Security Design with Status Concerns*

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Abstract

This paper provides a status-based explanation for convertible securities. An entrepreneur with status concerns inducing risk-taking decides how to finance the firm and how to dynamically manage it. Solving analytically for the optimal security, we find that it is substantially similar to a convertible security. Our model can explain why convertible securities are mainly issued by start-ups and small firms, as we show that their salient characteristics, higher volatility and dynamic flexibility, accentuate incentives to issue convertible securities. We also provide analytical results relevant to quantifying how status concerns affect credit risk, an established factor behind security choice.

1 Introduction

Financial securities play a fundamental role in the economy by facilitating interaction between entrepreneurs, those with project ideas, and financiers, those who wish to invest their resources. There is a voluminous security design and financial contracting literature examining how security choice depends on various considerations affecting the entrepreneur's or the financier's decision-making (see Biais, Mariotti, and Rochet (2013), and Sannikov (2012) for excellent literature reviews.) While this literature has made substantial progress, it appears that some salient factors affecting security issuance have not yet been identified.[1]

This paper introduces into a security design setting a feature that is universally considered to be a defining characteristic of entrepreneurs—their willingness to take risks in some situations (supporting evidence is discussed below). Throughout our analysis, we adopt the interpretation from a classic paper by Friedman and Savage (1948) that this behavior arises due to *status concerns* when an agent's wealth lies between levels associated with low and high status.

The idea is as follows. Status tends to increase in a discrete step when an indi-

[1]An influential study by Kaplan and Stromberg (2003) provides a detailed comparison of real-world financial contracts used in venture capital with theoretical counterparts. They find that "real-world contracts are more complex than existing theories predict," and note that "there is room for additional theory."

vidual purchases a good associated with higher status (e.g., a house in an upscale neighborhood). If one's wealth is below the target level at which one can afford a high-status good but not by much, the desire to possess the good induces risk-taking because higher wealth volatility implies a higher chance to exceed the target wealth level. An equivalent way to understand the emergence of risk-taking incentives is that they stem from an increase in the marginal utility around the level of wealth permitting the purchase of a status good (as elaborated in Section 2.2). Though status concerns are a commonly-noted factor behind risk-taking, especially in the context of entrepreneurial activity (as discussed later), other reasons could be behind this behavior such as concern for firm survival.[2]

Our contributions are as follows. We develop an analytically tractable dynamic framework for examining security design under non-standard preferences that capture status concerns via embedding both types of risk attitudes, risk seeking and aversion.[3] Our model provides a status-based explanation for convertible securities, as we find that a status-driven entrepreneur issues a "convertible-like" security to finance her firm.

The novel contribution of our work relative to existing theories of convertible securities is that our model provides an explanation for why convertible securities are mainly issued by riskier and more flexible firms, such as start-up and small firms. We show that incentives to issue convertible securities are positively related to firm riskiness and its dynamic flexibility. When the entrepreneur does not participate in the financing decision, we characterize analytically how status concerns affect the firm value dynamics and demonstrate that the effect can be substantial. This uncovers another channel—pertaining to credit risk—through which status concerns can affect security design, as well as be relevant for the pricing of convertible securities (Das and Sundaram (2007)).

We now preview our model and main results in more detail. We consider a continuous-time complete-information security design framework, as in Cadenillas, Cvitanic, and Zapatero (2007), in which an entrepreneur chooses how to finance her

[2]For example, when venture capital financing occurs in multiple rounds, securing the next round may depend upon achieving a certain performance target, causing the entrepreneur to take greater risks if she falls behind the target. Rather than a concern for status, this behavior is akin to a concern for survival or gambling for resurrection.

[3]Cuoco and Kaniel (2011) and Sotes-Paladino and Zapatero (2019), among others, examine preferences of this type in the context of delegated portfolio management.

firm and how to dynamically manage its operation. The first decision involves choosing a financial security to be issued to a financier, which is a risk sharing rule that specifies how the future risky firm value is shared between the two parties. The second decision involves dynamically choosing the expected growth rate ("return") and volatility of the firm value process ("risk"). The entrepreneur has status concerns and so, as explained above, she seeks risk when her wealth is between levels associated with low and high status, and is averse to risks when her status is low or high. The financier is risk averse; she buys the security from the entrepreneur if it provides her with the required reservation level of expected utility.

We solve analytically for the optimal security and find that it is considerably similar to a convertible security, in that it features distinct equity- and debt-like components. The optimal security without status concerns is equity-like, and so it is the debt-like component that emerges due to status concerns. The reason is as follows. The risk-taking incentives arising due to status concerns result in the entrepreneur's increasing the firm riskiness when high status is in sight. To insulate the risk averse financier from this risk, the entrepreneur introduces a debt-like segment. The entrepreneur essentially caters to the financier's risk preferences when designing the security because of the need to satisfy the financier's participation condition.

Researchers have been seeking to identify factors behind the decision to issue convertible securities. Given the evidence that start-up and small companies rely on such securities more often than other companies (see Section 3.4), the high risk of a firm is often viewed as a possible driver (Brennan and Schwartz (1998)). Our model provides formal support for this view, as we find that the incentive to issue a convertible security become more pronounced when the firm volatility increases. As discussed above, a key mechanism in our model generating a convertible security is the need to protect the financier from a status-induced increase in firm riskiness, and this need becomes stronger when the firm is more volatile.

Another characteristic of start-up and small firms that can be related to the use of convertible securities is their dynamic flexibility, the ability to adjust their characteristics over time at relatively little cost. Indeed, Biais and Casamata (1999) point out that firms relying on convertibles tend to be those for which "the ability to switch to riskier ventures is large." Motivated by this consideration, we examine the importance of this dynamic flexibility by studying how its absence affects the optimal security. We find that the optimal security in the resulting static model, in which the entrepreneur is not able to switch firm riskiness, is no longer similar to a convertible security. Hence, our model is consistent with Biais and Casamata's point: The optimal security in the static model features a segment providing a negative exposure to firm risk (i.e., a short position), instead of the debt segment in the optimal security of the dynamic setting. This segment allows the entrepreneur to satisfy her desire to take risks when approaching high status even though she is not able to achieve this by making the firm value riskier. Just as offering a positive stake in the firm allows the firm owner to reduce her risk (the classical risk sharing notion), offering a negative stake leads to the opposite result. Hence, the static and dynamic solutions are quite different.

We then consider a modified set-up in which the status-driven entrepreneur faces just one choice: how to manage the firm over time, and does not decide on what security to issue. This analysis can be applicable to firms, presumably larger ones, in which managing a firm and financing it are separate tasks undertaken in different divisions. We explicitly characterize the entrepreneur's dynamic strategy and find that, with status concerns, the firm volatility can substantially vary over time, while it is constant when status concerns are absent. Understanding the implications of time-varying firm volatility has been attracting growing attention (Choi and Richardson (2016), Du, Elkamhi, and Ericsson (2018)). The implications for security design also seem clear. Firm volatility and its dynamics are key inputs in structural credit risk modelling, and credit risk is, in turn, a well-documented factor affecting the process of security design and issuance.

Non-standard preferences are, by definition, less understood than standard ones, which makes the robustness of our main results to be a natural concern. We devote considerable attention to this issue in the Internet Appendix, in which we argue that our main results remain valid under alternative ways of modelling status concerns and under parameter values different from those considered in the main body of the paper.

It has long been recognized that people care about their status in society, and in particular about financial status (Frank (1985), Heffetz and Frank (2011)). Informally, how much someone cares about status is likely to be related to how actively she pursues opportunities that can propel her to a higher status and, by this measure, entrepreneurs' concern for status appears to be rather pronounced. There is considerable evidence supporting this point. According to the 2011 High Impact Entrepreneurship Global Report, a comprehensive cross-country study of entrepreneurship, the idea that successful entrepreneurs have high status has wide support among both entrepreneurs and non-entrepreneurs. Becker, Murphy, and Werning (2005) argue that entrepreneurship as an activity is especially appealing in countries in which entrepreneurial success leads to high status. Begley and Tan (2001) provide empirical support for this argument. It is generally accepted that another specific feature of entrepreneurs, besides status concerns, is their willingness to take risks. Begley and Boyd (1987) find that status concerns (in their language, "need for achievement") and risk-taking propensity are two of the three features distinguishing entrepreneurs from the rest (the third feature is tolerance of ambiguity; overconfidence is another trait associated with entrepreneurship, see Hayward, Shepherd, and Griffin (2006)).

The term "entrepreneur" in this paper can refer not only to an individual person but also to an established company considering how to finance its operations. In this case, it is not clear whether aggregating (possibly heterogeneous) status concerns of the company's multiple shareholders would lead to the objective function of the form considered in this paper. However, our model remains applicable as long as the company's risk-taking incentives are analogous to those of our entrepreneur, which seems to be the case empirically. There is extensive research on organizational economics initiated by the influential work of Cyert and March (1963). It challenges the view that all complex interactions within companies can be reduced to the standard assumption of profit maximization. It is argued that companies, when deciding how much risk to take, consider their current performance relative to a certain aspiration level, a target that a company tries to achieve (see Audia and Greve (2006) and the literature review therein). A common argument in this literature is that "managers seem to feel that risk taking is more warranted when faced with failure to meet targets than when targets were secure," and that "executives ... would not take risks where a failure could jeopardize the survival of the firm" (March and Shapira (1987)). This pattern—taking risks when below but near the target, and avoiding risks when either above or well behind the target—mirrors the idea of Friedman and Savage used in this paper.

Though this behavior may arise for alternative reasons, status concerns can well be a factor. Companies' important decisions, such as security issuance, are ultimately made by CEOs, and CEOs are likely to have pronounced status concerns. In addition to the obvious point that someone with little concern for status is not likely to become a CEO in the first place, there is also evidence direct evidence supporting this point.[4] This is also consistent with survey findings that wealthier people, such as those in charge of security issuance, tend to care more about status (McBride (2001), Dynan and Ravina (2007)).

Our paper contributes to the literature aiming to explain the use of convertible securities. A common theme of existing works is that convertible securities help to mitigate various agency problems, which typically arise under asymmetric information. In particular, convertible securities are shown to mitigate the asset substitution problem (Green (1984)), window-dressing behavior (Cornelli and Yosha (2003)), moral hazard in the presence of renegotiation (Dewatripont, Legros, and Matthews (2003)), inefficient investment (Schmidt (2003)), the underinvestment problem (Lyandres and Zhdanov (2014)), and other asymmetric information problems (Constantinides and Grundy (1989), Stein (1992), Repullo and Suarez (2004), Hellmann (2006), Chakraborty and Yilmaz (2011)). Our analysis shows that convertible securities also have an economic role under full information, as is the case in our model. Several studies in this area, such as Larsen (2005), Cadenillas, Cvitanic, and Zapatero (2007), Bolton and Harris (2013), and Miao and Zhang (2015) consider, like us, settings without asymmetric information, but they do not explain the use of convertible securities.

More broadly, our work also contributes to the growing literature investigating the role of status concerns in various areas of economics and finance. Examples include Becker, Murphy, and Werning (2005), Moldovanu, Sela, and Shi (2007), Auriol and Renault (2008), Besley and Ghatak (2008), Roussanov (2010), Dijk, Holmen, and Kirchler (2014), Georgarakos, Haliassos, and Pasini (2014), and Hong, Jiang, Wang, and Zhao (2014).

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the optimal security, describes how the entrepreneur manages the firm, and relates the findings to empirical evidence. Section 4 characterizes the

[4] Shemesh (2017) presents evidence that the CEOs' risk-taking behavior is affected by status concerns. Other works exploring the behavior of CEOs through the prism of status concerns include Wade, Porac, Pollock, and Graffin (2006), Malmendier and Tate (2009), and Goel and Thakor (2010).

optimal security in a static setting, solves a model without security issuance decision, and discusses limitations and robustness. Section 5 concludes. Appendix A presents all proofs. The Internet Appendix elaborates on several points related to the analysis in the main body of the paper.

2 Model

We start by describing the key elements of our model, after which we provide all the details. There are two agents in the economy: an *entrepreneur* with status concerns who owns a firm, and a *financier* who can fund the firm's operation. We build on the framework of Cadenillas, Cvitanic, and Zapatero (2007), in that we study security design in a continuous-time setting with complete markets, full information and no agency problems between the entrepreneur and the financier. On the other hand, we assume that these concerns affect outside investors to the extent that they do not consider buying the firm or investing in it. Therefore, outside investors or other participants of the financial markets are not considered in our analysis. In this sense, we are not in the perfect market world of Modigliani and Miller in which financing decisions are irrelevant. The firm value follows a random process, whose parameters are controlled by the entrepreneur. We assume that the financier—a venture capitalist, an angel investor, a bank, or the financial market—cannot manage the firm herself without the entrepreneur, and so she does not consider buying it from the entrepreneur. The entrepreneur is key to the firm's functioning and her preferences play a key role in the firm's optimal capital structure.

Status concerns are modeled following Friedman and Savage (1948)'s seminal insight: The idea is that when one's wealth is sufficiently high, but not yet at a level associated with high status, one is willing to take risks to increase the probability of reaching such a high status. Otherwise, when a status change is not likely, which is the case when wealth is either sufficiently low or high, one exhibits the normal aversion to risk. Such risk-taking patterns are exhibited not only at an individual level but also by companies, as discussed in the Introduction. Accordingly, we view the entrepreneur in our paper as referring not only to an individual entrepreneur or a small start-up firm but also to an established company.

The implicit assumption for a security design problem to be meaningful is that the firm does not have enough of its own resources to operate, and so the entrepreneur has to attract the required funding from the financier in exchange for an optimally chosen security, a claim on the firm's future value. The main goal of the paper is to examine how the presence of status concerns affects security design, and to show that accounting for status concerns helps explain the use of the convertible securities—this is the focus of Section 3. We also explore other aspects of security design with status concerns, which requires us to modify the main economic setting—these modifications are studied in Section 4.

We now provide a detailed description of our main setting.

2.1 Firm value dynamics

We consider an entrepreneur who owns a firm and dynamically controls its operation. We model this notion by positing that she dynamically chooses the process ϕ determining the evolution of the firm value V:

$$dV_t/V_t = \phi_t \mu dt + \phi_t \sigma d\omega_t, \tag{1}$$

where $\mu, \sigma > 0$ determine, jointly with the multiplicative parameter ϕ, the firm's mean growth rate and volatility, respectively, and ω is a standard Brownian motion representing the uncertainty. The value V cannot be realized by selling the firm in the financial markets; the entrepreneur's involvement in the firm is crucial.

The term ϕ in specification (1) that we refer to as the *firm riskiness* formalizes the idea of "nothing ventured, nothing gained." Specifically, if the entrepreneur wants to raise the firm's expected growth rate (first term), she needs to increase the parameter ϕ, implying that she is also raising the firm riskiness (second term). To elaborate further, suppose that the entrepreneur develops a certain product that she plans to sell in the future. In the process of development, she can dynamically choose how novel the product is going to be relative to existing ones. The more novel the product is the higher are the expected future profits due to lower competition, resulting in a higher expected growth rate of the firm value. At the same time, the future demand for novel products is less predictable, implying a higher firm riskiness. If the firm is relatively large and undertakes several projects at the same time, an increase in ϕ can be interpreted as shifting its focus to riskier projects with higher expected returns.

We note that the drift and diffusion terms in (1) cannot be chosen independently of each other as they both depend on the firm riskiness ϕ. If the two terms were independent, we would need to introduce a mechanism preventing the choice of a project with an infinitely high drift such as costly effort of increasing the drift as in Cadenillas, Cvitanic, and Zapatero (2007). This would complicate our analysis, and so is left outside this paper.

2.2 Status concerns

A key novelty of this paper, as compared to the existing security design literature, is that the entrepreneur is driven by her desire to achieve a higher financial status, a well-documented feature of human behavior that we refer to as status concerns. We model status concerns in line with the classical insight of Friedman and Savage (1948): preferences that are concave for low wealth levels—the low status region, convex for intermediate wealth levels—the middle status region, and concave for high wealth levels—the high status region.

Given the well-documented tendency of entrepreneurs to take risks, preferences with convexities seem especially appropriate for modeling entrepreneurs' behavior. As indicated by Becker, Murphy, and Werning (2005), "[S]tart-ups and other entrepreneurial efforts...are much more common and less well rewarded than would be expected from the usual assumptions of risk aversion and diminishing marginal utility of income." The convexities and the ensuing risk-loving behavior need not necessarily be related to status concerns, but can arise endogenously if one is inherently risk-averse, as shown by Patel and Subrahmanyam (1978), Gregory (1980), Robson (1992), Rosen (1997), Gollier, Koehl, and Rochet (1997), and Vereshchagina and Hopenhayn (2009). Let us briefly describe a mechanism in one of these papers, by Patel and Subrahmanyam, without taking a stand that it is more relevant than others in our context.

The traditional argument for decreasing marginal utility relies on the divisibility of consumption goods. Under divisibility, one can consume the same set of goods regardless of one's wealth, with a higher wealth level resulting simply in a higher consumption of each good. Consuming more of the same goods leads to satiation, and hence marginal utility decreases in wealth. Clearly, in reality, when the individual's wealth increases, she can well start consuming new types of goods that she could not afford hitherto because they are both expensive and non-divisible. Examples are "status" goods such as a private jet, a yacht, a membership of elite golf clubs, and so on. When one switches from "low" to "high" status goods, the satiation mechanism is not at work, and so the marginal utility may be increasing in a region of wealth in which the switching occurs.

Accordingly, we posit that the entrepreneur's utility function $u_E(\cdot)$ over her wealth $W_{E\tau}$ at some future date τ is

$$u_E(W_{E\tau}) = \begin{cases} \dfrac{(W_{E\tau})^{1-\gamma_E}}{1-\gamma_E} & W_{E\tau} < L, \\ \dfrac{(W_{E\tau}-\alpha)^{1-\gamma_E}}{1-\gamma_E} + B, & W_{E\tau} \geq L, \end{cases} \tag{2}$$

where γ_E, $L > 0$, $\alpha \in [0, L)$, and $B = (L^{1-\gamma_E} - (L-\alpha)^{1-\gamma_E})/(1-\gamma_E) \geq 0$ ensures continuity of preferences. The parameter α represents the *status concerns*—the higher α is, the stronger is the entrepreneur's desire to achieve high status, and so the more pronounced is the convexity region in the utility. The special case of $\alpha = 0$ corresponds to a standard CRRA utility function with no status concerns. Figure 1 presents typical shapes of utility functions with status concerns. Going from left to right in Figure 1, we first have the *low-status* region of wealth in which the utility is concave, then the *middle-status* region with convex utility, and finally the *high-status* region with concave utility. The position of the middle-status region is determined by the parameter L; henceforth, we refer to L as the *status level* (of wealth). The parameter γ_E represents the entrepreneur's *risk aversion* when her wealth is in the low or high-status region. Our subsequent results are robust to alternative preference specifications, as we discuss in Section 4.3.

The other agent is a financier who can provide funds and other resources required to keep the firm operational. Her preferences are given by a standard risk-averse CRRA utility function $u_F(\cdot)$ over her wealth W_{FT} at some future date T:

$$u_F(W_{FT}) = \frac{(W_{FT})^{1-\gamma_F}}{1-\gamma_F}, \tag{3}$$

where $\gamma_F > 0$ is the financier's relative risk aversion. Given the empirical evidence on

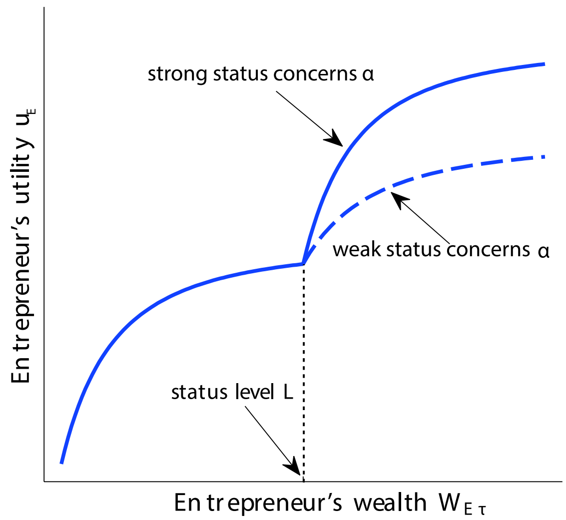


Figure 1: Entrepreneur's Utility Function $u_E(W_{E\tau})$

preference heterogeneity, we acknowledge that other utility functions for financiers are also possible, which may affect our results. We also note that convertible securities used in venture capital financing often have payment-in-kind coupon payments so that a start-up company does not actually have to pay interest until the maturity of the security. This motivates the absence of intermediate consumption in our model.

2.3 Security design problem

The main goal of this paper is to analyze the scenario in which the firm can be operational only if the entrepreneur is able to attract funding from the financier.[5] In return, the entrepreneur offers the financier a state-contingent claim, or a *security*, represented by a function $W_{FT}(V_T)$. The security specifies the amount W_{FT} that the financier will receive at date T for each possible realization of the firm value $V_T > 0$. We do not impose any restrictions on the function $W_{FT}(V_T)$, such as the monotonicity constraint present in numerous security design works. Moreover, the entrepreneur cannot run with the money before paying back the financier, which we

[5]The funding here can be interpreted broadly, in that it can refer not only to the monetary payment but also to other forms of the financier's involvement, such as sharing her experience and expertise, giving access to her network of contacts, and so on. It is widely believed that such non-monetary forms of support have a substantial value.

model by assuming $\tau > T$, meaning that the entrepreneur's horizon τ is longer than that of the financier T.

The financier agrees to finance the firm if her expected utility with the security offered to her is not lower than her (commonly known) reservation utility $\bar{u}_F$, which is likely to depend on the initial value of the funding provided by the financier, the financier's outside investment opportunities and bargaining power. We assume that this reservation utility is not prohibitively high from the entrepreneur's perspective, and so the financing transaction between the entrepreneur and the financier does take place. At the payoff date T, the entrepreneur pays the financier the required amount, which reduces the firm value by this amount. She continues managing the firm until her horizon τ, at which time she consumes the value V_τ.

The optimal security $W^*_{FT}(V_T)$ and the optimal firm riskiness process ϕ^*_t, $t \in [0, \tau)$ are such that the financier accepts the security, and the corresponding time-τ firm value V^*_τ maximizes the entrepreneur's expected utility (2), as formalized in Definition 1.

Definition 1 *The optimal security, $W^*_{FT}(V_T)$, and the firm riskiness process, $\phi^*_t > 0$, $t \in [0, \tau]$, are determined as the solution to the problem*

$$\max_{\phi_t, W_{FT}} E_0[u_E(V_\tau)] \tag{4}$$

$$\text{subject to} \quad dV_t = V_t\phi_t\mu dt + V_t\phi_t\sigma d\omega_t - W_{FT}dI_t,$$

$$E_0[u_F(W_{FT})] \geq \bar{u}_F, \tag{5}$$

where I_t is a step function $I_t \equiv 1_{\{t=T\}}$.

3 Security Design and Firm Riskiness

In this Section, we examine security design in a setting whose key feature is that the entrepreneur can either be seeking risks as she tries to achieve a higher status or averse to risks when status concerns are weak. Introducing preferences with both types of risk attitude have proved valuable in various areas of finance, however this has not yet been done, to our knowledge, in the security design and financial contracting literature. Our paper aims to make a step in this direction, and shows that the resulting model helps explaining the use of convertible securities.

Proposition 1 characterizes the optimal security in closed-form.

Proposition 1 *The optimal security $W^*_{FT}(V_T)$ is given parametrically through a pair of functions $(W_{FT}(x), V_T(x))$ where the parameter x varies from 0 to $+\infty$. The functions $W_{FT}(x)$ and $V_T(x)$ are*

$$W_{FT}(x) = (\bar{u}_F(1-\gamma_{_F}))^{-1/(\gamma_F - 1)}\, e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F}, \tag{6}$$

$$\begin{aligned} V_T(x) = {} & K_{1T}\, g(x)^{-1/\gamma_E} + \alpha N\left(\frac{\ln(B/\alpha) - \ln g(x) - K_{2T}}{K_{3T}}\right) \\ & + (\bar{u}_F(1-\gamma_{_F}))^{-1/(\gamma_F - 1)}\, e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F}, \end{aligned} \tag{7}$$

where $N(\cdot)$ is the standard normal cumulative distribution function, the constant B is as given in equation (2), the function $g(x)$ and the quantities K_{1T}, K_{2T}, and K_{3T} are provided in Appendix A.

The details of the derivation are provided in the Appendix, and here we briefly note that each value of the parameter x in equations (6)–(7) corresponds to a certain state of the world at date T, and all possible states occupy the interval $(0, +\infty)$. Therefore, the expressions (6)and (7) specify, respectively, the payoff to the financier W_{FT} and the project value V_T for every state of the world, thus defining parametrically the optimal security $W_{FT}(V_T)$.

It is, of course, not to be expected that a security whose representation is as complicated as presented in Proposition 1, or in subsequent Propositions, would be used exactly as is in real financing transactions. It is natural to expect that for a given optimal security obtained in a theoretical model, the security that is likely to be used in a real transaction is one of the standard securities, or a portfolio thereof, whose payoff structure is closest to that of the optimal one. For example, this can be because parties involved in real transactions prefer to deal with familiar securities whose properties are well understood. We rely on this argument throughout the paper in that we often comment on how our optimal security is similar to some standard security, which should be interpreted as suggesting that this standard security is likely to be issued in reality.

We also note that the optimal quantities reported in all of our Propositions are given by convoluted expressions, and so we examine their properties numerically

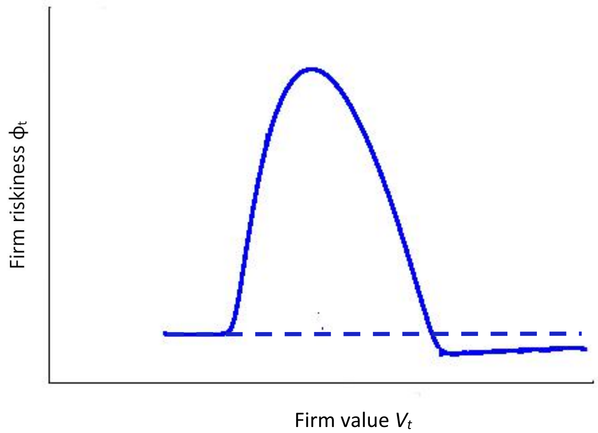


Figure 2: Optimal Firm Riskiness. The figure depicts the time-t firm riskiness, ϕ_t^*, with status concerns $\alpha > 0$ (solid line) and no status concerns $\alpha = 0$ (dashed line).The parameter values are $\alpha = 0.5$ for the solid line and $\alpha = 0$ for the dashed line, $\gamma_E = 3$, $L = 2$, $V_0 = 3$, $\mu = 0.1$, $\sigma = 0.8$, $t = 3.5$, $\tau = 4, T = 3$, and so $B = 0.0972$.

by calibrating the parameters to plausible values. In Section IA3 of the Internet Appendix, we show that our results are robust to alternative parameter values.

3.1 Firm riskiness

The choice of the security to be issued occurs concurrently with the choice of the dynamic firm riskiness, as stated in Definition 1, implying that the two are interrelated. Given this, we first discuss how the entrepreneur dynamically adjusts the firm riskiness as it enables us to better explain her choice of the security. It is also worth noting that understanding how the firm riskiness evolves over time can be valuable for other questions in financial economics beyond security design, as we elaborate later in Section 4.2.

The firm riskiness ϕ cannot be characterized analytically, however the explicit expression for the optimal security in Proposition 1 enables us to calculate ϕ numerically using a fairly straightforward procedure. Figure 2 plots the optimal firm riskiness when the entrepreneur has status concerns $\alpha > 0$ (solid lines), and in the benchmark case of no status concerns $\alpha = 0$ (dashed lines). We see a sharp distinction between the cases of status concerns and no status concerns, in terms of how actively the entrepreneur manages the riskiness. In particular, the status-driven entrepreneur adjusts the riskiness depending on the firm value, whereas without status concerns she simply chooses a constant firm riskiness. Recalling the earlier discussion of the three status regions (Section 2.2), we place $\underline{L}$ and $\overline{L}$ onto the x-axis to mark the boundaries of these regions, so that the low, middle, and high-status regions correspond to, respectively, $V_t < \underline{L}$, $\underline{L} \leq V_t \leq \overline{L}$, and $V_t > \overline{L}$.[6]

The main result of Figure 2 is that the entrepreneur takes substantial risks in the middle status region as she, driven by the convex part of her preferences, attempts to increase the likelihood of reaching high status via risk-taking. In the low and high status regions, on the other hand, the status change is unlikely, and so the entrepreneur reduces the riskiness to levels close to that without status concerns. This risk-taking behavior is qualitatively similar to that described in Friedman and Savage (1948); however this result is not entirely anticipated because our and their settings are different: in our setting, the entrepreneur chooses both how much risk to take and what security to issue, whereas their agent's only choice is the extent of risk taking.

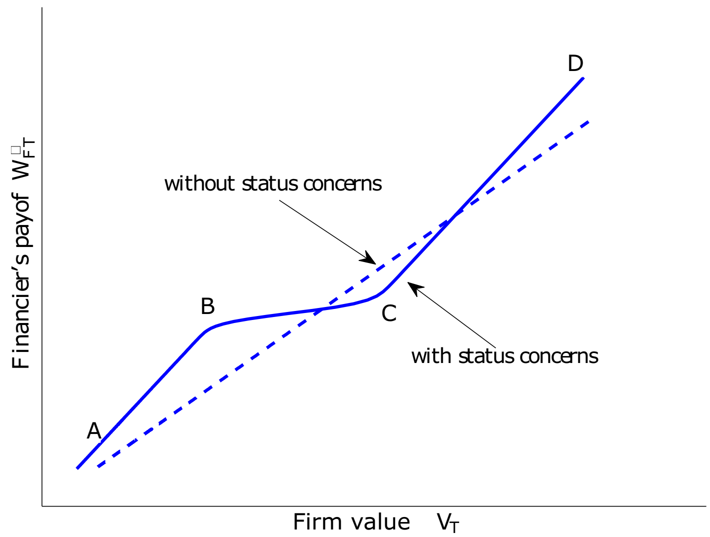
We see from Figure 2 that in the high status region the firm riskiness is lower than the level without status concerns (solid line is below dashed line). The reason is that the entrepreneur is more risk-averse in the high status region than in the low-status region. Indeed, computing the relative risk aversion coefficients of the two functions in (2), we get γ_E for low status and $\gamma_E W_{E\tau}/(W_{E\tau} - \alpha)$ for high status; the former is lower than the latter. We explore in more detail how the entrepreneur's risk taking with and without security issuance are related to each other in Section 4.2.

3.2 Optimal security

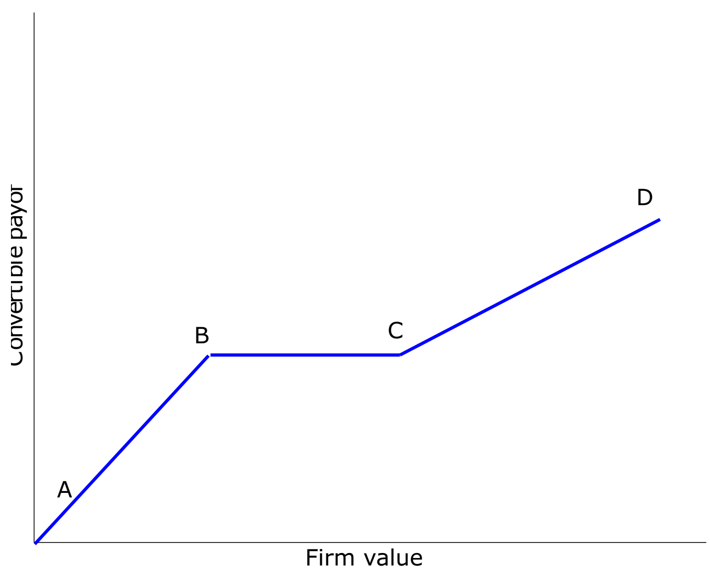
We now show that our model provides an explanation for the use of convertible securities, which is one of our key findings. To this end, we plot the optimal security characterized in Proposition 1.

Panel (a) of Figure 3 plots the optimal security when status concerns are present

[6] While the entrepreneur's status is realized at her horizon τ (when her consumption takes place), the entrepreneur can compute her *expected* status at any prior date $t < \tau$. Accordingly, we refer to the region $V_t < \underline{L}$ as the low-status region because, when $V_t < \underline{L}$, the entrepreneur expects to have low status at date τ, and analogously for the two other regions. The thresholds $\underline{L}$ and $\overline{L}$ are formally defined in equation (A20) in the proof of Proposition 3.



(a) Optimal security



(b) Payoff of an actual convertible security

Figure 3: Optimal Security and Convertible Security. Panel (a) depicts the optimal security W^*_{FT} in the external financing case of our model, while panel (b) depicts the payoff profile of an actual convertible security. In panel (a), the solid line corresponds to the case of status concerns and the dashed line to the case of no status concerns. The parameter values are $\alpha = 1.5$ for the solid line and $\alpha = 0$ for the dashed line, $\gamma_F = 3$, $u_F = -0.5$, $T = 3$, and the other parameters are as in Figure 2.

(solid line) and absent (dotted line). We see that the status-driven entrepreneur finances the firm by offering to the financier a security that is considerably similar to a convertible security, which has the payoff profile depicted in panel (b). Absent status concerns ($\alpha = 0$, dotted line in panel (a)), the optimal security is similar to equity; we discuss this case further in the next Subsection 3.3. A key feature of a convertible security is its hybrid nature in that it exhibits attributes of both equity and debt (but cannot be statically replicated by a mix of equity and debt): the A-B-C segment corresponds to debt and the C-D segment corresponds to equity. The slope of C-D is determined by the conversion ratio, namely the number of equity shares into which a convertible security can be converted.[7]

From panel (a), we see that a concern for status causes the entrepreneur to introduce a debt-like segment into an equity-like security, which she would choose without status concerns. The entrepreneur knows that she is going to increase the firm risk in the middle status region, as explained in Section 3.1, and includes the debt-like segment as a way to insulate the financier from this risk. Recall that the financier does not have status concerns, and so would find an equity-like security unattractive as it would pass on to its holder the higher risk. Aware of the need to make the security attractive to the financier to satisfy her participation constraint, the entrepreneur offers a convertible security with the debt-like segment—promising a payoff insensitive to firm value fluctuations—for middle status firm values when the entrepreneur intends to take considerable risks.

While we focus our discussion on convertible securities and entrepreneurial financing, our model is potentially relevant in other situations in which convertible-like payoff profiles are used. For example, if a firm issues a portfolio of securities with such or similar payoff profile (e.g., a bond and a call option), our model can rationalize this behavior. Another example concerns compensation schemes consisting of a fixed wage (akin to debt) and a performance-based bonus (akin to equity), which are widely used in reality. It may be that our model, once appropriately adapted to the context of determining the optimal compensation between a worker and an employer, can provide a status-based explanation for this type of compensation packages.

[7]For a more detailed description of convertible securities, see for example Section 24-6 in Brealey, Myers, and Allen (2010). Hereafter, we use a generic term "convertible security," rather than specifying a particular security, because there are several real-world instruments that have features of both equity and debt, e.g., convertible bonds and convertible preferred stocks.

3.3 Two special cases

It is of interest to take a closer look at two special cases of the model: i) when the financier is close to risk neutral, and ii) when there are no status concerns and the entrepreneur and the financier have different risk aversions.

The first case is motivated by a common argument in the literature that financiers, if well-diversified, can be close to risk-neutral or risk-neutral when evaluating investment opportunities. Accordingly, we depict in Figure 4 the optimal security for two relatively low values of the financier's risk aversion, $\gamma_F = 0.4$ and $\gamma_F = 0.2$. We see that the lower the financier's risk aversion is (going from solid to dashed line), the less pronounced is the debt-like segment in the middle of the payoff profile. This is consistent with the intuition presented above. The debt-like segment is needed to protect the financier from the status-driven increase in the firm riskiness, and a decrease in her risk aversion implies that she needs less protection. Therefore, the slope of the middle segment goes up when the risk aversion goes down. This means that, other things equal, the more risk tolerant the financier is the less likely the financing takes the form of a convertible security. We do not consider the perfect risk-neutrality case because it is at odds with empirical evidence that venture capital and private equity firms (real-world counterparts of our "financier") typically demand compensation for risk (Gompers, Gornall, Kaplan, and Strebulaev (2020), Gompers, Kaplan, and Mukharlyamov (2016)).[8]

In the second special case, we assume away status concerns but allow for different risk aversions for the entrepreneur and the financier. The goal is to establish that our main result cannot be generated through a simpler mechanism of risk aversion heterogeneity without relying on status-driven convexities. We do not plot the optimal securities in this case given that we can explain our findings by referring to results presented in Larsen (2005). When the financier is less (more) risk averse than the entrepreneur, the payoff profile of optimal security is given by an increasing convex (concave) function; recall that the function is linear when the risk aversions are the same (dashed line in Figure 3(a)).

[8]There are several aspects in which the reality of entrepreneurial financing is different from an idealized setting used to justify risk-neutrality. For example, start-up investments are not liquid and this can magnify risk aversion (Ang, Papanikolaou, and Westerfield (2014)). Second, general partners (GPs) of venture capital and private equity funds are often not well-diversified as they invest a non-trivial fraction of their own wealth into the funds, which again contributes to risk aversion.

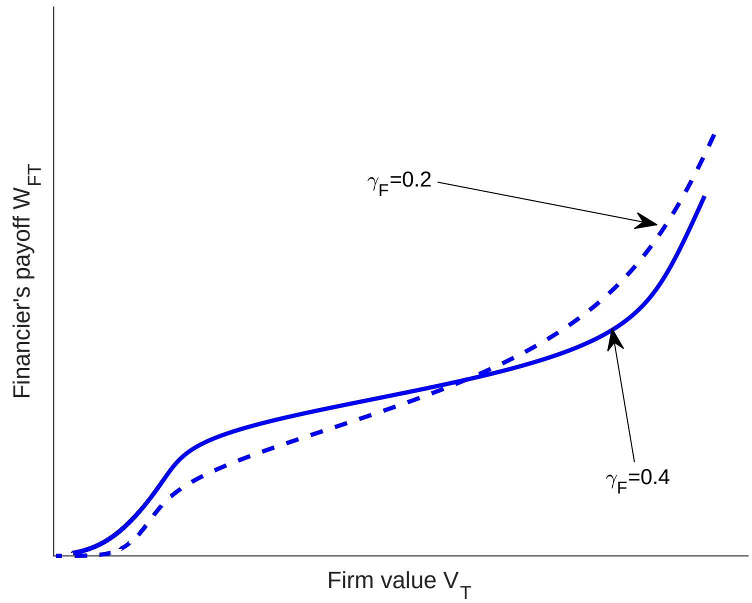


Figure 4: Optimal security when financier is close to risk neutral. This figure depicts the optimal security when the financier's relative risk aversion is 0.4 (solid line) and 0.2 (dashed line).

As argued in the beginning this Section, in reality entrepreneurs are likely to resort to a standard security, or a portfolio of such securities, whose payoff profile is the closest to the optimal one. Given this, when the two risk aversions are the same or sufficiently close, the entrepreneur is likely to issue an equity. When the risk aversion heterogeneity is large enough for the deviation from linearity to be non-trivial, the issued security is likely to consist of two linear segments. When the optimal security is convex ($\gamma_F < \gamma_E$), the payoff profile of its real-world counterpart can be as depicted in Figure 5 of Larsen (2005, p. 509). Such a profile can be obtained by creating a portfolio of an equity and a call option, whereby the number of units of each instrument are straightforwardly computed from the slopes of the two segments. Analogously, when the optimal security is concave ($\gamma_F > \gamma_E$), the payoff profile of the security used in reality can be as depicted in Figure 3 of Larsen (2005, p. 505), which corresponds to a portfolio of an equity and a short position in a call option.

With some stretching of the terminology, we refer in the paper to the optimal securities without a debt-like segment as equity-like even though they are not necessarily identical to equity. Given that it is only equity-like securities that can be generated under standard preferences, we see that status concerns is indeed a key ingredient for our explanation of convertible securities.

3.4 Convertible securities and risky firms

In this Section, our aim is to show that our model is consistent with the following observed patterns of the use of convertible securities: i) convertible securities are more likely to be used by start-up firms than by established companies (Sahlman (1990), Gompers (1999), Kaplan and Stromberg (2003)), ii) when a start-up firm is financed in multiple stages, convertibles tend to be used in earlier stages, and iii) for established public companies, smaller companies are more likely to issue convertible securities.[9] A simple explanation would be to claim that status concerns are more pronounced for the type of firms, in each of the three above cases, that are more likely to issue convertible securities. While a plausible story, we are not aware of any evidence concerning the strength of status concerns in different types of firms.

We instead pursue an alternative empirically motivated approach to explaining the above three patterns. Specifically, we view these patterns as a manifestation of a single underlying phenomenon: riskier firms are more likely to issue convertible securities. Indeed, in each of the three cases we can observe a positive link between the riskiness and the use of convertibles given that: i) start-up firms are riskier than established ones, ii) a start-up firm at an earlier stage is riskier than at later stages, and iii) smaller firms are riskier than larger ones. A similar point is made in Brennan and Schwartz (1988) who note that "companies issuing convertible bonds tend to be characterized by higher market and earnings variability, higher business and or financial risk."[10]

To see how our model fits the findings above, we examine how making our firm

[9] Noddings, Christoph, and Noddings (2001) consider publicly traded companies in the U.S. that have issued either convertible debt or convertible preferred stocks. Out of the companies using convertible debt, 58% are small-cap companies, 27% are middle-cap companies, and 15% are large-cap companies. For companies using convertible preferred stocks, the corresponding numbers are 47%, 39%, and 14%. A similar observation is made in Brealey, Myers, and Allen (2010), who note that "convertibles tend to be issued by the smaller and more speculative firms."

[10] This is also evidenced by the spate of issues of convertible bonds by technology firms, as reported in a recent article in the Financial Times ("US convertible debt splurge reflects tech shares rally," August 15, 2018).

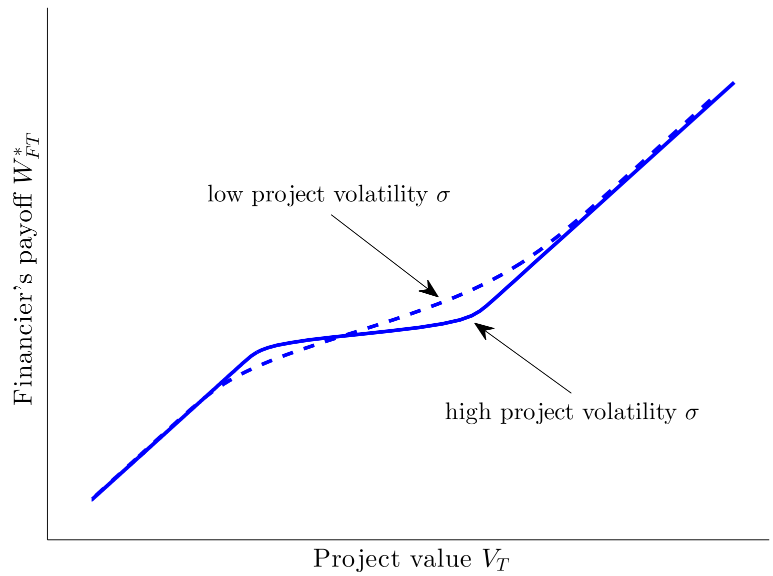


Figure 5: Effect of Firm Volatility on Optimal Security. This figure depicts the optimal security for relatively high firm volatility (solid line) and relatively low firm volatility (dashed line). The parameter values are $\sigma = 0.2$ for the dashed line and $\sigma = 0.8$ for the solid line, and the other parameters are as in Figure 3.

more volatile, through increasing the volatility parameter σ in specification (1), affects the structure of the optimal security. Figure 5 depicts the optimal security for varying levels of the firm volatility parameter σ. The figure reveals that, as the volatility increases (going from a dashed to a solid line), the similarity between the optimal security and a convertible security becomes more pronounced, in that the slope of the middle segment becomes lower and so closer to the fully flat middle segment of an actual convertible security. To understand why, recall that the role of the middle segment is to insulate the financier from an increase in the firm risk in the middle status region. The higher σ is, the riskier is the firm value other things equal (as seen from (1)), and so the higher is the need to protect the financier against the risk, resulting in a flatter middle segment for a higher volatility parameter.

Following the discussion in the paragraph immediately after Proposition 1, the greater similarity between our optimal security and an actual convertible security obtained as the firm volatility increases implies a higher likelihood of convertible securities being used for financing, consistent with the evidence.

4 Alternative Settings

In this Section, we consider several modifications of the above setting so as to shed light on additional questions concerning the economic role of status concerns. In Section 4.1, we show that the use of convertible securities is linked to the ability to dynamically adjust firm characteristics, as suggested in the literature, by showing that the optimal security in a static setting is notably different from a convertible security. In Section 4.2, we examine how the status-driven entrepreneur manages the firm when she is not making the financing choice. The analysis can be valuable for security design through the credit risk channel—credit risk is known to be one of the determinants behind security choice.

4.1 Security design with fixed firm riskiness

Young start-up firms and small established firms, which we focus on in this paper, are likely to be relatively flexible in choosing how to deploy their assets as compared to mature large companies. Given this, it seems appropriate to adopt a dynamic setting, as we do in our main analysis, in which the entrepreneur can change the firm riskiness over time, e.g., due to modifying the novelty of the product being developed. However, it also seems valuable to examine a static version of our model with a fixed riskiness so as to address the point that the incentives to issue convertible securities can be linked to the firm's dynamic behavior. For example, Biais and Casamata (1999) note: "Equity and convertible bond financing, provided by venture capitalists, play an important role for young, innovative, and high-tech firms, where the ability to switch to riskier ventures is large."

Our analysis below provides support to Biais and Casamata's argument, as we find that when the entrepreneur is not able to adjust the riskiness over time the optimal security has a different payoff structure than a convertible security.

We take the security design setting presented in Section 2, and specialize it by assuming that the entrepreneur is not able to change the firm riskiness ϕ. To solve the resulting model analytically, we need to assume that ϕ is not too low (as discussed in the proof of Proposition 2), and we also need to model the entrepreneur's status concerns using a different preference specification (IA-1) presented in the Internet Appendix. We have formally verified that neither of these features drive the results presented below.[11]

Proposition 2 defines the security design problem in the static setting and provides its analytical solution.

Proposition 2 *In a static setting with a constant firm riskiness ϕ, the optimal security $W^*_{FT}(V_T)$ is the solution of the problem:*

$$\begin{aligned} & \max_{W_{FT}} E_0[u_E(V_\tau)] && (8) \\ \textit{subject to} \quad & dV_t = V_t\phi\mu dt + V_t\phi\sigma d\omega_t - W_{FT}dI_t, \\ & E_0[u_F(W_{FT})] \geq \bar{u}_F, && (9) \end{aligned}$$

where I_t is a step function $I_t \equiv 1_{\{t=T\}}$. The optimal security $W^*_{FT}(V_T)$ is given implicitly by the equation

$$f(V_T - W_{FT}) + zW_{FT}^{-\gamma_F} = 0. \tag{10}$$

where the function $f(\cdot)$ is

$$\begin{aligned} f(x) =& x^{-\gamma_E}K_1 N\left(\frac{\ln(L/x) - K_2}{K_4}\right) - \frac{(x)^{-\gamma_E}}{1-\gamma_E}K_1 n\left(\frac{\ln(L/x) - K_2}{K_4}\right)/(K_4) \\ & + \alpha x^{-\gamma_E}K_1 N\left(\frac{-\ln(L/x) + K_2}{K_4}\right) + \alpha\frac{x^{-\gamma_E}}{1-\gamma_E}K_1 N\left(\frac{-\ln(L/x) + K_2}{K_4}\right)/(K_4) \\ & + Bn\left(\frac{-\ln(L/x) + K_3}{K_4}\right)/(xK_4). \end{aligned} \tag{11}$$

In the above, the quantities K_1, K_2, K_3, K_4, and z are computed as given in Appendix A.

Figure 6 depicts the optimal security in the static setting, revealing that it is notably different from a convertible security. In particular, we see that in place of where the debt-like segment is located in an actual convertible security, the static optimal security has a segment corresponding to a negative, or a short, equity position.

[11] In Section IA1 of the Internet Appendix, we present a numerical solution of the static model when the entrepreneur's preferences are given by the initial specification (2), and find that our predictions are unaffected.

The intuition is that the entrepreneur still wants to take risks in the middle status region driven by the convex part of her preferences. However, she is now unable to do so through increasing the firm riskiness, and so takes risks by offering the financier a negative stake in the firm in this region. Analogously to the classical risk sharing result, that offering a positive stake in the firm to another agent allows one to share risks and thus reduce her own risk exposure, offering a negative stake allows the entrepreneur to increases her risk exposure.

Comparing the results of the dynamic and the static settings reveals a connection between the ability to manipulate the firm riskiness and the monotonicity of the optimal security. When this ability is present, the optimal security is given by a monotonically increasing function; when absent, the function is non-monotonic. If we consider the static setting and add the monotonicity constraint, as often done in the security design literature, the optimal security is likely to have a convertible-like shape with a flat middle segment. However, the static intuition combined with the monotonicity constraint seems to be silent about another key prediction of our dynamic framework consistent with empirical evidence—that the incentive to issue a convertible security becomes weaker as the firm volatility decreases. Informally, taking the static optimal security depicted in Figure 6 and making it monotonic entails replacing the middle segment with a negative slope by a flat segment. The resulting security is convertible-like regardless of the firm volatility.

On the methodological front, the goal of the static analysis is to address a possible concern that adopting a dynamic framework overly complicates the analysis without producing more realistic implications. Contrary to this possibility, we find that a dynamic setting is more tractable than a static one: the former is solved explicitly for any parameter values, while the latter require imposing a parameter restriction, as noted before Proposition 2. As for the economic implications, a static model seems more problematic because offering a negative stake in the firm is hardly an empirically appealing property.

4.2 Firm risk dynamics without security issuance

In our model, the entrepreneur both manages the firm and chooses what type of a security to issue. In some actual companies, these two decisions can be, to some

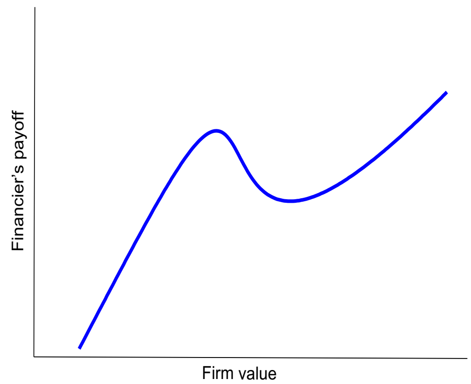


Figure 6: Optimal Security in a Static Setting. This figure shows the payoff of the optimal security in a static setting when the firm riskiness cannot be changed. The parameter values are as in panel (a) of Figure 3.

extent, independent. Entrepreneurs, who propose new projects and manage them, can be driven by status concerns, while investment officers, those who decide on security issuance, may have other considerations. Analyzing the security design problem in such a setting will require considerably changing the model and so is beyond the scope of this paper. What we can examine without much alteration in the framework is how the status-driven entrepreneur dynamically manages the firm riskiness when she is not involved in security issuance.

This analysis, while not explicitly modelling security issuance, is still likely to be valuable in the security design context. It is well documented that credit risk is one of the important factors behind the financing choice decision, and credit risk in turn is directly linked to the firm's risk-taking choices. Our analysis is instrumental for quantifying this risk, as we elaborate below. Our results can also be used in other contexts. Choi and Richardson (2016) note that "[U]nderstanding why asset volatility (i.e., volatility of firm value) changes through time is a fundamental issue in finance. This is because asset volatility plays a key role both in capital structure valuation and the standard return/risk tradeoff independent of financial leverage."

Proposition 3 formally defines the problem solved by the entrepreneur when she does not also decide on security issuance, and presents its solution in closed form.

Proposition 3 *When there is no security issuance by the entrepreneur, her problem is to dynamically choose the firm riskiness, $\phi_t^* > 0$, $t \in [0, \tau]$, such that it solves the* *problem*

$$\max_{\phi_t} E[u_E(V_\tau)] \tag{12}$$
$$\textit{subject to} \quad dV_t = V_t\phi_t\mu dt + V_t\phi_t\sigma d\omega_t.$$

The solution is

$$\phi_t^* = \frac{\mu}{\sigma^2 V_t^*}\left[\frac{K_{1t}}{\gamma_E}(y\xi_t)^{-1/\gamma_E} + \frac{\alpha}{K_{3t}} n\left(\frac{\ln\frac{B}{\alpha y\xi_t} - K_{2t}}{K_{3t}}\right)\right], \tag{13}$$

and the optimal firm value, V_t^*, is given by

$$V_t^* = K_{1t}(y\xi_t)^{-1/\gamma_E} + \alpha N\left(\frac{\ln\frac{B}{\alpha y\xi_t} - K_{2t}}{K_{3t}}\right), \tag{14}$$

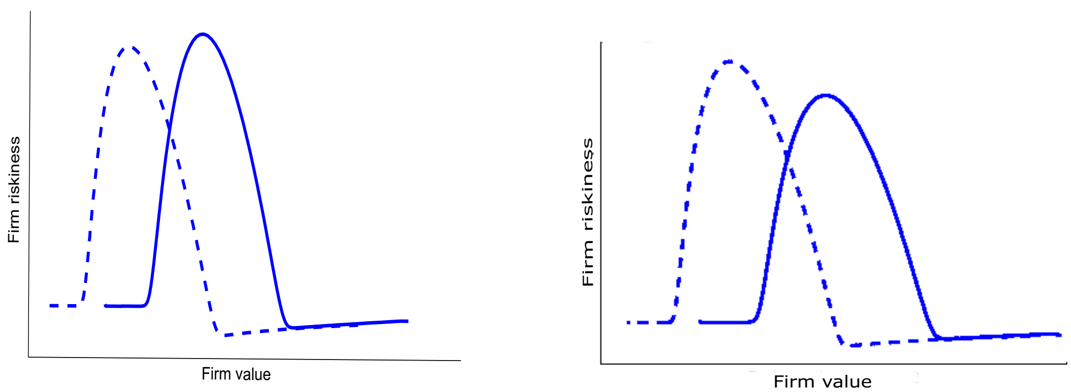
where $N(\cdot)$ and $n(\cdot)$ are the standard normal cumulative distribution function and probability density function, respectively, the constant B is as defined in equation (2), and the quantities K_{1t}, K_{2t}, K_{3t}, and y are provided in Appendix A.

Figure 7 depicts the behavior of firm riskiness for two calibrations, when the firm is relatively young (panel (a)) and mature (panel (b)). We see that the entrepreneur's behavior without security issuance (dashed lines in both panels) is qualitatively similar to that with the issuance (solid lines). In particular, the entrepreneur substantially increases the firm riskiness in the middle status region when status change is likely.[12]

Though the patterns of status-induced risk-taking are broadly similar in the two scenarios, the magnitudes of the effect are different and, more interestingly, the magnitude in the issuance scenario can be higher or lower than with no issuance. Indeed, the peak level of risk taking is higher with security issuance for a young firm (panel (a)), but is higher without the issuance for a mature firm (panel (b)). The issuance and no issuance cases differ in two main aspects, which have an opposite effects on risk-taking incentives. First, in the issuance case, the entrepreneur shares some firm risk with the financier, which induces more risk-taking. Second, issuing a security

[12]With security issuance, a certain amount is paid to the financier out of the firm value. Hence, the increase in volatility in that case occurs for higher firm values, and this is why the two humps are located at different positions on the x-axis.

to the risk-averse financier comes with the need to provide her with a certain reservation utility, which dampens the incentives to take risks. For a young firm, when the payment to the financier will take place relatively far in the future, the need to protect the financier is relatively weak; but it becomes strong as the payment date approaches. Accordingly, the entrepreneur increases the firm riskiness more in the security issuance case for a young firm, as seen in panel (a), but the opposite results obtains for a mature firm, as seen in panel (b).



(a) Young firm, $t = 1$

(b) Mature firm, $t = 2$

Figure 7: Firm Riskiness With and Without Security Issuance. Panel (a) depicts the optimal firm riskiness, ϕ_t^*, without (dashed line) and with (solid line) security issuance for a relatively young firm, $t = 1$. Analogously, panel (b) describes the firm riskiness for a relatively mature firm, $t = 2$. The parameter values are as in Figure 3.

While we do not model explicitly the financing decision in this Section, our results are likely to be relevant for understanding how these decisions are made, as well for other aspects. In the context of security design, the importance of understanding asset volatility dynamics can be motivated by empirical evidence (Marsh (1992)) and survey evidence (Graham and Harvey (2001)) that credit risk is one of the important determinants behind the choice of financing, and credit risk is clearly affected by the evolution of the asset volatility.

The analytical results of Proposition 3 enable one to quantify the link between credit risk and dynamic variation in risk taking of the form described in the Proposition which, as discussed in the Introduction, is consistent with the evidence for real companies. In particular, one can rely on a widely-used structural approach for credit risk modelling (pioneered by Merton (1974)), and use our specification for the firm value (14) to model firm value. The extent of risk taking and the performance level around which it occurs can be controlled through the status concern parameter α and the status level parameter L, respectively. We note that without status concerns, $\alpha = 0$, the firm value volatility is constant (dash-dotted line in Figure 7), and so our analysis obtains as a special case the behavior that is commonly assumed in structural credit risk modelling.[13]

5 Conclusion

This paper develops a dynamic security design framework in which a status-driven entrepreneur owning a firm decides how to finance and how to manage it. We characterize analytically the optimal security and find that it is considerably similar to a convertible security. We find that incentives to issue convertible securities are positively related to firm riskiness and its dynamic flexibility, which can explain why start-up and small firms possessing these characteristics are more likely to issue such securities. We also characterize analytically how the entrepreneur manages the firm when she does not choose what security to issue. The derived firm value process can be used to quantify the implications of status concerns for credit risk, which is known to be an important factor in firms' decisions regarding security issuance.

We have aimed to keep the model general enough to be applicable to studying security issuance by both young start-up firms and mature companies. To better understand the specifics of financing by either type of the firms, future work can specialize our setting by introducing relevant additional features. For example, to tailor our model to the context of start-up financing, one could allow the financing to be spread over multiple rounds rather than a single round as in our analysis. It would also be interesting to see how incentives arising under asymmetric information, extensively studied in existing works, interact with risk-taking incentives induced by status concerns.

[13] There is a growing interest in examining the role of time-varying asset volatility in credit risk modelling. A recent example is Du, Elkamhi, and Ericsson (2018).

Appendix A: Proofs

Proof of Proposition 1. In proving Propositions 1-3, we employ martingale methods in a continuous-time complete market setting. These methods are particularly popular in portfolio choice and asset pricing models. Accordingly, to make our analysis easier to relate to such familiar models, we consider an investment problem that is methodologically analogous to the problem faced by our entrepreneur. Specifically, consider an investor who dynamically allocates her wealth between two assets, cash and a risky asset following a geometric Brownian motion with a mean return μ and volatility σ. If we denote the investor's time-t wealth by V_t and the wealth share invested in the risky asset by ϕ_t, then, as is well-known in the continuous-time finance literature, the dynamic process for wealth V_t is given by the process (1). Hence, the investor with the same preferences as the entrepreneur optimally chooses the same risky wealth share as the optimal firm riskiness chosen by the entrepreneur.[14]

During the time period $(T, \tau]$, after the financier is paid, the setting is analogous to that without security issuance, as presented in Section 4.2. Hence, the entrepreneur's behavior after time T is as characterized in Proposition 3, and so we use some of its results here. Throughout the Appendix, we use $u_E(\cdot)$ to denote the concavified utility function of the entrepreneur given in (A22), and not the one with a local convexity given in (2). To compute the entrepreneur's time-T indirect utility function $v_E(W_{ET}) = E_T[u_E(V_\tau^*)]$, we use the optimality condition (A23) in which the Lagrange multiplier y is implicitly given by $E_T[\xi_\tau V_\tau^*] = \xi_T W_{ET}$ to ensure that time-τ firm value V_τ^* is feasible given the time-T firm value $V_\tau = W_{ET}$. This implies that the entrepreneur's time-T indirect utility is given by

$$\begin{aligned} v_E(W_{ET}) &= E_T\left[\frac{(y\xi_\tau)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E}\mathbb{1}_{\{y\xi_\tau > B/\alpha\}}\right] + E_T\left[\left(\frac{(y\xi_\tau)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E} + B\right)\mathbb{1}_{\{y\xi_\tau \le B/\alpha\}}\right] \\ &= \frac{(y\xi_T)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E}K_{1T} + B * N\left(\frac{\ln(B/\alpha) - \ln(y\xi_T) + K_{2T}}{K_{3T}}\right), \end{aligned} \tag{A1}$$

[14] Methodological aspects aside, ene can think of examples when the entrepreneur's problem is actually similar to that of an investor. For example, if a firm consists of several projects with different expected growth rates and levels of riskiness, the entrepreneur's job is essentially to dynamically manage a "portfolio" of projects.

where K_{1T}, K_{2T}, and K_{3T} are

$$K_{1T} \equiv e^{(1-\gamma_E)(\tau-T)\mu^2/(2\gamma_E^2\sigma^2)}, K_{2T} \equiv (\tau - T)\mu^2/(2\sigma^2),\ K_{3T} \equiv \sqrt{\tau - T}\mu/\sigma.$$

and y is defined implicitly by

$$\begin{aligned} W_{ET} &= E_T[\xi_\tau(y\xi_\tau)^{-1/\gamma_E}\mathbb{1}_{\{y\xi_\tau > B/\alpha\}}]/\xi_T + E_T[\xi_\tau((y\xi_\tau)^{-1/\gamma_E} + \alpha)\mathbb{1}_{\{y\xi_\tau \leq B/\alpha\}}]/\xi_T \\ &= (y\xi_T)^{-1/\gamma_E}K_{1T} + \alpha N\left(\frac{\ln(B/\alpha) - \ln(y\xi_T) - K_{2T}}{K_{3T}}\right). \end{aligned} \tag{A2}$$

In what follows, we will also need the expression for the marginal indirect utility function $v_E'(\cdot)$. Equations (A1) and (A2) define the indirect utility as a composite function $v_E(y(W_{ET})$, which means that

$$\frac{dv_E}{dW_{ET}} = \frac{dv_E}{dy}\frac{dy}{dW_{ET}}. \tag{A3}$$

Computing the two derivatives on the right-hand side of (A3) from (A1) and (A2), respectively, we get

$$\frac{dv_E}{dy} = -K_{1T}y^{-1/\gamma_E}\xi_T^{(\gamma_E-1)/\gamma_E}/\gamma_E - B * n\left(\frac{\ln(B/\alpha) - \ln(y\xi_T) + K_{2T}}{K_{3T}}\right)/(yK_{3T}), \tag{A4}$$

$$\frac{dy}{dW_{ET}} = \left(-K_{1T}\xi_T^{-1/\gamma_E}y^{-1-1/\gamma_E}/\gamma_E - \alpha n\left(\frac{\ln(B/\alpha) - \ln(y\xi_T) - K_{2T}}{K_{3T}}\right)/(yK_{3T})\right)^{-1}. \tag{A5}$$

Substituting (A4) and (A5) into (A3) and rearranging yields the marginal indirect utility

$$\frac{dv_E}{dW_{ET}} = \frac{K_{1T}K_{3T}(y\xi_T)^{(\gamma_E-1)/\gamma_E} + \gamma_E B * n\left(\frac{\ln(B/\alpha)-\ln(y\xi_T)+K_{2T}}{K_{3T}}\right)}{K_{1T}K_{3T}(y\xi_T)^{-1/\gamma_E} + \gamma_E \alpha n\left(\frac{\ln(B/\alpha)-\ln(y\xi_T)-K_{2T}}{K_{3T}}\right)}. \tag{A6}$$

Though it is straightforward to prove that $v_E(\cdot)$ is increasing, establishing that $v_E(\cdot)$ is concave appears daunting given its complicated functional form. However, we have verified the concavity for a large number of model calibrations. We proceed by treating $v_E(\cdot)$ as an increasing concave function, with $v_E'(\cdot) > 0$ and $v_E''(\cdot) < 0$.

Taking into account that the entrepreneur's wealth at time T, after she pays the financier, is $W_{ET} = V_T - W_{FT}$, we can equivalently write the entrepreneur's optimization problem (provided in Definition 1) as

$$\max_{\phi_t, W_{FT}} E[v_E(V_T - W_{FT})] \tag{A7}$$

$$\text{subject to} \quad dV_t = V_t\phi_t\mu dt + V_t\phi_t\sigma d\omega_t, \tag{A8}$$

$$E[u_F(W_{FT})] \geq \bar{u}_F. \tag{A9}$$

The first-order conditions for (A7), given the financier's utility function (3), are

$$v'_E(V_T^* - W_{FT}^*) = z\xi_T, \tag{A10}$$

$$-v'_E(V_T^* - W_{FT}^*) = z_1(W_{FT}^*)^{-\gamma_F}, \tag{A11}$$

where z and z_1 are the Lagrange multipliers associated with the constraints (A8) and (A9), respectively. From (A10) and (A11), we obtain

$$W_{FT}^* = (-z\xi_T/z_1)^{-1/\gamma_F}. \tag{A12}$$

Substituting (A12) into (A9) yields

$$\bar{u}_F = \frac{(-z/z_1)^{1-1/\gamma_F}}{1-\gamma_F} E[\xi_T^{1-1/\gamma_F}] = \frac{(-z/z_1)^{1-1/\gamma_F}}{1-\gamma_F} e^{(1-\gamma_F)\mu^2 T/(2\gamma_F^2\sigma^2)}. \tag{A13}$$

From (A13), the ratio $-z/z_1$ is

$$-z/z_1 = (\bar{u}_F(1-\gamma_F))^{\gamma_F/(\gamma_F-1)} e^{\mu^2 T/(2\gamma_F\sigma^2)}. \tag{A14}$$

Substituting (A14) into (A12) yields

$$W_{FT}^*(\xi_T) = (\bar{u}_F(1-\gamma_F))^{-1/(\gamma_F-1)}\, e^{-\mu^2/(2\gamma_F^2\sigma^2)} \xi_T^{-1/\gamma_F}. \tag{A15}$$

The optimal security specifies the entrepreneur's payoff as a function of the firm value V_T^*, and so for each state of the world ξ_T we need to compute the corresponding firm value $V_T^*(\xi_T)$. From (A10), (A2) and (A15), V_T^* is given by

$$V_T^*(\xi_T) = K_{1T}(y\xi_T)^{-1/\gamma_E} + \alpha N\left(\frac{\ln(B/\alpha) - \ln(y\xi_T) - K_{2T}}{K_{3T}}\right) + (\bar{u}_F(1-\gamma_F))^{-1/(\gamma_F-1)}\, e^{-\mu^2/(2\gamma_F^2\sigma^2)}\xi_T^{-1/\gamma_F}, \tag{A16}$$

where y is computed from (A6) by equating the right-hand side of (A6) to $z\xi_T$. The Lagrange multiplier z is such that the time-T firm value $V_T^*(\xi_T)$ is feasible given the initial value V_0:

$$E[\xi_T V_T^*(\xi_T)] = V_0. \tag{A17}$$

Equations (A15) and (A16) then provide the parametric characterization of the optimal security $W_{FT}^*(V_T)$, as stated in Proposition 1. *Q.E.D.*

Proof of Proposition 2. Given that ϕ is constant, the firm value V follows a geometric Brownian motion process

$$dV_t/V_t = \phi\mu dt + \phi\sigma d\omega_t.$$

Hence, sitting at time T, the logarithm of time-τ firm value is distributed as

$$\ln V_\tau \sim N(\ln V_T + (\phi\mu - \phi^2\sigma^2/2)(\tau - T), \phi^2\sigma^2(\tau - T)).$$

Using this expression, we compute the entrepreneur's time-T indirect utility function $v_E(V_T) = E_T[u_E(V_\tau^*)]$, where V_T is time-T firm value after the financier is paid. This yields

$$\begin{aligned} v_E(V_T) = \frac{(V_T)^{1-\gamma_E}}{1-\gamma_E} K_1 N\left(\frac{\ln(L/V_T) - K_2}{K_4}\right) + \alpha\frac{(V_T)^{1-\gamma_E}}{1-\gamma_E} K_1 N\left(\frac{-\ln(L/V_T) + K_2}{K_4}\right) \\ + BN\left(\frac{-\ln(L/V_T) + K_3}{K_4}\right), \end{aligned} \tag{A18}$$

where K_1, K_2, K_3, and K_4 are given by

$$K_1 \equiv e^{(1-\gamma_E)(\phi\mu - \gamma_E(\phi^*)^2\sigma^2/2)(\tau-T)}, K_2 \equiv (\phi\mu + (0.5 - \gamma_E)(\phi)^2\sigma^2)(\tau - T),$$
$$K_3 \equiv (\phi^*\mu - \phi^2\sigma^2/2)(\tau - T), K_4 \equiv \phi\sigma\sqrt{\tau - T}.$$

The optimal security is the solution of the problem:

$$\max_{W_{FT}} E[v_E(V_T - W_{FT}(V_T))],$$
$$E[u_F(W_{FT})] \geq \bar{u}_F.$$

We have numerically examined the shape of the function $v_E(V_T)$ for a large number of model parametrizations and have established that it is concave if the firm riskiness ϕ exceeds a certain threshold (whose value depends on the parametrization), and is concave-concave-convex otherwise. We are only able to solve this problem analytically in the standard case of a concave function $v_E(V_T)$, and so to achieve this we assume that the firm riskiness ϕ exceeds the threshold.

The first-order condition is

$$-v'_E(V_T - W_{FT}) = zW_{FT}^{-\gamma_F}. \tag{A19}$$

Differentiating $v_E(\cdot)$ given in (A18), we obtain that the marginal indirect utility is $v'_E(x) = f(x)$, where $f(\cdot)$ is as given in (11), and so the optimal security (10) obtains.

The Lagrange multiplier z is such that the financier's expected utility is equal to her reservation utility $\bar{u}_F$. *Q.E.D.*

Proof of Proposition 3. As discussed in the portfolio choice literature (see, e.g., Carpenter (2000), Basak, Pavlova, and Shapiro (2007)), to solve our non-concave optimization problem (12), we convert it into an equivalent concave problem by concavifying the entrepreneur's preferences. To do so, we replace the convex part of the utility function (corresponding to middle status) with a linear segment $a+b*W_{E\tau}$ that is tangent to both the low-status segment of the utility function (top line in specification (2)) and the high-status segment (bottom line in (2)). Denoting the tangency points by $\underline{L}$ and $\overline{L}$, respectively, the parameters a and b of the linear segment are obtained by solving the following system of equations:

$$
\begin{aligned}
\frac{\underline{L}^{1-\overline{\gamma}_E}}{1-\gamma_E} &= a + b\underline{\mathrm{L}}, \\
\frac{(\overline{L}-\alpha)^{1-\gamma_E}}{1-\gamma_E} + B &= a + b\overline{L}, \\
\underline{L}^{-\gamma_E} &= b, \\
(\overline{L}-\alpha)^{-\gamma_E} &= b.
\end{aligned} \tag{A20}
$$

The first and second equations in this system ensure that the concavified utility function is continuous at the points $\underline{L}$ and $\overline{L}$. The third and fourth equations ensure that the utility function is smooth at $\underline{L}$ and $\overline{L}$. Solving the system, we obtain

$$
a = \frac{\gamma_E (B/\alpha)^{1-1/\gamma_E}}{1-\gamma_E},\ b = B/\alpha,\ \underline{L} = (B/\alpha)^{-1/\gamma_E},\ \overline{L} = (B/\alpha)^{-1/\gamma_E} + \alpha. \tag{A21}
$$

Hence, the concavified utility function of the entrepreneur is

$$
u_E(W_{E\tau}) = \begin{cases} \dfrac{(W_{E\tau})^{1-\gamma_E}}{1-\gamma_E} & W_{E\tau} < (B/\alpha)^{-1/\gamma_E}, \\ W_{E\tau}(B/\alpha) + \frac{\gamma_E (B/\alpha)^{1-1/\gamma_E}}{1-\gamma_E}, & (B/\alpha)^{-1/\gamma_E} \leq W_{E\tau} \leq (B/\alpha)^{-1/\gamma_E} + \alpha, \\ \dfrac{(W_{E\tau}-\alpha)^{1-\gamma_E}}{1-\gamma_E} + B, & W_{E\tau} > (B/\alpha)^{-1/\gamma_E} + \alpha. \end{cases} \tag{A22}
$$

Given the utility function (A22), the first-order condition with respect to time-τ firm value V_τ, after some manipulation, is

$$
V_\tau^* = \begin{cases} (y\xi_\tau)^{-1/\gamma_E} & y\xi_\tau > B/\alpha, \\ (y\xi_\tau)^{-1/\gamma_E} + \alpha & y\xi_\tau \leq B/\alpha, \end{cases} \tag{A23}
$$

where y is the Lagrange multiplier computed from the condition that the firm value V_τ^* is feasible: $E[\xi_\tau V_\tau^*] = V_0$, and ξ is as given in Section 3. To compute the optimal time-t firm value V_t^* for $t < \tau$, we use the fact that the process for $\xi_t V_t^*$ is a martingale, and so $V_t^* = E_t[\xi_\tau V_\tau^*]/\xi_t$. Substituting herein expression (A23), we obtain

$$
V_t^* = E_t[(y\xi_\tau)^{-1/\gamma_E}\xi_\tau]/\xi_t + \alpha E_t[\xi_\tau \mathbb{1}_{\{\xi_\tau \leq B/\alpha\}}]/\xi_t. \tag{A24}
$$

To compute the two expectations in (A24), we use the fact that the state-price process ξ_t is lognormally distributed, implying that ξ_τ^{1-1/γ_E} is also lognormally distributed with mean growth rate $(1-\gamma_{_E})\mu^2/(2\gamma_{_E}^2\sigma^2)$, and so

$$E_t[(y\xi_\tau)^{-1/\gamma_E}\xi_\tau]/\xi_t = (y\xi_t)^{-1/\gamma_E} e^{(1-\gamma_{_E})(\tau-T)\mu^2/(2\gamma_{_E}^2\sigma^2)}. \tag{A25}$$

As ξ_τ is lognormally distributed, its truncated expected value can also be computed explicitly (e.g., Chapter 19 in Greene (2011)):

$$E_t[\xi_\tau \mathbb{1}_{\{\xi_\tau \le B/\alpha\}}]/\xi_t = N\left(\frac{\ln\frac{B}{\alpha y \xi_t} - (\tau-t)\mu^2/(2\sigma^2)}{\sqrt{\tau-t}\mu/\sigma}\right). \tag{A26}$$

Substituting (A25) and (A26) into (A24) yields (14) in which the quantities K_{1t}, K_{2t}, and K_{3t} are given by

$$K_{1t} \equiv e^{(1-\gamma_{_E})(\tau-t)\mu^2/(2\gamma_{_E}^2\sigma^2)}, K_{2t} \equiv (\tau-t)\mu^2/(2\sigma^2),\ K_{3t} \equiv \sqrt{\tau-t}\mu/\sigma.$$

Applying Itô's Lemma to (14), after some algebra, we obtain the diffusion term in the dynamic process for V_t^* as

$$-(\xi_t\mu/\sigma)\frac{\partial V_t^*}{\partial \xi_t}d\omega_t = (\xi_t\mu/\sigma)\left[\frac{K_1(t)}{\gamma_{_E}}(y\xi_t)^{-1/\gamma_E} + \frac{\alpha}{K_3(t)}n\left(\frac{\ln\frac{B}{\alpha y\xi_t} - K_2(t)}{K_3(t)}\right)\right]d\omega_t, \tag{A27}$$

From (1), the diffusion term is $\phi_t^*\sigma V_t^* d\omega_t$, which after equating with (A27) and rearranging yields (13). *Q.E.D.*

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Internet Appendix for "Security Design with Status Concerns"

In this Internet Appendix, we formally examine several points related to the analysis and discussions in the main paper. Hereafter, the term "paper" refers to the main article for which this document is a companion. Equations and figures appearing in this Internet Appendix have labels starting with "IA". Numerical labels without "IA" refer to equations and figures appearing in the main paper.

To ensure tractability, we have to abstract away from some pertinent factors behind real-world security issuance, such as factors related to asymmetric information (extensively studied in other security design models). Moreover, we also have to adopt a framework in which an actual convertible security is not admissible: our optimal security can be close to but can never perfectly coincide with a convertible security. Indeed, the holder of a convertible security receives the full firm value for sufficiently low firm values—the slope of A-B segment in Figure 3 (b) is 45 degrees. In our setting, however, the entrepreneur's preferences preclude her from paying out the full firm value because this would leave her with zero wealth, which cannot occur because her marginal utility tends to infinity as wealth tends to zero, as seen from (2). Given this, the optimal security always pays less than the firm value, and the slope of A-B segment in Figure 3 (a) is less than 45 degrees.[1] For actual convertible securities, the slope of A-B is higher than C-D; in our model this relation seems to hold only under the multiplicative status specification (see Figure IA-2) while the slopes are similar under the additive specification.

IA1 Numerical analysis of a static setting

As noted in Section 4.1 in the paper, we were able to solve analytically a static security design problem only under the multiplicative status specification (IA-1). To show that the results remain valid under the additive status specification (2), as used in the dynamic model, we now present results of a numerical analysis.

[1] There are ways to extend our model to address this issue. One can assume that the entrepreneur has other sources of income (e.g., housing wealth, savings), in which she would not be left with zero wealth if she chose to pay the full firm value. Alternatively, one could consider preferences that are well-defined at zero wealth. We leave these extensions for future work.

For a constant firm riskiness ϕ, the firm value process (1) is a geometric Brownian motion and so its realization at time T can take any value from a continuous set $(0, +\infty)$. Given the (uncountably) infinite number of realizations of the firm value, coupled with non-standard preferences, solving this model even numerically appears challenging. We therefore discretize the firm value process (1) so as to obtain a binomial tree. This is a standard approach to approximating a geometric Brownian motion, and is discussed in detail in Cox, Ross, and Rubinstein (1979).

We have solved the model with the discretized firm value process numerically under different parametrizations, and the results are similar across all parametrizations. Figure IA-1 depicts a typical shape of the optimal security. Given the discretization, the optimal security's payoff function is not continuous but is defined on a discrete finite set of future firm values. The continuous profile presented in Figure IA-1 is simply obtained via linear interpolation. We see that the optimal security is similar to that presented in Figure 6 in Section 4.1.

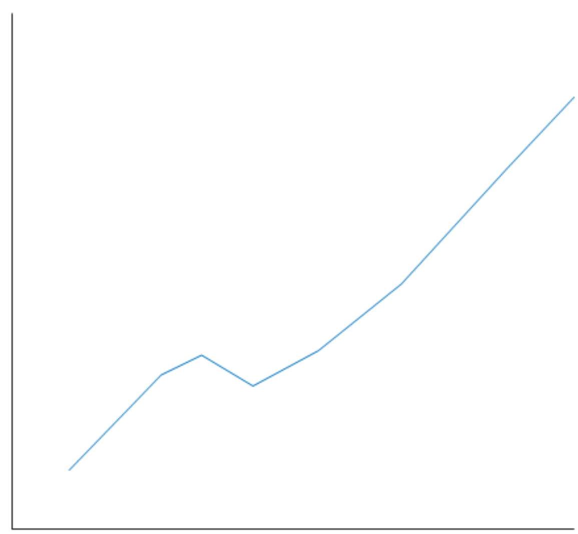


Figure IA-1: Optimal security in a binomial static setting. The model parameters are as in Figure 3. Given the parameters of the firm value process, we build the corresponding binomial tree with a quarterly time step.

IA2 Alternative status specifications

It is known that when one modifies a standard utility function to account for some aspect of human behavior, model implications may well differ depending on whether the modification is additive or multiplicative. For example, models with multiplicative habits (e.g., Abel (1990)) often generate different predictions from those with additive habits (e.g., Campbell and Cochrane (1999)).

Accordingly, we seek to understand whether our main implications remain valid if we replace the additive status specification (2) used in the main analysis by a multiplicative status specification. In particular, we assume the same setting as presented in Section 2, but the entrepreneur's utility function is now given by

$$U_E(W_{E\tau}) = \begin{cases} \frac{W_{E\tau}^{1-\gamma_E}}{1-\gamma_E} & W_{E\tau} < L, \\ \alpha \frac{W_{E\tau}^{1-\gamma_E}}{1-\gamma_E} + \mathrm{B} & W_{E\tau} \geq L, \end{cases} \tag{IA-1}$$

where $B = (1-\alpha)L^{1-\gamma_E}/(1-\gamma_E)$. All the parameters in (IA-1) have the same interpretations as they have in (2). The difference between the two specifications is that the parameter α capturing the strength of status concerns enters multiplicatively in (IA-1), and not additively as in (2). To obtain a concave-convex-concave utility function (IA-1), so that its shape is similar to that depicted in Figure 1, we set $\alpha > 1$. The case of $\alpha = 1$ corresponds to the (globally concave) CRRA utility function with no status concerns. We recall that we use this specification in the main text in Section 4.1 because of its tractability in the static version of the model.

The optimal security is as presented in Proposition IA-1, and the proof is provided at the end of this subsection.

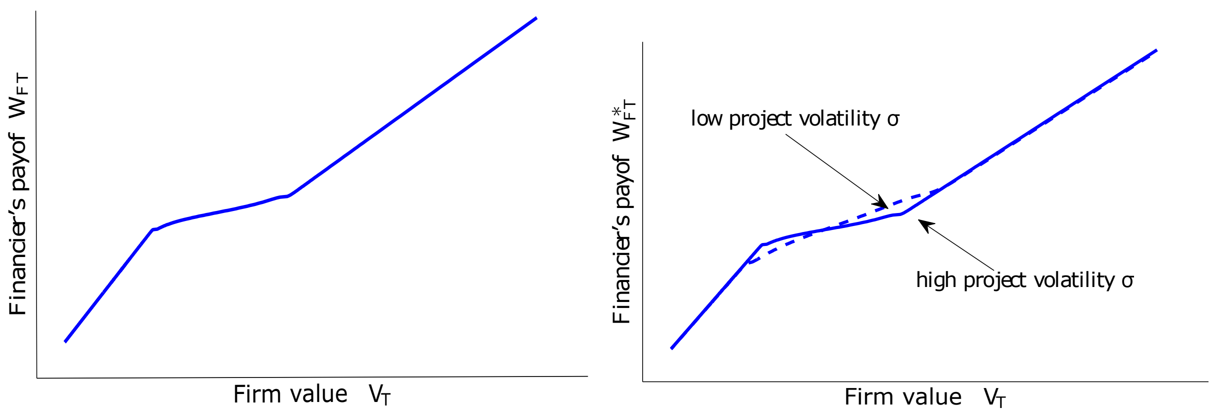
Proposition IA-1 *The optimal security $W^*_{FT}(V_T)$ for the multiplicative status specification (IA-1) is given parametrically through a pair of functions $(W_{FT}(x), V_T(x))$, where x varies from 0 to $+\infty$. The two functions are*

$$W_{FT}(x) = (\bar{u}_F(1-\gamma_{_F}))^{-1/(\gamma_F - 1)}\, e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F}, \tag{IA-2}$$

$$V_T(x) = g(x)^{-1/\gamma_E} K_{1T} N\left(\frac{-\ln b + \ln g(x)) - K_4}{K_{3T}}\right) + \alpha^{1/\gamma_E} g(x)^{-1/\gamma_E} K_{1T} N\left(\frac{\ln b - \ln g(x) + K_4}{K_{3T}}\right)$$
$$+ (\bar{u}_F(1-\gamma_{_F}))^{-1/(\gamma_F - 1)}\, e^{-\mu^2/(2\gamma_F^2\sigma^2)} x^{-1/\gamma_F}, \tag{IA-3}$$

where the function $g(x)$ is given in Appendix B.

Figure IA-2 depicts the optimal security (panel (a)), and examines how it is affected by the firm volatility (panel (b)).[2] We see that the results are analogous to those obtained under the additive status specification—see Figures 3 and 5.



(a) Optimal security (b) Varying firm volatility

Figure IA-2: Optimal security under multiplicative status specification. Panel (a) depicts the optimal security for the multiplicative status specification. Panel (b) depicts the optimal security for relatively high firm volatility (solid line) and relatively low firm volatility (dashed line).

Thinking along the lines of Friedman and Savage (1948), one may come up with other types of preferences capturing status concerns but different from those considered in this paper. For example, the entrepreneur's utility function may jump upwards as wealth crosses the status threshold L, or there can be multiple status thresholds

[2]The parameter values are $\gamma_{_E} = 3$, $\alpha = 15$, $L = 2$, $B = 1.74$, $V_0 = 3$, $\mu = 0.1$, $\sigma = 0.8$, $T = 3.5$, $\tau = 4$, $\gamma_{_F} = 3$, and $u_F = -0.5$. In panel (b), $\sigma = 0.2$ for the dashed line and $\sigma = 0.8$ for the solid line.

resulting in multiple convexity regions in the utility function. However, note that the first step in solving the resulting models is to concavify the utility function, as discussed in the proof of Proposition 3. For each of these two utility functions, it is easy to check there is a parameter region of positive measure such that the concavified function is indistinguishable from that obtained by concavifying a continuous utility function with a single convexity region. If this is the case, the discontinuity or the multiple status thresholds are not going to affect our main results. Examining settings when this is not the case is beyond the scope of this paper.

Proof of Proposition IA-1. Because the steps of the proof are similar to those used in the proof of Proposition 1, we provide only brief elaborations throughout the proof below.

Parameters a and b of the concavifying line $a + b * W_{E\tau}$ and the tangency points $\underline{L}$ and $\overline{L}$ are computed from the system

$$\begin{aligned}
\frac{\underline{L}^{1-\gamma_E}}{1-\gamma_E} &= a + b\underline{L}, \\
\frac{\overline{L}^{1-\gamma_E}}{1-\gamma_E}\alpha + B &= a + b\overline{L}, \\
\underline{L}^{-\gamma_E} &= b, \\
\alpha\overline{L}^{-\gamma_E} &= b,
\end{aligned} \tag{IA-4}$$

solving which yields

$$\begin{aligned}
&a = \frac{B}{1-\alpha^{1/\gamma_E}}, \; b = \left(\frac{(\gamma_E - 1)B}{\gamma_E(\alpha^{1/\gamma_E} - 1)}\right)^{\gamma_E/(\gamma_E - 1)}, \\
&\underline{L} = \left(\frac{(\gamma_E - 1)B}{\gamma_E(\alpha^{1/\gamma_E} - 1)}\right)^{1/(1-\gamma_E)}, \; \overline{L} = \alpha^{1/\gamma_E}\left(\frac{(\gamma_E - 1)B}{\gamma_E(\alpha^{1/\gamma_E} - 1)}\right)^{1/(1-\gamma_E)}.
\end{aligned} \tag{IA-5}$$

The concavified utility function of the entrepreneur is

$$u_E(W_{E\tau}) = \begin{cases} \dfrac{(W_{E\tau})^{1-\gamma_E}}{1-\gamma_E} & W_{E\tau} < \underline{L}, \\ a + b * W_{E\tau} & \underline{L} \leq W_{E\tau} \leq \overline{L}, \\ \alpha \dfrac{(W_{E\tau})^{1-\gamma_E}}{1-\gamma_E} + B & W_{E\tau} > \overline{L}. \end{cases} \tag{IA-6}$$

Using the first-order condition

$$V_\tau^* = \begin{cases} (y\xi_\tau)^{-1/\gamma_E} & y\xi_\tau > b, \\ (y\xi_\tau/\alpha)^{-1/\gamma_E} & y\xi_\tau \leq b, \end{cases} \tag{IA-7}$$

in which y satisfies $E_T[\xi_\tau V_\tau^*] = \xi_T W_{ET}$, we compute the indirect utility function $v_E(W_{ET}) = E_T[u_e(V_\tau^*)]$:

$$\begin{aligned} v_E(W_{ET}) &= \frac{(y\xi_T)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E} K_{1T} N\left(\frac{-\ln b + \ln(y\xi_T) - K_4}{K_{3T}}\right) \\ &+ \frac{\alpha^{1/\gamma_E}(y\xi_T)^{(\gamma_E-1)/\gamma_E}}{1-\gamma_E} K_{1T} N\left(\frac{\ln b - \ln(y\xi_T) + K_4}{K_{3T}}\right) + B * N\left(\frac{\ln b - \ln(y\xi_T) + K_{2T}}{K_{3T}}\right), \end{aligned} \tag{IA-8}$$

where $y\xi_T$ is given by

$$W_{ET} = (y\xi_T)^{-1/\gamma_E} K_{1T} N\left(\frac{-\ln b + \ln(y\xi_T) - K_4}{K_{3T}}\right) + \alpha^{1/\gamma_E}(y\xi_T)^{-1/\gamma_E} K_{1T} N\left(\frac{\ln b - \ln(y\xi_T) + K_4}{K_{3T}}\right). \tag{IA-9}$$

Differentiating (IA-8) and (IA-9) and rearranging, we obtain, respectively,

$$\frac{dv_E}{d(y\xi_T)} = (y\xi_T)^{-1/\gamma_E} K_{1T} C(y\xi_T, 1-\gamma_E) - \frac{B * n\left((\ln b - \ln(y\xi_T) + K_{2T})/K_{3T}\right)}{y\xi_T K_{3T}}, \tag{IA-10}$$

$$\frac{d(y\xi_T)}{W_{ET}} = \left((y\xi_T)^{-1-1/\gamma_E} K_{1T} C(y\xi_T, 1)\right)^{-1}. \tag{IA-11}$$

where $C(\cdot,\cdot)$ is

$$C(g(x),\beta) = -\frac{N\left((-\ln b + \ln g(x) - K_4)/K_{3T}\right)}{\gamma_E} + \frac{n\left((-\ln b + \ln g(x) - K_4)/K_{3T}\right)}{\beta K_{3T}} - \alpha^{1/\gamma_E}\frac{N\left((\ln b - \ln g(x) + K_4)/K_{3T}\right)}{\gamma_E} - \alpha^{1/\gamma_E}\frac{n\left((\ln b - \ln g(x) + K_4)/K_{3T}\right)}{\beta K_{3T}}, \tag{IA-12}$$

. Multiplying (IA-10) and (IA-11) yields, after some simple algebra, the marginal indirect utility

$$\frac{dv_E}{dW_{ET}} = \frac{(y\xi_T)^{(\gamma_E-1)/\gamma_E} K_{1T}K_{3T}C(y\xi_T, 1-\gamma_E) - B * n\left((\ln b - \ln(y\xi_T) + K_{2T})/K_{3T}\right)}{(y\xi_T)^{-1/\gamma_E} K_{1T}K_{3T}C(y\xi_T, 1)}. \tag{IA-13}$$

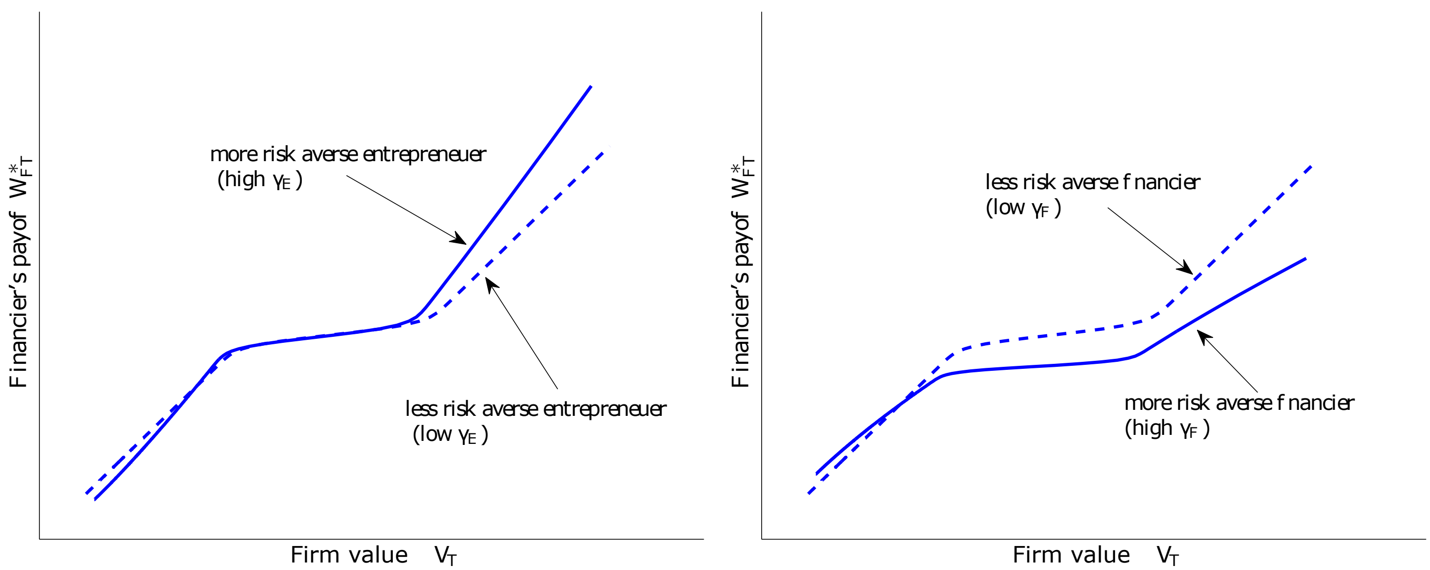
As is the case for the indirect utility function (A1), establishing analytically that $v_E(\cdot)$ given in (IA-13) is an increasing concave function does not appear possible. Therefore, we have verified numerically that this is the case for a large number of model calibrations.

The entrepreneur solves the optimization problem (A7) in which the indirect utility $v_E(\cdot)$ is now given by (IA-13). Modifying the solution of this problem presented in Proposition 1 appropriately so as to account for the different $v_E(\cdot)$ yields the optimal security presented in Proposition IA-1. *Q.E.D.*

IA3 Robustness to model parametrizations

To verify that a convertible-like shape of the optimal security is a general prediction of our model, and is not driven by a specific model parametrization, we have examined the shape of the optimal security under a large number of model parametrizations, and the optimal security has turned out to be similar to a convertible security in each case. In the interest of space, we only present some representative results of this analysis—see Figures IA-3 and IA-4 below.[3]

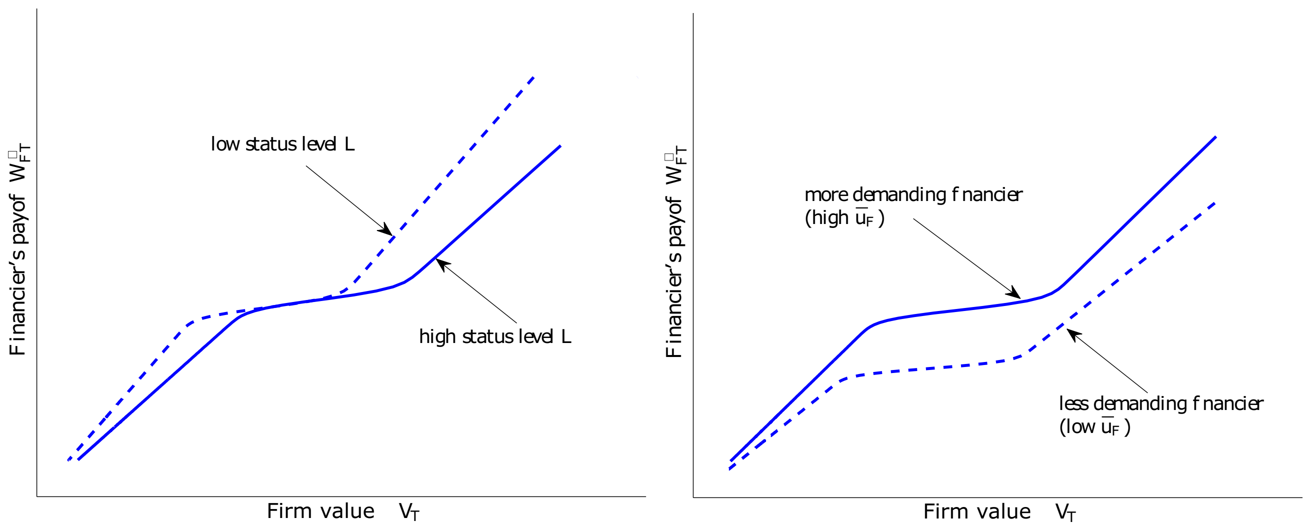
[3] In panel (a) of Figure IA-3, $\gamma_E = 3$ for the dashed line and $\gamma_E = 5$ for the solid line; in panel (b), $\gamma_F = 3$ for the dashed line and $\gamma_F = 5$ for the solid line. In panel (a) of Figure IA-4, $L = 2$ for the dashed line and $L = 2.5$ for the solid line; in panel (b), $u_F = -1$ for the dashed line and $u_F = -0.5$ for the solid line. The other parameter values are as in Figure 3.



(a) Varying entrepreneur's risk aversion

(b) Varying financier's risk aversion

Figure IA-3: Effect of Risk Aversion on Optimal Security. Panel (a) depicts the optimal security when the entrepreneur is relatively more risk averse (solid line) and relatively less risk averse (dashed line). Panel (b) depicts the optimal security when the financier is relatively more risk averse (solid line) and relatively less risk averse (dashed line).



(a) Effect of status level L (b) Effect of financier's reservation utility $\bar{u}_F$

Figure IA-4: Effect of Status Level and Reservation Utility on Optimal Security. Panel (a) depicts the optimal security when the status level (of wealth) is relatively high (solid line) and relatively low (dashed line). Panel (b) depicts the optimal security when the financier's reservation utility is relatively high (solid line) and relatively low (dashed line).

Additional References

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