{aligned} Z\left(k_{1,s}^{f}\right) &> \frac{1}{N} \\ \hat{\lambda}_{1,s}^{l,B\prime\prime}\left(k_{1,s}^{f}\right)\left(\chi\hat{\lambda}_{1,s}^{l,B}-(1-\alpha)\right) &< 2\chi\left(\hat{\lambda}_{1,s}^{l,B\prime}\left(k_{1,s}^{f}\right)\right)^2 \quad \text{for every } k_{1,s}^{f}\in(k_{1,s}^{T,f},k_{1,s}^{f,\max}],\ \text{where} \\ \hat{\lambda}_{1}^{l,B\prime\prime}\left(k_{1,s}^{f}\right) &= -\frac{\left[\begin{array}{c}\left(3F'''\left(k_{1,s}^{f}\right)+F''''\left(k_{1,s}^{f}\right)k_{1,s}^{f}\right)\hat{\lambda}_{1,s}^{l,B}\left(k_{1,s}^{f}\right)\\ +\left(2F''\left(k_{1,s}^{f}\right)+F'''\left(k_{1,s}^{f}\right)k_{1,s}^{f}\right)2\hat{\lambda}_{1,s}^{l,B\prime}\left(k_{1,s}^{f}\right)\end{array}\right]}{F''\left(k_{1,s}^{f}\right)+F'''\left(k_{1,s}^{f}\right)k_{1,s}^{f}}, \end{aligned}$$

then $M\left(k^f_{1,s}\right)$ *has a single discontinuity at* $k^f_{1,s}=k^{T,f}_{1,s}$, *where* $k^{T,f}_{1,s}$ *is defined by equation (65) below, and* $M\left(k^f_{1,s}\right)$ *is a continuos function on the intervals* $(0,k^{T,f}_{1,s})$ *and* $(k^{T,f}_{1,s},k^{f,\max}_{1,s}]$. *Also* $M'\left(k^f_{1,s}\right)<0$, $Z'\left(k^f_{1,s}\right)<0$ *on* $(0,k^{T,f}_{1,s})$ *and* $(k^{T,f}_{1,s},k^{f,\max}_{1,s}]$ *and* $\frac{\partial^2 T_{1,s}}{\partial\left(k^f_{1,s}\right)^2}<0$ *on* $(k^{T,f}_{1,s},k^{f,\max}_{1,s}]$ *and* $\frac{\partial^2 T_{1,s}}{\partial\left(k^f_{1,s}\right)^2}=0$ *on* $(0,k^{T,f}_{1,s})$.

Proof of Lemma 14: If $A_2\chi\leq 1$ then for some $k^f_{1,s}\geq 0$ it will be the case that $\tau_{1,s}=0$. There is a threshold fire sale, $k^{T,f}_{1,s}$, above which there is a positive bail-out and it is pinned down by

$$\hat{\lambda}^{l,B}_{1,s}\left(k^{T,f}_{1,s}\right)=\frac{A_2}{F'\left(k^{T,f}_{1,s}\right)+F''\left(k^{T,f}_{1,s}\right)k^{T,f}_{1,s}}=\frac{1}{\chi}. \tag{65}$$

The variable $F'\left(k^{T,f}_{1,s}\right)+F''\left(k^{T,f}_{1,s}\right)k^{T,f}_{1,s}$ is monotone and strictly decreasing with respect to $k^{T,f}_{1,s}$ and if $k^{T,f}_{1,s}\rightarrow\infty$ the expression goes to minus infinity and if $k^{T,f}_{1,s}=0$ it is equal to one. This is sufficient to guarantee the existence and uniqueness of $k^{T,f}_{1,s}$, given that $A_2\chi\leq 1$. Since

$$\begin{aligned}\lim_{k^f_{1,s}\rightarrow k^{T,f+}_{1,s}} T'_{1,s}\left(k^f_{1,s}\right) &= -\left(2F''\left(k^{T,f}_{1,s}\right)+F'''\left(k^{T,f}_{1,s}\right)k^{T,f}_{1,s}\right)\frac{1}{A_2}\frac{(1-\alpha)}{\alpha}a\left(\frac{\alpha a}{\omega}\right)^{\frac{\alpha}{1-\alpha}}>0 \qquad (66)\\ \lim_{k^f_{1,s}\rightarrow k^{T,f-}_{1,s}} T'_{1,s}\left(k^f_{1,s}\right) &= 0,\end{aligned}$$

then $\lim_{k^f_{1,s}\rightarrow k^{T,f+}_{1,s}} T'_{1,s}\left(k^f_{1,s}\right)>\lim_{k^f_{1,s}\rightarrow k^{T,f-}_{1,s}} T'_{1,s}\left(k^f_{1,s}\right)$. The function $M\left(k^f_{1,s}\right)$ has a single discontinuity on the interval $k^f_{1,s}\in[0,\infty)$ at $k^{T,f}_{1,s}$ and if $A_2\chi=1$ it is continuous on the whole interval as $k^{T,f}_{1,s}=0$ (i.e. there will be a bail-out for any level of the fire sale greater than zero).

Given that I assume that $F(\cdot)$ is a continuous function, this ensures that $M\left(k^f_{1,s}\right)$ and $Z\left(k^f_{1,s}\right)$are continuous function on the intervals $(0,k^{T,f}_{1,s})$ and $(k^{T,f}_{1,s},k^{f,\max}_{1,s}]$.

$$\begin{aligned}M'\left(k^f_{1,s}\right) &= F''\left(k^f_{1,s}\right)\left(1-\frac{1}{NZ\left(k^f_{1,s}\right)}\right)+\frac{1}{N}Z'\left(k^f_{1,s}\right)\frac{F'\left(k^f_{1,s}\right)}{\left(Z\left(k^f_{1,s}\right)\right)^2}\\ Z'\left(k^f_{1,s}\right) &= \left[2F''\left(k^f_{1,s}\right)+F'''\left(k^f_{1,s}\right)k^f_{1,s}+\frac{1}{\eta^f}\frac{\partial^2 T_{1,s}}{\partial\left(k^f_{1,s}\right)^2}\right]\end{aligned}$$

Given the assumptions made, $\left(1-\frac{1}{NZ\left(k^f_{1,s}\right)}\right)>0$, $F''\left(k^f_{1,s}\right)<0$ and $2F''\left(k^f_{1,s}\right)+F'''\left(k^f_{1,s}\right)k^f_{1,s}<0$. Therefore, in order to prove that $M'\left(k^f_{1,s}\right)<0$, it is sufficient to prove that the bail-out increases as the fire sale increases at a decreasing rate, $\frac{\partial^2 T_{1,s}}{\partial \left(k_{1,s}^f\right)^2} < 0$, where

$$\frac{\partial^2 T_{1,s}}{\partial \left(k_{1,s}^f\right)^2} = \frac{T_{1,s}\left[\left(1-\alpha-\tau_{1,s}\right)\tau_{1,s}''\left(k_{1,s}^f\right) - \alpha\frac{2(1-\alpha)-\tau_{1,s}}{(1-\alpha)(1-\tau_{1,s})}\left(\tau_{1,s}'\left(k_{1,s}^f\right)\right)^2\right]}{\tau_{1,s}\left(1-\tau_{1,s}\right)\left(1-\alpha\right)}$$

From Lemma 11, $\tau_{1,s}'\left(k_{1,s}^f\right) > 0$ and it is sufficient to prove that $\frac{\partial^2 \tau_{1,s}}{\partial \left(k_{1,s}^f\right)^2} < 0$ in order for $\frac{\partial^2 T_{1,s}}{\partial \left(k_{1,s}^f\right)^2} < 0$. If $\tau_{1,s} > 0$, conditional on assumption 6 being satisfied, then

$$\frac{\partial^2 \tau_{1,s}}{\partial \left(k_{1,s}^f\right)^2} = \left(\frac{\hat{\lambda}_{1,s}^{l,B\prime\prime}\left(k_{1,s}^f\right)}{\hat{\lambda}_{1,s}^{l,B\prime}\left(k_{1,s}^f\right)} - \frac{2\chi\hat{\lambda}_{1,s}^{l,B\prime}\left(k_{1,s}^f\right)}{\chi\hat{\lambda}_{1,s}^{l,B} - (1-\alpha)}\right)\tau_{1,s}'\left(k_{1,s}^f\right) < 0,$$

which also implies $Z'\left(k_{1,s}^f\right) < 0$ and $M'\left(k_{1,s}^f\right) < 0$ on $(0, k_{1,s}^{T,f})$ and $(k_{1,s}^{T,f}, k_{1,s}^{f,\max}]$.□

One can further re-write the first order condition with respect to k_0^i as

$$\lambda_0^l = \frac{\sum_s \pi_s\left[\lambda_{1,s}^l\left(A_{1,s}-q_{1,s}\right)+A_2\right]}{1-\sum_s \pi_s\left(q_{1,s}-\gamma\right)} = \tag{67}$$

$$\frac{A_2\left(\pi_g\left(A_{1,g}-1\right)+1\right)+\pi_b\lambda_{1,b}^l\left(A_{1,b}-F'\left(k_{1,b}^f\right)\right)}{\pi_b\left(1-F'\left(k_{1,b}^f\right)\right)+\gamma} \tag{68}$$

where $\lambda_{1,g}^l = A_2$ and

$$\lambda_{1,b}^l = \frac{A_2\frac{1}{N} + \frac{A_2}{F'\left(k_{1,b}^f\right)}\left(1-\frac{1}{N}\right)Z\left(k_{1,b}^f\right) - \frac{1}{\eta^f}\frac{1}{N}\mu_{1,b}^l F''\left(k_{1,b}^f\right)k_0}{Z\left(k_{1,b}^f\right)} \tag{69}$$

Notice that as $N \to \infty$, $\lambda_{1,s}^l = \lambda_{1,s}$ and $\lambda_0^l = \lambda_0$, which combined with the fact that the rest of the system of equations is the same in the case of small and large banks, it implies that, in the limiting case, the two problems coincide.

Consider all four types of equilibria. First, consider the case $\mu_{1,s}^l = 0$, which implies $\lambda_{1,b}^l = \lambda_{1,g}^l = \lambda_0^l$. The fact that $\lambda_{1,b}^l = \lambda_{1,g}^l$ implies $M\left(k_{1,b}^f\right) = 0$ and from equation (67), it follows that $E_0 A_{1,s} = \gamma$. However, from assumption 1, $E_0 A_{1,s} > 1 > \gamma$. Therefore, such an equilibrium does not exist. Next consider the case $\mu_{1,g}^l = 0, \mu_{1,b}^l > 0$ which implies $\lambda_{1,g}^l = \lambda_0^l > \lambda_{1,b}^l$. From the first order conditions, it follows that

$$\frac{A_2\left(\gamma - \pi_b F'\left(k_{1,b}^f\right) - \pi_g A_{1,g}\right)}{\left(A_{1,b} - F'\left(k_{1,b}^f\right)\right)\pi_b} = \lambda_{1,b}^l < \lambda_{1,g}^l = A_2$$

Since $A_{1,b} - F'\left(k^f_{1,b}\right) < 0$, this inequality implies $\gamma > E_0 A_{1,s}$, which violates assumption 1 and this case is not an equilibrium either. There are only two types of plausible equilibria.

Type 1 Equilibrium: $\mu^l_{1,g} > 0$, $\mu^l_{1,b} = 0$. This equilibrium implies $\lambda^l_{1,b} = \lambda^l_0 > \lambda^l_{1,g}$. Combining the first order conditions

$$\begin{aligned}
\lambda^l_{1,b} &= A_2\left(1 - \frac{M\left(k^f_{1,b}\right)}{F'\left(k^f_{1,b}\right)}\right); \lambda^l_{1,g} = A_2 \\
\frac{1}{G} &= 1 - \frac{M\left(k^f_{1,b}\right)}{F'\left(k^f_{1,b}\right)} \qquad (70)
\end{aligned}$$

where equation (70) determines the equilibrium fire sale. Assumption 1 implies that $\gamma \leq G < 1$. Therefore,

$$M\left(k^f_{1,b}\right) = F'\left(k^f_{1,b}\right)\frac{G-1}{G} < 0$$

where $M\left(k^f_{1,b}\right)$ is given by equation (64). From equation (67), it follows that $\lambda^l_0 > 1$ since the numerator is greater than one and the denominator is positive and less then one (The first statement follows from $A_2 \sum_s \pi_s A_{1,s} > 1$ which implies $\sum_s \pi_s \lambda^l_{1,s}\left(A_{1,s} - q_{1,s}\right) > \sum_s \pi_s A_2\left(A_{1,s} - 1\right) > 0 > -\left(A_2 - 1\right)$ while the second one follows from $\sum_s \pi_s\left(q_{1,s} - \gamma\right) > 0$). Therefore, $c_0 = 0$. The rest of the endogenous quantity variables, $d_{1,s}, k_{1,s}$ and k_0, are given by the same set of equations as in the type 1 equilibrium when banks are small.

Type 2 equilibrium: $\mu^l_{1,s} > 0$. The type 2 equilibrium is the same as in the case when banks are small as it is the corner equilibrium.

Another assumption is necessary in order to prove existence and uniqueness.

Assumption 7 *The following inequalities have to be satisfied to prove existence and uniqueness of the large banks' equilibrium and for the equilibrium to be such that there is a positive bail-out in the bad state*

$$\begin{aligned}
lim_{k^f_{1,b} \to k^{T,f+}_{1,b}} \Xi^l\left(k^f_{1,b}\right) &= F'\left(k^{T,f}_{1,b}\right)\frac{1}{G} - 1 + \frac{1}{N}\left(1 - \frac{F'\left(k^{T,f}_{1,b}\right)}{Z\left(k^{T,f}_{1,b}\right)}\right) > 0 \qquad (71) \\
lim_{k^f_{1,b} \to k^{T,f-}_{1,b}} \Xi^l\left(k^f_{1,b}\right) &= F'\left(k^{T,f}_{1,b}\right)\left(\frac{1}{G} - \frac{1}{N}\frac{1}{A_2\chi}\right) + \frac{1}{N} - 1 > 0, \qquad (72)
\end{aligned}$$

where $\Xi^l\left(k^f_{1,b}\right) = M\left(k^f_{1,b}\right) - F'\left(k^f_{1,b}\right)\frac{G-1}{G}$.

Existence and Uniqueness:

It is sufficient to solve for $k^f_{1,b}$ in order to solve the whole model. Consider the interior equilibrium where $k^f_{1,b}$ is determined by equation (70). Define $\Xi^l\left(k^f_{1,b}\right) = M\left(k^f_{1,b}\right) - F'\left(k^f_{1,b}\right)\frac{G-1}{G}$. From Lemma 14 and the assumptions made

$$\Xi^{l\prime}\left(k^f_{1,b}\right) = M'\left(k^f_{1,b}\right) - F''\left(k^f_{1,b}\right)\frac{G-1}{G} < 0 \text{ if } k^f_{1,b} > 0$$

From assumption 7, it follows that $lim_{k^f_{1,b}\to k^{T,f+}_{1,b}}\Xi^l\left(k^f_{1,b}\right) > 0$ and $lim_{k^f_{1,b}\to k^{T,f-}_{1,b}}\Xi^l\left(k^f_{1,b}\right) > 0$. If $\Xi^l\left(k^{f,\max}_{1,b}\right) \leq 0$, then there will be an unique solution to $\Xi^l\left(k^f_{1,b}\right) = 0$ and the equilibrium will be of type 1. Also the equilibrium fire sale will be in the range $(k^{T,f}_1, k^{f,\max}_1]$, which implies that there will be a positive bail-out. If $\Xi^l\left(k^{f,\max}_{1,b}\right) > 0$, the equilibrium will be of type 2. Therefore, a sufficient and necessary condition for the equilibrium to be of type 1 is given by

$$M\left(k^{f,\max}_{1,b}\right) - F'\left(k^{f,\max}_{1,b}\right)\frac{G-1}{G} \leq 0. \tag{73}$$

Finally, I derive the sufficient but not necessary conditions on m which guarantee that the consumer's consumption is always strictly positive in the decentralized equilibrium with no ex-ante regulation and large banks.

Assumption 8 *If $m > 0$ is such that $((\pi_b + \pi_g\gamma) + \pi_g(1-\gamma))\frac{T_{1,b}\left(k^{f,l,*}_{1,b}\right)+F'\left(k^{f,l,*}_{1,b}\right)k^{f,l,*}_{1,b}\eta^f+\frac{n_0}{\pi_b}}{\left(1+\frac{\gamma}{\pi_b}-A_{1,b}\right)} - n_0 < m$, where $T_{1,b}\left(k^{f,l,*}_{1,b}\right) = \chi\tau_{1,b}\left(k^{f,l,*}_{1,b}\right)a\left(\left(\frac{\left(1-\tau_{t,b}\left(k^{f,l,*}_{1,b}\right)\right)\alpha a}{\omega}\right)^{\frac{1}{1-\alpha}}\right)^{\alpha}$ and $\tau_{1,b}$ is given by equation (15), where $\lambda_{1,b}$ is replaced by $\lambda^l_{1,b}$, and $k^{f,l,*}_{1,b}$ is given by equation (70), then it will be always the case that the consumer has strictly positive consumption in $t = 0$ in the decentralized equilibrium with no ex-ante regulation and large banks. If $-\left(1+\frac{\pi_g}{\pi_b}\gamma\right)\frac{T_{1,b}\left(k^{f,l,*}_{1,b}\right)+F'\left(k^{f,l,*}_{1,b}\right)k^{f,l,*}_{1,b}\eta^f+\frac{n_0}{\pi_b}}{\left(1+\frac{\gamma}{\pi_b}-A_{1,b}\right)} + \frac{n_0}{\pi_b} < m$ and $-(1-\gamma)\frac{T_{1,b}\left(k^{f,l,*}_{1,b}\right)+F'\left(k^{f,l,*}_{1,b}\right)k^{f,l,*}_{1,b}\eta^f+\frac{n_0}{\pi_b}}{\left(1+\frac{\gamma}{\pi_b}-A_{1,b}\right)} < m$, then also the consumer's consumption in $t = 1$ is strictly positive.*□

The derivations and the intuition are equivalent to the ones under assumption 3.

6.B.2 Proof of Lemma 8

This Lemma states that if $S\left(k^{f,l,*}_{1,b}\right) < 0$, then $k^{f,l,*}_{1,b}(N<\infty) > k^{f,*}_{1,b}$, $q_{1,b}\left(k^{f,l,*}_{1,b}(N<\infty)\right) < q_{1,b}\left(k^{f,*}_{1,b}\right)$, $k^{l,*}_0(N<\infty) > k^*_0$, and $d^{l,*}_{1,b}(N<\infty) > d^*_{1,b}$. If $S\left(k^{f,l,*}_{1,b}\right) > 0$, the opposite is true. The proof regarding the relative size of the fire sale and the price of capital follows directly from equation (25) by comparing the $N \to \infty$ case (the small banks' case) to the $N < \infty$ case. The proof regarding comparing the size of the period zero investment is based on the fact that both k^*_0 and $k^{l,*}_0$ are determined by equation (40). From inequality (52), the derivative $\frac{\partial k_0}{\partial k^f_{1,b}}$ is positive, which proves that the larger the fire sale is, the larger the period zero investment is. Hence, the comparison between $k_0^{l,*}\,(N<\infty)$ and k_0^* mimics the one between $k_{1,b}^{f,l,*}\,(N<\infty)$ and $k_{1,b}^{f,*}$. Finally, since $d_{1,b}^*$ and $d_{1,b}^{l,*}$ are directly proportional to k_0^* and $k_0^{l,*}$ (both are determined by equation (41)), the comparison between $d_{1,b}^{l,*}\,(N<\infty)$ and $d_{1,b}^*$ mimics the one between $k_0^{l,*}\,(N<\infty)$ and k_0^*.□

6.B.3 Proof of Lemma 9

I prove that $k_{1,b}^{f,CP}<k_{1,b}^{f,l,*}$ and the proof that if $k_{1,b}^{f,CP}<k_{1,b}^{f,l,*}$, then $k_0^{CP}<k_0^{l,*}$, is exactly the same as in Lemma 4. First, I prove that if the equilibrium is of type 1, then $\lambda_{1,b}^{l\prime}\left(k_{1,b}^f\right)>0$, where $\lambda_{1,b}^l\left(k_{1,b}^f\right)$ is given by equation (62) and can be re-written as

$$\lambda_{1,b}^l\left(k_{1,b}^f\right)=\frac{1}{N}\frac{A_2}{Z\left(k_{1,b}^f\right)}+\frac{A_2}{F'\left(k_{1,b}^f\right)}\left(1-\frac{1}{N}\right)$$

From Lemma 14 and the assumptions made, $F''\left(k_{1,b}^f\right)<0$ and $Z'\left(k_{1,b}^f\right)<0$, which imply

$$\lambda_{1,b}^{l\prime}\left(k_{1,b}^f\right)=-Z'\left(k_{1,b}^f\right)\frac{1}{N}\frac{A_2}{\left(Z\left(k_{1,b}^f\right)\right)^2}-\frac{F''\left(k_{1,b}^f\right)A_2}{\left(F'\left(k_{1,b}^f\right)\right)^2}\left(1-\frac{1}{N}\right)>0$$

Next I prove that $\lambda_{1,b}^{CP}\left(k_{1,b}^f\right)-\lambda_{1,b}^l\left(k_{1,b}^f\right)>0$

$$\lambda_{1,b}^{CP}\left(k_{1,b}^f\right)-\lambda_{1,b}^l\left(k_{1,b}^f\right)=\left[\begin{array}{c}\left(1-\frac{F''\left(k_{1,b}^f\right)k_{1,b}^fN}{F'\left(k_{1,b}^f\right)+F''\left(k_{1,b}^f\right)k_{1,b}^f}\right)\frac{1}{N}\frac{1}{\eta^f}T_{1,b}'\left(k_{1,b}^f\right)\\-\left(1-\frac{1}{N}\right)F''\left(k_{1,b}^f\right)k_{1,b}^f\end{array}\right]\frac{A_2}{F'\left(k_{1,b}^f\right)Z\left(k_{1,b}^f\right)}>0,\quad(74)$$

where the inequality follows from the assumptions made and from Lemma 11. The rest of the proof is similar to the proof of Lemma 4. Consider the function $\psi^{CP}\left(k_{1,b}^f\right)$ defined by equation (49). Define

$$\psi^l\left(k_{1,b}^f\right)=\lambda_{1,b}^l-\lambda_0^l=\Omega_1\lambda_{1,b}^l-\Omega_2$$

$\lambda_{1,b}^{l\prime}\left(k_{1,b}^f\right)>0$ implies $\psi^{l\prime}\left(k_{1,b}^f\right)>0$ and in the Proof of Lemma 4, I proved that $\psi^{CP\prime}\left(k_{1,b}^f\right)>0$. Therefore, proving that $\psi^{CP}\left(k_{1,b}^f\right)>\psi^l\left(k_{1,b}^f\right)$ is sufficient to prove that $k_{1,b}^{f,CP}<k_{1,b}^{f,l,*}$, which follows directly from $\lambda_{1,b}^{CP}>\lambda_{1,b}^l$. □

6.B.4 Overinvestment and Overborrowing and Fiscal Capacity: Large Banks' Case

Lemma 15 *Consider the case where $N<\infty$. Larger fiscal capacity due to a more productive labor intensive sector or lower dis-utility of labor (i.e. larger tax base) leads to a larger equilibrium fire* *sale and period zero investment;* $\frac{\partial k_{1,b}^{f,l,*}}{\partial a} > 0$, $\frac{\partial k_{1,b}^{f,l,*}}{\partial \omega} < 0$, $\frac{\partial k_{0}^{l,*}}{\partial a} > 0$ *and* $\frac{\partial k_{0}^{l,*}}{\partial \omega} < 0$. *However, larger fiscal capacity due to the government being less corrupt or more efficient at collecting taxes implies lower fire sale,* $\frac{\partial k_{1,b}^{f,l,*}}{\partial \chi} < 0$, *while the effect on the period zero investment,* $\frac{\partial k_{0}^{l,*}}{\partial \chi}$, *depends on parameter assumptions.*

Proof of Lemma 15:

The equilibrium fire sale in the type 1 equilibrium $k_{1,b}^{f,l,*}$ is determined by the following equation:

$$F'\left(k_{1,b}^{f,l,*}\right) = \left(1 + \frac{S\left(k_{1,b}^{f,l,*}\right)}{Z\left(k_{1,b}^{f,l,*}\right)}\right) G = \left(1 + \frac{1}{N}\frac{F'\left(k_{1,b}^{f}\right)}{F'\left(k_{1,b}^{f}\right) + F''\left(k_{1,b}^{f}\right)k_{1,b}^{f} + \frac{1}{\eta^f}T'_{1,b}\left(k_{1,b}^{f}\right)} - \frac{1}{N}\right) G.$$

Totally differentiating the equation above with respect to $x \in \{a, \omega, \chi\}$ implies

$$\frac{\partial k_{1,b}^{f,l,*}}{\partial x} = \frac{\frac{1}{N\eta^f}\frac{\partial^2 T_{1,b}\left(x; k_{1,b}^{f} = k_{1,b}^{f,l,*}\right)}{\partial k_{1,b}^{f}\partial x}}{\left(\frac{1}{N} - \frac{1}{G}Z\left(k_{1,b}^{f,l,*}\right)\right)Z\left(k_{1,b}^{f,l,*}\right)\frac{F''\left(k_{1,b}^{f,l,*}\right)}{F'\left(k_{1,b}^{f,l,*}\right)} - \frac{1}{N}\left(2F''\left(k_{1,b}^{f,l,*}\right) + F'''\left(k_{1,b}^{f,l,*}\right)k_{1,b}^{f,l,*} + \frac{1}{\eta^f}T''_{1,b}\left(k_{1,b}^{f,l,*}\right)\right)}, \tag{75}$$

where $\frac{\partial^2 T_{1,b}\left(x; k_{1,b}^{f} = k_{1,b}^{f,l,*}\right)}{\partial k_{1,b}^{f}\partial x}$ is the cross partial derivative for a given $k_{1,b}^{f}$, evaluated at $k_{1,b}^{f} = k_{1,b}^{f,l,*}$.

Assumption 6 in the Appendix, which ensures that the large banks' problem is well behaved, implies $T''_{1,b}\left(k_{1,b}^{f,l,*}\right) < 0$. Also from assumption 2, $2F''\left(k_{1,b}^{f,l,*}\right) + F'''\left(k_{1,b}^{f,l,*}\right)k_{1,b}^{f,l,*} < 0$. Next I prove that $1 - \frac{N}{G}Z\left(k_{1,b}^{f,l,*}\right) \leq 0$. Since the equilibrium is of type 1, then

$$\frac{F'\left(k_{1,b}^{f,l,*}\right)}{N\left(S\left(k_{1,b}^{f,l,*}\right) + Z\left(k_{1,b}^{f,l,*}\right)\right)} Z\left(k_{1,b}^{f,l,*}\right) = \frac{G}{N} \leq Z\left(k_{1,b}^{f,l,*}\right)$$

which follows from

$$\frac{F'\left(k_{1,b}^{f,l,*}\right)}{N\left(S\left(k_{1,b}^{f,l,*}\right) + Z\left(k_{1,b}^{f,l,*}\right)\right)} = \frac{F'\left(k_{1,b}^{f,l,*}\right)}{N\left(F'\left(k_{1,b}^{f}\right) + \left(1 - \frac{1}{N}\right)F''\left(k_{1,b}^{f}\right)k_{1,b}^{f} + \left(1 - \frac{1}{N}\right)\frac{1}{\eta^f}T'_{1,b}\left(k_{1,b}^{f}\right)\right)} \leq 1.$$

Therefore, the denominator in equation (75) is positive and it will be the case that

$$\frac{\partial k_{1,b}^{f,l,*}}{\partial x} \propto \frac{\partial^2 T_{1,b}\left(x; k_{1,b}^{f} = k_{1,b}^{f,l,*}\right)}{\partial k_{1,b}^{f}\partial x}.$$

From Lemma 6, $\frac{\partial k_{1,b}^{f,l,*}(N<\infty)}{\partial a} > 0$, $\frac{\partial k_{1,b}^{f,l,*}(N<\infty)}{\partial \omega} < 0$, $\frac{\partial k_{1,b}^{f,l,*}(N<\infty)}{\partial \chi} < 0$, which completes the first part of the proof. Next, I derive the comparative statics of period zero investment with respect to fiscal capacity using equation (40)

$$\frac{\partial k_0^{l,*}}{\partial x}=\frac{\eta^f Z\left(k_{1,b}^{f,l,*}\right)\frac{\partial k_{1,b}^{f,l,*}}{\partial x}+\frac{\partial T_{1,b}\left(x;k_{1,b}^{f,l,*}\right)}{\partial x}}{\left(1+\frac{\gamma}{\pi_b}-A_{1,b}\right)}\text{ for }x\in\{a,\omega,\chi\}.$$

From the results above and Lemma 6, if $x\in\{a,\omega\}$, then $\frac{\partial k_{1,b}^{f,l,*}}{\partial x}$ and $\frac{\partial T_{1,b}\left(x;k_{1,b}^{f,l,*}\right)}{\partial x}$ have the same sign, which implies that the derivative $\frac{\partial k_0^{l,*}}{\partial x}$ can be signed as follows

$$\frac{\partial k_0^{l,*}}{\partial a}>0;\ \frac{\partial k_0^{l,*}}{\partial \omega}<0.$$

If $x=\chi$, $\frac{\partial T_{1,b}\left(x;k_{1,b}^{f,l,*}\right)}{\partial \chi}>0$ and $\frac{\partial k_{1,b}^{f,l,*}(N<\infty)}{\partial \chi}<0$ and whether the sign of $\frac{\partial k_0^{l,*}}{\partial \chi}$ is positive or negative will depend on their relative size.

Unlike the small banks' case, the comparative static of $k_0^{l,*}$ with respect to fiscal capacity is determined not only by how the optimal bail-out varies with respect to fiscal capacity, but also by how the equilibrium fire sale varies with respect to fiscal capacity. The direction of the two effects coincides when fiscal capacity is defined as the size of the tax base. However, the direction of the effects differ when the fiscal capacity is proxied with the degree of government efficiency, which is why one cannot clearly sign the derivative $\frac{\partial k_0^{l,*}}{\partial \chi}$. $\square$

Lemma 16 *Consider the case where $N<\infty$. Larger fiscal capacity due to a more productive labor intensive sector or lower dis-utility of labor (i.e. larger tax base) implies that the banker will overinvest by more relative to the Central Planner when banks are large; $\frac{\partial\left(k_0^{l,*}-k_0^{CP}\right)}{\partial a}>0$ and $\frac{\partial\left(k_0^{l,*}-k_0^{CP}\right)}{\partial \omega}<0$. However, larger fiscal capacity due to the government being less corrupt or more efficient at collecting taxes implies lower overinvestment; $\frac{\partial\left(k_0^{l,*}-k_0^{CP}\right)}{\partial \chi}<0$. The exact same comparative statics hold with respect to the degree of overborrowing against the bad state of nature.*

Proof of Lemma 16: Consider the case where $N<\infty$. From equation (40) and since $k_{1,b}^{f,CP}$ is not a function of the fiscal capacity, it follows that for $x\in\{a,\omega,\chi\}$

$$\frac{\partial\left(k_0^{l,*}-k_0^{CP}\right)}{\partial x}=\frac{\eta^f Z\left(k_{1,b}^{f,l,*}\right)\frac{\partial k_{1,b}^{f,l,*}}{\partial x}+\frac{\partial T_{1,b}\left(x;k_{1,b}^{f,l,*}\right)}{\partial x}-\frac{\partial T_{1,b}\left(x;k_{1,b}^{f,CP}\right)}{\partial x}}{\left(1+\frac{\gamma}{\pi_b}-A_{1,b}\right)}.$$

Since $k_{1,b}^{f,l,*}>k_{1,b}^{f,CP}$, from Lemmas 6 and 15 $\frac{\partial^2 T_{1,b}}{\partial\left(k_{1,b}^{f}\right)\partial a}>0$, $\frac{\partial k_{1,b}^{f,l,*}}{\partial a}>0$, it directly follows that $\frac{\partial\left(k_0^{l,*}-k_0^{CP}\right)}{\partial a}>0$. Since $\frac{\partial^2 T_{1,b}}{\partial\left(k_{1,b}^{f}\right)\partial \omega}<0$, $\frac{\partial k_{1,b}^{f,l,*}}{\partial \omega}<0$, then $\frac{\partial\left(k_0^{l,*}-k_0^{CP}\right)}{\partial \omega}<0$. Since $\frac{\partial^2 T_{1,b}}{\partial\left(k_{1,b}^{f}\right)\partial \chi}<0$, $\frac{\partial k_{1,b}^{f,l,*}}{\partial \chi}<0$, then $\frac{\partial\left(k_0^{l,*}-k_0^{CP}\right)}{\partial \chi}<0$. Since $d_{1,b}$ is directly proportional to k_0, the same comparative statics apply. $\square$

6.B.5 Lemma 17

The following Proposition makes the point that, unlike the case where banks are infinitesimally small, a minimum bank capital requirement might not be sufficient to replicate the CP's allocation when banks are large.

Lemma 17 *Imposing an exogenous minimum bank capital ratio such that* $\rho > \frac{n_0}{k_0^{l,*}}$ *and considering a symmetric equilibrium, the decentralized equilibrium can be one of the following four types:*

Type 1) $\lambda_{1,b}^l\left(k_{1,b}^f(\rho)\right) = \lambda_0^l\left(k_{1,b}^f(\rho)\right) > \lambda_{1,g}^l$ *if* $k_{1,b}^f(\rho) \in (\tilde{k}_{1,b}^f, k_{1,b}^{f,\max})$

Type 2) $\lambda_0^l\left(k_{1,b}^f(\rho)\right) > \lambda_{1,s}^l\left(k_{1,b}^f(\rho)\right)$ *if* $k_{1,b}^f(\rho) = k_{1,b}^{f,\max}$

Type 3) $\lambda_{1,b}^l\left(k_{1,b}^f(\rho)\right) = \lambda_0^l\left(k_{1,b}^f(\rho)\right) = \lambda_{1,g}^l$ *if* $k_{1,b}^f(\rho) = \tilde{k}_{1,b}^f$

Type 4) $\lambda_{1,g}^l = \lambda_0^l\left(k_{1,b}^f(\rho)\right) > \lambda_{1,b}^l\left(k_{1,b}^f(\rho)\right)$ *if* $k_{1,b}^f(\rho) \in [0, \tilde{k}_{1,b}^f)$

where $\tilde{k}_{1,b}^f$ *is given by* $M\left(\tilde{k}_{1,b}^f\right) = 0$ *if* $0 < \tilde{k}_{1,b}^f < k_{1,b}^{f,\max}$.

Proof of Lemma 17: The condition $\rho > \frac{n_0}{k_0^{l,*}}$ guarantees that the minimum bank capital requirement constraint is binding. Given the assumptions made

$$M\left(k_{1,b}^f\right) = F'\left(k_{1,b}^f\right) - 1 + \frac{1}{N}\left(\frac{F''\left(k_{1,b}^f\right)k_{1,b}^f}{F'\left(k_{1,b}^f\right) + F''\left(k_{1,b}^f\right)k_{1,b}^f}\right) < 0 \text{ if } k_{1,b}^f < k_{1,b}^{T,f}$$

$$\begin{aligned}
lim_{k_{1,b}^f \to k_{1,b}^{T,f}-} M\left(k_{1,b}^f\right) &= F'\left(k_{1,b}^{T,f}\right) - 1 + \frac{1}{N}\frac{F''\left(k_{1,b}^{T,f}\right)k_{1,b}^{T,f}}{A_2\chi} < 0 \\
lim_{k_{1,b}^f \to k_{1,b}^{T,f}+} M\left(k_{1,b}^f\right) &= -\left(1 - F'\left(k_{1,b}^{T,f}\right)\right)\left(1 - \frac{1}{N}\right) \\
&\quad + F'\left(k_{1,b}^{T,f}\right)\frac{1}{N}\left(1 - \frac{1}{A_2\chi + \frac{1}{\eta^f}\lim_{k_{1,b}^f \to k_{1,b}^{T,f}+} T_{1,b}'\left(k_{1,b}^f\right)}\right)
\end{aligned} \tag{76}$$

Since $\frac{\partial T_{1,b}}{\partial k_{1,b}^f} > 0$ then $lim_{k_{1,b}^f \to k_{1,b}^{T,f}+} M\left(k_{1,b}^f\right)$ can be smaller or greater than zero depending on the size of $\lim_{k_{1,b}^f \to k_{1,b}^{T,f}+} T_{1,b}'\left(k_{1,b}^f\right)$, which is given by equation (66). If $lim_{k_{1,b}^f \to k_{1,b}^{T,f}+} M\left(k_{1,b}^f\right) < 0$, since $M\left(k_{1,b}^f\right) < 0$, for any $k_{1,b}^f < k_{1,b}^{T,f}$ and $M\left(k_{1,b}^f\right)$ is decreasing on $[0, k_1^{T,f})$ and $(k_1^{T,f}, k_1^{f,\max}]$ then $\tilde{k}_{1,b}^f = 0$. If $lim_{k_{1,b}^f \to k_{1,b}^{T,f}+} M\left(k_{1,b}^f\right) > 0$ and $M\left(k_1^{f,\max}\right) < 0$, then $\tilde{k}_{1,b}^f$ is determined by $M\left(\tilde{k}_{1,b}^f\right) = 0$ where $\tilde{k}_{1,b}^f \in (k_{1,b}^{T,f}, k_1^{f,\max}]$. If $lim_{k_{1,b}^f \to k_{1,b}^{T,f}+} M\left(k_{1,b}^f\right) > 0$ and $M\left(k_1^{f,\max}\right) \geq 0$, then $k_{1,b}^f = k_{1,b}^{f,\max}$ and the equilibrium is of type 2. Given that the minimum bank capital requirement constraint is binding, it follows that $k_0 = \frac{n_0}{\rho}$. There are four types of equilibria.

Type 1 Equilibrium: $\mu_{1,g}^l > 0$ and $\mu_{1,b}^l = 0$ $\left(\lambda_{1,b}^l = \lambda_0^l > \lambda_{1,g}^l\right)$. The fire sale is determined by the budget constraint in the low state in $t = 1$

$$BC^1\left(k^f_{1,b}\right) = T_{1,b}\left(k^f_{1,b}\right) + F'\left(k^f_{1,b}\right)k^f_{1,b}\eta^f + \frac{n_0}{\pi_b} - \frac{n_0}{\rho}\left(1 + \frac{\gamma}{\pi_b} - A_{1,b}\right). \tag{77}$$

More precisely, if the equilibrium is of type 1, the equilibrium fire sale, $k^{f,l,*}_{1,b}(\rho)$, is determined by $BC^1\left(k^{f,l,*}_{1,b}(\rho)\right) = 0$. The rest of the endogenous variables can be expressed as a function of the fire sale as follows:

$$\begin{aligned} k_{1,b} &= \frac{n_0}{\rho} - k^f_{1,b}\eta^f; \quad k_{1,g} = A_{1,g}\frac{n_0}{\rho} + T_{1,g} \\ d_{1,g}(\rho) &= (1-\gamma)\frac{n_0}{\rho}; \quad d_{1,b}(\rho) = \left[\left(1 + \frac{\pi_g}{\pi_b}\gamma\right)\frac{1}{\rho} - \frac{1}{\pi_b}\right]n_0 \end{aligned}$$

In order for the equilibrium to be of type 1, the condition $\lambda^l_{1,b}\left(k^{f,l,*}_{1,b}(\rho)\right) > \lambda^l_{1,g}\left(k^{f,l,*}_{1,b}(\rho)\right)$ has to be satisfied, which implies $M\left(k^{f,l,*}_{1,b}(\rho)\right) < 0$ and that $k^{f,l,*}_{1,b}(\rho) \in (\tilde{k}^f_{1,b}, k^{f,\max}_{1,b})$. For the equilibrium to be of type 1 also the borrowing constraint in the low state has to be non-binding — i.e.,

$$d_{1,b}(\rho) = \left[\left(1 + \frac{\pi_g}{\pi_b}\gamma\right)\frac{1}{\rho} - \frac{1}{\pi_b}\right]n_0 < \left(F'\left(k^{f,l,*}_{1,b}(\rho)\right) - \gamma\right)\frac{n_0}{\rho}$$

Re-writing this inequality, it implies that the total amount of period zero borrowing has to be less than the maximum amount of borrowing possible

$$\frac{n_0}{\rho} - n_0 < \left[\pi_b\left(F'\left(k^{f,l,*}_{1,b}(\rho)\right) - \gamma\right) + \pi_g(1-\gamma)\right]\frac{n_0}{\rho} \tag{78}$$

Type 2 Equilibrium: $\mu^l_{1,s} > 0$, $\left(\lambda^l_0 > \lambda^l_{1,s}\right)$. If ρ is such that $k_0 = k^{\max}_0$, the allocation is given by the equations in the type 2 equilibrium of the problem of the small banks with no ex-ante regulation. If this case is an equilibrium then $k^{f,l,*}_{1,b}(\rho) = k^{f,\max}_{1,b}$.

Type 3 Equilibrium: $\mu^l_{1,s} = 0$, $\left(\lambda^l_0 = \lambda^l_{1,s}\right)$. $k^{f,l,*}_{1,b}(\rho)$ is pinned down by $M\left(k^{f,l,*}_{1,b}(\rho)\right) = 0$, where $k^{f,l,*}_{1,b}(\rho) = \tilde{k}^f_{1,b}$. The rest of the equations are

$$\begin{aligned} d_{1,g} &= \left(\frac{1}{\rho} - 1\right)\frac{n_0}{\pi_g} - \frac{\pi_b}{\pi_g}\left[\left(A_{1,b} - \gamma + F'\left(k^f_{1,b}\right)\right)\frac{n_0}{\rho} + T_{1,b} - F'\left(k^f_{1,b}\right)k^f_{1,b}\right] \\ k_{1,b} &= \frac{n_0}{\rho} - k^f_{1,b}\eta^f; \; k_{1,g} = (A_{1,g} - \gamma + 1)\frac{n_0}{\rho} + T_{1,g} - d_{1,g} \\ d_{1,b} &= \left(A_{1,b} - \gamma + F'\left(k^f_{1,b}\right)\right)\frac{n_0}{\rho} + T_{1,b} - F'\left(k^f_{1,b}\right)k^f_{1,b} \end{aligned}$$

In order for the equilibrium to be of type 3, the borrowing constraints in $t = 0$ against the good and bad states should not be binding.

$$d_{1,b} < \left(F'\left(k_{1,b}^{f,l,*}(\rho)\right) - \gamma\right)\frac{n_0}{\rho};\ d_{1,g} < (1-\gamma)\frac{n_0}{\rho}$$

Type 4 Equilibrium: $\mu_{1,g}^{l} = 0, \mu_{1,b}^{l} > 0\ \left(\lambda_{1,g}^{l} = \lambda_0^{l} > \lambda_{1,b}^{l}\right)$. $k_{1,b}^{f,l,*}(\rho)$ is pinned down by the budget constraint in $t=1$ in the low state given by

$$BC^4\left(k_{1,b}^{f}\right) = \left(A_{1,b} - F'\left(k_{1,b}^{f}\right)\right)\frac{n_0}{\rho} + F'\left(k_{1,b}^{f}\right)k_{1,b}^{f}\eta^{f} + T_{1,b}\left(k_{1,b}^{f}\right) \tag{79}$$

where $BC^4\left(k_{1,b}^{f,l,*}(\rho)\right) = 0$. The rest of the endogenous variables are determined by the following system of equations

$$\begin{aligned}
d_{1,g} &= \frac{n_0}{\pi_g}\left(\frac{1}{\rho} - 1\right) - \frac{\pi_b}{\pi_g}\frac{n_0}{\rho}\left(F'\left(k_{1,b}^{f}\right) - \gamma\right)\\
d_{1,b} &= \left(F'\left(k_{1,b}^{f}\right) - \gamma\right)\frac{n_0}{\rho};\ k_{1,b} = \frac{n_0}{\rho} - k_{1,b}^{f}\eta^{f}\\
k_{1,g} &= \left(A_{1,g} - \gamma + 1 + \frac{\pi_b}{\pi_g}\left(F'\left(k_{1,b}^{f}\right) - \gamma\right)\right)\frac{n_0}{\rho} + T_{1,g} - \frac{n_0}{\pi_g}\left(\frac{1}{\rho} - 1\right)
\end{aligned}$$

In order for the equilibrium to be of type 4, the following conditions also have to be satisfied $\lambda_{1,g}^{l}\left(k_{1,b}^{f,l,*}(\rho)\right) > \lambda_{1,b}^{l}\left(k_{1,b}^{f,l,*}(\rho)\right)$ which implies that $M\left(k_{1,b}^{f,l,*}(\rho)\right) > 0$ and $k_{1,b}^{f,l,*}(\rho) \in (0, \tilde{k}_{1,b}^{f})$. Furthermore, it has to be the case that $d_{1,g} < (1-\gamma)\frac{n_0}{\rho}$, where the latter inequality implies that the inequality (78) has to be satisfied.

Finally, $BC^{1\prime}\left(k_{1,b}^{f}\right) = T_{1,b}'\left(k_{1,b}^{f}\right) + \left(F'\left(k_{1,b}^{f}\right) + F''\left(k_{1,b}^{f}\right)k_{1,b}^{f}\right)\eta^{f} > 0$ and $BC^{4\prime}\left(k_{1,b}^{f}\right) = -F''\left(k_{1,b}^{f}\right)\frac{n_0}{\rho} + \left(F'\left(k_{1,b}^{f}\right) + F''\left(k_{1,b}^{f}\right)k_{1,b}^{f}\right)\eta^{f} + T_{1,b}'\left(k_{1,b}^{f}\right) > 0$.

Assume that $A_2\chi > G$ which guarantees that the CP's equilibrium is such that there is a positive optimal bail-out in the bad state which implies that $k_{1,b}^{f,CP} > k_{1,b}^{T,f}$. Consider $\rho = \rho^* = \frac{n_0}{k_0^{CP}}$, then

$$\begin{aligned}
\lim_{k_{1,b}^{f} \to k_{1,b}^{T,f+}} BC^1\left(k_{1,b}^{T,f};\rho^*\right) &= \lim_{k_{1,b}^{f} \to k_{1,b}^{T,f+}} T_{1,b}\left(k_{1,b}^{T,f}\right) + F'\left(k_{1,b}^{T,f}\right)k_{1,b}^{T,f}\eta^{f} &(80)\\
-T_{1,b}\left(k_{1,b}^{f,CP}\right) + F'\left(k_{1,b}^{f,CP}\right)k_{1,b}^{f,CP}\eta^{f} &< 0 &(81)
\end{aligned}$$

which follows from the fact that $T_{1,b}\left(k_{1,b}^{T,f}\right) + F'\left(k_{1,b}^{T,f}\right)k_{1,b}^{T,f}\eta^{f}$ is an increasing function of $k_{1,b}^{T,f}$ and from $k_{1,b}^{f,CP} > k_{1,b}^{T,f}$. Also for every, $k_{1,b}^{f}$ $BC^4\left(k_{1,b}^{f};\rho^*\right) < BC^1\left(k_{1,b}^{f};\rho^*\right)$ since $A_{1,b} < \gamma$ and $F'\left(k_{1,b}^{f}\right) \geq \gamma$. As a result, it is also the case that $\lim_{k_{1,b}^{f} \to k_{1,b}^{T,f+}} BC^4\left(k_{1,b}^{T,f};\rho^*\right) < 0$. Given the results above and since $BC^1\left(k_{1,b}^{f}\right)$ and $BC^4\left(k_{1,b}^{f}\right)$ are uniformly increasing, the only relevant region that needs to be considered is $k_{1,b}^{f} \in (k_{1,b}^{T,f}, k_1^{f,\max}]$ as the equilibrium allocation belongs to that region if $A_2\chi > G$. $\square$

6.B.6 Proof of Proposition 5

In Lemma 17 I showed that for any ρ, including $\rho = \frac{n_0}{k_0^{CP}}$, it could be the case that the equilibrium is such that the banker might choose to borrow against the bad state before he has exhausted his ability to borrow against the good state. Here I derive for what parametrization a minimum bank capital requirement is sufficient to decentralize the allocation. This is equivalent to deriving the parameter set for which the equilibrium is of type 1 (as defined in Lemma 17). Since $k_0^{CP} < k_0^{\max}$, if the equilibrium is not of type 1, it can be only of types 3 or 4. If when $\rho^* = \frac{n_0}{k_0^{CP}}$, the equilibrium, as defined in Lemma 17, is of type 1, then there will be no need for a second regulatory instrument. It will be the case that $BC^1\left(k_{1,b}^{f,l,*}(\rho^*)\right) = 0$ and $M'\left(k_{1,b}^{f,l,*}(\rho^*)\right) < 0$. Note also that if the equilibrium is of type 1, then $k_{1,b}^{f,l,*}(\rho^*) = k_{1,b}^{f,CP}$. However, if it's not of type 1 a second regulatory instrument will be needed.

The structure of the proof is the following. I will prove that if assumption (9), defined below, is not satisfied, then a single instrument controlling ex-ante investment will be needed to replicate the CP's allocation. If assumption (9) is satisfied, there is a threshold for each fiscal capacity variable, given the other fiscal capacity variables, below or above which the equilibrium will not be of type 1 and, hence, a second instrument will be needed to decentralize the constrained CP's allocation.

First, I start by examining equation (77). One can prove that since $\rho = \frac{n_0}{k_0^{CP}}$, then $\frac{\partial BC^1\left(x;k_{1,b}^{f,CP}\right)}{\partial x} = 0$ where $x \in \{a, \omega, \chi\}$. It follows from

$$\begin{aligned}
\frac{\partial BC^1\left(x;k_{1,b}^{f,CP}\right)}{\partial x} &= \frac{\partial T_{1,b}\left(k_{1,b}^{f,CP}\right)}{\partial x} - \frac{\partial k_0^{CP}(x)}{\partial x}\left(1 + \frac{\gamma}{\pi_b} - A_{1,b}\right) \\
&= \frac{\partial T_{1,b}\left(x;k_{1,b}^{f,CP}\right)}{\partial x} - \frac{\partial T_{1,b}\left(x;k_{1,b}^{f,CP}\right)}{\partial x} = 0.
\end{aligned}$$

This result is expected and simply reiterates the fact that the optimal fire sale in the constrained CP's problem, pinned down by equation (48), is not a function of the fiscal capacity.

Next I examine equation $M\left(k_{1,b}^{f}\right)$. At the end of Lemma 17, I proved that the relevant range for the equilibrium will be $k_{1,b}^{f} \in (k_{1,b}^{T,f}, k_1^{f,\max}]$. In Lemma 14, I proved that $M'\left(k_{1,b}^{f}\right) < 0$ over that region and in Lemma 17 I proved that the equilibrium will be of type 1, only if $M(k_{1,b}^{f,l,*}(\rho^*)) < 0$. If $lim_{k_{1,b}^{f} \to k_{1,b}^{T,f}+} M\left(k_{1,b}^{f}\right) < 0$, then no second ex-ante regulatory instrument is required. If the following assumption is satisfied

Assumption 9 *Assume the following*

$$lim_{k_{1,b}^{f} \to k_{1,b}^{T,f}+} M\left(k_{1,b}^{f}\right) > 0 \tag{82}$$

where $\lim_{k_{1,b}^f \to k_{1,b}^{T,f+}} M\left(k_{1,b}^f\right)$ *is given by equation (76), then* $M\left(k_{1,b}^f\right)$ *either crosses the zero line always once or is always above the zero line in the range* $k_{1,b}^f \in (0, k_{1,b}^{f,\max}]$.

Next I derive the comparative statics of $M\left(k_{1,b}^f\right)$ with respect to the fiscal capacity variables for a given $k_{1,b}^f$ and will prove that those are monotone. Therefore, the threshold above/below which the equilibrium will be of type 1 will be determined by $M\left(\tilde{x}; k_{1,b}^{f,CP}\right) = 0$ if assumption 9 is satisfied.

Consider the derivative of M with respect to the fiscal capacity variables for a given $k_{1,b}^f$. From Lemma 6, which applies in the large banks' case as well[28]

$$\frac{\partial M\left(\chi; k_{1,b}^f\right)}{\partial \chi} \propto \frac{\partial^2 T_{1,b}}{\partial\left(k_{1,b}^f\right)\partial\chi} < 0;$$
$$\frac{\partial M\left(\omega; k_{1,b}^f\right)}{\partial \omega} \propto \frac{\partial^2 T_{1,b}}{\partial\left(k_{1,b}^f\right)\partial\omega} < 0; \frac{\partial M\left(a; k_{1,b}^f\right)}{\partial a} \propto \frac{\partial^2 T_{1,b}}{\partial\left(k_{1,b}^f\right)\partial a} > 0.$$

Combing all these results implies that there exist thresholds determined as $M\left(k_{1,b}^{f,CP}, \tilde{a}(\omega,\chi)\right) = 0$, $M\left(k_{1,b}^{f,CP}, \tilde{\omega}(a,\chi)\right) = 0$, $M\left(k_{1,b}^{f,CP}, \tilde{\chi}(a,\omega)\right) = 0$ such that if $a > \tilde{a}(\omega,\chi)$ or if $\omega < \tilde{\omega}(a,\chi)$ or if $\chi < \tilde{\chi}(a,\omega)$, then $M\left(k_{1,b}^{f,CP}\right) > 0$ which would imply that the equilibrium will not be of type 1 and a second instrument will be required.

The focus so far was on ρ^i but the analysis can also be done with respect to $\tau_0^{k,i}$. The first order condition with respect to k_0^i, if a tax on investment was used instead of a minimum bank capital requirement, is equivalent to equation (63), with the only difference that $\tau_0^{k,i}$ enters in the place of $\xi^{l,i}$. This result is true as long as I assume that the total amount of lump sum rebates to banker i, $T_0^{k,i}$, are pre-determined at the beginning of period zero, so that banker i takes them as given. Since I assumed that the policy maker has a sufficient number of instruments to replicate the constrained CP's allocation and the CP's allocation is interior, then $\mu_{1,g}^{l,i} > 0$ and $\mu_{1,b}^{l,i} = 0$.

Similarly to the proof of Proposition 2, one can re-arrange equation (63) and it becomes equivalent to equation (57) with the only difference being that $\lambda_{1,b}^l$ enters in place of $\lambda_{1,b}$. The rest of the derivations are the same as in the proof of Proposition 2.

One can relax the assumption that $T_0^{k,i}$ is pre-determined in the beginning of period zero. All the results go through as long as $N > 1$. The period zero budget constraint of banker i becomes $k_0^i\left(1+\tau_0^{k,i}\right) - n_0 \leq \sum_s p_{1,s} d_{1,s}^i + \underbrace{\frac{1}{N}\sum_{i=1}^{N} k_0^i \tau_0^{k,i}}_{T_0^{k,i}}$. The new first order condition with respect to k_0^i is

[28] $\frac{\partial M\left(x; k_{1,b}^f\right)}{\partial x} = \left(\frac{1}{\eta^f}\frac{\partial^2 T_{1,b}}{\partial k_{1,b}^f \partial x}\right)\frac{1}{N}\frac{F'\left(k_{1,b}^f\right)}{\left(Z\left(k_{1,b}^f\right)\right)^2} \propto \frac{\partial^2\left(T_{1,b}\right)}{\partial\left(k_{1,b}^f\right)\partial(x)}$ where $x = \{a, \omega, \chi\}$.

the same with the exception that$\lambda_0^{l,i}\left(1+\tau_0^{k,i}\right)$ is replaced by $\lambda_0^{l,i}\left(1+\tau_0^{k,i}\left(1-\frac{1}{N}\right)\right)$ and the optimal tax on capital becomes $\tau_0^k=\frac{N}{N-1}\Phi\left(\frac{1}{\lambda_{1,b}\left(k_{1,b}^{f,CP}\right)}-\frac{1}{\lambda_{1,b}^{CP}\left(k_{1,b}^{f,CP}\right)}\right)$. The rest of the results go through. $\square$