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Mitigation of the Inefficiency in Imbalance Settlement Designs using Day-Ahead Prices

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***Abstract*--The design of electricity imbalance pricing mechanisms is internationally controversial. Policies on whether to permit virtual trading for market participants and whether, or how, to impose penalty incentives on the imbalance volumes vary widely. Furthermore, market designs vary depending whether the imbalance prices are obtained directly from real-time trading or based upon the offer and demand functions from the day-ahead energy markets. This paper develops an analytical framework for evaluating designs for imbalance settlement mechanisms and we have selected the Japanese electricity market, which has undergone several revisions in its imbalance mechanism, as a good example to assess such variations. We develop a predictive approach for the imbalance volumes and price densities using two-step quantile regressions and derive a new trading optimization for a virtual trader's arbitrage position. We construct supporting models to estimate prediction errors for renewable power and demand as drivers of imbalance volume. The empirical analysis reveals that even in a mechanism with imbalance penalties based upon day-ahead reference prices, virtual trading may still be beneficial to market participants as well as to the system operator. We also find that greater market transparency is crucial for increased benefits. The insights generalize beyond the Japan case study.**

***Index Terms*--Electricity Balancing, Imbalance Settlements, Forecasting, Market Design, Quantiles, Trading, Transparency, Statistical Arbitrage.**

I. Introduction

MARKET design and system transparency questions are increasingly becoming focused on electricity balancing arrangements and the determination of imbalance settlement prices. In most markets, wholesale trading and balancing services are successive stages in market clearing. The balancing arrangements play an important final role through which real-time deviations (the "imbalances") from prior market participant nominations are remedied by the "balancing actions" of the system operator (SO). The need for these actions is growing in volume and value as intermittent generation increases supply uncertainties and "behind the meter" embedded resources obscure demand forecasting. Essentially, balancing activities run in real-time, after the intraday power exchanges cease trading, for each successive delivery period and operate with the SO acting as a counterparty to the balancing services offered by the market participants. However, whilst they are receiving increasing interest, the design of imbalance settlement mechanisms, which may be market-based or administered, remain controversial (see, e.g., [1] and [2] for related policy discussions). This is partly due to a lack of clarity in terms of regulatory intent. This paper, therefore, aims to develop new insights into this issue.

In addition to the question of whether to permit arbitrage speculation (virtual trading) for market participants, there are different regulatory views on the incentives required in the imbalance behaviour. One regulatory view is that market participants should be required to make best endeavors ex ante to minimize deviations from their prior nominations, presumably through better forecasting. This view, therefore, often leads to imbalance settlement mechanisms that are designed to include penalties on those deviations. A good example was the "dual" imbalance pricing introduced in Britain in 2001, which had a punitive intent with different prices for the deficit or surplus status of individual participants. These prices were based on the separate buying and selling activities of the SO in each 30-min balancing period. However, a different view emerged in Britain by 2015 when the dual prices were replaced by a "single" price mechanism based on the net imbalance of the entire system in each 30-min period. Remarkably, the regulatory authority concluded that dual pricing "drives inefficiency in balancing by over-incentivizing parties to balance"[3]. With single imbalance pricing, market participants are not only able to hedge more effectively, but also speculate on the net imbalance position of the market and profit if they are out of balance in the opposite direction. By doing so, they may earn positive returns and may help the SO by reducing the balancing energy needed in each period. In [4] this was also shown to be effective in the Austrian system. However, it remains controversial whether participants should be permitted through single imbalance prices to take positions against the market, and some jurisdictions, for example Germany, disallow it, notwithstanding its potential efficiency gains. (Appendix A summarizes some country variations on single vs dual pricing.)

Furthermore, regardless of whether single or dual prices are used for imbalances, the source of these prices themselves varies widely according to different jurisdictions. In several markets a formula for the imbalance settlement price includes the day ahead energy price, or is obtained from separate day ahead, or intraday, auctions for the ancillary services which provide the anticipated capacity for frequency regulation and energy balancing. Elsewhere, markets clear in real time or close to real time (e.g., 5, 15 or 30 mins ahead) to balance the system. Whilst close to real-time price determination is more efficient in the sense of better representing current conditions, day ahead

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price setting persists in many countries. It is possible that regulators might want to see active and liquid intraday trading emerge before a real-time market becomes established, a view perhaps that a lack of liquidity leads to volatile and unreliable prices. However, intraday liquidity is not necessary as a matter of principle and there are examples in practice (eg Belgium) where real-time prices are used for imbalances despite a low intraday market liquidity. However, for whatever reasons, the evidence is that whilst real-time price determination appears most desirable, in many markets the imbalance prices are determined to a greater or lesser extent by the supply and demand functions clearing the day ahead auctions, particularly in markets that are slow to mature (see Appendix A).

This lack of a consensus in the arrangements for imbalance settlements is not helpful to countries that are more recently liberalizing their electricity markets (i.e., easing access for new entrants). Japan is an important example of a country facing this dilemma. In 2016, the Japanese power market was substantially liberalized with an imbalance settlement system based on a single price, influenced by the same motivations as the British reforms in 2015. The price was based upon the day ahead auction for wholesale energy, in the absence of intraday trading and any balancing markets, at the outset. However, in 2019, it moved to a dual price system, which penalizes positive and negative imbalances separately. The trigger for this revision was the rapid expansion of renewable energies, leading to large system imbalance due to forecast errors. Thus, the introduction of the penalties followed the intention to incentivize each individual operator to forecast better [5]. Nevertheless, the market design continued to be deliberated in Japan with the most recent policy review directed at considering a more efficient solution based on market principles, with particular reference to the greater provision of imbalance-related information in a timely manner [6].

Whilst the single price mechanism evidently provides an incentive for potentially beneficial arbitrage actions [4], but whether such an incentive is absent in the dual pricing system depends on the details of its implementation. Furthermore, if the settlement prices are inefficiently based upon day ahead prices, for whatever pragmatic reasons, does the possibility for arbitrage trading help to mitigate this inefficiency or exacerbate it? Considering this general question and using the Japanese deliberations as a case study, we develop an imbalance predictive model for the Japanese market and test trading strategies under its dual price imbalance settlement system with various penalties. We also consider how effective this may be under different conditions of market transparency.

The methodological approach we develop can be applied, with appropriate adjustment for local specificities, to most electricity markets that follow the “exchange model”, common in many European, S. American and Asian countries, “in which energy and related products are traded day-ahead and throughout the day at prices that clear the market”[7]. This is distinct from the “integrated model,” adopted in various US states, “in which a system operator centrally optimizes resources"[7]. In the latter case, whilst imbalance settlement procedures are required, and similar penalty issues arise, they are less material because of the central dispatch control and the shorter, more immediate, delivery periods (e.g., 5mins).

We develop a predictive approach for the imbalance volume and price densities using two-step quantile regressions and derive a new trading optimization for a virtual trader’s arbitrage position without the need for the discretization used in [4]. Furthermore, in constructing the predictive model for imbalance volume, where prediction error values for solar and demand are required as explanatory variables, we incorporate supporting predictive models for these. Since forecasts for demand and solar power generation were not made available to the Japanese market, our analysis has led to policy suggestions on the value of greater transparency in the provision of system data and forecasts to the market.

This paper is structured as follows. In the next section, we briefly review related research. In section III, we provide a Japanese market overview. In section IV, we formulate an effective arbitrage strategy, with the results reported in section V. Section VI concludes the paper.

II. Background Research

Various researchers have developed models for balancing markets and imbalance settlement procedures. A two-stage stochastic model for an integrated strategy of day-ahead offers and real-time operations was developed in [8] while arbitrage between the day-ahead and balancing markets has been investigated in [9] using storage and in [10] based on speculation. Potential strategic behavior in the German balancing activities was considered in [11] and [12], while [13] developed revenue-maximizing and risk-constrained strategies for a single price imbalance settlements. Research on profitable trading strategies by predicting the direction of the system imbalance includes [14] dealing with the German market and [15] in the Italian market. In the Nordic context, [16] concluded that balancing market prices were unpredictable before the closure of the day-ahead market, but in the Austrian market, [17] found that density forecasts developed from market data an hour before real-time substantially increased trading profitability and reduced risk. From a policy perspective, [18] compared the single and dual price systems, observing that in the dual price system with asymmetric penalties, participants such as wind power producers might tend to over contract in the wholesale market.

Distinct from these perspectives of optimal imbalance positions by market participants, [4] and [14] also investigate the benefits or otherwise to the SO in terms of managing the net imbalance at the system level. However, [14] evaluates only whether the direction of the arbitrage transaction and that of the system balance are consistent and does not consider the effect of its own transaction on the imbalance price. The analysis in [4] incorporates such an effect but is unusual, being applied to the Austrian zone of the German/Austrian electricity market, where there is a specific deterministic imbalance price calculation formula based on the imbalance volume and full transparency of system data. This facilitated a precise analysis of price effects and hence potential trading profits, but as such, has limited generalizability. It is an open question as to whether similar trading profits could be achieved when imbalance prices and volumes are subject to market uncertainties. We develop a more general and analytically precise optimization than in [4] and [14], and apply it to the Japanese market, as this is particularly challenging in its uncertainties and relative lack of market transparency. As a consequence, we believe this case study provides a more generalizable methodology.

III. JAPANESE MARKET

The Japan Electric Power Exchange (JEPX) started market trading in 2005. By 2019 it provided trading through three markets: the "forward market" for trading products up to one year, the "day-ahead market" auction, and the "intraday market" platform. Table I summarizes the day-ahead and intraday arrangements. These market arrangements are voluntary and similar to many power markets that have emerged worldwide (note that the intraday market was launched as recently as 2016, and the trading volume, shown in Fig 1, is about two orders of magnitude smaller than the day-ahead market, nor has it been increasing as steadily). Hence the emphasis remains upon the more representative day ahead prices. The operations requirements are that all power generators and retailers must submit a supply and demand plan to the SO by noon on the previous day ("next day plan"), and if necessary, provide an updated "current-day plan" by one hour before the delivery, the "gate closure" deadline.

TABLE I THE DAY AHEAD AND INTRADAY MARKETS IN JEPX

	Day ahead market (spot market)	Intraday market
Commodity	48 products dividing a day into 30-minute delivery periods	
Deadline for trading	10am in the day before the delivery date	One hour before delivery time
Trading method	Blind single price auction	Continuous session
	The price is determined by the intersection of all the cumulated buy/sell curves	Match buy and sell orders
Opening	Apr. 2005	Apr. 2016 (no intraday market before then)

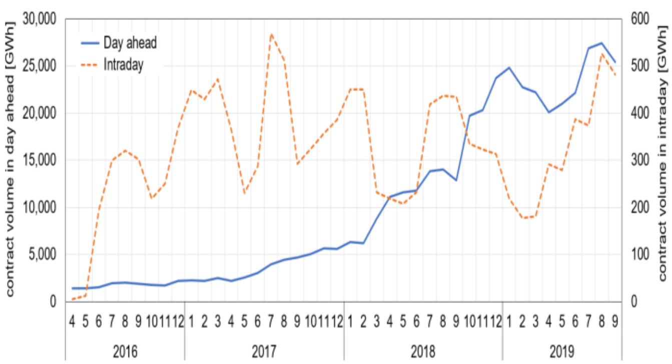

Fig. 1. Contract volume of the day ahead and the intraday market

The imbalance volumes caused by deviating in real-time from the submitted plans will be adjusted by the SOs (there are nine system control areas) and settled by the imbalance prices published ex post. As indicated, the imbalance pricing mechanism has been much deliberated and revised. Table II summarizes the main phases. At first, when the system was established in 2000 (the start of liberalization), it was a mechanism that imposed a relatively large penalty on each company when their imbalance volume exceeded 3% of their demand. However, from 2016, with the full liberalization of retail, a single imbalance price formula was set regardless of the imbalance position of individual businesses, out of consideration to reduce the burden on new market entrants. Thus, by linking the single price to the entire net system imbalance, participants had an incentive to improve the expected supply and demand balance of the entire system by going out of balance in the opposite direction. At the same time, by publishing imbalance prices with a substantial delay, the regulator had an additional (counteracting in a sense) policy intention to reduce predictability and limit this opportunistic imbalance strategy. When this system was introduced, the imbalance price formula was linked to the day-ahead price as in many countries (see Appendix A), based on its high transparency and the absence of balancing market [19][i]. Meanwhile, the imbalance volumes increased with the expansion of renewable resources [5].

Thereafter, from 2019, the dual price penalties were introduced to incentivize individual operators to maintain their supply and demand balance and not take these speculative or gaming positions against the direction of the market (virtual trading had not been (and even now) permitted and the new penalty incentive had the intention to strengthen such regulating intent [5]). Subsequently, a further review of the imbalance system has sought to "promote system users to appropriately forecast imbalance prices ... to ensure the supply-demand balance of the entire system" by "publishing information on imbalance status and prices in a timely manner" [6]. The interaction of market information and predictability is clearly crucial to any benefits that might accrue to imbalance optimization and further motivates our investigations.

The imbalance settlement price since 2019 has been determined by the following equation (note that the single price determined during 2016-19 excluded the k, l penalty terms):

$$RT_t = \alpha_t DI_t + \beta + \begin{cases} k \ (if\ player's\ position\ is\ deficit) \\ -l \ (if\ player's\ position\ is\ surplus\ ) \end{cases},\ k\ and\ l > 0. \qquad (1)$$

where RT_t is an imbalance price (note that it is not called real-time price in Japan, but is denoted RT_t to avoid confusion with the imbalance volume). DI_t is the weighted average price by transaction-amounts of the day-ahead market price DA_t and the intraday price ID_t (note that since the day-ahead market transaction volume predominates, DI_t is close to DA_t). The parameter α_t is a variable factor depending on the supply demand balance of the entire system, β is a constant term for area-specific correction, and k, l are penalty values depending on the player's imbalance position and location. Table III summarizes each imbalance price related parameter in (1).

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TABLE II EVOLUTION OF THE JAPANESE IMBALANCE SYSTEM

Year	Episode	Imbalance system	Price	Policy intent
2000-	Start of market liberalization	When shortage (surplus) exceeds 3% of demand, compensated at a high price (withdrawn for free).	Dual price	Secured incentives for grid users to comply with their plans.
2016-	Full liberalization for retailers	A single market price calculated by the adjustment factor α based on the system imbalance (note that the area correction term β was also added).	Single price	Considering the burden for new entrants, only the incentive of "macro adjustment" for the entire system was granted.
2019-	Review of imbalance system (Current system)	The fixed penalty term of k, l (as determined by the imbalance position of individual company) was added to the existing formula.	Dual price	Triggered by imbalance expansion due to renewable energy, the incentive of "micro adjustment," which urges appropriate planning (forecast), was added.

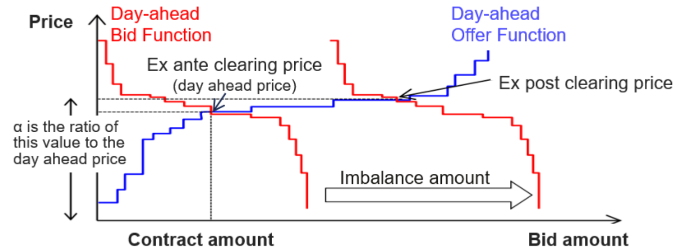
Source: METI [5][20] *Note: $\alpha, \beta, k,$ and l i are detailed below.*

TABLE III IMBALANCE PRICE RELATED PARAMETER OVERVIEW

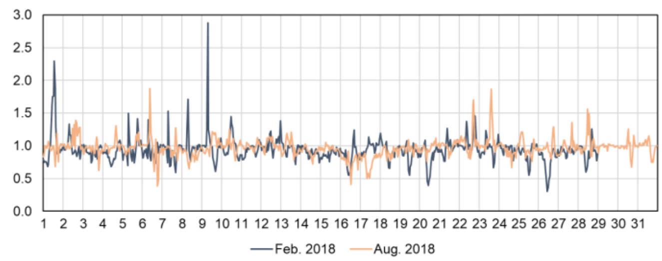
			α	β	k	l
Meaning			entire system balance	area-specific correction	deficit imbalance penalty	surplus imbalance penalty
Granularity (Update frequency)			half hourly	monthly	constant (unscheduled)	
Publication time			post	pre	pre	
Example value	Applicable period		-	Apr. 2019	from Apr.2019	
	Area	Hokkaido	variable value common to each area	2.28	2.98	1.49
		Tohoku		1.07	0.59	0.20
		Tokyo		1.10	0.64	0.17
		Chubu		-0.95	0.27	0.68
		Hokuriku		-0.95	0.28	0.68
		Kansai		-0.95	0.28	0.69
		Chugoku		-0.95	0.28	0.68
		Shikoku		-0.95	0.28	0.68
		Kyushu		-1.09	0.43	0.83

Data source: METI (https://www.meti.go.jp/)

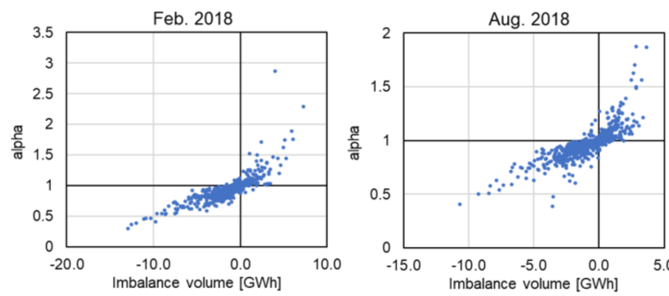
The parameter α denotes the ratio of what the market clearing price would have been, ex post, if the realized volumes had been traded in the day-ahead market to the actual, ex ante, day-ahead price [5]. Fig. 2 illustrates this for the situation where the retailers create a system imbalance by consuming more in real time than they contracted day-ahead. If that consumption had been bid day ahead, the bid function would have been moved to the right. If the generators stayed balanced, their offer function can be applied to the shifted bid function to create the clearing price, ex post, that would have been obtained had the retailers bid their actual consumption day ahead. Evidently the generators could also be out of balance and their offer function would similarly be shifted. However, as the bid and offer functions are not disclosed to market participants, and the imbalance volumes (and α) become available only 5 days later, this structural model is not useful in real-time. The historical time series of α is shown in Fig. 3. It is evident that α is distributed randomly around 1. In addition, as shown in Fig. 3, α has a downwardly convex relationship with the imbalance volume as a whole.



Source: [5]
Fig. 2. Calculation method of α



Data source: *JEPX (http://www.jepx.org/).*
Fig. 3. Time series of α Feb. 2018 and Aug. 2018)



Data source: *JEPX (http://www.jepx.org/).*
Fig. 4. Scatter plots of α vs imbalance amount (Feb. 2018 and Aug. 2018)

Next, the fixed penalty terms k, l are calculated from the costs of the supply and demand adjustments performed by the SO. They produce incentives by area [21]. They are generally set between 0.2 and 0.8, as shown in Table III, except for the higher values in Hokkaido (because the area's demand and supply are small). Note that because β is a value that compensates for the difference between each area price and the system price, its demand-weighted average is 0. These k, l, β parameters reflect a market design in which each area is subject to SO balancing [20]. However, in this study, we focus on the national average system price. The reason is that the intraday price is published only as an average price for all areas, and we want to consider arbitrage behavior at the national level through JEPX. Note that in Japan, there are no nodal prices, and the fixed wheeling rates that electricity utilities pay to the SO by each area are incurred separately. These depend on the actual grid usage fees (regardless of market procurement or imbalance settlement) and do not affect traders' decision-making.

Fig. 5 summarizes the information disclosure timing of parameters, prices, imbalance volumes, etc. However, based on the fact that the Japanese regulator introduced in 2019 a system plan (based on the EU transparency regulations) that would require SOs to publish the measured demand, renewable energy output, imbalance volume, and imbalance settlement price within 30-min at the latest [6], we model variations with

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potentially more timely and transparent market information.

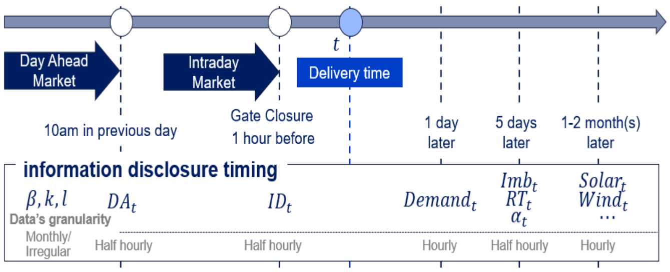

Fig. 5. Timeline of market transaction and information disclosure

IV. Optimal Positions

Whilst dual price systems with penalties are generally designed to discourage participants' virtual trading on the imbalance prices, it is possible that some profitable arbitrage incentives remain. Therefore, the research question in this paper is to understand more fully the effects of the remaining incentives for arbitrage profits and system imbalance changes due to variations in the penalties and to various information disclosure time lags (i.e., market transparency). After specifying a forecasting model for imbalance volumes and prices, we formulate a virtual trader's optimal strategy by assuming different scenarios for penalties and market transparency.

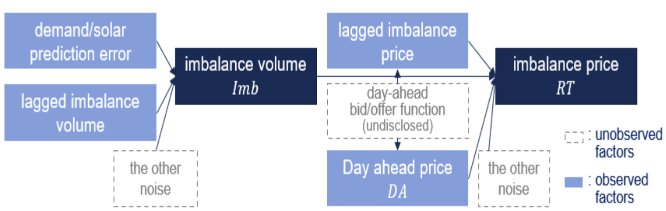

Fig. 6. Influencing factors of imbalance volume and imbalance price

For Japan, as with most markets, we need to treat the imbalance settlement price RT as a stochastic variable in addition to the imbalance volume Imb. Our structural scheme for predicting these is shown in Fig. 6. For the prediction of probability distributions for both RT and Imb, we use quantile regression (QR; e.g., [22]), based on factors indicated in Fig. 6. Note that by using these density forecast models instead of point estimation such as by OLS, it is possible to deal with the problems such as heteroscedasticity of RT with respect to each explanatory variable (need for density forecast of RT) and possible bias caused by non-linearity (downward-convex) of RT with respect to Imb (need for density forecast of Imb). In other words, the optimal position requires an expectation based upon the RT probability distribution. To justify using QR for forecast densities, we separately estimate an alternative standard approach using GARCH (G) and confirmed that the pinball loss (PL) score showed the superiority of density fitting of QR compared to G (see Appendix B)[ii].

A. Quantile prediction models: Imbalance volumes and prices

Recent research on imbalance forecasting demonstrated that higher wind and solar forecast errors increase the absolute values of imbalance volumes [23][iii]. In this study, considering that the amount of wind power generation in Japan is extremely small compared to solar power (the ratio of installed capacity of wind to that of solar power was 7.6% at the end of 2018), we only focus on solar power generation. Additionally, we incorporate the demand-prediction error. Because the forecasts of solar power and demand are not publicly available, we construct submodels that mimic how market participants might make those predictions for their own purposes. Although market participants may have their own solar and demand portfolio, since the imbalance volume that determines the imbalance price is the aggregate from all over Japan, even for each largest utility in each of the nine areas, the information value of their own portfolio prediction errors will not be substantial.

Using QR, Equation (2) below is estimated separately for discrete quantiles indexed through q. Note importantly that the model is estimated separately by hour $h \in \{0,1,\dots,23\}$, and information time lag $r \in \{1,2,\dots,8\}$ in a daily rolling manner; therefore, 192 models are estimated for each day, but hour h is omitted in the formula for simplicity of notation. Note also that hourly price (imbalance) data are obtained by averaging (summing) half hourly data to reduce dimensionality:

$$\widehat{Imb}_{t,q} = \gamma_{1,r,q} Imb_{t-r} + \gamma_{2,r,q} Imb_{t-24} + \gamma_{3,r,q} \Delta Solar_{t-r} + \gamma_{4,r,q} \Delta Demand_{t-r} + \gamma_{5,r,q}. \quad (2)$$

where $\widehat{Imb}_{t,q}$ is the forecast value of system imbalance (positive if shortage) at time $t \in \{1,2,\dots,T\}$. Imb_{t-r} (Imb_{t-24}) is the observed system imbalance volume with a time lag of r (24), $\Delta Solar_{t-r}$ and $\Delta Demand_{t-r}$ are solar and demand forecast errors calculated as the latest values measured at $t-r$ minus the day-ahead forecast, and γ is the coefficient estimated by the QR.

Next, we similarly construct quantile forecast models for the imbalance price as well. Considering that the JEPX price is always positive (there is a constraint that the prices in JEPX are 0.01 [JPY/kWh] or more) and imbalance volumes can be either plus or minus, we apply a logarithmic transformation to price data, and the following quantile forecast function can be obtained[iv]:

$$Q_p\big(ln(RT_t)\big) = \theta_{1,p} ln(DA_t) + \theta_{2,p} ln(RT_{t-r}) + \theta_{3,p} Imb_t. \quad (3)$$

where $Q_p(\cdot)$ means the forecast value on quantile p, RT_t (DA_t) is the imbalance (day-ahead) price at time t, RT_{t-r} is the latest observed imbalance price at $t-r$, and θ are coefficients estimated by the QR. Note that a constant term was not retained because small t-values were estimated. Furthermore, RT_t refers to $\alpha_t DI_t$ in (1), and the penalty term will be considered when solving the optimization below. Here, from the QR's property of "invariance to monotone transformations" (e.g., [22]), (3) is converted to the following formula:

$$\widehat{RT}_{t,p} \coloneqq Q_p(RT_t) = {DA_t}^{\theta_{1,p}} \times {RT_{t-r}}^{\theta_{2,p}} \times exp\big(\theta_{3,p} Imb_t\big). \quad (4)$$

where $\widehat{RT}_{t,p}$ is the forecasted imbalance price at time t on quantile p. In (4) because the explanatory variable Imb_t is unobserved at the forecasting time, by using (2) as input to (4), the following two-dimensional quantile forecast function can be constructed:

$$\widehat{RT_{t,p,q}} = DA_t^{\theta_{1,p}} \times RT_{t-r}^{\theta_{2,p}} \times exp\left(\theta_{3,p}\widehat{Imb_{t,q}}\right) \quad (5)$$

where $\widehat{RT_{t,p,q}}$ is the forecasted imbalance price at time t on quantile p on (4) and on quantile q of forecasted imbalance volume on (2).

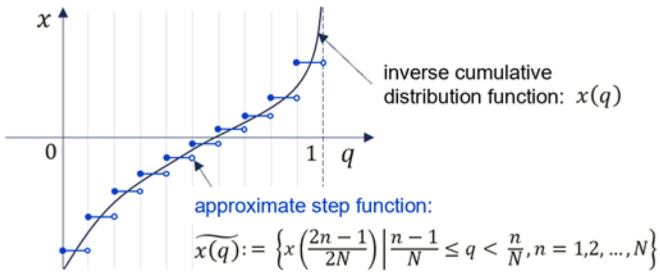

Fig. 7. The discrete approximation (univariate case)

To help with intuition, Fig. 7 shows a univariate cumulative distribution function $x(q)$ overlaid with a discrete step function $\widetilde{x(q)}$ with the N intervals of similar length as an approximate function. Here, the expected value of the stochastic variable X with the inverse cumulative distribution function $x(q)$, which is a smooth monotonic function on the domain $q \in [0, 1]$, can be approximated using the step function $\widetilde{x(q)}$ as follows:

$$E[X] = \int_0^1 x(q)dq \approx \frac{1}{N}\sum_{n=1}^{N} \widetilde{x(q)} \text{ s.t.}$$
$$\widetilde{x(q)} := \left\{x\left(\frac{2n-1}{2N}\right) \middle| \frac{n-1}{N} \le q < \frac{n}{N},\ n = 1,2,\dots,N\right\}. \quad (6)$$

where the approximation in (6) converges with N (N=10 was found to be adequate).

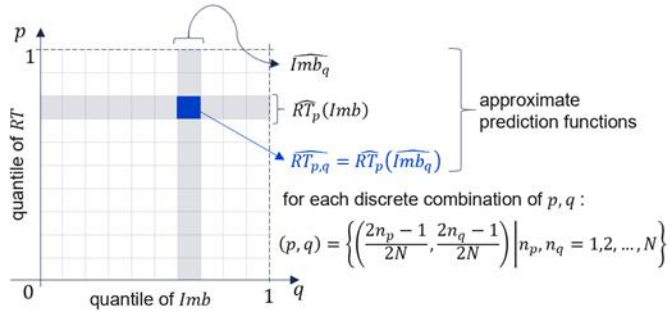

Fig. 8. Discrete approximation for the bivariate quantile forecast functions

Fig. 8 shows the concept of the discrete approximation extended to two dimensions and applied to the bivariate stochastic variables specified here (note that t is omitted for simplicity of notation). Here, we consider approximating the predicted value in each mesh with the value at the midpoint of the mesh when both quantiles of the imbalance volume and price are divided into N equal parts. Then, the prediction formula at the midpoint is obtained as $\widehat{RT_{p,q}} = \widehat{RT_p}\left(\widehat{Imb_q}\right)$ (i.e.,(5)) that simultaneously holds $\widehat{Imb_q}$ and $\widehat{RT_p}(Imb)$ (i.e., (2) and (4)), and the expected value of the imbalance price is obtained as the average value on each mesh as in (7).

$$\widehat{RT_t} = \frac{1}{N^2}\sum_{p,q} \widehat{RT_{t,p,q}}. \quad (7)$$

B. Optimization of Participant Imbalance Positions

We formulate an optimization problem whereby a participant makes an arbitrage transaction using the intraday market to speculate on the imbalance settlement price [v]. Evidently, this price varies depending on the imbalance volume; thus, if a participant has a deliberate long/short position (i.e., buy/sell electricity) in the intraday market, it contributes to a surplus/shortage of system imbalance. That is, if a participant has a long position x_t, since the imbalance volume in (5) becomes smaller by x_t the quantile forecast function is given as follows[vi]:

$$\widehat{RT_{t,p,q}}\left(\widehat{Imb_{t,q}}, x_t\right) = DA_t^{\theta_{1,p}} \times RT_{t-r}^{\theta_{2,p}} \times exp\left[\theta_{3,p}\left(\widehat{Imb_{t,q}} - x_t\right)\right]. \quad (8)$$

Conversely, for short positions. Here, the participant's profit is formulated as a product of the imbalance price spread with the intraday price ID_t and the transaction volume as follows:

$$\pi_{t,p,q}(x_t) = \left\{\widehat{RT_{t,p,q}}\left(\widehat{Imb_{t,q}}, x_t\right) - ID_t\right\} \times x_t. \quad (9)$$

Thus, the nonlinear optimization formulation is given by the following equation for the virtual trader (who may be a retailer or generator) to obtain the optimal position x_t that maximizes the expected profit of (9):

$$\max_{x_t} \left(\frac{1}{N^2}\sum_{p,q} \pi_{t,p,q}(x_t)\right) \quad \text{s.t.} \quad x_t \in [x_{min}, x_{max}]. \quad (10)$$

Note that in (10), decision variable x_t is defined as a "continuous variable" that can be taken within a certain range.

Next, the objective variable (i.e., expected profit $\widehat{\pi_t}$) of (10) is transformed as follows:

$$\widehat{\pi_t} = \frac{1}{N^2}\sum_{p,q} \pi_{t,p,q}(x_t)$$
$$= \frac{1}{N^2}\sum_{p,q} \left\{A_{t,p,q}x_t \times exp\left(-B_p x_t\right)\right\} - C_t x_t$$
$$\text{s.t.} \begin{cases} A_{t,p,q} := DA_t^{\theta_{1,p}} \times RT_{t-r}^{\theta_{2,p}} \times exp\left(\theta_{3,p}\widehat{Imb_{t,q}}\right) \\ B_p \quad := \theta_{3,p} \\ C_t \quad := ID_t \end{cases} \quad (11)$$

To understand the general structure of the problem, consider the following simplified equation with a profit variable $\tilde{\pi}(x)$, which does not consider the distinction of quantiles p and q:

$$\tilde{\pi}(x) := Rev(x) - Cost(x) = Ax \times exp(-Bx) - Cx.$$
$$\text{s.t.} \begin{cases} Rev(x) := Ax \times exp(-Bx) \\ Cost(x) := Cx \end{cases} \quad (12)$$

where $Rev(x)$ corresponds to imbalance settlement income (expenditure if negative), and $Cost(x)$ corresponds to the procurement cost at intraday (sales gain if negative). In relation to (11), A denotes a prediction of imbalance price indifferent to position x, B denotes the sensitivity of the imbalance settlement price to the volume, and C is the intraday price. Thus, $A, B,$ and C should all be positive values.

The functions $Rev(x)$ and $Cost(x)$ are plotted in Fig. 9,

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which displays the relationship between the optimal position x^* and the optimal profit $\tilde{\pi}^*(x^*)$. Similarly, the relationship between the first derivative of both functions and the optimal values (x^* and $\tilde{\pi}^*(x^*)$) is shown in Fig. 10, depending on the relationship between A and C. Note the optimal profit, calculated as $\tilde{\pi}^*(x^*) = \tilde{\pi}(0) + \int_0^{x^*} \frac{\partial \tilde{\pi}}{\partial x} dx = \int_0^{x^*} \left(\frac{\partial Rev(x)}{\partial x} - \frac{\partial cost(x)}{\partial x} \right) dx$, corresponds to the shaded area. In addition, the optimal decision $x^*$is found as the solution of the equation $\frac{\partial \pi}{\partial x} = \frac{\partial Rev(x)}{\partial x} - \frac{\partial cost(x)}{\partial x} = 0$ (i.e., the intersection of the two functions in Fig. 10), and there is always one solution within the range $(-\infty,\ 1/B)$. The relationship between the optimal decision and profit when $A, B,$ and C vary is shown in Table IV (the change in B in the case of $A = C$ is omitted because it always satisfies $x^* = 0$, and $\tilde{\pi}^*(x^*) = 0$ regardless of B).

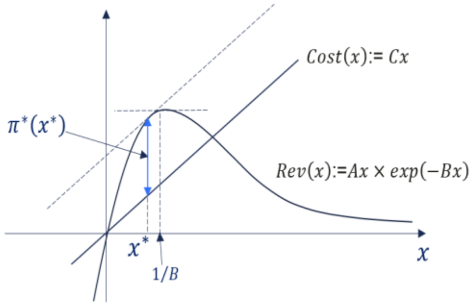

Fig. 9. Relationship between revenue/loss functions and optimal profit/decision

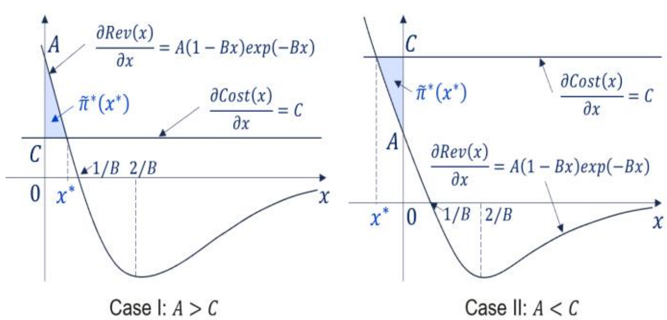

Fig. 10. Relationship between first derivatives of revenue/loss functions and optimal profit/decision

TABLE IV SENSITIVITY OF OPTIMAL DECISION AND PROFIT

	Depending on A					Depending on B						Depending on C				
						Case I: $A>C$			Case II: $C>A$							
	0	⋯	C	⋯	$+\infty$	0	⋯	$+\infty$	0	⋯	$+\infty$	0	⋯	A	⋯	$+\infty$
x^*	$-\infty$	↗	0	↗	$1/B$	$+\infty$	↘	0	$-\infty$	↗	0	$1/B$	↘	0	↘	$-\infty$
$\tilde{\pi}^*(x^*)$	$+\infty$	↘	0	↗	$+\infty$	$+\infty$	↘	0	$+\infty$	↘	0	A/eB	↘	0	↗	$+\infty$

Consider the following:

a. A (the forecast value of RT at x_t equal to zero). The optimal intraday long position x^* increases along with A, and the optimal profit $\tilde{\pi}^*(x^*)$ increases as A deviates from C. In other words, the higher/cheaper the forecasted imbalance settlement price is in comparison to the intraday price, the more the optimal long/short positions in the intraday market can be increased (the profit can be increased accordingly).

b. B (sensitivity of RT to Imb). If B is low, the impact on the imbalance settlement price given by player's transaction is small; therefore, the optimal absolute intraday position (long if $A > C$, and short otherwise) increases, which generates the profit. Conversely, if B is very high, the optimal intraday position and profit approach 0. This is because the chance of arbitrage is decreased because of the effect that the own transaction reduces the profit margin.

c. C (intraday price ID). If C is a certain price (i.e., imbalance settlement price forecast A), the optimal long position x^* and profit $\tilde{\pi}^*(x^*)$ equal 0 because there is no chance of arbitrage. The optimal long position x^* monotonously decreases in response to C, which means that the higher intraday price urges more short positions. Optimal profit $\pi^*(x^*)$ increases as C deviates from A because the profit margin increases with the price difference. (It should be noted here that if C is different from A, even if the system imbalance is zero, the trader will take a non-zero optimal arbitrage position. In other words, there is an incentive to promote "system-destabilizing arbitrage" as described later.)

Next, we consider an optimization problem (10) where multiple-quantile values are considered. Even in this case, the shape of the functions and the sensitivity analysis of the optimal position and profit are the same as the single-quantile case described above, as shown in Appendix C. Therefore, this problem corresponds to finding x_t when the first-order derivative of (11) (defined here as $f(x_t)$) is equal to 0; that is, to solve the following equation:

$$f(x_t) \coloneqq \frac{\partial \widehat{\pi_t}}{\partial x_t} = \frac{1}{N^2} \sum_{p,q} A_{t,p,q} \left(1 - B_p x_t\right) exp\left(-B_p x_t\right) - C_t = 0. \tag{13}$$

Since there is no closed form solution, we solve numerically using Newtonian approximation (see Algorithm 1). Note that $i_{max} = 5$ is used in the empirical analysis because sufficient convergence was confirmed[vii]. Furthermore, when the penalty term is considered, the algorithm should be as in Algorithm 2. For each long/short position scenario, we define the penalized marginal profit function $f_p(x_t)$ in advance by subtracting/adding the position-dependent penalty l/k from/to the marginal profit function $f(x_t)$ defined in (13). Then, for each scenario, the converged optimal solution should be judged to be consistent with the original condition of long/short scenario (i.e., the iterative solution $x_{t,i_{max}+1}$ of the long position scenario with the penalized marginal profit obtained by subtracting l from the original profit, must be positive from the assumption of long position, and vice versa). If that condition matches, the solution is valid; however, if none of the cases match, there is no optimal arbitrage transaction; that is, the optimal transaction volume is 0[viii].

Algorithm 1 Solution for the optimal decision using Newtonian approximation

Initialize: $x_{t,0} = 0$

for $i = 0 \colon i_{max}$ **do**

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$$x_{t,i+1} = x_{t,i} - f(x_{t,i})/f'(x_{t,i}) = x_{t,i} - \frac{\frac{1}{N^2}\sum_{p,q} A_{t,p,q}(1-B_p x_t)exp(-B_p x_t) - C_t}{\frac{1}{N^2}\sum_{p,q} -A_{t,p,q}B_p(2-B_p x_{t,i})exp(-B_p x_{t,i})}$$

end for
return $x_{t,i}$

Algorithm 2 Solution for the optimal decision using Newtonian approximation (considering penalty)

Initialize: $x_{t,0} = 0$
for $position \in \{"long,""short"\}$ **do**
  switch $(position)$
    case "$long$": define penalized marginal profit function $f_p(x_t) \coloneqq f(x_t) - l$
    case "$short$": define penalized marginal profit function $f_p(x_t) \coloneqq f(x_t) + k$
    for $i = 0{:}\, i_{max}$ **do**
      $x_{t,i+1} = x_{t,i} - f_p(x_{t,i})/f_p{}'(x_{t,i})$
    end for
    case "$long$": **if** $x_{t,i_{max}+1} > 0$ **return** $x_{t,i_{max}+1}$
    case "$short$": **if** $x_{t,i_{max}+1} < 0$ **return** $x_{t,i_{max}+1}$
  end switch
end for
return 0

V. BACKTESTING

In this section, actual JEPX data are used to backtest this trading strategy, namely,

- System imbalance volume Imb [MWh] from the sum of all nine areas in JAPAN[ix]
- Day-ahead price DA, Intraday price ID, Imbalance settlement price RT [JPY/kWh] from JEPX system price[x]
- Solar power forecast error $Solar$ (MWh) from a submodel using next-day weather forecasts.
- Demand forecast error $Demand$ [MWh] also from a submodel using next-day weather forecasts.

The solar and demand forecasting submodels are not described in detail here because of space limitations. They comprise state-of–the-art nonlinear, multiregional predictive models aligned to what market participants might develop for the same purpose. The training period is between December 2017 and November 2018, and for the backtest simulations, the out-of-sample data are from January 2018 to December 2018 (obtained from the forecast model estimated on the latest 30-day sample in a daily rolling manner). Note that the upper and lower limits of the player's positions were calibrated to the 90-percentiles of actual imbalance volumes in 2018. Regarding imbalance penalty scenarios, assuming that there is the same penalty for shortage and surplus, we set the following six scenarios: $k, l = \{0, 0.25, 0.5, 1.0, 2.0, 4.0\}$ [JPY/kWh]. Because the national averages (area demand weighted averages) of k and l are 0.53 and 0.51, respectively, $k, l = 0.5$ corresponding to the current system.

To predict imbalance settlement prices, we concluded that the following linear regression model is adequate (in the testing period of 2018, the R-squared was at least 0.997 for all 8,760 models for all days and hours):

$$ln(RT_t) = \vartheta_1 ln(DA_t) + \vartheta_2 ln(RT_{t-1}) + \vartheta_3 Imb_t + \epsilon_t. \quad (14)$$

Therefore, given a forecast of Imb_t, we have a function for profit based upon a trader's optimal decision x^*:

$$\pi_t(x^*) = \{\widehat{RT}_t(x^*) - ID_t\} \times x_t{}^* \text{ s.t.}$$
$$\widehat{RT}_t(x^*) = DA_t{}^{\vartheta_{1,p}} \times RT_{t-r}{}^{\vartheta_{2,p}} \times exp[\vartheta_{3,p}(Imb_{t,q} - x_t{}^*)]. \quad (15)$$

We use lagged values as predictors of Imb, with eight different time lag scenarios (1-8 hours delay). In addition, a perfect-foresight (PF) case is calculated for benchmark comparison.

The results are surprising for a dual imbalance settlement price mechanism. Fig. 11 shows the absolute value and standard deviation of the system imbalance volume when the trader is acting optimally. As the figure shows, the transparency with disclosure lag of 2 hours or less (in many markets, it is just several minutes) reduces imbalances, by allowing traders to conduct transactions in the direction of eliminating imbalances based on relatively accurate forecasts for real-time imbalance status. The penalty variations affect the relative results slightly, but the main observation is that, if information were disclosed within one hour, moderate penalties contribute to reduce the absolute value of the imbalance volume and its standard deviation; and in the case $k, l = 0.5$ (this is almost the same as the penalty strength of the current system), both the calculated values could be the smallest of all penalty scenarios, and the former could be reduced by 11.7%, and the latter could be reduced by 8.8%. Such a reduction in the system imbalance volume leads to a reduction of the idling and operating costs of power for supply and demand adjustments, and thus increases the social welfare[xi].

Fig. 12 shows that the trader's profit is substantial at all lags and under all penalties. Evidently, profit decreases with time lag and imbalance penalty. Interestingly, however, a small penalty (e.g., k and l range from 0.25 to 0.5) has little effect on the trader's profit, although it reduces imbalance volume, as demonstrated above. This indicates that a small penalty could restrain players from arbitrage transactions with small margins. As a result, it could reduce system imbalance volumes without reducing trader's profit[xii]. Note that, in the case of $k, l = 0.5$ with an hour lag, the absolute value of the arbitrage volume was 5,752 [GWh] and the profit was 14,304 [million JPY], so the average profit for the arbitrage volume was calculated to be 2.5 [JPY/kWh]. Considering that the absolute value of the actual spread of ID and RT was 1.5 [JPY/kWh] on average (with a standard deviation of 4.5 [JPY/kWh]), it indicates that arbitrage is conducted when the spread is relatively large. Furthermore, another interesting observation is that in the case of long information lag, the smaller penalties the more profits can be made but such transactions do not contribute to reducing system imbalances. To put it differently, it is suggested that the penalty has a function of suppressing undesired arbitrage in the direction of increasing system imbalance.

Note that we have referred to these actions as if they were from a single trader since our focus is upon the extent to which the market arrangements provide an incentive to act with optimal trades. We have not sought to address the issue of how multiple traders may interact amongst themselves. It is

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sufficient for our purposes to look at the market arrangements from the initial perspectives of whether and how they create profitable incentives. We discuss this further in the conclusions.

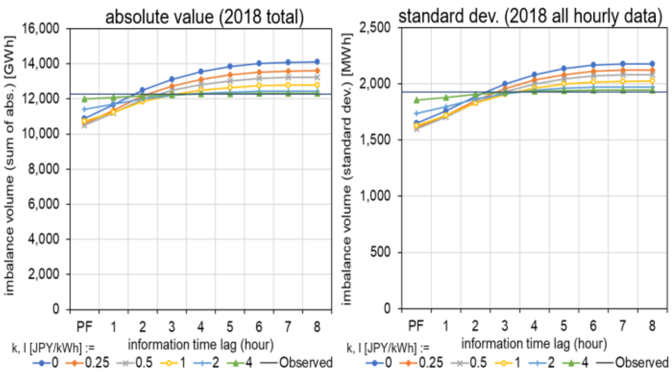

Fig. 11. Simulated imbalance volume changing with time lag and penalty

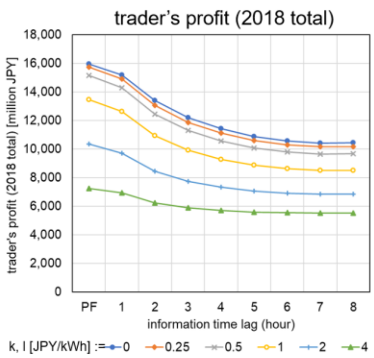

Fig. 12. Arbitrage profit changing with time lag and penalty (2018 total)

To confirm the impact of the penalty on the arbitrage volume with hour granularity data, Fig. 13 shows the simulated results for some weekdays in mid-February. The graph on the left shows the case without penalty, while the graph on the right shows the case where penalty is $k, l = 0.5$. In both cases, the information time lag was set as one hour. Overall, the trader's optimal decision when system imbalance is positive (i.e., shortage) tends to be positive (i.e., long position), and vice versa. On the other hand, there are some periods during which the directions are reversed, or excessive arbitrage is performed (e.g., the part with the arrow on the left graph) due to extremes in the intraday price (This is the "system-destabilizing arbitrage" referred to in the explanation part of Table IV[xiii]). Looking at the right graph with a penalty of 0.5, it is evident that such a reverse or excessive arbitrage transaction is slightly mitigated.

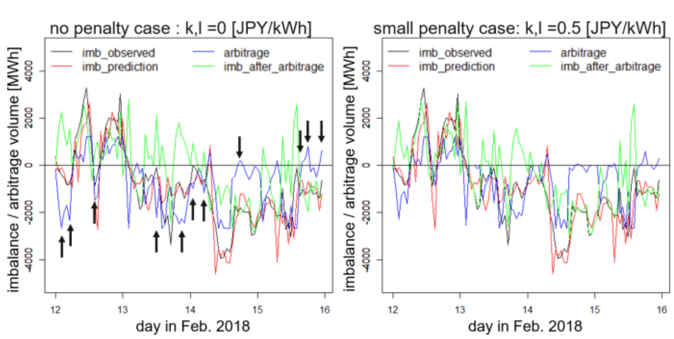

Fig. 13. Simulated imbalance volumes (mid Feb. 2018)

As demonstrated above, there is a strategic arbitrage position even under the dual price system, and depending on the strength of the penalties, the dual price has the potential to make the "system imbalance reduction effect" inherent in the single price even more efficient.

VI. Conclusions

This paper has provided both policy and methodological contributions to understanding potential behavior in imbalance settlement arrangements operating with dual pricing. Most surprisingly, despite the intent of dual pricing to limit the incentive for a participant to take arbitrage positions against the direction of imbalance, we find that it may still be profitable and would benefit the system in terms of average imbalance volumes if market transparency is increased. The case study is particularly relevant, as Japan has repeatedly reviewed its imbalance settlement design policies.

Thus, we observed that:

- Permitting a participant in Japan to optimize imbalance arbitrage may contribute to both its profits and system stabilization, as long as the imbalance-related information disclosure time-lag is sufficiently short.
- The imbalance penalty scheme introduced in 2019 in Japan actually enhances the effects of reducing system imbalance volume by means of virtual trading[xiv], although it was not intended to do so.

Importantly, regarding the previous empirical results for the single price markets [4][14] that the allowance of deliberate imbalance contributes to reduce system imbalances, our analysis revealed that the same result can hold even for the case of penalized settlement prices, and this provides meaningful implications for many markets with dual prices. Furthermore, we show that this beneficial effect of arbitrage may exist with dual imbalance prices based upon the day-ahead market prices. If we take the plausible view that the day-ahead reference prices are less efficient than real-time, or close to real-time traded prices, we can say that to some extent the arbitrage has mitigated this inefficiency. With a more efficient intraday market trading close to real-time, we would expect less arbitrage, and therefore a smaller effect. Nevertheless, for various reasons, many markets persist with the link to the day-ahead pricing. In these cases, the mitigation effect described here is a valuable second-best solution to efficient imbalance settlement pricing.

In terms of methodology, the formulation has potential applicability, and the optimization model is novel. In particular, our contributions:

- Analyzed a market mechanism where there is no deterministic relation of imbalance settlement price to the imbalance volume, as in most markets.
- Developed a two-step quantile regression method to incorporate correlated asymmetric probability distributions.
- Refined the optimization formulation, from that in [4], to allow the player's decision (trading position) to be treated as a continuous variable instead of an arbitrary discretization.

Finally, we should re-emphasize the perspective of incentives in this study which, like most analyses of arbitrage in various markets, looks at the opportunity in the market design for a participant to act in a self-beneficial way. It is a singular analysis of whether there is an incentive for a participant to profit. This is valuable for market design considerations of incentive compatibility. If the market is designed to discourage certain actions, the appropriate approach is to check whether a participant is indeed discouraged. How the market will evolve as multiple agents respond to such incentives is a different theoretical question that requires a dynamic gaming model leading potentially to collusive or non-collusive equilibria depending upon various behavioral assumptions. Yet our perspective of looking at a single participant and his optimal strategies under a given settlement mechanism is informative for the analysis of incentive-compatible market designs. Further analyses are still required to systematically investigate the benefits and risks of different design options regarding system-stabilizing and system-destabilizing incentives and their overall impact on system costs.

Appendix

A. Imbalance settlement designs for each country

Table V shows that even within Europe, where there has been a move towards a single energy market through harmonization, each country has so far evolved with different imbalance settlement designs. In addition to the single price vs dual price, they also vary with regard to using the day-ahead wholesale price in the imbalance settlement price formula. Notably, whilst countries with highly liquid markets (such as Germany, Britain and France) tend to adopt the real-time balancing energy costs, most countries use the day-ahead wholesale prices. Besides, Britain is planning to introduce reserve products that will be traded on day-ahead and may incorporate their prices, if activated, partially into the imbalance settlement price[xv].

Regarding the issue of which price source to use, there are various debates in the context of arbitrage incentives as with single price vs dual price. For example, there is a discussion of the distorted incentives in the Swiss market in which the imbalance settlement price was based on the day-ahead price multiplied by some coefficients [24]. That study concluded that "the pricing mechanism for imbalances should reflect real market prices or, if not, its prices must not be predictable," but the problem does not seem to be that simple. A typical example regarding this debate would be the German market, which adopts the price of balancing energy for the imbalance settlement. For instance, the study [11] reveals that the prices for balancing energy that are disconnected from spot prices can provide arbitrage opportunities and potentially lead to increased imbalances of the electricity system. In response to these problems, the German market has adopted an imbalance settlement design that sets a cap/floor based on the intraday price for the imbalance price (i.e., it ensures that the imbalance settlement price is always higher/lower than the intraday price for short/long systems). However, although it has been demonstrated that such measure can suppress harmful arbitrage to some extent, such transactions cannot be eliminated as long as the arbitrage is based on uncertain price forecasts [14]. Furthermore, even in Germany, the analysis by Eicke et al. [25] indicates that strategic arbitrage positions continue to be taken and that they are generally beneficial to the system with few adverse effects. Evidently, setting cap/floor by intraday price may be not appropriate when the trading volume of the intraday market is extremely small, as in Japan. The imbalance settlement design is clearly an issue that should be carefully considered according to the market idiosyncrasy of each country.

TABLE V IMBALANCE SETTLEMENT DESIGNS IN EUROPE

Single/Dual	Whether day-ahead prices are used[*1]	
	Yes	No
Single price	Austria, {Italy}, [Nordic markets[*2]]	Belgium, Britain, Germany, Netherlands[*3]
Dual price	Baltics (Estonia, Latvia, Lithuania), Hungary, Italy, Nordic markets, Spain, Switzerland, {Britain}, {France}	France

Note:
[*1] "Yes" include some cases where only the spot price is used (e.g., Baltics), the balancing energy cost is combined, etc. All "No" are based mainly on the balancing energy cost. Also, {} indicates the country applying it in the past, and [] does the country planning for future shift.
[*2] Nordic markets (Denmark, Finland, Norway, and Sweden) are planning to shift to single price from November 2021, but to continue using the day-ahead price as a temporary measure until May 22, 2023 [26]
[*3] In the Netherlands, dual prices may be applied depending on the "activation situations of Frequency Restoration Reserve" [27]

Source: Austria [4], Belgium [28], France [29], Germany [14], Italy [30], Netherlands [27], Nordic markets [26][31], Spain [32], other countries [33]

B. Goodness of density fitting – Pinball loss score

This study used QR for the density forecast of both stochastic variables Imb and RT. To verify its validity, an alternative model G is constructed separately, and the goodness of density fitting is compared by using the Pinball Loss (PL) score. The PL score is widely used to assess the calibration of density forecasts (e.g., [34]). In the G model, we first estimate OLS with the same terms as in (2) or (3), and then apply GARCH (1,1) to its prediction error (residual term) to obtain the density forecast. We calculated the following PL score $\bar{L}$ for each model as follows[xvi]:

$$\bar{L} = \frac{1}{24 \times D \times N} \sum_{h=0}^{23} \sum_{d=1}^{D} \sum_{n=1}^{N} L_d{}^{(h)} \left(\widehat{y_{d,q_n}^{(h)}}, y_d^{(h)} \right) \quad \text{s.t.}$$
$$L_d{}^{(h)} \left(\widehat{y_{d,q_n}^{(h)}}, y_d^{(h)} \right) = \begin{cases} (1 - q_n) \left(\widehat{y_{d,q_n}^{(h)}} - y_d^{(h)} \right) & if \ y_d^{(h)} < \widehat{y_{d,q_n}^{(h)}} \\ q_n \left(y_d^{(h)} - \widehat{y_{d,q_n}^{(h)}} \right) & if \ y_d^{(h)} \geq \widehat{y_{d,q_n}^{(h)}} \end{cases} . \tag{16}$$

where $y_d^{(h)}$ ($\widehat{y_{d,q_n}^{(h)}}$) is the observed target variable (forecasted quantile on $q_n \coloneqq \left\{ \frac{2n-1}{2N} \middle| n = 1,2,\dots,N \right\}$) at date d hour h. As a result of the calculation using out-of-sample data, QR was found to be significantly better (i.e., the PL score is smaller) than G in paired-sample t-test for both Imb and RT, as demonstrated in Table VI.

TABLE VI PL SCORE SUMMARY OF Imb AND RT

	QR	G	difference	p-value*	
Imb	2,218.0	2,257.4	-39.4	0.001218	**
RT	0.178	0.191	-0.012	3.95E-27	**

* The p-value is based on paired-sample t-tests.

C. Proof of the unique solution in the Multi-quantile case

Here, we prove that the sensitivity analysis of Table IV in one quantile case (as well as the shape of the functions in Fig. 8 and Fig. 9) are the same in multi-quantile case. First, when the revenue function is given by the average of $M(=N^2)$ quantiles, the expected return $Rev_M(x)$ is expressed by the following equation.

$$Rev_M(x) = \frac{1}{M}\sum_{j=1}^{M} A_j x \times exp(-B_j x). \tag{17}$$

If the function obtained by differentiating this function n times is written as $Rev_M{}^{(n)}(x)$, it is given by the following equation.

$$Rev_M{}^{(n)}(x) = \frac{1}{M}\sum_{j=1}^{M}(-1)^{n-1}A_j B_j{}^{n-1}(n - B_j x) \times exp(-B_j x) \tag{18}$$

Here, the solution of $Rev_M{}^{(n)}(x) = 0$ is x that satisfies the following equation.

$$x = \frac{n\sum_{j=1}^{M} A_j B_j{}^{n-1} \times exp(-B_j x)}{\sum_{j=1}^{M} A_j B_j{}^{n} \times exp(-B_j x)} \tag{19}$$

When the right side of (19) is written as $g_n(x)$, $g_n{}'(x)$, the first derivative of $g_n(x)$, is obtained as follows.

$$g_n{}'(x) = \frac{n\sum_{j1>j2} A_{j1}A_{j2}(B_{j1}B_{j2})^{n-1}(B_{j1}-B_{j2})^2 \times exp[-(B_{j1}+B_{j2})x]}{\left\{\sum_{j=1}^{M} A_j B_j{}^{n} \times exp(-B_j x)\right\}^2} \tag{20}$$

Therefore, $g_n{}'(x) > 0$. Also, considering $g_n(0) > 0$ and $g_n{}'(\infty) = 0$ (as there is always j such that $2B_j < B_{j1} + B_{j2}$, this equation can be confirmed from simple algebra by multiplying the denominator and numerator of (20) by $exp(2B_j x)$), so (19) has a unique solution in the range of $x > 0$. Considering this and the signs of $Rev_M{}^{(n)}(0)$ and the eqation $Rev_M{}^{(n)}(\infty) = 0$, the relationship shown in Table VII below is uniquely determined. That is, multi-quantile case has exactly the same function shape as the one quantile case[xvii].

TABLE VII THE SIGNS OF THE DIFFERENTIATED FUNCTIONS

x	$-\infty$	$\cdots$	0	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$+\infty$
$Rev_M(x)$	$-\infty$	$-$	0	$+$	$+$	$+$	$+$	$+$	$+$	$+$	0
$Rev_M{}^{(1)}(x)$	$+\infty$	$+$	$+$	$+$	0	$-$	$-$	$-$	$-$	$-$	0
$Rev_M{}^{(2)}(x)$	$-\infty$	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	0
$Rev_M{}^{(3)}(x)$	$+\infty$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	0	$-$	0

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[i] At this time, the measure to link the imbalance price to the real-time balancing cost was considered due to the absence of a balancing market. The alternative of the market participant submitting the real-time balancing cost was not adopted because it would not guarantee transparency [19]. Reference was also made to the German market, which discourages strategic imbalance settlement transactions by incorporating intraday price-based cap/floors into the imbalance settlement prices [11], but this approach was also not introduced in consideration of the extremely small trading volume of the intraday market and because "the strategy of the business operator is easily reflected in the price of intraday market that is conducted by continuous session", In other words, there was a possible situation where dominant players could manipulate the imbalance settlement price relatively easily with a small amount of intraday trading for them [19].

[ii] In addition to this, prediction methods based on computational intelligence (CI) such as machine learning can be considered, but the CI method generally requires a large amount of calculation and its ability to adapt to nonlinearities may not necessarily result in better forecast accuracy; while the statistical model has an advantage in that it is easy for practitioners to interpret [35]. Therefore, in this study, we decided to adopt the QR with an emphasis on the model transparency to practitioners, including the policy maker, and the ease of applying it to the analytical optimization problem described later.

[iii] The statement here is not necessarily universal, as there is research [36] that has demonstrated the fact that "the increasing energy production from variable renewable energy did not necessarily require an increase in demand for balancing power."

[iv] When applied to other markets where prices can be negative the logarithmic conversions are not usually applied. Even in this case, market analytical models can be constructed by the same approach, but the specific formulation is out of the scope of this study.

[v] It would be possible to use the day ahead market and/or the forward markets; however, as described in [16], it is difficult to predict the imbalance price before the day ahead auction, so these trading strategies were not considered.

[vi] The probability density of RT corresponds to the uncertainty of the shape of the bidding curves in the day ahead market, which is not observed at the time of forecasting or arbitrage trading (if observed, the imbalance price can be expressed as an explicit function of the imbalance volume). However, since these bidding curves are determined (despite unobserved) at that time, there is rationality in the assumption that arbitrage does not affect the density of imbalance price.

[vii] For reference, when comparing the optimal solution x^* when $i_{max} = 10$ and that of $i_{max} = 5$, the ratio of the difference between the two is less than 1.0×10^{-11} at the maximum, which ensures that $i_{max} = 5$ was sufficient for convergence.

[viii] There is no possibility that both scenarios will meet the conditions (that is, two convergent solutions never be obtained). This is clear when considering the fact that the optimum position under no penalty conditions always falls into either long/short, which corresponds to Case I/II in Fig. 10; where, whether the positive arbitrage profit can be obtained (i.e., whether the arbitrage is performed) even in the penalized case, corresponds to verifying whether the optimal position x^* is still positive/negative even if the horizontal line C shifts upward/downward by l/k.

[ix] Downloaded from each electric power company's web page (e.g. TEPCO: http://www.tepco.co.jp/pg/consignment/retailservice/imbalance/index-j.html)

[x] All prices are downloaded from http://www.jepx.org/market/index.html. In JEPX, the intraday market adopts continuous sessions, but because the prices for each contracting time points are not published, we use the average price for each delivery hour, which we were able to obtain, as the ID.

[xi] When quantitatively measuring social welfare, detailed technical information on power supplies for supply and demand adjustment is required; however, because we do not have them, pursuing this problem is out of scope for this study.

[xii] Whether or not similar results can be obtained in other markets cannot be known without empirical analysis using market-specific data and price calculation formulae. However, our empirical example may suggest that it

would be worth rethinking dual pricing for markets that have an imbalance price based on the day-ahead energy price.

[xiii] For example, the first hours on Feb. 12 and the last hours on Feb. 13 have excessive arbitrage (short position of ID). The reason for this is that ID at that time was significantly higher than DA.

[xiv] However, the current Japan's imbalance settlement price, which adds a uniform penalty for individual imbalances to the day-ahead-based price (Baltics countries also have similar dual price systems [33]) may be still inefficient from the reasons such that it has an incentive to prevent power producers from offering their flexible capacities to the balancing (reserve) market (note that in Japan, the balancing market was only recently started in April 2021). However, as a medium- to long-term response, Japanese regulators are currently considering the introduction of a dual price incentive based on the balancing energy costs, with a partial correction using wholesale electricity prices (depending on the consistency with system status) [37][38]. (However, the dramatic price surge that occurred in January 2021 [39] has prompted the regulators to reconsider the plan itself, and the debate is still ongoing [40].)

[xv] Britain has a plan to introduce "a standardized suite of upward and downward reserve products" from 2022, which will be traded day-ahead, from the policy intent to "ensure routes to market for all participants" and "create transparency" in the activities of the SO. In Britain, if reserve products are used to help with energy balancing, their costs are required to be included alongside the real-time balancing cost in the imbalance settlement prices (see the Imbalance Pricing Guide, https://www.elexon.co.uk/documents/training-guidance/bsc-guidance-notes/imbalance-pricing/)

[xvi] For the time lag, number of quantiles, and daily sample size, we used $r = 1$, $N = 10$, and $D = 365$ (i.e., all days of 2018). Note that the out-of-sample quantile are sorted to obtain monotonic quantile curves as with [34], which however does not affect the optimization problem and solution in this work. Also, to ensure robustness with small sample size, we used lasso penalized QR in the R package ‘quantreg’[22].

[xvii] When considering the sensitivity analysis shown in Table IV, the case A or B is 0 corresponds to when all A_j or B_j are 0, and the case it is $+\infty$ corresponds to when arbitrary A_j or B_j is $+\infty$, which can be confirmed from simple calculations.