Capital and Income Inequality: an Aggregate-Demand Complementarity[I]

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March 2021

Abstract

A novel complementarity between capital and income inequality leads to a significant amplification of the effects of aggregate-demand shocks on consumption. We characterize this finding using a simple model with heterogeneity in household saving and income, nominal rigidities, and capital. A fiscal policy that redistributes capital income causes further amplification, whereas redistributing profits generates dampening. After an interest rate shock, consumption inequality is more countercyclical than income inequality, consistent with the available empirical evidence.

JEL classification: E21, E22, E32, E44, E52

Keywords: capital, income inequality, aggregate demand, complementarity, monetary policy, heterogeneity.

[I]This supersedes the *November 2019* CEPR DP version "Capital, Income Inequality, and Consumption". We thank our discussants Thomas Drechsel, Benjamin Moll, and Jirka Slacalek, as well as Sushant Acharya, Adrien Auclert, Cristiano Cantore, Edouard Challe, Keshav Dogra, Stéphane Dupraz, Gaetano Gaballo, Brandon Kaplowitz, Lilia Maliar, Simon Mongey, Giorgio Primiceri, Ricardo Reis, Hélène Rey, Matthew Rognlie, Ludwig Straub, Andrea Tambalotti, Andreas Tischbirek, Gianluca Violante, and numerous conference and seminar participants for helpful comments. Surico gratefully acknowledges financial support from the European Research Council (Consolidator Grant 771976).

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1 Introduction

How do aggregate demand shocks transmit to the economy and what determines the magnitude of the response? In a seminal contribution, Samuelson (1939) already argued that the combination of a consumption function with an investment relation leads to an amplification of aggregate demand shocks: the celebrated *multiplier-accelerator*.

A recent literature reviewed below emphasizes the role of household heterogeneity as a microfoundation for a multiplier effect, in particular through an endogenous feedback between aggregate demand and income inequality in relation to the marginal propensity to consume (MPC), reminiscent of the Keynesian cross. Less attention, however, has been paid to the role of heterogeneity in the marginal propensity to save, and thus to investment, as a potential amplifier of demand-driven fluctuations.

In this paper, we show that income inequality together with heterogeneity in savings generates a strong *complementarity*: the impact of aggregate demand shocks on consumption when both heterogeneity dimensions are active is an order of magnitude larger than the mere addition of the effects of each heterogeneity in isolation. We elicit this novel amplification mechanism using an apparatus that distinguishes between two types of heterogeneity: heterogeneity in savings on the expenditure side, and income inequality on the resource side of the household budget constraint. We refer to the former as '*capital inequality*': a feature of any heterogeneous-agents model with a productive asset such as capital, that could be equally referred to as wealth inequality or capital market segmentation.

To each inequality corresponds one separate amplification channel. First, much in the spirit of Samuelson's (1939) multiplier-accelerator, capital inequality leads to amplification in and of itself. The intuition is that after an increase in aggregate demand, spenders consume all the additional income whereas savers invest a fraction of it, thus generating a further boost in aggregate income and a further round of aggregate demand effects. This *'capital inequality' channel* is an intrinsic feature of any heterogeneous-agent model with capital, as we discuss in the related literature section below.

This is distinct from the cyclical *'income inequality' channel*, which also leads to aggregate-demand amplification in and of itself under the condition that the income of high MPC spenders responds more than proportionally to changes in aggregate income, as emphasized by a literature reviewed below. Our key result is that when simultaneously present, capital and income inequality blend into a significant *complementarity* that we dub the *'multiplier of the multiplier'*.

We characterize our findings, first, in a simple saver-spender model that allows us to develop intuition and, then, in a richer (but still) tractable heterogeneous-agent New Keynesian model with investment in productive physical capital and idiosyncratic risk. The unconstrained households hold stocks, bonds and capital, while the constrained hand-to-mouth do not have access to asset markets and simply consume their labor income plus any transfers. Idiosyncratic uncertainty—captured by households changing state exogenously between these two states—gives rise to a precautionary, self-insurance saving motive. In our baseline model, stocks and capital are illiquid (cannot be used for self-insurance) and adjusting the capital stock is subject to a cost. Firms are subject to nominal rigidities. The government levies taxes on dividends and capital income, which it may choose to redistribute or not.

The effects of fiscal redistribution crucially depend on what type of income is targeted. The redistribution of monopoly profits dampens the aggregate-demand effects on consumption because profits in the model are countercyclical and thus redistributing them weakens the capital and income inequality channels. In contrast, capital income is highly pro-cyclical and thus its redistribution towards constrained households strongly amplifies the impact of aggregate-demand policies on household expenditure.

Our finding of a strong complementarity between capital and income inequality in the transmission of aggregate-demand shocks to consumption is robust to introducing idiosyncratic risk and sticky wages as well as to varying key model parameters—such as capital adjustment costs, the degree of nominal rigidities and the elasticity of intertemporal substitution—within a wide range of empirically plausible values.

A robust prediction of our framework that is testable concerns the cyclicality of consumption and income inequality. Our model predicts that, conditional on demand shocks, both consumption and income inequality are countercyclical but consumption inequality is more countercyclical than income inequality. This is in line with the available empirical evidence (see Coibion et al., 2017; Ampudia et al., 2018; Mumtaz and Theophilopoulou, 2017) and supports the empirical relevance of the channels we identify.

Related literature. Our analysis joins a burgeoning body of work that incorporates Heterogeneous Agents into the New Keynesian (HANK) framework. Because HANK models are typically complex, several studies have proposed tractable versions that help illustrate the transmission mechanisms at work.[1]

The "capital inequality" channel is a simple analytical generalization and microfoundation of Samuelson's (1939) celebrated multiplier-accelerator to a setting with household heterogeneity. It relies on a literal formalization of the saver-spender distinction based of physical capital holdings (or lack thereof) proposed by Mankiw (2000), following Campbell and Mankiw (1989), and first incorporated in a New Keynesian model by Galí et al. (2007) to study the effects of government spending. The same channel has been implicitly featured in other earlier contributions, including Kaplan, Moll, and Violante (2018), Gornemann et al. (2016), and Luetticke (forthcoming), and explicitly analyzed using a quantitative model by Alves et al. (2019). In independent and complementary work, Auclert, Rognlie, and Straub (2020) also emphasize the role of investment and heterogeneity in a model with sticky prices and wages, focusing on liquidity but abstracting from (cyclical variations in) income inequality, and therefore not featuring our complementarity. Finally, the income inequality channel and its role for aggregate-demand amplification in isolation has been studied by Bilbiie (2008, 2019), Auclert (2019), and Patterson (2019) in frameworks without capital.

Relative to these studies, we unveil a novel complementarity between capital and income inequality for aggregate-demand amplification. We characterize analytically these channels, in isolation and in combination, and then use a richer tractable heterogeneous-agent New Keynesian model with idiosyncratic risk—building on Bilbiie (2018), extended with capital investment—to quantify the contribution of the different assumptions to the transmission of monetary policy.

2 A Tale of Two Inequalities

In this section, we present a simple framework that serves to isolate the capital and income inequality channels and illustrate their complementarity. While our focus is on capital, the arguments hold for any productive asset that is in positive net supply in equilibrium, or more generally *wealth*. Let us start from a generic budget constraint of a household j:

$$C^j + S^j = Y^j,$$

[1]See for instance Oh and Reis 2012; Gornemann et al. 2016; McKay et al. 2016; Challe et al. 2017; Ravn and Sterk 2017; Kaplan et al. 2018; Hagedorn et al. 2019 for quantitative contributions and Galí et al. 2007; Bilbiie 2008, 2018, 2019; Debortoli and Galí 2018; Maliar and Naubert 2019; Acharya and Dogra 2020; Cantore and Freund 2020; Ravn and Sterk 2020 for tractable versions.

where C^j are consumption expenditures, S^j savings, and Y^j the household's total income that can include both labor and financial income. For now, we remain agnostic about the precise income sources—accounting for which, however, will play an important role in Section 3.

In this framework, we can identify two different types of heterogeneity. On the left hand side, households can differ in their *expenditures* depending on how much they save/invest (and consume); and on the right-hand side, they can differ in their *incomes*. We refer to these, respectively, as *capital* and *income inequality*; the former is akin to a stark form of *wealth* inequality. These inequalities are present in many heterogeneous-agent models with assets traded in equilibrium. Our aim is to make transparent their role for the transmission of aggregate shocks.

To that end, we propose a (to the best of our knowledge) novel way to elicit these two channels. To isolate the role of *capital inequality*, we assume that income is perfectly redistributed so that all households receive the same income Y:

$$C^j + S^j = Y.$$

In a model without net savings and capital, perfect income redistribution would imply that household heterogeneity is irrelevant for understanding aggregate dynamics. This is, however, no longer the case when differences in savings behavior are linked with MPC heterogeneity.

To isolate the role of *income inequality*, we assume that there is no savings vehicle in positive net supply, so that in equilibrium the budget constraint reads:

$$C^j = Y^j.$$

The crucial parameter is the elasticity of individual income with respect to aggregate income, $\chi_j = \frac{\partial \log Y^j}{\partial \log Y}$. When χ_j is higher for constrained households, income inequality (between unconstrained and constrained agents) becomes countercyclical and there is amplification of aggregate-demand shocks. This was shown by Bilbiie (2008) in a two-agent model, generalized by Auclert (2019) in a richer heterogeneous-agent model, and estimated using micro data on consumption and income by Patterson (2019). Conversely, if inequality is procyclical the effects of aggregate-demand shocks are dampened.

Given the empirical relevance of both channels, an important question is how much of the aggregate-demand effects on consumption they can account for. As we shall see, the two channels are complementary: their joint impact is much larger than the addition of their individual effects. We now characterize this finding analytically in a simple saver-spender model in the spirit of Mankiw (2000).

2.1 A Simple Saver-Spender Model

In this section, we outline a stylized model to isolate our main finding in the simplest, most transparent way. In the next section, we relax many of the simplifying assumptions and show that the main conclusions carry through in a fully-specified yet still tractable heterogeneous-agent model, whose closed-form solution echoes this one in a special case.

The economy consists of a continuum of households on the unit interval, of two types: a share $\lambda \in [0, 1)$ are *hand-to-mouth* spenders (H) and the rest $1 - \lambda$ are *savers* (S). Savers consume and save, while spenders live paycheck to paycheck, consuming all of their income. As our focus is on the demand side, we remain agnostic about the supply side and assume that the central bank controls the real interest rate. While our focus is on *monetary policy*, the insights apply to any kind of aggregate-demand policy. We sketch the model in log-linear form, where lowercase variables denote log-deviations from steady state. For a detailed derivation, see Appendix A.

Savers have access to two assets: bonds and physical capital. Their bond holding decision is characterized by a standard Euler equation:

$$c_t^S = E_t c_{t+1}^S - r_t, \tag{1}$$

where r_t is the real interest rate. Bonds are priced but not traded as we assume that they are in zero net supply in equilibrium.

Savers also invest in physical capital. To get tractability, we assume in this section, and in this section *only*, that their behavior can be characterized by a reduced-form investment rule $i_t = f(y_t, r_t, \ldots)$. We remain agnostic here about the exact underpinnings of this equilibrium equation; in Section 3 we study a fully microfounded version. As a leading example, we assume that investment is an isoelastic function of total income:

$$i_t = \eta y_t, \tag{2}$$

where $\eta > 0$ is the elasticity of investment with respect to output.[2] As shown in Appendix A.2, our analysis easily generalizes to include an elasticity to interest rates or future income as well.

[2]As is well known, the strong procyclicality of investment to output arises naturally as an equilibrium outcome of any neo-classical, RBC or NK model.

The budget constraint of savers (in log-linear form) reads:

$$C_Y c_t^S + \frac{I_Y}{1-\lambda} i_t = Y_Y^S y_t^S, \tag{3}$$

where y_t^S is the (post-transfer) income of the savers and $X_Y \equiv X/Y$ denotes the steady-state share of variable X in GDP (income) Y, for any $X \in \{C, I, Y^S\}$.[3]

Spenders just consume all their income in every period, i.e.:

$$c_t^H = y_t^H. \tag{4}$$

Goods market clearing requires that:

$$y_t = C_Y c_t + I_Y i_t. \tag{5}$$

Aggregate consumption and income are given by:

$$c_t = \lambda c_t^H + (1-\lambda) c_t^S \tag{6}$$

$$y_t = \lambda Y_Y^H y_t^H + (1-\lambda) Y_Y^S y_t^S. \tag{7}$$

To close the model, we have to specify how income is distributed. We assume that the income of the spenders moves with aggregate income according to:

$$y_t^H = \chi y_t, \tag{8}$$

where χ is the elasticity of *their* income to aggregate income. In Section 3, we use a richer microfounded framework where this elasticity is an equilibrium outcome of a structural model. Using the definition of aggregate income, savers' income is, combining (7) and (8): $y_t^S = \left(1 - \lambda\chi Y_Y^H\right) y_t / \left((1-\lambda) Y_Y^S\right)$.

2.2 The Multiplier of the Multiplier

We now analyze the two inequality channels, first in isolation and then in interaction. A useful benchmark is the representative-agent economy $\lambda = 0$, whereby a one-time real interest rate cut has a unit consumption multiplier $\partial c_t / \partial\left(-r_t\right) = 1$.

[3] We focus on a case with equal consumption in steady state across households, i.e. $C^S = C^H = C$, achieved by a fixed steady-state transfer explained in Appendix A. This simplifies the analytics but is not needed, as we show in the fully-specified model in Section 3 and Appendix C.3.

Income inequality. To isolate the role of income inequality, we assume that the savings rate is zero, i.e. $I_Y = 0$. The model then collapses to:

$$c_t^H = \chi y_t; \text{ and } c_t^S = \frac{1-\lambda\chi}{1-\lambda} y_t.$$

Using this together with market clearing in the Euler equation, we can derive the *aggregate Euler equation*:

$$c_t = E_t c_{t+1} - \frac{1-\lambda}{1-\lambda\chi} r_t.$$

The multiplier to a one time change in the real interest rate is:

$$\frac{\partial c_t}{\partial(-r_t)} = \frac{1-\lambda}{1-\lambda\chi}. \tag{9}$$

The effects of a change in the real rate are amplified iff $\chi > 1$, i.e. when spenders' income is more elastic to aggregate income than the savers', provided that $\lambda\chi < 1$. The reason is that an increase in aggregate demand, which leads to an increase in aggregate income, translates into an even larger increase in spenders' income; this causes aggregate demand to rise even further because spenders have unit MPC, and so on. This is the countercyclical inequality channel described in Bilbiie (2008, 2018), yielding a Keynesian-cross multiplier (in the spirit of Samuelson, 1948): a share λ agents have unit individual MPC and their income elasticity to aggregate income is χ, so the "aggregate MPC" out of aggregate income is approximately $\lambda\chi$. When households have proportional incomes $\chi = 1$, the case assumed by Campbell and Mankiw (1989), the multiplier is the same as in the representative-agent benchmark of $\lambda = 0$, i.e. $|\partial c_t / \partial r_t| = 1$.

Capital inequality: a reappraisal of Samuelson's (1939) Multiplier-Accelerator. To isolate the role of capital inequality, we assume instead that income is perfectly redistributed: $\chi = 1$, which implies proportional incomes $y_t^S = y_t^H = y_t$. Replacing in the budget constraints (3) and (4):

$$c_t^H = y_t \tag{10}$$

$$C_Y c_t^S + \frac{I_Y}{1-\lambda} i_t = Y_Y^S y_t.$$

We want to solve for savers' consumption as a function of aggregate consumption in order to obtain an aggregate Euler equation. To do so, first combine the investment function (2) with goods market clearing (5), obtaining:

$$i_t = \eta \frac{1 - I_Y}{1 - \eta I_Y} c_t. \tag{11}$$

Note that $\frac{1-I_Y}{1-\eta I_Y} > 1$ iff $\eta > 1$, provided that $\eta I_Y < 1$. Using (11) and (5) to replace (10) in the definition of aggregate consumption, we obtain:

$$c_t^S = \frac{1 - \lambda \frac{1-I_Y}{1-\eta I_Y}}{1 - \lambda} c_t, \tag{12}$$

which replaced in (1) delivers the aggregate Euler equation and Proposition 1:

$$c_t = E_t c_{t+1} - \frac{1 - \lambda}{1 - \lambda \frac{1-I_Y}{1-\eta I_Y}} r_t. \tag{13}$$

Proposition 1 (Amplification through capital) *The multiplier of a one time cut in the real interest rate is given by:*

$$\frac{\partial c_t}{\partial \left(-r_t\right)} = \frac{1 - \lambda}{1 - \lambda \frac{1-I_Y}{1-\eta I_Y}}. \tag{14}$$

If investment is more than one-to-one procyclical, i.e. $\eta > 1$*, then (i) the effect of a cut in the real rate is larger than one, i.e.* $\partial c_t / \partial \left(-r_t\right) > 1$*, and (ii) the multiplier is increasing in the share of spenders,* λ*, as long as* $0 < \lambda \frac{1-I_Y}{1-\eta I_Y} < 1$.

Proof. Follows immediately from $I_Y \in [0, 1)$. ■

Our analytical formalization provides a novel intuition for the amplification of monetary policy effects on consumption via investment: *the marginal propensity to save MPS (of savers) adds to the aggregate MPC through its indirect impact on the high-MPC spenders*, even if income is redistributed uniformly. When capital income gets redistributed to hand-to-mouth agents (either through market forces—capital augmenting the return on labor—or through fiscal redistribution), the latter increase their demand. This further boosts total income, part of which is saved and yields an increase in investment of ηI_Y, which generates further income, boosting the consumption of unit-MPC spenders, and so on—thereby triggering a distinct Keynesian-cross multiplier. This is summarized by the term $\frac{1-I_Y}{1-\eta I_Y}$, which magnifies the aggregate MPC through the above-described channel when investment is procyclical enough $\eta > 1$. The multiplier effect disappears without investment, since under full redistribution the model collapses to the representative-agent case. In the empirically plausible case $0 < \lambda \frac{1-I_Y}{1-\eta I_Y} < 1$, capital *amplifies* the monetary policy effects on consumption through heterogeneity.

We elaborate on the connection to Samuelson (1939), who studied the role of investment and consumption functions for spending multipliers, in Appendix A.3. Our capital inequality channel is a generalized and microfounded version of Samuelson's in a setting with MPC heterogeneity and segmented capital markets. This is a very general amplification mechanism operating in any heterogeneous-agent model with capital. Furthermore, it does not depend on our simple framework with a reduced-form investment equation; in Appendix A.1, we show that the only requirement is procyclical enough investment. Thus, any model with this feature automatically implies amplification of the consumption response through heterogeneity, even under proportional incomes.

Capital and income inequality. We now enable both channels, capital ($I_Y > 0$) and income inequality ($\chi > 1$). Replacing in the budget constraints (3) and (4):

$$c_t^H = \chi y_t \tag{15}$$
$$C_Y c_t^S + \frac{I_Y}{1-\lambda} i_t = \frac{1 - \lambda \chi Y_Y^H}{1-\lambda} y_t.$$

Following the same strategy as above, we solve again for savers' consumption:

$$c_t^S = \frac{1 - \lambda \chi \frac{1-I_Y}{1-\eta I_Y}}{1-\lambda} c_t, \tag{16}$$

to obtain the aggregate Euler equation and our next Proposition:

$$c_t = E_t c_{t+1} - \frac{1-\lambda}{1 - \lambda \chi \frac{1-I_Y}{1-\eta I_Y}} r_t. \tag{17}$$

Proposition 2 *The multiplier of an interest-rate cut when both channels are active is:*

$$\frac{\partial c_t}{\partial \left(-r_t\right)} = \frac{1-\lambda}{1 - \lambda \chi \frac{1-I_Y}{1-\eta I_Y}}. \tag{18}$$

If income inequality is countercyclical ($\chi > 1$) and investment is more than one-to-one procyclical ($\eta > 1$), this joint multiplier is larger than the product of the two individual multipliers:

$$\frac{\partial c_t}{\partial \left(-r_t\right)}\Big|_{K,\ no\ redist} > \frac{\partial c_t}{\partial \left(-r_t\right)}\Big|_{no\ K,\ no\ redist} \times \frac{\partial c_t}{\partial \left(-r_t\right)}\Big|_{K,\ redist}, \tag{19}$$

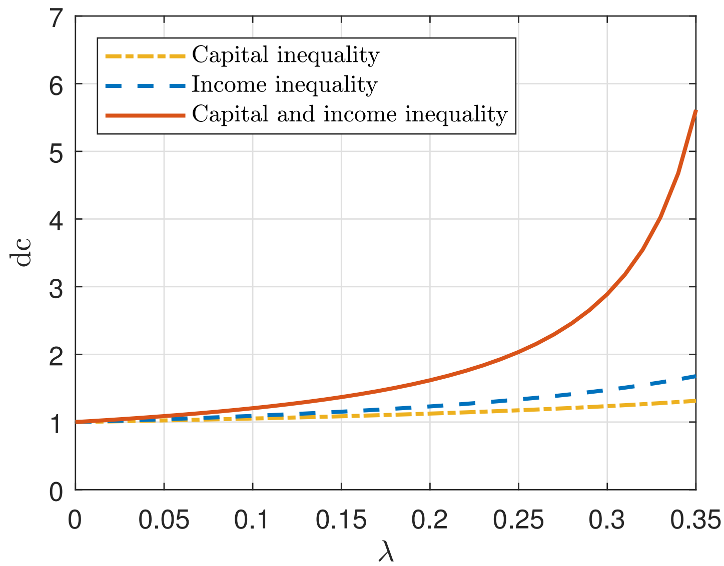
provided that $0 < \lambda \chi \frac{1-I_Y}{1-\eta I_Y} < 1$. Amplification ($\partial c_t / \partial \left(-r_t\right)$ increasing in λ) can occur even with procyclical income inequality ($\chi < 1$) iff $\chi \frac{1-I_Y}{1-\eta I_Y} > 1$.

Proof. Replacing the expressions for the respective multipliers from (9), (14) , and (18), the complementarity condition (19) becomes:

$$\frac{1-\lambda}{1-\lambda\chi\frac{1-I_Y}{1-\eta I_Y}} > \frac{1-\lambda}{1-\lambda\chi}\frac{1-\lambda}{1-\lambda\frac{1-I_Y}{1-\eta I_Y}}.$$

This holds if $\lambda(\chi-1)(\eta-1)\frac{I_Y}{1-\eta I_Y} > 0$, which is satisfied if $\chi > 1$, and $\eta > 1$. The final part follows from the derivative of (18) with respect to λ, i.e. $\left(\chi\frac{1-I_Y}{1-\eta I_Y}-1\right)\left(1-\lambda\chi\frac{1-I_Y}{1-\eta I_Y}\right)^{-2}$, which is positive even for $\chi < 1$ if $\chi\frac{1-I_Y}{1-\eta I_Y} > 1$. ■

Figure 1: The Complementarity



Notes: This figure shows the consumption multipliers as a function of the share of hand-to-mouth λ (using $I_Y = 0.235$, $\eta = 2$, and $\chi = 1.75$).

The model features are illustrated in Figure 1, which depicts the effect of a cut in the real rate on consumption as a function of the share of hand-to-mouth λ. When $\lambda = 0$, we are back to the representative-agent case and the multiplier is one in all models. The broken yellow line reveals that capital inequality, by itself, only leads to little amplification. This is almost by construction, as investment is undertaken by the savers and we have neutralized the feedback through the real interest rate. Income inequality alone, depicted as the dotted blue line, can lead to more amplification through the cyclical-inequality channel. But importantly, the model with capital and income inequality depicted as the red solid line delivers substantially more amplification than the mere product of the individual channels. Unequal capital expenditures lead to a multiplying effect of the multiplier associated with income inequality: *a multiplier of the multiplier*.

The intuition is most clearly seen by inspecting the multiplier under both capital and income inequality from expression (18). The numerator captures the automatic, direct effect: only $(1-\lambda)$ agents react directly to interest rates. While the denominator captures the multiplier, indirect effect(s). Turning off each channel in turn recovers the previous individual channels, each of which delivers a multiplier by scaling up the aggregate MPC, as described above. Putting the two channels together compounds the aggregate MPC and thus yields a double multiplier amplification: the two indirect effects interact non-linearly at each round, acting as multipliers of each other. Another way of appreciating the interaction of these two channels from expression (18) is to note that the multiplier due to the capital-inequality channel $\frac{1-I_Y}{1-\eta I_Y}$ appears as a multiplier "inside" (in the sense of multiplying the respective MPC of) the multiplier associated with the income-inequality channel, $\frac{1-\lambda}{1-\lambda\chi}$, and vice-versa.[4]

As we will show, this complementarity turns out to be very general and does not depend in any way on the simplifying assumptions adopted here. In the next section, we confirm our results in a fully-specified heterogeneous-agent model and verify the robustness of our findings with respect to different modeling assumptions and a wide range of empirically plausible parameterizations. Furthermore, in Appendix B.4, we reproduce all the analytical findings of this section in an analytically tractable case of the full model—illustrating again that none of the results here are driven by the simplifying assumptions on the income distribution and the savings technology.

3 A Tractable HANK Model with Capital

We propose a novel heterogeneous-agent model, drawing on elements from both the TANK and HANK literatures. Compared to the simple model from Section 2, this model will not only allow us to make a first step towards quantifying the complementarity and analyze its robustness with respect to different model features but will also enable us to study the role of different redistributive fiscal policies.

The economy comprises households, firms and a fiscal and monetary authority. The New Keynesian block is standard, therefore we focus here on the household side and the fiscal scheme. A full overview of the model including derivations can

[4]This interpretation links the two distinct channels that differentiate the early "TANK" contributions: investment in physical capital (Galí et al., 2007), versus income, i.e. receiving profits from holding shares in monopolistic firms, or not (Bilbiie, 2008). The current HANK literature focuses predominantly on the latter and its link with risk, self-insurance, and precautionary saving. We explore the former and its complementarity with the latter.

be found in Appendix B. As above, we denote variables in levels by uppercase and log-deviations by lowercase letters.

There are two types of households; a share $\lambda \in [0,1)$ *hand-to-mouth* (H) and a share $(1-\lambda)$ *savers* (S). All households have the same CRRA preferences in consumption and labor $U(C,N) = \frac{C^{1-\sigma^{-1}}}{1-\sigma^{-1}} - a\frac{N^{1+\varphi}}{1+\varphi}$, where the σ^{-1} is relative risk aversion and φ is the inverse labor elasticity. We incorporate idiosyncratic risk by assuming that households switch exogenously between types. In particular, the exogenous change of type follows a Markov chain: the probability to stay a saver is s and the probability to remain hand-to-mouth is h (with transition probabilities $1-s$ and $1-h$, respectively).

There is also limited asset market participation. The hand-to-mouth hold no assets, and thus consume their labor income and any redistributive transfers from the government:

$$C_t^H = \frac{W_t}{P_t} N_t^H + T_t^H, \tag{20}$$

where W_t is the nominal wage, P_t is the aggregate price level, N_t^H is hours worked and T_t^H are transfers from the government.

Savers hold and price all assets: risk-free bonds B_t^S, with a risk-free return of $\frac{1+r_{t-1}^n}{1+\pi_t}$ (in real terms); stocks ω_t, which are a claim to the firm dividends D_t (in real terms); physical capital K_t, which they rent out at rate R_t^K. Importantly, bonds are liquid and can be used to self-insure against idiosyncratic risk while stocks and capital are illiquid. This is reflected in the bond Euler equation:

$$(C_t^S)^{-\frac{1}{\sigma}} = \beta E_t \left\{ \frac{1+r_t^n}{1+\pi_{t+1}} \left[s(C_{t+1}^S)^{-\frac{1}{\sigma}} + (1-s)(C_{t+1}^H)^{-\frac{1}{\sigma}} \right] \right\}, \tag{21}$$

where β is the discount factor. In contrast, the Euler equations for illiquid capital and stocks are standard and relegated to the Appendix. This is a tractable way of introducing idiosyncratic risk and liquidity, key ingredients of full-blown HANK models. Note that the budget constraint also has to account for the flows of liquid assets between types, see Appendix B for details.

To facilitate the introduction of sticky wages in Section 3.3, we assume that the labor market is centralized: a union pools labor inputs and sets wages on behalf of both households. This results in a "labor-supply-like" wage schedule, which in log-linear form reads:

$$\varphi n_t = w_t - \sigma^{-1} c_t, \tag{22}$$

and a uniform allocation of hours $N_t^H = N_t^S = N_t$. While the choice of this labor market setting simplifies the analysis, it is not essential for any of our results.

The government taxes dividends and capital income at rates τ^D and τ^K, respectively, and redistributes all revenues from capital income and profits taxation, running a balanced budget in every period:

$$\lambda T_{H,t} = \tau^D D_t + \tau^K R_t^K K_t. \tag{23}$$

We close the model by assuming a monetary policy rule of the form $r_t^n = \phi_\pi \pi_t + \varepsilon_t$. The policy experiment we will consider is a shock, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, to this policy rule.[5]

The complete set of equilibrium conditions, log-linearized around the symmetric steady state $C^H = C^S = C$, can be found in Appendix B. We think the symmetric steady state is a reasonable benchmark, however, the assumption turns out to be inconsequential for all our results, see Appendix C.3.

The model nests the RANK model ($\lambda = 0$) and the simple TANK model ($s = h = 1$). Furthermore, it nests a version without capital by considering a version with infinite adjustment cost ($\omega = 0$) and no depreciation ($\delta = 0$).

3.1 Quantifying the Complementarity

We are now ready to study the channels identified in Section 2 by considering variants of our model with and without capital as well as under different redistribution schemes for fiscal policy. To isolate the role of income inequality, we shut down the capital inequality channel by considering a version of the model without capital and no redistribution ($\tau^D = \tau^K = 0$). To isolate the role of capital inequality, we assume that financial income is fully redistributed ($\tau^D = \tau^K = \lambda$) so that all households get the same total income and differ only on the expenditure side.[6] In this way, we quantify the marginal contribution of each channel as well as their complementarity. Throughout the analysis, we focus on the response of consumption to an expansionary monetary policy shock and use the multiplier in the RANK model without capital as benchmark.

We parameterize the model as follows. The time period is a quarter, implying a discount factor β of 0.99 and a depreciation rate δ of 0.025. We assume logarithmic

[5] Alternatively, we have also considered the case when the central bank controls the real rate, as in Section 2, which is equivalent to a Taylor rule with a coefficient of 1 on expected inflation. However, it turns out that the model is indeterminate under such a rule if we allow for idiosyncratic risk, at least in the countercyclical inequality region that we are interested in (for more information, see the modified Taylor principle in Bilbiie, 2018). Nevertheless, the results under real rate targeting—abstracting from idiosyncratic risk—turn out to be qualitatively robust to the simple Taylor rule considered here.

[6] Note that taxing capital not only affects the model dynamics but also the steady-state capital stock. In Appendix C.4, we show that our results are robust to keeping the steady-state capital stock fixed across specifications.

Table 1: Amplification of the Effects of Monetary Policy on Consumption

	Rep. agent	Heterogeneous agents		
		Prop. incomes	Inequality	Inequality and risk
No capital	1.00	1.00	1.51	1.60
Capital	0.66	1.11	2.25	2.62

Notes: Impact multipliers on aggregate consumption of an interest-rate cut in each model, relative to the representative agent-no capital benchmark. The heterogeneous-agent models are with: no income inequality and no risk in the second column; income inequality and no risk in the third; both income inequality and risk in the fourth column.

utility in consumption and unit labor supply elasticity ($\sigma = 1$, $\varphi = 1$), a capital share of $\alpha = 0.33$ and capital adjustment costs delivering an investment elasticity to marginal Q of 10. The Phillips curve is relatively flat with slope $\psi = 0.05$, the Taylor coefficient is 1.5, and the shock persistence is 0.6. All of these values are standard in the literature. We set the share of hand-to-mouth to $\lambda = 0.27$, in line with the estimates of Kaplan, Violante, and Weidner (2014) and Cloyne, Ferreira, and Surico (2020). We start by abstracting from idiosyncratic risk ($s = 1$) to underscore that the channels emphasized in this paper are present even in the absence of risk and precautionary behavior. Later, we turn idiosyncratic risk back on and analyze how our results are affected.

In Table 1, we record the values of the impact multipliers on consumption for an expansionary monetary policy shock across different specifications, relative to the response in the RANK model without capital. The first column reveals that introducing capital has a dampening effect in the representative-agent case, where the multiplier becomes just two-thirds of that in the model without capital. On the other hand, capital has an amplifying effect of 11% in the heterogeneous-agent model of column (2) with full income redistribution. This is the *capital inequality channel* that we have isolated in Section 2 at work.

In the model with no capital and no income redistribution in the third column, the effects of monetary policy on consumption are magnified by a factor of 1.51. This amplification works through the *cyclical income inequality channel* of Bilbiie (2008).[7] Finally, capital and income inequality *jointly* yield a multiplier of 2.25, which is substantially larger than the product of the two channels in isolation (1.11×1.51): the complementarity is quantitatively significant. This is the *multiplier of the multiplier*.

[7] Note that this model collapses to the representative agent model under full redistribution.

The previous analysis abstracts from idiosyncratic risk and different degrees of asset liquidity, which lie at the center of heterogeneous-agent models (i.a. Kaplan, Moll, and Violante 2018; Bayer et al. 2019). Our framework allows us to incorporate these features in a tractable way, where idiosyncratic uncertainty pertains to households' switching between types. We now turn these channels on, by assuming that savers face a 2% probability to become hand-to-mouth, $s = 0.98$.[8]

The results are depicted in the last column of Table 1. Idiosyncratic risk generates further amplification, especially in models with capital investment, thereby reinforcing the complementarity that we have identified ($2.62 > 1.15 \times 1.60$). It is also interesting to note that idiosyncratic risk amplifies the capital inequality channel even when income is perfectly redistributed. In contrast, in the model without capital, idiosyncratic risk only has an effect if incomes are not proportional. Finally, we note that capital and income inequality are still quantitatively important in shaping the amplification of the effects of monetary policy on consumption, even when compared to the idiosyncratic risk channel. An important difference, however, is that idiosyncratic risk magnifies not only the output and consumption responses but also the investment response, which in contrast gets dampened by the other channels.

3.2 Fiscal Redistribution

Our results suggest that the redistribution of income plays an important role in the transmission of aggregate-demand shocks. Yet so far, we have only analyzed two polar cases: full or no redistribution. An important question in models with multiple assets and different sources of financial income is how different types of income are redistributed and how this alters the transmission mechanism. In this section, we analyze two other relevant cases within the most general model specification with risk, capital and income inequality: (i) when only capital income is redistributed, and (ii) when only monopoly profits are redistributed.

The main finding is that redistributing only capital income amplifies further the effects of monetary policy shocks: the consumption multiplier becomes 4.34 instead of 2.62 (the case with no redistribution, bottom right entry of Table 1). The intuition for this result is that capital income is *highly procyclical* and therefore its redistribution towards constrained households makes their income more cyclical. This, in turn, increases the slope of the Keynesian cross and boosts the consumption

[8] Our stark notion of illiquidity implies that savers hit by a negative shock cannot take any capital and stocks with them. In Appendix B.5, we alternatively model capital as perfectly liquid: savers can *also* use capital to self-insure, and liquidity is in positive supply (which equals the capital stock). The results are comparable.

multiplier. In contrast, redistributing monopoly profits, which are countercyclical, dampens the income cyclicality of hand-to-mouth agents and can even reverse the aggregate-demand amplification: the effects of monetary policy on consumption goes now down to 0.5. For more details, see Appendix C.2.

3.3 Sticky Wages

We have shown that the redistribution of financial income can have large effects on the cyclical properties of the model. One potential concern, however, is that markups and thus profits are countercyclical herein. An avenue that the literature pursued to overstep this unappealing feature of the New-Keynesian framework are wage rigidities.[9] When wages are rigid, an aggregate demand expansion makes marginal costs increase by less, markups fall by less and sales increase by more, which can mitigate and even overturn the response of profits.

We introduce wage rigidities following Colciago (2011), assuming that the labor union faces wage-setting frictions: the nominal wage can only be re-optimized with a constant probability $1 - \theta_w$. This gives rise to a standard wage Phillips curve that connects nominal wage inflation to wage markups. We parameterize the slope of the wage Phillips curve to 0.075, which in a Calvo interpretation and given the other parameter values implies an average wage spell of slightly more than four quarters. The results of all models with sticky wages are recorded in Table 2, relative to the (sticky-wage) representative agent benchmark.

Table 2: The Role of Sticky Wages

	Rep. agent	Heterogeneous agents		
		Prop. incomes	Inequality	Inequality and risk
No capital	1.00	1.00	1.01	1.02
Capital	0.94	1.53	1.77	1.95

Notes: Impact multipliers on aggregate consumption of an interest-rate cut in each model with sticky wages, relative to the rep.-agent no-capital benchmark with sticky wages. The heterogeneous-agent models are with: no income inequality and no risk in the second column; income inequality and no risk in the third; both income inequality and risk in the fourth.

Two main results emerge from Table 2. First, the complementarity between capital and income inequality in amplifying the effects of aggregate-demand shocks to consumption is robust to introducing sticky wages, both without (1.77

[9] See for instance Colciago (2011) for an early two-agent model and Broer et al. (2020) in the context of the recent HANK literature.

$> 1.53 \times 1.01$) and with idiosyncratic risk ($1.95 > 1.61 \times 1.02$). Second, capital inequality and income inequality—on their own—generate modest additional amplification over and above sticky wages. While this is apparent for income inequality by moving across the columns of the first row of Table 2, it can be appreciated for capital inequality by comparing the first two columns of the second row with their flexible wage counterparts in Table 1. More specifically, the impact of sticky wages (relative to the flexible wage case) on the monetary transmission to consumption in the representative agent model with capital is as large as its *relative* impact in the proportional income model with capital (i.e. the ratio between the representative agent cases with sticky and flexible wages is $0.94/0.66 \approx 40\%$, which is very close to the ratio of 1.53/1.11 between sticky and flexible wage models under proportional incomes).

In summary, sticky wages—by introducing an additional source of non-neutrality—amplify significantly the effects of aggregate-demand shocks on consumption in both the representative agent case and in the model with proportional incomes. In the presence of both capital and income inequality, however, sticky wages alters the transmission of demand shocks only modestly (i.e. 1.95 and 1.77 in Table 2 under sticky wages are actually smaller than 2.62 and 2.25 in Table 1 under flexible wages) and the bulk of the monetary policy amplification still comes from the complementarity between capital and income inequality.

4 The Cyclicality of Consumption and Income Inequality

So far, we studied capital and income inequality as *transmission channels* for aggregate consumption dynamics. In this section, we analyze the implications of our framework for the distribution of income and consumption as *outcomes*.[10] We first derive theoretical predictions based on the simple model from Section 2, confirm them quantitatively in the fully-specified HANK model from Section 3 and then confront the predictions with the available empirical evidence.

In our framework, the dispersion in income and consumption across households is simply measured by the difference between savers' and spenders' (log) variables, which are: $x^S - x^H,\ x \in \{y, c\}$.[11] As we focus on one type of dis-

[10] The 2019 working paper version of this paper contains a more detailed analysis of the aggregate and inequality dynamics in comparison with the empirical evidence.

[11] This corresponds to the log-deviation of the ratio of the variables of savers and hand-to-mouth, as in Bilbiie (2018). In a model with two agents, this definition is equivalent to the Gini coefficient or measures of entropy.

turbances (i.e. demand shocks) only, throughout the paper we use the word 'cyclicality' to refer to cyclicality *conditional* on aggregate demand disturbances such as monetary policy shocks (as opposed to conditional on exogenous movements in aggregate supply, which we abstract from).[12]

In the simple saver-spender model, it is easy to show that income inequality is:

$$y_t^S - y_t^H = \frac{1-\chi}{(1-\lambda)Y_Y^S} y_t. \tag{24}$$

As explained in Section 2, income inequality is countercyclical iff $\chi > 1$, which is also the condition required for amplification.

Consumption inequality is instead given by:

$$c_t^S - c_t^H = \frac{1-\chi C_Y}{(1-\lambda)C_Y} y_t - \frac{I_Y}{(1-\lambda)C_Y} i_t. \tag{25}$$

Under our simplifying isoelastic investment function (2), this reduces to:

$$c_t^S - c_t^H = \frac{1-\chi C_Y - \eta I_Y}{(1-\lambda)C_Y} y_t. \tag{26}$$

Proposition 3 (Countercyclical consumption inequality). *Consumption inequality is countercyclical iff:*

$$C_Y(\chi - 1) + I_Y(\eta - 1) > 0. \tag{27}$$

If investment is more than one-to-one procyclical $\eta > 1$, then consumption inequality is more countercyclical than income inequality.

Proof. The first part follows automatically by rewriting $1 - \chi C_Y - \eta I_Y < 0$. For the second part, rewrite (25), using $(1-\lambda)Y_Y^S = 1 - \lambda C_Y$, as:

$$c_t^S - c_t^H = y_t^S - y_t^H + \frac{I_Y}{(1-\lambda)C_Y}\left(\frac{1-\lambda\chi C_Y}{1-\lambda C_Y} y_t - i_t\right)$$

For consumption inequality to be more countercyclical than income inequality, we need the term in brackets to be countercyclical, that is investment to be procyclical "enough". Replacing (2), the condition is:

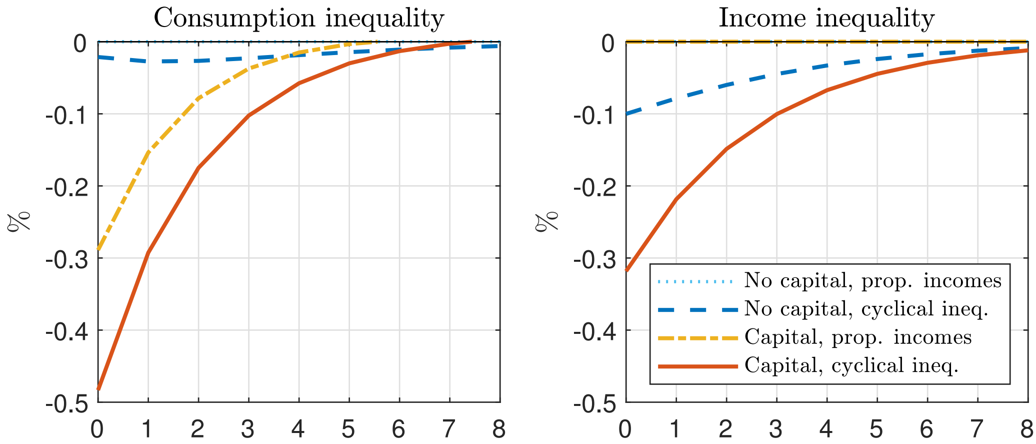
$$\lambda C_Y(\chi - 1) + (1 - \lambda C_Y)(\eta - 1) = \lambda\left[C_Y(\chi - 1) + I_Y(\eta - 1)\right] + (1-\lambda)(\eta - 1) > 0.$$

Since the term in square brackets is positive when consumption inequality is countercyclical, a *sufficient* condition for this to be satisfied is that $\eta > 1$. ■

[12] For a detailed treatment of the unconditional cyclicality of consumption and income inequality in TANK models, see Maliar and Naubert (2019).

It turns out that these results generalize to our richer setting with a fully-specified supply side, rigidities in prices and wages and idiosyncratic risk. Figure 2 shows the responses of consumption and income inequality under four different specifications of our HANK model: (i) no capital and proportional incomes (dotted light blue line), (ii) with capital and proportional incomes (broken dark blue line), (iii) no capital and cyclical inequality (broken and dotted yellow line) and (iv) with capital and cyclical income inequality (red solid line).

Figure 2: Distributional Effects of Monetary Policy



Notes: Impulse responses of consumption inequality and income inequality to an expansionary interest rate shock of 25 basis points in heterogeneous-agent models with and without capital inequality and with and without income inequality.

We can see that the model with capital and income inequality (red solid line) confirms the predictions of our simple framework: both consumption and income inequality are countercyclical and consumption inequality turns out to be more countercyclical.[13] The intuition goes as follows. From above, consumption inequality is countercyclical if and only if $C_Y\chi + I_Y\frac{\partial i_t}{\partial y_t} > 1$. Since in the model (as in the data), investment is more than one-to-one procyclical, consumption inequality is always more countercyclical than income inequality. Interestingly, consumption inequality can be countercyclical even if income inequality is acyclical or procyclical: this would be the case for any values of χ and η such that $C_Y(\chi - 1) + I_Y(\eta - 1) > 0$.

We can also see that both the capital and income inequality channels are instrumental for this result. First, the model without capital and proportional

[13] In Appendix C.5, we show that this is a robust implication of our model and does not depend on the specific parameterization used. Note that while we do not explicitly model the primitive source of income inequality and risk, our modeling approach can be thought as a parable for salient labor market features such as heterogeneity in productivity, skills and the divide between employed and unemployed workers.

incomes features no inequality dynamics as it collapses to the representative-agent model. If we only allow for the cyclical income inequality channel, both income and consumption inequality are countercyclical but consumption inequality is not more countercyclical than income inequality.[14] If on the other hand we allow only for the capital inequality channel, we observe a considerable drop in consumption inequality but income inequality does not change by construction.

How do these predictions compare with existing empirical evidence? A growing empirical literature studies the distributional effects of monetary policy using micro data on household consumption expenditure and income (see for instance Coibion et al. (2017) for the U.S., Ampudia et al. (2018) for the euro area and Mumtaz and Theophilopoulou (2017) for the U.K.). These studies show that following a cut in the interest rate both consumption and income inequality fall significantly. Importantly, consumption inequality robustly turns out to decline more than income inequality—in line with the predictions of our model. This illustrates the empirical relevance of the capital and income inequality channels, which we have shown to be instrumental in generating these predictions on the cyclicality of consumption and income inequality.

5 Sensitivity Analysis

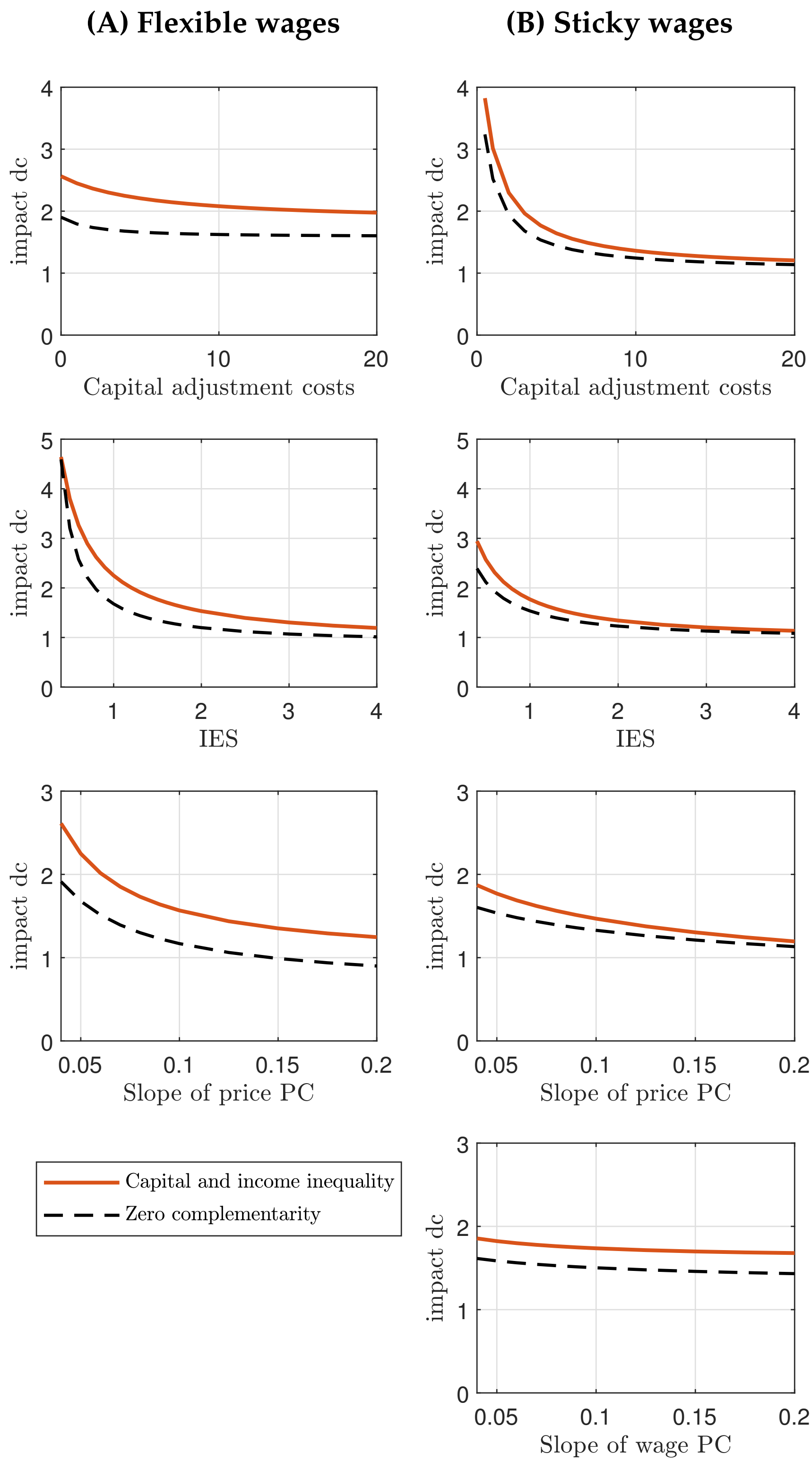
We have shown that capital and income inequality can lead to sizeable aggregate-demand amplification. In this section, we analyze the robustness of this finding quantitatively with respect to a wide range of empirically plausible values for key parameters such as capital adjustment costs[15], the intertemporal elasticity of substitution (IES) and price and wage stickiness.

The findings are illustrated in Figure 3. The column on the left (right) pertains to the case of flexible (sticky) wages. In each panel, we depict two multipliers as a function of the parameter of interest: the impact multiplier in the model with capital and income inequality (solid red line) and an artificial line capturing the multiplier that would obtain in the case of no complementarity (black dashed line labeled 'zero complementarity'). The latter is calculated as the product of the two multipliers in isolation. If the capital and income inequality line is above the zero complementarity line, this means that the two channels are complementary to

[14] In fact, income inequality turns out to be even more countercyclical than consumption inequality, however, this is an artifact of the constant, redistributive steady-state transfers that are used to equalize consumption in steady state. In the version of the model without the transfers, the two variables are equally cyclical, while all other implications are preserved (see Appendix C.3).

[15] Note that we express the multipliers here as a function of the capital adjustment cost parameter $\phi = 1/(\delta\omega)$ and not ω, the elasticity of investment to Tobin's Q.

Figure 3: Robustness of the Complementarity



Notes: Sensitivity of the consumption impact multipliers in the model with capital and income inequality together with the artificial multiplier in the zero complementarity case under different parameterizations for capital adjustment costs, IES, and price stickiness. The red solid line shows the multiplier of the model with capital and income inequality and the black dashed line shows the product of the two multipliers in isolation (i.e. the multiplier that would obtain if the channels were not complementary). Panel (A) shows the multipliers under flexible wages, Panel (B) under sticky wages.

each other. Along the horizontal axes we vary: capital adjustment costs in the first row, the IES in the second row, the slope of the price Phillips curve in the third row and the slope of the wage Phillips curve in the last row. In each row, we keep all other parameters fixed to the values used in the previous section and vary only the parameter of interest in that row.

The key takeaway from Figure 3 is that the complementarity of the capital and income inequality channels turns out to be robust within a wide range of empirically plausible values for the key parameters of interest. In Figure C.5 we also present the sensitivity analysis for the absolute impact responses of all our model specifications as opposed to the multipliers relative to the representative-agent benchmark. We can see that while the absolute responses are decreasing with capital adjustment costs and the frequency of prices and wages adjustments and increasing with the elasticity of intertemporal substitution, the relative multipliers are decreasing in all these parameters.

6 Conclusions

The idea that the combination of a consumption function and an investment function gives rise to amplification of aggregate demand fluctuations is an intuition that goes back to Samuelson (1939), who attributed it to Alvin Hansen in building the now famous multiplier-accelerator model.

In this paper, we explore this idea in a New Keynesian model with household heterogeneity in both income and savings and show that this gives rise to an aggregate-demand *complementarity* that is to the best of our knowledge novel to the literature. Namely, we isolate two key types of inequality, in capital and income, that each give rise to a distinct multiplier-like amplification channel. The former (segmentation in capital markets) leads to *amplification*, even when income is redistributed uniformly. This occurs as capital income is endogenously redistributed towards constrained households, who consume it and generate further demand, thus triggering a Keynesian-cross multiplier.

Counter-cyclical income inequality sets in motion further aggregate-demand amplification rounds as the income of constrained agents respond more than proportionally to fluctuations in aggregate income. We show that, together, the capital inequality and the income inequality channels engender aggregate-demand effects on consumption that are an order of magnitude larger than the mere addition of their individual effects in isolation: a strong complementarity that we call *'the multiplier of the multiplier'*.

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Online Appendix

A Analytical Model Derivations

In this appendix, we derive the stylized model from Section 2 from first principles. The model consists of two types of agents: a share of $1-\lambda$ savers S and a share λ hand-to-mouth spenders H. We remain agnostic about the supply-side and assume that the central bank can directly control the real interest rate (or, alternatively, prices are fixed).

Savers. Savers hold and price all assets. They have access to a risk-free bond and also invest in capital. Their behavior is characterized by a standard Euler equation for bonds:

$$(C_t^S)^{-1} = \beta E_t[(1+r_t)(C_{t+1}^S)^{-1}]. \tag{28}$$

We assume that savers invest in capital according to an investment function:

$$I_t = f(Y_t, r_t, ...). \tag{29}$$

For now, we remain agnostic about the exact functional form of the investment function. Later, we will consider two variants.

The budget constraint reads:

$$C_t^S + \frac{1}{1-\lambda} I_t = \tilde{Y}_t^S + T^S = Y_t^S, \tag{30}$$

where we have already imposed that bonds are in zero net supply. $\tilde{Y}_t^S$ is the income of the savers and T^S are *steady-state, constant* redistributive transfers that serve to control *steady-state* consumption across agents. We define Y_t^S as the post-transfer income of the savers.

Hand-to-Mouth. The hand-to-mouth spenders do not have access to bonds and capital markets. Their behavior is subject to their budget constraint:

$$C_t^H = \tilde{Y}_t^H + T^H = Y_t^H, \tag{31}$$

where T^H are again *steady-state, constant* redistributive transfers/taxes that serve to control the steady-state consumption distribution and Y_t^H is the hand-to-mouth's post-transfer income.

Market clearing and income distribution. Goods market clearing (the economy resource constraint) is the aggregation of the individual resource constraints (30) and (31) with weights λ and $1-\lambda$ respectively, i.e.:

$$C_t + I_t = Y_t, \tag{32}$$

where aggregate output, consumption, and investment are given by:

$$\begin{aligned} Y_t &= \lambda Y_t^H + (1-\lambda) Y_t^S, \\ C_t &= \lambda C_t^H + (1-\lambda) C_t^S, \\ I_t &= (1-\lambda) I_t^S. \end{aligned} \tag{33}$$

To close the model, we need to specify how the income distribution is determined; we will specify this in log-linear terms below and consider two cases: proportional incomes and cyclical income inequality.

Steady state. We focus on a steady state where both households have the same consumption. We achieve this by choosing the fixed, steady-state transfers T^S, T^H to ensure that $C^H = C^S = C$ under the restriction that the government budget is balanced, i.e. $\lambda T^H + (1-\lambda) T^S = 0$. From the budget constraint of the spenders, this further implies $Y^H = C$.

From the investment function, we obtain steady-state investment to output ratio, $I_Y \equiv \frac{I}{Y} = \frac{f(Y,r,...)}{Y}$ and from market clearing, we obtain the consumption to output ratio, $C_Y \equiv \frac{C}{Y} = 1 - \frac{I}{Y}$.

Loglinearized model. Log-linearizing the model equations around the symmetric steady state, we have

$$c_t^S = E_t c_{t+1}^S - r_t$$

for the Euler equation. The budget constraint of the savers becomes:

$$C_Y c_t^S + \frac{I_Y}{1-\lambda} i_t = Y_Y^S y_t^S,$$

where $Y_Y^S \equiv \frac{Y^S}{Y}$. For the hand-to-mouth, we have:

$$c_t^H = y_t^H.$$

The loglinearized market clearing condition is:

$$y_t = C_Y c_t + I_Y i_t.$$

Aggregate consumption and income are given by

$$c_t = \lambda c_t^H + (1-\lambda) c_t^S$$
$$y_t = \lambda Y_Y^H y_t^H + (1-\lambda) Y_Y^S y_t^S.$$

Finally, concerning the determination of changes in the income distribution, we assume directly that the (post-transfer) income of the hand-to-mouth responds to aggregate income with an elasticity χ, that is in loglinearized form:

$$y_t^H = \chi y_t.$$

Income of the savers is thus given by

$$y_t^S = \frac{1-\lambda\chi Y_Y^H}{(1-\lambda)Y_Y^S} y_t.$$

To turn off the cyclical income inequality channel, we let $\chi = 1$. In this case, both the income of spenders and savers is proportional to aggregate income, i.e. $y_t^H = y_t^S = y_t$. If $\chi > 1$, income inequality is countercyclical as discussed in the main body of the paper.

A.1 Isolating the Capital Inequality Channel

We study now the capital inequality channel in detail. To this end, we assume that incomes are proportional, i.e. $\chi = 1$. Our goal is to analyze the conditions under which this channel can generate amplification relative to the RANK benchmark.

Under proportional incomes, we have that

$$c_t^H = y_t.$$

Replacing this in the aggregate consumption and using the economy resource constraint $y_t = C_Y c_t + I_Y i_t$ gives:

$$\begin{aligned} c_t^S &= \frac{1}{1-\lambda} c_t - \frac{\lambda}{1-\lambda} \left(C_Y c_t + I_Y i_t \right) \\ &= \frac{1-\lambda C_Y}{1-\lambda} c_t - \frac{\lambda}{1-\lambda} I_Y i_t. \end{aligned}$$

By replacing the above expression in the savers' Euler equation for bonds we obtain:

$$c_t = E_t c_{t+1} + \frac{\lambda I_Y}{1-\lambda C_Y} \left(i_t - E_t i_{t+1} \right) - \frac{1-\lambda}{1-\lambda C_Y} r_t \tag{34}$$

There is amplification relative to RANK if investment is sufficiently responsive to an interest rate cut, that is:

$$\begin{aligned}
\frac{d\left(c_t - E_t c_{t+1}\right)}{d\left(-r_t\right)} &= \frac{\lambda I_Y}{1-\lambda C_Y}\frac{d\left(i_t - E_t i_{t+1}\right)}{d\left(-r_t\right)} + \frac{1-\lambda}{1-\lambda C_Y} > 1 \\
\text{or} \quad \frac{d\left(i_t - E_t i_{t+1}\right)}{d\left(-r_t\right)} &> 1
\end{aligned}$$

In other words, investment needs to be procyclical enough. The procyclicality of investment is one of the most salient feature of the data. Thus, in any empirically plausible model featuring investment, there will be amplification of the consumption response through heterogeneity—even under proportional incomes.

A.2 Isoelastic Investment

In the main body of the paper, we consider the case where investment is an isoelastic function of total income. Clearly, this is a stylized investment function that serves the purpose to illustrate the capital inequality channel in a simple and transparent way. In this appendix, we consider an extension, where we allow investment also to depend on the interest rate. Despite adding realism, this also serves the purpose to show that our result does not depend on the specifics of the investment function.

Investment is now an isoelastic function of total income and the interest rate:

$$i_t = \eta_y y_t - \eta_r r_t, \tag{35}$$

where $\eta_y > 0$ and $\eta_r > 0$ are the elasticities to output and the interest rate, respectively.

Substituting the economy resource constraint we get:

$$i_t = \frac{\eta_y\left(1 - I_Y\right)}{1-\eta_y I_Y} c_t - \frac{\eta_r}{1-\eta_y I_Y} r_t$$

Using this, we can solve for the savers' consumption from the definition of aggregate consumption, combined with the consumption of the spenders and and the resource constraint:

$$c_t^S = \frac{1-\lambda\chi\frac{1-I_Y}{1-\eta_y I_Y}}{1-\lambda} c_t + \frac{\lambda\chi\frac{\eta_r I_Y}{1-\eta_y I_Y}}{1-\lambda} r_t$$

Replacing this in the savers' Euler equation yields:

$$c_t = E_t c_{t+1} - \frac{\lambda \chi I_Y}{1 - \lambda \chi \frac{1-I_Y}{1-\eta_y I_Y}} \frac{\eta_r}{1 - \eta_y I_Y} \left(r_t - E_t r_{t+1} \right) - \frac{1-\lambda}{1 - \lambda \chi \frac{1-I_Y}{1-\eta_y I_Y}} r_t \tag{36}$$

The multiplier to a purely transitory shock is thus

$$\frac{d \left(c_t - E_t c_{t+1} \right)}{d \left(-r_t \right)} = \frac{1 - \lambda + \lambda \chi I_Y \frac{\eta_r}{1-\eta_y I_Y}}{1 - \lambda \chi \frac{1-I_Y}{1-\eta_y I_Y}}.$$

We can see that the multiplier is now higher relative to the case where investment is only a function of output. Furthermore, there is amplification even in the "Solow" case $\eta_y = 1$ with proportional incomes $\chi = 1$; the multiplier in that case is $1 + \frac{\lambda}{1-\lambda} \eta_r \frac{I_Y}{1-I_Y}$.

From the above, we can also see that it is straightforward to extend the present analysis to include other variables and in particular expectations about the future (e.g. future output) in the investment rule. Only the algebra would become a bit more cumbersome.

A.3 The Capital Inequality Channel as a Reappraisal of Samuelson (1939)

In this appendix, we make the relation to Samuelson (1939) transparent. Consider a static version of Samuelson's model (page 76),[16] whereby consumption is a fraction α_s of current (instead of lagged) income, and investment a fraction β_s of consumption (rather than the growth rate of consumption), i.e. in log-linear form:

$$\begin{aligned} y_t &= \left(1 - I_Y \right) c_t + I_Y i_t + \varepsilon_t \\ c_t &= \frac{\alpha_s}{1 - I_Y} y_t \\ i_t &= \beta_s \frac{1 - I_Y}{I_Y} c_t = \frac{\alpha_s \beta_s}{I_Y} y_t, \end{aligned} \tag{37}$$

where ε_t is an aggregate-demand shock, e.g. public spending. Solving for output, we get the expression for the multiplier:

$$y_t = \frac{1}{1 - \left(\alpha_s + \alpha_s \beta_s \right)} \varepsilon_t. \tag{38}$$

Consider now a variant of our simple saver-spender model under *proportional*

[16] Samuelson attributes the idea and model to Alvin Hansen.

incomes with the same aggregate-demand shock ε_t. As the MPC of the hand-to-mouth is one ($c_t^H = y_t$) and consumption of savers is fixed for simplicity by the Euler equation with fixed real interest rates[17], the short-run aggregate consumption function can be written as:

$$c_t = \lambda y_t. \tag{39}$$

Recall that aggregate investment is given by $i_t = \eta y_t$. Replacing this in the resource constraint $y_t = (1 - I_Y) c_t + I_Y i_t + \varepsilon_t$, we get the multiplier:

$$y_t = \frac{1}{1 - (\lambda C_Y + \eta I_Y)} \varepsilon_t, \tag{40}$$

which is essentially the same as in the static version of the Samuelson 1939 model (38). Most importantly, in the absence of (or for exogenous) investment, both cases boil down to the first-year undergraduate-textbook Keynesian-cross multiplier (also formalized by Samuelson, 1948), $1/(1 - MPC)$, where the MPC is given in the first case by α_s and in the second by λ the population share of unit-MPC spenders.

Thus, the capital inequality channel is observationally equivalent to (a static version of) Samuelson's multiplier-accelerator channel when it comes to aggregates. Relative to Samuelson, we generalize and microfound this channel in a setting with MPC heterogeneity and segmented capital markets. Most importantly, we study the interactions between the capital inequality and the cyclical income inequality channels and uncover a novel complementarity.

B Tractable HANK Model: Detailed Exposition and Derivations

The tractable HANK model (THANK) sketched out in Section 3 is a particular equilibrium of a more general model, which we outline here. Furthermore, we detail the assumptions under which it is possible derive the tractable equilibrium representation used in this paper.

[17]The loglinearized Euler equation for savers $c_t^S = E_t c_{t+1}^S - \sigma r_t$ trivially implies $c_t^S = 0$ when the real rate is fixed at all times, $r_{t+j} = 0$.

B.1 Model

The economy comprises households, firms and a government, consisting of a fiscal and a monetary authority. We discuss each sector in turn.

Households. There is a unitary mass of households, indexed by j. Households have the same CRRA preferences, $U(C, N) = \frac{C^{1-\sigma^{-1}}}{1-\sigma^{-1}} - a\frac{N^{1+\varphi}}{1+\varphi}$, and discount the future at rate β. Families have access to three assets: a risk-free bond, shares in imperfectly competitive firms, and physical capital. As discussed in the main text, we assume that the labor market is centralized: a union pools labor inputs and sets wages on behalf of both households. This results in a "labor-supply-like" wage schedule, in log-linear form:

$$\varphi n_t = w_t - \sigma^{-1} c_t,$$

and a uniform allocation of hours $N_t^H = N_t^S = N_t$.

Households participate infrequently in financial markets. When they do, they can freely adjust their portfolio and receive dividends from firms and capital income. We call this the savers' state (S). When agents do not participate in financial markets, they can use only bonds to smooth consumption. We call this the hand-to-mouth state (H). We denote by s the probability to keep participating in stock and capital markets in period $t+1$, conditional upon participating at t, i.e. $s = p(s_{t+1}^j = S|s_t^j = S)$, where s_t^j is the current state of household j. Similarly, we call h the probability to keep being excluded from financial markets, i.e. $h = p(s_{t+1}^j = H|s_t^j = H)$. Hence, the probability to become a financial market participant is $(1-h)$. The share of hand-to-mouth households thus evolves as $\lambda_{t+1} = h\lambda_t + (1-s)(1-\lambda_t)$. We focus on the stationary equilibrium with $\lambda = (1-s)/(2-s-h)$, which is the *unconditional* probability of being hand-to-mouth.

The requirement $s \geq 1-h$ ensures stationary and has a straightforward interpretation: the probability to remain in state S is larger than the probability to move to state S (the conditional probability is larger than the unconditional one). In the limit case of $s = 1-h = 1-\lambda$, idiosyncratic shocks are iid: being S or H tomorrow is independent on whether one is S or H today. At the other extreme stands TANK: idiosyncratic shocks are permanent ($s = h = 1$) and λ stays at its initial value (a free parameter).

We make two key assumptions to obtain a tractable representation. First, there is perfect insurance among the households in a particular state but not between households in different states. Accordingly, we can think of households as living on two different islands and that within each island all resources are pooled. Households on the same island will thus make the same consumption and saving choices. Second, however, we assume that stocks and capital are *illiquid*. When savers can no longer participate in financial markets, they cannot take their stock and capital holdings with them. Only bonds are liquid and can be transferred when switching between islands.

The timing is as follows. At the beginning of every period, resources within types are pooled. The aggregate shocks are revealed and households make their consumption and saving choices. Next, households learn their state in the next period and have to move to the corresponding island accordingly, taking an (equally-split) fraction of the bonds on the current island with them.

The flows across islands are as follows. The total measure of households leaving the H island each period is the number of households who participate next period: $\lambda(1-h)$. The measure of households staying on the island is thus λh. In addition, a measure $(1-\lambda)(1-s)$ leaves the S island for the H island at the end of each period. Recall that our assumptions regarding insurance imply symmetric consumption/saving choices for all households in a given island. Denote by B^S_{t+1} the per-capita beginning-of-period $t+1$ bonds of S (after the consumption-saving choice, and *also after* changing state and pooling). The end-of-period t per capita real values (after the consumption/saving choice but *before* agents move across islands) are Z^S_{t+1}. Likewise, B^H_{t+1} is the per capita beginning-of-period $t+1$ bonds in the H island (where the only asset is bonds). The end-of-period t values (before agents move across islands) are Z^H_{t+1}. We have the following relations:

$$\begin{aligned}
\mathbf{B}^S_{t+1} &= (1-\lambda)B^S_{t+1} = (1-\lambda)sZ^S_{t+1} + \lambda(1-h)Z^H_{t+1} \\
\mathbf{B}^H_{t+1} &= \lambda B^H_{t+1} = (1-\lambda)(1-s)Z^S_{t+1} + \lambda h Z^H_{t+1},
\end{aligned}$$

where $\mathbf{B}^i_{t+1}$, $i \in \{S, H\}$ denote the bond holdings of the entire island. As stocks and capital do not leave the S island, we do not have to keep track of them.

Capital accumulation is simply characterized by:

$$K_{t+1} = (1-\delta)\,K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t,$$

where δ is the depreciation rate and $\Phi(\cdot)$ the adjustment cost function satisfying the standard assumptions $\Phi' > 0$, $\Phi'' \leq 0$, $\Phi'(\delta) = 1$ and $\Phi(\delta) = \delta$.

The program of savers reads

$$V^S(\mathbf{B}_t^S, \omega_t, K_t) = \max_{C_t^S, Z_{t+1}^S, \omega_{t+1}, I_t, K_{t+1}} \quad \frac{(C_t^S)^{1-\sigma^{-1}}}{1-\sigma^{-1}} - a\frac{N_t^{1+\varphi}}{1+\varphi} + \beta E_t V^S(\mathbf{B}_{t+1}^S, \omega_{t+1}, K_{t+1})$$
$$+ \beta\frac{\lambda}{1-\lambda} E_t V^H(\mathbf{B}_{t+1}^H)$$

subject to

$$C_t^S + Z_{t+1}^S + \nu_t\frac{\omega_{t+1}}{1-\lambda} + \frac{I_t}{1-\lambda} = \frac{W_t}{P_t}N_t + \frac{1+r_{t-1}^n}{1+\pi_t}\frac{\mathbf{B}_t^S}{1-\lambda} + \left(\nu_t + (1-\tau^D)D_t\right)\frac{\omega_t}{1-\lambda} + (1-\tau^K)R_t^K\frac{K_t}{1-\lambda}$$
$$K_{t+1} = (1-\delta)\,K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t$$
$$\mathbf{B}_{t+1}^S = (1-\lambda)sZ_{t+1}^S + \lambda(1-h)Z_{t+1}^H$$
$$\mathbf{B}_{t+1}^H = (1-\lambda)(1-s)Z_{t+1}^S + \lambda h Z_{t+1}^H$$
$$Z_{t+1}^S \geq 0.$$

The household internalizes how aggregate bond holdings evolve according to households switching between types. Furthermore, the bond holdings a household takes from an island cannot be negative, i.e. borrowing is not possible.

The first-order conditions read

$$\begin{aligned}
(C_t^S)^{-\frac{1}{\sigma}} &= \Lambda_t^S \\
\Lambda_t^S &= \beta(1-\lambda)sE_t[V_B^S(\mathbf{B}_{t+1}^S, \omega_{t+1}, K_{t+1})] + \beta\lambda(1-s)E_t[V_B^H(\mathbf{B}_{t+1}^H)] + \Xi_t^S \\
\frac{\Lambda_t^S\nu_t}{1-\lambda} &= \beta E_t[V_\omega^S(\mathbf{B}_{t+1}^S, \omega_{t+1}, K_{t+1})] \\
\psi_t^S &= \beta E_t[V_K^S(\mathbf{B}_{t+1}^S, \omega_{t+1}, K_{t+1})] \\
\Lambda_t^S &= (1-\lambda)\psi_t^S\Phi'\left(\frac{I_t}{K_t}\right)
\end{aligned}$$

together with the complementary slackness condition:

$$Z_{t+1}^S\Xi_t^S = 0,$$

with $\Xi_t^S \geq 0$. Λ_t^S, ψ_t^S, and Ξ_t^S are Lagrange multipliers associated with the budget constraint, the capital accumulation equation and the inequality constraint, respectively.

From the Envelope theorem, we have

$$V_B^S(\mathbf{B}_t^S,\omega_t,K_t)=\frac{\Lambda_t^S}{1-\lambda}\frac{1+r_{t-1}^n}{1+\pi_t}$$
$$V_\omega^S(\mathbf{B}_t^S,\omega_t,K_t)=\frac{\Lambda_t^S}{1-\lambda}\left(\nu_t+(1-\tau^D)D_t\right)$$
$$V_K^S(\mathbf{B}_t^S,\omega_t,K_t)=\frac{\Lambda_t^S}{1-\lambda}(1-\tau^K)R_t^K+\psi_t^S\left[1-\delta+\Phi\left(\frac{I_t}{K_t}\right)-\Phi'\left(\frac{I_t}{K_t}\right)\frac{I_t}{K_t}\right].$$

Using this in the FOCs gives

$$(C_t^S)^{-\frac{1}{\sigma}}=\Lambda_t^S$$
$$\Lambda_t^S=\beta s E_t\left[\Lambda_{t+1}^S\frac{1+r_t^n}{1+\pi_{t+1}}\right]+\beta\lambda(1-s)E_t[V_B^H(\mathbf{B}_{t+1}^H)]+\Xi_t^S$$
$$\Lambda_t^S=\beta E_t\left[\Lambda_{t+1}^S\frac{\nu_{t+1}+(1-\tau^D)D_{t+1}}{\nu_t}\right]$$
$$(1-\lambda)\psi_t^S=\beta E_t\left[\Lambda_{t+1}^S(1-\tau^K)R_{t+1}^K+(1-\lambda)\psi_{t+1}^S\left[1-\delta+\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)-\Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right]\right]$$
$$\Lambda_t^S=(1-\lambda)\psi_t^S\Phi'\left(\frac{I_t}{K_t}\right).$$

The marginal Q is defined as the shadow value of installed capital in terms of consumption units, $Q_t=\frac{(1-\lambda)\psi_t^S}{\Lambda_t}$. Using this, we can rewrite the FOCs as

$$(C_t^S)^{-\frac{1}{\sigma}}=\beta s E_t\left[(C_{t+1}^S)^{-\frac{1}{\sigma}}\frac{1+r_t^n}{1+\pi_{t+1}}\right]+\beta\lambda(1-s)E_t[V_B^H(\mathbf{B}_{t+1}^H)]+\Xi_t^S$$
$$(C_t^S)^{-\frac{1}{\sigma}}=\beta E_t\left[(C_{t+1}^S)^{-\frac{1}{\sigma}}\frac{\nu_{t+1}+(1-\tau^D)D_{t+1}}{\nu_t}\right]$$
$$Q_t=\beta E_t\left\{\left(\frac{C_{t+1}^S}{C_t^S}\right)^{-\frac{1}{\sigma}}\left[(1-\tau^K)R_{t+1}^K+Q_{t+1}\left(1-\delta+\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right)-\Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right)\right]\right\}$$
$$1=Q_t\Phi'\left(\frac{I_t}{K_t}\right).$$

The only thing that remains to be determined is $V_B^H(\mathbf{B}_{t+1}^H)$. We can obtain this from the problem of the hand-to-mouth.

Their program reads

$$V^H(\mathbf{B}_t^H)=\max_{C_t^H,Z_{t+1}^H}\quad\frac{(C_t^S)^{1-\sigma^{-1}}}{1-\sigma^{-1}}-a\frac{N_t^{1+\varphi}}{1+\varphi}+\beta E_tV^H(\mathbf{B}_{t+1}^H)+\beta\frac{1-\lambda}{\lambda}E_tV^S(\mathbf{B}_{t+1}^S,\omega_{t+1},K_{t+1})$$

subject to

$$
\begin{aligned}
&C_t^H + Z_{t+1}^H = \frac{W_t}{P_t} N_t + \frac{1+r_{t-1}^n}{1+\pi_t} \frac{\mathbf{B}_t^H}{\lambda} + T_t^H \\
&\mathbf{B}_{t+1}^S = (1-\lambda) s Z_{t+1}^S + \lambda(1-h) Z_{t+1}^H \\
&\mathbf{B}_{t+1}^H = (1-\lambda)(1-s) Z_{t+1}^S + \lambda h Z_{t+1}^H \\
&Z_{t+1}^H \geq 0.
\end{aligned}
$$

The first-order conditions read

$$
\begin{aligned}
&(C_t^H)^{-\frac{1}{\sigma}} = \Lambda_t^H \\
&\quad \Lambda_t^H = \beta \lambda h E_t[V_B^H(\mathbf{B}_{t+1}^H)] + \beta(1-\lambda)(1-h) E_t[V_B^S(\mathbf{B}_{t+1}^S, \omega_{t+1}, K_{t+1})] + \Xi_t^H
\end{aligned}
$$

together with the complementary slackness condition:

$$
Z_{t+1}^H \Xi_t^H = 0,
$$

with $\Xi_t^H \geq 0$.

From the Envelope theorem, we have

$$
V_B^H(\mathbf{B}_t^H) = \frac{\Lambda_t^H}{\lambda} \frac{1+r_{t-1}^n}{1+\pi_t}.
$$

Thus, we can rewrite the Euler equations for bonds accordingly

$$
(C_t^H)^{-\frac{1}{\sigma}} = \beta E_t \left[\frac{1+r_t^n}{1+\pi_{t+1}} \left(h (C_{t+1}^H)^{-\frac{1}{\sigma}} + (1-h)(C_{t+1}^S)^{-\frac{1}{\sigma}} \right) \right] + \Xi_t^H
$$

and similarly for the savers:

$$
(C_t^S)^{-\frac{1}{\sigma}} = \beta E_t \left[\frac{1+r_t^n}{1+\pi_{t+1}} \left(s (C_{t+1}^S)^{-\frac{1}{\sigma}} + (1-s)(C_{t+1}^H)^{-\frac{1}{\sigma}} \right) \right] + \Xi_t^S.
$$

Note that the Euler equation for stocks and capital are isomorphic to the conditions in a representative-agent setting. There is no self-insurance motive, for they cannot be carried to the H state.[18]

In contrast, the bond Euler equations are of the same form as in fully-fledged incomplete-markets models of the Bewely-Huggett-Aiyagari type. In particular,

[18] As households pool resources when participating (which would be optimal with t=0 symmetric agents and $t = 0$ trading), they perceive a return conditional on participating next period. This exactly compensates for the probability of not participating next period, thus generating the same Euler equation as with a representative agent.

the probability $(1-s)$ measures the uninsurable risk to switch to a bad state next period, risk for which only bonds can be used to self-insure, thus generating a demand for bonds for "precautionary" purposes.

Two additional assumptions are required to deliver our simple equilibrium representation. First, we focus on equilibria where (whatever the reason) the constraint of H agents always binds (i.e. $\Xi^H > 0$) and their Euler equation is in fact a strict inequality (for instance, because the shock is a "liquidity" or impatience shock making them want to consume more today, or because their average income in that state is lower enough than in the S state, as would be the case if average profits were high enough; or simply because of a technological constraint preventing them from accessing any asset markets) and the constraint of S never binds ($\Xi^S = 0$) so that their Euler equation always holds with equality. Second, we focus on the zero-liquidity limit, that is we assume that even though the demand for bonds from S is well-defined (the constraint is not binding), the net supply of bonds is zero, so there are no bonds traded in equilibrium.

Under these assumptions, the H households are indeed hand-to-mouth as their budget constraint reads

$$C_t^H = \frac{W_t}{P_t} N_t + T_t^H.$$

The behavior of the savers is characterized by

$$(C_t^S)^{-\frac{1}{\sigma}} = \beta E_t \left[\frac{1+r_t^n}{1+\pi_{t+1}} \left(s(C_{t+1}^S)^{-\frac{1}{\sigma}} + (1-s)(C_{t+1}^H)^{-\frac{1}{\sigma}} \right) \right]$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}^S}{C_t^S} \right)^{-\frac{1}{\sigma}} \left[(1-\tau^K) R_{t+1}^K + Q_{t+1} \left(1 - \delta + \Phi\left(\frac{I_{t+1}}{K_{t+1}} \right) - \Phi'\left(\frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} \right) \right] \right\}$$

$$1 = Q_t \Phi'\left(\frac{I_t}{K_t} \right)$$

$$C_t^S + \frac{I_t}{1-\lambda} = \frac{W_t}{P_t} N_t + (1-\tau^D) \frac{D_t}{1-\lambda} + (1-\tau^K) R_t^K \frac{K_t}{1-\lambda}$$

$$K_{t+1} = (1-\delta) K_t + \Phi\left(\frac{I_t}{K_t} \right) K_t,$$

as market clearing implies that $\omega_t = \omega_{t+1} = 1$. Note that $Q_t = \left(\Phi'\left(\frac{I_t}{K_t} \right) \right)^{-1}$ corresponds to Tobin's marginal Q.

Firms. There is a continuum of monopolistically competitive firms producing differentiated goods $Y_t(j)$ using capital $K_t(j)$ and labor $N_t(j)$ according to a constant-returns production function $Y_t(j) = N_t(j)^{1-\alpha} K_t(j)^{\alpha}$, where α is the capital share. Firms rent labor and capital on competitive factor markets and set prices to maximize profits, subject to consumers' demand. However, firms face price-adjustment frictions, giving rise to a nominal rigidity (which can follow the Calvo or the Rotemberg specification).

Cost minimization delivers the optimal factor share and marginal cost:

$$\begin{aligned} \frac{K_t}{N_t} &= \frac{\alpha}{1-\alpha}\frac{W_t}{P_t R_t^K}; \\ \frac{MC_t}{P_t} &= (1-\alpha)^{\alpha-1}\alpha^{-\alpha}\left(R_t^K\right)^{\alpha}\left(\frac{W_t}{P_t}\right)^{1-\alpha}, \end{aligned}$$

which are common across firms in equilibrium because of constant returns to scale. The pricing problem delivers the standard Phillips curve for price inflation $\pi_t = \beta E_t \pi_{t+1} + \psi mc_t$ in log-linear form. The slope ψ is governed by the amount of price stickiness: when $\psi \to 0$, prices are completely fixed, while when $\psi \to \infty$ prices are flexible.

We also consider an extension featuring rigid wages, following Colciago (2011) and assuming that the labor union faces wage-setting frictions: the nominal wage can only be re-optimized with a constant probability $1-\theta_w$. By standard results, wage setting can be characterized by the following equations in log-linear form:

$$\begin{aligned} \pi_t^w &= \beta E_t \pi_{t+1}^w + \psi_w \mu_t^w \\ \mu_t^w &= \sigma^{-1} c_t + \varphi n_t - w_t \\ \pi_t^w &= w_t - w_{t-1} + \pi_t, \end{aligned}$$

where π_t^w represents nominal wage inflation, μ_t^w is a time-varying wage markup and ψ_w stands for the slope of the wage Phillips curve.

Government. The government implements both monetary and fiscal policy. Monetary policy follows a standard Taylor rule, $r_t^n = \phi_\pi \pi_t + \varepsilon_t$. The fiscal authority redistributes all revenues from capital income and profits taxation, running a balanced budget in every period: $\lambda T_{H,t} = \tau^D D_t + \tau^K R_t^K K_t$.

Market clearing. Finally, the resource constraint of the economy takes into account that part of output is used for investment:

$$Y_t = C_t + I_t.$$

B.2 Steady State

We consider a zero inflation steady state with $\pi = 0$. Steady-state real marginal cost is equal to the inverse of the flexible price markup $MC/P = \mathcal{M}^{-1}$. We will typically assume that there is an optimal subsidy in place to neutralize the steady-state markup such that $\mathcal{M} = 1$.

In our baseline simulations, we assume a symmetric steady state, i.e. $C^H = C^S = C$. This can be implemented by imposing a fixed steady state transfer from savers to hand-to-mouth, as explained in Appendix A. We believe that this is a reasonable benchmark and allows for better comparison to the analytical part, where we maintain this assumption throughout. Furthermore, it allows us to maintain the same steady state for both the flexible and sticky wage version of the model as discussed below. Importantly, however, this assumption turns out to be inconsequential for our quantitative results. Setting the steady-state transfer to zero and thus allowing consumptions to differ in steady state produces very similar results (see Appendix C.3).

The steady-state interest rate is then given by the Euler equation for bonds as $r^n = \beta^{-1} - 1$, which is equal to the rate of time preference. The steady-state rental rate of capital can be obtained from the investment Euler equation $R^K = (r^n + \delta)/(1 - \tau^K)$. The capital accumulation equation gives the steady-state investment to capital ratio $I/K = \delta$. The marginal cost equation implies that the real wage is $W/P = (1 - \alpha)\, \alpha^{\frac{\alpha}{1-\alpha}} \left(r^n + \delta\right)^{-\frac{\alpha}{1-\alpha}}$. The capital-labor ratio is therefore: $K/N = \left\{\alpha(1 - \tau^K)/\left[(r + \delta)\right]\right\}^{\frac{1}{1-\alpha}}$, which implies that the share of capital in output is $K/Y = \left(K/N\right)^{1-\alpha} = \alpha(1 - \tau^K)/\left(r^n + \delta\right)$. The steady state shares of investment and consumption in total output are hence:

$$\frac{I}{Y} = \alpha\frac{\delta(1 - \tau^K)}{r^n + \delta}$$
$$\frac{C}{Y} = 1 - \alpha\frac{\delta(1 - \tau^K)}{r^n + \delta}.$$

We can also get the wage and capital income shares as $WN/PY = 1 - \alpha$ and $R^K K/Y = \alpha$. Because of the optimal subsidy, steady-state profits are given by $D/Y = 0$. The steady-state transfer is thus given by $T^H/Y = \alpha\tau^K/\lambda$.

Sticky wages. For the sticky wages version of the model, we make a number of additional assumptions to ensure that the two models have the same steady state. In particular, we assume that wage inflation is zero as well, which equalizes the optimal reset wage and the level of real wages in steady state. Furthermore, we assume that there is a subsidy in place that neutralizes the steady-state wage markup. Under our assumption of equal consumptions in steady state, the steady-state real wage is the same as in the flexible wage model.

B.3 Log-linear Model

We consider a log-linear approximation of the THANK model around the deterministic steady state described above. We will express all variables as log deviations from steady state and denote them in lower case format ($x_t = \log(X_t) - \log(X)$). For rates, we log-linearize the gross rates, which will be approximately equal to the net rates. The two exceptions are transfers and dividends. This is because these variables can take zero value. We thus express these variables as absolute deviations from steady state, relative to steady state output, i.e. $x_t = \frac{X_t - X}{Y}$ for $X = \{D, T^H\}$. Table B.1 summarizes the log-linear equilibrium conditions.

Table B.1: Log-linear equilibrium conditions for the THANK model

No.	Name	Equation
1:	Wage markup	$\mu_t^w = \sigma^{-1} c_t + \varphi n_t - w_t$
2:	Phillips curve wages	$\pi_t^w = \beta E_t \pi_{t+1}^w + \psi_w \mu_t^w$
3:	Wage inflation	$\pi_t^w = w_t - w_{t-1} + \pi_t$
4:	Euler bonds, S	$c_t^S = s E_t c_{t+1}^S + (1-s) E_t c_{t+1}^H - \sigma(r_t^n - E_t \pi_{t+1})$
5:	Euler capital, S	$q_t = \beta E_t q_{t+1} + (1 - \beta(1-\delta)) E_t r_{t+1}^K - \sigma^{-1}(E_t c_{t+1}^S - c_t^S)$
6:	Tobins q, S	$\omega q_t = i_t - k_t$
7:	Capital accumulation	$k_{t+1} = (1-\delta) k_t + \delta i_t$
8:	Budget constraint, H	$\frac{C}{Y} c_t^H = (1-\alpha)(w_t + n_t) + t_t^H$
9:	Transfer, H	$t_t^H = \frac{\tau^D}{\lambda} d_t + \frac{\tau^K}{\lambda} \alpha (r_t^K + k_t)$
10:	Labor demand	$w_t = mc_t + y_t - n_t$
11:	Capital demand	$r_t^K = mc_t + y_t - k_t$
12:	Phillips curve	$\pi_t = \beta E_t \pi_{t+1} + \psi mc_t$
13:	Production function	$y_t = \alpha k_t + (1-\alpha) n_t$
14:	Profits	$d_t = y_t - (1-\alpha)(w_t + n_t) - \alpha(r_t^K + k_t)$
15:	Aggregate cons.	$c_t = \lambda c_t^H + (1-\lambda) c_t^S$
16:	Resource constraint	$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t$
17:	Taylor rule	$r_t^n = \phi_\pi \pi_t + \epsilon_t$

The model without capital essentially obtains if investment is inelastic to Q (infinite adjustment costs), $\omega = 0$, and if there is no depreciation $\delta = 0$, implying a fixed capital stock. The log-linearized equilibrium conditions in this case are:

Table B.2: Log-linear equilibrium conditions for the THANK model without capital

No.	Name	Equation
1:	Wage markup	$\mu_t^w = \sigma^{-1} c_t + \varphi n_t - w_t$
2:	Phillips curve wages	$\pi_t^w = \beta E_t \pi_{t+1}^w + \psi_w \mu_t^w$
3:	Wage inflation	$\pi_t^w = w_t - w_{t-1} + \pi_t$
4:	Euler bonds, S	$c_t^S = s E_t c_{t+1}^S + (1-s) E_t c_{t+1}^H - \sigma(r_t^n - E_t \pi_{t+1})$
5:	Budget constraint, H	$c_t^H = (1-\alpha)(w_t + n_t) + t_t^H$
6:	Transfer, H	$t_t^H = \frac{\tau^D}{\lambda} d_t$
7:	Labor demand	$w_t = mc_t + y_t - n_t$
8:	Phillips curve	$\pi_t = \beta E_t \pi_{t+1} + \psi mc_t$
9:	Production function	$y_t = (1-\alpha) n_t$
10:	Profits	$d_t = y_t - (1-\alpha)(w_t + n_t)$
11:	Aggregate cons.	$c_t = \lambda c_t^H + (1-\lambda) c_t^S$
12:	Resource constraint	$y_t = c_t$
13:	Taylor rule	$r_t^n = \phi_\pi \pi_t + \epsilon_t$

The parameterization of the model is discussed in the main text. In Table B.3, we summarize the calibrated parameters. The values of s, λ, τ^D, τ^K, and ψ_w depend on the particular model specification. The representative-agent model obtains when $\lambda = 0$, $s = 1$, and $\tau^D = \tau^K = 0$. The model with $\psi_w = \infty$ corresponds to the model with flexible wages.

Table B.3: Model parameterization

Parameter	Value	Description
α	0.33	Capital share of output
δ	0.025	Depreciation rate per quarter
ω	10	Elasticity of investment to Q
β	0.99	Discount factor
s	1 / 0.98	Probability of staying unconstrained
σ	1	Intertemporal elasticity of substitution
$1/\varphi$	1.00	Frisch elasticity
λ	0 / 0.27	Share of hand-to-mouth
τ^D, τ^K	$= \begin{cases} 0 & \text{no redistribution} \\ \lambda & \text{full redistribution} \end{cases}$	Taxes on profits and capital
ψ	0.050	Slope of PC
ψ_w	∞ / 0.075	Slope of PC wages
ϕ_π	1.50	Taylor rule coefficient
ϕ_i	0.00	Interest rate smoothing
ρ_i	0.60	Persistence MP shock

B.4 Analytical Results

An analytical solution of even the simplest representative-agent NK model with capital is, to the best of our knowledge, hitherto unavailable. Here, we make a number of simplifying assumptions to provide analytical closed-form solutions to the THANK model with capital, which is of independent interest. In particular, we adopt the following simplifying assumptions. First, we consider the case of full capital depreciation $\delta = 1$, as in D. Romer's textbook exposition of the RBC model, and no capital adjustment costs, $\omega^{-1} = 0$. Furthermore, we assume log utility in consumption ($\sigma = 1$) and infinitely elastic labor supply or equivalently indivisible labor ($\varphi = 0$).[19] On the supply side, we assume a contemporaneous Phillips curve $\pi_t = \psi mc_t$, as in Bilbiie (2018).[20] Finally, we assume a special monetary rule that just neutralizes inflation movements, i.e. just satisfies the Taylor principle $r_t^n = \pi_t + \varepsilon_t$, with $\phi_\pi = 1$. Finally, we assume that there is no idiosyncratic risk, i.e. $s = 1$.

Our aim is to characterize the response of consumption to a on-time monetary policy shock analytically. We will do so for each of the relevant models in turn.

Model without capital. The analytics for the model without capital are derived in Bilbiie (2018, 2019). Here we extend the results for the case with decreasing returns in labor. The aggregate Euler equation reads

$$c_t = E_t c_{t+1} - \frac{1-\lambda}{1-\lambda\chi_{noK}} \frac{1-\alpha}{1-\alpha+\frac{1-\lambda}{1-\lambda\chi_{noK}}\psi} \varepsilon_t,$$

where $\chi_{noK} = 1 + \left(1 - \frac{\tau^D}{\lambda}\right)(1-\alpha)$.

The effect of an expansionary monetary policy shock on consumption is thus given by:

$$\frac{\partial c_t}{\partial(-\varepsilon_t)} = \frac{1-\lambda}{1-\lambda\chi_{noK}} \frac{1-\alpha}{1-\alpha+\frac{1-\lambda}{1-\lambda\chi_{noK}}\psi}. \tag{41}$$

Capital under full redistribution. Let us now consider the model with capital. To start with, we focus on the case of full income redistribution, i.e. a version of the model in which both agents get the same income ($\chi = 1$, perfect redistribution of all forms of capital income). Recall that this can be achieved by setting $\tau^D =$

[19] Both of these assumptions are not necessary to obtain analytical results and can be relaxed.

[20] This can be microfounded by assuming that monopolistic firms have to pay a Rotemberg price adjustment cost relative to yesterday's market average price index, rather than relative to their own individual price.

$\tau^K = \lambda$. The aggregate Euler equation in this case becomes:

$$c_t = E_t c_{t+1} - \frac{\lambda}{1-\lambda}\frac{\alpha\beta}{1-\alpha\beta}\left(E_t k_{t+2} - k_{t+1}\right) - \sigma r_t.$$

We solve the model analytically to obtain:

$$k_{t+1} = \frac{\mu^{-1}}{\alpha\beta}\frac{\alpha}{1+\psi^{-1}}\frac{1}{1+\Lambda\left(Z+Q\right)}k_t - \frac{\mu^{-1}}{\alpha\beta}\frac{\left(1-\alpha\right)\psi^{-1}}{1+\psi^{-1}}\frac{1}{1+\Lambda\left(Z+Q\right)}\varepsilon_t,$$

with $\Lambda = \frac{\lambda}{1-\lambda}\frac{\alpha\beta}{1-\alpha\beta}$; $Z = \frac{1-\alpha^2\beta}{\alpha\beta\left(1+\psi^{-1}\right)}$; $Q = \frac{\psi^{-1}}{1+\psi^{-1}}\frac{2-\alpha(1+\beta)}{\alpha\beta}$ and the unstable, "forward" root of the system

$$\mu = \frac{1}{2}\left(B + \sqrt{B^2 - 4\frac{1}{\alpha\beta}\frac{\alpha}{1+\psi^{-1}}\frac{1}{1+\Lambda\left(Z+Q\right)}}\right) > 1$$

with $B = \left(\frac{1}{\alpha\beta} + \frac{\alpha}{1+\psi^{-1}} + \Lambda Z\right)\frac{1}{1+\Lambda(Z+Q)}$. Note that this nests the RANK case when $\lambda = 0$.

The effect of an expansionary monetary policy shock on consumption is:

$$\frac{\partial c_t}{\partial(-\varepsilon_t)} = 1 - \frac{1-\alpha^2\beta + \left(1-\alpha\right)\psi^{-1}\frac{\alpha^2\beta}{1+\psi^{-1}}\frac{\mu^{-1}}{\alpha\beta}\frac{1}{1+\Lambda(Z+Q)}}{\left(1-\alpha\right)\psi^{-1} + 1 - \alpha^2\beta} + \Lambda\frac{\left(1-\alpha\right)\psi^{-1}}{1+\psi^{-1}}\frac{\mu^{-1}}{\alpha\beta}\frac{1}{1+\Lambda\left(Z+Q\right)}\frac{\left(1-\alpha\right)\psi^{-1}}{\left(1-\alpha\right)\psi^{-1}+1-\alpha^2\beta}. \tag{42}$$

One can show that the multiplier is increasing with Λ and thus in the share of hand-to-mouth, λ. Thus, we confirm that household heterogeneity in combination with capital delivers amplification relative to RANK, even under perfect income redistribution.

Digression: RANK with capital. Of particular interest is the novel analytical expression for the multiplier in RANK, whereby $\lambda = 0$, which is:

$$\frac{\partial c_t}{\partial(-\varepsilon_t)} = 1 - \frac{1-\alpha^2\beta + \left(1-\alpha\right)\psi^{-1}\frac{\alpha^2\beta}{1+\psi^{-1}}}{\left(1-\alpha\right)\psi^{-1} + 1 - \alpha^2\beta} \leq 1 \tag{43}$$

As expected, the multiplier vanishes with flexible prices and is at its highest with fixed prices $\frac{\partial c_t}{\partial(-\varepsilon_t)} = 1$, when it in fact coincides with the one in a model without capital. Price flexibility lowers the consumption multiplier with capital because it implies an increase in inflation and the real rate, and an increase in investment.

Capital with cyclical inequality. We now add back the "cyclical inequality channel" by assuming that not all the asset income is redistributed. A natural benchmark is that none is redistributed, i.e. it all accrues to the savers who hold and price the assets.

Under our assumptions the consumption of the hand to mouth can be written as

$$c_t^H = \chi_K c_t + \frac{\alpha\beta}{1-\alpha\beta} k_{t+1} - \frac{(\chi_K - 1)\,\alpha}{1-\alpha} k_t,$$

where

$$\chi_K \equiv 1 + \frac{1-\alpha}{1-\alpha\beta}\left(1 - \frac{\tau}{\lambda}\right)$$

is the sufficient statistic for the cyclical inequality channel. Notice that we are back to the case of perfect redistribution when $\tau = \lambda$ while the case of no-investment amounts to setting the investment share to 0.

The aggregate consumption Euler equation becomes now:

$$\begin{aligned} c_t = {} & E_t c_{t+1} - \frac{1-\lambda}{1-\lambda\chi_K} r_t \\ & - \frac{\lambda}{1-\lambda\chi_K} \frac{\alpha\beta}{1-\alpha\beta} \left(E_t k_{t+2} - k_{t+1}\right) + \frac{\alpha}{1-\alpha} \frac{\lambda\,(\chi_K - 1)}{1-\lambda\chi_K} \left(k_{t+1} - k_t\right). \end{aligned}$$

The introduction of cyclical inequality affects the second and third term and introduces a fourth. The second term is independent of investment and has been discussed above. The third term, capturing the amplification of consumption through investment, is amplified (relative to the perfect-redistribution $\chi_K = 1$ case). The last term captures a novel dimension of amplification that has to do with the interaction of the two channels.

One can show that the effect of an expansionary monetary policy shock on consumption is now given by

$$\frac{\partial c_t}{\partial(-\varepsilon_t)} = \frac{1-\lambda}{1-\lambda\chi_K} \left\{ \begin{array}{c} 1 - \dfrac{\left(1-\alpha^2\beta\right)\frac{1-\lambda}{1-\lambda\chi_K} + (1-\alpha)\psi^{-1}\frac{\mu_{\chi_K}^{-1}}{\alpha\beta}\frac{\alpha^2\beta}{1+\psi^{-1}+(1-\alpha)\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)}\frac{1}{1+\Lambda\left(Z_{\chi_K}+Q_{\chi_K}\right)}}{(1-\alpha)\psi^{-1}+(1-\alpha^2\beta)\frac{1-\lambda}{1-\lambda\chi_K}} \\ +\Lambda\dfrac{\mu_{\chi_K}^{-1}}{\alpha\beta}\dfrac{(1-\alpha)\psi^{-1}}{1+\psi^{-1}+(1-\alpha)\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)}\dfrac{1}{1+\Lambda\left(Z_{\chi_K}+Q_{\chi_K}\right)}\dfrac{(1-\alpha)\psi^{-1}}{(1-\alpha)\psi^{-1}+(1-\alpha^2\beta)\frac{1-\lambda}{1-\lambda\chi_K}} \end{array} \right\} \tag{44}$$

where $Q_{\chi_K} = \frac{1-\lambda}{1-\lambda\chi_K}\frac{\psi^{-1}}{1+\psi^{-1}+(1-\alpha)\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)}\frac{2-\alpha(1+\beta)}{\alpha\beta}$ and $Z_{\chi_K} =$

$\frac{1-\lambda}{1-\lambda\chi_K}\frac{1}{1+\psi^{-1}+(1-\alpha)\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)}\frac{1-\alpha^2\beta}{\alpha\beta}$ and the root is

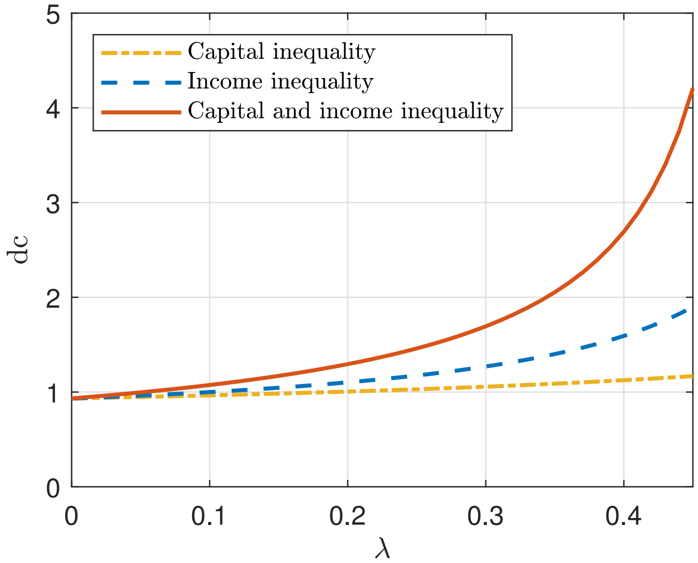
$$\mu_\tau = \frac{1}{2}\left(B_{\chi_K} + \sqrt{B_{\chi_K}^2 - 4\frac{1+\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)\frac{1-\alpha^2\beta}{1-\alpha}}{\beta\left(1+\psi^{-1}+(1-\alpha)\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)\right)}\frac{1}{1+\Lambda\left(Z_{\chi_K}+Q_{\chi_K}\right)}}\right)$$

with $B_{\chi_K} = \frac{\frac{1+\frac{1-\alpha^2\beta}{1-\alpha}\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)}{\alpha\beta}+\frac{\alpha}{1+\psi^{-1}+(1-\alpha)\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)}+\Lambda Z_{\chi_K}-\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)\frac{(1-\alpha)\psi^{-1}+\left(1-\alpha^2\beta\right)\frac{1-\lambda}{1-\lambda\chi_K}}{\alpha\beta\left(1+\psi^{-1}+(1-\alpha)\left(\frac{1-\lambda}{1-\lambda\chi_K}-1\right)\right)}}{1+\Lambda\left(Z_{\chi_K}+Q_{\chi_K}\right)}$.

Notice that the term outside the curly brackets is the multiplier without capital and without full redistribution, while the term inside is reminiscent of the expression for the multiplier with capital and with full income redistribution (it has the same form, but is a function of χ_K now).

Complementarity. Figure B.1 summarizes the amplification properties of the model. It plots the multiplier (the effect of a rate cut) on investment and consumption as a function of the share of hand-to-mouth. This is qualitatively very similar to what we obtain in the stylized model in Section 2.

Figure B.1: Analytical Multipliers in THANK



Notes: The consumption multipliers as a function of the share of hand-to-mouth λ in analytical New Keynesian models—with capital inequality, with income inequality and with both types of inequalities (baseline calibration).

It can be shown that the joint multiplier is larger than the product of the two, i.e $\frac{\partial c_t}{\partial(-\varepsilon_t)}\big|_{K,\text{no redist}} > \frac{\partial c_t}{\partial(-\varepsilon_t)}\big|_{noK,noredist} \times \frac{\partial c_t}{\partial(-\varepsilon_t)}\big|_{K,redist}$. We need to show that:

$$\frac{1-\lambda}{1-\lambda\chi_K}\left\{1+\frac{(1-\alpha)\frac{\lambda\alpha\beta}{(1-\lambda)(1-\alpha\beta)}}{1+\alpha\frac{\lambda(\chi_K-1)}{1-\lambda\chi_K}\left(1+\frac{1-\alpha\beta}{1-\alpha}\right)}\right\} > \frac{1-\lambda}{1-\lambda\chi_{noK}}\left(1+(1-\alpha)\frac{\lambda\alpha\beta}{(1-\lambda)(1-\alpha\beta)}\right)$$

Replacing the expressions for χ_{noK} and χ_K and rewriting we obtain:

$$\frac{\alpha\lambda\chi_K-\lambda(1-\alpha)\frac{\alpha\beta}{1-\alpha\beta}}{1-\lambda\chi_K+\lambda(1-\alpha)\frac{\alpha\beta}{1-\alpha\beta}} < \frac{\alpha\lambda\chi_K}{(1-\lambda\chi_K)\left(1+(1-\alpha)\frac{\lambda}{1-\lambda}\frac{\alpha\beta}{1-\alpha\beta}\right)}$$

The numerator of the left-hand side is always smaller, and the numerator is also always smaller under countercyclical income inequality $\chi_K > 1$, thus proving complementarity.

Furthermore, we show that there can be amplification (multiplier increasing in λ) even with procyclical income inequality $\chi_K < 1$. Taking the derivative of the multiplier with respect to λ we obtain:

$$\frac{(\chi_K-1)(1-\lambda)}{(1-\lambda\chi_K)^2}+\frac{\alpha\beta(1-\alpha)}{1-\alpha\beta}\frac{1}{\left(1-\lambda\chi_K+\alpha\lambda(\chi_K-1)\left(1+\frac{1-\alpha\beta}{1-\alpha}\right)\right)^2};$$

this is positive if:

$$\frac{\alpha\beta(1-\alpha)}{1-\alpha\beta} > (1-\chi_K)(1-\lambda)\left(1+\alpha\frac{\lambda(\chi_K-1)}{1-\lambda\chi_K}\left(1+\frac{1-\alpha\beta}{1-\alpha}\right)\right)^2.$$

This implicitly defines a threshold $\chi_K < 1$ beyond which amplification still occurs—although the expression is not as compact as for the stylized model in Section 2. The magnitude of this threshold under our baseline parameterization is 0.4.

Fixed-price limit. The above equations get quite unwieldy. For better comparison with the expressions in Section 2, it is instructive to look at the multipliers in the fixed-price limit. For the case without redistribution, the analytical expression is then given by

$$\frac{\partial c_t}{\partial(-\varepsilon_t)} = \frac{1-\lambda}{1-\lambda\chi_K}\left\{1+\frac{\lambda\alpha\beta}{(1-\lambda)(1-\alpha\beta)}\frac{(1-\alpha)}{1+\alpha\frac{\lambda(\chi_K-1)}{1-\lambda\chi_K}\left(1+\frac{1-\alpha\beta}{1-\alpha}\right)}\right\}$$

with $\chi_K = 1 + \frac{1-\alpha}{1-\alpha\beta}$.

Note that this joint multiplier nests the other two multipliers. For the model with capital under full redistribution ($\chi_K = 1$), this reads:

$$\frac{\partial c_t}{\partial\left(-\varepsilon_t\right)} = 1 + \frac{(1-\alpha)\,\lambda\alpha\beta}{(1-\lambda)\,(1-\alpha\beta)};$$

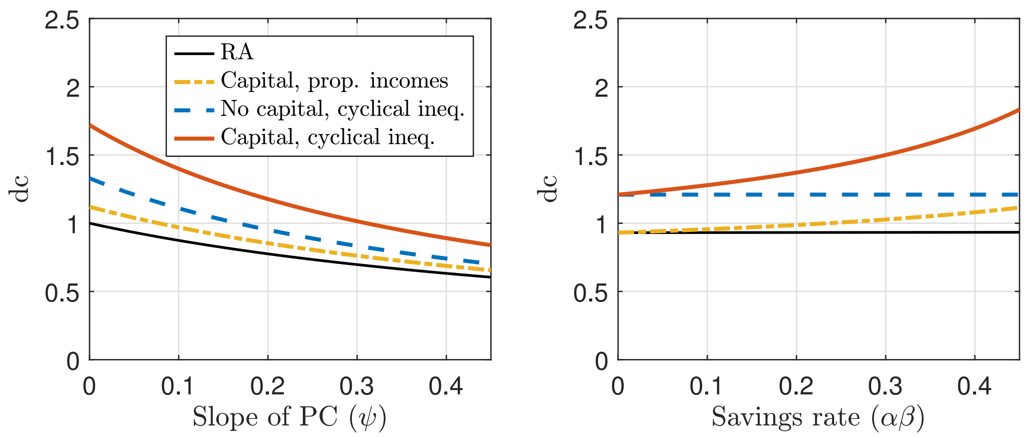
For the model with no capital investment $\alpha\beta = 0$, the multiplier becomes:

$$\frac{\partial c_t}{\partial\left(-\varepsilon_t\right)} = \frac{1-\lambda}{1-\lambda\chi_{noK}},$$

with the particular income distribution specification summarized by $\chi_{noK} = 2-\alpha$. We can think of these expressions as generalizations of the multipliers derived in Section 2.

Sensitivity. It is also instructive to look at how the multipliers change when we vary some key parameters. In the left panel of Figure B.2, we vary the slope of the Phillips curve, keeping all other parameters at their baseline values. As expected, the effects of monetary policy on consumption become less powerful when prices are less sticky (i.e. when the Phillips curve is steeper). Importantly, however, the capital and income inequality channels are still operative and the complementarity turns out to be robust as well.

Figure B.2: Sensitivity of Analytical Multipliers



Notes: The consumption multipliers as a function of the slope of the Phillips curve ψ and the savings rate $\alpha\beta$ in analytical representative-agent and heterogeneous-agent New Keynesian models with $\lambda = 0.27$.

In the right panel, we vary the savings rate ($\alpha\beta$), keeping everything else fixed. As expected, in the cyclical inequality model without capital, changing the savings rate has no effect. Interestingly, changing the savings rate has also virtually no effect in the representative-agent model with capital. This is because prices are sticky and we neutralize the feedback through the real interest rate. In the heterogeneous-agent models with capital, increasing the savings rate amplifies the effects of monetary policy through the capital inequality channel. When the savings rate approaches zero, the models converge to their no-capital counterparts.

Capital adjustment costs. We can also obtain an analytical solution for the model with capital adjustment costs in the limit case of fixed prices. In this case, we need to augment the model with the capital Euler equation (no-arbitrage) $q_t = \beta E_t q_{t+1} + E_t r^K_{t+1} - \varepsilon_t$, where ε_t is the de facto real-rate shock; and the investment function which under full depreciation is $k_{t+1} - k_t = \omega q_t$. Combining these with the other equilibrium conditions yields:

$$\beta\left(\omega^{-1} + \frac{\alpha}{1-\alpha}\right) E_t k_{t+2} - \left(\beta\omega^{-1} + \omega^{-1} + \frac{1}{1-\alpha}\right) k_{t+1} + \omega^{-1} k_t = \varepsilon_t - \left(1 + \frac{1-\alpha\beta}{1-\alpha}\right) E_t c_{t+1}$$

and the same equation as before:

$$\begin{aligned} c_t &= E_t c_{t+1} - \frac{\lambda}{1-\lambda\chi_K}\frac{\alpha\beta}{1-\alpha\beta} E_t k_{t+2} + \frac{\lambda}{1-\lambda\chi_K}\left(\frac{\alpha\beta}{1-\alpha\beta} + \frac{\alpha\left(\chi_K - 1\right)}{1-\alpha}\right) k_{t+1} \\ &\quad - \frac{\lambda}{1-\lambda\chi_K}\frac{\alpha\left(\chi_K - 1\right)}{1-\alpha} k_t - \frac{1-\lambda}{1-\lambda\chi_K}\varepsilon_t \end{aligned}$$

Combining these equations to leads to a second-order difference equation that can be solved by standard methods, e.g. factorization, as above; the stable root of the resulting characteristic polynomial being:

$$\mu_\omega = \frac{1}{2}\left(B_\omega - \sqrt{B_\omega^2 - \frac{4}{\omega X_\omega}}\right),$$

where $B_\omega = \frac{1}{X_\omega}\left[\beta\omega^{-1} + \omega^{-1} + \frac{1}{1-\alpha} + \left(1 + \frac{1-\alpha\beta}{1-\alpha}\right)\frac{\lambda}{1-\lambda\chi}\frac{\alpha\left(\chi_K - 1\right)}{1-\alpha}\right]$ and $X_\omega = \beta\left(\omega^{-1} + \frac{\alpha}{1-\alpha}\right) + \left(1 + \frac{1-\alpha\beta}{1-\alpha}\right)\frac{\lambda}{1-\lambda\chi_K}\frac{\alpha\beta}{1-\alpha\beta}$.

By solving the equation for an iid shock, we can obtain the following analytical expressions for the multipliers in the case of investment adjustment costs, using the same simplifying assumptions for the case of fixed prices. The effects of a one-time interest rate cut on capital and consumption are respectively:

$$\frac{\partial k_{t+1}}{\partial\left(-\varepsilon_t\right)} = \omega\mu_\omega;$$

$$\frac{\partial c_t}{\partial(-\varepsilon_t)} = \frac{1-\lambda}{1-\lambda\chi_K}\left(1+\omega\mu_\omega\frac{\lambda}{1-\lambda}\frac{\alpha\beta}{1-\alpha\beta}\right).$$

where μ_ω is the stable root (as defined above) of the second-order difference equation governing equilibrium capital dynamics under adjustment costs, and the AR(1) coefficient in the closed-form solution for capital.

A main insight from this analytical solution is that adjustment costs are crucial for the investment response, which collapses to zero when the elasticity of investment to Q tends to zero (infinite adjustment costs) and reaches a maximum when adjustment costs tend to zero (ω tends to infinity). Yet the response of consumption is *not similarly magnified*, because in the consumption multiplier the response of investment is weighed down by the share of hand to mouth agents, λ, times the savings rate $\alpha\beta$. Accordingly, for reasonable values of these parameters, the induced amplification of consumption is an order of magnitude lower than the amplification on investment.[21]

In summary, the analytical results in Section 2 readily generalize to a broad range of parameter values and do not depend on the simplifying assumptions on the investment technology or the supply side of the model.

Sticky wages. Adding sticky wages adds another layer of complication but we can obtain an analytical solution assuming a static wage Phillips curve $\pi_t^w = \psi_w\mu_t^w$ and fixed prices. The wage equation (Phillips curve) in the static case, having substituted our simplifying assumptions (fixed prices, $\varphi = 0$, etc.) becomes

$$w_t = \frac{1}{1+\psi_w}w_{t-1} + \frac{\psi_w}{1+\psi_w}c_t.$$

For the model **with capital** inequality but proportional incomes, combining this with the same equations used before under the assumption of an iid shock yields a second-order equation, denoting $X_w \equiv \frac{1-\alpha\beta}{\alpha\beta}\frac{1-\lambda}{(1-\alpha)\lambda\psi_w+(1+\psi_w)(1-\alpha\beta)}$:

$$E_t w_{t+1} - X_w\left(\frac{\alpha\beta}{1-\lambda}+1+\psi_w\right)w_t + X_w w_{t-1} = \psi_w X_w\left(1+\frac{(1-\alpha)\lambda\alpha\beta}{(1-\lambda)(1-\alpha\beta)}\right)\varepsilon_t$$

The smaller root is

$$\mu_w = \frac{1}{2}\left[X_w\left(\frac{\alpha\beta}{1-\lambda}+1+\psi_w\right) - \sqrt{X_w^2\left(\frac{\alpha\beta}{1-\lambda}+1+\psi_w\right)^2 - 4X_w}\right]$$

[21] These analytical results provide a complementary intuition for a numerical result of Alves et al. (2019), which finds little difference in the consumption responses across the cases with and without adjustment costs.

and it is stable ($\mu_w < 1$) whenever: $\lambda < \frac{(1-\alpha\beta)^2}{1-\alpha^2\beta} < 1$.

Factorizing the equation (the other root is $X_w \mu_w^{-1}$) we obtain the solution, given iid real rate:

$$w_t = \mu_w w_{t-1} - \psi_w \mu_w \left[1 + (1-\alpha) \frac{\lambda}{1-\lambda} \frac{\alpha\beta}{1-\alpha\beta}\right] \varepsilon_t.$$

The AR(1) coefficient in the closed-form solution for wages is equal to the stable root μ_ω. Intuitively, the stickier are wages, the larger this root and the more persistent are real wages.

The expression for capital then follows directly replacing this in the rest of the model, obtaining:

$$k_{t+1} = \frac{1+\psi_w}{\psi_w} \frac{1-\lambda}{\lambda} \frac{1-\alpha\beta}{\alpha\beta} w_t - \frac{1}{\psi_w} \frac{1-\lambda}{\lambda} \frac{1-\alpha\beta}{\alpha\beta} w_{t-1} + \frac{1-\lambda}{\lambda} \frac{1-\alpha\beta}{\alpha\beta} r_t.$$

The (proportional-incomes) multipliers on consumption and investment respectively are thus given by:

$$\begin{aligned} \frac{dc_t}{d(-\varepsilon_t)} &= (1+\psi_w)\mu_w \left(1 + (1-\alpha) \frac{\lambda}{1-\lambda} \frac{\alpha\beta}{1-\alpha\beta}\right); \\ \frac{dk_{t+1}}{d(-\varepsilon_t)} &= \frac{1-\lambda}{\lambda} \frac{1-\alpha\beta}{\alpha\beta} \left[(1+\psi_w)\mu_w \left(1 + (1-\alpha) \frac{\lambda}{1-\lambda} \frac{\alpha\beta}{1-\alpha\beta}\right) - 1\right]. \end{aligned} \tag{45}$$

These expressions illustrate that the combination of sticky wages and capital inequality leads to amplification even under proportional incomes. To start with, there is a standard amplifying effect of wage stickiness because of an additional failure of monetary neutrality (that obtains also in a representative-agent model). This is now amplified with heterogeneity because it also implies an increase in investment, and thus further amplification under proportional incomes through what we dub the capital inequality channel.[22]

Finally, we show that introducing sticky wages dampens the effects of monetary policy on consumption in the TANK model **without capital**. The aggregate

[22]Under our simplifying assumptions, the consumption multiplier is in fact non-monotonic in wage stickiness: it tends to the same value when wages are flexible ($\psi_w \to \infty$) as when they are fixed ($\psi_w = 0$), and thus exhibits a hump-shape. The intuition is that when wages become almost fixed, a further increase in stickiness dampens the responses of wages and investment, so income expands by less and the "capital inequality" feedback loop is weakened. This is, however, ar artifact of the analytical simplifying assumption: In our quantitative model, empirically-realistic parameterizations lie in the region where more stickiness leads to more amplification (even though the level of stickiness is already high)—see Figure 3.

Euler equation (with fixed prices) is given by:

$$c_t = E_t c_{t+1} - \frac{\lambda}{1-\lambda\chi_{noK,sw}}\left(1-\alpha\right)\left(1-\frac{\tau^D}{\lambda}\right) f\left(w_t - w_{t-1}\right) - \frac{1-\lambda}{1-\lambda\chi_{noK,sw}} r_t,$$

where $f = \frac{1}{1+\psi_w}$ can be interpreted as the fraction of fixed wages and

$$\chi_{noK,sw} = 1 + \left(1-\frac{\tau^D}{\lambda}\right)\left(1-f\right)\left(1-\alpha\right)$$

Plugging in for the wage equation and solving the model forward (the backward solution can be ruled out) we get

$$c_t = \frac{1-\lambda}{1-\lambda\chi_{noK,sw}} \sum_{j=0}^{\infty} r_{t+j} + f\left(1-\frac{\tau^D}{\lambda}\right)\left(1-\alpha\right)\frac{\lambda}{1-\lambda\chi_{noK,sw}} w_{t-1}$$

Thus, we have that

$$\frac{\partial c_t}{\partial(-\varepsilon_t)} = \frac{1-\lambda}{1-\lambda\chi_{noK,sw}}.$$

Proposition 4 *Wage stickiness dampens the effect relative to flex-wage, but it still leads to amplification.*

Proof. Because $f \in (0,1)$, we have $\chi_{noK,sw}$ and $\chi_{noK} > \chi_{noK,sw}$ and the result follows. ■

B.5 Liquid Capital

Thus far, we have assumed that physical capital is *illiquid*; this is reasonable insofar as our notion of capital encompasses machines and equipment, but also land, real estate, and any form of illiquid wealth *largo sensu*.

In this appendix, we consider the case when (some) capital is instead *liquid*. To simplify things, we assume that capital is *entirely liquid*. However, it is straightforward to extend the analysis to the partially liquid case. We model this by assuming that capital enters the portfolio of liquid assets: households choosing to invest in capital can use it to self-insure against the risk of becoming constrained in the future.

The resulting THANK model is identical to the one outlined above, except that now liquidity is in positive supply and will be held in equilibrium. In particular, we assume that the total supply of liquid assets is equal to the capital stock. More specifically, since we focus on equilibria where H do not hold any liquid assets at the end of the period, we have $Z^H_{t+1} = 0$ which implies:

$$K_{t+1} = (1-\lambda)\, Z^S_{t+1};$$

Beginning-of period liquid assets in island H will thus equal assets brought over from the S island, formally:

$$B^H_{t+1} = (1-h)\, Z^S_{t+1} = \frac{1-h}{1-\lambda} K_{t+1} = \frac{1-s}{\lambda} K_{t+1}$$

where the first equality used the stationary distribution $\frac{(1-\lambda)(1-s)}{\lambda} = 1-h$. Similarly, beginning-of-period assets in island S are

$$B^S_{t+1} = s Z^S_{t+1} = \frac{s}{1-\lambda} K_{t+1}$$

Replacing these asset-market clearing conditions in individual budget constraints (assuming no adjustment costs to ease notation) we have:

$$\begin{aligned} C^S_t + \frac{1}{1-\lambda} K_{t+1} &= \hat{Y}^S_t + \frac{s}{1-\lambda} \left(1 + R^K_t - \delta\right) K_t \\ C^H_t &= \hat{Y}^H_t + \left(1 + R^K_t - \delta\right) \frac{1-s}{\lambda} K_t \end{aligned}$$

where $\hat{Y}^j_t$ denotes any non-physical-capital income, net of taxes and transfers. The capital accumulation equation is standard $K_{t+1} = (1-\delta)\, K_t + I_t$.

We can see that the hand-to-mouth have now two sources of funds: the first term is as before labor income after any redistribution, and the second term consists of the per-capita payoff (net of depreciation) on the total stock of capital brought over by agents moving from the S state, that they decided to hold for precautionary purposes.

The Euler equation for holding capital is thus akin to that of an Aiyagari economy (replacing $Q_t = 1$ as implied by the lack of adjustment costs and ignoring complementary slackness):

$$(C^S_t)^{-\frac{1}{\sigma}} = \beta E_t \left\{ (1 + R^K_{t+1} - \delta) \left[s (C^S_{t+1})^{-\frac{1}{\sigma}} + (1-s)(C^H_{t+1})^{-\frac{1}{\sigma}} \right] \right\}.$$

We can see that the Euler equation for holding physical capital now features a self-insurance, precautionary-saving motive since capital is liquid. In other words, the Euler equation for liquid capital looks like the Euler equation for liquid bonds (21) (the expected returns on these two assets are equated by no-arbitrage).

To isolate the role of liquid capital, we focus on the case with proportional incomes in order to strip down the cyclical income inequality channel. A loglinear approximation of $H's$ budget constraint around a steady state with symmetric consumption delivers

$$\frac{C}{Y} c_t^H = y_t + \frac{1-s}{\lambda} \beta^{-1} k_t + \frac{1-s}{\lambda} \alpha r_t^K.$$

This illustrates most transparently, starting from a benchmark with proportional incomes, that having liquid capital acts "as if" there was direct fiscal redistribution of (illiquid) capital income.

Table B.4: Liquid Capital and Idiosyncratic Risk

Risk	$s = 1$	$s = 0.98$	$s = 0.95$	$s = 0.9$	$s = 0.8$
	1.11	1.16	1.27	1.50	2.15

Notes: Impact multipliers on aggregate consumption of an interest-rate cut in the THANK model with liquid capital for different levels of idiosyncratic risk. The multipliers are expressed relative to the representative agent-no capital benchmark.

With sufficiently high idiosyncratic risk and enough liquidity, this has thus a similar flavor as the fiscal redistribution of physical capital studied in the previous Section 3.2.[23] We illustrate this quantitatively in the full model with capital adjustment costs. Table B.4 shows the impact multipliers on aggregate consumption for different levels of idiosyncratic risk. Note that to get strong amplifying effects, we need quite high levels of risk.

Importantly, our complementarity turns out to be robust to the liquidity of capital. Table B.5 shows the consumption multipliers for the different models we consider. Clearly, the assumption of liquid capital only affects the multipliers in the models with capital. The capital inequality channel now leads to some more amplification (the multiplier goes from 1.11 to 1.16). The income inequality channel can still lead to quite substantial amplification itself. Importantly, the joint multiplier is again much larger than the product of the two individual multipliers.

[23] In terms of reduced-form dynamics, this amounts in equilibrium to a version of H agents' income being disproportionately cyclical $\chi > 1$, although the foundation of that is now the liquidity of capital.

Table B.5: Amplification under Liquid Capital and Idiosyncratic Risk

	Rep. agent	Heterogeneous agents	
		Prop. incomes	Inequality
No capital	1.00	1.00	1.60
Capital	0.66	1.16	2.51

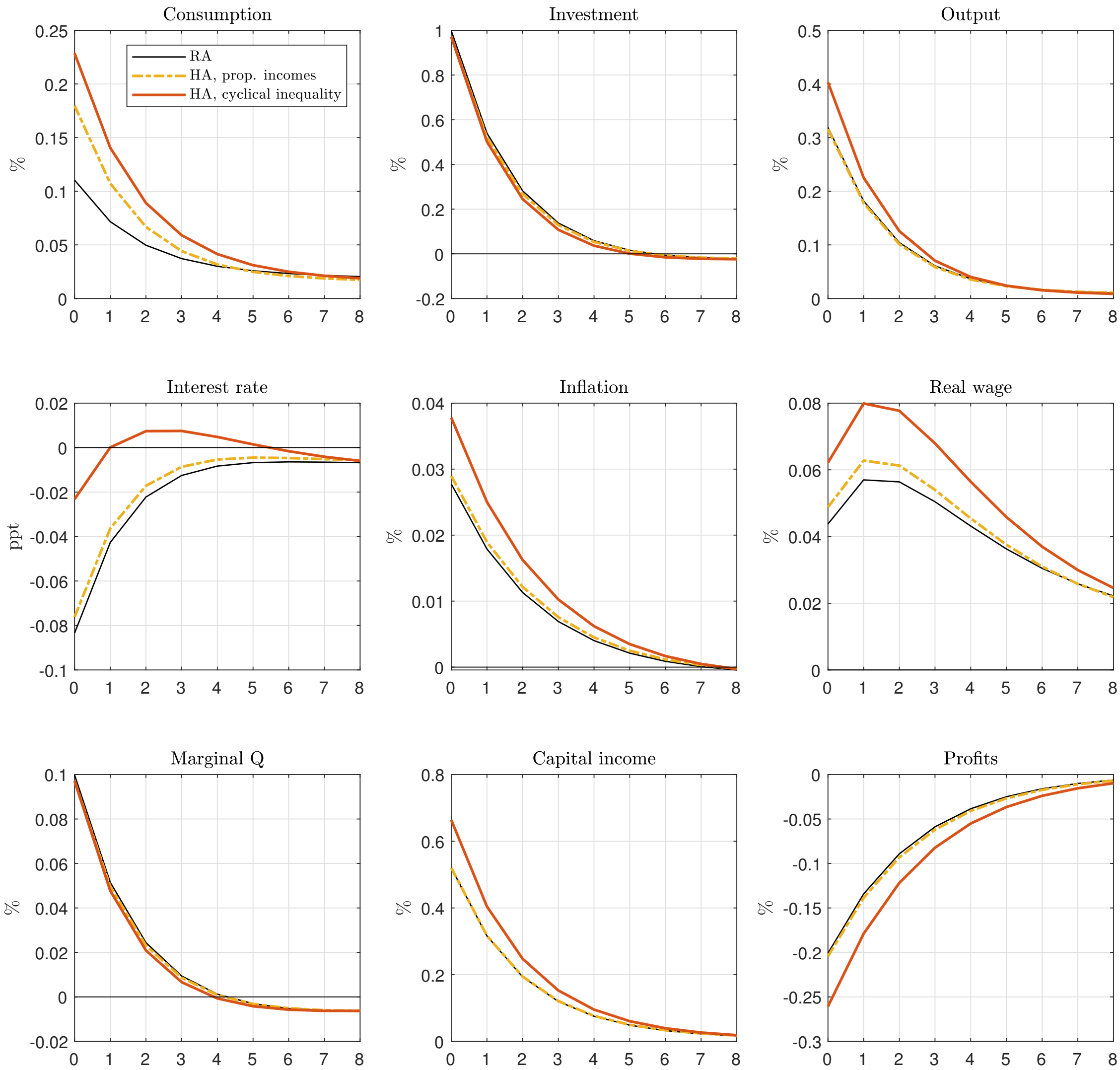
Notes: Impact multipliers on aggregate consumption of an interest-rate cut in each model, relative to the representative agent-no capital benchmark. For the heterogeneous-agent models, we assume moderate idiosyncratic risk ($s = 0.98$) and liquid capital. The second column shows the case without and the third column with income inequality.

C Additional Tables and Figures

C.1 Full Set of Impulse Responses

For completeness, in Figure C.1, we present the impulse responses of the main variables of interest to an interest rate shock of 25 basis points. The responses are based on the most general model of Section 3.3, with sticky wages and idiosyncratic risk.

Figure C.1: Impulse Responses to Monetary Policy Shock



Notes: Impulse responses of selected model variables to an expansionary interest rate shock of 25 basis points in the representative-agent model and heterogeneous-agent models with and without income inequality, under sticky wages.

The interest rate shock leads to an increase in marginal costs and thus inflation. As a consequence, the nominal interest rate falls but the fall is significantly smaller than the initial shock of 25 basis points because of the endogenous response to higher inflation. Real wages increase as well and because of wage rigidities the response features a hump-shaped behavior. Tobin's marginal Q also increases: as the expected return on capital increases, the value of installed capital increases as well. Finally, the response of capital income is highly procyclical while profits are slightly countercyclical.

Notice that sticky wages are key for this latter result. Under flexible wages, profits will be much more strongly countercyclical. At the same time, capital income is even more procyclical, which explains why only redistributing capital income has even more powerful effects in the flexible wage case. Another important difference concerns the investment response. With flexible wages, the investment response is no longer as similar across the different models. In particular, in the model with countercyclical inequality, the investment response gets dampened quite substantially, which is in line with the findings by Luetticke (forthcoming) in a full-blown HANK model with flexible wages. Thus, in this model, the amplification of the consumption response comes, at least to some extent, at the expense of a weaker investment response, which is absent in the presence of sticky wages. Finally, by construction, the response of real wages is a magnitude larger than in the model with flexible wages. Sticky wages help to mitigate all these issues and make the model more empirically relevant.

C.2 Further Results on Redistribution

In this appendix we present further results of the role of redistribution. Table C.1 shows the multipliers under different forms of redistribution in different model specifications with and without idiosyncratic risk and sticky wages.

Two results emerge from this comparison. First, note that the results with and without idiosyncratic risk are qualitatively very similar: redistributing capital income only has strong amplifying effects whereas redistributing profits only leads to dampening. Quantitatively, redistributing capital income has an even stronger magnifying effect in the presence of idiosyncratic risk. Second, also in models with sticky wages, the redistribution of capital and profit income have very different effects. Only redistributing profit income still has a dampening effect relative to the full- and no-redistribution benchmarks. However, because profits are less countercyclical in the model with sticky wages, the dampening is less stark. Similarly, only redistributing capital income has still amplifying effects but they turn out to be a bit less pronounced than in the flexible wage case.[24]

[24]Note that we can make this comparison only in the TANK model, as redistributing only capital

Table C.1: Redistribution under Different Model Specifications

Panel A: Flexible wages

TANK		Profit income	
		Yes	No
Capital income	Yes	1.11	3.31
	No	0.51	2.25

THANK		Profit income	
		Yes	No
Capital income	Yes	1.15	4.34
	No	0.50	2.62

Panel B: Sticky wages

TANK		Profit income	
		Yes	No
Capital income	Yes	1.53	2.12
	No	1.16	1.77

THANK		Profit income	
		Yes	No
Capital income	Yes	1.61	Indet.
	No	1.18	1.95

Notes: Impact responses of aggregate consumption to an expansionary monetary policy shock in heterogeneous-agent models with and without idiosyncratic risk and sticky wages relative to the representative-agent, no-capital benchmark under different schemes of income redistribution.

C.3 No Steady-State Transfers

Until now, we maintained the assumption that consumption of spenders and savers are equalized in steady state. We implemented this using a fixed, steady-state transfer. In this appendix, we show that this assumption is inconsequential for our results. To this end, we solve the model without the steady-state transfer, allowing for unequal consumptions in steady state. The consumption to output ratios are then given by $\frac{C^H}{Y} = (1-\alpha) + \frac{\tau^D}{\lambda}\frac{D}{Y} + \frac{\tau^K}{\lambda}\alpha$ and $\frac{C^S}{Y} = \frac{1}{1-\lambda}\left(\frac{C}{Y} - \lambda\frac{C^H}{Y}\right)$.

Note that this has consequences for the conditions characterizing the optimal behavior of the labor union. In particular, the wage markup is now given by

$$\mu_t^w = \sigma^{-1}\tilde{c}_t + \varphi n_t - w_t,$$

where $\tilde{c}_t = \frac{\lambda(c^H)^{-\frac{1}{\sigma}}}{\lambda(c^H)^{-\frac{1}{\sigma}}+(1-\lambda)(c^S)^{-\frac{1}{\sigma}}}\hat{c}_t^H + \frac{(1-\lambda)(c^S)^{-\frac{1}{\sigma}}}{\lambda(c^H)^{-\frac{1}{\sigma}}+(1-\lambda)(c^S)^{-\frac{1}{\sigma}}}\hat{c}_t^S$, as we can no longer substitute individual consumptions for aggregate consumption.

Tables C.2-C.3 and Figure C.2 show our main results under this alternative steady state. We can see that the results turn out to be very similar to the baseline case. This shows that the steady state transfers used to equalize consumption

income in the THANK model gives rise to indeterminacy.

across agents in steady state is not driving any of our results. Importantly, note that income and consumption inequality are now equally countercyclical in the no capital-proportional incomes case, as expected.

Table C.2: Amplification in Models without Steady-state Transfers

	Rep. agent	Heterogeneous agents		
		Prop. incomes	Inequality	Inequality and risk
No capital	1.00	1.00	1.49	1.67
Capital	0.66	1.12	2.19	2.55

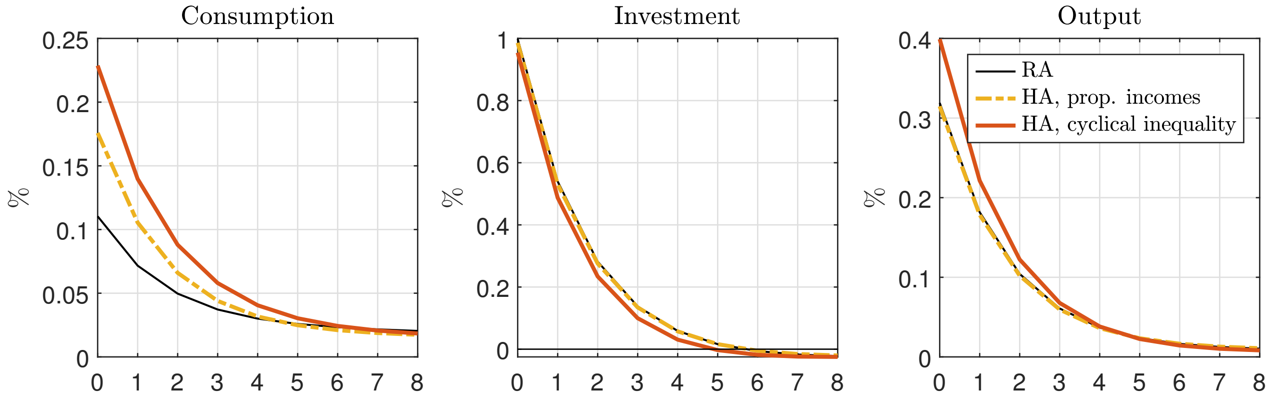
Notes: Impact multipliers on aggregate consumption of an interest-rate cut in each model, relative to the representative agent-no capital benchmark. The heterogeneous-agent models are with: no income inequality and no risk in the second column; income inequality and no risk in the third; both income inequality and risk in the fourth column. In the heterogeneous-agent models, there is no steady-state transfer, allowing for unequal consumptions in steady state.

Table C.3: Amplification in Sticky-wage Models without Steady-state Transfers

	Rep. agent	Heterogeneous agents		
		Prop. incomes	Inequality	Inequality and risk
No capital	1.00	1.00	1.07	1.10
Capital	0.94	1.50	1.77	1.95

Notes: Impact multipliers on aggregate consumption of an interest-rate cut in each model with sticky wages, relative to the rep.-agent no-capital benchmark. The heterogeneous-agent models are with: no income inequality and no risk in the second column; income inequality and no risk in the third; both income inequality and risk in the fourth. In the heterogeneous-agent models, there is no steady-state transfer, allowing for unequal consumptions in steady state.

Figure C.2: Aggregate Effects without Steady-state Transfers



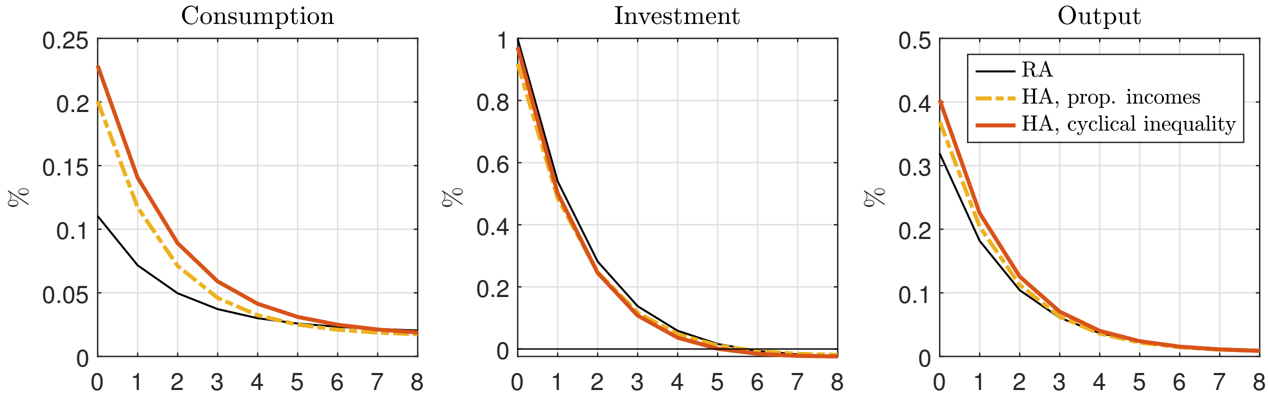
Notes: Impulse responses of aggregate consumption, investment and output to an expansionary interest rate shock of 25 basis points in the representative-agent model and in heterogeneous-agent models with and without income inequality. In the heterogeneous-agent models, there is no steady-state transfer, allowing for unequal consumptions in steady state.

C.4 Keeping Steady State fixed when Equalizing Incomes

To isolate the role of capital inequality, we redistribute financial income fully by taxing capital income and dividends at rate λ. However, taxing capital income not only changes the model dynamics but also the steady-state capital stock, see Appendix B.2. To show that our results are not driven by the change in the steady-state capital stock, we alternatively log-linearize the model around the same steady state (setting tax rate on capital to zero in steady state). Note that this only affects the results of the model with capital and proportional incomes. Under flexible wages the multiplier increases from 1.11 to 1.15. Under sticky wages, the multiplier rises from 1.53 to 1.71. Importantly, however, the complementarity is robust to this change.

Figure C.3 shows the impulse responses of consumption, investment and output. We can see that the responses of the model with proportional incomes are somewhat more pronounced, however, overall the responses are very similar.

Figure C.3: Aggregate Effects keeping Steady State fixed



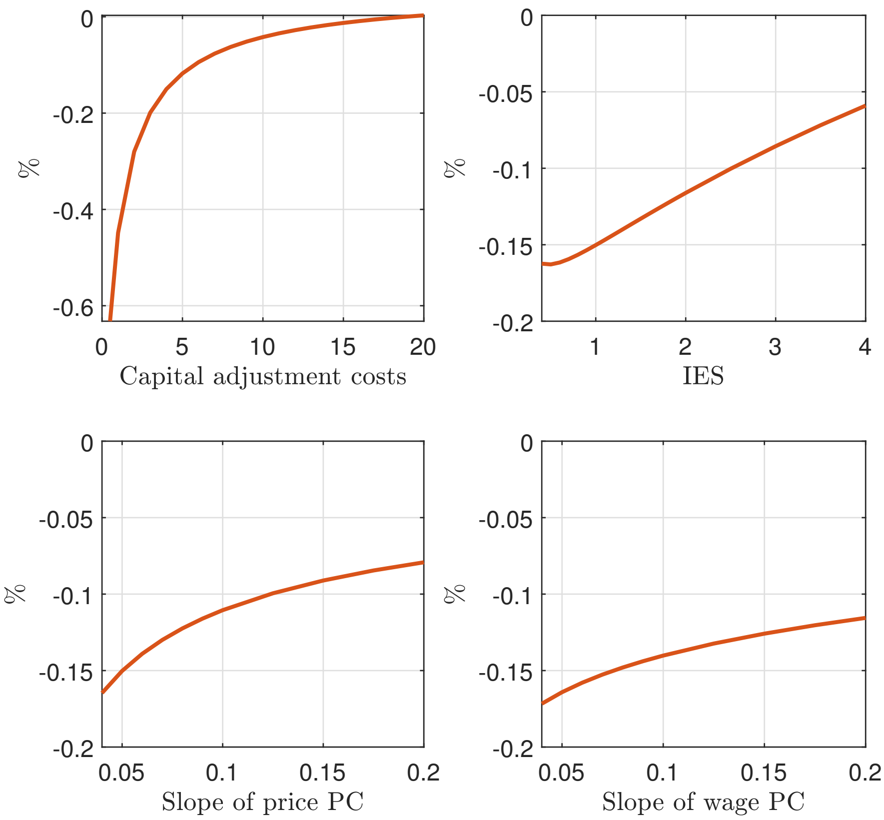
Notes: Impulse responses of aggregate consumption, investment and output to an expansionary interest rate shock of 25 basis points in the representative-agent model and in heterogeneous-agent models with and without income inequality. In the heterogeneous-agent models, we keep the steady state fixed by assuming that the steady-state capital tax is zero in steady state.

C.5 Cyclicality of Consumption and Income Inequality

In Section 4, we have shown that only the model with capital and income inequality is able to match the stylized facts on the cyclicality of consumption and income inequality: (1) both consumption and income inequality are countercyclical; (2) consumption inequality is more countercyclical than income inequality.

Here, we show that this is a robust implication of our model and not just the result of a specific calibration. Figure C.4 depicts the impact response of the difference between consumption and income inequality to a 25 basis points interest-rate cut for different parameterizations for capital adjustment costs, IES, and price and wages stickiness. A negative response indicates that consumption inequality is more countercyclical than income inequality. We can see that the response is consistently negative, implying that the model robustly predicts consumption inequality to be more countercyclical than income inequality.

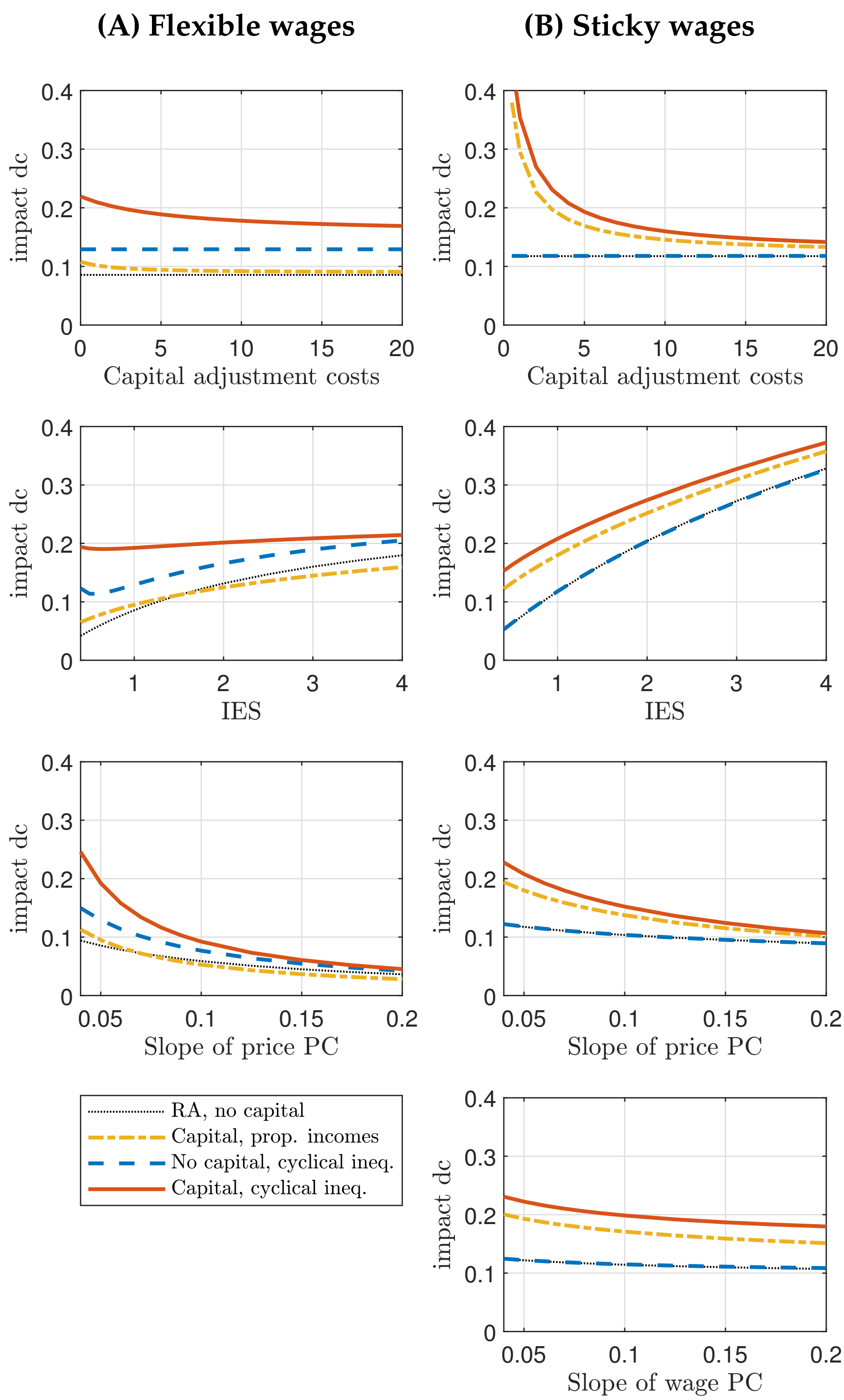
Figure C.4: Consumption and Income Inequality Differential



Notes: The figure shows the impact response of the difference between consumption and income inequality to a 25 basis points interest-rate cut under different parameterizations for capital adjustment costs, IES, and price and wages stickiness.

C.6 Sensitivity of Impact Responses

Figure C.5: Sensitivity Analysis



Notes: Sensitivity of the consumption impact multipliers of a 25 basis points interest-rate cut under different parameterizations for capital adjustment costs, IES, and price stickiness. Panel (A): models under flexible wages. Panel (B): models under sticky wages.