Feedback Effects, Asymmetric Trading, and the Limits to Arbitrage*

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Abstract

We analyze strategic speculators' incentives to trade on information in a model where firm value is endogenous to trading, due to feedback from the financial market to corporate decisions. Trading reveals private information to managers and improves their real decisions, enhancing fundamental value. This feedback effect has an asymmetric effect on trading behavior: it increases (reduces) the profitability of buying (selling) on good (bad) news. This gives rise to an endogenous limit to arbitrage, whereby investors may refrain from trading on negative information. Thus, bad news is incorporated more slowly into prices than good news, potentially leading to overinvestment.

KEYWORDS: feedback effect, asymmetric trading, limits to arbitrage

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1 Introduction

One of the core tenets of financial economics is the informativeness of market prices. The basic argument is that profit opportunities in the financial market will lead speculators to trade on their information, incorporating it into prices and eliminating any mispricing. For example, if speculators have negative private information about a stock, they will find it profitable to sell the stock. This action will push down the price, reflecting the speculators' information.

A key reason why price informativeness is deemed important is that prices can affect real decisions – the feedback effect. Indeed, if prices are informative, it is natural to expect decision makers, such as managers, directors, and activist investors, to use the information in prices to guide actions that affect firm value (such as investment). This paper shows that, if real decision makers take advantage of price informativeness by learning from prices, this affects speculators' incentives to trade on information and thus changes price informativeness in the first place.

The basic idea is as follows. If decision makers use the information in the price to take more informed actions, they will increase the value of the underlying asset. This increased asset value raises a speculator's profits from buying on positive information and lowers her profit from selling on negative information, in some cases causing her to suffer a loss. Taking this effect into account, the speculator may trade less on negative information, thus changing price informativeness in an asymmetric way. In particular, bad news is less likely to be incorporated into prices and affect real decisions. Therefore, the market is not strong-form efficient in the Fama (1970) sense, in that private information is not reflected in the price. However, it is strong-form efficient in the Jensen (1978) sense, in that a privately-informed investor cannot earn profits by trading on her information.

A classic example of how information from the stock market can shape real decisions is Coca-Cola's attempted acquisition of Quaker Oats. On November 20, 2000, the Wall Street Journal reported that Coca-Cola was in talks to acquire Quaker Oats. Shortly thereafter, Coca-Cola confirmed such discussions. The market reacted negatively, sending Coca-Cola's shares down 8% on November 20 and 2% on November 21. Coca-Cola's board rejected the acquisition later on November 21, potentially due to the negative market reaction. The following day, Coca-Cola's shares rebounded 8%. Thus, speculators who had short-sold on the initial merger announcement, based on the belief that the acquisition would destroy value, lost money –

precisely the effect modeled by this paper.[1] In Section 3.5, we discuss another similar example involving Hewlett Packard's (HP) acquisition of Compaq.

We formalize and analyze this intuition in a tractable model that delivers closed-form solutions, allowing the economic forces to be transparent. In particular, by studying versions of the model both with and without feedback, we can understand precisely how the feedback effect changes trading behavior. Our model features a manager, who can either increase investment (i.e., invest), decrease it (i.e., disinvest), or maintain the status quo. If the state of nature is good (bad), the optimal action is to invest (disinvest). While the state is unobserved by the manager, a speculator (such as a hedge fund) may be present in the market; if present, she observes the state and may choose to trade on her private information. As in Kyle (1985), also present is a noise trader and a market maker. The manager observes the trading in the market and uses it to update his prior on the state. If his posterior is sufficiently positive (negative), he invests (disinvests); if his prior is little changed, he maintains the status quo.

Our key result is that, in the presence of the feedback effect – i.e., when financial market trading is sufficiently informative to change the manager's decision – there is an asymmetry between the speculator's trading on positive and negative information. While the feedback effect reduces a speculator's incentive to sell if the state is bad, it increases her incentive to buy if the state is good. The intuition is that, when a speculator trades on information, she improves the efficiency of the firm's decisions, regardless of the direction of her trade. If she sells on negative information, she pushes down the price and conveys to the firm that its investment opportunities are poor. As a result, the firm may disinvest, boosting its value by avoiding overinvestment and reducing the profitability of selling. In contrast, buying on positive information reveals that investment is profitable, persuading the manager to invest more. This also increases firm value, since expansion is the correct decision upon good investment opportunities, and thus increases the profitability of buying.

Formally, in the presence of feedback, there is a clear asymmetry in equilibrium outcomes, whereby the range of parameters giving rise to an equilibrium where the speculator trades on good news and not on bad news is a strict superset of the range giving rise to an equilibrium

[1] Our model predicts that speculators refrain from selling in expectation of deal cancelation, the direct evidence of which is not empirically detectable. In the above example, speculators who sold might have expected that the acquisition would go through due to managerial private benefits. Hence, the example should be used to demonstrate the losses incurred by speculators when a corrective action was unexpectedly adopted in response to their selling.

where she trades on bad news and not on good news. Moreover, there is a range for which the equilibrium is unique and involves the speculator buying on good news and not trading on bad news. This equilibrium is no longer unique when feedback is not present, i.e. trading in the financial market is not sufficiently informative to change the manager's investment decision. In this no-feedback case, the equilibrium with buying and no selling exists over a smaller range and always coincides with the equilibrium that features selling and no buying.

Even though the speculator's trading behavior is asymmetric, it is not automatic that the impact on prices will be. The market maker is rational and takes into account that the speculator trades less on negative information, and so he adjusts his pricing function accordingly. Therefore, it may seem that negative information should have the same absolute price impact as positive information: the market maker knows that a moderate order flow can stem from the speculator having negative information but choosing not to trade, and therefore should decrease the price accordingly. We show that asymmetry in trading behavior does translate into asymmetry in price impact. The crux is that the market maker cannot distinguish the case of a speculator who has negative information but chooses to withhold it, from the case in which she is absent. Thus, a moderate order flow – which is consistent both with the speculator being absent, and with her being negatively-informed and not trading – does not lead to a large stock price decrease, and so negative information has a smaller effect on prices. Defining "news" as information received by the speculator (i.e. the speculator being present), our model implies that bad news travels slowly: it leads to a smaller short-term price impact and potentially larger long-run drift than good news. A common explanation for this phenomenon is that managers possess value-relevant information and publicize good news more readily than bad news, because they wish to boost the stock price (Hong, Lim, and Stein (2000)). Here, key information is held by a firm's investors rather than its managers, who "publicize" it not through public news releases, but by trading on it. Investors choose to disseminate bad news less readily than good news due to the feedback effect and its implications for trading profits.[2]

These stock return effects are most likely around major corporate events, when important decisions are taken such as an acquisition, a new product launch, or a change in strategy. That these events, and thus the stock return effects, do not necessarily happen on a day-to-day

[2] Another difference is that Hong, Lim, and Stein (2000) empirically find a profitable trading strategy inconsistent with market efficiency. In our model, the market is semi-strong-form efficient and so there is no profitable trading strategy. While bad news can lead to a larger long-run drift than good news, this result is conditional upon the speculator being present, which is unobservable to a potential trader.

basis does not take away from their importance. This is because these effects occur exactly at times when the stock price performs its utmost important role of affecting real decisions and allocating resources. Indeed, the asymmetric trading captured in our model generates important real consequences. Since negative information is less incorporated into prices, it has a lower effect on management decisions. Thus, while positive net present value ("NPV") projects will be encouraged, some negative-NPV projects will not be canceled, leading to overinvestment overall. In contrast to standard overinvestment theories based on the manager having private benefits (e.g., Jensen (1986), Stulz (1990), Zwiebel (1996)), here the manager is fully aligned with firm value and there are no agency problems. The manager wishes to maximize firm value by learning from prices, but is unable to do so since speculators refrain from revealing negative information. Applied to M&A as well as organic investment, the theory may explain why M&A appear to be "excessive" and a large fraction of acquisitions destroy value (see, e.g., Andrade, Mitchell, and Stafford (2001)). While intuition would suggest that the market can prevent bad acquisitions by communicating negative information to the manager, our model shows that it may fail to do so due to the feedback effect.

Our mechanism is based on the presence of a feedback effect – decision makers learn from the market when deciding their actions. A common perception is that managers know more about their own firms than outsiders (e.g. Myers and Majluf (1984)). While this perception is plausible for internal information about the firm in isolation, optimal decisions also depend on external information (such as market demand for a firm's products, the future prospects of the industry, or potential synergies with a target) which outsiders may possess more of. For example, a potential acquirer hires investment bank advisors even though they have less internal information, because they can add value on target selection. More importantly, we only require that outside investors possess some information that the manager does not have; they need not be more informed than the manager on an absolute basis. Luo (2005) provides large-sample evidence that an acquisition is more likely to be canceled if the market reacts negatively, particularly in cases where learning is more probable. Relatedly, Edmans, Goldstein, and Jiang (2012) demonstrate that a firm's market price affects the likelihood that it becomes a takeover target, which may arise because potential acquirers learn from the market price. More broadly, Chen, Goldstein, and Jiang (2007) show that the sensitivity of investment to price is higher when the price contains more private information not known to managers.

The model also applies to decision makers other than the manager who aim to maximize firm value, such as a board or an activist investor: a low stock price may induce them to block a bad investment or fire an underperforming manager. In addition to corporate decision makers, the model can also apply to regulators or policymakers who also affect security values: low stock or bond prices may trigger a bailout. Moreover, the applicability of our theory goes beyond financial markets to other economic contexts such as prediction markets, which can provide key information to policymakers (Wolfers and Zitzewitz (2004)).

This paper contributes to the literature exploring the theoretical implications of the feedback effect: see Bond, Edmans, and Goldstein (2012) for a survey. To our knowledge, we are the first to point out that feedback leads to an asymmetry between buying on good news and selling on bad news. A key ingredient for our result is that the speculator is acting strategically, i.e., she takes into account her impact on the price and the firm's decision. In reality, the most informed speculators are likely to be large traders (such as hedge funds); indeed, it is their ability to make large trades that incentivizes information acquisition. While strategic behavior and price impact are common in the broader literature on financial markets without feedback (e.g. Kyle (1985)), they are missing from most papers analyzing the implications of feedback for price informativeness. For example, the financial market is modeled as a "black box" in Bond, Goldstein, and Prescott (2010) as the price simply equals expected value given fundamentals, and there is no account of how speculators incorporate their information into the price via trading. Dow, Goldstein, and Guembel (2010), Goldstein, Ozdenoren, and Yuan (2013), and Bond and Goldstein (2014) feature a continuum of traders who effectively act as price takers.

Another feedback paper that does feature a strategic trader is Goldstein and Guembel (2008). Their paper analyzes how feedback provides an incentive for an uninformed speculator to manipulate the stock price by short-selling the stock. This reduces the stock price and induces incorrect disinvestment, thus generating a profit on the speculator's short position. Their model does not explore the potential asymmetry between trading on good versus bad news.[3] More recently, Boleslavsky, Kelly, and Taylor (2014) build on our analysis and develop another model where feedback leads to asymmetric trading by a strategic investor. Their paper demonstrates the broader applicability of the mechanism in our paper to the context of policymakers learning from the price to guide a bailout or monetary stimulus, as well as its

[3] The Goldstein and Guembel (2008) framework would not be appropriate to explore this asymmetry, given its other complexities. It needs to track the behavior of uninformed speculators, the core of the manipulation story, and to deal with multiple rounds of trade, which are essential for the manipulation strategy to work.

robustness to other modeling approaches. We discuss their paper further in Section 3.5.5.

Finally, the paper contributes to the large literature on limits to arbitrage[4], which analyzes why speculators do not trade fully on their information. We present a new source of limits to arbitrage, which arises endogenously as part of the arbitrage process – the feedback effect. It stems from the fact that the value of the asset being arbitraged is endogenous to the act of exploiting the arbitrage. Campbell and Kyle (1993) focus on fundamental risk, i.e., the risk that firm fundamentals will change while the arbitrage strategy is being pursued. In their model, such changes are unrelated to speculators' arbitrage activities. De Long, Shleifer, Summers, and Waldmann (1990) study noise trader risk, i.e., the risk that noise trading will increase the degree of mispricing. Noise trading only affects the asset's market price and not its fundamental value, which is again exogenous to the act of arbitrage. Shleifer and Vishny (1997) show that, even if an arbitrage strategy is sure to converge in the long-run, the possibility that mispricing may widen in the short-run may deter speculators from pursuing it, if they are concerned with short-run redemptions by their own investors. Similarly, Kondor (2009) demonstrates that financially-constrained arbitrageurs may stay out of a trade if they believe that it will become more profitable in the future. Many authors (e.g., Pontiff (1996), Mitchell and Pulvino (2001), and Mitchell, Pulvino, and Stafford (2002)) focus on the transaction and holding costs that arbitrageurs incur while pursuing an arbitrage strategy. Others (Geczy, Musto, and Reed (2002) and Lamont and Thaler (2003)) discuss the importance of short-sales constraints.

While many of these papers emphasize market frictions as the source of limits to arbitrage, the limit to arbitrage we uncover arises precisely when the market performs its utmost efficient role: guiding the allocation of real resources. Thus, while limits to arbitrage based on market frictions tend to attenuate with the development of financial markets, the effect identified by this paper may strengthen: as investors become more sophisticated, managers will learn from them to a greater degree. A natural limit to arbitrage featured in Kyle (1985) and the vast subsequent literature is price impact – trades move prices closer to fundamental value, and so speculators reduce their trading volumes to lessen this impact. In contrast, the feedback effect constitutes a limit to arbitrage by moving the fundamental value closer to the price.

This paper proceeds as follows. Section 2 presents the model. Section 3 contains the core analysis, demonstrating the asymmetric limit to arbitrage. Section 4 investigates the extent

[4] Here, we use "arbitrage" to refer to investors trading on their private information. This notion of "arbitrage" is broader than the traditional textbook notion of risk-free arbitrage when trading two identical securities.

to which information affects beliefs and prices. Section 5 concludes. Appendix A contains all proofs not in the main text.

2 The Model

The model has three dates, $t \in \{0, 1, 2\}$. There is a firm whose stock is traded in the financial market. The firm's manager needs to take a decision on whether to keep the current level of investment, increase it, or reduce it. The manager's goal is to maximize expected firm value; since there are no agency problems between the manager and the firm, we will use these two terms interchangeably. At $t = 0$, a risk-neutral speculator may be present in the financial market. If present, she is informed about the state of nature θ that determines both the value of the firm under the current investment level, and also the profitability of increasing or decreasing investment. She rationally anticipates the effect of her trading on the manager's investment level. Trading in the financial market occurs at $t = 1$. In addition to the speculator, two other agents participate in the financial market: a noise trader whose trades are unrelated to the realization of θ, and a risk-neutral market maker. The latter collects the orders from the speculator and noise trader, and sets a price at which he executes the orders out of his inventory. This price rationally anticipates the manager's investment decision. At $t = 2$, the manager takes the decision, which may be affected by the trading in the financial market at $t = 1$. Finally, all uncertainty is resolved and payoffs are realized. We now describe the firm's investment problem and the trading process in more detail.

2.1 The Firm's Decision

At $t = 2$, the manager takes an investment decision denoted by $d \in \{-1, 0, 1\}$, where $d = 0$ represents maintaining the current level of investment, $d = 1$ represents increasing investment (which we will often simply refer to as "investment"), and $d = -1$ represents reducing investment ("disinvestment"). Changing the level of investment in either direction (i.e., choosing $d \in \{-1, 1\}$) costs the firm $c \geq 0$. As we will discuss in Section 3.5, all of the model's results regarding the feedback effect hold with $c = 0$. The case of $c > 0$ allows for the possibility of no feedback effect, thus enabling us to understand the role of the feedback effect in our results.

The value of the firm, realized at $t = 2$, is denoted by $v(\theta, d)$. It depends on both the manager's action d and the state of nature $\theta \in \Theta \equiv \{H, L\}$ ("high" and "low"), and is summarized in Table 1. If the firm chooses $d = 0$, it is worth $v(H, 0) = R_H$ in state H and $v(L, 0) = R_L < R_H$ in state L. In state H, the correct action is to increase investment; doing so creates additional value of $x > 0$ (gross of the cost $c < x$) and so $v(H, 1) = R_H + x - c$. Reducing investment is the incorrect action and reduces firm value by x, and so $v(H, -1) = R_H - x - c$. Conversely, in state L, choosing $d = -1$ creates additional value of x, yielding a value of $v(L, -1) = R_L + x - c$; choosing $d = 1$ costs the firm x, yielding a value of $v(L, 1) = R_L - x - c$. We deliberately set the value created by correct investment in state H to equal the value created by correct disinvestment in state L, and to be the negative of the value destroyed by an incorrect investment decision, to avoid baking any asymmetries into the model. Instead, the asymmetric limit to arbitrage will stem entirely from the feedback effect.

		Investment d		
		1	0	-1
State θ	H	$R_H + x - c$	R_H	$R_H - x - c$
	L	$R_L - x - c$	R_L	$R_L + x - c$

Table 1: Firm value

Note that the above specification implies that:

$$v(H, 1) - v(L, 1) > v(H, 0) - v(L, 0) > v(H, -1) - v(L, -1). \tag{1}$$

Inequality (1) is the driving force behind our results. It means that increasing (reducing) investment increases (reduces) the dependence of firm value on the state. Thus, the speculator's private information on the state is less useful, the lower the investment level chosen by the manager. In turn, inequality (1) incorporates two cases, depending on whether firm value is monotonic in the underlying state:

Case 1: $v(H, -1) > v(L, -1)$, i.e. $R_H - x > R_L + x$. In this case, state H entails higher firm value, no matter what action has been taken by the firm. Hence, disinvestment attenuates, but does not eliminate, the effect of the state on firm value. For example, state H (L) can represent high (low) demand for the firm's products. Whether the firm increases or reduces its level of production, its value will be lower in state L, but the negative effect of low demand is attenuated if the firm operates at a lower scale. Note that $R_H - x > R_L + x$ is equivalent to

$R_H - R_L > 2x$, i.e. the speculator's private information over assets in place is relatively more important than the manager's investment decision, and thus the feedback effect.[5]

Case 2: $v(H,-1) < v(L,-1)$, i.e. $R_H - x < R_L + x$. In this case, if disinvestment occurs, firm value is higher in state L. The investment decision is sufficiently powerful to overturn the effect of the state on firm value. Firm value is non-monotonic in the state: one state does not dominate the other. For example, consider the case where $d = 1$ implies proceeding with a takeover decision, $d = -1$ implies selling assets for cash, and $d = 0$ implies doing nothing. State H corresponds to a state in which current acquisition opportunities dominate future ones, and state L refers to the reverse. If the firm does nothing or makes an acquisition, its value is higher in state H. In contrast, if the firm sells assets to raise cash, its value is higher in state L since it can use the cash raised to exploit future acquisition opportunities. Another example is related to Aghion and Stein (2008): $d = 1$ corresponds to a growth strategy, and $d = -1$ corresponds to a strategy focused on current profit margins. Growth prospects are good if $\theta = H$ and bad if $\theta = L$. If the firm eschews the growth strategy ($d = -1$), its value is higher in the low state in which there are no growth opportunities. In contrast, in the high state its rivals could pursue the growth opportunities, in turn worsening its competitive position.

Case 1, where a "high" state dominates a "low" state, is the common assumption in the literature (including the prior limits-to-arbitrage literature where firm value is exogenous) and will be the focus of our analyses. Section 3.4 will briefly discuss Case 2 and explain how the fundamental intuition for our asymmetric limit to arbitrage becomes even stronger; the full analysis is in Appendix B.1.

The prior probability that the state is $\theta = H$ is $y = \frac{1}{2}$, which is common knowledge. The manager uses information from trades in the financial market to update his prior to form a posterior q, which then guides his investment decision. Let γ_1 denote the posterior belief that the state is H such that the manager is indifferent between investing and doing nothing, i.e.:

$$\gamma_1 R_H + (1-\gamma_1) R_L = \gamma_1 (R_H + x) + (1-\gamma_1)(R_L - x) - c, \tag{2}$$

[5] The importance of the feedback effect is given by the gross gain in firm value from correct (dis)investment x, rather than the net gain $x - c$. It is true that inducing the manager to take the correct action increases firm value by $x - c$. However, the feedback effect can also deter the manager from taking the incorrect action, which would lead to firm value changing by $-x - c$. Thus, the gain in firm value from avoiding the incorrect decision is $x + c$, and so the cost c nets out.

which yields

$$\gamma_1 = \frac{1}{2} + \frac{c}{2x}.$$

Similarly, let γ_{-1} be the posterior belief on state H such that the manager is indifferent between disinvesting and doing nothing, i.e.:

$$\gamma_{-1} R_H + (1 - \gamma_{-1}) R_L = \gamma_{-1} (R_H - x) + (1 - \gamma_{-1}) (R_L + x) - c,$$

which yields

$$\gamma_{-1} = \frac{1}{2} - \frac{c}{2x}.$$

For completeness and without loss of generality, if the manager is indifferent between doing nothing and changing the investment level, we will assume that he will maintain the status quo. The values of γ_1 and $\gamma_{-1} < \gamma_1$ represent "cutoffs" that determine the manager's action. If and only if $q > \gamma_1$, he will increase investment; if and only if $q < \gamma_{-1}$, he will reduce investment. For $\gamma_{-1} \leq q \leq \gamma_1$, he will maintain the current investment level.

Since $y = \frac{1}{2}$, the ex-ante net firm value created by changing investment in either direction is $\frac{1}{2}(x - c) + \frac{1}{2}(-x - c) = -c \leq 0$, and so the ex-ante optimal decision is to do nothing. As long as the information in the market does not change the manager's prior much ($\gamma_{-1} \leq q \leq \gamma_1$), he will maintain the current investment level. As we can see from the definitions of γ_{-1} and γ_1, the range of posteriors for which the firm remains with the status quo is increasing in the adjustment cost c and decreasing in the value created from optimizing investment x.

2.2 Trade in the Financial Market

At $t = 0$, a speculator arrives in the financial market with probability λ, where $0 < \lambda < 1$. Whether she is present is unknown to anyone else.[6] If present, she observes the state of nature θ with certainty. We will use the term "positively- (negatively-) informed speculator" to describe a speculator who observes $\theta = H$ ($\theta = L$). The variable λ is a measure of market sophistication or the informedness of outside investors, and will generate a number of comparative statics. The speculator has no initial position in the firm. Section 3.5 will discuss how the key intuition and results continue to hold under a positive initial stake; the full analysis is in Appendix B.2.

[6]Since private information is not public knowledge, its existence is also unlikely to be public knowledge. Chakraborty and Yilmaz (2004) also feature uncertainty on whether the speculator is present, in an equilibrium in which informed insiders manipulate the market by trading in the wrong direction.

Trading in the financial market happens at $t = 1$. Always present is a noise trader, who trades $z \in \{-1, 0, 1\}$ with equal probability. If the speculator is present, she makes an endogenous trading choice $s \in \{-1, 0, 1\}$. Trading either -1 or 1 costs the speculator κ. The trading cost κ should be interpreted broadly. While direct transaction costs from commissions are typically small, other indirect costs can be large. These include borrowing costs (for short sales) and the opportunity costs of capital commitment (for purchases). These costs may differ between buying and selling, but the relative size is a priori unclear. Given our interest in exploring the endogenous asymmetry between buying and selling due to the feedback effect, we assume the same trading cost κ in both directions to avoid generating any asymmetry mechanically. Unless otherwise specified, we refer to trading profits and losses gross of the cost κ. If the speculator is indifferent between trading and not trading, we assume that she will not trade.

Following Kyle (1985), market orders are submitted simultaneously to a competitive market maker who absorbs orders out of his inventory and sets the price equal to expected asset value, given the information contained in the order flow. The market maker can only observe total order flow $X = s + z$, but not its individual components s and z. Possible order flows are $X \in \{-2, -1, 0, 1, 2\}$ and the pricing function is $p(X) = E(v|X)$. A critical departure from Kyle (1985) is that firm value here is endogenous, because it depends on the manager's action which is in turn based on information revealed by trading.

Specifically, the manager observes total order flow X and uses it to form his posterior q, which then guides his investment decision. Allowing the manager to observe order flow X, rather than just the price p, simplifies the analysis without affecting its economic content. In the equilibria that we analyze, there is a one-to-one correspondence between the price and the order flow in most cases; in the few cases where two order flows correspond to the same price, the manager's decision is the same for both order flows. Under the alternative assumption that the manager observes p, other equilibria can arise, in which the market maker sets a price that is consistent with a different managerial decision (one that is suboptimal given the information in the order flow) and this becomes self-fulfilling due to the dependence of the manager's decision on the price. Since our interest is in the feedback effect, we focus on equilibria where the manager's decision responds optimally to the information in the order flow.[7]

[7] Moreover, it seems reasonable to assume that managers have access to information about trading quantities. First, market making is competitive and so there is little secrecy in the order flow; second, microstructure databases (such as TAQ) provide such information at a short lag – rapidly enough to guide investment decisions.

As is standard in the feedback literature, we assume that the speculator cannot credibly communicate her information directly to the manager, since it is non-verifiable. Instead, she uses her information to maximize her trading profits (as in the theories of governance through trading/"exit" by Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011)). The trade-off between using private information to trade or intervene has been studied by Maug (1998) and Kahn and Winton (1998).

2.3 Equilibrium

The equilibrium concept we use is Perfect Bayesian Equilibrium. Here, it is defined as follows: (i) A trading strategy by the speculator: $S : \Theta \to \{-1, 0, 1\}$ that maximizes her expected final payoff $s(v - p) - |s|\kappa$, given the price setting rule, the strategy of the manager, and her information about the realization of θ. (ii) An investment strategy by the manager $D : \mathcal{Q} \to \{-1, 0, 1\}$ (where $\mathcal{Q} = \{-2, -1, 0, 1, 2\}$), that maximizes expected firm value v given the information in the order flow and all other strategies. (iii) A price setting strategy by the market maker $p : \mathcal{Q} \to \mathbb{R}$ that allows him to break even in expectation, given the information in the order flow and all other strategies. Moreover, (iv) the firm and the market maker use Bayes' rule to update their beliefs from the orders they observe in the financial market, and (v) beliefs on outcomes not observed on the equilibrium path satisfy the Cho and Kreps (1987) Intuitive Criterion. Finally, (vi) all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium.

3 Feedback Effect and Asymmetric Trading

In this section, we characterize the pure-strategy equilibria in our model and demonstrate the asymmetric limits to arbitrage that result from the feedback effect. We focus on Case 1 ($R_H - x > R_L + x$), where firm value is monotonic in the state. Case 2 is briefly discussed in Section 3.4 and fully analyzed in Appendix B.1.

3.1 Overview of equilibria when firm value is monotonic in states

The equilibrium will depend on whether order flow is sufficiently informative to overturn the ex-ante optimal decision of $d = 0$. Hence, we distinguish between two cases. In the first ("feedback") case, $\frac{1}{2-\lambda} > \gamma_1$. As we will show, $\frac{1}{2-\lambda}$ represents the posterior probability of state H under an order flow of $X = 1$ in some equilibria. When $\frac{1}{2-\lambda} > \gamma_1$, the probability λ that the speculator is present is high enough that $X = 1$ is sufficiently informative to induce the manager to invest. Thus, there is feedback from the market to real decisions. Since $\gamma_{-1}+\gamma_1 = 1$, $\frac{1}{2-\lambda} > \gamma_1$ is equivalent to $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$. In some equilibria, $\frac{1-\lambda}{2-\lambda}$ represents the posterior probability of state H under an order flow of $X = -1$. When $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$, the posterior is sufficiently low to induce the manager to disinvest. In the second ("no feedback") case, $\frac{1}{2-\lambda} \leq \gamma_1$ and $\frac{1-\lambda}{2-\lambda} \geq \gamma_{-1}$. Here, there is no feedback effect for these posteriors: the order flow is not sufficiently informative to change the manager's decision from the status quo.

As we will show, depending on the values of κ, four equilibrium outcomes can arise:

1. No Trade Equilibrium NT: the speculator does not trade,

2. Trade Equilibrium T: the speculator buys when she knows that $\theta = H$ and sells when she knows that $\theta = L$,

3. Partial Trade Equilibrium BNS (Buy – Not Sell): the speculator buys when she knows that $\theta = H$ and does not trade when she knows that $\theta = L$,

4. Partial Trade Equilibrium SNB (Sell – Not Buy): the speculator does not trade when she knows that $\theta = H$ and sells when she knows that $\theta = L$.

3.2 No feedback equilibria

Lemma 1 provides the characterization of equilibrium outcomes in the case of no feedback.

Lemma 1 *(Equilibrium, firm value is monotone in the state, no feedback). Suppose that $R_H - x > R_L + x$ and $\frac{1}{2-\lambda} \leq \gamma_1$ ($\Leftrightarrow \frac{1-\lambda}{2-\lambda} \geq \gamma_{-1}$). There exist cutoffs $\kappa_{NF} < \kappa_{NT}$ (defined in the proof) such that the trading game has the following pure-strategy equilibria:*

(a) When $\kappa < \kappa_{NF}$, the only pure-strategy equilibrium is T.

(b) When $\kappa \geq \kappa_{NT}$, the only pure-strategy equilibrium is NT.

(c) When $\kappa_{NF} \leq \kappa < \kappa_{NT}$, the two pure strategy equilibria are BNS and SNB.

There is no range of parameter values for which the BNS equilibrium exists and the SNB equilibrium does not exist, or vice versa.

Proof. This proof is incorporated in the proof of Proposition 1. ■

Two sources of limits to arbitrage are present in the no-feedback case, both of which are standard in the literature, and both of which are symmetric. The first source is the trading cost κ. As κ increases, we move to equilibria in which speculators trade less on their private information. κ_{NT} is the threshold for no trading: when $\kappa \geq \kappa_{NT}$ there is no trading in either direction. Unsurprisingly, greater transaction costs deter speculators from trading. At the other extreme, when the trading cost is sufficiently low ($\kappa < \kappa_{NF}$, where the subscript indexes the "no feedback" regime), the speculator always trades on her private information.

The second source of limits to arbitrage is the price impact that speculators exert when they trade on their information: Knowing that trading might move the price against them, speculators might refrain from trading. In our model, price impact leads to partial trade equilibria in the intermediate region $\kappa_{NF} \leq \kappa < \kappa_{NT}$. In these equilibria, the speculator trades on one type of information but not the other. While these equilibria are asymmetric – the speculator either buys on good news and does not trade on bad news, or she sells on bad news and does not trade on good news – there is symmetry in that both types of asymmetric equilibria, BNS and SNB, are possible in exactly the same range of parameters.

To understand the intuition behind this pair of asymmetric equilibria, consider the BNS equilibrium (the SNB equilibrium is analogous). In this equilibrium, the market maker believes that the speculator buys on good news and does not trade on bad news. Given that the market maker believes that the speculator buys on good news, a negative order flow is very revealing that the speculator is negatively informed and the price moves sharply to reflect this. Specifically, $X = -1$ is inconsistent with the speculator having positive information (as she would have bought), and so the price is only $\frac{1-\lambda}{2-\lambda} R_H + \frac{1}{2-\lambda} R_L$. Thus, the speculator makes little profit from selling on bad news; knowing this, she chooses not to trade on bad news. Conversely, given that the market maker believes that the speculator does not sell on bad news, a positive order flow of $X = 1$ is consistent with the speculator being negatively informed and choosing not to trade. As a result, the market maker sets a relatively low price of $\frac{1}{2} R_H + \frac{1}{2} R_L$, which allows the speculator to make high profits by buying. Thus, the equilibrium is sustainable.

These partial trade equilibria are an interesting feature of our no-feedback case. To our knowledge, they have not been previously discussed in the literature. However, they are driven by the well-known economic force of price impact. In many theories, price impact causes speculators to scale down their trading, and this is manifested in different ways in different models.

In our model, price impact is manifested in asymmetric partial trade equilibria: The order flow in the direction in which the speculator does not trade becomes particularly informative, leading to a larger price impact which reduces the potential trading profits. Importantly, in the absence of feedback, this force is symmetric: There is no value of κ in which one partial trade equilibrium exists but the other does not. The same force that deters the speculator from selling in the BNS equilibrium also deters her from buying in the SNB equilibrium, and the two forces are equally strong. Thus, the two equilibria are possible in exactly the same range of parameter values, and there is no range of parameter values for which either equilibrium is unique. In addition, there is no obvious way to select between these two equilibria. Under both BNS and SNB, expected firm value is $\frac{1}{2}(R_H + R_L) + \frac{1}{6}(x - c)$ and the speculator's expected trading profit is $\frac{1}{6}(R_H - R_L) - \frac{1}{2}\kappa$ (implying the same losses for noise traders). Hence, we cannot rank these equilibria based on the Pareto criterion.

3.3 Feedback equilibria

3.3.1 Characterization of equilibrium outcomes

Proposition 1 provides the characterization of equilibrium outcomes in the case of feedback.

Proposition 1 *(Equilibrium, firm value is monotone in the state, feedback). Suppose that $R_H - x > R_L + x$ and $\frac{1}{2-\lambda} > \gamma_1$ ($\Leftrightarrow \frac{1-\lambda}{2-\lambda} < \gamma_{-1}$). There exist cutoffs κ_{SNB}, κ_{NT}, and κ_T (defined in the proof), where $\kappa_T < \kappa_{SNB}$ and $\kappa_T < \kappa_{NT}$, such that the trading game has the following pure-strategy equilibria:*

(a) When $\kappa < \kappa_T$, the only pure-strategy equilibrium is T.

(b) When $\kappa \geq \kappa_{NT}$, the only pure-strategy equilibrium is NT.

(c) When $\kappa_T \leq \kappa < \kappa_{NT}$, BNS is an equilibrium.

(d) If $\kappa_{SNB} < \kappa_{NT}$, SNB is also an equilibrium in the range $\kappa_{SNB} \leq \kappa < \kappa_{NT}$.

There is a strictly positive range of parameter values ($\kappa_T \leq \kappa < \min(\kappa_{SNB}, \kappa_{NT})$) for which BNS is the only pure-strategy equilibrium. There is no range of parameter values for which the SNB equilibrium exists but the BNS equilibrium does not exist. Equilibrium results are depicted in Figure 1, which also contrasts them with the equilibrium results in the case of no feedback.

Proof. (This proof also incorporates the proof of Lemma 1 for ease of comparison. More details behind the calculations below are in Appendix A.) Since firm value is always higher when $\theta = H$ than when $\theta = L$, it is straightforward to show that the speculator will never buy when she knows that $\theta = L$ and never sell when she knows that $\theta = H$. Then, the only possible pure-strategy equilibria are NT, T, BNS, and SNB. Below, we identify the conditions under which each of these equilibria holds. If an order flow of $X = -2$ ($X = 2$) is observed off the equilibrium path, we assume that the market maker and manager believe that the speculator knows that the state is L (H). Since speculators always lose if they trade against their information, this is the only belief that is consistent with the Intuitive Criterion.

No Trade Equilibrium NT:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table (see Appendix A for the full calculations):

X	-2	-1	0	1	2
q	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
d	-1	0	0	0	1
p	$R_L + x - c$	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$R_H + x - c$

As shown in Appendix A, the gain to the negatively-informed speculator (gross of the transaction cost κ) from deviating to selling is $\kappa_{NT} \equiv \frac{1}{3}(R_H - R_L)$, and this is also the gain to the positively-informed speculator from deviating to buying. Thus, this equilibrium holds if and only if $\kappa \geq \kappa_{NT}$.

Partial Trade Equilibrium BNS:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:

X	-2	-1	0
q	0	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$
d	-1	$\begin{cases} -1 & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma_{-1} \\ 0 & \text{if } \frac{1-\lambda}{2-\lambda} \geq \gamma_{-1} \end{cases}$	0
p	$R_L + x - c$	$\begin{cases} \frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma_{-1} \\ \frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L & \text{if } \frac{1-\lambda}{2-\lambda} \geq \gamma_{-1} \end{cases}$	$\frac{1}{2}R_H + \frac{1}{2}R_L$

X	1	2
q	$\frac{1}{2}$	1
d	0	1
p	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$R_H + x - c$

Calculating the gain to the negatively-informed speculator from deviating to selling and to the positively-informed speculator from deviating to not trading, we can see that this equilibrium holds if and only if $\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right] \equiv \kappa_T \leq \kappa < \kappa_{NT} \equiv \frac{1}{3}(R_H - R_L)$ for the case of feedback and if and only if $\frac{1}{3}\left[\left(\frac{1-\lambda}{2-\lambda} + \frac{1}{2}\right)(R_H - R_L)\right] \equiv \kappa_{NF} \leq \kappa < \kappa_{NT} \equiv \frac{1}{3}(R_H - R_L)$ for the case of no feedback.

Partial Trade Equilibrium SNB:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:

X	-2	-1	0
q	0	$\frac{1}{2}$	$\frac{1}{2}$
d	-1	0	0
p	$R_L + x - c$	$\frac{1}{2}R_H + \frac{1}{2}R_L$	$\frac{1}{2}R_H + \frac{1}{2}R_L$

X	1	2
q	$\frac{1}{2-\lambda}$	1
d	$\begin{cases} 0 & \text{if } \frac{1}{2-\lambda} \leq \gamma_1 \\ 1 & \text{if } \frac{1}{2-\lambda} > \gamma_1 \end{cases}$	1
p	$\begin{cases} \frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L & \text{if } \frac{1}{2-\lambda} \leq \gamma_1 \\ \frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c & \text{if } \frac{1}{2-\lambda} > \gamma_1 \end{cases}$	$R_H + x - c$

Calculating the gain to the negatively-informed speculator from deviating to not trading and to the positively-informed speculator from deviating to buying, we can see that this equilibrium holds if and only if $\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) + \frac{1}{2}(R_H - R_L)\right] \equiv \kappa_{SNB} \leq \kappa < \kappa_{NT}$ for the case of feedback and if and only if $\kappa_{NF} \leq \kappa < \kappa_{NT}$ for the case of no feedback.

Trade Equilibrium T:

For a given order flow X, the posterior q, the manager's decision d and the price p are given by the following table:

X	-2	-1	0
q	0	$\frac{1-\lambda}{2-\lambda}$	$\frac{1}{2}$
d	-1	$\begin{cases} -1 & \text{if } \frac{1-\lambda}{2-\lambda} < \gamma_{-1} \\ 0 & \text{if } \frac{1-\lambda}{2-\lambda} \geq \gamma_{-1} \end{cases}$	0
p	$R_L + x - c$	$\begin{cases} \frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c & \text{if } \frac{1-\lambda}{2-\lambda} \leq \gamma_{-1} \\ \frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L & \text{if } \frac{1-\lambda}{2-\lambda} > \gamma_{-1} \end{cases}$	$\frac{1}{2}R_H + \frac{1}{2}R_L$

X	1	2
q	$\frac{1}{2-\lambda}$	1
d	$\begin{cases} 0 & \text{if } \frac{1}{2-\lambda} \leq \gamma_1 \\ 1 & \text{if } \frac{1}{2-\lambda} > \gamma_1 \end{cases}$	1
p	$\begin{cases} \frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L & \text{if } \frac{1}{2-\lambda} \leq \gamma_1 \\ \frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c & \text{if } \frac{1}{2-\lambda} > \gamma_1 \end{cases}$	$R_H + x - c$

Calculating the gain to both the positively-informed and negatively-informed speculator from deviating to not trading, we can see that this equilibrium holds if and only if $\kappa < \kappa_T$ for the case of feedback and if and only if $\kappa < \kappa_{NF}$ for the case of no feedback. ■

3.3.2 Discussion of equilibria and comparison with the case of no feedback

Figure 1 demonstrates the contrast in possible equilibrium outcomes between the feedback case of Lemma 1 and the no-feedback case of Proposition 1. There are two differences. First, consider the range $\kappa_T \leq \kappa < \kappa_{NF}$. In this range, the unique equilibrium without feedback is the T equilibrium where the speculator buys on good news and sells on bad news. With feedback, the unique equilibrium is instead the Partial Trade Equilibrium BNS, where the speculator buys on good news, but does not trade on bad news. Hence, for $\kappa_T \leq \kappa < \kappa_{NF}$, the feedback effect generates a limit to arbitrage whereby the speculator no longer trades on bad news. Second, consider the range $\kappa_{NF} \leq \kappa < \min(\kappa_{SNB}, \kappa_{NT})$. In this range, no-feedback case yields two Partial Trade Equilibria BNS and SNB, which cannot be distinguished by any standard criterion. With feedback, SNB is no longer an equilibrium, and the unique equilibrium is BNS. Hence, for $\kappa_{NF} \leq \kappa < \min(\kappa_{SNB}, \kappa_{NT})$, the feedback effect leads to asymmetric trading

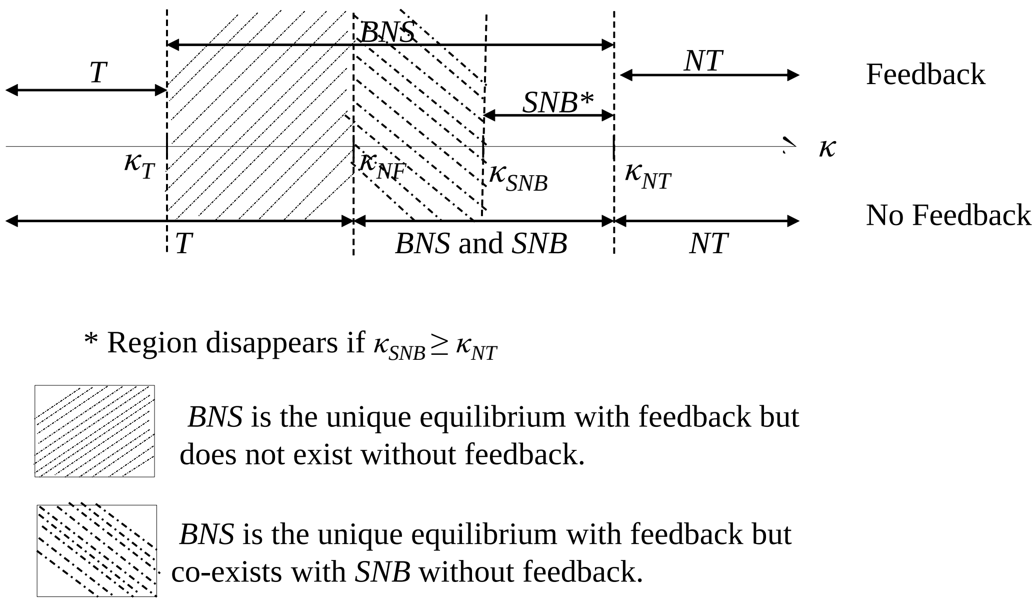

Figure 1: Parameter ranges for equilibria with and without feedback

in which buying is more common than selling (instead of both Partial Trade Equilibria holding for the same range of κ).

Overall, combining the two above parameter ranges, we see that feedback expands the range of parameters that supports the BNS equilibrium and contracts the range that supports the SNB equilibrium. In one range, BNS replaces T as the unique equilibrium; in the other range SNB disappears, leaving BNS as the unique equilibrium. Combining these two regions, there is a strictly positive range of parameters ($\kappa_T \leq \kappa < \min(\kappa_{SNB}, \kappa_{NT})$) for which BNS is the only pure-strategy equilibrium under feedback, as stated in Proposition 1. In contrast, there is no range of parameter values for which SNB exists but BNS does not. This is unlike the no-feedback case, where the BNS equilibrium is never unique and always coexists with the SNB equilibrium.

We now explain the intuition for why feedback makes the BNS equilibrium more prevalent and the SNB equilibrium less so. We start with BNS. Consider the realization of state L. If the negatively-informed speculator deviates to selling and the noise trader does not trade, we have $X = -1$, which provides sufficient negative information to induce the manager to disinvest in the case of feedback, but not in the case of no feedback. Disinvestment is the optimal decision in state L and improves firm value, reducing the profit of a selling speculator in the node of $X = -1$ from $\frac{1-\lambda}{2-\lambda}(R_H - R_L)$ (under no feedback) to $\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$. Hence, while a transaction cost of $\kappa \geq \kappa_{NF}$ is necessary and sufficient to deter the negatively-informed speculator from selling under no feedback, a transaction cost of only $\kappa \geq \kappa_T$ ($< \kappa_{NF}$) is necessary and sufficient to deter selling under feedback, and so the BNS equilibrium is easier to sustain. The difference between κ_{NF} and κ_T is $\frac{1}{3}\frac{1-\lambda}{2-\lambda}2x$, the probability of $X = -1$ ($\frac{1}{3}$) multiplied by the decrease in trading profits in this node under feedback ($\frac{1-\lambda}{2-\lambda}2x$). Due to feedback, the T equilibrium is replaced by the BNS equilibrium for $\kappa_T \leq \kappa < \kappa_{NF}$. The feedback effect thus provides an endogenous limit to arbitrage distinct from those identified in prior literature – *arbitrage is limited because the value of the asset being arbitraged is endogenous to the act of arbitrage.*

As shown in Appendix A, the transaction cost required to deter selling in the BNS equilibrium is $\kappa_T \equiv \frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right]$. As is intuitive, a smaller transaction cost is needed if the feedback effect on firm value x is important relative to the speculator's private information $R_H - R_L$. The required transaction cost is also lower if the probability of private information λ is high, as then the speculator's price impact is greater. Note that the transaction cost required to deter informed selling is strictly positive in Case 1, as the feedback effect reduces but does not eliminate the profits from informed selling. As discussed in Section 3.4, in Case 2 the feedback effect can be sufficiently strong to rule out informed selling even without a transaction cost. Finally, one may wonder if it is is reasonable to expect κ to be as large as κ_T so as to deter selling in the BNS equilibrium in Case 1. Recall that our leading interpretation of κ is that it captures the opportunity cost of trading other assets. If these other opportunities have similar information asymmetry (parameterized by $R_H - R_L$) to the firm in question, then the expected profit from the alternative trading opportunity (in the absence of feedback) is $\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L) + \frac{1}{2}(R_H - R_L)\right]$, which is higher than κ_T.

We now move to the SNB equilibrium. Consider the realization of state H. The critical order flow is now $X = 1$, which provides enough positive information to induce the manager to invest under feedback. Investment is the optimal decision in state H and improves firm value, *increasing* the profit of a buying speculator in the node of $X = 1$ from $\frac{1-\lambda}{2-\lambda}(R_H - R_L)$ (under no feedback) to $\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x)$, and so the SNB equilibrium is harder to sustain. While a transaction cost of $\kappa \geq \kappa_{NF}$ is necessary and sufficient to deter the positively-informed speculator from buying under no feedback, a higher transaction cost of $\kappa \geq \kappa_{SNB}$ ($> \kappa_{NF}$) is necessary and sufficient to deter buying under feedback. The difference between κ_{NF} and κ_{SNB} is $\frac{1}{3}\frac{1-\lambda}{2-\lambda}2x$, the probability of $X = 1$ ($\frac{1}{3}$) multiplied by increase in trading profits in this node under feedback ($\frac{1-\lambda}{2-\lambda}2x$). Moreover, if $x > \frac{\lambda}{4(1-\lambda)}(R_H - R_L)$, then $\kappa_{SNB} \geq \kappa_{NT}$ and the

SNB equilibrium is never sustainable with feedback. The first inequality is satisfied if x is large, so that the feedback effect creates significant value and thus markedly reduces (increases) the profitability of selling (buying). Even if $\kappa_{SNB} < \kappa_{NT}$, there is still a nonempty region $\kappa_T \leq \kappa < \kappa_{SNB}$, where BNS is sustainable even when SNB is not. The width of this range is $\kappa_{SNB} - \kappa_T = \frac{4}{3}\frac{1-\lambda}{2-\lambda}x$ and thus is increasing in x, the strength of the feedback effect.

In sum, due to the feedback effect, trading on information in either direction – buying on positive information or selling on negative information – puts information into prices, improving the manager's investment decision. This increases firm value, raising the profitability of informed buying relative to informed selling, and thus leads to asymmetric trading.

There is an important nuance in why the feedback effect reduces trading profits. Intuition may suggest that the market maker's pricing function will "undo" the feedback effect: since he is rational, the price he sets for a given order flow takes into account the order flow's effect on the manager's decision. Thus, the price received by the speculator will always reflect the manager's action d, and so it seems that the action should not affect her profits. Such intuition turns out to be incorrect. The source of the speculator's profits is not superior knowledge of the manager's action d, since the market maker can indeed perfectly predict this action from the order flow. The speculator's superior knowledge concerns the state – she directly observes θ, but the market maker can only imperfectly infer it from the order flow. In turn, the manager's action d (and thus the feedback effect on the manager's action) affects trading profits because it affects how important the state is for firm value. From (1), firm value is more sensitive to the state – and thus the speculator makes greater profits from her information on the state – the greater the level of investment. Hence, buying and causing the manager to invest increases the profitability of buying, whereas selling and causing the manager to disinvest reduces the profitability of selling.

3.3.3 Implications for real efficiency

We now discuss the implications of asymmetric trading for real efficiency. The feedback effect increases real efficiency by providing the manager information to improve his investment decision. However, the limit to arbitrage induced by the feedback effect deters the speculator from trading on her information, reducing price informativeness and thus the net gains from the feedback effect. Suppose the trading cost κ changes from $\kappa_T - \varepsilon$ to $\kappa_T + \varepsilon$ for an arbitrarily small positive ε. The equilibrium, in the case of feedback, will switch from T to BNS, which reduces the efficiency of the investment decision and thus firm value. Simple calculations show that firm value is higher in the T equilibrium by $\frac{1}{3}(x-c)$, which reflects that correct decisions occur more frequently under T due to informed selling by the speculator.[8]

Note that firm values in both equilibria remain higher than if the manager never learns from the market (e.g. because there is no informed speculator, or the manager ignores the information in prices).[9] Hence, the feedback effect directly adds value by informing the manager's decision. However, the feedback effect also indirectly reduces firm value by inducing the limit to arbitrage identified by this paper. This reduces the speculator's incentive to trade on bad news, lowering – but not eliminating – the extent to which the market informs the manager's decision. The overall effect of learning from the market on firm value remains positive.

3.4 Equilibria when firm value is non-monotonic in states

For completeness, we discuss the nature of the equilibria that arise when firm value is non-monotonic in the state, and outline the underlying intuition (the full analysis is in Appendix B.1). Under Case 2 ($R_H - x < R_L + x$), disinvestment not only mitigates the effect of the low state but is sufficiently powerful to overturn it, so that firm value is higher in the low state than in the high state. As a result, the asymmetric trading result becomes stronger. Now, if the speculator sells on negative information and we have $X = -1$ so that the manager disinvests, the speculator can suffer a loss (rather than just a smaller profit) even before transaction costs. As in Case 1, both the speculator and market maker will know that disinvestment will occur if $X = -1$, but have differing views on firm value conditional on disinvestment. The speculator knows that disinvestment will occur *and* that $\theta = L$. Unlike in Case 1, here firm value is highest under disinvestment when $\theta = L$. Thus, the speculator's knowledge that $\theta = L$ leads her to assign the highest possible value to a disinvesting firm ($v = R_L + x - c$). As in Case 1, the market maker does not know that $\theta = L$ and prices the firm taking into account the possibility that $\theta = H$. Unlike in Case 1, firm value is lower when $\theta = H$, and so the price set by the

[8] The calculation of firm value in both equilibria is as follows. With probability $\frac{1}{2}$, $\theta = H$. In the T equilibrium, the manager invests unless $X = 0$, and so $v(H) = R_H + \frac{2}{3}(x-c)$; in the BNS equilibrium, the manager only invests when $X = 2$, so $v(H) = R_H + \frac{1}{3}(x-c)$. With probability $\frac{1}{2}$, $\theta = L$. In the T equilibrium, $X \in \{-2,-1,0\}$ and so the manager correctly disinvests unless $X = 0$, so $v(L) = R_L + \frac{2}{3}(x-c)$. In the BNS equilibrium, $X \in \{-1,0,1\}$ and the manager correctly disinvests only if $X = -1$. Thus, $v(L) = R_L + \frac{1}{3}(x-c)$. Regardless of whether $\theta = \{H, L\}$, firm value is higher in the T equilibrium by $\frac{1}{3}(x-c)$.

[9] In this case, $v(H) = R_H$ and $v(L) = R_L$.

market maker ($\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c$) is *less* than the true value of the firm. Thus, the speculator's profit (before transaction costs) is negative ($\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$). This result contrasts standard informed trading models where a speculator can never make a loss (before transactions costs) if she trades in the direction of her information. The key to this loss is the feedback effect. As a result, the minimum transaction cost required to deter informed selling in the BNS equilibrium, $\kappa_T \equiv \frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right]$, is lower in Case 2 as the first term is now negative. Indeed, κ_T may be negative overall, in which case a negatively-informed speculator will not sell even if transactions costs are zero.

The non-monotonicity in Case 2 also introduces a new force: when the feedback effect is sufficiently strong, the positively-informed speculator may wish to manipulate the price by deviating (from her equilibrium action of buying in BNS or T, or no trade in SNB or NT) to selling.[10] If she sells when $\theta = H$, she potentially misleads the manager to believe that $\theta = L$ and disinvest. Since disinvestment is suboptimal when $\theta = H$, this decision reduces firm value and so the speculator may profit from her short position. Hence, for each of the four equilibria, an additional condition must be satisfied to rule out manipulation. A sufficient condition to prevent manipulation in all four equilibria is $R_H - R_L > \frac{4}{3}x$: the loss from trading against good news (which is proportional to $R_H - R_L$) is sufficiently high relative to the benefit from manipulation (which is proportional to x). The same issue does not arise with the negatively-informed speculator, as she never has an incentive to deviate to buying. If she does so, she misleads the manager to believe that $\theta = H$ and incorrectly invest. This decision reduces firm value, causing the speculator to incur a loss on her long position.[11]

3.5 Discussion of Model Assumptions and Applicability

The above analysis has shown that the feedback effect discourages informed selling relative to informed buying. This section discusses which features of our setting are necessary for this result and which can be relaxed, thus highlighting the conditions under which asymmetric trading due to the feedback effect likely exists in the real world.

[10] The positively-informed speculator will never sell *in equilibrium* because, if the market maker and manager believe that she is manipulating the price, she cannot profit from doing so, and so the set of pure-strategy equilibria remains unchanged at NT, T, SNB, and BNS. However, stronger conditions are required to ensure that she is not tempted to *deviate* to selling in the above equilibria.

[11] This analysis is related to Goldstein and Guembel (2008), who analyze the possibility of manipulative trading in the presence of feedback effects.

3.5.1 Condition for the feedback effect to exist

Our main result about the larger range of parameters where the BNS equilibrium holds, and the smaller range of parameters where the SNB equilibrium holds, requires feedback from the financial market to real decisions. This in turn arises if financial market trading conveys sufficient information to influence the manager's decision. Specifically, the asymmetry between the BNS and SNB equilibria in Proposition 1 requires $\frac{1}{2-\lambda} > \gamma_1 = \frac{1}{2} + \frac{c}{2x} \iff \frac{1-\lambda}{2-\lambda} < \gamma_{-1} = \frac{1}{2} - \frac{c}{2x}$. These inequalities are more likely to be satisfied if x is large relative to c – the value created by improving the manager's investment decision is high relative to the cost of doing so – because then the feedback effect is more important. Note that the asymmetry holds most clearly when $c = 0$, as then the feedback effect always exists. The role of $c > 0$ is to give rise to cases in which the feedback effect is absent, allowing us to compare the equilibria in the feedback and no-feedback cases, and thus highlight the role of the feedback effect in generating asymmetric trading.

They are also more likely to be satisfied if λ, the probability that the speculator is present, is high, so that the order flow is sufficiently informative to change managerial decisions. The extent to which the manager will change his decision in response to trading will also depend on additional factors outside the model. If the investment is difficult to reverse (e.g., an M&A deal in which there is a formal merger agreement or a termination fee, or an irreversible physical investment), or the manager is less likely to reverse it due to agency problems (e.g., weak governance allows him to pursue negative-NPV investment to maximize his private benefits), the feedback effect will be weaker and so the result on reduced selling relative to buying may not arise.

Hewlett Packard's (HP) acquisition of Compaq illustrates a circumstance under which the feedback effect arises. HP's stock price fell 19% upon announcement on September 4, 2001. That HP's CEO conveyed the unanimous support of its high-profile board for the deal contributed to the magnitude of the decline, as traders did not fear that their selling would lead to deal cancellation. To everyone's surprise, Walter Hewlett, who earlier voted in favor of the deal as a board member, announced opposition on behalf of the Hewlett Foundation in the wake of the stock price drop. As chairman of the second-largest shareholder and the son of the company's founder, he posed a credible threat to the deal. Shares of HP rose 17% in response, suggesting that the speculators would not have sold so aggressively had they known that the negative price impact could trigger a corrective action. The combination of rational investor expectation at the time of deal announcement and the expectation being ex post incorrect (due to the unexpected behavior of Walter Hewlett) offers a unique opportunity to observe the feedback effect.

3.5.2 Uncertainty regarding the presence of a speculator ($\lambda < 1$)

Another important assumption in our model is $\lambda < 1$, so that there is uncertainty on whether there is an informed speculator in the market. To see this, note that the feedback effect only affects profits for the nodes of $X = \{-1, 1\}$. If $X = \{-2, 2\}$, the speculator is fully revealed and makes zero trading profits; if $X = 0$, there is no feedback effect as the price is uninformative. Thus, the profits from informed buying equal the profits from informed selling, and again there is no asymmetry. In turn, $\lambda < 1$ is necessary for the speculator not to be fully revealed when $X = \{-1, 1\}$ and thus for trading profits to be non-zero. For example, consider the market maker's inference from seeing $X = -1$ in the BNS equilibrium. This order flow is consistent with either the speculator being absent (in which case the state may be H or L), or present and negatively informed. If $\lambda = 1$, the first case is ruled out, and so the market maker knows for certain that $\theta = L$. Thus, $X = -1$ is fully revealing: the market maker knows *both* that disinvestment will occur, *and* that the state is L, and so sets the price exactly equal to the fundamental value of $R_L + x - c$. The speculator's profits are zero, and thus automatically unaffected by the manager's decision and the feedback effect. Indeed, if $\lambda = 1$, then $\kappa_T = \kappa_{SNB}$ and there is no range of parameter values in which there is a BNS equilibrium but no SNB equilibrium.

In contrast, if $\lambda < 1$, the market maker predicts the manager's action but does not know the state. Since $X = -1$ can be consistent with the speculator being absent and the state being H, the market maker allows for the possibility that $\theta = H$ and sets a price of $\frac{1-\lambda}{2-\lambda} v(H, d) + \frac{1}{2-\lambda} v(L, d)$. Because the speculator knows the state in addition to the action, she makes a profit of $\frac{1-\lambda}{2-\lambda} (v(H, d) - v(L, d))$.

The core interpretation of the parameter λ is the probability that an informed speculator is present in the market. Another interpretation is that the speculator is always present, but can only trade with probability λ. For example, with probability $1 - \lambda$ she receives a liquidity shock that prevents her from trading: buying a share requires capital, and shorting a share requires posting margin. A third framework is that the speculator is always present and can trade, but is informed only with probability λ. This alternative scenario, however, requires us to consider the possibility that the uninformed speculator will choose to sell to manipulate the price, as in Goldstein and Guembel (2008), because doing so may dupe the manager into disinvesting. Since $d = 0$ is optimal in the absence of information, such manipulation will enable the speculator to profit on a short position. To keep the paper focused on its primary contribution, we do not analyze this framework here.

3.5.3 Zero initial position

The core model assumes that the speculator has a zero initial stake in the firm. Appendix B.2 fully analyzes the case in which the speculator owns an initial stake of $\alpha > 0$ (i.e. is a blockholder) and shows that the key results continue to hold. The fundamental force of the model – the feedback effect increases the profitability of buying on positive information relative to selling on negative information – is independent of the speculator's initial stake. It remains the case that there is a strictly positive range of transaction costs for which the BNS equilibrium exists and the SNB equilibrium does not, and that there is no range for which the SNB equilibrium exists but the BNS equilibrium does not. Moreover, the width of the range of transaction costs for which BNS exists and SNB does not ($\kappa_{SNB} - \kappa_T$ in the core model) is $\frac{1}{3}\frac{1-\lambda}{2-\lambda}4x$ and independent of the initial stake α.

The intuition for the irrelevance of the initial stake is as follows. A positive initial stake increases a negatively-informed speculator's incentive to sell, because if selling leads to (correct) disinvestment, it increases the value of the speculator's initial stake. However, it also increases the positively-informed speculator's incentive to buy, because if buying leads to (correct) investment, it increases the value of the speculator's initial stake by the same margin. Specifically, if a negatively-informed speculator trades -1, she ends up with a final position of $\alpha - 1$. If a positively-informed speculator trades $+1$, she ends up with $\alpha + 1$. The incentive to trade on information to increase the value of her initial stake α (through the feedback effect) is symmetric across buying and selling, and so cancels out. We are thus left with the difference between trading -1 on negative information and trading $+1$ on positive information, which is the same as in the core model with $\alpha = 0$. Hence, the asymmetry between buying on good news and selling on bad news remains despite the fact that both trading directions become more attractive when the speculator has an initial position.

3.5.4 Corrective action

In our model, the real decision is a corrective action in that it improves firm value in the low state. This case arises when the decision maker maximizes firm value. While we model a manager who attempts to maximize firm value via an investment decision, other potential applications include a board of directors firing an underperforming manager in the bad state or an outside investor engaging in activism to restore shareholder value. An alternative real decision is an amplifying action, where the decision maker's objective is something other than firm value, and maximizing this objective leads him to worsen firm value in the low state. For example, capital providers may withdraw their investment in the low state, reducing firm value further (Goldstein, Ozdenoren, and Yuan (2013)), or customers or employees could terminate their relationship with a troubled firm (Subrahmanyam and Titman (2001)). Our model provides distinctive insights on the feedback effect when real decisions are of the corrective nature. In a model with amplifying actions, the speculator will no longer be reluctant to sell on bad news if she has a zero initial stake, since the information will reduce firm value further, enabling her to profit more on her short position.

3.5.5 Other assumptions

Several other assumptions are made only for tractability and can be substantially weakened at the cost of complicating the model with little additional insight. The first is that the manager has no signal and the speculator has a perfect signal about the state of nature θ. We only require that the speculator has some important decision-relevant information that the manager does not have – it is not even necessary that the speculator be more informed than the manager.[12]

Another non-critical assumption is discrete trading volumes (i.e., the speculator cannot trade an amount between 0 and 1). We conjecture that our results will continue to hold in more complex models with continuous trading volumes. Our intuition is that in such a model the speculator would sell a small amount (rather than zero) on negative information without

[12] For example, assume that the optimal decision d depends on both an internal state variable θ_i about the firm, and an external state variable θ_e about the industry's future prospects. Assume also that the manager has a perfect signal about θ_i and the speculator is completely uninformed about θ_i. In addition, the manager has a noisy signal about θ_e and the speculator has a less precise signal about θ_e which is conditionally uncorrelated with the manager's signal. Even though the manager is more informed than the speculator about both θ_i and θ_e, as the speculator's information about θ_e is still incremental and relevant for his decision.

significantly increasing the probability of disinvestment, but she will buy a greater amount upon good information and so the asymmetry of trading strategies would remain and that is likely to cause asymmetry in the updating of the manager. In fact, our conjecture is confirmed in a subsequent paper by Boleslavsky, Kelly, and Taylor (2014).

Relatedly, the role of the transaction cost is to demonstrate how the feedback effect changes incentives to trade in a tractable and stark way: rather than changing the speculator's trading volume (which requires a significantly more complex model with continuous trading volumes), the feedback effect changes the range of transaction costs under which the speculator is willing to trade a given volume. Here, the transaction cost is necessary to deter informed selling in the BNS equilibrium in Case 1 because the feedback effect attenuates, but does not eliminate, trading profits. Thus, the feedback effect alone does not induce the speculator to change her trading volume from -1 to 0 (the only other non-positive trading amount). As Boleslavsky et al. (2014) also show, transactions costs are not necessary in a continuous trading framework, because the feedback effect leads to the negatively-informed speculator trading a smaller amount, rather than not trading at all.[13]

Finally, while we assume that there is only one speculator, the results will likely continue to hold in a model with multiple speculators as long as each of them is large enough to have an effect on the total order flow (and hence on the firm's decision). The key ingredient in our model is that speculators are strategic, which does not require them to be monopolistic.

4 Effect of Information on Beliefs and Prices

The previous section demonstrated that the feedback effect increases the prevalence of the BNS equilibrium, in which a speculator buys on good news and does not trade on bad news. In this section, we study the implications of the BNS equilibrium in the case of feedback ($\frac{1}{2-\lambda} > \gamma_1 \Leftrightarrow \frac{1-\lambda}{2-\lambda} < \gamma_{-1}$). Section 4.1 calculates the effect of good and bad news about the state on the posterior beliefs q, to study the extent to which information reaches the manager and affects real decisions. Section 4.2 analyzes the impact of news on prices to generate stock return predictions.

[13] Other than added complexity, another difference is that the equilibrium in Boleslavsky et al. (2014) is only in mixed strategies. Thus, the real decision maker is always indifferent between the different actions he can take, and so does not gain from using the information in the market.

4.1 Beliefs

Since the manager uses the posterior belief q to guide his investment decision, we can interpret q as measuring the extent to which information reaches the manager and affects his actions. In a world in which no agent observes the state, or in which the manager does not learn from prices or order flows, the posterior q would equal the prior $y = \frac{1}{2}$. Conversely, in a world of perfect information transmission, $q = 1$ if $\theta = H$ and $q = 0$ if $\theta = L$. Our model, in which information is partially revealed through prices, lies in between these two polar cases. The absolute distance between q and $\frac{1}{2}$ measures the extent to which information reaches the manager.

Thus far, we have shown that good news received by the speculator has a different impact on her trades (and thus the total order flow) than bad news. However, it is not obvious that this difference will translate into a differential impact on the manager's beliefs. The manager is rational and takes into account the fact that the speculator does not sell on negative information: Indeed, in the analysis of the BNS equilibrium in the proof of Proposition 1, the manager recognizes that $X = 1$ could be consistent with a negatively-informed speculator who chooses not to trade, and so $q(1)$ equals $q(0)$ (where $q(X)$ denotes the posterior at $t = 1$ upon observing order flow X). Put differently, although negative information does not cause a negative order flow (on average), it can still have a negative effect on beliefs and be fully conveyed to the manager. Thus, it may still seem possible for good and bad news to be conveyed symmetrically to the manager – by taking into account the speculator's asymmetric trading strategy, he can "undo" the asymmetry. Indeed, we start by showing that, if we do not condition on the presence of the speculator, the effects on beliefs of the high and low states being realized are symmetric. This is a direct consequence of the law of iterated expectations: the expected posterior must equal the prior.

Lemma 2 *(Symmetric effect of high and low state on beliefs at $t = 1$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1$ (and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$). (i) If $\theta = H$, the expected posterior probability of the high state is $q^H = \frac{(1-\lambda)^2}{6-3\lambda} + \frac{1}{3} + \frac{\lambda}{3}$ and is increasing in λ. (ii) If $\theta = L$, the expected posterior probability of the high state is $q^L = \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}$ and is decreasing in λ. (iii) We have $\frac{q^H+q^L}{2} = \frac{1}{2}$: thus, the realization of state H has the same absolute impact on beliefs as the realization of state L.*

Proof. See Appendix A. ■

Of greater interest is to study the effect of the state realization conditional upon the speculator being present. We use the term "good news" to refer to $\theta = H$ being realized and the speculator being present, since in this case there is an agent in the economy who directly receives news on the state; "bad news" is defined analogously. While the above analysis studied the effect of the state being realized (regardless of whether the state is learned by any agent in the economy), this analysis studies the impact of the speculator receiving information about the state. The goal is to investigate the extent to which the speculator's good and bad news is conveyed to the manager at $t = 1$. The results are given in Proposition 2 below:

Proposition 2 *(Asymmetric effect of good and bad news on beliefs at $t = 1$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1$ (and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$). (i) If $\theta = H$ and the speculator is present, the expected posterior probability of the high state is $q^{H,spec} = \frac{2}{3}$ and is independent of λ. (ii) If $\theta = L$ and the speculator is present, the expected posterior probability of the high state is $q^{L,spec} = \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}$ and is decreasing in λ. (iii) We have*

$$\frac{q^{H,spec} + q^{L,spec}}{2} = \frac{1 + \frac{1-\lambda}{6-3\lambda}}{2}, \tag{3}$$

which is decreasing in λ. Since $\frac{1+\frac{1-\lambda}{6-3\lambda}}{2} > \frac{1}{2}$, (3) implies that $\left|q^{H,spec} - y\right| - \left|q^{L,spec} - y\right| > 0$, i.e. the absolute increase in the manager's posterior if the speculator receives good news exceeds the absolute decrease in his posterior if the speculator receives bad news. The difference is decreasing in λ.

Proof. See Appendix A. ■

Proposition 2 shows that, conditional upon the speculator being present, the impact on beliefs of good news is greater in absolute terms than the impact of bad news, and the asymmetry is monotonically decreasing in the probability of the speculator's presence λ. Even though the manager takes the speculator's asymmetric trading strategy into account, he cannot distinguish the case of a negatively-informed (and non-trading) speculator from that of an absent speculator (i.e. no information) – both cases lead to the order flow being $\{-1, 0, 1\}$ with equal probability. Thus, negative information has a smaller effect on his belief. If the speculator is always present ($\lambda = 1$), the manager has no such inference problem and there is no asymmetry.

In sum, due to the reduced incentive to sell that results from the feedback effect, negative information received by the speculator is transmitted to the manager to a lesser extent than positive information. As a result, the manager cannot use this information to guide his investment decision, with negative real consequences. In particular, even if there is an agent in the economy (the speculator) who knows for certain that disinvestment is optimal, because $\theta = L$, disinvestment may not occur. The failure to disinvest does not occur because the manager is pursuing private benefits, as in the standard theories of Jensen (1986), Stulz (1990) and Zwiebel (1996). In contrast, the manager is fully aligned with firm value and there are no agency problems. The manager wishes to maximize firm value by learning from prices, but is unable to do so since speculators refrain from impounding their information into prices. Even though he takes into account the fact that the speculator does not trade on negative information when updating his beliefs, he cannot fully undo the asymmetry of her trading behavior.

The above analysis considered the change in the manager's posterior at $t = 1$. At $t = 2$, the state is realized and the posterior becomes either 1 (if $\theta = H$) or 0 (if $\theta = L$). Since bad news is conveyed to the manager to a lesser extent at $t = 1$, it seeps out to a greater extent ex post, between $t = 1$ and $t = 2$. Thus, bad news causes a greater change in the posterior between $t = 1$ and $t = 2$ than good news. This result is stated in Corollary 1 below:

Corollary 1 *(Asymmetric effect of high and low state on beliefs at $t = 2$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1 \Leftrightarrow \frac{1-\lambda}{2-\lambda} < \gamma_{-1}$. When the speculator is present, the absolute impact on beliefs between $t = 1$ and $t = 2$ of the realization of the state is greater for $\theta = L$ than for $\theta = H$, i.e.*

$$\left|0 - q^{L,spec}\right| - \left|1 - q^{H,spec}\right| > 0.$$

The asymmetry is monotonically decreasing in the frequency of the speculator's presence λ.

Proof. Follows from simple calculations ■

The smaller effect of bad news on the posterior at $t = 1$ is counterbalanced by its larger effect at $t = 2$. As we will show in Section 4.2, surprisingly this result need not hold when we examine the effect of news on prices rather than posteriors.

4.2 Stock Returns

We now calculate the impact of the state realization and news on prices, to generate stock return implications. We study short-run stock returns between $t = 0$ and $t = 1$, and long-run drift between $t = 1$ and $t = 2$. While this analysis is similar to Section 4.1 but studying prices rather than beliefs, we will show that not all the results remain the same.

4.2.1 Short-Run Stock Returns

Lemma 3 is analogous to Lemma 2 and shows that, unconditionally, the good and bad states have the same absolute impact on prices, since the market maker takes the speculator's asymmetric trading strategy into account when devising his pricing function. Let p_0 denote the "ex ante" stock price at $t = 0$, before the state has been realized.

Lemma 3 *(Symmetric effect of high and low state on returns between $t = 0$ and $t = 1$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1$ (and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$):*

(i) The stock price impact of the high state being realized is $p_1^H - p_0 = \frac{\lambda}{6}\left[p(2) - p(-1)\right] > 0$.

(ii) The stock price impact of the low state being realized is $p_1^L - p_0 = \frac{\lambda}{6}\left[p(-1) - p(2)\right] = -\left(p_1^H - p_0\right) < 0$.

Proof. See Appendix A. ■

We have $p_1^H - p_0 = -\left(p_1^L - p_0\right)$: the negative effect of the low state equals the positive effect of the high state. Thus, the unconditional expected return is zero. This is an inevitable consequence of market efficiency. The price at $t = 0$ is an unbiased expectation of the $t = 1$ expected price in the high state and the $t = 1$ expected price in the low state. Since both states are equally likely, the absolute effect of the high state must equal that of the low state.

Proposition 3 is analogous to Proposition 2 and shows that, conditional on the speculator being present, good news has a greater effect than bad news:

Proposition 3 *(Asymmetric effect of good and bad news on returns between $t = 0$ and $t = 1$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1$ (and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$):*

(i) If $\theta = H$ and the speculator is present, the average return between $t = 0$ and $t = 1$ is $p_1^{H,spec} - p_0 = \frac{1}{3}\left(1 - \frac{\lambda}{2}\right)\left(p(2) - p(-1)\right) > 0$.

(ii) If $\theta = L$ and the speculator is present, the average return between $t = 0$ and $t = 1$ is $p_1^{L,spec} - p_0 = \frac{\lambda}{6}\left(p(-1) - p(2)\right) < 0$.

(iii) The difference in the absolute average returns between the speculator learning $\theta = H$ and $\theta = L$ is given by:

$$\left|p_1^{H,spec} - p_0\right| - \left|p_1^{L,spec} - p_0\right| = \frac{1}{3}\left(1 - \lambda\right)\left(p(2) - p(-1)\right) > 0, \tag{4}$$

i.e. the stock price increase upon good news exceeds the stock price decrease upon bad news. This difference is decreasing in λ.

(iv) The average return, conditional on the speculator being present, is positive:

$$p_1^{spec} - p_0 = \frac{1}{3}\frac{1-\lambda}{2}\left(p\left(2\right) - p\left(-1\right)\right) > 0. \tag{5}$$

This difference is decreasing in λ.

Proof. See Appendix A. ■

Proposition 3 states that the average return, *conditional* on the speculator being present, is positive – i.e., the stock price increase upon positive information exceeds the stock price decrease upon negative information (part (iii)). Put differently, if the speculator receives positive news, this is impounded into prices to a greater degree than if she receives negative news. Since good and bad news are equally likely, this means that the average return, conditional on the speculator being present, is positive (part (iv)). As with Proposition 2, the key to this result is that, even though the market maker is rational, he cannot distinguish the case of a negatively-informed speculator from that of an absent speculator (i.e., no information). If $\lambda = 1$, equations (4) and (5) become zero and there is no asymmetry; the asymmetry is monotonically decreasing in λ. Note that the positive average return given in part (iv) is not inconsistent with market efficiency, because it is conditional upon the speculator being present, which is private information. An uninformed investor cannot buy the stock at $t = 0$ and expect to earn a positive return at $t = 1$, because she will not know whether the speculator is present.[14]

4.2.2 Long-Run Drift

We now move from short-run returns to calculating the long-run drift of the stock price, to analyze the stock return analog of Corollary 1, i.e., the impact of the state realization on prices between $t = 1$ and $t = 2$.

Corollary 2 *(Asymmetric effect of good and bad news on returns between $t = 1$ and $t = 2$). Consider the BNS equilibrium where $\frac{1}{2-\lambda} > \gamma_1$ (and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$):*

[14] In contrast, Hong, Lim, and Stein (2000) find that bad news is impounded into prices to a lesser degree than good news, in a way that is inconsistent with market efficiency. Thus, their results imply an actionable trading strategy that does not require the trader to condition upon the speculator's presence.

(i) If $\theta = H$ and the speculator is present, the average return between $t = 1$ and $t = 2$ is $p_2^{H,spec} - p_1^{H,spec} = \frac{1}{3}(R_H - R_L) > 0$.

(ii) If $\theta = L$ and the speculator is present, the average return between $t = 1$ and $t = 2$ is

$$p_2^{L,spec} - p_1^{L,spec} = \frac{(3-2\lambda)(R_L - R_H) + 2(1-\lambda)x}{3(2-\lambda)}, \tag{6}$$

which is negative in Case 1, but can be positive or negative in Case 2.

(iii) If (6) < 0, the difference in the absolute average returns between the speculator learning $\theta = H$ and $\theta = L$ is given by:

$$\left|p_2^{H,spec} - p_1^{H,spec}\right| - \left|p_2^{L,spec} - p_1^{L,spec}\right| = \frac{(1-\lambda)(R_L - R_H + 2x)}{3(2-\lambda)},$$

which is positive in Case 2 and negative in Case 1. The magnitude of the difference is decreasing in λ.

(iv) Expected firm value at $t = 2$, conditional upon the speculator being present, is:

$$p_2^{spec} = \frac{1}{2}(R_H + R_L) + \frac{1}{3}(x - c),$$

and the average return between $t = 1$ and $t = 2$ if the speculator is present is:

$$p_2^{spec} - p_1^{spec} = \frac{1}{6}\frac{1-\lambda}{2-\lambda}(R_L - R_H + 2x),$$

which is positive in Case 2 and negative in Case 1. The magnitude of the difference is decreasing in λ.

Proof. See Appendix A. ■

Corollary 1 showed that the smaller effect of bad news on beliefs at $t = 1$ is counterbalanced by a larger effect on beliefs at $t = 2$, and so the average increase in beliefs in the short-run is reversed by an average decrease in beliefs in the long-run. Corollary 2 shows that this need not be the case for returns: it is possible for bad news to have a smaller effect than good news at both $t = 1$ and $t = 2$, and so the speculator's presence can lead to positive average returns in both the short-run and long-run.

In Case 1, we do have the same result for prices as we do for beliefs – the smaller effect of bad news on prices at $t = 1$ is counterbalanced by a larger effect on prices at $t = 2$. This is because firm value is monotonic in the state. Thus, the large fall in the beliefs, that arises when the low state is realized at $t = 2$, translates into a large fall in the stock price – the low state is bad for firm value. As a result, prices are too high at $t = 1$, conditional upon the speculator being present. Miller (1977) similarly shows that prices are too high if bad news is not traded upon. However, in his model, the lack of trading on bad news results from exogenous short-sales constraints; here, the reluctance to short-sell is generated endogenously. Note that the long-term drift in returns does not violate market efficiency. The key to reconciling this result with market efficiency is that firm value is endogenous to trading. If the speculator sold aggressively upon observing $\theta = L$, the decline in the stock price would lead to disinvestment occurring. The market is not strong-form efficient in the Fama (1970) sense, since the speculator's private information is not incorporated into prices, but is strong-form efficient in the Jensen (1978) sense as the speculator cannot make profits on her information. Since she does not trade on her information, the negative effect of $\theta = L$ on firm value must manifest predominantly at $t = 2$.

In contrast, for Case 2, firm value is not monotonic in the state. Thus, while beliefs fall significantly at $t = 2$ when $\theta = L$ is realized, this does not lead to a large fall in the stock price. The initial fall in beliefs at $t = 1$ may lead to the manager disinvesting, and firm value under disinvestment is higher when $\theta = L$ than when $\theta = H$. Thus, the realization of $\theta = L$ at $t = 2$ becomes good news for the stock price. Thus, bad news leads to a smaller decline in prices at $t = 2$ as well as $t = 1$. Put differently, bad news about the state is not necessarily bad news about firm value, because the manager can take a corrective action that is sufficiently powerful to overturn the effect of the state on firm value.

5 Conclusion

This paper has analyzed the effect of feedback from financial markets to corporate decisions on a speculator's incentives to trade on information. Even if a speculator has negative information on economic conditions, she may strategically refrain from trading on it, because doing so conveys her information to the manager. The manager may then optimally disinvest, which improves firm value but reduces the profits from the speculator's sell order. While the feedback effect reduces the incentive to sell on negative information, it reinforces the incentive to buy on positive information. Doing so induces the manager to optimally increase investment, enhancing firm value and thus the profitability of her buy order.

Overall, the feedback effect causes strategic speculators to trade asymmetrically on information. By deterring them from selling on negative information, it creates a limit to arbitrage that reduces the informativeness of prices. Unlike the limits to arbitrage identified by prior literature, our effect is asymmetric. In addition, it does not rely on exogenous frictions or agency problems, but is instead generated endogenously as part of the arbitrage process. Thus, even if speculators have perfect private information and no wealth constraints or trading restrictions, they may choose not to trade on their information. In addition, our model identifies the settings in which the feedback effect, and thus asymmetric trading, is most likely to exist in practice. The asymmetry should be stronger if the value created by correct investment decisions is large, or financial market trading is more informative. It should be weaker if investment is irreversible (e.g. due to a termination fee or firm commitment for an M&A deal), or the manager's investment decisions are motivated by private benefits rather than firm value maximization.

Asymmetric trading has implications for both stock returns and real investment. In terms of stock returns, bad news has a smaller effect on short-run prices than good news, even though the market maker is rational and takes the speculator's trading strategy into account when devising his pricing function. Interestingly, in contrast to underreaction models, the smaller short-run reaction to bad news may also coincide with smaller long-run drift, since the manager can disinvest to attenuate the effect of bad economic conditions on firm value. In terms of real investment, the manager may overinvest in negative-NPV projects, even though there are no agency problems and he is attempting to learn from the market to take the efficient decision. Even though there is an agent in the economy who knows with certainty that the investment is undesirable, and the manager is aware of the speculator's asymmetric trading strategy, this information is not conveyed to the manager and so the desired disinvestment does not occur.

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A Proofs

Proof of Proposition 1

This proof only provides material supplementary to what is in the main text.

No Trade Equilibrium NT. The order flows of $X = -2$ and $X = 2$ are off the equilibrium path and the posteriors are given by 0 and 1, respectively, as these are the only posteriors that satisfy the Intuitive Criterion (as stated in the main text). The order flows of $X \in \{-1, 0, 1\}$ are on the equilibrium path and so the posteriors can be calculated by Bayes' rule:

$$\begin{aligned} q(X) &= \Pr(H|X) \\ &= \frac{\Pr(X|H)}{\Pr(X|H) + \Pr(X|L)}. \end{aligned}$$

We thus have:

$$\begin{aligned} q(-1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} \\ &= \frac{1}{2}, \end{aligned}$$

and $q\,(0)$ and $q\,(1)$ are calculated in exactly the same way. Sequential rationality leads to the decisions d and prices p as given by the Table in the proof in the main text.

We now turn to calculating the speculator's payoff (gross of the transaction cost κ) under different trading strategies, which comprises of the value of her final stake (of -1, 0, or 1 share), plus (minus) the price received (paid) for any share sold (bought). Under the positively-informed speculator's equilibrium strategy of not trading, we have $X \in \{-1, 0, 1\}$ and so her payoff is 0. If she deviates to buying:

- With probability (w.p.) $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}\,(R_H - R_L)$.

Thus, her expected gross gain from deviating to buying is given by:

$$\frac{1}{3}\,(R_H - R_L) \equiv \kappa_{NT}. \tag{7}$$

A similar calculation shows that, if the negatively-informed speculator sells, her expected gross gain is also given by (7). Thus, if and only if $\kappa \geq \kappa_{NT}$, the no-trade equilibrium is sustainable. The above calculations apply both in the case of feedback ($\frac{1}{2-\lambda} > \gamma_1$ and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$) and no feedback ($\frac{1}{2-\lambda} \leq \gamma_1$ and $\frac{1-\lambda}{2-\lambda} \geq \gamma_{-1}$).

Partial Trade Equilibrium BNS. The order flow of $X = -2$ is off the equilibrium path and the posterior is given by 0. The posteriors of the other order flows are given as follows:

$$\begin{aligned}
q(-1) &= \frac{(1-\lambda)(1/3)}{(1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1-\lambda}{2-\lambda}, \\
q(0) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2}, \\
q(1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2}, \\
q(2) &= \frac{\lambda(1/3)}{\lambda(1/3)} = 1.
\end{aligned}$$

Under the positively-informed speculator's equilibrium strategy of buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

If she deviates to not trading, her payoff is 0. Thus, her expected gross gain from deviating to not trading is $-\kappa_{NT}$ (as given by (7)) in the cases of both feedback and no feedback.

Under the positively-informed speculator's equilibrium strategy of not trading, her payoff is 0. If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{1}{3}$, $X = -1$. In the case of feedback, she receives $\frac{1-\lambda}{2-\lambda}(R_H - x - c) + \frac{1}{2-\lambda}(R_L + x - c)$ per share, and so her payoff is $-(R_L + x - c) + (\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c) = \frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$. In the case of no feedback, she receives $\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L$ per share, and so her payoff is $-R_L + (\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L) = \frac{1-\lambda}{2-\lambda}(R_H - R_L)$.
- W.p. $\frac{1}{3}$, $X = 0$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_L + (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

Thus, her expected gross gain from deviating to selling is given by:

$$\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right] \equiv \kappa_T \tag{8}$$

in the case of feedback, and

$$\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda} + \frac{1}{2}\right](R_H - R_L) \equiv \kappa_{NF} \tag{9}$$

in the case of no feedback.

Thus, the BNS equilibrium is sustainable if and only if $\kappa_T \leq \kappa < \kappa_{NT}$ in the case of feedback, and $\kappa_{NF} \leq \kappa < \kappa_{NT}$ in the case of no feedback.

Partial Trade Equilibrium SNB. The order flow of $X = 2$ is off the equilibrium path and the posterior is given by 1. The posteriors of the other order flows are given as follows:

$$\begin{aligned}
q(-2) &= \frac{0}{\lambda\,(1/3)} = 0,\\
q\,(-1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2},\\
q\,(0) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2},\\
q(1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + (1-\lambda)(1/3)} = \frac{1}{2-\lambda}.
\end{aligned}$$

Under the negatively-informed speculator's equilibrium strategy of selling:

- W.p. $\frac{1}{3}$, $X = -2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_L + \left(\frac{1}{2}R_H + \frac{1}{2}R_L\right) = \frac{1}{2}(R_H - R_L)$.

If she deviates to not trading, her payoff is 0. Thus, her expected gross gain from deviating to not trading is $-\kappa_{NT}$ (as given by (7)) in the cases of both feedback and no feedback.

Under the positively-informed speculator's equilibrium strategy of not trading, her payoff is 0. If she deviates to buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0.

- W.p. $\frac{1}{3}$, $X=1$. In the case of feedback, she pays $\frac{1}{2-\lambda}(R_H+x)+\frac{1-\lambda}{2-\lambda}(R_L-x)-c$ per share, and so her payoff is $(R_H+x-c)-(\frac{1}{2-\lambda}(R_H+x)+\frac{1-\lambda}{2-\lambda}(R_L-x)-c)=\frac{1-\lambda}{2-\lambda}(R_H-R_L+2x)$. In the case of no feedback, she pays $\frac{1}{2-\lambda}R_H+\frac{1-\lambda}{2-\lambda}R_L$ per share, and so her payoff is $R_H-(\frac{1}{2-\lambda}R_H+\frac{1-\lambda}{2-\lambda}R_L)=\frac{1-\lambda}{2-\lambda}(R_H-R_L)$.

- W.p. $\frac{1}{3}$, $X=0$, and she pays $\frac{1}{2}R_H+\frac{1}{2}R_L$ per share. Her payoff is $R_H-(\frac{1}{2}R_H+\frac{1}{2}R_L)=\frac{1}{2}(R_H-R_L)$.

Thus, her expected gross gain from deviating to buying is given by:

$$\frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H-R_L+2x)+\frac{1}{2}(R_H-R_L)\right]\equiv\kappa_{SNB}$$

in the case of feedback, and κ_{NF} (as given by (9)) in the case of no feedback.

Thus, the SNB equilibrium is sustainable if and only if $\kappa_{SNB}\leq\kappa<\kappa_{NT}$ in the case of feedback, and $\kappa_{NF}\leq\kappa<\kappa_{NT}$ in the case of no feedback.

Trade Equilibrium T. All order flows are on the equilibrium path and so the posteriors are given as follows:

$$\begin{aligned}
q(-2) &= \frac{0}{\lambda(1/3)}=0,\\
q(-1) &= \frac{(1-\lambda)(1/3)}{(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)}=\frac{1-\lambda}{2-\lambda},\\
q(0) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+\lambda(1/3)+(1-\lambda)(1/3)}=\frac{1}{2},\\
q(1) &= \frac{\lambda(1/3)+(1-\lambda)(1/3)}{\lambda(1/3)+(1-\lambda)(1/3)+(1-\lambda)(1/3)}=\frac{1}{2-\lambda},\\
q(2) &= \frac{\lambda(1/3)}{\lambda(1/3)}=1.
\end{aligned}$$

Under the negatively-informed speculator's equilibrium strategy of selling:

- W.p. $\frac{1}{3}$, $X=-2$, and she is fully revealed. Her payoff is 0.

- W.p. $\frac{1}{3}$, $X=-1$. In the case of feedback, she receives $\frac{1-\lambda}{2-\lambda}(R_H-x)+\frac{1}{2-\lambda}(R_L+x)-c$ per share, and so her payoff is $-(R_L+x-c)+(\frac{1-\lambda}{2-\lambda}(R_H-x)+\frac{1}{2-\lambda}(R_L+x)-c)=\frac{1-\lambda}{2-\lambda}(R_H-R_L-2x)$. In the case of no feedback, she receives $\frac{1-\lambda}{2-\lambda}R_H+\frac{1}{2-\lambda}R_L$ per share, and so her payoff is $-R_L+(\frac{1-\lambda}{2-\lambda}R_H+\frac{1}{2-\lambda}R_L)=\frac{1-\lambda}{2-\lambda}(R_H-R_L)$.

- W.p. $\frac{1}{3}$, $X=0$, and she receives $\frac{1}{2}R_H+\frac{1}{2}R_L$ per share. Her payoff is $-R_L+(\frac{1}{2}R_H+\frac{1}{2}R_L)=\frac{1}{2}(R_H-R_L)$.

If she deviates to not trading, her payoff is 0. Thus, her expected gross gain from deviating to not trading is $-\kappa_T$ (as given by (8)) in the case of feedback, and $-\kappa_{NF}$ (as given by (9)) in the case of no feedback.

A similar calculation shows that, if the positively-informed speculator deviates to not trading, her gross gain is $-\kappa_{SNB}$ ($\kappa_{SNB}>\kappa_T$) in the case of feedback and $-\kappa_{NF}$ in the case of no feedback. Thus, the trade equilibrium is sustainable if and only if $\kappa<\kappa_T$ in the case of feedback, and $\kappa<\kappa_{NF}$ in the case of no feedback.

We now turn to the range of parameter values in which BNS is the only pure-strategy equilibrium in the case of feedback. If $\kappa_T\leq\kappa<\kappa_{NT}$, then the conditions for both the NT and T equilibrium to exist are violated. In addition, this is also the range where BNS equilibrium exists. We thus must derive conditions under which the SNB equilibrium does not hold, so that BNS is the unique equilibrium. There are two cases to consider. (i) If $\kappa_{SNB}\geq\kappa_{NT}$, the SNB equilibrium never exists, and so $\kappa_T\leq\kappa<\kappa_{NT}$ is sufficient for BNS to be the unique equilibrium. (ii) If $\kappa_T<\kappa_{SNB}\leq\kappa_{NT}$, the SNB equilibrium exists unless $\kappa<\kappa_{SNB}$. Thus, BNS is the unique equilibrium if $\kappa_T\leq\kappa<\kappa_{SNB}$. Combining the two cases gives the range $\kappa_T\leq\kappa<\min(\kappa_{SNB},\kappa_{NT})$ in the Proposition.

Proof of Lemma 2

For part (i), if $\theta=H$, the expected posterior is given by:

$$\begin{aligned} q^H &= (1-\lambda)\left[\frac{1}{3}q(-1)+\frac{1}{3}q(0)+\frac{1}{3}q(1)\right]+\lambda\left(\frac{1}{3}q(0)+\frac{1}{3}q(1)+\frac{1}{3}q(2)\right) \\ &= \frac{1-\lambda}{3}q(-1)+\frac{1}{3}q(0)+\frac{1}{3}q(1)+\frac{\lambda}{3}q(2) \\ &= \frac{(1-\lambda)^2}{6-3\lambda}+\frac{1}{3}+\frac{\lambda}{3}. \end{aligned} \tag{10}$$

We have:

$$
\begin{aligned}
\frac{\partial q^H}{\partial \lambda} &= \frac{1}{3} + \frac{1}{3}\left[\frac{-2(1-\lambda)(2-\lambda) + (1-\lambda)^2}{(2-\lambda)^2}\right] \\
&= \frac{1}{3}\left[1 + \left(\frac{1-\lambda}{2-\lambda}\right)^2 - 2\frac{1-\lambda}{2-\lambda}\right] \\
&= \frac{1}{3}\left[1 - \left(\frac{1-\lambda}{2-\lambda}\right)\right]^2 \\
&> 0.
\end{aligned}
$$

The expected posterior is increasing in λ: if the speculator is more likely to be present, she is more likely to impound her information into prices by trading.

Moving to part (ii), if $\theta = L$, we have:

$$
\begin{aligned}
q^L &= \frac{1}{3}\left(q\left(-1\right) + q\left(0\right) + q\left(1\right)\right) \\
&= \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}.
\end{aligned} \tag{11}
$$

This quantity is decreasing in λ. Even though the speculator does not trade upon $\theta = L$ if she is present, her information is still partially incorporated into prices. With $\theta = L$, there is a $\frac{1}{3}$ probability that the order flow is $X = -1$. This is consistent with the speculator being absent (in which case the state may be either H or L) or her being present and observing $\theta = L$; it is not consistent with the speculator observing $\theta = H$. The greater the likelihood that the speculator is present, the greater the likelihood that $X = -1$ stems from $\theta = L$, and thus the greater the decrease in the market maker's posterior. Part (iii) follows from simple calculations.

Proof of Proposition 2

For parts (i) and (ii), we have:

$$
\begin{aligned}
q^{H,spec} &= \frac{1}{3}\left(q\left(0\right) + q\left(1\right) + q\left(2\right)\right) \\
&= \frac{2}{3},
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
q^{L,spec} &= \frac{1}{3}\left(q\left(-1\right) + q\left(0\right) + q\left(1\right)\right) \\
&= \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}.
\end{aligned} \tag{13}
$$

Note that $q^{H,spec}$ is independent of λ, but $q^{L,spec}$ is decreasing in λ. The variable λ can affect the expected posterior in two ways: first, it can change the relative likelihood of the different order flows, and second, it can change the actual posterior given a certain order flow. Since we are conditioning on the speculator being present, the first channel is ruled out: conditional on the speculator being present and $\theta = H$, $X \in \{0, 1, 2\}$ with uniform probability regardless of λ; conditional on the speculator being present and $\theta = L$, $X \in \{-1, 0, 1\}$ with uniform probability regardless of λ. Turning to the second channel, the only posterior that depends on λ is $q(-1)$: since $X = -1$ is inconsistent with the speculator being present and seeing $\theta = H$, it has a particularly negative impact on the likelihood of $\theta = H$ if the speculator is more likely to be present. In contrast, $X \in \{-2, 2\}$ is fully revealing and so the posterior is independent of λ; $X \in \{0, 1\}$ is completely uninformative and so the posterior is again independent of λ. Since $X = -1$ can only occur in the presence of a speculator if she has received bad news, only $q^{L,spec}$ depends on λ but $q^{H,spec}$ does not. Part (iii) follows from simple calculations.

Proof of Lemma 3

We start by calculating p_0. With probability $\frac{1}{2}$, the state will be $\theta = L$ and there is no trade, regardless of whether the speculator is present. Thus, $X \in \{-1, 0, 1\}$ with equal probability. With probability $\frac{1}{2}$, the state will be $\theta = H$. If the speculator is absent (w.p. $(1 - \lambda)$), there is no trade and we again have $X \in \{-1, 0, 1\}$. If the speculator is present, $X \in \{0, 1, 2\}$. Letting $p(X)$ denote the stock price set by the market maker after observing order flow X at $t = 1$, the price at $t = 0$ will be the expectation over all possible future prices at $t = 1$, and is given as follows:

$$
\begin{aligned}
p_0 &= \frac{\lambda}{2}\left(\frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{1}{3}p(2)\right) + \left(1 - \frac{\lambda}{2}\right)\left(\frac{1}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1)\right) \\
&= \frac{1}{3}\left(\left(1 - \frac{\lambda}{2}\right)p(-1) + p(0) + p(1) + \frac{\lambda}{2}p(2)\right) \\
&= \frac{1}{6}\left[3R_H + 3R_L - 2c + 2\lambda x\right]. \qquad (14)
\end{aligned}
$$

Even though the initial belief y is independent of λ, the initial stock price p_0 is increasing in λ, because the speculator provides information to improve the manager's decision.

For part (i), if $\theta = H$ is realized, the expected price at $t = 1$ is given by:

$$\begin{aligned}
p_1^H &= (1-\lambda)\left[\frac{1}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1)\right] + \lambda\left(\frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{1}{3}p(2)\right) \\
&= \frac{1-\lambda}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{\lambda}{3}p(2) \\
&= \frac{(3-\lambda)R_H + (3-2\lambda)(R_L + \lambda x)}{3(2-\lambda)} - \frac{c}{3}.
\end{aligned} \tag{15}$$

Note that:

$$\begin{aligned}
\frac{\partial p_1^H}{\partial \lambda} &= \frac{1}{3}p(2) - \frac{1}{3}p(-1) + \frac{1-\lambda}{3}\frac{\partial p(-1)}{\partial \lambda} \\
&= \frac{R_H - R_L + 2(3 - 4\lambda + \lambda^2)x}{3(2-\lambda)^2} > 0,
\end{aligned}$$

i.e., p_1^H is increasing in λ, since the speculator impounds information about the high state into prices.

Turning to part (ii), if $\theta = L$ is realized, the expected price at $t = 1$ is given by:

$$p_1^L = \frac{1}{3}\left(p(-1) + p(0) + p(1)\right). \tag{16}$$

We have $\frac{\partial p_1^L}{\partial \lambda} = \frac{R_L - R_H + 2x}{3(2-\lambda)^2}$. If the speculator is more likely to be present, then $X = -1$ is more likely to result from $\theta = L$. Thus, the price is higher if and only if firm value is higher in this state, i.e., $R_L + x > R_H - x$ (Case 2).

The calculations of $p_1^H - p_0$ and $p_1^L - p_0$ follow automatically.

Proof of Proposition 3

For part (i), if the speculator receives positive information, she will buy one share and so the expected price becomes:

$$p_1^{H,spec} = \frac{1}{3}\left(p(0) + p(1) + p(2)\right). \tag{17}$$

Unlike p_1^H (equation (15)), this quantity is independent of λ, for the same reasons that $q^{H,spec}$ (equation (12)) is independent of λ. The stock return realized when the speculator receives good information is thus given by:

$$
\begin{aligned}
p_1^{H,spec} - p_0 &= \frac{1}{3}\left(p\left(0\right)+p\left(1\right)+p\left(2\right)\right) - \frac{1}{3}\left(\left(1-\frac{\lambda}{2}\right)p\left(-1\right)+p\left(0\right)+p\left(1\right)+\frac{\lambda}{2}p\left(2\right)\right) \\
&= \frac{1}{3}\left(1-\frac{\lambda}{2}\right)\left(p\left(2\right)-p\left(-1\right)\right) \\
&= \frac{1}{6}\left(R_H - R_L + 2(1-\lambda)x\right) > 0, \qquad (18)
\end{aligned}
$$

and we have

$$\frac{\partial\left(p_1^{H,spec} - p_0\right)}{\partial\lambda} = -\frac{1}{3}x < 0.$$

Equation (18) is decreasing in λ, whereas the stock return not conditioning on the speculator's presence, $p_1^H - p_0$, was increasing in λ. This reversal is because p_0 is increasing in λ, but $p_1^{H,spec}$ is independent of λ.

For part (ii), if the speculator is present and receives negative information, we have:

$$p_1^{L,spec} = \frac{1}{3}\left(p\left(-1\right)+p\left(0\right)+p\left(1\right)\right) = p_1^L, \qquad (19)$$

and

$$
\begin{aligned}
p_1^{L,spec} - p_0 &= \frac{1}{3}\left(p\left(-1\right)+p\left(0\right)+p\left(1\right)\right) - \frac{1}{3}\left(\left(1-\frac{\lambda}{2}\right)p\left(-1\right)+p\left(0\right)+p\left(1\right)+\frac{\lambda}{2}p\left(2\right)\right) \\
&= \frac{\lambda}{6}\left(p\left(-1\right)-p\left(2\right)\right) = p_1^L - p_0 < 0.
\end{aligned}
$$

Parts (iii) and (iv) follow from simple calculations.

Dropping constants, both equation (4) (the asymmetry between the price impact of good and bad news) and equation (5) (the average return, conditional on the speculator being present) become:

$$(1-\lambda)\left(\frac{R_H - R_L + 2(1-\lambda)x}{2-\lambda}\right).$$

Differentiating with respect to λ gives:

$$\frac{R_L - R_H - 2(3-4\lambda+\lambda^2)x}{(2-\lambda)^2} < 0.$$

Thus, both equations (4) and (5) are decreasing in λ.

Proof of Corollary 2

We start with part (i). If the speculator receives good news, she will buy and the investment will be undertaken only if the noise trader buys. We thus have $p_2^{H,spec} = \frac{1}{3}(R_H + x - c) + \frac{2}{3}R_H$. This observation yields:

$$p_2^{H,spec} - p_1^{H,spec} = R_H + \frac{1}{3}(x-c) - \frac{1}{3}\left(p\left(0\right) + p\left(1\right) + p\left(2\right)\right)$$
$$= \frac{1}{3}\left(R_H - R_L\right).$$

Moving to part (ii), if the speculator receives bad news, she will not trade. The firm reduces investment only if the noise trader sells. We thus have $p_2^{L,spec} = \frac{1}{3}(R_L + x - c) + \frac{2}{3}R_L$. This yields:

$$p_2^{L,spec} - p_1^{L,spec} = R_L + \frac{1}{3}(x-c) - \frac{1}{3}\left(p\left(-1\right) + p\left(0\right) + p\left(1\right)\right)$$
$$= \frac{(3-2\lambda)(R_L - R_H) + 2(1-\lambda)x}{3(2-\lambda)},$$

which is negative in Case 1, but can be positive or negative in Case 2. Part (iii) follows from simple calculations. For part (iv), we first calculate the expected firm value at $t = 2$ if the speculator is present, not conditioning on the state. If $\theta = H$, investment depends on the order flow: if $X = 2$, we have $d = 1$ and so firm value is $v = R_H + x - c$; if $X \in \{0, 1\}$, we have $d = 0$ and so $v = R_H$. If $\theta = L$, disinvestment depends on the order flow: if $X = -1$, we have $d = -1$ and so $v = R_L + x - c$; if $X \in \{0, 1\}$, we have $d = 0$ and so $v = R_L$. Expected firm value at $t = 2$ is thus given by:

$$p_2^{spec} = \frac{1}{2}(R_H + R_L) + \frac{1}{3}(x-c),$$

and so we have

$$p_2^{spec} - p_1^{spec} = \frac{1}{6}\frac{1-\lambda}{2-\lambda}(R_L - R_H + 2x),$$

which is positive if we are in Case 2 and negative if we are in Case 1.

B Supplementary Appendix: Not For Publication

B.1 Equilibria when firm value is non-monotonic in states: Full analysis

In this subsection, we consider the case where, if the firm disinvests, its value is higher in state $\theta = L$ ($R_H - x < R_L + x$). Hence, disinvestment is sufficiently powerful to outweigh the effect of the state on firm value and lead to a higher value in the low state.

The analysis of equilibrium outcomes becomes more complicated in the case of non-monotonicity. In the core model, where firm value is monotone in the state, a positively-informed speculator always loses money by selling and a negatively-informed speculator always loses money by buying, since firm value is always higher in state H than in state L. However, now that firm value may be higher in state L, a positively-informed speculator may find it optimal to sell and a negatively-informed speculator may find it optimal to buy. Hence, there are nine possible pure-strategy equilibria (each type of speculator – positively-informed and negatively-informed – may either buy, sell, or not trade). The following Lemma simplifies the equilibrium analysis, moving us closer to the analysis conducted in the core model.

Lemma 4 *(No equilibrium with trading against information). Suppose that $R_H - x < R_L + x$. Then:*

(i) The trading game has no pure-strategy equilibrium where the speculator sells when she knows that $\theta = H$.

(ii) The trading game has no pure-strategy equilibrium where the speculator buys when she knows that $\theta = L$.

Proof. See Appendix B.3. ■

Following the Lemma, there are four possible pure-strategy equilibria, just as in the previous subsection: NT, T, SNB, and BNS. However, the conditions for these equilibria to hold are now tighter. The reason that the positively-informed speculator never sells *in equilibrium* is that if the market maker and the manager believe that she sells, she cannot make a profit from selling. However, she still might be tempted to *deviate* to selling in any of the four equilibria mentioned above. When she sells, she potentially misleads the market maker and the manager to believe that the negatively-informed speculator is present, and so to disinvest. Since disinvestment is suboptimal if $\theta = H$, this decision reduces firm value and causes the speculator to make a profit on her short position. Hence, for any of the above four equilibria to hold, an additional condition must be satisfied to ensure that the positively-informed speculator does not have an incentive to deviate to selling. Interestingly, the same issue does not arise with the negatively-informed speculator, as she never has an incentive to deviate to buying. If she does so, she misleads the market maker and the manager to believe that the positively-informed speculator is present, and so to (incorrectly) take the investment. This decision reduces firm value, causing the speculator to incur a loss from selling.[15]

In analyzing deviations from the equilibrium, another issue that arises in this subsection is the specification of off-equilibrium beliefs. In Case 1, due to monotonicity, the only assumption that satisfied the Intuitive Criterion was that an off-equilibrium order flow of $X = 2$ is due to the positively-informed speculator (and so the posterior is $q = 1$), while an off-equilibrium order flow of $X = -2$ is due to the negatively-informed speculator (and so the posterior is $q = 0$). In this subsection, however, the Intuitive Criterion is not sufficient to rule out other off-equilibrium beliefs. We nevertheless retain this assumption regarding off-equilibrium beliefs, which is reasonable given the possible equilibria in our model. Our results remain the same for any other off-equilibrium beliefs that are monotone in the order flow.[16]

Proposition 4 provides the characterization of equilibrium outcomes.

Proposition 4 *(Equilibrium, firm value is non-monotone in the state). Suppose that $R_H - x < R_L + x$, and suppose that the belief of the market maker and the manager is that an off-equilibrium order flow of $X = -2$ ($X = 2$) is associated with the presence of negatively-informed (positively-informed) speculator. Then, if $R_H - R_L$ is sufficiently high compared to x (formally, $R_H - R_L > \frac{4}{3}x$), the characterization of equilibrium outcomes is identical to that in Lemma 1 for the case of feedback and Proposition 1 for the case of no feedback.*

More specifically, the following ***additional*** *conditions are required for the various equilibria to hold:*

Equilibrium NT: $\kappa \geq \frac{2}{3}(R_L - R_H + x)$.

Equilibrium SNB: in the case of feedback, $\kappa \geq \frac{2}{3}(R_L - R_H + x)$; in the case of no feedback, $\kappa \geq \frac{2}{3}(R_L - R_H + x)$.

[15] Goldstein and Guembel (2008) also derive conditions to ensure that the speculator does not deviate from the equilibrium to trade against her information.

[16] Other papers that use similar monotonicity assumptions for off-equilibrium beliefs include Gul and Sonnenschein (1988) and Bikhchandani (1992).

Equilibrium BNS: in the case of feedback, $\frac{6-2\lambda}{2-\lambda}x < \frac{12-5\lambda}{4-2\lambda}(R_H - R_L)$; in the case of no feedback, $\frac{2}{3}x < \frac{12-5\lambda}{12-6\lambda}(R_H - R_L)$.

Equilibrium T: in the case of feedback, $\frac{4}{2-\lambda}x < 3(R_H - R_L)$; in the case of no feedback, $\frac{2}{3}x < (R_H - R_L)$.

The condition $R_H - R_L > \frac{4}{3}x$ is sufficient for all of the above conditions to be satisfied.

Proof. The calculations of the posterior q, the manager's decision d and the price p for different order flows X in the various possible equilibria are identical to those provided in the proof of Proposition 1. Hence, the conditions for the positively-informed speculator to choose between buying and not trading and for the negatively-informed speculator to choose between selling and not trading are identical to those derived in the proof of Proposition 1. Analyzing the possible trading profits for the negatively-informed speculator from deviating to buying in each of the four possible equilibria, it is straightforward to see that she always loses from buying and hence will never deviate. Appendix B.3 calculates the possible trading profits for the positively-informed speculator from deviating to selling in each of the four possible equilibria, which yields the additional conditions stated in the body of the proposition. These conditions are automatically satisfied when $R_H - R_L > \frac{4}{3}x$. ■

As Proposition 4 demonstrates, the main force identified in the previous subsection for the case where $R_H - x > R_L + x$, exists also in the case where $R_H - x < R_L + x$. That is, the feedback effect deters informed selling relative to informed buying. In this subsection, this force is even stronger because the minimum transaction cost required to deter the negatively-informed speculator from selling in the BNS equilibrium, $\kappa_T \equiv \frac{1}{3}\left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right]$, is lower when $R_H - x < R_L + x$: the first term in the expression for κ_T is negative. A strong feedback effect, in which disinvestment not only mitigates the effect of the low state but also overturns it, implies that the negatively-informed speculator can make a loss from selling – even before transaction costs. This result is in contrast to standard informed trading models where a speculator can never make a loss (before transactions costs) if she trades in the direction of her information. This loss occurs at the $X = -1$ node. As in the core model, the key to this result is $\lambda < 1$. Even though both the speculator and market maker know that disinvestment will occur if $X = -1$, they have differing views on firm value conditional on disinvestment. The speculator knows that disinvestment will occur, *and* that disinvestment is desirable for firm value (since she knows that $\theta = L$), and so firm value is $R_L + x - c$. In contrast, the market maker knows the disinvestment will occur but is not certain that it is optimal, because she is unsure of the underlying state θ. Order flow $X = -1$ is consistent with a negatively-informed speculator, but also with an absent speculator and selling by the noise trader. Hence, it is possible that $\theta = H$, in which case disinvestment is undesirable, leading to firm value of $R_H - x - c$. Therefore, the price set by the market maker is only $\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c$, since he puts weight on the possibility that disinvestment may be undesirable, and so the speculator loses $\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$ before transaction costs.

Moreover, Proposition 4 also shows that the feedback effect generates an additional force in this subsection: the desire of the positively-informed speculator to deviate and manipulate the price by selling. She can potentially profit from leading the manager to divest incorrectly, which enables her to gain on her short position. The manipulation incentive is not strong enough to interfere with equilibrium conditions as long as $R_H - R_L$ is sufficiently high relative to x; a sufficient condition is $R_H - R_L > \frac{4}{3}x$. In this case, the loss from trading against good news (which is proportional to $R_H - R_L$) is high relative to the benefit from manipulation (which is proportional to x, the value destroyed by inducing the manager to divest incorrectly). Otherwise, additional conditions are required to sustain the various possible equilibria.

B.2 Positive initial position: Full analysis

We now assume that the speculator starts off with a stake of α and can trade $s \in \{-1, 0, 1\}$. Thus, her final position becomes $\{\alpha - 1, \alpha, \alpha + 1\}$. Our main result from the core model is that, under feedback, the range of κ that supports the BNS equilibrium is strictly greater than that which supports the SNB equilibrium. Thus, for conciseness, we consider the case of feedback ($\frac{1}{2-\lambda} > \gamma_1 = \frac{1}{2} + \frac{c}{2x} \iff \frac{1-\lambda}{2-\lambda} < \gamma_{-1} = \frac{1}{2} - \frac{c}{2x}$) and analyze only the BNS and SNB equilibria, rather than the T and NT equilibria.

Buy Not Sell Equilibrium BNS.

Under the positively-informed speculator's equilibrium strategy of buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is $(\alpha+1)(R_H + x - c) - (R_H + x - c) = \alpha(R_H + x - c)$.

- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $(\alpha + 1)R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \alpha R_H + \frac{1}{2}(R_H - R_L)$.

If she deviates to not trading:

- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and her payoff is αR_H.
- W.p. $\frac{1}{3}$, $X = -1$, and her payoff is $\alpha (R_H - x - c)$.

Her expected gross gain from deviating to not trading is:

$$\frac{1}{3}[2\alpha x + (R_H - R_L)] \equiv \kappa_{NT}^{\alpha}.$$

Under the negatively-informed speculator's equilibrium strategy of not trading:

- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and her payoff is αR_L.
- W.p. $\frac{1}{3}$, $X = -1$ and her payoff is $\alpha (R_L + x - c)$.

If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$ and she is fully revealed. Thus her payoff is $(\alpha - 1)(R_L + x - c) + (R_L + x - c) = \alpha (R_L + x - c)$.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1-\lambda}{2-\lambda}(R_H - x - c) + \frac{1}{2-\lambda}(R_L + x - c)$ per share. Her payoff is $(\alpha - 1)(R_L + x - c) + (\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c) = \alpha (R_L + x - c) + \frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$.
- W.p. $\frac{1}{3}$, $X = 0$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $(\alpha - 1)R_L + (\frac{1}{2}R_H + \frac{1}{2}R_L) = \alpha R_L + \frac{1}{2}(R_H - R_L)$.

Her expected gross gain from deviating to selling is:

$$\frac{1}{3}\left[\alpha(x - c) + \frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L)\right] \equiv \kappa_T^{\alpha}.$$

The BNS equilibrium holds for $\kappa \in [\kappa_T^{\alpha}, \kappa_{NT}^{\alpha}]$.

Sell Not Buy Equilibrium SNB.

Under the positively-informed speculator's equilibrium strategy of not trading:

- W.p. $\frac{1}{3}$, $X = 1$ and her payoff is $\alpha (R_H + x - c)$.

- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$ and her payoff is αR_H.

If she deviates to buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is $(\alpha+1)(R_H + x - c) - (R_H + x - c) = \alpha (R_H + x - c)$.
- W.p. $\frac{1}{3}$, $X = 1$, and she pays $\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c$ per share. Her payoff is $(\alpha + 1)(R_H + x - c) - (\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c) = \alpha (R_H + x - c) + \frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x)$
- W.p. $\frac{1}{3}$, $X = 0$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $(\alpha + 1)R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \alpha R_H + \frac{1}{2}(R_H - R_L)$.

Thus, her expected gross gain from deviating to buying is:

$$\frac{1}{3}\left[\alpha (x - c) + \frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) + \frac{1}{2}(R_H - R_L)\right] \equiv \kappa_{SNB}^{\alpha}.$$

Under the negatively-informed speculator's equilibrium strategy of selling:

- W.p. $\frac{1}{3}$, $X = -2$, and she is fully revealed. Her payoff is $(\alpha - 1)(R_L + x - c) + (R_L + x - c) = \alpha (R_L + x - c)$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $(\alpha - 1)R_L + \left(\frac{1}{2}R_H + \frac{1}{2}R_L\right) = \alpha R_L + \frac{1}{2}(R_H - R_L)$.

If she deviates to not selling:

- W.p. $\frac{1}{3}$, $X = 1$, and her payoff is $\alpha (R_L - x - c)$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and her payoff is αR_L.

Thus, her expected gross gain from deviating to not trading is:

$$\frac{1}{3}[2\alpha x + (R_H - R_L)] \equiv \kappa_{NT}^{\alpha}.$$

The SNB equilibrium holds for $\kappa \in [\kappa_{SNB}^{\alpha}, \kappa_{NT}^{\alpha}]$.

The maximum value of κ that supports the BNS and SNB equilibria is the same for both equilibria (κ_{NT}^{α}). The transaction cost must be sufficiently small to deter the positively-informed speculator from deviating to not trading in BNS, and the negatively-informed speculator from deviating to not trading in SNB. Under BNS, the positively-informed speculator's motive to play her equilibrium strategy of buying is that doing so leads to correct investment. Under BNS, the negatively-informed speculator's motive to play her equilibrium strategy of selling is that doing so leads to correct disinvestment. Since the value created by correct investment equals the value created by correct disinvestment, these motives are equally strong, thus leading to the same threshold.

While the maximum value of κ must be sufficiently low not to deter the positively-informed speculator from deviating to not buying (under BNS) and the negatively-informed speculator from deviating not selling (under SNB), the minimum value of κ must be sufficiently high to deter the negatively-informed speculator from deviating to selling (under BNS) and positively-informed speculator from deviating to buying (under SNB). While this minimum is is κ_T^{α} for BNS, it is κ_{SNB}^{α} for SNB. We have

$$\begin{aligned}\kappa_{SNB}^{\alpha} - \kappa_T^{\alpha} &= \frac{1}{3}\left\{\alpha[(x-c)-(x-c)] + \frac{1-\lambda}{2-\lambda}[(R_H - R_L + 2x) - (R_H - R_L - 2x)]\right\} \\ &= \frac{1}{3}\frac{1-\lambda}{2-\lambda}4x > 0\end{aligned} \tag{20}$$

Thus, the minimum value is strictly smaller for BNS, and so the range of κ that support BNS is a strict superset of the range that supports SNB – just as in the core model where the speculator has a zero initial stake.

To understand the intuition, the first term in the difference (20) is zero, since the value created by correct investment equals the value created by correct disinvestment. The second term is positive due to the feedback effect: investment increases the sensitivity of firm value to the state of nature (which becomes now $R_H - R_L + 2x$), and disinvestment decreases the sensitivity of firm value to the state of nature (which becomes now $R_H - R_L - 2x$). Due to the feedback effect, the negatively-informed speculator's motive to deviate to selling under BNS is relatively low, as it may induce the manager to efficiently disinvest and thus reduces the profits on the share she sells. In contrast, due to the same feedback effect, the positively-informed speculator's motive to deivate to buying under SNB is relatively high, as it may induce the manager to efficiently invest and thus increase the profits on the share she buys. Thus, the minimum transaction cost to deter deviation to trading is higher in SNB than BNS, and the SNB equilibrium is strictly more difficult to sustain.

Note that the difference $\kappa^{\alpha}_{SNB} - \kappa^{\alpha}_{T}$ is *independent* of the initial stake α. While a higher α increases the speculator's incentives to sell on negative information, since doing so increases the value of her block, it equally increases her incentives to buy on positive information for the same reason. These two effects cancel out, and so the only difference is the gain on the one share that the speculator trades. Thus, her initial position does not matter.

B.3 Additional Proofs

Proof of Lemma 4

For part (i), suppose that the speculator sells when she knows that $\theta = H$: then $X \in \{-2, -1, 0\}$ when $\theta = H$. In each of these nodes, the posterior probability q of state H is at least $\frac{1}{2}$ (since these nodes are consistent with the action of the positively-informed speculator and may or may not be consistent with the action of the negatively-informed speculator, depending on her equilibrium action). Then the manager will choose $d \in \{0, 1\}$ and so firm value is either R_H or $R_H + x - c$. The price, however, will incorporate the possibility that $\theta = L$. For $d \in \{0, 1\}$, firm value is lower under $\theta = L$ than under $\theta = H$. Thus, the price the speculator receives will be lower than firm value, and so the speculator makes a loss from selling.

For part (ii), suppose that the speculator buys when she knows that $\theta = L$: then $X \in \{0, 1, 2\}$ when $\theta = L$. Given that the positively-informed speculator does not sell, the posterior probability q is $\frac{1}{2}$ at $X \in \{0, 1\}$. Thus, the manager chooses $d = 0$ and so firm value is R_L. Since the price is $\frac{1}{2}R_H + \frac{1}{2}R_L$, the speculator will lose money on these nodes. When $X = 2$, there are two possibilities. If the positively-informed speculator buys in equilibrium, then the outcome is the same as on the other nodes. If she does not trade in equilibrium, then the negatively-informed speculator is revealed, buying a security worth $R_L + x - c$ for a price of $R_L + x - c$. Thus, in expectation she makes a loss, given she loses at $X \in \{0, 1\}$.

Proof of Proposition 4

This proof only provides material supplementary to Appendix B.1. As discussed in the main text, it is straightforward to show that the negatively-informed speculator will not deviate to buying. Here, we calculate the profits made if the positively-informed speculator deviates to selling, to derive the necessary conditions to prevent such a deviation.

No Trade Equilibrium NT. Under the positively-informed speculator's equilibrium strategy of not trading, her payoff is 0. If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and her (gross) payoff is $-(R_H - x - c) + (R_L + x - c) = R_L - R_H + 2x$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_H + \frac{1}{2}R_H + \frac{1}{2}R_L = \frac{1}{2}(R_L - R_H)$.

Thus, her overall gross gain from deviating to selling is given by:

$$\frac{2}{3}(R_L - R_H + x) \equiv \kappa^M_{NT}.$$

Thus, if and only if $\kappa \geq \kappa^M_{NT}$, she will not deviate to selling. The above calculations apply both in the case of feedback and no feedback. The sufficient condition $R_H - R_L > \frac{4}{3}x$ implies $R_L - R_H + x < 0$, and hence the additional equilibrium condition is satisfied.

Partial Trade Equilibrium BNS. Under the positively-informed speculator's equilibrium strategy of buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and her payoff is $-(R_H - x - c) + R_L + x - c = R_L - R_H + 2x$.
- W.p. $\frac{1}{3}$, $X = -1$, and her payoff is $-(R_H - x - c) + \frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c = \frac{1}{2-\lambda}(R_L - R_H + 2x)$ in the case of feedback and $-R_H + \frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L = \frac{1}{2-\lambda}(R_L - R_H)$ in the case of no feedback.
- W.p. $\frac{1}{3}$, $X = 0$, and her payoff is $-R_H + \frac{1}{2}R_H + \frac{1}{2}R_L = \frac{1}{2}(R_L - R_H)$.

Thus, she will not deviate if

$$\frac{6 - 2\lambda}{2 - \lambda}x < \frac{12 - 5\lambda}{4 - 2\lambda}(R_H - R_L)$$

in the case of feedback and

$$\frac{2}{3}x < \frac{12-5\lambda}{12-6\lambda}(R_H - R_L)$$

in the case of no feedback.

In the case of feedback, the condition is equivalent to

$$R_H - R_L > \frac{12-4\lambda}{12-5\lambda}x.$$

It is straightforward to show that

$$\frac{4}{3} > \frac{12-4\lambda}{12-5\lambda},$$

and hence the sufficient condition $R_H - R_L > \frac{4}{3}x$ implies the additional equilibrium condition in the case of feedback. A similar argument shows that the additional equilibrium condition is also satisfied in the case of no feedback.

Partial Trade Equilibrium SNB. Under the positively-informed speculator's equilibrium strategy of not trading, her payoff is 0. If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and her payoff is $-(R_H - x - c) + R_L + x - c = R_L - R_H + 2x$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and her payoff is $-R_H + \frac{1}{2}R_H + \frac{1}{2}R_L = \frac{1}{2}(R_L - R_H)$.

Thus, her expected gross gain from deviating to selling is given by:

$$\frac{2}{3}[R_L - R_H + x] \equiv \kappa^M_{SNB}.$$

Thus, she will not deviate to selling if and only if $\kappa \geq \kappa^M_{SNB}$. It is straightforward to verify that the condition $R_H - R_L > \frac{4}{3}x$ is sufficient for κ^M_{SNB} to be negative and for the additional equilibrium conditions to be satisfied.

Trade Equilibrium T. Under the positively-informed speculator's equilibrium strategy of buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{1}{3}$, $X = 1$. In the case of feedback, she pays $\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c$ per share, and so her payoff is $R_H + x - c - (\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c) = \frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x)$.

In the case of no feedback, she pays $\frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L$ per share, and so her payoff is $R_H - (\frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L) = \frac{1-\lambda}{2-\lambda}(R_H - R_L)$.

- W.p. $\frac{1}{3}$, $X = 0$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and her payoff is $-(R_H - x - c) + R_L + x - c = R_L - R_H + 2x$.

- W.p. $\frac{1}{3}$, $X = -1$. In the case of feedback, she receives $\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c$ per share, and so her payoff is $-(R_H - x - c) + \frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c - \kappa = \frac{1}{2-\lambda}(R_L - R_H + 2x)$. In the case of no feedback, she receives $\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L$ per share, and so her payoff is $-R_H + \frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L = \frac{1}{2-\lambda}(R_L - R_H)$.

- W.p. $\frac{1}{3}$, $X = 0$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_H + \frac{1}{2}R_H + \frac{1}{2}R_L = \frac{1}{2}(R_L - R_H)$.

Thus, she will not deviate if

$$\frac{4}{2-\lambda}x < 3(R_H - R_L)$$

in the case of feedback and

$$\frac{2}{3}x < (R_H - R_L)$$

in the case of no feedback. The condition $R_H - R_L > \frac{4}{3}x$ implies $3(R_H - R_L) > 4x\frac{1}{2-\lambda}$ and $R_H - R_L > \frac{2}{3}x$, and thus the additional equilibrium conditions.

References

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