Stock Market and No-Dividend Stocks*

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Abstract

We develop a stationary model of the aggregate stock market featuring both dividend-paying and no-dividend stocks within a familiar, parsimonious consumption-based equilibrium framework. We find that such a simple feature leads to profound implications supporting several stock market empirical regularities that leading consumption-based asset pricing models have difficulty reconciling. We show that the presence of no-dividend stocks in the stock market leads to a lower correlation between the stock market return and aggregate consumption growth rate, a non-monotonic and even a negative relation between the stock market risk premium and its volatility, and a downward sloping term structure of equity risk premia. When we quantify these effects, we find them to be economically significant. We also find that no-dividend stocks command lower mean returns but have higher return volatilities and higher market betas than comparable dividend-paying stocks, consistently with empirical evidence. We provide straightforward intuition for all these results and the underlying economic mechanisms at play.

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Keywords: Stock market, no-dividend stocks, dynamic asset pricing, incomplete information, stock market correlation with consumption, market risk premium-volatility relation, term structure of equity premia.

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1 Introduction

The aggregate stock market contains both dividend-paying and no-dividend stocks.[1] Existing asset pricing models, however, overlook this composition and typically model the stock market as being made up of only dividend-paying stocks. We provide the first general equilibrium model of the stock market in which both types of stocks coexist. We uncover such a simple feature leads to profound implications that support several empirical regularities on the stock market which leading consumption-based asset pricing models, such as the habit-formation model (Campbell and Cochrane (1999)), the long-run risk model (Bansal and Yaron (2004)), and the rare disaster models (Rietz (1988), Barro (2006)) all have difficulty in reconciling. Namely, i) the low correlation between the stock market return and the aggregate consumption growth rate (Cochrane and Hansen (1992), Campbell and Cochrane (1999), Albuquerque, Eichenbaum, Luo, and Rebelo (2016), Heyerdahl-Larsen and Illeditsch (2017)), ii) the mixed evidence on the relation between the stock market conditional risk premium and volatility, as numerous works find this relation to be negative (Campbell (1987), Glosten, Jagannathan, and Runkle (1993), Whitelaw (2000), Harvey (2001), Brandt and Kang (2004)), while many others find it to be positive (French, Schwert, and Stambaugh (1987), Scruggs (1998), Ghysels, Santa-Clara, and Valkanov (2005), Bali and Peng (2006), Guo and Whitelaw (2006), Ludvigson and Ng (2007)), iii) the downward sloping term structure of equity risk premia (van Binsbergen, Brandt, and Koijen (2012), van Binsbergen and Koijen (2017)).

Why does the stock market composition matter? After all, it should not matter much if both types of stocks have similar risk-return profiles. Because then the aggregate stock market behavior would be similar to that of dividend-paying stocks as in standard models. However, much empirical evidence (highlighted below) reveals considerable differences between the risk-return behavior of dividend-paying and no-dividend stock returns: stocks that pay no dividends have lower risk premia but have higher return volatilities and higher market betas than comparable stocks that pay dividends. In this paper, we argue that the presence of dividend-paying stocks along with their risk-return differences from the no-dividend stock returns could well be (at least partially) behind the somewhat puzzling empirical regularities on the stock

[1] For example, Hartzmark and Solomon (2013) find that over the long-sample of 1927-2011, the average proportion of no-dividend stocks is around 35% and accounts for 21.3% of the aggregate US stock market capitalization. Similarly, by taking into account of rising share repurchase programs since the mid-1980ies, Boudoukh et al. (2007) report that over the 1984-2003 period, the average proportion of no-dividend stocks is 64% and no-payout stocks, i.e., no dividends or no share repurchases, is 51% with the relative market capitalizations of 16.4% and 14.2%, respectively. See Section 6 for further discussion on firms' payout policy in the data.

market discussed above. Towards that, we develop a model of the aggregate stock market featuring both dividend-paying and no-dividend stocks within a familiar consumption-based general equilibrium framework. In addition to supporting all of the empirical evidence above on the aggregate stock market, the model also supports the evidence on the cross-sectional differences between the typical dividend-paying and no-dividend stock returns, and provides simple intuition for the underlying economic mechanisms at play. We obtain closed-form solutions for all quantities of interest. On the other hand, our simple framework does not generate a sufficiently rich stochastic discount factor, limiting its ability to quantitatively match the stock market moments that the leading consumption-based asset pricing models (listed earlier) can do.

Our stock market model is based on recognizing that, differently from dividend-paying stocks, there are several noteworthy features of no-dividend stocks investors would need to take into account while determining their prices. First, the absence of their current dividends introduces information incompleteness, and hence necessities the estimation of their eventual future dividends using other relevant fundamental information. Second, the absence of their dividends also leads to additional uncertainty about their future dividend payment periods. Third, no-dividend stocks do not contribute directly to aggregate consumption, and hence to the stochastic discount factor. We accordingly adopt a standard, workhorse, infinite-horizon, dynamic pure-exchange economy in which the stock market contains both dividend-paying and no-dividend stocks at all times, as in reality. We achieve a stationary setting by considering standard dividend stocks that pay dividends at all times, and additionally two other types of stocks that pay dividends in alternating random periods that are governed by a Poisson process. In the absence of its dividends, we employ standard Bayesian filtering theory to estimate the future dividend distribution of the no-dividend stock using other relevant fundamental information and obtain an estimated pseudo-dividend process. This necessary filtering process induces additional variation in the estimated pseudo-dividend by making it more volatile than the corresponding underlying process, which would have been used under complete information. To better illustrate these features and our intuition, our multi-stock model is deliberately simple and parsimonious in the sense that there is a single investor with standard constant relative risk aversion (CRRA) preferences and the aggregate consumption growth rate has a constant mean and volatility.

Our model generates rich equilibrium implications. We first show that while a dividend-paying stock price is driven by its dividend, a no-dividend stock price is driven by its estimated pseudo-dividend in the absence of its dividends. Moreover, the presence of no-dividend stocks generates a novel spillover effect in that the expected future dividend payment times of no-dividend stocks affect the prices of all stocks, including the stocks that pay dividends at all times. This is because the expected dividend payment times are also when the stochastic discount factor shocks are anticipated to change, and what portion of the future dividends are expected to be discounted under which stochastic discount factor matters for all stock prices.

We then demonstrate that the presence of no-dividend stocks in the stock market leads to a lower correlation between the stock market return and the aggregate consumption growth rate. This occurs because the stock market consists of stocks that currently do not pay dividends and hence do not contribute to the current aggregate consumption while contributing to the fluctuations in the stock market returns. When we quantify this effect in our model, we see that its magnitude can be economically large. In particular, if we set the no-dividend relative stock size in our model to equal to the long-run average of the no-dividend relative stock size in the US data during 1927-2011 as reported by Hartzmark and Solomon (2013), 21.3%, this correlation is 0.53. When we set it to the average relative size of the no-payout stocks (no dividends or no share repurchases) during 1984-2003 in the US as reported by Boudoukh et al. (2007), 14.2%, this correlation is 0.75. In contrast, this correlation is typically close to one in leading asset pricing models. Our result then illustrates that the presence of no-dividend stocks can help explain a meaningful fraction of the relatively low correlation documented by empirical works, as highlighted earlier.

To study the relation between the stock market conditional risk premium and volatility, we first look at the price dynamics of the individual stocks that make up the stock market. We show that the risk premium of a no-dividend stock is lower than that of an otherwise identical dividend-paying stock. This is because in the absence of its dividends a no-dividend stock price is driven by its estimated pseudo-dividend, which does not contribute directly to aggregate consumption and hence comoves less with it as compared to a dividend-paying stock. Therefore the investor requires a lower risk premium to hold a no-dividend stock in equilibrium, since in our model, as also in standard consumption-based models, a stock risk premia is proportional to the covariance of its returns with the aggregate consumption growth rate. This result is consistent with much cross-sectional empirical evidence, which documents that stocks that pay no dividends (or no payouts including share repurchases) have lower average returns than comparable dividend-paying stocks (Christie (1990), Naranjo, Nimalendran, and Ryngaert (1998), Fuller and Goldstein (2011), Hartzmark and Solomon (2013)).

We next demonstrate that a no-dividend stock commanding a lower risk premium does not necessarily imply that its returns are less volatile or it has a lower market beta than a comparable dividend stock. On the contrary, we show that a no-dividend stock has a more volatile return and a higher market beta than a comparable dividend-paying stock. This is due to a no-dividend stock price being driven by its estimated pseudo-dividend, and the estimation process, necessitated by the absence of dividends, inducing additional variability. This additional variation in a no-dividend stock return also makes its return contribute to and comove with the aggregate stock market return more than a comparable dividend-paying stock, leading to a higher market beta for it. These results are also consistent with the cross-sectional empirical evidence, which documents that stocks that pay no dividends have higher return volatility (Naranjo, Nimalendran, and Ryngaert (1998), Pástor and Veronesi (2003), Hartzmark and Solomon (2013)), and higher market beta (Boudoukh, Michaely, Richardson, and Roberts (2007), Fuller and Goldstein (2011)) than comparable stocks that pay dividends. We also offer an alternative interpretation of the no-dividend (dividend) stocks in our model as the growth (value) stocks. This is because a typical growth stock is one with a low fundamental to price ratio while also sharing the three key features of no-dividend stocks in our model. With this interpretation, our findings are also consistent with the documented empirical regularities for growth and value stocks, since growth stocks have lower mean returns, higher return volatilities, and higher market betas than value stocks (e.g., Lettau and Wachter (2007)).

We then show that the presence of no-dividend stocks in the stock market leads to a non-monotonic and even a negative relation between the conditional risk premium and volatility of the stock market. This is because the stock market risk premium is a weighted-average of the risk premia of stocks that make up the stock market. With no-dividend stocks, which command low risk premia but high volatility, being part of the stock market, the stock market risk premium is non-monotonically related to, and in particular, is decreasing in its volatility for sufficiently high relative-size of the no-dividend stocks. This result sheds light on the decidedly mixed vast empirical findings on this relation, referred to as also "the risk-return tradeoff" (discussed earlier). Indeed, our result is in line with the empirical findings of Rossi and Timmermann (2010), who find a non-monotonic relation between the conditional risk premium and volatility by showing a positive relation for low and medium levels of volatility and a negative relation for high levels of volatility, and hence argue that the lack of consensus in the earlier empirical literature may be due to this non-monotonic relation.

Finally, we demonstrate that the presence of no-dividend stocks in the stock market can lead to a downward sloping term structure of the stock market equity risk premia by showing that short-term assets, claims to short-term aggregate dividends, tend to command a higher mean return than the stock market. This is because a short-term asset is more like a dividend stock since the no-dividend stocks begin paying out dividends only after some time (which may even be after the short-term asset maturity). Since the mean return of a dividend stock is higher than that of a comparable no-dividend stock, this leads to a higher mean return for the short-term asset as compared to the stock market, supporting the empirical evidence discussed earlier.

In summary, our model generates the following new testable predictions for the aggregate stock market returns, which are not in the existing literature. The higher the relative market capitalization of the no-dividend stocks in the stock market, (i) the lower the correlation between the stock market return and the aggregate consumption growth rate, (ii) the more likely the relation between the conditional risk premium and volatility of the stock market return to be negative, (iii) the more likely the term structure of equity risk premia to be downward sloping (i.e., the short-term asset has a higher mean return than the stock market). When we quantify these effects in our model, we find them to be economically significant.

Our work is related to the literature on the correlation between the stock market return and the aggregate consumption growth rate. As discussed earlier, this correlation appears to be weak in the data and leading consumption-based asset pricing models all have difficulty in reconciling this evidence. Hence, this finding is sometimes referred to as the "low correlation puzzle," and even pointed out by behavioral theories as one of the main shortcomings of the consumption-based asset pricing framework (e.g., Barberis, Huang, and Santos (2001)). Recently, Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Heyerdahl-Larsen and Illeditsch (2017) develop consumption-based models with a single stock and demand shocks that arise from the time variation in investors' rate of time preference to reconcile this finding. Our result here complements these works by offering an alternative, simple, but yet novel, possible explanation for this apparent low correlation. A similar low correlation may also arise in models where the aggregate consumption is partially funded by labor income, such as that in Santos and Veronesi (2006). Our result here demonstrates that in addition to labor income, the relative size of no-dividend stocks also matters for this correlation.

Our paper is also related to the vast literature studying the relation between the conditional risk premium and volatility of the stock market. As discussed earlier, numerous works empirically study this relation, but the conclusions on the sign of the relation are mixed. On the theory side, a number of works, using a single-stock setup, demonstrate that a non-monotonic relation can arise in equilibrium if there is time-variation in state variables or investment opportunities (Abel (1988), Backus and Gregory (1993), Veronesi (2000), Whitelaw (2000)). Our contribution here is to illustrate that a non-monotonic relation can arise in equilibrium for an alternative, simple reason. The stock market also consists of no-dividend stocks that have relatively low mean returns but high return volatility.

Our analysis is also related to the recently growing literature on the shape of the term structure of equity risk premia. As discussed earlier, this term structure appears to be downward sloping, which is considered somewhat puzzling since it goes against the implications of numerous leading asset pricing models. Indeed, van Binsbergen, Brandt, and Koijen show that the term structure of equity risk premia is upward sloping in the habit-formation model of Campbell and Cochrane (1999) and the long-run risk model of Bansal and Yaron (2004), and it is flat in the rare disaster model of Gabaix (2012). Several recent theoretical works reconcile this finding by generating a downward sloping term structure of equity risk premia via alternative mechanisms (Belo, Collin-Dufresne, and Goldstein (2015), Croce, Lettau, and Ludvigson (2015), Hasler and Marfè (2016)). We complement this literature by demonstrating that a downward sloping term structure can easily arise when the stock market consists of stocks that do not pay dividends.

Related works that study no-dividend stocks from an asset pricing perspective are Pástor and Veronesi (2003) and Choi et al. (2013). Our methodology and modeling of no-dividend stocks differ considerably from both these works, and hence do many of our results, even though each paper contains one result similar to one of our main cross-sectional results. In particular, Pástor and Veronesi study the effects of parameter uncertainty about a firm's average profitability and primarily focus on its market-to-book ratio, and find that firms that pay no dividends have more volatile returns due to learning effects, a finding similar to ours. However, differently from our setting, their framework is a partial equilibrium one in which the stochastic discount factor is specified exogenously (hence whether a firm pays dividends or not has no effect on it). Moreover, it is not possible to consider the aggregate stock market consisting of both the dividend-paying and no-dividend stocks as in our model. Therefore, in their framework, it is not possible to obtain our key implications on the aggregate stock market nor our cross-sectional implications for the stock price spillovers, stock mean returns, and market beta. On the other hand, Choi et al. consider a production economy in which managers choose the firm payout policy while facing non-convex costs in adjusting dividends and investments. They solve their model numerically and show that firms with a low probability of paying dividends in the near term command risk premia close to zero, a result similar to our finding that the no-dividend stock mean return is lower than that of a dividend-paying stock. Even though our framework differs from theirs in several major aspects, one key difference is that we explore the information incompleteness necessitated by the absence of dividends and show how it leads to higher return volatility and market betas for no-dividend stocks, as well as providing implications of these for the aggregate stock market returns.

The corporate finance literature, on the other hand, primarily focuses on the optimality of firms' dividend policies. In this literature, there are various theories as to why some firms do not pay dividends, including the roles of taxes, life-cycle of firms, catering to investor demands, and asymmetric information (e.g., signaling and agency problems) (see Allen and Michaely (2002)). Given these potential reasons are mutually exclusive, in our model, we intentionally do not specify why stocks pay or do not pay dividends so that we do not commit to one particular reason while ignoring others. In that sense, our model does not determine endogenously why firms pay or do not pay dividends. However, in return, it generates rich general equilibrium implications arising from the presence of no-dividend stocks in the stock market.

The remainder of the paper is organized as follows. Section 2 presents our model of the stock market that features both dividend-paying and no-dividend stocks. Section 3 provides our results on the correlation of the stock market with aggregate consumption, while Section 4 on the stock market risk premium-volatility relation, and Section 5 on the term structure of equity risk premia. Section 6 provides a discussion of the model parameter values employed in our figures and quantitative statements. Section 7 concludes. Appendix A contains the proofs, while Appendix B discusses the effects of differences in firm characteristics.

2 Stock Market with No-Dividend Stocks

In this section, we present our model of the aggregate stock market, consisting of dividend-paying and no-dividend stocks. Our model is based on recognizing that, differently from dividend-paying stocks, there are several noteworthy features of no-dividend stocks investors would need to take into account while determining their prices. First, the absence of their dividends introduces information incompleteness, and hence necessities the estimation of their future dividends using other relevant fundamental information. Second, the absence of their dividends also leads to additional uncertainty about their future dividend payment periods. Third, no-dividend stocks do not contribute directly to aggregate consumption, and hence to the stochastic discount factor. Since no-dividend stocks are part of the aggregate stock market, these features turn out to have important implications for the stock market returns, as we demonstrate in subsequent sections. In the following, we provide the details of our model with the above features.

2.1 Securities Market

The economy is cast in a stationary setting in the sense that there are dividend-paying and no-dividend stocks at all times as in reality, and our implications are not transitory and hold in all periods of the economy. The stationary structure admits much tractability in the analysis and is achieved by considering a stock market that contains the standard type of stocks that pay dividends throughout and also two other types of stocks that pay dividends in an alternating manner.[2]

In particular, we consider a continuous-time pure-exchange economy with infinite horizon, denoted by $\mathcal{T} = [0, \infty)$. We model the stock market as consisting of three types of risky stocks, each type being in positive net supply of one unit. The first type pays out dividends D_{1t} at all times $t \in \mathcal{T}$ with dynamics

$$\frac{dD_{1t}}{D_{1t}} = \mu_1 dt + \sigma_1 d\omega_{1t}, \tag{1}$$

where μ_1 and σ_1 are constants representing the mean and volatility of the stock dividend growth rate, and ω_1 is a Brownian motion. The second and third stock types pay out dividends D_2 and D_3, respectively, in alternating periods that are some non-overlapping random subperiods of $\mathcal{T}$, with dynamics

$$\frac{dD_{2t}}{D_{2t}} = \mu_2 dt + \sigma_2 d\omega_{2t}, \tag{2}$$

$$\frac{dD_{3t}}{D_{3t}} = \mu_3 dt + \sigma_3 d\omega_{3t}, \tag{3}$$

where μ_i and σ_i, $i = 2, 3$, are constants representing the mean and volatility of the dividend growth rates. All Brownian motions ω_i have a common pairwise correlation among them $d\omega_{1t} d\omega_{2t} = d\omega_{1t} d\omega_{3t} = d\omega_{2t} d\omega_{3t} = \rho dt$, with the correlation coefficient $\rho \in (-1, 1)$.[3]

The alternating periods are governed by the arrival times $0 < \tau_1 < \tau_2 < \ldots$, of the independent Poisson process N_t with intensity λ. Without loss of generality, we assume that stock 2 pays dividends during the random periods $[\tau_n, \tau_{n+1})$ where n is an odd number and denote its set

[2] Alternatively, one could consider a setting where there is just one standard type of stock that pays dividends throughout and one no-dividend stock. However, since the no-dividend stock eventually needs to pay dividends at some point in the future to have a non-zero current price, this would imply a non-stationary structure for this alternative setting. That is, in some periods all stocks would pay dividends with the model implications of no-dividend stocks not holding for these periods of the economy.

[3] For generality and realism, we allow the dividend processes to be possibly correlated. However, all our main results and mechanisms are equally valid when this correlation is zero, as demonstrated in Propositions 1–6.

of dividend-paying odd-numbered periods by $\mathcal{T}_o \equiv [\tau_1, \tau_2) \cup [\tau_3, \tau_4) \cup \ldots$. Similarly, stock 3 pays dividends during $[\tau_n, \tau_{n+1})$ where n is an even number and its set of dividend-paying even-numbered periods is denoted by $\mathcal{T}_e \equiv [0, \tau_1) \cup [\tau_2, \tau_3) \cup \ldots$. We refer to stock i that is currently not paying dividends, where $i = 2$ in $\mathcal{T}_e$ and $i = 3$ in $\mathcal{T}_o$, as the *no-dividend stock* and the unobservable processes D_i with dynamics (2) or (3) during these periods as *pseudo-dividends.* We note that since $\mathcal{T}_o \cup \mathcal{T}_e = \mathcal{T}$ and $\mathcal{T}_o \cap \mathcal{T}_e = \emptyset$, at any given time $t \in \mathcal{T}$, there is one no-dividend stock and two dividend (paying) stocks in our model, leading to a stationary dividend-payment structure. Moreover, this way, in our model and as in reality, the no-dividend stock only accumulates capital gains/losses as opposed to a dividend stock that also delivers a dividend yield.[4] The individual stock prices S_i, $i = 1, 2, 3$, and hence the stock market level $S_t = \sum_{i=1}^{3} S_{it}$, are to be determined endogenously in equilibrium. Also available for trading is a riskless bond that is in zero net supply.

Remark 1 (Non-random dividend periods). In our specification, the investor does not know when exactly the current no-dividend stock starts paying dividends and how long each dividend period is since the alternating periods are determined by the arrival times of a Poisson process. In particular, during any period $[\tau_n, \tau_{n+1})$ the investor views the next arrival time τ_{n+1} as an independent random variable with an exponential distribution. Moreover, due to the well-known memoryless property of the exponential distribution, the dividend alternating time becomes time independent, which in turn leads to stationary, constant discount terms for the stock prices as we discuss in Section 3. That being said, we note that all our main results continue to hold in an alternative specification in which the dividend alternating times τ_n are non-random known constants. However, in this case, we lose the stationary property as the discount terms for the stock prices would depend on time.

2.2 Absence of Dividends, Incomplete Information and Learning

In the stock market, the absence of dividends on the no-dividend stocks introduces information incompleteness while estimating the distribution of their future dividends, an issue that does not exist for dividend stocks.[5] This necessities using other relevant observable (albeit noisier)

[4]Furthermore, the no-dividend stock in our model can also be mapped into a real world stock that can be distinguished in the data as the "currently non-dividend paying stock that paid dividends previously" as in Fama and French (2001). Similarly, the alternating dividend stock can be distinguished in the data as the "currently dividend-paying stock that did not pay dividends in the previous period".

[5]To see this note that the dividend level of a no-dividend stock, say stock 2, at a future time u when paying dividends can be written as $\ln D_{2u} = \ln D_{2t} + (\mu_2 - \frac{1}{2}\sigma_2^2)\,(u - t) + \sigma_2(\omega_{2u} - \omega_{2t})$. Hence while forming a rational

information for estimating the no-dividend stock's future dividends. Towards that, we consider fundamental news processes F_i, $i = 1, 2, 3$, that contain valuable information about the future dividends of each stock.[6] Since the no-dividend stock could in principle start paying dividends very far in the future and the fundamental news process needs to contain valuable information about those dividends, we assume a long-run dependency between D_i and F_i by imposing simple mean-reverting (stationary) dynamics for their logarithmic difference as follows

$$d\left(\ln F_{it} - \ln D_{it}\right) = \kappa_i \left[\zeta_i - \left(\ln F_{it} - \ln D_{it}\right)\right] dt + \nu_i d\omega^*_{it}, \tag{4}$$

where $\kappa_i > 0$, ζ_i, and ν_i are constants representing the mean-reversion, long-run mean, and the volatility of $\ln F_{it} - \ln D_{it}$, respectively, and ω^*_i is a Brownian motion independent from all other Brownian motions introduced earlier. In economic terms, the long-run dependency (the cointegration between $\ln F_{it}$ and $\ln D_{it}$) is equivalent to assuming neither the fundamental news process F_i nor the dividend D_i grow to be infinitely larger than the other in the long-run.[7] Note that the dynamics of the fundamental news process F_i itself is readily deduced from the dynamics (1)–(4) as

$$d \ln F_{it} = (\mu_i - \tfrac{1}{2}\sigma_i^2 + \kappa_i\zeta_i + \kappa_i \ln D_{it} - \kappa_i \ln F_{it})dt + \sigma_i d\omega_{it} + \nu_i d\omega^*_{it}. \tag{5}$$

As (5) reveals, the mean growth rate of the observable fundamental news process F_i contains information about the unobserved pseudo-dividend D_i during the period stock i does not pay dividends.

We employ the standard Bayesian filtering theory to estimate the unobserved pseudo-dividend

estimate of future dividends, in addition to the known parameters μ_2 and σ_2, the investor needs to know the current level D_{2t}, which is not available currently. The future estimates of D_2 are then used in the determination of the stock price in equilibrium via $S_{2t} = \mathrm{E}_t\left[\int_t^\infty \xi_{t,u} D_{2u} \mathbf{1}_{u\in\mathcal{T}_o} du\right]$, where $\xi_{t,u}$ is the stochastic discount factor.

[6] For simplicity, we assume there is only one fundamental news process for each stock as this is sufficient for our purposes. In reality, there are numerous financial and accounting news series, such as cash-flows and earnings news/announcements, which contain valuable information about a stock's future prospects in the absence of its dividends. Such series would be good candidates for our fundamental news processes. Moreover, for symmetry, the fundamental news process is assumed to exist irrespective of whether the stock currently pays dividends or not. However, since there is no information incompleteness about the first stock's future dividend distribution at all times, the information contained in the fundamental news F_1 is redundant in our analysis.

[7] For instance, in the simpler special case of $\zeta_i = 0$, the expected long-term (logarithmic) fundamental news gives the expected long-term (logarithmic) dividend, that is, $\lim_{u\to\infty} \mathrm{E}_t\left[\ln F_{iu}\right] = \lim_{u\to\infty} \mathrm{E}_t\left[\ln D_{iu}\right]$. In general, the long-term relation between the growth rates of the fundamental news process F_i and the dividend D_i in our model is in line with the behavior of the steady-state of the Gordon growth model in which dividends, earnings, and book equities all grow at the same rate under the so-called clean-surplus accounting (Campbell (2017, p. 131)).

D_i during the period stock i does not pay dividends ($i = 2$ in $\mathcal{T}_e$, $i = 3$ in $\mathcal{T}_o$). At the beginning of a no-dividend period $[\tau_n, \tau_{n+1})$, the prior distribution of the (logarithmic) pseudo-dividend is assumed to be normally distributed with mean $\widehat{\ln D}_{i\tau_n} = \lim_{t\to\tau_n} \ln D_{it}$, i.e., the last available observation of $\ln D_{it}$, and variance $V_{i\tau_n}$.[8] The Bayesian updating rule then implies that for any $t \in [\tau_n, \tau_{n+1})$, the time-$t$ posterior distribution conditional on the information set $\mathcal{G}_{it} = \sigma\{(D_{1s}, D_{js}, F_{is}) : \tau_n \leq s \leq t\}$, where $j = 2, 3$ and $j \neq i$, is also normally distributed as presented in the following Lemma 1. The Bayesian estimation for stock i ends once the next dividend alternating time τ_{n+1} arrives as its dividends become observable (estimation begins again with the above features when τ_{n+2} arrives, and so on).

Lemma 1. *During the random period $[\tau_n, \tau_{n+1})$, in which stock i, $i = 2, 3$, is the no-dividend stock, let the prior of the (logarithmic) pseudo-dividend $\ln D_i$ at time τ_n be normally distributed with mean $\widehat{\ln D}_{i\tau_n}$ and variance $V_{i\tau_n}$. Then the posterior of $\ln D_i$ during this period $t \in [\tau_n, \tau_{n+1})$, conditional on the information $\mathcal{G}_{it} = \sigma\{(D_{1s}, D_{js}, F_{is}) : \tau_n \leq s \leq t\}$, where $j = 2, 3$ and $j \neq i$, is also normally distributed with mean $\widehat{\ln D}_{it}$ and variance V_{it} such that the mean estimate of the pseudo-dividend $\widehat{D}_{it} = \mathrm{E}[D_{it}|\mathcal{G}_{it}] = \exp(\widehat{\ln D}_{it} + \frac{1}{2}V_{it})$, henceforth the estimated pseudo-dividend, satisfies the dynamics*

$$\frac{d\widehat{D}_{it}}{\widehat{D}_{it}} = \mu_i dt + \frac{\rho\sigma_i}{1+\rho} d\omega_{1t} + \frac{\rho\sigma_i}{1+\rho} d\omega_{jt} + \frac{(1+\rho)\left(\sigma_i^2 + \kappa_i V_{it}\right) - 2\rho^2\sigma_i^2}{\sqrt{(1+\rho)^2\left(\sigma_i^2 + \nu_i^2\right) - 2(1+\rho)\rho^2\sigma_i^2}} d\widehat{\omega}_{it}, \tag{6}$$

$$dV_{it} = -\left[\frac{\Big((1+\rho)\left(\sigma_i^2 + \kappa_i V_{it}\right) - 2\rho^2\sigma_i^2\Big)^2}{(1+\rho)^2\left(\sigma_i^2 + \nu_i^2\right) - 2(1+\rho)\rho^2\sigma_i^2} - \Big(1 - \frac{2\rho^2}{1+\rho}\Big)\sigma_i^2\right] dt, \tag{7}$$

where $\widehat{\omega}_i$ is a $\mathcal{G}_{it}$-Brownian motion independent of the other Brownian motions ω_1 and ω_j, where $j = 2, 3$ and $j \neq i$.

The posterior variance V_{it} of the no-dividend stock (stock 2 during $\mathcal{T}_e$, stock 3 during $\mathcal{T}_o$), is deterministic and converges to its constant non-zero steady-state value, denoted by $V_{i\infty}$, in the long-run (see (A.8) in Appendix A). To ensure that learning is optimal in the sense

[8] We note that, unlike the prior mean, we cannot set the prior variance to its limiting value that would have been zero, leading to zero posterior variance and dogmatic beliefs (no Bayesian learning). To ensure that the investor behaves in a rational way and does not discard valuable information when faced with information incompleteness in the absence of dividends, we set the prior variance to a positive value. Moreover, to also ensure stationarity, for all periods, we set the same positive common value for the prior variance $V_{i\tau_n}$. That being said, we note that our main results do not depend critically on the latter specification and our results continue to hold if alternatively we set the prior variance for each period differently (e.g., decreasing over time). Though, in this case, we would lose the stationary structure of our framework.

that it leads to more precise estimates over time, we set the exogenous prior variance to be greater than the steady-state posterior variance, that is, $V_{i\tau_n} > V_{i\infty}$. One notable implication of Lemma 1 is that the estimation, necessitated by the absence of dividends, induces additional variability in the estimated pseudo-dividend $\widehat{D}_i$ dynamics (6), making it more volatile than the underlying pseudo-dividend D_i (see also (A.10)–(A.11) in Appendix A). This is because there is an additional uncertainty about the mean estimate of the pseudo-dividend $\widehat{D}_i$ captured by the posterior variance V_{it}, which would not be present had the dividends been observable. This additional uncertainty amplifies the shocks to the fundamental news process during the estimation and leads to a more volatile estimate of the pseudo-dividend.

The steady-state posterior variance being less than the prior variance and the estimation leading to a more volatile process not only make sense economically, but is also present in the models of learning about a constant parameter, say the expected dividend growth rate μ_i. In these models, the posterior variance declines over time and converges to zero in the steady-state since the investor eventually learns about the true parameter value (e.g., Brennan (1998), Pástor and Veronesi (2003), Cvitanić, Lazrak, Martellini, and Zapatero (2006), Collin-Dufresne, Johannes, and Lochstoer (2016)). However, differently from these models of parameter uncertainty, in our setting the investor learns about a non-stationary stochastic process, D_i, and moreover stops learning at a (random) time τ_{n+1} once the dividends become observable, since this leads to complete information.[9] Nevertheless, our model implications would go through under parameter uncertainty if alternatively we were to assume that the investors can observe the dividend levels D_i but not their expected growth rates μ_i. However, we believe it would be hard to justify the assumption of observable dividends for no-dividend stocks, which is our main focus. Moreover, since the posterior mean, i.e., the estimated expected growth rate, is typically time-varying in the case of parameter uncertainty, we would lose the simple Gordon-growth model like stock price structures of Proposition 1 in Section 3.

Remark 2 (Alternative fundamental news process). We specify the fundamental news process (5) so that there is a long-run dependency between between D_i and F_i for the economic reasons discussed above. However, this specification is not necessary for our mechanism and results, which also obtain under an alternative familiar, but somewhat less plausible in our

[9]Note that our learning is also different from the models in which the investor learns about a stationary process, e.g., an Ornstein-Uhlenbeck process, that results with smoother estimated processes (e.g., Brennan and Xia (2001), Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010)).

setting, specification of fundamental news that takes the form of a "signal plus noise" process

$$d \ln F_{it} = \ln D_{it} dt + \nu_i d\omega^*_{it}. \tag{8}$$

This is because, under this alternative formulation, the investor again uses noisier information when estimating the distribution of future dividends, and this procedure induces additional variability in the estimated pseudo-dividend for the same reasons as discussed above. Therefore all our subsequent results remain valid with this specification. We provide the details of the analysis with this alternative formulation in Appendix A.

2.3 Preferences and Endowments

There is a single investor in the economy who chooses a non-negative consumption C and a portfolio strategy in the three risky stocks that make up the stock market and the riskless bond so as to maximize her CRRA preferences from intertemporal consumption

$$u(C_t, t) = e^{-\beta t} \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{9}$$

where β is her rate of time preference, γ is the constant relative risk-aversion coefficient, subject to the appropriate budget constraint. The investor is endowed with all the wealth in the economy, which is a claim against the exogenously specified aggregate consumption (endowment) Y with dynamics at all times $t \in \mathcal{T}$ given by

$$\frac{dY_t}{Y_t} = \mu_Y dt + \sum_k \alpha_k \left(\frac{dD_{kt}}{D_{kt}} - \mu_k dt \right), \tag{10}$$

where $\mu_Y = \sum_k \alpha_k \mu_k$ with the summation without a superscript (throughout the paper), $\sum_k$, indicates that the summation is taken only over the stocks that currently pay dividends, and α_k are the appropriate constants representing the sensitivities of the aggregate consumption growth rate to each dividend shock. Economically these sensitivities can be thought of as the average relative share of dividends in the aggregate consumption (see also Remark 3).

As (10) illustrates, the fluctuations in aggregate consumption are driven by current dividend shocks. In particular, during the odd-numbered periods $\mathcal{T}_o$, a positive (negative) shock to any dividend D_1 or D_2 increases (decreases) the aggregate consumption. The magnitude of the increase/decrease is determined by the sensitivity parameters, α_1 and α_2, respectively. Similarly, during the even-numbered periods $\mathcal{T}_e$, shocks in aggregate consumption only arise from the shocks to dividends D_1 and D_3 with the sensitivity parameters, α_1 and α_3, respectively. In sum, we can rewrite the aggregate consumption dynamics explicitly in terms of the constant sensitivities and the dividend dynamics (1)–(3) as

$$\frac{dY_t}{Y_t} = \begin{cases} (\alpha_1\mu_1 + \alpha_2\mu_2)dt + \alpha_1\sigma_1 d\omega_{1t} + \alpha_2\sigma_2 d\omega_{2t}, & t \in \mathcal{T}_o, \\ (\alpha_1\mu_1 + \alpha_3\mu_3)dt + \alpha_1\sigma_1 d\omega_{1t} + \alpha_3\sigma_3 d\omega_{3t}, & t \in \mathcal{T}_e. \end{cases} \tag{11}$$

We see that the aggregate consumption growth rate in (11) has the same constant mean and volatility for all times $t \in \mathcal{T}$ when the second and third stocks are *otherwise identical*, namely $\mu_2 = \mu_3$, $\sigma_2 = \sigma_3$, $\alpha_2 = \alpha_3$. We adopt this natural specification along with identical relative stock sizes, $S_{2t}/S_t = S_{3t}/S_t$, for our comparative statistics results throughout.

We note that in our specification the aggregate consumption Y_t does not necessarily coincide with the aggregate dividends paid out by the stock market, which is $D_{1t} + D_{2t}$ during $\mathcal{T}_o$ and $D_{1t} + D_{3t}$ during $\mathcal{T}_e$, where their difference can be thought of as the investor's implicit non-financial income (such as labor and government transfers). This specification is not only consistent with the data, since the aggregate dividend is only a fraction of the aggregate consumption (Santos and Veronesi (2006)), but is also present in numerous asset pricing models, including Campbell and Cochrane (1999), Brennan and Xia (2001), Bansal and Yaron (2004), Barberis, Greenwood, Jin, and Shleifer (2015). As a final note, we sometimes make comparisons with a comparable benchmark economy in which all three stocks pay dividends with dynamics (1)–(3). In what follows, we denote the benchmark economy quantities with an upper bar ($^-$). Following (10), the aggregate consumption dynamics in this benchmark economy becomes $dY_t/Y_t = \sum_{i=1}^{3} \bar{\alpha}_i\mu_i dt + \sum_{i=1}^{3} \bar{\alpha}_i\sigma_i d\omega_{it}$, at all times. To make the benchmark economy comparable to our economy, we equate the sum of the sensitivities across economies, $\sum_{i=1}^{3} \bar{\alpha}_i = \sum_k \alpha_k$, along with the sensitivity parameter for the first stock, $\bar{\alpha}_1 = \alpha_1$, since this stock has an identical role across both economies.[10]

Remark 3 (Stock dividends, output, and aggregate consumption). Our specification of the aggregate consumption dynamics is in the spirit of Lucas (1978), in which stocks are claims to the trees whose output (dividends) are perishable and must be consumed in that period. This way if a stock currently does not pay dividends, it does not contribute to the current aggregate

[10]By doing so, we capture the feature that the sensitivity of the aggregate consumption growth rate to the aggregate dividend is the same both in our main economy and the benchmark economy. Moreover, this way we also ensure that when the mean dividend growth rates are the same, $\mu_1 = \mu_2 = \mu_3$, the aggregate consumption mean growth rate remains the same across economies.

consumption. We capture this economic mechanism, which is key to our analysis, in a tractable way through the constant sensitivities, which in turn lead to constant mean and volatility of the aggregate consumption growth rate in our economy, as (11) illustrates. This simplifies the analysis leading to the stock prices being as in the Gordon growth model in the comparable benchmark economy, as discussed in Section 3.

In our model, the aggregate consumption growth rate (10) is i.i.d. since it loads on two i.i.d. dividend growth rates with constant weights α_k. This is in contrast to the somewhat similar "two-trees" model of Cochrane, Longstaff, and Santa-Clara (2007), in which the aggregate consumption growth rate is non-i.i.d. since it loads on two i.i.d. dividend growth rates with time-varying weights being equal to the relative shares of two dividends. This specification can generate additional aggregate consumption dynamics but it also leads to an analytically intractable equilibrium and possibly a non-stationarity one in the long-run since one tree can dominate in the limit. On the other hand, by assuming constant weights (sensitivities), our setting does not account for the effects of time-variation in the relative shares of dividends in aggregate consumption. However, it in turn ensures much tractability and leads to a stationary consumption process over time. Moreover, one may be tempted to deduce that the no-dividend stock in our model can also be thought of as the limiting case of a low-dividend paying stock in the two-tree model. This is, however, not correct since in the two-tree model no matter how small the dividend is, it still provides a valuable signal and there is no information incompleteness.

3 Stock Market Correlation with Consumption

In this section, we investigate how the presence of no-dividend stocks in the stock market affects the correlation of the stock market with the aggregate consumption in equilibrium. We first demonstrate that their presence generates a novel spillover effect in that the expected dividend payment period affects the prices of all stocks. We then show that their presence leads to a lower correlation between the stock market return and the aggregate consumption growth rate, consistent with the well-known empirical regularity.

Equilibrium in our economy is defined in a standard way. The economy is said to be in *equilibrium* if the equilibrium consumption, portfolio strategy, stock and bond prices are such that the investor chooses her optimal consumption and portfolio strategy, and the good, stocks and bond markets clear. One major difficulty obtaining the equilibrium stock prices in our model is that the stochastic discount factor shocks alternate countably infinite times at the random arrival times of the Poisson process N_t. We solve for the equilibrium stock prices by considering the countably finite case and then taking the limit. While doing so we also make use of the stationary increments property of Poisson processes, which leads to a recursive relationship among each random period's contribution to the stock price. The tractability of our model leads to closed-form solutions for all economic quantities, as presented in our Propositions.

Proposition 1 presents the equilibrium stock prices.[11] In what follows, the stock mean returns r_i, $i = 1, 2, 3$, are defined as $\mathrm{E}_t\left[(dS_{it} + D_{it}dt)/S_{it}dt\right]$ for dividend stocks (i.e., $i = 1, 3$ in $\mathcal{T}_e$, $i = 1, 2$ in $\mathcal{T}_o$) and $\mathrm{E}_t\left[dS_{it}/S_{it}dt\right]$ for no-dividend stocks (i.e., $i = 2$ in $\mathcal{T}_e$, $i = 3$ in $\mathcal{T}_o$).

Proposition 1 (Equilibrium stock prices). *The equilibrium stock market level and individual stock prices $i = 1, 2, 3$, in the benchmark economy with all dividend stocks are given by*

$$\bar{S}_t = \sum_{i=1}^{3} \bar{S}_{it}, \tag{12}$$

$$\bar{S}_{it} = \frac{1}{\bar{r}_i - \mu_i} D_{it}, \tag{13}$$

and in the economy with no-dividend stocks by

$$S_t = \sum_{i=1}^{3} S_{it}, \tag{14}$$

$$S_{it} = \begin{cases} \frac{(\tilde{r}_i - \mu_i + \lambda) + \lambda \mathbf{1}_{i=1}}{(r_i - \mu_i + \lambda)(\tilde{r}_i - \mu_i + \lambda) - \lambda^2} D_{it} & \text{for a dividend stock,} \\ \frac{\lambda}{(r_i - \mu_i + \lambda)(\tilde{r}_i - \mu_i + \lambda) - \lambda^2} \widehat{D}_{it} & \text{for a no-dividend stock,} \end{cases} \tag{15}$$

where the estimated pseudo-dividend $\widehat{D}_{it}$ is as in Lemma 1, and the equilibrium mean returns of the individual stocks $\bar{r}_i$ and r_i are provided in Proposition 3 with $\tilde{r}_i$ denoting the mean returns in the other alternating period.

In the benchmark economy in which all stocks in the stock market pay dividends, each individual equilibrium stock price is driven by its current dividends D_{it}, as in standard asset pricing models. In our setup, these prices follow the simple Gordon growth model with the constant discount terms given by the stock mean returns net of dividend growth rates. In our economy in which there are stocks that do not pay dividends, the individual equilibrium

[11] The usual parameter restrictions that are necessary to ensure that the stock prices are well defined and finite in our model are provided in the proof of Proposition 1 in Appendix A.

stock prices still have simple structures, though differ in two major ways. First, while the prices of dividend-paying stocks are still driven by their current dividends D_{it}, the price of a no-dividend stock is now driven by its estimated pseudo-dividend $\widehat{D}_{it}$ in the absence of its dividends. Second, all individual stock prices now have additional terms adjusting for the changes in the equilibrium stochastic discount factor shocks at the random dividend alternating times governed by the arrival times τ_n of the Poisson process N_t with intensity λ. Naturally, these differences in individual stock prices are reflected in the stock market level as it is the sum of the individual stock prices.

The new additional terms in our economy reveal that the no-dividend stock's expected dividend payment times that are determined by λ not only affects its own price but also spills over to all other stock prices including the first stock that pays dividends at all times. This is because the expected dividend payment times are also the times when the aggregate consumption, and hence the stochastic discount factor, shocks are anticipated to change. Since stock prices are the total expected discounted future dividends, what portion of the future dividends are expected to be discounted under the odd-numbered period $\mathcal{T}_o$ and even-numbered period $\mathcal{T}_e$ stochastic discount factors matters for their prices. This spillover effect is noteworthy since it is not present in the benchmark economy, in which each stock pays dividends at all times and its price depends only on its own parameters, apart from the obvious indirect dependence through its endogenous equilibrium mean return (Proposition 3). Moreover, even though a no-dividend stock's expected dividend payment times spilling over to the other stock prices is due to a simple economic mechanism, to the best of our knowledge this is a novel result and has not been demonstrated previously in the literature.[12]

Proposition 1 also reveals that at the random dividend alternating times τ_n there are discrete changes in stock prices following the changes in price structures when moving from an odd-numbered period $\mathcal{T}_o$ to an even-numbered period $\mathcal{T}_e$, and vice versa, as equations (15) reveal. In particular, at each point in time the expected discrete change for an individual stock i is $\lambda\Delta_i dt$

[12]At this point, we believe it is helpful to highlight that Proposition 1 does not necessarily imply that one individual stock price is higher than the other. In particular, for suitable parameter choices of the dividend processes (1)–(3), any equilibrium price ratio, S_{it}/D_{it} or $S_{it}/\widehat{D}_{it}$, can be greater, less, or equal to another. Indeed, it is straightforward to show that there exist a unique mean growth rate $\mu_i^* > \mu_j$ as a function of λ such that the no-dividend stock i and the alternating dividend stock j have the same price ratios. This also illustrates that our model does not necessarily contradict the classic Miller and Modigliani (1961) finding that a firm's dividend policy does not affect its value. Our comparative statics results in this paper are ceteris paribus (all else being fixed), being valid for otherwise identical stocks with the same parameter values.

over the next instant, where the constant

$$\Delta_i = \begin{cases} \frac{(r_i - \mu_i + \lambda)\mathbf{1}_{i=1} + \lambda}{(\tilde{r}_i - \mu_i + \lambda) + \lambda \mathbf{1}_{i=1}} - 1 & \text{for a dividend stock,} \\ \frac{r_i - \mu_i + \lambda}{\lambda} - 1 & \text{for a no-dividend stock.} \end{cases} \tag{16}$$

As we demonstrate in Section 4, these (rare) discrete changes at random times τ_n contribute to the individual stock volatilities but do not affect their mean returns and risk premia since there are no associated discrete changes in the aggregate consumption levels, and hence no discrete changes in the state price density, leading to a zero risk premium for the discrete covariance.

We now examine our model's implication for the correlation of the stock market with the aggregate consumption, $\rho_{SYt} = \mathrm{Cov}_t\left[(dS_t/S_t), (dY_t/Y_t)\right] / \sqrt{\mathrm{Var}_t\left[dS_t/S_t\right] \mathrm{Var}_t\left[dY_t/Y_t\right]}$.

Proposition 2 (**Equilibrium stock market correlation with consumption**). *The equilibrium correlation of the stock market return with the aggregate consumption growth rate in the benchmark economy is given by*

$$\bar{\rho}_{SYt} = \frac{\sum_{i=1}^{3} \bar{\alpha}_i \sigma_i^2 \frac{\bar{S}_{it}}{\bar{S}_t} + \sum_{i=1}^{3} \sum_{j \neq i}^{3} \bar{\alpha}_i \rho \sigma_i \sigma_j \frac{\bar{S}_{jt}}{\bar{S}_t}}{\sqrt{\left(\sum_{i=1}^{3} \bar{\sigma}_{S_i t}^2 \left(\frac{\bar{S}_{it}}{\bar{S}_t}\right)^2 + \sum_{i=1}^{3} \sum_{j \neq i}^{3} \rho \sigma_i \sigma_j \frac{\bar{S}_{it}}{\bar{S}_t} \frac{\bar{S}_{jt}}{\bar{S}_t}\right) \left(\sum_{i=1}^{3} \bar{\alpha}_i^2 \sigma_i^2 + \sum_{i=1}^{3} \sum_{j \neq i}^{3} \bar{\alpha}_i \bar{\alpha}_j \rho \sigma_i \sigma_j\right)}}, \tag{17}$$

and in the economy with no-dividend stocks by

$$\rho_{SYt} = \frac{\sum_k \alpha_k \sigma_k^2 \frac{S_{kt}}{S_t} + \sum_k \sum_{j \neq k}^{3} \alpha_k \rho \sigma_k \sigma_j \frac{S_{jt}}{S_t}}{\sqrt{\left(\sum_{i=1}^{3} \sigma_{S_i t}^2 \left(\frac{S_{it}}{S_t}\right)^2 + \sum_{i=1}^{3} \sum_{j \neq i}^{3} \left(\rho \sigma_i \sigma_j + \lambda \Delta_i \Delta_j\right) \frac{S_{it}}{S_t} \frac{S_{jt}}{S_t}\right) \left(\sum_k \alpha_k^2 \sigma_k^2 + \sum_k \sum_{\ell \neq k} \alpha_k \alpha_\ell \rho \sigma_k \sigma_\ell\right)}}, \tag{18}$$

where the equilibrium stock volatilities $\bar{\sigma}_{S_i t}$ and $\sigma_{S_i t}$ are provided in Proposition 4, the stock market levels $\bar{S}_t$ and S_t, and the individual stock prices $\bar{S}_{it}$ and S_{it} are as in Proposition 1, and the constant discrete changes in the stock prices Δ_i at Poisson arrival times are as in (16)*.*

Consequently, in the economy with no-dividend stocks, the correlation of the stock market return with the aggregate consumption growth rate is lower than that of in the comparable benchmark economy with the same relative stock sizes $S_{it}/S_t = \bar{S}_{it}/\bar{S}_t$, $i = 1, 2, 3$.

In the benchmark economy with all dividend stocks, a shock to any dividend D_i, $i = 1, 2, 3$, causes fluctuations in both the aggregate consumption and the stock market returns. This leads the covariance, and hence the correlation, of the stock market return with the aggregate

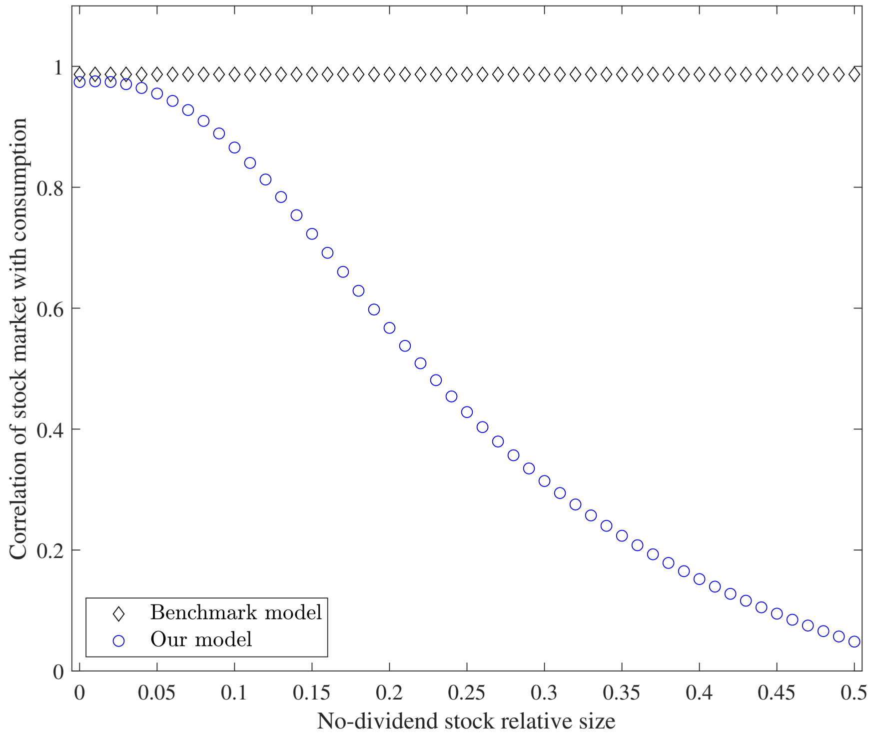


Figure 1: **Correlation of stock market return with consumption growth rate.** This figure plots the equilibrium correlation of the stock market return with the aggregate consumption growth rate by varying the no-dividend relative stock size S_{it}/S_t in our economy (blue circles). The corresponding correlation in the benchmark economy in which all stocks pay dividends, is obtained by setting the no-dividend stock relative size to zero (black diamonds). The parameter values follow from Table 1 of Section 6.

consumption growth rate to depend on all dividend growth rate variances and covariances (the numerator of (17)). However, in our economy with no-dividend stocks, the shocks in aggregate consumption arise only from the shocks to the dividend stocks, whereas the stock market returns are additionally driven by the no-dividend stock estimated pseudo-dividends. Hence the variance term in the numerator of (18) only has the dividend growth rate variances of the dividend stocks.[13]

The notable implication here is that the correlation of the stock market return with the aggregate consumption growth rate in our economy with no-dividend stocks is lower than that of in the benchmark economy. This result is intuitive as it simply says that when the stocks that

[13]As highlighted in Section 2.3, recall that a summation without a superscript indicates that the summation is over dividend-paying stocks, e.g., in $\sum_{k=1}$ the index is $k = 1, 2$ during $\mathcal{T}_o$ and $k = 1, 3$ during $\mathcal{T}_e$.

do not contribute to the current aggregate consumption are also part of the stock market, the stock market return is less correlated with the aggregate consumption. Figure 1 illustrates the low correlation in our economy by plotting the equilibrium correlation of the aggregate stock market return with the aggregate consumption growth rate against the no-dividend relative stock size S_{it}/S_t. We see that as the no-dividend stocks become more dominant in the stock market, this correlation progressively becomes smaller. In particular, if we set the no-dividend relative stock size in our model to equal to the long-run average of the no-dividend relative stock size in the US data during 1927-2011 as reported by Hartzmark and Solomon (2013), 21.3%, this correlation is 0.53. When we set it to the average relative size of the no-payout stocks (no dividends or no share repurchases) during 1984-2003 in the US as reported by Boudoukh et al. (2007), 14.2%, this correlation is 0.75. In contrast, this correlation is typically close to one in leading asset pricing models (as discussed below) and also in the comparable benchmark economy without no-dividend stocks as depicted in Figure 1.

As discussed in the Introduction, this correlation appears to be weak in the data (Cochrane and Hansen (1992), Campbell and Cochrane (1999), Albuquerque, Eichenbaum, Luo, and Rebelo (2016), Heyerdahl-Larsen and Illeditsch (2017)), and leading consumption-based asset pricing models have difficulty in reconciling this evidence, hence this finding is sometimes referred to as the "low correlation puzzle". Our contribution here is to demonstrate that a significant portion of this low correlation may be due to a very simple reason that is typically not considered in standard consumption-based asset pricing models. That is, the stock market consists of many stocks that currently do not pay dividends and hence do not contribute to the current aggregate consumption or dividends, while contributing to the fluctuations in the aggregate stock market returns. Therefore, it naturally follows that the stock market returns, which are partially driven by the fluctuations in no-dividend stocks, correlate less with the current aggregate consumption growth rate. Moreover, as we illustrate in Figure 1, this effect can be quantitatively significant.

4 Risk Premium-Volatility Relation

In this section, we are primarily interested in investigating how the presence of no-dividend stocks in the stock market affects the relation between the stock market conditional risk premium and volatility, "the risk-return tradeoff" in equilibrium. Towards that, we first show that a no-dividend stock's risk premium is lower, but its return volatility and market beta are higher than those of an otherwise identical dividend stock, consistent with the empirical evidence.

More notably, we demonstrate that the presence of no-dividend stocks in the stock market can generate a non-monotonic and even a negative relation between the stock market risk premium and its volatility, again consistent with the empirical evidence.

Proposition 3 presents the stock mean returns and their properties in equilibrium.

Proposition 3 (Equilibrium stock mean returns). *The equilibrium mean returns of the stock market and individual stocks $i = 1, 2, 3$, in the benchmark economy are given by*

$$\bar{r}_{S_t} = \sum_{i=1}^{3} \bar{r}_i \frac{\bar{S}_{it}}{\bar{S}_t}, \tag{19}$$

$$\bar{r}_i = \bar{r} + \gamma\Big(\bar{\alpha}_i\sigma_i^2 + \sum_{j\neq i}^{3} \bar{\alpha}_j\rho\sigma_i\sigma_j\Big), \tag{20}$$

and in the economy with no-dividend stocks by

$$r_{S_t} = \sum_{i=1}^{3} r_i \frac{S_{it}}{S_t}, \tag{21}$$

$$r_i = \begin{cases} r + \gamma\Big(\alpha_i\sigma_i^2 + \sum_{j\neq i} \alpha_j\rho\sigma_i\sigma_j\Big) & \text{for a dividend stock,} \\ r + \gamma \sum_k \alpha_k\rho\sigma_i\sigma_k & \text{for a no-dividend stock,} \end{cases} \tag{22}$$

where the interest rates are given by $\bar{r} = \beta+\gamma\sum_{i=1}^{3}\bar{\alpha}_i\mu_i - \frac{1}{2}\gamma(\gamma+1)\left(\sum_{i=1}^{3}\bar{\alpha}_i^2\sigma_i^2 + \sum_{i=1}^{3}\sum_{j\neq i}^{3}\bar{\alpha}_i\bar{\alpha}_j\rho\sigma_i\sigma_j\right)$, *and* $r = \beta + \gamma\sum_k \alpha_k\mu_k - \frac{1}{2}\gamma(\gamma+1)\left(\sum_k \alpha_k^2\sigma_k^2 + \sum_k\sum_{\ell\neq k}\alpha_k\alpha_\ell\rho\sigma_k\sigma_\ell\right)$, *and the stock market levels* $\bar{S}_t$ *and* S_t*, and the individual stock prices* $\bar{S}_{it}$ *and* S_{it} *are as in Proposition 1.*

Consequently, in the economy with no-dividend stocks, the risk premium of a no-dividend stock is lower than that of an otherwise identical dividend stock.

In both the benchmark and our economies, the equilibrium stock market risk premia, $r_{S_t} - r$, are simple weighted averages of the dividend and no-dividend stock risk premia. The weights are the relative sizes of the individual stocks, and each individual stock risk premium is proportional to the covariance of its stock return with the aggregate consumption growth rate, as in standard consumption-based asset pricing models.[14] In the benchmark economy when all stocks

[14] In Proposition 3 we report the equilibrium stock mean returns, which consist of the interest rate (the first terms) and the risk premium (the second terms). In our analysis we focus primarily on the risk premium, since the interest rate is a common component across stocks. Naturally, our results for the risk premia also hold for the mean returns also.

in the stock market pay dividends, each individual stock risk premium is made up of a variance component and a covariance component. The variance component is due to the fact that each stock dividend, which drives the stock price, contributes directly to the current aggregate consumption with the sensitivity $\bar{\alpha}_i$, thereby requiring the risk premium $\gamma\bar{\alpha}_i\sigma_i^2$. The covariance component is due to the fact that each stock is driven by its dividend, which (potentially) comoves with the other stocks' dividends, thereby requiring the risk premium $\gamma\sum_{j\neq i}^{3}\bar{\alpha}_j\rho\sigma_i\sigma_j$. In our economy, dividend stock risk premium is again made up of a variance and a covariance components. However, a no-dividend stock risk premium has only a covariance component since its estimated pseudo-dividend, which drives its price, does not directly contribute to the aggregate consumption shocks, but only (potentially) comoves with it.

A notable implication is that a no-dividend stock risk premium is lower than that of an otherwise identical dividend stock. This is intuitive because as discussed earlier the no-dividend stock price is driven by its estimated pseudo-dividend, which does not contribute directly to the aggregate consumption, and hence comoves less with the aggregate consumption growth rate as opposed to a comparable dividend stock. Therefore the investor requires a lower risk premium to hold the no-dividend stock in equilibrium.[15] This result is consistent with much cross-sectional empirical evidence, which documents that stocks that pay no dividends (or no payouts including share repurchases) have lower average returns than comparable stocks that pay dividends (Christie (1990), Naranjo, Nimalendran, and Ryngaert (1998), Fuller and Goldstein (2011), Hartzmark and Solomon (2013)).

Proposition 4 reports the equilibrium stock return volatilities for the stock market $\sigma_{S_t} = \sqrt{\mathrm{Var}_t\left[dS_t/S_t dt\right]}$ and the individual stocks $\sigma_{S_it} = \sqrt{\mathrm{Var}_t\left[dS_{it}/S_{it}dt\right]}$ for $i = 1, 2, 3$.

Proposition 4 (**Equilibrium stock return volatilities**). *The equilibrium volatilities of the stock market and individual stocks* $i = 1, 2, 3$, *in the benchmark economy are given by*

$$\bar{\sigma}_{S_t} = \sqrt{\sum_{i=1}^{3}\bar{\sigma}_{S_it}^2\Big(\frac{\bar{S}_{it}}{\bar{S}_t}\Big)^2 + \sum_{i=1}^{3}\sum_{j\neq i}^{3}\rho\sigma_i\sigma_j\frac{\bar{S}_{it}}{\bar{S}_t}\frac{\bar{S}_{jt}}{\bar{S}_t}}, \tag{23}$$

$$\bar{\sigma}_{S_it} = \sigma_i, \tag{24}$$

[15] In the special case of no-correlation, $\rho = 0$, the investor in fact does not require any risk premium to hold the no-dividend stock in equilibrium.

and in the economy with no-dividend stocks by

$$\sigma_{S_t} = \sqrt{\sum_{i=1}^{3} \sigma_{S_i t}^2 \Big(\frac{S_{it}}{S_t}\Big)^2 + \sum_{i=1}^{3}\sum_{j\neq i}^{3} \left(\rho\sigma_i\sigma_j + \lambda\Delta_i\Delta_j\right) \frac{S_{it}}{S_t}\frac{S_{jt}}{S_t}}, \tag{25}$$

$$\sigma_{S_i t} = \begin{cases} \sqrt{\sigma_i^2 + \lambda\Delta_i^2} & \textit{for a dividend stock,} \\ \sqrt{\frac{2\rho^2\sigma_i^2}{1+\rho} + \frac{\left[(1+\rho)\left(\sigma_i^2+\kappa_i V_{it}\right)-2\rho^2\sigma_i^2\right]^2}{(1+\rho)^2\left(\sigma_i^2+\nu_i^2\right)-2(1+\rho)\rho^2\sigma_i^2} + \lambda\Delta_i^2} & \textit{for a no-dividend stock,} \end{cases} \tag{26}$$

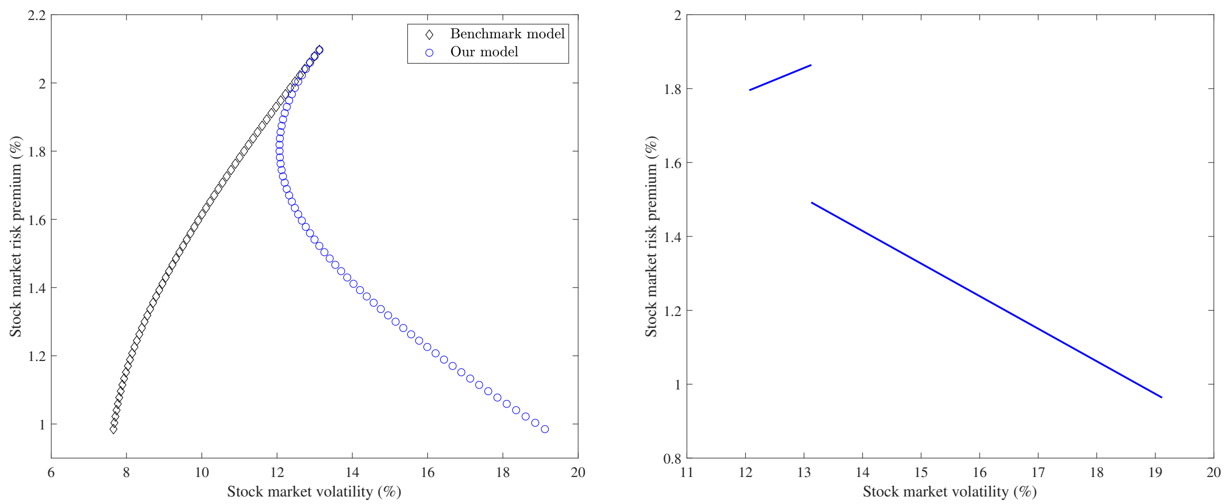
where the posterior variance V_{it} is as in Lemma 1, the stock market levels $\bar{S}_t$ and S_t, and the individual stock prices $\bar{S}_{it}$ and S_{it} are as in Proposition 1, and the discrete changes in the stock prices Δ_i at Poisson arrival times are as in (16)*.*

Consequently, in the economy with no-dividend stocks, the volatility of a no-dividend stock is higher than that of an otherwise identical dividend stock.

The equilibrium stock market volatility is driven by the individual relative stock sizes as they determine the extent to which each stock's volatility contributes to the stock market return fluctuations. In the benchmark economy when all stocks pay dividends, the return volatility of each stock i is constant and equals to the volatility of its dividend growth rate, σ_i. In our economy with no-dividend stocks, in addition to the dividend growth rate volatility σ_i, the return volatility of a dividend stock is also affected by the uncertainty about the arrival of the dividend alternating times due to the discrete changes in their prices during these times. On the other hand, in the absence of its dividends, the return volatility of a no-dividend stock is driven by the volatility of its estimated pseudo-dividend growth rate and the uncertainty about the arrival of the dividend alternating times. Therefore, the posterior variance V_{it} along with the parameters of the fundamental news process κ_i, v_i all affect the no-dividend stock volatility.

Consequently, we show that a no-dividend stock return volatility is higher than that of an otherwise identical dividend stock.[16] This is intuitive because as discussed earlier, a no-dividend stock price is driven by its estimated pseudo-dividend, and the estimation process, necessitated by the absence of dividends, induces additional variability, which is reflected in the stock returns. This result is also consistent with the empirical evidence, which documents that stocks that pay no dividends have higher return volatility than comparable stocks that pay

[16] We note that both the diffusion and discrete-change components of a no-dividend stock return volatility are higher than those of an otherwise identical dividend stock. In our discussion we focus on the diffusion component as it is the dominant volatility term. The component due to the discrete price changes at the arrival of the (rare) dividend alternating times are economically much smaller.



Panel A. Risk premium versus volatility　　Panel B. Risk premium versus volatility: Fitted values

Figure 2: **Stock market risk premium versus volatility.** Panel A plots the equilibrium relation between the conditional risk premium and volatility of the stock market return for different levels of no-dividend relative stock size S_{it}/S_t within the range of [0, 0.30], in our economy (blue circles). The corresponding relation in the benchmark economy is denoted by the black diamonds. Panel B plots the fitted values of the two linear regressions of the conditional risk premium on the stock market return volatility when the no-dividend relative stock size S_{it}/S_t is in the range of [0, 0.15] (low volatility region) and in the range of [0.15, 0.30] (high volatility region). The parameter values follow from Table 1 of Section 6.

dividends (Naranjo, Nimalendran, and Ryngaert (1998), Pástor and Veronesi (2003), Hartzmark and Solomon (2013)).

Having determined the aggregate stock market (conditional) risk premium and volatility in Propositions 3–4, we next investigate our model implications for the relation between these two quantities. Figure 2 Panel A presents our findings with a scatter plot of the stock market (conditional) risk premium and volatility, where each point represents a different no-dividend relative stock size S_{it}/S_t within an empirically relevant range of [0, 0.30]. To illustrate this relation more clearly, Panel B plots the fitted values of the linear regressions of the stock market risk premium on its return volatility in our model by dividing the no-dividend relative stock size range into two: [0, 0.15] (low volatility region) and [0.15, 0.30] (high volatility region).

As Figure 2 Panel A illustrates, in the benchmark economy when all stocks pay dividends, the relation between the stock market risk premium and volatility is monotonically positive, consistent with the standard intuition. However, as Figure 2 depicts, in our economy with no-dividend stocks, this relation becomes non-monotonic and even negative. This is because, as discussed above, the stock market risk premium is the (relative size) weighted-average of the corresponding risk premia of stocks that make up the stock market. Therefore, when the no-dividend stocks, the stocks that command low risk premia but high volatility, are also part of the stock market, the stock market risk premium is non-monotonically related to its volatility. In particular, the market risk premium is decreasing in its volatility for high volatility levels, corresponding to high relative-size of the no-dividend stocks. Our prediction is in line with the empirical evidence in Rossi and Timmermann (2010), who find a non-monotonic relation between the conditional risk premium and volatility by showing a positive relation for low and medium levels of volatility and a negative relation for high levels of volatility. As Figure 2 clearly demonstrates, this result and its mechanism can be economically significant. In terms of magnitudes of the risk premium and volatility, we find that when the no-dividend relative stock size in our model is equal to its long-run average in the data, 21.3%, for the same risk premium of 1.31% the stock market volatility is 8.51% in benchmark economy but it is 15.07% in our economy. Similarly, when the relative size of the no-dividend stocks is equal to the average relative size of the no-payout stocks (no dividends or no share repurchases), 14.2%, for the same risk premium of 1.57% the stock market volatility is 9.76% in benchmark economy but it is 12.81% in our economy.[17]

As discussed in the Introduction, numerous empirical works find a negative relation between the stock market conditional risk premium and volatility (e.g., Campbell (1987), Glosten, Jagannathan, and Runkle (1993), Whitelaw (2000), Harvey (2001), Brandt and Kang (2004)), while many others, consistent with the basic intuition, find this relation to be positive, (e.g., French, Schwert, and Stambaugh (1987), Scruggs (1998), Ghysels, Santa-Clara, and Valkanov (2005), Bali and Peng (2006), Guo and Whitelaw (2006), Ludvigson and Ng (2007)). On the theory side, a number of works, using a single stock setup, demonstrate that a non-monotonic and a negative relation can arise in equilibrium if there is time-variation in state variables or investment opportunities. Our contribution here is to illustrate that, using a simple multiple-stocks setup, a non-monotonic and a negative relation can also arise in equilibrium for a very simple

[17]We note that as can be seen from the y-axis of Figure 2, our model generates a somewhat low risk premium for the stock market for plausible parameter values. This is to be expected given our simplistic setting, e.g., a single investor, standard CRRA preferences, constant mean and volatility for the aggregate consumption growth rate, which is very similar to the settings of the original "equity premium puzzle" literature (e.g., Mehra and Prescott (1985)). It is well-known in this literature that models with these simplistic features yield fairly low risk premium for reasonable parameter values as opposed to what is observed in the data (typically around 6%). In order to preserve simplicity and tractability, in this paper we refrain from introducing other features that are typically employed in the literature to obtain a more realistic equity premium, and leave that for future research.

reason, that the stock market also consists of no-dividend stocks, whose mean return-volatility relation goes against the standard intuition (low mean return but high return volatility).

In addition to its return volatility, the market beta of an individual stock is another popular measure of risk for a stock. To examine whether the no-dividend stock is also riskier than the comparable dividend stock when the risk is measured by the market beta, in Proposition 5, we present our model implications for the equilibrium market betas β_{S_i} for each individual stock $i = 1, 2, 3$, defined as $\beta_{S_it} = \mathrm{Cov}_t\left[(dS_{it}/S_{it}), (dS_t/S_t)\right]/\mathrm{Var}_t\left[dS_t/S_t\right]$.

Proposition 5 (**Equilibrium market betas of individual stocks**). *The equilibrium market betas of individual stocks $i = 1, 2, 3$, in the benchmark economy are given by*

$$\bar{\beta}_{S_it} = \frac{\sigma_i^2 \frac{\bar{S}_{it}}{\bar{S}_t} + \sum_{j\neq i}^3 \rho\sigma_i\sigma_j \frac{\bar{S}_{jt}}{\bar{S}_t}}{\sum_{i=1}^3 \bar{\sigma}_{S_it}^2 \left(\frac{\bar{S}_{it}}{\bar{S}_t}\right)^2 + \sum_{i=1}^3 \sum_{j\neq i}^3 \rho\sigma_i\sigma_j \frac{\bar{S}_{it}}{\bar{S}_t}\frac{\bar{S}_{jt}}{\bar{S}_t}}, \tag{27}$$

and in the economy with no-dividend stocks by

$$\beta_{S_it} = \begin{cases} \dfrac{\sigma_i^2 \frac{S_{it}}{S_t} + \sum_{j\neq i}^3 \rho\sigma_i\sigma_j \frac{S_{jt}}{S_t} + \lambda\sum_{j=1}^3 \Delta_i\Delta_j \frac{S_{jt}}{S_t}}{\sum_{i=1}^3 \sigma_{S_it}^2\left(\frac{S_{it}}{S_t}\right)^2 + \sum_{i=1}^3\sum_{j\neq i}^3\left(\rho\sigma_i\sigma_j + \lambda\Delta_i\Delta_j\right)\frac{S_{it}}{S_t}\frac{S_{jt}}{S_t}} & \textit{for a dividend stock,} \\[2ex] \dfrac{\left(\frac{2\rho^2\sigma_i^2}{1+\rho} + \frac{\left[(1+\rho)\left(\sigma_i^2+\kappa_i V_{it}\right) - 2\rho^2\sigma_i^2\right]^2}{(1+\rho)^2\left(\sigma_i^2+\nu_i^2\right) - 2(1+\rho)\rho^2\sigma_i^2}\right)\frac{S_{it}}{S_t} + \sum_{j\neq i}^3 \rho\sigma_i\sigma_j\frac{S_{jt}}{S_t} + \lambda\sum_{j=1}^3\Delta_i\Delta_j\frac{S_{jt}}{S_t}}{\sum_{i=1}^3 \sigma_{S_it}^2\left(\frac{S_{it}}{S_t}\right)^2 + \sum_{i=1}^3\sum_{j\neq i}^3\left(\rho\sigma_i\sigma_j + \lambda\Delta_i\Delta_j\right)\frac{S_{it}}{S_t}\frac{S_{jt}}{S_t}} & \textit{for a no-dividend stock,} \end{cases} \tag{28}$$

where the posterior variance V_{it} is as in Lemma 1, the stock market levels $\bar{S}_t$ and S_t, and the individual stock prices $\bar{S}_{it}$ and S_{it} are as in Proposition 1, the individual stock volatilities $\bar{\sigma}_{S_it}$ and σ_{S_it} are as in Proposition 4, and the discrete changes in the stock prices Δ_i at Poisson arrival times are as in (16)*.*

Consequently, in the economy with no-dividend stocks, the market beta of a no-dividend stock is higher than that of an otherwise identical dividend stock.

In the benchmark economy, the equilibrium market betas are in terms of the underlying risks σ_i and relative stock sizes $\bar{S}_{it}/\bar{S}_t$. When two stocks are otherwise identical (i.e., same underlying risks and relative stock sizes), they have the same market beta. In our economy with no-dividend stocks, the market beta of a dividend stock is additionally affected by the uncertainty about the arrival of the dividend alternating times due to the discrete changes in stock prices during these times. The market beta of a no-dividend stock is further affected by its posterior variance V_{it} along with the parameters κ_i, v_i through their effects on its return volatility.

Consequently, a no-dividend stock market beta is higher than that of a comparable dividend stock. This is because a no-dividend stock return is more volatile (Proposition 4), and hence it contributes to and comoves with the aggregate stock market return more as compared to a comparable dividend stock. This result is also consistent with the empirical evidence, which documents that stocks that pay no dividends (or no payouts including share repurchases) have higher market betas than comparable stocks that pay dividends (Boudoukh, Michaely, Richardson, and Roberts (2007), Fuller and Goldstein (2011)).

Remark 4 (Our model's relation to value vs growth stocks). In our model, we refer to the stock that is currently not paying dividends as a no-dividend stock since it has zero dividend yield. Therefore, one could also think of no-dividend (dividend) stocks in our model as the *growth (value)* stocks, since in the literature a typical growth (value) stock is one with a low (high) fundamental to price ratio, where this ratio typically is the book-to-market, earnings yield, *dividend yield*, or the ratio of cash flows to price (Lettau and Wachter (2007)). Moreover, the three key features of no-dividend stocks in our model are also valid for growth stocks. For example, growth stocks too share the element of estimation of their true future dividends, since their current low fundamentals are not representative of their eventual significant future dividends. Second, growth stocks also share the element of having additional uncertainty about their main future dividend payment period. Third, due to their low current fundamentals, the growth stocks currently contribute little to aggregate consumption, and hence to the stochastic discount factor. With this interpretation our cross-sectional model implications are also consistent with the documented empirical regularities for growth and value stocks, since as summarized in Lettau and Wachter (2007), in the data, growth stocks have lower mean returns, and yet they have higher return volatilities and higher market betas as compared to value stocks.

5 Term Structure of Equity Risk Premia

Finally, we investigate our model implications for the shape of the term structure of stock market equity risk premia. There has been growing interest in this term structure following the findings of van Binsbergen, Brandt, and Koijen (2012), who study a claim on the dividends of the S&P 500 index in the near future, i.e., the short-term asset, and find that the short-term asset commands a higher average return (and Sharpe ratio) than the underlying index, and conclude that the term structure of equity risk premia is downward sloping. As discussed in the Introduction, this empirical finding is considered somewhat puzzling since it goes against the implications of several leading asset pricing models. We here demonstrate that the presence of no-dividend stocks in the stock market can generate this downward sloping term structure of equity risk premia. Towards that, we define the short-term asset following van Binsbergen, Brandt, and Koijen (2012) as a claim to the aggregate dividends up to maturity T at a time t, and then present its equilibrium mean return, denoted by $r_{S_{t,T}}$, in Proposition 6.[18]

Proposition 6 (Equilibrium short-term asset mean return). *The equilibrium mean return of the short-term asset in the benchmark economy is given by*

$$\bar{r}_{S_{t,T}} = \sum_{i=1}^{3} \frac{\bar{h}_{it,T}\bar{S}_{it}}{\sum_{j=1}^{3}\bar{h}_{jt,T}\bar{S}_{jt}}\bar{r}_i, \tag{29}$$

and in the economy with no-dividend stocks by

$$r_{S_{t,T}} = \sum_{i=1}^{3} \frac{h_{it,T}S_{it}}{\sum_{j=1}^{3}h_{jt,T}S_{jt}}r_i, \tag{30}$$

where $\bar{h}_{it,T} = 1 - e^{-(\bar{r}_i-\mu_i)(T-t)}$, $i = 1,2,3$, *and*

$$h_{it,T} = \begin{cases} \lambda \frac{(r_i-\mu_i+\lambda)(\tilde{r}_i-\mu_i+\lambda)-\lambda^2}{(\tilde{r}_i-\mu_i)(\tilde{r}_i-\mu_i+2\lambda)}\left[\frac{1-e^{-(r_i-\mu_i+\lambda)(T-t)}}{r_i-\mu_i+\lambda} - \frac{e^{-(\tilde{r}_i-\mu_i)(T-t)}-e^{-(r_i-\mu_i+\lambda)(T-t)}}{r_i-\tilde{r}_i+\lambda}\right]\mathbf{1}_{i=1} & \\ +\frac{(r_i-\mu_i+\lambda)(\tilde{r}_i-\mu_i+\lambda)-\lambda^2}{(r_i-\mu_i)(\tilde{r}_i-\mu_i+\lambda+\lambda\mathbf{1}_{i=1})}\left[1-e^{-(r_i-\mu_i+\lambda)(T-t)}-\lambda\frac{1-e^{-(r_i-\mu_i+\lambda)(T-t)}}{r_i-\mu_i+\lambda}\right] & \text{for a dividend stock,} \\ \frac{(r_i-\mu_i+\lambda)(\tilde{r}_i-\mu_i+\lambda)-\lambda^2}{\tilde{r}_i-\mu_i}\left[\frac{1-e^{-(r_i-\mu_i+\lambda)(T-t)}}{r_i-\mu_i+\lambda} - \frac{e^{-(\tilde{r}_i-\mu_i)(T-t)}-e^{-(r_i-\mu_i+\lambda)(T-t)}}{r_i-\tilde{r}_i+\lambda}\right] & \text{for a no-dividend stock,} \end{cases} \tag{31}$$

where the individual stock prices $\bar{S}_{it}$ *and* S_{it} *are as in Proposition 1, and the mean returns* $\bar{r}_i$ *and* r_i *are as in Proposition 3.*

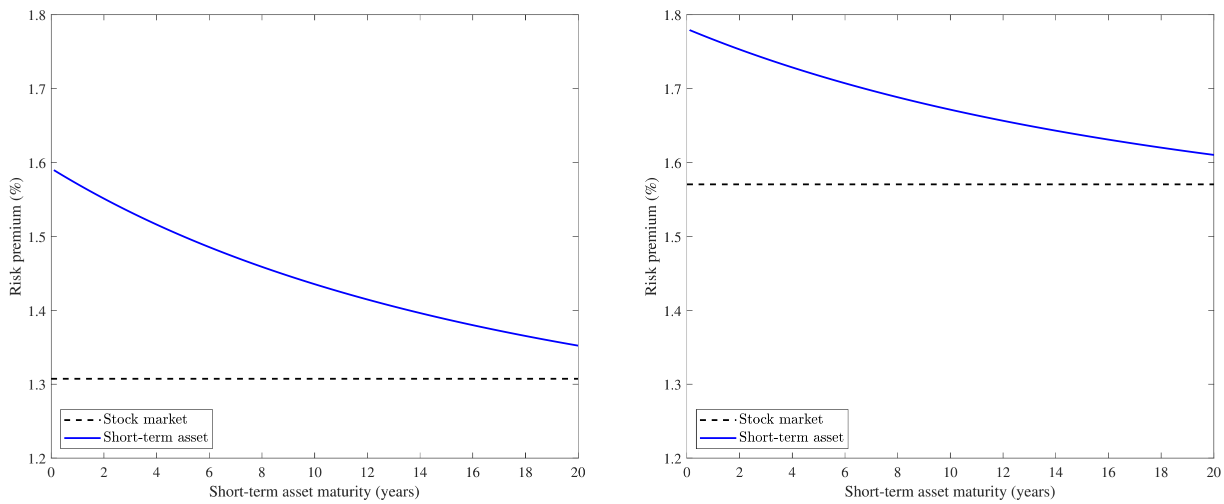
Consequently, in the economy with no-dividend stocks, the mean return of the short-term asset is higher than that of the stock market if the deterministic term $h_{it,T}$ *for a dividend stock is greater than that of a no-dividend stock.*

[18] In our analysis, we restrict ourselves to the case of there being at most one arrival of the dividend alternating times in the life of the short-term asset. That is, if currently $t \in [\tau_n, \tau_{n+1})$, the short-term asset maturity is either $T \in [t, \tau_{n+1})$ or $T \in [\tau_{n+1}, \tau_{n+2})$. This specification is sufficient for us to make our point. Moreover, it is also economically more plausible since the maturity of a short-term asset considered in the pertinent studies is typically upto 2 years (van Binsbergen, Brandt, and Koijen (2012)), and therefore it is not very likely for a stock to change its payout policy and switch from being classified as a dividend-paying to a no-dividend stock, and vice versa, twice or more, in such a short time interval.

Similar to the structure of the equilibrium stock market mean return of Proposition 3, the short-term asset equilibrium mean return is a weighted average of the mean returns of the individual stocks. For the stock market these weights are simply the relative stock sizes, whereas for the short-term asset the weights also include the deterministic terms $h_{it,T}$, which represent the fraction of each stock in the short-term asset. In particular, in the benchmark economy, these fractions have simple forms and are driven by the dividend yield $\bar{r}_i - \mu_i$ and the short-term asset maturity $T-t$. In our economy with no-dividend stocks, as (31) illustrates, these fractions are more involved and are additionally affected by the no-dividend stock's expected dividend payment time that is governed by λ. These more complicated forms for the fractions $h_{it,T}$ arise because the short-term asset is a claim to the aggregate dividends upto $T-t$, during which the aggregate dividends (and the stochastic discount factor) shocks may remain the same or change when the no-dividend stock starts paying dividends.

Importantly, we find that in our economy with no-dividend stocks, the mean return of the short-term asset is higher than that of the stock market if the fraction of the dividend stocks in the short-term asset is greater than the corresponding fraction for the no-dividend stocks. This condition is satisfied for plausible parameter values since the short-term asset is more like a dividend stock than a no-dividend stock. This is because the value of the short-term asset only depends on the aggregate dividends upto its maturity, during which the no-dividend stock may not start paying dividends, and hence it is represented less in the short-term asset. Moreover, a dividend stock mean return is higher than that of an otherwise identical no-dividend stock (Proposition 3), and hence by giving higher weights to the stocks with higher mean returns, the short-term asset mean return becomes higher than that of the stock market.

This result also implies a downward sloping term structure of equity risk premia as illustrated in Figure 3, which plots the equilibrium risk premium of the short-term asset and the stock market against the maturity date of the short-term asset $T-t$ in our economy. We see that the shorter the maturity of the short-term asset, the higher its risk premium, which approaches monotonically to the stock market risk premium as its maturity increases, consistent with the empirical evidence (van Binsbergen, Brandt, and Koijen (2012), van Binsbergen and Koijen (2017)). For the economic magnitude of the effects, we see that when the no-dividend relative stock size in our model is equal to its long-run average in the data, 21.3%, (Panel A) or is equal to the average relative size of the no-payout stocks (no dividends or no share repurchases), 14.2%, (Panel B), the risk premium of the short-term asset with maturity up to 2 years is 18% higher (1.55% vs 1.31% in Panel A) and 12% higher (1.75% vs 1.57% in Panel B) than the risk



Panel A. No-dividend stock relative size: 21.3% Panel B. No-dividend stock relative size: 14.2%

Figure 3: **Term structure of equity risk premia.** These panels plot the equilibrium risk premium of the short-term asset and the stock market against the maturity date of the short-term asset $T - t$ in our economy in which the no-dividend stock relative size is 21.3% (Panel A) and 14.2% (Panel B). The parameter values follow from Table 1 of Section 6.

premium of the stock market, respectively. We note that even though we do not provide it for brevity, the corresponding Sharpe ratio for the short-term asset is higher than that of the stock market in our model as also in the data.

Finally, we note that our model implications are for the unconditional slope of the term structure of equity risk premia. There are recent empirical works looking at the shape of the conditional term structure by studying the time-variation and trends in this term structure (e.g., Gormsen (2018), Gormsen and Lazarus (2020)). Due to the simplicity of our model (i.e., having standard CRRA preferences and aggregate consumption growth rate being i.i.d.), it is beyond the scope of our current setting to meaningfully study the time-variation and trends in this term structure. We leave these interesting features for future research.

6 Parameter Values

To quantify the effects in our model, particularly the key results involving the aggregate stock market, in this section, we discuss the determination of the parameter values employed in our figures. We do so by primarily matching the aggregate consumption growth rate mean and volatility in our economy to the corresponding ones in the US data in Campbell (2017). We also use the US stock market data from Robert Shiller's website to estimate the fundamental news process parameters.[19] Table 1 summarizes the parameter values used. We note that the behavior of the equilibrium quantities depicted in our figures is typical and does not vary much with alternative plausible values of parameters.

We start by setting the sum of the sensitivities, $\sum_k \alpha_k = \alpha_1 + \alpha_2 = \alpha_1 + \alpha_3$, which captures the sensitivity of the aggregate consumption growth rate to the aggregate dividend in our model to the relative-share of the aggregate dividend in the aggregate consumption in the data, as 15% (Santos and Veronesi (2006)). We decompose the sum of the sensitivities to capture the fact that dividend-paying stocks are several times larger than no-dividend stocks in reality. Indeed, Fama and French (2001, Table 3) reports that the assets of a typical dividend-paying firm is 5.3 times (1,389 vs 262) that of a no-dividend stock that formerly paid dividends in their full sample. Accordingly, we set the ratio of the total sensitivity of the dividend-paying stocks to the sensitivity of the no-dividend stock, which paid dividends previously in our model, as $0.15/\alpha_2 = 0.15/\alpha_3$, to 5.3, yielding $\alpha_1 = 0.122$ and $\alpha_2 = \alpha_3 = 0.028$.[20] We then set the common correlation coefficient to simply $\rho = 0$ and treat each dividend growth rate symmetrically, and match the mean and volatility of the aggregate consumption growth rate in our economy to the corresponding ones in the data, 1.74% and 1.64%, respectively, as reported in Campbell (2017). This gives the mean and volatility of each dividend growth rate as $\mu_i = 11.6\%$ and $\sigma_i = 13.1\%$, for $i = 1, 2, 3$. We note that the values for μ_i are somewhat large but their values do not affect our main results. To choose the intensity parameter for the Poisson process λ, we use the average propensity of a no-dividend stock start to pay dividends next year in the data, which is reported to be 10.1% in the full sample (1927-1999) of Fama and French (2001, Table 2). Equating this value to the corresponding propensity in our model $0.101 = 1 - e^{-\lambda}$, yields $\lambda = 0.106$. With this choice, the expected next dividend payment time $(1/\lambda)$ is around 10 years in our economy.

For the fundamental news process parameters, we use the monthly US stock market data from Robert Shiller's website (footnote 19), which provides the time-series of the aggregate real dividends and earnings, among other quantities. We take the difference between the logarithm of the real earnings and the logarithm of the real dividends as a proxy for the process $\ln F_{it} - \ln D_{it}$

[19] Source: `http://www.econ.yale.edu/~shiller/data/ie_data.xls`

[20] As we discuss in Section 2.3, to make the benchmark economy comparable to our economy, we equate the sum of the sensitivities across economies, $\sum_{i=1}^{3} \bar{\alpha}_i = \sum_k \alpha_k$, along with the sensitivity parameter for the first stock, $\bar{\alpha}_1 = \alpha_1$, which always pays dividends. We also treat the remaining stock sensitivities in the benchmark economy as equal, which yields $\bar{\alpha}_1 = 0.122$ and $\bar{\alpha}_2 = \bar{\alpha}_3 = 0.014$.

Table 1: **Parameter values.** This table reports the parameter values used in our Figures.

Parameter	Symbol	Value
Stock i, $i = 1, 2, 3$, dividend growth rate mean	μ_i	0.116
Stock i, $i = 1, 2, 3$, dividend growth rate volatility	σ_i	0.131
Stock i, $i = 1, 2, 3$, fundamental news mean reversion	κ_i	0.082
Stock i, $i = 1, 2, 3$, fundamental news long-run mean	ζ_i	0.550
Stock i, $i = 1, 2, 3$, fundamental news volatility	ν_i	0.127
No-dividend stock i, $i = 2, 3$, prior variance	$V_{i\tau_n}$	5.480
Intensity of the Poisson process N	λ	0.106
Correlation coefficient	ρ	0
Investor's relative risk aversion coefficient	γ	10
Investor's time preference coefficient	β	0.001
Sensitivity parameters	$(\alpha_1, \alpha_2, \alpha_3)$	$(0.122, 0.028, 0.028)$

with Ornstein-Uhlenbeck dynamics (4) in our model. This monthly series in the data, for the period January 1871-January 2020, has a sample mean of 0.55, sample standard deviation of 0.315, and the first-order autocorrelation of 0.9932. Using the well-known properties of Ornstein-Uhlenbeck processes, matching to our model, these values imply the long-run mean to be $\zeta_i = 0.55$, the mean-reversion speed $\kappa_i = -12 \times \ln(0.993) = 0.082$, and the volatility parameter $\nu_i = 0.315\sqrt{2 \times 0.082} = 0.127$, for $i = 1, 2, 3$. We next choose the common prior variance $V_{i\tau_n}$ for the pseudo-dividend level for each random period $[\tau_n, \tau_{n+1})$ sufficiently high to ensure that learning is optimal. We evaluate the posterior variance at its average value denoted by $V_i^A \equiv \frac{1}{\tau_{n+1}-\tau_n} \int_{\tau_n}^{\tau_{n+1}} V_{it} dt = \frac{1}{\tau_!} \int_0^{\tau_1} V_{it} dt$, where the second equality follows from the stationarity of Poisson processes, which has a closed-form solution in our model as

$$V_i^A = V_{i\infty} \left[1 + \left(1 + \frac{p_i}{q_i} \right) \frac{1}{m_i \tau_1} \ln \left(\frac{1 - q_i e^{-m_i \tau_1}}{1 - q_i} \right) \right],$$

where the constants $V_{i\infty}$, p_i, q_i, and m_i are provided in the proof of Lemma 1 in Appendix A. Evaluating the first arrival time τ_1 at its expected arrival time of $1/\lambda$ leads to the average posterior variance of $V_i^A = 1.12$ for the prior variance of $V_{i\tau_n} = 5.48$. Finally, for the investor-level parameters, we set the relative risk aversion coefficient to $\gamma = 10$ and a relatively low subjective time preference $\beta = 0.001$, consistent with leading asset pricing models (e.g., Bansal and Yaron (2004)) so that the stock prices are finite, and hence well-defined.

To make our quantitative statements corresponding to our Figures more relevant, we choose the no-dividend relative stock sizes S_{it}/S_t, $i = 2, 3$, as the average value in the data. However, choosing the right no-dividend relative stock sizes is not straightforward, as numerous empirical works illustrate there are trends in the firms' payout policies. For instance, dividends were the major payout choice until mid-1980ies, but since then share repurchases have become more prevalent (e.g., Grullon and Michaely (2002). See also the recent survey of Farre-Mensa, Michaely, and Schmalz (2014) for more on trends in payout policy). For this reason, we use two sources of evidence. First, we make use of the long-sample evidence in Hartzmark and Solomon (2013), who find that over 1927-2011, during most of which dividends are the main payout choice, the no-dividend stocks account for 21.3% of the aggregate stock market capitalization in the US. Second, we use the evidence in Boudoukh et al. (2007), who report that over the 1984-2003 period, during which share repurchases are also a significant fraction of the total payouts, the no-payout stocks, i.e., no dividends or no share repurchases, have an average relative market capitalization of 14.2%.[21] With our baseline parameter values in Table 1, the stock market has a risk premium of 1.31%, return volatility of 15.07%, and correlation with the aggregate consumption of 0.53 when the no-dividend relative stock size in our model is equal to the long-run average of the no-dividend relative stock size in the data, 21.3%, as reported by Hartzmark and Solomon (2013). Similarly, when the no-dividend relative stock size in our model is equal to the average relative size of the no-payout stocks (no dividends or no share repurchases) in the data, 14.2%, as reported by Boudoukh et al. (2007), the stock market risk premium increases to 1.57%, its return volatility decreases to 12.81%, and its correlation with the aggregate consumption increases to 0.75 (these can also be seen in our Figures 1 and 2).

[21] To be more specific, using monthly data from January 1927 to December 2011, Hartzmark and Solomon (2013, Table 1) report $718,726$ no-dividend firms with average market capitalization of 894 millions of dollars. The corresponding numbers for the dividend-paying stocks are $1,359,690$ and $1,739$ millions of dollars, implying the no-dividend stock average relative size to be $(718,726 \times 894)/(718,726 \times 894 + 1,359,690 \times 1,739) = 21.3\%$. On the other hand, Boudoukh et al. (2007) use monthly data from July 1984 to December 2003 and two measures of payout yield, one based on the statement of cash flows (Table 3 Panel B), the other based on the change in Treasury stock (Table 3 Panel C), and we report the average relative stock size across the two measures. For instance, based on the change in Treasury stock, they report $1,986$ zero-payout firms with average (log) market capitalization of 3.71 millions of dollars, implying the no-payout stock average relative stock size to be $(1,986 \times e^{3.71})/(1,986 \times e^{3.71} + \sum_{i=1}^{10} N_i \times e^{\ln(ME_i)})$, where N_i and $\ln(ME_i)$ denote the corresponding numbers for the decile i payout yield.

7 Conclusion

In this paper, we provide an analysis of the aggregate stock market that features both dividend-paying and no-dividend stocks within a familiar consumption-based general equilibrium framework. Our analysis leads to closed-form solutions for quantities of interest and profound qualitative implications that support several empirical regularities on the aggregate stock market while providing simple intuition for the underlying economic mechanisms at play. Most notably, we show that the presence of no-dividend stocks in the stock market leads to a lower correlation between the stock market return and the consumption growth rate, a non-monotonic and even a negative relation between the stock market risk premium and its volatility, and a downward sloping term structure of equity risk premia. In terms of new testable predictions for the aggregate stock market returns, these findings translate into: the higher the relative market capitalization of the no-dividend stocks in the stock market, (i) the lower the correlation between the stock market return and the aggregate consumption growth rate, (ii) the more likely the relation between the conditional risk premium and volatility of the stock market return to be negative, (iii) the more likely the term structure of equity risk premia to be downward sloping.

We also show that the presence of no-dividend stocks in the stock market generates a novel spillover effect in that the expected future dividend payment times of no-dividend stocks also affect the prices of all other stocks. Furthermore, consistently with much cross-sectional empirical evidence, we find that no-dividend stocks command lower mean returns while having higher return volatilities and higher market betas than comparable stocks that pay dividends.

The framework we consider in this paper is parsimonious in the sense that there is a single investor with standard CRRA preferences, and the aggregate consumption growth rate has a constant mean and volatility. Therefore, this framework can be extended in several different dimensions to study other potentially important issues such as, heterogeneous investors, more exotic preferences, and more general aggregate consumption process. For instance, considering decreasing relative risk aversion (DRRA) preferences rather than CRRA may yield interesting implications in our framework. This is because the investor's relative risk aversion would be more sensitive to the shocks of the dividend stocks than to the shocks of the no-dividend stocks, which may help explain the findings of Fuller and Goldstein (2011) that no-dividend stocks command lower mean returns even more in declining markets. We leave these considerations for future research.

Appendix A: Proofs

Proof of Lemma 1. We employ the standard Bayesian filtering theory (e.g., Liptser and Shiryaev (2001), Theorem 12.7) to estimate the unobserved pseudo-dividend D_{it} during the random period $[\tau_n, \tau_{n+1})$, in which stock i, $i = 2, 3$, is the no-dividend stock. We first denote the vector of relevant observable processes by $X_t \equiv \begin{bmatrix} \ln D_{1t} & \ln D_{jt} & \ln F_{it} \end{bmatrix}^\intercal$, where $j = 2, 3$, and $j \neq i$, and the relevant vectors for their drift terms by

$$A_0 \equiv \begin{bmatrix} \mu_1 - \frac{1}{2}\sigma_1^2 \\ \mu_j - \frac{1}{2}\sigma_j^2 \\ \mu_i - \frac{1}{2}\sigma_i^2 + \kappa_i \zeta_i - \kappa_i \ln F_{it} \end{bmatrix}, \qquad A_1 \equiv \begin{bmatrix} 0 \\ 0 \\ \kappa_i \end{bmatrix}, \tag{A.1}$$

and the variance and covariance matrices of observable and unobservable processes by

$$\Sigma_{oo} \equiv \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_j & \rho\sigma_1\sigma_i \\ \rho\sigma_1\sigma_j & \sigma_j^2 & \rho\sigma_j\sigma_i \\ \rho\sigma_1\sigma_i & \rho\sigma_j\sigma_i & (\sigma_i^2 + \nu_i^2) \end{bmatrix}, \qquad \Sigma_{uo} \equiv \begin{bmatrix} \rho\sigma_1\sigma_i & \rho\sigma_j\sigma_i & \sigma_i^2 \end{bmatrix}. \tag{A.2}$$

The filtering theory then implies that if the prior of the $\ln D_i$ at time τ_n is normally distributed with mean $\widehat{\ln D}_{i\tau_n}$ and variance $V_{i\tau_n}$, then the posterior of $\ln D_i$ during the period $t \in [\tau_n, \tau_{n+1})$ conditional on the information $\mathcal{G}_{it} = \sigma\{(D_{1s}, D_{js}, F_{is}) : \tau_n \leq s \leq t\}$ is also normally distributed with mean $\widehat{\ln D}_{it} = \mathrm{E}[\ln D_{it} | \mathcal{G}_{it}]$ and variance $V_{it} = \mathrm{E}\left[(\ln D_{it} - \widehat{\ln D}_{it})^2 | \mathcal{G}_{it}\right]$ with dynamics

$$d\widehat{\ln D}_{it} = (\mu_i - \frac{1}{2}\sigma_i^2)dt + (\Sigma_{uo} + V_{it} A_1^\intercal)\, \Sigma_{oo}^{-1} \left[dX_t - (A_0 + A_1 \widehat{\ln D}_{it})dt\right], \tag{A.3}$$

$$dV_{it} = -\left[(\Sigma_{uo} + V_{it} A_1^\intercal)\, \Sigma_{oo}^{-1}\, (\Sigma_{uo} + V_{it} A_1^\intercal)^\intercal - \sigma_i^2\right] dt. \tag{A.4}$$

Substituting (A.1)–(A.2) into the posterior mean dynamics (A.3) and rearranging after some algebra yields

$$d\widehat{\ln D}_{it} = (\mu_i - \frac{1}{2}\sigma_i^2)dt + \frac{\rho\sigma_i\,(\nu_i^2 - \kappa_i V_{it})}{(1+\rho)\,(\sigma_i^2 + \nu_i^2) - 2\rho^2\sigma_i^2} d\omega_{1t} + \frac{\rho\sigma_i\,(\nu_i^2 - \kappa_i V_{it})}{(1+\rho)\,(\sigma_i^2 + \nu_i^2) - 2\rho^2\sigma_i^2} d\omega_{jt}$$
$$+ \frac{(1+\rho)\,(\sigma_i^2 + \kappa_i V_{it}) - 2\rho^2\sigma_i^2}{(1+\rho)\,(\sigma_i^2 + \nu_i^2) - 2\rho^2\sigma_i^2} \sqrt{\sigma_i^2 + \nu_i^2}\, d\widehat{\omega}_{it}^*, \tag{A.5}$$

where the innovation process is given by $d\widehat{\omega}_{it}^* = \left[d\ln F_{it} - (\mu_i - \frac{1}{2}\sigma_i^2 + \kappa_i\zeta_i - \kappa_i \ln F_{it} + \kappa_i \widehat{\ln D}_{it})dt\right] / \sqrt{\sigma_i^2 + \nu_i^2}$, with the correlations $d\omega_{1t} d\widehat{\omega}_{it}^* = d\omega_{jt} d\widehat{\omega}_{it}^* = (\rho\sigma_i / \sqrt{\sigma_i^2 + \nu_i^2})dt$. Since it is typically more convenient to work with independent (uncorrelated) Brownian motions, we define a new Brownian motion $\widehat{\omega}_i$ that is independent of the Brownian motions ω_1 and ω_j through the relation

$$d\widehat{\omega}^*_{it} = \frac{1}{\sqrt{\sigma_i^2+\nu_i^2}}\left[\frac{\rho\sigma_i}{1+\rho}d\omega_{1t} + \frac{\rho\sigma_i}{1+\rho}d\omega_{jt} + \sqrt{\sigma_i^2+\nu_i^2 - 2\frac{\rho^2\sigma_i^2}{1+\rho}}d\widehat{\omega}_{it}\right],$$

which after substituting into (A.5) yields the dynamics

$$d\widehat{\ln D}_{it} = (\mu_i - \frac{1}{2}\sigma_i^2)dt + \frac{\rho\sigma_i}{1+\rho}d\omega_{1t} + \frac{\rho\sigma_i}{1+\rho}d\omega_{jt} + \frac{(1+\rho)\left(\sigma_i^2+\kappa_i V_{it}\right) - 2\rho^2\sigma_i^2}{\sqrt{(1+\rho)^2\left(\sigma_i^2+\nu_i^2\right) - 2\left(1+\rho\right)\rho^2\sigma_i^2}}d\widehat{\omega}_{it}. \quad \text{(A.6)}$$

We next substitute (A.1)–(A.2) into the posterior variance dynamics (A.4) and obtain

$$dV_{it} = -\left[\frac{\left((1+\rho)\left(\sigma_i^2+\kappa_i V_{it}\right) - 2\rho^2\sigma_i^2\right)^2}{(1+\rho)^2\left(\sigma_i^2+\nu_i^2\right) - 2\left(1+\rho\right)\rho^2\sigma_i^2} - \left(1 - \frac{2\rho^2}{1+\rho}\right)\sigma_i^2\right]dt, \quad \text{(A.7)}$$

as in (7). The steady-state value of the posterior variance $V_{i\infty}$ is the constant which solves the quadratic equation by setting $dV_{it} = 0$ in (A.7), and given by

$$V_{i\infty} = \frac{1}{\kappa_i}\left[\sqrt{((1-\hat{\rho}^2)\sigma_i^2+\nu_i^2)(1-\hat{\rho}^2)\sigma_i^2} - (1-\hat{\rho}^2)\sigma_i^2\right], \quad \text{(A.8)}$$

where we have defined the constant $\hat{\rho}^2 \equiv 2\rho^2/(1+\rho)$. Moreover, the closed-form solution for the posterior variance at all times $t \in [\tau_n, \tau_{n+1})$ follows from the well-known solution to the Riccati equation and is given by

$$V_{it} = V_{i\infty}\frac{1+p_i e^{-m_i(t-\tau_n)}}{1-q_i e^{-m_i(t-\tau_n)}}, \quad \text{(A.9)}$$

where we have defined the constants

$$m_i \equiv 2\kappa_i\sqrt{\frac{(1-\hat{\rho}^2)\sigma_i^2}{(1-\hat{\rho}^2)\sigma_i^2+\nu_i^2}}, \qquad q_i \equiv \frac{\kappa_i V_{i\tau_n} - \sqrt{((1-\hat{\rho}^2)\sigma_i^2+\nu_i^2)(1-\hat{\rho}^2)\sigma_i^2} + (1-\hat{\rho}^2)\sigma_i^2}{\kappa_i V_{i\tau_n} + \sqrt{((1-\hat{\rho}^2)\sigma_i^2+\nu_i^2)(1-\hat{\rho}^2)\sigma_i^2} + (1-\hat{\rho}^2)\sigma_i^2},$$

$$p_i \equiv \frac{\kappa_i V_{i\tau_n}\left[\sqrt{((1-\hat{\rho}^2)\sigma_i^2+\nu_i^2)(1-\hat{\rho}^2)\sigma_i^2} + (1-\hat{\rho}^2)\sigma_i^2\right] - (1-\hat{\rho}^2)\sigma_i^2\nu_i^2}{\kappa_i V_{i\tau_n}\left[\sqrt{((1-\hat{\rho}^2)\sigma_i^2+\nu_i^2)(1-\hat{\rho}^2)\sigma_i^2} - (1-\hat{\rho}^2)\sigma_i^2\right] + (1-\hat{\rho}^2)\sigma_i^2\nu_i^2}.$$

It is also easy to see from (A.9) that the prior variance $V_{i\tau_n}$ is greater than the steady-state posterior variance, $V_{i\tau_n} > V_{i\infty}$, if and only if $p_i + q_i > 0$.[22]

Finally, applying Itô's Lemma to the estimated pseudo-dividend relation $\widehat{D}_{it} = \exp(\widehat{\ln D}_{it} + \frac{1}{2}V_{it})$ gives its dynamics as

$$\frac{d\widehat{D}_{it}}{\widehat{D}_{it}} = d\widehat{\ln D}_{it} + \frac{1}{2}\left(d\widehat{\ln D}_{it} d\widehat{\ln D}_{it} + dV_{it}\right) = d\widehat{\ln D}_{it} + \frac{1}{2}\sigma_i^2 dt,$$

where the second equality follows from (A.6)–(A.7). After substituting (A.6) into the last equality above we obtain (6).

The volatility of the estimated pseudo-dividend is readily given by the dynamics (6) as

$$\sigma_{\widehat{D}_i t} = \sqrt{2\frac{\rho^2\sigma_i^2}{1+\rho} + \frac{\left((1+\rho)\left(\sigma_i^2 + \kappa_i V_{it}\right) - 2\rho^2\sigma_i^2\right)^2}{(1+\rho)^2\left(\sigma_i^2 + \nu_i^2\right) - 2\left(1+\rho\right)\rho^2\sigma_i^2}}. \tag{A.10}$$

Since $V_{i\tau_n} > V_{it} > V_{i\infty}$ at all times t, (A.10) takes its minimum value, σ_i, when the posterior variance is at its steady-state (A.8) implying

$$\sigma_{\widehat{D}_i t} > \sigma_i, \tag{A.11}$$

that is, the estimated pseudo-dividend is indeed more volatile than the pseudo-dividend at all times $t \in [\tau_n, \tau_{n+1})$. □

Proof of statements in Remark 2. Under the alternative "signal plus noise" specification (8), the corresponding quantities of (A.1)–(A.2) in the proof of Lemma 1 become

$$A_0 \equiv \begin{bmatrix} \mu_1 - \frac{1}{2}\sigma_1^2 \\ \mu_j - \frac{1}{2}\sigma_j^2 \\ 0 \end{bmatrix}, \quad A_1 \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \Sigma_{oo} \equiv \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_j & 0 \\ \rho\sigma_1\sigma_j & \sigma_j^2 & 0 \\ 0 & 0 & \nu_i^2 \end{bmatrix}, \quad \Sigma_{uo} \equiv \begin{bmatrix} \rho\sigma_1\sigma_i & \rho\sigma_j\sigma_i & 0 \end{bmatrix}.$$

Following similar steps to those for the filtering in the proof of Lemma 1 yield the dynamics for the posterior mean and variance as

$$d\widehat{\ln D}_{it} = (\mu_i - \frac{1}{2}\sigma_i^2)dt + \frac{\rho\sigma_i}{1+\rho}d\omega_{1t} + \frac{\rho\sigma_i}{1+\rho}d\omega_{jt} + \frac{V_{it}}{\nu_i}d\widehat{\omega}_{it}^*, \quad dV_{it} = -\left[\frac{V_{it}^2}{\nu_i^2} - \left(1 - \frac{2\rho^2}{1+\rho}\right)\sigma_i^2\right]dt,$$

[22] A simpler sufficient condition for $V_{i\tau_n} > V_{i\infty}$ to hold is given by $V_{i\tau_n} \geq \frac{\nu_i}{\kappa_i}\sqrt{(1-\hat{\rho}^2)\sigma_i^2}$, which is satisfied for an appropriate choice of an initial prior.

where the innovation process $\widehat{\omega}_i^*$ is given by $d\widehat{\omega}_{it}^* = \frac{1}{\nu_i}\left[d\ln F_{it} - \widehat{\ln D}_{it}dt\right]$, with the correlations $d\omega_{1t}d\widehat{\omega}_{it}^* = d\omega_{jt}d\widehat{\omega}_{it}^* = 0$. In this case, the steady-state value of the posterior variance $V_{i\infty}$ is simply given by $V_{i\infty} = \nu_i\sigma_i\sqrt{1-(2\rho^2/(1+\rho))}$, and the volatility of the estimated pseudo-dividend by $\sigma_{\widehat{D}_it} = \sqrt{(2\rho^2/(1+\rho))\sigma_i^2 + (V_{it}^2/\nu_i^2)}$, from which we again obtain $\sigma_{\widehat{D}_it} > \sigma_i$, that is, the estimated pseudo-dividend is more volatile than the pseudo-dividend under this specification also. □

Proof of Proposition 1. We proceed by determining the equilibrium state price density process. We then recover the equilibrium stock prices, and hence the stock market level, first in the comparable benchmark economy and then in our economy.

The equilibrium state price density process ξ at all times is given by the marginal utility of the representative investor evaluated at the aggregate consumption

$$\xi_t = e^{-\beta t}Y_t^{-\gamma}. \tag{A.12}$$

In the benchmark economy in which all three stocks pay dividends, applying Itô's Lemma to (A.12) using the aggregate consumption dynamics $dY_t/Y_t = \sum_{i=1}^3 \bar{\alpha}_i\mu_i dt + \sum_{i=1}^3 \bar{\alpha}_i\sigma_i d\omega_{it}$, gives the state price density dynamics as

$$\frac{d\xi_t}{\xi_t} = -\bar{r}dt - \gamma\bar{\alpha}_1\sigma_1 d\omega_{1t} - \gamma\bar{\alpha}_2\sigma_2 d\omega_{2t} - \gamma\bar{\alpha}_3\sigma_3 d\omega_{3t}, \tag{A.13}$$

where $\bar{r}$ is the equilibrium interest rate in this economy as provided in Proposition 3. In this economy, by no arbitrage, the stock prices are given by

$$\bar{S}_{it} = \frac{1}{\xi_t}\mathrm{E}_t\left[\int_t^\infty \xi_u D_{iu}du\right], \qquad \text{for } i = 1, 2, 3. \tag{A.14}$$

Applying Itô's Lemma to ξD_i, using the dividend dynamics (1)–(3) and (A.13) yields the drift terms $-(\bar{r}_i - \mu_i)$ where the constants $\bar{r}_1$, $\bar{r}_2$, and $\bar{r}_3$ are given by

$$\bar{r}_1 = \bar{r} + \gamma(\bar{\alpha}_1\sigma_1^2 + \bar{\alpha}_2\rho\sigma_1\sigma_2 + \bar{\alpha}_3\rho\sigma_1\sigma_3), \tag{A.15}$$

$$\bar{r}_2 = \bar{r} + \gamma(\bar{\alpha}_1\rho\sigma_1\sigma_2 + \bar{\alpha}_2\sigma_2^2 + \bar{\alpha}_3\rho\sigma_2\sigma_3), \tag{A.16}$$

$$\bar{r}_3 = \bar{r} + \gamma(\bar{\alpha}_1\rho\sigma_1\sigma_3 + \bar{\alpha}_2\rho\sigma_2\sigma_3 + \bar{\alpha}_3\sigma_3^2). \tag{A.17}$$

Since the process ξD_i, for $i = 1, 2, 3$, has a constant drift of $-(\bar{r}_i - \mu_i)$, we have the expectation $\mathrm{E}_t[\xi_u D_{iu}] = e^{-(\bar{r}_i-\mu_i)(u-t)}\xi_t D_{it}$, which after substituting into (A.14) yields $\bar{S}_{it} = \int_t^\infty e^{-(\bar{r}_i-\mu_i)(u-t)}du D_{it}$. Evaluating the simple integral (under the parameter restriction of $\bar{r}_i - \mu_i > 0$, so that the stock price is finite, and hence, well-defined) leads to the individual stock price expressions (13), and hence the stock market level (12), in the benchmark economy. Moreover, applying Itô's Lemma to these price expressions gives the benchmark economy individual stock price dynamics as

$$\frac{d\bar{S}_{it}}{\bar{S}_{it}} + \frac{D_{it}}{\bar{S}_{it}}dt = \bar{r}_i dt + \sigma_i d\omega_{it}, \qquad \text{for } i = 1, 2, 3, \tag{A.18}$$

and the stock market dynamics as

$$\frac{d\bar{S}_t}{\bar{S}_t} + \frac{\sum_{i=1}^{3} D_{it}}{\bar{S}_t}dt = \sum_{i=1}^{3}\left(\frac{d\bar{S}_{it}}{\bar{S}_{it}} + \frac{D_{it}}{\bar{S}_{it}}dt\right)\frac{\bar{S}_{it}}{\bar{S}_t} = \sum_{i=1}^{3}\bar{r}_i\frac{\bar{S}_{it}}{\bar{S}_t}dt + \sum_{i=1}^{3}\sigma_i\frac{\bar{S}_{it}}{\bar{S}_t}d\omega_{it}. \tag{A.19}$$

In our economy with no-dividend stocks, applying Itô's Lemma to (A.12) using the aggregate consumption dynamics (11) gives the state price density dynamics as

$$\frac{d\xi_t}{\xi_t} = \begin{cases} -rdt - \gamma\alpha_1\sigma_1 d\omega_{1t} - \gamma\alpha_2\sigma_2 d\omega_{2t}, & t \in \mathcal{T}_o, \\ -rdt - \gamma\alpha_1\sigma_1 d\omega_{1t} - \gamma\alpha_3\sigma_3 d\omega_{3t}, & t \in \mathcal{T}_e, \end{cases} \tag{A.20}$$

where r is the equilibrium interest rate in this economy as provided in Proposition 3. In our economy, by no arbitrage, the stock prices are given by

$$S_{it} = \frac{1}{\xi_t}\mathrm{E}_t\left[\int_{[t,\infty)\cap\mathcal{T}_i}\xi_u D_{iu}du\right], \qquad \text{for } i = 1, 2, 3,$$

where $\mathcal{T}_i$ denotes the period stock i pays dividend, which is $[0,\infty)$ for stock 1, $\mathcal{T}_o$ for stock 2, and $\mathcal{T}_e$ for stock 3. To be able to solve for the equilibrium stock prices in this economy in which stochastic discount factor shocks alternate infinite times, we partition the time horizon $[0,\infty)$ into $J+1$ random periods

$$[0,\infty) = [0,\tau_1) \cup [\tau_1,\tau_2) \cup [\tau_2,\tau_3) \cup \ldots \cup [\tau_{J-1},\tau_J) \cup [\tau_J,\infty),$$

where τ_j are the arrival times of the Poisson process N_t. Without loss of generality, we assume J is an odd number. We observe that in the limit $J \to \infty$ the last period $[\tau_J,\infty)$ vanishes, and for this reason we simply assume that during the last period all stocks pay dividends, leading to the benchmark economy stock price expressions (13) for the period $[\tau_J,\infty)$.[23]

[23]Alternatively, one could simply set the stock prices in the last period $[\tau_J,\infty)$ to be all zero without affecting the equilibrium stock prices since in the limit $J \to \infty$ the contribution of this period to the stock prices is zero.

Working backwards, we note that period $[\tau_{J-1}, \tau_J)$ is a subset of $\mathcal{T}_e$ during which stocks 1 and 3 pay dividends, and hence the stock prices during this period satisfy

$$S_{1t} = \frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^{\tau_J} \xi_u D_{1u} du + \xi_{\tau_J} S_{1\tau_J} \right], \tag{A.21}$$

$$S_{2t} = \frac{1}{\xi_t} \mathrm{E}_t \left[\xi_{\tau_J} S_{2\tau_J} \right], \tag{A.22}$$

$$S_{3t} = \frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^{\tau_J} \xi_u D_{3u} du + \xi_{\tau_J} S_{3\tau_J} \right]. \tag{A.23}$$

Using the well-known properties of Poisson processes that during any time $t \in [\tau_{j-1}, \tau_j)$, the next arrival time τ_j is an exponential random variable that is independent from all Brownian motions with its distribution function given by

$$G(u-t) = \mathbb{P}(\tau_j \leq u | \tau_j > t) = \mathbb{P}(\tau_1 \leq u - t) = 1 - e^{-\lambda(u-t)}, \tag{A.24}$$

and its corresponding density function by

$$g(u-t) = \lambda e^{-\lambda(u-t)}, \tag{A.25}$$

we obtain the first term in (A.21) as

$$\mathrm{E}_t \left[\int_t^{\tau_J} \xi_u D_{1u} du \right] = \mathrm{E}_t \left[\int_t^{\infty} \xi_u D_{1u} \mathbf{1}_{\{u<\tau_J\}} du \right] = \mathrm{E}_t \left[\int_t^{\infty} \xi_u D_{1u} \mathbb{P}(u < \tau_J | \tau_J > t)\, du \right],$$

where the last equality follows from taking the expectation with respect to τ and the property of indicator functions. Substituting the right tail probability $\mathbb{P}(u < \tau_J | \tau_J > t) = 1 - G(u-t) = e^{-\lambda(u-t)}$ gives

$$\mathrm{E}_t \left[\int_t^{\tau_J} \xi_u D_{1u} du \right] = \mathrm{E}_t \left[\int_t^{\infty} \xi_u D_{1u} e^{-\lambda(u-t)} du \right], \tag{A.26}$$

where now the expectation needs to be taken with respect to the Brownian motions only. Applying Itô's Lemma to ξD_1, using (1) and (A.20) for this period yields the drift term $-(r_{1,e} - \mu_1)$ where the constant $r_{1,e}$ is given by

$$r_{1,e} = r + \gamma \alpha_1 \sigma_1^2 + \gamma \alpha_3 \rho \sigma_1 \sigma_3. \tag{A.27}$$

Since the process ξD_1 has a constant drift of $-(r_{1,e} - \mu_1)$ during this period, we have $\mathrm{E}_t[\xi_u D_{1u}] =$

$e^{-(r_{1,e}-\mu_1)(u-t)}\xi_t D_{1t}$, which after substituting into (A.26) yields

$$\mathrm{E}_t\left[\int_t^{\tau_J}\xi_u D_{1u}du\right]=\int_t^{\infty}e^{-(r_{1,e}-\mu_1+\lambda)(u-t)}du\xi_t D_{1t}=\frac{1}{r_{1,e}-\mu_1+\lambda}\xi_t D_{1t},$$

where the last equality follows from solving the simple integral (under the parameter restriction of $r_{1.e}-\mu_1>0$, so that the stock price is finite, and hence well-defined for any value of λ). For the second term in (A.21), we substitute the stock 1 price at time τ_J given by $S_{1\tau_J}=A_{1,J}D_{1\tau_J}$ where the constant $A_{1,J}=1/\left(\bar{r}_1-\mu_1\right)$, to obtain $\mathrm{E}_t\left[\xi_{\tau_J}S_{1\tau_J}\right]=A_{1,J}\mathrm{E}_t\left[\xi_{\tau_J}D_{1\tau_J}\right]$. Taking the expectation with respect to τ_J gives

$$\mathrm{E}_t\left[\xi_{\tau_J}D_{1\tau_J}\right]=\mathrm{E}_t\left[\int_t^{\infty}\xi_u D_{1u}g\left(u-t\right)du\right]=\mathrm{E}_t\left[\int_t^{\infty}\xi_u D_{1u}\lambda e^{-\lambda(u-t)}du\right],$$

and using the conditional expectation result again we obtain

$$\mathrm{E}_t\left[\xi_{\tau_J}D_{1\tau_J}\right]=\int_t^{\infty}e^{-(r_{1,e}-\mu_1+\lambda)(u-t)}du\lambda\xi_t D_{1t}=\frac{\lambda}{r_{1,e}-\mu_1+\lambda}\xi_t D_{1t}.$$

Substituting the first and second terms into (A.21) gives the stock 1 price during $[\tau_{J-1},\tau_J)$ as

$$S_{1t}=\frac{1}{r_{1,e}-\mu_1+\lambda}D_{1t}+\frac{\lambda}{r_{1,e}-\mu_1+\lambda}A_{1,J}D_{1t}.$$

Following similar steps for the dividend-paying stock 3 leads to its price during $[\tau_{J-1},\tau_J)$ as

$$S_{3t}=\frac{1}{r_{3,e}-\mu_3+\lambda}D_{3t}+\frac{\lambda}{r_{3,e}-\mu_3+\lambda}A_{3,J}D_{3t},$$

where the constants

$$r_{3,e}=r+\gamma\alpha_1\rho\sigma_1\sigma_3+\gamma\alpha_3\sigma_3^2, \tag{A.28}$$

and $A_{3,J}=1/\left(\bar{r}_3-\mu_3\right)$. For the no-dividend stock 2 during this period, we substitute the stock 2 price at time τ_J given by $S_{2\tau_J}=A_{2,J}D_{2\tau_J}$ where the constant $A_{2,J}=1/\left(\bar{r}_2-\mu_2\right)$ into (A.22) and obtain $\mathrm{E}_t\left[\xi_{\tau_J}S_{2\tau_J}\right]=A_{2,J}\mathrm{E}_t\left[\xi_{\tau_J}D_{2\tau_J}\right]$. Since in the absence of its dividends, the investor uses the estimated pseudo-dividend $\widehat{D}_2$ (Lemma 1) to estimate the distribution of future dividends, we simply substitute $\widehat{D}_{2\tau_J}$ for $D_{2\tau_J}$ in the above expectation to obtain

$$\mathrm{E}_t\left[\xi_{\tau_J}D_{2\tau_J}\right]=\mathrm{E}_t\left[\xi_{\tau_J}\widehat{D}_{2\tau_J}\right]=\mathrm{E}_t\left[\int_t^{\infty}\xi_u\widehat{D}_{2u}g\left(u-t\right)du\right]=\mathrm{E}_t\left[\int_t^{\infty}\xi_u\widehat{D}_{2u}\lambda e^{-\lambda(u-t)}du\right],$$

where again the second equality follows from taking the expectation with respect to τ_J.[24] Applying Itô's Lemma to $\xi \widehat{D}_2$, using the dynamics of the estimated pseudo-dividend (6) and (A.20) for this period yields the drift term $-(r_{2,e} - \mu_2)$ where the constant $r_{2,e}$ is given by

$$r_{2,e} = r + \gamma\alpha_1\rho\sigma_1\sigma_2 + \gamma\alpha_3\rho\sigma_2\sigma_3. \tag{A.29}$$

Since $\xi \widehat{D}_2$ has a constant drift of $-(r_{2,e} - \mu_2)$ during this period, we have the expectation $\mathrm{E}_t\left[\xi_u \widehat{D}_{2u}\right] = e^{-(r_{2,e}-\mu_2)(u-t)}\xi_t \widehat{D}_{2t}$, which yields

$$\mathrm{E}_t\left[\xi_{\tau_J} D_{2\tau_J}\right] = \int_t^\infty e^{-(r_{2,e}-\mu_2+\lambda)(u-t)} du \lambda \xi_t \widehat{D}_{2t} = \frac{\lambda}{r_{2,e} - \mu_2 + \lambda}\xi_t \widehat{D}_{2t},$$

where the last equality follows from solving the simple integral (under the parameter restriction of $r_{2,e} - \mu_2 > 0$). This gives the stock 2 price during the period $[\tau_{J-1}, \tau_J)$ as

$$S_{2t} = \frac{\lambda}{r_{2,e} - \mu_2 + \lambda} A_{2,J} \widehat{D}_{2t}.$$

Moving one period backwards to the period $[\tau_{J-2}, \tau_{J-1})$, which is a subset of $\mathcal{T}_o$ during which stocks 1 and 2 pay dividends, the stock prices during this period satisfy

$$S_{1t} = \frac{1}{\xi_t}\mathrm{E}_t\left[\int_t^{\tau_{J-1}} \xi_u D_{1u} du + \xi_{\tau_{J-1}} S_{1\tau_{J-1}}\right], \tag{A.30}$$

$$S_{2t} = \frac{1}{\xi_t}\mathrm{E}_t\left[\int_t^{\tau_{J-1}} \xi_u D_{2u} du + \xi_{\tau_{J-1}} S_{2\tau_{J-1}}\right], \tag{A.31}$$

$$S_{3t} = \frac{1}{\xi_t}\mathrm{E}_t\left[\xi_{\tau_{J-1}} S_{3\tau_{J-1}}\right]. \tag{A.32}$$

Using the distribution function (A.24) for the next arrival time during this period, and following similar steps as in the previous case, we obtain the first term in (A.30) as $\mathrm{E}_t\left[\int_t^{\tau_{J-1}} \xi_u D_{1u} du\right] = \mathrm{E}_t\left[\int_t^\infty \xi_u D_{1u} e^{-\lambda(u-t)} du\right]$. Applying Itô's Lemma to ξD_1, using (1) and (A.20) for this period yields the drift term $-(r_{1,o} - \mu_1)$ where the constant $r_{1,o}$ is given by

$$r_{1,o} = r + \gamma\alpha_1\sigma_1^2 + \gamma\alpha_2\rho\sigma_1\sigma_2. \tag{A.33}$$

Since the process ξD_1 has a constant drift of $-(r_{1,o} - \mu_1)$ during this period, we have $\mathrm{E}_t\left[\xi_u D_{1u}\right] =$

[24]Note that during this period, the estimation of D_2 does not affect the aggregate consumption, and hence the state price density ξ, since the fluctuations in aggregate consumption are driven by current dividend shocks.

$e^{-(r_{1,o}-\mu_1)(u-t)}\xi_t D_{1t}$, which yields

$$\mathrm{E}_t\left[\int_t^{\tau_{J-1}} \xi_u D_{1u} du\right] = \int_t^{\infty} e^{-(r_{1,o}-\mu_1+\lambda)(u-t)} du \xi_t D_{1t} = \frac{1}{r_{1,o}-\mu_1+\lambda}\xi_t D_{1t}.$$

For the second term in (A.30), we substitute the stock 1 price at time τ_{J-1} given by $S_{1\tau_{J-1}} = A_{1,(J-1)} D_{1\tau_{J-1}}$, where we have defined the constant

$$A_{1,(J-1)} = \frac{1}{r_{1,e}-\mu_1+\lambda} + \frac{\lambda}{r_{1,e}-\mu_1+\lambda} A_{1,J},$$

to obtain $\mathrm{E}_t\left[\xi_{\tau_{J-1}} S_{1\tau_{J-1}}\right] = A_{1,(J-1)}\mathrm{E}_t\left[\xi_{\tau_{J-1}} D_{1\tau_{J-1}}\right]$. Taking the expectation with respect to τ_{J-1} gives

$$\mathrm{E}_t\left[\xi_{\tau_{J-1}} D_{1\tau_{J-1}}\right] = \mathrm{E}_t\left[\int_t^{\infty} \xi_u D_{1u} g\left(u-t\right) du\right] = \mathrm{E}_t\left[\int_t^{\infty} \xi_u D_{1u} \lambda e^{-\lambda(u-t)} du\right],$$

and using the conditional expectation result for this period, we obtain

$$\mathrm{E}_t\left[\xi_{\tau_{J-1}} D_{1\tau_{J-1}}\right] = \int_t^{\infty} e^{-(r_{1,o}-\mu_1+\lambda)(u-t)} du \lambda \xi_t D_{1t} = \frac{\lambda}{r_{1,o}-\mu_1+\lambda}\xi_t D_{1t}.$$

Substituting the first and second terms into (A.30) gives the stock 1 price during $[\tau_{J-2}, \tau_{J-1})$ as

$$S_{1t} = \frac{1}{r_{1,o}-\mu_1+\lambda} D_{1t} + \frac{\lambda}{r_{1,o}-\mu_1+\lambda} A_{1,(J-1)} D_{1t}.$$

Following similar steps for the other dividend stock 2 leads to the first term in (A.31) as $\mathrm{E}_t\left[\int_t^{\tau_{J-1}} \xi_u D_{2u} du\right] = \xi_t D_{2t}/(r_{2,o}-\mu_2+\lambda)$, where the constant

$$r_{2,o} = r + \gamma\alpha_1\rho\sigma_1\sigma_2 + \gamma\alpha_2\sigma_2^2. \tag{A.34}$$

For the second term in (A.31), we substitute the stock 2 price at time τ_{J-1} given by $S_{2\tau_{J-1}} = A_{2,(J-1)} \widehat{D}_{2\tau_{J-1}}$, where we have defined the constant

$$A_{2,(J-1)} = \frac{\lambda}{r_{2,e}-\mu_2+\lambda} A_{2,J},$$

to obtain $\mathrm{E}_t\left[\xi_{\tau_{J-1}} S_{2\tau_{J-1}}\right] = A_{2,(J-1)}\mathrm{E}_t\left[\xi_{\tau_{J-1}} \widehat{D}_{2\tau_{J-1}}\right] = A_{2,(J-1)}\mathrm{E}_t\left[\xi_{\tau_{J-1}} D_{2\tau_{J-1}}\right]$, with the last equality following from the fact that $\widehat{\ln D_{2\tau_n}} = \lim_{t\to\tau_n} \ln D_{2t}$ for all τ_n. Taking the expectation with respect to τ_{J-1} and using the conditional expectation result for this period, we obtain

$\mathrm{E}_t\left[\xi_{\tau_{J-1}} D_{2\tau_{J-1}}\right] = \int_t^\infty e^{-(r_{2,o}-\mu_2+\lambda)(u-t)} du \lambda \xi_t D_{2t} = \lambda \xi_t D_{2t}/(r_{2,o} - \mu_2 + \lambda)$. Substituting the first and second terms into (A.31) gives the stock 2 price during $[\tau_{J-2}, \tau_{J-1})$ as

$$S_{2t} = \frac{1}{r_{2,o} - \mu_2 + \lambda} D_{2t} + \frac{\lambda}{r_{2,o} - \mu_2 + \lambda} A_{2,(J-1)} D_{2t}.$$

For the no-dividend stock 3 during this period, we substitute the stock 3 price at time τ_{J-1} given by $S_{3\tau_{J-1}} = A_{3,(J-1)} D_{3\tau_{J-1}}$, where we have defined the constant

$$A_{3,(J-1)} = \frac{1}{r_{3,e} - \mu_3 + \lambda} + \frac{\lambda}{r_{3,e} - \mu_3 + \lambda} A_{3,J},$$

to obtain $\mathrm{E}_t\left[\xi_{\tau_{J-1}} D_{3\tau_{J-1}}\right] = \mathrm{E}_t\left[\xi_{\tau_{J-1}} \widehat{D}_{3\tau_{J-1}}\right] = \mathrm{E}_t\left[\int_t^\infty \xi_u \widehat{D}_{3u} \lambda e^{-\lambda(u-t)} du\right]$ where again the last equality following from taking the expectation with respect to τ_{J-1}. Applying Itô's Lemma to $\xi \widehat{D}_3$, using the dynamics of the estimated pseudo-dividend (6) and (A.20) for this period yields the drift term $-(r_{3,o} - \mu_3)$ where the constant $r_{3,o}$ is given by

$$r_{3,o} = r + \gamma \alpha_1 \rho \sigma_1 \sigma_3 + \gamma \alpha_2 \rho \sigma_2 \sigma_3. \tag{A.35}$$

Since $\xi \widehat{D}_3$ has a constant drift of $-(r_{3,o} - \mu_3)$ during this period, we have the expectation $\mathrm{E}_t\left[\xi_u \widehat{D}_{3u}\right] = e^{-(r_{3,o}-\mu_3)(u-t)} \xi_t \widehat{D}_{3t}$, which yields $\mathrm{E}_t\left[\xi_{\tau_{J-1}} D_{3\tau_{J-1}}\right] = \int_t^\infty e^{-(r_{3,o}-\mu_3+\lambda)(u-t)} du \lambda \xi_t \widehat{D}_{3t}$, and the stock 3 price during $[\tau_{J-2}, \tau_{J-1})$ as

$$S_{3t} = \frac{\lambda}{r_{3,o} - \mu_3 + \lambda} A_{3,(J-1)} \widehat{D}_{3t}.$$

Following similar steps as above, by working backwards, we obtain the stock prices in any period $[\tau_{J-M}, \tau_{J-M+1})$ where M is an odd number such that $1 < M < J$, so that this period is a subset of $\mathcal{T}_e$, as

$$\begin{aligned}
S_{1t} &= \frac{1 + \lambda A_{1,(J-M+1)}}{r_{1,e} - \mu_1 + \lambda} D_{1t}, &\quad \text{where} \quad A_{1,(J-M+1)} &= \frac{1 + \lambda A_{1,(J-M+2)}}{r_{1,o} - \mu_1 + \lambda}, \\
S_{2t} &= \frac{\lambda A_{2,(J-M+1)}}{r_{2,e} - \mu_2 + \lambda} \widehat{D}_{2t}, &\quad \text{where} \quad A_{2,(J-M+1)} &= \frac{1 + \lambda A_{2,(J-M+2)}}{r_{2,o} - \mu_2 + \lambda}, \\
S_{3t} &= \frac{1 + \lambda A_{3,(J-M+1)}}{r_{3,e} - \mu_3 + \lambda} D_{3t}, &\quad \text{where} \quad A_{3,(J-M+1)} &= \frac{\lambda A_{3,(J-M+2)}}{r_{3,o} - \mu_3 + \lambda},
\end{aligned}$$

and in any period $[\tau_{J-M}, \tau_{J-M+1})$ where M is an even number such that $1 < M < J$, so that this period is a subset of $\mathcal{T}_o$, as

$$S_{1t} = \frac{1+\lambda A_{1,(J-M+1)}}{r_{1,o}-\mu_1+\lambda} D_{1t}, \quad \text{where} \quad A_{1,(J-M+1)} = \frac{1+\lambda A_{1,(J-M+2)}}{r_{1,e}-\mu_1+\lambda},$$
$$S_{2t} = \frac{1+\lambda A_{2,(J-M+1)}}{r_{2,o}-\mu_2+\lambda} D_{2t}, \quad \text{where} \quad A_{2,(J-M+1)} = \frac{\lambda A_{2,(J-M+2)}}{r_{2,e}-\mu_2+\lambda},$$
$$S_{3t} = \frac{\lambda A_{3,(J-M+1)}}{r_{3,o}-\mu_3+\lambda} \widehat{D}_{3t}, \quad \text{where} \quad A_{3,(J-M+1)} = \frac{1+\lambda A_{3,(J-M+2)}}{r_{3,e}-\mu_3+\lambda}.$$

Finally, to obtain the stock prices in closed-form, we solve the difference equations A_i. We first consider $A_{1,(J-M+1)}$ where M is an odd number ($t \in \mathcal{T}_e$) and recursively substitute its next period value to obtain

$$\begin{aligned} A_{1,(J-M+1)} &= \left[1+k_1+k_1^2+k_1^3+....+k_1^{(M-3)/2}\right] \frac{\lambda}{r_{1,o}-\mu_1+\lambda} \frac{1}{r_{1,e}-\mu_1+\lambda} \\ &+ \left[1+k_1+k_1^2+k_1^3+....+k_1^{(M-3)/2}\right] \frac{1}{r_{1,o}-\mu_1+\lambda} + k_1^{(M-1)/2} \frac{1}{\bar{r}_1-\mu_1}, \end{aligned} \tag{A.36}$$

where the positive constant $k_1 = \lambda^2/\left((r_{1,e}-\mu_1+\lambda)(r_{1,o}-\mu_1+\lambda)\right) < 1$. In the limit $M \to \infty$, also $J \to \infty$, the square bracket terms in (A.36) becomes a geometric sum $1/(1-k_1)$ while the last term converges to zero resulting with the constant that does not vary for any period in $\mathcal{T}_e$

$$A_{1,e} = \frac{r_{1,e}-\mu_1+2\lambda}{(r_{1,e}-\mu_1+\lambda)(r_{1,o}-\mu_1+\lambda)-\lambda^2},$$

yielding stock 1 price during $t \in \mathcal{T}_e$ as

$$S_{1t} = \frac{1+\lambda A_{1,e}}{r_{1,e}-\mu_1+\lambda} D_{1t} = \frac{(r_{1,o}-\mu_1+\lambda)+\lambda}{(r_{1,e}-\mu_1+\lambda)(r_{1,o}-\mu_1+\lambda)-\lambda^2} D_{1t}. \tag{A.37}$$

Similar steps as above for stock 2 difference equation $A_{2,(J-M+1)}$ also leads to

$$A_{2,(J-M+1)} = \left[1+k_2+k_2^2+k_2^3+...+k_2^{(M-3)/2}\right] \frac{1}{r_{2,o}-\mu_2+\lambda} + k_2^{(M-1)/2} \frac{1}{\bar{r}_2-\mu_2},$$

where $k_2 = \lambda^2/\left((r_{2,e}-\mu_2+\lambda)(r_{2,o}-\mu_2+\lambda)\right) < 1$, and in the limit $M \to \infty$, we obtain

$$A_{2,e} = \frac{r_{2,e}-\mu_2+\lambda}{(r_{2,e}-\mu_2+\lambda)(r_{2,o}-\mu_2+\lambda)-\lambda^2},$$

yielding stock 2 price during $t \in \mathcal{T}_e$ as

$$S_{2t} = \frac{\lambda A_{2,e}}{r_{2,e} - \mu_2 + \lambda} \widehat{D}_{2t} = \frac{\lambda}{\left(r_{2,e} - \mu_2 + \lambda\right)\left(r_{2,o} - \mu_2 + \lambda\right) - \lambda^2} \widehat{D}_{2t}. \tag{A.38}$$

Similar steps as above again for stock 3 difference equation $A_{3,(J-M+1)}$ leads to

$$A_{3,(J-M+1)} = \left[1 + k_3 + k_3^2 + k_3^3 + ... + k_3^{(M-3)/2}\right] \frac{\lambda}{r_{3,e} - \mu_3 + \lambda} \frac{1}{r_{3,o} - \mu_3 + \lambda} + k_3^{(M-1)/2} \frac{1}{\bar{r}_3 - \mu_3},$$

where $k_3 = \lambda^2 / \left((r_{3,e} - \mu_3 + \lambda)(r_{3,o} - \mu_3 + \lambda)\right) < 1$, and in the limit $M \to \infty$, we obtain

$$A_{3,e} = \frac{\lambda}{\left(r_{3,e} - \mu_3 + \lambda\right)\left(r_{3,o} - \mu_3 + \lambda\right) - \lambda^2},$$

yielding stock 3 price during $t \in \mathcal{T}_e$ as

$$S_{3t} = \frac{1 + \lambda A_{3,e}}{r_{3,e} - \mu_3 + \lambda} D_{3t} = \frac{r_{3,o} - \mu_3 + \lambda}{\left(r_{3,e} - \mu_3 + \lambda\right)\left(r_{3,o} - \mu_3 + \lambda\right) - \lambda^2} D_{3t}. \tag{A.39}$$

Similar arguments also leads to the stock prices during $\mathcal{T}_o$, as

$$S_{1t} = \frac{\left(r_{1,e} - \mu_1 + \lambda\right) + \lambda}{\left(r_{1,o} - \mu_1 + \lambda\right)\left(r_{1,e} - \mu_1 + \lambda\right) - \lambda^2} D_{1t}, \tag{A.40}$$

$$S_{2t} = \frac{r_{2,e} - \mu_2 + \lambda}{\left(r_{2,o} - \mu_2 + \lambda\right)\left(r_{2,e} - \mu_2 + \lambda\right) - \lambda^2} D_{2t}, \tag{A.41}$$

$$S_{3t} = \frac{\lambda}{\left(r_{3,o} - \mu_3 + \lambda\right)\left(r_{3,e} - \mu_3 + \lambda\right) - \lambda^2} \widehat{D}_{3t}. \tag{A.42}$$

Using the stock prices in (A.37)–(A.39) for $t \in \mathcal{T}_e$ and (A.40)–(A.42) for $t \in \mathcal{T}_o$, we obtain the compact stock price expressions for any $t \in [0, \infty)$ in (15). Moreover, applying Itô's Lemma to these price expressions gives the dynamics of a dividend stock i by

$$\frac{dS_{it}}{S_{it}} + \frac{D_{it}}{S_{it}} dt = r_i dt + \sigma_i d\omega_{it} + \Delta_i \left(dN_t - \lambda dt\right), \tag{A.43}$$

and of a no-dividend stock i by

$$\frac{dS_{it}}{S_{it}} = r_i dt + \sum_k \frac{\rho \sigma_i}{1 + \rho} d\omega_{kt} + \frac{\left(1 + \rho\right)\left(\sigma_i^2 + \kappa_i V_{it}\right) - 2\rho^2 \sigma_i^2}{\sqrt{\left(1 + \rho\right)^2 \left(\sigma_i^2 + \nu_i^2\right) - 2\left(1 + \rho\right)\rho^2 \sigma_i^2}} d\widehat{\omega}_{it} + \Delta_i \left(dN_t - \lambda dt\right), \tag{A.44}$$

where the equilibrium mean returns of the individual stocks r_i, $i = 1, 2, 3$, are as in (A.27)–

(A.29) for $t \in \mathcal{T}_e$ and (A.33)–(A.35) for $t \in \mathcal{T}_o$, and also reported in a compact way for any $t \in [0, \infty)$ in (22), and the expected discrete changes Δ_i, $i = 1, 2, 3$, are as in (16). We recall that the summation without a superscript, $\sum_k$, indicates that the summation is taken only over the stocks that currently pay dividends (across $k = 1, 3$ in $\mathcal{T}_e$ and across $k = 1, 2$ in $\mathcal{T}_o$), and it is also understood that all price levels S_{it} above denote the left-limit prices, $\lim_{u \to t} S_{iu}$, which coincides with its right-limit level at all times except at the Poisson arrival times τ_n. The stock market dynamics in our economy with no-dividend stocks is given by

$$\frac{dS_t}{S_t} + \frac{\sum_k D_{kt}}{S_t} dt = \sum_k \left(\frac{dS_{kt}}{S_{kt}} + \frac{D_{kt}}{S_{kt}} dt \right) \frac{S_{kt}}{S_t} + \frac{dS_{it}}{S_{it}} \frac{S_{it}}{S_t}, \tag{A.45}$$

and substituting (A.43)–(A.44) into (A.45) and rearranging yields the stock market dynamics

$$\begin{aligned} \frac{dS_t}{S_t} + \frac{\sum_k D_{kt}}{S_t} dt &= \sum_{i=1}^{3} r_i \frac{S_{it}}{S_t} dt + \sum_k \left(\sigma_k \frac{S_{kt}}{S_t} + \frac{\rho \sigma_i}{1 + \rho} \frac{S_{it}}{S_t} \right) d\omega_{kt} \\ &+ \frac{(1+\rho)\left(\sigma_i^2 + \kappa_i V_{it}\right) - 2\rho^2 \sigma_i^2}{\sqrt{(1+\rho)^2 \left(\sigma_i^2 + \nu_i^2\right) - 2(1+\rho)\rho^2 \sigma_i^2}} \frac{S_{it}}{S_t} d\widehat{\omega}_{it} + \sum_{i=1}^{3} \Delta_i \frac{S_{it}}{S_t} \left(dN_t - \lambda dt\right), \end{aligned} \tag{A.46}$$

when the current no-dividend stock is stock i ($i = 2$ in $\mathcal{T}_e$, $i = 3$ in $\mathcal{T}_o$). □

Proof of Proposition 2. In the benchmark economy, using the dynamics of the stock market (A.19) and the aggregate consumption $dY_t/Y_t = \sum_{i=1}^{3} \bar{\alpha}_i \mu_i dt + \sum_{i=1}^{3} \bar{\alpha}_i \sigma_i d\omega_{it}$, we obtain the equilibrium covariance of the stock market return with the aggregate consumption growth rate as

$$\mathrm{Cov}_t \left[\frac{d\bar{S}_t}{\bar{S}_t}, \frac{dY_t}{Y_t} \right] \frac{1}{dt} = \sum_{i=1}^{3} \bar{\alpha}_i \sigma_i^2 \frac{\bar{S}_{it}}{\bar{S}_t} + \sum_{i=1}^{3} \sum_{j \neq i}^{3} \bar{\alpha}_i \rho \sigma_i \sigma_j \frac{\bar{S}_{jt}}{\bar{S}_t}.$$

Using the stock market dynamics (A.19), we also obtain its return variance as

$$\mathrm{Var}_t \left[\frac{d\bar{S}_t}{\bar{S}_t} \right] \frac{1}{dt} = \sum_{i=1}^{3} \bar{\sigma}_{S_i t}^2 \left(\frac{\bar{S}_{it}}{\bar{S}_t} \right)^2 + \sum_{i=1}^{3} \sum_{j \neq i}^{3} \rho \sigma_i \sigma_j \frac{\bar{S}_{it}}{\bar{S}_t} \frac{\bar{S}_{jt}}{\bar{S}_t}, \tag{A.47}$$

where the individual stocks volatilities $\bar{\sigma}_{S_i t}$ are as in Proposition 4. Substituting these along with the consumption growth rate variance $\mathrm{Var}_t [dY_t/Y_t] / dt = \sum_{i=1}^{3} \bar{\alpha}_i^2 \sigma_i^2 + \sum_{i=1}^{3} \sum_{j \neq i}^{3} \bar{\alpha}_i \bar{\alpha}_j \rho \sigma_i \sigma_j$, into the correlation definition $\bar{\rho}_{SYt} = \mathrm{Cov}_t \left[d\bar{S}_t/\bar{S}_t, dY_t/Y_t \right] / \sqrt{\mathrm{Var}_t \left[d\bar{S}_t/\bar{S}_t \right] \mathrm{Var}_t \left[dY_t/Y_t \right]}$ gives the equilibrium correlation of the stock market return with the aggregate consumption growth rate in the benchmark economy as (17).

In our economy with no-dividend stocks, using the dynamics of the stock market (A.46) and the aggregate consumption $dY_t/Y_t = \sum_k \alpha_k \mu_k dt + \sum_k \alpha_k \sigma_k d\omega_{kt}$, we obtain the equilibrium covariance of the stock market return with the aggregate consumption growth rate as

$$\mathrm{Cov}_t\left[\frac{dS_t}{S_t}, \frac{dY_t}{Y_t}\right]\frac{1}{dt} = \sum_k \alpha_k \sigma_k^2 \frac{S_{kt}}{S_t} + \sum_k \sum_{j\neq k}^{3} \alpha_k \rho \sigma_k \sigma_j \frac{S_{jt}}{S_t}.$$

Using the stock market dynamics (A.46), we also obtain its return variance as

$$\mathrm{Var}_t\left[\frac{dS_t}{S_t}\right]\frac{1}{dt} = \sum_{i=1}^{3} \sigma_{S_i t}^2 \Big(\frac{S_{it}}{S_t}\Big)^2 + \sum_{i=1}^{3}\sum_{j\neq i}^{3} \left(\rho\sigma_i\sigma_j + \lambda\Delta_i\Delta_j\right)\frac{S_{it}}{S_t}\frac{S_{jt}}{S_t}, \tag{A.48}$$

where the individual stocks volatilities $\sigma_{S_i t}$ are as in Proposition 4. Substituting these along with the consumption growth rate variance $\mathrm{Var}_t\left[dY_t/Y_t\right]/dt = \sum_k \alpha_k^2\sigma_k^2 + \sum_k \sum_{\ell\neq k} \alpha_k \alpha_\ell \rho \sigma_k \sigma_\ell$, into the correlation definition $\rho_{SYt} = \mathrm{Cov}_t\left[dS_t/S_t, dY_t/Y_t\right]/\sqrt{\mathrm{Var}_t\left[dS_t/S_t\right]\mathrm{Var}_t\left[dY_t/Y_t\right]}$ gives the equilibrium correlation of the stock market return with the aggregate consumption growth rate in our economy as (18).

The property that the correlation of the stock market return with the aggregate consumption growth rate in our economy with no-dividend stocks is lower than that of in the comparable benchmark economy follows from the facts that in the correlation (18) the covariance term in its numerator is not greater than while the variance terms in its denominator are greater than the corresponding quantities in the benchmark correlation (17) when the stocks have the same relative sizes $S_{it}/S_t = \bar{S}_{it}/\bar{S}_t$ (along with our comparable benchmark economy assumptions $\sum_{i=1}^3 \bar{\alpha}_i = \sum_k \alpha_k$, $\bar{\alpha}_1 = \alpha_1$, and the second and third stocks being otherwise identical, $\mu_2 = \mu_3$, $\sigma_2 = \sigma_3$, $\alpha_2 = \alpha_3$, $S_{2t}/S_t = S_{3t}/S_t$, as discussed in Section 2.3). □

Proof of Proposition 3. In the benchmark economy, the stock market dynamics is as in (A.19), whose drift term immediately gives its mean return as reported in (19). Similarly, the drift term of (A.18) gives the individual stock mean returns, which are derived in (A.15)–(A.17) and as reported in (20).

In our economy with no-dividend stocks, the stock market dynamics is as in (A.46), whose drift term immediately gives its mean return as reported in (21). Similarly, the drift terms of (A.43) and (A.44) give the dividend and no-dividend stock mean returns, respectively, which are derived in (A.27)–(A.29) for $t \in \mathcal{T}_e$ and (A.33)–(A.35) for $t \in \mathcal{T}_o$, and reported compactly for any $t \in [0, \infty)$ in (22).

The property that the risk premium, $r_i - r$, of a no-dividend stock is lower than that of an otherwise identical dividend stock in our economy follows immediately by comparing the quantities in (22). $\square$

Proof of Proposition 4. In benchmark economy, the equilibrium stock market volatility (23) is given by the square root of (A.47), and the individual stock return volatilities (24) are by the diffusion terms in (A.18).

In our economy with no-dividend stocks, the equilibrium stock market volatility (25) is given by the square root of (A.48), and the individual stock return volatilities (26) are the square roots of the individual stock return variances, which are obtained by using (A.43)–(A.44).

The property that the volatility of a no-dividend stock is higher than that of an otherwise identical dividend stock in our economy follows immediately by comparing the quantities in (26). We also observe that both the diffusion (using (A.10)–(A.11)) and the discrete change component Δ_i given by (16) of no-dividend stocks are larger than those of dividend stocks. $\square$

Proof of Proposition 5. Using the dynamics in (A.18) and (A.19), we obtain the covariance between the individual stock i and the stock market returns in the benchmark economy as

$$\mathrm{Cov}_t\left[\frac{d\bar{S}_{it}}{\bar{S}_{it}}, \frac{d\bar{S}_t}{\bar{S}_t}\right]\frac{1}{dt} = \sigma_i^2\frac{\bar{S}_{it}}{\bar{S}_t} + \sum_{j\neq i}^{3}\rho\sigma_i\sigma_j\frac{\bar{S}_{jt}}{\bar{S}_t}.$$

Substituting these into the market beta definition $\bar{\beta}_{S_it} = \mathrm{Cov}_t\left[d\bar{S}_{it}/\bar{S}_{it}, d\bar{S}_t/\bar{S}_t\right]/\mathrm{Var}_t\left[d\bar{S}_t/\bar{S}_t\right]$, along with (A.47) gives the equilibrium market betas of individual stocks as reported in (27).

Similarly, using the dynamics (A.43)–(A.44) and (A.46), we obtain the covariance between the individual stock i and the stock market returns in our economy as

$$\mathrm{Cov}_t\left[\frac{dS_{it}}{S_{it}}, \frac{dS_t}{S_t}\right]\frac{1}{dt} = \sigma_i^2\frac{S_{it}}{S_t} + \sum_{j\neq i}^{3}\rho\sigma_i\sigma_j\frac{S_{jt}}{S_t} + \lambda\sum_{j=1}^{3}\Delta_i\Delta_j\frac{S_{jt}}{S_t},$$

for a dividend stock, and as

$$\mathrm{Cov}_t\left[\frac{dS_{it}}{S_{it}}, \frac{dS_t}{S_t}\right]\frac{1}{dt} = \left(\frac{2\rho^2\sigma_i^2}{1+\rho} + \frac{\left[(1+\rho)\left(\sigma_i^2+\kappa_i V_{it}\right) - 2\rho^2\sigma_i^2\right]^2}{(1+\rho)^2\left(\sigma_i^2+\nu_i^2\right) - 2\left(1+\rho\right)\rho^2\sigma_i^2}\right)\frac{S_{it}}{S_t} + \sum_{j\neq i}^{3}\rho\sigma_i\sigma_j\frac{S_{jt}}{S_t} + \lambda\sum_{j=1}^{3}\Delta_i\Delta_j\frac{S_{jt}}{S_t},$$

for a no-dividend stock. Substituting these into the market beta definition $\beta_{S_it} = \mathrm{Cov}_t\left[dS_{it}/S_{it}, dS_t/S_t\right]/\mathrm{Var}_t\left[dS_t/S_t\right]$, along with (A.48) gives the equilibrium market betas of individual stocks as reported in (28).

The property that the market beta of a no-dividend stock is higher than that of an otherwise identical dividend stock in our economy follows from comparing the quantities in (28). Since in their numerators, the middle terms $\sum_{j\neq i}^{3}\rho\sigma_i\sigma_j(S_{jt}/S_t)$ are identical and in the last terms the discrete change component Δ_i for the no-dividend stock is always larger than that of the dividend stock, this property holds if

$$\sigma_i^2 < \frac{2\rho^2\sigma_i^2}{1+\rho} + \frac{\left[(1+\rho)\left(\sigma_i^2+\kappa_i V_{it}\right) - 2\rho^2\sigma_i^2\right]^2}{(1+\rho)^2\left(\sigma_i^2+\nu_i^2\right) - 2\left(1+\rho\right)\rho^2\sigma_i^2},$$

and this inequality always holds due to the relation (A.10)–(A.11). □

Proof of Proposition 6. In benchmark economy, by no-arbitrage, the short-term asset price is given by $\bar{S}_{t,T} = \mathrm{E}_t\left[\int_t^T \xi_u D_u du\right]/\xi_t$, where the aggregate dividend is $D_u = \sum_{i=1}^3 D_{iu}$ for all $u \geq t$, and the state price density is as in (A.13). Using the individual stock results in the proof of Proposition 1 we obtain

$$\bar{S}_{t,T} = \sum_{i=1}^{3} \frac{1}{\xi_t}\mathrm{E}_t\left[\int_t^T \xi_u D_{iu} du\right] = \sum_{i=1}^3 \int_t^T e^{-(\bar{r}_i-\mu_i)(u-t)} du D_{it} = \sum_{i=1}^{3} \frac{1-e^{-(\bar{r}_i-\mu_i)(T-t)}}{\bar{r}_i-\mu_i} D_{it},$$

where $\bar{r}_i$ is as in (20), and the benchmark economy short-term asset price as $\bar{S}_{t,T} = \sum_{i=1}^3 \bar{h}_{it,T}\bar{S}_{it}$, where the deterministic process $\bar{h}_{it,T} = 1 - e^{-(\bar{r}_i-\mu_i)(T-t)}$ and the stock price $\bar{S}_{it}$ is as in (13) for $i = 1, 2, 3$. The risk premium of the short-term asset in the benchmark economy $\bar{r}_{S_{t,T}} - \bar{r}$ is given by

$$\bar{r}_{S_{t,T}} - \bar{r} = -\frac{d\xi_t}{\xi_t}\frac{d\bar{S}_{t,T}}{\bar{S}_{t,T}}\frac{1}{dt}. \tag{A.49}$$

Applying Itô's Lemma to the short-term asset price $\bar{S}_{t,T} = \sum_{i=1}^3 \bar{h}_{it,T}\bar{S}_{it}$ leads to the dynamics

$$\frac{d\bar{S}_{t,T}}{\bar{S}_{t,T}} + \frac{D_t}{\bar{S}_{t,T}}dt = \sum_{i=1}^{3} \frac{\bar{h}_{it,T}\bar{S}_{it}}{\sum_{j=1}^3 \bar{h}_{jt,T}\bar{S}_{jt}}\left(\bar{r}_i dt + \sigma_i d\omega_{it}\right),$$

which after substituting into (A.49) along with the state price density dynamics (A.13) gives

$$\bar{r}_{S_{t,T}} - \bar{r} = \sum_{i=1}^{3} \frac{\bar{h}_{it,T}\bar{S}_{it}}{\sum_{j=1}^3 \bar{h}_{jt,T}\bar{S}_{jt}}\left(\bar{r}_i - \bar{r}\right),$$

where the individual stock mean returns $\bar{r}_i$ are as in (20). Canceling out the interest rates in the above expression gives the equilibrium short-term asset mean return as in (29).

In our economy with no-dividend stocks, by no-arbitrage, the short-term asset price is given by $S_{t,T} = \mathrm{E}_t \left[\int_t^T \xi_u D_u du \right] / \xi_t$, where the aggregate dividend is $D_u = \sum_k D_{ku}$, where again the summation without a superscript, $\sum_k$, indicates that the summation is taken over the stocks that pay dividends (across $k = 1, 3$ in $\mathcal{T}_e$ and across $k = 1, 2$ in $\mathcal{T}_o$). We restrict ourselves to the case of there being at most one arrival of the dividend alternating times in the life of the short-term asset. That is, if currently $t \in [\tau_n, \tau_{n+1})$, the short-term asset maturity is either $T \in [t, \tau_{n+1})$ or $T \in [\tau_{n+1}, \tau_{n+2})$. As we discuss in footnote 18, this specification is sufficient for us to make our point while also being economically plausible. Without loss of generality, we assume that currently we are in period $\mathcal{T}_o$, i.e., in $[\tau_n, \tau_{n+1})$ where n is an odd number, so that currently stocks 1 and 2 are dividend stocks and stock 3 is the no-dividend stock. To determine the short-term asset price, we first consider a fixed τ_{n+1} for the next alternating period arrival time, and for simplicity drop the subscripts and denote it simply by τ. We also denote the short-term asset price when τ is fixed by $S^{\tau}_{t,T}$, which is decomposed into two cases, whether the dividend-paying stocks (hence the stochastic discount factor shocks) alternate within the life of the short-term asset or not, i.e., $T \leq \tau$ and $T > \tau$, by considering

$$S^{\tau}_{t,T} = S^{\tau}_{t,T} \mathbf{1}_{\{T \leq \tau\}} + S^{\tau}_{t,T} \mathbf{1}_{\{T > \tau\}}. \tag{A.50}$$

Then by taking the expectation of $S^{\tau}_{t,T}$ with respect to the uncertainty about the next arrival time τ, we determine the short-term asset price $S_{t,T}$.

In the first case $T \leq \tau$, the dividend-paying stocks do not alternate within the life of the short-term asset so that the aggregate dividend is $D_u = D_{1u} + D_{2u}$ for all $t \leq u < T$, and the state price density is as in (A.20). In this case, we have $S^{\tau}_{t,T} = \sum_{i=1}^{2} \mathrm{E}_t \left[\int_t^T \xi_u D_{iu} du \right] / \xi_t$, and using the individual stock results in the proof of Proposition 1 we obtain

$$S^{\tau}_{t,T} = \sum_{i=1}^{2} \frac{1}{\xi_t} \mathrm{E}_t \left[\int_t^T \xi_u D_{iu} du \right] = \sum_{i=1}^{2} \int_t^T e^{-\left(r_{i,o} - \mu_i\right)(u-t)} du D_{it} = \sum_{i=1}^{2} \frac{1 - e^{-\left(r_{i,o} - \mu_i\right)(T-t)}}{r_{i,o} - \mu_i} D_{it}, \tag{A.51}$$

where $r_{i,o}$, $i = 1, 2$, is as in (A.33)–(A.34).

In the second case $T > \tau$, the dividend-paying stocks do alternate within the life of the short-term asset so that the aggregate dividend is $D_u = D_{1u} + D_{2u}$ for all $t \leq u < \tau$ and $D_u = D_{1u} + D_{3u}$ for all $\tau \leq u < T$ and the state price density is as in (A.20). In this case, we have $S^{\tau}_{t,T} = \mathrm{E}_t \left[\int_t^{\tau} \xi_u \sum_{i=1}^{2} D_{iu} du + \xi_{\tau} S^{\tau}_{\tau,T} \right] / \xi_t$, and using similar steps as in (A.51) gives the first term as

$$\sum_{i=1}^{2}\frac{1}{\xi_t}\mathrm{E}_t\left[\int_t^{\tau}\xi_u D_{iu}du\right]=\sum_{i=1}^{2}\frac{1-e^{-\left(r_{i,o}-\mu_i\right)(\tau-t)}}{r_{i,o}-\mu_i}D_{it},$$

and the second term component $S^{\tau}_{\tau,T}$ as

$$S^{\tau}_{\tau,T}=\frac{1-e^{-\left(r_{1,e}-\mu_1\right)(T-\tau)}}{r_{1,e}-\mu_1}D_{1\tau}+\frac{1-e^{-\left(r_{3,e}-\mu_3\right)(T-\tau)}}{r_{3,e}-\mu_3}D_{3\tau},$$

which in turn yields

$$\begin{aligned}\frac{1}{\xi_t}\mathrm{E}_t\left[\xi_\tau S^{\tau}_{\tau,T}\right] &= \frac{1-e^{-\left(r_{1,e}-\mu_1\right)(T-\tau)}}{r_{1,e}-\mu_1}\frac{1}{\xi_t}\mathrm{E}_t\left[\xi_\tau D_{1\tau}\right]+\frac{1-e^{-\left(r_{3,e}-\mu_3\right)(T-\tau)}}{r_{3,e}-\mu_3}\frac{1}{\xi_t}\mathrm{E}_t\left[\xi_\tau D_{3\tau}\right]\\ &= \frac{1-e^{-\left(r_{1,e}-\mu_1\right)(T-\tau)}}{r_{1,e}-\mu_1}e^{-\left(r_{1,o}-\mu_1\right)(\tau-t)}D_{1t}+\frac{1-e^{-\left(r_{3,e}-\mu_3\right)(T-\tau)}}{r_{3,e}-\mu_3}e^{-\left(r_{3,o}-\mu_3\right)(\tau-t)}\widehat{D}_{3t},\end{aligned}$$

and putting these together we obtain the short-term asset price for a fixed τ in the second case

$$\begin{aligned}S^{\tau}_{t,T} &= \sum_{i=1}^{2}\frac{1-e^{-\left(r_{i,o}-\mu_i\right)(\tau-t)}}{r_{i,o}-\mu_i}D_{it}\\ &+\frac{1-e^{-\left(r_{1,e}-\mu_1\right)(T-\tau)}}{r_{1,e}-\mu_1}e^{-\left(r_{1,o}-\mu_1\right)(\tau-t)}D_{1t}+\frac{1-e^{-\left(r_{3,e}-\mu_3\right)(T-\tau)}}{r_{3,e}-\mu_3}e^{-\left(r_{3,o}-\mu_3\right)(\tau-t)}\widehat{D}_{3t}. \qquad \text{(A.52)}\end{aligned}$$

Finally, substituting the first and second case prices (A.51)–(A.52) into (A.50), and taking the expectation with respect to τ, using the independent exponential distribution for its density given by (A.25), we determine the short-term asset price $S_{t,T}$ as

$$\begin{aligned}S_{t,T} &= \sum_{i=1}^{2}\frac{1-e^{-\left(r_{i,o}-\mu_i\right)(T-t)}}{r_{i,o}-\mu_i}D_{it}\lambda\int_0^{\infty}\mathbf{1}_{\{T-t\le u\}}e^{-\lambda u}du+\lambda\int_0^{\infty}\sum_{i=1}^{2}\frac{1-e^{-\left(r_{i,o}-\mu_i\right)u}}{r_{i,o}-\mu_i}D_{it}\mathbf{1}_{\{T-t>u\}}e^{-\lambda u}du\\ &+\lambda\int_0^{\infty}\frac{1-e^{-\left(r_{1,e}-\mu_1\right)(T-t-u)}}{r_{1,e}-\mu_1}e^{-\left(r_{1,o}-\mu_1\right)u}D_{1t}\mathbf{1}_{\{T-t>u\}}e^{-\lambda u}du\\ &+\lambda\int_0^{\infty}\frac{1-e^{-\left(r_{3,e}-\mu_3\right)(T-t-u)}}{r_{3,e}-\mu_3}e^{-\left(r_{3,o}-\mu_3\right)u}\widehat{D}_{3t}\mathbf{1}_{\{T-t>u\}}e^{-\lambda u}du,\end{aligned}$$

which after removing the indicator functions becomes

$$S_{t,T} = \sum_{i=1}^{2} \frac{1-e^{-\left(r_{i,o}-\mu_i\right)(T-t)}}{r_{i,o}-\mu_i} D_{it} \lambda \int_{T-t}^{\infty} e^{-\lambda u} du + \lambda \int_0^{T-t} \sum_{i=1}^{2} \frac{1-e^{-\left(r_{i,o}-\mu_i\right)u}}{r_{i,o}-\mu_i} D_{it} e^{-\lambda u} du$$
$$+\lambda \int_0^{T-t} \left[\frac{1-e^{-\left(r_{1,e}-\mu_1\right)(T-t-u)}}{r_{1,e}-\mu_1} e^{-\left(r_{1,o}-\mu_1\right)u} D_{1t} + \frac{1-e^{-\left(r_{3,e}-\mu_3\right)(T-t-u)}}{r_{3,e}-\mu_3} e^{-\left(r_{3,o}-\mu_3\right)u} \widehat{D}_{3t} \right] e^{-\lambda u} du.$$

Evaluating the simple exponential integrals and rearranging yields $S_{t,T} = \sum_{i=1}^{3} h_{it,T} S_{it}$, where the stock prices S_{it} are as in (15) and the deterministic processes $h_{it,T}$ are as in (31). Following similar steps also leads to the same short-term asset price when we are currently in period $\mathcal{T}_e$. The risk premium of the short-term asset $r_{S_{t,T}} - r$ in our economy is given by

$$r_{S_{t,T}} - r = -\frac{d\xi_t}{\xi_t} \frac{dS_{t,T}}{S_{t,T}} \frac{1}{dt}. \tag{A.53}$$

Applying Itô's Lemma to the short-term asset price leads to the continuous dynamics

$$\frac{dS_{t,T}}{S_{t,T}} = \ldots dt + \sum_{i=1}^{3} \frac{h_{it,T} S_{it}}{\sum_{j=1}^{3} h_{jt,T} S_{jt}} \frac{dS_{it}}{S_{it}},$$

which after substituting into (A.53) along with the state price density dynamics (A.20) gives

$$r_{S_{t,T}} - r = \sum_{i=1}^{3} \frac{h_{it,T} S_{it}}{\sum_{j=1}^{3} h_{jt,T} S_{jt}} \left(r_i - r\right),$$

where the individual stock mean returns r_i are as in (22). Canceling out the interest rates in the above expression gives the equilibrium short-term asset mean return as in (30).

The property that the mean return of the short-term asset is higher than that of the stock market in the economy with no-dividend stocks holds if and only if

$$\sum_{i=1}^{3} \frac{h_{it,T} S_{it}}{\sum_{j=1}^{3} h_{jt,T} S_{jt}} r_i > \sum_{i=1}^{3} \frac{S_{it}}{\sum_{j=1}^{3} S_{jt}} r_i.$$

Since the no-dividend stock mean return is lower than that of dividend stocks, this property holds if the no-dividend stock i weight in the short-term asset mean return is less than or equal to its weight in the stock market mean return, that is $h_{it,T} S_{it} / (\sum_{j=1}^{3} h_{jt,T} S_{jt}) \leq S_{it} / (\sum_{j=1}^{3} S_{jt})$. Since this is equivalent to $0 \leq \sum_k \left(h_{kt,T} - h_{it,T}\right) S_{kt}$, we see that when the deterministic term $h_{kt,T}$ for a dividend stock is greater than that of a no-dividend stock this condition holds. $\square$

Appendix B: Differences in Firm Characteristics

Thus far, we have quantified the effects in our model under the assumption that dividend and no-dividend stocks have the same characteristics. In reality, several firm characteristics are shown to affect the decision to pay dividends or not. In particular, Fama and French (2001) and Denis and Osobov (2008) show that more profitable firms, firms with less investments and growth opportunities, and larger firms are more likely to pay dividends. Since these firm characteristics are bound to affect an individual stock mean return and volatility, it would be of interest to decompose our cross-sectional results to see to what extent they are due to our mechanisms and to what extent they are due to differences in firm characteristics. In this Appendix, we discuss the effects of different firm characteristics, namely the differences in mean growth rates, volatilities, and sizes of their fundamentals, on our results.[25] We find that these differences in firm characteristics can capture a significant portion of the difference in individual stock risk premia but can only explain a small portion of the differences in stock volatilities and market betas between the dividend and no-dividend stocks. We also find that our key results involving the aggregate stock market become more pronounced under these different characteristics.

To quantify our results under different firm characteristics, we modify our main parameter values determined in Section 6 and summarized in Table 1 as follows. We keep the sensitivity parameters, which essentially capture the average size of fundamentals in our model, α_i, $i = 1, 2, 3$, as before since they already take into account of the fact that a typical dividend-paying stock fundamental is (5.3 times) larger than that of a typical no-dividend stock. We choose the mean growth μ_i and volatility σ_i parameters so that the no-dividend stock fundamentals have higher mean growth rates and volatilities than those of the dividend stock, while ensuring that the aggregate consumption growth rate mean and volatility are still as in the data, 1.74% and 1.64%, respectively. Towards that we consider the case when $\mu_i = 1.5\mu_1$ while $\sigma_i = 1.5\sigma_1$ or $\sigma_i = 2\sigma_1$, for no-dividend stocks $i = 2, 3$.[26] This leads to the mean growth rates of $\mu_1 = 0.106$ and $\mu_2 = \mu_3 = 0.159$, and the growth rate volatilities of $\sigma_1 = 0.127$ and $\sigma_2 = \sigma_3 = 0.191$ in the case of $\sigma_i = 1.5\sigma_1$, and $\sigma_1 = 0.122$ and $\sigma_2 = \sigma_3 = 0.244$ in the case of $\sigma_i = 2\sigma_1$. All other parameter values are as before and presented in Table 1. We conduct our analysis for the no-dividend relative

[25] Due to the pure-exchange economy setting of our model, we cannot meaningfully consider the effects of other characteristics such as firms' different investment rates and profitability.

[26] We are unable to consider the case of $\mu_i = 2\mu_1$, since such a high mean growth rate would violate the usual parameter restriction of $\bar{r}_i - \mu_i > 0$ in the stock price expression (13), leading to negative, not-well-defined prices in equilibrium even in the benchmark economy. That being said, our analysis shows that the differences in mean growth rates do not play much of a role in our results.

Table 2: **Effects of different firm characteristics on our cross-sectional results.** This table reports what fraction of the cross-sectional results in our economy are due to our mechanisms as opposed to the differences in firm characteristics that are also present in the benchmark economy, for varying no-dividend relative stock sizes S_{it}/S_t, and no-dividend stock relative fundamental volatility σ_i/σ_1. The mean growth rate of the no-dividend stock fundamental is fixed at $\mu_i = 1.5\mu_1$. The reported effects for the benchmark economy are computed using $(\bar{r}_i - \bar{r}_1) / (r_i - r_1)$ (risk premium), $(\bar{\sigma}_{S_it} - \bar{\sigma}_{S_1t}) / (\sigma_{S_it} - \sigma_{S_1t})$ (return volatility), and $(\bar{\beta}_{S_it} - \bar{\beta}_{S_1t})/(\beta_{S_it} - \beta_{S_1t})$ (market beta). The reported effects for our model are one minus the effects for the benchmark economy. The parameter values are $\mu_1 = 0.106$, $\mu_2 = \mu_3 = 0.159$, and $\sigma_1 = 0.127$, $\sigma_2 = \sigma_3 = 0.191$ in the case of $\sigma_i/\sigma_1 = 1.5$, and $\sigma_1 = 0.122$, $\sigma_2 = \sigma_3 = 0.244$ in the case of $\sigma_i/\sigma_1 = 2$ for $i = 2, 3$. The other parameter values are as in Table 1 of Section 6.

		Risk premium		Return volatility		Market beta	
S_{it}/S_t	σ_i/σ_1	Benchmark	Our model	Benchmark	Our model	Benchmark	Our model
21.3%	1.5	73.8%	26.2%	12.2%	87.8%	-5.8%	105.8%
	2	53.5%	46.5%	21.7%	78.3%	12.7%	87.3%
14.2%	1.5	73.8%	26.2%	12.2%	87.8%	-24.0%	124.0%
	2	53.5%	46.5%	21.7%	78.3%	-7.2%	107.2%

stock size in our model being equal to its average value in the data, 21.3% or 14.2%.

Table 2 reports what fraction of the differences in stock risk premium, volatility, and market betas in our economy are due to our mechanisms as opposed to the differences in the fundamental mean growth rates and volatilities that are also present in the benchmark economy. Table 2 reveals that the differences in firm characteristics can play a role at varying degrees in explaining our cross-sectional results. We see that a significant portion of our result that the risk premium of a no-dividend stock is lower than that of a dividend stock, $r_i - r_1 < 0$, can be attributed to the differences in firm characteristics, particularly the dividend-paying firms being larger than the no-dividend ones. That being said, we also see that as the fundamental volatility difference increases, our model mechanism contribution to the risk premium difference increases. For example, when $\sigma_i = 2\sigma_1$, a no-dividend stock risk premium is 1.82% lower than that of a dividend stock in our model, whereas the corresponding difference is only 0.97% in the benchmark economy. This implies that $0.97/1.82 = 53.5\%$ of our risk premium result can be attributed to the different characteristics, namely dividend-paying stocks being large with less volatile fundamentals, and the remaining 46.5% due to our mechanisms, namely the no-dividend stock not contributing directly to the aggregate consumption.[27]

However, looking at our return volatility and market beta results we see that the differences in firm characteristics can only explain small fractions of them. For instance, even when $\sigma_i = 2\sigma_1$, only 21.7% of our result that the return volatility of a no-dividend stock is higher than that of a dividend stock, $\sigma_{S_i t} - \sigma_{S_1 t} > 0$ can be attributed to the different characteristics, namely no-dividend stocks having more volatile fundamentals, and the remaining 78.3% due to our mechanisms, namely the no-dividend stock price being driven by its estimated pseudo-dividend, which requires estimation. Interestingly, we also see that our result that the market beta of a no-dividend stock is higher than that of a dividend stock, $\beta_{S_i t} - \beta_{S_1 t} > 0$, cannot even be immediately generated in the benchmark economy, which typically generates the wrong sign for this difference, leading to negative values in Table 2. This occurs since in the absence of our mechanism, large stocks (dividend stocks) contribute to and comove with the aggregate stock market return more, leading to their market betas to be higher than smaller stocks.

Finally, we also look at the effects of different firm characteristics on our aggregate stock market results. We find that our main results continue to hold and in fact quantitatively become more pronounced under the different firm characteristics that we consider. In particular, when $\sigma_i = 1.5\sigma_1$, we find that the correlation between the stock market return and the aggregate consumption growth rate becomes 0.51 and 0.72 (previously, 0.53 and 0.75) when the no-dividend relative stock size is equal to 21.3% and 14.2%, respectively. We also find the presence of no-dividend stocks in the stock market still generates a non-monotonic and particularly negative relation between the stock market risk premium and its volatility for relatively high volatility periods, and downward sloping term structure of equity risk premia. For the latter result, the economic magnitude of the effects are significant. For instance, we find that the risk premium of the short-term asset is 41% and 24% higher than the risk premium of the stock market (previously, 18% and 12% higher) when the no-dividend relative stock size is equal to 21.3% and 14.2%, respectively.

[27] We find it helpful to present the decomposition in relative terms in percentages rather than in absolute differences in Table 2, since as also discussed in footnote 17 our model does not generate realistic equity risk premium so their difference may not be indicative of the contribution of different firm characteristics in reality.

References

Abel, Andrew B., 1988, Stock prices under time-varying dividend risk: An exact solution in an infinite-horizon general equilibrium model, *Journal of Monetary Economics* 22, 375–393.

Albuquerque, Rui, Martin Eichenbaum, Victor Xi Luo, and Sergio Rebelo, 2016, Valuation risk and asset pricing, *Journal of Finance* 71, 2861–2904.

Allen, Franklin, and Roni Michaely, 2002, Payout policy, *Handbook of the Economics of Finance* 1, 337–429.

Backus, David K., and Allan W. Gregory, 1993, Theoretical relations between risk premiums and conditional variances, *Journal of Business & Economic Statistics* 11, 177–185.

Bali, Turan G., and Lin Peng, 2006, Is there a risk-return trade-off? Evidence from high-frequency data, *Journal of Applied Econometrics* 21, 1169–1198.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Barberis, Nicholas, Robin Greenwood, Lawrence Jin, and Andrei Shleifer, 2015, X-CAPM: An extrapolative capital asset pricing model, *Journal of Financial Economics* 115, 1–24.

Barberis, Nicholas, Ming Huang, and Tano Santos, 2001, Prospect theory and asset prices, *Quarterly Journal of Economics* 116, 1–53.

Barro, Robert J., 2006, Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 823–866.

Belo, Frederico, Pierre Collin-Dufresne, and Robert S. Goldstein, 2015, Dividend dynamics and the term structure of dividend strips, *Journal of Finance* 70, 1115–1160.

Boudoukh, Jacob, Roni Michaely, Matthew Richardson, and Michael R. Roberts, 2007, On the importance of measuring payout yield: Implications for empirical asset pricing, *Journal of Finance* 62, 877–915.

Brandt, Michael W., and Qiang Kang, 2004, On the relationship between the conditional mean and volatility of stock returns: A latent VAR approach, *Journal of Financial Economics* 72, 217–257.

Brennan, Michael J., 1998, The role of learning in dynamic portfolio decisions, *Review of Finance* 1, 295–306.

Brennan, Michael J., and Yihong Xia, 2001, Stock price volatility and equity premium, *Journal of Monetary Economics* 47, 249–283.

Campbell, John Y., 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373–399.

Campbell, John Y., 2017, *Financial Decisions and Markets: A Course in Asset Pricing* (Princeton University Press).

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Choi, Seung Mo, Shane Johnson, Hwagyun Kim, and Changwoo Nam, 2013, Dividend policy, investment, and stock returns, Working paper, Texas A&M University.

Christie, William G., 1990, Dividend yield and expected returns, *Journal of Financial Economics* 28, 95–125.

Cochrane, John H., and Lars Peter Hansen, 1992, Asset pricing explorations for macroeconomics, *NBER Macroeconomics Annual* 7, 115–165.

Cochrane, John H., Francis A. Longstaff, and Pedro Santa-Clara, 2007, Two Trees, *Review of Financial Studies* 21, 347–385.

Collin-Dufresne, Pierre, Michael Johannes, and Lars A. Lochstoer, 2016, Parameter learning in general equilibrium: The asset pricing implications, *American Economic Review* 106, 664–698.

Croce, Mariano M., Martin Lettau, and Sydney C. Ludvigson, 2015, Investor information, long-run risk, and the term structure of equity, *Review of Financial Studies* 28, 706–742.

Cvitanić, Jakša, Ali Lazrak, Lionel Martellini, and Fernando Zapatero, 2006, Dynamic portfolio choice with parameter uncertainty and the economic value of analysts' recommendations, *Review of Financial Studies* 19, 1113–1156.

Denis, David J., and Igor Osobov, 2008, Why do firms pay dividends? International evidence on the determinants of dividend policy, *Journal of Financial Economics* 89, 62–82.

Dumas, Bernard, Alexander Kurshev, and Raman Uppal, 2009, Equilibrium portfolio strategies in the presence of sentiment risk and excess volatility, *Journal of Finance* 64, 579–629.

Fama, Eugene F., and Kenneth R. French, 2001, Disappearing dividends: Changing firm characteristics or lower propensity to pay?, *Journal of Financial Economics* 60, 3–43.

Farre-Mensa, Joan, Roni Michaely, and Martin Schmalz, 2014, Payout policy, *Annual Review of Financial Economics* 6, 75–134.

French, Kenneth R., G. William Schwert, and Robert F. Stambaugh, 1987, Expected stock returns and volatility, *Journal of Financial Economics* 19, 3–29.

Fuller, Kathleen P., and Michael A. Goldstein, 2011, Do dividends matter more in declining markets?, *Journal of Corporate Finance* 17, 457–473.

Gabaix, Xavier, 2012, Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance, *Quarterly Journal of Economics* 127, 645–700.

Ghysels, Eric, Pedro Santa-Clara, and Rossen Valkanov, 2005, There is a risk-return trade-off after all, *Journal of Financial Economics* 76, 509–548.

Glosten, Lawrence R., Ravi Jagannathan, and David E. Runkle, 1993, On the relation between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance* 48, 1779–1801.

Gormsen, Niels Joachim, 2018, Time variation of the equity term structure, Working paper, University of Chicago.

Gormsen, Niels Joachim, and Eben Lazarus, 2020, Duration-driven returns, Working paper, University of Chicago.

Grullon, Gustavo, and Roni Michaely, 2002, Dividends, share repurchases, and the substitution hypothesis, *Journal of Finance* 57, 1649–1684.

Guo, Hui, and Robert F. Whitelaw, 2006, Uncovering the risk-return relation in the stock market, *Journal of Finance* 61, 1433–1463.

Hartzmark, Samuel M., and David H. Solomon, 2013, The dividend month premium, *Journal of Financial Economics* 109, 640–660.

Harvey, Campbell R., 2001, The specification of conditional expectations, *Journal of Empirical Finance* 8, 573–637.

Hasler, Michael, and Roberto Marfè, 2016, Disaster recovery and the term structure of dividend strips, *Journal of Financial Economics* 122, 116–134.

Heyerdahl-Larsen, Christian, and Philipp Illeditsch, 2017, Demand disagreement, Working paper, London Business School.

Lettau, Martin, and Jessica A. Wachter, 2007, Why is long-horizon equity less risky? A duration-based explanation of the value premium, *Journal of Finance* 62, 55–62.

Liptser, Robert S., and Albert N. Shiryaev, 2001, *Statistics of Random Processes: II. Applications*, volume 2 (Springer).

Lucas, Robert E., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.

Ludvigson, Sydney C., and Serena Ng, 2007, The empirical risk-return relation: A factor analysis approach, *Journal of Financial Economics* 83, 171–222.

Mehra, R., and E.C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Miller, Merton H., and Franco Modigliani, 1961, Dividend policy, growth, and the valuation of shares, *Journal of Business* 34, 411–433.

Naranjo, Andy, M. Nimalendran, and Mike Ryngaert, 1998, Stock returns, dividend yields, and taxes, *Journal of Finance* 53, 2029–2057.

Pástor, Luboš, and Pietro Veronesi, 2003, Stock valuation and learning about profitability, *Journal of Finance* 58, 1749–1789.

Rietz, Thomas A., 1988, The equity risk premium: A solution, *Journal of Monetary Economics* 22, 117–131.

Rossi, Alberto, and Allan Timmermann, 2010, What is the shape of the risk-return relation?, AFA 2010 Atlanta Meetings Paper.

Santos, Tano, and Pietro Veronesi, 2006, Labor income and predictable stock returns, *Review of Financial Studies* 19, 1–44.

Scheinkman, José A., and Wei Xiong, 2003, Overconfidence and speculative bubbles, *Journal of Political Economy* 111, 1183–1220.

Scruggs, John T., 1998, Resolving the puzzling intertemporal relation between the market risk premium and conditional market variance: A two-factor approach, *Journal of Finance* 53, 575–603.

van Binsbergen, Jules, Michael Brandt, and Ralph Koijen, 2012, On the timing and pricing of dividends, *American Economic Review* 102, 1596–1618.

van Binsbergen, Jules H., and Ralph S. J. Koijen, 2017, The term structure of returns: Facts and theory, *Journal of Financial Economics* 124, 1–21.

Veronesi, Pietro, 2000, How does information quality affect stock returns?, *Journal of Finance* 55, 807–837.

Whitelaw, Robert F., 2000, Stock market risk and return: An equilibrium approach, *Review of Financial Studies* 13, 521–547.

Xiong, Wei, and Hongjun Yan, 2010, Heterogeneous expectations and bond markets, *Review of Financial Studies* 23, 1433–1466.