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Structuring Mortgages for Macroeconomic Stability

John Y. Campbell, Nuno Clara and João F. Cocco *

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ABSTRACT

We study mortgage design features aimed at stabilizing the macroeconomy. We model overlapping generations of mortgage borrowers and an infinitely lived risk-averse representative mortgage lender. Mortgages are priced using an equilibrium pricing kernel derived from the lender's endogenous consumption. We consider an adjustable-rate mortgage (ARM) with an option that during recessions allows borrowers to pay only interest on their loan and extend its maturity. We

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find that this maturity extension option stabilizes consumption growth over the business cycle, shifts defaults to expansions, and is welfare enhancing. The cyclical properties of the maturity extension ARM are attractive to a risk-averse lender so the mortgage can be provided at a relatively low cost.

I. Introduction

Events in the last decade have shown that adjustable-rate mortgages (ARMs) have advantages over fixed-rate mortgages (FRMs) in stabilizing the economy, at least when the central bank has monetary independence and can lower the short-term interest rate in a recession (Eberly and Krishnamurthy (2014)). A lower short rate provides automatic budget relief for ARM borrowers and helps to support their spending while reducing the incidence of mortgage default. A lower short rate can also provide some relief to FRM borrowers, but this requires both a decline in the long-term mortgage rate and refinancing, which may be constrained by declining house prices and tightening credit standards. In the aftermath of the Great Recession, mortgage payment rigidity and barriers to FRM refinancing were of concern to US policymakers and motivated the introduction of the Home Affordable Refinance Program (HARP) (Agarwal et al. (2020), Di Maggio et al. (2017)).

We argue that the stabilizing properties of plain-vanilla ARMs can be enhanced by adding an interest-only option that applies only during recessions, and that allows borrowers to extend loan maturity. This option is included in the contract ex-ante and is available to all borrowers.¹ During a recession, any borrower who decides to take advantage of the option pays only loan interest, with principal loan repayments restarting after the recession ends, and with loan maturity extended. This proposal provides additional budget relief to distressed borrowers.

We use a quantitative model to evaluate our proposal and to compare it with other mortgage designs considered in the recent literature (Eberly and Krishnamurthy (2014), Guren et al. (2020)). Our model has several important features. First, the demand for mortgage loans comes from households who use them to purchase houses or to refinance existing mortgages. Borrowers can default on their mortgages and do so if their income and house prices fall substantially. Household income is subject to both economy-wide and individual specific shocks, as in Guvenen et al. (2014). Recessions are characterized by lower expected income growth, a higher probability of a large decline in income, and a higher probability of a decline in house prices. Therefore, the fundamental drivers of mortgage risk increase in recessions, and mortgages originated in recessions are riskier than those originated in expansions.

Second, loans are provided by an infinite-horizon risk-averse representative lender that chooses consumption to maximize expected utility, subject to a borrowing constraint. In addition to the cash flows from the mortgages,

¹The inclusion of the option in the original contract avoids barriers to ex-post renegotiation of the sort documented by Piskorski et al. (2010) and Adelino et al. (2013). Piskorski and Seru (2018) emphasize the tradeoff between the flexibility of ex-post mortgage modification and the barriers that can inhibit its implementation.

the lender receives a risky income stream. We use the equilibrium consumption choices of the lender to derive the stochastic discount factor that we use to price the mortgages. The lender's income process is parameterized so that our model generates reasonable asset pricing moments, including higher Sharpe ratios in recessions than in expansions. This, combined with the fact that mortgages originated in recessions are riskier, leads to increases in loan premia for loans originated in bad times, similar to those in the data. Since the choices of borrowers and the cash flows from the mortgages depend on loan premia, and the consumption choices of the representative lender depend on mortgage cash flows, the equilibrium solution of our model requires that we solve for a fixed point in the borrowers' and lender's problems.

There is evidence that credit standards are looser in expansions than in recessions (for example Keys et al. (2010) and Corbae and Quintin (2015)). With this evidence in mind, a third feature of our model is that mortgage lending criteria are tighter in bad times. Specifically, the maximum loan-to-value ratio declines during recessions, which constrains borrowers' ability to refinance during bad times.

It is equally important for our model to capture what happens in the years prior to a recession. During the boom years of the mid-2000s, high levels of mortgage cash-out refinancing increased household leverage at the onset of the financial crisis (Khandani et al. (2013), Chen et al. (2020)). A fourth feature of our model is that it allows cash-out refinancing. In each period households can prepay their existing loan and take out a new loan with a higher principal value, subject to current loan-to-value constraints. These debt market dynamics affect the benefits to borrowers of an option to

extend loan maturity, the impact that the option has on lender cash flows, and mortgage rates.

Finally, our model abstracts from inflation dynamics. To economize on state variables, we consider a real economy in which either all mortgages are inflation-indexed, or the price level is constant. While this is an obvious limitation of our analysis, we believe our results are empirically relevant given the limited variability in realized and expected inflation over the last 15 years.

Our model delivers the following results about an interest-only option added to a standard adjustable-rate mortgage. Not all borrowers exercise the option. Some borrowers keep on making loan principal repayments during bad times, motivated by precautionary motives and a desire to deleverage. However, because some borrowers do exercise the option, it leads to a smaller drop in borrowers' consumption and a lower mortgage default rate during recessions. Those individuals who exercise the option reach the end of the recession with higher debt levels than would otherwise be the case, leading to an increase in defaults during expansions.

The exercise of the interest-only option leads to a decline in the mortgage cash flows received by the lender during recessions. However, the lender cares not only about mortgage cash flows, but also the value of its loan portfolio. The interest-only option reduces defaults and the resulting losses of loan values in recessions. Therefore, in equilibrium, the adding of an interestonly option to the ARM contract leads to a small decline in required mortgage premia. In order to cope with the reduced mortgage cash flows arising from the exercise of the interest-only option by borrowers, the lender increases its precautionary savings, and cuts back on consumption during bad times. This reduces the benefits of the policy for consumption stabilization, but we show that aggregate consumption is more stable when we add the interest-only option to the ARM contract. We also show that the interest-only option is welfare improving.

We also use our model to evaluate FRMs. We model the choices of borrowers, the resulting mortgage cash flows, and equilibrium mortgage premia for FRMs. We find that FRMs are less effective than ARMs in stabilizing the economy during bad times. Because our model abstracts from inflation uncertainty, borrowers are actually better off with FRMs than with plainvanilla ARMs, because they benefit from stable real mortgage payments. However, borrowers prefer an ARM with an interest-only option to a FRM.

Another contract design we consider is a FRM with an option to switch to an ARM during recessions, as proposed by Eberly and Krishnamurthy (2014) and Guren et al. (2020). In our model the switching option does have a stabilizing effect in the economy during bad times. However, borrowers switch when interest rates are low during recessions, reducing the payments to and the wealth of the risk-averse lender at times when its consumption is already low. The lender needs to be compensated for this ex-ante, in the form of a higher mortgage rate which makes the option to switch expensive and hence relatively unattractive for borrowers.

These results reflect an interaction between the decisions of mortgage borrowers, including decisions to default, and the pricing of mortgages by risk-averse lenders. Our paper is novel in incorporating both default and lender risk aversion; many papers that model default, such as Piskorski and Tchistyi (2010), Piskorski and Tchistyi (2017), and Guren et al. (2020), assume risk-neutral lenders, while Favilukis et al. (2017) and Beraja et al. (2018) model lender risk aversion but ignore default.

Our focus on default and lender risk aversion does require us to keep our model simple in several other dimensions. First, unlike Piskorski and Tchistyi (2010) and Piskorski and Tchistyi (2017) we propose simple modifications to standard mortgage contracts rather than deriving optimal contracts from first principles. However, our proposed contract shares similarities with the constrained efficient contract derived by Piskorski and Tchistyi (2010) that resembles an option ARM. Second, we consider mortgage systems that contain only a single type of mortgage contract, rather than allowing multiple contracts to coexist in equilibrium. Third, unlike Piskorski and Tchistyi (2017), Favilukis et al. (2017), and Guren et al. (2020) we treat house prices as exogenous and therefore ignore potential feedback effects from household spending and mortgage defaults to house prices. In other words, we stop short of building a general equilibrium model with endogenous house prices. Instead we look at the response of consumption and defaults to given income and house price shocks. These are the initial effects that might in general equilibrium feed back into income and house prices, creating a downward spiral in the worst case. Although general equilibrium feedback effects are certainly of interest, we believe there is value in carefully assessing initial effects because they will be robust to errors in modeling the general equilibrium. Fourth, our simple approach to the lender problem differs from that of Greenwald et al. (2020) who model an intermediary sector with capital requirements, bank failures, and bailouts. Their focus is on shared appreciation mortgages whereas we study interest indexed loans with flexible payments.

The paper is structured as follows. In section II we present our model and its parameterization. Section III compares the macroeconomic stabilization effects and the welfare benefits to borrowers and lenders of an ARM with an interest only option and a FRM with an option to switch to an ARM. In section IV we characterize borrowers' use of these options. Section V shows results for alternative maturity extension policies, including one in which the interest-only option is permanently available. Sections VI and VII explore the robustness of our results to alternative environments. Section VI varies the specification of the mortgage market and borrower behavior, while section VII varies the macroeconomic environment. The final section concludes and briefly relates our analysis to mortgage forbearance provisions enacted in March 2020 during the COVID-19 recession.

II. The Model

We model the decisions of borrowers, who demand loans, and of a representative lender who supplies these loans. On the borrowers' side, and in each period t, a new set of agents enters our economy and stays in it for Tperiods. Therefore our economy has, on the demand side, a stationary overlapping generations structure. Even though borrowers face expected positive real income growth during the periods in which they are in our economy, in each period a new cohort of borrowers is born and an old cohort drops out. The agents that are born face the same (initial) level of house prices relative to income. One possible interpretation is that there are common long-term trends in real house prices and aggregate income, which we abstract from, to focus on cyclical fluctuations. In addition, our model economy is real, and it captures the behavior of the group of individuals who use a mortgage loan to buy a house.

On the supply side, we assume that there is an infinite-horizon representative lender, who is endowed with a risky income stream. The lender originates loans at the initial date, when borrowers first enter the economy, and in later periods when there is refinancing. It provides the funds and it receives the payments from the loans. The lender decides in each period how much to consume and save. We use the lender's optimal consumption choices to derive a pricing kernel, that we use to price the loans. Even though we solve endogenously for some equilibrium prices (the loan premium), in several other dimensions our model is partial equilibrium. We make these modeling choices so as to be able to model with more realism several of the features of the mortgage contracts that are the focus of our paper.

A. Baseline model setup

A.1. Aggregate state

In each period t the economy may be in either an expansion or a recession. An indicator variable I_t equals one in an expansion, and zero otherwise. An exogenous transition probability matrix governs the evolution between these states. Persistence in the aggregate state of the economy is captured by the parameterization of this matrix.

The risk-free real interest rate is also exogenous, but stochastic and corre-

lated with the business cycle. Let $r_{1t} = \log(1 + R_{1t})$ denote the log real rate, the log of the gross real return on a default-free one-period bond held from time t to time t + 1. In each period the log real rate is either high or low, with probabilities that depend on whether the economy is in an expansion or recession. We write the unconditional mean and standard deviation of r_{1t} as μ_r and σ_r , respectively.

We model house price variation in a similar fashion. The change in the log real price of housing, Δp_t^H , is either high or low with probabilities that depend on the state of the economy. We write the unconditional mean and standard deviation of Δp_t^H as μ_H and σ_H , respectively. We set μ_H equal to zero, but since house price increases are more likely to occur if the economy is in an expansion, and there is persistence in the business cycle, the conditional expectation of house price changes is higher during an expansion than during a recession.

A.2. Demand for mortgage loans

The demand for mortgage loans comes from overlapping generations of agents entering the economy, as well as from existing agents refinancing their mortgages. All agents entering the economy are initially identical, with identical wealth and permanent income, but they subsequently experience idiosyncratic labor income shocks that imply cross-sectional heterogeneity increasing with age.

Initial home purchase. At the time that agent *i* initially enters the economy (denoted t_i) he or she buys a house of size H_{i,t_i} using a downpayment

financed from an initial wealth endowment W_{i,t_i} and a mortgage loan with maturity T. The house size that the agent buys depends on the prevailing level of house prices $(P_{t_i}^H)$ at time of entry.

We let d_{i,t_i} denote the downpayment as a proportion of the house value. It is indexed by t_i to allow for the possibility that it depends on the state of the economy. The initial loan amount the agent takes, K_{i,t_i} , is given by:

$$K_{i,t_i} = (1 - d_{i,t_i}) P_{t_i}^H H_{i,t_i}.$$
(1)

A higher proportional downpayment implies that agents use more of their previously accumulated savings to buy the house and therefore take on a loan with a lower loan-to-value (LTV) ratio $K_{i,t_i}/(P_{t_i}^H H_{i,t_i}) = (1 - d_{i,t_i}).$

To ensure stationarity we assume that initial wealth and house size vary in proportion to the level of initial house prices. That is, we assume that agents who enter the economy after a period of house price increases (decreases) buy a smaller (larger) house, so that the initial loan amount is invariant to the level of initial house prices and is proportional to initial income. This assumption simplifies the model solution since, in combination with the assumptions we make on preferences, it implies that we do not need to keep track of the level of house prices at the time of a home purchase.

Finally, we further simplify the model by assuming that the downpayment ratio and hence the LTV ratio vary exogenously with the state of the economy, but that the initial loan-to-income (LTI) ratio does not change from expansions to recessions. The sizes of new houses purchased vary with the business cycle to make this possible. This assumption ensures that the sizes of new mortgages are constant over time and over the cycle in relation to house prices and initial income, which again economizes on state variables when we solve our model. However, in section VI we solve the model, and price mortgages, for alternative values for the initial LTV and LTI.

Preferences. As in Campbell and Cocco (2015) we assume preference separability between housing and non-housing consumption and that house size remains fixed throughout the time during which agent i is in our economy. Under these assumptions we can drop housing from the preference specification. Our agents choose non-durable consumption and manage their mortgages to maximize

$$E_{t_i} \sum_{t=t_i}^{t_i+T} \beta_i^{t-t_i} \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + \beta_i^T b_i \frac{W_{i,t_i+T+1}^{1-\gamma_i}}{1-\gamma_i},$$
(2)

where W_{i,t_i+T+1} denotes terminal wealth that includes both financial and housing wealth. If agents have positive outstanding debt at the terminal date, we calculate terminal wealth net of the debt outstanding. The parameter b_i measures the relative importance of utility derived from terminal wealth. It controls the incentives of individuals to accumulate longer-term savings. These preferences give rise to a precautionary savings motive with relative prudence equal to $\gamma_i + 1$.

Labor income. In each period agents' labor income (Y_{it}) evolves according to the process estimated by Guvenen et al. (2014). Recessions are characterized by a smaller probability of a large increase and an increased probability of a large drop in labor income. As usual, we use a lower case letter to denote the natural log of the variable, so that $y_{it} \equiv \log(Y_{it})$. Log real labor income is the sum of a transitory (ϵ_{it}) and a persistent (z_{it}) component. Innovations to the persistent component feature a mixture of normals:

$$y_{it} = z_{it} + \epsilon_{it},\tag{3}$$

$$z_{it} = \rho z_{i,t-1} + \eta_{it},\tag{4}$$

where $\epsilon_{it} \sim \mathcal{N}(0, \sigma_{\epsilon})$ and:

$$\eta_{it} = \begin{cases} \eta_{it}^1 \sim \mathcal{N}(\mu_{1,I_t}, \sigma_1), & \text{with probability } p_1 \\ \eta_{it}^2 \sim \mathcal{N}(\mu_{2,I_t}, \sigma_2), & \text{with probability } 1 - p_1, \end{cases}$$
(5)

where recall the subscript I_t indicates whether period t is an expansion or a recession. This setup allows us to capture important deviations of labor income growth from normality, including negative skewness and excess kurtosis, and business cycle variation in expected labor income growth through the different means of the normal distributions. The higher probability of a large drop in labor income in recessions affects borrowers' incentives to default on mortgage loans. Although realistic in these respects, we note that the income process we assume does not fully capture unemployment spells or the short-term income protection provided by unemployment insurance (Hsu et al. (2018)).

We model the tax code in the simplest possible way, by considering a linear taxation rule. Gross labor income and interest earned are taxed at the constant tax rate ϕ .

A.3. Terms of mortgage loans

We study two types of mortgage contracts that differ in the interest rate risk that agents face, adjustable-rate mortgages (ARMs) and fixed-rate mortgages (FRMs). Since our model abstracts from inflation risk, the fixed-rate mortgages we model are implicitly inflation-indexed and not the nominal contracts observed in reality.

Adjustable-rate mortgages. The interest rate on ARMs is the short-term interest rate plus a mortgage premium ψ_{i,t_i}^{ARM} :

$$R_{it}^{ARM} = R_{1t} + \psi_{i,t_i}^{ARM}.$$
(6)

The mortgage premium compensates lenders for prepayment and for default risk. The subscripts i and t_i allow for the possibility that the premium depends on borrower characteristics and on the aggregate state of the economy at the time that the loan begins. The loan premium remains fixed over the life of the loan, but the loan rate fluctuates with the level of short rates.

The period t payment due on the mortgage taken by agent i is given by:

$$L_{it}^{ARM} = R_{it}^{ARM} D_{it} + \Delta D_{i,t+1},\tag{7}$$

where D_{it} is the principal amount outstanding on the loan at the beginning of period t before any mortgage payments are made in that period and $\Delta D_{i,t+1}$ is the loan principal repayment due in period t. To economize on state variables we assume that in each period the principal reduction is the same that would occur in a fixed-rate loan with an exogenously specified mortgage rate. This allows us to link principal outstanding to the loan period.

Fixed-rate mortgages. The interest rate on FRMs is fixed over the life of the loan. It is equal to the long-term bond rate at the time that the loan begins plus a mortgage premium ψ_{i,t_i}^{FRM} :

$$R_i^{FRM} = R_{T,t_i} + \psi_{i,t_i}^{FRM}.$$
(8)

To model long-term bond rates we assume that the log expectations hypothesis of the term structure holds, so that expected log returns on bonds of all maturities are equal. By specifying the expectations hypothesis in logs, we ensure that it is consistent across all holding periods and allow for long bonds to have somewhat higher simple average returns resulting from their greater return volatility.

Refinancing, default, and prepayment options. We model three options that borrowers have in mortgage contracts: to refinance, to default, and to prepay. The option to refinance the loan, i.e. to prepay the existing loan and simultaneously take out a new one, has a monetary cost Θ_R , equal to a proportion θ_R of the initial house value. Refinancing allows agents to extract additional cash from their accumulated home equity. They choose how much additional equity to extract subject to the downpayment constraint prevailing at the time that the refinancing takes place. We maintain the function that maps loan amount to maturity for non-refinanced mortgages: therefore, if agents refinance to the initial loan amount the new loan has maturity T, but we allow agents to refinance to larger or smaller loan amounts than the initial one, with longer or shorter maturities accordingly. Borrowers also have an option to default on the loan. In case of default they lose the house in foreclosure, are excluded from credit markets, and become renters for the remaining time horizon. In addition default carries a utility penalty in the period that the agent defaults equal to λ which can be interpreted as a social stigma cost (Guiso et al. (2013)). Mortgage loans are non-recourse, so lenders have no claim on labor income in the event of default. We model a lower bound on consumption which can be interpreted as arising from social security benefits or other transfers, and which ensures that borrowers' decisions are not dominated by extremely unlikely states with extremely high marginal utility of consumption.

Finally, borrowers with positive home equity have the option to sell their house, prepay their loan, and become renters for the remaining time horizon. We assume that the house rented is the same size as the one previously owned. The rental cost is equal to the user cost of housing plus a rental premium of ε . We follow Campbell and Cocco (2015) and define the date t rental cost RC_{it} for a house of size H_{i,t_i} as:

$$RC_{it} = [R_{1t} - E_t[\exp(\Delta p_{t+1}^H) - 1] + \tau_p + m_p + \varepsilon]P_t^H H_{i,t_i},$$
(9)

where R_{1t} is the one-period real-rate, $E_t[\exp(\Delta p_{t+1}^H - 1)]$ is the expected real house price change from period t to period t + 1, and τ_p and m_p are the property tax rate and the proportional housing maintenance cost respectively.

A.4. Supply of mortgage loans

An infinite-horizon representative lender originates loans at the initial date when agents enter the economy, and in later periods when there is refinancing. In periods subsequent to loan origination, the lender receives the mortgage payments, unless borrowers decide to default or to refinance. In case of default, the lender takes possession of the house and sells it in the same period at current prices, and receives this amount net of foreclosure costs. In case of refinancing, the lender receives the balance outstanding on the current mortgage and writes a new mortgage contract with a new principal amount.

The loan premium compensates the lender for default, prepayment, and the costs of originating and servicing loans. It depends on the type of loan and on the state of the economy at the time that the loan is originated, and it reflects differences in expected default/prepayment behavior and in discount rates.

We also model the possibility that the lender uses tighter lending criteria in recessions or has tighter criteria imposed on it by regulators. We do this by specifying a maximum LTV, denoted LTV^{max} , and setting this maximum lower for loans originated in bad times than for loans originated in good times. This makes it more difficult for borrowers to refinance their loans and extract home equity during recessions. We assume a competitive market for the supply of loans and solve for the loan premia demanded by the representative lender.

Preferences. The representative lender has power utility and an infinite-

horizon. This contrasts with our modeling of borrowers as finite horizon agents. We use the subscript l to denote variables of the lender problem. The lender chooses non-durable consumption to maximize

$$E_t \sum_{t=0}^{\infty} \left[\beta_l^t \frac{C_{lt}^{1-\gamma_l}}{1-\gamma_l} \right] \tag{10}$$

where β_l denotes the degree of time preference and γ_l is the coefficient of relative risk aversion.

Income. In addition to a portfolio of mortgages, the lender is endowed with a diversified portfolio of other assets (and it has no debt outstanding). This portfolio of assets provides an infinitely-lived income stream that we denote by Y_{lt} , or $y_{lt} \equiv ln(Y_{lt})$. It is subject to permanent shocks so that:

$$y_{lt} = \bar{y}(I_t) + q_{lt},\tag{11}$$

where \bar{y} denotes the average log income level, which may be different in recessions/expansions, and $q_{lt} \equiv ln(Q_{lt})$ is an aggregate permanent component:

$$q_{lt} = q_{l,t-1} + v_{lt}.$$
 (12)

Shocks to the lender's log permanent income, v_{lt} , are assumed to be normally distributed with mean $-\frac{\sigma_v^2}{2}$ and variance σ_v^2 . The Jensen's adjustment term for the mean ensures that the expected income level, Y_{lt} , does not grow over time.

It is important to contrast our modeling of the lender as an infinite horizon agent with constant expected income, and of individual borrowers as finite horizon agents, facing an expected increasing income profile, and mortgage debt outstanding. The permanent shocks that the income of the lender is subject to contribute to generating volatility in the stochastic discount factor. In their absence, the infinite-horizon lender would accumulate sufficient savings to smooth consumption over time. We assume that the representative lender cannot borrow against its future income.

Mortgage cash flows. As in Beraja et al. (2018), the lender holds the portfolio of mortgages and in each period t receives the corresponding stream of cash flows (denoted CF_t). They are equal to the sum of the mortgage cash flows received from all the borrowers. Naturally, they depend on borrowers' decisions, and can be negative if many borrowers decide to refinance their loans and to draw down additional home equity, or positive if most borrowers simply decide to make their mortgage payments. In the benchmark scenario, mortgage cash flows are higher in recessions than in expansions, primarily because borrowers are much more likely to remortgage and extract home equity in good times. Although recessions increase current mortgage cash flows, they reduce the lender's overall wealth since the loan portfolio shrinks as a result of default.

Mortgage cash flows $CF_t(I_t, R_{1t}, LoanType, D_t)$ depend on the aggregate state of the economy (the business cycle I_t and the level of interest rates R_{1t}), on the type of mortgage that is being modeled ($LoanType \in \{FRM, ARM\}$), and on the amount of outstanding loans at the beginning of period t (denoted by D_t):

$$D_t = \sum_{i=1}^N D_{it} \tag{13}$$

where D_{it} denotes the beginning of period t outstanding loan balance of borrower i and N is the total number of borrowers in the economy. For the FRM, in addition to the above variables mortgage cash flows also depend on the proportion of borrowers that took a loan when interest rates were low/high (as this affects the interest rate on the loans and the cash flows from the mortgages currently outstanding).

In order to generate volatility in the stochastic discount factor, we have modeled the income of the infinite-horizon lender as being subject to permanent shocks. However, without further assumptions, this would mean that the income could grow over time, and the mortgages become an infinitesimal part of the lender's portfolio (similarly, the income of the lender could decline over time, and the mortgages become essentially the whole portfolio of the lender). In order to prevent these degenerate outcomes, and to ensure the stability of the system, we make the assumption that the size of the mortgage market scales up/down with the permanent income of the lender. In other words, we assume that the mortgage cash flows received by the lender (CF_{lt}) are equal to:

$$CF_{lt} = CF_t \times Q_{lt} \tag{14}$$

Similarly, the amount of loans outstanding also scales up and down with the permanent income of the lender, so that $D_{lt} = D_t \times Q_{lt}$. The assumption that the permanent income of the lender and the size of the mortgage market grow at the same rate is also important for tractability. It implies that the problem of the lender can be scaled by its permanent income, allowing us to reduces the dimensionality of the model. We explain further below, when we describe the model solution. In each period, the total income received by the lender is the sum of the income from its diversified portfolio of assets Y_{lt} (other than the mortgages) and the cash flows from mortgages CF_{lt} .

A.5. Equilibrium mortgage premia

We use the equilibrium consumption choices of the representative lender to derive the stochastic discount factor and the discount rates that we use to calculate the expected present discounted value of the loan cash flows. That is, in a first step we calculate the stochastic discount factor (denoted M) using

$$M_{t,t+1} = \beta_l \left(\frac{C_{l,t+1}}{C_{lt}}\right)^{-\gamma_l}.$$
(15)

For a loan initiated at t, and given C_{lt} and the values for the state variables of the lender problem, it is straightforward to use (15) to calculate the time t discount rates for time t + 1 cash flows. These discount rates depend through consumption on the values of the state variables of the lender problem (including the real rate and the recession/expansion indicator, that are also state variables of the borrower problem and that affect default and prepayment behavior).

Given the multi-period nature of the loans, we also need discount rates for cash flows that occur further out in the future. That is: for a loan originated at t, we need not only $M_{t,t+1}$ but also $M_{t+1,t+2}$ and so on up to $M_{t+T-1,t+T}$ (the initial loan maturity is T). In order to derive these, we first calculate, for each combination of recession/expansion and of low/high interest rate, the consumption distribution of the representative lender. We then use the probabilities of future recessions/expansions, of low/high interest rates, and the associated consumption distribution, to calculate the discount rates for the future periods. Naturally, the probabilities of future recession/expansion and of low/high interest rate depend on the state of the economy at time t, when the loan is initiated.

With the probabilities of future recession/expansion and low/high rate, the corresponding discount rates, and mortgage cash flows, we are able to calculate the expected discounted value of future cash flows for loans originated at time t. When comparing across mortgage contracts, we determine loan premia endogenously to equate the expected discounted value to a given break-even value.

Equilibrium mortgage premia could in principle depend on all of the state variables of the problem. However, given their large number, this would be intractable. Therefore, for most of the cases considered, we assume that mortgage premia depend only on the loan type (ARM, FRM, ARM with an option to extend maturity, and FRM with an option to convert to ARM), on loan parameters (such as initial LTI and LTV), and on the state of the business cycle at loan origination.

The calculation of equilibrium mortgage premia requires that we solve for a fixed point: we iterate on candidate loan premia until the mortgage cash flows and the lender's optimal consumption choices and associated stochastic discount factor yield present values of loan cash flows and mortgage premia that match those in the solution of the borrowers' problem. Since borrowers may default/refinance their loans in expansions or recessions, borrowers' choices, mortgage cash flows and the loan premium for loans initiated in a recession depend on the loan premium for loans initiated in an expansion (and vice versa). This means that we need to solve simultaneously for the recession and expansion premia. Given the large number of state variables, this is computationally very intensive.

In summary, for each type of loan that we consider, we follow an iterative procedure with the following steps:

- 1. Make an initial guess for the recession/expansion mortgage premia.
- 2. Given mortgage premia, solve for the borrowers' optimal choices.
- 3. Using the borrowers' optimal choices, obtain the aggregate mortgage cash flows and the value of the outstanding loans, CF_t and D_t , respectively.
- 4. With the mortgage cash flows and the value of the outstanding loans as an input, solve for the lender's optimal consumption choices, and use them to calculate the stochastic discount factor/discount rates.
- 5. Use the discount rates and the expected loan cash flows to calculate the expected present discounted value of the originated loans.
- If the present value is higher (lower) than the break-even value, decrease (increase) the mortgage premia and solve for the borrowers' optimal choices.

7. Repeat until the mortgage premia, the associated borrowers' choices, and the discount rates calculated from the lender's optimal consumption choices yield a present discounted value for the mortgage cash flows that is equal to the break-even value.

B. Model timing and solution

B.1. Timing, choice and state variables

The timing of the problem is such that at the beginning of each period t the state of the economy (I_t) , interest rates (R_{1t}) , house prices (P_t^H) and labor income of borrowers (Y_{it}) and of the representative lender (Y_{lt}) are realized.

Borrowers' problem. We define borrower cash-on-hand in period t, denoted X_{it} , as the sum of the beginning of period financial assets and realized income. The remaining state variables of the borrower problem are the level of permanent income Z_{it} , the level of debt outstanding/loan period D_{it} , the loan premia ψ_{it} (equivalently, the state of the economy when the agent's mortgage was originated), and whether the agent has previously moved to the rental market I_{it}^R . We denote the state space for borrower i at time t by $\Omega_{it} \equiv \{I_t, R_{1t}, P_t^H, X_{it}, Z_{it}, D_{it}, \psi_{it}, I_{it}^R\}$.

The level of debt, D_{it} , and loan premia, ψ_{it} , pin down home equity and the mortgage payments due (for the ARM). For the FRM we also need to keep track of the level of interest rates at mortgage initiation since this determines the loan rate and required mortgage payments (thus the set Ω_{it} has one additional state variable in the FRM case). For both contracts, loan premia are endogenously determined at origination and remain unchanged until loan termination.

After the realization of the random variables is observed, borrowers decide whether to make the scheduled mortgage payments, refinance, default, or prepay the loan. If they refinance, borrowers need to decide the new loan amount, subject to the prevailing downpayment constraint. In addition, they decide in each period their consumption of non-durable goods. The problem is simpler for borrowers who have previously defaulted, and need only choose how much to consume and save in each period.

We set up the problem recursively and define two distinct value functions: V is the value of repaying the loan or refinancing and V^R is the value of moving to the rental market (either through default or through mortgage prepayment). If the agent has a loan outstanding the Bellman equation is given by:

$$V_{it}(\Omega_{it}) = \max\{U(C_{it}) + \beta E_t \max[V_{i,t+1}(\cdot), V_{i,t+1}^R(\cdot)]\},$$
(16)

where V^R denotes the value obtained from moving to the rental market. The Bellman equation for an agent in the rental market is given by:

$$V_{it}^{R}(X_{i,t}, Z_{it}, I_t, R_{1t}, P_t^{H}) = \max\{U(C_{it}) + \beta E_t V_{i,t+1}^{R}(\cdot)\}.$$
 (17)

In periods when the agent does not move to the rental market and does not refinance his or her loan, cash-on-hand evolves according to:

$$X_{i,t+1} = [X_{it} - C_{it} - L_{it}^{LoanType} - PC_t + \phi R_{it}^{LoanType} D_{i,t-1}](1 + (1 - \phi)R_{it}) + (1 - \phi)Y_{i,t+1},$$

where $LoanType \in \{FRM, ARM\}$. Cash-on-hand in period t + 1 is equal to cash-on-hand in period t, minus consumption (C_{it}) , mortgage payments $(L_{it}^{LoanType})$, property maintenance and tax costs (PC_t) , plus the interest tax shield, the interest on savings and realized labor income (net of income taxes).²

If the agent decides to tap into home equity through loan refinancing, he or she must choose a new loan amount D'_{it} , prepay the outstanding amount of the old loan (D_{it}) , and pay a refinancing cost of Θ_R . In such a situation cash-on-hand evolves according to:

$$X_{i,t+1} = [X_{it} - C_{it} - (1 + R_{it}^{LoanType})D_{it} + \phi R_{it}^{LoanType}D_{i,t-1} + D'_{it} - PC_t - \Theta_R]$$

(1 + (1 - \phi)R_{1t}) + (1 - \phi)Y_{i,t+1}. (19)

The choice of the new loan amount (D'_{it}) is subject to a LTV constraint such that $D'_{it} \leq LTV_t^{max}P_t^HH_{i,t_i}$, where we allow LTV^{max} to depend on the business cycle I_t .

If the agent has positive home equity, he or she can decide to move to the

²Property maintenance and tax costs are a proportion of the house value, i.e., $PC_t \equiv (m_p + \tau_p(1-\phi))H_{i,t_i}P_t^H$. We give further details in the parameterization section.

rental market. Such a decision happens at the beginning of the period. In such a case the agent receives the net proceeds from selling the house (net of transaction costs θ_c) minus the outstanding loan amount (which is prepaid). The law of motion for cash-on-hand is:

$$X_{i,t+1} = [X_{it} - C_{it} + (1 - \theta_c)P_t^H H_{i,t_i} - (1 + R_{it}^{LoanType})D_{it} + \phi R_{it}^{LoanType}D_{i,t-1} - RC_{it}](1 + (1 - \phi)R_{1t}) + (1 - \phi)Y_{i,t+1}(20)$$

i.e. agents receive the net proceeds from selling the house but need to start paying the rental cost RC_{it} . Finally, cash-on-hand for agents already in the rental market or for agents who default is given by:

$$X_{i,t+1} = [X_{it} - C_{it} - RC_{it}](1 + (1 - \phi)R_{1t}) + (1 - \phi)Y_{i,t+1}.$$
 (21)

Lender's problem. The cash-on-hand of the lender, denoted X_{lt} , is equal to the beginning of period financial savings plus realized income and mortgage cash flows. The state space of the lender at time t is defined by $\Omega_{lt} \equiv$ $\{I_t, R_{1t}, X_{lt}, Q_{lt}, LoanType, D_{lt}\}$. For the FRM loan we also need to keep track of the proportion of existing borrowers who took a mortgage when interest rates were low/high, so that the problem has an additional state variable. We define the lender's problem recursively:

$$V_{lt}(\Omega_{tl}) = \max \frac{C_{lt}^{1-\gamma_l}}{1-\gamma_l} + \beta_l E_t V_{l,t+1}(.)$$
(22)

In each period, the lender chooses consumption/saving subject to the budget

constraint, the stochastic processes for income and for the mortgage portfolio, and subject to non-negativity constraints on consumption and on cash-onhand:

$$X_{l,t+1} = (X_{lt} - C_{lt})R_{1t} + Y_{l,t+1} + CF_{l,t+1}(I_{t+1}, R_{1,t+1}, LoanType, D_{l,t+1})$$
(23)

$$C_{lt} \le X_{lt} \tag{24}$$

$$C_{lt} \ge 0, X_{lt} \ge 0 \tag{25}$$

The beginning of period t + 1 cash-on-hand is equal to financial savings from the previous period, that are invested in one year treasuries, plus the lender's income and the cash flows from the mortgage portfolio. Since permanent income follows a random walk, the problem above is not stationary. We can make it stationary, and eliminate one state variable from the lender's problem (Q_{lt}) , by scaling the variables by permanent income: $\tilde{X}_{lt} \equiv \frac{X_{lt}}{Q_{lt}}$, $\tilde{C}_{lt} \equiv \frac{C_{lt}}{Q_{lt}}$, $\tilde{CF}_{lt} \equiv \frac{CF_{lt}}{Q_{lt}}$ and $\tilde{D}_{lt} \equiv \frac{D_{lt}}{Q_{lt}}$

Given a stream of cash flows from the mortgages (CF_{lt}) and a transition matrix for the sum of loan principal outstanding (a transition matrix for D_{lt}), we can solve the lender's infinite-horizon problem using value function iteration. We approximate the distribution of D_{lt} using five equidistant points and define a transition for it that depends on the business cycle, the level of interest rates and the type of mortgage. The solution to this problem gives the optimal consumption responses of the lender as a function of the state variables of the problem.

B.2. Numerical solution and simulated data

We provide a brief description of the numerical solution methodology, and give further details in Appendix IA.A. We solve the borrowers' finite-horizon problem by backwards induction, for given loan premia and maximum LTV. We use the optimal policy functions, four hundred different paths for the aggregate variables (recession/expansion, house prices and interest rates), and the realizations of individual earnings to generate simulated data, over a forty-year period.

In each period a new set of borrowers enters the economy and stays in it for twenty years. We discard the first twenty periods as burn-in and calculate the statistics for the last twenty periods of our simulated economy. This ensures that in each period a new set of agents enters our economy at the same time that a set of agents drops out from our sample. For each aggregate state and at each point in time there are 550 agents in our data (i.e. 25 agents for each age cohort).

In reality, a smaller number of loans for house purchase are originated in recessions than in expansions. To capture this in the model, we generate simulated data for an equal number of agents entering the economy in each period, but assume that if they enter the economy in a recession, only a fraction of them, those with higher initial income, buy a house. This may be interpreted as affordability constraints preventing some individuals from taking a mortgage and becoming homeowners. We give further details in the parameterization section.

We use the simulated data to calculate the cash flows from the mortgages (CF_{lt}) , how they depend on the state variables, and the associated transition probability matrix for the outstanding debt (D_{lt}) . These are an input for the lender's infinite-horizon problem, which we solve by value function iteration until convergence.

We use the lender's optimal consumption choices to derive a pricing kernel and the discount factors that allow us to calculate the expected present discounted value of the cash flows of the loans initiated in a recession/expansion. We generate simulated data for the different experiments that we carry out, but the realizations for the random variables are the same throughout so that different experiments are comparable.

C. Alternative mortgage structures

We study two main alternatives to the standard mortgages described so far. First, we augment ARM loans with a maturity extension option that gives borrowers the choice to pay only interest and to extend loan maturity during recessions. We allow all borrowers, including those with negative home equity, to take advantage of this option if they wish to do so. There is no monetary cost of exercising the option. If borrowers extend maturity, debt service temporarily comprises only interest, with principal repayments restarting the following period and loan maturity extended by one period. For multi-year recessions, borrowers choose whether to exercise the option in each of the recession years.

In this extended model there is an additional choice variable in recession

years, the borrowers' decision of whether to extend maturity. The model solution does not require an additional state variable. The mortgage payments for the maturity extension ARM ($L_{it}^{\text{Mat Ext}}$), in periods in which borrower *i* exercises the option include only interest so that

$$L_{it}^{\text{MatExt}} = D_{it} R_{it}^{\text{MatExt}},\tag{26}$$

where R_{it}^{MatExt} denotes the endogenously determined interest rate on the maturity extension ARM. Since loan principal repayments are postponed, the option to extend loan maturity provides cash-flow relief to borrowers during bad times. We solve endogenously for the equilibrium mortgage premium required by the representative lender when all borrowers have the option to extend maturity. For tractability, we consider each loan type (baseline ARM and maturity extension ARM) separately, i.e. we do not model an economy where both loan types co-exist in equilibrium and borrowers choose between the two.

A second main alternative mortgage structure we study combines the FRM loan with an option that allows borrowers to costlessly switch during recessions to an ARM loan with the same level of principal outstanding. Since interest rates are more likely to be low during bad times, the switch to an ARM allows borrowers to take advantage of low rates to reduce their required mortgage payments.

Both these changes to mortgage structure have an impact on the lender's mortgage cash flows and optimal consumption choices. We take these into account when we solve for the corresponding equilibrium mortgage premia.

D. Parameterization

We use several data sources and estimates from the literature to parameterize our model. In this section we briefly describe these sources and give further details in Appendix IA.B. Tables I and II summarize our parameter choices.

D.1. Aggregate state variables

We use NBER business cycle dates to parameterize our model. The unconditional probability of a recession is 0.22. The conditional transition probabilities capture the persistence in the aggregate state of the economy. Expansions are more persistent than recessions: the probability that an expansion continues from one period to the next is 0.82, while the probability that a recession continues is only 0.37 (as reported in Panel A of Table I).

Panel B summarizes our parameter choices for real interest rates. We calculate the expected real interest rate using quarterly data on 1-year nominal Treasury bond yields and on expected inflation from the Michigan survey from 1977Q4 to 2014Q3. Over the whole sample period the real interest rate was on average higher in recessions than in expansions: 1.59% compared to 2.44%, respectively. However, this was driven mainly by the recessions of the early 1980s. If one focuses on the period after 1985 the average real interest rate was on average higher in expansions than in recessions: 1.12% compared to 0.04%, respectively. The unconditional mean over this period was 1% and the standard deviation was 2.5%.

In our model the real interest rate can either be low or high. We set

the unconditional probabilities of low and high rates to be equal, so for a mean of 1% and a standard deviation of 2.5% the two possible values for the real rate are -1.5% and 3.6%. We adjust the conditional probabilities of low and high rates to match the post-1985 means during expansion and recession, which implies a 0.48 probability of a low rate in an expansion, and a 0.62 probability in a recession. This real interest rate process inherits the persistence of the business cycle variable.

Panel B of Table I also reports our parameter choices for house prices. We match the unconditional mean and standard deviation of log house price changes from Campbell and Cocco (2015). To parameterize the relation between house price changes and the aggregate state of the economy we use Case-Shiller house price data between 1981 and 2014. In our model house prices can either increase or decrease by 16% each period. We calculate the conditional probabilities of house price declines in expansion and recession to match the average house price increase of 1% in an expansion and decline of 3% in a recession observed in the S&P/Case-Shiller US National Home Price Index data. During an expansion, the probability of a house price increase is 0.52, whereas this probability is only 0.39 in a recession.

D.2. Mortgage and housing parameters

Panel C of Table I reports the parameters we use to model the mortgage and housing markets. In the base model the initial loan-to-income (LTI) ratio is constant at 3.5. The maximum LTV is 0.9 for loans initiated in expansions and 0.8 for loans initiated in recessions. These constraints restrict agents' ability to refinance their loans. We assume that all new agents entering the economy take out loans with maximum LTV and therefore set downpayments for entering agents equal to the minimum values of 0.1 and 0.2 implied by the maximum LTVs. In Section VI we solve our model for other values for these parameters.

Mortgages have an initial maturity of 20 years, shorter than the 30-year maturity of the typical US mortgage. We make this choice for computational tractability, as the size of the state space of the borrowers' problem grows exponentially with the number of model periods.³ However, the main effect of the assumed maturity is on the debt burden and we can vary this by considering alternative values of the initial LTI ratio.

In reality, fewer people buy houses in recessions than in expansions. We have used HMDA data from 2004 to 2014 to calculate the value of mortgages originated for house purchase in each year. The ratio of originations in recession years to originations in expansion years is 0.65. In the model we assume that only this fraction of the agents that enter our economy in a recession buy houses. Homebuyers are the agents with the highest income; other agents move to the rental market and stay in it for the remainder of their lifetime.

There are no charges to borrowers for prepaying mortgages, but refinancing incurs a cost equal to 1.5% of house value (Chen et al. (2020)). There is a 6% commission paid on house value when a mortgage is prepaid and the house is sold. In event of default, there is a loss to lenders of 27% of the

³We have considered an alternative, of modeling 15 periods and letting each model period be 2 years, so that mortgage maturity would be 30 years. However, this alternative implies that the minimum duration of a recession is counterfactually long at 2 years.

value of the house, reflecting a combination of the costs of selling the house and foreclosure losses (Campbell et al. (2011)).

Finally, our model has three housing parameters: property taxes at 1.5% of value per year, maintenance expenses at 2.5% of value per year (both taken from Campbell and Cocco (2015)), and a rental premium of 1%. In Section VII and in Appendix IA.C we perform robustness to several of the parameters shown in Table I.

D.3. Preferences and labor income process of the borrowers

Panel A.1 of Table II reports the borrowers' preference parameters. We set the subjective time discount factor to 0.98, the coefficient of relative risk aversion to 2, and the bequest parameter b so that agents in our model accumulate financial savings at a rate similar to that observed in the data. More precisely we target a terminal value for financial wealth that roughly matches the average level of \$20,400 observed for individuals aged between 35 and 44 in the 2013 wave of the Survey of Consumer Finances (SCF).

Beyond the financial implications of default, our model assumes that default creates disutility for borrowers through a "stigma" effect (Guiso et al. (2013)). In the base case we set the value for the stigma parameter λ to 0.1 so that average mortgage default rates generated by the model match those in the data (discussed in Appendix IA.B). A high value for stigma reduces the incentives for individuals to default for strategic reasons, so that when default happens it is more likely to occur when borrowers have low income. Ganong and Noel (2020) use monthly administrative data linking income and mortgage default to study the motives behind default decisions, distinguishing between strategic and adverse cash-flow events. They find that almost all defaults are associated with cash-flow shocks, and their data are consistent with a model with a stigma parameter of the sort we assume.

We take the earnings process parameter values from Guvenen et al. (2014), and report them in the appendix. We assume a flat income tax rate of 20%, with mortgage interest tax deductible at this rate.

D.4. Preferences and income process of the lender

We assume that the representative lender has the same preference parameters as borrowers do: a time discount factor of 0.98 and a risk aversion coefficient of 2. However, the representative lender differs from borrowers along several important dimensions. First, the lender has an infinite horizon, compared to the finite horizon of borrowers. Second, individual borrowers face an increasing income profile that creates a motive to borrow, and they have outstanding mortgage debt while the lender has no debt.

We calibrate the income process of the representative lender so that the model generates a more volatile stochastic discount factor and a higher Sharpe ratio in recessions than in expansions, and ARM mortgage premia that match the data.

We have obtained monthly data on ARM effective rates from the Federal Housing Finance Agency, covering the period 1986–2008 (the series was discontinued at this point). From these effective rates, we subtract the one-year bond yield and calculate the average mortgage premia during NBER recession and expansion months. The values are 3% and 1.7%, respectively.⁴

⁴If we use the Freddie Mac interest rate survey, the corresponding ARM recessions

We set the ARM premium in the model to 3% in a recession and 1.7% in an expansion, and choose the ratio of the average lender income in expansions to recessions so that the net present value per loan for loans initiated in recessions and expansions is the same. The net present value obtained in this way is the base case. The implied lender income process has a ratio of average income in expansions to average income in recessions of 2.7, and a standard deviation of permanent income shocks of 10%.⁵ Panel A.2 of Table II reports these parameters.

The parameter in the last row of Panel A.2 measures the importance of the mortgage cash flows relative to the income that the lender has from other assets. One possible way to interpret the lender in our model is as a financial intermediary. We obtain data from the Federal Reserve Board, from 2004 to 2018, on the value of real estate loans in the balance sheets of commercial banks. The ratio of the value of these loans relative to other bank assets is 19.4%, so we set the unconditional mean of mortgage cash flows relative to the income from other assets equal to 20%. This means that in our model mortgages are a significant part of the lender's wealth; they have an impact on its consumption choices and the equilibrium stochastic discount factor (which would not be the case if the mortgages were marginal, i.e. an infinitesimal part of the lender's wealth). This parameter also provides a scale between the borrowers' and lender side of the model.

and expansions premia are 2.7% and 1.4%, respectively.

⁵We have also investigated counter-cyclical income risk as a potential source of the dispersion in Sharpe ratios across the cycle, but its quantitative effects are significantly smaller than the drop in average income in recessions.

The average lender income level that leads to an expected ratio of mortgage cash flows to income of 20% depends on the equilibrium choices of borrowers. We determine this income level for the base case plain-vanilla ARM contract by solving for an additional fixed point. More precisely, we iterate on the income level so that, in equilibrium, the ratio of mortgage cash flows to lender income is 20%. The lender's income level remains at the same value in all the experiments that we carry out, even if a change in the type of mortgage contract leads to a change in the expected ratio of mortgage cash flows to income. This ensures that all policies are comparable to the benchmark plain-vanilla ARM contract.

D.5. Asset pricing moments

In Panel B of Table II we report values for some of the equilibrium asset pricing moments and other quantities generated by our model. In Panel B.1 we do so for the case in which the mortgages are marginal in the lender's portfolio. This is simply to help us understand the role that mortgage cash flows play in the model. The first two lines report the expected values for the inverse of the stochastic discount factor conditional on low/high interest rate and recession/expansion. We see that their values are equal to the level of the risk-free rate in expansions, but are somewhat higher in recessions. This happens because there is a small proportion of observations, equal to 0.8% of the total, for which our representative lender is borrowing constrained. All of these observations correspond to recession periods, and to high values of the inverse of the stochastic discount factor.⁶ They may be interpreted as times of stress or scarcity of capital in the financial system, among the agents providing the loans.

The next two rows show that the volatility of the stochastic discount factor and the maximum Sharpe ratios are roughly twice as large in recessions than in expansions. And the last row of Panel B.1 reports the average ratio of the lender cash-on-hand relative to income. The value is higher in recessions than in expansions, but this is because of the fall in income in bad times.

In Panel B.2 of Table II we report the model results for the baseline ARMs scenario, i.e. for the case in which the mortgages are a significant part of the lender's portfolio. The mortgages play a stabilizing role in the lender's wealth, and contribute to an increased ability to smooth consumption. This can be seen in several ways. First, the lender accumulates less cash-on-hand due to a reduced precautionary savings motive. This in turn leads to an increase in the proportion of observations for which the lender borrowing constraint binds, which is now equal to 2% of the total. Second, in spite of the reduced savings, the volatility of the stochastic discount factor and the maximum Sharpe ratio are slightly lower.

D.6. Default in the model versus the data

Before we evaluate alternative mortgage structures, we briefly compare to the data some model predictions that were not directly targeted in our calibration. We focus on default, given its importance for our analysis, and

⁶The expected values of the inverse of the stochastic discount factor calculated without these observations are always equal to the risk-free rate.

in particular on time-series and cross-sectional patterns in default since the overall average default rate is targeted by our choice for the value of the stigma parameter.

The model endogenously generates a pattern for the dispersion in default rates across the cycle that is similar to that observed in the data, with higher default rates in recessions than in expansions. For our baseline ARM contract, the default rate is 0.018 in recessions and 0.012 in expansions Therefore, default in our model is counter-cyclical with a difference between expansions and recessions of -0.006. This dispersion is not targeted by our parameter choices; it is due to the income and house price shocks that borrowers face and the endogenous model quantities and choices.

In the data, there is considerable variation in both the average level and the cyclical dispersion of default rates that depends on the particular default measure (charge-off rates, mortgage delinquency, foreclosure rates), the type of mortgage, and the sample period. For instance, Federal Reserve data on charge-off rates on residential mortgage loans, available since the first quarter of 1991 and described in Appendix IA.B, are equal to 0.009 in recession years and 0.004 in expansion years. The degree of cyclicality is -0.005.

Data from the Mortgage Bankers Association show foreclosure rates for prime ARMs (for subprime ARMs) of around 0.005 (0.02) in the expansion years before the Great Recession, rising to around 0.02 (0.06) during the Great Recession. This is a higher degree of cyclicality than in our model, but the fairly high foreclosure rates during the Great Recession are not representative of those of previous recession years.

The model also generates predictions for cross-sectional variation in de-

fault with LTV and with residual income (defined as the income left after making required mortgage payments, as in Gerardi et al. (2015)), that resemble those in the data. Figure 1 plots the model's implied default rates for different LTV ranges and levels of residual income, measured at the point of default. It shows that the probability of default is increasing in LTV and decreasing in residual income. These patterns are similar to those shown by Gerardi et al. (2015).

III. Mortgage Structure and Macroeconomic Stabilization

In this section we focus on the implications of alternative mortgage structures for macroeconomic stabilization, and in section IV we characterize the use that the agents make of the options. In both sections, we first evaluate the impact of augmenting an ARM with an option to extend maturity during a recession, and then turn our attention to a FRM with an option to convert to an ARM during a recession.

A. Alternative adjustable rate mortgages

A.1. Baseline ARMs

We first describe the results for baseline ARMs, the benchmark mortgages to which we compare all other mortgage types.

Table III is the main results table. The first two columns report the results for baseline ARMs, without an option to extend loan maturity. The first column reports the mean values for several variables of interest, and the second column reports their cyclicality (the difference between the values in expansions and those in recessions). The first row shows that the average ARM loan premium is 0.020 and is countercyclical with a cyclicality of -0.013. The second row shows that the average default rate is 0.013 and is again countercyclical with a cyclicality of -0.006.

Individual borrowers experience positive and procyclical income growth, 0.027 on average with a cyclicality of 0.046. The fourth row of Table III shows that individual borrowers' consumption growth is on average higher and more procyclical than their income growth, 0.040 on average with a cyclicality of 0.073. This is due to the effects of leverage and fluctuations in collateral value. When a recession hits and income drops, levered agents are forced to cut consumption proportionally more than income to meet their mortgage payments.

Even though individual borrowers face expected income and consumption growth while they are in our economy, in each period a new set of agents enters the economy and another drops out, so aggregate income and consumption of borrowers in our model are stationary. In other words, the log change in the aggregate consumption of borrowers is on average zero, but its cyclicality is similar to that of the log change in individual borrower consumption.

The next two rows of Table III report lender side variables. The unconditional mean of the lender's cash-on-hand to income is equal to 2.363, with a cyclicality of -2.030. The countercyclicality of this ratio largely reflects the fact that the denominator of the ratio increases in good times. The consumption of the representative lender is strongly procyclical with a cyclicality of 0.107. This reflects our calibration of the lender's income process, needed to generate a volatile stochastic discount factor and realistic cyclicality in mortgage loan premia.

A risk-neutral lender. To illustrate the importance of lender risk aversion, we have calculated model results for baseline ARMs for the case of a riskneutral lender. In this case we calculate the expected present value of the loan cash flows using the risk-free rate for discounting. We then solve for the equilibrium mortgage premia so that the expected present discounted value of the loan cash flows is the same as in the base case.

Appendix Table IA.II reports the results. The equilibrium values are a recession (expansion) premium of 0.016 (0.011), implying a cyclicality of -0.005. Therefore, in the model with a risk-neutral lender there is much less variation in mortgage premia over the business cycle. With risk neutrality, mortgage premia vary only because of differences in the expected cash flows of loans originated at different points in time and because of the correlation between interest rates and the business cycle. In our model with lender risk aversion and an endogenous stochastic discount factor, both expected cash flows and time variation in the volatility of the stochastic discount factor matter for mortgage pricing. The model with a risk-neutral lender also implies default rates that are lower on average at 0.011 and slightly less countercyclical with a cyclicality of -0.005.

A.2. Maturity extension ARMs

In the next two columns of Table III we evaluate an ARM with an option that allows borrowers to pay only interest and extend loan maturity in recessions. This option cannot be replicated through cash-out mortgage refinancing for two reasons. First, the option to extend maturity is available to all borrowers, including those with low or negative home equity. In other words, we assume that the leverage constraint does not apply in the case of maturity extension.⁷ Second, unlike for mortgage refinancing for which there is a loan origination cost, we assume that there is no monetary cost associated with the exercise of the option to extend maturity.

When we introduce the maturity extension option in the model, its exercise by borrowers leads to an increase in the average duration of mortgage loans and a decrease in the number of loans that are originated. The increase in average duration and the reduction in the number of new loans are larger for loans originated in recessions. The duration effect leads to an increase in the net present value per loan: the loans are outstanding for more periods and the lender receives the premium for longer. However, the effects on the total NPV of the lender are much smaller, due to the counteracting effect of the reduction in the number of loans granted. Since we want to take into account the latter effect, when solving for the mortgage premia for maturity extension ARMs, we find the mortgage premia that generate the same unconditional total net present value of the loans as in the base case ARMs.

⁷DeFusco and Mondragon (2020) document frictions to refinancing during recessions in the US mortgage system.

This implies that in the maturity extension scenario the lender has the same total expected net present value of mortgage loans but a higher net present value per loan.⁸

Equating the total net present value of mortgage loans translates, in equilibrium, into a reduction in mean loan premia of 0.002 relative to baseline ARMs. The main reason is a lower probability of default and lender losses in bad times. As can be seen in the second row of the table, default is now procyclical with a cyclicality of 0.011. Default rates decrease in recessions, as borrowers exercise the option to extend maturity rather than default on their mortgages. Some of these borrowers end up defaulting when the recession ends and they need to start making principal repayments once again. Overall (across expansions and recessions) there is no significant change in average default rates relative to the base case. The option to extend loan maturity shifts defaults from recessions to expansions, which may have benefits in stabilizing the macroeconomy, rather than reducing the unconditional average default rate.

The option to extend loan maturity allows agents to defer payments in recessions and in this way to better smooth consumption over the business cycle. During a recession a proportion of 0.61 of borrowers exercise the option (this is the negative of the -0.606 cyclicality in the option exercise

⁸The alternative, of adjusting mortgage premia so that the net present value per loan is the same as in the baseline case, leads to a larger reduction in mortgage premia, particularly during recessions, with additional macroeconomic stabilization benefits. However, in this case the lender's total net present value of mortgage loans would be lower than in the base case.

rate since the option is only available during a recession.) The cyclicality in borrower consumption decreases from 0.073 for the baseline ARMs to 0.065 for the maturity extension ARMs. In addition, there is a small decline in the unconditional mean of individual consumption growth, from 0.040 in the base case to 0.039, due to a reduced precautionary savings motive. This implies that in the maturity extension scenario borrowers' consumption is higher when they enter the economy and they are able to better smooth consumption over the life cycle.

The maturity extension option has two main effects on the lender reported in Table III. First, the lender accumulates significantly more cash-on-hand, 3.163 compared to 2.363 for the baseline case, due to a precautionary savings motive. Second, the lender's consumption is more procyclical, with a cyclicality of 0.137 as compared with 0.107 for the baseline case. This means that some of the consumption stabilization benefits to borrowers are counteracted by the effects on the lender's consumption. However, there still is an aggregate benefit, as the cyclicality of aggregate consumption is reduced from 0.082 in the baseline case to 0.078 for maturity extension ARMs. Naturally, the benefit of aggregate consumption stabilization would be greater if we were to assume that a fraction of the lenders are abroad (so that we would not take them into account in the aggregation).

The last two rows of the table report the welfare gains of the maturity extension option relative to baseline ARMs, in the form of consumptionequivalent variations. Both borrowers and the representative lender are better off, but the gains for borrowers are an order of magnitude larger, equal to 0.015 of annual consumption, compared to 0.001 for the lender. The reason why the lender is better off is that, although mortgages are priced using marginal utility, the lender benefits from infra-marginal lending.

A risk-neutral lender. We have also solved our model for the ARM contract with an option to extend maturity under the alternative assumption of lender risk-neutrality. Appendix Table IA.II reports the results. When the lender is risk-neutral, the stochastic discount factor does not endogenously vary with the type of mortgage contract under consideration; only differences in the timing of cash-flows and default drive loan profitability. We find that the option to extend maturity leads to a tiny increase in loan premia of only 2 basis points, which is explained by a slight increase in average default. More importantly, the maturity extension contract generates positive welfare gains for borrowers and stabilizes their consumption, consistent with the results for a risk-averse lender.

B. Alternative fixed rate mortgages

B.1. Baseline FRMs

We begin our study of FRMs by pricing a standard FRM with no options other than the standard refinancing option and the option to default. Recall that since our model has zero inflation, FRMs in our model should be interpreted as inflation-indexed FRMs and not the commonly observed nominal FRMs. We determine FRM premia endogenously, by iterating on the borrowers' and lender's problems until we find the premia that generate the same net present value as in the baseline ARMs. To facilitate comparison with ARMs, we report FRM premia relative to the short-term interest rate. The results in the first row of Table III show that mortgage premia for FRMs are slightly lower on average and more strongly countercyclical than for baseline ARMs. FRM premia are higher than ARM premia in recessions and lower than ARM premia in expansions. This primarily reflects shifts in the term structure of riskless interest rates, which tends to be upward sloping in recessions and downward sloping in expansions. Since the FRM is a long-term fixed-income security, its yield tends to be higher relative to the short rate during recessions.

FRMs have a similar average default rate to ARMs, but FRM defaults are more strongly countercyclical with a cyclicality of -0.009 compared to -0.006 for baseline ARMs. FRM borrowers in a situation of negative equity and unable to refinance cannot take advantage of the decline in interest rates that tends to occur during bad times, and are therefore more likely to default. Those unable to refinance but able to meet mortgage payments are more likely to need to cut consumption. This explains why FRM borrowers have more strongly procyclical consumption growth with a cyclicality of 0.075 as compared to 0.073 for baseline ARMs.

The higher default rates of FRMs during recessions create losses to lenders that generate higher precautionary savings and more strongly procyclical lender consumption. Despite these macroeconomic disadvantages of FRMs, in our model agents prefer them to baseline ARMs: the welfare gain of a FRM relative to a baseline ARM is equivalent to 0.004 and 0.022 of annual consumption for borrowers and for the lender, respectively.

B.2. FRMs with an ARM conversion option

Eberly and Krishnamurthy (2014) and Guren et al. (2020) emphasize the benefits for macroeconomic stability of switching borrowers from a FRM to an ARM when a recession hits. The switch allows borrowers to benefit from lower rates and mortgage payments. We investigate in the context of our model the benefits of giving borrowers an option to switch to an ARM during bad times. We assume that such a switch is costless and that all borrowers, including those with negative home equity, are allowed to switch. When they do so, they switch to a plain vanilla ARM with the same principal outstanding and mortgage premia as the baseline ARMs, and they stay on the ARM contract for the rest of their lives unless they default.

We note some important differences between our setting and that of Guren et al. (2020). We treat the switch as a borrower option, we incorporate lender risk aversion in our analysis, we model lender consumption choices, and we solve for the equilibrium mortgage premia that generate the same net present value for mortgage loans. On the other hand, we do not solve for equilibrium in the housing market, so that we are only able to capture the first-round effects of mortgage design.

The results in the last two columns of Table III show that borrowers do take advantage of the option to switch to the ARM. Conditional on having a FRM without having defaulted, a proportion of almost 0.55 of borrowers exercise the option (this is the negative of the cyclicality of -0.546 in option exercise since the option is only available in recessions.) The recession periods when borrowers exercise the option are those when interest rates are low.

The optimal use of the option by borrowers imposes significant losses on the lender, precisely at those times when its marginal utility of consumption is at the highest level (in recession periods with a low return on savings). This means that mortgage premia must increase so as to maintain the same net present value of the loans. The average mortgage premium is 0.025 for the FRM with the option compared to 0.019 for the baseline FRM.

The switching option does have a stabilizing effect in the economy during bad times. As Table III shows, the default rate is less countercyclical and the growth rates of borrower and lender consumption are less procyclical than for the plain vanilla FRM. However, all these variables are more strongly cyclical than they are in the case of the maturity extension ARM. In addition, the switching option reduces the welfare of both borrowers and the representative lender once endogenous adjustments in mortgage pricing are taken into account.

C. Interest rate reduction during recessions

Monetary authorities may be able to provide cash-flow relief to borrowers by reducing interest rates during recessions. To further characterize the consumption stabilization provided by the alternative mortgage contracts, we calculate the average log changes in the consumption of borrowers during recessions for different movements of the risk-free rate.

The results are shown in Table IV. The first row reports the results for the base case ARM. The first column reports the average log consumption change when interest rates are low in the period before the recession and they stay low during the recession, the second column when they are high before the

recession and they are reduced in the recession period, and so on. The results confirm the notion that a decline in interest rates during recessions helps ARM borrowers and stimulates their consumption, with a model-implied increase of 0.003. However, this requires that monetary authorities do have the possibility of reducing rates when the recession hits. If rates are already low and stay low in a recession, the consumption of ARM borrowers declines by 0.012.

In the second row of Table IV we report similar statistics for ARMs combined with an interest-only option to extend maturity during recessions. The policy option has a sizeable effect on the consumption of borrowers, reducing the consumption decline when rates are already low at the onset of a recession to only 0.006.

The results for the baseline FRMs are reported in the third row. FRM consumption declines are greater than ARM consumption declines whenever interest rates are low during the recession (whether or not they were already low at the start of the recession). FRM borrowers with insufficient home equity to refinance are unable to take advantage of low interest rates and are forced to cut consumption by more.

The final row of Table IV shows that the FRM with the option to convert to ARM during recessions allows borrowers to take advantage of low interest rates during recessions, since their consumption declines are lower than for the baseline. However, the ARM with an interest-only option has a much lower consumption decline during recessions. This is true regardless of the movements in interest rates that occur during the recession, as shown by Table IV.

D. Summary comparison of mortgage contracts

We briefly summarize the comparison of the mortgage contracts that we have studied, focusing on their implications for macroeconomic stability. The top two panels of Figure 2 show the cyclicality of the default rate and of aggregate consumption, respectively. The blue bars refer to ARMs and the red bars to FRMs. The solid bars refer to the base mortgages, and striped bars to the mortgages with options.

Panel A shows that defaults are much more likely to be low in recessions when borrowers have ARMs with interest-only options (the default rate becomes procyclical in the presence of the option). In comparison, FRMs with switching options are much less effective at shifting default from recessions to expansions.

Panel B shows the cyclicality of aggregate consumption growth (the difference between log consumption in expansions and recessions). Consumption growth is more stable when borrowers have ARMs with interest-only options. The procyclicality of consumption growth is highest for plain vanilla FRMs. And although adding an option to switch to an ARM to the FRM helps to stabilize consumption growth, its procyclicality still is considerably higher than that of the ARM with an interest-only option available in recessions.

The bottom two subplots of Figure 2 report the consumption-equivalent welfare gains for alternative mortgage contracts, for borrowers and for the representative lender, using the plain-vanilla ARM as the base case. The interest-only option generates significant welfare gains for borrowers, and the representative lender is also better off. Borrowers prefer standard FRMs to standard ARMs, but they lose when the option to switch to the ARM is added to the FRM contract due to the increase in equilibrium mortgage premia.

IV. Mortgage Structure and Borrower Decisions

In this section we describe in greater detail how borrowers manage their mortgages under alternative mortgage structures.

A. ARM borrower decisions

The first four columns of Table V report results for the baseline ARMs and the maturity extension ARMs. The different rows report statistics for subgroups of borrowers who default on their mortgages, refinance them, make required mortgage payments, and exercise the option to extend maturity. (Prepayments are relatively rare and are omitted from the table for simplicity.) For each mortgage type, the first and second columns report results conditional on recession and expansion, respectively.

We start by briefly discussing baseline ARMs. Default is relatively rare but more common in a recession (0.018) than in an expansion (0.012). Refinancing is rare in a recession (0.015) but common in an expansion (0.109). Regular mortgage payments are made by a proportion of 0.951 of agents in recessions, and 0.869 in expansions. The remaining borrowers, 0.016 in a recession and 0.01 in an expansion, prepay their mortgages and move to rental housing. The remainder of the table reports summary statistics for the three groups of defaulters, refinancers, and mortgage payers. Defaulters have much higher LTI ratios and their LTV ratios are above one, while refinancers have low LTVs before refinancing that remain relatively moderate after refinancing. Related to this, and although not shown in the table, defaulters have experienced declining house prices while refinancers have experienced recent increases in their house prices (we show these statistics in Appendix Table IA.III). The appendix also shows that defaulters and refinancers have significantly lower average labor income and that they have recently experienced declines in labor income

There are several reasons why mortgage refinancing is procyclical in our model. Borrowers refinance when they have positive home equity after an increase in house prices. This is more likely to be the case in an expansion than in a recession. Furthermore, expected income growth is higher and income risk lower in expansions than in recessions: agents with precautionary savings motives respond by levering more aggressively. Finally, mortgage rates tend to be lower in expansions due to the lower mortgage premium on loans initiated at such times (which more than offsets the effect of a higher short-term interest rate). Borrowers who refinance extract home equity ("cash out"): in expansions, average LTVs increase at refinancing from 0.65 to 0.80.

Pro-cyclical cash-out refinancing increases household leverage at the time that a recession hits. The recession then causes larger declines in consumption, more defaults, and greater losses given default for lenders. This type of borrower behavior has been previously studied by Khandani et al. (2013), who call it the "ratchet effect" on leverage, and Chen et al. (2020).

The next two columns of Table V report results for the ARM with an interest-only option during recessions. The statistics on incidence show that the option to extend maturity reduces the default rate in recessions, from 0.018 in the base case to 0.004, but increases it in expansions. Comparing the characteristics of defaulters to those for the baseline ARM, we see that default now takes place at higher LTI and LTV ratios. The increases in these ratios are significantly larger in recessions than in expansions. The increases in LTI and LTV for agents who default in expansions are in part explained by the fact that those who exercise the option to extend maturity reach the end of a recession with higher leverage than they would have had if the option to extend had not been available and they had made the scheduled loan principal repayments.

Table V shows that those borrowers who decide to extend maturity have higher LTIs than those who decide to pay, but still considerably lower than those who default. Relatedly, in appendix IA.C we show that borrowers who decide to extend maturity have income that is lower than average, but still higher than the income of those who decide to default. The average LTV ratio for extended loans is 0.97, so many of the borrowers who extend maturity would not be able to refinance their loans due to the leverage constraint. In the appendix we also characterize the decisions of borrowers in the period after they have exercised the maturity extension option. We show that the vast majority of them go back to making the regular mortgage payments in case the economy moves into an expansion.

B. FRM borrower decisions

The last four columns of Table V show the characteristics of the borrowers who make each decision for the baseline FRM and the FRM with the option to switch to an ARM. (We provide additional statistics in Appendix Table IA.V.) The refinancing rates for the baseline FRM loans are greater than was the case for the baseline ARMs. FRM borrowers refinance to extract home equity, but also to take advantage of low interest rates. The short-term rates at the time of FRM refinancing are on average 0.8% in recessions and 1% in expansions (shown in Appendix Table IA.V). For ARMs the corresponding values are 1.4% and 2%, respectively.

The last two columns of the table show the results for the FRM with the option to convert to an ARM. The overall proportion of observations for which borrowers use the option 0.211, rising to 0.546 when we restrict the sample to those periods and borrowers who are on the FRM (i.e. they have not switched to an ARM).

Table V shows that those borrowers who exercise the option to switch have average LTVs of 0.87, so these are agents who are unable to refinance due to the LTV constraint. The exercise of the option allows them to lower mortgage payments to income from 0.28 to 0.16, and in this way it provides cash-flow relief (these additional statistics are shown in the appendix Table IA.V). The immediate cash-flow relief is however lower than the relief provided by the ARM with the maturity extension option (which allows agents to lower mortgage payments to income from 0.27 to 0.10). Furthermore, since the option to switch involves a permanent move to an ARM contract, it only provides cash-flow relief in the period of the switch, and not in subsequent recession periods.

The option to switch to an ARM is exercised in recessions when the risk-free interest rate is at its lowest level, of -1.5%. It is at these times that the incentives to switch to an ARM are highest. But it also imposes significant losses for the representative lender: the mortgage cash flows are reduced precisely at those times when the interest on savings are low (and the lender income is also low since it is a recession period). In comparison, for the ARM maturity extension scenario, the average risk-free interest rate in periods when the option to extend is exercised is 0.6%.

V. Alternative Maturity Extension Policies

In this section we consider several variations of the option to extend maturity of an ARM.

A. Permanent option to extend maturity

The first alternative is a permanent option to costlessly extend maturity regardless of business cycle conditions. Under this alternative the option to extend maturity is available both in expansions and in recessions. As before, we solve for the loan premia so that in equilibrium the net present value of the loans is the same as for the baseline ARMs. The first two columns of Table VI show the results.

The permanent option to extend maturity reduces some of the economic stabilization benefits of the standard maturity extension ARM (previously shown in Table III). The default rate is now acyclical, and equal to 0.014 in both recessions and expansions. The average default rate is slightly higher than when the maturity extension option is only available in recessions.

The reason is straightforward. There is a significant increase in the average proportion of borrowers exercising the option. The degree of cyclicality in the fraction of those doing so is -0.078, so that the option is more likely to be exercised in recessions, but the option is also frequently exercised in expansions. Some of those who exercise the option are borrowers who would not gave been able to refinance due to the LTV constraint. When a recession arrives, these more levered agents are more likely to default. However, the recession default rate is still lower than the 0.018 recession default rate of the baseline ARM.

The permanent option to extend mortgage maturity allows borrowers to better smooth consumption over the life-cycle. The mean log change in individual borrowers' consumption with a permanent option is 0.037, compared to 0.039 when the option is only available in recessions. Agents increase their consumption early on in the life-cycle, which translates into fairly large welfare gains, of 0.041 in consumption equivalent units, compared to the plain vanilla ARM. The representative lender also benefits, due to higher outstanding loan amounts and interest received on the loans.

Despite these large welfare benefits, the permanent option to extend maturity is not as effective as the standard maturity extension option at stabilizing consumption over the business cycle. The cyclicality in borrowers' and in aggregate consumption changes are equal to 0.067 and 0.085, respectively, compared to 0.065 and 0.078 for the case when the maturity extension is only available in recessions. In the case of a permanent option, borrowers who exercise the option in expansions enter recessions with more debt outstanding and accordingly are forced to make a greater reduction in their consumption.

B. Permanent option to extend maturity, costly in expansions

The large welfare gains for borrowers of a permanent option to extend maturity are partially due to the reduction in the refinancing costs incurred by agents. In our model, borrowers with positive home equity can access cash, but need to pay a refinancing cost. They want to do so more often in expansions. We interpret these refinancing costs as transactions costs of property valuation and the writing of new mortgage contracts, costs that do not exist when maturity is extended. When agents are given a permanent interest-only option, they increase leverage by exercising the option instead of by refinancing their loans: the unconditional average refinancing rate declines from 0.088 to 0.049.

To investigate further the role of refinancing costs, we have solved the model allowing agents to always extend the maturity of their loans, but assuming that in expansions they incur a monetary cost equal to the refinancing cost (the option is still free in recessions). The results are reported in the third and fourth columns of Table VI.

The cost deters agents from exercising the option in expansions. Its cyclicality is now -0.550, equal to the difference between the 0.057 and 0.607 fractions of agents who exercise the option in expansions and recessions, respectively. This also leads to a reduction in the default rate during recessions and to consumption changes that are less strongly procyclical. Therefore, as one might have expected, the permanent option to extend maturity which is costly in expansions is more similar to the case in which the option is only available in recessions.

C. Costly option to extend maturity in recessions

A natural question to ask is what happens if there is also a cost of exercising the option to extend maturity in recessions (and when the option is not available in expansions). This is the penultimate case reported in Table VI. The option becomes significantly less attractive: the probability that agents exercise it in recessions decreases from 0.606 for the baseline maturity extension scenario to 0.073. The agents who benefited the most from the free interest-only option were cash-constrained ones, who now have to pay a cost to use the policy. The postponing of principal repayments does not give many of them a sufficiently large incentive to incur the cost.

Naturally, since many borrowers do not make use of the option, its stabilization benefits are smaller than when the option is free. The default rate is acyclical and the cyclicality in borrower consumption is 0.072. There are marginal welfare gains for borrowers, equal to 0.001 compared to the baseline ARM.

D. Costless option in recessions but with affordability restriction

In the last two columns of Table VI we assume that it is costless to exercise the option in recessions (as in the base case maturity extension policy), but we impose the restriction that only those borrowers whose ratio of mortgage payments to permanent income (MTI) is above a threshold of 0.15 are allowed to exercise the option.⁹ This additional restriction prevents those borrowers with low MTI, who are more likely to be exercising the option for strategic reasons, from doing so.

The last column of Table VI shows that the MTI restriction leads to a decline in the proportion of those exercising the option in recessions to 0.43 (from 0.61 for the base case maturity extension policy, as shown in Table III). In spite of the option preventing those borrowers who are more likely to exercise it for strategic reasons from doing so, it does not reduce equilibrium loan premia. The representative lender reduces its precautionary savings and accumulates less cash-on-hand, which has a counteracting effect on premia.

Default rates are similar to the base case maturity extension policy. This is a reflection of the fact that those borrowers who are prevented from exercising the option to extend maturity due to the MTI restriction are those who would have been less likely to default in the first place. There is, however, a small increase in the cyclicality of borrowers' consumption, which shows that the policy does have an impact on borrowers. This increase is offset by a decrease in the cyclicality of the representative lender's consumption, so the cyclicality of aggregate consumption, shown in the last row of Table VI, is similar to that of the base case maturity extension policy (shown in Table III).

Overall, the results in the last two columns of Table VI show that a maturity extension option with an MTI restriction is effective in stabilizing the

⁹We use the ratio of mortgage payments to permanent income since the level of temporary income is not a state variable of the problem.

economy. This case is particularly interesting since it resembles some of the private forbearance agreements between lenders and borrowers who face difficulties meeting mortgage payments. It is important to note that we have assumed that the policy is costless to borrowers, and that it does not involve additional costs for the representative lender. But the practical implementation of the MTI restriction would require borrowers' income verification. If the costs of such income verification are substantial, or if borrowers can hide their income from lenders, then making the option freely available without an affordability requirement may be more effective.

VI. Alternative Mortgage Market Specifications

We study the benefits of the maturity extension option (available in recessions for free) for other loan parameterizations, including higher initial LTV in recessions and higher initial LTI, and for other borrower characteristics, including borrowers who suffer from inertia. For each of these cases, we first report the results for the baseline ARMs and then consider the effects of the option.

A. Loan to value and loan to income

In the first four columns of Table VII, we report results for the case of a higher initial LTV in recessions, equal to 0.9. This is also the maximum LTV limit in subsequent recession periods. In this table, in the first two columns we report the results for the baseline ARMs. There is an increase in the

mean loan premium, and in particular in the recession one (as can be seen by the reduction in cyclicality from -0.013 when the maximum recession LTV is 0.8 to -0.026). In addition, there is an increase in the mean default rate, and in particular the recession one (the cyclicality is reduced from -0.006 to -0.008). Also compared to the case of a lower recession LTV shown in the first column of Table III, there is only a marginal increase in the procyclicality of aggregate consumption. This is mainly due to an increase in the cyclicality of lender consumption.

The next two columns of Table VII report the results for the corresponding maturity extension scenario. The option is effective in stabilizing the economy. There is a reduction in the required mean loan premium that yields the same net present value of loans as the baseline ARMs. There is a shift in default from recessions to expansions (and a marginal decrease in its average incidence). Borrowers are better able to smooth consumption over the cycle. The representative lender accumulates more precautionary savings and its consumption becomes more pro-cyclical. However, the cyclicality of aggregate consumption is reduced from 0.083 to 0.077. Finally, the option has welfare benefits, particularly significant for borrowers.

The last four columns of Table VII show the results for a higher initial LTI, equal to 5 (with the maximum LTV in recessions equal to the base value of 0.8). The higher initial LTI could be the result of households buying a larger house or of higher house prices relative to income at the time that they enter the economy and purchase their house. As before, the first two columns report the results for the baseline ARMs, that serve as a benchmark for studying the effects of adding the interest-only option.

The option is beneficial. In addition to shifting default away from recessions, it leads to a small decrease in average default rates. The cyclicality of borrowers' consumption is significantly reduced, but at the expense of an increase in the cyclicality of the representative lender's consumption. Overall, there is a small decrease in the cyclicality of aggregate consumption.

B. Borrower inertia

There is evidence that borrowers are often slow to exercise options. Andersen et al. (2020) show this in an environment where FRMs can be refinanced regardless of income, credit score, or home equity. It has also been documented in the US for prequalified refinancing offers by Johnson et al. (2019) and Keys et al. (2016).¹⁰ Earlier work on this sluggishness includes Schwartz (2006) and Campbell (2006).

With this evidence in mind, in Table VIII we show the results for the case of borrower inertia. We assume that, in each period, with 0.5 probability, borrowers are inattentive, and they simply make mortgage payments. That is, inattentive borrowers do not refinance, default or extend the maturity of their mortgages. However, these borrowers may still be forced to default, if their cash-on-hand is below what they need to meet the mortgage payments (and they are in a situation of negative equity).

Borrower inertia makes, ceteris paribus, the loans more profitable. For the plain vanilla ARMs, the mean equilibrium loan premium that generates the same net present value of loan cash flows is equal to 0.017, compared

 $^{^{10}\}mathrm{See}$ also Agarwal et al. (2016) and Agarwal et al. (2020).

to 0.02 for the case in which there is no borrower inertia (shown in the first column of Table III). In addition, the average default rate is smaller and equal to 0.012 for the case of inert borrowers compared to 0.013 in the absence of inertia.

We calculate the default rate based on all periods with outstanding loans, including periods in which borrowers suffer from inertia. The small impact of borrower inertia on default rates reflects the fact that most borrowers default for cash-flow motives. In addition, our inert households are rational, in the sense that they know that they will be at times inattentive. They react by taking action and defaulting more often in periods in which they are not constrained. They also adjust along other dimensions. For instance, the average log change in individual borrower consumption is 0.042, compared to 0.04 in the absence of inertia. Inert households cut back on consumption early in life due a precautionary savings motive.

The next two columns of Table VIII show the results for the maturity extension ARMs, for the case of borrower inertia. We calculate the fraction of borrowers using the option taking into account the periods for which the loans are outstanding, regardless of whether borrowers are inattentive in the period. Naturally, the incidence of option use is lower than in the case in which borrowers are not inert. In spite of this, the maturity extension option is still useful in stabilizing the economy. It leads to a small decline in the mean default rate and a shift in default from recessions to expansions. Borrower consumption becomes less procyclical, but the benefits in terms of aggregate consumption are small. The welfare gains to borrowers of the option are also smaller than in the absence of borrower inertia, but they are still positive and significant. The representative lender is marginally better off. These results show that the benefits of the maturity extension policy are reduced, but they are still meaningful in the presence of large borrower inertia.

C. Borrowers with higher stigma of default

Ganong and Noel (2020) study the motives behind default decisions, distinguishing between strategic and adverse cash-flow events, and find that almost all defaults are driven by the latter. Therefore, in the last four columns of Table VIII, we report the results for a higher utility cost of default, or stigma, equal to twice the base value. As expected, the default rate decreases, from an average of 0.013 for the base value of stigma to 0.008 for the higher value. The equilibrium mortgage premium also decreases from a mean of 0.020 for the base stigma to 0.017 for the higher utility cost of default. As before, the maturity extension option shifts default from recessions to expansions, and it leads to a smaller drop in aggregate consumption during bad times. It has welfare benefits for borrowers, but lenders are marginally worse off due to the reduction in equilibrium mortgage premia.

D. Multiple borrower types

In the analyses that we have carried out so far, for tractability, we have assumed that there is only one borrower type present in the economy. We now relax this assumption and show results for a case in which there are two borrower types, with different subjective discount factors, simultaneously present in the economy.

We focus on the subjective discount factor, given the work of Corbae and Quintin (2015) and Iacoviello and Pavan (2013) and the natural interpretation of the resulting borrower behavior in terms of credit risk. The parameter values are taken from Iacoviello and Pavan (2013). Patient (impatient) borrowers have a discount factor of 0.999 (0.941) and the fraction of impatient borrowers is 2/3.

The representative lender is able to identify the borrower type when granting the loan (there is complete information), and prices the loans given to the two types differently. The optimal consumption choices of the representative lender, and the stochastic discount factor depend on the mortgage cash flows and outstanding loans that result from the aggregation of the cash flows and loans given to the two borrower types. The case with multiple borrower types is much more difficult to solve since the loan premia and the expected present value of the cash flows of the loans granted to patient borrowers depend on the market conditions for impatient borrowers (and vice versa, through their effects on the pricing kernel). This means that we need to iterate on the problems of each of the borrower types.

Table IX shows the results. The two top panels show statistics for patient and impatient borrowers, respectively, and the bottom panel for the aggregate of the borrowers and the representative lender.

Focusing first on the results for the plain vanilla ARMs, we see that, as expected, impatient borrowers pay higher mortgage premia and default more. Furthermore, since they save less and consume more early in life, their average consumption growth is smaller, and the drops in their consumption during recessions are larger. The last two columns of Table IX show the results for the maturity extension option. Its stabilization benefits are similar to before, confirming that the effects that we have previously identified also hold in an economy with heterogeneous borrowers, with different degrees of impatience (which may also be interpreted as differential credit risk). In terms of welfare, impatient borrowers benefit more, but the gains are significant also for patient borrowers.

VII. Alternative Macroeconomic Environments

We now explore the way in which mortgage market structure interacts with different specifications for the macroeconomic environment.

A. Inflation

We consider the scenario of deterministic 2% inflation. The main results are shown in Panel A of Table X. Positive inflation generates a tilt in the real mortgage payments on nominal mortgages, which become relatively higher in the initial years of the loan. The higher initial real mortgage payments increase the likelihood of loan termination through refinancing: the unconditional probability increases from 0.088 for the case of zero inflation to 0.092 for the 2% inflation scenario. On the other hand, the unconditional incidence of default decreases, although only slightly, from 0.013 for zero inflation to 0.012 for positive inflation. In the latter case, real mortgage payments are higher early on, when lenders are protected against the risk of default through the down payment. If and when house prices decline, real outstanding debt is lower which explains the reduction in default rates. The higher initial real mortgage payments are also the reason for the slightly higher average consumption growth rate for individual borrowers in the positive inflation economy (0.041 compared to 0.040 for the zero inflation scenario).

We now turn to the analysis of the option to extend loan maturity. In case it is exercised borrowers make only the interest payments due in that period, but must repay the remaining real loan balance from the following period onward. In other words, when the loan maturity option is exercised the remaining nominal loan balance is increased by 2%, with a corresponding increase in the remaining nominal mortgage payments, to compensate lenders for the effects of inflation on their cash flows. The results in the last two columns of Panel A of Table X show that the maturity extension option is effective in stabilizing the economy with positive inflation: it leads to a shift in defaults from recessions to expansions and a smaller drop in aggregate consumption during bad times. Both borrowers and the representative lender are better off in welfare terms.

B. Zero rental premium

In the benchmark calibration we set the rental premium to 1%, which could reflect the compensation required by property owners for the moral hazard costs associated with renting (e.g. the property may require higher maintenance compared to an owner-occupied unit). We evaluate the effectiveness of the maturity extension option in an economy with a zero rental premium. The default option becomes more attractive and borrowers exercise it much more often: the mean default probability increases to 0.020 as shown in Panel B of Table X. Loan premia also increase to 0.024. Borrowers reduce their precautionary savings: consumption is higher in the early periods, and individual borrower consumption growth is smaller. The last two columns of Panel B show the results for the corresponding maturity extension ARMs. As before, the option is effective in stabilizing the economy in recessions.

C. A low-interest-rate environment

Baseline ARMs do not help to stabilize the economy when the real interest rate is low before the recession and constrained by the zero lower bound during the recession. To quantify this and to investigate the extent to which the option to extend maturity helps, we have simulated our baseline model, but setting the realization of interest rates to low for ten consecutive periods. This is a very low but positive probability event (0.001 unconditional probability). It is a different and simpler experiment than those previously reported in this section, since it does not require a new solution of the model. Panel C from Table X reports model summary statistics calculated using data only for these ten periods of low rates.

The results for baseline ARMs are shown in the first two columns. The average default rate is equal to 0.011, which is lower than the average value of 0.013 for the base case simulation. This naturally is a direct consequence of the low interest rates, that reduce the debt burden and the incentive to default. The difference in default rates between recessions and expansions is also smaller than for the base case simulation, reflecting the lower average default rate.

The average log change in individual borrower consumption is 0.047, higher than the value for the base case simulation of 0.040. Borrowers benefit from the persistently low realized interest rate, and they respond by increasing consumption by more over the sample. Borrower consumption is more procyclical, reflecting the fact that interest rates are persistently low and no longer provide additional debt relief in recessions compared to expansions. The representative lender is negatively affected by the persistently low realizations of the interest rate; it responds by reducing its savings and consumption over these ten periods. This explains the negative value for the average log change in lender consumption.

The last two columns of Panel C show the results for maturity extension ARMs. The option to extend maturity leads to a small decline in the average default rate and it shifts default from recessions to expansions. The consumption of borrowers becomes less procyclical, but at the expense of an increase in the procyclicality of the lender's consumption. Overall, the option helps to stabilize aggregate consumption.

The welfare calculations that we have reported throughout the paper are based on the value functions of the agents and therefore are ex-ante calculations. For the simulation with persistently low interest rates, we perform a different type of welfare calculation, an ex-post one based on realized consumption. More precisely, we calculate the ex-post utility of borrowers (and lender) over the ten years, based on their realized consumption in each of the years, for the simulations with and without the option to extend maturity. We use these realized utilities to measure the ex-post welfare gains of the option. The last two rows of Panel C report the results. The option is ex-post beneficial for borrowers and lender.

VIII. Conclusion

We have used a quantitative dynamic model of borrower and lender behavior to evaluate changes to the design of mortgage contracts aimed at increasing macroeconomic stability. In our model the demand for loans comes from borrowers who purchase a house using a mortgage that is a given multiple of income. After the initial period, borrowers decide in each period how much to consume and save, and whether to refinance to a new mortgage, to default, or to prepay their mortgage and move to rental housing. Mortgage loans are supplied by a representative infinite-horizon risk-averse lender, who in addition to the mortgage cash flows receives a risky income stream. We have solved for a stochastic equilibrium where agents anticipate the occurrence of individual and aggregate shocks, but these shocks (to income, interest rates, and house prices) are exogenous in our model. Loan premia are determined endogenously using a pricing kernel derived from the equilibrium consumption choices of the lender.

We have analyzed several changes to mortgage contract design. The two most important are an ARM contract combined with a maturity extension option to pay only interest and extend loan maturity in recessions, and a FRM contract combined with an option to switch to an ARM during recessions. The maturity extension ARM has several advantages. Relative to a standard ARM, it stabilizes consumption growth over the business cycle, it shifts defaults to expansions, and it is welfare enhancing. The cyclical properties of the maturity extension ARM are attractive to risk-averse lenders so they are willing to offer these mortgages at relatively low cost.

Results are less promising for a FRM with an option to switch to an ARM. Relative to a standard FRM, this mortgage modestly stabilizes borrower consumption growth over the business cycle and modestly reduces defaults in recessions. The option is typically exercised in recessions when interest rates are low, imposing losses on lenders in bad times; this makes these mortgages relatively expensive in equilibrium.

The contrast between these results illustrates the importance of taking into account the impact of mortgage contracts on risk-averse lenders. Mortgages that impose risk on lenders tend to be relatively expensive, so the form in which cash-flow relief is given to borrowers during bad times matters.

The maturity extension ARM is appealing because it is simple and involves only a relatively small change to existing contracts. However its implementation does require the availability of a "recession" index. In the US one could use recession dates determined by the business cycle dating committee of the National Bureau of Economic Research. A difficulty is that recessions are usually called with a lag, due to the time that it takes to gather and process data on the state of the economy. We expect, however, that in the future, due to advances in information technology, the time required for data gathering will be reduced. For example, the COVID-19 recession that started in the US in February 2020 was announced roughly three months later, on June 8, 2020. A delay in calling a recession could reduce the benefits of the maturity extension option. On the other hand, it may also take some time for the effects of a recession to be felt on household incomes and house prices, which may counteract the fact that the maturity extension option is not available at the onset of the recession.

Finally, we note that our maturity extension option has some similarities to mortgage forbearance provisions in the US Coronavirus Aid, Relief, and Economic Security (CARES) Act of March 2020. Under the CARES Act, any borrower with a mortgage funded or backed by the US government or one of the government-sponsored enterprises (GSEs) has the right to declare a pandemic-related loss of income, without needing documentary evidence, and can obtain a six-month suspension of mortgage payments, renewable for six months. While the payment reduction under the CARES Act is greater than the one we consider (since both principal and interest payments can be suspended), the subsequent resumption of payments takes a variety of forms and maturity extension is not universally available. The provision of a standardized maturity extension option would give both borrowers and lenders greater clarity about the future availability and terms of mortgage forbearance, facilitating risk management and the appropriate pricing of residential mortgages while assisting the central bank in its task of macroeconomic stabilization.

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Figure 1. Cross-section of defaults predicted by the model

This figure uses model simulated data to study its cross-sectional default predictions. The simulated data is for the plain vanilla ARMs, and the baseline parameterization of the model. The left panel plots the default probability (blue line) as a function of current loan-to-value (LTV). The red (yellow) line plots the default probability as a function of LTV, for borrowers with a low (high) proportion of residual income relative to total income. Residual income is equal to the income of the borrower net of the required mortgage payments. The right panel plots the probability of default as a function of residual income in the x-axis (on an inverted scale). The blue line plots the overall default probability, and the red (yellow) line plot the default probability conditional on the current LTV being higher (lower) than 1.2.

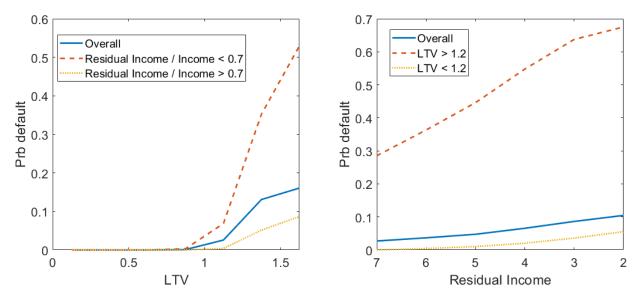


Figure 2. Cyclicality of consumption, default rates and welfare gains across scenarios

This figure plots model statistics for the several mortgage contracts studied in Section III and the baseline parameterization. The data is obtained from simulating the model. The cyclicality of a variable is given by the difference in the expansion and recession mean values. Panel A (Panel B) shows the cyclicality of the default rate (of the log change in aggregate consumption, i.e. $\Delta \log C_t \equiv \log C_t - \log C_{t-1}$). Aggregate consumption is calculated by adding the consumption of all of the borrowers and lender in the economy, i.e. $C_t = \sum_i C_{it} + C_{lt}$. Panel C (Panel D) shows the welfare gain of borrowers (the representative lender) of the different mortgage contracts relative to the plain vanilla ARMs shown in the first column. The welfare gains are measured as consumption equivalent variations.

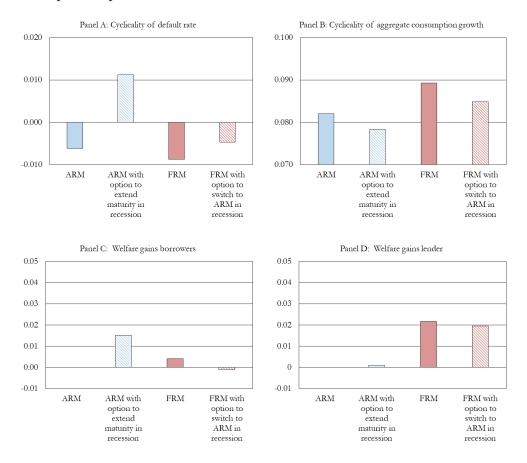


Table I. Baseline parameters

Panel A reports the unconditional probability of a recession and business cycle transition probabilities, parameterized using the NBER business cycle dates. Panel B reports our calibration of the real interest rate and house prices processes. For the former, we use data from the FED and the Michigan survey of inflation expectations (further details are included in Appendix IA.B.1). The mean and standard deviation of the house price process are from Campbell and Cocco (2015). We use the NBER dates and S&P Case-Shiller U.S. National Home Price Index data to parameterize the correlation between house price changes and the business cycle (further details are given in Appendix IA.B.3). We use data from FHFA Monthly Interest Rate Survey, the Freddie Mac Single Family Loan-Level Dataset, and the Census Bureau to parameterize the initial LTV and LTI (further details in Appendix IA.B.3). The remaining loan and housing parameter values are taken from several papers in the literature (the details are given in the paper).

Description	Parameter	Value
Panel A: Business cycle proba	bilities	
P(recession)		0.22
P(recession recession)		0.37
P(recession expansion)		0.18
Panel B: Real interest rate and h	ouse prices	
Mean log real rate	μ_r	0.01
St. dev. of real rate	σ_r	0.025
P(high rate recession)		0.38
P(high rate expansion)		0.52
Mean log house price change	μ_H	0.00
St dev log house price change	σ_{e}	0.16
P(increase in house prices recession)		0.39
P(increase in house prices expansion)		0.52
Panel C: Loan and housing n	narket	
Initial loan to income	lti	3.50
Initial loan to value in recession	ltv	0.80
Initial loan to value in expansion	ltv	0.90
Loan maturity (years)	au	20
Prepayment cost	$ heta_P$	0.00
Refinancing cost	$ heta_R$	0.015
House sale commission	$ heta_c$	0.06
Property taxes	$ au_p$	0.015
Maintenance expenses	m_p	0.025
Rental premium	ε	0.01
Foreclosure loss	O_l	0.27

 Table II. Preference and income process parameters and asset pricing moments

Panel A reports the preference and income process parameter values. The degrees of impatience and risk-aversion are standard in the literature. We set the borrowers' utility of terminal wealth so that agents accumulate average financial wealth similar to those aged 35-44 in the 2013 wave of the Survey of Consumer Finances. The default utility penalty is chosen to match the average default rate in the data (details in appendix IA.B.2). The borrowers' income process parameters are from Guvenen et al. (2014). The ratio of average income across the cycle and the standard deviation of permanent income shocks of the lender are chosen so that our baseline model generates ARM recession and expansion premia that match the data (details in Appendix IA.B.2). The ratio of mortgage cash-flows to income matches the ratio of real estate assets to other assets in banks' balance sheets. Panels B.1 and B.2 report the model equilibrium asset pricing moments when the mortgage cash-flows are infinitesimal in the lender's portfolio and for the baseline value, respectively. The last row reports the equilibrium cash-on-hand/income of the lender.

Description	Parameter	Value
Panel A.1: B	orrowers	
Subjective discount factor	β_i	0.98
Risk aversion		2.00
Number of periods	${\gamma_i \over T}$	20
Utility of terminal wealth	b_i	10.0
Default utility penalty	λ	0.10
Income process parameters		Guvenen et al.
Tax rate	ϕ	0.20
Panel A.2: Represe	entative lender	
Subjective discount factor	β_l	0.98
Risk aversion	γ_l	2.00
Number of periods		∞
Ratio of average inc. across the cycle	$\bar{y}(Exp)/\bar{y}(Rec)$	2.67
St. dev. of income shocks	σ_v	0.10
Ratio of mortgage cash-flows to inc.	$E[CF_{lt}/Y_{lt}]$	0.20
Description	Recession	Expansion
Panel B.1: Marg	ginal ARMs	
1/E[sdf low rate]	0.003	-0.015
1/E[sdf high rate]	0.052	0.036
St. dev. of the sdf	0.163	0.074
Maximum Sharpe ratio	0.166	0.075
Lender cash-on-hand/income	5.46	2.80
Panel B.2: Base	eline ARMs	
1/E[sdf low rate]	0.029	-0.015
1/E[sdf high rate] 82	0.039	0.036
St. dev. of the sdf	0.141	0.066
Maximum Sharpe ratio	0.146	0.067
Lender cash-on-hand/income	3.94	1.91

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an option to extend maturity in recessions, the baseline FRM, and the FRM contract with an option to convert to The fifth (sixth) row reports the ratio of lender cash-on-hand to income (the log change in the consumption of the This table reports key moments for the mortgage contracts studied, including the baseline ARM, the ARM with an ARM in recessions. For each contract, the first (second) column reports the unconditional mean (cyclicality) of the variable of interest. The cyclicality is the difference between the expansion and recession means. The data are $\Delta \log C_t \equiv \log C_t - \log C_{t-1}$. Aggregate consumption is calculated by adding the consumption of all of the borrowers obtained by simulating the model for each of the contract types. The first (second) row reports the equilibrium loan The fourth row reports the log change in the consumption of individual borrowers, i.e. $\Delta \log C_{it} \equiv \log C_{it} - \log C_{it-1}$. ender, i.e. $\Delta \log C_{lt} \equiv \log C_{lt} - \log C_{l,t-1}$). The seventh row reports the log change in aggregate consumption, i.e. and lender in the economy, i.e. $C_t = \sum_i C_{it} + C_{it}$. The last two rows report the welfare gain of borrowers and lender, espectively, in each of the contracts relative to the baseline ARM. The welfare gains are measured in equivalent premium (default rate). The third row reports the proportion of borrowers who exercise the option (when available). consumption variations.

	Basel	ine ARM	Mat e	ext. ARM	Basel	ine FRM	FRM	vith option
Description	Mean	Cyclicality	Mean	<u>Mean</u> Cyclicality	Mean	Cyclicality	\overline{Mean}	Cyclicality
<u>Loan premi</u> um	0.020	-0.013	0.018	-0.013	0.019	-0.018	0.025	-0.018
Default rate	0.013	-0.006	0.013	0.011	0.013	-0.009	0.013	-0.005
Fraction using option	n/a	n/a	0.134	-0.606	n/a	n/a	0.108	-0.546
Log change in ind. borrower cons.	0.040	0.073	0.039	0.065	0.040	0.075	0.040	0.072
Lender cash-on-hand/income	2.363	-2.030	3.163	-2.555	2.521	-2.163	2.375	-1.962
Log change in lender cons.	0.000	0.107	0.000	0.137	0.000	0.128	0.000	0.119
Log change in aggregate cons.	0.000	0.082	0.000	0.078	0.000	0.089	0.000	0.085
Welfare gain for borrowers			0.015		0.004		-0.001	
Welfare gain for lender			0.001		0.022		0.020	

Table IV. Changes in borrower log consumption conditional on interest rate changes

This table reports the average change in individual borrower log consumption conditional on the economy moving to a recession and conditional on the movement of the risk-free real interest rate. The first column reports the results for the change in log consumption when interest rates are low and remain low. The second (third) column the results for when there is a decrease (increase) in interest rates. The last column the results when interest rates are kept high. The first row of the table has the results for the baseline ARM, the second row for the maturity extension ARM contract, the third row for a standard FRM contract, and the last row for a FRM contract with the option to convert to an ARM contract in recessions. The data are obtained by simulating the model for each of the contract types.

		$\Delta \log C_{it}$	recession	
Mortgage contract	Low to Low	High to Low	Low to High	High to High
Baseline ARM	-0.012	0.003	-0.048	-0.036
Maturity extension ARM	-0.006	0.010	-0.040	-0.027
Baseline FRM	-0.022	-0.008	-0.038	-0.027
FRM w/ opt to convert to ARM $$	-0.013	0.002	-0.045	-0.032

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(expansion). The first five rows report the incidence of the different decisions. The remaining rows report the payments and use the option (if available)). The data are obtained by simulating the model for each of the contract average loan to income and loan to value conditional on these decisions (default, mortgage refinance, make mortgage This table describes the borrowers' decisions for the the baseline ARM, the maturity extension ARM contract (first four columns), the FRM contract and the FRM with an option to convert to ARM in recession (last four columns). For each contract type, the first (second) column reports moments conditional on the economy being in a recession types.

	Baselin	ie ARM	Mat ext. ARN	: ARM	Baselir	Baseline FRM	FRM with option	ch option
	Recession	Expansion	Recession	Expansion	Recession	Expansion	Recession	Expansion
Incidence		4		4		4		4
Default	0.018	0.012	0.004	0.015	0.020	0.011	0.017	0.012
Refinance	0.015	0.109	0.006	0.104	0.017	0.115	0.014	0.111
Pay	0.951	0.869	0.374	0.871	0.948	0.864	0.743	0.866
Use the option	n/a	n/a	0.606	n/a	n/a	n/a	0.211	n/a
Use the option (cond. on FRM)	n/a	n/a	n/a	n/a	n/a	n/a	0.546	n/a
Loan to Income								
Default	7.826	7.448	8.907	7.741	7.753	7.509	7.848	7.462
Refinance	4.059	5.033	4.201	5.163	4.222	5.027	3.954	5.034
Pay	3.120	2.781	1.429	2.967	3.141	2.808	2.884	2.770
Use the option	n/a	n/a	4.713	n/a	n/a	n/a	3.987	n/a
$\underline{\text{Loan to Value}}$								
Default	1.376	1.366	1.546	1.488	1.377	1.373	1.382	1.362
Refinance (before ref)	0.378	0.650	0.334	0.669	0.389	0.661	0.369	0.652
Refinance (after ref)	0.476	0.799	0.430	0.808	0.484	0.801	0.470	0.798
Pay	0.791	0.769	0.682	0.826	0.798	0.777	0.770	0.769
Use the option	n/a	n/a	0.973	n/a	n/a	n/a	0.870	n/a

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required mortgage payments relative to permanent income over 0.15 are allowed to do so. The table follows the This table reports results for the ARM contract with alternative maturity extension options: (i) permanent option to extend, i.e. borrowers are allowed to freely extend maturity both in recessions and expansions; (ii) permanent option to extend maturity, but costly in expansions; (iii) maturity extension is only possible in recessions at a cost identical to the refinancing cost; (iv) free option to extend maturity in recessions, but only those borrowers with same format as Table III. The data are obtained by simulating the model for each of the contract types.

	Per	manent	Perm.,	costly in exp.	Rec	., costly	Rec. ar	MTI > 0.15
Description	<u>Mean</u>	0	<u>Mean</u>	Cyclicality	Mean	\circ	Mean	\mathcal{O}
<u>Loan premi</u> um	0.019		0.018	I	0.020		0.018	
Default rate	0.014		0.013		0.013		0.013	
Fraction using option	0.634		0.179		0.016		0.095	
Log change in ind. borrower cons.	0.037	0.067	0.039	0.065	0.040	0.072	0.039	0.066
Lender cash-on-hand/income	2.993		3.108		2.465		2.924	
Log change in lender cons.	0.000		0.000		0.000		0.000	
Log change in aggregate cons.	0.000		0.000		0.000		0.000	
Welfare gain for borrowers	0.041		0.016		0.001		0.013	
Welfare gain for lender	0.017		0.002		0.000		0.001	

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rameterizations, that include: (i) a higher initial LTV in recessions equal to 0.9, which is also the maximum LTV limit; and (ii) a higher initial LTI of 5.0 (for the baseline parameterzation of a maximum LTV limit in recessions This table reports the results for the baseline ARMs and the maturity extension ARMs, for alternative loan paequal to 0.8). The data are obtained by simulating the model for each of the contract types and alternative loan parameterizations. The table follows the same format as Table III.

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	Hi	gher LTV lin	nit in red	cessions		Higher ir	nitial LTI	
	Base	Baseline ARM	Mat.	ext. ARM	[Base]	Baseline <u>ARM</u> Mat		ext. ARM
Description	Mean	Cyclicality	Mean	Cyclicality	Mean	Cyclicality	_	Cyclicality
<u>Loan premi</u> um	0.023	-0.026	0.021	-0.026	0.021	-0.018	_	-0.018
Default rate	0.016	-0.008	0.015	0.013	0.021	-0.008		0.017
Fraction using option	n/a	n/a	0.136	-0.614	n/a	n/a	_	-0.670
Log change in ind. borrower cons.	0.046	0.074	0.045	0.065	0.039	0.070	~	0.059
Lender cash-on-hand/income	2.390	-2.036	3.207	-2.587	2.139	-2.014	. –	-2.641
Log change in lender cons.	0.000	0.112	0.000	0.138	0.000	0.091		0.135
Log change in aggregate cons.	0.000	0.083	0.000	0.077	0.000	0.078		0.077
Welfare gain for borrowers			0.016				0.025	
Welfare gain for lender			0.003				0.002	

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what they need to meet the mortgage payments). The higher stigma of default considered is 0.20, which is twice the baseline value. The data are obtained by simulating the model for each of the contract types and alternative This table reports the results for the baseline ARMs and the maturity extension ARMs, for other borrower charutility cost of default. For the case of borrower inertia, in each period, with probability 0.5, borrowers are inattentive and they simply make mortgage payments (but may still be forced to default if their cash-on-hand is below acteristics including a parameterization in which borrowers suffer from inertia and one in which they have a higher borrower characteristics. The table follows the same format as Table III.

		Borrowe	er inertia			Higher	stigma	
	\underline{Base}	line ARM	Mat.	ext. ARM	Basel	ine ARM	Mat.	ext. ARM
Description	Mean	<u> Iean</u> Cyclicality	Mean	Cyclicality	Mean	/ <u>Mean</u> Cyclicality	Mean	Cyclicality
<u>Loan premi</u> um	0.017	-0.011	0.015	-0.011	0.017	-0.013	0.015	-0.013
Default rate	0.012	-0.005	0.011	0.012	0.008	-0.004	0.008	0.007
Fraction using option	n/a	n/a	0.072	-0.325	n/a	n/a	0.128	-0.579
Log change in ind. borrower cons.	0.042	0.071	0.042	0.068	0.040	0.074	0.039	0.065
Lender cash-on-hand/income	2.426	-2.065	2.833	-2.331	2.385	-2.044	3.135	-2.535
Log change in lender cons.	0.000	0.110	0.000	0.141	0.000	0.108	0.000	0.136
Log change in aggregate cons.	0.000	0.082	0.000	0.081	0.000	0.083	0.000	0.079
Welfare gain for borrowers			0.009				0.016	
Welfare gain for lender			0.001				-0.001	

Table IX. Multiple borrower types, with different discount factors

This table reports results for the baseline ARM and the maturity extension ARM, for the case in which there are two types of borrowers in the economy who differ in their subjective discount factor. The parameterization of borrower types follows Iacovello and Pavan (2013). The patient (impatient) borrowers have a discount factor of 0.999 (0.941), and the fraction of impatient borrowers is 2/3. The lender is able to observe the borrower type and prices the loans given to each type differently. Panel A (Panel B) reports data for the patient (impatient) borrowers. Panel C shows the results for all the borrowers and the representative lender. The table includes the same variables as Table III.

	Base	line ARM	Mat.	ext. ARM		
Description	Mean	Cyclicality	Mean	Cyclicality		
Panel A: Patient borrowers						
Loan premium	0.019	-0.012	0.018	-0.012		
Default rate	0.012	-0.005	0.012	0.010		
Fraction using option	n/a	n/a	0.124	-0.559		
Log change in ind. borrower cons.	0.046	0.072	0.045	0.065		
Panel B: Impatient borrowers						
Loan premium	0.021	-0.014	0.018	-0.014		
Default rate	0.016	-0.008	0.015	0.013		
Fraction using option	n/a	n/a	0.156	-0.702		
Log change in ind. borrower cons.	0.028	0.075	0.028	0.063		
Panel C: All borrowers and lender						
Default rate	0.014	-0.007	0.014	0.012		
Fraction using option	n/a	n/a	0.142	-0.643		
Log change in ind. borrower cons.	0.036	0.074	0.035	0.064		
Lender cash-on-hand/income	2.374	-2.040	3.240	-2.608		
Log change in lender cons.	0.000	0.109	0.000	0.143		
Log change in aggregate cons.	0.000	0.084	0.000	0.081		
Welfare gain for patient borrowers			0.014			
Welfare gain for impatient borrowers			0.019			
Welfare gain for lender			0.000			

Table X. Alternative macro parameterizations

This table reports results for the baseline ARM and the maturity extension ARM, for alternative parameterizations and simulations of the macro environment. Panel A reports the model results assuming a constant inflation rate of 2%. In Panel B, we set the rental premium to zero. Panel C reports results for the baseline parameters but for a low interest rate spell. We simulate the model and keep aggregate paths where the interest rate is low for ten consecutive periods. The panel reports statistics for these ten consecutive periods of low interest rate. The welfare gains that we calculate in Panel C are ex-post welfare gains of the maturity extension option over these ten periods, calculated using the realized consumption choices of borrowers and lender over the ten periods.

	Base	eline ARM	Mat.	ext. ARM		
Description	Mean	Cyclicality	Mean	Cyclicality		
Panel A: Cor	nstant in	flation rate				
Loan premium	0.020	-0.013	0.018	-0.013		
Default rate	0.012	-0.006	0.012	0.012		
Fraction using option	n/a	n/a	0.127	-0.572		
Log change in ind. borrower cons.	0.041	0.074	0.040	0.064		
Lender cash-on-hand/income	2.284	-1.986	3.160	-2.531		
Log change in lender cons.	0.000	0.105	0.000	0.138		
Log change in aggregate cons.	0.000	0.082	0.000	0.079		
Welfare gain for borrowers			0.014			
Welfare gain for lender			0.003			
Panel B: N	o rental	premium				
Loan premium	0.024	-0.015	0.022	-0.015		
Default rate	0.020	-0.010	0.019	0.012		
Fraction using option	n/a	n/a	0.127	-0.575		
Log change in ind. borrower cons.	0.037	0.070	0.037	0.065		
Lender cash-on-hand/income	2.273	-1.961	3.007	-2.415		
Log change in lender cons.	0.000	0.108	0.000	0.133		
Log change in aggregate cons.	0.000	0.081	0.000	0.077		
Welfare gain for borrowers			0.011			
Welfare gain for lender			0.006			
Panel C: 10-year low interest rate spell						
Default rate	0.011	-0.005	0.010	0.011		
Fraction using option	n/a	n/a	0.121	-0.543		
Log change in ind. borrower cons.	0.047	0.077	0.046	0.069		
Lender cash-on-hand/income	2.013	-1.571	2.662	-1.912		
Log change in lender cons.	-0.011	0.209	-0.011	0.243		
Log change in aggregate cons.	-0.002	0.109	-0.002	0.099		
Ex-post welf. gain for borrowers			0.008			
Ex-post welf. gain for lender			0.002			