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Using cross-impact analysis for probabilistic risk assessment

Ahti Salo[1] | Edoardo Tosoni[1] | Juho Roponen[1] | Derek W. Bunn[2]

[1]Department of Mathematics and Systems Analysis, Systems Analysis Laboratory, Aalto University School of Science, Aalto, Finland

[2]London Business School, London, UK

Correspondence
Ahti Salo, Department of Mathematics and Systems Analysis, Systems Analysis Laboratory, Aalto University School of Science, 00076 Aalto, Finland.
Email: ahti.salo@aalto.fi

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Abstract

Cross-impact analysis is widely employed to inform management and policy decisions based on the formulation of scenarios, defined as combinations of outcomes of relevant uncertainty factors. In this paper, we argue that the use of nonprobabilistic variants of cross-impact analysis is problematic in the context of risk assessment where the usual aim is to produce conservative risk estimates which may exceed but are not smaller than the actual risk level. Then, building on the characterization of probabilistic dependencies, we develop an approach to probabilistic cross-impact analysis which (i) admits several kinds of probabilistic statements about the outcomes of relevant uncertainty factors and their dependencies; (ii) maps such statements into constraints on the joint probability distribution over all possible scenarios; (iii) provides support for preserving the consistency of elicited statements; and (iv) uses mathematical optimization to compute lower and upper bounds on the overall risk level. This approach—which is illustrated with an example from the context of nuclear waste repositories—is useful in that it retains the informativeness of cross-impact statements while ensuring that these statements are interpreted within the coherent framework of probability theory.

KEYWORDS

cross-impact analysis, probabilistic risk assessment, scenario analysis

1 | INTRODUCTION

In its many variants, scenario analysis is widely employed to support strategic decisions whose impacts depend on key uncertainties (Bunn & Salo, 1993; Lord et al., 2016). In such situations, the systematic identification of relevant uncertainty factors; the characterization of outcomes which depict possible realizations of these factors; and the formulation of scenarios as different combinations of such outcomes provides support for organizational learning, fosters managerial insights and provides an improved basis for strategic decisions through a systematic analysis of uncertainties (Schoemaker, 1993; A. Wright, 2005).

Yet, a practical challenge in scenario analysis is that the number of possible scenarios grows very rapidly with the number of uncertainty factors and their outcomes. This is because for every combination of outcomes of these uncertainty factors, there exists a distinct scenario that could be generated (Carlsen et al., 2016; Tietje, 2005). Thus, if there are 10 factors with five possible outcomes for each, for example, the total number of possible scenarios which can be defined by such outcome combinations is $5^{10} \approx 9.7$ million. Understandably, the number of scenarios which are usually elaborated is typically much smaller, given that resources for developing scenarios by engaging experts or by consulting other sources of information are limited. Moreover, the elaboration of scenarios and the assimilation of their implications is constrained by the amount of time and attention that decision and policy makers can devote to the scenario process. Thus, in many public policy and corporate scenario analyses which are developed primarily by consulting experts and other respondents, the number of scenarios is in the range between four and eight (see, e.g., Lord et al., 2016; Wiebe et al., 2018).

In this setting, methods of cross-impact analysis provide a structured approach to choosing those outcome combinations for which scenarios are built, based on statements concerning the logical relationships between the factors and their outcomes. Such statements are typically elicited by asking the respondent to characterize which pairs of outcomes are consistent in the sense that these outcomes are likely to occur jointly. Typically, these cross-impact statements are expressed verbally and then mapped to corresponding numerical parameters. For instance, Scholz and Tietje (2001) present a 7-point numerical scale from −3 to 3 such that, for example, −3 indicates that the two outcomes are *strongly inconsistent* in the sense that they are very unlikely to occur together; 0 represents *independence*; and 3 indicates that the outcomes are *strongly consistent* so that the occurrence of an outcome induces the other. Finally, the elicited statements are synthesized algorithmically to provide suggestions for which combinations of outcomes scenarios should be built (see, e.g., Salo & Bunn, 1995; Seeve & Vilkkumaa, 2021; Tietje, 2005).

As one of the important application areas of scenario analysis, risk assessment covers both risk analysis (which helps identify, characterize, and analyze future events and developments that can negatively impact individuals, assets or the environment) and risk evaluation (which supports judgments about the extent to which these risks can be tolerated) (Rausand, 2013). In risk assessment, the demands on the rigor, quality and transparency of methodological support are particularly stringent. In part, this is because risk management decisions can have far-reaching consequences, especially in the context of safety-critical systems whose failures can cause human casualties, irreversible environmental damages, and major financial losses. Thus, for example, in the assessment of the safety of nuclear waste repositories, it is necessary to account for the full range of relevant uncertainty factors (called features, events, and processes [FEPs]; see Tosoni et al., 2018) and their implications for regulatory decisions. Methodological rigor is also needed in assessing risks due to the impacts of climate change, healthcare interventions, and environmental regulations (see, e.g., Hirabayashi et al., 2013). In all these areas, the possibility of rare but extremely serious events is of much concern. These events have usually very low probabilities which can be notoriously difficult to estimate because of scarce empirical evidence and paucity of relevant data based (see, e.g., Goodwin & Wright, 2010).

Within the field of risk assessment, probabilistic risk analysis (PRA) constitutes a theoretically coherent framework which is compatible with well-established statistical techniques for data analysis; it also provides support for synthesizing expert judgments (Bedford & Cooke, 2001). In the analysis of safety-critical systems, it is often required that the PRA estimates—which reflect both the *probability* and the *severity* of negative impacts—should be *conservative* so that the actual risk level is not underestimated (see, e.g., Aven & Zio, 2011). This requirement is justified by the recognition that in safety-critical systems, errors due to "false negatives"—the failure to take appropriate risk management decisions in response to risks which were deemed tolerable but were actually too high—can be far greater than errors arising from "false positives"—the cost of unnecessarily implementing risk management actions in response to assessed risks which, in reality, were not big enough to warrant such actions. Even more generally, such conservatism is widely called for in situations where there are significant uncertainties. For example, the "precautionary principle" (Science for Environment Policy, 2017) has been invoked to guide the public response to risks in contexts such as climate change mitigation (Stern, 2007). Also the "minimax regret" decision rule, which has been proposed as an approach for ensuring the resource adequacy of electricity systems (National Grid, 2020), is motivated by the desire to limit the amount of harm that could be experienced ex post. If the impacts can be characterized in terms of real-valued consequences (for instance through monetization), information about the tail risk represented by the least preferred consequences can be provided through risk measures such as Value-at-Risk and conditional Value-at-Risk, defined at appropriate confidence levels (see Liesiö & Salo, 2012).

The above remarks motivate our central observation on the use of cross-impact analysis in risk assessment and the ensuing decision making. That is, to the extent that cross-impact analysis focuses on a small subset of all possible scenarios, there is a real possibility that the resulting estimates about the overall risk level will *not* be conservative, because the risks associated with all the other "non-constructed" scenarios may be underestimated or even neglected. This may not be of major concern in contexts where the stakes are not very high or where "softer" process objectives such as organizational learning are dominant. However, if the analysis serves as an essential input to safety-critical risk management decisions, it is possible that the sufficient conservatism required by regulatory decision making is not being upheld. Indeed, while all model-based analyses are simplifications and there is always some "model risk," in safety-critical applications, due-diligence requires that this should be minimized.

Against this backdrop, we examine cross-impact analysis from the PRA perspective, with the aim of clarifying how cross-impact analysis can be employed to support risk management decisions. This perspective is motivated by the recognition that (i) risk assessment is, by definition, focused on the identification, characterization, and analysis of relevant uncertainties and their impacts, and that (ii) PRA is often endorsed and in many cases even required as the only appropriate coherent framework for addressing these uncertainties (see, e.g., Helton & Sallaberry, 2009; USEPA, 2014; USNRC, 2016). As a motivating prelude to our methodological development, we point out limitations in nonprobabilistic cross-impact approaches by examining the cross-impact balances (CIB) method (Weimer-Jehle, 2006, 2008). We have chosen this method due its visibility in the literature and the attention that it has recently received in the context of climate change mitigation (Kemp-Benedict et al., 2010; Panula-Ontto et al., 2018; Schweizer, 2020; Weimer-Jehle et al., 2020).

Furthermore, by building on formulations for capturing probabilistic dependencies, we develop a probabilistic method of cross-impact analysis which combines methodological coherence with the expressiveness of cross-impact statements for characterizing dependencies between pairs of outcomes for uncertainty factors. These statements are translated into constraints on the joint probability distribution over the set of *all* possible scenarios (which, by design, are assumed to be mutually exclusive and collectively exhaustive; see, e.g., the early work of Duperrin & Godet, 1975 and citations to it). In addition to cross-impact statements, our method accommodates many other kinds of probabilistic statements, such as lower or upper bounds on the marginal and conditional probabilities of the joint probability distribution. Throughout the elicitation process, the method can offer support for preserving the consistency of the elicited statements so that the corresponding constraints are satisfied by at least some scenario probabilities.

In the context of risk assessment, our method can also be employed together with measures of risk importance to identify the scenarios which matter most from the risk management perspective (see, e.g., Salo et al., 2021; Tosoni, 2021). A precondition for this is that estimates about the expected consequences in every possible scenario can be assessed. While the generation of such estimates can be supported by computational models in some contexts (cf. the case study in Section 4), this assessment task may be challenging if the number of possible scenarios is large and the required estimates have to be elicited from experts (see, e.g., Dias et al., 2018). This task may be less onerous if the consequences depend primarily on few uncertainty factors, because it may suffice to assess consequences by conditioning these on, say, pairs or triplets of outcomes for two or three uncertainty factors. It may also be possible to estimate scenario-specific consequences by using mathematical models in which the consequences are expressed as functions of the outcomes that define the scenarios. One possibility is to apply the rank nodes method (Fenton et al., 2016; Laitila & Virtanen, 2016) which has been successfully employed to support the development of conditional probability tables for Bayesian networks. This method appears particularly relevant thanks to its flexibility which is achieved by associating weighting parameters with each uncertainty factor.

More generally, even if scenario-specific consequences are not formally assessed, the proposed approach to the elicitation of cross-impact statements and their conversion into constraints on the underlying joint probability distribution provides a structured and systematic way for characterizing this distribution. In this regard, it serves similar purposes as approaches for modeling dependencies between continuous random variables with real-valued outcomes (see, e.g., Van Dorp, 2005).

While our emphasis is on probabilistic approaches, we note that nonprobabilistic approaches such as CIB do not automatically lead to excessively permissive conclusions about system safety, provided that deliberate attempts are made to select those scenarios which pose significant risks while also accounting for the impacts of those scenarios which are not elaborated. This notwithstanding, a major shortcoming of these nonprobabilistic approaches is that they are not founded on a coherent theoretical framework within which the adequacy, appropriateness, and sufficiency of these kinds of adjustments could be formally assessed. This makes it hard if not impossible to ascertain if such adjustments warrant valid conclusions about system safety. Thus, there is a striking contrast with PRA which, due to its probabilistic foundations, builds on a coherent framework within which such an assessment can be made.

The rest of this paper is structured as follows. Section 2 discusses methods of cross-impact analysis and remarks on nonprobabilistic approches in light of the CIB method. Section 3 shows how cross-impact statements can be converted into constraints on the joint probability distribution over all possible scenarios. It also formulates maximization problems which can be solved to infer conservative risk estimates, based on all the elicited information. Section 4 presents a numerical example. Section 5 concludes.

2 | METHODS OF SCENARIO AND CROSS-IMPACT ANALYSIS

Of the variety of methods in scenario analysis, most are associated with one of the three main schools which are commonly referred to as the intuitive logics school; the probabilistic/modified trends school; and La Prospective (Bradfield et al., 2005; Bunn & Salo, 1993). The first, intuitive logics, is least quantitative in that it adopts a top-down inductive approach in seeking to formulate descriptive scenarios which represent possible futures and thus help generate actionable insights (Bowman, 2016; G. Wright et al., 2013). The second school consists of methods such as Trend-Impact Analysis and Cross-Impact Analysis which employ techniques for quantifying expert judgments, for example by characterizing possible deviations from historical averages or prior expectations (Bradfield et al., 2005). The third school, La Prospective, can be viewed as a “blend of tools and systems analysis” (Godet, 2000) or even as a mixture of methods from intuitive logics and probabilistic analysis (Bradfield et al., 2005).

Regardless of the school, it is useful to consider the determinants of the scenario quality (Bunn & Salo, 1993). In particular, scenarios should be *comprehensive*, meaning that they represent the full range of possible futures that are relevant to decision making or the broader objectives of the scenario process; *consistent*, meaning that the outcome combinations are plausible in light of available knowledge about the reality which they seek to depict; and *coherent*, meaning that the development of scenarios is founded on sound theories for reasoning about uncertainties. In practice, the pursuit of these qualities involves inevitable trade-offs. For example, increasing the number of scenarios to ensure comprehensiveness would, at some stage, result in the generation of scenarios which are less plausible and therefore less consistent, too.

In our methodological development, we focus on probabilistic approaches in which uncertainty factors are modeled as random variables $X^i, i = 1, \ldots, n$ such that the ith uncertainty factor has n_i possible realizations (called outcomes) $x^i_k, k = 1, \ldots, n_i$ represented by the set $S^i = \left\{x^i_1, \ldots, x^i_{n_i}\right\}$. A *scenario* $s = (x^1, \ldots, x^n)$ is defined as a combination of outcomes $x^i \in S^i$ for all uncertainty factors $i = 1, \ldots, n$. Thus, mathematically, the set of all scenarios is the Cartesian product $S = \bigtimes_{i=1}^{n} S^i$ which has $|S| = \prod_{i=1}^{n} n_i$ elements. For example, if there are 5 factors with three outcomes for each, there are $3^5 = 243$ distinct scenarios that can be generated.

Much of the early methodological development of cross-impact analysis took place in the 1970s and 1980s. One of the major aims was to support inferences about which scenarios could be deemed more plausible than others, based on cross-impact statements about the consistency of outcomes for pairs of uncertainty factors. The proposed methods were largely developed within the framework of probability theory by interpreting the elicited cross-impact judgments in terms of statements about conditional probabilities and by translating these statements into corresponding constraints on the joint probability distribution over the set of all possible scenarios (for an early review, see tab. 1 in Salo & Bunn, 1995).

In the elicitation of cross-impact statements, one notable challenge is that when several cross-impact statements are elicited without explicit guidance, the resulting set of elicited statements may be inconsistent so that the corresponding constraints will not be satisfied by any probability distribution over the set of scenarios (G. Wright et al., 1988). In this case, it would be necessary to revise earlier statements, either by removing some of them or, alternatively, by relaxing the bounds of those statements which have been encoded as intervals. Both cases are problematic in that it can be challenging to identify which one(s) of the many earlier statements are more “wrong” than others.

In recent years, the literature on cross-impact analysis has continued to diversify. There are now approaches in which the assessed cross-impact evaluations are no longer linked to probabilities. One of these approaches is the CIB method (Weimer-Jehle, 2006, 2008) which is a structured technique for identifying consistent scenarios based on cross-impact assessments about causal dependencies between uncertainty factors. In CIB, specifically, the respondent is invited to use a scale ranging from −3 to 3 to assess what impact the outcome $x_k^i \in S^i$ of the ith factor will have on the outcome $x_l^j \in S^j$ of the jth factor. These statements are assessed for all pairs of outcomes $(x_k^i, x_l^j), x_k^i \in S^i, x_l^j \in S^j$ and pairs of uncertainty factors $i \neq j$, resulting in responses $C_{kl}^{ij}, i \neq j, k = 1, \ldots, n_i, l = 1, \ldots, n_j$. These responses form the elements of the cross-impact matrix C.

In the selection of scenarios, CIB focuses exclusively on consistent scenarios which are defined as combinations of outcomes $\left(x_{k_1^*}^1, \ldots, x_{k_n^*}^n\right)$ such that (see eq. 1 in Weimer-Jehle, 2008)

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} C_{k_i^* k_j^*}^{ij} \geq \sum_{\substack{i=1 \\ i \neq j}}^{n} C_{k_i^* l}^{ij}, j = 1, \ldots, n, l = 1, \ldots, n_j. \tag{1}$$

In other words, the scenario $\left(x_{k_1^*}^1, \ldots, x_{k_n^*}^n\right), x_{k_i^*}^i \in S^i$ is consistent in the sense that the sum of corresponding cross-impact terms in each column $x_{k_j^*}^j, j = 1, \ldots, n_j$ of the aggregate matrix is not less than what would be obtained by adding the terms in the column for some other outcome $x_l^j \neq x_{k_j^*}^j$ instead.

Even if this requirement seems plausible, it is highly restrictive in that the number of scenarios which satisfies the condition (1) can be very small, which undermines the objective of generating a *comprehensive* set of scenarios. For instance, in the example in tab. 3 of Weimer-Jehle (2006) with five factors (four with three possible outcomes and one with four), only three out of the $3^4 \times 4 = 324$ scenarios are consistent, because none of the 321 other scenarios fulfill the consistency requirement (1).

Alarmingly, it is also possible to construct cross-impact matrices such that the consistency requirement in (1) is not satisfied by *any* scenario. For example, consider the cross-impact matrix in Figure 1 which is based on two uncertainty factors such that the possible outcomes of the first factor are $\{a, b, c\}$ and those of the second factors are $\{x, y, z\}$. Then, condition (1) means that for example, the scenario (k_1^*, k_2^*) would be consistent if and only if $C_{k_2^* k_1^*}^{21} \geq C_{k_2^* l}^{21}, l \neq k_1^*$ for $j = 1$ in (1), and $C_{k_1^* k_2^*}^{12} \geq C_{k_1^* l}^{12}, l \neq k_2^*$ for $j = 2$ in (1).

Yet the following nine inequalities show that for *any* scenario there exists some other column such that at least one of these conditions is violated:

$$C_{ax}^{12} = 0 < 1 = C_{ay}^{12}, C_{ya}^{21} = -3 < 3 = C_{yc}^{21}, C_{az}^{12} = -1 < 1 = C_{ay}^{12}$$
$$C_{bx}^{12} = -3 < 3 = C_{bz}^{12}, C_{by}^{12} = 0 < 3 = C_{bz}^{12}, C_{zb}^{21} = 0 < 1 = C_{za}^{21}$$
$$C_{xc}^{21} = -1 < 1 = C_{xb}^{21}, C_{cy}^{12} = 0 < 1 = C_{cx}^{12}, C_{cz}^{12} = -2 < 1 = C_{cx}^{12}.$$

Even if the numerical values in the cross-impact matrix in Figure 1 are hypothetical, this example shows that there can be data sets of cross-impacts statements such that no scenarios satisfy the condition (1). Admittedly, the absence of consistent scenarios can be attributed to the lack of consistency in the statements. However, to the extent that the elicitation process offers no structured guidance for the specification of statements, there is a risk that the set of scenarios which are screened for further elaboration becomes too small, thus undermining the attainment of the comprehensiveness as a quality attribute. In other words, the strong emphasis on the consistency criterion based on a dichotomous “yes-no” assessment may, depending on the elicited cross-impact statements, be so stringent that the number of consistent scenarios is too small to ensure the comprehensiveness of the generated scenarios, all the more so because the extent to which the scenarios are comprehensive is not formally defined. From this perspective, we find that among nonprobabilistic

		Factor 1			Factor 2		
		a	*b*	*c*	*x*	*y*	*z*
Factor 1	*a*				0	1	-1
	b				-3	0	3
	c				1	0	-2
Factor 2	*x*	0	1	-1			
	y	-3	0	3			
	z	1	0	-2			

FIGURE 1 An example of inconsistencies

cross-impact methods there are significant advantages to adopting approaches which (i) employ quantitative measures for concepts such as consistency and comprehensiveness and (ii) provide suggestions for the selection of scenarios by solving corresponding optimization problems. One such approach for generating scenarios which are both plausible and diverse is presented in (Seeve & Vilkkumaa, 2021).

In Figure 1, the cross-impact terms are not monotonic in the sense that transitions to a higher index (e.g., moving first from a to b and then proceeding to c) would be associated with systematic increases or decreases in the assessed cross-impacts. In effect, the monotonicity of such changes makes sense only on condition that there exists a corresponding metric or ordinal scale such that there is a sense of direction ranging from outcomes on the lower levels to those on the higher levels (as opposed to a nominal scale which merely indicates selections from the set of outcomes without such directionality, for example, choices among political parties; see Carlsen et al., 2016).

Uncertainty factors which are assessed using metric scales (e.g., temperature) can be discretized to formulate corresponding ordinal scales. Then, assuming that there are such ordinal scales for all uncertainty factors, the outcomes for each factor can be ordered with a transitive, antisymmetric, and total binary relation $\prec_i, i = 1, \ldots, n$. In this case, the monotonicity property can be stated as

$$x^i_k \prec_i x^i_{k'} \prec_i x^i_{k''} \Rightarrow \left(\left[C^{ij}_{kl} \le C^{ij}_{k'l} \le C^{ij}_{k''l}\right] \vee \left[C^{ij}_{kl} \ge C^{ij}_{k'l} \ge C^{ij}_{k''l}\right]\right) \wedge \left(\left[C^{ji}_{lk} \le C^{ji}_{lk'} \le C^{ji}_{lk''}\right] \vee \left[C^{ji}_{lk} \ge C^{ji}_{lk'} \ge C^{ji}_{lk''}\right]\right). \quad (2)$$

In risk assessment, one should be wary of assuming that the lowest and highest risks would be attained at the endpoints of any such ordinal scale. For example, if departures from the normal conditions in a production facility are measured on a natural ordinal scale (or even an interval scale, as in the case of, e.g., temperature), deviations into either direction can contribute to increased risks.

Still, even with monotonic cross-impacts, it is possible that there are no CIB-consistent scenarios. One such example is in Figure 2 where there are three uncertainty factors whose outcomes belong to the sets $\{a, b, c\}$, $\{i, j, k\}$, and $\{x, y, z\}$, respectively. The shaded rows indicate the selection of the scenario (a, i, x) which is also indicated by the upward pointing arrows and the digits "1" in the second row at the bottom of the figure. The numbers in the first row under the downward arrows show the sums for those columns which have the highest column sum for the selection of outcomes for each uncertainty factor. For factor 3, this sum is the highest $2 = 0 + 2$ (obtained from matrix entries $C^{13}_{az} = 0$ and $C^{23}_{iz} = 2$) while the corresponding sum associated with the scenario (a, i, x) is $-4 = -3 + (-1)$, based on $C^{13}_{ax} = -3$ and $C^{23}_{ix} = -1$. Thus, scenario (a, i, x) is not consistent, because condition (1) would be violated by replacing the outcome x by z. It straightforward to check that none of the 27 scenarios are consistent.

In view of these examples, the procedures of the CIB method seem excessively restrictive in that there are examples of numerical inputs such that the consistency requirements hold either for very few or, at the limit, no scenarios at all. As a result, it appears that in the case of nonprobabilistic cross-impact analysis, approaches which are based on the formulation of optimization problems towards the identification of a set of consistent and diverse scenarios should be preferred. For example Seeve and Vilkkumaa (2021) present a structured approach which was applied to generate scenarios for the National Emergency Supply Agency in Finland. In what follows, however, we explore how the probabilistic interpretation of cross-impact statements can be employed to establish a coherent methodological foundation for using cross-impact analysis in the context of risk assessment, in particular.

2.1 | Probabilistic dependencies

There is an extensive literature on the characterization of probabilistic dependencies between events. Such dependencies will arise if there are causal relationships between the events; but they may very well exist even in the absence of such relationships. Specifically, research on the topic of probabilistic causation has sought to characterize what causation means in probabilistic terms (see, e.g., Williamson, 2009 for an overview as well as contributions by Pearl, 2013; Suppes, 1970). In general, there is wide agreement that a

		Factor 1			Factor 2			Factor 3		
		a	b	c	i	j	k	x	y	z
Factor 1	a				1	-3	-3	-3	0	0
	b				2	0	-2	3	0	-1
	c				3	2	1	3	-2	-2
Factor 2	i	3	3	-2				-1	0	2
	j	0	0	3				-1	0	1
	k	-3	0	3				-3	-2	-1
Factor 3	x	3	2	-2	1	0	0			
	y	3	0	-2	3	0	0			
	z	-3	-3	3	3	3	0			
		↓			↓					↓
		6	5	-4	2	-3	-3	-4	0	2
		1	0	0	1	0	0	1	0	0
		↑			↑			↑		

FIGURE 2 An example of inconsistencies with monotonic cross-impacts

statement such as "the event A may have been caused by the event B" can be interpreted as meaning that the occurrence of A is more likely if the event B has occurred, that is,

$$\mathbb{P}(A|B) > \mathbb{P}(A). \tag{3}$$

Here, the qualification "may have been" is warranted, because the inequality (3) lacks any contextual knowledge. For instance, it does not consider *when* the events occur, even if the attribution of causality would be possible only on condition that the event B occurs before A. Moreover, even if this were to be the case, it could be that the event A can be more meaningfully attributed to intermediate events which occur after B but before A. There are even parallels to empirical econometrics where the notion of "Granger causality" (Granger, 1969) is defined so that B is said to cause A if the regression $A(t) = a + bB(t-1)$ (where t refers to points in time) has a significant regression coefficient b but $B(t) = a + bA(t-1)$ does not.

In view of (3), we interpret the ratio $\mathbb{P}(A|B)/\mathbb{P}(A)$ as an indication of the degree of probabilistic dependency between the occurrence of events B and A, noting that this ratio need not be reflect causal relationships between the events. In keeping with this interpretation, we suggest that the cross-impacts are linked to ratios between conditional and marginal probabilities, as defined by

$$C_{kl}^{ij} := \frac{p_{k|l}^{i|j}}{p_k^i} = \frac{p_{kl}^{ij}}{p_k^i p_l^j}, \tag{4}$$

where $p_k^i = \mathbb{P}(X^i = x_k^i)$, $p_l^j = \mathbb{P}(X^j = x_l^j)$, $p_{k|l}^{i|j} = \mathbb{P}(X^i = x_k^i | X^j = x_l^j)$, and $p_{kl}^{ij} = \mathbb{P}(X^i = x_k^i \wedge X^j = x_l^j)$. In particular, C_{kl}^{ij} thus provides an answer to the question "How many times more likely does the outcome x_k^i of the ith uncertainty factor become if it is known that the outcome of the jth uncertainty factor is x_l^j?" This question invites intuitively meaningful and theoretically well-defined answers on a ratio scale. Such answers can be encoded with the help of verbal descriptors that can be calibrated through experiments (see Pöyhönen et al., 1997). Note that if the outcomes x_j^i are x_l^j are independent, then $p_{k|l}^{i|j} = p_k^i$ and $C_{kl}^{ij} = 1$.

Based on the interpretation (3), the cross-impact terms are symmetric, because (4) implies $C_{kl}^{ij} = C_{lk}^{ji}$. This property is desirable in that symmetry is aligned with the nondirectional relational structure of (in) consistencies. That is, stating that the events A and B are "inconsistent" does not involve causal judgments about why the joint occurrence is very unlikely or, in particular, whether or not it is the occurrence of one which is preventing the other from occurring. Furthermore, this property also makes it easier to elicit the cross-impacts terms, because evaluations are needed only for *unordered pairs* of outcomes (i.e., $\sum_{i,j=1,i\neq j}^{n}(n_i \times n_j)/2$) instead for all ordered pairs (i.e., $\sum_{i,j=1,i\neq j}^{n} n_i \times n_j$).

The following result shows that the relation (3) implies $\mathbb{P}(A|B) > \mathbb{P}(A|\neg B)$ and vice versa.

Theorem 1. *Assume that events A, B are such that* $0 < \mathbb{P}(B) < 1$. *Then*

$$\mathbb{P}(A|B) > \mathbb{P}(A) \iff \mathbb{P}(A|B) > \mathbb{P}(A|\neg B). \tag{5}$$

Proof. "$\Rightarrow$": If (3) holds, then

$$\begin{aligned}\mathbb{P}(A) &= \mathbb{P}(A|B)\mathbb{P}(B) + \mathbb{P}(A|\neg B)\mathbb{P}(\neg B) > \mathbb{P}(A)\mathbb{P}(B) \\ &\quad + \mathbb{P}(A|\neg B)\mathbb{P}(\neg B) \iff \mathbb{P}(A)(1-\mathbb{P}(B)) > \mathbb{P}(A|\neg B)\mathbb{P}(\neg B),\end{aligned}$$

where the first inequality follows from (3) and the last inequality can be divided by $\mathbb{P}(\neg B) = 1 - \mathbb{P}(B) > 0$ to obtain $P(A) > P(A|\neg B)$, which together with (3) implies $\mathbb{P}(A|B) > \mathbb{P}(A|\neg B)$. "$\Leftarrow$": Because $\mathbb{P}(A|\neg B) < \mathbb{P}(A|B)$, this follows from

$$\begin{aligned}\mathbb{P}(A) &= \mathbb{P}(A|B)\mathbb{P}(B) + \mathbb{P}(A|\neg B)\mathbb{P}(\neg B) < \mathbb{P}(A|B)[\mathbb{P}(B) + \mathbb{P}(\neg B)] \\ &= \mathbb{P}(A|B).\end{aligned}$$

□

However, if the ratio $\mathbb{P}(A|B)/\mathbb{P}(A|\neg B)$ were to be taken as a point of departure for evaluating cross-impacts, the resulting ratios would be asymmetric and consequently the number of parameters in the model would become much higher. Moreover, it could be cognitively more challenging for the respondent to specify statements involving comparisons in which the event A is conditioned on the *non-occurrence* of B.

The interpretation of cross-impacts in (4) implies that

$$\frac{C_{kl}^{ij}}{C_{kl'}^{ij}} = \frac{p_{kl}^{ij}}{p_k^i p_l^j} \times \frac{p_k^i p_{l'}^j}{p_{kl'}^{ij}} = \frac{p_{k|l}^{i|j}}{p_{k|l'}^{i|j}}. \tag{6}$$

Thus, the ratio between two different cross-impact terms provides information about "How many times more probable is the occurrence of x_k^i when x_l^j occurs, as opposed to when $x_{l'}^j$ occurs?" (cf. the discussion of Bayes factors; Kass & Raftery, 1995).

More generally, an important benefit of this probabilistic interpretation of cross-impact assessments is that the accuracy of such statements can be tested empirically, for instance by carrying out experiments with controlled subjects or by revisiting earlier cross-impact studies and examining how frequently the observed outcomes match those implied by the stated cross-impact ratios. These kinds of empirical studies help assess to what extent the statements may need to be calibrated to ensure a better fit with empirically observed marginal and conditional probabilities (see, e.g., Hora, 2007; O'Hagan et al., 2006).

2.2 | Relationship between cross-impact statements and scenario probabilities

The elicitation of statements about the ratio (4) for several pairs of uncertainty factors and their outcomes constitutes an approach to the elicitation of a dependency structure Werner et al. (2017). In this process, it is possible to employ discrete scales which translate numerical or verbal statements about how strongly the outcomes being assessed enforce each other into corresponding ranges of probability ratios (see Theil, 2002). To ensure the validity of assessments, these translations need to be properly justified and clearly communicated so that they can be understood by respondents.

The cross-impact ratio between the outcomes indexed by k, l of factors i and j is related to the joint probability distribution $p(\cdot): S \mapsto [0, 1]$ through

$$C_{kl}^{ij} = \frac{p_{kl}^{ij}}{p_k^i p_l^j} = \frac{\sum_{s \in S_{kl}^{ij}} p(s)}{\left(\sum_{s \in S_k^i} p(s)\right)\left(\sum_{s \in S_l^j} p(s)\right)}, \tag{7}$$

where $p(s) := \mathbb{P}(s), s \in S$ denote scenarios probabilities, the set S_{kl}^{ij} contains those scenarios in which the outcomes of factors i and j are x_k^i and x_l^j, respectively, and the set S_k^i consists of those scenarios in which the outcome of the ith uncertainty factor is x_k^i (and similarly for the scenario set S_l^j).

We assume that all outcomes of uncertainty factors occur with a probability that is strictly positive, that is, $p_k^i > 0, i = 1, \ldots, n$, $k = 1, \ldots, n_i$. This assumption is plausible, because otherwise the "impossible" outcome x_k^i such that $p_k^i = 0$ could be removed from the analysis. Technically, this assumption can be introduced through the constraint $p_k^i \geq \varepsilon$ where $\varepsilon > 0$ is a very small number.

Because the expression (7) is nonlinear in $p(s), s \in S$ with quadratic terms in the denominator, it is not possible to convert upper and lower bounds on this ratio into linear constraints on scenario probabilities. This is in contrast to bounds on marginal or conditional probabilities which both can be modeled through linear constraints on scenario probabilities (see Salo & Bunn, 1995).

The expression (7) can be written in matrix notation as follows. Let the set of all n scenarios be $S = \{s_1, s_2, \ldots, s_{|S|}\}$ and let $|S|$ denote the cardinality of S, that is the total number of scenarios. Furthermore, let the vector $p \in \mathbb{R}^{|S|}$ contain all the scenario probabilities so that probability of the ith scenario is $p_i = \mathbb{P}(s_i)$.

To link scenarios to the specific outcomes of uncertainty factors, we employ $m \times 1$ dimensional binary vectors $\sigma_k^i \in \{0, 1\}^{|S|}$ so that the mth element of this vector is 1 if the realization of the ith uncertainty factor in scenario s_m is x_k^i and zero otherwise. Then, the probability of the outcome x_k^i can be derived from the joint probability distribution over scenarios through

$$p_k^i = \mathbb{P}\left(X^i = x_k^i\right) = \sum_{j=1}^{|S|} p_j \left(\sigma_k^i\right)_j = \left(\sigma_k^i\right)^\intercal p, \tag{8}$$

where $^\intercal$ denotes the transpose of a matrix. The conditional probability $p_{k|l}^{i|j}$ in (4), in turn, can be written as

$$p_{k|l}^{i|j} = \frac{p_{kl}^{ij}}{p_l^j} = \frac{\left(\sigma_k^i \circ \sigma_l^j\right)^\intercal p}{\left(\sigma_l^j\right)^\intercal p}, \tag{9}$$

where the Hadamard product $\circ$ is defined as $(\sigma_k^i \circ \sigma_l^j)_m = (\sigma_k^i)_m (\sigma_l^j)_m$, $m = 1, \ldots, |S|$. Thus, the entry for the mth scenario in the vector $\sigma_k^i \circ \sigma_l^j$ is equal to 1 if and only if the outcomes of the ith and jth are equal to x_k^i and x_l^j. Placing lower and upper bounds $p_k^i \in \left[\underline{p}_k^i, \bar{p}_k^i\right]$ on the expression (8) leads to linear constraints on scenario probabilities. The linear fractional expression in (9) is the ratio between sums of those scenario probabilities which are picked by the vectors $\sigma_k^i \circ \sigma_l^j$ and σ_l^j, respectively. Thus, bounding this ratio through bounds $p_{k|l}^{i|j} = \left[\underline{p}_{k|l}^{i|j}, \bar{p}_{k|l}^{i|j}\right]$ can be transformed into linear constraints by multiplying these bounds by the denominator $(\sigma_l^j)^\intercal p$. For instance, the constraint $p_{k|l}^{i|j} \leq \bar{p}_{k|l}^{i|j}$ is equivalent to $(\sigma_k^i \circ \sigma_l^j)^\intercal p \leq \bar{p}_{k|l}^{i|j}[(\sigma_l^j)^\intercal p]$.

The cross-impact ratio (7) can be written as

$$C_{kl}^{ij} = \frac{p_{kl}^{ij}}{p_k^i p_l^j} = \frac{\left(\sigma_k^i \circ \sigma_l^j\right)^\intercal p}{\left[\left(\sigma_k^i\right)^\intercal p\right]\left[\left(\sigma_l^j\right)^\intercal p\right]}, \tag{10}$$

which is the same as the equality $C_{kl}^{ij}[(\sigma_k^i)^\intercal p][(\sigma_l^j)^\intercal p] = (\sigma_k^i \circ \sigma_l^j)^\intercal p$, which, in turn, is equivalent to the quadratic constraint

$$C_{kl}^{ij} \frac{1}{2} p^\intercal \mathbf{Q}_{kl}^{ij} p - \left(\sigma_k^i \circ \sigma_l^j\right)^\intercal p = 0, \tag{11}$$

where $\mathbf{Q}_{kl}^{ij} = (\sigma_k^i (\sigma_l^j)^\intercal + \sigma_l^j (\sigma_k^i)^\intercal)$ is a symmetric matrix.

Thus, the modeling of cross-impact statements about the (4) leads to *quadratic* constraints on the scenario probabilities. As in the case of marginals and conditionals, these constraints can be introduced by eliciting lower and upper bounds on the cross-impact terms (i.e., $C_{kl}^{ij} \in \left[\underline{C}_{kl}^{ij}, \bar{C}_{kl}^{ij}\right]$, $\underline{C}_{kl}^{ij} \leq \bar{C}_{kl}^{ij}$) which impose inequality constraints on the underlying scenario probabilities. Yet, because the matrix $\mathbf{Q}_{kl}^{ij}$ can be indefinite, this set of scenario probabilities may be nonconvex, making it computationally more challenging to explore the implications of cross-impact statements for probabilistic inference. There are, however, specialized algorithms for optimization problems with quadratic terms in the objective function or in the constraints (see, e.g., Audet et al., 2000). These algorithms have been incorporated in commercial optimization solvers which are capable of handling problems such as the example in Section 4.

2.3 | Consistency implications of probabilistic statements

Because cross-impact statements refer to the same set of underlying scenario probabilities based on the ratio (4), these statements are interdependent in the sense that a given statement about any cross-impact term imposes constraints on the values of the cross-impact term for other pairs of uncertainty factors and their outcomes. One such example is the ratio (6) which connects pairs of cross-impact terms.

Specifically, if the implications of the earlier statements are not observed when introducing new ones, the constraints implied by the new statements may conflict with the constraints derived from the earlier ones. In this case, there are no feasible scenario probabilities which satisfy the full set of constraints that are associated with all the earlier and the newer statements.

To prevent this possibility, we strongly recommend that the consistency of the model should be maintained throughout the elicitation process so that new statements are introduced only on the condition that the resulting augmented set of constraints continues to be satisfied by at least some feasible scenario probabilities. One reason for this is that resolving a complex set of mutually inconsistent constraints can pose conceptual and computational difficulties. That is, it would call for the identification of those statements that are more "wrong" than others, leading to either the removal or relaxation of constraints that are associated with earlier statements.

In practice, the consistency of the statements can be supported so that the expression for the new statement to be added (i.e., marginal probability (8), conditional probability (9), or cross-impact statement (10)) is employed as the objective function which is then minimized and maximized subject to the constraints implied by the earlier statements. That is, the interval defined by these lower and upper consistency bounds indicates for which values the new statement is consistent with the earlier ones. The new statement will eliminate some previously feasible scenario probabilities from further consideration if and only if it excludes some values from the interval defined by the consistency bounds.

For example, consider the situation in which the cross-impact term C_{kl}^{ij} is about to be specified in terms of its lower and upper bounds $\left[\underline{C}_{kl}^{ij}, \overline{C}_{kl}^{ij}\right]$. Then, if the minimum of the difference on the left side in (11) is strictly positive for the cross-impact term $\underline{C}_{kl}^{ij}$, the new constraint will be excessively restrictive in that none of the feasible probabilities will satisfy the constraint based on $\underline{C}_{kl}^{ij}$. Conversely, if the maximum of this difference is strictly negative for the constraint based on $\overline{C}_{kl}^{ij}$, this upper bound is too restrictive. In this way, optimization problems can be solved to ensure the consistency of statements.

There are also further consistency checks that can be readily carried out by checking inequality expressions. First, note that the equality $\sum_{l=1}^{n_j} p_{k|l}^{i|j} p_l^j = p_k^i$ can be divided by p_k^i to obtain

$$\sum_{l=1}^{n_j} \frac{p_{k|l}^{i|j}}{p_k^i} p_l^j = \sum_{l=1}^{n_j} C_{kl}^{ij} p_l^j = 1,$$

which shows that the *probability-weighted average of cross-impact terms on any row of the cross-impact matrix for uncertainty factorsiandjmust equal one*. Thus, if $\underline{C}_{kl}^{ij}, \overline{C}_{kl}^{ij}$ are the lower and upper bounds on the next cross-impact ratio C_{kl}^{ij} which is being elicited, there must exist some feasible vector p of scenario probabilities such that the corresponding marginal probabilities p_l^j satisfy the inequalities $\sum_{l=1}^{n_j} \underline{C}_{kl}^{ij} p_l^j \le 1 \le \sum_{l=1}^{n_j} \overline{C}_{kl}^{ij} p_l^j$. Similarly, examining the marginals p_l^j leads to the equality $\sum_{k=1}^{n_i} C_{kl}^{ij} p_k^i = 1$ so that the probability-weighted average of cross-impact terms in any *column* must be equal to one. Thus $\sum_{k=1}^{n_i} \underline{C}_{kl}^{ij} p_k^i \le 1 \le \sum_{k=1}^{n_i} \overline{C}_{kl}^{ij} p_k^i$ for any $l = 1, \ldots, n_j$.

Even further relationships between the marginal and conditional probabilities and the cross-impact ratios can be established. For example, because $\max\{p_k^i, p_l^j\} \le 1$, it follows that $\overline{C}_{kl}^{ij} \ge C_{kl}^{ij} = p_{kl}^{ij}/(p_k^i p_l^j) \ge p_{kl}^{ij}/\min\{p_k^i, p_l^j\}$ and hence $p_{kl}^{ij} \le \overline{C}_{kl}^{ij} \min\{p_k^i, p_l^j\}$. Thus, if the upper bound on the cross-impact term is small, then the probability p_{kl}^{ij} of the joint event will be low relative to the marginal probabilities. In the same vein, using the inequality $p_{kl}^{ij} \le \min\{p_k^i, p_l^j\}$ gives $\underline{C}_{kl}^{ij} \le C_{kl}^{ij} = p_{kl}^{ij}/(p_k^i p_l^j) \le \min\{p_k^i, p_l^j\}/(p_k^i p_l^j) = 1/\max\{p_k^i, p_l^j\}$ so that $\max\{p_k^i, p_l^j\} \le 1/\underline{C}_{kl}^{ij}$. In other words, having a very large lower bound on the cross-impact term will place an upper bound on the marginal probabilities.

If consistency bounds are not systematically employed in the elicitation process, there are strategies which can be applied to preserve the consistency of the model. That is, if it is only the most recently elicited statement that is found to be inconsistent with the earlier statements, then it is possible to backtrack by omitting this statement from consideration. More constructively, the respondent can also be asked to revise the lower and upper bounds of this statement so that consistency is preserved. In principle, one could also seek to identify those subsets of statements that are mutually consistent and contain as many statements as possible (for a discussion of analogous approaches in the case of constraints on marginal and conditional probability statements, see Salo & Bunn, 1995). However, in the case of cross-impact statements, this strategy would call for a considerable amount of computational effort and, in addition, require that the respondent is prepared to indicate which one(s) of the earlier statements should be omitted.

We also note that the assessment of inconsistencies in nonprobabilistic cross-impact analysis differs from our approach. In the CIB method, for example, all cross-impact statements are elicited at the outset, whereafter an algorithm is applied to identify the scenarios that satisfy the consistency criterion. By construction, the application of this criterion in the CIB method presumes that all the statements have been elicited (i.e., it is not possible to exclude inconsistent scenarios based on a subset of cross-impact statements). Also, because this criteria lacks a formal theoretical foundation, it appears that nonprobabilistic approaches in which the consistency of scenarios is not treated as a dichotomous "yes-no" criterion but, rather, quantified by providing a more systemic measure of consistency, are more defensible. One such approach is developed by Seeve and Vilkkumaa (2021) who generate sets of plausible scenarios which are diverse, too, as measured by how different the scenarios are from each other.

In this context, it is worth noting that "comprehensiveness" has different connotations in nonprobabilistic and probabilistic approaches. In nonprobabilistic approaches, comprehensiveness refers to the extent to which the set of generated scenarios represents the entire range of possible futures (which, as a criterion, does not require that *all* the possible futures would have to be generated). In probabilistic approaches, and especially in the context of safety-critical systems, comprehensiveness commonly refers to the extent to which the residual uncertainties concerning the attainment of the safety requirements permit conclusive statements about the safety of the system (for a review and discussion, see Tosoni et al., 2018).

Furthermore, we remark that "consistency" has a somewhat different meaning in the CIB method than in our approach. In the former, consistencies are associated with individual entries of the cross-impact matrix (with 3 indicating *strong consistency* and −3 representing *strong inconsistency*) as well as with those *scenarios* that fulfill the criterion in Equation (1). In our approach, consistencies refer to *sets of statements* such that the corresponding constraints are fulfilled by some joint probability distribution over the set of all possible scenarios. That is, the scenarios are not treated as inconsistent as such but, rather, they are more or less probable, depending on the logical implications of the elicited probability statements. Also from this perspective of offering insights into what these statements signify, there are advantages to maintaining the consistency of the probability model, because this permits many kinds of probabilistic inferences, such as deriving bounds on those marginal and conditional probabilities that have not yet been elicited.

3 | CONDITIONING CONSEQUENCES ON SCENARIOS

In risk assessment, the aim is to characterize the magnitude of risks, as measured by the severity and probability of harmful consequences. These consequences can differ considerably in terms of what kinds of impacts they pertain to (e.g., human casualties, environmental damages, financial losses).

We first consider the situation where these consequences are represented by a real-valued random variable Z (e.g., amount of released radioactivity from a nuclear facility) whose realization depends on which one of the $|S|$ scenarios occurs. Because the scenarios are mutually exclusive and collectively exhaustive, the probability for the event that the consequences exceed a given threshold level $\theta \in \mathbb{R}$ (e.g., a regulatory limit) is obtained by conditioning Z on these scenarios $s \in S$ so that

$$\mathbb{P}(Z > \theta) = \sum_{s \in S} \mathbb{P}(Z > \theta | s)\mathbb{P}(s). \tag{12}$$

In risk assessment, one relevant rationale for the development of scenarios is that the approach of assessing the conditional probabilities $\mathbb{P}(Z > \theta | s)$ for the different scenarios separately can lead to a more structured and defensible elicitation process than seeking to obtain a single holistic estimate $\mathbb{P}(Z > \theta)$ (for an overview of structured elicitation methods, see Dias et al., 2018).

The expression (12) can be also employed to shed light on the question about which one(s) out of further candidates for additional uncertainty factors $X^{n+1}, X^{n+2}, \ldots$ should be introduced to complement the n uncertainty factors $X^1, \ldots, X^n$, on the basis of which scenarios have already been formulated. Toward this end, the scenario-based conditioning of $\mathbb{P}(Z > \theta | s)$ can be extended to include the additional uncertainty factor X^{n+1} so that

$$\mathbb{P}(Z > \theta | s) = \sum_{x_k^{n+1} \in S^{n+1}} \mathbb{P}\left(Z > \theta | s, X^{n+1} = x_k^{n+1}\right)\mathbb{P}\left(s \wedge \left\{X^{n+1} = x_k^{n+1}\right\}\right).$$

In particular, this expression suggests that the inclusion of the additional uncertainty factor X^{n+1} is unlikely to be very useful if (i) the conditional probabilities $\mathbb{P}(Z > \theta | s, X^{n+1} = x_k^{n+1})$ are the same for different outcomes $x_k^{n+1} \in S^{n+1}$ (i.e., the first term in the sum is the same for all outcomes of the uncertainty factor X^{n+1}) or if (ii) the factor X^{n+1} is perfectly correlated with any one of the n factors that are included in the scenarios $s \in S$ (i.e., there exists some other factor X^i, $i = 1, \ldots, n$ such that the outcomes of X^{n+1} are implied by the states of X^i). These conditions, together with an assessment of how much extra effort is required to elicit the additional parameters $\mathbb{P}(Z > \theta | s, X^{n+1} = x_k^{n+1})$ and $\mathbb{P}(s \wedge \{X^{n+1} = x_k^{n+1}\})$, help evaluate which additional uncertainty factors should be included in the analysis.

Furthermore, the expression (12) implies that if some scenarios are omitted from the sum on the right side, the assessed probability for the event $Z > \theta$ will be lower than the actual probability, unless this omission is compensated through an upward adjustment in the other terms in the sum. Furthermore, if the aim is to establish a conservative upper bound for $\mathbb{P}(Z > \theta)$, then the estimates employed for the terms $\mathbb{P}(Z > \theta | s)$ should be upper bounds on these scenario-specific probabilities.

The expression (12) can also be generalized to situations where Z is not necessarily real-valued but takes on values in the set of possible consequences $\mathbb{C}$. An appropriate disutility function $U : \mathbb{C} \mapsto \mathbb{R}$ can then be defined so that the value of this function is highest for the least preferred consequences and lowest for most preferred consequences. Such a disutility function can be also defined to characterize the probability with which these consequences will be unacceptable. That is, let the set $\mathbb{C}^{\text{fail}}$ consist of all unacceptable consequences and define the disutility function so that

$$U(Z) = \begin{cases} 1, & Z \in \mathbb{C}^{\text{fail}} \\ 0, & Z \notin \mathbb{C}^{\text{fail}}. \end{cases}$$

For this disutility function, the expression $\sum_{s \in S} \mathbb{E}[U(Z)|s]p(s)$ gives the probability with which the consequences will be unacceptable. More generally, we assume that the risk assessment process is required to provide conservative estimates for the expressions

$$\mathbb{E}[Z] = \sum_{s \in S} \mathbb{E}[Z|s]p(s), \tag{13}$$

$$\mathbb{E}[U(Z)] = \sum_{s \in S} \mathbb{E}[U(Z)|s]p(s), \tag{14}$$

where in (13) the term Z representing consequence is assumed to be real-valued and the disutility function in (14) makes it possible to handle other types of consequences as well.

Using the notations $u(s) = \mathbb{E}[U(Z)|s]$, the above formulations can be combined with the results of the preceding section to state the following optimization problem

$$\begin{array}{ll} \max/\min_{p(s)} & \sum_{s \in S} u(s)p(s) \\ \text{subject to} & \sum_{s \in S} p(s) = 1, \\ & p \geq 0, \end{array} \tag{15}$$

plus all the constraints that correspond to the elicited statements about the marginal probabilities, conditional probabilities, and cross-impact terms. Thus, lower and upper bounds for the risk level can be estimated by solving the optimization problem as a minimization and a maximization, respectively, of the objective function.

Building on the above, the main phases of probabilistic cross-impact analysis for assessing risks can now be outlined as follows:

1. Define the scenarios $s \in S$ by specifying the uncertainty factors and their possible outcomes.
2. Assess bounds for the expected scenario-specific consequences $\mathbb{E}[Z|s]$ or their expected disutilities $\mathbb{E}[U(Z)|s]$.
3. Obtain information about the joint probability distribution over scenarios by eliciting cross-impact statements about the ratio (4) and/or statements about the marginal and conditional probability distributions (see Salo & Bunn, 1995). These statements can be elicited by employing interval valued statements defined by lower and upper bounds.
4. Compute lower and upper bounds for the aggregate risk level (as expressed in (12), (13), or (14)) based on information about corresponding scenario-specific expectations and the joint probability distribution over scenarios.
5. Once the maximum tolerable risk level has been determined, assess the risk management implications of the available information by considering the following possibilities (Tosoni et al., 2019):
   - If the upper bound of the aggregate risk level, obtained by maximizing (15), is below the maximum tolerable risk level, the system can be deemed safe.
   - If the lower bound of the aggregate risk level, obtained by minimizing (15), is higher than the maximum tolerable risk level, the system can be deemed unsafe.
   - Otherwise, return to steps 2 and 3 to obtain additional information with the aim of deriving tighter bounds on the aggregate risk level.

From the viewpoint of data analysis and generation, solving the problem (15) presumes that the expected scenario-specific disutilities $u(s)$, $s \in S$ are available for all scenarios. There are, however, problem contexts in which estimates about these disutilities can be generated with the help of computational models, as illustrated by the example in the next section. The maximization problem (15) can also be solved based on conservative upper bound estimates about these disutilities. One can also explore just how large these disutilities would have to be so that the maximum tolerable risk level would be reached.

Because the cross-impact statements are interpreted as constraints on the joint probabilities, it is conceptually and computationally straightforward to integrate the use of such statement in Monte Carlo simulations in which vectors representing joint probabilities are generated. That is, computational results reflecting cross-impact statements can be produced by retaining only those probability vectors that satisfy the constraints implied the cross-impact statements. In particular, this makes it possible to benefit from cross-impact statements when using other approaches for the exploration dependencies in safety risk models (see, e.g., Harrison & Cheng, 2011).

4 | CASE STUDY

The risk assessment of nuclear waste management facilities is an important application context of scenario analysis (Tosoni et al., 2018). In this context, the uncertainty factors consist of so-called FEPs which include, for instance, physical and chemical variables that affect the lifetime of the facility and its surrounding environment. The FEP outcomes can be represented through discretized states such as *low*, *medium*, and *high*.

In this section, we revisit the case study (Tosoni et al., 2019) on the nuclear waste repository at Dessel (Belgium) in which the Bayesian network in Figure 3 was developed to represent dependencies between nine FEPs. As shown in Table 1, there are two possible outcomes for the first five FEPs while the two last ones have three possible outcomes.

In this setting, scenarios are defined as combinations of outcomes for each FEP. Thus, for example, there is a scenario which represents the following combination of FEP states: a *beyond-design-basis* Earthquake (BDBE), *low* Water flux, *micro* crack Aperture, *low* Diffusion coefficient, *low* Distribution coefficient, *slow* Chemical degradation, *fast* Concrete degradation, *slow* Monolith degradation, and *low* Hydraulic conductivity. Given the nine FEPs and their two or three outcomes, the total number of scenarios is $2^7 \times 3^2 = 1152$.

The scenarios differ from each other in terms of how probable it is that radioactive particles will be released into the environment, causing human exposure to radiation. For each scenario, this impact is quantified by the conditional probability that the subsequent dose rate to humans exceeds a predefined safety threshold level. Aggregating these conditional probabilities over all scenarios based on (12) thus gives an estimate about the radiological risk, which is measured by the total probability with which this threshold θ is violated.

For each scenario s, the corresponding conditional probability $\mathbb{P}(Z > \theta|s)$ in (12) of violating the threshold θ was computed as the average of three numbers, that is, (i) the prior value in Tosoni et al. (2020) and (ii) the lower and upper bounds in Tosoni et al. (2019). This approach was adopted, because it serves to illustrate how results concerning the total violation probability $\mathbb{P}(Z > \theta)$ in (12) changes as a result of providing additional information about the probabilities. These conditional violation probabilities are not reported here due to the large number of scenarios, but they are available from the authors upon request. For instance, the conditional violation probability for the scenario described in the second paragraph of this section was 0.678.

In the following illustrative analysis, we build on the model and data in papers Tosoni et al. (2019, 2020) which represent the nuclear waste repository as a Bayesian network (Pearl & Russel, 2003). In this network, the nodes represent the FEPs, whereas directed arcs indicate cause dependencies between the FEPs. The uncertainties associated with the FEP outcomes are modeled as the feasible sets of marginal and conditional probabilities (Tosoni et al., 2019).

Specifically, we consider three steps in which increasingly detailed information about scenario probabilities are provided. The first step uses only marginal probabilities of FEP outcomes. In the second step, the dependencies between those FEPs which are linked by arcs in the Bayesian network are approximated with cross-impact statements. In the third step, it is stated that the six FEPs in Figure 3 (i.e., Water flux, Earthquake, Crack aperture, Diffusion coefficient, Distribution coefficient, Chemical degradation) from which there are only outgoing arcs are almost independent. This statement is introduced by allowing the cross-impact ratio (4) to assume value in the

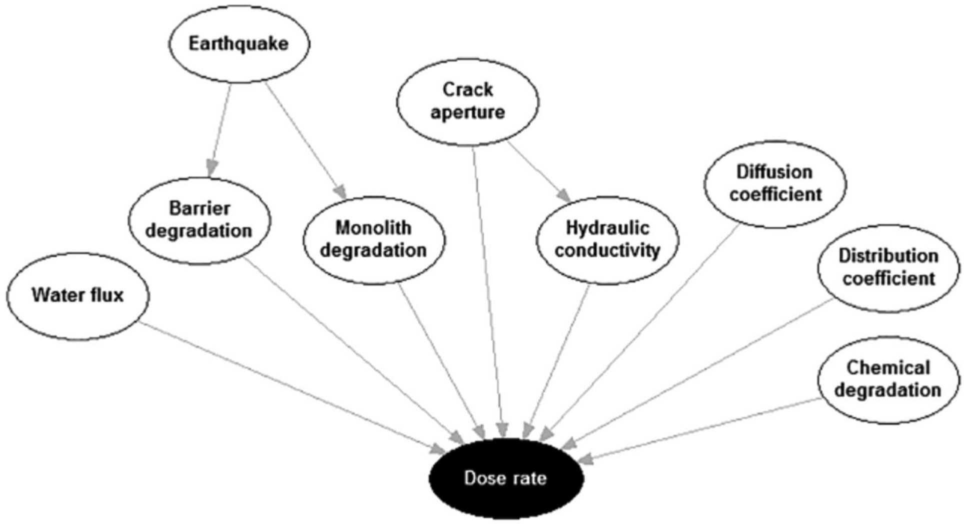

FIGURE 3 The Bayesian network for the case study (Tosoni et al., 2019)

interval [0.9950–1.0050]. Note that this assumption is weaker than the full independence assumption which is embedded in the structure of the Bayesian network and corresponds to the requirement that all cross-impacts between the outcomes of these six FEPs are equal to 1. Thus, the introduction of the relatively narrow interval [0.9950–1.0050] helps explore how the results would change if it were to be the case that the Bayesian network in Figure 3 is *not* a valid model of the dependencies between the FEPs. Furthermore, because the statements in second step do not yet limit these dependencies, the introduction of these intervals in the third step provides a significant amount of additional information. This leads to much tighter constraints on the scenario probabilities so that a reduction in the violation probability $\mathbb{P}(Z > \theta)$ can be expected.

For the first step, the lower and upper bounds for the marginal probabilities of FEP outcomes in Table 1 were computed by sampling the feasible sets of marginal and conditional probabilities in the Bayesian network, leading to corresponding sample distributions over FEP outcomes. The marginals in Table 1 were taken from these distributions by employing their 5% and 95% quantiles.

For the second step, the characterization of dependencies between selected pairs of FEP outcomes was also based on the model in Tosoni et al. (2019) as above, except that the sample distributions were established for the cross-impact ratio in (7) (rather than for the marginal probability distributions). Moreover, the bounds for cross-impact terms were established by using the more conservative 0.5% and 99.5% quantiles (as opposed to 5% and 95% quantiles) to allow for more imprecision in the characterization of cross-impacts. The resulting bounds on the cross-impact ratios are reported in Tables 2 and 3.

Looking at the ratios in Table 2, it is instructive to see that if a *major* outcome of the FEP Earthquake does occur, the probability of *fast* Barrier degradation becomes much higher (i.e., [1.5755–6.9785] times) in comparison with the situation where there is no information about the probability of an Earthquake. On the other hand, if the outcome for the Earthquake is *BDBE* (i.e., of a lower magnitude than a major earthquake, but still beyond what the repository barriers are designed to withstand),

TABLE 1 FEPs and their outcomes in Tosoni et al. (2019) and corresponding on bounds marginal probabilities

FEP	Outcome	Probability bounds
Earthquake	BDBE	[0.9912–0.9950]
	Major	[0.0050–0.0088]
Water flux	Low	[0.6525–0.8428]
	High	[0.1572–0.3475]
Crack aperture	Micro	[0.8148–0.8874]
	Macro	[0.1126–0.1852]
Diffusion coefficient	Low	[0.5209–0.7275]
	High	[0.2725–0.4791]
Distribution coefficient	Low	[0.5215–0.7268]
	High	[0.2732–0.4785]
Chemical degradation	Fast	[0.5361–0.6694]
	Slow	[0.3306–0.4639]
Barrier degradation	Fast	[0.0787–0.2337]
	Slow	[0.7663–0.9213]
Monolith degradation	Very fast	[0.0293–0.2678]
	Fast	[0.0594–0.2695]
	Slow	[0.4627–0.9114]
Hydraulic conductivity	Low	[0.5993–0.7066]
	Medium	[0.2016–0.2715]
	High	[0.0872–0.1342]

Abbreviation: BDBE, beyond-design-basis earthquake; FEPs, features, events, and processes.

the probability of *slow* Barrier degradation will grow, albeit marginally. This is in keeping with the recognition that a *major* Earthquake can have an impact on the speed of Barrier degradation; but its absence does not have a comparable impact.

TABLE 2 Bounds on the cross-impact ratios for pairs of outcomes for the FEPs Earthquake, Barrier degradation, and Monolith degradation

		Barrier degradation		Monolith degradation		
		Fast	Slow	Very fast	Fast	Slow
Earthquake	BDBE	[0.9544–0.09963]	[1.0011–1.0036]	[0.9950–1.0050]	[0.9329–0.9982]	[0.9975–1.0052]
	Major	[1.5755–6.9785]	[0.5660–0.8174]	[0.9950–1.0050]	[1.2769–10.1853]	[0.3406–1.3606]

Abbreviations: BDBE, beyond-design-basis earthquake; FEPs, features, events, and processes.

TABLE 3 Bounds to the cross-impact ratios for pairs outcomes of the FEPs Crack aperture and Hydraulic conductivity

		Hydraulic conductivity		
		Low	Medium	High
Crack	Micro	[1.0896–1.1880]	[0.4941–0.7490]	[0.9950–1.0050]
Aperture	Macro	[0.1628–0.3017]	[2.5666–3.9985]	[0.9950–1.0050]

Abbreviation: FEPs, features, events, and processes.

The bounds in Tables 2 and 3 specify no restrictions on dependencies between the six FEPs which are independent in Figure 3 as they have only outgoing arcs in this Bayesian network. Thus, the independence between these six FEPs is introduced in the third step. As noted above, however, this independence assumption is quite strong, so that we relax it by allowing for minor deviations from independence by bounding the cross-impact ratios to the interval [0.995–1.005]. Moreover, in Tables 2 and 3 there are two columns (i.e., *very fast* Monolith degradation in Table 2, *high* Hydraulic conductivity in Table 3) in which the independence assumption contained in the Bayesian data has been relaxed similarly.

Based on the probability information for the three steps above, the following conservative upper bounds for the level of radiological risk can now be computed by solving the maximization problem (15) subject to the corresponding constraints on scenario probabilities.

1. *Marginals only*: When there is information about the marginals only, the upper bound on the maximum level of risk is 0.576.
2. *Cross-impacts bounds for arcs between FEPs in* Tables 2 and 3: When the constraints based on these bounds are added to the information in the first step, the upper bound is reduced to 0.571.
3. *Cross-impact bounds for independent FEPs*: When the narrow intervals [0.995–1.005] are introduced for pairs of outcomes for independent FEPs in the Bayesian network, the upper bound becomes 0.427.

TABLE 4 Upper bounds on the risk level for different settings of probabilistic information

	Setting	1	2	3
Constraints	1. Marginals	✓	✓	✓
	2. CI ratios for designated FEP dependencies (Tables 2, 3)	✗	✓	✓
	3. CI ratios for independent FEPs	✗	✗	✓
	Upper bound on risk level	0.576	0.571	0.427

Abbreviation: FEPs, features, events, and processes.

The results are summarized in Table 4. The greatest reduction in the upper bound is attained as a result of introducing the assumption of near-independence when moving from the second step to the third. This can be explained by noting that the number of such constraints is high (i.e., lower and upper bound constraints for every combination of outcomes for all pairs of the six FEPs) and because these intervals are relatively tight. This can be contrasted with the shift from the first step to the second step which leads to a much smaller reduction in the total violation probability.

More generally, this example shows how probabilistic cross-impact analysis can be interfaced with other models. Specifically, scenario-specific estimates concerning radiological risk were inferred from Tosoni et al. (2019, 2020). Parameters of the Bayesian network (Tosoni et al., 2019) were employed to generate information about the marginal probabilities. Analogously, information about conditional dependencies was provided through cross-impact ratios stated in terms of lower and upper bounds. We emphasize that all this information about probabilities and dependencies could have been introduced directly without explicit reference to the Bayesian network (which has been employed as a useful tool for generating such information). This notwithstanding, we stress that the numerical results are illustrative and do not provide any indications as to the safety of the nuclear waste repository at Dessel.

5 | DISCUSSION AND CONCLUSIONS

In this paper, we have considered the limitations of nonprobabilistic cross-impact analyses in risk management and, specifically, in the risk assessment of safety critical systems for which the aim is produce conservative estimates that provide an upper bound on the overall risk level. Importantly, we have shown that instead of limiting attention to the most consistent scenarios only, it is pertinent to account for all the scenarios that can make a nonnegligible contribution to the overall risk level, even if some of these scenarios are quite improbable. That is, neglecting these scenarios may lead to risk estimates which are too small, as the actual risk will be higher than what is suggested by the analysis. This, in turn, may lead to the selection of inadequate and insufficient risk mitigation actions.

We have also advocated the probabilistic interpretation of cross-impacts, because this helps establish precise and empirically testable mappings between the qualitative verbal expressions employed in the elicitation process and their numerical counterparts. This interpretation also makes it possible to integrate the scenario process with other approaches for analyzing probabilistic inputs, for instance by carrying out statistical analyses or by synthesizing them with judgmental forecasts (see, e.g., G. Wright et al., 2009). Furthermore, probabilistic models are appealing not least because they can be adapted to assess the attractiveness and effectiveness of insurance as one of the quantitative risk management options.

We have also developed a probabilistic cross-impact method which is capable of accommodating and synthesizing many kinds of probability elicitation statements (including both marginal and conditional probabilities as well as cross-impacts statements). All these statements are converted into corresponding linear or quadratic constraints in the optimization models which can be solved to (i) guide the elicitation of further statements which are consistent with the statements that have been elicited earlier and (ii) compute lower and upper bounds on the overall risk level at any stage of the elicitation process. Results such as these are useful for reaching conclusions about the safety of the system, which provides support for risk management decisions. There are also promising avenues for future work, for example by employing cross-impact statements together with other methods for assessing dependencies and their impacts (see, e.g., Harrison & Cheng, 2011). One could also assess how the cross-impact statements and therefore scenario probabilities, too, would be impacted by alternative risk management actions. This would make it possible to accommodate endogenously dependent scenario probabilities (for a case study with decision-dependent scenario probabilities, see Vilkkumaa et al., 2018).

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CONFLICT OF INTERESTS

The authors declare that there are no conflict of interests.

DATA AVAILABILITY STATEMENT

Data are available on request from the authors.

ORCID

Ahti Salo http://orcid.org/0000-0003-1526-7923

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