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Capital and income inequality: An aggregate-demand complementarity☆

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ABSTRACT

A novel complementarity between capital and income inequality leads to a significant amplification of the effects of aggregate-demand shocks on consumption. We characterize this finding using a simple model with heterogeneity in household saving and income, nominal rigidities, and capital. A fiscal policy that redistributes capital income causes further amplification, whereas redistributing profits generates dampening. After an interest rate shock, consumption inequality is more countercyclical than income inequality, consistent with the available empirical evidence. Procyclical investment also requires a more aggressive Taylor rule in order to attain determinacy, and aggravates the forward guidance puzzle.

1. Introduction

How do aggregate demand shocks transmit to the economy and what determines the magnitude of the response? In a seminal contribution, Samuelson (1939) already argued that the combination of a consumption function with an investment relation leads to an amplification of aggregate demand shocks: the celebrated *multiplier-accelerator*. A recent literature reviewed below emphasizes the role of household heterogeneity as a microfoundation for a multiplier effect, in particular through an endogenous feedback between aggregate demand and income inequality in relation to the marginal propensity

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to consume (MPC), reminiscent of the Keynesian cross. Less attention, however, has been paid to the role of heterogeneity in the marginal propensity to save, and thus to investment, as a potential amplifier of demand-driven fluctuations.

In this paper, we show that income inequality together with heterogeneity in savings generates a strong *complementarity*: the impact of aggregate demand shocks on consumption when both heterogeneity dimensions are active is an order of magnitude larger than the mere addition of the effects of each heterogeneity in isolation. We elicit this novel amplification mechanism using an apparatus that distinguishes between two types of heterogeneity: heterogeneity in savings on the expenditure side, and income inequality on the resource side of the household budget constraint. We refer to the former as '*capital inequality*': a feature of any heterogeneous-agents model with a productive asset such as capital, that could be equally referred to as wealth inequality or capital market segmentation.

To each inequality corresponds one separate amplification channel. First, much in the spirit of Samuelson's (1939) multiplier-accelerator, capital inequality leads to amplification in and of itself. The intuition is that after an increase in aggregate demand, spenders consume all the additional income whereas savers invest a fraction of it, thus generating a further boost in aggregate income and a further round of aggregate demand effects. This *'capital inequality' channel* is an intrinsic feature of any heterogeneous-agent model with capital, as we discuss in the literature review below.

This is distinct from the cyclical *'income inequality' channel*, which also leads to aggregate-demand amplification in and of itself under the condition that the income of high MPC spenders responds more than proportionally to changes in aggregate income, as emphasized by a literature reviewed below. Our key result is that when simultaneously present, capital and income inequality blend into a significant *complementarity* that we dub the *'multiplier of the multiplier'*.

We characterize our findings, first, in a simple saver-spender model that allows us to develop intuition and, then, in a richer (but still) tractable heterogeneous-agent New Keynesian model with investment in productive physical capital and idiosyncratic risk. The unconstrained households hold stocks, bonds and capital, while the constrained hand-to-mouth do not have access to asset markets and simply consume their labor income plus any transfers. Idiosyncratic uncertainty—captured by households changing state exogenously between these two states—gives rise to a precautionary, self-insurance saving motive. In our baseline model, stocks and capital are illiquid (cannot be used for self-insurance) and adjusting the capital stock is subject to a cost. Firms are subject to nominal rigidities. The government levies taxes on dividends and capital income, which it may choose to redistribute or not.

The effects of fiscal redistribution crucially depend on what type of income is targeted. The redistribution of monopoly profits dampens the aggregate-demand effects on consumption because profits in the model are countercyclical, so redistributing them weakens the capital and income inequality channels. In contrast, capital income is highly procyclical so its redistribution towards constrained households strongly amplifies the aggregate-demand effects.

Our finding of a strong complementarity between capital and income inequality is robust to introducing idiosyncratic risk and sticky wages and to varying key parameters—such as capital adjustment costs, the degree of nominal rigidities and the elasticity of intertemporal substitution—within a wide empirically plausible range.

A robust testable prediction of our model concerns the cyclicality of consumption and income inequality: conditional on demand shocks, both are countercyclical but the former is more countercyclical than the latter. This is in line with the available empirical evidence (see Ampudia et al., 2018; Coibion et al., 2017; Mumtaz and Theophilopoulou, 2017) and supports the empirical relevance of the channels we identify. Lastly, our mechanism has stark policy implications: procyclical investment leads to intertemporal aggregate-demand amplification that requires a more aggressive interest rate response in order to ensure determinacy, and aggravates the forward guidance puzzle.

Related literature Our analysis joins a burgeoning body of work that incorporates Heterogeneous Agents into the New Keynesian (HANK) framework. Because HANK models are typically complex, several studies have proposed tractable versions that help illustrate the transmission mechanisms at work.[1]

The "capital inequality" channel is a simple analytical generalization and microfoundation of Samuelson's (1939) celebrated multiplier-accelerator to a setting with household heterogeneity. It relies on a literal formalization of the saver-spender distinction based of physical capital holdings (or lack thereof) proposed by Mankiw (2000), following Campbell and Mankiw (1989), and first incorporated in a New Keynesian model by Galí et al. (2007) to study the effects of government spending. The same channel has been implicitly featured in other earlier contributions, including Kaplan et al. (2018), Gornemann et al. (2016), and Luetticke (2021), and explicitly analyzed using a quantitative model by Alves et al. (2019). In independent and complementary work, Auclert et al. (2020) also emphasize the role of investment and heterogeneity in a model with sticky prices and wages, focusing on liquidity but abstracting from cyclical variations in income inequality, and therefore not featuring our complementarity. Finally, the income inequality channel and its role for aggregate-demand amplification in isolation has been studied by Bilbiie (2008, 2020), Auclert (2019), and Patterson (2019) in frameworks without capital.

Relative to these studies, we unveil a novel complementarity between capital and income inequality for aggregate-demand amplification. We characterize analytically these channels, in isolation and in combination, and then use a richer

[1] See for instance Auclert et al. 2018; Bayer et al. 2019; Challe et al. 2017; Gornemann et al. 2016; Hagedorn et al. 2019; Kaplan et al. 2018; McKay et al. 2016; Oh and Reis 2012; Ravn and Sterk 2017 for quantitative contributions and Acharya and Dogra 2020; Bilbiie 2008; 2018; 2020; Cantore and Freund 2021; Debortoli and Galí 2018; Eggertsson and Krugman 2012; Galí et al. 2007; Maliar and Naubert 2019; Ravn and Sterk 2020; Werning 2015 for tractable versions.

tractable heterogeneous-agent New Keynesian model with idiosyncratic risk—building on Bilbiie (2018), extended with capital investment—to quantify the contribution of the different assumptions to the transmission of monetary policy.

2. A tale of two inequalities

In this section, we present a simple framework that serves to isolate the capital and income inequality channels and illustrate their complementarity. While we focus is on capital, the arguments hold for any productive asset in positive net supply, or generally *wealth*. Let us start from a generic budget constraint of a household j:

$$C^j + S^j = Y^j,$$

where C^j are consumption expenditures, S^j savings, and Y^j the household's total income that can include both labor and financial income (accounting for the distinction between the two will play an important role in Section 3).

In this framework, we can identify two different types of heterogeneity. On the left hand side, households can differ in their *expenditures* depending on how much they save/invest (and consume); and on the right-hand side, they can differ in their *incomes*. We refer to these, respectively, as *capital* and *income inequality*; the former is akin to a stark form of *wealth* inequality. These inequalities are present in many heterogeneous-agent models with assets traded in equilibrium. Our aim is to make transparent their role for the transmission of aggregate shocks.

To that end, we propose a (to the best of our knowledge) novel way to elicit these two channels. To isolate the role of *capital inequality*, we assume that income is perfectly redistributed so that all households receive the same income Y:

$$C^j + S^j = Y.$$

In a model without net savings and capital, perfect income redistribution would imply that heterogeneity is irrelevant for aggregate dynamics. This is, however, no longer the case when differences in savings behavior are linked with MPC heterogeneity.

To isolate the role of *income inequality*, we assume that there is no savings vehicle in positive net supply, so that in equilibrium the budget constraint reads:

$$C^j = Y^j.$$

The crucial parameter is the elasticity of individual income with respect to aggregate income, $\chi_j = \frac{\partial \log Y^j}{\partial \log Y}$. When χ_j is higher for constrained households, income inequality (between unconstrained and constrained agents) becomes countercyclical and there is amplification of aggregate-demand shocks. This was shown by Bilbiie (2008) in a two-agent model, generalized by Auclert (2019) in a richer heterogeneous-agent model, and estimated using micro data on consumption and income by Patterson (2019). Conversely, procyclical inequality implies dampening.

Given the empirical relevance of both channels, an important question is how much of the aggregate-demand effects on consumption they can account for. As we shall see, the two channels are complementary: their joint impact is much larger than the addition of their individual effects. We now characterize this finding analytically in a simple saver-spender model in the spirit of Mankiw (2000).

2.1. A simple saver-spender model

In this section we outline a stylized model to isolate our main finding in the most transparent way. Next, we relax many of the simplifying assumptions to show that the main conclusions carry through in a fully-specified yet still tractable heterogeneous-agent model whose closed-form solution echoes this one in a special case.

The economy consists of a continuum of households on the unit interval, of two types: a share $\lambda \in [0, 1)$ are *hand-to-mouth* spenders (H) and the rest $1 - \lambda$ are *savers* (S). Savers consume and save, while spenders live paycheck to paycheck, consuming all of their income. As our focus is on the demand side, we remain agnostic about the supply side and assume that the central bank controls the real interest rate. While our focus is on *monetary policy*, the insights apply to any kind of aggregate-demand policy. We sketch the model in log-linear form, where lowercase variables denote log-deviations from steady state. For a detailed derivation, see Appendix A.

Savers have access to two assets: bonds and physical capital. Their bond holding decision is characterized by a standard Euler equation:

$$c_t^S = E_t c_{t+1}^S - r_t, \tag{1}$$

where r_t is the real interest rate. Bonds are priced but not traded as we assume that they are in zero net supply in equilibrium.

Savers also invest in physical capital. To get tractability, we assume in this section, and in this section *only*, that their behavior can be characterized by a reduced-form investment rule $i_t = f(y_t, r_t, \ldots)$. We remain agnostic here about the exact underpinnings of this equilibrium equation; in Section 3 we study a fully microfounded version. As a leading example, we assume that investment is an isoelastic function of total income:

$$i_t = \eta y_t, \tag{2}$$

where $\eta > 0$ is the elasticity of investment to output.[2] We generalize this in Appendix A.2 to include an elasticity to interest rates or future income too.

The budget constraint of savers (in log-linear form) reads:

$$C_Y c_t^S + \frac{I_Y}{1-\lambda} i_t = Y_Y^S y_t^S, \tag{3}$$

where y_t^S is the (post-transfer) income of the savers and $X_Y \equiv X/Y$ denotes the steady-state share of variable X in GDP (income) Y, for any $X \in \{C, I, Y^S\}$.[3]

Spenders just consume all their income in every period, i.e.:

$$c_t^H = y_t^H. \tag{4}$$

Goods market clearing requires that:

$$y_t = C_Y c_t + I_Y i_t. \tag{5}$$

Aggregate consumption and income are given by:

$$c_t = \lambda c_t^H + (1-\lambda) c_t^S \tag{6}$$

$$y_t = \lambda Y_Y^H y_t^H + (1-\lambda) Y_Y^S y_t^S. \tag{7}$$

To close the model, we have to specify how income is distributed. We assume that the income of the spenders moves with aggregate income according to:

$$y_t^H = \chi y_t, \tag{8}$$

where χ is the elasticity of *their* income to aggregate income. In Section 3, we use a richer microfounded framework where this elasticity is an equilibrium outcome of a structural model. Using the definition of aggregate income, savers' income is, combining (7) and (8): $y_t^S = \left(1 - \lambda \chi Y_Y^H\right) y_t / \left((1-\lambda) Y_Y^S\right)$.

2.2. The multiplier of the multiplier

We now analyze the two inequality channels, first in isolation and then in interaction. A useful benchmark is the representative-agent economy $\lambda = 0$, whereby a one-time real interest rate cut has a unit consumption multiplier $\partial c_t / \partial(-r_t) = 1$.

Income inequality

To isolate the role of income inequality, we assume that the savings rate is zero, i.e. $I_Y = 0$. The model then collapses to:

$$c_t^H = \chi y_t; \text{ and } c_t^S = \frac{1-\lambda\chi}{1-\lambda} y_t.$$

Using this together with market clearing in the Euler equation, we can derive the *aggregate Euler equation*:

$$c_t = E_t c_{t+1} - \frac{1-\lambda}{1-\lambda\chi} r_t.$$

The multiplier to a one time change in the real interest rate is:

$$\frac{\partial c_t}{\partial(-r_t)} = \frac{1-\lambda}{1-\lambda\chi}. \tag{9}$$

The effects of a change in the real rate are amplified iff $\chi > 1$, i.e. when spenders' income is more elastic to aggregate income than the savers', provided that $\lambda\chi < 1$. The reason is that an increase in aggregate demand, which leads to an increase in aggregate income, translates into an even larger increase in spenders' income; this causes aggregate demand to rise even further because spenders have unit MPC, and so on. This is the countercyclical inequality channel described in Bilbiie (2008, 2018), yielding a Keynesian-cross multiplier (in the spirit of Samuelson, 1948): a share λ agents have unit individual MPC and their income elasticity to aggregate income is χ, so the "aggregate MPC" out of aggregate income is approximately $\lambda\chi$. When households have proportional incomes $\chi = 1$, the case assumed by Campbell and Mankiw (1989), the multiplier is the same as in the representative-agent benchmark of $\lambda = 0$, $|\partial c_t / \partial r_t| = 1$.

Capital inequality: a reappraisal of Samuelson's (1939) Multiplier-Accelerator

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[2] As is well known, the strong procyclicality of investment to output arises naturally as an equilibrium outcome of any neoclassical, RBC or NK model.

[3] We focus on a case with equal consumption in steady state across households, i.e. $C^S = C^H = C$, achieved by a fixed steady-state transfer explained in Appendix A. This simplifies the analytics but is not needed, as we show in the fully-specified model in Section 3 and Appendix C.5.

To isolate the role of capital inequality, we assume instead that income is perfectly redistributed: $\chi = 1$, which implies proportional incomes $y_t^S = y_t^H = y_t$. Replacing in the budget constraints (3) and (4):

$$\begin{aligned} c_t^H &= y_t \\ C_Y c_t^S + \frac{I_Y}{1-\lambda} i_t &= Y_Y^S y_t. \end{aligned} \tag{10}$$

We want to solve for savers' consumption as a function of aggregate consumption in order to obtain an aggregate Euler equation. To do so, first combine the investment function (2) with goods market clearing (5), obtaining:

$$i_t = \eta \frac{1 - I_Y}{1 - \eta I_Y} c_t. \tag{11}$$

Note that $\frac{1-I_Y}{1-\eta I_Y} > 1$ iff $\eta > 1$, provided that $\eta I_Y < 1$. Using (11) and (5) to replace (10) in the definition of aggregate consumption, we obtain:

$$c_t^S = \frac{1 - \lambda \frac{1-I_Y}{1-\eta I_Y}}{1-\lambda} c_t, \tag{12}$$

which replaced in (1) delivers the aggregate Euler equation and Proposition 1:

$$c_t = E_t c_{t+1} - \frac{1-\lambda}{1 - \lambda \frac{1-I_Y}{1-\eta I_Y}} r_t. \tag{13}$$

Proposition 1 (Amplification through capital)**.** *The multiplier of a one time cut in the real interest rate is given by:*

$$\frac{\partial c_t}{\partial(-r_t)} = \frac{1-\lambda}{1 - \lambda \frac{1-I_Y}{1-\eta I_Y}}. \tag{14}$$

If investment is more than one-to-one procyclical, i.e. $\eta > 1$, then (i) the effect of a cut in the real rate is larger than one, i.e. $\partial c_t / \partial(-r_t) > 1$, and (ii) the multiplier is increasing in the share of spenders, λ, as long as $0 < \lambda \frac{1-I_Y}{1-\eta I_Y} < 1$.

Proof. Follows immediately from $I_Y \in [0, 1)$. □

Our analytical formalization provides a novel intuition for the amplification of monetary policy effects on consumption via investment: *the marginal propensity to save MPS (of savers) adds to the aggregate MPC through its indirect impact on the high-MPC spenders*, even if income is redistributed uniformly. When capital income gets redistributed to hand-to-mouth agents (either through market forces—capital augmenting the return on labor—or through fiscal redistribution), the latter increase their demand. This further boosts total income, part of which is saved and yields an increase in investment of ηI_Y, which generates further income, boosting the consumption of unit-MPC spenders, and so on—thereby triggering a distinct Keynesian-cross multiplier. This is summarized by the term $\frac{1-I_Y}{1-\eta I_Y}$, which magnifies the aggregate MPC through the above-described channel when investment is procyclical enough $\eta > 1$. The multiplier effect disappears without investment, since under full redistribution the model collapses to the representative-agent case. In the empirically plausible case $0 < \lambda \frac{1-I_Y}{1-\eta I_Y} < 1$, capital *amplifies* the monetary policy effects on consumption through heterogeneity.

We elaborate on the connection to Samuelson (1939), who studied the role of investment and consumption functions for spending multipliers, in Appendix A.3. Our capital inequality channel is a generalized, microfounded version of Samuelson's in a setting with MPC heterogeneity and segmented capital markets. This general amplification mechanism operates in *any* heterogeneous-agent model with capital. Furthermore, it does not depend on our simple framework with a reduced-form investment equation; in Appendix A.1, we show that the only requirement is procyclical enough investment. Thus, any model with this feature automatically implies amplification of the consumption response through heterogeneity, even under proportional incomes.

Capital and income inequality

We now enable both channels, capital ($I_Y > 0$) and income inequality ($\chi > 1$). Replacing in the budget constraints (3) and (4):

$$\begin{aligned} c_t^H &= \chi y_t \\ C_Y c_t^S + \frac{I_Y}{1-\lambda} i_t &= \frac{1 - \lambda \chi Y_Y^H}{1-\lambda} y_t. \end{aligned} \tag{15}$$

Following the same strategy as above, we solve again for savers' consumption:

$$c_t^S = \frac{1 - \lambda \chi \frac{1-I_Y}{1-\eta I_Y}}{1-\lambda} c_t, \tag{16}$$

to obtain the aggregate Euler equation and our next Proposition:

$$c_t = E_t c_{t+1} - \frac{1-\lambda}{1 - \lambda \chi \frac{1-I_Y}{1-\eta I_Y}} r_t. \tag{17}$$

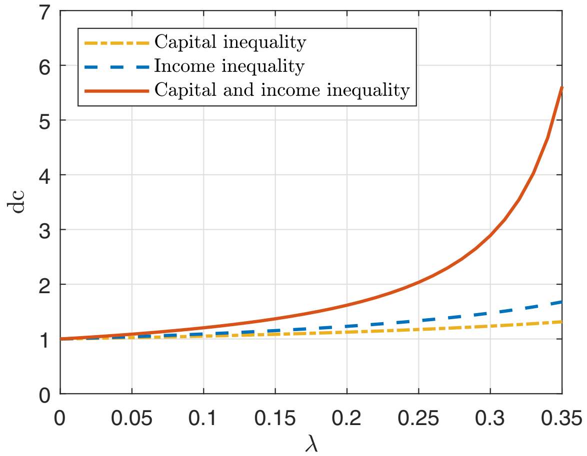

Fig. 1. The Complementarity. *Notes*: This figure shows the consumption multipliers as a function of the share of hand-to-mouth λ (using $I_Y = 0.235$, $\eta = 2$, and $\chi = 1.75$).

Proposition 2. *The multiplier of an interest-rate cut when both channels are active is:*

$$\frac{\partial c_t}{\partial(-r_t)} = \Omega \equiv \frac{1-\lambda}{1-\lambda\chi\frac{1-I_Y}{1-\eta I_Y}}. \tag{18}$$

If income inequality is countercyclical $\chi > 1$ and investment more than one-to-one procyclical $\eta > 1$, the joint multiplier Ω is larger than the product of the two individual multipliers:

$$\Omega = \frac{\partial c_t}{\partial(-r_t)}\Big|_{K,\ \text{no redist}} > \frac{\partial c_t}{\partial(-r_t)}\Big|_{no\ K,\ \text{no redist}} \times \frac{\partial c_t}{\partial(-r_t)}\Big|_{K,\ redist} \tag{19}$$

provided that $0 < \lambda\chi\frac{1-I_Y}{1-\eta I_Y} < 1$. Amplification ($\partial c_t/\partial(-r_t)$ increasing in λ) can occur even with procyclical income inequality ($\chi < 1$) iff $\chi\frac{1-I_Y}{1-\eta I_Y} > 1$.

Proof. Replacing the expressions for the respective multipliers from (9), (14), and (18), the complementarity condition (19) becomes:

$$\frac{1-\lambda}{1-\lambda\chi\frac{1-I_Y}{1-\eta I_Y}} > \frac{1-\lambda}{1-\lambda\chi}\frac{1-\lambda}{1-\lambda\frac{1-I_Y}{1-\eta I_Y}}.$$

This holds if $\lambda(\chi-1)(\eta-1)\frac{I_Y}{1-\eta I_Y} > 0$, which is satisfied if $\chi > 1$, and $\eta > 1$. The final part follows from the derivative of (18) with respect to λ, i.e. $\left(\chi\frac{1-I_Y}{1-\eta I_Y} - 1\right)\left(1-\lambda\chi\frac{1-I_Y}{1-\eta I_Y}\right)^{-2}$, which is positive even for $\chi < 1$ if $\chi\frac{1-I_Y}{1-\eta I_Y} > 1$. □

The model features are illustrated in Fig. 1, which depicts the effect of a cut in the real rate on consumption as a function of the share of hand-to-mouth λ. When $\lambda = 0$, we are back to the representative-agent case: the multiplier is one in all models. The broken yellow line reveals that capital inequality by itself only leads to little amplification. This is almost by construction, as investment is undertaken by the savers and we have neutralized the feedback through the real interest rate. Income inequality alone, depicted as the dotted blue line, can lead to more amplification (the cyclical-inequality channel). But importantly, the model with capital *and* income inequality depicted as the red solid line delivers substantially more amplification than the mere product of the individual channels. Unequal capital expenditures lead to a multiplying effect of the multiplier associated with income inequality: *a multiplier of the multiplier*.

The intuition is most clearly seen by inspecting the multiplier under both capital and income inequality from expression (18). The numerator captures the automatic, direct effect: only $(1-\lambda)$ agents react directly to interest rates. While the denominator captures the multiplier, indirect effect(s). Turning off each channel in turn recovers the previous individual channels, each of which delivers a multiplier by scaling up the aggregate MPC, as described above. Putting the two channels together compounds the aggregate MPC and thus yields a double multiplier amplification: the two indirect effects interact non-linearly at each round, acting as multipliers of each other. Another way of appreciating the interaction of these two channels from expression (18) is to note that the multiplier due to the capital-inequality channel $\frac{1-I_Y}{1-\eta I_Y}$ appears as a multiplier "inside" (in the sense of multiplying the respective MPC of) the multiplier associated with the income-inequality channel, $\frac{1-\lambda}{1-\lambda\chi}$, and vice-versa.[4]

As we will show, this complementarity turns out to be very general and does not depend in any way on the simplifying assumptions adopted here. In the next section, we confirm our results in a fully-specified heterogeneous-agent model and verify the robustness of our findings with respect to different modeling assumptions and a wide range of empirically plausible parameterizations. Furthermore, in Appendix B.4, we reproduce all the analytical findings of this section in an analytically tractable case of the full model—illustrating again that none of the results here are driven by the simplifying assumptions on the income distribution and the savings technology.

2.3. Testable predictions: The cyclicality of inequality

So far, we studied capital and income inequality as *transmission channels* for aggregate consumption dynamics. However, it is equally interesting to study the model implications for the distribution of income and consumption as *outcomes*. In this section, we use our framework to derive some testable predictions, that we will later confront with the available empirical evidence.

We measure the dispersion in income and consumption across households by the difference between savers' and spenders' (log) variables, which are: $x^S - x^H, \ x \in \{y, c\}$.[5] As we focus on one type of disturbances (i.e. demand shocks) only, throughout the paper we use the word 'cyclicality' to refer to cyclicality *conditional* on aggregate demand disturbances such as monetary policy shocks (as opposed to conditional on exogenous movements in aggregate supply, which we abstract from).[6]

In our simple saver-spender model, it is easy to show that income inequality is:

$$y_t^S - y_t^H = \frac{1-\chi}{(1-\lambda)Y_Y^S} y_t. \tag{20}$$

As explained in Section 2, income inequality is countercyclical iff $\chi > 1$, which is also the condition required for amplification. Consumption inequality is instead given by:

$$c_t^S - c_t^H = \frac{1-\chi C_Y}{(1-\lambda)C_Y} y_t - \frac{I_Y}{(1-\lambda)C_Y} i_t. \tag{21}$$

Under our simplifying isoelastic investment function (2), this reduces to:

$$c_t^S - c_t^H = \frac{1-\chi C_Y - \eta I_Y}{(1-\lambda)C_Y} y_t. \tag{22}$$

Proposition 3 (Countercyclical consumption inequality)**.** *Consumption inequality is countercyclical iff:*

$$C_Y(\chi - 1) + I_Y(\eta - 1) > 0. \tag{23}$$

If investment is more than one-to-one procyclical $\eta > 1$, then consumption inequality is more countercyclical than income inequality.

Proof. The first part follows automatically by rewriting $1 - \chi C_Y - \eta I_Y < 0$. For the second part, rewrite (21), using $(1-\lambda)Y_Y^S = 1 - \lambda C_Y$, as:

$$c_t^S - c_t^H = y_t^S - y_t^H + \frac{I_Y}{(1-\lambda)C_Y}\left(\frac{1-\lambda\chi C_Y}{1-\lambda C_Y} y_t - i_t\right).$$

For consumption inequality to be more countercyclical than income inequality, we need the term in brackets to be countercyclical, that is investment to be procyclical "enough". Replacing (2), the condition is:

$$\lambda C_Y(\chi - 1) + (1-\lambda C_Y)(\eta - 1) = \lambda[C_Y(\chi - 1) + I_Y(\eta - 1)] + (1-\lambda)(\eta - 1) > 0.$$

Since the term in square brackets is positive when consumption inequality is countercyclical, a *sufficient* condition for this to be satisfied is that $\eta > 1$. □

As we show in Section 4.2, this result generalizes to richer settings with nominal rigidities and idiosyncratic risk. The intuition is that, from (21), consumption inequality is countercyclical if and only if $C_Y \chi + I_Y \frac{\partial i_t}{\partial y_t} > 1$; Since investment is more than one-to-one procyclical, consumption inequality is always more countercyclical than income inequality. In Section 4.2, we confront these theoretical predictions with the available empirical evidence.

[4] This interpretation links the two distinct channels that differentiate the early "TANK" contributions: investment in physical capital (Galí et al., 2007), versus income, i.e. receiving profits from holding shares in monopolistic firms, or not (Bilbiie, 2008). The current HANK literature focuses predominantly on the latter and its link with risk, self-insurance, and precautionary saving. We explore the former and its complementarity with the latter.

[5] This is the log-deviation of the ratio of savers' and hand-to-mouth's x, as in Bilbiie (2018). With two agents, this definition is proportional to the Gini coefficient or measures of entropy.

[6] For a detailed treatment of the unconditional cyclicality of consumption and income inequality in TANK models, see Maliar and Naubert (2019).

2.4. Policy implications: Determinacy and forward guidance

The amplification mechanism we unconvered has important implications for the monetary authority's ability to stabilize the economy (in the sense of ruling out expectation-driven fluctuations) via interest rate rules, and for the power of forward guidance (FG). In a nutshell, the cyclicality of investment makes it harder to ensure equilibrium determinacy via a Taylor rule, requiring a larger response to inflation or real activity, more so than in a model with heterogeneity but without investment. And second, it magnifies FG power and aggravates the "FG puzzle", resuscitating it even when other incomplete-market forces are such that it would be ruled out without investment.

To substantiate these points, we need to extend the previous simple model to include idiosyncratic risk and self-insurance; in particular, the savers' loglinearized Euler equation for (now, *liquid*) bonds takes into account the risk of transitioning to the constrained H state next period with probability $1-s$:

$$c_t^S = sE_t c_{t+1}^S + (1-s)E_t c_{t+1}^H - r_t. \tag{24}$$

We derive this from a fully specified model in the next Section, see Eq. (30). Replacing individual consumptions (15) and (16) in (24) delivers the next Proposition.

Proposition 4. *(**Aggregate Euler compounding, determinacy, and forward guidance**) The aggregate Euler equation with idiosyncratic risk and illiquid capital investment is:*

$$c_t = \Theta E_t c_{t+1} - \Omega r_t,\ \ \Theta \equiv 1 + (1-s)\frac{\chi\frac{1-I_Y}{1-\eta I_Y} - 1}{1-\lambda\chi\frac{1-I_Y}{1-\eta I_Y}}; \tag{25}$$

*There is **compounding** $\Theta > 1$ (for $s < 1$) iff investment is procyclical **enough**, specifically:*

$$\chi\frac{1-I_Y}{1-\eta I_Y} > 1 \rightarrow \eta > 1 + (1-\chi)\frac{1-I_Y}{I_Y}, \tag{26}$$

which makes the Taylor principle insufficient for determinacy and aggravates the forward guidance puzzle.

Procyclical *enough* investment in the sense of (26) generates Euler compounding even when income inequality is procyclical $\chi < 1$ and would *by itself* generate discounting $\Theta < 1$. The compounding intuition is similar to the one stemming from countercyclical inequality and risk, previously emphasized by Bilbiie (2018, 2020), Acharya and Dogra (2020), and Ravn and Sterk (2020). To isolate our channel, consider acyclical inequality and risk $\chi = 1$; future good news of aggregate income are now correctly anticipated to lead to more future saving, investment, and thus (through redistribution) disproportionately more future income in the constrained state. As such, they trigger the reverse of self-insurance: a fall in (precautionary) saving and an increase in consumption today that is higher than it would be in a representative-agent or no-risk economy: that is, Euler *compounding*. Adding countercyclical inequality $\chi > 1$ of course magnifies this (as it magnifies the static amplification discussed above), but the key point is that compounding may even occur with procyclical inequality $\chi < 1$.[7] To prove the part about FG is immediate, solving (25) forward for the effect of a real rate cut at $t+T$:

$$\frac{\partial c_t}{\partial(-r_{t+T})} = \Theta^T\Omega. \tag{27}$$

The power of FG is increasing with time if $\Theta > 1$, restoring the FG puzzle (Del Negro et al., 2015) even with procyclical income inequality or risk $\chi < 1$ when (26) holds.[8]

To find the determinacy condition in the simplest possible case, consider without loss of generality a static Phillips curve $\pi_t = \kappa c_t$ (results carry through to the more standard forward-looking form) and a Taylor rule setting the nominal rate as a function of inflation $r_t^n \equiv r_t + E_t\pi_{t+1} = \phi_\pi\pi_t$, so that r_t is now endogenous.[9] Replacing these in (25) we obtain the difference equation $c_t = \frac{\Theta+\kappa\Omega}{1+\phi_\pi\kappa\Omega}E_t c_{t+1}$, which by standard results is determinate iff the modified, HANK-with-investment Taylor principle holds (a generalization of Bilbiie, 2018):

$$\phi_\pi > 1 + \frac{\Theta-1}{\kappa\Omega}. \tag{28}$$

Procyclical investment, by generating Euler-equation compounding in the presence of heterogeneity and idiosyncratic risk, makes it more difficult for the central bank to stabilize the economy in the sense of ruling out expectation-driven fluctuations by ensuring determinacy with a Taylor rule. The reason is that, as described above, it creates a further aggregate-demand amplification loop that makes it harder to rule out sunspot expansions and requires the monetary authority to be more aggressive in increasing the real rate to counteract non-fundamental aggregate demand expectations.[10]

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[7] Note that (26) is the same as the condition for countercyclical consumption inequality (23).

[8] Illustrations of the survival of the FG puzzle with procyclical or acyclical income inequality have been previously noted by quantitative examples in earlier versions of Bilbiie (2018) and Auclert et al. (2020). Our Proposition shows this as a general property and finds a closed-form parameter condition in our simple model.

[9] Notice that assuming instead $\pi_t = \tilde{\kappa}y_t$ and using (11), we obtain the same PC redefining $\kappa = \tilde{\kappa}\frac{1-I_Y}{1-\eta I_Y}$.

[10] Responding to inflation is an indirect way to address this "real" demand amplification and the threshold response (28) becomes very large when prices very sticky. As discussed in Bilbiie (2018), without investment this can be circumvented by responding to output, with a rule $i_t = \phi_\pi\pi_t + \phi_y y_t$; when

Our results thus add to the analytical literature emphasizing how countercyclical income inequality and risk generate Euler compounding, make the Taylor principle insufficient for determinacy, and aggravate the FG puzzle (see references above); put differently, our results show that procyclical income inequality/risk, a property implicitly satisfied in McKay et al. (2016), is *insufficient* in the presence of investment to guarantee Euler discounting, Taylor principle sufficiency, and rule out the FG puzzle.

3. A tractable HANK model with capital

We propose a novel heterogeneous-agent model, drawing on elements from both the TANK and HANK literatures. Compared to the simple model from Section 2, this model will not only allow us to make a first step towards quantifying the complementarity and analyze its robustness with respect to different model features but will also enable us to study the role of different redistributive fiscal policies.

The economy comprises households, firms and a fiscal and monetary authority. The New Keynesian block is standard, so we focus on the household side and the fiscal scheme (the full model including derivations is in Appendix B). As above, we denote variables in levels by uppercase and log-deviations by lowercase letters.

There are two types of households; a share $\lambda \in [0, 1)$ *hand-to-mouth* (H) and a share $(1-\lambda)$ *savers* (S). All households have the same CRRA preferences in consumption and labor $U(C, N) = \frac{C^{1-\sigma^{-1}}}{1-\sigma^{-1}} - a\frac{N^{1+\varphi}}{1+\varphi}$, where the σ^{-1} is relative risk aversion and φ is the inverse labor elasticity. We incorporate idiosyncratic risk by assuming that households switch exogenously between types. In particular, the exogenous change of type follows a Markov chain: the probability to stay a saver is s and the probability to remain hand-to-mouth is h (with transition probabilities $1-s$ and $1-h$, respectively).

There is limited asset market participation. The hand-to-mouth hold no assets, and thus consume their labor income and any redistributive government transfers:

$$C_t^H = \frac{W_t}{P_t} N_t^H + T_t^H, \tag{29}$$

where W_t is the nominal wage, P_t the price level, N_t^H hours worked and T_t^H transfers.

Savers hold and price all assets: risk-free bonds B_t^S, with a risk-free return of $\frac{1+r_{t-1}^n}{1+\pi_t}$ (in real terms); stocks ω_t, which are a claim to the firm dividends D_t (in real terms); physical capital K_t, which they rent out at rate R_t^K. Importantly, bonds are liquid and can be used to self-insure against idiosyncratic risk while stocks and capital are illiquid. This is reflected in the bond Euler equation (of which (24) above is the loglinearized version around a symmetric steady state):

$$(C_t^S)^{-\frac{1}{\sigma}} = \beta E_t \left\{ \frac{1+r_t^n}{1+\pi_{t+1}} \left[s(C_{t+1}^S)^{-\frac{1}{\sigma}} + (1-s)(C_{t+1}^H)^{-\frac{1}{\sigma}} \right] \right\}, \tag{30}$$

where β is the discount factor. In contrast, the Euler equations for illiquid capital and stocks are standard and relegated to the Appendix. This is a tractable way of introducing idiosyncratic risk and liquidity, key ingredients of full-blown HANK models. Note that the budget constraint also has to account for the flows of liquid assets between types, see Appendix B for details.

To facilitate the introduction of sticky wages in Section 3.3, we assume that the labor market is centralized: a union pools labor inputs and sets wages on behalf of both households. This results in a "labor-supply-like" wage schedule (in log-linear form):

$$\varphi n_t = w_t - \sigma^{-1} c_t, \tag{31}$$

and a uniform allocation of hours $N_t^H = N_t^S = N_t$. While this labor market setting simplifies the analysis, it is not essential for any of our results, see Appendix C.7.

The government taxes dividends and capital income at rates τ^D and τ^K, respectively, and redistributes all revenues from capital income and profits taxation, running a balanced budget in every period:

$$\lambda T_{H,t} = \tau^D D_t + \tau^K R_t^K K_t. \tag{32}$$

We close the model by assuming a monetary policy rule of the form $r_t^n = \phi_\pi \pi_t + \varepsilon_t$. The policy experiment we will consider is a shock, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, to this policy rule.

The complete set of equilibrium conditions, log-linearized around the symmetric steady state $C^H = C^S = C$, can be found in Appendix B. We think the symmetric steady state is a reasonable benchmark, however, the assumption turns out to be inconsequential for all our results, see Appendix C.5.

The model nests the RANK model ($\lambda = 0$) and the simple TANK model ($s = h = 1$). Furthermore, it nests a version without capital by considering a version with infinite adjustment cost ($\omega = 0$) and no depreciation ($\delta = 0$).

$\phi_\pi = 1$, the output response necessary to ensure determinacy needs to satisfy:

$$\phi_y > \frac{1-s}{1-\lambda}\left(\chi - \frac{1-\eta I_Y}{1-I_Y}\right),$$

and is intuitively larger when there is procyclical investment.

Table 1
Amplification of the effects of monetary policy on consumption.

	Rep. agent	Heterogeneous agents		
		Prop. incomes	Inequality	Inequality and risk
No capital	1.00	1.00	1.51	1.60
Capital	0.66	1.11	2.25	2.62

Notes: Impact multipliers on aggregate consumption of an interest-rate cut in each model, relative to the representative agent-no capital benchmark. The heterogeneous-agent models are with: no income inequality and no risk in the second column; income inequality and no risk in the third; both income inequality and risk in the fourth column.

3.1. Quantifying the complementarity

We are now ready to study the channels identified in Section 2 by considering variants of our model with and without capital as well as under different redistribution schemes for fiscal policy. To isolate the role of income inequality, we shut down the capital inequality channel by considering a version of the model without capital and no redistribution ($\tau^D = \tau^K = 0$). To isolate the role of capital inequality, we assume that financial income is fully redistributed ($\tau^D = \tau^K = \lambda$) so that all households get the same total income and differ only on the expenditure side.[11] In this way, we quantify the marginal contribution of each channel as well as their complementarity. Throughout the analysis, we focus on the response of consumption to an expansionary monetary policy shock and use the multiplier in the RANK model without capital as benchmark.

We parameterize the model as follows. The time period is a quarter, implying a discount factor β of 0.99 and a depreciation rate δ of 0.025. We assume logarithmic utility in consumption and unit labor supply elasticity ($\sigma = 1$, $\varphi = 1$), a capital share of $\alpha = 0.33$ and capital adjustment costs delivering an investment elasticity to marginal Q of 10. The Phillips curve is relatively flat with slope $\psi = 0.05$, the Taylor coefficient is 1.5, and the shock persistence is 0.6. All of these values are standard in the literature. We set the share of hand-to-mouth to $\lambda = 0.27$, in line with the estimates of Kaplan et al. (2014) and Cloyne et al. (2020). We start by abstracting from idiosyncratic risk ($s = 1$) to underscore that the channels emphasized in this paper are present even in the absence of risk and precautionary behavior. Later, we turn idiosyncratic risk back on and analyze how our results are affected.

In Table 1, we record the impact multipliers on consumption for an expansionary monetary policy shock across different specifications, relative to the response in RANK without capital. The first column reveals that introducing capital has a dampening effect in the representative-agent case: the multiplier becomes just two-thirds of that in the model without capital. On the other hand, capital has an amplifying effect of 11% in the heterogeneous-agent model of column (2) with full income redistribution. This is the *capital inequality channel* that we have isolated in Section 2 at work.

In the model with no capital and no income redistribution in the third column, the effects of monetary policy on consumption are magnified by a factor of 1.51. This amplification works through the *cyclical income inequality channel* of Bilbiie (2008).[12] Finally, capital and income inequality *jointly* yield a multiplier of 2.25, which is substantially larger than the product of the two channels in isolation (1.11×1.51): the complementarity is quantitatively significant. This is the *multiplier of the multiplier*.

The previous analysis abstracts from idiosyncratic risk and different degrees of asset liquidity, which lie at the center of heterogeneous-agent models (i.a. Kaplan et al. 2018; Bayer et al. 2019). Our framework allows us to incorporate these features in a tractable way, where idiosyncratic uncertainty pertains to households' switching between types. We now turn these channels on, by assuming that savers face a 2% probability to become hand-to-mouth, $s = 0.98$.[13]

The results are depicted in the last column of Table 1. Idiosyncratic risk generates further amplification, especially in models with capital investment, thereby reinforcing the complementarity that we have identified ($2.62 > 1.11 \times 1.60$).[14] It is also interesting to note that idiosyncratic risk amplifies the capital inequality channel even when income is perfectly redistributed. In contrast, in the model without capital, idiosyncratic risk only has an effect if incomes are not proportional. Finally, we note that capital and income inequality are still quantitatively important in shaping the amplification of the effects of monetary policy on consumption, even when compared to the idiosyncratic risk channel. An important difference, however, is that idiosyncratic risk magnifies not only the output and consumption responses but also the investment response, which in contrast gets dampened by the other channels.

[11] Taxing capital affects both the dynamics and the steady-state capital stock. In Appendix C.6, we show that our results are robust to keeping the latter fixed across specifications.

[12] Note that this model collapses to the representative agent model under full redistribution.

[13] Our stark notion of illiquidity implies that savers hit by a negative shock cannot take any capital and stocks with them. In Appendix B.5, we alternatively model capital as perfectly liquid: savers can *also* use capital to self-insure, so that liquidity is in positive supply. The results are comparable.

[14] Strictly speaking, evaluating the complementarity under idiosyncratic risk actually requires the multiplier of the model with proportional incomes and risk, which is slightly larger, 1.15 instead of 1.11. To avoid repetition in the no capital case (where risk is irrelevant), we did not include these multipliers in Table 1 but present them in Table C.1 in the Appendix.

Table 2
The role of sticky wages.

	Rep. agent	Heterogeneous agents		
		Prop. incomes	Inequality	Inequality and risk
No capital	1.00	1.00	1.01	1.02
Capital	0.94	1.53	1.77	1.95

Notes: Impact multipliers on aggregate consumption of an interest-rate cut in each model with sticky wages, relative to the rep.-agent no-capital benchmark with sticky wages. The heterogeneous-agent models are with: no income inequality and no risk in the second column; income inequality and no risk in the third; both income inequality and risk in the fourth.

3.2. Fiscal redistribution

Our results suggest that the redistribution of income plays an important role in the transmission of aggregate-demand shocks. Yet so far, we have only analyzed two polar cases: full or no redistribution. An important question in models with multiple assets and different sources of financial income is how different types of income are redistributed and how this alters the transmission mechanism. In this section, we analyze two other relevant cases within the most general model specification with risk, capital and income inequality: (i) when only capital income is redistributed, and (ii) when only monopoly profits are redistributed.

The main finding is that redistributing only capital income amplifies further the effects of monetary policy shocks: the consumption multiplier becomes 4.34 instead of 2.62 (the case with no redistribution, bottom right entry of Table 1). The intuition is that capital income is *highly procyclical*, hence its redistribution towards constrained households makes their income more cyclical. This, in turn, increases the slope of the Keynesian cross and boosts the consumption multiplier. In contrast, redistributing monopoly profits, which are countercyclical, dampens the income cyclicality of hand-to-mouth agents and can even reverse the aggregate-demand amplification: the effect of monetary policy on consumption goes down to 0.5. See Appendix C.2 for details.

3.3. Sticky wages

We have shown that the redistribution of financial income can have large effects on the cyclical properties of the model. One potential concern, however, is that markups and thus profits are countercyclical herein. An avenue that the literature pursued to overstep this unappealing feature of the New-Keynesian framework are wage rigidities.[15] With rigid wages, a demand expansion makes marginal costs increase by less, markups fall by less and sales increase by more, which mitigates the response of profits.

We introduce wage rigidities following Colciago (2011), assuming that the labor union faces wage-setting frictions: the nominal wage can only be re-optimized with a constant probability $1 - \theta_w$. This gives rise to a standard wage Phillips curve that connects nominal wage inflation to wage markups. We parameterize the slope of the wage Phillips curve to 0.075, which in a Calvo interpretation and given the other parameter values implies an average wage spell of slightly more than four quarters. The results of all models with sticky wages are recorded in Table 2, relative to the (sticky-wage) representative agent benchmark.

Two main results emerge from Table 2. First, the complementarity between capital and income inequality is robust to introducing sticky wages, both without ($1.77 > 1.53 \times 1.01$) and with idiosyncratic risk ($1.95 > 1.61 \times 1.02$, where $1.61 > 1.53$ is the multiplier with proportional incomes and risk from Table C.1, see also footnote [14]). Second, capital inequality and income inequality, on their own, generate modest additional amplification over and above sticky wages. While this is apparent for income inequality by moving across the columns of the first row of Table 2, it can be appreciated for capital inequality by comparing the first two columns of the second row with their flexible wage counterparts in Table 1. Specifically, the impact of sticky wages (relative to the flexible wage case) on the monetary transmission to consumption in the representative agent model with capital is as large as its *relative* impact in the proportional income model with capital (i.e. the ratio between the representative agent cases with sticky and flexible wages is $0.94/0.66 \approx 40\%$, which is very close to the ratio of $1.53/1.11$ between sticky and flexible wage models under proportional incomes).

In summary, sticky wages, by introducing an additional source of non-neutrality, amplify significantly the effects of aggregate-demand shocks on consumption in both the representative-agent and proportional-incomes cases. In the presence of both capital and income inequality, however, sticky wages alter transmission only modestly (i.e. 1.95 and 1.77 in Table 2 under sticky wages are actually smaller than 2.62 and 2.25 in Table 1 under flexible wages) and the bulk of the monetary policy amplification still comes from the complementarity between capital and income inequality.

[15] See for instance Colciago (2011) for an early two-agent model and Broer et al. (2020) in the context of the recent HANK literature.

3.4. Sensitivity analysis

In this section, we analyze the robustness of our amplification mechanism quantitatively with respect to a wide range of empirically plausible values for capital adjustment costs[16], the intertemporal elasticity of substitution (IES) and price and wage stickiness.

The findings are illustrated in Fig. 2. The column on the left (right) pertains to the case of flexible (sticky) wages. In each panel, we depict two multipliers as a function of the parameter of interest: the impact multiplier in the model with capital and income inequality (solid red line) and an artificial line capturing the multiplier that would obtain in the case of no complementarity (black dashed line labeled 'zero complementarity'). The latter is calculated as the product of the two multipliers in isolation. If the capital and income inequality line is above the zero complementarity line, this means that the two channels are complementary to each other. The key takeaway is that the complementarity is robust within a wide empirically plausible range for the key parameters.[17]

Finally, in Appendix C we also perform a number of other sensitivity analyses, including checks concerning the role of steady-state transfers, alternative labor market settings, liquidity through government bonds, the returns to scale in labor and the sensitivity with respect to the specification of the Taylor rule. While some of these alternative model specifications can change the absolute magnitudes of the multipliers, our result of a strong complementarity between capital and income inequality turns out to be robust along all these dimensions.

4. Empirical relevance

We have shown that capital and countercyclical income inequality can amplify the effects of monetary policy on consumption substantially. In this section, we discuss the empirical relevance of our findings. We start by discussing how the model can help reconcile the empirical evidence on the aggregate effects of monetary policy. Next, we confront the theoretical predictions of our framework on the cyclicality of inequality with the available empirical evidence. Throughout, we focus on the richest version of our framework, featuring idiosyncratic risk, precautionary saving and sticky wages.

4.1. Aggregate effects

A large empirical literature studies the effects of monetary policy shocks on the macroeconomy. This literature typically finds that monetary policy has sizeable effects on output, consumption and investment. More specifically, an expansionary interest rate shock of 25 basis points typically leads to an increase in output by 0.4-0.5%, an expansion in consumption by around 0.2-0.25% and an increase in investment in the range of 0.8-1%, at the peak of the responses (see for instance Christiano et al., 2005 for the U.S., Smets and Wouters, 2003 for the euro area, and Harrison and Oomen, 2010, for the U.K.).

Fig. 3 shows the dynamic effects of a monetary policy shock on consumption, investment and output in our model. For comparison, we also report the reponses of the model with proportional incomes and the representative agent benchmark. Throughout, we define a monetary policy shock as an unexpected, mean-reverting innovation in the Taylor rule of -25 basis points (in annualized terms). We can see that the model with cyclical income inequality is able to match the empirical responses relatively well: the peak responses of consumption, investment and output are all in the same ballpark as their empirical counterparts.

Importantly, we can also see that the models without cyclical income inequality fare less well in that respect. In the representative agent model in particular, the investment response turns out to be way too responsive relative to the consumption (and output) response. This is a well-known problem in the New Keynesian literature. Introducing capital in the representative agent New Keynesian model can lead to large amplification of the effects of monetary policy on output, driven by an unrealistically large investment response (see e.g. Carlstrom and Fuerst, 2005; Dupor, 2001; Rupert and Šustek, 2019). The complementarity between capital and income inequality that we uncover in this paper helps to bring the relative consumption and investment responses closer to what we observe in the data, without resorting to implausibly high capital or investment adjustment costs. The beauty of our mechanism is that it does so by amplifying the consumption response while the investment response is only slightly attenuated.

4.2. Inequality dynamics

In Section 2.3, we derive two key theoretical predictions of our framework: both consumption and income inequality are countercyclical and consumption inequality turns out to be more countercyclical. Here, we show that these predictions

[16] Note that we express the multipliers here as a function of the capital adjustment cost parameter $\phi = 1/(\delta\omega)$ and not ω, the elasticity of investment to Tobin's Q.

[17] In Figure C.3 in the Appendix, we also present the sensitivity analysis for the absolute impact responses of all our model specifications as opposed to the multipliers relative to the representative-agent benchmark. We can see that while the absolute responses are decreasing with capital adjustment costs and the frequency of prices and wages adjustments and increasing with the elasticity of intertemporal substitution, the relative multipliers are decreasing in all these parameters.

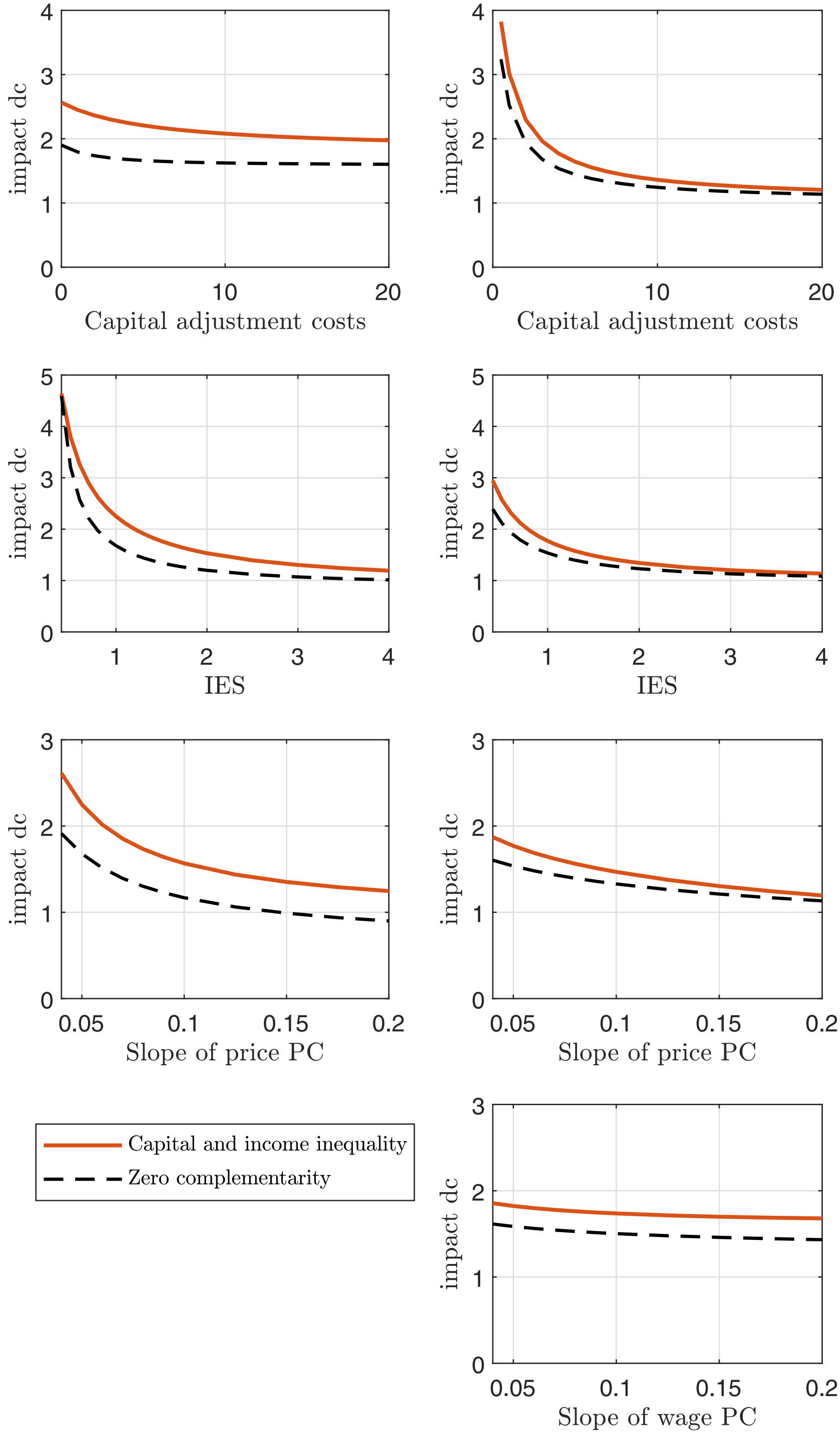

Fig. 2. Robustness of the complementarity. *Notes*: Sensitivity of the consumption impact multipliers in the model with capital and income inequality together with the artificial multiplier in the zero complementarity case under different parameterizations for capital adjustment costs, IES, and price stickiness. The red solid line shows the multiplier of the model with capital and income inequality and the black dashed line shows the product of the two multipliers in isolation (i.e. the multiplier that would obtain if the channels were not complementary). Panel (A) shows the multipliers under flexible wages, Panel (B) under sticky wages. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

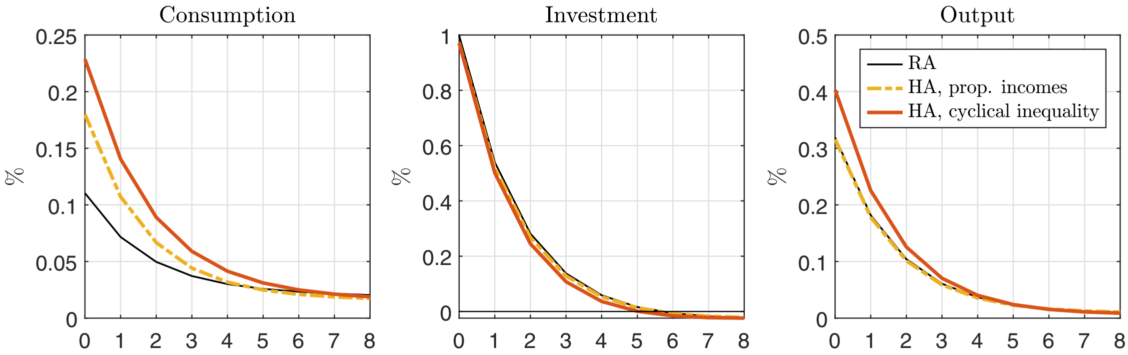

Fig. 3. Aggregate effects of monetary policy. *Notes*: Impulse responses of aggregate consumption, investment and output to an expansionary interest rate shock of 25 basis points in the representative-agent model and in heterogeneous-agent models with and without income inequality.

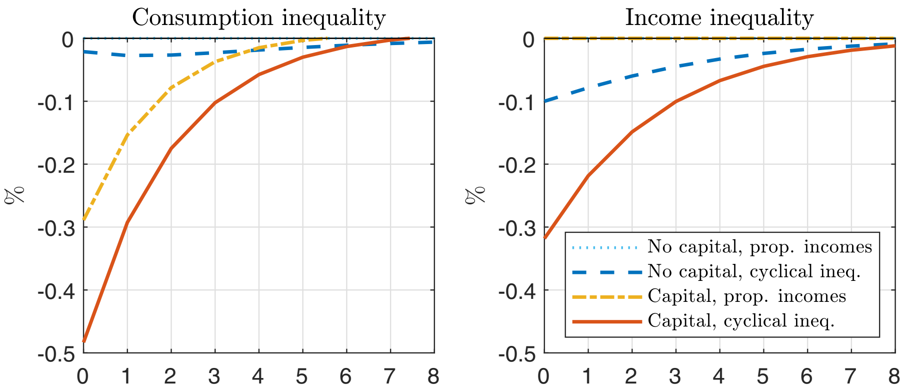

Fig. 4. Distributional effects of monetary policy. *Notes*: Impulse responses of consumption inequality and income inequality to an expansionary interest rate shock of 25 basis points in heterogeneous-agent models with and without capital inequality and with and without income inequality.

readily generalize to our richer setting with a fully-specified supply side, rigidities in prices and wages and idiosyncratic risk.

Fig. 4 shows the responses of consumption and income inequality in our model. For comparison, we also show the responses in the representative agent benchmark (no capital and proportional incomes), the model with cyclical income inequality but no capital, and the model with capital but with proportional incomes.

We see that the fully-specified model confirms our simple framework's predictions. Consumption and income inequality are both countercyclical but consumption inequality is more countercyclical. In Appendix C.3, we show that this implication of our quantitative model is robust to a wide range of plausible parameterizations.

Comparing the responses under the different model variants, we also see that having both the capital and income inequality channels is instrumental for this result. First, the model without capital and proportional incomes features no inequality dynamics since it collapses to the representative-agent model. If we only allow for cyclical income inequality, both income and consumption inequality are countercyclical but the latter is *not* more countercyclical than the former.[18] If on the other hand we allow only for the capital inequality channel, we observe a considerable drop in consumption inequality but income inequality does not change by construction.

How do these predictions compare with existing empirical evidence? A growing empirical literature studies the distributional effects of monetary policy using micro data on household consumption expenditure and income (see for instance Coibion et al. (2017) for the U.S., Ampudia et al. (2018) for the euro area and Mumtaz and Theophilopoulou (2017) for the U.K.). These studies show that following a cut in the interest rate both consumption and income inequality fall significantly. Importantly, consumption inequality robustly turns out to decline more than income inequality—in line with the predictions of our model. This illustrates the empirical relevance of the capital and income inequality channels, which we have shown to be instrumental in generating these predictions on the cyclicality of consumption and income inequality.

[18] In fact, income inequality turns out to be even more countercyclical than consumption inequality, however, this is an artifact of the constant, redistributive steady-state transfers that are used to equalize consumption in steady state. In the version of the model without the transfers, the two variables are equally cyclical, while all other implications are preserved (see Appendix C.5).

5. Conclusions

The idea that the combination of a consumption function and an investment function gives rise to amplification of aggregate demand fluctuations is an intuition that goes back to Samuelson (1939), who attributed it to Alvin Hansen in building the now famous multiplier-accelerator model.

In this paper, we explore this idea in a New Keynesian model with household heterogeneity in both income and savings and show that this gives rise to an aggregate-demand *complementarity* that is to the best of our knowledge novel to the literature. Namely, we isolate two key types of inequality, in capital and income, that each give rise to a distinct multiplier-like amplification channel. The former (segmentation in capital markets) leads to *amplification*, even when income is redistributed uniformly. This occurs as capital income is endogenously redistributed towards constrained households, who consume it and generate further demand, thus triggering a Keynesian-cross multiplier.

Counter-cyclical income inequality sets in motion further aggregate-demand amplification rounds as the income of constrained agents respond more than proportionally to fluctuations in aggregate income. We show that, together, the capital inequality and the income inequality channels engender aggregate-demand effects on consumption that are an order of magnitude larger than the mere addition of their individual effects in isolation: a strong complementarity that we call *'the multiplier of the multiplier'*.

Our theoretical framework makes predictions regarding the aggregate and distributional effects of monetary policy that are aligned with existing empirical evidence. It also has stark policy implications regarding the monetary authority's ability to stabilize the economy: when both investment and heterogeneity are of the essence, the central bank needs to be more aggressive to anchor expectations and the forward guidance puzzle is aggravated.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jmoneco.2022.01.002

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