Optimal Transmission Investment with Regulated Incentives based upon Forward Considerations of Firm and Intermittent Resources with Batteries

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***Abstract*—Regulatory support for transmission investment by private investors follows a complex process of dialogue, information sharing, commitments, and incentives. Whilst this process has attracted substantial research, the formulation of efficient solutions still remains an open question and this has become more complex with the emergence of new intermittent and battery-storage resources. We propose an optimal incentive solution in which the private transmission company earns congestion rents and an incentive fee, set by agreement with the regulator. This fee is set in the context of assessing the impact of current and future generation resources (conventional, intermittent, and batteries) on the congestion rents. The welfare-optimizing solution follows from a mixed integer, nonlinear, bilevel optimization problem, which is computationally challenging. We re-cast the problem for tractability and devise a disjunctive-based decomposition algorithm. Its performance has been successfully demonstrated on two standard IEEE case studies.**

***Index Terms*—Forward-looking transmission investment, Battery-storage system, Incentive regulation.**

I. Introduction

ELECTRICITY regulators are facing an increasing requirement to ensure efficient investment in transmission assets through market incentives, rather than via direct control. State ownerships of the networks are now quite widely regarded as unnecessary and expensive claims on public funds, when there is an effective Independent System Operator. Thus, merchant investors in transmission lines have been motivated by the prospect of capturing congestion rents and/or regulated returns within a wholesale power market, or arbitrage rents across interconnectors between separate markets. However, it is recognized that congestion or arbitrage rents are not enough to stimulate adequate transmission investment and further incentives are required [1]. But developing a formal process for efficient investment signals has been elusive, not least because there is an endogenous relationship between transmission and generation investments. Generators will not take final investment decisions to build new plants unless they have a connection agreement for entry into the transmission system, whilst transmission companies will not build new lines unless there are power flow benefits that can be monetised. Thus, efficient incentives for merchant transmission investment are inter-related with economic signals for generation new-build. Furthermore, generation resources are becoming increasingly complicated to model with the widespread penetration of renewable resources, such as wind [2], and the emerging adoption of large-scale batteries [3]. Designing efficient incentives in this context remains an open question in both theory and practice. Thus, in 2016, a FERC report [4], in reference to a previous Order issued in 2011 which included new discriminatory incentives to stimulate non-incumbent generator and inter-regional transmission investments, said, “It is difficult to assess [as a consequence] whether the investments made are more efficient or cost-effective”. Elsewhere, in the world there is active consideration of new transmission remuneration and charging principles with an emphasis upon efficient forward signals for investment (e.g., Britain [5], New Zealand [6]). This widespread re-evaluation of the transmission investment has been largely motivated by the technological disruptions of new large-scale renewable facilities and distributed energy resources, both of which place substantial new requirements upon network resources and their re-configurations. In this paper we therefore address and propose a solution to this open question of how regulators can apply efficient incentives to transmission owners whilst having regard to the changing circumstances of optimal generation investment.

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A. Modelling Perspective and Background Research

We characterise the main feature of the problem in a stylized but realistic model which consists of a Transmission Company (Transco) responding to investment signals from the congestion rents and regulatory incentives in an energy market with conventional incumbent generators, new investors in wind-generation and battery-storage, the latter in particular presenting substantial modeling challenges. Wind captures the intermittency aspect and is also a proxy for solar in our analysis, whilst battery storage is both an increasingly important hedge for intermittent resources as well as a flexibility resource for network operators, offsetting to some extent the need for new transmission investment. Many jurisdictions do not permit network operators to own battery resources and so optimal independent battery investment must be assumed. We assume that the regulatory body seeks to provide incentives to the Transco within the context of a welfare-maximizing perspective on the energy market evolution as a whole. Thus, in our model, it considers incumbent behaviour as well optimal wind and battery investment.

Coordination of transmission and generation investment is not a new problem and was often posed in terms of whether the transmission decision should lead or follow the generation decision [7] and [8]. The concept of investment coordination is further discussed in [9] and [10], whilst [11] and [12] propose game theoretic models for proactive and reactive coordination. In [13] proactive coordination under imperfect competition is assumed, leading to an analysis of investment coordination between transmission networks and different types of energy storage assets. All these papers support the proactive view for efficient coordination [14]. In [15] the issue of forward-looking transmission investment coordination is considered alongside investment in wind-generation capacities. Although we can rely on market prices to deliver the optimal investment in the battery-storage and the wind-generation capacities, the market-driven transmission investment remains in theoretical debate[1]. This means that transmission investors do not have a proper regulatory mechanism to incentivize their optimal investment in transmission capacity, which in turn would support the optimal market-wide investment in battery-storage and wind-generation. Price-cap regulation is proposed in [16] and [17] and under certain conditions, these regulatory mechanisms lead to transmission investment plans which maximizes social welfare [18]. Alternatively, [19] proposes a reward/penalty regulatory mechanism in which the regulator rewards the transmission investor when the transmission is expanded to reduce the congestion rents. An out-turn regulatory variation was proposed by [20]. The out-turn is defined as the difference between the actual electricity price and the price without transmission congestion, such that the transmission investor is responsible for total out-turn cost and any transmission losses. In [18], [21], and [22] the transmission investor maximizes its profit (sum of merchandising surplus and a fixed charge) subject to the price-cap constraint introduced in [16]. From a different perspective, [23] proposes an incremental surplus subsidy (ISS) mechanism for regulating a firm when the regulator does not have cost information of the regulated firm. In other words, when a monopolist carries out a relevant regulatory activity, the ISS awards the monopolist the gain from any social-welfare increase. The ISS mechanism has a number of attractive features [24] and it has been argued that implementing the ISS mechanism in Australia can maximise the social-welfare of transmission network investments. Motivated by this, we reformulate and extend the ISS approach to demonstrate that it can efficiently coordinate the greater complexity of jointly optimal investments in battery-storage, wind-generation, and the transmission network. The lack of a comprehensive paper addressing engineering, regulatory economics and computational aspects of the transmission investment coordination is clear from Table I.

B. Contributions

The current paper contributes to the relevant literature as follows:

(a) It develops a merchant-regulatory incentive mechanism (ISS based mechanism) for forward-looking transmission investment coordinated with the optimal investment in battery-storage and wind-generation capacity.
We extend [23] and [24], proving analytically that the ISS mechanism can be efficiently used for electricity network investments which also involve new battery-storage and wind-generation facilities. This has been done in Section II and through Lemma 1. In particular, Lemma 1 proves that our extended ISS mechanism to consider transmission investment, wind-generation investment, and battery-storage investment results in social-welfare maximizing outcomes. The extension of the ISS mechanism to fully co-ordinate transmission investment, battery-storage investment and wind-generation investment are original contributions of our paper.

(b) The proposed optimal investment framework is modeled as a mixed-integer nonlinear bilevel program which is difficult to solve, but we develop a novel transformation to a more computationally tractable mixed integer linear program (MILP).
The novelty of the transformation lies in the combination of duality conditions and tailored linearization techniques. Linarization techniques are used to eliminate bilinear terms or replace them with equivalent linear terms. The mathematical proof of the Lemma 2 shows that the energy storage charge and discharge variables can be dropped from the problem without affecting the solution space. Although various previous researchers have used similar relaxations in their formulations, none of them were able to provide a mathematical proof that such relaxations will not affect the solution space of their problems. Furthermore, we proposed Lemma 3 and Lemma 4 in which some nonlinear terms are expressed equivalently as linear terms. Both Lemma 3 and Lemma 4 are derived after careful consideration of our problem structure. These novel reformulation techniques are original contributions of our paper.

(c) A disjunctive-based decomposition (DBD) algorithm is proposed and carefully studied to further improve the computational tractability of the problem.
The proposed decomposition algorithm employs ideas presented in [25] to eliminate disjunctive parameters from master problem and proposes a technique to eliminate disjunctive parameters from the sub-problem. But the algorithm presented in [25] still contains disjunctive parameters, whilst our proposed DBD algorithm eliminates the presence of such parameters completely. Furthermore, the DBD algorithm proposes a unique multi-cut acceleration technique which creates a fast convergence in the algorithm. The combination of the sub-problem reformulation technique and multi-cut acceleration are original contributions of our paper.

II. Proposed mechanism

A profit-maximizing Transmission Company (Transco) recovers its investment costs by collecting the congestion rent and

[1]Merchant transmission investment where the investor relies on inter-locational price differences (congestion rent) has a tendency to deliver sub-optimal investment both in theory and in practice [1].

Table I
THE COMPARATIVE LITERATURE REVIEW

Papers	I	II	III	IV	V	VI	VII
[16], [17], [18], [21]	Yes	No	No	No	No	No	No
[22], [23], [24]	Yes	No	No	Yes	No	No	No
[7], [9], [10], [11], [15], [26]	No	Yes	No	No	Yes	Yes	No
[27], [28], [29]	Yes	Yes	No	No	Yes	No	No
Our paper	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**

(I): Incentive mechanism proposed, (II): Can be applied to investment coordination, (III): Supports coordinated investment in more than two assets, (IV): Social-welfare maximum is guaranteed, (V): Comprehensive mathematical model is proposed, (VI): Comprehensive mathematical model is reduced to an equivalent MILP model, (VII): Novel solution methodology is proposed

receiving a regulated fixed-fee from the regulator. The fixed-fee ensures that the Transco is compensated for its contribution to positive social welfare. The forward-looking Transco forecasts market-wide changes in generation and storage capacities in its optimal transmission investment decision. The regulated fixed-fee is calculated using an ISS mechanism. The ISS mechanism awards the monopolist the gain from the social-welfare increase at each investment period. Under the ISS incentive mechanism the regulator collects the social welfare increase through transmission tariffs or taxes and pays them to a transmission company as a fixed fee. Such fixed fee incentivizes a transmission company to perform investments in social-welfare maximizing way. The original ISS mechanism [23] was proposed to address regulated transmission investment problems. The process is that the regulator will set a fixed fee, Φ_t, computed for each investment planning period (typically a year). The fixed fee is set by the regulator who has a goal to achieve social welfare maximising investments, and it was theoretically proven in [23] that the original ISS mechanism can achieve this. Thus, it could be efficiently applied to incentivize socially optimal transmission investments. However, the ISS mechanism has not yet considered battery-storage or wind-generation investments and that is one of our main contributions. As with most transmission charging, the fixed fee, Φ_t, is paid to the Transco by the consumers (for example from grid tariffs). We, therefore, formulate a new fixed fee calculation as (1):

$$\Phi_t = \Delta\pi_t^G + \Delta\pi_t^E + \Delta\pi_t^W + \Delta\pi_t^L - \pi_{t-1}^T + \overline{C}_{t-1}^{\intercal} - \tilde{C}_t^E - \widehat{C}_t^W. \quad (1)$$

In (1), $\Delta\pi_t^G$ is the difference in the expected surplus of the conventional generators, π_t^G, between time period t and the previous period. In the expected surplus for the status quo period, $t = 1$, where only the existing system is considered, the fixed fee will be equal to zero. Similarly, $\Delta\pi_t^E$, $\Delta\pi_t^W$, and $\Delta\pi_t^L$ are the differences of the expected surpluses for Battery Storage System (BSS), wind generator and loads respectively[2]. The π_{t-1}^T is the expected merchandising surplus of the Transco in the previous period and $\overline{C}_{t-1}^{\intercal}$ is the transmission investment costs in the previous period. The $\tilde{C}_t^E$ and $\widehat{C}_t^W$ are the total investments in the battery-storage and wind generator capacity for the period at hand.

The objective of the proposed incentive mechanism is to maximize the social welfare, which is defined as the expected surpluses of all participants minus all investment costs:

$$SW_t = \pi_t^G + \pi_t^E + \pi_t^W + \pi_t^L + \pi_t^T - \overline{C}_t^{\intercal} - \tilde{C}_t^E - \widehat{C}_t^W. \quad (2)$$

Given the fixed fee defined above, the Transco will maximize its total surplus for each planning period as defined in (3):

$$\text{Maximize } \textstyle\sum_{t\in\mathcal{T}}(\pi_t^T + \Phi_t - \overline{C}_t^{\intercal}) \quad (3a)$$

$$\text{Subject to: } \Phi_t = \Delta\pi_t^G + \Delta\pi_t^E + \Delta\pi_t^W + \Delta\pi_t^L - \pi_{t-1}^T + \overline{C}_{t-1}^{\intercal} - \Delta\tilde{C}_t^E - \Delta\widehat{C}_t^W, \forall t > 1 \quad (3b)$$

$$\text{Where } \pi_t^W, \pi_t^G, \pi_t^L, \pi_t^E, \tilde{C}_t^E, \widehat{C}_t^W \in SOL^3\{\text{Maximize } SW\} \quad (3c)$$

In (3), and throughout the rest of the paper, SOL is short for "Solution of the following mathematical problem". In this case it refers to the solution of the social welfare maximization problem. The following lemma shows that the proposed incentive mechanism provides social-welfare maximizing investments.

Lemma 1. *The optimization problem in (3) results in social-welfare maximizing investments.*

Proof. We assume two periods, status-quo period, $t = 1$ and investment period, $t = 2$. If we substitute the fixed fee variable in the objective function (3a) with the right-hand side of the constraint (3b), we can re-write the objective function as:

$$\sum_{t\in\mathcal{T}}(\pi_t^T + \Phi_t - \overline{C}_t^{\intercal}) = \sum_{t\in\mathcal{T}}(\pi_t^T + \Delta\pi_t^G + \Delta\pi_t^E + \Delta\pi_t^W + \Delta\pi_t^L - \pi_{t-1}^T + \overline{C}_{t-1}^{\intercal} - \Delta\tilde{C}_t^E - \Delta\widehat{C}_t^W - \overline{C}_t^{\intercal}) = \sum_{t\in\mathcal{T}}(\Delta\pi_t^G + \Delta\pi_t^W + \Delta\pi_t^E + \Delta\pi_t^L + \pi_{t=T}^T - \pi_{t=1}^T + \overline{C}_{t=1}^{\intercal} - \overline{C}_T^{\intercal} - \Delta\tilde{C}_t^E - \Delta\widehat{C}_t^W) = \Delta SW_t \quad (4)$$

This is the same expression as the definition of the social welfare (2). Hence, the objective function of the Transco is equivalent to maximizing the social welfare over the planning period. Thus, if a global solution for problem (3) is reached it will guarantee social welfare maximum investments. □

It is assumed that the regulator and the Transco agree on the parameters involved in calculating expression (4). These parameters are those which are used to calculate the surpluses π_t^T, π_t^G,π_t^W, π_t^E, and π_t^L. In practice, that would be the basis of their regulatory dialogue. Therefore, the forward-looking investment decision of the Transco is modeled through the stochastic bilevel program in (5).

In the optimization problem (5), the objective function of the Transco is modeled in (5a) as the total expected congestion rent π_t^T plus fixed fee Φ_t minus total investment cost. $\pi_t^T = \sum_{snk}\lambda_{ntks}(\sum_{ik} I_n^{(i)} d_{itks} + \sum_{nb} E_n^{(b)}(\tilde{d}_{btks} - \tilde{g}_{btks}) - \sum_{jk} J_n^{(j)} g_{jtks} - \sum_{wk} W_n^{(w)} \widehat{g}_{wtks})$ is the total expected congestion rent of the Transco. In this formulation, the congestion

[2] Under the load we assume aggregated load which includes retail, distribution and end consumers of the electricity at each node of the electricity network. Load surplus is measured as the difference between the net value of its electricity consumption and total costs of the consumption. Under load net value we refer to total value load receives by consuming electricity at a given price.

rent is calculated as the difference between the money which is paid by units which withdraw power from the network (i.e. load and battery charging) and the money which is paid to units which inject power to the network (i.e. generating units and discharging batteries). A formal definition of congestion rents is provided in reference [30]. The investment in transmission lines is modeled through the variable y_{mt}. The investment decision is irreversible and ensured through an auxiliary variable z_{mt} and the investment constraint (5c).

$$\underset{z_{mt},y_{mt}\in\{0,1\}}{\text{Maximize}} \ \sum_t \frac{\Phi_t+\pi_t^T-\sum_m C_{mt}^{(T)}y_{mt}}{(1+r)^{t-1}} \tag{5a}$$

Subject to:

$$z_{mt}=\sum_{\widehat{t}\leq t} y_{m,\widehat{t}} \ \forall m,t\geq 2 \tag{5b}$$

$$\sum_t y_{mt}\leq 1 \ \ \forall m \tag{5c}$$

$$z_{mt}=0 \ \forall m,t{=}1 \tag{5d}$$

$$\text{Where } \Phi_t\in SOL\{(6)\} \text{ and } \pi_t^T\in SOL\{(7)\} \tag{5e}$$

In the optimization problem (5), the fixed fee Φ_t is decided by the regulator through (6) whilst the congested rent π_t^T depends on the wind and battery capacity investments as well as the hourly dispatch of the system, which is simulated through (7). $SOL\{(6)\}$ and $SOL\{(7)\}$ refer to solutions of the models (6) and (7), respectively.

The regulator's decision problem is therefore formulated as in (6). In the regulatory equation (6c) of the problem (6), $\pi_t^L = \sum_{sik}(\Psi A_i d_{itks} - \sum_n I_n^{(i)}\lambda_{ntks}d_{itks})$, $\pi_t^G = \sum_{sjk}(\sum_n J_n^{(j)}\lambda_{ntks}g_{jtks} - \Psi C_j g_{jtks})$, $\pi_t^W = \sum_{snwk} W_n^{(w)}\lambda_{ntks}\widehat{g}_{wtks}$, and $\pi_t^E = \sum_{snbk}(E_n^{(b)}\lambda_{ntks}(\tilde{g}_{btks}-\tilde{d}_{btks})+\Psi(C_b^{(dh)}\tilde{d}_{btks}-C_b^{(ch)}\ \tilde{g}_{btks}))$ are expected load surplus, generation surplus, wind-generation surplus and battery-storage surplus in the wholesale market. Here two separate time indices t and k are used for investment periods (in years) and operational periods (in hours), respectively. The regulatory constraint is modeled in (6c), which is not applied to the first planning period $t=1$ ($\Phi_{(t=1)}=0$ in (6b)).

$$\text{Find } \Phi_t \text{ such that:} \tag{6a}$$

$$\Phi_{(t=1)}=0 \tag{6b}$$

$$\begin{aligned}\Phi_t &= \Delta\pi_t^L+\Delta\pi_t^G+\Delta\pi_t^W+\Delta\pi_t^E-\pi_{t-1}^T+\\ &\tfrac{1}{(1+r)^{t-1}}\big(\textstyle\sum_m C_{mt}^{(T)}y_{m(t-1)}-\sum_w C_{wt}^{(W)}(u_{wt}-u_{w(t-1)})-\\ &\textstyle\sum_b C_{bt}^{(E)}(e_{bt}-e_{b(t-1)})-\sum_b C_{bt}^{(P)}(p_{bt}-p_{b(t-1)})\big) \ \forall t\geq 2\end{aligned} \tag{6c}$$

$$\text{Where } \pi_t^T\in SOL\{(7)\} \tag{6d}$$

The fixed fee Φ_t can be calculated through a set of equations (regulatory constraints) without any explicit objective function. Thus, the regulator's decision problem (6) can be merged with the Transco's decision problem (5) and solved simultaneously. On the other hand, the battery-storage and wind-generator capacity investment can be modeled as a maximization problem in (7). Nodal prices (λ_{ntks} (\$/MWh)), power flows ($f_{ltks},\widehat{f}_{mtks}$ (MW)), voltage angles (θ_{ntks} (rad)), dispatched load (d_{itks} (MW)), generations ($g_{jtks},\widehat{g}_{wtks}$ (MW)), and battery-storage charge and discharge ($\tilde{d}_{btks},\tilde{g}_{btks}$ (MW)) as well as battery-storage and wind-generation investments are calculated in (7). Objective function (7a) represents maximization of the social welfare including the demand utility function, generation operational cost, wind-generation investment cost, and battery-storage investment cost. The non-decreasing properties of wind-generation invested capacities (u_{wt} (MW)) and battery-storage invested capacities (e_{bt} (MWh),p_{bt} (MW)) are modeled in (7b)-(7d). Investments in wind-generation and battery-storage are modeled as continuous variables and are not reversible. Investments in battery-storage can be divided into two separate investments: energy capacity e_{bt} and power capacity p_{bt}. Energy capacity determines the maximum limit of the state of charge whilst power capacity sets the limits on charge and discharge rates. $C_b^{(ch)}$ (\$/MWh) and $C_b^{(dh)}$ (\$/MWh) are the marginal operational costs of a battery which occur due to degradation and limited number of cycles of an asset[4]. The Active power balance constraint for node n is written in (7e). The active power flow constraints for existing lines are formulated in (7f). Similarly (7g) and (7h) are used for candidate lines. Here the active power flow constraint will be enforced only when we invest in the corresponding candidate line m ($z_{mt}=1$). The flow of the candidate line m is forced to be zero when there is no investment in this line according to constraint (7i). 100 MVA is used to convert reactance (X_l) to per unit values in the active power flow constraints (7f), (7g), and (7h). The energy balance constraint for battery-storage b is modeled in (7j) to keep track of the state of charge of the battery-storage and its available energy capacity. The upper and lower bounds on generation, load, battery-storage power, and energy, as well as on the thermal limits of existing and candidate transmission lines are modeled in (7k)-(7r). The simultaneous charge and discharge in battery-storage is not possible due to its technical limitations. Thus, following the traditional battery-storage operational models in [31] and [32] an auxiliary integer variable a_{btks} is introduced into the upper and lower limit constraints of battery-storage (7o)-(7p) to prevent the simultaneous charge and discharge during operation. Constraint (7s) sets node 1 as the reference node with zero voltage angle. $\Omega=\{u_{wt},\ e_{bt},\ p_{bt},\ \tilde{d}_{btks},\ \tilde{g}_{btks},\ q_{btks},\ d_{itks},\ g_{jtks},\ \widehat{g}_{wtks},\ f_{ltks},\ \widehat{f}_{mtks},\ \theta_{ntks}\}$ is the set of decision variables of the lower-level problem. Lagrange multipliers are assigned to each constraint and presented in parentheses separated by a colon. $\Re$ is the set of real numbers.

$$\begin{aligned}\underset{\Omega}{\text{Maximize}}&\sum_t \tfrac{1}{(1+r)^{t-1}}\Big(\textstyle\sum_{sk}\Psi P_s(\sum_i A_i d_{itks}-\sum_j C_j\ g_{jtks}-\\ &\textstyle\sum_b(C_b^{(dh)}\tilde{d}_{btks}+C_b^{(ch)}\ \tilde{g}_{btks}))-(\sum_b C_{bt}^{(E)}(e_{bt}-e_{b(t-1)})-\\ &\textstyle\sum_b C_{bt}^{(P)}(p_{bt}-p_{b(t-1)})-\sum_w C_{wt}^{(W)}(u_{wt}-u_{w(t-1)})\Big)\end{aligned} \tag{7a}$$

Subject to:

$$u_{wt}-u_{w(t-1)}\geq 0 \ :(\eta_{wt}) \ \forall w,t \tag{7b}$$

$$e_{bt}-e_{b(t-1)}\geq 0 \ :(\kappa_{bt}) \ \forall b,t \tag{7c}$$

$$p_{bt}-p_{b(t-1)}\geq 0 \ :(\vartheta_{bt}) \ \forall b,t \tag{7d}$$

$$\begin{aligned}&\textstyle\sum_j J_n^{(j)}g_{jtks}+\sum_w W_n^{(w)}\widehat{g}_{wtks}-\sum_i I_n^{(i)}d_{itks}+\sum_b E_n^{(b)}\tilde{g}_{btks}-\\ &\textstyle\sum_b E_n^{(b)}\tilde{d}_{btks}-\sum_l S_n^{(l)}f_{ltks}+\sum_l R_n^{(l)}f_{ltks}-\sum_m \overline{S}_n^{(m)}\widehat{f}_{mtks}+\\ &\textstyle\sum_m \overline{R}_n^{(m)}\widehat{f}_{mtks}=0 \ :(\lambda_{ntks}) \ \forall n,t,k,s\end{aligned} \tag{7e}$$

[4] $C_b^{(ch)}$ and $C_b^{(dh)}$ are calculated as a share of investment cost per operational cycle.

$$-\tfrac{100}{X_l}\big(\textstyle\sum_n S_n^{(l)}\theta_{ntks} - \sum_n R_n^{(l)}\theta_{ntks}\big) + f_{ltks} = 0 \ : (\sigma_{ltks}) \tag{7f}$$

$$\widehat{f}_{mtks} - \tfrac{100}{X_m}\big(\textstyle\sum_n \overline{S}_n^{(m)}\theta_{ntks} - \sum_n \overline{R}_n^{(m)}\theta_{ntks}\big) \le \Xi_m(1-z_{mt}) : (\overline{\sigma}_{mtks}) \ \forall m,t,k,s \tag{7g}$$

$$\widehat{f}_{mtks} - \tfrac{100}{X_m}\big(\textstyle\sum_n \overline{S}_n^{(m)}\theta_{ntks} - \sum_n \overline{R}_n^{(m)}\theta_{ntks}\big) \ge -\Xi_m(1-z_{mt}) \ : (\underline{\sigma}_{mtks}) \ \forall m,t,k,s \tag{7h}$$

$$-z_{mt}\Xi_m \le \widehat{f}_{mtks} \le z_{mt}\Xi_m : (\underline{\zeta}_{mtks}, \overline{\zeta}_{mtks}) \ \forall m,t,k,s \tag{7i}$$

$$q_{btks} = q_{bt(k-1)s} - \tilde{g}_{btks} + \tilde{d}_{btks} \ : (\tau_{btks}) \ \forall b,t,k,s \tag{7j}$$

$$-\widehat{F}_m \le \widehat{f}_{mtks} \le \widehat{F}_m \ : (\underline{\gamma}_{mtks}, \overline{\gamma}_{mtks}) \ \forall m,t,k,s \tag{7k}$$

$$-F_l \le f_{ltks} \le F_l \ : (\underline{\mu}_{ltks}, \overline{\mu}_{ltks}) \ \forall l,t,k,s \tag{7l}$$

$$0 \le g_{jtks} \le G_j \ : (\underline{\nu}_{jtks}, \overline{\nu}_{jtks}) \ \forall j,s,t,k \tag{7m}$$

$$0 \le \widehat{g}_{wtks} \le (\widehat{G}_w + u_{wt})\varrho_{wtks} : (\underline{\eta}_{wtks}, \overline{\eta}_{wtks}) \ \forall w,t,k,s \tag{7n}$$

$$0 \le \tilde{g}_{btks} \le p_{bt}a_{btks} \ : (\underline{\kappa}_{btks}, \overline{\kappa}_{btks}) \ \forall b,t,k,s \tag{7o}$$

$$0 \le \tilde{d}_{btks} \le p_{bt}(1-a_{btks}) \ : (\underline{\vartheta}_{btks}, \overline{\vartheta}_{btks}) \ \forall b,t,k,s \tag{7p}$$

$$0 \le q_{btks} \le e_{bt} \ : (\underline{\rho}_{btks}, \overline{\rho}_{btks}) \ \forall b,t,k,s \tag{7q}$$

$$0 \le d_{itks} \le D_{it} \ : (\underline{\omega}_{itks}, \overline{\omega}_{itks}) \ \forall i,t,k,s \tag{7r}$$

$$\theta_{(n=1)tks} = 0 \ : (\xi_{ntks}) \ \forall t,k,s,n=1 \tag{7s}$$

$$a_{btks} \in \{0,1\} \tag{7t}$$

$$d_{itks}, \tilde{d}_{btks}, \tilde{g}_{btks}, q_{btks}, g_{jtks}, \widehat{g}_{wtks}, f_{ltks}, \widehat{f}_{mtks}, p_{bt}, e_{bt}, \theta_{ntks}, u_{wt} \in \Re \tag{7u}$$

The optimization problem (5)-(7) is a mixed-integer nonlinear bilevel program and accordingly computationally challenging. We proposed the following reformulation technique.

A. The linear programming (LP) equivalent

We have the following Lemma 2 for the battery-storage and wind-generation capacity investment optimization model (7).

Lemma 2. *The binary variables in the MILP model (7) can be omitted if simultaneous charge and discharge operations of the battery-storage system are excluded. This means that the relaxed LP formulation of (7) without charge and discharge binary variables is equivalent to the original MILP formulation.*

Proof. Assume that the binary variables are not in place (the problem (7) is formulated as an LP problem) and the charge and discharge happen simultaneously, i.e. $\tilde{d}_{btks} > 0$ and $\tilde{g}_{btks} > 0$. This implies that we can derive the KKT optimality conditions for the relaxed LP formulation of (7). The Lagrangian multipliers of constraints (7o) and (7p) will be equal to zero, $\underline{\kappa}_{btks} = 0$ and $\underline{\vartheta}_{btks} = 0$. Using the stationarity conditions (8a) and (8b) of the relaxed LP model of (7)[5]:

$$\tfrac{-\Psi}{(1+r)^{t-1}} P_s C_b^{(ch)} - \textstyle\sum_n E_n^{(b)}\lambda_{ntks} + \tau_{btks} + \underline{\vartheta}_{btks} - \overline{\vartheta}_{btks} = 0 \tag{8a}$$

$$\tfrac{-\Psi}{(1+r)^{t-1}} P_s C_b^{(dh)} + \textstyle\sum_n E_n^{(b)}\lambda_{ntks} - \tau_{btks} + \underline{\kappa}_{btks} - \overline{\kappa}_{btks} = 0 \tag{8b}$$

we can derive

$$\textstyle\sum_n E_n^{(b)}\lambda_{ntks} \overset{(8a)}{=} -\tfrac{\Psi}{(1+r)^{t-1}} P_s C_b^{(ch)} + \tau_{btks} + \underline{\vartheta}_{btks} - \overline{\vartheta}_{btks} \overset{(8b)}{=} \frac{\Psi}{(1+r)^{t-1}} P_s C_b^{(dh)} + \tau_{btks} - \underline{\kappa}_{btks} + \overline{\kappa}_{btks} \quad \forall b,t,k,s. \tag{9a}$$

Previously we assumed that $\tilde{d}_{btks} > 0$ and $\tilde{g}_{btks} > 0$ which leads to (9b).

$$-\frac{\Psi P_s}{(1+r)^{t-1}}(C_b^{(dh)} + C_b^{(ch)}) = \overline{\vartheta}_{btks} + \overline{\kappa}_{btks} \ \forall b,t,k,s \tag{9b}$$

Under the assumption $\tilde{d}_{btks} > 0$ and $\tilde{g}_{btks} > 0$ the sum of $\overline{\vartheta}_{btks} + \overline{\kappa}_{btks}$ on the right-hand side of the equation (9b) will be either 0 or strictly positive, whilst the expression $-P_t\Psi P_s C_b^{(dh)} - P_t\Psi P_s C_b^{(ch)}$ on the left-hand side is strictly negative. This leads to a contradiction and to the conclusion that the assumptions $\tilde{d}_{btks} > 0$ and $\tilde{g}_{btks} > 0$ cannot hold. Thus, battery-storage operations will not charge and discharge at the same time and at least one of the variables $\tilde{d}_{btks}$ or $\tilde{g}_{btks}$ should be equal to zero in the optimal solution. Furthermore, the LP equivalent reformulation is a relaxation of the original MILP, meaning that the solution of the LP equivalent ($SW_{LP}(\boldsymbol{y}^*)$) is greater than or equal to the original MILP solution ($SW_{MILP}(\boldsymbol{x}^*)$), where $\boldsymbol{y}^*$ and $\boldsymbol{x}^*$ are optimal solution vectors of the original MILP and the LP equivalent, respectively. On the other hand, since we have proved that the disjunctive property of constraints (7o) and (7p) are maintained in $\boldsymbol{y}^*$, we have $SW_{MILP}(\boldsymbol{y}^*) \le SW_{MILP}(\boldsymbol{x}^*)$). Therefore, $SW_{MILP}(\boldsymbol{y}^*) \le SW_{MILP}(\boldsymbol{x}^*) \le SW_{LP}(\boldsymbol{y}^*)$). Moreover, since the SW_{MILP} and SW_{LP} are linear functions, $SW_{MILP}(\boldsymbol{x}^*) = SW_{LP}(\boldsymbol{y}^*)$ and $\boldsymbol{x}^* = \boldsymbol{y}^*$. This means that the optimal solutions of variables $\tilde{d}_{btks}$ and $\tilde{g}_{btks}$ achieved under the original MILP problem and the relaxed LP problem are the same. □

B. The one-level linear equivalent

Using Lemma 2, the lower-level MILP model (7) is transformed to an equivalent LP model which can then be replaced by the primal feasibility conditions, dual feasibility conditions and the strong-duality conditions [33], [34]. The optimization problem (5)-(7) becomes a one-level nonlinear problem [6]. Using Lemma 3 and Lemma 4 the nonlinear one-level problem can be equivalently reformulated into a mixed-integer linear problem.

Lemma 3. *If the disjunctive parameters (Ξ_m)*[7] *are tuned properly then bilinear terms $T1 = z_{mt}\Xi_m(\underline{\zeta}_{mtks} + \overline{\zeta}_{mtks})$ and $T2 = (1 - z_{mt})(\overline{\sigma}_{mtks} + \underline{\sigma}_{mtks})$*[8] *are always equal to zero and can be linearized as:*

$$-\Xi_m z_{mt} \le \overline{\sigma}_{mtks} + \underline{\sigma}_{mtks} \le \Xi_m z_{mt} \tag{10a}$$

$$-\Xi_m(1 - z_{mt}) \le \overline{\zeta}_{mtks} + \underline{\zeta}_{mtks} \le \Xi_m(1 - z_{mt}) \tag{10b}$$

Proof. We prove this in (31) for $z_{mt} = 0$ and $z_{mt} = 1$.

$$\text{if } z_{mt} = 0 \Rightarrow \overline{\sigma}_{mtks} + \underline{\sigma}_{mtks} = 0 \Rightarrow T1 = 0, T2 = 0 \tag{11a}$$

$$\text{if } z_{mt} = 1 \Rightarrow \overline{\zeta}_{mtks} + \underline{\zeta}_{mtks} = 0 \Rightarrow T1 = 0, T2 = 0 \quad \square \tag{11b}$$

Lemma 4. *The Non-linear congestion rents calculation can be equivalently expressed as a linear sum of the physical upper- and lower-bounds of the existing and new transmission lines multiplied by their corresponding Lagrange multipliers.*

[5]Full derivations of stationary conditions and complementary slackness conditions are presented in the Appendix B.

[6]Full derivations of the dual feasibility conditions and the strong-duality conditions are presented in the Appendix B.

[7]As a disjunctive parameter, we define a parameter Ξ_m used in the linearization of disjunctive constraints. This parameter should be big enough to include whole feasible region and small enough not too affect computational tractability of the problem.

[8]Terms T1 and T2 appear in the dual feasibility conditions. Please refer to Appendix B for full derivations.

Proof. We start with the initial bilinear expression of congestion rent for a given operation period:

$$\textstyle\sum_{n,i} I_n^{(i)} \lambda_{ntks} d_{itks} - \sum_{n,j} J_n^{(j)} \lambda_{ntks} g_{jtks} - \sum_{n,w} W_n^{(w)} \lambda_{ntks} + \sum_{nb} E_n^{(b)} \lambda_{ntks} (\tilde{d}_{btks} - \tilde{g}_{btks}) \widehat{g}_{wtks} \quad (12)$$

The nodal prices can be extracted from these terms, i.e.

$$\sum_n \lambda_{ntks} \underbrace{\begin{pmatrix} \sum_i I_n^{(i)} d_{itks} + \sum_{nb} E_n^{(b)} (\tilde{d}_{btks} - \tilde{g}_{btks}) - \\ \sum_j J_n^{(j)} g_{jtks} - \sum_w W_n^{(w)} \widehat{g}_{wtks} \end{pmatrix}}_{L1}. \quad (13)$$

The expression L1 also appears in the power flow constraint (7e) and can thus be replaced by the sum of the power flows:

$$\sum_l f_{ltks} \underbrace{(-\textstyle\sum_n S_n^{(l)} \lambda_{ntks} + \sum_n R_n^{(l)} \lambda_{ntks})}_{L2} + \sum_m \widehat{f}_{mtks} \underbrace{(-\textstyle\sum_n \overline{S}_n^{(m)} \lambda_{ntks} + \sum_n \overline{R}_n^{(m)} \lambda_{ntks})}_{L3} \quad (14)$$

Expressions L2 and L3 are parts of the stationarity condition constraints [9]. Thus L2 and L3 can be represented equivalently as a linear combination of dual variables. Using complementary slackness conditions, stationarity conditions and Lemma 3, constraint (29) can be equivalently reformulated as:

$$\textstyle\sum_l F_l (\overline{\mu}_{ltks} + \underline{\mu}_{ltks}) + \sum_m \widehat{F}_m (\overline{\gamma}_{mtks} + \underline{\gamma}_{mtks}) \quad \square \quad (15)$$

Employing Lemma 3 and Lemma 4, the strong-duality condition of the lower-level problem is derived using linearized bilinear terms as set out in (34).

$$\begin{aligned} &\textstyle\sum_t \frac{1}{(1+r)^{t-1}} \Big(\sum_{s,k} \Psi P_s (\sum_i A_i d_{itks} - \sum_j C_j \; g_{jtks} - \\ &\textstyle\sum_b (C_b^{(dh)} \tilde{d}_{btks} + C_b^{(ch)} \; \tilde{g}_{btks})) - (\sum_b C_{bt}^{(E)} (e_{bt} - e_{b(t-1)}) - \\ &\textstyle\sum_b C_{bt}^{(P)} (p_{bt} - p_{b(t-1)}) - \sum_w C_{wt}^{(W)} (u_{wt} - u_{w(t-1)})) \Big) = \\ &\textstyle\sum_t \big(\sum_i D_{it} \overline{\omega}_{itks} + \sum_j G_j \overline{\nu}_{jtks} + \sum_l F_l (\underline{\mu}_{ltks} + \overline{\mu}_{ltks}) + \\ &\textstyle\sum_m \widehat{F}_m \overline{\gamma}_{mtks} \big) \end{aligned} \quad (16a)$$

$$-\Xi_m z_{mt} \le \overline{\sigma}_{mtks} + \underline{\sigma}_{mtks} \le \Xi_m z_{mt} \quad (16b)$$

$$-\Xi_m (1 - z_{mt}) \le \overline{\zeta}_{mtks} + \underline{\zeta}_{mtks} \le \Xi_m (1 - z_{mt}) \quad (16c)$$

Using Lemmas 2-4, the initial mixed-integer nonlinear bilevel model (5)-(7) is transformed into a MILP model (17), where Ω' is the set of all primal and dual variables of (17).

$$\begin{aligned} &\underset{z_{mt} \in \{0,1\}, y_{mt} \in \{0,1\}, \Omega'}{\text{Maximize}} \quad \textstyle\sum_t \langle \sum_s (\sum_l F_l (\overline{\mu}_{ltks} + \underline{\mu}_{ltks}) + \\ &\textstyle\sum_m \widehat{F}_m (\overline{\gamma}_{mtks} + \underline{\mu}_{ltks})) + \Phi_t - \frac{1}{(1+r)^{t-1}} \sum_m C_{mt}^{(T)} y_{mt} \rangle \end{aligned} \quad (17a)$$

Subject to:

$$z_{m,(t=1)} = 0, \; \textstyle\sum_t y_{mt} \le 1, \; \forall m \quad (17b)$$

$$z_{mt} = \textstyle\sum_{\widehat{t} \le t} y_{m,\widehat{t}} \; \forall m, \; \forall t \ge 2 \quad (17c)$$

$$\begin{aligned} &\Phi_t = \textstyle\sum_s P_s \sum_k (\frac{\Psi}{(1+r)^{t-1}} (\sum_i A_i d_{itks} - \sum_j C_j g_{jtks} + \\ &\textstyle\sum_b (C_b^{(ch)} \tilde{g}_{btks} + C_b^{(dh)} \tilde{d}_{btks}) - \sum_i A_i d_{i(t-1)ks} - \\ &\textstyle\sum_j C_j g_{j(t-1)ks} + \sum_{b,k} (C_b^{(ch)} \tilde{g}_{b(t-1)ks} + C_b^{(dh)} \tilde{d}_{b(t-1)ks}) - \\ &\textstyle\sum_l F_l (\overline{\mu}_{lt-1ks} + \underline{\mu}_{lt-1ks}) - \sum_m \widehat{F}_m (\overline{\gamma}_{mt-1ks} + \underline{\gamma}_{mt-1ks})) + \\ &\textstyle\frac{1}{(1+r)^{t-1}} (\sum_m C_{mt}^{(T)} y_{m(t-1)} - \sum_w C_{wt}^{(W)} (u_{wt} - u_{w(t-1)}) - \\ &\textstyle\sum_b C_{bt}^{(E)} (e_{bt} - e_{b(t-1)}) - \sum_b C_{bt}^{(P)} (p_{bt} - p_{b(t-1)}))) \; \forall t \ge 2 \end{aligned} \quad (17d)$$

[9]Please refer to equations (22g) and (22h) in the Appendix B

$$(6b), (7b)–(7n), (7q)–(7u), (8), (34) \quad (17e)$$

$$0 \le \tilde{g}_{btks} \le p_{bt} \;\; : (\underline{\kappa}_{btks}, \overline{\kappa}_{btks}) \; \forall b, t, k, s \quad (17f)$$

$$0 \le \tilde{d}_{btks} \le p_{bt} \;\; : (\underline{\vartheta}_{btks}, \overline{\vartheta}_{btks}) \; \forall b, t, k, s \quad (17g)$$

$$\underline{\omega}_{itks}, \overline{\omega}_{itks}, \underline{\nu}_{jtks}, \overline{\nu}_{jtks}, \underline{\eta}_{wtks}, \overline{\eta}_{wtks}, \sigma_{ltks}, \overline{\zeta}_{mtks} \ge 0 \quad (17h)$$

III. Disjunctive-based decomposition (DBD)

The reformulated and linearized optimal investment model (17) is a linear disjunctive program with disjunctive properties occurring in the transmission investment constraints (7g)-(7i). The disjunctive structure of the problem can be exploited to improve computational tractability of the problem using a decomposition algorithm. In this section, we propose a modified and accelerated decomposition algorithm based on the initial decomposition framework presented in [25]. We show that the proposed algorithm in Fig. 1 and Fig. 2 not only exploits the disjunctive structure of the problem but removes the need for tuning any additional arbitrary parameters used in the disjunctive constraint formulation. For simplicity, a general form is used in this section to explain the proposed DBD algorithm. The vectors of variables and proper parameter matrices are employed to re-state (17) in the general form as (18).

$$\underset{\boldsymbol{u} \ge 0, \boldsymbol{y}}{\text{Maximize}} \quad R^\intercal \boldsymbol{u} \quad (18a)$$

$$\text{Subject to: } A\boldsymbol{u} \le B \;\; : (\boldsymbol{\mu}) \quad (18b)$$

$$V\boldsymbol{u} \le C - \Xi(\mathcal{I} - \boldsymbol{y}) \;\; : (\boldsymbol{\nu}) \quad (18c)$$

$$E\mathbf{y} = Z, \; y \in \{0,1\}, \; \forall y \in \mathbf{y} \quad (18d)$$

Here $\mathbf{u} = [\underline{\omega}_{itks}, \overline{\omega}_{itks}, \underline{\nu}_{jtks}, \overline{\nu}_{jtks}, \underline{\eta}_{wtks}, \overline{\eta}_{wtks}, \sigma_{ltks}, \overline{\zeta}_{mtks}, \overline{\xi}_{ntks}, \underline{\xi}_{ntks}]$ is vector of continuous variables and $\mathbf{y} = [z_{mt}, y_{mt}]$ is vector of binary variables. The problem in (17) could be written in the general form in (18) using proper parameter matrices R, A, B, V, C, E, and Z. $\mathcal{I}$ is an all-ones vector. The (18c) is employed to enforce $V\mathbf{u} \le C$ when the corresponding binary variables are equal to one. The $\Xi \in \Re$ is a sufficiently large constant that satisfies $V\mathbf{u} \le C - \Xi$ when $y \in \mathbf{y}$ is zero. Furthermore, $V\mathbf{u} \le C$ is enforced when corresponding $y \in \mathbf{y}$ is one. The disjunctive structure can be used to split the problem into a master problem (MP) and a subproblem (SP). The SP is formulated based on the dual of the original problem (18) where the binary variables are fixed to some values as in (19).

$$\underset{\boldsymbol{\mu} \ge 0, \boldsymbol{\nu} \ge 0}{\text{Minimize}} \quad K_v = B^\intercal \boldsymbol{\mu} + C^\intercal \boldsymbol{\nu} - \Xi(\mathcal{I} - \boldsymbol{y}_v)^\intercal \boldsymbol{\nu} \quad (19a)$$

$$\text{Subject to: } A^\intercal \boldsymbol{\mu} + V^\intercal \boldsymbol{\nu} \ge R \;\; : (\mathbf{u}) \quad (19b)$$

The objective function contains the disjunctive parameters in the term $-\Xi(\mathcal{I} - \boldsymbol{y}_v)^\intercal \boldsymbol{\nu}$. However, the complementary slackness conditions of constraints (18c) guarantee that the term $(\mathcal{I} - \boldsymbol{y}_v)^\intercal \boldsymbol{\nu}$ is equal to zero for the optimal solution. Therefore, $-\Xi(\mathcal{I} - \boldsymbol{y}_v)^\intercal \boldsymbol{\nu}$ could be removed from the objective function if an additional constraint which represents complementary slackness condition is added to the formulation. The problem (19) is thus equivalently reformulated as SP in (20) without the disjunctive parameters.

$$\textbf{SP:} \underset{\boldsymbol{\mu} \ge 0, \boldsymbol{\nu} \ge 0}{\text{Minimize}} \quad K_v = B^\intercal \boldsymbol{\mu} + C^\intercal \boldsymbol{\nu} \quad (20a)$$

$$\text{Subject to: } A^\intercal \boldsymbol{\mu} + V^\intercal \boldsymbol{\nu} \ge R \;\; : (\mathbf{u}) \quad (20b)$$

$$(\mathcal{I} - \boldsymbol{y}_v)^\intercal \boldsymbol{\nu} = 0 \quad (20c)$$

In each iteration, $\boldsymbol{\nu}_v$ forms the new set Ω_v; this set is used to represent index sets for extreme points corresponding to the constraints with integer variables (18c). MP is formulated in (21).

$$\textbf{MP:} \underset{x_v, \boldsymbol{y}_v}{\text{Maximize}} \ \sum_v K_v x_v \tag{21a}$$

$$\text{Subject to:} \sum_{m,t\in\Omega_v} y_{m,t} \leq |\Omega_v| - 1 + \sum_{v'\ni(K_{v'}\geq K_v)} x_{v'} \tag{21b}$$

$$\sum_{v=0}^{K} x_v = 1, E\mathbf{y} = Z, y_{m,t} \in \{0,1\} \ \forall y_{m,t} \tag{21c}$$

Feasibility cuts are obtained according to the approach presented in [35] for the constraint (21b). Ω_v is set of indices of the dual variables in the SP (20) which have positive values in iteration v. $|\Omega_v|$ is the cardinality (number of members) of the set Ω_v. The binary variable x_v is employed to activate the corresponding feasibility cut according to $y_{m,t}$. Also x_0 is used to prevent unbounded solutions. The solution of MP provides a set of binary variables $y_{m,t} \in \mathbf{y}_v$ which is used to update the fixed values of the complicating variables in the SP ($\boldsymbol{y}_v$). The MP (21) has cuts which are as tight, or tighter than, cuts of a Benders decomposition. However, the cuts in (21b) do not contain disjunctive parameters (unlike the original Benders cuts) [35]. The proposed DBD algorithm solves the SP (20) and the MP (21) while increasing the number of iterations until the optimality gap is satisfied. The DBD decomposition algorithms allows the elimination of the disjunctive parameters from the cuts of the master problem. The DBD algorithm in combination with the reformulation technique proposed in Lemma 3 facilitates the complete removal of disjunctive parameters from both MP and SP and eliminates any numerical stability problem present in the original model formulation. The proposed DBD algorithm is detailed in Fig. 2. Here, a Benders decomposition approach and our modified DBD approach are shown for two iterations. The MP during the initial iterations might have multiple optimal solutions. At each iteration, these multiple solutions are found and then the associated SPs to these optimal solutions are solved in parallel. A simple numerical example is provided in Appendix D to demonstrate the proposed DBD approach.

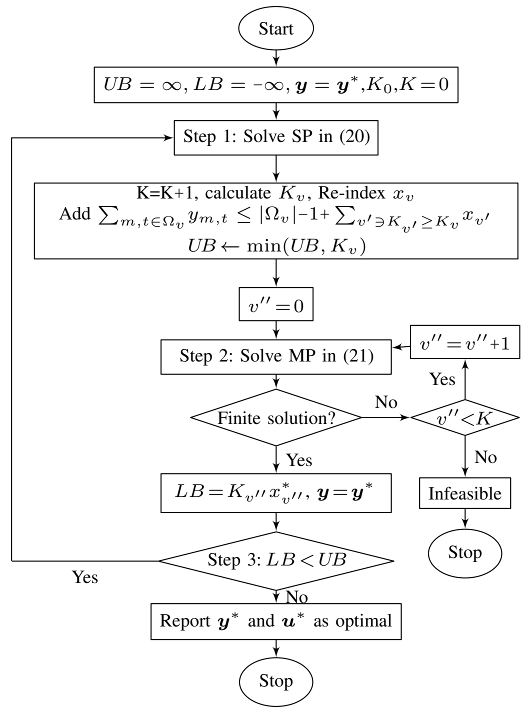

Figure 1. DBD algorithm

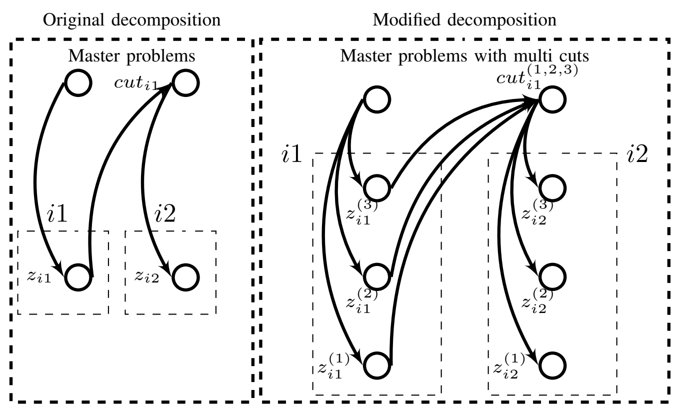

Figure 2. The DBD algorithm in Fig. 1 without multi-cut acceleration technique (original decomposition) and with multi-cut acceleration technique (modified decomposition)

IV. Case studies

Our proposed investment mechanism has been applied to three case study systems.

A. A six-node illustrative example

For illustrative purposes and in order to validate the proposed model, a six-node system is tested. Two periods are considered with period 1 representing the status quo. The profit-maximizing Transco has four candidate transmission lines (6,3), (5,3), (5,4) and (6,4), where each pair (x, y) represents a line from node x to node y. The wind generators $W1$ and $W2$ are considering to invest in nodes 6 and 5, respectively. The outputs of the wind generators are stochastic and scenarios of wind-generation output are made using the moment-matching technique proposed in [36]. Three sites are considered for battery-storage investments in nodes 4, 5, and 6. The system data are presented in Tables II and III. The results of the transmission investment under our proposed mechanism are

Table II
The data of loads in the six-node system

Load	Node	Short-run Marginal Value ($/MWh)	Capacity (MW) at $t=1$
D1	Node 1	250	95
D2	Node 2	245	75
D3	Node 3	235	82
D4	Node 4	730	105
D5	Node 5	225	55

reported in Table IV and they are compared to the case when no regulation is applied (which is simulated using the same model but setting the fixed fee equal to zero for all time periods) and the benchmark case which refers to the system investments under the social-welfare maximizing objective. The comparisons are presented in Appendix C.

Based on the general theoretical proof provided in the Lemma 1 it is expected that our proposed mechanism should result

Table III
THE DATA OF GENERATORS AND BATTERIES IN THE SIX-NODE SYSTEM

Generator	Node	Short-run marginal cost ($/MWh)	Capacity (MW)	Expansion cost
G1	Node 1	600	60	-
G2	Node 3	500	50	-
G3	Node 6	1,000	40	-
W1	Node 6	50	0	80,000
W2	Node 5	50	0	7,000
BSS1	Node 5	0	0	800/500
BSS2	Node 4	0	0	300/600
BSS3	Node 6	0	0	500/300

Table IV
THE INVESTMENT RESULTS IN THE SIX-NODE SYSTEM

	MRM[i]	NRG[ii]	SWM[iii]
M1	**1**	0	**1**
M2	0	**1**	0
M3	**1**	**1**	**1**
M4	**1**	0	**1**
W1 (MW)	1,229	1,309	1,229
W2 (MW)	235	136	235
BSS1 (MW/MWh)	0/0	0/0	0/0
BSS2 (MW/MWh)	78/78	43/29	78/78
BSS3 (MW/MWh)	157/56	132/40	157/56
Wind spillage [viii](%)	0.5%	3%	0.5%
Fixed Fee (M$)	4,830.84	-	-
Congestion Rent (M$)	661.9	759.5	-
Trans.[vi] Inv.[vii] Cost (M$)	48.5	5.8	-

i: our proposed merchant-regulatory mechanism, ii: No regulation, iii: Social welfare maximising results, iv: battery-storage surplus, v: Wind-generator surplus, vi: Transmission, vii: Investment viii: wind spillage is calculated as $\frac{(\widehat{G}_w+u_{wt})\varrho_{wtks}-\widehat{g}_{wtks}}{(\widehat{G}_w+u_{wt})\varrho_{wtks}}$

in socially optimal investments[10]. Indeed, under our proposed investment mechanism, the Transco invests in lines M1, M3, and M4 but never in line M2 which is identical to investment decisions in the benchmark case (social welfare maximum investments). In contrast, the investments under the no regulatory mechanism are completely different from the benchmark case. Therefore, they cannot deliver investments which maximize social-welfare. Our proposed mechanism also supports social-welfare maximizing investments in wind-generation and battery-storage capacities. Consequently wind-energy spillage is substantially reduced. The investment mechanism resulted in the same investments as the social-welfare maximization ones also for larger case studies such as the IEEE 118- and 300-node systems, as reported in Tables VI and VIII

B. Performance of the proposed DBD algorithm

The proposed model and algorithm are further verified with the larger-scale case studies (IEEE 118- and 300-node systems from MATPOWER software [37]) on a computer with two 2.3GHz processors and 128GB of RAM. Maximum demand is increased by 50%. As before, the scenarios of wind-generation outputs are generated using the moment-matching technique proposed in [36].The rest of the input data used in respective case studies is provided in Table V. The performance of DBD algorithm is compared to the performance of the off-the-shelf solvers, CPLEX and Gurobi, as well as Benders decomposition. Disjunctive parameters are still present in the master problem of the Benders decomposition as well as in the formulation which is solved by the CPLEX and Gurobi. These disjunctive parameters can be divided into two groups. First, disjunctive parameters are present in constraints where involved variables do not have any natural upper limits (i.e., in (32) natural upper limits cannot be defined since the Lagrange multipliers do not have any upper bound). These disjunctive parameters are tuned using a trial and error method. The Disjunctive parameters are iteratively increased until they no longer affect the solution of the model. Second, disjunctive parameters are present in constraints where involved variables have natural upper limits (i.e., (7g)-(7i) with a natural upper limit defined by transmission capacity, $\widehat{F}_m$). These disjunctive parameters are set to be equal to natural limits.

As we can see in Table VI and Table VIII, the DBD algorithm can find the optimal solution, the Benders decomposition algorithm and the off-the-shelf solvers CPLEX and Gurobi fail to report any solution [11]. This improvement of computational tractability is result of three major contributions of the proposed decomposition algorithm. First, the disjunctive nature of the the MILP model is fully exploited. Second, our proposed algorithm does not have disjunctive parameters in either master problem or subproblem. Third, convergence of our proposed algorithm was accelerated using parallel computation techniques and multiple cut generation.

Table V
THE INPUT DATA FOR CONSIDERED CASE STUDIES

	IEEE 118-node	IEEE 300-node
Number of candidate lines	30	60
Number of existing lines	175	411
Conventional Generation (MW)	4,300	20,678
Wind Generation (MW)	2,500	12,000
Battery storage (MWh)	100	100
Number of scenarios	20	20
Number of operation subperiods	105	72
Maximum Load (MW)	4,242	23,526
Number of periods	4	4

Table VI
THE COMPUTATIONAL RESULTS FOR THE IEEE 118-NODE CASE STUDY

	Objective Function ($)	Computation Time (h)	iterations
CPLEX solver	*	*	*
Gurobi solver	*	*	*
Benders decomposition	*	*	*
Proposed DBD	3,859	8.01	1,105

"*": No solution after 24 hours of simulation

Table VII
THE INVESTMENT RESULTS FOR THE IEEE 118-NODE CASE STUDY

	MRM	NRG	SWM
Social Welfare	3,859	3,570	3,859
Total Investment in transmission	530	250	530
Total Investment in battery-storage	153	53	153

[10]Socially optimal investments are obtained using a central planning approach which is mathematically modeled in Appendix B.

[11]The Cplex and Gurobi solvers are parts of GAMS package 25.0.

Table VIII
THE COMPUTATIONAL RESULTS FOR THE IEEE 300-NODE CASE STUDY

	Objective Function ($)	Computation Time (h)	Iterations
CPLEX solver	*	*	*
Gurobi solver	*	*	*
Benders decomposition	*	*	*
Proposed DBD	6,712	9.5	10,139

Table IX
THE INVESTMENT RESULTS FOR THE IEEE 300-NODE CASE STUDY

	MRM	NRG	SWM
Social Welfare	6,712	6,150	6,712
Total Investment in transmission	700	550	700
Total Investment in battery-storage	355	155	355

V. CONCLUSIONS

This paper develops and tests a regulated incentive design for efficient investments by a Transco. In particular, a mathematical model for the coordinated investment in the battery-storage, wind-generation and transmission-network capacities is proposed and implemented. This incentive design takes account of optimal changes in the energy market resources. We used wind-generation and battery-storage as topical examples but the principles can extend to other resources. Battery-storage operations as well as wind-generation are becoming increasingly important and poses computational complexity. The proposed investment mechanism results in the maximum social-welfare investments in the whole system. The mathematical model of the proposed investment mechanism is a large-scale mixed integer bilevel program and which is computationally challenging. This bilevel program is converted into a one-level equivalent stochastic MILP through four Lemmas. This reformulated MILP model is solved using a proposed disjunctive-based decomposition algorithm. The numerical results support the efficiency of the proposed investment mechanism, mathematical model, and associated algorithm for efficient coordinated investments. For larger-scale applications stronger computers with parallel-processing capability will be required, however, this should be further investigated. In addition, the model proposed in the paper does not account for an end-effect of an investment planing period and for time delays connected to construction. A further limitation of the model is a simplified battery operation representation (efficiency is considered to be equal to 1) and degradation cost assumptions. These limitations should be addressed in the future work.

Notwithstanding the potential for model improvements, we have presented a detailed model of considerable complexity and shown how it can provide the basis for an alternative regulatory incentive approach. There is currently a gap in the regulatory analysis of transmission incentives with respect to the overall competitive market evolution. The use of the extended ISS approach here to provide optimal fixed fees to enter into the financial planning of merchant transmission investors is practical and optimal. Furthermore it is likely to appeal to the banks and other lenders as it is a transparent formula. The regulation of network companies is increasingly following a process of dialogue to agree scenarios and parameters, followed by incentives for compliance. This approach fits in with that paradigm.

APPENDIX A: NOMENCLATURE [12]

Binary Variables

a_{btks} Battery storage charge/discharge indicator;

z_{mt},y_{mt} Transmission investment decision variables;

Incidence matrices

$I_n^{(i)}$ Incidence matrix element of load i, node n;

$J_n^{(j)}$ Incidence matrix element of generator j, node n;

$R_n^{(l)}$ Incidence matrix element of receiving node n, line l;

$\overline{R}_n^{(m)}$ Incidence matrix element of receiving node n, line m;

$S_n^{(l)}$ Incidence matrix element of sending node n, line l;

$\overline{S}_n^{(m)}$ Incidence matrix element of sending node n, line m;

$W_n^{(w)}$ Incidence matrix element of generator w, node n;

Parameters

A_i Load i marginal value ($/MWh);

$C_b^{(ch)}$ Marginal operational cost of charging battery-storage b ($/MWh);

$C_b^{(dh)}$ Marginal operational cost of discharging battery-storage b ($/MWh);

$C_{bt}^{(E)}$ Marginal investment cost of battery-storage energy capacity for candidate b at period t ($/MWh);

C_j Marginal operation cost of generator j ($/MWh);

$C_{bt}^{(P)}$ Marginal investment cost of battery-storage power capacity for candidate b at period t ($/MW);

$C_{mt}^{(T)}$ Investment cost of transmission line m at period t ($/line);

$C_{wt}^{(W)}$ Marginal investment cost of wind generator w at period t ($/MW);

D_{it} Maximum capacity of load i at period t (MW);

F_l Maximum capacity of existing transmission line l, (MW);

$\widehat{F}_m$ Maximum capacity of candidate transmission line m, (MW);

G_j Maximum capacity of generator j (MW);

$\widehat{G}_w$ Maximum capacity of wind generator w (MW);

P_s Probability of scenario s;

Ψ Number of operational periods in an investment period;

r Interest rate;

ϱ_{wtks} Stochastic output of wind generator w at period t, k, scenario s (MW);

Ξ_m, Ξ Sufficiently large constants;

X_l Reactance of existing transmission line l (p.u.);

X_m Reactance of candidate transmission line m (p.u.);

Indices and Sets

$b \in \mathcal{E}$	Battery storages	$i \in \mathcal{D}$	Loads
$j \in \mathcal{G}$	Generators	$k \in \mathcal{K}$	Operation periods
$l \in \mathcal{L}$	Existing lines	$m \in \mathcal{M}$	Candidate lines
$n \in \mathcal{N}$	Nodes	$s \in \mathcal{S}$	Scenarios
$t \in \mathcal{T}$	Investment periods	$w \in \mathcal{W}$	wind generators

APPENDIX B: DERIVATIONS

[12] All continuous variables are introduced directly in the text of the manuscript

The mathematical formulation of battery-storage and wind-energy investment planning problem (7) is a linear program, for which the Karush-Kuhn-Taker (KKT) optimality conditions are necessary and sufficient [38]. Thus, the optimal solution of the battery-storage and wind-energy investment planning problem can be equivalently described by the solution of its KKT conditions. The stationarity and complementary slackness of the problem (7) are derived in (22) and (23) respectively.

$$\tfrac{\Psi P_s}{(1+r)^{t-1}}A_i-\sum_{n\in\mathcal{N}}I_n^{(i)}\lambda_{ntks}+\underline{\omega}_{itks}-\overline{\omega}_{itks}=0,\ \forall i,t,k,s \tag{22a}$$

$$\tfrac{-\Psi P_s}{(1+r)^{t-1}}C_j+\sum_{n\in\mathcal{N}}J_n^{(j)}\lambda_{ntks}+\underline{\nu}_{jtks}-\overline{\nu}_{jtks}=0,\ \forall j,t,k,s \tag{22b}$$

$$\tfrac{-\Psi P_s}{(1+r)^{t-1}}C_b^{(ch)}-\sum_n E_n^{(b)}\lambda_{ntks}+\tau_{btks}+\underline{\vartheta}_{btks}-\overline{\vartheta}_{btks}=0,\ \forall b,t,k,s \tag{22c}$$

$$\tfrac{-\Psi P_s}{(1+r)^{t-1}}C_b^{(dh)}+\sum_n E_n^{(b)}\lambda_{ntks}-\tau_{btks}+\underline{\kappa}_{btks}-\overline{\kappa}_{btks}=0,\ \forall b,t,k,s \tag{22d}$$

$$-\tau_{btks}+\tau_{bt(k+1)s}+\underline{\rho}_{btks}-\overline{\rho}_{btks}=0\ \ \forall b,t,k,s \tag{22e}$$

$$\sum_n W_n^{(w)}\lambda_{ntks}+\underline{\eta}_{wtks}-\overline{\eta}_{wtks}=0,\ \forall w,t,k,s \tag{22f}$$

$$\sum_n R_n^{(l)}\lambda_{ntks}-\sum_n S_n^{(l)}\lambda_{ntks}+\sigma_{ltks}+\underline{\mu}_{ltks}-\overline{\mu}_{ltks}=0,\ \forall l,t,k,s \tag{22g}$$

$$\begin{aligned}&-\sum_n\overline{S}_n^{(m)}\lambda_{ntks}+\sum_n\overline{R}_n^{(m)}\lambda_{ntks}+\underline{\sigma}_{mtks}-\overline{\sigma}_{mtks}+\\&\underline{\gamma}_{mtks}-\overline{\gamma}_{mtks}+\underline{\zeta}_{mtks}-\overline{\zeta}_{mtks}=0\ \ \forall m,t,k,s\end{aligned} \tag{22h}$$

$$\begin{aligned}&-\tfrac{100}{X_l}\sum_{l\in\mathcal{L}}S_n^{(l)}\sigma_{ltks}+\tfrac{100}{X_l}\sum_l R_n^{(l)}\sigma_{ltks}+\xi_{n=1tks}-\\&\tfrac{100}{X_m}\sum_{m\in\mathcal{M}}\overline{S}_n^{(m)}\underline{\sigma}_{mtks}+\tfrac{100}{X_m}\sum_{m\in\mathcal{M}}\overline{R}_n^{(m)}\underline{\sigma}_{mtks}+\\&\tfrac{100}{X_m}\sum_{m\in\mathcal{M}}\overline{S}_n^{(m)}\overline{\sigma}_{mtks}-\tfrac{100}{X_m}\sum_{m\in\mathcal{M}}\overline{R}_n^{(m)}\overline{\sigma}_{mtks}=0,\ \forall n,t,k,s\end{aligned} \tag{22i}$$

$$\tfrac{-1}{(1+r)^{t-1}}C_{bt}^{(E)}+\kappa_{bt}-\kappa_{bt+1}+\sum_{k,s}\overline{\rho}_{btks}=0\ \ \forall b,t \tag{22j}$$

$$\tfrac{-1}{(1+r)^{t-1}}C_{bt}^{(P)}+\vartheta_{bt}-\vartheta_{bt+1}+\sum_{k,s}\overline{\kappa}_{btks}+\sum_{k,s}\overline{\vartheta}_{btks}=0,\ \forall b,t \tag{22k}$$

$$\tfrac{-1}{(1+r)^{t-1}}C_{wt}^{(W)}+\eta_{wt}-\eta_{wt+1}+\sum_{k,s}\overline{\eta}_{wtks}\varrho_{wtks}=0\ \ \forall w,t \tag{22l}$$

$$u_{wt}\eta_{wt}=u_{w(t-1)}\eta_{wt}\ \forall w,t \tag{23a}$$

$$e_{bt}\kappa_{bt}=e_{b(t-1)}\kappa_{bt}\ \forall b,t \tag{23b}$$

$$p_{bt}\vartheta_{bt}=p_{b(t-1)}\vartheta_{bt}\ \forall b,t \tag{23c}$$

$$\begin{aligned}&(\widehat{f}_{mtks}-\tfrac{100}{X_m}(\sum_n\overline{S}_n^{(m)}\theta_{ntks}-\sum_n\overline{R}_n^{(m)}\theta_{ntks}))\overline{\sigma}_{mtks}=\\&\Xi_m(1-z_{mt})\overline{\sigma}_{mtks}\ \forall m,t,k,s\end{aligned} \tag{23d}$$

$$\begin{aligned}&-(\widehat{f}_{mtks}-\tfrac{100}{X_m}(\sum_n\overline{S}_n^{(m)}\theta_{ntks}-\sum_n\overline{R}_n^{(m)}\theta_{ntks}))\underline{\sigma}_{mtks}=\\&\Xi_m(1-z_{mt})\underline{\sigma}_{mtks}\ \forall m,t,k,s\end{aligned} \tag{23e}$$

$$f_{ltks}\overline{\mu}_{ltks}=F_l\overline{\mu}_{ltks}\ \forall l,t,k,s \tag{23f}$$

$$-f_{ltks}\underline{\mu}_{ltks}=F_l\underline{\mu}_{ltks}\ \forall l,t,k,s \tag{23g}$$

$$\widehat{f}_{mtks}\overline{\gamma}_{mtks}=\widehat{F}_m\overline{\gamma}_{mtks}\ \forall m,t,k,s \tag{23h}$$

$$-\widehat{f}_{mtks}\underline{\gamma}_{mtks}=\widehat{F}_m\underline{\gamma}_{mtks}\ \forall m,t,k,s \tag{23i}$$

$$-\widehat{f}_{mtks}\underline{\zeta}_{mtks}=z_{mt}\Xi_m\underline{\zeta}_{mtks}\ \forall m,t,k,s \tag{23j}$$

$$\widehat{f}_{mtks}\overline{\zeta}_{mtks}=z_{mt}\Xi_m\underline{\zeta}_{mtks}\ \forall m,t,k,s \tag{23k}$$

$$g_{jtks}\underline{\nu}_{jtks}=0\ \ \forall j,s,t,k \tag{23l}$$

$$g_{jtks}\overline{\nu}_{jtks}=G_j\overline{\nu}_{jtks}\ \ \forall j,s,t,k \tag{23m}$$

$$\widehat{g}_{wtks}\underline{\eta}_{wtks}=0\ \ \forall w,t,k,s \tag{23n}$$

$$\widehat{g}_{wtks}\overline{\eta}_{wtks}=(\widehat{G}_w+u_{wt})\varrho_{wtks}\overline{\eta}_{wtks}\ \ \forall w,t,k,s \tag{23o}$$

$$\tilde{g}_{btks}\underline{\kappa}_{btks}=0\ \ \forall b,t,k,s \tag{23p}$$

$$\tilde{g}_{btks}\overline{\kappa}_{btks}=p_{bt}\overline{\kappa}_{btks}\ \ \forall b,t,k,s \tag{23q}$$

$$\tilde{d}_{btks}\underline{\vartheta}_{btks}=0\ \ \forall b,t,k,s \tag{23r}$$

$$\tilde{d}_{btks}\overline{\vartheta}_{btks}=p_{bt}\overline{\vartheta}_{btks}\ \ \forall b,t,k,s \tag{23s}$$

$$q_{btks}\underline{\rho}_{btks}=0\ \ \forall b,t,k,s \tag{23t}$$

$$q_{btks}\overline{\rho}_{btks}=e_{bt}\overline{\rho}_{btks}\ \ \forall b,t,k,s \tag{23u}$$

$$d_{itks}\underline{\omega}_{itks}=0\ \ \forall i,t,k,s \tag{23v}$$

$$d_{itks}\overline{\omega}_{itks}=D_{it}\overline{\omega}_{itks}\ \ \forall i,t,k,s \tag{23w}$$

Using KKT conditions of the problem (3) we can prove that the term

$$\begin{aligned}&\sum_{ni}I_n^{(i)}\lambda_{ntks}d_{itks}-\sum_{nj}J_n^{(j)}\lambda_{ntks}g_{jtks}+\\&\sum_{nb}E_n^{(b)}\lambda_{ntks}(\tilde{d}_{btks}-\tilde{g}_{btks})-\sum_{nw}W_n^{(w)}\lambda_{ntks}\widehat{g}_{wtks}\end{aligned} \tag{24}$$

is equal to

$$\sum_l F_l(\overline{\mu}_{ltks}+\underline{\mu}_{ltks})+\sum_m\widehat{F}_m(\overline{\gamma}_{mtks}+\underline{\gamma}_{mtks}) \tag{25}$$

We start with the initial bilinear expression:

$$\begin{aligned}&\sum_{n,i}I_n^{(i)}\lambda_{ntks}d_{itks}-\sum_{n,j}J_n^{(j)}\lambda_{ntks}g_{jtks}-\\&\sum_{n,w}W_n^{(w)}\lambda_{ntks}+\sum_{nb}E_n^{(b)}\lambda_{ntks}(\tilde{d}_{btks}-\tilde{g}_{btks})\widehat{g}_{wtks}\end{aligned} \tag{26}$$

The nodal prices can be extracted from these terms, i.e.

$$\sum_n\lambda_{ntks}\underbrace{\begin{pmatrix}\sum_i I_n^{(i)}d_{itks}+\sum_{nb}E_n^{(b)}(\tilde{d}_{btks}-\tilde{g}_{btks})-\\ \sum_j J_n^{(j)}g_{jtks}-\sum_w W_n^{(w)}\widehat{g}_{wtks}\end{pmatrix}}_{L1}. \tag{27}$$

The term L1 also appears in the power flow constraint (7e) and can thus be replaced by the sum of the power flows:

$$\begin{aligned}&\sum_l\lambda_{ntks}(-\sum_n S_n^{(l)}f_{ltks}+\sum_n R_n^{(l)}f_{ltks})+\\&\sum_m\lambda_{ntks}(-\sum_n\overline{S}_n^{(m)}\widehat{f}_{mtks}+\sum_n\overline{R}_n^{(m)}\widehat{f}_{mtks})=\\&\sum_l f_{ltks}\underbrace{(-\sum_n S_n^{(l)}\lambda_{ntks}+\sum_n R_n^{(l)}\lambda_{ntks})}_{L2}+\\&\sum_m\widehat{f}_{mtks}\underbrace{(-\sum_n\overline{S}_n^{(m)}\lambda_{ntks}+\sum_n\overline{R}_n^{(m)}\lambda_{ntks})}_{L3}\end{aligned} \tag{28}$$

Terms L2 and L3 are parts of stationary condition constraints (22g) and (22h) respectively. Thus L2 and L3 equivalently can be represented as a linear combination of dual variables from constraints (22g) and (22h):

$$\begin{aligned}&\sum_l f_{ltks}(\overline{\mu}_{ltks}-\underline{\mu}_{ltks}-\sigma_{ltks})+\sum_m\widehat{f}_{mtks}(\underline{\sigma}_{mtks}-\overline{\sigma}_{mtks}+\\&\underline{\gamma}_{mtks}-\overline{\gamma}_{mtks}+\underline{\zeta}_{mtks}-\overline{\zeta}_{mtks})\end{aligned} \tag{29}$$

Using complementary slackness conditions (23d)-(23k) and stationary condition (22i) constraint (29) can be equivalently reformulated as:

$$\begin{aligned}&\sum_l F_l(\overline{\mu}_{ltks}+\underline{\mu}_{ltks})+\sum_m\widehat{F}_m(\overline{\gamma}_{mtks}+\underline{\gamma}_{mtks})+\\&\sum_m\underbrace{z_{mt}\Xi_m(\underline{\zeta}_{mtks}+\overline{\zeta}_{mtks})}_{T1}+\\&\sum_m\underbrace{\Xi_m(1-z_{mt})(\overline{\sigma}_{mtks}+\underline{\sigma}_{mtks})}_{T2}\end{aligned} \tag{30}$$

The terms $T1=z_{mt}\Xi_m(\underline{\zeta}_{mtks}+\overline{\zeta}_{mtks})$ and $T2=(1-z_{mt})(\overline{\sigma}_{mtks}+\underline{\sigma}_{mtks})$ include the disjunctive parameters Ξ_m and used to formulate power flow constraints of candidate transmission lines (7g)-(7i) are complicated because they include variables both from the upper and lower level problems and are thus non-linear. However, it can be shown that each of these terms are always equal to zero. If the disjunctive parameters are tuned properly, i.e., large enough that they do not limit power flows on selected candidate lines but small enough to avoid poorly conditioned matrices, then the constraints (7i) will never be binding.

$$\text{If } z_{mt}=0 \Rightarrow \overline{\sigma}_{mtks}+\underline{\sigma}_{mtks}=0 \Rightarrow T1=0, T2=0. \quad (31a)$$

$$\text{If } z_{mt}=1 \Rightarrow \overline{\zeta}_{mtks}+\underline{\zeta}_{mtks}=0 \Rightarrow T1=0, T2=0. \quad (31b)$$

If z_{mt} is equal to zero then the Lagrangian multipliers $\overline{\sigma}_{mtks}$ and $\underline{\sigma}_{mtks}$ are equal to zero due to the complementary slackness condition, resulting in both expression T1 and T2 to be equal to zero. By analogy when z_{mt} is equal to one then $(1-z_{mt})$ is equal to zero and using complimentary slackness conditions $\underline{\zeta}_{mtks}$ and $\overline{\zeta}_{mtks}$ are zero leading to T1 and T2 equal to zero. Thus we can remove the T1 and T2 terms from the strong duality constraint (30) and enforce T1=0 and T2=0 as a separate linear constraints:

$$-\Xi_m z_{mt} \le \overline{\sigma}_{mtks}+\underline{\sigma}_{mtks} \le \Xi_m z_{mt} \quad (32a)$$

$$-\Xi_m(1-z_{mt}) \le \overline{\zeta}_{mtks}+\underline{\zeta}_{mtks} \le \Xi_m(1-z_{mt}) \quad (32b)$$

Such reformulation will remove bilinear terms and will not affect the decision space.

$$\sum_t \frac{1}{(1+r)^{t-1}}\Big(\sum_{s,k}\Psi P_s(\sum_i A_i d_{itks} - \sum_j C_j\ g_{jtks} - \sum_b (C_b^{(dh)}\tilde{d}_{btks} + C_b^{(ch)}\ \tilde{g}_{btks})) - (\sum_b C_{bt}^{(E)}(e_{bt}-e_{b(t-1)}) - \sum_b C_{bt}^{(P)}(p_{bt}-p_{b(t-1)}) - \sum_w C_{wt}^{(W)}(u_{wt}-u_{w(t-1)})\Big) = \sum_t\Big(\sum_i D_{it}\overline{\omega}_{itks}+\sum_j G_j\overline{\nu}_{jtks}+\sum_l F_l(\underline{\mu}_{ltks}+\overline{\mu}_{ltks})+ \sum_m \widehat{F}_m(\overline{\gamma}_{mtks})+T1+T2\Big) \quad (33a)$$

Similarly, the strong duality (33) of the problem (7) can be reformulated using (31) as in (34)

$$\sum_t \frac{1}{(1+r)^{t-1}}\Big(\sum_{s,k}\Psi P_s(\sum_i A_i d_{itks} - \sum_j C_j\ g_{jtks} - \sum_b (C_b^{(dh)}\tilde{d}_{btks} + C_b^{(ch)}\ \tilde{g}_{btks})) - (\sum_b C_{bt}^{(E)}(e_{bt}-e_{b(t-1)}) - \sum_b C_{bt}^{(P)}(p_{bt}-p_{b(t-1)}) - \sum_w C_{wt}^{(W)}(u_{wt}-u_{w(t-1)})\Big) = \sum_t\Big(\sum_i D_{it}\overline{\omega}_{itks}+\sum_j G_j\overline{\nu}_{jtks}+\sum_l F_l(\underline{\mu}_{ltks}+\overline{\mu}_{ltks})+ \sum_m \widehat{F}_m(\overline{\gamma}_{mtks})\Big) \quad (34a)$$

$$-\Xi_m z_{mt} \le \overline{\sigma}_{mtks}+\underline{\sigma}_{mtks} \le \Xi_m z_{mt} \quad (34b)$$

$$-\Xi_m(1-z_{mt}) \le \overline{\zeta}_{mtks}+\underline{\zeta}_{mtks} \le \Xi_m(1-z_{mt}) \quad (34c)$$

Appendix C: Benchmark optimization model

$$\underset{\Omega}{\text{Maximize}}\sum_t \frac{1}{(1+r)^{t-1}}\Big(\sum_{sk}\Psi P_s(\sum_i A_i d_{itks}-\sum_j C_j\ g_{jtks} - \sum_b (C_b^{(dh)}\tilde{d}_{btks}+C_b^{(ch)}\ \tilde{g}_{btks})) - (\sum_b C_{bt}^{(E)}(e_{bt}-e_{b(t-1)}) - \sum_b C_{bt}^{(P)}(p_{bt}-p_{b(t-1)}) - \sum_w C_{wt}^{(W)}(u_{wt}-u_{w(t-1)}) - \sum_m C_{mt}^{(T)} y_{mt}\Big) \quad (35a)$$

$$\text{Subject to:} (7b)-(7u) \quad (35b)$$

$$(17c)-(17b) \quad (35c)$$

Appendix D: A simple numerical example

A simple numerical example for the generalized disjunctive program (9) is demonstrated in this section which is minimization problem in (36) which is a MILP problem.

$$\underset{u_1,u_2\ge 0, y_1\in\{0,1\}}{\text{Minimize}} \quad u_1+u_2 \quad (36a)$$

$$\text{Subject to: } u_1 \ge 8\ :\mu_1 \quad (36b)$$

$$u_2 \ge 2\ :\mu_2 \quad (36c)$$

$$u_2-u_1 \ge 2-\Xi y_1\ :\nu_1 \quad (36d)$$

$$u_1-u_2 \ge 1-\Xi(1-y_1)\ :\nu_2 \quad (36e)$$

Two positive continuous variables u_1 and u_2 are used along with a binary variable y_1 to model the disjunctive feature. μ_1, μ_2, ν_1, and ν_2 are Lagrangian variables. The binary variable y_1 makes sure that one and only one of constraints $u_2-u_1\ge 2$ and $u_1-u_2\ge 1$ are enforced. The parameter Ξ should be adjusted to implement this disjunctive feature properly. Feasibility region of this problem for different values of the parameter Ξ and the binary variable y_1 is shown in (3).

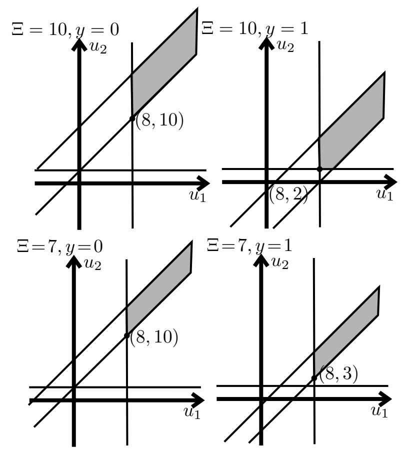

Figure 3. Feasibility region of problem (36)

Different feasibility regions of (36) are shown for four cases with $(\Xi=10, y_1=0)$, $(\Xi=10, y_1=1)$, $(\Xi=7, y_1=0)$, and $(\Xi=7, y_1=1)$. Here diferent values for the disjunctive parameter Ξ and the integer variable y_1 are investigated. The optimal solutions are shown in Fig. (3). As the objective is to minimize u_1+u_2, the optimal solutions for $\Xi=10$ are $y_1=1$, $u_1=8$, and $u_2=2$ with optimal value $u_1+u_2=10$. Furthermore, the optimal solutions for $\Xi=7$ are $y_1=1$, $u_1=8$, and $u_2=3$ with optimal value $u_1+u_2=11$. Therefore, employing new values for the disjunctive variable Ξ could change results (from 13 to 11 in this example). Finding the right value for disjunctive parameters is extremely hard (if not impossible) in large-scale problems with more integer variables. Solving this problem with Benders algorithm will also face this issue of finding the right value for Ξ. For given values for y_1, the remaining variables are continuous and we can use the duality theorem for remaining LP problem. Dual of the problem (36) is demonstrated in (37) after fixing the binary variable y_1.

$$\textbf{SPD:} \underset{\mu_1,\mu_2,\nu_1,\nu_2\ge 0}{\text{Maximize}}\ 8\mu_1+2\mu_2+2\nu_1+\nu_2 \quad (37a)$$

$$\text{Subject to:} \mu_1-\nu_1+\nu_2 \le 1 : u_1 \quad (37b)$$

$$\mu_2+\nu_1-\nu_2 \le 1 : u_2 \quad (37c)$$

$$y_1^*\nu_1 = 0 \quad (37d)$$

Table X
SOLUTIONS OF (37) FOR ALL COMBINATIONS OF y_1

y_1	μ_1	μ_2	ν_1	ν_2	K
0	2	0	1	0	18
1	1	1	0	0	10

$$(1-y_1^*)\nu_2 = 0 \tag{37e}$$

Solutions of the problem (37) for different combinations of the fixed binary variable y_1 are shown in Table X. We recognize that the SPD does not have any disjunctive parameter Ξ.

A. The DBD Algorithm:

The simple numerical example in (36) could be solved using the proposed DBD algorithm in Figures 1 and 2 without employing any disjunctive parameter (Ξ) in two iterations as follows:
Initialize: The algorithm is initialized first with $UB = 1000$, $LB = -1000$, $\boldsymbol{y}^* = [y_1^*] = [0]$, $K_0 = 1000$, and $K = 0$.
Iteration 1: Step 1 starts with updating $K \leftarrow K+1 = 1$. Then, the SPD in (37) is solved for given $y_1^* = 0$. From Table X we have $K_1 = 18$. As $\nu_1 = 1 > 0$, set of indices is $\Omega_1 = \{1\}$ with cardinality (number of elements in set) $|\Omega_1| = 1$. Indices 0 and 1 are changed since $K_0 = 1000$ and $K_1 = 18$ are decreasing in v (as the value of K is increasing the index of K should also be increasing). With new indices, $K_0 = 18$ and $K_1 = 1000$ are non-decreasing in v (in this new arrangement, we see that $18 < 1000$ and $0 < 1$, this property between K values and K indices should always hold. If it does not hold we do re-arrangement similar to what we did here). In Step 2, from (21b) we have to add the constraint ($y_1 \leq x_1$) to MPD-1 in (38). The UB is updated to $UB \leftarrow \min(UB = 1000, 18) = 18$ and the following equation holds -1000 $\leq$ global solution $\leq$ 18.

$$\textbf{MPD-1: } \underset{y_1, x_1 \in \{0,1\}}{\text{Minimize}} \; 18x_0 + 1000x_1 \tag{38a}$$

$$\text{Subject to:} y_1 \leq x_1 \tag{38b}$$

$$x_0 + x_1 = 1 \tag{38c}$$

Optimal solutions of MPD-1 in (38) are $x_0^* = 0$, $x_1^* = 1$, and $y_1^* = 1$. The LB remains the same since $LB = 18x_0^* + 1000x_1 = 1000$. In Step 3, we go to the next iteration since $UB < LB$.
Iteration 2: Step 1 starts with updating $K \leftarrow K+1 = 2$. Then, the SPD in (37) is solved for given binary variable $y_1^* = 1$. From Table X we have $K_2 = 10$. As $\nu_1 = \nu_2 = 0$, set of indices is an empty set $\Omega_2 = \{\}$ with cardinality $|\Omega_2| = 0$. Indices v are changed since $K_0 = 18$, $K_1 = 1000$ and $K_2 = 10$ are decreasing from index 1 to index 2. With new indices we have, $K_0 = 10$, $K_1 = 18$, and $K_2 = 1000$ which are non-decreasing in v. In Step 2, from (21b) we have to add the constraint ($1 \leq x_0 + x_2$) to the MPD-1 in (38). The UB is updated to $UB \leftarrow \min(UB = 18, 10) = 10$ and the following equation holds -1000 $\leq$ global solution $\leq$ 10.

$$\textbf{MPD-2: } \underset{y_1, x_1 \in \{0,1\}}{\text{Minimize}} \; 10x_0 + 18x_1 + 1000x_2 \tag{39a}$$

$$\text{Subject to:} y_1 \leq x_2 \tag{39b}$$

$$1 \leq x_0 + x_2 \tag{39c}$$

$$x_0 + x_1 + x_2 = 1 \tag{39d}$$

Optimal solutions of MPD-2 are $x_0^* = 1$, $x_1^* = 0$, $x_2^* = 0$, and $y_1^* = 0$. The LB is updated $LB = 10x_0^* + 18x_1^* + 1000x_2 = 10$. In Step 3, there is no need for more iterations since $UB = LB$. Marginal value of constraints (37b) and (37b) are 8 and 2 respectively. Therefore, $y_1 = 1$, $u_1 = 8$ and $u_2 = 2$ are reported as optimal solution which ends the algorithm. Note that in Iterations 1 and 2, we did not use the disjunctive parameter Ξ.

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