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Online Appendix: Supply Chains and Antitrust Governance

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Appendix A: Proofs

Proof of Proposition 1. The proof consists of two parts. In the first part we show that, for each contract T, the manufacturers' strategies for choosing payment terms $\{\mathtt{v}_{-j}, \mathtt{v}_j\} = \{\mathtt{v}^C, \mathtt{v}^C\}$ is a Nash equilibrium (Claim EC.1). In the second part, we show that $\{\mathtt{v}^C, \mathtt{v}^C\}$ is a unique equilibrium (Claim EC.5).

Claim EC.1. *For all T, there exists no $\mathtt{v}' \neq \mathtt{v}^C$ such that $\Pi_j([\mathtt{v}_{-j}, \mathtt{v}_j] = [\mathtt{v}^C, \mathtt{v}']) > \Pi_j([\mathtt{v}_{-j}, \mathtt{v}_j] = [\mathtt{v}^C, \mathtt{v}^C])$; here $\mathtt{v}_j$ denotes manufacturer j's payment terms.*

Proof: We first establish the two retailers' equilibrium choices of payment terms when the manufacturers payment strategy is $\{\mathtt{v}^C, \mathtt{v}'\}$. For each contract T, we show that the retailers' equilibrium choices are $\{\mathtt{v}_{-i}, \mathtt{v}_i\} = \{\mathtt{v}^C, \mathtt{v}^C\}$, where $\mathtt{v}_i$ denotes retailer i's choice of payment terms. This implies that $\hat{q}_{1j} + \hat{q}_{2j} = 0$ and so $\Pi_j([\mathtt{v}^C, \mathtt{v}']) \ngtr \Pi_j[\mathtt{v}^C, \mathtt{v}^C]$. Here we work with the extended contract structure $\overline{T}(w, f, \alpha, \xi) = f + wq + \alpha R(q) - \xi/2q^2$, where $R(q)$ denotes the revenue generated by the purchased quantity q. Note that $\overline{T}$ encompasses the wholesale price (WP), revenue-sharing (RS), quantity discount (QD), and wholesale price plus fixed fee (WPFF) contract structures: WP $= \overline{T}(w, 0, 0, 0)$, RS $= \overline{T}(w, 0, \alpha, 0)$, QD $= \overline{T}(w, 0, 0, \xi)$, and WPFF $= \overline{T}(w, f, 0, 0)$. Note also that, similar to Cachon and Kök (2010), we need to only include the quadratic part of the QD contract; the reason is that, if the retailer orders $\hat{q} > (w - c)/\xi$, then the manufacturer can increase w so that $\hat{q} = (w - c)/\xi$ – that is, increase its profit by raising w without affecting the retailer's decision. The rest of this proof proceeds in three steps. First we show that, given $\overline{T}$, the payment strategy $\{\mathtt{v}_{-i}, \mathtt{v}_i\} = \{\mathtt{v}', \mathtt{v}'\}$ is not an equilibrium outcome for the retailers because retailer i can profitably deviate from $\mathtt{v}'$ to $\mathtt{v}^C$; formally, $\pi_i([\mathtt{v}_{-i}, \mathtt{v}_i] = [\mathtt{v}', \mathtt{v}']) < \pi_i([\mathtt{v}_{-i}, \mathtt{v}_i] = [\mathtt{v}', \mathtt{v}^C])$ (Claim EC.2). Next, we show that $\{\mathtt{v}_{-i}, \mathtt{v}_i\} = \{\mathtt{v}^C, \mathtt{v}^C\}$ is an equilibrium outcome because retailer i would earn less profit by deviating from $\mathtt{v}^C$ to $\mathtt{v}'$: $\pi_i([\mathtt{v}_{-i}, \mathtt{v}_i] = [\mathtt{v}^C, \mathtt{v}']) < \pi_i([\mathtt{v}_{-i}, \mathtt{v}_i] = [\mathtt{v}^C, \mathtt{v}^C])$; this is Claim EC.3. The combination of Claim EC.2 and Claim EC.3 establishes that, for each retailer, $\mathtt{v}^C$ is an equilibrium choice under $\overline{T}$. Finally, we prove the same result for the minimum order quantity (MOQ) contract structure (Claim EC.4).

Claim EC.2. *Under $\overline{T}$, we have $\pi_i([\mathtt{v}_{-i}, \mathtt{v}_i] = [\mathtt{v}', \mathtt{v}']) < \pi_i([\mathtt{v}_{-i}, \mathtt{v}_i] = [\mathtt{v}', \mathtt{v}^C])$.*

Proof: We first characterize the optimal purchase order quantities $\hat{q}_j$and $\hat{q}_{\text{-}j}$ under $\overline{T}$, after which we complete the proof by showing that $G([\mathtt{v}^C, \mathtt{v}']) = \Pi_j[\mathtt{v}', \mathtt{v}'] + \pi_{i=j}([\mathtt{v}', \mathtt{v}']) - \pi_{i=j}([\mathtt{v}', \mathtt{v}^C]) < 0$. Note that the inequality $G([\mathtt{v}^C, \mathtt{v}']) < 0$ implies $\pi_i([\mathtt{v}', \mathtt{v}']) < \pi_i([\mathtt{v}', \mathtt{v}^C])$ because, by definition, $\Pi_j \geq 0$. Here on wards, we use $\Psi(q_i, q_{\text{-}i}, \mathtt{v})$, to denote the retailers' profit as a function of quantities q and contract payment terms $\mathtt{v}$. Under $\{\mathtt{v}_{-i}, \mathtt{v}_i\} = \{\mathtt{v}', \mathtt{v}'\}$, the profit functions and $G([\mathtt{v}^C, \mathtt{v}'])$ can be written as follows:

$$\Psi_{-i} = -f + (1 - \alpha)(a - b(q_{\text{-}i} + q_i))q_{\text{-}i} - wq_{-i} + \frac{\xi}{2}q_{-i}^2, \tag{EC.1}$$

$$\Psi_i = -f + (1 - \alpha)(a - b(q_{\text{-}i} + q_i))q_i - wq_i + \frac{\xi}{2}q_i^2, \tag{EC.2}$$

$$\Pi_j = f + \alpha(a - b(q_{\text{-}i} + q_i))q_{i=j} + wq_{i=j} - \frac{\xi}{2}q_{i=j}^2 - cq_{i=j}, \tag{EC.3}$$

$$G = (a - c - b(\hat{q}_{\text{-}i} + \hat{q}_i))\hat{q}_i - \pi_{i=j}([\mathtt{v}', \mathtt{v}^C]). \tag{EC.4}$$

It follows from equations (EC.1) and (EC.2) that $\partial^2_{q_{-i}}\Psi_{-i} = \partial^2_{q_i}\Psi_i = 2b(-1+\alpha+\xi/2b) < 0$ for each of the four focal contracts: WP ($\alpha=0$, $\xi=0$), RS ($\alpha\in[0,1)$, $\xi=0$), QD ($\alpha=0$, $\xi\in[0,2b)$), and WPFF ($\alpha=0$, $\xi=0$). Solving the retailer's first-order conditions for a symmetric solution yields

$$\hat{q}_i = \frac{a-w'}{b(3-\xi')}, \tag{EC.5}$$

where $w' = w/(1-\alpha)$ and $\xi' = \xi/(b-b\alpha)$. Since the retailer's selection of payment terms is endogenous, it follows that a necessary condition for $\{\mathtt{v}_{-i},\mathtt{v}_i\} = \{\mathtt{v}',\mathtt{v}'\}$ to be an equilibrium switch from the status quo $\{\mathtt{v}_{-i},\mathtt{v}_i\} = \{\mathtt{v}^C,\mathtt{v}^C\}$ is that $\pi_i([\mathtt{v}',\mathtt{v}']) > \pi([\mathtt{v}^C,\mathtt{v}^C])$. Satisfying that condition requires the manufacturer j to set $\mathtt{v}'$ such that $\Omega[\mathtt{v}',\mathtt{v}'] > \Omega[\mathtt{v}^C,\mathtt{v}^C]$, where Ω denotes the combined manufacturers and retailers profit (equivalently, the supply-chain profit) under the given payment terms. Note that if $\Omega[\mathtt{v}',\mathtt{v}'] \not> \Omega[\mathtt{v}^C,\mathtt{v}^C]$ then both $\pi_i([\mathtt{v}',\mathtt{v}']) > \pi_i([\mathtt{v}^C,\mathtt{v}^C])$ and $\Pi_j[\mathtt{v}',\mathtt{v}'] \geq \Pi_j[\mathtt{v}^C,\mathtt{v}^C] = 0$ cannot jointly hold together. For the payment terms $\{\mathtt{v}_{-j},\mathtt{v}_j\} = \{\mathtt{v}^C,\mathtt{v}^C\}$, we obtain $\hat{q}_i = (a-c)/3b$ (using EC.5) and the total supply-chain quantity as $Q^C = Q([\mathtt{v}^C,\mathtt{v}^C]) = 2\hat{q}_i = 2(a-c)/3b$. Observe that $Q^C > Q^* = (a-c)/2b$, where Q^* is the supply chain's profit-maximizing quantity (i.e., $Q^* = \arg\max(a-bQ)Q - cQ$). This implies a necessary condition for $\Omega([\mathtt{v}',\mathtt{v}']) > \Omega([\mathtt{v}^C,\mathtt{v}^C])$ is $Q([\mathtt{v}',\mathtt{v}']) < Q([\mathtt{v}^C,\mathtt{v}^C])$ because $\Omega(Q)$ is concave in Q and $Q([\mathtt{v}^C,\mathtt{v}^C]) > Q^*$. Therefore,

$$\frac{a-w'}{b(3-\xi')} \leq \frac{a-c}{3b},$$
$$3a-3w' \leq 3a-3c-\xi'(a-c),$$
$$3(w'-c)-\xi'(a-c) \geq 0. \tag{EC.6}$$

Next we solve for retailer i's profit by deviating from $\mathtt{v}'$ to $\mathtt{v}^C$. The retailer's profit functions and corresponding first-order conditions are then

$$\Psi_{-i} = -f + (1-\alpha)(a-b(q_{-i}+q_i))q_{-i} - w'(1-\alpha)q_{-i} + \frac{\xi'(b-b\alpha)}{2}q_{-i}^2, \tag{EC.7}$$

$$\partial_{q_{-i}}\Psi_{-i} = (1-\alpha)(a-w'-b(2-\xi')q_{-i}-bq_i), \tag{EC.8}$$

$$\Psi_i = (a-b(q_{-i}+q_i))q_i - cq_i, \tag{EC.9}$$

$$\partial_{q_i}\Psi_i = a-c-2bq_i-bq_{-i}. \tag{EC.10}$$

Solving $\partial_{q_i}\Psi_i = 0$ gives $\hat{q}_i = (a-c-b\hat{q}_{-i})/2b$. Substituting $\hat{q}_i$ into $\partial_{q_{-i}}\Psi_{-i} = 0$, we have

$$\hat{q}_{-i} = \frac{a-2w'+c}{b(3-2\xi')}. \tag{EC.11}$$

Now substituting $\hat{q}_{-i}$ back into $\partial_{q_i}\Psi_i = 0$, we derive q_i and $Q([\mathtt{v}^C,\mathtt{v}'])$ as follows:

$$q_i = \frac{a-2c+w'-(a-c)\xi'}{b(3-2\xi')}, \tag{EC.12}$$

$$Q(\overline{T}([\mathtt{v}^C,\mathtt{v}'])) = \frac{a-2c+w'-(a-c)\xi'}{3(b-2\xi')} + \frac{a-2w'+c}{b(3-2\xi')}$$
$$= \frac{2a-c-w'-(a-c)\xi'}{b(3-2\xi')}. \tag{EC.13}$$

Using equations (EC.4), (EC.5), (EC.12), and (EC.13), we can write $G(\mathtt{v}^C,\mathtt{v}')$ as

$$G = \frac{(a-3c+2w'-(a-c)\xi')\,(a-w')(3-2\xi')^2 - (a-2c+w'-(a-c)\xi')^2\,(3-\xi')^2}{b(3-\xi')^2(3-2\xi')^2}. \tag{EC.14}$$

Next we show that $G<0$ for all $\xi' \in [0, 3(w'-c)/(a-c)]$ (cf. equation (EC.6)). We start by putting $G(\xi') = D(\xi')N(\xi')$, where $D(\xi')$ is equal to $1/b(3-\xi')^2(3-2\xi')^2$ and $N(\xi')$ is the numerator in equation (EC.14). Since $D(\xi')>0$ for all ξ', it is sufficient to show $N(\xi')<0$. We establish this inequality by demonstrating that N is increasing in ξ' (which follows from $\partial^2_{\xi'}N<0$, $\partial_{\xi'}N|_{\xi'=0}>0$, and $\partial_{\xi'}N|_{\xi'=3(w'-c)/(a-c)}>0$) and that $N(\xi'=3(w'-c)/(a-c))=0$. We now solve for $\partial_{\xi'}N(\xi')$:

$$\begin{aligned}
\frac{\partial N}{\partial \xi'} &= 3(a-c)^2+30(w'-c)^2+27(w'-c)(a-c)\\
&\quad -\xi'^3 4(a-c)^2+\xi'^2\left(12(a-c)^2+18(a-c)(w'-c)\right)\\
&\quad -\xi'\left(12(a-c)^2+18(w'-c)^2+44(a-c)(w'-c)\right), \qquad \text{(EC.15)}\\
\frac{\partial^2 N}{\partial \xi'^2} &= -12\xi'^2(a-c)^2+2\xi'\left(12(a-c)^2+18(a-c)(w'-c)\right)\\
&\quad -12(a-c)^2-18(w'-c)^2-44(a-c)(w'-c)\\
&= -12(a-2c+w'-\xi'(a-c))^2-10(2-\xi')(a-c)(w'-c)\\
&\quad -2(w'-c)\left(3(w'-c)-\xi'(a-c)\right)\\
&< 0
\end{aligned}$$

by equation (EC.6) and $\xi'<3/2$ (by equation EC.12) . Equation (EC.15) now yields $\partial_{\xi'}N(\xi'=0)>0$ and

$$\begin{aligned}
\left.\frac{\partial N}{\partial \xi'}\right|_{\xi'=3(w'-c)/(a-c)} &= 3(a-c)^2+30(w'-c)^2+27(w'-c)(a-c)\\
&\quad -\xi'^3 4(a-c)^2+\xi'^2\left(12(a-c)^2+18(a-c)(w'-c)\right)\\
&\quad -\xi'\left(12(a-c)^2+18(w'-c)^2+44(a-c)(w'-c)\right)\\
&= 4\xi'^2(a-c)(3(w'-c)-\xi'(a-c))\\
&\quad +10(w'-c)(3(w'-c)-\xi'(a-c))\\
&\quad +\xi'^2(12(a-c)^2+6(a-c)(w'-c))+3(a-c)^2\\
&\quad -\xi'(3(a-c)^2+18(w'-c)^2+34(a-c)(w'-c))\\
&= 3(a-w')(a-2w'+c)\\
&> 0,
\end{aligned}$$

since $a-2w'+c>0$ by equation (EC.12). Finally,

$$\begin{aligned}
N\left(\xi'=\frac{3(w'-c)}{a-c}\right) &= (a-3c+2w'-(a-c)\xi')(a-w')(3-2\xi')^2-(a-2c+w'-(a-c)\xi')^2(3-\xi')^2\\
&= 9(a-w')^2(a-2w'+c)^2-9(a-2w'+c)^2(a-w')^2\\
&= 0.
\end{aligned}$$

Claim EC.3. *Under* $\overline{T}$, *we have* $\pi_i([\mathtt{v}_{-i},\mathtt{v}_i]=[\mathtt{v}^C,\mathtt{v}'])<\pi_i([\mathtt{v}_{-i},\mathtt{v}_i]=[\mathtt{v}^C,\mathtt{v}^C])$.

Proof: Following the steps used to prove Claim EC.2, we can show that $H([\mathtt{v}^C,\mathtt{v}'])=\Pi_j([\mathtt{v}',\mathtt{v}^C])+\pi_{i=j}([\mathtt{v}',\mathtt{v}^C])-\pi_{i=j}([\mathtt{v}^C,\mathtt{v}^C])<0$. From equations (EC.11), (EC.13), and (EC.5) for $\{\mathtt{v}_{-j},\mathtt{v}_j\}=\{\mathtt{v}^C,\mathtt{v}^C\}$, we obtain

$$H([\mathtt{v}^C,\mathtt{v}'])=\left(\frac{9\,(a-2c+w'-(a-c)\xi')\,(a-2w'+c)-(a-c)^2(3-2\xi')^2}{9b^2(3-2\xi')^2}\right). \qquad \text{(EC.16)}$$

Because the denominator of H is greater than zero, it is sufficient to show that the numerator of H ($\equiv \overline{N}$) is less than zero for all $\xi' \in [0, 3(w'-c)/(a-c)]$. For this purpose, we establish that $\partial_{\xi'}\overline{N} > 0$ and $\overline{N}(\xi' = 3(w-c)/(a-c)) = 0$. From equation (EC.16) it follows that

$$\begin{aligned}\frac{\partial \overline{N}}{\partial \xi'} &= -9(a-c)(a-2w'+c) + 4(a-c)^2(3-2\xi') \\ &= (a-c)\left(3\left(a-2w'+c\right) + 8\left(3(w'-c) - \xi'(a-c)\right)\right) \\ &> 0,\end{aligned}$$

by equation (EC.6) and equation (EC.12). Finally, it is easy to verify that $\overline{N}(\xi' = 3(w'-c)/(a-c)) = 9\left(a-2w'+c\right)^2 - 9(a-2w'-c)^2 = 0$.

Claim EC.4. *For the MOQ contract structure, $\{\mathtt{v}^C, \mathtt{v}^C\}$ is an equilibrium payment strategy.*

Proof: Under any deviation, retailer i either chooses the optimal purchase quantity based on the offered w or is constrained to select a higher quantity $q_{\min}$ that, in turn, results in higher Q. In the former case, Claim EC.2 results apply as it is. In the latter case, a higher Q results in a lower supply chain profit (see the discussion in the proof of Claim EC.2); hence either the manufacturer or the retailers will incur a loss, thus making deviation unprofitable. As this result is true for all values of $q_{\min}$, we set $q_{\min} = q^*$ similar to channel coordinating MOQ contract terms (Tuncel et al. 2019).

Claim EC.5. *For each contract T, $\{\mathtt{v}^C, \mathtt{v}^C\}$ is an unique equilibrium.*

Proof: Because this setting exhibits symmetry, no asymmetric equilibrium will exist for the manufacturers' decisions about payment terms. Now suppose there exists a payment term vector $\{\mathtt{v}', \mathtt{v}'\} \neq \{\mathtt{v}^C, \mathtt{v}^C\}$ that constitutes an equilibrium strategy for the manufacturers. For contracts WP, RS, and MOQ, the existence of that vector implies that $\Pi_{j=\{1,2\}}([\mathtt{v}', \mathtt{v}']) > 0$ because $|\{\mathtt{v} \mid \Pi([\mathtt{v}, \mathtt{v}]) = 0\}| = 1$, where $|\mathbb{S}|$ denotes the cardinality of set $\mathbb{S}$. The implication is that manufacturer j can profitably deviate from $\mathtt{v}'$ to $\mathtt{v}'^-$, a strategy that passes a share ϵ ($0 < \epsilon \ll \pi_i([\mathtt{v}', \mathtt{v}'])$) of $\Pi([\mathtt{v}', \mathtt{v}'])$ to retailer i and, in turn, secures orders from both the retailers. The combined orders from the two retailers compensate for the loss of ϵ and yield the inequality $\Pi_j([\mathtt{v}', \mathtt{v}'^-]) > \Pi_j([\mathtt{v}', \mathtt{v}'])$. Under QD and WPFF, if the manufacturers choose $\mathtt{v}'$ such that $\Pi_{j=\{1,2\}}([\mathtt{v}', \mathtt{v}']) > 0$ then − by the foregoing arguments − it is profitable for manufacturer j to deviate. Finally, we note that $|\{\mathtt{v} \mid \Pi([\mathtt{v}, \mathtt{v}]) = 0\}| > 1$ for the QD and WPFF contract structures; hence $\{\mathtt{v}', \mathtt{v}'\}$ under these contracts may represent an equilibrium strategy with zero-profit outcome but with $w' > c$. By Claim EC.2 we have $\pi_i([\mathtt{v}', \mathtt{v}']) < \pi_i([\mathtt{v}', \mathtt{v}^C])$, where $\mathtt{v}^C = \{c, 0\}$. Since retailer i's individual profit function π_i is continuous in w, the preceding inequality implies the existence of a $c < w'' < w'$ such that $\pi_i([\mathtt{v}', \mathtt{v}']) < \pi_i([\mathtt{v}', \mathtt{v}'']) < \pi_i([\mathtt{v}', \mathtt{v}^C])$. Hence the payment vector $\mathtt{v}'' = \{w'', 0\}$ amounts to a profitable deviation for manufacturer j from $\{\mathtt{v}', \mathtt{v}'\}$; that is, $\Pi_j([\mathtt{v}', \mathtt{v}'']) > \Pi_j([\mathtt{v}', \mathtt{v}'])$. ■

Proof of Proposition 2. This proof has two parts. First we show that collusion can occur only if the retailers' combined order quantity under collusion Q^A is lower than that in the competitive scenario Q^C (Claim EC.6). We shall refer to this requirement as condition C4. Second, we analyze the joint maximization problem under collusion, along with the necessary conditions C1, C2, and C4, to establish the result for each contract structure separately.

Claim EC.6. *Collusion can occur only if $Q^A < Q^C$.*

Proof: Note that conditions C1 and C2 (see Section 3.2) imply that a collusion can occur *only if* the total supply chain profit under collusion is higher than that under competition. Optimizing the supply chain's profit ($\Omega(Q) = (a - bQ)Q - cQ$), we derive the optimal quantity Q^* as $(a-c)/2b$. Note that $Q^* < Q^C = 2(a-c)/3b$, which implies that $\Omega(Q) > \Omega(Q^C)$ only if $Q \in [Q^*, Q^C)$ – that is, because Ω is concave in Q (since $\partial^2_Q \Omega = -2b$)

Next, in Claim EC.7 we give results for WP, RS, and QD contracts using the extended contract structure $\overline{T}(w,0,\alpha,\xi)$ (see proof of Proposition 1 for the definition of $\overline{T}$). We conclude the proof by establishing results separately for the WPFF (Claim EC.8) and MOQ (Claim EC.9) contract structures.

Claim EC.7. *For $\overline{T}(w,0,\alpha,\xi)$, there exist no ξ' that satisfy conditions C1, C2, and C4.*

Proof: Suppose there is a $\{w,\alpha,\xi\} \neq \{c,0,0\}$ such that C4 and C2 both hold. We can then use equation (EC.5) to evaluate C2 as follows:

$$\begin{aligned}
(1-\alpha)(a-2b\hat{q})\hat{q} - w\hat{q} + \frac{\xi}{2}\hat{q}^2 &\geq \frac{(a-c)^2}{9b},\\
(1-\alpha)\left(1-\frac{\xi'}{2}\right)\frac{(a-w')^2}{b(3-\xi')^2} &\geq \frac{(a-c)^2}{9b},\\
(1-\alpha)\left(1-\frac{\xi'}{2}\right)\left(\frac{a-w}{b(3-\xi')}\right)^2 &\geq \left(\frac{a-c}{3b}\right)^2,\\
\Longrightarrow \left(\frac{a-w'}{b(3-\xi')}\right) &\geq \left(\frac{a-c}{3b}\right)\Big/\sqrt{(1-\alpha)\left(1-\frac{\xi'}{2}\right)},\\
\frac{a-w'}{b(3-\xi')}(\equiv \hat{q}(\mathtt{v}')) &\geq \frac{a-c}{3b}(\equiv \hat{q}(\mathtt{v}^C)) \quad \text{as } \xi\in[0,2b) \text{ and } \alpha\in[0,1);
\end{aligned}$$

thus, by C4 we have a contradiction.

Claim EC.8. *The manufacturers can collude using the WPFF contract structure.*

Proof: We rewrite the manufacturers collusion problem using the Lagrange multiplier for C2 and expression for $\hat{q}$ under WPFF as

$$\begin{aligned}
\max_{w,f,\lambda} 2F + \frac{2(w-c)(a-w)}{3b} + \lambda_1\left(\frac{(a-w)^2}{9b} - F - \frac{(a-c)^2}{9b}\right), \qquad \text{(EC.17)}\\
\text{s.t. } \Pi^A > 0,
\end{aligned}$$

since $\Pi^C_i = 0$ for $i = 1,2$. We denote the modified objective function by $G(\mathtt{v},\lambda)$. Solving for the first-order conditions yields

$$\frac{\partial G}{\partial w} = \frac{(6-2\lambda_1)a - w(12-2\lambda) + 6c}{9b}, \qquad \text{(EC.18)}$$

$$\frac{\partial G}{\partial F} = 2-\lambda. \qquad \text{(EC.19)}$$

We can now use equations (EC.18) and (EC.19) to obtain $\lambda = 2$ and $w^* = \frac{a+3c}{4}$. Given the wholesale price w^*, each retailer would order $q^A = \frac{a-(a+3c)/4}{3b} = \frac{a-c}{4b}$. Furthermore, the profit of each manufacturer is $\Pi = f + \frac{(a-c)^2}{16b}$ and that of each retailer is $\pi = \frac{(a-c)^2}{16b} - f$. Finally, the conditions C1 ($\Pi^A > \Pi^C$, where $\Pi^C = 0$) and C2 ($\pi^A \geq \pi^C$, where $\pi^C = (a-c)^2/9b$) imply that $f \in \left(-\frac{(a-c)^2}{16b}, -\frac{7(a-c)^2}{144b}\right]$.

Claim EC.9. *For the MOQ contract, there exist no* $\mathtt{v}'$ *that satisfy conditions C1, C2, and C4.*

Proof: Under collusion, (a) the manufacturers jointly decide on the values of w and $q_{\min}$ and (b) the retailers' $\hat{q}$ is equal to $\max\{q_{\min}, (a-w)/3b\}$. As a consequence, if w is set such that $q_{\min} < (a-w)/3b$, then the retailers' quantity equals the quantity under a WP contract and the manufacturers' profit maximization problem therefore reduces to the problem under a WP contract. For such w, collusion cannot occur (by Claim EC.7). So for collusion to occur, there must exist a $w < a$ and a $q_{\min} \in [0, a/2b]$ such that: (i) $q_{\min} > (a-w)/3b$; (ii) $\Psi(q_{\min}, q_{\min}, w) \geq \pi^C$ (C2); and (iii) $q_{\min} < \hat{q}^C$ (C4). Suppose there does exist such a pair $\{w, q_{\min}\}$; then solving for $\Psi(q_{\min}, q_{\min}, w) - \pi^C$ gives

$$
\begin{aligned}
\Psi(q_{\min}, q_{\min}, w) - \pi^C &= (a - 2bq_{\min})q_{\min} - wq_{\min} - (a-c)^2/9b, \\
&= (a - w - 2bq_{\min})q_{\min} - b\hat{q}(\mathtt{v}^C)^2 \quad \text{since } \hat{q}(\mathtt{v}^C) = (a-c)/3b, \\
&< (3bq_{\min} - 2bq_{\min})q_{\min} - b\hat{q}(\mathtt{v}^C)^2 \quad \text{since } a - w < 3bq_{\min} \text{ by (i)}, \\
&< b(q_{\min}^2 - \hat{q}(\mathtt{v}^C)^2), \\
&< 0 \quad \text{by C4},
\end{aligned}
$$

Thus C2 fails to hold – a contradiction. ■

Proof of Proposition 3. That the WPFF contract structure facilitates collusion through slotting fees ($f < 0$), follows directly from Claim EC.8. Further, we can use $\hat{q}^C$ (derived from equation (EC.5) with $w = c$, $\xi = 0$, and $\alpha = 0$) and $\hat{q}^A$ (from Claim EC.8) to obtain the consumer surplus CS and the total surplus TS under competition and collusion. Thus, we have $\mathrm{CS}^C = 2(a-c)^2/9b$, $\mathrm{CS}^A = (a-c)^2/8b$, $\mathrm{TS}^C = 4(a-c)^2/9b$, and $\mathrm{TS}^A = 3(a-c)^2/8b$. ■

Proof of Proposition 4. In the absence of the IB ruling scenario, the manufacturers have to satisfy conditions C1, C2, and C3 (see Section 3.2). In addition, for collusion to be beneficial, the manufacturers have to also satisfy condition C4 (Claim EC.6). Following claims 7 and 9, we know that contracts WP, RS, QD, and MOQ cannot satisfy the conditions C1, C2 and C4 simultaneously. Though the WPFF contract satisfy these conditions, from Proposition 3 we know it fail to satisfy the condition C3 as consumer surplus under collusive decision making is lower compared to that under the competitive scenario. ■

Proof of Proposition 5. Following the same steps as in the proof of Claim EC.8, we can show that the two manufacturers will set a w^* such that the combined retailers' quantity is equal to the expected profit-maximizing quantity of the centralized supply chain. The manufacturers will compensate for any loss in expected profits via the slotting fees mechanism. Using equations (EC.25) and (EC.29) from the proof of Proposition 6 to follow, we have $w^* = \int_{\frac{3}{2}bQ^A}^{\infty} \left\{a - \frac{3}{2}Q^A b\right\} f(a)\, da$; here Q^A is defined by the implicit function (EC.29). ■

Proof of Proposition 6. (a) Optimal Order Quantity under Competition. Solving for $\tilde{q}$ using the Stage 3 (see Figure 1) order quantity–constrained revenue maximization problem, we have $\tilde{q}(q) =$

$\min(a/3b, q)$. Substituting this equality into the retailer's Stage 2 expected profit maximization problem now yields

$$\hat{q}_i = \arg\max_q E_a\left[\{a - b\left(\min(a/3b, q) + \min(a/3b, \hat{q}_{-i})\right)\}\min(a/3b, q)\right] - cq, \tag{EC.20}$$

$$= \arg\max_q \frac{1}{9}\int_{-\infty}^{3bq} \frac{a^2}{b} f(a)\, da + \int_{3bq}^{\infty} \{a - b\left(q + \hat{q}_{-i}\right)\} q f(a)\, da - cq, \tag{EC.21}$$

Applying the Leibniz integral rule (Flanders 1973) to the expanded expected profit function, equation (EC.21), we obtain the first- and second-order derivatives as follows:

$$\frac{\partial \Psi^C}{\partial q} = \frac{1}{9}\frac{9b^2q^2}{b} f(3bq)3b + \int_{3bq}^{\infty} \{a - b\left(2q + \hat{q}_{-i}\right)\} f(a)\, da - \{3bq - b\left(q + \hat{q}_{-i}\right)\} q f(3bq)3b - c$$

$$= 3b^2 q\left(-q + \hat{q}_{-i}\right) f(3bq) + \int_{3bq}^{\infty} \{a - b\left(2q + \hat{q}_{-i}\right)\} f(a)\, da - c, \tag{EC.22}$$

$$\frac{\partial^2 \Psi^C}{\partial q^2} = 3b^2\left(-q + \hat{q}_{-i}\right) f(3bq) - 3b^2 q f(3bq) + 3b^2 q\left(-q + \hat{q}_{-i}\right) f'(3bq)3b$$

$$- \int_{3bq}^{\infty} 2b f(a)\, da - \{3bq - b\left(2q + \hat{q}_{-i}\right)\} f(3bq)3b \tag{EC.23}$$

$$= -9b^2 q f(3bq) + 6b^2 \hat{q}_{-i} f(3bq) + 3b^2 q\left(-q + \hat{q}_{-i}\right) f'(3bq)3b - 2b\left[1 - F(3bq)\right] \tag{EC.24}$$

$$< 0;$$

under the symmetric outcome of the retailers' purchase quantity decisions, $\hat{q}_i = \hat{q}_{-i}$. Thus, the optimal order quantity under competition, $\hat{q}^C = \hat{q}_i^C = \hat{q}_{-i}^C$, is characterized by the first order condition $\frac{\partial \Psi^C}{\partial q} = 0$:

$$\int_{3bq^C}^{\infty} \left\{a - 3q^C b\right\} f(a)\, da - c = 0,$$

$$\int_{\frac{3}{2}bQ^C}^{\infty} \left\{a - \frac{3}{2}Q^C b\right\} f(a)\, da - c = 0, \tag{EC.25}$$

where $Q^C = 2\hat{q}^C$ is the combined order quantity placed by the two retailers under the competitive scenario.

Optimal Order Quantity under Collusion. Proposition 2 implies that, to maximize their collusion benefit, the manufacturers set payment terms such that the retailers' combined order quantity Q^A equals that of a centralized firm. By following the preceding proof steps for the competition scenario, we obtain the centralized firm's Stage 3 order quantity as $\tilde{q}(q) = \min(a/2b, Q)$. Substituting this inequality into the retailers' Stage 2 problem, equation (6), gives

$$\hat{Q} = \arg\max_Q E_a\left[\left(a - b\min(a/2b, Q)\right)\min(a/2b, Q)\right] - cQ$$

$$= \arg\max_Q \frac{1}{4}\int_{-\infty}^{2bQ} \frac{a^2}{b} f(a)\, da + \int_{2bQ}^{\infty} \{a - bQ\} Q f(a)\, da - cQ. \tag{EC.26}$$

Applying the Leibniz integral rule (Flanders 1973) to the expanded expected profit function Ψ^A, equation (EC.26), we obtain the first- and second-order derivatives as

$$\frac{\partial \Psi^A}{\partial Q} = \frac{1}{4} 4b^2 \frac{Q^2}{b} f(2bQ)2b + \int_{2bQ}^{\infty} \{a - 2bQ\} f(a)\, da - \{2bQ - bQ\} Q f(2bQ)2b - c$$

$$= \int_{2bQ}^{\infty} \{a - 2bQ\} f(a)\, da - c, \tag{EC.27}$$

$$\frac{\partial^2 \Psi^A}{\partial Q^2} = \int_{2bQ}^{\infty} -2b f(a)\, da = -2b\left[1 - F(2bQ)\right] < 0. \tag{EC.28}$$

Solving $\frac{\partial \Psi^A}{\partial Q} = 0$ to characterize Q^A, we have

$$\int_{2bQ^A}^{\infty} \left\{a - 2bQ^A\right\} f(a)\, da - c = 0. \tag{EC.29}$$

(b) Comparing equations (EC.25) and (EC.29), we can see that using $Q^A = 3/4Q^C$ in the latter reduces it to the former.

(c) We use our results from part (a) to solve for the retailers' expected price $p = E_a\left[\{a - b\left(\tilde{q}_i(a) + \tilde{q}_{-i}(a)\right)\}\right]$ under the competition and collusion scenarios. Using the Stage 3 revenue maximization problem, we compute the difference in expected price under the collusion and competition scenario as

$$\begin{aligned}
p^A - p^C &= E_a\left[\left\{a - b\min(a/2b, Q^A)\right\}\right] - E_a\left[\left\{a - 2b\left(\min(a/3b, q^C)\right)\right\}\right], \\
&= E_a\left[2b\left(\min(a/3b, q^C)\right) - b\min(a/2b, Q^A)\right], \\
&= E_a\left[b\min(2a/3b, Q^C) - b\min(a/2b, 3/4Q^C)\right] \quad \text{since } Q^c = 2q^C \text{ and } Q^A = 3/4Q^C, \\
&= E_a\left[1/4b\min(2a/3b, Q^C)\right].
\end{aligned} \tag{EC.30}$$

■

Proof of Proposition 7. Following our steps in the proof of Proposition 2, we write the two manufacturers' joint objective in a market with N retailers as

$$G(\mathtt{v}, \lambda) = NF + \frac{N(w-c)(a-c)}{(N+1)b} + \lambda\left(\frac{(a-w)^2}{b(N+1)^2} - F - \pi^{C(N)}\right), \tag{EC.31}$$

where $\pi^{C(N)}$ denotes each retailer's profit in this market under the competitive scenario. Irrespective of the number of retailers, in the competitive case the manufacturers will set the WPFF contract's payment term vector as $\mathtt{v}^C = \{w = c,\ f = 0\}$. Solving the retailer's quantity maximization problem for $\mathtt{v}^C$ gives $q^{C(N)} = \frac{a-c}{(N+1)b}$, from which it follows that the retailer earns the profit $\pi^{C(N)} = \frac{(a-c)^2}{b(N+1)^2}$. Substituting $\pi^{C(N)}$ into equation (EC.31) and then solving for the first-order conditions, we obtain

$$\frac{\partial G}{\partial w} = \frac{N(N+1)(a-2w+c) - 2\lambda(a-w)}{(N+1)b}, \tag{EC.32}$$

$$\frac{\partial G}{\partial F} = N - \lambda. \tag{EC.33}$$

Together, equations (EC.33) and (EC.32) yield $\lambda = N$ and $w^* = \frac{(N-1)a+(N+1)c}{2N}$. Given the wholesale price w^*, each retailer orders the quantity $q^{A(N)} = \frac{a-((N-1)a+(N+1)c)/2N}{(N+1)b} = \frac{a-c}{2Nb}$. Furthermore, the profit of each manufacturer is $\Pi = \frac{(N-1)(a-c)^2}{8Nb} + \frac{N}{2}f$ while each retailer's profit is $\pi = \frac{(a-c)^2}{4N^2b} - f$. The conditions C1 and C2 together determine the range of $f \in \left(-\frac{(N-1)(a-c)^2}{4N^2b}, -\frac{(a-c)^2(N-1)(3N+1)}{4N^2(N+1)^2b}\right]$. ■

Proof of Proposition 8. Following the steps used to prove Proposition 7, we can derive the each manufacturer's profit, based on the collective order of the N/M retailers, and the fixed-payment term as $\Pi_j^A = \frac{(N-1)(a-c)^2}{4MNb} + \frac{N}{M}f$ for $j = 1, 2$ and $f \in \left(-\frac{(N-1)(a-c)^2}{4N^2b}, -\frac{(a-c)^2(N-1)(3N+1)}{4N^2(N+1)^2b}\right]$. Substituting f, we get $\Pi_j^A = \frac{(N-1)(a-c)^2}{4MNb}\left(1 - \frac{3N+1}{(N+1)^2}\right)$ which is decreasing in M. ■

Proposition EC.1. *When the competing manufacturers offer differentiated products, the equilibrium payment terms are as follows:*

a. wholesale price, $\mathtt{v}^C = \left\{w = \frac{\theta(1-\gamma)+c}{2-\gamma}\right\}$*;*

b. revenue sharing, $\mathtt{v}^C = \left\{w = \frac{\gamma^2(\theta(1-\gamma)^2/(3-\gamma^2)+c)}{(2-\gamma)}, \alpha = \frac{3(1-\gamma^2)}{3-\gamma^2}\right\}$*;*

c. quantity discount, $\mathtt{v}^C = \left\{w = \frac{3\theta(1+2\gamma-3\gamma^2)+c(1+7\gamma+10\gamma^2)}{4+13\gamma+\gamma^2}, \xi = 2(1-\gamma)\right\}$*;*

d. wholesale price plus fixed fee, $\mathtt{v}^C = \{w = \frac{\theta(1-\gamma)+3c}{(4-\gamma)},\ f = \frac{(\theta-c)^2(1-\gamma)}{(4-\gamma)^2(1+\gamma)}\}$*;*

e. minimum order quantity, $\mathtt{v}^C = \{w = g(\gamma),\ q_{\min} = \frac{\theta-c}{4(1+\gamma)}\}$*;*

where $\gamma \in [0,1)$ *captures the degree of partial differentiation between the offered products, and* $g(\gamma)$ *is an implicit function of* γ*.*

Proof of Proposition EC.1. When the competing manufacturers offer differentiated products, they need to meet the retailers' innate reservation value for carrying a product in their respective assortment. As a result, following Cachon and Kök (2010), we modify the competing manufacturers' problem, in the deterministic demand scenario, to

$$\mathtt{v}_j^C = \arg\max_{\mathtt{v}_j} \Pi_j(\mathbf{v}) = \sum_{i=1,2} \left(T(\mathtt{v}_j, \hat{q}_{ij}, \tilde{r}_{ij}) - c\hat{q}_{ij}\right), \qquad \text{s.t. } \pi_{1j}(\mathbf{v}) \geq \pi_j^\circ(T), \pi_{2j}(\mathbf{v}) \geq \pi_j^\circ(T); \tag{EC.34}$$

where $\pi_j^\circ(T)$ denotes the retailers' reservation value for product j under the contract T. For a given payment terms $\mathbf{v}$ under T, $\pi_j^\circ(T)$ is defined as the profit a retailer can earn by carrying only the product -j in its assortment (Cachon and Kök 2010). Henceforth, we use $^\circ$ to denote reservation value related functions and variables. Likewise, we define the retailers' profit function as:

$$\Psi(q_{ij}, q_{i\text{-}j}, q_{\text{-}ij}, q_{\text{-}i\text{-}j}, \mathbf{v}) = \sum_{j=1,2} (\theta - (q_{ij} + q_{\text{-}ij}) - \gamma(q_{i\text{-}j} + q_{\text{-}i\text{-}j}))q_{ij} - T(\mathtt{v}_j, q_{ij}, r_{ij}), \tag{EC.35}$$

where $r_{ij} = (\theta - (q_{ij} + q_{\text{-}ij}) - \gamma(q_{i\text{-}j} + q_{\text{-}i\text{-}j}))q_{ij}$. Below, we analyze each of the five contracts separately.

<u>WP</u>: The Hessian matrix $\mathtt{H}$ of function Ψ is negative definite (with $\partial_{q_{11}}\Psi = -2$, and $\partial_{q_{11}}\partial_{q_{22}}\Psi - \partial_{q_{12}}\partial_{q_{21}}\Psi = 4(1-\gamma^2) > 0$ since $\gamma \in (0,1)$. Thus, using the first-order conditions ($\partial_{q_{11}}\Psi = 0$, $\partial_{q_{12}}\Psi = 0$), we characterize the optimal order quantities as

$$\hat{q}_{ij} = \frac{\theta(1-\gamma) - w_j + w_{\text{-}j}\gamma}{3(1-\gamma^2)}. \tag{EC.36}$$

Substituting $\hat{q}_{ij}$ in equation (EC.34), we get

$$\Pi_j = \frac{(w_j - c)(\theta(1-\gamma) - w_j + w_{\text{-}j}\gamma)}{3(1-\gamma^2)}. \tag{EC.37}$$

Note that Π is concave in w ($\partial_w^2\Pi = -4/3(1-\gamma^2) < 0$). By solving $\partial_w\Pi = 0$, we get the interior solution $w^\ddagger$ as

$$w^\ddagger = \frac{\theta(1-\gamma) + c}{2-\gamma}. \tag{EC.38}$$

Next, we compute $\pi_j^{\circ}(T=\texttt{WP})$ by deriving $\max \Psi(0, q_{i\text{-}j}, 0, q_{\text{-}i\text{-}j}, \mathbf{v}=[0, w^{\ddagger}])$. Solving the first-order condition to obtain the optimal quantity and substituting it back in Ψ gives

$$\hat{q}^{\circ}_{\text{-}j} = \frac{\theta - c}{6 - 3\gamma}, \tag{EC.39}$$

$$\pi_j^{\circ}(T=\texttt{WP}) = \frac{(\theta - c)^2}{9(2-\gamma)^2}. \tag{EC.40}$$

Finally, using (EC.35), (EC.36), and (EC.38) we get

$$\begin{aligned}\pi_j(\mathbf{v}=[w^{\ddagger}, w^{\ddagger}]) &= \sum_{j=1,2} \left(\theta - (\hat{q}_{ij} + \hat{q}_{\text{-}ij}) - \gamma(\hat{q}_{i\text{-}j} + \hat{q}_{\text{-}i\text{-}j})\right) \hat{q}_{ij} - w_j \hat{q}_{ij}, \\ &= \left(\theta - \frac{2(\theta-c)(1-\gamma)}{6+3\gamma-3\gamma^2} - \frac{\theta(1-\gamma)+c}{2-\gamma}\right) \frac{2(\theta-c)}{6+3\gamma-3\gamma^2}, \\ &= \frac{2(\theta-c)^2}{9(2-\gamma)^2(1+\gamma)}. \end{aligned} \tag{EC.41}$$

Comparing (EC.41) and (EC.40), we get $\pi_{ij}(\mathbf{v}=[w^{\ddagger}, w^{\ddagger}]) - \pi_j^{\circ}(T=\texttt{WP}) = \frac{(\theta-c)^2(1-\gamma)}{9(2-\gamma)^2(1+\gamma)} > 0$. Thus, $w^C = w^{\ddagger}$.

<u>RS</u>: The Hessian matrix $\texttt{H}$ of function Ψ under the RS contract is negative definite under symmetry (with $\partial_{q_{11}}\Psi = -2(1-\alpha) < 0$ since $\alpha \in [0,1)$, and $\partial_{q_{11}}\partial_{q_{22}}\Psi - \partial_{q_{12}}\partial_{q_{21}}\Psi = \sum_{j=1,2} 4(1-\alpha)^2(1-\gamma^2) > 0$ since $\gamma \in (0,1)$. Thus, using the first-order conditions we characterize the optimal order quantities as

$$\hat{q}_{ij} = \frac{3(1-\alpha_{\text{-}j})(\theta(1-\alpha_j) - w_j) - (\theta(1-\alpha_{\text{-}j}) - w_{\text{-}j})(3 - 2\alpha_j - \alpha_{\text{-}j})\gamma}{9(1-\alpha_1)(1-\alpha_2)(1-\gamma^2) - 2\gamma^2(\alpha_1 - \alpha_2)^2}. \tag{EC.42}$$

Next, we substitute $\hat{q}_{ij}$ in the manufacturers' profit functions to compute the internal solution for the payment vector $\texttt{v}_j = \{w^{\ddagger}, \alpha^{\ddagger}\}$ as

$$w^{\ddagger} = \frac{\gamma^2(\theta(1-\gamma)^2/(3-\gamma^2) + c)}{(2-\gamma)}, \tag{EC.43}$$

$$\alpha^{\ddagger} = \frac{3(1-\gamma^2)}{3-\gamma^2}. \tag{EC.44}$$

The $\texttt{H}$ for the manufacturers profit function at $\{w^{\ddagger}, \alpha^{\ddagger}\}$ is semi-negative definite with $\partial_w^2 \Pi|_{w^{\ddagger},\alpha^{\ddagger}} = -\frac{2(3-\gamma^2)}{3\gamma^4(1-\gamma^2)}$ and $\partial_{q_{11}}\partial_{q_{22}}\Pi - \partial_{q_{12}}\partial_{q_{21}}\Pi|_{w^{\ddagger},\alpha^{\ddagger}} = 0$. Next, following the steps described in the WP contract analysis, we compute

$$\hat{q}^{\circ}_{\text{-}j} = \frac{(\theta-c)^2(3-\gamma^2)}{9(2-\gamma)^2(1+\gamma)}, \tag{EC.45}$$

$$\pi_j^{\circ}(T=\texttt{RS}) = \frac{(\theta-c)^2\gamma^2(3-\gamma^2)}{18(2-\gamma)^2}, \tag{EC.46}$$

$$\pi_j(\mathbf{v}=[\{w^{\ddagger}, \alpha^{\ddagger}\}, \{w^{\ddagger}, \alpha^{\ddagger}\}]) = \frac{(\theta-c)^2\gamma^2(3-\gamma^2)}{9(2-\gamma)^2(1+\gamma)}. \tag{EC.47}$$

Comparing (EC.47) and (EC.46), we get $\pi_j(\mathbf{v}=[\{w^{\ddagger}, \alpha^{\ddagger}\}, \{w^{\ddagger}, \alpha^{\ddagger}\}]) - \pi_j^{\circ}(T=\texttt{RS}) = \frac{(\theta-c)^2\gamma^2(3-\gamma^2)(1-\gamma)}{9(2-\gamma)^2(1+\gamma)} > 0$. Thus, $\{w^C = w^{\ddagger}, \alpha^C = \alpha^{\ddagger}\}$.

<u>QD</u>: Analogous to the $\xi \in [0, 2b)$ condition in the undifferentiated products setting, we obtain $\xi \in [0, 2(1-\gamma))$ condition for the differentiated products setting. These conditions ensure a positive order quantity, determined by the FOC, in the equivalent centralized supply chain setting. Hence, for the subsequent analysis in this proposition, we use $\xi \in [0, 2(1-\gamma))$. The Hessian matrix $\texttt{H}$ of Ψ under the QD contract is negative

definite under symmetry (with $\partial_{q_{11}}\Psi = -(2-\xi) < 0$ and $\partial_{q_{11}}\partial_{q_{22}}\Psi - \partial_{q_{12}}\partial_{q_{21}}\Psi = \sum_{j=1,2}(2(1-\gamma)-\xi)(2(1+\gamma)-\xi) > 0)\forall\xi \in [0, 2(1-\gamma)$ and $\gamma \in (0,1)$. Solving for the first-order conditions, we get

$$\hat{q}_{ij} = \frac{\theta(3(1-\gamma)-\xi_{-j}) - w_j(3-\xi_{-j}) + 3w_{-j}\gamma}{9(1-\gamma^2) - 3(\xi_j + \xi_{-j}) + \xi_j\xi_{-j}}. \tag{EC.48}$$

Next, we substitute $\hat{q}_{ij}$ in the manufacturers' profit functions to compute the internal solution for the parameter w

$$w^{\ddagger} = \frac{3\theta(3-\xi+3\gamma^2) - c(3-\xi)(\xi - 3(1+\gamma))}{\xi^2 - 3\xi(3+\gamma) + 9(2+\gamma-\gamma^2)}. \tag{EC.49}$$

By substituting $w^{\ddagger}$ in $\partial_{\xi}\Pi$, we get

$$\partial_{\xi}\Pi|_{w^{\ddagger}} = \frac{(\theta-c)^2(3-\xi)^2}{(\xi^2 - 3\xi(3+\gamma) + 9(2+\gamma-\gamma^2))^2}. \tag{EC.50}$$

Equation (EC.50) implies $\partial_{\xi}\Pi|_{w^{\ddagger}} > 0\ \forall\xi \in [0, 2(1-\gamma))$. Thus, $\xi^{\ddagger} = 2(1-\gamma)$. Substituting $\xi^{\ddagger} = 2(1-\gamma)$ in equation (EC.49) gives

$$w^{\ddagger} = \frac{3\theta(1+2\gamma-3\gamma^2) + c(1+7\gamma+10\gamma^2)}{4+13\gamma+\gamma^2}. \tag{EC.51}$$

Note that $\partial_w^2\Pi|_{\alpha^{\ddagger}} = -\frac{4(1+2\gamma)(2+7\gamma)}{(1-\gamma)(1+5\gamma)^2} < 0$. Next, following the steps described in the WP contract analysis, we compute

$$\hat{q}^{\circ}_{-j} = \frac{(\theta-c)(1+5\gamma)}{4+13\gamma+\gamma^2}, \tag{EC.52}$$

$$\pi^{\circ}_j(T = \mathtt{QD}) = \frac{(\theta-c)^2\gamma(1+5\gamma)^2}{(4+13\gamma+\gamma^2)^2}, \tag{EC.53}$$

$$\pi_j(\mathbf{v} = [w^{\ddagger}, \xi^{\ddagger}, w^{\ddagger}, \xi^{\ddagger}]\}) = \frac{4(\theta-c)^2\gamma(1+2\gamma)^2}{(4+13\gamma+\gamma^2)^2}. \tag{EC.54}$$

Comparing (EC.54) and (EC.53), we get $\pi_j(\mathbf{v} = [\{w^{\ddagger}, \xi^{\ddagger}\}, \{w^{\ddagger}, \xi^{\ddagger}\}]) - \pi^{\circ}_j(T = \mathtt{QD}) = \frac{27\gamma(1+2\gamma-3\gamma^2)}{(4+13\gamma+\gamma^2)^2} > 0$. Thus, $\{w^C = w^{\ddagger}, \xi^C = \alpha^{\ddagger}\}$.

MOQ: Given a MOQ contract $\mathtt{v} = \{w, q_{\min}\}$, the retailers' are bound to select $q_{\min}$ if the w-induced optimal purchase quantity, $\hat{q}(w)$, is lower than $q_{\min}$. If $\hat{q}(w) \geq q_{\min}$ the MOQ contract effectively reduces to the WP contract and the above analysis for the WP contract will apply as it is. Following the literature, we set $q^C_{\min} = \frac{\theta-c}{4(1+\gamma)}$ to the channel coordinating quantity in the case of differentiated products. Below, we first show that $\hat{q}(w) \geq q^C_{\min}$ if $\gamma \in [2/3, 1)$ (Case I). Thus, for these γ values, we have $\{w^c = \frac{\theta(1-\gamma)+c}{2-\gamma}, q_{\min} = \frac{\theta-c}{4(1+\gamma)}\}$. We conclude the proof by extending analysis to $\gamma \in (0, 2/3)$ case (Case II).

Case I ($\gamma \in [2/3, 1)$): Under the WP contract, using (EC.36) and (EC.38) we get the retailers combined quantity as

$$\begin{aligned} Q^C_j &= q_{1j} + q_{2j}, \\ &= \frac{2(\theta-c)}{6+3\gamma-3\gamma^2}. \end{aligned} \tag{EC.55}$$

Next, using the centralized supply chain profit function ($\Omega = (\theta - q_j - \gamma q_{-j})q_j + (\theta - q_{-j} - \gamma q_j)q_{-j} - c(q_j + q_{-j})$), we get the channel coordinating optimal Q^* by solving for the first-order condition ($\partial^2_{q_j}\Omega = -2 < 0$)

$$Q^*_j = \frac{\theta-c}{2(1+\gamma)}. \tag{EC.56}$$

Comparing (EC.55) and (EC.56), we get

$$Q_j^C - Q_j^* = \frac{2(\theta-c)}{6+3\gamma-3\gamma^2} - \frac{\theta-c}{2(1+\gamma)}, \tag{EC.57}$$
$$= \frac{(\theta-c)(3\gamma-2)}{6(2-\gamma)(1+\gamma)}; \tag{EC.58}$$

thus, $Q_j^C \geq Q_j^*$ ($\equiv \hat{q}(w) \geq q_{\min}^C$) for $\gamma > 2/3$.

Case II ($\gamma \in (0, 2/3)$): In this case since $\hat{q}(w^\ddagger)$, where $w^\ddagger$ is defined in (EC.38), is lower than $q_{\min}^C$ the retailers set $\hat{q}_{ij} = q_{\min}^C$. Accordingly, the manufacturers set w to maximize their profits from the retailers subject to the reservation constraint, and the w-induced quantity to be lower than $q_{\min}^C$ as otherwise the retailers deviate to the order quantity $\hat{q}(w)$. Using the Lagrange multiplier for the retailers' reservation constraint, we define the manufacturers' new problem as

$$\begin{aligned} w_j^C = \arg\max_{\mathtt{w}_j} \sum_{i=1,2} \Big(&(w_j - c) q_{\min} \\ &-\lambda_i(\Psi(0, \hat{q}_{1\text{-}j}^\circ(w_{\text{-}j}), 0, \hat{q}_{2\text{-}j}^\circ(w_{\text{-}j}), \mathbf{v}=[0, w_{\text{-}j}]) - \Psi(q_{\min}, q_{\min}, q_{\min}, q_{\min}, \mathbf{v}=[w_j, w_{\text{-}j}])) \Big), \\ &\text{s.t. } \hat{q}_{1j}(w_j) \leq q_{\min}^c, \hat{q}_{2j}(w_j) \leq q_{\min}^c. \end{aligned} \tag{EC.59}$$

where $\hat{q}(w)$ denote the competing retailers optimal order quantity when offered the wholesale price w and $\hat{q}^\circ(w)$ denotes the optimal order quantity when computing the reservation value. We denote the manufacturers modified objective function by $G(w_j, \lambda)$. Using (EC.59), we get

$$\partial_{w_j} G = \frac{(1-\lambda_1)(a-c)}{4(1+\gamma)}. \tag{EC.60}$$

Thus, we get $\lambda_1 = 1$ from $\partial_{w_j} G = 0$ which implies the manufacturers set $w^\ddagger$ such that it binds the reservation constraint. We get

$$w^\ddagger = \frac{(3-\sqrt{5-4\gamma})(\theta\sqrt{5-4\gamma}+3c)}{4(1+\gamma)}. \tag{EC.61}$$

Using (EC.61) and (EC.36), we get $\hat{q}(w^\ddagger) - q_{\min}^c = \frac{(\theta-c)(2-\gamma-\sqrt{5-4\gamma})}{4(1+\gamma)^2} < 0$ for $\gamma \in (0, 2/3)$.

Thus, based on the Case I and Case II, the MOQ payment vector equals $\{w^c = g(\gamma), q_{\min} = \frac{\theta-c}{4(1+\gamma)}\}$ where $g(\gamma) = \mathbb{I}_{[\gamma\in(0,2/3)]} \frac{(3-\sqrt{5-4\gamma})(\theta\sqrt{5-4\gamma}+3c)}{4(1+\gamma)} + \mathbb{I}_{[\gamma\in[2/3,1)]} \frac{\theta(1-\gamma)+c}{2-\gamma}$.

<u>WPFF</u> Under the WPFF contract, the retailers' order quantities as a function of w equals that under the WP contract because $\partial_f \partial_q \pi = 0$. Using a Lagrangian multiplier for the retailers' reservation constraint, and expression for $\hat{q}_{ij}$ (EC.36) we rewrite the manufacturer j's objective function in (EC.34 for a WPFF contract $\mathrm{v}_j = \{w_j, f_j\}$ as

$$G_j(\mathbf{v}) = \sum_{i=1,2} f_j + (w_j - c)\left(\frac{\theta(1-\gamma) - w_j + w_{\text{-}j}\gamma}{3(1-\gamma^2)}\right) + \lambda_i(\pi_{ij}(\mathbf{v}) - \pi_j^\circ[T]). \tag{EC.62}$$

The first-order conditions are

$$\frac{\partial G_j}{\partial w_j} = \frac{2(3(\gamma w_{\text{-}j} + c - 2w_j) + (\lambda_j + \lambda_{\text{-}j})(w_j - w_{\text{-}j}) + a(1-\gamma)(3 - \lambda_j - \lambda_{\text{-}j})}{9(1-\gamma^2)}, \tag{EC.63}$$
$$\frac{\partial G_j}{\partial f_j} = 2 - \lambda_j - \lambda_{\text{-}j}. \tag{EC.64}$$

Solving for the first-order conditions, we get $\lambda_1 = \lambda_2 = 1$ and $w^C = \frac{\theta(1-\gamma)+3c}{(4-\gamma)}$. As $\lambda's \neq 0$, we obtain value of f by binding the reservation constraint ($\pi_{ij}(\mathbf{v}) = \pi_j^{\circ}[T]$). Solving for $\max \Psi(0, q_{i\text{-}j}, 0, q_{\text{-}i\text{-}j}, \mathbf{v} = [\{0,0\}, \{w^C, f\}])$ gives

$$\hat{q}_{\text{-}j}^{\circ} = \frac{\theta - c}{4-\gamma}, \tag{EC.65}$$

$$\pi_j^{\circ}[T = \texttt{WPFF}] = -f + \frac{(\theta-c)^2}{(4-\gamma)^2}. \tag{EC.66}$$

Using (EC.35), (EC.36), and (EC.66), we solve for $\pi_{ij}(\mathbf{v}) = \pi_j^{\circ}[T]$ and obtain $f^C = \frac{(\theta-c)^2(1-\gamma)}{(4-\gamma)^2(1+\gamma)}$. ■

Proof of Proposition 9. We structure the proof in two parts. First, we show that the manufacturers cannot collude using the WP (Claim EC.10), RS (Claim EC.11), QD (Claim EC.12), and MOQ contract structures (Claim EC.13). Next, in (Claim EC.14) we characterize the WPFF terms that can facilitate collusion.

Claim EC.10. *For the WP contract, there exists no $\mathbf{v}'$ such that conditions C1 and C2 holds simultaneously.*

Proof: Consider there exists a $\mathbf{v}'$ such that $\Pi(\mathbf{v}') + \pi(\mathbf{v}') > \Pi(\mathbf{v}^C) + \pi(\mathbf{v}^C)$ (necessary condition for C1 and C2 to hold jointly). Using (EC.36), (EC.38), and (EC.56), we get

$$\hat{Q}^C|_{\texttt{WP}} - Q^* = \frac{(\theta-c)(3\gamma-2)}{3(2-\gamma)(1+\gamma)}, \tag{EC.67}$$

By (EC.67) we get $\hat{Q}^C|_{\texttt{WP}} \geq Q^*$ for $\gamma \in [2/3, 1)$. For these values $\Pi(\mathbf{v}') + \pi(\mathbf{v}') > \Pi(\mathbf{v}^C) + \pi(\mathbf{v}^C)$ holds only if $Q^A < Q^C$ since the supply chain profit function is concave in Q and attains maximum at Q^*. Using the equilibrium payment terms specified in Proposition EC.1, we evaluate condition C2 as

$$\begin{aligned} \frac{2(\theta-w)^2}{9(1+\gamma)} &\geq \frac{2(\theta-c)^2}{9(2-\gamma)^2(1+\gamma)}, \\ \left(\frac{\theta-w}{3+3\gamma}\right)^2 \left(\frac{3(2-\gamma)(1+\gamma)}{6+3\gamma-3\gamma^2}\right)^2 &\geq \left(\frac{\theta-c}{6+3\gamma-3\gamma^2}\right)^2, \\ \implies q^A &\geq q^C, \end{aligned} \tag{EC.68}$$

which contradicts $Q^A < Q^C$. Next, for $\gamma \in (0, 2/3)$, $\Pi(\mathbf{v}') + \pi(\mathbf{v}') > \Pi(\mathbf{v}^C) + \pi(\mathbf{v}^C)$ holds only if $Q^A > Q^C$, which evaluates to

$$\begin{aligned} \frac{(\theta-w)}{(3+3\gamma)} &> \frac{(\theta-c)}{6+3\gamma-3\gamma^2}, \\ \frac{\theta-w}{\theta-c} &> \frac{3(1+\gamma)}{6+3\gamma-3\gamma^2}. \end{aligned} \tag{EC.69}$$

Next, using the equilibrium payment terms, we evaluate C1 as

$$\begin{aligned} \frac{2(\theta-w)(w-c)}{3(1+\gamma)} &> \frac{2(\theta-c)^2(1-\gamma)}{3(1+\gamma)(2-\gamma)^2}, \\ \left(\frac{\theta-w}{\theta-c}\right)\left(1 - \left(\frac{\theta-w}{\theta-c}\right)\right) - \frac{1-\gamma}{(2-\gamma)^2} &> 0, \\ m(x) &> 0, \end{aligned} \tag{EC.70}$$

where $m(x,\gamma) = x(1-x) - \frac{1-\gamma}{(2-\gamma)^2}$. Note, $m(x,\gamma)$ is decreasing in x for $x > 1/2$ as $\partial_x m = 1 - 2x$. Thus, (EC.70) contradicts (EC.69) since (i) $g(\gamma) = (3(1+\gamma)/(6+3\gamma-3\gamma^2)$ is increasing in γ as $\partial_\gamma g = 1/(2-x)^2$, (ii) g(0) = 1/2, (iii) $m(g(\gamma),\gamma) = 0$. ■

Claim EC.11. *For the RS contractual structure, there exists no* $\mathbf{v}'$ *such that conditions C1 and C2 holds simultaneously.*

Proof: Consider there exists a $\mathbf{v}'$ such that $\Pi(\mathbf{v}')+\pi(\mathbf{v}')>\Pi(\mathbf{v}^C)+\pi(\mathbf{v}^C)$ (necessary condition for C1 and C2 to hold jointly). Using (EC.42), (EC.43), and (EC.56), we get

$$\hat{Q}^C|_{\texttt{RS}}-Q^*=\frac{((\theta-c))\gamma(3-2\gamma)}{3(2-\gamma)(1+\gamma)}. \tag{EC.71}$$

By (EC.71) we get $\hat{Q}^C|_{\texttt{RS}}\geq Q^*$ for $\gamma\in(0,1)$. Thus, for these cases $\Pi(\mathbf{v}')+\pi(\mathbf{v}')>\Pi(\mathbf{v}^C)+\pi(\mathbf{v}^C)$ holds only if $Q^A<Q^C$. Using the equilibrium payment terms specified in Proposition EC.1 and where $w'=w/(1-\alpha)$, we evaluate (i) condition C2

$$\begin{aligned}\frac{2(\theta-w')^2(1-\alpha)}{9(1+\gamma)}&\geq\frac{(\theta-c)^2\gamma^2(3-\gamma^2)}{9(2-\gamma)^2(1+\gamma)},\\ \frac{(\theta-w')^2}{(\theta-c)^2}&\geq\frac{\gamma^2(3-\gamma^2)}{2(1-\alpha)(2-\gamma)^2},\end{aligned} \tag{EC.72}$$

(ii) $q^A<q^c$

$$\begin{aligned}\frac{(\theta-w')}{3(1+\gamma)}&<\frac{(\theta-c)^2(3-\gamma^2)}{6(2-\gamma)(1+\gamma)},\\ \frac{(\theta-w')}{(\theta-c)}&<\frac{(3-\gamma^2)}{2(2-\gamma)},\end{aligned} \tag{EC.73}$$

and (iii) condition C1

$$\begin{aligned}\frac{2(\theta-w')(3(w'-c)+\alpha(a-w'))}{9(1+\gamma)}&>\frac{(\theta-c)^2(3-\gamma^2)(1-\gamma)}{3(2-\gamma)^2(1+\gamma)},\\ \frac{(\theta-w')}{(\theta-c)^2}(3(w'-c)+\alpha(\theta-w'))&>\frac{2(1-\gamma)(3-\gamma^2)}{2(2-\gamma)^2},\\ \frac{3(\theta-w')}{(\theta-c)}-(3-\alpha)\frac{(\theta-w')^2}{(\theta-c)^2}&>\frac{3(1-\gamma)(3-\gamma^2)}{2(2-\gamma)^2},\\ \frac{(\theta-w')^2}{(\theta-c)^2}&<-\frac{3(3-\gamma^2)}{2(2-\gamma)^2(3-\alpha)}.\end{aligned} \tag{EC.74}$$

by (EC.73). Note that (EC.74), contradicts (EC.72) since $\gamma\in(0,1)$ and $\alpha\in(0,1)$. ■

Claim EC.12. *For the QD contract, there exists no* $\mathbf{v}'$ *such that conditions C1 and C2 holds simultaneously.*

Proof Consider there exists a $\mathbf{v}'=\{w,\xi\}$ such that $\Pi(\mathbf{v}')+\pi(\mathbf{v}')>\Pi(\mathbf{v}^C)+\pi(\mathbf{v}^C)$ (necessary condition for C1 and C2 to hold jointly). Using (EC.48), (EC.51), and (EC.56), we get

$$\hat{Q}^C|_{\texttt{QD}}-Q^*=\frac{(\theta-c)\gamma(7\gamma-1)}{(1+\gamma)(\gamma^2+13\gamma+4})), \tag{EC.75}$$

By (EC.75) we get $\hat{Q}^C|_{\texttt{QD}}\geq Q^*$ for $\gamma\in[1/7,1)$. For these values $\Pi(\mathbf{v}')+\pi(\mathbf{v}')>\Pi(\mathbf{v}^C)+\pi(\mathbf{v}^C)$ holds only if $Q^A<Q^C$. Using the equilibrium payment terms specified in Proposition EC.1, we evaluate (i) condition C2

$$\begin{aligned}\frac{(\theta-w)^2(2(1+\gamma)-\xi)}{(3(1+\gamma)-\xi)^2}&\geq\frac{4(\theta-c)^2\gamma(1+2\gamma)^2}{(\gamma^2+13\gamma+4)^2},\\ \frac{(\theta-w)^2}{(\theta-c)^2}&\geq h(\xi,\gamma)=\frac{4(3(1+\gamma)-\xi)^2\gamma(1+2\gamma)^2}{(2(1+\gamma)-\xi)(\gamma^2+13\gamma+4)^2},\end{aligned} \tag{EC.76}$$

(ii) $q^A < q^C$

$$\begin{aligned}\frac{\theta - w}{3(1+\gamma)-\xi} &< \frac{(\theta-c)(1+2\gamma)}{(\gamma^2+13\gamma+4)^2},\\ \frac{\theta-w}{\theta-c} &< \frac{(3(1+\gamma)-\xi)(1+2\gamma)}{(\gamma^2+13\gamma+4)^2},\end{aligned} \tag{EC.77}$$

and (iii) C1

$$\begin{aligned}\frac{(\theta-w)((6(1+\gamma)-\xi)(w-c)-\xi(\theta-c))}{(\xi-3(1+\gamma))^2} &> \frac{2(\theta-c)^2(2+9\gamma+3\gamma^2-14\gamma^3)}{(\gamma^2+13\gamma+4)^2},\\ \frac{(\theta-w)}{(\theta-c)^2}((6(1+\gamma)-\xi)(w-c)-\xi(\theta-c)) &> \frac{(\xi-3(1+\gamma))^2 2(2+9\gamma+3\gamma^2-14\gamma^3)}{(\gamma^2+13\gamma+4)^2},\\ \frac{(\theta-w)^2}{(\theta-c)^2} < g(\xi,\gamma) &= \frac{2(14\gamma^3-3\gamma^2-7\gamma+1)(\xi-3(1+\gamma))^2}{(\gamma^2+13\gamma+4)^2(6(1+\gamma)-\xi)},\end{aligned} \tag{EC.78}$$

by (EC.80). Comparing (EC.78) and (EC.76), we get

$$\begin{aligned}h(\xi,\gamma)-g(\xi,\gamma) &= \frac{2(1+2\gamma)(3(1+\gamma)-\xi)^2(2+24\gamma+32\gamma^2+10\gamma^3-\xi(1+\gamma(7-3\gamma))}{(\gamma^2+13\gamma+4)^2(6(1+\gamma)-\xi)(2(1+\gamma)-\xi)},\\ &= k(\xi,\gamma)(2+24\gamma+32\gamma^2+10\gamma^3-\xi(1+\gamma(7-3\gamma))>0,\end{aligned} \tag{EC.79}$$

since $k(\xi,\gamma)>0$ for $\gamma\in(1/7,1)$, $\xi\in(0,2(1-\gamma)$ and $l(\xi,\gamma)=2+24\gamma+32\gamma^2+10\gamma^3-\xi(1+\gamma(7-3\gamma)>0$ as $\partial_\xi h=-(1+\gamma(7-3\gamma))<0$ for $\gamma\in(0,1)$ and $h(2(1-\gamma),\gamma)=4\gamma(\gamma^2+13\gamma+3)>0$. Thus, (EC.76) contradicts (EC.79).

Next, for $\gamma\in(0,1/7)$ the necessary condition $\Pi(\mathbf{v}')+\pi(\mathbf{v}')>\Pi(\mathbf{v}^C)+\pi(\mathbf{v}^C)$ will hold only if $Q^A>Q^C$ since $Q^C<Q^*$. Thus, we have (i) $q^A>q^C$

$$\begin{aligned}\frac{\theta - w}{3(1+\gamma)-\xi} &> \frac{(\theta-c)(1+2\gamma)}{(\gamma^2+13\gamma+4)^2},\\ \frac{\theta-w}{\theta-c} &> \frac{(3(1+\gamma)-\xi)(1+2\gamma)}{(\gamma^2+13\gamma+4)^2},\end{aligned} \tag{EC.80}$$

and (ii) C1

$$\begin{aligned}\frac{\theta-w}{(\theta-c)^2}((6(1+\gamma)-\xi)(w-c)-\xi(\theta-c)) &> \frac{(3(1+\gamma)-\xi)^2 2(2+9\gamma+3\gamma^2-14\gamma^3)}{(\gamma^2+13\gamma+4)^2},\\ \left(\frac{\theta-w}{\theta-c}\right)\left((6(1+\gamma)-2\xi)-\left(\frac{\theta-w}{\theta-c}\right)(6(1+\gamma)-\xi)\right) &> \frac{(3(1+\gamma)-\xi)^2 2(2+9\gamma+3\gamma^2-14\gamma^3)}{(\gamma^2+13\gamma+4)^2},\\ h(x,\xi,\gamma)-\frac{(3(1+\gamma)-\xi)^2 2(2+9\gamma+3\gamma^2-14\gamma^3)}{(\gamma^2+13\gamma+4)^2} &> 0,\end{aligned} \tag{EC.81}$$

where $h(x,\xi,\gamma)=x(6(1+\gamma-2\xi)-x(6(1+\gamma)-\xi))$ is concave in x (as $\partial_x^2 h=-2(6(1+\gamma)-\xi)<0\,\forall\gamma\in(0,1/7))$ with maximum at $x^\ddagger=\frac{3(1+\gamma)-\xi}{6(1+\gamma)-\xi}$. This gives

$$\begin{aligned}&h(x^\ddagger,\xi,\gamma)-\frac{(3(1+\gamma)-\xi)^2 2(2+9\gamma+3\gamma^2-14\gamma^3)}{(\gamma^2+13\gamma+4)^2}\\ &\quad=-\frac{(3(1+\gamma)-\xi)^2}{(\gamma^2+13\gamma+4)^4(6(1+\gamma)-\xi)}\bigg(8(29+37\gamma)-169(1+\gamma)^4-(55-518)(1+\gamma)^2\\ &\qquad+\xi(14\gamma(1+\gamma)^2-31(1+\gamma)^2+39(1+\gamma)-10\bigg),\\ &\quad=k(\xi,\gamma)*n(\xi,\gamma),\end{aligned} \tag{EC.82}$$

where $k(\xi,\gamma) = -\frac{(3(1+\gamma)-\xi)^2}{(\gamma^2+13\gamma+4)^4(6(1+\gamma)-\xi)} < 0$ for $\gamma \in (0,1/7)$ and $\xi \in (0,2(1-\gamma))$. Also, $n(\xi,\gamma) > 0\,\forall \xi \in (0,\,2(1-\gamma))$ as it is decreasing in ξ (with $\partial_\xi n = -(1+\gamma)^2(62-28\gamma)+78(1+\gamma)-20 < 0$), and $n(0,\gamma) = 8(29+37\gamma) - 169(1+\gamma)^4 - (55-518)(1+\gamma)^2 > 0\,\forall \gamma \in (0,1/7)$. Thus, (EC.82) contradicts (EC.81).

■

Claim EC.13. *For the MOQ contract, there exists no* $\mathbf{v}'$ *such that conditions C1 and C2 holds simultaneously.*

Proof In the proof of Proposition EC.1, we show that for $\gamma \in [2/3,1)$ the MOQ contract is effectively reduced to the WP contract. Thus, for these values, Claim EC.10 analysis apply as it is. Further, for $\gamma \in (0,2/3)$ each retailer orders $\hat{q}^C = q_{\min} = \frac{\theta-1}{4(1+\gamma)}$ so the retailers combined quantity under competition equals Q^*; thus, we have $\Pi^c + \pi^c = \Omega^*$. As a result, for any $\mathbf{v}' \neq \mathbf{v}^C$ either condition C1 or C2 will be violated because $\Pi(\mathbf{v}') + \pi(\mathbf{v}') \leq \Omega^* = \Pi(\mathbf{v}^C) + \pi(\mathbf{v}^C)$. ■

Claim EC.14. *For the WPFF contract, there exists a* $\mathbf{v}'$ *such that conditions C1 and C2 holds simultaneously.*

Proof Following the steps of Claim EC.8, one can show that the manufacturers under collusion sets w^A such that the $Q^A = Q^*$. Using EC.36 and EC.56, we get

$$\frac{\theta - w^A}{3(1+\gamma)} = \frac{\theta - c}{4(1+\gamma)},$$
$$w^A = \frac{\theta+3c}{4}. \tag{EC.83}$$

Next using the competition payment terms specified in the Proposition EC.1, (EC.35 and (EC.83, we obtain lower-bound on f^A by binding the condition C2

$$\frac{(\theta-c)^2 - 16\overline{f}^A(1+\gamma)}{8(1+\gamma)} = \frac{2(\theta-c)^2\gamma}{(4-\gamma)^2(1+\gamma)},$$
$$\Longrightarrow \overline{f}^A = \frac{(\theta-c)^2(16-24\gamma+\gamma^2)}{16(4-\gamma)^2(1+\gamma)}. \tag{EC.84}$$

Finally, the lower-bound on f^A is obtained by binding the condition C1

$$\frac{(\theta-c)^2 + 16\underline{f}^A(1+\gamma)}{8(1+\gamma)} = \frac{4(\theta-c)^2(1-\gamma)}{(4-\gamma)^2(1+\gamma)},$$
$$\Longrightarrow \underline{f}^A = \frac{(\theta-c)^2(16-24\gamma-\gamma^2)}{16(4-\gamma)^2(1+\gamma)}. \tag{EC.85}$$

Thus, $\mathtt{v}^A = \left\{ w^A = \frac{\theta+3c}{4}, f^A \in \left(\frac{(\theta-c)^2(16-24\gamma-\gamma^2)}{16(4-\gamma)^2(1+\gamma)}, \frac{(\theta-c)^2(16-24\gamma+\gamma^2)}{16(4-\gamma)^2(1+\gamma)} \right] \right\}$. It is easy to verify that under $\mathtt{v}^A$ the reservation constraint is also satisfied. Finally, note that $\Omega^* - \Omega^C = 2(\theta-c)^2(2-\gamma)/(4-\gamma)^2(1+\gamma)$ is increasing in γ with $\partial_\gamma(\Omega^* - \Omega^C) = (\theta-c)^2\gamma(\gamma^2+4\gamma+8)/(2(4-\gamma)^3(1+\gamma)^2 > 0$. ■

Proof of Proposition 10. Consider that the manufacturer j offers an extended contract $\overline{T}$ (for definition see the proof of Proposition 1) with terms $\mathtt{v}_j = \{w_j,\, f_j,\, \alpha_j,\, \xi_j\}$. Let $\overline{T}(\mathtt{v}_{\text{-}j})$ capture the manufacturer -j's best response to $\overline{T}(\mathtt{v}_j)$. Note that $\mathtt{v}_{\text{-}j}$ encapsulates -j's response through the WP, RS, QD, and WPFF contractual structures. Below, we show that the manufacturer -j can emulate the retailers' order quantities, and associated profit outcomes, under its best response with the WP, RS, and QD contracts by using a WPFF contract. Thus, $\Pi_{\text{-}j}(T=\texttt{WPFF})\geq \Pi_{\text{-}j}(T\in\{\texttt{WP},\texttt{WPRS},\texttt{QD}\})$. We conclude the proof by covering the MOQ contract. Note that the retailers, when faced with $[\mathtt{v}_j,\mathtt{v}_{\text{-}j}]$ payment terms, may either opt to accept the payment term of the respective manufacturer (i.e., $i=j$) or accept the payment term of -j^{th} manufacturer. The proof steps below analyse the former case. Repeating the analysis with both the retailers accepting payment terms $\mathtt{v}_{\text{-}j}$ provides analysis for the latter case. Under $\mathbf{v}=[\mathtt{v}_j,\mathtt{v}_{\text{-}j}]$, by solving the first-order conditions ($\partial_{q_i}\pi_i=0$ and $\partial_{q_{\text{-}i}}\pi_{\text{-}i}=0$), we get

$$\hat{q}_{i=j}[\mathtt{v}_j,\mathtt{v}_{\text{-}j}]=\frac{a(1-\xi'_{\text{-}j})-(2-\xi'_{\text{-}j})w'_j+w'_{\text{-}j}}{b(3-2\xi'_j-2\xi'_{\text{-}j}+\xi'_j\xi'_{\text{-}j})}, \tag{EC.86}$$

$$=-(2-\xi'_{\text{-}j})\hat{q}_{\text{-}i=\text{-}j}+\frac{a-w'_{\text{-}j}}{b}, \tag{EC.87}$$

where $w'_j=w_j/(1-\alpha_j)$, and $\xi'_j=\xi_j/b(1-\alpha_j)$. For brevity, in the remaining part of the proof we use $\hat{q}_j$ to denote $\hat{q}_{i=j}$ and $\hat{q}_{\text{-}j}$ to denote $\hat{q}_{i=\text{-}j}$. Next, we characterize the optimal order quantities when the manufacturer -j respond using a WPFF contract $\{w^\circ,\ f^\circ\}$:

$$\hat{q}_j[\mathtt{v}_j,\{w^\circ,f^\circ\}]=\frac{a+w^\circ-2w'_j}{b(3-2\xi'_j)}, \tag{EC.88}$$

$$\hat{q}_{\text{-}j}[\mathtt{v}_j,\{w^\circ,f^\circ\}]=\frac{a(1-\xi'_j)-(2-\xi'_j)w^\circ+w'_j}{b(3-2\xi'_j)}, \tag{EC.89}$$

$$=-(2-\xi'_j)\hat{q}_j+\frac{a-w'_j}{b}. \tag{EC.90}$$

Equation (EC.88) imply that, for a given T_j, the $\hat{q}_j[\mathtt{v}_j,\{w^\circ,f^\circ\}]$ is a one-to-one function of w°. Further, equations (EC.87) and (EC.90) imply that $\hat{q}_{\text{-}j}[\mathtt{v}_j,\{w^\circ,f^\circ\}]$ can be expressed as a function of the optimal $\hat{q}_j$ and contract parameters of the manufacturer j ($\mathtt{v}_j=[\{w_j,\alpha_j,\xi_j\}]$). Taken together, the manufacturer -j can emulate the retailers' optimal order-quantities under $[\mathtt{v}_j,\mathtt{v}_{\text{-}j}]$ by setting w° such that $\hat{q}_j[\mathtt{v}_j,\mathtt{v}_{\text{-}j}]=\hat{q}_j[\mathtt{v}_j,\{w^\circ,f^\circ\}]$. Furthermore, using the flexibility of parameter f° the manufacturer -j can replicate the profit outcomes obtained under $[\mathtt{v}_j,\mathtt{v}_{\text{-}j}]$. Equating eqs (EC.86) and (EC.88), we get

$$w^\circ_j=\frac{a\xi'_j(\xi'_{\text{-}j}-1)-(2\xi'_{\text{-}j}-3)w'_j-\xi'_jw'_{\text{-}j}}{3-2\xi'_{\text{-}j}-\xi'_j(2-\xi'_{\text{-}j})}. \tag{EC.91}$$

In the case of MOQ, say the manufacturer -j responds to the manufacturer j's $\overline{T}$ using the following MOQ contract: $\{w^\dagger,\ q^\dagger_{\min}\}$. Note that if the $\hat{q}_{\text{-}j}>q^\dagger_{\min}$ then the above analysis applies as it is since the MOQ contract effectively reduces to the WP contract. When $\hat{q}_{\text{-}j}=q_{\min}$, we have $\partial_{q_{\text{-}i}}\pi_{\text{-}i}[\mathtt{v}_j,\{w^\dagger,q^\dagger_{\min}\}]|_{\{\hat{q}_j,\hat{q}_{\text{-}j}=q_{\min}\}}\leq 0$). In this case, by setting w° such that $\partial_{q_{\text{-}i}}\pi_{\text{-}i}[\mathtt{v}_i,\{w^\circ,f^\circ\}]|_{\{\hat{q}_j,\hat{q}_{\text{-}j}=q_{\min}\}}=0$, the manufacturer -$j$'s optimal order-quantity will be $q^\dagger_{\min}$. Setting such a w° is feasible since, for a given T_j, the $\hat{q}_{\text{-}j}$ is a one-to-one function of w° (see equation EC.89). Now, substituting $q^\dagger_{\min}$ in equation (EC.90) and re-arranging the terms, we get $a-w'_j-b(2-\xi'_j)\hat{q}_j+bq^\dagger_{\min}=0$ which is equivalent to the first-order condition of the manufacturer j when $\hat{q}_{\text{-}j}=q^\dagger_{\min}$. Finally, say the manufacturer -j responds to the manufacturer j's MOQ contract that results in $\hat{q}_j=q^j_{\min}$ for $j=1,2$. In this case, we have $\partial_{q_i}\pi_i=a-2bq^j_{\min}-w_j-bq^{\text{-}j}_{\min}\leq 0$. Note that for a given $q^{\text{-}j}_{\min}$, $\partial_i\pi_i$ is a one-to-one function of w_j. Thus, the manufacturer -j can emulate the MOQ contracts induced minimum order quantities by setting w° such that $\partial_{q_{\text{-}i}}\pi_{\text{-}i}[\mathtt{v}_i,\{w^\circ,f^\circ\}]|_{\{\hat{q}_j=q^j_{\min},\hat{q}_{\text{-}j}=q^{\text{-}j}_{\min}\}}=0$. ■

Proof of Proposition 11. We present the proof in three parts. First, we show that if the manufacturers' can collude for a ϕ (say, ϕ') value then they can also collude for all $\phi<\phi'$ (Claim EC.15). Next, we characterize the maximum ϕ $(=\overline{\phi})$ for which the manufacturers' can collude using the WP, RS, QD, and MOQ. It follows from Claim EC.6 that collusion is only sustainable if $Q^A<Q^C$. This, in turn, implies that the MOQ contract effectively reduces to the WP contract under the collusive decision making scenario. Based on this observation, we analyze the aforementioned contracts collectively using the extended contract structure $\overline{T}$ (see the proof of Proposition 1). Finally, we conclude this proof by extending the analysis to cover the WPFF contract.

Claim EC.15. *The manufacturers can collude $\forall\phi<\phi'$ if they can collude for $\phi=\phi'$.*

Proof. Note that $\partial_\phi\Pi<0$ and $\partial_\phi\Pi^C=0$. Thus, the optimal solution $\mathbf{v}^A$ that enables the manufacturers' to collude for $\phi=\phi'$ is a feasible point for $\forall\phi<\phi'$. ■

Under the contract $\overline{T}$ the manufacturers' first-order conditions, after substituting the optimal order quantity $\hat{q}=(a-w')/3(b-\xi')$ (see EC.5), for the modified collusion problem (8) are:

$$\frac{\partial\Pi}{\partial w'}=\frac{2(c(3-\xi')+a(3-\alpha(2-\xi')(1-\phi)-(2-\xi')\phi)-w'(6+2\alpha(\phi-1)-2\phi+\xi'((1-\alpha)(\phi-1)))}{b(3-\xi')^2}, \tag{EC.92}$$

$$\frac{\partial\Pi}{\partial\xi'}=\frac{(a-w')(2c(3-\xi')+a(3-\alpha-\xi'(\alpha(\phi-1)-\phi-1)-\phi(1-\alpha))-w'(9-\alpha+(1-\alpha)(\phi-\xi'(\phi-1))))}{b(3-\xi')^3}, \tag{EC.93}$$

$$\frac{\partial\Pi}{\partial\alpha}=\frac{(\xi'-2)(a-w')^2(\phi-1)}{b(3-\xi')^2}. \tag{EC.94}$$

Using the equation (EC.92), we solve for the FOC under the WP contract ($\alpha=0,\xi'=0$) to get $w^\ddagger=\frac{3(a+c)-2a\phi}{2(3-\phi)}$, and $\Pi(w\ddagger)=(a-c)^2(3-2\phi)^2/18b(3-\phi)$. Note that condition C1 is satisfied for $\phi<3$ as $\Pi^C=0$. Also, the individual retailer profit equals $\pi(w^\ddagger)=(a-c)^2/4b(3-\phi)^2>0\ \forall\phi\le 3$. Further, for these values of ϕ, the second-order derivative is negative (with $\partial^2_{w'}\Pi=4(\phi-3)/9b)$). So, under the WP contract collusion is feasible for all $\phi\le\overline{\phi}_{\texttt{WP}}=3$, with $w^A=W^\ddagger$. Next, solving for the RS contract ($\xi'=0$), note that $\partial_\alpha\Pi<0$ for $\phi>1$. This implies for $\phi>1$, the manufacturers' set $\alpha=0$, thus the RS contract also mimics the WP contract in this region and enables collusion for $\phi\le 3$. For the QD contract ($\alpha=0$), using eq (EC.92) the FOC w.r.t ξ' can be re-written, by substituting for the optimal w' from eq (EC.93), as $\partial_{\xi'}\Pi=\frac{-(a-c)^2(\phi-1)}{b(6+\xi'(\phi-1)-2\phi)^2}$. Again, note that $\partial_{\xi'}\Pi<0$ for $\phi>1$ which implies over this region the manufacturers' set $\xi'=0$; effectively reducing the QD contract to WP. Finally, under the WPFF contract, the manufacturers can collude for all values of ϕ (i.e., $\overline{\phi}_{\texttt{WPFF}}=\infty$). This is a direct outcome of the Claim EC.8 which shows that using the slotting fee feature the colluding manufacturers can ensure $\pi^A=\pi^C$. ■

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