London Business School

Sources of Risk in the Foreign Exchange Market

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A thesis submitted to the London Business School
for the degree of Doctor of Philosophy

April, 2013

Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London Business School is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

Abstract

I quantify alternative sources of risk in currency returns. Firstly, in a joint work with Mikhail Chernov and Jeremy Graveline, we focus on crash risk. We develop and estimate an empirical model of currency returns that includes normal shocks with stochastic variance, jumps up and down in the exchange rate, and jumps in the variance. We identify these components using daily data on exchange rates and at-the-money implied variances. We find that crash risk is time-varying. The probability of a jump in the exchange rate, associated with depreciation (appreciation) of the US dollar, is increasing in the domestic (foreign) interest rate. The probability of a jump in variance is increasing in the variance. Many of the jumps in exchange rates are associated with macroeconomic and political news, but jumps in variance are not. On average, jumps account for 25% of total currency risk, as measured by the entropy of exchange rate changes, over horizons of one to three months. Preliminary analysis suggests that jump risk is priced.

Secondly, I quantify the risk-return relationship in the foreign exchange market in cross-section and across investment horizons by focusing on the role of multiple sources of consumption risk. I estimate a flexible structural model of the joint dynamics of aggregate consumption, inflation, nominal interest rate, and stochastic variance with cross-equation restrictions implied by recursive preferences. I identify short-run, long-run, variance consumption risks and inflation risk. I find that the long-run consumption risk plays a prominent role: it carries a Sharpe ratio of 0.66 and contributes the most to the level and spread of excess returns between high and low interest rate currencies at alternative investment horizons. The short-run consumption risk has an effect at the horizon of one quarter only, where it explains at least 26% of the corresponding spread in excess returns.

Acknowledgements

I am deeply indebted to my advisors, Mikhail Chernov and Francisco Gomes, for continuous support throughout the years of my study at the London Business School. Mikhail has become a wonderful mentor for me from the very beginning of the PhD program. His willingness to share knowledge, patience and encouragement helped me to overcome difficult times and reach the final point of the program. He was my source of inspiration that helped me moving forward.

Also I owe a lot to Francisco who courageously took me on board in the middle of the program. The decision was courageous because occasionally he had to listen to me speaking about beauty of asset pricing and Bayesian econometrics *in Russian*. I am very grateful for his patience, support and smile and I thank him for generously sharing with me his opinion about my work and his expertise. Mikhail and Francisco were the very best aspect of my PhD studies. I was very lucky to have an opportunity to learn from these people.

My thanks also go to many people who were giving me helpful advice at different stages of the PhD program, including David Backus, Jaroslav Borovička, Magnus Dalhquist, Jack Favilukis, Vito Gala, Christian Heyerdahl-Larsen, Christian Julliard, Igor Makarov, Stavros Panageas, Anna Pavlova, Hélène Rey, Paolo Surico, and Stanley Zin. In addition, I would like to thank my friends and colleagues – Jose Carabias, Afonso Januario, Sara de Masi, Alessandro Graniero, Sapnoti Eswar, Anya Kleymenova, Sunyoung Lee, Lusine Mkrtchyan, Mauricio Prado, Boris Radnaev, Oleg Shibanov, and Yili Zhang, for their kind support and encouragement.

I thank the participants of the 2012 annual meeting of the Society of Economic Dynamics and seminars at Bocconi University, HEC Paris, London Business School, London School of Economics, New Economic School, Rutgers Business School, Stockholm School of Economics, UCLA Anderson School of Management, and UNC Kenan-Flagler Business School for their helpful feedback, comments and suggestions.

This work was funded by the doctorate fellowship generously provided by the AXA research fund. I gratefully acknowledge this financial support.

Finally, I cannot but mention that I dedicate this work to my beloved mother who always believes in me and whose support has made all this possible.

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Chapter 1

Introduction

This thesis systematically studies the risk-return relationship in the foreign exchange market. High average profitability of carry trades (manifestation of the violation of the uncovered interest rate parity) and apparent variation in the currency risk premium have stimulated a lot of research in the recent past. A large body of theoretical literature is dedicated to design realistic equilibrium models rationalising the forward premium anomaly. Examples include, but not limited to, models with habits (Heyerdahl-Larsen, 2012; Verdelhan, 2010), long-run risks (Bansal and Shaliastovich, 2013; Colacito, 2009), disasters (Farhi and Gabaix, 2008; Gourio, Siemer, and Verdelhan, 2012; Guo, 2007), limited market participation (Alvarez, Atkeson, and Kehoe, 2009; Bacchetta and Wincoop, 2010), learning (Yu, 2013), and investor sentiment (Burnside, Han, Hirshleifer, and Wang, 2011; Ilut, 2012). Additionally, researchers empirically evaluate quantitative success of various macro-based and return-based factors trying to account for the cross-sectional properties of observed currency returns, thereby providing a risk-based explanation for high average profitability of carry trades (e.g., Lustig and Verdelhan, 2007; Lettau, Maggiori, and Weber, 2012).

Understanding the nature of risk in the foreign exchange market is instrumental both for the theoretical general equilibrium literature and empirical studies. On the one hand, structure of risk, i.e., assumptions about fundamental shocks, is the necessary part of any theoretical model. Therefore, direct evidence on types of shocks that matter for currency returns can serve as a first step towards testing any theory of currency risk premium. Alternatively, the structure of risk in currency markets sheds light on decomposition of currency risk premium and helps to interpret findings of empirical literature at a fundamental level. Ultimately, in the world with multiple sources of risk researchers are interested in measuring currency risk premiums due to different shocks in isolation, for example, (1) real versus nominal shock, or (2) normal versus crash risk.

Chapter 2, based on joint work with Mikhail Chernov and Jeremy Graveline, quantifies normal and crash risk in the distribution of exchange rates. Several authors propose to seek the origins of currency risk premia in the currencies' exposure to crash risk (e.g., Brunnermeier, Nagel, and Pedersen, 2008). The merit of this explanation critically depends on metrics associated with crashes, i.e., probabilities and magnitudes of crash events. Our study contributes to the existing literature by estimating the characteristics of crash risk, describing the determinants of crash events and documenting when these events occurred historically.

To characterize crash risk, we develop and estimate an empirical model of exchange rate evolution that allows for normal and jump risk. The model incorporates a stochastic variance with jumps as well as upward and downward jumps in the currency price. The primer accommodates an apparently high kurtosis of the FX implied volatility. The latter captures mild skewness of currency returns along with the presence of dramatic moves in currency prices in either direction on a distinct day.

We estimate the model on daily data for foreign exchange rates and at-the-money volatilities for four currency pairs: the Australian dollar, the British pound, the Swiss franc, and the Japanese yen versus the US dollar. We document a number of interesting findings: (1) jumps in currency price and its variance are time-varying: the probability of jump up (down) in the exchange rate, associated with a depreciation (appreciation) of the US dollar, is positively related to the US (foreign) interest rate, whereas the probability of a jump in currency variance is positively controlled by variance itself (2) many jumps in currency price can be linked to specific macro-economic or political events and announcements, whereas jumps in variance are associated with uncertainty, market anxiety and unrevealed expectations, (3) jumps are quantitatively important: on average jumps account for 25% (and can be as high as 40%) of total currency risk, as measured by the entropy of exchange rate changes over horizon of one quarter; (4) contribution of jumps in variance to the total risk increases with the investment horizon.

Additionally, we use our model to perform a back-of-envelope computation that could suggest whether jump risk is priced in currency markets. To this end, we explore the shape of the volatility smiles derived under the null of the model. Under the assumption of zero jump risk premia, the model captures the asymmetry of the volatility smile in the data but cannot replicate its curvature even when we account for statistical uncertainty. We think this evidence suggests that jump risk may be priced in the foreign exchange market and leave a careful investigation of the issue for future research.

As it has become evident in Chapter 2, normal or regular risk is the most prominent component of total currency risk regardless of investment horizon. In Chapter 3, I thoroughly study the origins of normal risk in currency markets through the lens of a structural model. In particular, I investigate the role of multiple sources of US consumption risk in the cross-section of currency baskets across alternative investment horizons.

The motivation for my study starts from the following two observations. First, Lustig and Verdelhan (2007) document that (1) sorting currencies based on their respective interest rates in baskets forms an interesting cross-section of currency returns and (2) variation in realized consumption growth is responsible for the cross-section of currency returns at a fixed investment horizon that corresponds to the decision interval of the representative agent. Second, even though studying the role of multiple sources of consumption risk on the cross-section of equity returns is a popular area of interest (Bansal, Dittmar, and Lundblad, 2005; Campbell, Giglio, Polk, and Turley, 2012; Hansen, Heaton, and Li, 2008), there has been no research yet on the cross-section of currency returns. In a nutshell, my contribution is in expanding analysis of Lustig and Verdelhan (2007) to alternative investment horizons and characterizing multiple sources of consumption risk.

To identify multiple sources of consumption risk, I estimate a flexible structural model of the joint dynamics of aggregate consumption, inflation, nominal interest rate, and stochastic variance with cross-equation restrictions implied by recursive preferences. The model captures the spirit of the long-run risk models. I employ an important innovation – instead of writing down a model of consumption growth as an explicit function of latent states (e.g., as it is done in Bansal and Yaron, 2004), I model consumption growth jointly with an asset price in a vector autoregression. In particular, I exploit the fact that a theoretical equilibrium asset price is a function of unobservable factors of consumption growth. As a result, I gain flexibility and a possibility to estimate expected consumption growth more precise than it would be possible otherwise. The cost of these improvements is reflected in the cross-equation and identifying restrictions that I have to impose in order to guarantee that the model is internally consistent and to meaningfully interpret consumption risks, respectively.

I gauge the importance of multiple sources of consumption risk by computing term-structure of marginal prices and quantities of risk associated with the cross-section of currency baskets (shock-exposure and shock-price elasticity of Borovička and Hansen, 2011). I find that the risk that exerts the dominant cumulative impact on consumption growth in the long-run (long-run consumption risk) plays the most prominent role in the FX market. Firstly, baskets of high and low interest rate currencies have significantly different exposures to the risk across horizons from one quarter to ten years. Secondly, the multi-period price of the risk is economically meaningful and statistically significant (e.g., the one-period average Sharpe ratio is 0.66). Therefore, there is a non-trivial cross-section of currency risk premia associated with the long-run consumption risk at short and medium-term horizons.

Inflation risk matters both in the cross-section of currencies and across alternative horizons if consumption growth reacts contemporaneously to the inflation shock. However, the corresponding cross-section of currency risk premia is less pronounced than one associated with the long-run consumption risk: the multi-period price of the inflation risk is less than half of that for the long-run consumption risk. Alternatively, the short-run consumption risk generates the cross-section of currency risk premia only at a single investment horizon corresponding to the decision interval of the representative agent (i.e., one quarter in my study). At multiple horizons, the risk is priced in the economy but currencies are immune to it. Finally, currencies universally exhibit high sensitivity to the variance risk at horizons longer than three years, however these risk exposures are associated with small prices of risk.

Chapter 2

Crash risk in currency returns

2.1 Introduction

The time variation and high magnitude of returns to currency speculation have attracted a lot of recent attention. Much of the literature has focused on measuring risk premiums, or expected excess returns, in this market (e.g., Lustig and Verdelhan, 2007). However, expected returns alone do not tell the whole story. Investors also care about the risks that they must bear to earn these returns. Therefore, the distribution of risks is an important ingredient in understanding currency premiums.

A number of papers have suggested that investors in currency markets require high returns on average as a compensation for crash risk. The merit of this explanation hinges on the magnitude and probability of large moves in currency markets. Our paper is the first empirical study that systematically quantifies crashes, documents when they occurred historically and what their determinants were.

It is impossible to characterize crash risk without modelling regular, or normal, risk. This is because one has to be able to establish whether a large move in the exchange rate takes place because of a crash, or because the conditional variance of a normal shock is high. Option-implied volatilities are particularly helpful here because they provide information

[0]This chapter is based on Chernov, Graveline, and Zviadadze (2012).

about the market's view on conditional variance. This is why we develop and estimate a model that incorporates both types of risks and information from both exchange rates and implied volatilities.

We establish the relative importance of three key modelling elements. First, it is well-documented that currency returns are heteroscedastic (e.g., Baillie and Bollerslev, 1989; Engel and Hamilton, 1990; Engle, Ito, and Lin, 1990; Jorion, 1988; Harvey and Huang, 1991). Casual observation of time-series variation in option-implied exchange rate volatility also confirms this point. We capture this feature of the data with a standard stochastic volatility component in our model.

Second, there is also strong empirical evidence that daily changes in exchange rates are not conditionally Gaussian (as would approximately be the case in a model with only stochastic volatility). To account for this feature of the data, our model includes jump risks in exchange rates. We allow the probability of these jumps to be time-varying, in order to capture the variation in conditional skewness that has been previously documented (e.g., Bakshi, Carr, and Wu, 2008; Brunnermeier, Nagel, and Pedersen, 2008; Carr and Wu, 2007; Johnson, 2002).

Third, changes in the at-the-money implied volatility of a typical exchange rate exhibit unconditional skewness of 1 and kurtosis of 10 or more. To accommodate this property, our model allows for jumps in the variance of Gaussian shocks to exchange rates. The importance of such jumps for modelling equity returns has been emphasized in Broadie, Chernov, and Johannes (2007); Duffie, Pan, and Singleton (2000); Eraker, Johannes, and Polson (2003), among others. To our knowledge, our paper is the first to investigate the role of jumps in the volatility of exchange rates.

A jump in an exchange rate is qualitatively different from a jump in its variance. Almost by definition, large jumps are rare events. Therefore, when there is a direct jump in the exchange rate, one doesn't necessarily expect there to be many subsequent jumps in the near future. By contrast, when there is a jump in the variance of the Gaussian shock to an exchange rate, one expects there to be many large subsequent moves in the exchange rate because of the high new level of variance. We use our model and empirical analysis to determine whether these qualitative distinctions lead to materially quantitative differences.

We use daily joint data for exchange rates and implied volatilities from 1986 to 2010 (the options data start in 1994) on four spot exchange rates: Australian dollar, Swiss franc, British pound, and Japanese yen. Crashes are rare and volatility is persistent, so it is important to use a long time span of data. Short samples are likely to either over- or under-represent jumps and periods of high or low volatility leading to biased estimates of the required probabilities.

We employ Bayesian MCMC to estimate candidate models. One of the key advantages of this approach is that it provides estimates of the conditional distribution of currency returns, as well as estimates of the realized shocks. This feature allows us to link large shocks, or jumps, to important macro-finance events and thereby illuminate the potential economic channels that are responsible for crash risk in currencies.

Our statistical tests strongly favour both jumps in exchange rates and in their variances. This conclusion is similar to the one in the equity literature. However, this is where the similarity ends. In contrast to equity models that favour one Poisson (counting) process controlling the arrival rate of all jumps, we find three such processes in FX. The three types of jumps arise via different mechanisms. The arrival rate of a jump in the variance of currency returns is positively related to the variance itself. Thus, this component belongs to the class of self-exciting processes. The probability of a jump up in the exchange rate, which corresponds to a depreciation of the US dollar, is positively related to the domestic (US) interest rate. The probability of a jump down, which corresponds to an appreciation of the US dollar, is positively related to the foreign interest rate.

Although jumps in currencies and in variance are alternative channels for large currency returns, we find that economically they are quite distinct. We can connect most of the jumps in FX to important macro or political announcements. In contrast, jumps in variance cluster at the moments of high uncertainty in the markets, which are captured by comments on current events, political speculation and overall anxiety about upcoming events.

We use entropy (a generalized measure of variance) of changes in an exchange rate to measure the amount of risk associated with currency positions and to decompose this risk into the contributions from different sources of shocks (Alvarez and Jermann, 2005; Backus, Chernov, and Martin, 2011). Appropriately scaled entropy is equal to the variance of an exchange rate return if it is normally distributed, but otherwise includes high-order cumulants. Therefore, entropy is a convenient measure that captures both normal and tail risk in one number. We find that, depending on the currency, the time-series average of the joint contribution of the three types of jumps can be as high as 25% of the total risk and on individual days this contribution can be up to 40%. Jumps in variance contribute about a third to the average contribution and can be as high as 15% of the total risk on individual days. Also, the contribution of jumps in variance to the total risk increases with investment horizon.

Given the large contribution of jumps to the overall risk, it is natural to ask whether the jump risk is priced. The full answer to this question requires an explicit model of the pricing kernel and the use of assets, such as out-of-the-money options, that are particularly sensitive to jumps for estimation. While such analysis is outside of the scope of this paper, we carry out a limited option valuation exercise. We select representative implied volatility smiles for currencies with positive and negative interest rate differential. Such smiles exhibit positive and negative skewness, respectively, in the data. Our model can replicate the same sign of skewness even when we assume zero premiums for jump risk. However, these theoretical smiles cannot match the curvature of the smile observed in the data, even after accounting for statistical uncertainty. In our view, this initial evidence suggests that jump risk may be priced.

Related literature

We limit our discussion of related literature to papers that highlight the importance of jumps for understanding the properties of exchange rate returns. One exception are the works of Brandt and Santa-Clara (2002) and Graveline (2006). These papers are early antecedents of our paper in terms of methods and research questions. These authors also estimate a time-series model of exchange rates using the time-series of FX and implied variance. However, they do not allow for jumps.

Our paper is related to recent empirical papers that investigate whether the high currency returns can be explained as compensation for jump, or crash, risk. Brunnermeier, Nagel, and Pedersen (2008) provide evidence consistent with a hypothesis that large exchange rate moves are related to funding constraints of speculators enagaged in carry trades. In particular, they relate the sign and magnitude of skewness of various exchange rates relative to the USD to those of the respective interest rate differentials. Jurek (2009) analyzes the returns on carry trade portfolios in which the exposure to currency crashes is hedged with options. He concludes that exposure to currency crashes account for 15% to 35% of the excess returns on unhedged carry trade portfolios. Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) investigate whether carry trade returns reflect a "peso problem" (i.e., a low probability event that did not occur in the sample). They use carry returns hedged with options to argue that any such peso event must be a modest negative return on the carry trade combined with an extremely large value of the stochastic discount factor (i.e., the marginal utility of a representative investor must be very high in the, as yet, unobserved peso state). Jordà and Taylor (2012) propose to manage the risk of carry positions by conditioning on macro information instead of options, but the resulting strategy still yields a very high Sharpe ratio. The common thread in these papers is that they provide indirect evidence on the magnitude of jump risk. Our paper aims to complement this previous work with a formal statistical model and analysis.

Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2012) use an explicit model of exchange rates that allows for both normal and jump risks. Under the model assumptions, short-dated at-the-money options are not exposed to crash risk. Therefore, hedged carry trades are exposed to normal risk alone. In contrast, carry trades are exposed to both types of risk. This property allows the authors to quantify the contribution of jump risk by observing returns on hedged and unhedged portfolios. However, similar to the aforementioned papers, the authors do not test the assumed model directly.

Our paper is also related to the option pricing literature, which has focused on modeling the risk-adjusted (risk-neutral) distribution of exchange rates. By construction, these papers do not consider risk premiums. However, the shock structures under the risk-adjusted and actual (true) distributions are usually modelled to be similar. Bates (1996) considers option prices on the Deutsche Mark and is the earliest paper that argues for the inclusion of jumps in currencies. He considers a single normally distributed jump in FX with a constant probability. Carr and Wu (2007) distinguish jumps up and down in FX and also allow for time-varying jump probabilities controlled by unobservable states. Bakshi, Carr, and Wu (2008) extend the Carr-Wu model to a triangle of currencies (GBP, JPY, and USD) and estimate it using 2.25 years of data on exchange rates and option prices. Our analysis provides additional economic intuition, as time variation in jump probabilities are driven by observable interest rates. None of these papers consider jumps in variance or estimate jump times and sizes.

There is also an important literature that attempts to explain the behaviour of exchange rates in macro-founded equilibrium.[1] Our paper is silent about the prices of risk, but it may have implications for how to best model the fundamentals in an equilibrium setting. Gourio, Siemer, and Verdelhan (2012) and Guo (2007) propose production-based models with recursive preferences. Productivity is allowed to experience a disastrous decline with time-varying probability. Farhi and Gabaix (2008) consider a pure exchange economy with additive preferences and a similar assumption of time-varying probability of a disaster in consumption. Disasters are modelled as jumps down, and all three papers allow unobservable processes to drive disaster probabilities. Exchange rates inherit these properties. Our results suggest that it may also be important to allow for jumps in the volatility of these processes and for the process driving probability of jumps in consumption to be related to interest rates in equilibrium.

Our results speak also to the frictions-based equilibrium model of Plantin and Shin (2011). These authors focus on endogenously generated dynamics of a carry trade. A carry trade gets started in a high-liquidity environment, such as accommodative monetary policy. It is self-enforcing because of the speculators' belief that others will join the trade. The trade crashes when the speculators hit funding constraints. As a result, extended periods of slow appreciations of a high interest rate currency are randomly interrupted by endogenous crashes. Because our analysis is implemented at the daily frequency, we are able to capture, in reduced form, related phenomena.

[1] Examples include, but not limited to Bekaert (1996); Backus, Gavazzoni, Telmer, and Zin (2010); Bansal and Shaliastovich (2013); Colacito (2009); Colacito and Croce (2010).

2.2 Preliminaries

This section motivates our analysis and highlights properties of the data that our model is designed to capture.

2.2.1 Excess returns

Let r_t be the continuously-compounded domestic (e.g., USD) interest rate, $\tilde{r}_t$ be the analogous foreign (e.g., GBP) interest rate, and S_t be the exchange rate expressed as units of domestic currency per unit of foreign currency. Then borrowed $\exp(-r_t)$ units of the domestic currency buys $1/S_t \cdot \exp(-r_t)$ units of the foreign currency at time t, which grows at the foreign risk free interest rate to $1/S_t \cdot \exp(\tilde{r}_t - r_t)$ units at time $t+1$, and can be exchanged for $S_{t+1}/S_t \cdot \exp(\tilde{r}_t - r_t)$. Then the amount borrowed in domestic currency (with interest) can be repaid. Thus, the log excess return to investing in the foreign currency is

$$y_{t+1} = (s_{t+1} - s_t) - (r_t - \tilde{r}_t),$$

where $s_t = \ln S_t$. In this paper, we will always treat USD as the domestic currency.

Figures 2.1 - 2.4 display the time series of log excess returns, y_{t+1} (panel (a)), and implied volatilities (panel (b)) for the currencies we consider in this paper. We have selected four currencies - Australian Dollar (AUD), Swiss Franc (CHF), British Pound (GBP), and Japanese Yen (JPY) based on the availability of daily data, and cross-sectional and time-series variation in the interest rate differential. We use one-month LIBOR to proxy for interest rates. Using one-month rather than overnight rates implicitly assumes a flat term structure at the very short end of the LIBOR curve and allows us to abstract from potential high-frequency idiosyncratic effects associated with fixed-income markets. Because we treat USD as a domestic currency, the movements up correspond to depreciation in the USD.

2.2.2 Properties of excess returns

We provide summary statistics of daily log excess returns and changes in the one-month at-the-money implied volatility in Table 2.1. Means are close to zero at daily frequency. Therefore, these summary statistics inform us primarily about the properties of shocks.

All currencies have volatility of about 10% per year. There is evidence of substantial kurtosis (AUD and JPY are the most notable in this regard), which is suggestive of non-normalities. Skewness of all currencies is mild. It turns out that this is a manifestation of time-varying and sign-switching conditional skewness. We produce a rough estimate of conditional skewness by computing a six-month rolling window. The time-series of these estimates are displayed in panels (a) of Figures 2.1 - 2.4. Depending on the currency, conditional skewness ranges from -2 to 2. Thus, excess returns are not only fat-tailed, but also asymmetric with the degree of asymmetry changing over time.

The implied volatility is itself quite variable at about 60% per year (the number in the table multiplied by $\sqrt{252}$) and highly non-normal with skewness and kurtosis much higher than that of the currency returns themselves. The implied volatility from the short-dated options should be very close to the true volatility of exchange rates (which is unobservable) and therefore its properties provide insight into the features that a realistic model of variance must require.

As a reference, we report the same summary statistics for S&P 500 whose risks were thoroughly studied in the literature. The index returns are more volatile and exhibit much stronger departures from normality as compared to currencies. In particular, negative unconditional skewness is evident (in fact, a measure of conditional skewness becomes positive rarely). In contrast, changes in VIX, a cousin of implied variance, display weaker non-normalities than currencies. These statistics suggest that a model of currency risks could be substantively different from that of equity risks even though one clearly has to use similar building blocks.

2.2.3 Risks and expected excess returns

We can generically represent excess returns as:

$$y_{t+1} = E_t(y_{t+1}) + \text{shocks}. \tag{2.2.1}$$

Most of the research is focused on conditional expected excess returns $E_t(y_{t+1})$. For example, if currencies do not carry a risk premium, then uncovered interest rate parity (UIP) holds and $E_t(y_{t+1}) = 0$. However, Bilson (1981), Fama (1984), and Tryon (1979) establish that the regression

$$s_{t+1} - s_t = a_1 + a_2(f_t - s_t) + \text{shocks}, \tag{2.2.2}$$

where f_t is is the log of the one-month forward exchange rate, typically yields estimates of a_2 of approximately -2. If covered interest rate parity (i.e., no-arbitrage) holds, then the log forward exchange rate is given by $f_t = s_t + r_t - \tilde{r}_t$, therefore this result is equivalent to:

$$y_{t+1} = a_1 + (a_2 - 1)(r_t - \tilde{r}_t) + \text{shocks}, \tag{2.2.3}$$

with a slope coefficient of about -3. Subsequent research has extended the specification of risk premiums $E_t(y_{t+1})$ (e.g., Beber, Breedon, and Buraschi, 2010; Bekaert and Hodrick, 1992; Lustig, Roussanov, and Verdelhan, 2011; Menkhoff, Sarno, Schmeling, and Schrimpf, 2011, among others).

Asset pricing theory relates expected excess returns to compensation for bearing risks, that is, it relates "$E_t\left(y_{t+1}\right)$" to the "shocks" in equation (2.2.1). In the language of pricing kernels, expected excess returns are determined by the covariation of currency risks with a pricing kernel. We don't empirically test any specific asset pricing theories (pricing kernels) in this paper, but a thorough analysis of the shocks is a necessary ingredient for full testing of any dynamic asset pricing model. To illustrate this point, we provide an example of two theories that can lead to identical expected excess returns despite the different shock structures (Appendix A.1.1). In this situation, the distribution of shocks is the only element that can distinguish one theory from the other.

To measure shocks, we need to model conditional means as well. We use a simple specification that encompasses the UIP regressions result by allowing for linear dependence on the domestic and foreign interest rates, and includes variance of FX returns as an extra variable.[2] Because we are working with daily returns, the magnitude of the drift term is much smaller than the higher order moments and so any omitted variables that might affect expected returns are not likely to introduce much bias in our results. Verdelhan (2011) provides direct evidence supporting this conjecture. As such, to avoid overfitting, we did not include any other variables in the drift of the exchange rate.[3] While our focus is on careful modelling of risks of currencies themselves, our conclusions should have implications for modelling of economic channels leading to the observed risk premiums. As highlighted by our examples in Appendix A.1.1, successful equilibrium models should be able to replicate not only the measured risk premiums, but the distribution of currency shocks as well. To this end, our model can be used to construct portfolios that isolate jump risks and serve as inputs to traditional factor models that examine the pricing of these risks. Moreover, our extensive analysis of the shocks to currency returns provides useful guidance for specifying shocks to fundamentals in equilibrium models.

2.3 Empirical model

We start by presenting our empirical model in Section 2.3.1. Section 2.3.2 discusses how we arrived at the assumed functional forms.

2.3.1 Currency dynamics

In this paper we model each exchange rate in isolation from others. A large fraction of currency analysis, such as UIP regressions or equilibrium modelling is conducted on a currency-by-currency basis. This approach is able to identify the normal and non-normal shocks, and

[2]This addition can be supported in various theoretical settings (Bacchetta and van Wincoop, 2006; Brennan and Xia, 2006). Empirical work with such a term includes Bekaert and Hodrick (1993), Bekaert (1995), Brandt and Santa-Clara (2002), Domowitz and Hakkio (1985), Lustig, Roussanov, and Verdelhan (2011), and Menkhoff, Sarno, Schmeling, and Schrimpf (2011).

[3]A recent literature suggests improving inference about conditional mean of excess returns by considering portfolios of currencies (e.g., Barroso and Santa-Clara, 2011; Lustig, Roussanov, and Verdelhan, 2011; Lustig and Verdelhan, 2007; Menkhoff, Sarno, Schmeling, and Schrimpf, 2011).

how they should be modelled. However, we cannot say which fraction of shocks can be explained by common variation in the exchange rates, and which fraction is country-pair specific. The important question of modelling the joint distribution of currency risks goes hand-in-hand with modelling of the pricing kernel and we leave this investigation for future research.[4]

We model log excess FX returns as

$$y_{t+1} \equiv (s_{t+1} - s_t) - (r_t - \tilde{r}_t) = \mu_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \,, \tag{2.3.1}$$

where w_{t+1}^s is a standard Gaussian shock (i.e., zero mean and unit variance), z_{t+1}^u is a jump up (i.e., depreciation of USD) and the negative of z_{t+1}^d is a jump down (i.e., appreciation of USD). The conditional spot variance is v_t and the jump intensities of z_{t+1}^u and z_{t+1}^d are h_t^u and h_t^d respectively.[5] The discussion of μ_t is postponed until we have further described these three shocks.

The conditional spot variance v_t is assumed to follow a mean-reverting "square-root" process,

$$v_{t+1} = (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \,, \tag{2.3.2}$$

which itself can jump with intensity h_t^v.[6] The shocks to excess returns w^s and to conditional spot variance w^v have a correlation coefficient $\text{corr}\,(w^s, w^v) = \rho$. Finally, to ensure positivity of the variance when jumps are present, we only allow for upward jumps so that z_{t+1}^v has non-negative support.

[4] Lustig, Roussanov, and Verdelhan (2011), Sarno, Schneider, and Wagner (2012) perform such modelling allowing normal shocks only. Bakshi, Carr, and Wu (2008) model a triangle of currencies (GBP, JPY, and USD) allowing for jumps in FX.

[5] This specification can be viewed as a discrete-time model or as a Euler discretization of a continuous-time model (see, e.g., Platen and Rebolledo, 1985 for semimartingales). In any case, a discrete-time model is required at the estimation stage, and which is why we omit explicit continuous-time formulation. Formally, all shocks, even the Gaussian variables, are jumps in discrete time. We model small jumps via Gaussian shocks and large jumps via the compound Poisson process. We distinguish the small and the large jumps by imposing the respective priors at the estimation stage. We apply the term jump to the large component only for the ease of referral.

[6] In continuous time, the Feller condition $\sigma_v^2 < 2v(1-\nu)$ ensures that the variance stays positive if there are no jumps. A formal modelling of this process in discrete time is achieved via a Poisson mixture of Gamma distributions (e.g., Gourieroux and Jasiak, 2006; Le, Singleton, and Dai, 2010). We use a direct discretization of the continuous-time counterpart so that the model parameters can be easily interpreted. We ensure that the variance stays positive at the estimation stage by a careful design of the simulation strategy.

The jump arrival rate is controlled by a Poisson distribution. The assumed jump intensities imply that the number of jumps takes non-negative integer values j with probabilities

$$\mathrm{Prob}(j_{t+1}^k = j) = e^{-h_t^k}(h_t^k)^j/j!\,, \quad k = u, d, v\,. \tag{2.3.3}$$

We allow all of the jump intensities to depend on the domestic and foreign interest rates, as well as on the conditional spot variance,

$$h_t^k = h_0^k + h_r^k r_t + \tilde{h}_r^k \tilde{r}_t + h_v^k v_t\,, \quad k = u, d, v\,. \tag{2.3.4}$$

For a given number of jumps per period, the magnitude of a jump size is assumed to be random with a Gamma distribution,

$$z_t^k | j \sim \mathcal{G}\mathrm{amma}(j, \theta_k)\,, \quad k = u, d, v\,. \tag{2.3.5}$$

Intuitively, because we consider daily data, a Bernoulli distribution is a very good approximation to our model as it is reasonable to assume no more than one jump per day. Then, the probability of a jump is $1 - e^{-h_t^k} \approx h_t^k$ and the distribution of the jump size is exponential with mean parameter θ_k.[7]

We complement our data on exchange rate rates with variances implied from option prices. In this respect we follow the rich options literature that highlights the importance of using information in options for model estimation (e.g., see Aït-Sahalia and Kimmel, 2007; Brandt and Santa-Clara, 2002; Chernov and Ghysels, 2000; Jones, 2003; Pan, 2002; Pastorello, Renault, and Touzi, 2000). Many authors use implied variance in empirical work by interpreting it as a very accurate approximation of the risk-adjusted expectation of the average future variance realized over an option's lifetime. This is certainly true for models with stochastic volatility only. If this is the case, one can derive α_{iv} and β_{iv} as explicit functions of risk-adjusted parameters (e.g., Chernov, 2007, and Jones, 2003). The one-for-one relationship between implied variance and risk-adjusted expected variance may break down

[7]Our choice of the variance jump size distribution is frequently used when modelling variance to ensure its positivity as discussed above. The model of variance is also capable of generating quite rapid variance declines after jumps. A jump leads to a large deviation from the long-run mean v, and mean-reversion controlled by parameter ν ensures that the variance is pulled back.

in the presence of jumps. For example, Chernov (2007) has to assume that the risk-adjusted mean of jumps in FX is equal to zero to retain the simple relationship. Importance of careful accounting for jumps is manifested more clearly in the literature on model-free implied variance, such as VIX for S&P 500, where analytic expressions are feasible. Martin (2011) shows that, in the presence of jumps, VIX is equal to risk-adjusted expected variance plus additional terms reflecting the higher order risk-adjusted cumulants of returns.

We treat the Black-Scholes implied variance of a short-term (one-month) at-the-money option, IV_t, as a noisy and biased observation of the conditional spot variance v_t. Such a view allows us to avoid the aforementioned difficulties in explicit connection between implied variance and risk-adjusted expected future variance. The cost of such approach is our inability to estimate risk-adjusted parameters of the model. Specifically,

$$IV_t = \alpha_{iv} + \beta_{iv} v_t + \sigma_{iv} v_t \sqrt{\lambda_t}\, \varepsilon_t \,, \tag{2.3.6}$$

where IV_t is expressed in daily terms, ε_t is $\mathcal{N}(0,1)$ and λ_t is $\mathcal{IG}(\nu/2, \nu/2)$, so the product $\sqrt{\lambda_t}\varepsilon_t$ is $t_\nu-$distributed (Cheung, 2008; Jacquier, Polson, and Rossi, 2004).[8] We have considered a version of (2.3.6) with non-zero loadings on r_t and $\tilde{r}_t$, but this specification did not find empirical support.[9]

The model implies that expected log excess return is equal to

$$E_t\left[y_{t+1}\right] = \mu_t + \underbrace{h_t^u \theta_u}_{E_t\left[z_{t+1}^u\right]} - \underbrace{h_t^d \theta_d}_{E_t\left[z_{t+1}^d\right]} \,. \tag{2.3.7}$$

As discussed in the previous section, we assume that

$$\mu_t = \mu_0 + \mu_r r_t + \tilde{\mu}_r \tilde{r}_t + \mu_v v_t. \tag{2.3.8}$$

[8] Jones (2003) makes a strong case for heteroscedastic measurement errors in implied variance. His specification sets $\lambda_t = 1$. Cheung (2008) generalizes the specification to the Student $t-$error. We tried using a normal error with volatility σ_{iv}, a normal error with volatility $\sigma_{iv} v_t$, and the Student $t-$error described above. We find that heavy-tailed t_3 works very well.

[9] The error specification in (2.3.6) is very flexible. Therefore, it could be the case that the contribution of interest rates to the variation in implied variance cannot be empirically distinguished from the error, if the former is reasonably small.

The resulting expected excess return is

$$E_t\left[y_{t+1}\right] = \mu_0^* + \mu_r^* r_t + \tilde{\mu}_r^* \tilde{r}_t + \mu_v^* v_t \tag{2.3.9}$$

where

$$\mu_0^* = \mu_0 + h_0^u \theta_u - h_0^d \theta_d\,, \tag{2.3.10a}$$

$$\mu_r^* = \mu_r + h_r^u \theta_u - h_r^d \theta_d\,, \tag{2.3.10b}$$

$$\tilde{\mu}_r^* = \tilde{\mu}_r + \tilde{h}_r^u \theta_u - \tilde{h}_r^d \theta_d\,, \tag{2.3.10c}$$

$$\mu_v^* = \mu_v + h_v^u \theta_u - h_v^d \theta_d\,. \tag{2.3.10d}$$

Thus, our risk premium encompasses the UIP regressions which set

$$\tilde{\mu}_r^* = -\mu_r^*\,, \tag{2.3.11}$$

$$\mu_v^* = 0\,. \tag{2.3.12}$$

We conclude with a discussion of our approach to modelling interest rates. We do not need an explicit model of interest rates to estimate our model of FX excess returns if we are willing to assume that one-day r_t and $\tilde{r}_t$ can be reasonably proxied with short-term yields. We view this feature as a strength of our approach because explicitly modelling the behaviour of spot interest rates entails a massive effort. There is a separate literature dedicated to this task and the state-of-the-art models rely on five factors for capturing the interest rate dynamics. These studies are typically conducted with monthly or quarterly data, so they do not take into account the higher-frequency movements in interest rates which are susceptible to jumps themselves (e.g., Johannes, 2004; Piazzesi, 2005). Moreover, interest rates and currencies have low conditional correlation and variability in interest rates is much smaller than that in currencies. In summary, elaborate modelling and estimation of interest rates does not appear to be worthwhile in our case.

Nonetheless, we use the estimated model to compute some useful objects (expectations of future variance, or expected excess returns over multiple horizon) that depend on the distribution of interest rates. In order to obtain reasonable quantities, we assume the simplest possible model for the interest rates:

$$r_{t+1} = (1-b_r)a_r + b_r r_t + \sigma_r r_t^{1/2} w_{t+1}^r, \tag{2.3.13a}$$

$$\tilde{r}_{t+1} = (1-\tilde{b}_r)\tilde{a}_r + \tilde{b}_r r_t + \tilde{\sigma}_r \tilde{r}_t^{1/2} \tilde{w}_{t+1}^r. \tag{2.3.13b}$$

As in the case with the variance process, a square root process for interest rates is subject to caveats in discrete time. We calibrate the models to match the mean, variance and serial correlation of the respective observed short-term interest rates. Our computations with reasonable variation in parameters confirm our intuition that they have minimal impact on the role of normal and non-normal currency risks.

2.3.2 Qualitative features of the model

In this section we explain how we arrived at the specified functional form of the model. We evaluated too many models to provide a detailed account of our analysis, so we briefly summarize the results that led us to the above specification. Our initial specifications were motivated by the well-developed literature on equity returns (Andersen, Benzoni, and Lund, 2002; Chernov, Gallant, Ghysels, and Tauchen, 2003; Eraker, Johannes, and Polson, 2003; Eraker, 2004; Jones, 2003) and some of the few models of currencies (Bates, 1996; Johnson, 2002; Jorion, 1988; Maheu and McCurdy, 2008). The salient features of equity data are presence of substantial moves up and down and a pronounced negative skewness in the return distribution. Therefore, jumps in equity returns are often modelled via a single compound Poisson process with a normally distributed size of non-zero mean. However, in contrast to equity returns, currency returns have very mild skewness over long samples, which suggests a zero-mean normal distribution for jump sizes.

Further, Bates (1996), Campa, Chang, and Reider (1998), Carr and Wu (2007), and Johnson (2002) emphasize the time-varying and sign-switching nature of the risk-adjusted skewness of exchange rates. The key to modelling this feature successfully is to allow the conditional expected jump to vary over time. A single jump process with a zero mean jump size implies a zero conditional expected jump. Two jump processes have a potential to generate the requisite variation either via time-varying jump intensities, or time-varying jump size distributions, or both. We do not explore time-varying jump means as such specifications do not allow for tractable option valuation in the affine framework, and we eventually want our model to be used for option analysis. As can be seen from the expression for the currency risk premium (2.3.7), the conditional jump expectation is $h_t^u\theta_u - h_t^d\theta_d$, and is capable of producing the needed variation. We have also considered normally distributed jump sizes in excess returns with means of the jump size distribution having opposing signs. However, because normal distributions have infinite support, it was hard to distinguish empirically the down and up components. The exponential distribution does not have this issue because the support is on the positive line.

Another interesting feature of our specification is that we allow not only for two different Poisson processes in currency returns, but also for a third one in the variance. Our starting point was again in the equity literature where all jumps in returns and variance are guided by the same (or at least correlated) Poisson processes. We found that the model with correlated Poisson processes fitted the data poorly.

2.4 Empirical approach

We employ the Bayesian MCMC approach to estimate the model. This method was successfully implemented in many applications (see Johannes and Polson, 2009 for a review). For our purposes, the key advantage of this approach over other methodologies is that estimation of unobserved variance and jump times and sizes is a natural by-product of the procedure. Appendix A.1.2 describes all the details of the implementation.

It is worth pointing out how we distinguish jumps and normal shocks in the model. Formally, all shocks are discontinuous in our discrete-time formulation. We think of jumps as relatively infrequent events with relatively large variance. We use priors on jump arrival and jump size parameters to express this view.

It proved to be extremely fruitful to use option implied variances in our estimation. Ignoring information in option prices made it very hard to settle on a particular model. Parameters were estimated imprecisely and the algorithm had poor convergence properties – both are manifestations of the data being not sufficiently informative about the model. We had a similar experience when estimating the most general model, even when using the options data. Complicated dependencies of jump intensities on state variables, and the sheer number of separate Poisson processes was too much for the available data.

As such, we pursue the following model selection strategy. First, we treat implied variances as observed spot variances and estimate the model of variance (2.3.2). At this stage we select the best model by checking the significance of parameters on the basis of both confidence intervals and Bayes odds ratios. Specifically, the parameters of concern are the ones controlling the jump intensity in (2.3.4) for $k = v$. It turns out that, regardless of the currency, only the loading on variance is significant. In other words, the probability of jumps in the variance is affected by the variance itself. Thus, jumps in the variance are self-exciting (Hawkes, 1971).[10] Pinning down the model of variance is an extremely useful step in our estimation procedure.

Second, we use the lessons from the estimation exercise on the basis of implied variance alone to guide us in a formal search in the context of our full model. That is, we take the model (2.3.1), (2.3.2) and combine it with equation (2.3.6) that recognizes implied variances as noisy observations of the spot variance. As a benchmark, we estimate the stochastic variance model with no jumps. Next, we estimate a model with jumps in variance but no jumps in exchange rates ($h_t^u = h_t^d = 0$). We refer to this model as stochastic variance with jumps.

Finally, we allow for the full model with jumps in both exchange rates and variance. Here, we focus on the significance of the parameters controlling the jump intensities in (2.3.4) for $k = u$ and d. We are not reporting all the details here, but we find that $\tilde{h}_r^u$, h_v^u, h_r^d and h_v^d are insignificant. Thus, the probability of jumps up in the exchange rate is driven by the domestic rates only, and the probability of jumps down in the exchange rate is driven by the foreign rate only. We also test if some interesting parameters, or combinations of parameters, are equal to zero. First, we can test the UIP regression restrictions on the risk premiums in Eq. (2.3.11) (whether interest rates affect the risk premium as a differential) and Eq.

[10] The recent literature on equity returns also finds support for self-exciting jumps. See, for example, Aït-Sahalia, Cacho-Diaz, and Laeven (2011); Carr and Wu (2011); Nowotny (2011); Santa-Clara and Yan (2010).

(2.3.12) (whether the variance affects the risk premium). As we noted earlier, the behaviour of the FX skewness is dramatically affected by expected effect of jumps, which is equal to $h_t^u\theta_u - h_t^d\theta_d$. Here we are interested in testing whether $\theta_u = \theta_d = \theta$, $h_0^u = h_0^d = h_0$, and $h_r^u = \tilde{h}_r^d = h_r$. These hypotheses are interesting because if they cannot be jointly rejected then expected jump would be equal to $\theta h_r(r_t - \tilde{r}_t)$. Thus, the excess return asymmetries will be directly driven by the interest rate differential as noted in Brunnermeier, Nagel, and Pedersen (2008). The final version of this model that incorporates all the unrejected null hypotheses is referred to as the preferred.

We implement a series of informal diagnostics and specification tests to establish the preferred model. The diagnostics test the null hypothesis that the shocks to the observable excess return, w^s, and implied variance, ε, should be normal under the null of a given model. We can construct the posterior distribution of these shocks and evaluate how they change from model to model and whether they are normal. Appendix A.1.3 describes the procedure.

One has to exhibit caution when interpreting the evidence on normality of ε. The variance of the error term in the implied variance equation (2.3.6), $\sigma_{iv}^2 v_t^2 \lambda_t$ is very flexible. If a model is misspecified, λ_t will adjust so that the ε is close to a normal variable. Therefore, diagnostics of ε are not enough. We should be tracking the size of the variance of the error term. A better specified model should have smaller variance. We keep track of the time-series average of this variance – which we refer to as $IVvar$ – and report its posterior distribution.

Bayes odds ratios offer a formal specification test of the models. The test produces a number that measures the relative odds of two models given the data (the posterior distribution of the null model is in the denominator of the ratio). Following Kass and Raftery (1995), we interpret a log odds ratio that is greater than 3 as strong evidence against the null. Odds ratios do not necessarily select more complex models because the ratios contain a penalty for using more parameters (so-called automatic Occam's razor). Appendix A.1.4 details the computations.

2.5 Results

We start by highlighting statistical properties of the estimated models. Next, we study economic implications.

2.5.1 Statistical properties of currency risks

Tables 2.2 - 2.5 report the parameter estimates and Tables 2.6 - 2.9 report the corresponding model diagnostics. Table 2.10 displays the results of specification tests on the basis of Bayes odds ratios. Table 2.11 summarises parameters of the calibrated interest rate processes.

The results exhibit a lot of similarities across the different currencies. As we move from models with stochastic variance to stochastic variance with jumps, we observe a change in two key parameters: both the persistence ν of variance and the long-run mean of its conditionally normal component v decline. Taking AUD as an example, ν declines from 0.9943 to 0.9855. This seemingly small change translates into drop in the half-life of the conditionally normal component, $\log 2/(1-\nu)$, from 122 to 48 days. The high persistence of variance in the model without jumps is a sign of misspecification. Variance has to take high values occasionally to generate the observed exchange rates in the data. In the absence of jumps, variance builds up to the high values gradually via the high persistent channel. Additionally, in the case of GBP only, the volatility of variance σ_v declines significantly from 0.0321 to 0.0272. High σ_v helps the misspecified model with stochastic variance in generating high values of variance. The diagnostics support this interpretation. $IVvar$ drops by 50% across all currencies; this change is statistically significant. As expected, diagnostics for ε show that it is close to a normal variable for both models because of the flexibility in λ_t. Bayes odds ratios strongly favour stochastic variance with jumps.

Continuing with AUD, its volatility ($\sqrt{v}$) declines from 0.70% to 0.53% per day (11.19% to 8.43% per year). This happens because the total variance has contributions from the regular and jump components in the model with stochastic variance with jumps. When there are no jumps in FX, the long-run variance is equal to $v_J = [(1-\nu)v + h_0^v\theta_v]/[1-\nu-h_v^v\theta_v]$. See

Appendix A.1.5 for more details. This expression produces the average volatility of 0.65%, much closer to the figure in the model with stochastic variance.

To aid in interpreting the parameters controlling jumps in variance, consider the impact of a jump in variance. Suppose the current variance is at its long-run mean and the variance jumps by the average amount θ_v. Then in the case of AUD, the resulting volatility will move from 0.65% to $(v_J + \theta_v)^{1/2} = 0.90\%$, a nearly 40% increase in volatility (this increase ranges from 20% to 40% for the different currencies). The average jump intensity is equal to $h_0^v + h_v^v v_J = 0.0053$ jumps per day, or 1.34 per year (this number ranges from 1.34 to 2.61 for the the different currencies). Jumps in variance are self-exciting, so that a jump increases the likelihood of another jump. When the variance jumps by θ_v, intensity changes to 1.71 for AUD (the range is from 1.71 to 3.41 for all the currencies).

Also note that ρ, the "leverage effect," has the same sign as the average interest rate differential. It is positive for JPY and CHF, and negative for AUD and GBP. This result is consistent with the analysis in Brunnermeier, Nagel, and Pedersen (2008) and the common wisdom among market participants that investors who are long carry are essentially short volatility. For example, consider the position of a carry trade investor who borrows money in USD and invests in AUD. This investor can loose money when the AUD depreciates against the USD. We estimate that ρ is negative for this currency pair, so the volatility of this exchange rate tends to increase during times when the AUD depreciates.

The preferred version of the full model is the one with all of the aforementioned restrictions imposed ($\theta_u = \theta_d = \theta$, $h_0^u = h_0^d = h_0$, and $h_r^u = \tilde{h}_r^d = h_r$). That is, the size of the jumps in FX up and down are symmetric and their intensities have numerically identical functional form (but they depend on different interest rates). As a result, the overall structure of jump arrivals differs from the one used in popular models of S&P 500 returns, where jumps in variance and the index are simultaneous.

Parameters reflecting the average jump size have a different interpretation as compared to jump in variance. The latter is a jump in the level of the variable, while the former is the jump in return. Therefore, it is scale-free: it is not daily or annual, it reflects by how much the return changes at the moment of the jump. Thus, on average, AUD returns increase (decline) by 1.69% when there is a jump up (down). Average intensities of down and up jumps are similar to each other and to those of variances for a given currency: they range from 0.87 to 3.35 jumps per year (we use sample averages of interest rates to compute average h_t^u and h_t^d).

The diagnostics of residuals w^s indicate that the major improvement in moving from stochastic variance with jumps to the preferred model comes from a statistically significant drop in kurtosis from roughly 4 to 3.5 across all currencies. The absolute value of skewness of w experiences a significant drop for all currencies except for GBP, where it was insignificantly different from zero in the model with stochastic variance with jumps already. Serial correlation is slightly negative for all currencies except for GBP (where it is zero in the model with stochastic variance with jumps already), and the change from one model to another is insignificant. $IVvar$ does not change appreciably because we did not change our model for variance. Bayes odds ratios strongly favor the preferred model. In summary, the preferred is clearly a superior model, but there are some residual non-normalities left in the fitted shocks to exchange rates. We leave improvements to future research.

The expected excess return in (2.3.9) can be simplified for the preferred model to

$$E_t(y_{t+1}) = \mu_0 + (\mu_r + h_r\theta)r_t + (\tilde{\mu}_r - h_r\theta)\tilde{r}_t + \mu_v v_t. \tag{2.5.1}$$

Thus, by testing if $\mu_r = -\tilde{\mu}_r$ and $\mu_v = 0$, we test the UIP regression specification (2.3.11) - (2.3.12) of currency excess returns across all three models. For all currency pairs, we cannot reject that $\mu_r = -\tilde{\mu}_r$ at the conventional significance levels. Moreover, $\mu_r \approx -3$ for all currencies that is consistent with our earlier discussion of UIP regression results. In addition, the loading on the variance μ_v is significantly negative in all currencies except for JPY which has a significantly positive estimate. The tiny serial correlation of the residuals w^s suggests that this model is adequate in capturing conditional mean of excess returns and, therefore, potentially omitted variables cannot affect materially our conclusions about the structure of currency risks.

The probability of USD depreciation, h_t^u, depends positively on the US interest rate. This result seems to contradict basic intuition about the relationship between changes in FX and interest rates. It is important to note that this connection is applicable to jumps only. Parameter estimates and expression (2.5.1) imply that, all else equal, the expected excess return on the USD is higher when the difference between the U.S. and foreign interest rate is higher. However, when the interest rate differential is positive, our model says that the probability of a large depreciation in the USD (a jump up) is higher than the probability of a large appreciation (a jump down).

2.5.2 Jumps and events

In this section we study the economic properties of the documented jumps. We ask basic questions regarding the structure of the jump components, examine whether jumps can be related to important economic events, and gauge their impact on the overall risk of currency trades. Our discussion is based on Figures 2.1 - 2.4 and Table 2.12.

For each exchange rate, the figures display the time series of data (excess returns and implied volatilities) complemented with the estimated unobservable model components: spot volatility $v_t^{1/2}$ and jumps. Regarding the latter, the model produces an estimate of a jump size and an ex-post probability that a jump actually took place on a specific day. However, all of this information is not easy to digest as there are a lot of small jumps and jumps with small ex-post probability of taking place. Thus, to simplify the reporting, we stratify the jump probabilities by assigning them a value of one if their actual value is 0.5 or higher, and zero otherwise. Then we plot the estimated jumps sizes on the days with the assigned value of one. Our strategy yields 219 jumps overall across all currencies. Approximately 25% of these jumps take place simultaneously in at least two currencies. We call such jumps international. There are only 8 episodes when FX and variance jumped at the same time. We overlay the plots of the estimated jumps sizes with the state-dependent ex-ante jump probabilities h_t^v, h_t^u, and h_t^d.

To interpret the plots better, we have to reference the jump magnitudes against the summary statistics available in Table 2.1. Let us use JPY in Figure 2.4 as an example. Table 2.1 tells us that the volatility of JPY is 0.7% per day and the mean is approximately zero. Thus, a "regular" excess return can be within the range of ±2% ($\mu \pm 3\sigma$). The upper left panel of Figure 2.4 shows that there are quite a few days when excess returns are outside of this range. In practice, the volatility is time-varying and unobserved. Therefore, the "regular" range is time-varying also and uncertain. The estimation procedure takes this uncertainty into account by producing ex-post probability of a jump taking place. We arbitrarily select the level of uncertainty about jumps that we are comfortable with by discarding the jumps with such probabilities less than 50%. The bottom left panel confirms this by showing the estimated jumps in excess returns. Their magnitude ranges from 2% to 6% in absolute value. Interestingly, the larger jumps coincide with spikes in the moving-window estimates of skewness across all currencies.

However, not all of the big spikes in excess returns are attributable to jumps in FX. For example, on October 28, 2008, the plot of excess returns shows a big drop of 5.5%. The model tells us that there were no jumps on that day. The model is capable of generating such a big move via a normal component because of the jump in variance. Volatility jumped by $(v_{t-1} + z_t^v)^{1/2} - v_{t-1}^{1/2} = 0.18\%$ (2.9% annualized), on average, on each of three days between October 22 and 24. Each day the jump in variance was increasing the probability of a jump the following day. Over these three days volatility moved from 1.35% (21.5% annualized) to 1.8% (28% annualized). To put this number into a perspective, the long-run volatility mean is $v_J^{1/2} = 0.66\%$ (10.4% annualized). Thus, the increase in volatility over these three days was roughly equal to the average level of volatility. Moreover, there is no significant news associated with either October 22-24 or October 28. Thus, we attribute these events to pure uncertainty in the markets.

GBP generates large movements via jumps in variance in the most transparent way. The exchange rate itself exhibits only 11 jumps throughout the sample, none of which take place after 2000. In fact, even the famous "Black Wednesday" – the GBP devaluation on September 16, 1992 – is attributed primarily to a jump in variance on September 8. The volatility moved from 0.93% (14.7% annualised) to 1.13% (17.9% annualised), then it drifted up to 1.15% (18.3% annualised) on the day of the crash. These values of volatility are high, as the average volatility of GBP is $v_J^{1/2} = 0.55\%$. Nonetheless, this level of volatility is still insufficient to generate the whole drop of -4.10%. Of course, these rough computations assume that v_t is known with certainty. It is not, and a small deviation in the estimate may be able to attribute the whole return to a normal shock in FX. This is why the estimated probability of a jump is only 26% on this day (and the estimated jump size is -0.42%).

As a next step, we check if the jumps we detect are related to economic, political or financial events. For each day a currency has experienced a jump, we check if there were significant news. This strategy is effectively opposite to the one employed in studies of the news impact on financial assets (see, e.g., Andersen, Bollerslev, Diebold, and Vega, 2003 for FX). Usually one measures news surprise by computing a standardised difference between an announcement's expectation and realisation and then checks, usually at intra-day frequency, if the surprise had an impact on values of financial assets. Our approach does not require us taking a stand on measuring a surprise. In addition, we are careful in distinguishing announcements, a clear public release of a fact, from uncertainty: anticipation, comments on the current economic situation and overall market anxiety that is sometimes evident in the news. Table 2.12 provides a summary of the types of events associated with jumps. Appendix A.1.6 provides a jump-by-jump description of all events.

Consistent with Figures 2.1 - 2.4, we see that there are many more jumps in variance than in the exchange rates. Almost all jumps are associated with important events, however there is a critical difference between jumps in FX and those in variance. The former are most commonly associated with announcements and the latter are related to uncertainty. Interestingly, Harvey and Huang (1991) relate elevated volatility levels to public news announcements. Our observations focus on jumps rather than regular increases in volatility.

While we do not formally model the joint behaviour of exchange rates, we find a lot of simultaneous jumps across the currencies, particularly jumps in variance. Therefore, it appears promising to extend the existing research on common and currency-specific factors affecting risk premiums (e.g., Lustig, Roussanov, and Verdelhan, 2013) by allowing for common and currency-specific jump risk.

The plots of jump intensities provide a partial insight into why jumps in variance are so prominent. Probabilities of jumps in FX are moving together with interest rates, which experienced secular decline in our sample in all countries. As we highlighted earlier, the probability of a jump in variance is primarily driven by the variance itself. This probability went through a couple of cycles of high and low values as volatility is less persistent than interest rates.

2.5.3 Decomposing the total risk

Are these risks important quantitatively? Jumps in FX and variance should affect the tail events the most. The properties of tails are captured by high-order moments or cumulants. One could report each of these statistics separately and measure how they are affected by the various shocks present in our model. We choose to summarize all this information by one number and measure the total risk corresponding to investment horizon n using entropy of changes in FX:

$$L_t(S_{t+n}/S_t) = \log E_t(e^{s_{t+n}-s_t}) - E_t(s_{t+n} - s_t). \tag{2.5.2}$$

Entropy is a loaded term. Our use of entropy is similar to that of Backus, Chernov, and Martin (2011) and Backus, Chernov, and Zin (2012), who characterise entropy of the pricing kernel, and the closest to the way it is used in Martin (2011) who uses entropy of equity index returns. Entropy's connection to the cumulants of log FX returns makes it attractive for our purposes.

Specifically, the definition (2.5.2) implies that

$$\begin{aligned} L_t(S_{t+n}/S_t) &= k_t(1; s_{t+n} - s_t) - \kappa_{1t}(s_{t+n} - s_t) && (2.5.3) \\ &= \kappa_{2t}(s_{t+n} - s_t)/2! + \kappa_{3t}(s_{t+n} - s_t)/3! + \kappa_{4t}(s_{t+n} - s_t)/4! + \ldots && (2.5.4) \end{aligned}$$

where $k_t(1; s_{t+n} - s_t)$ is the conditional cumulant-generating function of $s_{t+n} - s_t$ evaluated at the argument equal to 1, $\kappa_{jt}(s_{t+n} - s_t)$ is the jth conditional cumulant of $s_{t+n} - s_t$, and we used the fact that $k_t(1; s_{t+n} - s_t)$ is equal to the infinite sum of $\kappa_{jt}(s_{t+n} - s_t)/j!$ over j starting with $j = 1$. The significance of the property (2.5.4) is that if currency changes are normally distributed, then entropy is equal to a half of their variance (the first term in the sum). All the higher-order terms arise from non-normalities. Thus, entropy captures tail behaviour of returns in a compact form. As a result, we view entropy as a natural generalisation of variance as a risk measure. For this reason, Alvarez and Jermann (2005) explicitly refer to L_t as generalised variance.

We decompose the total risk of currency returns into the contribution of the jump and normal components. Appendix A.1.7 explains how we compute the full entropy on the basis of equation (2.5.3). We can compute the individual components by zeroing out the rest.[11] Figures 2.5 and 2.6 display the contributions of these components for the investment horizons of 1 months ($n = 21$) and 3 months ($n = 63$). We overlay these contributions with a time-series plot of entropy computed treating USD as a domestic currency. We scale entropy to ensure that it is equal to variance in the normal case and to adjust for the horizon. Finally, to make the number easily interpretable, we take the square-root and express it in percent. Thus, we plot $\sqrt{2L_t/n}$. Finally, Table 2.13 supports the plots by reporting summary statistics of the relative contributions of the different components.

We start by characterising the contribution of the different components at a given point in time. We see that the regular, or normal, risk is the most prominent regardless of the horizon. The average total contribution of jumps at a one-month horizon ranges from 11.03 % for GBP to 20.19% for AUD. The risk of jump in FX (up or down) [the range is between 6.82% and 9.17%] is higher than that of the jump in variance [the range is between 3.76% and 4.83%] and has higher time-variation at a one-month horizon (GBP is the exception as jumps in variance contribute 4.37% as compared to 2.98% for jumps up and 3.68% for jumps down). Therefore, in the short-term the risk of the jump in variance has the smallest contribution to the overall currency risk. However, this conclusion changes as we extend the investment horizon to three months. The average total contribution of jumps at this horizon increases – the range is from 17.71% for GBP to 25.19% for JPY. In this case individual contribution of jumps in variance [the range is 8.94% to 11.48%] is higher than those of jumps in FX [the range is 2.78% to 8.57%] (in the case of GBP the contribution of the jump in variance is greater than the sum of jumps up and down).

[11]If two variables x_s and y_s are conditionally independent, then $L_t(x_s y_s) = L_t(x_s) + L_t(y_s)$. Therefore, our decomposition approach correctly separates the contributions of the two jumps in currencies. Because probability of the jump in variance depends on the variance itself, the normal shock to variance and the jump are conditionally independent only over one period, $n = 1$. When $n > 1$ our procedure attributes all the covariance terms, which are positive because of the estimated functional form of jump probabilities, to the jump in variance. We think that this approach is sensible because the presence of these covariance terms is due to jumps.

The contribution of jumps in FX declines towards the end of our sample thereby making the contribution of jumps in variance more important. We connect this result to the secular decline in the probability of FX jumps that we highlighted earlier. This effect diminishes expected contribution of such jumps to the overall risk. In contrast, the probability of jumps in variance is less persistent and, therefore, exhibits mean-reversion in our sample.

The time-series variation in total risk resembles the time-series variation in the spot variance v_t. This is not surprising because L_t is a linear function of v_t (Appendix A.1.7). Thus, whenever spot variance moves, especially jumps, we observe a clear move in entropy. We conclude that the risk of jumps in variance are at least as important as the risk of jumps in FX, but the two types of jumps serve a different purpose.

2.5.4 Preliminary evidence of priced jump risk

The large amount of jump risk prompts a natural question of whether this risk is priced. Answering this question has important implications for the carry trade literature. In particular, one should be able to attribute a specific fraction of carry risk premium to compensation for bearing crash risk.

The full answer to this question requires an explicit model of the pricing kernel and the use of assets that are sensitive to jump risk for estimation of risk premiums. In this regard, out-of-the-money options are particularly informative about the price of jump risks (i.e., the covariance of the pricing kernel with jumps). However, such analysis is outside of the scope of this paper. Instead, we provide a back-of-the-envelope computation, which we view as preliminary evidence of priced jump risk.

Our idea is to explore the shape of the implied volatility smile that is derived from our model. If the jump risk is not priced, then we would be able to replicate the smile without additional assumptions about risk premiums. To capture the diversity of the smile shapes, we consider examples of a currency with a positive average interest rate differential (GBP) and a negative one (JPY). The reason we focus on the interest rate differential is that our model connects it to asymmetry of the conditional distribution of an exchange rate.

Indeed, the third conditional cumulant can be obtained by taking the third derivative of the cumulant-generating function of log currency returns (provided in Appendix A.1.7). For example,

$$\kappa_{3t}(s_{t+1} - s_t) = 6\theta^3 h_r(r_t - \tilde{r}_t)$$

in the preferred model.

For both currencies we pick a day in which the variance and interest rate differential are roughly equal to the sample averages: November 12, 2007 (GBP) and April 20, 2004 (JPY). Figure 2.7 displays the implied volatility smiles observed on these days. Consistent with our expectations, GBP exhibits positive asymmetry (defined as the difference between implied volatility corresponding to moneyness less than one and the one with moneyness greater than one, with moneyness symmetric around at-the-money), and JPY exhibits negative asymmetry. The solid black lines in Figure 2.7 are the option-implied volatilities that correspond to our model estimates with no risk premiums. The model can generate both the smile and the correct direction of asymmetry. However, the level and curvature of the smile that are implied by the model cannot match those observed in the data. The natural question is whether the omitted risk premiums are responsible for this disparity.

Before we conclude that the disparity is due to risk premiums, we first evaluate whether statistical uncertainty about parameter values and the unobserved spot variance could account for the difference in levels and curvatures. The theoretical implied volatility is a function of observable states, option contract characteristics (strike and time to maturity), model parameters, and the unobservable variance: $IV_t^{1/2} = F(S_t, r_t, \tilde{r}_t, K, T-t, \Theta, v_t)$. The solid black lines in Figure 2.7 display $F(S_t, r_t, \tilde{r}_t, K, T-t, \hat{\Theta}, \hat{v}_t)$. We can take the uncertainty about these estimated values into account by computing confidence bounds. One of the by-products of our estimation procedure is a set of draws $\{\Theta^{(g)}, v_t^{(g)}\}_{g=1}^{250{,}000}$ from the posterior distribution $p(\Theta, v_t|\text{full dataset})$. We obtain the corresponding set of draws from the posterior distribution of implied volatilities by evaluating $F(S_t, r_t, \tilde{r}_t, K, T-t, \Theta^{(g)}, v_t^{(g)})$. The blue dashed lines in Figure 2.7 display the (2.5%, 97.5%) posterior coverage interval for theoretical implied volatilities.

The only case where the posterior interval covers observed implied volatilities corresponds to six-month options on GBP. Thus, statistical uncertainty, on its own, cannot explain the gap between the model and the data. The fact that the curvature is much more pronounced in the data suggests that jump risk premiums are present (variance risk premiums may be helpful in adjusting the level of the smile but not its curvature). See, for instance, Backus, Chernov, and Martin (2011) who provide a detailed discussion of how risk adjustment in the jump parameters affects the shape of the smile (Figure 3). While by no means conclusive, this initial evidence warrants further more detailed investigation of the magnitude of jump risk premiums.

2.6 Conclusion

We explore sources of risk affecting currency returns. We find a large time-varying component that is attributable to jump risk. The most interesting part of this finding is that jumps in currency variance play an important role, especially at long (quarterly) investment horizons, yet there is no obvious link between macroeconomic fundamentals and these jumps. We interpret this evidence as a manifestation of economic uncertainty.

We see at least two important directions in which our analysis can be extended. First, we should use prices of financial assets associated with currencies (e.g., bonds, options) to estimate the pricing kernel. This would allow us to characterize how the risks documented in this paper are valued in the marketplace. Second, existing research presents evidence of common and currency-specific factors affecting risk premiums. Our evidence suggests informally that common jump risks are shared across the different currencies. It would be useful to establish a model of joint currency behaviour that explicitly incorporates common and country-specific jump components and how they contribute to risk premiums.

2.7 Tables and graphs

Table 2.1: Properties of excess log currency and S&P 500 returns and changes in implied volatility

		Mean	Std Dev	Skewness	Kurtosis	Nobs
AUD	Excess return	0.0186	0.7435	-0.3870	13.7202	6332
	$\Delta\sqrt{IV}$	0.0109	3.7661	0.9077	9.7290	3933
CHF	Excess return	0.0057	0.7232	0.1194	4.7841	6521
	$\Delta\sqrt{IV}$	0.0073	3.8057	0.9966	9.8095	3823
GBP	Excess return	0.0096	0.6197	-0.2337	5.6832	6521
	$\Delta\sqrt{IV}$	0.0142	4.0001	1.3884	44.2683	3823
JPY	Excess return	0.0003	0.6950	0.3626	8.0878	6393
	$\Delta\sqrt{IV}$	-0.0045	4.8277	1.0395	10.7764	3934
S&P 500	Excess return	0.0090	1.1803	-1.3584	32.9968	6521
	$\Delta\sqrt{VIX}$	0.0089	5.8997	0.5096	6.7502	3914

Notes. Descriptive statistics for daily log currency and log S&P 500 excess returns and changes in implied volatility, in percent, per day: AUD return from September 25, 1986, to December 31, 2010, and AUD IV from December 6, 1995, to December 31, 2010; CHF return from January 3, 1986, to December 31, 2010, and CHF IV from May 8, 1996, to December 31, 2010; GBP return from January 3, 1986, to December 31, 2010, and GBP IV from May 8, 1996, to December 31, 2010; JPY return from July 2, 1986, to December 31, 2010, and JPY IV from December 5, 1995, to December 31, 2010; S&P 500 return from January 3, 1986, to December 31, 2010 and VIX from January 2, 1996, to December 31, 2010. The interest rates used to compute returns are one-month LIBOR rates. Source: Bloomberg.

Table 2.2: AUD parameter estimates

	SV ($\theta = 0$, $\theta_v = 0$)	SVJ ($\theta = 0$)	Preferred
μ_0	-0.0004	0.0003	0.0014
	(-0.0181,0.0173)	(-0.0175, 0.0182)	(-0.0166, 0.0194)
μ_r	-2.4893	-2.5200	-2.7643
	(-3.5895, -1.3910)	(-3.6170, -1.4190)	(-3.8613, -1.6801)
μ_v	-0.0150	-0.0152	-0.0147
	(-0.0247, -0.0053)	(-0.0249, -0.0056)	(-0.0244, -0.0051)
v	0.4968	0.2819	0.2819
	(0.2903, 0.8984)	(0.2101, 0.3728)	(0.2110, 0.3734)
ν	0.9943	0.9855	0.9857
	(0.9925, 0.9961)	(0.9837, 0.9873)	(0.9838, 0.9875)
σ_v	0.0391	0.0343	0.0342
	(0.0379, 0.0404)	(0.0330, 0.0357)	(0.0329, 0.0356)
ρ	-0.2924	-0.2770	-0.2839
	(-0.3279, -0.2563)	(-0.3156, -0.2378)	(-0.3237, -0.2435)
θ_v		0.3864	0.3837
		(0.3392, 0.4406)	(0.3367, 0.4362)
θ			1.6910
			(1.5208, 1.8779)
h_0^v		0.0037	0.0036
		(0.0029, 0.0040)	(0.0028, 0.0040)
h_v		0.0038	0.0038
		(0.0031, 0.0040)	(0.0031, 0.0040)
h_0			0.0017
			(0.0000, 0.0038)
h_r			0.1737
			(0.1177, 0.1992)
α_{iv}	0.0033	0.0027	0.0033
	(0.0009, 0.0064)	(-0.0002, 0.0056)	(0.0005, 0.0063)
β_{iv}	1.0006	1.0021	1.0022
	(0.9958, 1.0054)	(0.9983, 1.0059)	(0.9984, 1.0061)

Notes. The estimates correspond to daily excess currency returns, in percent. The 95% confidence intervals are reported in parentheses. The preferred model is:

$$\begin{aligned}
y_{t+1} &= \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \\
v_{t+1} &= (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \\
h_t^u &= h_0 + h_r r_t,\ h_t^d = h_0 + h_r \tilde{r}_t,\ h_t^v = h_0^v + h_v v_t \\
z_t^u|j &\sim \mathcal{G}\text{amma}(j,\theta),\ z_t^d|j \sim \mathcal{G}\text{amma}(j,\theta),\ z_t^v|j \sim \mathcal{G}\text{amma}(j,\theta_v)
\end{aligned}$$

Table 2.3: CHF parameter estimates

	SV ($\theta = 0$, $\theta_v = 0$)	SVJ ($\theta = 0$)	Preferred
μ_0	0.0340	0.0353	0.0324
	(0.0150, 0.0531)	(0.0163, 0.0543)	(0.0132, 0.0516)
μ_r	-2.9851	-3.0674	-3.2501
	(-4.3345, -1.6354)	(-4.4174, -1.7169)	(-4.5952, -1.8996)
μ_v	-0.0198	-0.0199	-0.0199
	(-0.0333, -0.0064)	(-0.0335, -0.0065)	(-0.0334, -0.0064)
v	0.5088	0.3502	0.3427
	(0.3136, 0.8364)	(0.2564, 0.4741)	(0.2507, 0.4639)
ν	0.9891	0.9789	0.9785
	(0.9863, 0.9919)	(0.9758, 0.9818)	(0.9755, 0.9815)
σ_v	0.0388	0.0337	0.0337
	(0.0373, 0.0404)	(0.0321, 0.0352)	(0.0321, 0.0352)
ρ	0.0875	0.0856	0.0856
	(0.0480, 0.1271)	(0.0439, 0.1273)	(0.0416, 0.1298)
θ_v		0.2205	0.2145
		(0.1845, 0.2622)	(0.1804, 0.2550)
θ			1.3582
			(1.1771, 1.5744)
h_0^v		0.0037	0.0037
		(0.0029, 0.0040)	(0.0028, 0.0040)
h_v		0.0145	0.0144
		(0.0131, 0.0150)	(0.0130, 0.0150)
h_0			0.0051
			(0.0011, 0.0078)
h_r			0.2175
			(0.0615, 0.2973)
α_{iv}	0.0061	0.0041	0.0056
	(0.0009, 0.0113)	(-0.0013, 0.0093)	(0.0003, 0.0104)
β_{iv}	0.9919	0.9934	0.9946
	(0.9753, 1.0091)	(0.9795, 1.0071)	(0.9802, 1.0096)

Notes. The estimates correspond to daily excess currency returns, in percent. The 95% confidence intervals are reported in parentheses. The preferred model is:

$$
\begin{aligned}
y_{t+1} &= \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \\
v_{t+1} &= (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \\
h_t^u &= h_0 + h_r r_t,\ h_t^d = h_0 + h_r \tilde{r}_t,\ h_t^v = h_0^v + h_v v_t \\
z_t^u | j &\sim \mathcal{G}\text{amma}(j,\theta),\ z_t^d | j \sim \mathcal{G}\text{amma}(j,\theta),\ z_t^v | j \sim \mathcal{G}\text{amma}(j,\theta_v)
\end{aligned}
$$

Table 2.4: GBP parameter estimate

	SV ($\theta = 0$, $\theta_v = 0$)	SVJ ($\theta = 0$)	Preferred
μ_0	0.0348	0.0360	0.0337
	(0.0128, 0.0565)	(0.0138, 0.0584)	(0.0116, 0.0556)
μ_r	-3.0632	-3.1692	-3.1897
	(-4.4127, -1.7209)	(-4.5046, -1.8351)	(-4.5138, -1.8673)
μ_v	-0.1326	-0.1377	-0.1341
	(-0.1928, -0.0720)	(-0.1986, -0.0773)	(-0.1952, -0.0731)
v	0.3773	0.2227	0.2180
	(0.1855, 0.8002)	(0.1631, 0.2989)	(0.1619, 0.2909)
ν	0.9941	0.9810	0.9809
	(0.9919, 0.9963)	(0.9786, 0.9834)	(0.9786, 0.9833)
σ_v	0.0321	0.0272	0.0273
	(0.0311, 0.0332)	(0.0262, 0.0283)	(0.0263, 0.0284)
ρ	-0.1341	-0.1295	-0.1303
	(-0.1713, -0.0965)	(-0.1692, -0.0896)	(-0.1709, -0.0895)
θ_v		0.1953	0.1959
		(0.1728, 0.2206)	(0.1731, 0.2211)
θ			1.1680
			(0.9593, 1.4127)
h_0^v		0.0038	0.0038
		(0.0033, 0.0040)	(0.0033, 0.0040)
h_v		0.0121	0.0121
		(0.0110, 0.0125)	(0.0110, 0.0125)
h_0			0.0012
			(0.0001, 0.0020)
h_r			0.1223
			(0.0634, 0.1491)
α_{iv}	0.0109	0.0063	0.0089
	(0.0063, 0.0155)	(0.0009, 0.0109)	(0.0039, 0.0137)
β_{iv}	0.9905	0.9951	0.9940
	(0.9855, 0.9955)	(0.9905, 0.9996)	(0.9895, 0.9985)

Notes. The estimates correspond to daily excess currency returns, in percent. The 95% confidence intervals are reported in parentheses. The preferred model is:

$$
\begin{aligned}
y_{t+1} &= \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \\
v_{t+1} &= (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \\
h_t^u &= h_0 + h_r r_t,\ h_t^d = h_0 + h_r \tilde{r}_t,\ h_t^v = h_0^v + h_v v_t \\
z_t^u | j &\sim \mathcal{G}\text{amma}(j, \theta),\ z_t^d | j \sim \mathcal{G}\text{amma}(j, \theta),\ z_t^v | j \sim \mathcal{G}\text{amma}(j, \theta_v)
\end{aligned}
$$

Table 2.5: JPY parameter estimates

	SV ($\theta = 0$, $\theta_v = 0$)	SVJ ($\theta = 0$)	Preferred
μ_0	0.0253	0.0253	0.0203
	(0.0064, 0.0441)	(0.0062, 0.0443)	(0.0018, 0.0389)
μ_r	-3.1861	-3.2046	-3.4590
	(-4.4200, -1.9526)	(-4.4540, -1.9531)	(-4.6992, -2.2266)
μ_v	0.0151	0.0152	0.0152
	(0.0054, 0.0248)	(0.0055, 0.0249)	(0.0054, 0.0248)
v	0.4816	0.3143	0.3012
	(0.2926, 0.8111)	(0.2328, 0.4223)	(0.2207, 0.4079)
ν	0.9896	0.9762	0.9771
	(0.9868, 0.9924)	(0.9730, 0.9794)	(0.9739, 0.9802)
σ_v	0.0496	0.0438	0.0436
	(0.0476, 0.0516)	(0.0419, 0.0458)	(0.0417, 0.0455)
ρ	0.3681	0.3505	0.3631
	(0.3316, 0.4040)	(0.3098, 0.3902)	(0.3205, 0.4047)
θ_v		0.3917	0.3771
		(0.3313, 0.4622)	(0.3198, 0.4447)
θ			1.2351
			(1.0847, 1.4071)
h_0^v		0.0037	0.0037
		(0.0031, 0.0040)	(0.0029, 0.0040)
h_v		0.0077	0.0076
		(0.0068, 0.0080)	(0.0067, 0.0080)
h_0			0.0052
			(0.0034, 0.0060)
h_r			0.4447
			(0.3133, 0.4984)
α_{iv}	0.0140	0.0116	0.0159
	(0.0086, 0.0193)	(0.0062, 0.0169)	(0.0099, 0.0214)
β_{iv}	1.0052	1.0083	1.0248
	(0.9871, 1.0248)	(0.9916, 1.0256)	(1.0069, 1.0431)

Notes. The estimates correspond to daily excess currency returns, in percent. The 95% confidence intervals are reported in parentheses. The preferred model is:

$$
\begin{aligned}
y_{t+1} &= \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \\
v_{t+1} &= (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \\
h_t^u &= h_0 + h_r r_t,\ h_t^d = h_0 + h_r \tilde{r}_t,\ h_t^v = h_0^v + h_v v_t \\
z_t^u | j &\sim \mathcal{G}\text{amma}(j,\theta),\ z_t^d | j \sim \mathcal{G}\text{amma}(j,\theta),\ z_t^v | j \sim \mathcal{G}\text{amma}(j,\theta_v)
\end{aligned}
$$

Table 2.6: Model diagnostics for AUD

	SV ($\theta = 0$, $\theta_v = 0$)	SVJ ($\theta = 0$)	Preferred
skewness^C	-0.3080	-0.3074	-0.2004
	(-0.3308, -0.2860)	(-0.3304, -0.2855)	(-0.2408, -0.1599)
kurtosis^C	4.1472	4.0822	3.4892
	(4.0677, 4.2366)	(4.0006, 4.1810)	(3.3802, 3.6055)
autocorrelation^C	-0.0281	-0.0271	-0.0324
	(-0.0311, -0.0252)	(-0.0303, -0.0241)	(-0.0406, -0.0242)
skewness^{IV}	0.0402	0.0303	0.0310
	(-0.0373, 0.1181)	(-0.0466, 0.1070)	(-0.0459, 0.1080)
kurtosis^{IV}	3.0618	3.0385	3.0375
	(2.9103, 3.2314)	(2.8902, 3.2034)	(2.8896, 3.2033)
autocorrelation^{IV}	0.1043	0.0634	0.0637
	(0.0749, 0.1336)	(0.0331, 0.0937)	(0.0334, 0.0940)
$IVvar$	0.0064	0.0034	0.0034
	(0.0041, 0.0122)	(0.0021, 0.0070)	(0.0021, 0.0070)

Notes. Posterior means and 95% confidence intervals (reported in parentheses) for the residuals from the currency return return and from the IV equations. Superscript C stands for the residuals from the currency return equation, superscript IV stands for the residuals from the IV equation.

Table 2.7: Model diagnostics for CHF

	SV ($\theta = 0$, $\theta_v = 0$)	SVJ ($\theta = 0$)	Preferred
skewness^C	0.1178	0.1282	0.0586
	(0.0994, 0.1365)	(0.1078, 0.1486)	(0.0182, 0.0983)
kurtosis^C	3.9497	3.9438	3.4333
	(3.8825, 4.0198)	(3.8919, 4.0011)	(3.3373, 3.5405)
autocorrelation^C	-0.0203	-0.0198	-0.0272
	(-0.0227, -0.0179)	(-0.0226, -0.0170)	(-0.0352, -0.0192)
skewness^{IV}	0.0224	0.0201	0.0210
	(-0.0574, 0.1022)	(-0.0585, 0.0985)	(-0.0573, 0.0995)
kurtosis^{IV}	3.0648	3.0399	3.0406
	(2.9091, 3.2378)	(2.8887, 3.2097)	(2.8890, 3.2094)
autocorrelation^{IV}	0.0777	0.0565	0.0564
	(0.0459, 0.1094)	(0.0247, 0.0883)	(0.0246, 0.0881)
$IVvar$	0.0010	0.0006	0.0006
	(0.0007, 0.0017)	(0.0004, 0.0011)	(0.0004, 0.0011)

Notes. Posterior means and 95% confidence intervals (reported in parentheses) for the residuals from the currency return and from the IV equations. C stands for the residuals from the currency return equation, superscript IV stands for the residuals from the IV equation.

Table 2.8: Model diagnostics for GBP

	SV ($\theta = 0$, $\theta_v = 0$)	SVJ ($\theta = 0$)	Preferred
skewness^C	-0.0407	-0.0211	-0.0232
	(-0.0606, -0.0202)	(-0.0436, 0.0012)	(-0.0609, 0.0143)
kurtosis^C	3.9181	3.8540	3.4947
	(3.8427, 4.0061)	(3.7784, 3.9423)	(3.4006, 3.5969)
autocorrelation^C	0.0009	0.0006	-0.0027
	(-0.0024, 0.0040)	(-0.0038, 0.0047)	(-0.0094, 0.0037)
skewness^{IV}	0.0352	0.0212	0.0215
	(-0.0443, 0.1146)	(-0.0565, 0.0995)	(-0.0568, 0.0998)
kurtosis^{IV}	3.0710	3.0293	3.0296
	(2.9160, 3.2461)	(2.8798, 3.1972)	(2.8786, 3.1977)
autocorrelation^{IV}	0.0791	0.0510	0.0510
	(0.0483, 0.1096)	(0.0204, 0.0814)	(0.0204, 0.0815)
$IVvar$	0.0011	0.0004	0.0004
	(0.0007, 0.0019)	(0.0003, 0.0008)	(0.0003, 0.0008)

Notes. Posterior means and 95% confidence intervals (reported in parentheses) for the residuals from the currency return and from the IV equations. C stands for the residuals from the currency return equation, superscript IV stands for the residuals from the IV equation.

Table 2.9: Model diagnostics for JPY

	SV ($\theta = 0$, $\theta_v = 0$)	SVJ ($\theta = 0$)	Preferred
skewness^C	0.3348	0.3360	0.1298
	(0.3060, 0.3650)	(0.3038, 0.3668)	(0.0799, 0.1800)
kurtosis^C	4.8254	4.7148	3.6054
	(4.7109, 4.9645)	(4.5982, 4.8361)	(3.4829, 3.7445)
autocorrelation^C	-0.0146	-0.0140	-0.0221
	(-0.0176 -0.0116)	(-0.0174, -0.0108)	(-0.0312, -0.0131)
skewness^{IV}	0.0568	0.0278	0.0311
	(-0.0210, 0.1349)	(-0.0495, 0.1054)	(-0.0465, 0.1087)
kurtosis^{IV}	3.0707	3.0430	3.0423
	(2.9175, 3.2420)	(2.8940, 3.2100)	(2.8923, 3.2098)
autocorrelation^{IV}	0.1042	0.0758	0.0768
	(0.0733, 0.1349)	(0.0443, 0.1070)	(0.0453, 0.1083)
$IVvar$	0.0061	0.0029	0.0037
	(0.0036, 0.0125)	(0.0017, 0.0059)	(0.0021, 0.0078)

Notes. Posterior means and 95% confidence intervals (reported in parentheses) for the residuals from the currency return and from the IV equations. C stands for the residuals from the currency return equation, superscript IV stands for the residuals from the IV equation.

Table 2.10: Log-Bayes-Odds Ratios

	AUD	CHF	GBP	JPY
SVJ/SV	22.03	52.05	44.50	34.89
Preferred/SVJ	26.36	18.77	13.43	61.22

Notes. Formal model comparison. We compare the SV ($\theta = 0$, $\theta_v = 0$), SVJ ($\theta = 0$) and the preferred models pairwise. In the first row, we consider the SV and SVJ models and quantify evidence against the SV model; in the second row, we consider the SVJ and the preferred models and quantify evidence against the SVJ model.

Table 2.11: Calibration of the interest rates

	AUD	CHF	GBP	JPY
a_r	0.0181	0.0184	0.0184	0.0182
$\tilde{a}_r$	0.0291	0.0121	0.0269	0.0077
b_r	0.9999	0.9997	0.9997	0.9998
$\tilde{b}_r$	0.9991	0.9995	0.9998	0.9997
σ_r	0.0012	0.0016	0.0016	0.0014
$\tilde{\sigma}_r$	0.0035	0.0030	0.0018	0.0027

Notes. We calibrate processes for domestic (US)

$$r_{t+1} = (1 - b_r)a_r + b_r r_t + \sigma_r r_t^{1/2} w_{t+1}^r$$

and foreign interest rates

$$\tilde{r}_{t+1} = (1 - \tilde{b}_r)\tilde{a}_r + \tilde{b}_r \tilde{r}_t + \tilde{\sigma}_r \tilde{r}_t^{1/2} \tilde{w}_{t+1}^r$$

Parameters correspond to daily interest rates in percent. There are four versions of the parameters corresponding to the US interest rate. This is because the foreign data have different starting dates, and we calibrated the US rate in the matching samples.

Table 2.12: Summary of events associated with jumps

Type of event/uncertainty	Jump Up	Jump Down	Jump in Vol
		AUD	
Trade	1	1	–
Macro-Economic	2	2	9
Intervention	–	4	1
Monetary policy	2	7	10
Spillover from financial markets	1	5	20
Other	1	3	3
Total	6	21	34
International	2	5	17
		CHF	
Trade	6	–	–
Macro-Economic	1	2	11
Intervention	6	2	2
Monetary policy	1	1	10
Spillover from financial markets	4	1	18
Other	2	–	12
Total	17	6	45
International	9	3	23
		GBP	
Trade	–	–	1
Macro-economic	1	2	13
Intervention	2	1	3
Monetary policy	2	2	17
Spillover from financial markets	1	–	19
Other	–	2	14
Total	5	6	56
International	3	1	25
		JPY	
Trade	11	3	6
Macro-Economic	4	3	11
Intervention	8	4	4
Monetary policy	2	–	16
Spillover from financial markets	7	1	15
Other	3	3	7
Total	34	13	52
International	12	4	19

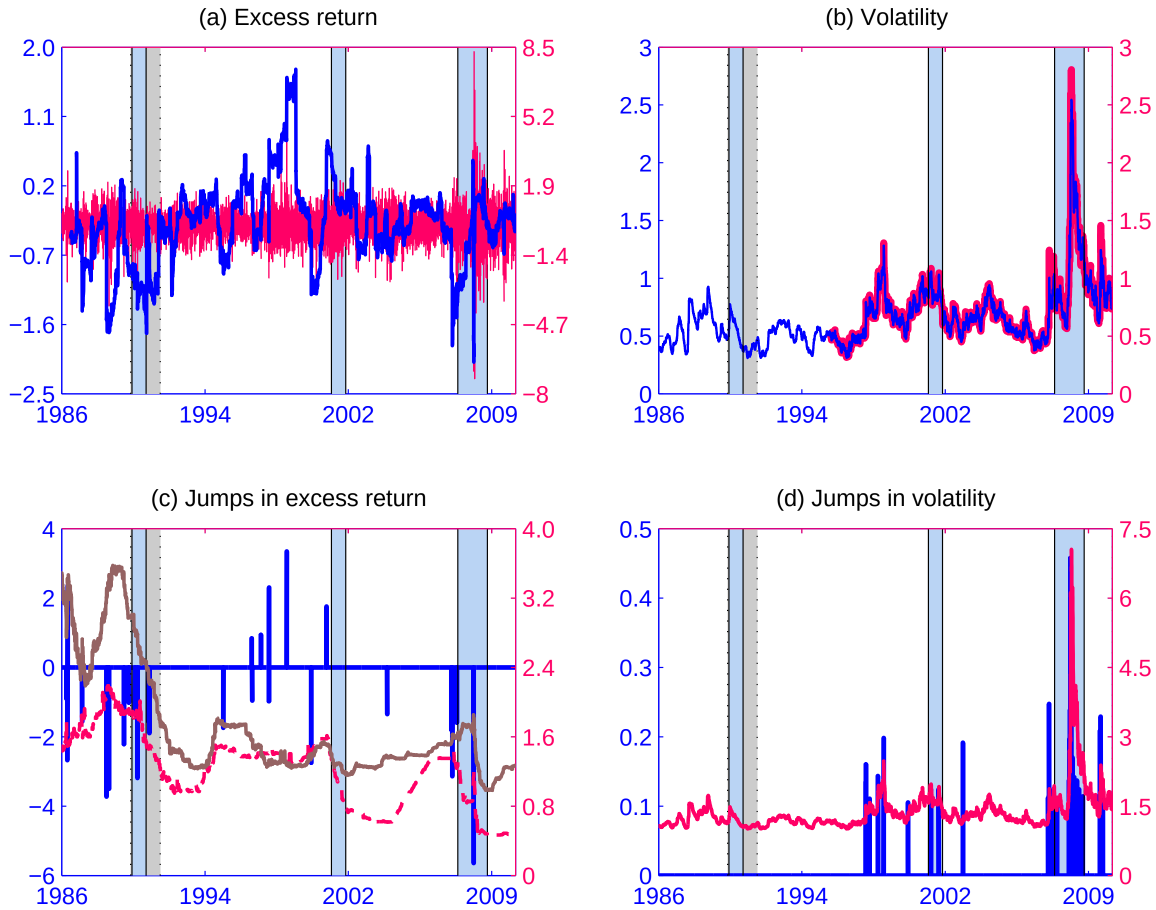
Notes: We match each jump in the preferred model with economic, political or financial events. If we cannot attribute a jump to a specific event then we indicate type of uncertainty dominating FX markets on that date. We compute how many jumps correspond to every economic source of risk. We distinguish trade, intervention, and monetary policy events (inflation, interest rate policy, monetary union) from events connected to other macro-economic factors (growth, employment, sales, payroll, etc.) We group episodes of dramatic movements in stock and commodity markets under the "Spillover from financial markets." All remaining episodes fall under rubric "Other". We report total number of jumps in prices (up and down) and volatility in the row "Total". We provide the number of jumps that occur simultaneously in two or more currencies in the row "International". Every jump episode can be generated by multiple sources of economic uncertainty. In such a case, we assign the jump to every important source of risk. Thus in our table the number in the row "Total" can be lower than the columnwise sum of the inputs.

Table 2.13: Decomposition of the total risk

	Jump Up		Jump Down		Jump in Vol		Normal	
	$n = 21$	$n = 63$	$n = 21$	$n = 63$	$n = 21$	$n = 63$	$n = 21$	$n = 63$
				AUD				
Mean	6.90	6.37	9.17	8.40	4.12	10.18	79.81	75.05
Std	3.70	2.97	5.22	4.13	1.10	1.99	9.60	8.38
Min	0.31	0.38	0.60	0.74	1.58	4.45	55.17	56.33
Max	15.84	12.41	26.24	21.34	7.14	14.73	97.45	94.32
				CHF				
Mean	6.82	6.66	5.48	5.37	3.76	8.94	83.94	79.04
Std	2.56	2.03	1.82	1.50	0.27	0.39	4.32	3.34
Min	1.02	1.31	0.98	1.26	3.13	8.01	66.78	69.49
Max	16.34	11.88	12.04	10.55	4.84	9.91	94.87	89.38
				GBP				
Mean	2.98	2.78	3.68	3.45	4.37	11.48	88.96	82.29
Std	1.51	1.19	1.68	1.38	0.80	1.35	3.73	3.42
Min	0.18	0.23	0.25	0.32	2.64	7.79	74.21	72.60
Max	8.84	5.83	9.87	8.45	7.70	15.48	96.87	91.58
				JPY				
Mean	9.10	8.57	5.80	5.43	4.83	11.19	80.27	74.82
Std	4.18	3.32	3.53	2.96	0.71	1.08	7.56	6.10
Min	0.93	1.23	0.69	0.91	3.05	7.96	54.70	57.34
Max	22.04	18.05	17.68	13.89	6.82	13.61	65.20	89.67

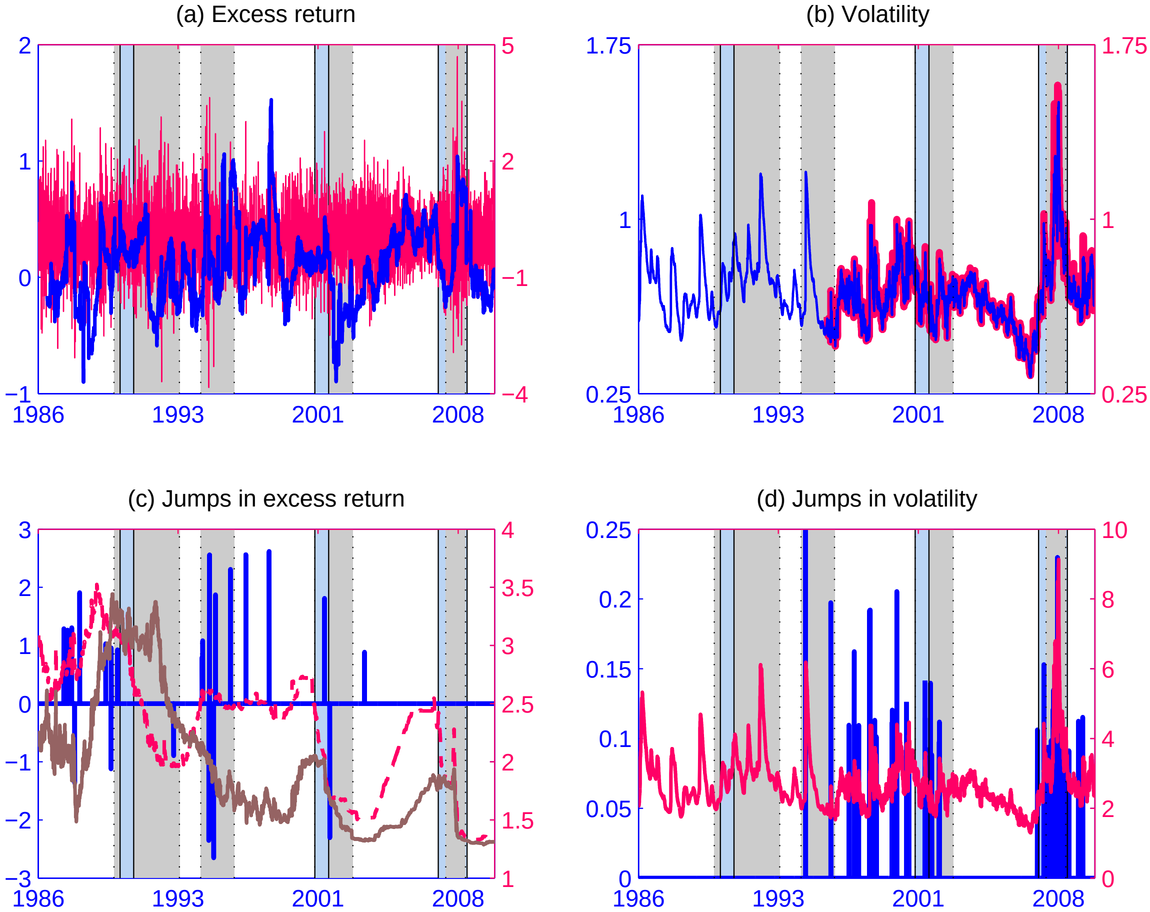
Notes. We report summary statistics of the percentage contribution of the different risks to the total risk of currency returns (horizon $n = 21$ and $n = 63$ days).

Figure 2.1: AUD data and estimated states



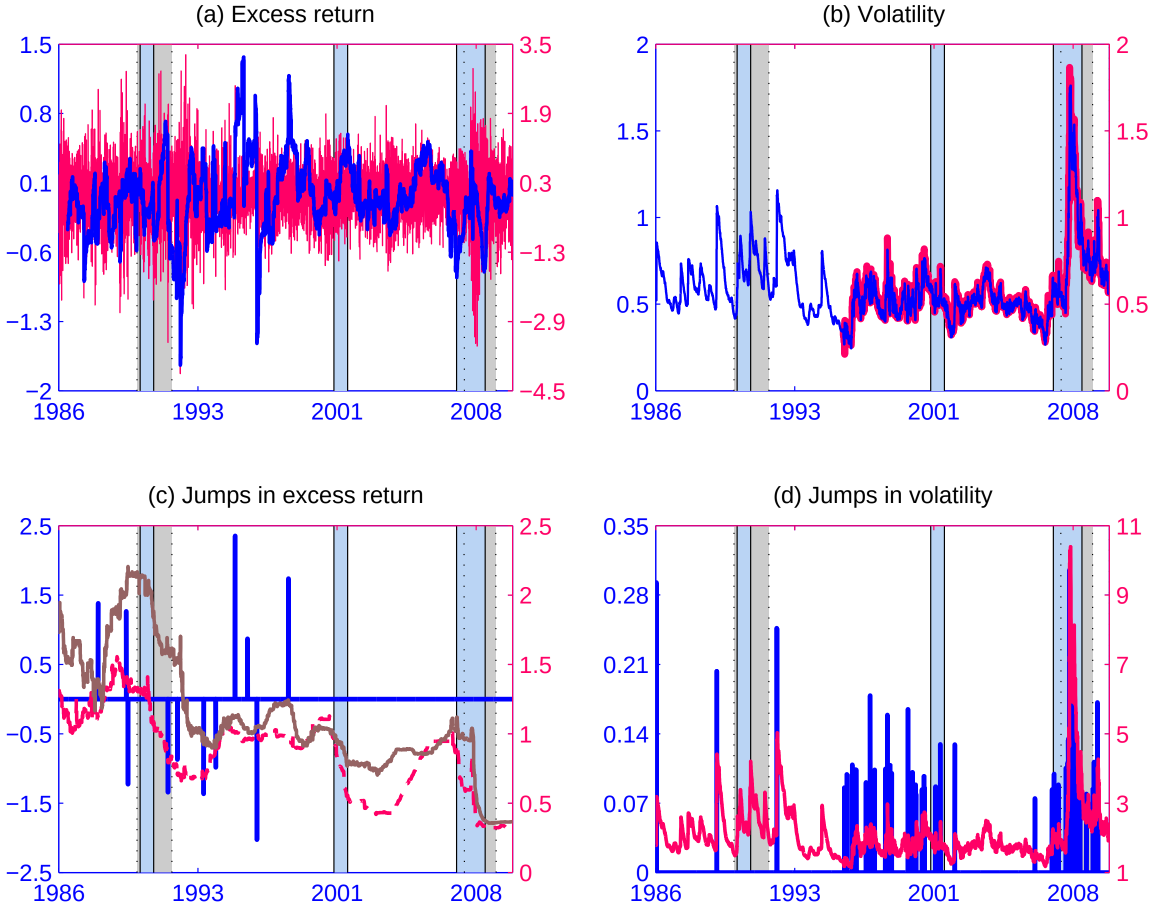
Panel (a): thin red line displays observed log currency returns; thick blue line displays the conditional 6-months skewness of the log currency returns. Panel (b): thick red line shows observed one-month at-the-money implied volatility; thin blue line shows the estimated spot volatility $\sqrt{v_t}$. The bottom panels display estimated jump sizes in returns (c) and volatility (d) with jump intensities. Panel (c): brown solid line is the intensity of the jump down; the red dashed line is the intensity of the jump up; blue bars are jumps themselves. Panel (d): the red line is the intensity of the jump in volatility; blue bars are jumps. We say that there was a jump at time t if the estimated probability of a jump on that day was above 50%. Gray vertical bars with the dashed border indicate recessions in Australia; light blue vertical lines with the thin solid border indicate recessions in the US.

Figure 2.2: CHF data and estimated states



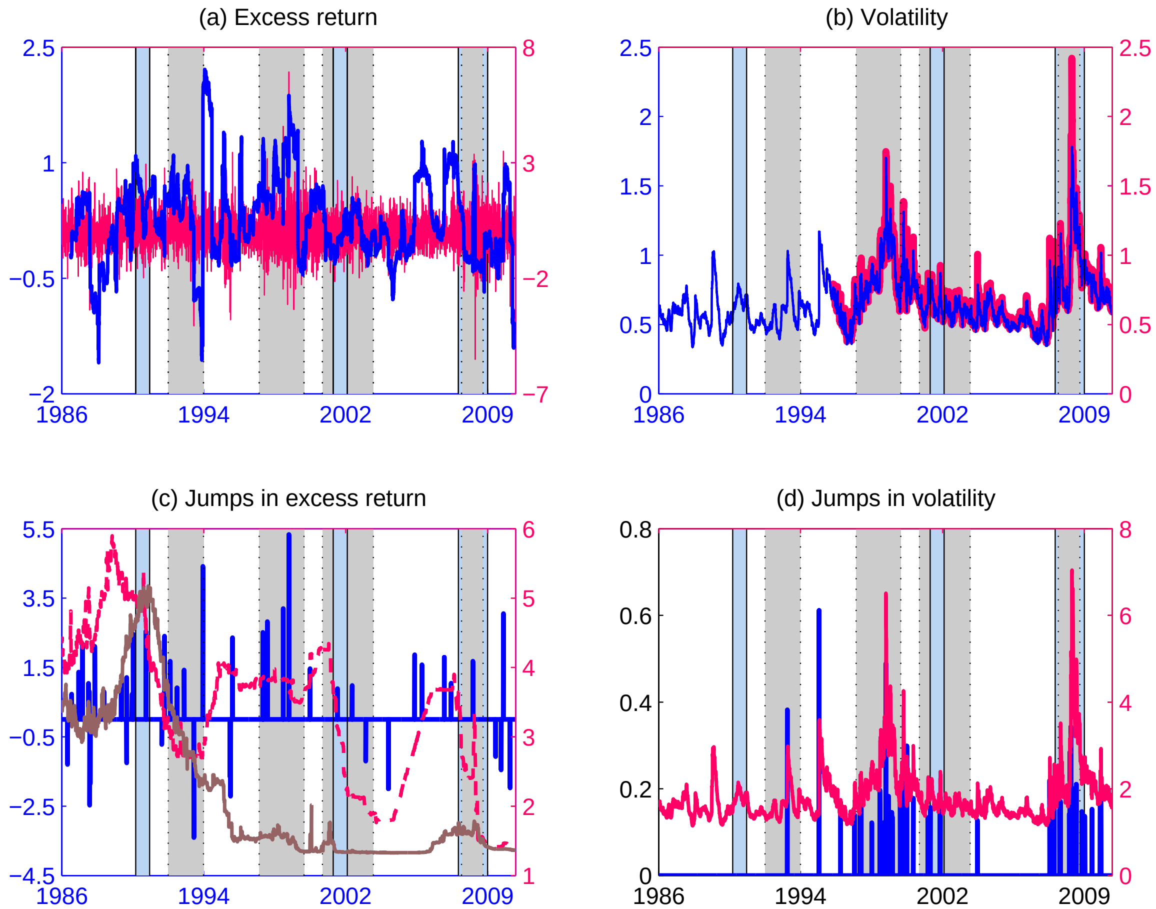
Notes. Panel (a): thin red line displays observed log currency returns; thick blue line displays the conditional 6-months skewness of the log currency returns. Panel (b): thick red line shows observed one-month at-the-money implied volatility; thin blue line shows the estimated spot volatility $\sqrt{v_t}$. The bottom panels display estimated jump sizes in returns (c) and volatility (d) with jump intensities. Panel (c): brown solid line is the intensity of the jump down; the red dashed line is the intensity of the jump up; blue bars are jumps themselves. Panel (d): the red line is the intensity of the jump in volatility; blue bars are jumps. We say that there was a jump at time t if the estimated probability of a jump on that day was above 50%. Gray vertical bars with the dashed border indicate recessions in Switzerland; light blue vertical lines with the thin solid border indicate recessions in the US.

Figure 2.3: GBP data and estimated states



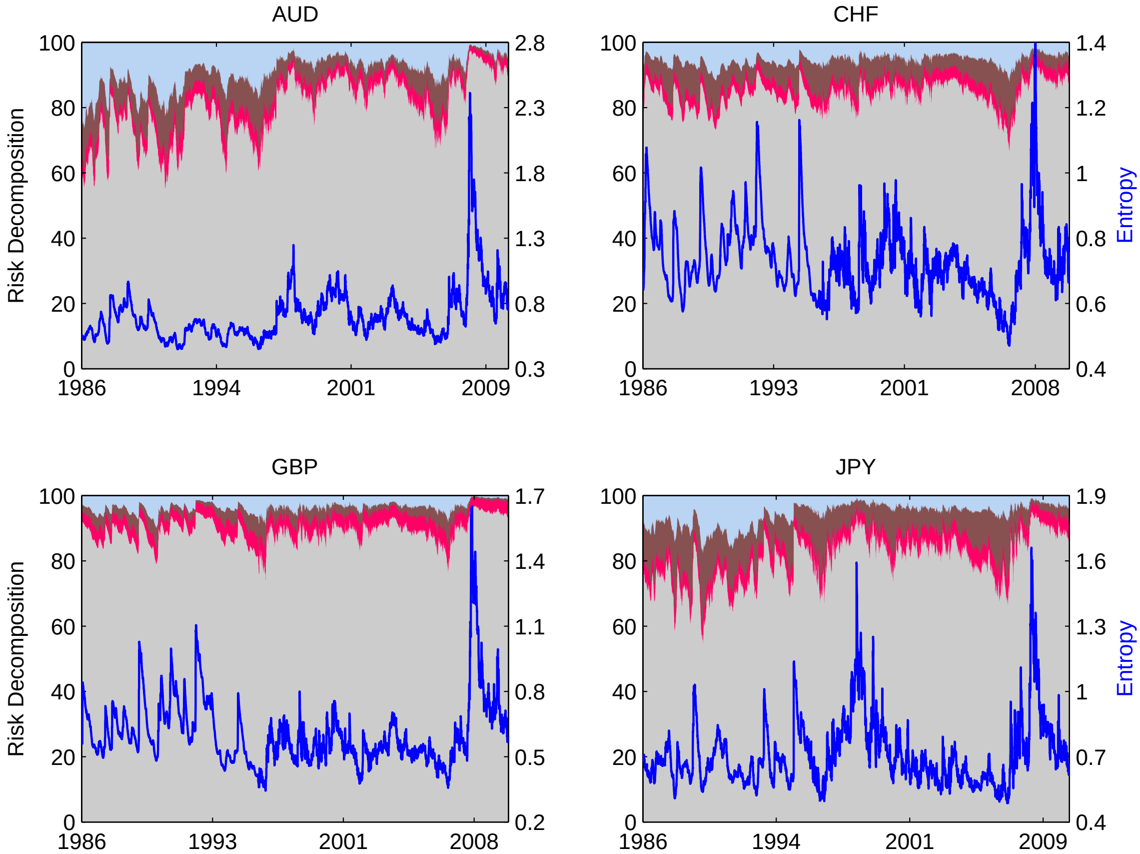
Notes. Panel (a): thin red line displays observed log currency returns; thick blue line displays the conditional 6-months skewness of the log currency returns. Panel (b): thick red line shows observed one-month at-the-money implied volatility; thin blue line shows the estimated spot volatility $\sqrt{v_t}$. The bottom panels display estimated jump sizes in returns (c) and volatility (d) with jump intensities. Panel (c): brown solid line is the intensity of the jump down; the red dashed line is the intensity of the jump up; blue bars are jumps themselves. Panel (d): the red line is the intensity of the jump in volatility; blue bars are jumps. We say that there was a jump at time t if the estimated probability of a jump on that day was above 50%. Gray vertical bars with the dashed border indicate recessions in the UK; light blue vertical lines with the thin solid border indicate recessions in the US.

Figure 2.4: JPY data and estimated states



Notes. Panel (a): thin red line displays observed log currency returns; thick blue line displays the conditional 6-months skewness of the log currency returns. Panel (b): thick red line shows observed one-month at-the-money implied volatility; thin blue line shows the estimated spot volatility $\sqrt{v_t}$. The bottom panels display estimated jump sizes in returns (c) and volatility (d) with jump intensities. Panel (c): brown solid line is the intensity of the jump down; the red dashed line is the intensity of the jump up; blue bars are jumps themselves. Panel (d): the red line is the intensity of the jump in volatility; blue bars are jumps. We say that there was a jump at time t if the estimated probability of a jump on that day was above 50%. Gray vertical bars with the dashed border indicate recessions in Japan; light blue vertical lines with the thin solid border indicate recessions in the US.

Figure 2.5: Conditional decomposition of the total risk for monthly returns

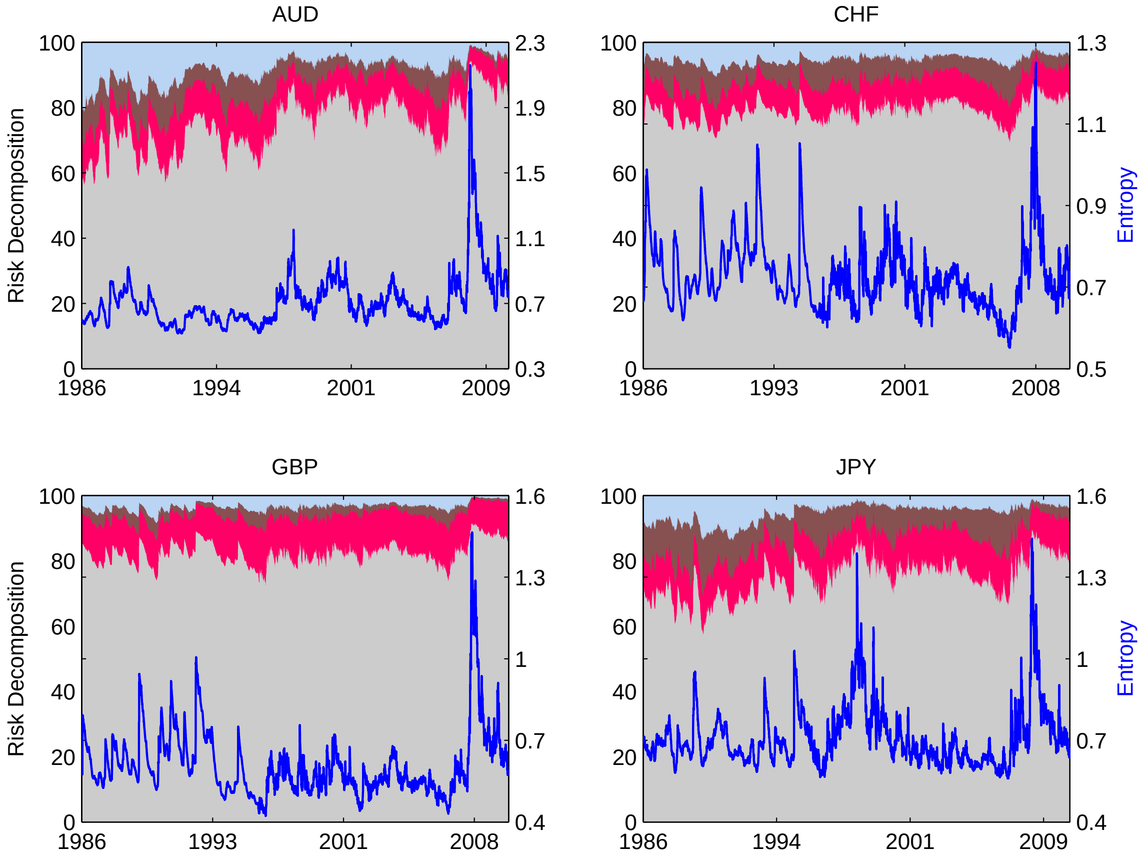


Notes. We display cumulative contribution of the different risks to the total risk of excess returns (the left axis). We measure the total amount of risk using entropy

$$L_t(S_{t+n}/S_t) = \kappa_{2t}(s_{t+n} - s_t)/2! + \kappa_{3t}(s_{t+n} - s_t)/3! + \kappa_{4t}(s_{t+n} - s_t)/4! + \dots,$$

where $\kappa_{jt}(s_{t+n} - s_t)$ is the jth cumulant of log FX returns. Investment horizon is $n = 21$ days. The contribution of the down jumps in FX is displayed in light blue, contribution of the up jumps in FX is in brown, and the contribution of the jumps in variance is in red. Gray area is the contribution of the normal shocks. The blue line shows $\sqrt{2L_t/n}$ in percent (the right axis). This quantity has an interpretation of one-period volatility in the case of normally distributed returns.

Figure 2.6: Conditional decomposition of the total risk for quarterly returns

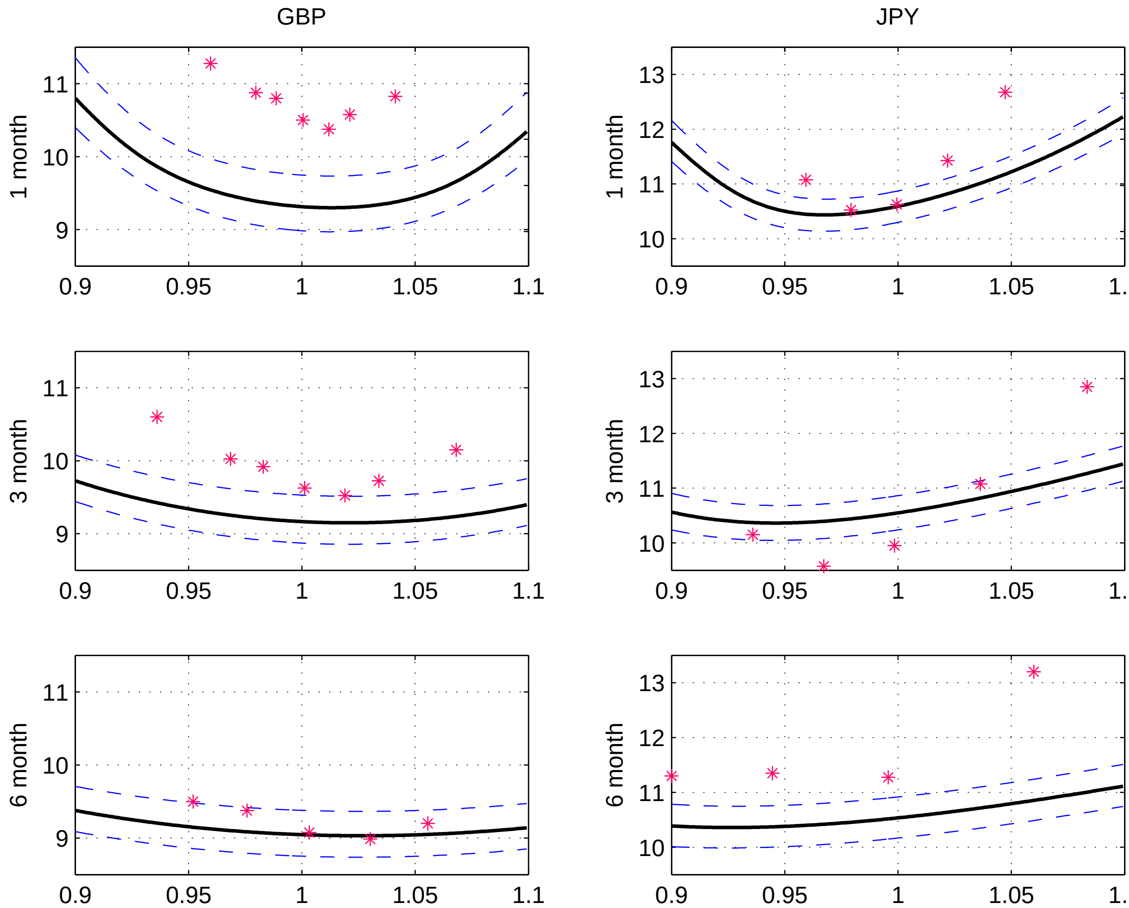


Notes. We display cumulative contribution of the different risks to the total risk of excess returns (the left axis). We measure the total amount of risk using entropy

$$L_t(S_{t+n}/S_t) = \kappa_{2t}(s_{t+n} - s_t)/2! + \kappa_{3t}(s_{t+n} - s_t)/3! + \kappa_{4t}(s_{t+n} - s_t)/4! + \dots,$$

where $\kappa_{jt}(s_{t+n} - s_t)$ is the jth cumulant of log FX returns. Investment horizon is $n = 63$ days. The contribution of the down jumps in FX is displayed in light blue, contribution of the up jumps in FX is in brown, and the contribution of the jumps in variance is in red. Gray area is the contribution of the normal shocks. The blue line shows $\sqrt{2L_t/n}$ in percent (the right axis). This quantity has an interpretation of one-period volatility in the case of normally distributed returns.

Figure 2.7: Implied volatility



Notes. We check the ability of the model to generate the implied volatility (IV) smiles and varying IV skewness. We select a currency with a positive average interest rate differential (GBP) and a negative one (JPY). For both currencies we pick a day with an approximately average variance and interest rate differential: November 12, 2007 (GBP) and April 20, 2004 (JPY). The asterisks indicate observed implied volatilities. The solid black lines depict theoretical implied volatilities evaluated at the estimated parameters and spot variance under the assumption that investors do not demand compensation for the variance or jump risks. The dashed blue lines show the 95% posterior coverage intervals for the theoretical smiles.

Chapter 3

Term-structure of consumption risk premia in the cross-section of currency returns

3.1 Introduction

In this paper, I quantify the risk-return relationship in the foreign exchange market in the cross-section and across investment horizons. I perform the analysis from the perspective of a US representative agent with recursive preferences over consumption. As in the long-run risk literature, I allow for the possibility that there are multiple sources of risk affecting consumption growth, such as shocks to expected consumption growth, stochastic variance of consumption growth, and consumption growth itself. My focus is on identifying such shocks, measuring their impact on currency prices in the cross-section and at different investment horizons, and understanding their relative importance for multi-period currency risk premia.

An influential paper by Lustig and Verdelhan (2007) shows that sorting currencies by their respective interest rates generates baskets with different exposures to realized consumption growth, which can explain the cross-sectional differences in one-period currency risk premia. The authors limit their attention to a fixed investment horizon that corresponds to the decision interval of the representative agent with recursive preferences. My contribution is in expanding the analysis to alternative horizons and characterizing multiple sources of consumption risk.

My interest lies in describing empirical properties of consumption risks. Therefore, instead of taking a stand on a specific process for consumption growth, I estimate a flexible vector autoregression (VAR) with stochastic variance that captures the spirit of the long-run risk models. I account for stochastic variance because as it has been widely documented in the literature (e.g., Bansal, Kiku, Shaliastovich, and Yaron, 2012; Campbell, Giglio, Polk, and Turley, 2012), the time-variation in the variance of consumption growth has first order implications for the macro dynamics and properties of asset prices.

An important feature of my approach is that I use additional information about the consumption process contained in macro variables and asset prices. An asset price is an appealing source of information about consumption because in equilibrium it reflects the unobservable components of the consumption growth process that are difficult to estimate on the basis of consumption data alone. Specifically, I learn about the consumption growth process through the joint dynamics of consumption growth, inflation, and the nominal yield.

I choose nominal bond as an asset because the nominal yield reflects risks relevant for exchange rates, as the theoretical literature (e.g., Bansal and Shaliastovich, 2013) has emphasized. In addition, the use of the yield, as opposed to another asset price, is convenient because it does not require the modeling of any cash flow dynamics. I incorporate inflation for two reasons. First, inflation has forecasting ability for future consumption growth (Piazzesi and Schneider, 2006). Second, I need the inflation dynamics to convert the model implied real risk-free rate to the observed nominal interest rate.

The pricing kernel derived by applying recursive preferences to the consumption growth process depends on the nominal yield because it is one of the states of the model. On the other hand, the pricing kernel must value all assets, including the nominal yield. The twofold role of the nominal yield in the model implies a set of pricing restrictions on the VAR parameters.

In summary, I specify my model of consumption growth in the form of a vector autoregression with stochastic variance and structural restrictions derived under recursive preferences.

This approach is novel, and it adds power to identify expected consumption growth better. I estimate the model by using quarterly data for US consumption growth, inflation, and a three-month nominal yield from the second quarter of 1947 until the fourth quarter of 2011. For estimation, I employ the Bayesian Markov Chain Monte Carlo (MCMC) methods. The key advantage of this approach is that it allows me imposing the required pricing restrictions directly; in addition, this approach delivers the estimated time series of stochastic variance.

I identify structural shocks from the estimated reduced-form innovations choosing to work with globally identified systems. I show that I have a choice of only two systems because of various restrictions based on economic intuition and regularity conditions (Rubio-Ramirez, Waggoner, and Zha, 2010). My model features the following four structural shocks to consumption: (1) the short-run consumption risk, (2) the inflation risk, (3) the long-run consumption risk, and (4) the variance risk. The only difference between the identification schemes is in the underlying identifying assumptions about the short-run consumption and inflation risks. I label the identification schemes "Fast Inflation" and "Fast Consumption". Under "Fast Inflation", inflation reacts to the short-run consumption shock contemporaneously, whereas consumption growth reacts to the inflation shock with a one-quarter delay (inflation is a faster variable). In contrast, under "Fast Consumption", consumption growth reacts to the inflation shock contemporaneously, whereas inflation reacts to the consumption growth shock with a one-quarter delay (consumption growth is a faster variable).

I use data on twelve currencies of developed economies over the period from 1986 to 2011 at a quarterly frequency. The choice of currencies is limited by the availability of the term-structure data required for the cross-sectional sorting. I sort currencies into three currency baskets based on the level of the average foreign yield.

I find that the model fits the macroeconomic data and data on asset prices well. First, the model captures important economic episodes such as the Great Moderation, recessions, and the recent financial crisis. Second, diagnostics of fitting errors do not exhibit noticeable misspecification. This provides a realistic setup for examining the model's asset pricing implications. I perform such an analysis across forty investment horizons, from one quarter to ten years. I use the shock-exposure and shock-price elasticities of Borovička, Hansen, Hendricks, and Scheinkman (2011) and Borovička and Hansen (2011) to characterize the term-structure of consumption risks and their prices in the cross-section of currency baskets at alternative horizons. Shock elasticities measure the sensitivity of expected cash flows and returns with respect to the change in the amount of the underlying risk and account for the presence of stochastic variance. The elasticities represent marginal quantities and marginal prices of risk (marginal Sharpe ratios).

I document the prominent role of the long-run risk for currency pricing in the cross-section and across multiple horizons. This is the main finding of my paper. First, there is a stable cross-sectional pattern in the term-structure of quantities of risk. At all horizons, the low interest rate currencies act as a hedge against the long-run risk, whereas the high interest rate currencies display a positive exposure to the risk. Differences are statistically significant and economically meaningful. Second, the long-run risk is associated with the highest risk compensation: its one-period log Sharpe ratio is 0.66. At the horizon of one quarter, the long-run risk explains at least 48% of the cross-sectional spread in excess returns.

The other shocks contribute to risk premia less prominently. The short-run consumption risk is priced in the cross-section of currency baskets at the horizon of one quarter only. This finding is based on the fact that only the contemporaneous difference in exposures of the low and the high interest rate currencies to the short-run consumption risk is statistically significant. At longer horizons, currencies are mostly immune to the short-run consumption risk or their risk exposures are insignificantly different from each other. The short-run consumption risk carries a high one-period log Sharpe ratio of 0.58 (0.52) under the "Fast Inflation" ("Fast Consumption") identification scheme, and, as a result, it explains at least 26% of the spread in one-period excess returns between the corner baskets.

At most investment horizons, currency baskets have significantly different exposures to the inflation risk. Similarly to the case of the long-run risk, low interest rate currencies act as a hedge against the risk, while the high interest rate currencies have a positive exposure to the risk. However, the price of the inflation risk is statistically significant only if consumption growth is a fast variable, i.e., reacts contemporaneously to the inflation risk. In this case, the inflation risk explains 26% of the spread in the one-period excess returns. The contribution of the inflation risk in explaining the cross-sectional spread in excess returns is smaller than the contribution of the long-run risk because its Sharpe ratio of 0.26 is almost a half of that for the long-run risk.

Finally, the loadings of FX cash flows on the variance risk are not significantly different in the cross-section at any horizon. The variance risk matters in a different respect. All currency baskets are highly sensitive to the risk at horizons longer than three years: the impact of a positive variance shock gradually increases with time and eventually causes substantial declines in cash flows of all currency baskets. However, the marginal price of this exposure is small. For example, at a three year horizon the marginal Sharpe ratio associated with the low interest rate currencies is 0.07, with the intermediate interest rate currencies is 0.06, and with the high interest rate currencies is 0.08. The marginal prices are positive suggesting that the currency baskets act as marginal hedges against an unfavorable variance shock to the representative agent.

Related literature

My paper is related to two strands of international macro-finance literature that examines time-series and cross-sectional properties of currency risk premia. I limit my discussion to papers that interpret currency risk premia as compensation for consumption risks. On the empirical front, Sarkissian (2003) and Lustig and Verdelhan (2007) study the ability of the consumption growth factor to explain the cross-section of currency returns. Sarkissian (2003) adapts the framework of Constantinides and Duffie (1996) to a multi-country setting and documents that the cross-country variance of consumption growth exhibits explanatory power for cross-sectional differences in returns on individual currencies, whereas the consumption growth itself does not. Lustig and Verdelhan (2007) establish in the framework of the durable CCAPM of Yogo (2006) that the consumption growth is a priced factor in the cross-section of returns on currency baskets formed by sorting currencies by respective interest rates. There are two common features in these papers. First, both studies recognize the presence of multiple sources of consumption risk but do not describe them explicitly. Second, both papers do not extend the analysis beyond a fixed horizon which is a decision interval of the representative agent (one quarter in the case of Sarkissian, 2003, and one year in the case of Lustig and Verdelhan, 2007).

Part of the theoretical literature features different consumption-based models dedicated to rationalizing the time-series behavior of currency risk premia, e.g., the violation of the uncovered interest rate parity. Models include but not limited to settings with habits (Heyerdahl-Larsen, 2012; Verdelhan, 2010), long-run risks (Bansal and Shaliastovich, 2013; Colacito, 2009; Colacito and Croce, 2013), and disasters (Farhi and Gabaix, 2008). My paper is closely related to the international long-run risk literature, but my focus is different. Theoretical international long-run risk studies model a joint distribution of domestic and foreign macroeconomic quantities to pin down a theoretical equilibrium exchange rate consistent with the forward premium anomaly. Instead, I model multiple sources of consumption risk of the US representative agent, estimate them, and establish their relative importance for currency risk premia in the cross-section of currencies and across alternative investment horizons.

My paper is also related to Hansen, Heaton, and Li (2008), who provide evidence on the importance of the permanent shock to consumption growth in account for the value premium. The similarity is in terms of approach, that is establishing the importance of consumption risks for explaining the cross-section of asset prices by joint modeling the stochastic discount factor (under the assumption of recursive preferences) and cash flow processes. My study differs from Hansen, Heaton, and Li (2008) in three principal dimensions. First, I study the foreign exchange market, which has been less examined than the US equity market. Second, my model has stochastic variance, so I account for variation in volatility of consumption growth, and therefore, in risk premia. Third, I quantify the relative importance of consumption risks at short and medium horizons, rather than at infinite horizons.

3.2 The model

Similarly to Lustig and Verdelhan (2007), I study the relative importance of consumption risks for currency pricing from the viewpoint of the US representative agent. In other words, I examine how currency cash flows covary with the US consumption risks and how this covariation is priced. My key modeling ingredients are: (1) the stochastic discount factor implied by the preferences of the representative agent and the dynamics of the consumption growth process and (2) currency cash flow. I proceed by describing each component in turn.

3.2.1 Recursive preferences

I use a standard framework of the representative agent model with recursive preferences. The recursive utility of Epstein and Zin (1989) and Weil (1989) is designed to account for the temporal distribution of risks; therefore, it is a natural setting for studying the role of multiple sources of risk. Notable applications of this framework for understanding the joint dynamics of exchange rates, macro quantities, and asset prices include but not limited to Backus, Gavazzoni, Telmer, and Zin (2010), Bansal and Shaliastovich (2013), Colacito and Croce (2011), Colacito and Croce (2013), Colacito (2009), Tretvoll (2011a), and Tretvoll (2011b).

The recursive utility is a constant elasticity of substitution recursion,

$$U_t = [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}, \tag{3.2.1}$$

with the certainty equivalent function,

$$\mu_t(U_{t+1}) = [E_t(U_{t+1}^\alpha)]^{1/\alpha}, \tag{3.2.2}$$

where c_t is consumption at time t, U_t is utility from time t onwards, $(1-\alpha)$ is the coefficient of relative risk aversion, $1/(1-\rho)$ is the elasticity of intertemporal substitution (EIS), and β is the subjective discount factor.

Under recursive preferences, the stochastic discount factor $m_{t,t+1}$ has two components, consumption growth and a forward looking component:

$$m_{t,t+1} = \beta(c_{t+1}/c_t)^{\rho-1}(U_{t+1}/\mu_t(U_{t+1}))^{\alpha-\rho}. \tag{3.2.3}$$

Appendix A.5 of the NBER version of Backus, Chernov, and Zin (2012) provides the derivation. There are two alternative ways to consider the component $U_{t+1}/\mu_t(U_{t+1})$ and to further derive the pricing kernel. One possibility is to use the connection between U_t and the equilibrium value of the aggregate consumption stream. This link would imply that the log stochastic discount factor is a function of consumption growth, $\log g_{t,t+1} = \log(c_{t+1}/c_t)$, and the return to a claim on future wealth, $r^w_{t,t+1}$, (Epstein and Zin, 1991):

$$\log m_{t,t+1} = \alpha/\rho \cdot \log \beta - \alpha(1-\rho)/\rho \cdot \log g_{t,t+1} - (1-\alpha/\rho) \cdot \log r^w_{t,t+1}. \quad (3.2.4)$$

The other possibility is to specify the process for consumption growth explicitly and derive this component of the pricing kernel as a function of the model's states and fundamental shocks (e.g., Backus, Chernov, and Zin, 2012; Hansen, Heaton, and Li, 2008).

The latter approach serves my purpose of describing the relative importance of multiple sources of consumption risk for currency pricing across multiple horizons. First, under the null of a structural model, the multi-period objects (consumption growth, stochastic discount factor, and cash flow) directly follow from the dynamics of the corresponding one-period objects. Therefore, a multi-period characterization of currency risk exposures and corresponding prices of risk does not require more data than its one-period counterpart. Second, this setting allows the decomposition of the total risk premium into the contributions of different sources of risk across multiple horizons (Borovička and Hansen, 2011).

3.2.2 Consumption growth process

It is a well known problem in asset pricing that high-quality consumption data are available at low frequency, and consequently, the identification of multiple sources of consumption risk is a challenging task. As a result, most studies of the joint behaviour of macro economic quantities and asset prices are theoretical. Authors calibrate various empirically plausible processes for consumption growth and study the implications of these models for asset prices.

The common critique of this approach is that different consumption growth processes are observationally equivalent given small sample sizes. Nonetheless, they have very different implications for asset prices. This observation has two implications. On the one hand, an econometrician working with consumption based models faces a serious challenge. On the other hand, this observation suggests that theoretical asset prices are informative about the consumption growth process. Indeed, in equilibrium asset prices are functions of observable consumption growth and unobservable states. As a result, one can learn about the data-generating process for consumption growth by observing asset prices. I exploit this implication to identify consumption growth empirically.

I specify a parsimonious yet flexible model of consumption growth. I posit a vector autoregressive process for $Y_{t+1} = (\log g_{t,t+1},\ \log \pi_{t,t+1},\ i^1_{t+1},\ \sigma^2_{t+1})'$ that includes consumption growth $\log g_{t,t+1}$, inflation $\log \pi_{t,t+1}$, short-term nominal yield i^1_{t+1}, and the stochastic variance σ^2_{t+1}

$$Y_{t+1} \quad = \quad F + GY_t + H\sigma_t\varepsilon_{t+1}, \tag{3.2.5}$$

where F is a four-by-one column-vector, and G and H are four-by-four matrices.[1] The vector ε_{t+1} contains four structural shocks, $\varepsilon_{t+1} = (\varepsilon_{g,t+1},\ \varepsilon_{\pi,t+1},\ \varepsilon_{i,t+1},\ \varepsilon_{\sigma,t+1})'$. Shocks $\varepsilon_{g,t+1}$, $\varepsilon_{\pi,t+1}$, and $\varepsilon_{\sigma,t+1}$ are the consumption risk, the inflation risk, and the variance risk, respectively. I interpret the fourth shock $\varepsilon_{i,t+1}$ later, once I have obtained the estimation results. I impose six parameter restrictions $G_{41} = G_{42} = G_{43} = H_{41} = H_{42} = H_{43} = 0$ to ensure that the stochastic variance follows the discretized version of the continuous-time square-root process.[2]

I select a variable to be included in Y_t if the variable has forecasting power for the future consumption growth. Hall (1983) and Hansen and Singleton (1983) show that lagged consumption growth is useful in predicting future US consumption growth. Piazzesi and Schneider (2006) argue that inflation is a leading recession indicator. Bansal, Kiku, and Yaron (2012b), Constantinides and Ghosh (2011), and Colacito and Croce (2011) argue

[1] I use double subscripts for log consumption growth and inflation to indicate the time period of the corresponding change in consumption or price level. For example, $\log \pi_{t,t+\tau}$ is a $\tau-$period inflation from t to $t+\tau$. I use superscripts for interest rates to indicate the type of the corresponding yields. For example, i^τ_t corresponds to the yield of the τ-period nominal bond at time t. σ^2_t is a one-period stochastic variance, σ_t is a one-period stochastic volatility.

[2] In continuous time, the Feller condition $2F_{41} > H^2_{44}$ guarantees that the variance stays strictly positive. A formal modeling of this process in discrete time is achieved via a Poisson mixture of Gamma distributions (Gourieroux and Jasiak, 2006; Le, Singleton, and Dai, 2010). I use a direct discretization of the continuous-time square-root process to streamline the estimation of the model: I draw all parameters of the model together because the vector ε_{t+1} follows the multivariate normal distribution. I ensure that the variance remains positive by drawing it in logs.

that the real risk-free rate serves as a direct measure of the predictable component in future consumption growth. Instead of including the real risk-free rate in Y_t, I use a short-term nominal yield and inflation.[3]

Among the possible asset prices, I use the nominal yield for a number of reasons. First, the extant empirical and theoretical literature on the violation of the uncovered interest rate parity has documented that risks in exchange rates and interest rates are related (e.g., Bansal and Shaliastovich, 2013; Heyerdahl-Larsen, 2012; Verdelhan, 2010). At a later stage in my paper, I project the currency prices on the US stochastic discount factor. Therefore, it is critical to ensure that important sources of exchange rate risks are captured by the model of the stochastic discount factor. Second, the use of the yield does not require modeling of an extra cash flow process, e.g., the dividend process, or taking a stand on whether the stock market return is a good proxy for the return on the aggregate consumption claim.

I introduce stochastic variance to the model because the time-variation in the volatility of consumption growth is a salient feature of consumption data, which in its turn serves as a direct source of time variation in risk premia (Bansal and Shaliastovich, 2013; Drechsler and Yaron, 2011). Recently, Bansal, Kiku, Shaliastovich, and Yaron (2012) and Campbell, Giglio, Polk, and Turley (2012) have revisited the importance of the stochastic variance of consumption growth and emphasized its first-order implications for understanding the macro dynamics, as well as the time-series and cross-sectional properties of asset prices. In general, it is a challenging task to identify the stochastic variance in consumption data. My strategy of using a multi-variate system of consumption growth, inflation, and nominal yield to do so has a higher power because several variables have a stronger information content regarding the common unobserved variance.[4]

The pricing kernel derived by applying preferences (3.2.1)-(3.2.2) to the consumption growth

[3]Price-dividend ratio and default premia are other variables used in consumption growth predictive regressions. See Colacito and Croce (2011) for details. I do not use these variables because connecting them to the pricing kernel requires an additional modeling effort. For example, the use of the price-dividend ratio must be accompanied by the model of the dividend growth process.

[4]Carriero, Clark, and Marcellino (2012) document that a vector autoregression with common stochastic volatility factor efficiently summarizes the information content of several macroeconomic variables, such as GDP growth, consumption growth, growth of payroll employment, the unemployment rate, GDP inflation, the 10-year Treasury bond yield, the federal funds rate, and growth of business fixed investment. The authors justify this modeling approach using the observation that the pattern of estimated volatilities is often similar across variables.

process (3.2.5) is

$$\log m_{t,t+1} \quad = \quad \log m + \eta' Y_t + q' \sigma_t \varepsilon_{t+1}, \tag{3.2.6}$$

where $\eta = (\eta_g,\ \eta_\pi,\ \eta_i,\ \eta_\sigma)'$ and $q = (q_g,\ q_\pi,\ q_i,\ q_\sigma)'$. The parameters of the vectors η and q are functions of the structural parameters of the model (Appendix A.2.1). Note that the pricing kernel naturally depends on all the states Y_t, but one of the states i_t^1 is a transformed asset price. Because the pricing kernel must value all assets in the economy, including the nominal bond, I impose a number of cross-equation restrictions on the VAR (3.2.5). As a result, the requirement of internal consistency of my model leads to a constrained vector autoregression. Except for these restrictions, I do not impose any other parameter constraints.

The equilibrium nominal yield is an affine function of all the model's states,

$$i_t^1 = A \log g_{t-1,t} + B \log \pi_{t-1,t} + C i_t^1 + D \sigma_t^2 + E,$$

where A, B, C, D, and E are the functions of the structural parameters describing the dynamics of the consumption growth and the preference parameters. I provide the full derivation of the equilibrium nominal yield in the Appendix A.2.1. To ensure that the price of the nominal bond is internally consistent, I require

$$A = B = D = E = 0, \tag{3.2.7}$$

$$C = 1. \tag{3.2.8}$$

The restrictions $A = B = E = 0$ and $C = 1$ are linear

$$\frac{G_{21}}{G_{11}} = \frac{G_{22}}{G_{12}} = \frac{G_{23} - 1}{G_{13}} = \frac{F_2 - \log \beta}{F_1} = \rho - 1, \tag{3.2.9}$$

whereas the restriction $D = 0$ is nonlinear

$$\alpha(\alpha-\rho)(P+e_1)'HH'(P+e_1)/2+e_2'Ge_4-e_2'HH'e_2/2$$
$$-[(\alpha-\rho)P+e_1(\alpha-1)]'HH'[(\alpha-\rho)P+e_1(\alpha-1)]/2$$
$$+e_2'HH'[(\alpha-\rho)P+e_1(\alpha-1)]-(\rho-1)e_1'Ge_4=0, \tag{3.2.10}$$

and depends on the endogenous parameters $P = (p_g,\ p_\pi,\ p_i,\ p_\sigma)'$ that show up in the solution of the value function

$$\log u_t = \log(U_t/c_t) = \log u + p_g \log g_{t-1,t} + p_\pi \log \pi_{t-1,t} + p_i i_t^1 + p_\sigma \sigma_t^2. \tag{3.2.11}$$

The parameters of the vector P are functions of the preference parameters and the parameters governing the dynamics of consumption growth. Vectors e_i that enter the nonlinear restriction (3.2.10) are the corresponding coordinate vectors in a four-dimensional space. The nonlinear nature of the restriction (3.2.10), combined with the presence of the endogenous parameters, represents a serious challenge for the estimation. See Appendix A.2.1 for the model's solution and further details.

In summary, I model consumption growth via its joint dynamics with inflation and nominal interest rate. I allow for common stochastic variance and impose restrictions required for internal pricing consistency. This process for consumption growth, combined with recursive preferences, leads me to a fully articulated model of the pricing kernel.

3.2.3 Foreign exchange cash flow

To illustrate the basic risk-return relationship in the foreign exchange market, I consider the following investment strategy. At time t, the US representative investor buys a zero-coupon foreign bond of maturity τ and pays $\exp(-\tilde{i}_t^\tau)s_t/p_t$ US dollars (USD) in real terms. At a future date $t+\tau$, the foreign bond pays back one unit of the foreign currency, i.e., $s_{t+\tau}/p_{t+\tau}$ USD in real terms. The excess τ- period log real return on this strategy,

$$\log rx_{t,t+\tau} = [\log s_{t+\tau} - \log s_t + \tilde{i}_t^\tau - i_t^\tau - \log \pi_{t,t+\tau}]/\tau,$$

is called a currency return because the currency price is a risky part of the strategy.[5] I use the following notation: exchange rate s_t is the price of one unit of foreign currency in terms of USD; i_t^τ ($\tilde{i}_t^\tau$) is the US (foreign) nominal yield of maturity τ; p_t is the US price index.

Next, I introduce the notion of the FX cash flow: $\delta_{t,t+\tau} = s_{t+\tau}/[s_t \pi_{t,t+\tau}]$. Strictly speaking, it is the real normalized cash flow of a foreign bond, or cash flow growth. I prefer to work with $\delta_{t,t+\tau}$ rather that with the original cash flow $s_{t+\tau}/p_{t+\tau}$ because the primer is stationary. The law of one price shows that the price of the foreign bond of maturity τ reflects the future currency risk at the horizon τ:

$$e^{-\tilde{i}_t^\tau} = E_t[m_{t,t+\tau} s_{t+\tau}/[s_t \pi_{t,t+\tau}]] = E_t[m_{t,t+\tau} \delta_{t,t+\tau}], \tag{3.2.12}$$

where $m_{t,t+\tau}$ is the τ-period US (domestic) stochastic discount factor.

Since the study by Lustig and Verdelhan (2007), it is a standard practice in the literature to sort currencies into baskets depending on the level of the respective short interest rates (equivalently, on interest rate differentials) and to examine the covariance of the baskets' returns with some macroeconomic variables or return-based factors.[6] According to the law of one price (3.2.12), this sorting assigns currencies into baskets based on the price of the future currency exposure to risks at a fixed horizon τ. Equivalently, the existing literature has focused on understanding the nature of the risk-return relationship in the cross-section of currency baskets at a fixed horizon.

In contrast, I aim to understanding how the exposure of FX cash flows to the multiple sources of risk is priced at alternative horizons. Instead of sorting FX cash flows multiple times by the corresponding yields of maturity τ, I sort currencies into baskets based on the average yield in the corresponding foreign term-structures

$$\tilde{y}_t = \sum_{\tau=1}^{T} \tilde{i}_t^\tau.$$

[5] I assume that the investor holds foreign bond until maturity.

[6] Examples include but not limited to Burnside (2011), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Della Corte, Riddiough, and Sarno (2012), Lustig, Roussanov, and Verdelhan (2011), Lettau, Maggiori, and Weber (2012), Menkhoff, Sarno, Schmeling, and Schrimpf (2011), Mueller, Stathopoulos, and Vedolin (2012).

Thus, I build a cross-section of currencies with different exposure to the risks across multiple horizons. Other sorting strategies are possible. My view is that the average yield is a good proxy for the price of the multi-period exposure of FX cash flow to the risks. I perform robustness check and sort currencies into baskets based on the respective short interest rates as in the rest of the literature. My results remain similar.[7]

To characterize the risk-return relationship in the foreign exchange market at alternative horizons, I need the model of the joint dynamics of the pricing kernel and FX cash flows. Under the null of the model, I can perform analysis at any horizon without requiring more data at longer horizons, and, more importantly, I can deduce the role of every specific consumption risk for currency pricing. I augment the dynamics of the pricing kernel described in the previous section with the law of motion of the FX cash flow. To do so, I project the FX cash flow on the information set of the representative agent and the structural shocks and omit the orthogonal component:

$$\log \delta_{t,t+1} \quad = \quad \log \delta + \mu' Y_t + \xi' \sigma_t \varepsilon_{t+1} + \xi_v \sigma_t v_{t+1}, \tag{3.2.13}$$

where $\mu = (\mu_g,\ \mu_\pi,\ \mu_i,\ \mu_\sigma)'$, $\xi = (\xi_g,\ \xi_\pi,\ \xi_i,\ \xi_\sigma)'$ and v_{t+1} is an idiosyncratic shock. The omitted orthogonal component is irrelevant for the US pricing and does not affect statistical inference. The latter acts similarly to a linear regression, with an omitted variable that is orthogonal to regressors. By using the process (3.2.13), I make an additional assumption that world economies share the same volatility factor. Having a separate volatility factor for a foreign economy is appealing; however, at the estimation stage the FX data at a quarterly frequency are not informative enough about it.

3.3 Data

3.3.1 Macro data

I use quarterly data on consumption growth, inflation, and three-month nominal yield from the second quarter of 1947 to the fourth quarter of 2011. In total, there are 259

[7] Appendix A.2.2 provides these results.

observations. I collect consumption and price data from the NIPA tables of the Bureau of Economic Analysis. The nominal yield comes from CRSP. Appendix A.2.3 contains detailed data description.

Table 3.1 provides basic descriptive statistics. The unconditional standard deviation of consumption growth is slightly higher than 1% annualized. This value is at least twice as low as the value over a longer time interval, including the Great Depression. Panels (a)-(c) of Figure 3.1 displays the dynamics of these variables. It is clear from Panels (b) and (c) that inflation and nominal rate tend to decrease during recessions. This observation is useful for the interpretation of empirical evidence later in the paper.

3.3.2 Currency and interest rate data

I collect daily data on twelve spot exchange rates from Thomson Reuters provided by Datastream. The sample contains the price of the Australian dollar, the British pound, the Canadian dollar, the Danish krone, the Euro, the Deutsche mark, the Japanese yen, the New Zealand dollar, the Norwegian krone, the South African rand, the Swedish krone, and the Swiss frank in terms of USD. The sample runs from the beginning of 1986 until the end of 2011. According to the latest report of the Bank of International Settlements, these currencies are among the twenty two currencies with the highest daily turnover, as of April 2010.

I use fixed income data from Datastream, Bloomberg and the dataset of Wright (2011). Wright (2011) provides detailed term-structure data for Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, and the UK until the first quarter of 2009. From the first quarter of 2009 until the last quarter of 2011, I compute swap implied interest rates for all of these countries but Germany. For Denmark, the Euro area, and South Africa, I compute the swap implied interest rates for the entire time interval. The term-structure data contain yields of forty maturities, from one quarter to ten years. Appendix A.2.3 (Table A.6) describes data availability and sources of data for every country.

I choose currencies of developed countries that are used elsewhere in the literature.[8] Because

[8] See Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), Lustig, Roussanov, and Verdelhan (2011),

of data availability on the term-structure of interest rates, my sample contains a smaller number of currencies. I work with quarterly currency quotes sampled at the end of the corresponding quarter. The choice of the data frequency corresponds to the frequency of consumption growth data.[9]

At the end of each quarter, I sort currencies into three baskets by the average yield in the foreign term-structure $\tilde{y}_t$. Because the number of currencies is small, I use only three portfolios. Basket "Low" contains the low interest rates currencies, basket "Intermediate" contains the intermediate interest rate currencies, and basket "High" contains the high interest rate currencies. Table 3.2 provides descriptive statistics of currency portfolio returns. The average return is monotonically increasing from basket "Low" to basket "High". Similar to Lustig and Verdelhan (2007), I find a spread in excess returns between basket "High" and basket "Low" of approximately 4.5% per year. Table 3.3 displays the dynamic composition of the baskets. Evidently, some currencies, e.g., the Japanese yen or the Swiss franc, remain in the same basket during the entire time period, so the basket re-balancing does not affect them. Other currencies, for example, the Canadian dollar or the Swedish krone, belong to each basket for several quarters.

3.4 Methodology

In this section, I describe my estimation approach. My ultimate goal is to estimate the joint dynamics of the stochastic discount factor and the FX cash flow. It suffices, however, to estimate the joint process for consumption growth and the FX cash flow. Recursive preferences applied to the dynamics of consumption growth pin down the dynamics of the stochastic discount factor. I assume that the idiosyncratic foreign exchange shock does not affect the dynamics of consumption growth; therefore, the estimation of the joint process is equivalent to a three-stage procedure: (1) estimation of the consumption growth process (3.2.5) with pricing restrictions (3.2.9) and (3.2.10), (2) identification of the structural

Lustig and Verdelhan (2007), Menkhoff, Sarno, Schmeling, and Schrimpf (2011), Rafferty (2011) among others.

[9] US consumption data are available at monthly, quarterly, and annual frequencies. It is well known that annual data are preferable but there are few observations to carry empirical work. I choose consumption data at a quarterly frequency as a compromise of quality and the number of available observations.

shocks ε_{t+1} from the estimated reduced-form innovations in the vector autoregression, and (3) estimation of the cash flow process, taking into account identified structural shocks ε_{t+1}. My approach is free of the generated regressors' problem and provides the full distribution for all parameter estimates, stochastic variance, and structural shocks because I use the Bayesian methods. Below, I explain every stage in detail.

3.4.1 Estimation of the consumption growth process

I employ the Bayesian MCMC methods to estimate a vector autoregressive model of consumption growth,

$$Y_{t+1} = F + GY_t + \Sigma^{1/2}\sigma_t w_{t+1}, \tag{3.4.14}$$

with restrictions (3.2.9) and (3.2.10) and stochastic variance, where w_{t+1} are reduced-form innovations that are unknown linear functions of structural shocks: $H\varepsilon_{t+1} = \Sigma^{1/2}w_{t+1}$. Matrix $\Sigma^{1/2}$ is the Cholesky lower triangular matrix and vectors of shocks $w_{t+1} \sim \mathcal{N}(0, I)$ and $\varepsilon_{t+1} \sim \mathcal{N}(0, I)$ follow the multivariate normal distribution.

The key advantage of this estimation approach is that it allows to impose the required pricing restrictions (3.2.9) and (3.2.10) directly and delivers the estimated time-series of stochastic variance as a byproduct of the estimation routine. I carefully design the simulation method for the stochastic variance. In particular, I draw the log of variance; therefore, the variance itself never becomes negative or zero. My approach to estimating a vector autoregression with stochastic variance is different from standard methods used in applied macroeconomics.[10] In particular, I draw all the parameters of the vector autoregression jointly because the stochastic variance is a part of the vector of state variables.

The pricing consistency restrictions (3.2.9) and (3.2.10) are functions of the structural parameters governing the dynamics of consumption growth (3.2.5) and the preference parameters α, β, and ρ. Therefore, in addition to the twenty two parameters of the consumption

[10] See, for example, Cogley and Sargent (2005), Justiniano and Primiceri (2008), Primiceri (2005), Sargent and Surico (2010).

growth process, there are three more preference parameters to estimate. This is a very challenging task, considering that the restriction (3.2.10) is nonlinear and requires a solution to the fixed point problem. I discuss the nature of the fixed point problem in Appendix A.2.4.

Parameters ρ and β appear in the linear restrictions (3.2.9) so that if I estimate F_1, F_2, G_{11}, G_{12}, G_{13}, and ρ (and this is a straightforward procedure), I can pin down $\log \beta$, G_{21}, G_{22}, and G_{23}. Instead, the parameter α enters only the nonlinear restriction (3.2.10). For this reason, it is not clear how to set up a prior for α and how to characterize its posterior distribution. Therefore, in this study I follow an easier yet still challenging route; that is, I assume the preference parameters and estimate the remaining twenty two parameters of the dynamics of consumption growth.[11] I account for the linear restrictions (3.2.9) by incorporating them directly in the parameter posterior distribution. I reject all the MCMC draws that violate the nonlinear restriction (3.2.10). To evaluate the nonlinear restriction, I solve the fixed-point problem at each draw; this process makes the estimation problem very time-consuming. If I had to estimate the preference parameters as well, in particular, α, the problem would be even more complicated.

I assume the following values for the preference parameters: $\alpha = -9$, $\beta = 0.9924$, and $\rho = 1/3$. The parameters α and ρ imply the preference for early resolution of uncertainty and have been extensively used in the literature to address a number of asset pricing puzzles. For example, by utilizing these preference parameters, Bansal and Yaron (2004) explain salient features of the equity market in an equilibrium framework of endowment economy; Hansen, Heaton, and Li (2008) empirically explain the value premium puzzle; whereas Bansal and Shaliastovich (2013) rationalize properties of the term-structure of nominal interest rates and the violation of the uncovered interest rate parity. In addition, in the international setting Colacito and Croce (2013) advocate EIS=3/2 ($\rho = 1/3$) as a value supported by empirical evidence gained through the lens of their structural model. Finally, I borrow the value of the subjective discount factor β from Hansen, Heaton, and Li (2008).

[11] I leave estimation of the preference parameters for the future research.

3.4.2 Identification

To recover structural shocks ε_{t+1} from the reduced-form innovations w_{t+1}, I augment the model with a number of economically motivated identifying restrictions as is usually done in structural vector autoregressions in applied macroeconomics.[12] The natural question is the number and the type of restrictions that should be imposed. Rothenberg (1971) provides a necessary condition, also known as the order condition, which says that to identify a system of n equations there must be at least $n(n-1)/2$ restrictions imposed. I have a system of four equations. Therefore, to identify the structural shocks ε_{t+1}, it is necessary to impose six restrictions. Theorem 1 in Rubio-Ramirez, Waggoner, and Zha (2010) provides a sufficient condition, also known as the rank condition, stating that the location of restrictions in the matrix H matters.

I choose to work with zero restrictions. The necessary and sufficient conditions together with several additional considerations guide me towards particular identification schemes. Firstly, the stochastic variance σ_t^2 follows the square root process, meaning that three restrictions on matrix H have been imposed from the beginning: $H_{41} = H_{42} = H_{43} = 0$. Next, economically, there must be no zero restrictions on the elements of the third row of the matrix H. The third variable in the system is the nominal rate. It is an equilibrium outcome, and hence, an affine function of the model's states. In principle, the nominal rate might not load materially on one state or another, but a priori, it would be unreasonable to restrict the model in any possible way. Finally, two equations and three more restrictions remain. Here, I follow Theorem 1 from Rubio-Ramirez, Waggoner, and Zha (2010) and find that the only two combinations of three zero restrictions (1) $H_{12} = H_{13} = H_{23} = 0$ and (2) $H_{13} = H_{21} = H_{23} = 0$ guarantee that the model is globally identified.

Identification $H_{12} = H_{13} = H_{23} = 0$ is labeled "Fast Inflation" because inflation reacts contemporaneously to a direct consumption shock, whereas consumption growth reacts to a current inflation shock with a one-quarter delay. Table 3.4 displays the corresponding location of zero restrictions. Identification $H_{13} = H_{21} = H_{23} = 0$ is labeled "Fast Con-

[12] See Blanchard and Quah (1989), Cochrane (1994), Christiano, Eichenbaum, and Evans (1999), Eichenbaum and Evans (1995), Leeper, Sims, Zha, Hall, and Bernanke (1996), Stock and Watson (2012), Uhlig (2005) among others.

sumption" because consumption reacts contemporaneously to an inflation shock, whereas inflation reacts to a direct consumption growth shock with a one-quarter delay. Table 3.5 displays the corresponding location of zero restrictions. I borrow the terminology from structural VARs in applied macroeconomics.

I name the direct consumption shock $\varepsilon_{g,t+1}$ the short-run consumption shock and the shock $\varepsilon_{i,t+1}$ the long-run risk shock, based on their estimated properties. In particular, the impact of the shock $\varepsilon_{g,t+1}$ on consumption growth is concentrated in the short-run, whereas the cumulative impact of the shock $\varepsilon_{i,t+1}$ on consumption growth dominates at long horizons.

The identification of the long-run risk shock $\varepsilon_{i,t+1}$ and the variance shock $\varepsilon_{\sigma,t+1}$ is exactly the same in both identification schemes. The shock $\varepsilon_{i,t+1}$ is identified in the spirit of Bansal and Yaron (2004), i.e., the long-run risk shock affects expected consumption growth but not consumption growth itself, and does not feed into the variance process. The identification of the variance shock $\varepsilon_{\sigma,t+1}$ has a flavor of the structural assumptions of Colacito (2009) who allows for non-zero conditional correlations between consumption growth and stochastic variance and expected consumption growth and stochastic variance. Identification of the short-run consumption shock $\varepsilon_{g,t+1}$ and the inflation shock $\varepsilon_{\pi,t+1}$ is different across the identification schemes, as discussed above.

3.4.3 Estimation of the FX cash flow process

The estimation of the FX cash flow process (3.2.13) becomes straightforward once the structural shocks ε_{t+1} from the VAR (3.2.5) are identified. Intuitively, the cash flow process is a part of the vector autoregression that also includes the dynamics of consumption growth, inflation, nominal yield, and stochastic variance. The FX cash flow does not Granger-cause the economic states, whereas the economic states do cause the FX cash flow. In other words, there is nothing new to learn about the economy from the dynamics of foreign exchange cash flow that is not already contained in the dynamics of economic states. Given this property, the estimation of the joint distribution of the economic states and foreign exchange cash flow can be performed in two steps, as follows: (1) estimate the model of consumption growth (3.2.5) and (2) use the results from (1) to estimate the foreign exchange cash flow, i.e., measure the loadings of the corresponding cash flow on economic states and structural shocks. Because a two-stage estimation is equivalent to the estimation of the joint process, the problem of generated regressors does not arise.

Effectively, estimating the FX cash flow process is almost identical to running a linear regression because the full distribution of the stochastic variance σ_t^2 and structural shocks ε_{t+1} are already known, as a byproduct of the Bayesian MCMC approach. Components such as $\sigma_t \varepsilon_{g,t+1}$ in the process (3.2.13) act as additional regressors to the economic states. I use the Bayesian MCMC methods to estimate the FX cash flow process. I provide the details of the estimation algorithm and discuss my choice of priors in Appendix A.2.5.

3.4.4 Shock elasticity

In this section, I describe how I quantify prices and quantities of consumption risks in the cross-section of currency baskets at alternative horizons. I follow the idea of dynamic value decomposition of Hansen (2012) and, in particular, I use shock-exposure and shock-price elasticities of Borovička and Hansen (2011) and Borovička, Hansen, Hendricks, and Scheinkman (2011). Shock-exposure elasticity and shock-price elasticity are marginal metrics of quantity and price of risk, respectively.

The importance of a distinct source of risk for a cash flow is measured by the magnitude of the risk premium earned because of the cash flow's exposure to the risk. Two metrics matter: quantity of risk (exposure) and price of risk (compensation per unit of exposure). In a dynamic world with multiple sources of risk, the total risk premium associated with a cash flow is a compensation for exposure to all the sources of risk at many horizons. Thus, to shed light on the relative importance of one source of risk, it is necessary to isolate one shock of that type and study its pricing implications for cash flow $\delta_{t,t+\tau}$ across different horizons τ. In this case, the quantity and price of risk depend on the time gap $\tau - 1$ between the moment when the shock is realized and the moment when the shock impacts the cash flow. This dependence on time creates a term-structure of risks and their prices.

Borovička and Hansen (2011) describe in detail how to characterize the term-structure of risks and their prices in a structural model with stochastic variance in discrete time. I illustrate their approach in a simple example by examining the role of the variance risk. Appendix A.2.6 provides the formal derivation of shock elasticities in the context of my model.

To characterize the role of the variance risk ε_σ, Borovička and Hansen (2011) propose to undertake the following steps. First, they change the exposure of the cash flow $\log \delta_{t,t+\tau}$ to the risk $\sigma_t \varepsilon_{\sigma,t+1}$.[13] To do so, they introduce a perturbation

$$\log h(\mathrm{v}) = \gamma(\mathrm{v}, \sigma_t) + \mathrm{v}\sigma_t \varepsilon_{\sigma,t+1},$$

where the functional form of $\gamma(\mathrm{v}, \sigma_t)$ is not important and v is a scalar, and add this perturbation to the original multi-period cash flow $\delta_{t,t+\tau}$:

$$\log \bar{\delta}_{t,t+\tau} \quad = \quad \log \delta_{t,t+\tau} + \log h(\mathrm{v}).$$

As a result, they change the amount of the variance risk in the cashflow by the value v at time $t+1$. Next, the authors study how the log of the expected cash flow changes in response to a change in the amount of risk, when the change is marginal, i.e., they compute the following derivative

$$\ell_\delta(Y_t, \tau) = \left. \frac{\mathrm{d} \log E_t[\delta_{t,t+\tau}]}{\mathrm{d} \log h(\mathrm{v})} \right|_{\mathrm{v}=0} = \left. \frac{\mathrm{d} \log E_t[\bar{\delta}_{t,t+\tau}]}{\mathrm{d}\mathrm{v}} \right|_{\mathrm{v}=0} \tag{3.4.15}$$

and call the result the shock-exposure elasticity. Similarly, they study how the log risk premium changes in response to a change in the amount of risk, when the change is marginal, i.e., they compute the following derivative

$$\ell_p(Y_t, \tau) = \left. \frac{\mathrm{d} \log E_t[rx_{t,t+\tau}]}{\mathrm{d} \log h(\mathrm{v})} \right|_{\mathrm{v}=0} = \left. \frac{\mathrm{d} \log E_t[\bar{\delta}_{t,t+\tau}]}{\mathrm{d}\mathrm{v}} \right|_{\mathrm{v}=0} - \left. \frac{\mathrm{d} \log E_t[\bar{\delta}_{t,t+\tau} m_{t,t+\tau}]}{\mathrm{d}\mathrm{v}} \right|_{\mathrm{v}=0} \tag{3.4.16}$$

and call this object the shock-price elasticity. The derivative with respect to $\log h(\mathrm{v})$ is effectively a derivative with respect to the random variable $\mathrm{v}\sigma_t \varepsilon_{\sigma,t+1}$. In continuous time, such a derivative is known as the Malliavin derivative. Borovička, Hansen, Hendricks, and Scheinkman (2011) show that it is equal to the directional derivative in the right-hand side

[13] Without loss of generality, assume that $\sigma_t \varepsilon_{\sigma,t+1}$ has a unit standard deviation.

of (3.4.15) or (3.4.16) in continuous time. Borovička and Hansen (2011) simply adopt the directional derivative as a definition of the shock elasticity in discrete time.

The shock-exposure elasticity $\ell_\delta(Y_t, \tau)$ is marginal quantity of risk, whereas the shock-prices elasticity $\ell_p(Y_t, \tau)$ is marginal price of risk, or the marginal Sharpe ratio. The elasticities depend on the time elapsed since the shock has been realized until it impacts the cash flow and on the information set Y_t. These marginal metrics can be viewed as asset pricing counterparts to cumulative impulse response functions. Shock elasticities are specifically designed to study asset pricing implications of structural models with stochastic variance (or other types of nonlinearity).

In a linear model, marginal metrics of quantity and price of risk correspond to their average counterparts. Therefore, the shock-exposure elasticity is the cumulative impulse response function of the multi-period log cash flow (multi-period quantity of risk), whereas the shock-price elasticity is the cumulative impulse response function of the negative of the multi-period log stochastic discount factor (average multi-period Sharpe ratio). Section 2.2 of Borovička and Hansen (2011) illustrates this equivalence.[14] However, in a model with stochastic variance (or other types of nonlinearities), shock elasticities do not coincide with cumulative impulse response functions, and have a different interpretation. This difference is critical because only shock elasticities can describe risks in isolations in the presence of nonlinearities.

My model contains three types of risk, namely, the short-run consumption risk, the inflation risk, and the long-run consumption risk, which enter the model linearly. Therefore, the shock-exposure and shock-price elasticity for these risks have a standard interpretation of average quantity and price of risk. Shock elasticities for the variance risk has interpretation of marginal quantity and price of risk. In this case, prices of risk associated with different currency baskets are different, i.e., they are basket specific. This is a direct manifestation of nonlinearity.

[14]Intuitively, the equivalence holds because the nonlinearities, for which shock elasticities additionally account, are absent. Roughly speaking, the cumulative impulse response function requires computing the expectation of the log, whereas the shock elasticity requires computing the opposite, i.e., the log of the expectation. In a linear model the order does not matter.

3.5 Results

I present my findings in the following order. I start with a discussion of the estimated dynamics of the structural VAR. Next, I analyze how foreign exchange cash flows are sensitive to the four identified sources of consumption risk at alternative horizons. Finally, I examine how these risk exposures are priced at alternative horizons.

3.5.1 Macro dynamics

I use the data displayed in panels (a)-(c) of Figure 3.1 to estimate the model (3.4.14) with the consistency restrictions (3.2.9) and (3.2.10). Appendix A.2.7 summarizes the diagnostics of fitting errors based on which I conclude that the model has a good fit. One of the outputs of the estimation procedure is the estimated path of the unobservable stochastic variance σ_t^2, another output of the estimation procedure is the expected consumption growth $E_t \log g_{t,t+1}$ displayed in panel (a) of Figure 3.1.

I take the square root of σ_t^2 and scale it appropriately, so that the series represents the stochastic volatility of consumption growth. I display this series in panel (d) of Figure 3.1. The annualized volatility of consumption growth varies from 0.6% to 2.12%. It captures the important economic periods: the volatility is high after the Second World War, during the oil crises, the monetary experiment, and the recent financial crisis, and volatility is low during the Great Moderation.

Table 3.6 reports the parameter estimates for the elements of the matrices F, G, and Σ. The element G_{44} is of special interest because it characterizes the persistence of the stochastic variance. The estimated half-life of the variance component is $\log 2/(1-G_{44}) = 13$ quarters. It is particularly interesting to compare the estimate of G_{44} with the corresponding values used in calibrations elsewhere in the literature. Similar to the specification of the consumption growth process in Bansal and Yaron (2004), my model has only one stochastic variance factor.[15] I proceed by comparing the estimate of G_{44} with the corresponding

[15]In contrast to the theoretical long-run risk literature, I specify the stochastic variance not as an autoregressive process but as a discretized version of the square-root process.

parameter values used in different calibrations of the Bansal and Yaron (2004) model, e.g., in Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012a) and Bansal, Kiku, and Yaron (2012b). These values are 0.9615, 0.9949, and 0.997 on a quarterly basis, respectively; they are higher than my point estimate of G_{44} which is 0.9476. However, the persistence parameter used by Bansal and Yaron (2004) is within the confidence interval of the estimated parameter G_{44}.

The estimated persistence of the expected consumption growth is 0.81 with the 95% confidence interval from 0.71 to 0.90.[16] These magnitudes are somewhat smaller than the values used in standard calibrations of the long-run risk models.[17] The expected consumption growth loads significantly on all the observables used in the estimation with the largest in absolute terms loading on the nominal yield ($G_{13} = 0.38$).[18] Because of the dominant role of the nominal yield, the cyclical properties of the expected growth and the nominal yield are similar. Occasionally, however, the expected consumption growth mirrors the dynamics of other variables. For example, during the recent financial crisis the dynamics of the expected consumption growth is mostly related to the dynamics of inflation with a negative sign, whereas during the economic downturn of 1958 the expected consumption growth closely tracks the evolution of the realized consumption growth.

Table 3.7 contains the estimates for the parameters of the matrix H. Under both identification schemes, I find that a positive variance shock leads to a positive contemporaneous move in inflation, whereas a positive short-run consumption shock leads to a positive contemporaneous move in the nominal yield. Additionally, under "Fast Inflation" a positive short-run consumption shock leads to an increase in inflation, whereas under "Fast Consumption", a positive inflation shock increases consumption growth. This impact of the structural shocks on the states of the model affects the one-period prices of risks attached to them.

[16] I compute the persistence parameter as an autocorrelation of the expected consumption growth $corr(E_t \log g_{t,t+1}, E_{t-1} \log g_{t-1,t})$.

[17] For example, Bansal and Yaron (2004) use the autoregressive parameter of 0.94, whereas Bansal, Kiku, and Yaron (2012a) use the value of 0.93. I refer to the parameter values corresponding to the consumption dynamics at a quarterly frequency.

[18] The loading of the expected consumption growth on realized consumption growth is $G_{11} = 0.2$, the loading on inflation is $G_{12} = -0.2$, and the loading on consumption variance is G_{13}/Σ_{11} =5.7. Note that the consumption variance is several order lower than consumption growth or inflation.

3.5.2 Term-structure of exposures of FX cash flows to the multiple sources of consumption risk

Table 3.8 and Table 3.9 describe the distribution of the parameters of the cash flow process estimated for all currency baskets under both identification schemes. For the one-period exposures, the parameters ξ_g, ξ_π, ξ_i, and ξ_σ are of central interest. These parameters are the loadings on the vector of structural shocks $\sigma_t \varepsilon_{t+1}$ in the cash flow process, and, therefore, can be interpreted as the quantity of the short-run risk, inflation risk, long-run risk, and variance risk, respectively. Under both identification schemes, the cash flow of basket "Low" loads negatively on the long-run risk shock, the cash flow of basket "Intermediate" loads positively on the short-run consumption shock and inflation shock and negatively on the long-run risk shock, and the cash flow of basket "High" loads positively on the short-run consumption shock, inflation shock, and long-run risk shock. Thus, at horizon of one quarter, the cash flows of basket "Low" and basket "Intermediate" serve as hedges against the long-run risk shock; in other words, cash flows increase after a negative long-run risk shock.

For multi-period horizons, the parameters μ_g, μ_π, μ_i, and μ_σ become important. In conjunction with the parameters of the matrices G and H, they determine how shocks propagate across time in the cross-section of FX cash flows. Under both identification schemes, cash flows are predictable. Consumption growth, inflation and stochastic variance have forecasting power for basket "Low"; inflation and nominal rate have forecasting power for basket "Intermediate"; and consumption growth, inflation and nominal rate have forecasting power for basket "High".

In many cases, the contemporaneous and future effects of the same shock are opposite. For example, a positive short-run consumption shock $\varepsilon_{g,t+1}$ contemporaneously decreases the cash flow of basket "Low" ($\xi_g < 0$) but increases the corresponding future one-period cash flow ($\mu_g > 0$). Therefore, it is hard to gauge whether the cumulative effect of $\varepsilon_{g,t+1}$ on the multi-period cash flow of basket "Low" is positive or negative on the basis of the estimated parameters alone. Shock-exposure elasticities are helpful in this regard.

Figure 3.2 and Figure 3.3 display the shock-exposure elasticity under the "Fast Inflation" identification and the "Fast Consumption" identification, respectively. To plot the graphs, I set the stochastic variance σ_t^2 to be equal to 1, i.e., to its long-run mean.[19] Shock-exposure elasticities for short-run consumption shock, inflation shock, and long-run risk shock can be interpreted as quantities of risk in a standard sense (for example, $\xi_g \sigma_t$ is a one-period quantity of the short-run risk associated with some FX cash flow). These shocks do not feed into the stochastic variance process; therefore, the average metrics of price and quantity of risk coincide with their marginal counterparts. In contrast, shock exposure elasticity for the variance shock has an interpretation of the marginal quantity of risk: marginal change in the expected cash flow due to a marginal change in the volatility of the underlying shock.

To highlight the difference between average and marginal quantity of risk, I interpret a currency with a negative exposure elasticity to the short-run consumption shock, inflation shock, or long-run risk shock as an average hedge against the corresponding shock, whereas a currency with a negative cash flow exposure elasticity to the variance shock as a marginal hedge against the shock. Bearing this in mind, I proceed by looking at the cross-sectional implications of the exposure elasticities.

Under both identification schemes, there is significant cross-sectional heterogeneity of the currency exposure elasticities to the long-run risk shock and inflation shock across all horizons from one quarter to ten years. The sensitivity to the long-run risk shock is lowest for basket "Low" and highest for basket "High", whereas the sensitivity to the inflation shock is lowest for basket "Intermediate" and highest for basket "High". Differences in the exposure elasticities of basket "Low" and basket "Intermediate" to the long-run risk and inflation shocks are not statistically significant for multi-period horizons, although the differences are economically meaningful in case of the elasticity exposure to the long-run risk shock. Pair-wise differences in the exposure elasticities of basket "Low" and basket "High" and basket "Intermediate" and "basket High" to the long-run risk and inflation shocks are economically and statistically significant at all horizons.[20] Thus, the low and intermediate interest rate currencies are average hedges against the long-run and inflation risks.

[19] The shock-exposure elasticities scale up and down depending on the magnitude of the stochastic variance.

[20] Under the "Fast Consumption" identification, the differences in the exposure elasticities of basket "Low" and basket "High" to the inflation risk are significant only at horizons from three quarters to five years. To avoid overcrowding the figures, I do not display the confidence bounds for shocks elasticities. Results are available upon request.

Differences in the shock-exposure elasticities for the short-run consumption shock across currency baskets are not statistically significant. However, one observation is worth mentioning. Under the "Fast Consumption" identification, the exposure elasticities of the cash flows of basket "Low" and basket "High" are economically different from each other. The quantity of risk associated with the low interest rate currencies is higher than the quantity of risk associated with the high interest rate currencies.

The loadings of FX cash flows on the variance risk are not significantly different in the cross-section. The variance risk matters in a different respect. Under both identification schemes at horizons longer than three years, FX cash flows are the most sensitive to the variance shock. To gauge the cumulative impact of the variance shock on the cash flows, it is helpful to consider an example. A sensitivity of the high interest rate currencies to the variance risk at horizon of ten years is -0.034 (a sensitivity to the long-run risk is 0.026, to the inflation shock – 0.015, to the short-run consumption shock – 0.02), whereas the corresponding metric on a one-period horizon is $\xi_\sigma = 0.005$. The variance shock has a long-lasting impact on FX cash flows. At long horizons, all currency baskets are marginal hedges against the variance shock; that is, FX cash flow marginally decreases as a result of a marginal increase in the exposure to the variance shock.

3.5.3 Term-structure of prices of the multiple sources of consumption risk

In the previous section, I have documented the following findings: (1) there are economically and statistically significant differences in exposures of currencies to the inflation and long-run risks at multiple horizons and to the short-run consumption risk at a one-period horizon only and (2) the sensitivity of currency baskets to the variance shock is large in absolute value and negative at long horizons without cross-sectional differences among the baskets. The next natural question concerns how the currency exposure to the consumption risks is priced at different horizons. Namely, it is important to understand if the cross-sectional differences in quantity of short-run consumption risk, long-run risk, or inflation risk across the currency baskets lead to a material difference in risk premia in the cross-section at different horizons.

I start characterizing the prices of risks from a one-period perspective. Table 3.10 describes the distribution of p_g, p_π, p_i, and p_σ that are the parameters of the value function (3.2.11). Parameters p_g and p_i are positive and statistically significant, whereas the confidence intervals for the parameters p_π and p_σ include zero. Standard calibrations of the long-run risk models (see, for example, Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2012a; Drechsler and Yaron, 2011) produce a negative value for p_σ and a positive loading on the $\sigma_t\varepsilon_{\sigma,t+1}$ in the stochastic discount factor (negative price of the variance risk).

At this stage, it is important to make three remarks. First, the preference parameters and the parameters p_g, p_π, p_i, and p_σ are not the only determinants of the signs of the prices of risk. The negative of the vector of the one-period prices of risks, q, depends on H: $q = H'[(\alpha - \rho)P + e_1(\alpha - 1)]$ (see Appendix A.2.1). The matrix H is not diagonal, and therefore, the interaction between H and P matters. Second, in my model σ_t^2 can play the following two roles: (1) the variance factor and (2) the predictability factor of the future consumption growth (similar to Backus, Routledge, and Zin, 2010). Higher variance today could be associated with higher expected consumption growth in the future, so in general, the sign of p_σ is undetermined on the basis of economic intuition alone. Finally, as Appendix A.2.8 shows for the Bansal and Yaron (2004) model, it could be difficult to precisely identify the parameter p_σ from the data.

Table 3.11 describes the distribution of q_g, q_π, q_i, and q_σ (elements of the vector q) under both identification schemes. The absolute value of the price of the short-run consumption shock is higher under the "Fast Inflation" identification. The inflation shock carries a statistically significant price of risk only under the "Fast Consumption" identification. The distribution of q_i and q_σ is identical across the schemes because these risks are identified in exactly the same manner. The long-run risk shock carries a statistically significant positive price of risk ($-q_\sigma$). The confidence interval for the price of the variance risk includes zero. High uncertainty about p_σ leads to a high uncertainty about the price of the variance risk.

I take into account the properties of the prices of risks and one-period exposures of FX cash flows to the risks and analyze the model implications for the cross-section of one-period risk premia in Table 3.12 and Table 3.13. Basket "Low" is associated with a negative risk premium (approximately -2% annualized) because it pays well in bad states of the world when a negative long-run risk shock is realized. The average historical return on basket "Low" is -2.52% which falls within the confidence interval of the one-period total risk premium attached to this basket. Basket "High" is associated with a positive risk premium (approximately 3.2% annualized) because its cash flow is positively exposed to all the risks which carry positive prices. The average historical return on basket "High" is 2%, within the confidence interval of the one-period total risk premium attached to this basket. Finally, basket "Intermediate" earns a negative risk premium of approximately -0.35% annualized because its cash flow is more sensitive to the long-run risk shock, and this sensitivity is negative. Similar to the other baskets, the historical average return on basket "Intermediate" (-0.71%) is within the model implied 95% confidence interval of the one-period total risk premium.

To summarize, the level and the spread of the excess returns in the cross-section of currencies is fully explained by the exposure of currencies to the priced sources of consumption risk. Under the "Fast Inflation" identification, exposure to the long-run risk shock accounts for 52% of the one-period spread of excess returns between the high and low interest rate currencies, whereas the remaining 48% are due to the different exposure of the currency baskets to the short-run consumption shock. Under the "Fast Consumption" identification, exposure to the long-run risk shock, the short-run consumption shock, and the inflation shock contribute 48%, 26%, and 26%, respectively, to the spread of real excess returns.

I analyze the multi-period prices of risks by examining the shock price elasticities displayed in Figure 3.4 and Figure 3.5.[21] The price elasticity of the short-run consumption shock, the inflation shock and the long-run risk shock corresponds to the negative of the cumulative impulse response function of the multi-period log stochastic discount. This works similarly to a linear model without stochastic variance because these shocks do not feed into the process for stochastic variance. Therefore, the marginal price of risk associated with these shocks is also the average price of risk, or average Sharpe ratio for log returns.

Such an interpretation is not appropriate for the price elasticity for the variance shock. The variance shock feeds into the variance process, and therefore, it is associated with important nonlinearities in the model. The price elasticity of the variance shock is a marginal change

[21] As in the case of exposure elasticity, I plot price elasticity by setting $\sigma_t^2 = 1$.

in the risk premium caused by the marginal change in the exposure to the source of risk, i.e., a marginal Sharpe ratio for log returns. The price elasticity for the variance shock is cash flow dependent because its marginal price of risk is not equal to its average price of risk.

To put the magnitudes displayed in Figure 3.4 and Figure 3.5 into perspective, I refer to a number of studies that report Sharpe ratios for different currency strategies. Table 3 in Ang and Chen (2010) reports an annualized Sharpe ratio of 0.64 for a currency portfolio based on the level of the yield curve and 0.81 for a currency portfolio based on the slope of the yield curve; Table 1 in Burnside (2011) reports an annualized Sharpe ratio of 0.90 for the equally-weighted carry trade and 0.63 for the HML carry trade; Table 1 in Lustig, Roussanov, and Verdelhan (2013) documents an annualized Sharpe ratio of 0.66 for the dollar carry trade.[22]

These numbers are not exact counterparts to the prices of risk that I document in the paper. In particular, I report Sharpe ratios for log returns, consider different strategies, and use different data. However, I believe these numbers are still informative and could be used as a rough benchmark. The one period log Sharpe ratios for the short-run consumption shock and long-run risk shock are smaller than their multi-period counterparts but already substantial enough against the numbers quoted for currency strategies elsewhere in the literature (see above). For example, the annualized Sharpe ratio due to the short-run consumption shock is approximately 0.52 or 0.58 (depending on the identification strategy) and due to the predictability shock is 0.66.

The risk premium of all currency baskets at all investment horizons is especially sensitive to the long-run risk under both identification schemes. This funding, in conjunction with the substantial spread in quantity of the long-run risk across currency baskets, is the main result of the paper. Currency baskets carry significantly different compensation for the long-run risk at all horizons from one quarter to ten years. The spread in compensation

[22] Ang and Chen (2010) describe a currency strategy based on the level (slope) of the yield curve as one that entails going long in a currency with a high level factor (low term spread) and short in a currency with a low level factor (high term spread); Burnside (2011) defines the equally weighted carry trade as the average of up to twenty individual currency carry trades against the US dollar; Lustig, Roussanov, and Verdelhan (2013) determine dollar carry trade as a strategy of going long in all available one-month currency forward contracts when the average forward discount of developed countries is positive and short otherwise.

decreases with the investment horizon: basket "Low" acts as a weaker hedge, whereas basket "Intermediate" loses its hedging capability completely. Nonetheless, the spread between the corner baskets remains statistically significant.

The price of inflation risk is statistically significant at all horizons under the "Fast Consumption" identification only. In this case, the significant difference in exposure to inflation risk between basket "Low" and basket "High" (basket "Intermediate" and basket "High") leads to a significant spread of excess returns at all horizons (at horizons shorter than five years). The cross-sectional spread of the inflation risk premia is smaller than the cross-sectional spread of the long-run risk premia because the price of the long-run risk is more than double that of the inflation risk.

Finally, the sensitivity of the currency risk premia to the variance risk is relatively small at all investment horizons. This finding demonstrates that a high sensitivity of a cash flow to a specific source of risk does not necessarily lead to a high risk compensation. Moreover, the positive marginal price of the variance risk, suggests that all currency baskets act as a marginal hedge against the unfavorable variance shock.

3.6 Conclusion

In this paper, I provide novel evidence of how multiple sources of consumption risk are priced in the foreign exchange market at short and medium horizons, from one quarter to ten years. I accomplish the task by examining the role of the consumption risks through the lens of the vector autoregressive process of the joint dynamics of consumption growth, inflation, and a three-month nominal yield with stochastic variance and structural restrictions derived under recursive preferences.

I establish four structural consumption shocks, including the short-run consumption risk, the inflation risk, the long-run consumption risk, and the variance risk. I find that the compensation for currency exposure to these risks at the horizon of one quarter matches both the level and the cross-sectional spread of currency risk premia. I document the prominent role of the long-run consumption risk: (1) it carries the highest price of risk (annualized average log Sharpe ratio is 0.66 at the horizon of one quarter and higher at longer horizons), and (2) it contributes the most to the level and to the spread of excess returns between baskets of high and low interest rate currencies at short and medium horizons (at the horizon of one quarter, this risk explains at least 42% of the spread).

The role of other sources of risk is limited. The short-run consumption risk is priced in the cross-section of currency returns at the horizon of one quarter only, where it explains at least 26% of the corresponding spread of excess returns between high and low interest rate currencies. The inflation risk matters at multiple horizons if consumption growth is a faster variable than inflation (consumption growth reacts to the inflation shock within a quarter whereas inflation reacts to the short-run consumption shock with a delay of one quarter). This risk explains a lower fraction of the spread in excess returns in comparison with the long-run risk because its price of risk is less than a half of that for the long-run risk (annualized average log Sharpe ratio of the inflation risk at the horizon of one quarter is 0.26). Finally, I find that all currency baskets are uniformly highly sensitive to the variance risk at horizons longer than three years, although the compensation for this exposure is small.

I leave at least two interesting avenues for the future research. The first question is the estimation of the preference parameters, perhaps starting with the parameter of the elasticity of intertemporal substitution and the subjective discount factor. The second direction of research is further exploration of the role of the variance risk in macroeconomy and asset markets by utilizing assets that are informative about this type of risk at the estimation stage.

3.7 Tables and figures

Table 3.1: Properties of macro economic variables

	Mean	Std Dev	Skewness	Kurtosis	N observations
$\log g_{t,t+1}$	0.0048	0.0052	-0.45	4.04	259
$\log \pi_{t,t+1}$	0.0083	0.0076	0.81	5.30	259
i_t^1	0.0113	0.0076	0.93	4.13	259

Notes. Descriptive statistics for consumption growth, inflation, and nominal yield. Sample period: second quarter of 1947 – fourth quarter of 2011. Quarterly.

Table 3.2: Properties of real log excess returns

	Mean	Std Dev	Skewness	Kurtosis	Autocorrelation
Basket "Low"	-0.0063	0.0517	0.38	3.07	0.01
Basket "Intermediate"	-0.0018	0.0432	0.09	3.81	0.15
Basket "High"	0.0050	0.0502	0.03	3.62	0.12

Notes. The three currency baskets are formed by sorting currencies by their corresponding average yields at a quarterly basis. Average yields are computed for each currency's term-structure at each point of time. Sample period: 1986 – 2011. Quarterly.

Table 3.3: Composition of currency baskets

Currency	Basket "Low"	Basket "Intermediate"	Basket "High"
Australia	0	23	76
Canada	20	75	8
Denmark	11	70	12
Germany	34	16	2
Euro area	17	12	0
Japan	103	0	0
Norway	1	24	30
New Zealand	4	10	73
Sweden	32	29	15
Switzerland	95	0	0
UK	5	50	48
South Africa	0	0	58

Notes. Table entry shows the number of periods each currency belongs to each basket. Sample period: 1986 – 2011, at a quarterly frequency.

Table 3.4: Identification "Fast Inflation"

	$\varepsilon_{g,t+1}$	$\varepsilon_{\pi,t+1}$	$\varepsilon_{i,t+1}$	$\varepsilon_{\sigma,t+1}$
Consumption eq	H_{11}	0	0	H_{14}
Inflation eq	H_{21}	H_{22}	0	H_{24}
Interest rate eq	H_{31}	H_{32}	H_{33}	H_{34}
Variance eq	0	0	0	H_{44}

Notes. A globally identified system. Inflation reacts to a consumption shock ε_g contemporaneously, whereas consumption growth reacts to an inflation shock ε_π with a delay of one period.

Table 3.5: Identification "Fast Consumption"

	$\varepsilon_{g,t+1}$	$\varepsilon_{\pi,t+1}$	$\varepsilon_{i,t+1}$	$\varepsilon_{\sigma,t+1}$
Consumption eq	H_{11}	H_{12}	0	H_{14}
Inflation eq	0	H_{22}	0	H_{24}
Interest rate eq	H_{31}	H_{32}	H_{33}	H_{34}
Variance eq	0	0	0	H_{44}

Notes. A globally identified system. Consumption growth reacts to an inflation shock ε_π contemporaneously, whereas inflation reacts to a consumption shock ε_g with a delay of one period.

Table 3.6: The model of consumption growth. Parameter estimates

Parameter	Estimate	Confidence interval, 95%
F_1	-0.0004	(-0.0019, 0.0011)
F_2	-0.0074	(-0.0008, 0.0013)
F_3	-0.0002	(-0.0006, 0.0002)
F_4	0.0525	(0.0229, 0.0843)
G_{11}	0.2009	(0.1051, 0.3029)
G_{12}	-0.2000	(-0.2827, -0.1204)
G_{13}	0.3792	(0.2896, 0.4654)
G_{14}	0.0017	(0.0009, 0.0026)
G_{21}	-0.1339	(-0.2019, -0.0701)
G_{22}	0.1333	(0.0803, 0.1885)
G_{23}	0.7472	(0.6897, 0.8069)
G_{24}	0.0046	(0.0036, 0.0057)
G_{31}	0.0721	(0.0378, 0.1060)
G_{32}	0.0183	(-0.0061, 0.0422)
G_{33}	0.9635	(0.9425, 0.9845)
G_{34}	0.0003	(3.66e-5, 0.0006)
G_{44}	0.9476	(0.9156, 0.9771)
Σ_{11}	3.10e-5	(2.15e-5, 4.53e-5)
Σ_{12}	8.85e-6	(3.06e-6, 1.60e-5)
Σ_{13}	2.68e-6	(1.07e-6, 5.05e-6)
Σ_{14}	-0.0002	(-0.0004, 2.51e-5)
Σ_{22}	3.87e-5	(2.80e-5, 5.35e-6)
Σ_{23}	2.90e-6	(9.22e-7, 5.85e-6)
Σ_{24}	0.0003	(4.80e-5, 0.0005)
Σ_{33}	2.71e-6	(1.92e-6, 3.87e-6)
Σ_{32}	3.73e-5	(-3.01e-5, 0.0001)
Σ_{44}	0.0310	(0.0175, 0.0499)

Notes. I estimate a vector autoregression with stochastic variance

$$Y_{t+1} = F + GY_t + \sigma_t \Sigma^{1/2} w_{t+1}$$

and restrictions: (1) $G_{21}/G_{11} = G_{22}/G_{12} = (G_{23} - 1)/G_{13} = (F_2 - \log\beta)/F_1 = \rho - 1$ and (2) $\alpha(\alpha-\rho)(P+e_1)'\Sigma(P+e_1)/2 + e_2'Ge_4 - e_2'\Sigma e_2/2 - [(\alpha-\rho)P + e_1(\alpha-1)]'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)]/2 + e_2'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)] - (\rho-1)e_1'Ge_4 = 0$. Note that $\Sigma = HH'$, where H is from (2.5).

Vector $Y_t = (\log g_{t-1,t},\ \log \pi_{t-1,t},\ i_t^1,\ \sigma_t^2)'$ includes US consumption growth, inflation, one-period nominal yield, and stochastic variance.

To save space, I do not duplicate the symmetric entries of the matrix Σ. Sample period: second quarter of 1947 – fourth quarter of 2011. Quarterly.

Table 3.7: Global identification

Identification "Fast Inflation"			Identification "Fast Consumption"		
Parameter	Estimate	Confidence interval, 95%	Parameter	Estimate	Confidence interval, 95%
H_{11}	0.0054	(0.0045, 0.0065)	H_{11}	0.0051	(0.0043, 0.0061)
H_{14}	-0.0011	(-0.0024, 0.0002)	H_{12}	0.0017	(0.0007, 0.0028)
H_{21}	0.0019	(0.0009, 0.0030)	H_{14}	-0.0011	(-0.0024, 0.0002)
H_{22}	0.0057	(0.0047, 0.0069)	H_{22}	0.0060	(0.0050, 0.0072)
H_{24}	0.0014	(0.0003, 0.0025)	H_{24}	0.0014	(0.0003, 0.0025)
H_{31}	0.0005	(0.0002, 0.0009)	H_{31}	0.0004	(0.0002, 0.0007)
H_{32}	0.0003	(-4.8e-5, 0.0007)	H_{32}	0.0004	(9.3e-5, 0.0008)
H_{33}	0.0015	(0.0012, 0.0017)	H_{33}	0.0015	(0.0012, 0.0017)
H_{34}	0.0002	(-0.0002, 0.0006)	H_{34}	0.0002	(-0.0002, 0.0006)
H_{44}	0.1747	(0.1324, 0.2233)	H_{44}	0.1747	(0.1324, 0.2233)

Notes. I identify structural shocks ε_{t+1} from the reduced form innovations w_{t+1}: $\Sigma^{1/2} w_{t+1} = H\varepsilon_{t+1}$. I consider two globally exactly identified models. Identification "Fast Inflation" is determined by the following zero restrictions: $H_{12} = H_{13} = H_{23} = H_{41} = H_{42} = H_{43} = 0$. Identification "Fast Consumption" is determined by the following zero restrictions: $H_{13} = H_{21} = H_{23} = H_{41} = H_{42} = H_{43} = 0$. Quarterly.

Table 3.8: Estimated FX cash flow process (identification “Fast Inflation”)

Parameter	Basket “Low”	Basket “Intermediate”	Basket “High”
$\log \delta$	-0.0011	-0.0176	-0.0077
	(-0.0161, 0.0144)	(-0.0355, -0.0038)	(-0.0289, 0.0080)
μ_g	1.6033	-0.1360	-1.4884
	(0.9346, 2.2631)	(-0.7566, 0.4897)	(-2.2656, -0.7477)
μ_π	-0.5243	-2.7934	-1.9153
	(-0.9637, -0.1050)	(-3.2375, -2.3639)	(-2.4630, -1.3713)
μ_i	0.0698	2.4242	2.0582
	(-0.3352, 0.4634)	(2.0549, 2.8050)	(1.5984, 2.5358)
μ_σ	-0.0110	0.0012	-0.0020
	(-0.0207, -0.0017)	(-0.0099, 0.0090)	(-0.0185, 0.0081)
ξ_g	-0.0034	0.0072	0.0163
	(-0.0074, 0.0003)	(0.0036, 0.0110)	(0.0106, 0.0222)
ξ_π	-0.0030	0.0070	0.0157
	(-0.0086, 0.0023)	(0.0020, 0.0121)	(0.0101, 0.0217)
ξ_i	-0.0120	-0.0100	0.0064
	(-0.0152, -0.0089)	(-0.0128, -0.0072)	(0.0036, 0.0091)
ξ_σ	$-7.18 \cdot 10^{-5}$	0.0003	0.0055
	(-0.0099, 0.0104)	(-0.0085, 0.0088)	(-0.0053, 0.0169)

Notes. For each currency basket, I estimate the FX cash flow process:

$$\log \delta_{t,t+1} = \log \delta + \mu' Y_t + \sigma_t \xi' \varepsilon_{t+1} + \xi_v \sigma_t v_{t+1},$$

where $\mu = (\mu_g,\ \mu_\pi,\ \mu_i,\ \mu_\sigma)'$ and $\xi = (\xi_g,\ \xi_\pi,\ \xi_i,\ \xi_\sigma)'$. Quarterly. There are 95% confidence intervals in the brackets.

Table 3.9: Estimated FX cash flow process (identification "Fast Consumption")

Parameter	Basket "Low"	Basket "Intermediate"	Basket "High"
$\log\delta$	-0.0007	-0.0173	-0.0082
	(-0.0152, 0.0149)	(-0.0363, -0.0035)	(-0.0305, 0.0076)
μ_g	1.6081	-0.1311	-1.4615
	(0.9475, 2.2560)	(-0.7724, 0.5232)	(-2.2080, -0.6957)
μ_π	-0.5177	-2.8045	-1.9180
	(-0.9744, -0.0887)	(-3.2364, -2.3927)	(-2.4375, -1.4041)
μ_i	0.0632	2.4308	2.0496
	(-0.3225, 0.4349)	(2.0674, 2.7973)	(1.5791, 2.5139)
μ_σ	-0.0109	0.0010	-0.0020
	(-0.0202, -0.0025)	(-0.0106, 0.0088)	(-0.0195, 0.0078)
ξ_g	-0.0022	0.0047	0.0106
	(-0.0068, 0.0023)	(0.0005, 0.0086)	(0.0054, 0.0157)
ξ_π	-0.0041	0.0088	0.0202
	(-0.0089, 0.0008)	(0.0043, 0.0136)	(0.0146, 0.0263)
ξ_i	-0.0120	-0.0100	0.0064
	(-0.0151, -0.0088)	(-0.0126, -0.0073)	(0.0036, 0.0091)
ξ_σ	$-1.6 \cdot 10^{-5}$	0.0003	0.0053
	(-0.0106, 0.0102)	(-0.0087, 0.0093)	(-0.0054, 0.0171)

Notes. For each currency basket, I estimate the FX cash flow process:

$$\log\delta_{t,t+1} = \log\delta + \mu' Y_t + \sigma_t \xi' \varepsilon_{t+1} + \xi_v \sigma_t v_{t+1},$$

where $\mu = (\mu_g,\ \mu_\pi,\ \mu_i,\ \mu_\sigma)'$ and $\xi = (\xi_g,\ \xi_\pi,\ \xi_i,\ \xi_\sigma)'$. Quarterly. There are 95% confidence intervals in the brackets.

Table 3.10: Parameters of the fixed point problem

Parameter	p_g	p_π	p_i	p_σ	b_0	b_1
Estimate	2.46	-0.29	24.26	0.01	-4e-4	0.9912
Conf inter	(1.34, 3.68)	(-1.12, 0.48)	(18.17, 30.60)	(-0.11, 0.13)	(-1e-3, 0)	(0.9901, 0.9933)

Notes. I solve the approximate equation:

$$\log u_t \approx b_0 + b_1 \log \mu_t(u_{t+1} g_{t+1})$$

The value function is $\log u_t = \log u + p_g \log g_{t-1,t} + p_\pi \log \pi_{t-1,t} + p_i i_t^1 + p_\sigma \sigma_{t,1}^2$. Quarterly.

Table 3.11: Parameters q

	Identification "Fast Inflation"		Identification "Fast Consumption"	
Parameter	Estimate	Confidence interval, 95%	Estimate	Confidence interval, 95%
q_g	-0.29	(-0.36, -0.22)	-0.26	(-0.34, -0.18)
q_π	-0.04	(-0.14, 0.04)	-0.13	(-0.22, -0.04)
q_i	-0.33	(-0.43, -0.25)	-0.33	(-0.43, -0.25)
q_σ	-0.03	(-0.23, 0.19)	-0.03	(-0.23, 0.19)

Notes. Vector q is the vector of loadings on the structural shocks $\sigma_t \varepsilon_{t+1}$ in the pricing kernel $\log m_{t,t+1}$:

$$\log m_{t,t+1} = \log m + \eta' Y_t + q' \sigma_t \varepsilon_{t+1}, \qquad (2.6)$$

where $q = H'((\alpha - \rho)P + e_1(\alpha - 1))$, $q = (q_g,\ q_\pi,\ q_i,\ q_\sigma)'$. Preference parameters: $\alpha = -9$, $\rho = 1/3$, $\beta = 0.9924$. Quarterly.

Table 3.12: One-period risk premia (identification "Fast Inflation")

	Basket "Low"	Basket "Intermediate"	Basket "High"
Short-run risk	-0.0010	0.0021	0.0048
	(-0.0026, 0.0001)	(0.0008, 0.0042)	(0.0021, 0.0087)
Inflation risk	-0.0001	0.0003	0.0007
	(-0.0008, 0.0002)	(-0.0003, 0.0012)	(-0.0006, 0.0024)
Long-run risk	-0.0040	-0.0034	0.0022
	(-0.0072, -0.0019)	(-0.0061, -0.0016)	(0.0009, 0.0041)
Variance risk	$5.3 \cdot 10^{-5}$	$6.3 \cdot 10^{-5}$	0.0003
	(-0.0015, 0.0017)	(-0.0011, 0.0014)	(-0.0016, 0.0027)
Total	-0.0052	-0.0009	0.0079
	[0.23]	[0.19]	[0.2]
Data	-0.0063	-0.0018	0.0050

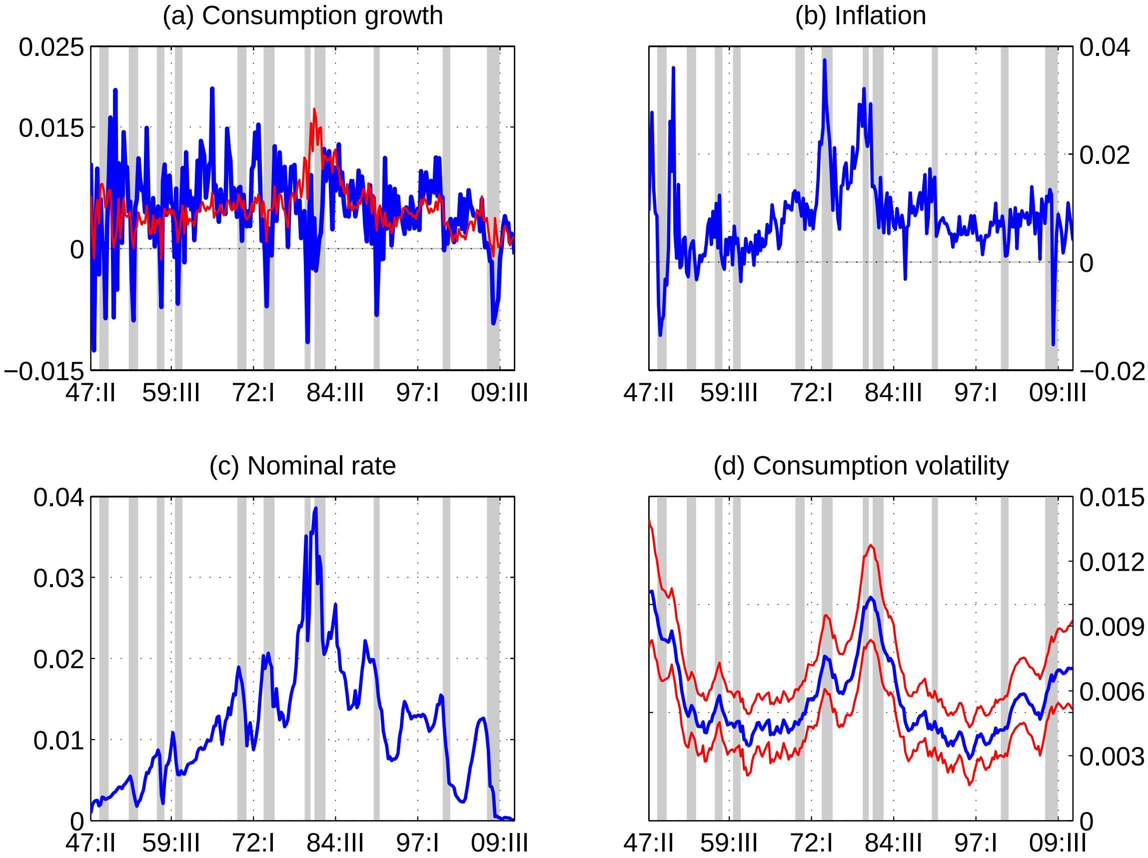
Notes: One period risk premia associated with multiple sources of risk. Stochastic variance σ_t^2 is set to be equal 1. Quarterly. I report p-values in the square brackets and 95% confidence intervals in the round brackets. The last row "Data" reports the level of the observed average excess returns.

Table 3.13: One-period risk premia (identification “Fast Consumption”)

	Basket “Low”	Basket “Intermediate”	Basket “High”
Short-run risk	-0.0006	0.0012	0.0028
	(-0.0020, 0.0006)	(0.0001, 0.0028)	(0.0010, 0.0055)
Inflation risk	-0.0006	0.0012	0.0028
	(-0.0017, 0.0001)	(0.0003, 0.0027)	(0.0008, 0.0057)
Long-run risk	-0.0040	-0.0034	0.0022
	(-0.0069, -0.0019)	(-0.0060, -0.0016)	(0.0008, 0.0041)
Variance risk	$4.8 \cdot 10^{-5}$	$4.3 \cdot 10^{-5}$	0.0003
	(-0.0014, 0.0016)	(-0.0010, 0.0012)	(-0.0014, 0.0026)
Total	-0.0051	-0.0009	0.0080
	[0.22]	[0.19]	[0.11]
Data	-0.0063	-0.0018	0.0050

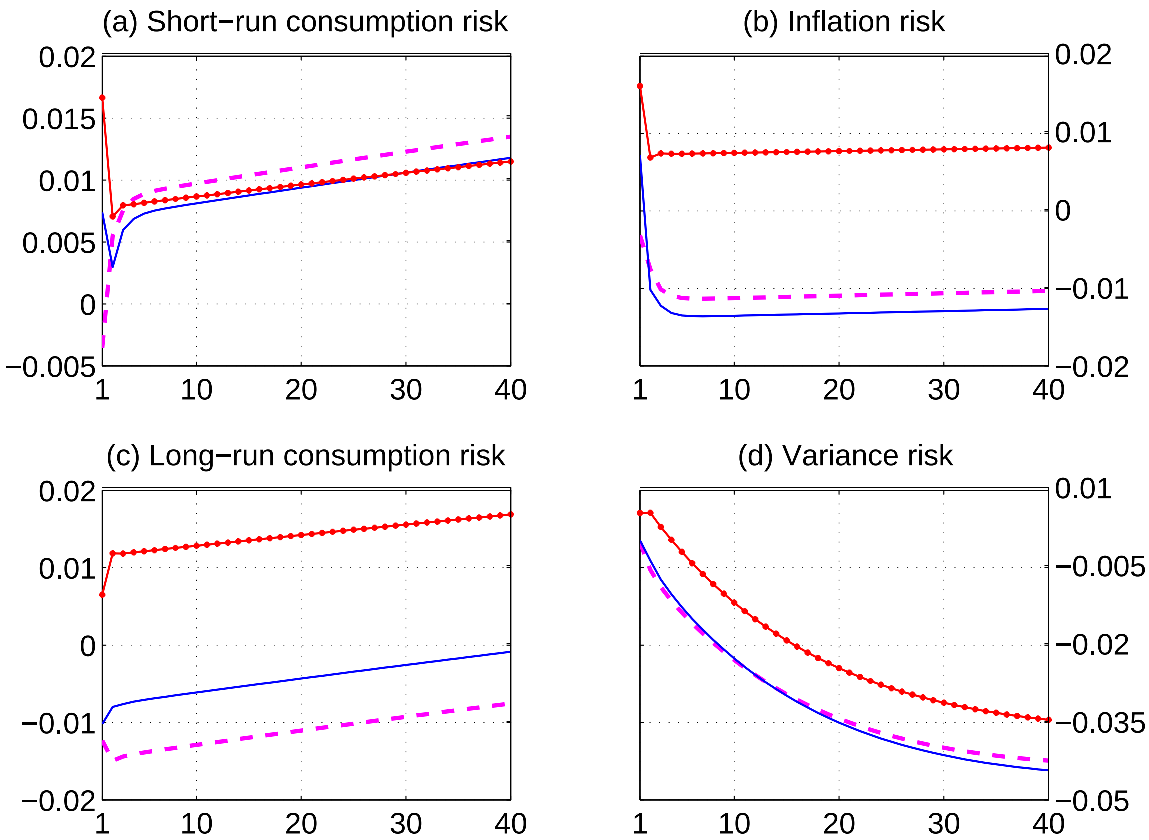
Notes: One period risk premia associated with multiple sources of risk. Stochastic variance σ_t^2 is set to be equal 1. Quarterly. I report p-values in the square brackets and 95% confidence intervals in the round brackets. The last row “Data” reports the level of the observed average excess returns.

Figure 3.1: Dynamics of the model's states



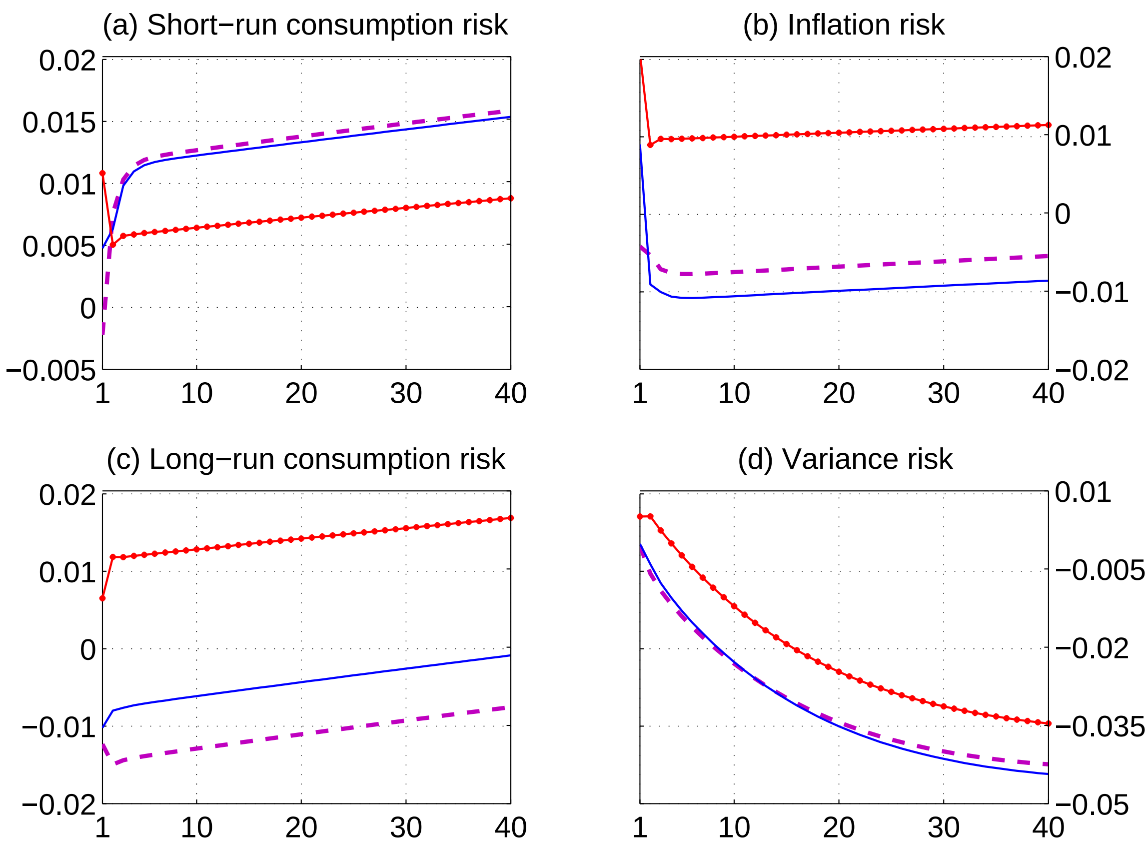
Panel (a) displays quarterly log consumption growth (thick blue line) and estimated expected consumption growth (thin red line). Panel (b) displays quarterly inflation. Panel (c) displays the 3-month nominal yield, quarterly. Panel (d) displays consumption volatility $\sqrt{\Sigma_{11}}\sigma_t$, quarterly. Blue line is the mean path of volatility, red lines correspond to the 95% confidence bounds. Grey bars are the NBER recessions.

Figure 3.2: Shock-exposure elasticity (identification "Fast Inflation")



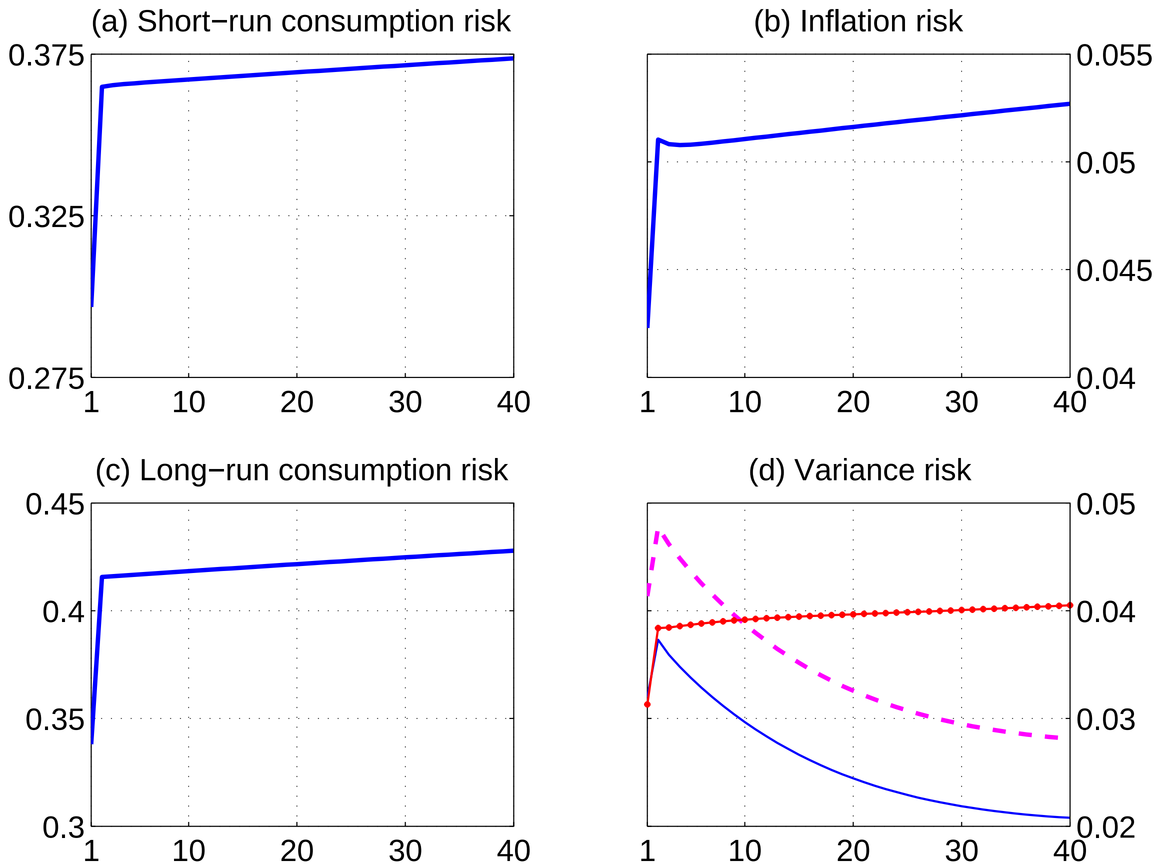
Panel (a) displays shock-exposure elasticity for the short-run consumption risk. Panel (b) displays shock-exposure elasticity for the inflation risk. Panel (c) displays shock-exposure elasticity for the long-run risk. Panel (d) displays shock-exposure elasticity for the variance risk. Identification "Fast Inflation". Quarterly. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". The horizontal axes: from 1 quarter to 10 years.

Figure 3.3: Shock-exposure elasticity (identification "Fast consumption")



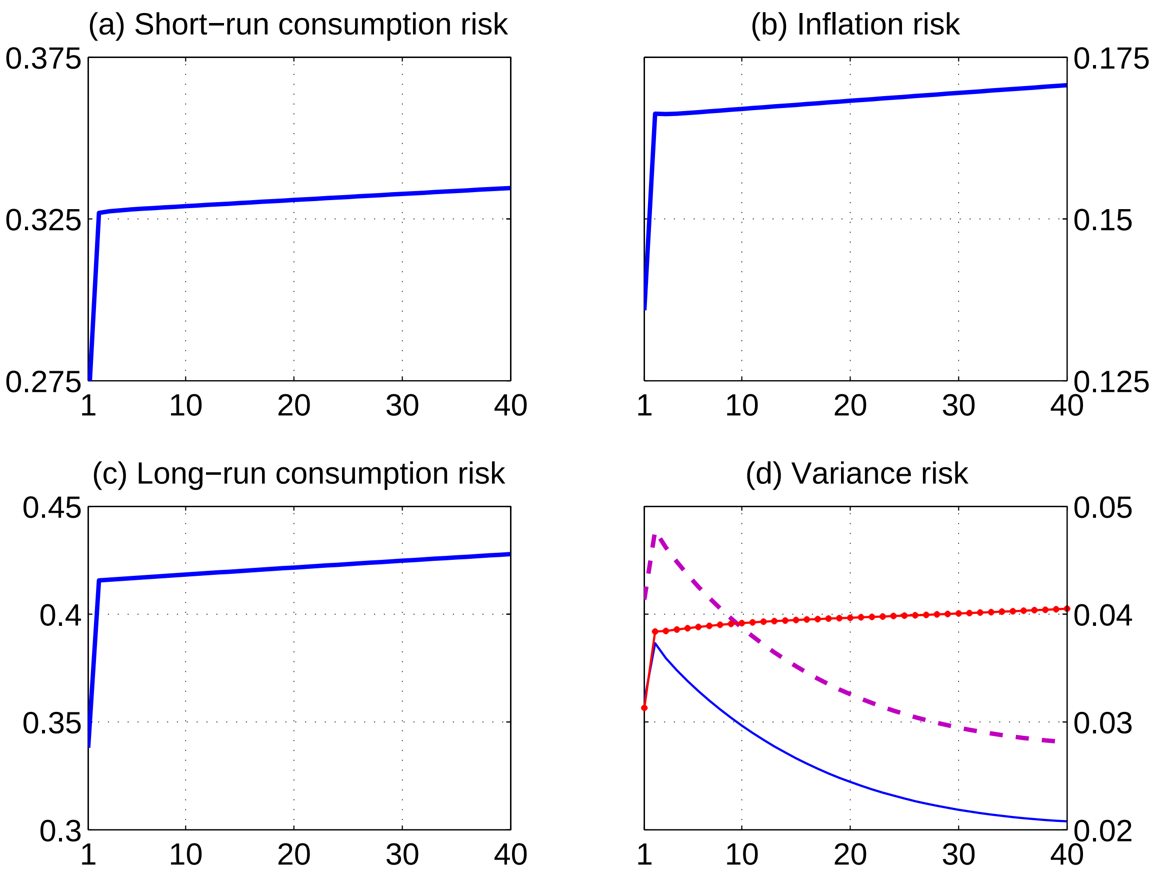
Panel (a) displays shock-exposure elasticity for the short-run consumption risk. Panel (b) displays shock-exposure elasticity for the inflation risk. Panel (c) displays shock-exposure elasticity for the long-run risk. Panel (d) displays shock-exposure elasticity for the variance risk. Identification "Fast Consumption". Quarterly. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". The horizontal axes: from 1 quarter to 10 years.

Figure 3.4: Shock-price elasticity (identification "Fast Inflation")



Panel (a) displays shock-price elasticity for the short-run consumption risk. Panel (b) displays shock-price elasticity for the inflation risk. Panel (c) displays shock-price elasticity for the long-run risk. Panel (d) displays shock-price elasticity for the variance risk. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". The horizontal axes: from 1 quarter to 10 years. Identification "Fast Consumption". Quarterly.

Figure 3.5: Shock-price elasticity (identification "Fast Consumption")



Panel (a) displays shock-price elasticity for the short-run consumption shock. Panel (b) displays shock-price elasticity for the inflation shock. Panel (c) displays shock-price elasticity for the long-run risk shock. Panel (d) displays shock-price elasticity for the variance shock. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". Identification "Fast Inflation". Quarterly.

Chapter 4

Conclusion

This thesis studies the risk-return relationship in the foreign exchange market. Chapter 2 develops a statistical model of currency price evolution with the purpose of quantifying the relative importance of crash risk versus normal risk and the metrics associated with crash risk (probabilities and sizes of crash events and their determinants). Chapter 3 investigates the sources of normal risk in currency markets and asks if the multiple sources of the US consumption risk are reflected in the cross-section of currencies at different investment horizons.

The main findings of the monograph can be summarized as follows: (1) crash risk in currency markets is quantitatively important, (2) there are three distinct sources of time-varying crash risk – upward and downward jumps in foreign exchange rates and jumps in the variance of currency returns, (3) normal risk is quantitatively the largest chunk of risk regardless of the investment horizons but the importance of jumps in volatility increases with the investment horizon, (4) many jump events in currency price are associated with important macroeconomic and political news, whereas jumps in variance are not, (5) probability of jumps in currency prices is driven by the interest rate differential, whereas the probability of jumps in variance is controlled by the variance itself, (6) multiple sources of US consumption risk matter in the foreign exchange market, (7) the long-run consumption risk is the most prominent source of risk – it matters both in the cross-section of currency returns and across alternative investment horizons, (8) the short-run consumption risk is priced in the cross-section of currency returns only at the one-period investment horizon.

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Appendix A

Appendix

A.1 Supplementary material for Chapter 2

A.1.1 Long-run risk models

In this section we provide two examples of models with critically different shocks to the respective endowment processes that, nonetheless, yield the same functional dependence of currency excess returns on observable variables. We rely on the Long-Run Risk framework of Bansal and Yaron (2004) and use various modelling elements inspired by Bansal and Shaliastovich (2013); Benzoni, Collin-Dufresne, and Goldstein (2011); Drechsler and Yaron (2011); Wachter (2013).

We neither make any claims about realism of these models nor attempt to distinguish them empirically. In fact, we try to construct the simplest models possible that deliver risk premiums dependent on the interest rate differential and the variance of changes in the exchange rate. Moreover, the models have implications for real exchange rates while we are studying the empirical behaviour of nominal exchange rates. Thus, these models serve for pure illustrative purposes. We use recursive preferences and define utility from date t on

$$U_t = [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}, \tag{A.1.1}$$

and certainty equivalent function,

$$\mu_t(U_{t+1}) \quad = \quad \left[E_t(U_{t+1}^{\alpha})\right]^{1/\alpha}.$$

In standard terminology, $\rho < 1$ captures time preference (with intertemporal elasticity of substitution $1/(1-\rho)$) and $\alpha < 1$ captures risk aversion (with coefficient of relative risk aversion $1-\alpha$). The time aggregator and certainty equivalent functions are both homogeneous of degree one, which allows us to scale everything by current consumption. If we define scaled utility $u_t = U_t/c_t$, equation (A.1.1) becomes

$$u_t \quad = \quad [(1-\beta) + \beta\mu_t(g_{t+1}u_{t+1})^{\rho}]^{1/\rho}, \tag{A.1.2}$$

where $g_{t+1} = c_{t+1}/c_t$ is consumption growth. The pricing kernel is

$$\begin{aligned} m_{t+1} &= \beta(c_{t+1}/c_t)^{\rho-1}\left[U_{t+1}/\mu_t(U_{t+1})\right]^{\alpha-\rho} \\ &= \beta g_{t+1}^{\rho-1}\left[g_{t+1}u_{t+1}/\mu_t(g_{t+1}u_{t+1})\right]^{\alpha-\rho}. \end{aligned}$$

The relationship (A.1.2) serves, essentially, as a Bellman equation. Its loglinear approximation

$$\begin{aligned} \log u_t &= \rho^{-1}\log\left[(1-\beta) + \beta\mu_t(g_{t+1}u_{t+1})^{\rho}\right] \\ &= \rho^{-1}\log\left[(1-\beta) + \beta e^{\rho\log\mu_t(g_{t+1}u_{t+1})}\right] \\ &\approx b_0 + b_1\log\mu_t(g_{t+1}u_{t+1}) \end{aligned} \tag{A.1.3}$$

gives us transparent closed-form expressions for pricing kernels (Hansen, Heaton, and Li, 2008). The last line is a first-order approximation of $\log u_t$ in $\log\mu_t$ around the point $\log\mu_t = \log\mu$, with

$$\begin{aligned} b_1 &= \beta e^{\rho\log\mu}/[(1-\beta) + \beta e^{\rho\log\mu}] \\ b_0 &= \rho^{-1}\log[(1-\beta) + \beta e^{\rho\log\mu}] - b_1\log\mu. \end{aligned}$$

The equation is exact when $\rho = 0$, in which case $b_0 = 0$ and $b_1 = \beta$. We focus on this case for the simplicity sake and to avoid the debate on the accuracy of the log-linear approximation as this subject is not the focus of our paper.

We extend this setting to two countries that we refer to as home (US) and foreign. The representative agents in each country have different risk aversion: $1-\alpha$ and $1-\tilde{\alpha}$, respectively. Similarly, all other foreign-country-specific objects, such as consumption growth, or pricing kernel are denoted by tilde $\tilde{}$.

Model 1: Stochastic Variance

The domestic consumption growth is

$$\begin{aligned}
\log g_{t+1} &= \log g + x_t + k\sigma_{g,t}\eta_{t+1}, \\
x_{t+1} &= \gamma x_t + \sigma_{x,t}e_{t+1}, \\
\sigma^2_{g,t+1} &= (1-\nu_g)v_g + \nu_g\sigma^2_{g,t} + \sigma_{gw}\sigma_{g,t}w_{g,t+1}, \\
\sigma^2_{x,t+1} &= (1-\nu_x)v_x + \nu_x\sigma^2_{x,t} + \sigma_{xw}\sigma_{x,t}w_{x,t+1}.
\end{aligned}$$

The foreign consumption growth is similar, except for the loading of the variance of consumption growth on $\sigma^2_{g,t}$:

$$\begin{aligned}
\log \tilde{g}_{t+1} &= \log g + x_t + \tilde{k}\sigma_{g,t}\tilde{\eta}_{t+1}, \\
x_{t+1} &= \gamma x_t + \sigma_{x,t}e_{t+1}, \\
\sigma^2_{g,t+1} &= (1-\nu_g)v_g + \nu_g\sigma^2_{g,t} + \sigma_{gw}\sigma_{g,t}w_{g,t+1}, \\
\sigma^2_{x,t+1} &= (1-\nu_x)v_x + \nu_x\sigma^2_{x,t} + \sigma_{xw}\sigma_{x,t}w_{x,t+1}.
\end{aligned}$$

For simplicity, we assume that all the shocks, η_t, $\tilde{\eta}_t$, e_t, $w_{g,t}$, and $w_{x,t}$, are independent.

To solve the model, guess the domestic value function:

$$\log u_t = \log u + p_x x_t + p_{\sigma g}\sigma^2_{g,t} + p_{\sigma x}\sigma^2_{x,t}.$$

Compute:

$$\begin{aligned}
\log(u_{t+1}g_{t+1}) &= \log ug + x_t + k\sigma_{g,t}\eta_{t+1} + p_x x_{t+1} + p_{\sigma g}\sigma^2_{g,t+1} + p_{\sigma x}\sigma^2_{x,t+1} \\
&= \log ug + p_{\sigma g}(1-\nu_g)v_g + p_{\sigma x}(1-\nu_x)v_x + (1+p_x\gamma)x_t + p_{\sigma g}\nu_g\sigma^2_{g,t} + p_{\sigma x}\nu_x\sigma^2_{x,t} \\
&+ k\sigma_{g,t}\eta_{t+1} + p_x\sigma_{x,t}e_{t+1} + p_{\sigma g}\sigma_{gw}\sigma_{g,t}w_{g,t+1} + p_{\sigma x}\sigma_{xw}\sigma_{x,t}w_{x,t+1}, \\
\log\mu_t(u_{t+1}g_{t+1}) &= [\log ug + p_{\sigma g}(1-\nu_g)v_g + p_{\sigma x}(1-\nu_x)v_x] + (1+p_x\gamma)x_t \\
&+ (p_{\sigma g}\nu_g + \alpha k^2/2 + \alpha p^2_{\sigma g}\sigma^2_{gw}/2)\sigma^2_{g,t} + (p_{\sigma x}\nu_x + \alpha p^2_x/2 + \alpha p^2_{\sigma x}\sigma^2_{xw}/2)\sigma^2_{x,t}.
\end{aligned}$$

Plug $\log\mu_t(u_{t+1}g_{t+1})$ into the Bellman equation (A.1.3) and match coefficients:

$$\begin{aligned}
\text{constant}: &\quad \log u = \beta(\log ug + p_{\sigma g}(1-\nu_g)v_g + p_{\sigma x}(1-\nu_x)v_x) \\
x_t: &\quad p_x = \beta(1+p_x\gamma) \\
\sigma^2_{g,t}: &\quad p_{\sigma g} = \beta(p_{\sigma g}\nu_g + \alpha k^2/2 + \alpha p^2_{\sigma g}\sigma^2_{gw}/2) \\
\sigma^2_{x,t}: &\quad p_{\sigma x} = \beta(p_{\sigma x}\nu_x + \alpha p^2_x/2 + \alpha p^2_{\sigma x}\sigma^2_{xw}/2)
\end{aligned}$$

These equations imply that

$$\begin{aligned}
\log u &= \beta(\log g + p_{\sigma g}(1-\nu_g)v_g + p_{\sigma x}(1-\nu_x)v_x)/(1-\beta) \\
p_x &= \beta/(1-\beta\gamma)
\end{aligned}$$

and $p_{\sigma g}$ and $p_{\sigma x}$ are the smallest roots of the following quadratic equations:

$$\begin{aligned}
\alpha\beta\sigma^2_{gw}p^2_{\sigma g} + 2(\beta\nu_g - 1)p_{\sigma g} + \alpha\beta k^2 &= 0 \\
\alpha\beta\sigma^2_{xw}p^2_{\sigma x} + 2(\beta\nu_x - 1)p_{\sigma x} + \alpha\beta p^2_x &= 0.
\end{aligned}$$

We select the smallest roots because they ensure that the corresponding risk premium is zero when variance is zero. The foreign value function is computed following identical steps. The log pricing kernel at home is

$$\begin{aligned}
\log m_{t+1} &= \log\beta - \log g_{t+1} + \alpha[\log g_{t+1}u_{t+1} - \log\mu_t(g_{t+1}u_{t+1})] \\
&= \log\beta - \log g - x_t - \alpha^2(k^2 + p_{\sigma g}^2\sigma_{gw}^2)\sigma_{g,t}^2/2 - \alpha^2(p_x^2 + p_{\sigma x}^2\sigma_{xw}^2)\sigma_{x,t}^2/2 \\
&+ (\alpha-1)k\sigma_{g,t}\eta_{t+1} + \alpha p_x\sigma_{x,t}e_{t+1} + \alpha p_{\sigma g}\sigma_{gw}\sigma_{g,t}w_{g,t+1} + \alpha p_{\sigma x}\sigma_{xw}\sigma_{x,t}w_{x,t+1}.
\end{aligned}$$

The log pricing kernel abroad $\tilde{m}_t$ has a similar expression.

Domestic and foreign interest rates are:

$$\begin{aligned}
r_t &= -\log\beta + \log g + x_t + (2\alpha-1)k^2\sigma_{g,t}^2/2, \\
\tilde{r}_t &= -\log\beta + \log g + x_t + (2\tilde{\alpha}-1)\tilde{k}^2\sigma_{g,t}^2/2.
\end{aligned}$$

Interest rate differential is

$$r_t - \tilde{r}_t = [(2\alpha-1)k^2 - (2\tilde{\alpha}-1)\tilde{k}^2]\sigma_{g,t}^2/2. \tag{A.1.4}$$

By no-arbitrage:

$$s_{t+1} - s_t = \log\tilde{m}_{t+1} - \log m_{t+1}. \tag{A.1.5}$$

Thus, exchange rate growth process is

$$\begin{aligned}
s_{t+1} - s_t &= [\alpha^2(k^2 + \sigma_{gw}^2p_{\sigma g}^2) - \tilde{\alpha}^2(\tilde{k}^2 + \sigma_{gw}^2\tilde{p}_{\sigma g}^2)]\sigma_{g,t}^2/2 \\
&+ [\alpha^2(p_x^2 + p_{\sigma x}^2\sigma_{xw}^2) - \tilde{\alpha}^2(p_x^2 + \tilde{p}_{\sigma x}^2\sigma_{xw}^2)]\sigma_{x,t}^2/2 \\
&+ [(\tilde{\alpha}-1)\tilde{k}\tilde{\eta}_{t+1} - (\alpha-1)k\eta_{t+1}]\sigma_{g,t} + p_x(\tilde{\alpha}-\alpha)\sigma_{x,t}e_{t+1} \\
&+ (\tilde{\alpha}\tilde{p}_{\sigma g} - \alpha p_{\sigma g})\sigma_{gw}\sigma_{g,t}w_{g,t+1} + (\tilde{\alpha}\tilde{p}_{\sigma x} - \alpha p_{\sigma x})\sigma_{xw}\sigma_{x,t}w_{x,t+1}.
\end{aligned}$$

and we can compute expected excess returns $E_t(s_{t+1}-s_t-(r_t-\tilde{r}_t))$ and conditional variance of the exchange rate changes $var_t(s_{t+1}-s_t)$.

The conditional variance of the exchange rate growth is

$$
\begin{aligned}
var_t(s_{t+1}-s_t) &= [(\tilde{\alpha}-1)^2\tilde{k}^2+(\alpha-1)^2k^2+(\tilde{\alpha}\tilde{p}_{\sigma g}-\alpha p_{\sigma g})^2\sigma_{gw}^2]\sigma_{g,t}^2 \\
&+ [p_x^2(\tilde{\alpha}-\alpha)^2+(\tilde{\alpha}\tilde{p}_{\sigma x}-\alpha p_{\sigma x})^2\sigma_{xw}^2]\sigma_{x,t}^2. \qquad \text{(A.1.6)}
\end{aligned}
$$

The expected excess log currency return is

$$
\begin{aligned}
E_t(s_{t+1}-s_t-(r_t-\tilde{r}_t)) &= (var_t(\log m_{t+1})-var_t(\log\tilde{m}_{t+1}))/2 \\
&= [(\alpha-1)^2k^2-(\tilde{\alpha}-1)^2\tilde{k}^2+\alpha^2p_{\sigma g}^2\sigma_{gw}^2-\tilde{\alpha}^2\tilde{p}_{\sigma g}^2\sigma_{gw}^2]\sigma_{g,t}^2/2 \\
&+ [p_x^2(\alpha^2-\tilde{\alpha}^2)+\alpha^2p_{\sigma x}^2\sigma_{xw}^2-\tilde{\alpha}^2\tilde{p}_{\sigma x}^2\sigma_{xw}^2]\sigma_{x,t}^2/2. \qquad \text{(A.1.7)}
\end{aligned}
$$

Interest rate differential and conditional variance of the exchange rate changes depend on $\sigma_{g,t}^2$ and $\sigma_{x,t}^2$. Therefore, one can express the dependence of expected excess returns on $\sigma_{g,t}^2$ and $\sigma_{x,t}^2$ as a dependence on the interest rate differential and conditional variance of the exchange rate changes.

Solve the system of equations (A.1.4-A.1.6) for the stochastic variances:

$$
\begin{aligned}
\sigma_{g,t}^2 &= \frac{B_v(r_t-\tilde{r}_t)-B_r var_t(s_{t+1}-s_t)}{A_rB_v}, \qquad \text{(A.1.8)} \\
\sigma_{x,t}^2 &= \frac{-A_v(r_t-\tilde{r}_t)+A_r var_t(s_{t+1}-s_t)}{A_rB_v}, \qquad \text{(A.1.9)}
\end{aligned}
$$

where

$$
\begin{aligned}
A_r &= [(2\alpha-1)k^2-(2\tilde{\alpha}-1)\tilde{k}^2]/2 \\
A_v &= (\tilde{\alpha}-1)^2\tilde{k}^2+(\alpha-1)^2k^2+(\tilde{\alpha}\tilde{p}_{\sigma g}-\alpha p_{\sigma g})^2\sigma_{gw}^2, \\
B_v &= p_x^2(\tilde{\alpha}-\alpha)^2+(\tilde{\alpha}\tilde{p}_{\sigma x}-\alpha p_{\sigma x})^2\sigma_{xw}^2.
\end{aligned}
$$

Expressions (A.1.7)-(A.1.9) imply that the log expected excess currency return is a linear function of interest rate differential and the variance of exchange rate growth:

$$
E_ty_{t+1} = E_t(s_{t+1}-s_t-(r_t-\tilde{r}_t)) = \delta_r(r_t-\tilde{r}_t)+\delta_v var_t(s_{t+1}-s_t),
$$

where

$$
\begin{aligned}
\delta_r &= (B_v s_g - A_v s_x)/(A_r B_v), \\
\delta_v &= s_x / B_v, \\
s_g &= [(\alpha-1)^2 k^2 - (\tilde{\alpha}-1)^2 \tilde{k}^2 + \alpha^2 p_{\sigma g}^2 \sigma_{gw}^2 - \tilde{\alpha}^2 \tilde{p}_{\sigma g}^2 \sigma_{gw}^2]/2, \\
s_x &= [p_x^2(\alpha^2 - \tilde{\alpha}^2) + \alpha^2 p_{\sigma x}^2 \sigma_{xw}^2 - \tilde{\alpha}^2 \tilde{p}_{\sigma x}^2 \sigma_{xw}^2]/2.
\end{aligned}
$$

Model 2: Disasters

The domestic and foreign consumption growths are

$$
\begin{aligned}
\log g_{t+1} &= \log g + x_t + \sigma_g \eta_{t+1} + z_{g,t+1}, \\
\log \tilde{g}_{t+1} &= \log g + x_t + \sigma_g \tilde{\eta}_{t+1} + z_{g,t+1}, \\
x_{t+1} &= \gamma x_t + \sigma_x e_{t+1} + z_{x,t+1},
\end{aligned}
$$

where the jump sizes are drawn from the normal distributions

$$
\begin{aligned}
z_{g,t+1}|j &\sim \mathcal{N}(j\mu_g, j\sigma_g^2), \\
z_{x,t+1}|j &\sim \mathcal{N}(j\mu_x, j\sigma_x^2),
\end{aligned}
$$

and the jump arrival rate is controlled by a Poisson distribution

$$
Prob(j_{t+1} = j) = \exp(-h_{k,t}) h_{k,t}^j / j!, \; k = g, x.
$$

The jump intensities $h_{g,t}$ and $h_{x,t}$ are time-varying:

$$
\begin{aligned}
h_{g,t+1} &= (1-\nu_{hg}) v_{hg} + \nu_{hg} h_{g,t} + \sigma_{hg} h_{g,t}^{1/2} \varepsilon_{hg,t+1}, \\
h_{x,t+1} &= (1-\nu_{hx}) v_{hx} + \nu_{hx} h_{x,t} + \sigma_{hx} h_{x,t}^{1/2} \varepsilon_{hx,t+1}.
\end{aligned}
$$

The model is solved by guessing the value function for each country. For example, guess the domestic value function:

$$\log u_t = \log u + p_x x_t + p_{hg} h_{g,t} + p_{hx} h_{x,t}.$$

Compute

$$\begin{aligned}
\log u_{t+1} g_{t+1} &= [\log ug + p_{hg}(1-\nu_{hg})v_{hg} + p_{hx}(1-\nu_{hx})v_{hx}] + (p_x\gamma + 1)x_t + p_{hg}\nu_{hg}h_{g,t} \\
&+ p_{hx}\nu_{hx}h_{x,t} + \sigma_g\eta_{t+1} + p_x\sigma_x e_{t+1} + p_{hg}\sigma_{hg}h_{g,t}^{1/2}\varepsilon_{hg,t+1} + p_{hx}\sigma_{hx}h_{x,t}^{1/2}\varepsilon_{hx,t+1}, \\
&+ p_x z_{x,t+1} + z_{g,t+1}, \\
\log \mu_t(u_{t+1}g_{t+1}) &= [\log ug + p_{hg}(1-\nu_{hg})v_{hg} + p_{hx}(1-\nu_{hx})v_{hx}] + (p_x\gamma + 1)x_t + p_{hg}\nu_{hg}h_{g,t} \\
&+ p_{hx}\nu_{hx}h_{x,t} + \alpha\sigma_g^2/2 + \alpha p_x^2\sigma_x^2/2 + \alpha p_{hg}^2\sigma_{hg}^2 h_{g,t}/2 + \alpha p_{hx}^2\sigma_{hx}^2 h_{x,t}/2 \\
&+ (e^{\alpha p_x\mu_x + (\alpha p_x\sigma_x)^2/2} - 1)h_{x,t}/\alpha + (e^{\alpha\mu_g + (\alpha\sigma_g)^2/2} - 1)h_{g,t}/\alpha.
\end{aligned}$$

Plug $\log \mu_t(u_{t+1}g_{t+1})$ into the Bellman equation (A.1.3) and match coefficients:

$$\begin{aligned}
\text{constant}: \quad \log u &= \beta(\log ug + p_{hg}(1-\nu_{hg})v_{hg} + p_{hx}(1-\nu_{hx})v_{hx} + \alpha\sigma_g^2/2 + \alpha p_x^2\sigma_x^2/2) \\
x_t: \quad p_x &= \beta(1 + p_x\gamma) \\
h_{g,t}: \quad p_{hg} &= \beta(p_{hg}\nu_{hg} + \alpha p_{hg}^2\sigma_{hg}^2/2 + (e^{\alpha\mu_g + (\alpha\sigma_g)^2/2} - 1)/\alpha) \\
h_{x,t}: \quad p_{hx} &= \beta(p_{hx}\nu_{hx} + \alpha p_{hx}^2\sigma_{hx}^2/2 + (e^{\alpha p_x\mu_x + (\alpha p_x\sigma_x)^2/2} - 1)/\alpha)
\end{aligned}$$

These equations imply that

$$\begin{aligned}
\log u &= \beta(\log g + p_{hg}(1-\nu_{hg})v_{hg} + p_{hx}(1-\nu_{hx})v_{hx} + \alpha\sigma_g^2/2 + \alpha p_x^2\sigma_x^2/2)/(1-\beta) \\
p_x &= \beta/(1-\beta\gamma)
\end{aligned}$$

and p_{hg} and p_{hx} are the smallest roots of the following quadratic equations:

$$\begin{aligned}
\alpha^2\sigma_{hg}^2\beta p_{hg}^2 + 2\alpha(\beta\nu_{hg} - 1)p_{hg} + 2\beta(e^{\alpha\mu_g + (\alpha\sigma_g)^2/2} - 1) &= 0, \\
\alpha^2\sigma_{hx}^2\beta p_{hx}^2 + 2\alpha(\beta\nu_{hx} - 1)p_{hx} + 2\beta(e^{\alpha p_x\mu_x + (\alpha p_x\sigma_x)^2/2} - 1) &= 0.
\end{aligned}$$

We select the smallest roots because they ensure that the corresponding risk premium is zero when variance is zero. The foreign value function is computed following identical steps.

The log pricing kernel at home is

$$\begin{aligned}\log m_{t+1} &= [\log\beta-\log g-\alpha^2\sigma_g^2/2-\alpha^2p_x^2\sigma_x^2/2]-x_t-\alpha^2p_{hg}^2\sigma_{hg}^2h_{g,t}/2-\alpha^2p_{hx}^2\sigma_{hx}^2h_{x,t}/2\\ &- (e^{\alpha\mu_g+(\alpha\sigma_g)^2/2}-1)h_{g,t}-(e^{\alpha p_x\mu_x+(\alpha p_x\sigma_x)^2/2}-1)h_{x,t}+(\alpha-1)\sigma_g\eta_{t+1}+\alpha p_x\sigma_x e_{t+1}\\ &+ \alpha p_{hg}\sigma_{hg}h_{g,t}^{1/2}\varepsilon_{hg,t+1}+\alpha p_{hx}\sigma_{hx}h_{xt}^{1/2}\varepsilon_{hx,t+1}+(\alpha-1)z_{g,t+1}+\alpha p_x z_{x,t+1}.\end{aligned}$$

The log pricing kernel abroad has a similar expression.

We can compute domestic interest rate $r_t=-\log E_t(m_{t+1})$, a similar expression applies to the foreign interest rate $\tilde{r}_t$:

$$\begin{aligned}r_t &= -\log E_t m_{t+1}=[-\log\beta+\log g+\alpha^2\sigma_g^2/2-(\alpha-1)^2\sigma_g^2/2]+x_t\\ &+ (e^{\alpha\mu_g+(\alpha\sigma_g)^2/2}-e^{(\alpha-1)\mu_g+((\alpha-1)\sigma_g)^2/2})h_{g,t},\\ \tilde{r}_t &= -\log E_t\tilde{m}_{t+1}=[-\log\beta+\log g+\tilde{\alpha}^2\sigma_g^2/2-(\tilde{\alpha}-1)^2\sigma_g^2/2]+x_t\\ &+ (e^{\tilde{\alpha}\mu_g+(\tilde{\alpha}\sigma_g)^2/2}-e^{(\tilde{\alpha}-1)\mu_g+((\tilde{\alpha}-1)\sigma_g)^2/2})h_{g,t}.\end{aligned}\tag{A.1.10}$$

Thus, the interest rate differential is

$$\begin{aligned}r_t-\tilde{r}_t &= r_0+[e^{\alpha\mu_g+(\alpha\sigma_g)^2/2}-e^{\tilde{\alpha}\mu_g+(\tilde{\alpha}\sigma_g)^2/2}]h_{g,t}\\ &+ (e^{(\tilde{\alpha}-1)\mu_g+((\tilde{\alpha}-1)\sigma_g)^2/2}-e^{(\alpha-1)\mu_g+((\alpha-1)\sigma_g)^2/2})h_{g,t},\end{aligned}\tag{A.1.11}$$

where

$$r_0 = (\alpha-\tilde{\alpha})\sigma_g^2.$$

By no-arbitrage:

$$s_{t+1}-s_t=\log\tilde{m}_{t+1}-\log m_{t+1},\tag{A.1.12}$$

so that the exchange rate growth process:

$$
\begin{aligned}
s_{t+1} - s_t &= (\alpha^2 - \tilde{\alpha}^2)\sigma_g^2/2 + p_x^2(\alpha^2 - \tilde{\alpha}^2)\sigma_x^2/2 + (\alpha^2 p_{hg}^2 - \tilde{\alpha}^2 \tilde{p}_{hg}^2)\sigma_{hg}^2 h_{g,t}/2 \\
&+ (\alpha^2 p_{hx}^2 - \tilde{\alpha}^2 \tilde{p}_{hx}^2)\sigma_{hx}^2 h_{x,t}/2 + (e^{\alpha\mu_g + (\alpha\sigma_g)^2/2} - e^{\tilde{\alpha}\mu_g + (\tilde{\alpha}\sigma_g)^2/2})h_{g,t} \\
&+ (e^{\alpha p_x \mu_x + (\alpha p_x \sigma_x)^2/2} - e^{\tilde{\alpha} p_x \mu_x + (\tilde{\alpha} p_x \sigma_x)^2/2})h_{x,t} + (\tilde{\alpha} - 1)\sigma_g \tilde{\eta}_{t+1} - (\alpha - 1)\sigma_g \eta_{t+1} \\
&+ p_x(\tilde{\alpha} - \alpha)\sigma_x e_{t+1} + (\tilde{\alpha}\tilde{p}_{hg} - \alpha p_{hg})\sigma_{hg} h_{g,t}^{1/2} \varepsilon_{hg,t+1} \\
&+ (\tilde{\alpha}\tilde{p}_{hx} - \alpha p_{hx})\sigma_{hx} h_{x,t}^{1/2} \varepsilon_{hx,t+1} + (\tilde{\alpha} - \alpha) z_{g,t+1} + p_x(\tilde{\alpha} - \alpha) z_{x,t+1}.
\end{aligned}
$$

Therefore, we can compute expected excess returns $E_t(s_{t+1} - s_t - (r_t - \tilde{r}_t))$ and conditional variance of the exchange rate changes $var_t(s_{t+1} - s_t)$. All of these objects depend on $h_{g,t}$ and $h_{x,t}$. Therefore, one can express the dependence of expected excess returns on $h_{g,t}$ and $h_{x,t}$ as a dependence on the interest rate differential and conditional variance of the exchange rate changes.

The conditional variance of the exchange rate growth is

$$
\begin{aligned}
var_t(s_{t+1} - s_t) &= v_0 + (\tilde{\alpha}\tilde{p}_{hg} - \alpha p_{hg})^2 \sigma_{hg}^2 h_{g,t} + (\tilde{\alpha}\tilde{p}_{hx} - \alpha p_{hx})^2 \sigma_{hx}^2 h_{x,t} \\
&+ ((\tilde{\alpha} - \alpha)^2 \mu_g^2 + \sigma_g^2) h_{g,t} + ((\tilde{\alpha}\tilde{p}_x - \alpha p_x)^2 \mu_x^2 + \sigma_x^2) h_{x,t}. \quad \text{(A.1.13)}
\end{aligned}
$$

where

$$
v_0 = (\tilde{\alpha} - 1)^2 \sigma_g^2 + (\alpha - 1)^2 \sigma_g^2 + p_x^2 (\tilde{\alpha} - \alpha)^2 \sigma_x^2.
$$

The expected log excess currency return is

$$
\begin{aligned}
E_t(s_{t+1} - s_t - (r_t - \tilde{r}_t)) &= rx_0 + (\alpha^2 p_{hg}^2 - \tilde{\alpha}^2 \tilde{p}_{hg}^2)\sigma_{hg}^2 h_{g,t}/2 + (\alpha^2 p_{hx}^2 - \tilde{\alpha}^2 \tilde{p}_{hx}^2)\sigma_{hx}^2 h_{x,t}/2 \\
&+ \mu_g(\tilde{\alpha} - \alpha) h_{g,t} + \mu_x p_x (\tilde{\alpha} - \alpha) h_{x,t} \\
&+ (e^{(\alpha-1)\mu_g + ((\alpha-1)\sigma_g)^2/2} - e^{(\tilde{\alpha}-1)\mu_g + ((\tilde{\alpha}-1)\sigma_g)^2/2}) h_{g,t} \\
&+ (e^{\alpha p_x \mu_x + (\alpha p_x \sigma_x)^2/2} - e^{\tilde{\alpha} p_x \mu_x + (\tilde{\alpha} p_x \sigma_x)^2/2}) h_{x,t}, \quad \text{(A.1.14)}
\end{aligned}
$$

where

$$rx_0 = ((\alpha-1)^2-(\tilde{\alpha}-1)^2)\sigma_g^2/2+p_x^2(\alpha^2-\tilde{\alpha}^2)\sigma_x^2/2.$$

Solve the system of equations (A.1.11)-(A.1.13) for the jump intensities:

$$h_{g,t} = \frac{B_v(r_t-\tilde{r}_t)-B_r var_t(s_{t+1}-s_t)+B_r v_0-B_v r_0}{A_r B_v}, \tag{A.1.15}$$

$$h_{x,t} = \frac{-A_v(r_t-\tilde{r}_t)+A_r var_t(s_{t+1}-s_t)+A_v r_0-A_r v_0}{A_r B_v}, \tag{A.1.16}$$

where

$$\begin{aligned}
A_r &= e^{\alpha\mu_g+(\alpha\sigma_g)^2/2}-e^{\tilde{\alpha}\mu_g+(\tilde{\alpha}\sigma_g)^2/2}+e^{(\tilde{\alpha}-1)\mu_g+((\tilde{\alpha}-1)\sigma_g)^2/2}-e^{(\alpha-1)\mu_g+((\alpha-1)\sigma_g)^2/2}\\
A_v &= (\tilde{\alpha}\tilde{p}_{hg}-\alpha p_{hg})^2\sigma_{hg}^2+(\tilde{\alpha}-\alpha)^2\mu_g^2+\sigma_g^2,\\
B_v &= (\tilde{\alpha}\tilde{p}_{hx}-\alpha p_{hx})^2\sigma_{hx}^2+p_x^2(\tilde{\alpha}-\alpha)^2\mu_x^2+\sigma_x^2.
\end{aligned}$$

Expressions (A.1.14)-(A.1.16) imply that the log expected excess currency return is a linear function of interest rate differential and the variance of exchange rate growth:

$$E_t y_{t+1} = E_t(s_{t+1}-s_t-(r_t-\tilde{r}_t)) = \delta_0+\delta_r(r_t-\tilde{r}_t)+\delta_v var_t(s_{t+1}-s_t),$$

where

$$\begin{aligned}
\delta_0 &= rx_0-s_g r_0/A_r-s_x(A_r v_0-A_v r_0)/(A_r B_v),\\
\delta_r &= (-A_v s_x+B_v s_g)/(A_r B_v),\\
\delta_v &= s_x/B_v,\\
s_g &= (\alpha^2 p_{hg}^2-\tilde{\alpha}^2\tilde{p}_{hg}^2)\sigma_{hg}^2/2+(e^{\mu_g(\alpha-1)+((\alpha-1)\sigma_g)^2/2}-e^{(\tilde{\alpha}-1)\mu_g+((\tilde{\alpha}-1)\sigma_g)^2/2})+\mu_g(\tilde{\alpha}-\alpha),\\
s_x &= (\alpha^2 p_{hx}^2-\tilde{\alpha}^2\tilde{p}_{hx}^2)\sigma_{hx}^2/2+e^{\alpha p_x\mu_x+(\alpha p_x\sigma_x)^2/2}-e^{\tilde{\alpha}p_x\mu_x+(\tilde{\alpha}p_x\sigma_x)^2/2}+\mu_x p_x(\tilde{\alpha}-\alpha).
\end{aligned}$$

A.1.2 The estimation algorithm

In this section we outline the estimation algorithm for the Preferred model. We estimate the discrete time model on the basis of daily data. We assume that there is no more than one jump per day. We re-write our model using notation that is more convenient for estimation purposes:

$$y_{t+1} = \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + \bar{z}_{t+1}^u \bar{j}_{t+1}^u - \bar{z}_{t+1}^d \bar{j}_{t+1}^d, \quad \text{(A.1.17)}$$

$$v_{t+1} = (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + \bar{z}_{t+1}^v \bar{j}_{t+1}^v, \quad \text{(A.1.18)}$$

$$IV_t = \alpha_{iv} + \beta_{iv} v_t + \sigma_{iv} v_t \sqrt{\lambda_t} \varepsilon_t. \quad \text{(A.1.19)}$$

Indicator $\bar{j}_t^k$, $k = \{u, d, v\}$, is equal to one if there is a jump at t, and zero otherwise. Correspondingly, $\bar{z}_t^k$ is a jump size:

$$\bar{z}_t^u \sim \mathcal{E}xp(\theta), \quad \text{(A.1.20)}$$

$$\bar{z}_t^d \sim \mathcal{E}xp(\theta), \quad \text{(A.1.21)}$$

$$\bar{z}_t^v \sim \mathcal{E}xp(\theta_v). \quad \text{(A.1.22)}$$

Introduce new notations: $\psi = \rho\sigma_v$, $\eta = \sigma_v^2(1-\rho^2)$, $\alpha = (1-\nu)v$, $\beta = \nu$, and Θ is the collection of all parameters. Denote the full history of excess returns, variance, implied variance, domestic and foreign interest rates, jump times and sizes by Y, V, IV, R, $\tilde{R}$, $\bar{J}^k$, $\bar{Z}^k$ ($k = \{u, d, v\}$), respectively. All the data are available on the interval $t \in [1, T]$, except for the implied variance which is available on the interval $t \in [T_2 + 1, T]$, $T_2 > 0$.

Posterior distributions for the parameters

- Assume a normal prior for μ_0: $\mu_0 \sim N(a, A)$.

  Posterior distribution is

  $$p(\mu_0 | Y, V, \bar{Z}^u, \bar{Z}^d, \bar{Z}^v, \bar{J}^u, \bar{J}^d, \bar{J}^v, R, \tilde{R}, \Theta_{\{-\mu_0\}}) \propto N(\hat{a}, \hat{A}),$$

  where

$$\hat{A} = \left(\frac{1}{A} + \left(\frac{\psi^2}{\eta} + 1\right)\sum_{t=0}^{T-1}\frac{1}{v_t}\right)^{-1},$$

$$\hat{a} = \hat{A}\left(\frac{a}{A} + \left(\frac{\psi^2}{\eta} + 1\right)\sum_{t=0}^{T-1}\frac{y_{t+1} - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t - \bar{z}^u_{t+1}\bar{j}^u_{t+1} + \bar{z}^d_{t+1}\bar{j}^d_{t+1}}{v_t}\right) -$$

$$-\hat{A}\left(\frac{\psi}{\eta}\sum_{t=0}^{T-1}\frac{(v_{t+1} - \alpha - \beta v_t - \bar{z}^v_{t+1}\bar{j}^v_{t+1})}{v_t}\right).$$

- Assume a normal prior for μ_v: $\mu_v \sim N(a, A)$.

  Posterior distribution is

$$p(\mu_v|Y, V, \bar{Z}^u, \bar{Z}^d, \bar{Z}^v, \bar{J}^u, \bar{J}^d, \bar{J}^v, R, \tilde{R}, \Theta_{\{-\mu_v\}}) \propto N(\hat{a}, \hat{A}),$$

  where

$$\hat{A} = \left(\frac{1}{A} + \left(\frac{\psi^2}{\eta} + 1\right)\sum_{t=0}^{T-1} v_t\right)^{-1},$$

$$\hat{a} = \hat{A}\left(\frac{a}{A} + \left(\frac{\psi^2}{\eta} + 1\right)\sum_{t=0}^{T-1}(y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \bar{z}^u_{t+1}\bar{j}^u_{t+1} + \bar{z}^d_{t+1}\bar{j}^d_{t+1})\right) -$$

$$-\hat{A}\left(\frac{\psi}{\eta}\sum_{t=0}^{T-1}(v_{t+1} - \alpha - \beta v_t - \bar{z}^v_{t+1}\bar{j}^v_{t+1})\right).$$

- Assume a normal prior for μ_r: $\mu_r \sim N(a, A)$.

  Posterior distribution is

$$p(\mu_r|Y, V, \bar{Z}^u, \bar{Z}^d, \bar{Z}^v, \bar{J}^u, \bar{J}^d, \bar{J}^v, R, \tilde{R}, \Theta_{\{-\mu_r\}}) \propto N(\hat{a}, \hat{A})$$

where

$$\hat{A} = \left(\frac{1}{A} + \left(\frac{\psi^2}{\eta} + 1\right)\sum_{t=0}^{T-1}\frac{(r_t - \tilde{r}_t)^2}{v_t}\right)^{-1},$$

$$\hat{a} = \hat{A}\left(\frac{a}{A} + \left(\frac{\psi^2}{\eta} + 1\right)\sum_{t=0}^{T-1}\frac{(y_{t+1} - \mu_0 - \mu_v v_t - \bar{z}^u_{t+1}\bar{j}^u_{t+1} + \bar{z}^d_{t+1}\bar{j}^d_{t+1})(r_t - \tilde{r}_t)}{v_t}\right) -$$

$$-\hat{A}\left(\frac{\psi}{\eta}\sum_{t=0}^{T-1}\frac{(r_t - \tilde{r}_t)(v_{t+1} - \alpha - \beta v_t - \bar{z}^v_{t+1}\bar{j}^v_{t+1})}{v_t}\right).$$

- Assume a normal prior for α: $\alpha \sim N(a, A)$.

  Posterior distribution is

$$p(\alpha|Y, V, \bar{Z}^u, \bar{Z}^d, \bar{Z}^v, \bar{J}^u, \bar{J}^d, \bar{J}^v, \Theta_{\{-\alpha\}}) \propto N(\hat{a}, \hat{A}),$$

  where

$$\hat{A} = \left(\frac{1}{A} + \frac{1}{\eta}\sum_{t=0}^{T-1}\frac{1}{v_t}\right)^{-1},$$

$$\hat{a} = \hat{A}\left(\frac{a}{A} + \frac{1}{\eta}\sum_{t=0}^{T-1}\frac{v_{t+1} - \beta v_t - \bar{j}^v_{t+1}\bar{z}^v_{t+1}}{v_t}\right) -$$

$$-\hat{A}\left(\frac{\psi}{\eta}\sum_{t=0}^{T-1}\frac{(y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t - \bar{j}^u_{t+1}\bar{z}^u_{t+1} + \bar{j}^d_{t+1}\bar{z}^d_{t+1})}{v_t}\right).$$

- Assume a normal prior for β: $\beta \sim N(a, A)$.

  Posterior distribution is

$$p(\beta|Y, V, \bar{Z}^u, \bar{Z}^d, \bar{Z}^v, \bar{J}^u, \bar{J}^d, \bar{J}^v, \Theta_{\{-\beta\}}) \propto N(\hat{a}, \hat{A}),$$

where

$$
\begin{aligned}
\hat{A} &= \left(\frac{1}{A} + \frac{1}{\eta}\sum_{t=0}^{T-1} v_t\right)^{-1}, \\
\hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=0}^{T-1} \frac{v_{t+1} - \alpha - \bar{j}^v_{t+1}\bar{z}^v_{t+1}}{\eta}\right) - \\
&-\hat{A}\left(\frac{\psi}{\eta}\sum_{t=0}^{T-1}(y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t - \bar{j}^u_{t+1}\bar{z}^u_{t+1} + \bar{j}^d_{t+1}\bar{z}^d_{t+1})\right).
\end{aligned}
$$

- Assume dependent normal-inverse gamma priors for ψ and η:

$$
\begin{aligned}
&\psi|\eta \sim N(a, A\eta), \\
&\eta \sim IG(b, B).
\end{aligned}
$$

Posterior distributions are

$$
\begin{aligned}
&p(\psi|Y, V, \bar{Z}^u, \bar{Z}^d, \bar{Z}^v, \bar{J}^u, \bar{J}^d, \bar{J}^v, R, \tilde{R}, \Theta_{\{-\psi\}}) \propto N(\hat{a}, \hat{A}\eta), \\
&p(\eta|Y, V, \bar{Z}^v, \bar{J}^v, R, \tilde{R}, \Theta_{\{-\eta\}}) \propto IG(\hat{b}, \hat{B}),
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{A} &= \left(\sum_{t=0}^{T-1}(w^s_{t+1})^2 + \frac{1}{A}\right)^{-1}, \\
\hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=0}^{T-1}\xi_{t+1}w^s_{t+1}\right), \\
\hat{b} &= b + \frac{T}{2}, \\
\hat{B} &= B + \frac{1}{2}\sum_{t=0}^{T-1}\xi^2_{t+1} + \frac{a^2}{2A} - \frac{\hat{a}^2}{2\hat{A}}, \\
\xi_{t+1} &= \frac{v_{t+1} - \alpha - \beta v_t - \bar{j}^v_{t+1}\bar{z}^v_{t+1}}{\sqrt{v_t}}.
\end{aligned}
$$

- Assume a normal prior for α_{iv}: $\alpha_{iv} \sim N(a, A)$.

Posterior distribution is

$$p(\alpha_{iv}|\beta_{iv},\sigma_{iv},IV,\{\lambda_t\}_{t=T_2+1}^{T},\{v_t\}_{t=T_2+1}^{T}) \propto N(\hat{a},\hat{A}),$$

where

$$\hat{A} = \left(\frac{1}{A} + \sum_{t=T_2+1}^{T} \frac{1}{\sigma_{iv}^2 v_t^2 \lambda_t}\right)^{-1},$$

$$\hat{a} = \hat{A}\left(\frac{1}{\sigma_{iv}^2}\sum_{t=T_2+1}^{T} \frac{IV_t - \beta_{iv} v_t}{v_t^2 \lambda_t} + \frac{a}{A}\right).$$

- Assume a normal prior for β_{iv}: $\beta_{iv} \sim N(a, A)$.

  Posterior distribution is

$$p(\beta_{iv}|\alpha_{iv},\sigma_{iv},IV,\{\lambda_t\}_{t=T_2+1}^{T},\{v_t\}_{t=T_2+1}^{T}) \propto N(\hat{a},\hat{A}),$$

  where

$$\hat{A} = \left(\frac{1}{A} + \sum_{t=T_2+1}^{T} \frac{1}{\sigma_{iv}^2 \lambda_t}\right)^{-1},$$

$$\hat{a} = \hat{A}\left(\frac{1}{\sigma_{iv}^2}\sum_{t=T_2+1}^{T} \frac{IV_t - \alpha_{iv}}{v_t \lambda_t} + \frac{a}{A}\right).$$

- Assume an inverse-gamma prior for σ_{iv}^2: $\sigma_{iv}^2 \sim IG(b, B)$.

  Posterior distribution is

$$p(\sigma_{iv}^2|\alpha_{iv},\beta_{iv},\{v_t\}_{t=T_2+1}^{T},IV,\{\lambda_t\}_{t=T_2+1}^{T}) \propto IG(\hat{b},\hat{B}),$$

  where

$$\hat{b} = b + \frac{T - T_2}{2},$$
$$\hat{B} = B + \sum_{t=T_2+1}^{T} \frac{(IV_t - \alpha_{iv} - \beta_{iv} v_t)^2}{2\lambda_t v_t^2}.$$

- Assume an inverse-gamma prior for θ_v: $\theta_v \sim IG(b, B)$.

  Posterior distribution is

$$p(\theta_v | \bar{Z}^v) \propto p(\bar{Z}^v | \theta_v) p(\theta_v) \propto IG(\hat{b}, \hat{B}),$$

  where

$$\hat{b} = b + T,$$
$$\hat{B} = B + \sum_{t=1}^{T} \bar{z}_t^v.$$

- Assume an inverse-gamma prior for θ: $\theta \sim IG(b, B)$.

  Posterior distribution is

$$p(\theta | \bar{Z}^u, \bar{Z}^d) \propto p(\bar{Z}^u, \bar{Z}^d | \theta) p(\theta) \propto IG(\hat{b}, \hat{B}),$$

  where

$$\hat{b} = b + 2T,$$
$$\hat{B} = B + \sum_{t=1}^{T} (\bar{z}_t^u - \bar{z}_t^d).$$

- We use the Metropolis-Hastings Random Walk algorithm to estimate the parameters of the jump intensities. In particular, we draw parameters in pairs – h_0^v and h_v; h_0 and h_r. Also, we draw these parameters in logs to guarantee that jump intensities stay strictly positive.

Posterior distributions for the latent variables

We have eight unobservable objects in the model: variance, three paths of the jump times, three paths of the jump sizes, and λ_t.

For each $t \in [T_2 + 1, T]$:

- Prior distribution for λ_t is $IG(\frac{\nu}{2}, \frac{\nu}{2})$.

  The posterior distribution is

$$p(\lambda_t | IV_t, v_t, \alpha_{iv}, \beta_{iv}, \sigma_{iv}, \nu) \propto IG\left(\frac{\nu}{2} + \frac{1}{2}, \frac{\nu}{2} + \frac{(IV_t - \alpha_{iv} - \beta_{iv} v_t)^2}{2\sigma_{iv}^2 v_t^2}\right).$$

For each $t \in [1, T]$:

- Jumps in variance arrive with a time-varying intensity $h_t^v = h_0^v + h_v v_t$, i.e., $p(\bar{j}_{t+1}^v = 1) = h_t^v$. The posterior distribution for the jump in variance is the Bernoulli distribution with the success probability equal to $b^v = \frac{p}{p+q}$, where

$$\begin{aligned}
p &= h_t^v \exp\left(-\frac{X_{t+1}' \Sigma^{-1} X_{t+1}}{2}\right), \\
q &= (1 - h_t^v) \exp\left(-\frac{Y_{t+1}' \Sigma^{-1} Y_{t+1}}{2}\right), \\
X_{1,t+1} = Y_{1,t+1} &= \frac{y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t - \bar{j}_{t+1}^u \bar{z}_{t+1}^u + \bar{j}_{t+1}^d \bar{z}_{t+1}^d}{\sqrt{v_t}}, \\
X_{2,t+1} &= \frac{v_{t+1} - \alpha - \beta v_t - \bar{z}_{t+1}^v}{\sqrt{v_t}}, \\
Y_{2,t+1} &= \frac{v_{t+1} - \alpha - \beta v_t}{\sqrt{v_t}}.
\end{aligned}$$

  and Σ denotes the variance-covariance matrix of $X_{t+1} = (X_{1,t+1},\ X_{2,t+1})'$ and $Y_{t+1} = (Y_{1,t+1},\ Y_{2,t+1})'$.

- The prior distribution for the size of the jump in variance $\bar{z}^v_{t+1}$ is the exponential distribution with mean θ_v. Note that:

$$\begin{aligned}
&p(\bar{z}^v_{t+1}|y_{t+1}, v_{t+1}, v_t, \bar{z}^u_{t+1}, \bar{z}^d_{t+1}, \bar{j}^u_{t+1}, \bar{j}^d_{t+1}, \bar{j}^v_{t+1} = 1, r_t, \tilde{r}_t, \Theta) \\
&\propto p(y_{t+1}, v_{t+1}|v_t, \bar{z}^u_{t+1}, \bar{z}^d_{t+1}, \bar{z}^v_{t+1}, \bar{j}^u_{t+1}, \bar{j}^d_{t+1}, \bar{j}^v_{t+1} = 1, r_t, \tilde{r}_t, \Theta)p(\bar{z}^v_{t+1}) \\
&\propto \exp\left(-\frac{X'_{t+1}\Sigma^{-1}X_{t+1}}{2}\right)\frac{1}{\theta_v}\exp\left(-\frac{\bar{z}^v_{t+1}}{\theta_v}\right)I_{(\bar{z}^v_{t+1}>0)} \\
&\propto \exp\left(\frac{\psi}{\eta}X_{1,t+1}X_{2,t+1} - \frac{1}{2\eta}X^2_{2,t+1}\right)\exp\left(-\frac{\bar{z}^v_{t+1}}{\theta_v}\right)I_{(\bar{z}^v_{t+1}>0)} \\
&\propto \exp\left(-\frac{(\bar{z}^v_{t+1} - m_{t+1})^2}{2M_{t+1}}\right)I_{(\bar{z}^v_{t+1}>0)},
\end{aligned}$$

where

$$\begin{aligned}
X_{1,t+1} &= \frac{y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t - \bar{j}^u_{t+1}\bar{z}^u_{t+1} + \bar{j}^d_{t+1}\bar{z}^d_{t+1}}{\sqrt{v_t}}, \\
X_{2,t+1} &= \frac{v_{t+1} - \alpha - \beta v_t - \bar{z}^v_{t+1}}{\sqrt{v_t}}.
\end{aligned}$$

Thus, the posterior distribution for $\bar{z}^v_{t+1}$ is the truncated normal distribution with the parameters m_{t+1} (mean) and M_{t+1} (variance):

$$\begin{aligned}
&M_{t+1} = \eta v_t, \\
&m_{t+1} = -\psi(y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t - \bar{j}^u_{t+1}\bar{z}^u_{t+1} + \bar{j}^d_{t+1}\bar{z}^d_{t+1}) \\
&+v_{t+1} - \alpha - \beta v_t - \frac{M_{t+1}}{\mu_z}.
\end{aligned}$$

Correspondingly, $p(\bar{z}^v_{t+1}|\bar{j}^v_{t+1} = 0, \theta_v) \sim \mathcal{E}xp(\theta_v)$.

- Upward jumps in excess returns arrive with a time-varying intensity $h^u_t = h_0 + h_r r_t$, i.e., $p(\bar{j}^u_{t+1} = 1) = h^u_t$. The posterior distribution for the upward jump in excess returns is the Bernoulli distribution with the success probability $b^u = \frac{p}{p+q}$, where

$$
\begin{aligned}
p &= h_t^u \exp\left(-\frac{X_{t+1}'\Sigma^{-1}X_{t+1}}{2}\right),\\
q &= (1-h_t^u)\exp\left(-\frac{Y_{t+1}'\Sigma^{-1}Y_{t+1}}{2}\right),\\
X_{1,t+1} &= \frac{y_{t+1}-\mu_0-\mu_r(r_t-\tilde{r}_t)-\mu_v v_t-\bar{z}_{t+1}^u+\bar{j}_{t+1}^d\bar{z}_{t+1}^d}{\sqrt{v_t}},\\
Y_{1,t+1} &= \frac{y_{t+1}-\mu_0-\mu_r(r_t-\tilde{r}_t)-\mu_v v_t+\bar{j}_{t+1}^d\bar{z}_{t+1}^d}{\sqrt{v_t}},\\
X_{2,t+1} &= Y_{2,t+1} = \frac{v_{t+1}-\alpha-\beta v_t-\bar{j}_{t+1}^v\bar{z}_{t+1}^v}{\sqrt{v_t}},
\end{aligned}
$$

and Σ denotes the variance-covariance matrix of $X_{t+1}=(X_{1,t+1},\ X_{2,t+1})'$ and $Y_{t+1}=(Y_{1,t+1},\ Y_{2,t+1})'$.

- The prior distribution for the size of the upward jump in excess returns $\bar{z}_{t+1}^u$ is the exponential distribution with the mean θ. Note that:

$$
\begin{aligned}
&p(\bar{z}_{t+1}^u|y_{t+1},v_{t+1},v_t,\bar{z}_{t+1}^d,\bar{z}_{t+1}^v,\bar{j}_{t+1}^u=1,\bar{j}_{t+1}^d,\bar{j}_{t+1}^v,r_t,\tilde{r}_t,\Theta)\\
&\propto p(y_{t+1},v_{t+1}|\bar{z}_{t+1}^u,\bar{z}_{t+1}^d,\bar{z}_{t+1}^v,\bar{j}_{t+1}^u=1,\bar{j}_{t+1}^d,\bar{j}_{t+1}^v,v_t,r_t,\tilde{r}_t,\Theta)p(\bar{z}_{t+1}^u)\\
&\propto \exp\left(-\frac{X_{t+1}'\Sigma^{-1}X_{t+1}}{2}\right)\frac{1}{\theta}\exp\left(-\frac{\bar{z}_{t+1}^u}{\theta}\right)I_{(\bar{z}_{t+1}^u>0)}\\
&\propto \exp\left(-\frac{1}{2}\left(1+\frac{\psi^2}{\eta}\right)X_{1,t+1}^2+\frac{\psi}{\eta}X_{1,t+1}X_{2,t+1}\right)\exp\left(-\frac{\bar{z}_{t+1}^u}{\theta}\right)I_{(\bar{z}_{t+1}^u>0)}\\
&\propto \exp\left(-\frac{(\bar{z}_{t+1}^u-m_{t+1})^2}{2M_{t+1}}\right)I_{(\bar{z}_{t+1}^u>0)},
\end{aligned}
$$

where

$$
\begin{aligned}
X_{1,t+1} &= \frac{y_{t+1}-\mu_0-\mu_r(r_t-\tilde{r}_t)-\mu_v v_t-\bar{z}_{t+1}^u+\bar{j}_{t+1}^d\bar{z}_{t+1}^d}{\sqrt{v_t}},\\
X_{2,t+1} &= \frac{v_{t+1}-\alpha-\beta v_t-\bar{j}_{t+1}^v\bar{z}_{t+1}^v}{\sqrt{v_t}}.
\end{aligned}
$$

Thus, the posterior distribution for $\bar{z}^u_{t+1}$ is the truncated normal distribution with the parameters m_{t+1} (mean) and M_{t+1} (variance):

$$
\begin{aligned}
M_{t+1} &= \frac{v_t}{\left(1+\frac{\psi^2}{\eta}\right)}, \\
m_{t+1} &= (y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t + \bar{j}^d_{t+1}\bar{z}^d_{t+1}) \\
&-\frac{\psi}{(\eta+\psi^2)}(v_{t+1} - \alpha - \beta v_t - \bar{j}^v_{t+1}\bar{z}^v_{t+1}) - \frac{M_{t+1}}{\theta}.
\end{aligned}
$$

Correspondingly, $p(\bar{z}^u_{t+1}|\bar{j}^u_{t+1} = 0, \theta) \sim \mathcal{E}xp(\theta)$.

- Downward jumps in excess returns arrive with a time-varying intensity $h^d_t = h_0 + h_r\tilde{r}_t$, i.e., $p(\bar{j}^d_{t+1} = 1) = h^d_t$. The posterior distribution for the downward jump in excess returns is the Bernoulli distribution with the success probability $b^d = \frac{p}{p+q}$, where

$$
\begin{aligned}
p &= h^d_t \exp\left(-\frac{X'_{t+1}\Sigma^{-1}X_{t+1}}{2}\right), \\
q &= (1-h^d_t)\exp\left(-\frac{Y'_{t+1}\Sigma^{-1}Y_{t+1}}{2}\right), \\
X_{1,t+1} &= \frac{y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t + \bar{z}^d_{t+1} - \bar{j}^u_{t+1}\bar{z}^u_{t+1}}{\sqrt{v_t}}, \\
Y_{1,t+1} &= \frac{y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t - \bar{j}^u_{t+1}\bar{z}^u_{t+1}}{\sqrt{v_t}}, \\
X_{2,t+1} &= Y_{2,t+1} = \frac{v_{t+1} - \alpha - \beta v_t - \bar{j}^v_{t+1}\bar{z}^v_{t+1}}{\sqrt{v_t}},
\end{aligned}
$$

  and Σ denotes the variance-covariance matrix of $X_{t+1} = (X_{1,t+1},\ X_{2,t+1})'$ and $Y_{t+1} = (Y_{1,t+1},\ Y_{2,t+1})'$.

- The prior distribution for the the size of the downward jump in excess returns is the exponential distribution with mean θ.

  Note that

$$
\begin{aligned}
&p(\bar{z}^d_{t+1}|y_{t+1}, v_{t+1}, v_t, \bar{z}^u_{t+1}, \bar{z}^v_{t+1}, \bar{j}^u_{t+1}, \bar{j}^d_{t+1} = 1, \bar{j}^v_{t+1}, r_t, \tilde{r}_t, \Theta)\\
&\propto p(y_{t+1}, v_{t+1}|\bar{z}^u_{t+1}, \bar{z}^d_{t+1}, \bar{z}^v_{t+1}, \bar{j}^u_{t+1}, \bar{j}^d_{t+1} = 1, \bar{j}^v_{t+1}, v_t, r_t, \tilde{r}_t, \Theta) p(\bar{z}^d_{t+1})\\
&\propto \exp\left(-\frac{X'_{t+1}\Sigma^{-1}X_{t+1}}{2}\right)\frac{1}{\theta}\exp\left(-\frac{\bar{z}^d_{t+1}}{\theta}\right) I_{(\bar{z}^d_{t+1}>0)}\\
&\propto \exp\left(-\frac{1}{2}\left(1+\frac{\psi^2}{\eta}\right)X^2_{1,t+1} + \frac{\psi}{\eta}X_{1,t+1}X_{2,t+1}\right)\exp\left(-\frac{\bar{z}^d_{t+1}}{\theta}\right) I_{(\bar{z}^d_{t+1}>0)}\\
&\propto \exp\left(-\frac{(\bar{z}^d_{t+1} - m_{t+1})^2}{2M_{t+1}}\right) I_{(\bar{z}^d_{t+1}>0)},
\end{aligned}
$$

where

$$
\begin{aligned}
X_{1,t+1} &= \frac{y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t + \bar{z}^d_{t+1} - \bar{j}^u_{t+1}\bar{z}^u_{t+1}}{\sqrt{v_t}},\\
X_{2,t+1} &= \frac{v_{t+1} - \alpha - \beta v_t - \bar{j}^v_{t+1}\bar{z}^v_{t+1}}{\sqrt{v_t}}.
\end{aligned}
$$

Thus, the posterior distribution for $\bar{z}^d_{t+1}$ is the truncated normal distribution with the parameters m_{t+1} (mean) and M_{t+1} (variance):

$$
\begin{aligned}
&M_{t+1} = \frac{v_t}{\left(1+\frac{\psi^2}{\eta}\right)},\\
&m_{t+1} = -(y_{t+1} - \mu_0 - \mu_r(r_t - \tilde{r}_t) - \mu_v v_t - \bar{j}^u_{t+1}\bar{z}^u_{t+1})\\
&+\frac{\psi}{(\eta+\psi^2)}(v_{t+1} - \alpha - \beta v_t - \bar{j}^v_{t+1}\bar{z}^v_{t+1}) - \frac{M_{t+1}}{\theta}.
\end{aligned}
$$

Correspondingly, $p(\bar{z}^d_{t+1}|\bar{j}^d_{t+1} = 0, \theta) \sim \mathcal{E}xp(\theta)$.

- To guarantee that the estimated variance is strictly positive, we draw it in logs. The posterior distribution for the variance differs depending on whether IV data are available ($t > T_2$) or not ($t \leq T_2$).

If IV is not available, the posterior distribution for the spot variance is

$$
\begin{aligned}
& p(\log v_t | v_{t-1}, v_{t+1}, \bar{j}^u_t, \bar{j}^u_{t+1}, \bar{j}^d_t, \bar{j}^d_{t+1}, \bar{j}^v_t, \bar{j}^v_{t+1}, \bar{z}^u_t, \bar{z}^u_{t+1}, \bar{z}^d_t, \bar{z}^d_{t+1}, \bar{z}^v_t, \bar{z}^v_{t+1}, r_{t-1}, r_t, \tilde{r}_{t-1}, \tilde{r}_t, \Theta) \\
\propto \quad & \exp\left(-\frac{1}{2}\left(\frac{\psi^2}{\eta}+1\right)\left(\frac{(y_{t+1}-\mu_0-\mu_r(r_t-\tilde{r}_t)-\bar{j}^u_{t+1}\bar{z}^u_{t+1}+\bar{j}^d_{t+1}\bar{z}^d_{t+1})^2}{v_t}+\mu_v^2 v_t\right)\right) \\
\times \quad & \exp\left(\frac{\psi}{\eta}\left(\mu_v\beta v_t+\frac{(y_{t+1}-\mu_0-\mu_r(r_t-\tilde{r}_t)-\bar{j}^u_{t+1}\bar{z}^u_{t+1}+\bar{j}^d_{t+1}\bar{z}^d_{t+1})(v_{t+1}-\alpha-\bar{j}^v_{t+1}\bar{z}^v_{t+1})}{v_t}\right)\right) \\
\times \quad & \exp\left(\frac{\psi}{\eta}\frac{(y_t-\mu_0-\mu_r(r_{t-1}-\tilde{r}_{t-1})-\mu_v v_{t-1}-\bar{j}^u_t\bar{z}^u_t+\bar{j}^d_t\bar{z}^d_t)v_t}{v_{t-1}}\right) \\
\times \quad & \exp\left(-\frac{1}{2\eta}\left(\frac{(v_{t+1}-\alpha-\bar{j}^v_{t+1}\bar{z}^v_{t+1})^2}{v_t}+\beta^2 v_t+\frac{v_t^2-2v_t(\alpha+\beta v_{t-1}+\bar{j}^v_t\bar{z}^v_t)}{v_{t-1}}\right)\right).
\end{aligned}
$$

If IV is available, the posterior distribution for the spot variance is

$$
\begin{aligned}
\propto \quad & \exp\left(-\frac{1}{2}\left(\frac{\psi^2}{\eta}+1\right)\left(\frac{(y_{t+1}-\mu_0-\mu_r(r_t-\tilde{r}_t)-\bar{j}^u_{t+1}\bar{z}^u_{t+1}+\bar{j}^d_{t+1}\bar{z}^d_{t+1})^2}{v_t}+\mu_v^2 v_t\right)\right) \\
\times \quad & \exp\left(\frac{\psi}{\eta}\left(\mu_v\beta v_t+\frac{(y_{t+1}-\mu_0-\mu_r(r_t-\tilde{r}_t)-\bar{j}^u_{t+1}\bar{z}^u_{t+1}+\bar{j}^d_{t+1}\bar{z}^d_{t+1})(v_{t+1}-\alpha-\bar{j}^v_{t+1}\bar{z}^v_{t+1})}{v_t}\right)\right) \\
\times \quad & \exp\left(\frac{\psi}{\eta}\frac{(y_t-\mu_0-\mu_r(r_{t-1}-\tilde{r}_{t-1})-\mu_v v_{t-1}-\bar{j}^u_t\bar{z}^u_t+\bar{j}^d_t\bar{z}^d_t)v_t}{v_{t-1}}\right) \\
\times \quad & \exp\left(-\frac{1}{2\eta}\left(\frac{(v_{t+1}-\alpha-\bar{j}^v_{t+1}\bar{z}^v_{t+1})^2}{v_t}+\beta^2 v_t+\frac{v_t^2-2v_t(\alpha+\beta v_{t-1}+\bar{j}^v_t\bar{z}^v_t)}{v_{t-1}}\right)\right) \\
\times \quad & \frac{1}{v_t}\exp\left(-\frac{(IV_t-\alpha_{iv}-\beta_{iv}v_t)^2}{2\sigma_{iv}^2\lambda_t v_t^2}\right).
\end{aligned}
$$

Thus, if implied variance is observed, the posterior distribution for the spot variance has one additional component (the last multiplier).

A.1.3 Model diagnostics

The Bayesian MCMC approach provides output that is useful for the model diagnostics purposes. In particular, we estimate a system

$$y_{t+1} = \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + \bar{z}_{t+1}^u \bar{j}_{t+1}^u - \bar{z}_{t+1}^d \bar{j}_{t+1}^d, \qquad \text{(A.1.23)}$$

$$v_{t+1} = (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + \bar{z}_{t+1}^v \bar{j}_{t+1}^v,$$

$$IV_t = \alpha_{iv} + \beta_{iv} v_t + \sigma_{iv} v_t \sqrt{\lambda_t} \varepsilon_t. \qquad \text{(A.1.24)}$$

and construct distributions for the residuals $\{w_t^{s,g}\}$ and $\{\varepsilon_t^g\}$ (the superscript g stands for a simulation path). Our model implies that the residuals from equations (A.1.23) and (A.1.24), w_t^s and ε_t, are iid standard normal, i.e., skewness=0, kurtosis=3, and no serial correlation.

For each g, we construct fitted residuals,

$$\hat{w}_{t+1}^{s,g} = \frac{y_{t+1} - \hat{\mu}_0^g - \hat{\mu}_r^g(r_t - \tilde{r}_t) - \hat{\mu}_v^g \hat{v}_t^g - \hat{\bar{z}}_{t+1}^{u,g} \hat{\bar{j}}_{t+1}^{u,g} + \hat{\bar{z}}_{t+1}^{d,g} \hat{\bar{j}}_{t+1}^{d,g}}{\sqrt{\hat{v}_t^i}},$$

$$\hat{\varepsilon}_{t+1}^g = \frac{IV_t - \hat{\alpha}_{iv}^g - \hat{\beta}_{iv}^g \hat{v}_t^g}{\hat{\sigma}_{iv}^g \hat{v}_t^g \sqrt{\hat{\lambda}_t^g}},$$

and we compute their third and fourth moments, and autocorrelations: skew($\hat{w}^{s,g}$), skew($\hat{\varepsilon}^g$), kurt($\hat{w}^{s,g}$), kurt($\hat{\varepsilon}^g$), autocorr($\hat{w}^{s,g}$), and autocorr($\hat{\varepsilon}^g$). Therefore, as a natural by-product of our estimation, we have distributions of skewness, kurtosis, and autocorrelation for $\{w^s\}$ and $\{\varepsilon\}$:

$$M = \{skew(w^{s,g}), kurt(w^{s,g}), autocorr(w^{s,g}), skew(\varepsilon^g), kurt(\varepsilon^g), autocorr(\varepsilon^g)\}_{g=1}^G,$$

where G is the number of executed simulations. Hence, we can easily construct confidence intervals for these six components of M and check whether they contain skewness of zero, kurtosis of 3, and serial correlation of zero. We report the corresponding model diagnostics tables in the main text of the paper.

Additionally, we keep track of the variance of the innovations from equation (A.1.19),

$\sigma_{iv}^2 v_t^2 \lambda_t$. A better fitting model should have a lower variance. Similar to other diagnostics, we store the whole distribution of $\{\sigma_{iv}^{2,g} v_t^{2,g} \lambda_t^g\}_{g=1}^G$ and report its mean and 95% confidence bound in the main text of the paper.

A.1.4 Bayes odds ratios

In Bayesian statistics, a common formal approach to model selection is a comparison of the posterior model probabilities. If the prior model probabilities are uniformly distributed, the posterior model probabilities collapse to the Bayes factor (for a detailed discussion, see Gamerman and Lopes, 2006). The Bayes factor simplifies in the case of nested models with similar priors for common parameters. It equals to the ratio of the posterior and the prior under the encompassing model. This ratio is known as the Savage-Dickey density ratio (Verdinelli and Wasserman, 1995).

SV versus SVJ

In this section, we are evaluating two models: stochastic volatility model (SV)

$$y_{t+1} = \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s, \tag{A.1.25}$$

$$v_{t+1} = (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v, \tag{A.1.26}$$

$$IV_t = \alpha_{iv} + \beta_{iv} v_t + \sigma_{iv} v_t \sqrt{\lambda_t} \varepsilon_t \tag{A.1.27}$$

and stochastic volatility model with jumps in variance (SVJ)

$$y_{t+1} = \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s, \tag{A.1.28}$$

$$v_{t+1} = (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + \bar{z}_{t+1}^v \bar{j}_{t+1}^v, \tag{A.1.29}$$

$$IV_t = \alpha_{iv} + \beta_{iv} v_t + \sigma_{iv} v_t \sqrt{\lambda_t} \varepsilon_t. \tag{A.1.30}$$

Let Ω denote the collection of the latent variables and parameters of the models, i.e., $\Omega = \{\Theta, \bar{J}^v, \bar{Z}^v, \Lambda\}$ ($\Lambda = \{\lambda_t\}_{t=T_2+1}^T$). We treat variance as observable in this case (this subsection only). First, variance in the model with jumps in variance has an unknown unconditional distribution. Second, in our model the intensity of the jumps in variance is governed by the variance itself. These two observations mean that evaluation of the Bayes factor would involve the use of an intractable distribution if variance is latent. We view this simplification as reasonable because in order to estimate variance we use information embedded in ATM options, i.e., implied variance tells us very accurately what the spot variance is.

We compare two nested models; if $\bar{j}_t^v = 0$ for any $t \in [1, T]$ then the SVJ model is equivalent to the SV model. Therefore, we have the following identity for predictive densities:

$$p(Y, IV|\Omega, R, \tilde{R}, V, \mathrm{SV}) = p(Y, IV|\Omega, R, \tilde{R}, \bar{J}^v = 0, V, \mathrm{SVJ}) \tag{A.1.31}$$

We make an additional assumption that models share the same prior distributions for the common parameters, i.e, $p(\Omega|\mathrm{SV}) = p(\Omega|\bar{J}^v = 0, \mathrm{SVJ})$. Thereby, we work with the Bayes factor in the form of the Savage-Dickey density ratio. We follow Eraker, Johannes, and Polson (2003) to show this.

Start with the predictive density for the SV model and use two facts: (1) the SV model is nested in the SVJ model, and (2) models have identical priors for the common parameters:

$$\begin{aligned}
&p(Y, IV|R, \tilde{R}, V, \mathrm{SV}) = \int p(Y, IV|\Omega, R, \tilde{R}, V, \mathrm{SV}) p(\Omega|SV) d\Omega \\
&= \int p(Y, IV|\Omega, R, \tilde{R}, V, \bar{J}^v = 0, \mathrm{SVJ}) p(\Omega|\mathrm{SV}) d\Omega \\
&= \int p(Y, IV|\Omega, R, \tilde{R}, V, \bar{J}^v = 0, \mathrm{SVJ}) p(\Omega|\bar{J}^v = 0, \mathrm{SVJ}) d\Omega = p(Y, IV|\bar{J}^v = 0, R, \tilde{R}, V, \mathrm{SVJ}).
\end{aligned}$$

The posterior odds ratio of the model SV to the model SVJ is

$$\begin{aligned}
Odds(\mathrm{SV}, \mathrm{SVJ}) &= \frac{Pr(\mathrm{SV}|Y, IV, V, R, \tilde{R})}{Pr(\mathrm{SVJ}|Y, IV, V, R, \tilde{R})} = \frac{p(Y, IV|R, \tilde{R}, V, \mathrm{SV})}{p(Y, IV|R, \tilde{R}, V, \mathrm{SVJ})} \\
&= \frac{p(Y, IV|\bar{J}^v = 0, R, \tilde{R}, V, \mathrm{SVJ})}{p(Y, IV|R, \tilde{R}, V, \mathrm{SVJ})} = \frac{Pr(\bar{J}^v = 0|Y, IV, R, \tilde{R}, V, \mathrm{SVJ})}{Pr(\bar{J}^v = 0|R, \tilde{R}, V, \mathrm{SVJ})}.
\end{aligned}$$

Consider the denominator. Let $x = \{h_0^v, h_v\}$ and X to be the domain of x.

$$Pr(\bar{J}^v = 0|R, \tilde{R}, V, \text{SVJ}) = \int\limits_{x\in X} Pr(\bar{J}^v = 0|h_0^v, h_v, V, \text{SVJ})p(h_0^v, h_v|\text{SVJ})dx$$

$$= \int\limits_{x\in X} \prod_{t=1}^{T} (1 - h_0^v - h_v v_{t-1})p(h_0^v, h_v|\text{SVJ})dx = \int\limits_{x\in X} \prod_{t=1}^{T} (1 - h_0^v - h_v v_{t-1})p(h_0^v)p(h_v)dx$$

$$= \frac{1}{K}\sum_{k=1}^{K}\left(\prod_{t=1}^{T}(1 - h_0^{v,k} - h_v^k v_{t-1})\right). \tag{A.1.32}$$

Thereby, we evaluate a prior ordinate numerically by fixing a large number K, drawing independently $\{h_0^{v,k}\}_{k=1}^K$ and $\{h_v^k\}_{k=1}^K$ from the uniform distributions with domains $[\underline{h}_0^v, \overline{h}_0^v]$ and $[\underline{h}_v, \overline{h}_v]$, respectively, and approximating the integral by a sum.

Consider the numerator

$$\begin{aligned} &Pr(\bar{J}^v = 0|Y, IV, R, \tilde{R}, V, \text{SVJ}) \\ &= \int_{x\in X} Pr(\bar{J}^v = 0|h_0^v, h_v, V, Y, IV, \text{SVJ})p(h_0^v, h_v|Y, IV, V, \text{SVJ})dx. \end{aligned} \tag{A.1.33}$$

Work with the second component in $(A.1.33)$:

$$p(h_0^v, h_v|Y, IV, V, \text{SVJ}) = \int_{\bar{J}^v} p(h_0^v, h_v|\bar{J}^v, V, \text{SVJ})p(\bar{J}^v|Y, IV)d\bar{J}^v$$

$$= \int\limits_{\bar{j}^v}\left(\prod_{t=1}^{T}(h_0^v + h_v v_{t-1})^{\bar{j}_t^v}(1 - h_0^v - h_v v_{t-1})^{1-\bar{j}_t^v}/C^m\right)p(\bar{J}^v|Y, IV)d\bar{J}^v. \tag{A.1.34}$$

C^m is a normalization constant which guarantees that the first multiplier under the integral in $(A.1.32)$ is a density function:

$$C^m = \int\limits_{x\in X}\prod_{t=1}^{T}(h_0^v + h_v v_{t-1})^{\bar{j}_t^v}(1 - h_0^v - h_v v_{t-1})^{1-\bar{j}_t^v}dx$$

$$\approx \frac{1}{K}\sum_{k=1}^{K}\prod_{t=1}^{T}(h_0^{v,k} + h_v^k v_{t-1})^{\bar{j}_t^v}(1 - h_0^{v,k} - h_v^k v_{t-1})^{1-\bar{j}_t^v}.$$

Component $(A.1.34)$ becomes

$$p(h_0^v, h_v|Y, IV, V, \text{SVJ}) = \frac{1}{M}\sum_{m=1}^{M}\prod_{t=1}^{T}(h_0^v + h_v v_{t-1})^{\bar{j}_t^{v,m}}(1 - h_0^v - h_v v_{t-1})^{1-\bar{j}_t^{v,m}}/C^m$$

Finally, we compute the posterior ordinate $(A.1.33)$:

$$\begin{aligned} & Pr(\bar{J}^v = 0|Y, IV, R, \tilde{R}, V, \text{SVJ}) \\ \approx\ & \frac{1}{KM}\sum_{k=1}^{K}\left(\prod_{t=1}^{T}(1 - h_0^{v,k} - h_v^k v_{t-1})\right)\sum_{m=1}^{M}\prod_{t=1}^{T}(h_0^{v,k} + h_v^k v_{t-1})^{\bar{j}_t^{v,m}}(1 - h_0^{v,k} - h_v^k v_{t-1})^{1-\bar{j}_t^{v,m}}/C^m. \end{aligned}$$

SVJ versus Preferred

In this section, we are working with the SVJ model (A.1.28-A.1.30) and our preferred model given by

$$y_{t+1} = \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + \bar{z}_{t+1}^u \bar{j}_{t+1}^u - \bar{z}_{t+1}^d \bar{j}_{t+1}^d, \qquad \text{(A.1.35)}$$

$$v_{t+1} = (1 - \nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + \bar{z}_{t+1}^v \bar{j}_{t+1}^v, \qquad \text{(A.1.36)}$$

$$IV_t = \alpha_{iv} + \beta_{iv} v_t + \sigma_{iv} v_t \sqrt{\lambda_t}\varepsilon_t. \qquad \text{(A.1.37)}$$

Similar arguments tell us that the Bayes factor takes the form of the Savage-Dickey density ratio if we assume identical priors for common parameters, i.e., $p(\Omega|\text{SVJ}) = p(\Omega|\bar{J}^u = 0, \bar{J}^d = 0, \text{Preferred})$. Note that Ω includes latent variables corresponding to jump times and jump sizes in currency returns. Here we do not have to assume that variance is observable because it cancels out in the final expression (see below). The posterior odds ratio of the model SVJ to the model Preferred is

$$\begin{aligned} Odds(\text{SVJ}, \text{Preferred}) &= \frac{Pr(\text{SVJ}|Y, IV, V, R, \tilde{R})}{Pr(\text{Preferred}|Y, IV, V, R, \tilde{R})} \\ &= \frac{Pr(\bar{J}^u = 0, \bar{J}^d = 0|Y, IV, R, \tilde{R}, V, \text{Preferred})}{Pr(\bar{J}^u = 0, \bar{J}^d = 0|R, \tilde{R}, V, \text{Preferred})}. \qquad \text{(A.1.38)} \end{aligned}$$

We start with the denominator:

$$
\begin{aligned}
& Pr(\bar{J}^u = 0, \bar{J}^d = 0 | R, \tilde{R}, V, \text{Preferred}) \\
&= \int\limits_{x \in X} Pr(\bar{J}^u = 0, \bar{J}^d = 0 | h_0, h_r, R, \tilde{R}, \text{Preferred}) p(h_0, h_r) dx \\
&\approx \frac{1}{K} \sum_{k=1}^{K} \prod_{t=1}^{T} (1 - h_0^k - h_r^k r_{t-1})(1 - h_0^k - h_r^k \tilde{r}_{t-1}).
\end{aligned} \tag{A.1.39}
$$

We denote $x = (h_0, h_r)$ and X is the domain of x.

For the numerator, we have

$$
\begin{aligned}
& Pr(\bar{J}^u = 0, \bar{J}^d = 0 | R, \tilde{R}, Y, IV, V, \text{Preferred}) \\
& \int\limits_{x \in X} Pr(\bar{J}^u = 0, \bar{J}^d = 0 | h_0, h_r, R, \tilde{R}) p(h_0, h_r | Y, IV, R, \tilde{R}, V, \text{Preferred}) dx.
\end{aligned} \tag{A.1.40}
$$

Work with the second component of (A.1.40):

$$
\begin{aligned}
& p(h_0, h_r | Y, IV, R, \tilde{R}, V, \text{Preferred}) \\
&= \int\limits_{\bar{J}^u,\ \bar{J}^d} p(h_0, h_r | \bar{J}^u, \bar{J}^d, R, \tilde{R}, \text{Preferred}) p(\bar{J}^u, \bar{J}^d | Y, IV, \text{Preferred}) d\bar{J}^u d\bar{J}^d \\
&= \int\limits_{\bar{J}^u,\ \bar{J}^d} p(h_0, h_r | \bar{J}^u, \bar{J}^d, R, \tilde{R}, \text{Preferred}) p(\bar{J}^u | Y) p(\bar{J}^d | Y) d\bar{J}^u d\bar{J}^d.
\end{aligned}
$$

We approximate $p(\bar{J}^u|Y)$ and $p(\bar{J}^d|Y)$ by using the MCMC draws for the jump times. Therefore, to complete our derivation all we need is to evaluate the conditional joint density function of the parameters of the jumps' intensities:

$$
\begin{aligned}
& p(h_0, h_r | \bar{J}^u, \bar{J}^d, R, \tilde{R}, \text{Preferred}) \\
&= \prod_{t=1}^{T} (h_0 + h_r r_{t-1})^{\bar{j}_t^u} (h_0 + h_r \tilde{r}_{t-1})^{\bar{j}_t^d} (1 - h_0 - h_r r_{t-1})^{1 - \bar{j}_t^u} (1 - h_0 - h_r \tilde{r}_{t-1})^{1 - \bar{j}_t^d} / C^m,
\end{aligned}
$$

where

$$
\begin{aligned}
C^m &= \int_{x\in X} \prod_{t=1}^{T} (h_0 + h_r r_{t-1})^{\bar{j}_t^u} (h_0 + h_r \tilde{r}_{t-1})^{\bar{j}_t^d} (1 - h_0 - h_r r_{t-1})^{1-\bar{j}_t^u} (1 - h_0 - h_r \tilde{r}_{t-1})^{1-\bar{j}_t^d} dx \\
&= \frac{1}{K} \sum_{k=1}^{K} \prod_{t=1}^{T} (h_0^k + h_r^k r_{t-1})^{\bar{j}_t^u} (h_0^k + h_r^k \tilde{r}_{t-1})^{\bar{j}_t^d} (1 - h_0^k - h_r^k r_{t-1})^{1-\bar{j}_t^u} (1 - h_0^k - h_r^k \tilde{r}_{t-1})^{1-\bar{j}_t^d}.
\end{aligned}
$$

Thereby,

$$
\begin{aligned}
&p(h_0, h_r | Y, IV, R, \tilde{R}, V, \text{Preferred}) \\
&\approx \frac{1}{M} \sum_{m=1}^{M} \prod_{t=1}^{T} (h_0 + h_r r_{t-1})^{\bar{j}_t^{u,m}} (h_0 + h_r \tilde{r}_{t-1})^{\bar{j}_t^{d,m}} (1 - h_0 - h_r r_{t-1})^{1-\bar{j}_t^{u,m}} (1 - h_0 - h_r \tilde{r}_{t-1})^{1-\bar{j}_t^{d,m}} / C^m.
\end{aligned}
$$

The numerator in (A.1.38) is as follows

$$
\begin{aligned}
&Pr(\bar{J}^u = 0, \bar{J}^d = 0 | R, \tilde{R}, Y, IV, V, \text{Preferred}) \\
&\approx \frac{1}{KM} \sum_{k=1}^{K} \prod_{t=1}^{T} (1 - h_0^k - h_r^k r_{t-1})(1 - h_0^k - h_r^k \tilde{r}_{t-1}) \\
&= \sum_{m=1}^{M} \prod_{t=1}^{T} (h_0^k + h_r^k r_{t-1})^{\bar{j}_t^{u,m}} (h_0^k + h_r^k \tilde{r}_{t-1})^{\bar{j}_t^{d,m}} (1 - h_0^k - h_r^k r_{t-1})^{1-\bar{j}_t^{u,m}} (1 - h_0^k - h_r^k \tilde{r}_{t-1})^{1-\bar{j}_t^{d,m}} / C^m.
\end{aligned}
$$

This completes our derivation.

A.1.5 Expected future variance

We do not consider the most general model to streamline the presentation. We focus on the empirically relevant case where intensity of jumps in variance depends on variance only, and jumps up (down) in FX depend on domestic (foreign) interest rate only. We start by computing expectation of the variance process in (2.3.2). Conditional expectation $E_t(v_{t+i}) \equiv v_{t,i}$ can be computed via a recursion. Note that $v_{t,0} = v_t$. Suppose we know $v_{t,i-1}$. Then

$$\begin{aligned} v_{t,i} &= (1-\nu)v + \nu v_{t,i-1} + \sigma_v E_t(E_{t+i-1}(v_{t+i-1}^{1/2} w_{t+i}^v)) + E_t(E_{t+i-1} z_{t+i}^v) \\ &= (1-\nu)v + \nu v_{t,i-1} + \theta_v h_0^v + \theta_v h_v^v v_{t,i-1} = (1-\nu)v + \theta_v h_0^v + (\nu + \theta_v h_v^v) v_{t,i-1}. \end{aligned}$$

We can solve this recursion analytically:

$$\begin{aligned} v_{t,i} &= [(1-\nu)v + \theta_v h_0^v](1 + (\nu + \theta_v h_v^v)) + (\nu + \theta_v h_v^v)^2 v_{t,i-2} \\ &= [(1-\nu)v + \theta_v h_0^v](1 - (\nu + \theta_v h_v^v)^i)/(1 - (\nu + \theta_v h_v^v)) + (\nu + \theta_v h_v^v)^i v_t. \end{aligned}$$

Next, we can compute expectation of average future v :

$$\begin{aligned} & E_t\left(\sum_{i=1}^n v_{t+i}\right)/n = 1/n \sum_{i=1}^n E_t v_{t+i} = 1/n \sum_{i=1}^n v_{t,i} \\ =\ & 1/n \sum_{i=1}^n [(1-\nu)v + \theta_v h_0^v](1 - (\nu + \theta_v h_v^v)^i)/(1 - (\nu + \theta_v h_v^v)) + 1/n \sum_{i=1}^n (\nu + \theta_v h_v^v)^i v_t \\ =\ & \frac{(1-\nu)v + \theta_v h_0^v}{1 - (\nu + \theta_v h_v^v)} \left[1 - \frac{\nu + \theta_v h_v^v}{n} \frac{1 - (\nu + \theta_v h_v^v)^n}{1 - (\nu + \theta_v h_v^v)}\right] + \frac{\nu + \theta_v h_v^v}{n} \frac{1 - (\nu + \theta_v h_v^v)^n}{1 - (\nu + \theta_v h_v^v)} v_t \\ \equiv\ & \underbrace{\frac{(1-\nu)v + \theta_v h_0^v}{1 - (\nu + \theta_v h_v^v)} [1 - \beta_n]}_{\alpha_n} + \beta_n v_t. \end{aligned}$$

Similarly, we can obtain conditional expectations of future interest rates:

$$r_{t,i} \equiv E_t(r_{t+i}) = a_r(1 - b_r^i) + b_r^i r_t,$$

and the expectations of average future interest rates:

$$\begin{aligned} & E_t\left(\sum_{i=1}^n r_{t+i}\right)/n = 1/n \sum_{i=1}^n E_t r_{t+i} = 1/n \sum_{i=1}^n r_{t,i} \\ =\ & a_r\left[1 - \frac{b_r}{n} \frac{1 - b_r^n}{1 - b_r}\right] + \frac{b_r}{n} \frac{1 - b_r^n}{1 - b_r} r_t \end{aligned}$$

and the similar expression holds for expectations associated with $\tilde{r}_t$.

Now, we can characterize the variance of excess returns:

$$v_t^y \equiv var_t(y_{t+1}) = v_t + 2h_t^u\theta_u^2 + 2h_t^d\theta_d^2.$$

Therefore, the conditional expectation of this variance can be computed on the basis of our results for the variance of the normal component v_t and the expectations of interest rates:

$$v_{t,i}^y \equiv E_t(v_{t+i}^y) = v_{t,i} + 2\theta_u^2 h_0^u + 2\theta_u^2 h_r^u E_t(r_{t+i}) + 2\theta_d^2 h_0^d + 2\theta_d^2 \tilde{h}_r^d E_t(\tilde{r}_{t+i}).$$

This expression implies that the unconditional expectation, or long-run mean, of the variance is:

$$\begin{aligned} v_J = \lim_{i\to\infty} v_{t,i}^y &= [(1-\nu)v + \theta_v h_0^v]/(1-(\nu+\theta_v h_v^v)) \\ &+ 2\theta_u^2 h_0^u + 2\theta_u^2 h_r^u a_r + 2\theta_d^2 h_0^d + 2\theta_d^2 \tilde{h}_r^d \tilde{a}_r. \end{aligned}$$

When there are no jumps, that is, $\theta_v = 0$, $\theta_u = 0$, and $\theta_d = 0$, then $v_J = v$.

Next, we compute $E_t(\sum_{i=1}^n v_{t+i}^y)/n$

$$\begin{aligned} & E_t\left(\sum_{i=1}^n v_{t+i}^y\right)/n = 1/n\sum_{i=1}^n E_t v_{t+i}^y = 1/n\sum_{i=1}^n v_{t,i}^y \\ = \quad & \alpha_n + 2\theta_u^2 h_0^u + 2\theta_u^2 h_r^u a_r\left[1 - \frac{b_r}{n}\frac{1-b_r^n}{1-b_r}\right] + 2\theta_d^2 h_0^d + 2\theta_d^2 \tilde{h}_r^d \tilde{a}_r\left[1 - \frac{\tilde{b}_r}{n}\frac{1-\tilde{b}_r^n}{1-\tilde{b}_r}\right] \\ + \quad & \beta_n v_t + 2\theta_u^2 h_r^u \frac{b_r}{n}\frac{1-b_r^n}{1-b_r} r_t + 2\theta_d^2 \tilde{h}_r^d \frac{\tilde{b}_r}{n}\frac{1-\tilde{b}_r^n}{1-\tilde{b}_r}\tilde{r}_t. \end{aligned}$$

A.1.6 Jumps and news

For each day a currency has experienced a jump according to our model, we search Factiva if there were significant news. Table A.1 displays a detailed account of this news.

A.1.7 Computing entropy

Entropy of currency changes over a horizon of n days is equal to:

$$L_t(S_{t+n}/S_t) = \log E_t(e^{x_{t,n}}) - E_t(x_{t,n}) = k_t(1; x_{t,n}) - \kappa_{1t}(x_{t,n}),$$

where $x_{t,n} = \log(S_{t+n}/S_t) = \sum_{i=t}^{t+n}(s_{i+1} - s_i)$, $k_t(s; x_{t,n})$ is a cumulant-generating function of $x_{t,n}$, and $\kappa_{1t}(x_{t,n})$ is the first cumulant of $x_{t,n}$. Thus, we need to compute the cumulant-generating function of $x_{t,n}$:

$$k_t(s; x_{t,n}) = \log E_t e^{s x_{t,n}}.$$

The first cumulant can be recovered as $\partial k_t(s; x_{t,n})/\partial s$ at $s = 0$. Denote the the drift of log currency changes by $\bar{\mu}_t = \mu_t + (r_t - \tilde{r}_t)$.

Guess

$$k_t(s; x_{t,n}) = A(n) + B_v(n)v_t + B_r(n)r_t + \tilde{B}_r(n)\tilde{r}_t.$$

Then

$$\begin{aligned}
& A(n) + B_v(n)v_t + B_r(n)r_t + \tilde{B}_r(n)\tilde{r}_t \\
= \;& k(s; x_{t,n}) = \log E_t[e^{s x_{t,1}} E_{t+1} e^{s x_{t+1,n-1}}] \\
= \;& \log E_t[e^{s x_{t,1}} e^{A(n-1)+B_v(n-1)v_{t+1}+B_r(n-1)r_{t+1}+\tilde{B}_r(n-1)\tilde{r}_{t+1}}] \\
= \;& A(n-1) + \log E_t e^{s x_{t,1}+B_v(n-1)v_{t+1}} + \log E_t e^{B_r(n-1)r_{t+1}+\tilde{B}_r(n-1)\tilde{r}_{t+1}} \\
= \;& A(n-1) + s\bar{\mu}_t + B_v(n-1)((1-\nu)v + \nu v_t) \\
+ \;& B_r(n-1)((1-b_r)a_r + b_r r_t) + \tilde{B}_r(n-1)((1-\tilde{b}_r)\tilde{a}_r + \tilde{b}_r\tilde{r}_t) \\
+ \;& \log E_t e^{s(1-\rho^2)^{1/2} v_t^{1/2} w_{t+1}^s + s\rho v_t^{1/2} w_{t+1}^v + s z_{t+1}^u + s z_{t+1}^d + B_v(n-1)\sigma_v v_t^{1/2} w_{t+1}^v + B_v(n-1) z_{t+1}^v} \\
+ \;& \log E_t e^{B_r(n-1) r_t^{1/2} \sigma_r w_{t+1}^r + \tilde{B}_r(n-1)\tilde{r}_t^{1/2}\tilde{\sigma}_r \tilde{w}_{t+1}^r} \\
= \;& A(n-1) + s\bar{\mu}_t + B_v(n-1)((1-\nu)v + \nu v_t) \\
+ \;& B_r(n-1)((1-b_r)a_r + b_r r_t) + \tilde{B}_r(n-1)((1-\tilde{b}_r)\tilde{a}_r + \tilde{b}_r\tilde{r}_t)
\end{aligned}$$

$$
\begin{aligned}
&+ \quad s^2 v_t/2 + v_t s\rho\sigma_v B_v(n-1) + B_v^2(n-1)\sigma_v^2 v_t/2 + h_t^u((1-s\theta_u)^{-1}-1) + h_t^d((1+s\theta_d)^{-1}-1) \\
&+ \quad h_t^v((1-B_v(n-1)\theta_v)^{-1}-1) + B_r^2(n-1)\sigma_r^2 r_t/2 + \tilde{B}_r^2(n-1)\tilde{\sigma}_r^2\tilde{r}_t/2 \\
&= \quad A(n-1) + s(\mu + (\mu_r+1)(r_t - \tilde{r}_t) + \mu_v v_t) + B_v(n-1)((1-\nu)v + \nu v_t) \\
&+ \quad B_r(n-1)((1-b_r)a_r + b_r r_t) + \tilde{B}_r(n-1)((1-\tilde{b}_r)\tilde{a}_r + \tilde{b}_r\tilde{r}_t) \\
&+ \quad s^2 v_t/2 + v_t s\rho\sigma_v B_v(n-1) + B_v^2(n-1)\sigma_v^2 v_t/2 + s\theta_u(h_0^u + h_r^u r_t)/(1-s\theta_u) \\
&- \quad s\theta_d(h_0^d + \tilde{h}_r^d\tilde{r}_t)/(1+s\theta_d) + (h_0^v + h_v^v v_t)B_v(n-1)\theta_v/(1-B_v(n-1)\theta_v) \\
&+ \quad B_r^2(n-1)\sigma_r^2 r_t/2 + \tilde{B}_r^2(n-1)\tilde{\sigma}_r^2\tilde{r}_t/2.
\end{aligned}
$$

Collect terms, match them with the corresponding terms in the first line, solve for the coefficients:

$$
\begin{aligned}
A(n) &= A(n-1) + s\mu + B_v(n-1)(1-\nu)v + s\theta_u h_0^u/(1-s\theta_u) - s\theta_d h_0^d/(1+s\theta_d) \\
&+ h_0^v B_v(n-1)\theta_v/(1-\theta_v B_v(n-1)) + B_r(n-1)(1-b_r)a_r + \tilde{B}_r(n-1)(1-\tilde{b}_r)\tilde{a}_r \\
B_v(n) &= s\mu_v + B_v(n-1)\nu + s^2/2 + s\rho\sigma_v B_v(n-1) + B_v^2(n-1)\sigma_v^2/2 \\
&+ h_v^v B_v(n-1)\theta_v/(1-B_v(n-1)\theta_v), \\
B_r(n) &= s(\mu_r+1) + B_r(n-1)b_r + s\theta_u h_r^u/(1-s\theta_u) + B_r^2(n-1)\sigma_r^2/2, \\
\tilde{B}_r(n) &= -s(\mu_r+1) + \tilde{B}_r(n-1)\tilde{b}_r - s\theta_d\tilde{h}_r^d/(1+s\theta_d) + \tilde{B}_r^2(n-1)\tilde{\sigma}_r^2/2.
\end{aligned}
$$

To compute initial conditions for the above recursion, write down the cumulant generating function of a one-period return:

$$
k_t(s; x_{t,1}) = s\bar{\mu}_t + s^2 v_t/2 + (h_0^u + h_r^u r_t)\frac{s\theta_u}{1-s\theta_u} - (h_0^d + \tilde{h}_r^d\tilde{r}_t)\frac{s\theta_d}{1+s\theta_d}.
$$

Therefore,

$$
\begin{aligned}
A(1) &= s\mu + h_0^u\frac{s\theta_u}{1-s\theta_u} - h_0^d\frac{s\theta_d}{1+s\theta_d}, \\
B_v(1) &= s\mu_v + s^2/2, \\
B_r(1) &= s(\mu_r+1) + s\theta_u h_r^u/(1-s\theta_u), \\
\tilde{B}_r(1) &= -s(\mu_r+1) - s\theta_d\tilde{h}_r^d/(1+s\theta_d).
\end{aligned}
$$

A.2 Supplementary material for Chapter 3

A.2.1 Model's solution and pricing restrictions

In this Appendix, I derive the solution to my model. I briefly repeat the main building blocks for the ease of explicating.

The representative agent has recursive preferences

$$U_t = [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho} \tag{A.2.41}$$

with the certainty equivalent function

$$\mu_t(U_{t+1}) = [E_t(U_{t+1}^\alpha)]^{1/\alpha}, \tag{A.2.42}$$

and preference parameters α (risk aversion is $1-\alpha$), β (subjective discount factor), and ρ ($1/(1-\rho)$ is the elasticity of intertemporal substitution).

The consumption growth process is described by a vector autoregressive system

$$Y_{t+1} = F + GY_t + H\sigma_t\varepsilon_{t+1}, \tag{A.2.43}$$

where $Y_{t+1} = (\log g_{t,t+1},\ \log \pi_{t,t+1},\ i^1_{t+1},\ \sigma^2_{t+1})'$.

To solve the model, I follow closely the solution method of Backus, Chernov, and Zin (2012). Since the utility U_t is determined by a constant elasticity of substitution recursion (A.2.41) and the certainty equivalent function is also homogenous of degree one, I scale (A.2.41) by consumption c_t:

$$u_t = [(1-\beta) + \beta\mu_t(u_{t+1}g_{t,t+1})^\rho]^{1/\rho}, \tag{A.2.44}$$

where $u_t = U_t/c_t$, and $g_{t,t+1} = c_{t+1}/c_t$.

The log pricing kernel under the recursive utility is

$$\log m_{t,t+1} = \log \beta + (\rho - 1) \log g_{t,t+1} + (\alpha - \rho)(\log (u_{t+1} g_{t,t+1}) - \log \mu_t(u_{t+1} g_{t,t+1})) \tag{A.2.45}$$

Appendix A.5 of the NBER version of Backus, Chernov, and Zin (2012) provides the corresponding derivation.

To derive the pricing kernel, I need to solve the equation (A.2.44). I use a log-linear approximation of (A.2.44) to obtain a closed-form solution to the value function $\log u_t$ and to the pricing kernel:

$$\log u_t \approx b_0 + b_1 \log \mu_t(g_{t,t+1} u_{t+1}), \tag{A.2.46}$$

where

$$b_1 = \beta e^{\rho \log \mu} / [(1 - \beta) + \beta e^{\rho \log \mu}], \tag{A.2.47}$$

$$b_0 = \rho^{-1} \log [(1 - \beta) + \beta e^{\rho \log \mu}] - b_1 \log \mu. \tag{A.2.48}$$

The equation is exact if the elasticity of intertemporal substitution is equal to one. In such a case $b_0 = 0$ and $b_1 = \beta$. See Section III in Hansen, Heaton, and Li (2008) and Appendix A.7 in Backus, Chernov, and Zin (2012) for details about the log-linear approximation and its accuracy.

I guess that the solution to the equation (A.2.46) is an affine function of the four model's states:

$$\log u_t = \log u + P' Y_t, \tag{A.2.49}$$

where P is a vector of loadings $P = (p_g,\ p_\pi,\ p_i,\ p_\sigma)'$.

Next, I verify my guess. I compute the log of the certainty equivalent function

$$\log \mu_t(u_{t+1}g_{t,t+1}) = \underbrace{[\log u + e_1'F + P'F]}_{\log \mu} + [P'G + e_1'G]Y_t + \alpha[P+e_1]'\Sigma[P+e_1]\sigma_t^2/2, \tag{A.2.50}$$

where $\Sigma = HH'$ and e_1 is a coordinate vector with the first element equal to 1. Then I substitute (A.2.49) and (A.2.50) to the equation (A.2.46) and collect and match the corresponding terms. The equation (A.2.46) has a constant term and four variables, hence I obtain the system of five equations:

$$\log u = b_0 + b_1 \log u + b_1 e_1'F + b_1 P'F, \tag{A.2.51}$$

$$p_g = b_1(P+e_1)'Ge_1 \tag{A.2.52}$$

$$p_\pi = b_1(P+e_1)'Ge_2, \tag{A.2.53}$$

$$p_i = b_1(P+e_1)'Ge_3, \tag{A.2.54}$$

$$p_\sigma = b_1(P+e_1)'Ge_4 + \alpha b_1(P+e_1)'\Sigma(P+e_1)/2, \tag{A.2.55}$$

where e_i are the corresponding coordinate vectors.

Equations for p_g, p_π, and p_i are linear and therefore they result in unique solutions:

$$\begin{aligned} p_g &= A_g/B_g, \\ p_\pi &= A_\pi/B_\pi, \\ p_i &= A_i/B_i, \end{aligned}$$

where

$$
\begin{aligned}
A_g &= -(G_{11}b_1 - G_{11}G_{22}b1^2 + G_{12}G_{21}b_1^2 - G_{11}G_{33}b_1^2 + G_{13}G_{31}b_1^2 + G_{11}G_{22}G_{33}b_1^3 \\
&- G_{11}G_{23}G_{32}b_1^3 - G_{12}G_{21}G_{33}b_1^3 + G_{12}G_{23}G_{31}b_1^3 + G_{13}G_{21}G_{32}b_1^3 - G_{13}G_{22}G_{31}b_1^2), \\
A_\pi &= -(G_{13}b_1 + G_{12}G_{23}b_1^2 - G_{13}G_{22}b_1^2), \\
A_i &= -(G_{12}b_1 + G_{13}G_{32}b_1^2 - G_{12}G_{33}b_1^2), \\
B_g &= B_\pi = B_i \\
&= G_{11}b_1 + G_{22}b_1 + G_{33}b_1 - G_{11}G_{22}b_1^2 + G_{12}G_{21}b_1^2 - G_{11}G_{33}b_1^2 + G_{13}G_{31}b_1^2 \\
&- G_{22}G_{33}b_1^2 + G_{23}G_{32}b_1^2 + G_{11}G_{22}G_{33}b_1^3 - G_{11}G_{23}G_{32}b_1^3 - G_{12}G_{21}G_{33}b_1^3 \\
&+ G_{12}G_{23}G_{31}b_1^3 + G_{13}G_{21}G_{32}b_1^3 - G_{13}G_{22}G_{31}b_1^3 - 1
\end{aligned}
$$

The equation for p_σ is quadratic:

$$A_\sigma p_\sigma^2 + B_\sigma p_\sigma + C_\sigma = 0,$$

where

$$
\begin{aligned}
A_\sigma &= \alpha b_1 \Sigma_{44}/2, \\
B_\sigma &= \alpha b_1(\Sigma_{34}p_i + \Sigma_{24}p_\pi + \Sigma_{14}(p_g+1)) + b_1G_{44} - 1, \\
C_\sigma &= \alpha b_1((p_g+1)(\Sigma_{13}p_i + \Sigma_{12}p_\pi + \Sigma_{11}(p_g+1)) + p_i(\Sigma_{33}p_i + \Sigma_{23}p_\pi + \Sigma_{13}(p_g+1)) \\
&+ p_\pi(\Sigma_{23}p_i + \Sigma_{22}p_\pi + \Sigma_{12}(p_g+1)))/2 + (b_1p_gG_{14} + b_1p_\pi G_{24} + b_1p_iG_{34} + b_1G_{14}).
\end{aligned}
$$

This equation has two real roots if its discriminant Discr $= (B_\sigma^2 - 4A_\sigma C_\sigma)$ is positive. Only one real root is good, however. It has to be selected based on the property of stochastic stability (Hansen (2012)),

$$p_\sigma = \frac{-B_\sigma + \mathrm{sign}(B_\sigma)\mathrm{Discr}^{1/2}}{2A_\sigma}.$$

Finally, $\log u$ follows as

$$\log u = [b_0 + b_1e_1'F + b_1P'F]/[1 - b_1].$$

I plug the solution $\log u_t$ into (A.2.45) and obtain the final expression for the pricing kernel

$$\begin{aligned}\log m_{t,t+1} &= [\log\beta + (\rho-1)e_1'F] + (\rho-1)e_1'GY_t - \alpha(\alpha-\rho)(P+e_1)'\Sigma(P+e_1)\sigma_t^2/2 \\ &+ [(\alpha-\rho)P + e_1(\alpha-1)]'H\sigma_t\varepsilon_{t+1} \qquad \text{(A.2.56)}\end{aligned}$$

or

$$\log m_{t,t+1} = \log m + \eta' Y_t + q'\sigma_t\varepsilon_{t+1},$$

where

$$\begin{aligned}\eta &= (\rho-1)G'e_1 - \alpha(\alpha-\rho)e_4(P+e_1)'\Sigma(P+e_1)/2, \\ q &= H'[(\alpha-\rho)P + e_1(\alpha-1)].\end{aligned}$$

Next, I derive a one-period real risk-free rate

$$\begin{aligned}r_{f,t}^1 &= -E_t(\log m_{t,t+1}) - Var_t(\log m_{t,t+1})/2 \\ &= -\log\beta - (\rho-1)e_1'F - (\rho-1)e_1'GY_t + \alpha(\alpha-\rho)(P+e_1)'\Sigma(P+e_1)\sigma_t^2/2 \\ &- [(\alpha-\rho)P + e_1(\alpha-1)]'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)]\sigma_t^2/2. \qquad \text{(A.2.57)}\end{aligned}$$

Finally, the nominal one-period rate is

$$\begin{aligned}i_t^1 &= r_{f,t}^1 + E_t(\log\pi_{t,t+1}) - Var_t(\log\pi_{t,t+1})/2 + cov_t(\log m_{t,t+1}, \log\pi_{t,t+1}) \\ &= -\log\beta - (\rho-1)e_1'F + e_2'F - (\rho-1)e_1'GY_t + e_2'GY_t + \alpha(\alpha-\rho)(P+e_1)'\Sigma(P+e_1)\sigma_t^2/2 \\ &- [(\alpha-\rho)P + e_1(\alpha-1)]'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)]\sigma_t^2/2 - e_2'\Sigma e_2\sigma_t^2/2 \\ &+ e_2'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)]\sigma_t^2. \qquad \text{(A.2.58)}\end{aligned}$$

Note that i_t^1 enters both the left-hand side and the right-hand side of (A.2.58), because the nominal yield i_t^1 is a part of the state-vector Y_t.

$$i_t^1 = A\log g_{t-1,t} + B\log\pi_{t-1,t} + Ci_t^1 + D\sigma_t^2 + E,$$

where

$$
\begin{aligned}
&A = -\log\beta - (\rho-1)e_1'F + e_2'F,\\
&B = -(\rho-1)e_1'Ge_1 + e_2'Ge_1\\
&C = -(\rho-1)e_1'Ge_2 + e_2'Ge_2,\\
&D = -(\rho-1)e_1'Ge_3 + e_2'Ge_3,\\
&E = -[(\alpha-\rho)P + e_1(\alpha-1)]'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)]/2 - e_2'\Sigma e_2/2 + e_2'Ge_4\\
&+e_2'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)] + \alpha(\alpha-\rho)(P+e_1)'\Sigma(P+e_1)/2\\
&-(\rho-1)e_1'Ge_4.
\end{aligned}
$$

The expression (A.2.58) is not an equation which nails down the nominal rate, it is an identity. Therefore, to guarantee consistent pricing of the nominal yield, the following five restrictions must be satisfied:

$$A = 0, \quad B = 0, \quad C = 1, \quad D = 0, \quad E = 0.$$

Four restrictions $A = B = E = 0$, $C = 1$ are linear and can be written as

$$\frac{G_{21}}{G_{11}} = \frac{G_{22}}{G_{12}} = \frac{G_{23}-1}{G_{13}} = \frac{F_2 - \log\beta}{F_1} = \rho - 1.$$

The other restriction is nonlinear and it involves the endogenous parameters p_g, p_π, p_i, and p_σ:

$$
\begin{aligned}
&-[(\alpha-\rho)P + e_1(\alpha-1)]'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)]/2 - e_2'\Sigma e_2/2 + e_2'Ge_4\\
&+e_2'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)] + \alpha(\alpha-\rho)(P+e_1)'\Sigma(P+e_1)/2\\
&-(\rho-1)e_1'Ge_4 = 0.
\end{aligned}
\tag{A.2.59}
$$

A.2.2 Estimation results. Different cross-section of currencies

In this section, I analyze how multiple sources of consumption risk are reflected in multi-period log excess returns on currency baskets, formed by sorting currencies by their respective nominal short interest rates.

I use the same sample of twelve currencies. At the end of each period (quarter), I sort currencies by their respective short yields into “Low”, “Intermediate”, and “High” currency baskets. Table A.2 provides some basic descriptive statistics for the cross-section of log excess returns. Average spread in returns between the high and low interest currencies exceeds 4% per year. Table A.3 shows the dynamic decomposition of the three currency baskets.

Table A.4 and Table A.5 provide the parameter estimates together with the corresponding 95% confidence intervals for the FX cash flow processes for “Low”, “Intermediate, and “High” currency baskets under both identification schemes. I find that sources of consumption risk matter at least at a one period horizon. For example, under identification “Fast Inflation”, all currency baskets load significantly on the long-run risk, whereas “Intermediate” and “High” currency baskets in addition significantly load on the short-run consumption and inflation risks. Shock elasticities provide additional evidence of how currency baskets are sensitive to the underlying risks and how corresponding exposures are priced at multiple investment horizons.

Figure A.1 and Figure A.2 display the shock-exposure elasticity under the “Fast Inflation” identification and “Fast Consumption” identification, respectively. Figure A.3 and Figure A.4 display the shock-price elasticity under the “Fast Inflation” identification and “Fast Consumption” identification, respectively.

The long-run consumption risk plays the most prominent role under both identification schemes. Firstly, it carries the highest price of risk across all investment horizons from one quarter to ten years. Secondly, currencies exhibit significantly different sensitivity to the risk: the high interest rate currencies have positive exposure, whereas the low interest rate currencies have negative exposure to the risk at all investment horizons.

The other sources of risk play less important role. The short-run risk matters only at a one-period horizon and only under the “Fast Inflation” identification scheme. The inflation risk is priced only under the ”Fast Consumption” identification scheme. In this case, “Low” and “High” currency baskets have significantly different exposure to the risk at most investment horizons but the price of inflation risk is less than a half of that for the long-run consumption risk. For the latter reason, a larger fraction of spread in excess returns between baskets of high and low interest rate currencies can be attributed to the currencies' exposure to the long-run consumption risk rather than to the inflation risk. Finally, all currencies are highly sensitive to the variance risk at long horizons, however the compensation for this risk exposure is small.

A.2.3 Data description

Macro data come from the NIPA tables of the Bureau of Economic Analysis and CRSP. I use Table 2.1 (Personal income and its disposition), Table 2.3.4 (Personal indexes for personal consumption expenditures by major type of product) and Table 2.3.5 (Personal consumption expenditures by major type of product). I measure real consumption as per capita expenditure on non-durable goods and services. Non-durables and services is the sum of entries of the row 8 from Table 2.3.5 divided by entries of the row 8 from Table 2.3.4 and components of row 13 from Table 2.3.5 divided by components of row 13 from Table 2.3.4. I construct price index associated with personal consumption expenditures. Row 40 of the Table 2.3.1 provides population data.

Table A.6 describes sources and availability of currency and fixed income data.

A.2.4 Fixed point problem

In this Appendix, I sketch the fixed point problem embedded in the equation (A.2.46).

1. I guess b_0 and b_1 and solve equations (A.2.51)-(A.2.55).
2. I compute $\log \mu$ from (A.2.50). Next, I evaluate (A.2.47) and (A.2.48) to obtain $b_0^{'}$ and $b_1^{'}$:

$$\begin{aligned} b_1^{'} &= \beta e^{\rho \log \mu} / [(1-\beta) + \beta e^{\rho \log \mu}], \\ b_0^{'} &= \rho^{-1} \log [(1-\beta) + \beta e^{\rho \log \mu}] - b_1 \log \mu. \end{aligned}$$

3. If $b_0^{'}$ and $b_1^{'}$ are not close enough to the initial values of b_0 and b_1, I set $b_0 = b_0^{'}$ and $b_1 = b_1^{'}$ and return to Stage 2.

I iterate until I achieve convergence. I set the following convergence criterion: $(b_0 - b_0^{'})^2 + (b_1 - b_1^{'})^2 < 10^{-18}$.

A.2.5 Estimation algorithm and choice of priors

VAR with cross-equation restrictions

This section provides details of the estimation algorithm for the VAR with stochastic variance and cross-equation restrictions.

I estimate a vector autoregression for $Y_{t+1} = (\log g_{t,t+1},\ \log \pi_{t,t+1},\ i_{t+1}^{1},\ \sigma_{t+1}^{2})'$

$$Y_{t+1} = F + GY_t + \Sigma^{1/2} \sigma_t w_{t+1}, \tag{A.2.60}$$

with restrictions

$$\frac{G_{21}}{G_{11}} = \frac{G_{22}}{G_{12}} = \frac{G_{23}-1}{G_{13}} = \frac{F_2 - \log\beta}{F_1} = \rho - 1, \tag{A.2.61}$$

$$\begin{aligned}&\alpha(\alpha-\rho)(P+e_1)'HH'(P+e_1)/2 + e_2'Ge_4 - e_2'HH'e_2/2\\ &-[(\alpha-\rho)P + e_1(\alpha-1)]'HH'[(\alpha-\rho)P + e_1(\alpha-1)]/2\\ &+e_2'HH'[(\alpha-\rho)P + e_1(\alpha-1)] - (\rho-1)e_1'Ge_4 = 0,\end{aligned} \tag{A.2.62}$$

where e_i are the coordinate vectors in a four-dimensional space with a unit element in the i-th position and P is a vector of state loadings of the value function.

I impose the linear restrictions (A.2.61) directly in the estimation algorithm, whereas I check whether the restriction (A.2.62) is satisfied after every MCMC draw. I discard draws if the restriction (A.2.62) is violated and keep draws if the restriction is satisfied.[1] I assume preference parameters: $\alpha = -9$ (risk aversion is 10), $\rho = 1/3$ (EIS=3/2), and $\beta = 0.9924$ (subjective discount factor); and estimate the twenty two parameters of F, G, and Σ that describe the dynamics of the macroeconomic system.

To account for the linear restriction (A.2.61), it is more convenient to work with the model re-written in the following form:

$$\bar{Y}_{t+1} = F + GY_t + \Sigma^{1/2}\sigma_t w_{t+1}, \tag{A.2.63}$$

where $\bar{Y}_{t+1} = (\log g_{t,t+1},\ \log\pi_{t,t+1} - i_t^1 - \log\beta,\ i_{t+1}^1,\ \sigma_{t+1}^2 - (1-G_{44}))'$. Note that I normalize the stochastic variance σ_t^2 to have the unit mean, so that $F_4 = 1 - G_{44}$. I denote a vector

[1] It is a very time consuming procedure to draw many MCMC simulations and evaluate the nonlinear restriction (A.2.62) in each of them. The need to solve a fixed point problem makes the problem computationally intense. I overcome some of the difficulties by implementing the following short cut. I draw $M = 2500K$ simulations, evaluate the nonlinear restriction (A.2.62) in each of them and select $J = 10K$ simulations that deliver the minimal value to $\mathrm{r}(F, G, \Sigma, \alpha, \rho, \beta) = (-[(\alpha-\rho)P + e_1(\alpha-1)]'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)]/2 - e_2'\Sigma e_2/2 + e_2'Ge_4 + e_2'\Sigma[(\alpha-\rho)P + e_1(\alpha-1)] + \alpha(\alpha-\rho)(P+e_1)'\Sigma(P+e_1)/2 - (\rho-1)e_1'Ge_4)^2$. Next, I perform a low scale optimization: minimize $\mathrm{r}(F, G, \Sigma, \alpha, \rho, \beta)$ with respect to F, G, Σ starting from each of J simulations. I limit the number of iterations by $K = 300$ so that the solution to the optimization problem is very close to the starting MCMC parameter set for each simulation. The proximity of initial and final parameter sets are evaluated in terms of the difference in the variances of the pricing kernel evaluated in both sets. If the difference in the variance is less than 0.25% of the initial variance of the pricing kernel then the final parameter set is feasible. Additionally, I consider that the nonlinear restriction (A.2.62) is satisfied if $\mathrm{r}(F, G, \Sigma, \alpha, \rho, \beta) < \bar{r} = 10^{-8}$. I set $\bar{r}$ such that the maximal mispricing of the nominal yield is negligible – it is less than 0.1 basis point annualized.

of the unknown parameters of matrices F and G by $\Theta = (\theta_1,\ \theta_2,\ ...,\theta_{12})'$:

$$\Theta = \begin{pmatrix} F_1, & G_{11}, & G_{12}, & G_{13}, & G_{14}, & G_{24}, & F_3, & G_{31}, & G_{32}, & G_{33}, & G_{34}, & G_{44} \end{pmatrix}'.$$

For the stochastic variance, I use the discretized version of the square root model. Therefore, I set $G_{41} = G_{42} = G_{43} = 0$.

Posterior distributions for Θ and Σ follow immediately after I re-write the vector autoregression as a linear regression with $4(T-1)$ observations

$$\bar{y} \;=\; y\Theta + w$$

by stacking vectors $\bar{Y}_{t+1}$ into a large column-vector $\bar{y}$ and forming the matrix y.[2]

Vector of disturbances w has $4(T-1)$ elements and follows the multivariate normal distribution $\mathcal{N}(0,\Omega)$. The variance-covariance matrix Ω has a block-diagonal structure:

$$\Omega = \begin{pmatrix} \Sigma_1 & 0 & ... & 0 \\ 0 & \Sigma_2 & 0 & ... \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & ... & \Sigma_{T-1} \end{pmatrix},$$

where $\Sigma_t = \sigma_t^2 \Sigma$.

I assume a multivariate normal prior for Θ: $\Theta \sim \mathcal{N}(a, A)$; and an inverse Wishart prior for Σ: $\Sigma \sim \mathcal{IW}(b, B)$. The posterior distributions for Θ and Σ follow

[2] The component of the matrix y that corresponds to the dynamics of the system at time t is $y_t =$

$$\begin{pmatrix} 1 & \log g_{t-1,t} & \log \pi_{t-1,t} & i_t^1 & \sigma_t^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (\rho-1) & (\rho-1)\log g_{t-1,t} & (\rho-1)\log \pi_{t-1,t} & (\rho-1)i_t^1 & 0 & \sigma_t^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \log g_{t-1,t} & \log \pi_{t-1,t} & i_t & \sigma_t^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_t^2. \end{pmatrix}$$

$$
\begin{aligned}
&p(\Theta|\bar{y}, y, \alpha, \beta, \rho, \Sigma) \\
&= p(\bar{y}|y, \alpha, \beta, \rho, \Sigma, \Theta)p(\Theta) \propto \exp\left(-(\bar{y} - y\Theta)'\Omega^{-1}(\bar{y} - y\Theta)/2\right)\exp\left(-(\Theta - a)'A^{-1}(\Theta - a)/2\right) \\
&\propto \exp\left(-(\Theta - \hat{a})'\hat{A}^{-1}(\Theta - \hat{a})/2\right) \propto \mathcal{N}(\hat{a}, \hat{A}), \\
&p(\Sigma|\bar{y}, y, \alpha, \beta, \rho, \Theta) = p(\bar{y}|y, \alpha, \beta, \rho, \Theta, \Sigma)p(\Sigma) \\
&\propto |\Omega|^{-(T-1)/2}\exp\left(-(\bar{y} - y\Theta)'\Omega^{-1}(\bar{y} - y\Theta)/2\right)|\Sigma|^{-(B+p+1)/2}\exp\left(-tr(b\Sigma^{-1})/2\right) \\
&\propto |\Sigma|^{-(\hat{B}+p+1)/2}\exp\left(-tr(\hat{b}\Sigma^{-1})/2\right) \propto \mathcal{IW}(\hat{b}, \hat{B}),
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{A} &= \left(\sum_{t=2}^{T} y'_{t-1}\Sigma^{-1}y_{t-1}/\sigma^2_{t-1} + A^{-1}\right)^{-1}, \\
\hat{a} &= \hat{A}\left(\sum_{t=2}^{T} y'_{t-1}\Sigma^{-1}\bar{y}_t/\sigma^2_{t-1} + A^{-1}a\right), \\
\hat{b} &= b + \frac{1}{2}\sum_{t=2}^{T}\frac{(\bar{y}_t - y_{t-1}\Theta)(\bar{y}_t - y_{t-1}\Theta)'}{\sigma^2_{t-1}}, \\
\hat{B} &= B + T - 1.
\end{aligned}
$$

I factorize the posterior distribution for the stochastic variance as follows:

$$
p(\sigma^2_t|\bar{Y}_{t+1}, Y_{t\{-\sigma^2_t\}}, Y_{t-1}, \alpha, \rho, \beta, \Theta, \Sigma) \propto p(\bar{Y}_{t+1}|Y_t, \alpha, \rho, \beta, \Theta, \Sigma)p(\bar{Y}_t|Y_{t-1}, \alpha, \rho, \beta, \Theta, \Sigma).
$$

I employ the Metropolis-Hastings Random Walk algorithm to draw the logarithm of stochastic variance $\log\sigma^2_t$. Transformation to logs guarantees the positivity of the stochastic variance.

I draw $\log\sigma^2_t$ (if $1 < t < T$) from the following posterior distribution

$$
\begin{aligned}
p(\log\sigma^2_t|\bar{Y}_{t+1}, Y_{t\{-\sigma^2_t\}}, Y_{t-1}, \alpha, \rho, \beta, \Theta, \Sigma) \quad \propto \quad & \frac{1}{\sigma^2_t}\exp\left(-\frac{(\bar{Y}_{t+1} - F - GY_t)'\Sigma^{-1}(\bar{Y}_{t+1} - F - GY_t)}{2\sigma^2_t}\right) \\
& \exp\left(-\frac{(\bar{Y}_t - F - GY_{t-1})'\Sigma^{-1}(\bar{Y}_t - F - GY_{t-1})}{2\sigma^2_{t-1}}\right).
\end{aligned}
$$

I adjust the posterior distributions for $\log \sigma_1^2$ and $\log \sigma_T^2$ accordingly:

$$
\begin{aligned}
p(\log \sigma_T^2 | \bar{Y}_{T\{-\sigma_T^2\}}, Y_{T-1}, \alpha, \rho, \beta, \Theta, \Sigma) &\propto \sigma_T^2 \exp\left(-\frac{(Y_T - F - GY_{T-1})'\Sigma^{-1}(\bar{Y}_T - F - GY_{T-1})}{2\sigma_{T-1}^2} \right), \\
p(\log \sigma_1^2 | \bar{Y}_2, Y_{1\{-\sigma_1^2\}}, \alpha, \rho, \beta, \Theta, \Sigma) &\propto \frac{1}{\sigma_1^2} \exp\left(-\frac{(\bar{Y}_2 - F - GY_1)'\Sigma^{-1}(\bar{Y}_2 - F - GY_1)}{2\sigma_1^2} \right).
\end{aligned}
$$

FX cash flow process

The cash-flow process is

$$
\begin{aligned}
\log \delta_{t,t+1} &= \log s_{t+1} - \log s_t - \log \pi_{t,t+1} = \log \delta + \mu' Y_t + \xi' \sigma_t \varepsilon_{t+1} + \xi_v \sigma_t v_{t+1} \\
&= \log \delta + \mu_g \log g_{t-1,t} + \mu_\pi \log \pi_{t-1,t} + \mu_i i_t^1 + \mu_\sigma \sigma_t^2 + \xi_g \sigma_t \varepsilon_{g,t+1} + \xi_\pi \sigma_t \varepsilon_{\pi,t+1} \\
&+ \xi_i \sigma_t \varepsilon_{i,t+1} + \xi_\sigma \sigma_t \varepsilon_{\sigma,t+1} + \xi_v \sigma_t v_{t+1}. \qquad \text{(A.2.64)}
\end{aligned}
$$

For each currency basket, I estimate nine parameters μ_g, μ_π, μ_i, μ_σ, ξ_g, ξ_π, ξ_i, ξ_σ, ξ_v.

Denote

$$
\begin{aligned}
\zeta_t &= \log \delta + \mu_g \log g_{t-1,t} + \mu_\pi \log \pi_{t-1,t} + \mu_i i_t^1 + \mu_\sigma \sigma_t^2 + \xi_g \sigma_t \varepsilon_{g,t+1} + \xi_\pi \sigma_t \varepsilon_{\pi,t+1} \\
&+ \xi_i \sigma_t \varepsilon_{i,t+1} + \xi_\sigma \sigma_t \varepsilon_{\sigma,t+1}.
\end{aligned}
$$

I use conjugate priors: I assume normal independent priors for parameters $\log \delta$, μ_g, μ_π, μ_i, and μ_σ; I assume inverse gamma priors for ξ_g^2, ξ_π^2, ξ_i^2, ξ_σ^2, and ξ_v^2. The posterior distributions follow immediately.

- If $\log \delta$ has a normal prior $\mathcal{N}(a, A)$, then the posterior distribution is $\mathcal{N}(\hat{a}, \hat{A})$, where

$$
\begin{aligned}
\hat{A} &= \left(\frac{1}{A} + \sum_{t=1}^{T-1} \frac{1}{\sigma_t^2 \xi_v^2} \right)^{-1}, \\
\hat{a} &= \hat{A} \left(\frac{a}{A} + \sum_{t=1}^{T-1} \frac{(\log \delta_{t,t+1} - \zeta_t + \log \delta)}{\sigma_t^2 \xi_v^2} \right).
\end{aligned}
$$

- If μ_g has a normal prior $\mathcal{N}(a, A)$, then the posterior distribution is $\mathcal{N}(\hat{a}, \hat{A})$, where

$$\begin{aligned}\hat{A} &= \left(\frac{1}{A} + \sum_{t=1}^{T-1} \frac{(\log g_{t-1,t})^2}{\sigma_t^2 \xi_v^2}\right)^{-1}, \\ \hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=1}^{T-1} \frac{\log g_{t-1,t}(\log \delta_{t,t+1} - \zeta_t + \mu_g \log g_{t-1,t})}{\sigma_t^2 \xi_v^2}\right).\end{aligned}$$

- If μ_π has a normal prior $\mathcal{N}(a, A)$, then the posterior distribution is $\mathcal{N}(\hat{a}, \hat{A})$, where

$$\begin{aligned}\hat{A} &= \left(\frac{1}{A} + \sum_{t=0}^{T-1} \frac{(\log \pi_{t-1,t})^2}{\sigma_t^2 \xi_v^2}\right)^{-1}, \\ \hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=1}^{T-1} \frac{\log \pi_{t-1,t}(\log \delta_{t,t+1} - \zeta_t + \mu_\pi \log \pi_{t-1,t})}{\sigma_t^2 \xi_v^2}\right).\end{aligned}$$

- If μ_i has a normal prior $\mathcal{N}(a, A)$, then the posterior distribution is $\mathcal{N}(\hat{a}, \hat{A})$, where

$$\begin{aligned}\hat{A} &= \left(\frac{1}{A} + \sum_{t=1}^{T-1} \frac{(i_t^1)^2}{\sigma_t^2 \xi_v^2}\right)^{-1}, \\ \hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=1}^{T-1} \frac{i_t^1(\log \delta_{t,t+1} - \zeta_t + \mu_i i_t^1)}{\sigma_t^2 \xi_v^2}\right).\end{aligned}$$

- If μ_σ has a normal prior $\mathcal{N}(a, A)$, then the posterior distribution is $\mathcal{N}(\hat{a}, \hat{A})$, where

$$\begin{aligned}\hat{A} &= \left(\frac{1}{A} + \sum_{t=1}^{T-1} \frac{\sigma_t^2}{\xi_v^2}\right)^{-1}, \\ \hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=1}^{T-1} \frac{(\log \delta_{t,t+1} - \zeta_t + \mu_\sigma \sigma_t^2)}{\xi_v^2}\right).\end{aligned}$$

- If ξ_g has a normal prior $\mathcal{N}(a, A)$, then the posterior distribution is $\mathcal{N}(\hat{a}, \hat{A})$, where

$$\begin{aligned}
\hat{A} &= \left(\frac{1}{A} + \sum_{t=1}^{T-1} \frac{(\varepsilon_{g,t+1})^2}{\xi_v^2}\right)^{-1}, \\
\hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=1}^{T-1} \frac{\varepsilon_{g,t+1}(\log\delta_{t,t+1} - \zeta_t + \xi_g \sigma_t \varepsilon_{g,t+1})}{\sigma_t \xi_v^2}\right).
\end{aligned}$$

- If ξ_π has a normal prior $\mathcal{N}(a, A)$, then the posterior distribution is $\mathcal{N}(\hat{a}, \hat{A})$, where

$$\begin{aligned}
\hat{A} &= \left(\frac{1}{A} + \sum_{t=1}^{T-1} \frac{(\varepsilon_{\pi,t+1})^2}{\xi_v^2}\right)^{-1}, \\
\hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=1}^{T-1} \frac{\varepsilon_{\pi,t+1}(\log\delta_{t,t+1} - \zeta_t + \xi_\pi \sigma_t \varepsilon_{\pi,t+1})}{\sigma_t \xi_v^2}\right).
\end{aligned}$$

- If ξ_i has a normal prior $\mathcal{N}(a, A)$, then the posterior distribution is $\mathcal{N}(\hat{a}, \hat{A})$, where

$$\begin{aligned}
\hat{A} &= \left(\frac{1}{A} + \sum_{t=1}^{T-1} \frac{(\varepsilon_{i,t+1})^2}{\xi_v^2}\right)^{-1}, \\
\hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=1}^{T-1} \frac{\varepsilon_{i,t+1}(\log\delta_{t+1} - \zeta_t + \xi_i \sigma_t \varepsilon_{i,t+1})}{\sigma_t \xi_v^2}\right).
\end{aligned}$$

- If ξ_σ has a normal prior $\mathcal{N}(a, A)$, then the posterior distribution is $\mathcal{N}(\hat{a}, \hat{A})$, where

$$\begin{aligned}
\hat{A} &= \left(\frac{1}{A} + \sum_{t=1}^{T-1} \frac{(\varepsilon_{\sigma,t+1})^2}{\xi_v^2}\right)^{-1}, \\
\hat{a} &= \hat{A}\left(\frac{a}{A} + \sum_{t=1}^{T-1} \frac{\varepsilon_{\sigma,t+1}(\log\delta_{t,t+1} - \zeta_t + \xi_\sigma \sigma_t \varepsilon_{\sigma,t+1})}{\sigma_t \xi_v^2}\right).
\end{aligned}$$

- If ξ_v has an inverse gamma prior $\mathcal{IG}(b, B)$, then the posterior distribution is $\mathcal{IG}(\hat{b}, \hat{B})$, where

$$\begin{aligned}
\hat{b} &= (T-1)/2 + b, \\
\hat{B} &= B + \sum_{t=0}^{T-1} \frac{(\log\delta_{t,t+1} - \zeta_t)^2}{2\sigma_t^2}.
\end{aligned}$$

Choice of priors

Table A.7 lists the prior distributions for the parameters of the vector-autoregression (A.2.63) with cross-equation restrictions (A.2.61) and (A.2.62). Table A.8 lists the prior distributions for the parameters of the FX cash flow process for the "Low", "Intermediate", and "High" baskets (here I consider the cross section of currency returns as defined in the main body of the paper). To set up these priors, I run P regressions

$$\log \delta_{t,t+1} \quad = \quad \log \delta + \mu' Y_t + \sigma_t \xi' \varepsilon_{t+1} + \xi_v v_{t+1},$$

for each currency basket. P is the number of paths of the stochastic variance σ_t^2 and shocks ε_{t+1} in their estimated posterior distributions. I take one path of σ_t^2 and ε_{t+1} at a time and estimate parameters $\log \delta$ and ξ_v and the elements of the vectors μ and ξ. For each parameter, I use the average and standard deviation computed across all parameter estimates of the $P-$ regressions as the mean and standard deviation of the corresponding prior distribution.

A.2.6 Shock elasticity

In this section, I follow lead of Borovička and Hansen (2011) and derive the shock-exposure and the shock-price elasticity for the four sources of consumption risk ε_{t+1}.

Shock-exposure elasticity

The shock-exposure elasticity quantifies the term-structure of marginal quantities of risk. It depends on the functional form of the cash flow process and the evolution of the model's states.

The cash flow process is

$$\log \delta_{t,t+1} \quad = \quad \log \delta + \mu' Y_t + \xi' \sigma_t \varepsilon_{t+1},$$

where without loss of generality, I omit the idiosyncratic shock v_{t+1}.

The dynamics of the model's states is summarized in the vector autoregression:

$$Y_{t+1} \quad = \quad F + GY_t + H\sigma_t\varepsilon_{t+1}.$$

The shock-exposure elasticity has the following mathematical representation

$$\ell_\delta(Y_t,\tau) = \left.\frac{\mathrm{d}\log E[\tilde{\delta}_{t,t+\tau}|Y_t]}{\mathrm{dv}}\right|_{\mathrm{v}=0} = \alpha_h(Y_t)\cdot\tilde{E}_\delta(\varepsilon_{t+1}|Y_t),$$

where $\alpha_h(Y_t)$ is a vector which selects one source of risk ($\alpha_h(Y_t)\cdot\varepsilon_{t+1}$ has a unit standard deviation) and $\tilde{E}_\delta$ is an operator of the mathematical expectation under the change of measure represented by the random variable[3]

$$L^\delta_{t,\tau} = \frac{\delta_{t,t+1}E(\delta_{t,t+\tau}/\delta_{t,t+1}|Y_{t+1})}{E(\delta_{t,t+1}E(\delta_{t,t+\tau}/\delta_{t,t+1}|Y_{t+1})|Y_t)}.$$

I derive the shock exposure elasticity by using the multiplicative factorization of the multi-period cash flow and applying the law of iterated expectations a number of times.

First, I compute $L^\delta_{t,1}$:

$$L^\delta_{t,1} = \frac{\delta_{t,t+1}}{E(\delta_{t,t+1}|Y_t)} = \frac{\exp\left(\xi'\varepsilon_{t+1}\sigma_t\right)}{\exp\left(\xi'\xi\sigma_t^2/2\right)} = \frac{\exp\left(\tilde{e}_\delta'(0,Y_t)\varepsilon_{t+1}\right)}{\exp\left(\tilde{e}_\delta'(0,Y_t)\tilde{e}_\delta(0,Y_t)/2\right)},$$

where $\tilde{E}_\delta(\varepsilon_{t+1}|Y_t) = \tilde{e}_\delta(0,Y_t)$ and note that

$$\ell_\delta(Y_t,1) = \alpha_h(Y_t)\cdot\xi\sigma_t.$$

Next, I use the law of iterated expectations

$$\begin{aligned} E(\delta_{t,t+\tau}|Y_t) &= E(\delta_{t,t+1}\delta_{t+1,t+2}\cdots\delta_{t+\tau-1,t+\tau}|Y_t) \\ &= E(\delta_{t,t+1}E(\delta_{t+1,t+2}\cdots E(\delta_{t+\tau-1,t+\tau}|Y_{t+\tau-1})|\cdots|Y_{t+1})|Y_t) \end{aligned}$$

and compute $E(\delta_{t,t+\tau}|Y_t)$ recursively.

[3]For example, $\alpha_h(Y_t) = (1,\ 0,\ 0,\ 0)'\sigma_t$, where $E(\sigma_t^2) = 1$, or $\alpha_h(Y_t) = (1,\ 0,\ 0,\ 0)'$ selects the short-run consumption shock. Other specifications of $\alpha_h(Y_t)$ are possible.

I start with

$$\begin{aligned}
&E(\delta_{t+\tau-1,t+\tau}|Y_{t+\tau-1}) = \exp\left(\log\delta + \mu' Y_{t+\tau-1} + \xi'\xi\sigma^2_{t+\tau-1}/2\right) \\
&= \exp\left(\mathcal{A}_0(1) + \mathcal{A}_g(1)\log g_{t+\tau-2,t+\tau-1} + \mathcal{A}_\pi(1)\log\pi_{t+\tau-2,t+\tau-1} + \mathcal{A}_i(1) i^1_{t+\tau-1} + \mathcal{A}_\sigma(1)\sigma^2_{t+\tau-1}\right),
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{A}_0(1) &= \log\delta, \\
\mathcal{A}_g(1) &= \mu_g, \\
\mathcal{A}_\pi(1) &= \mu_\pi, \\
\mathcal{A}_i(1) &= \mu_i, \\
\mathcal{A}_\sigma(1) &= \mu_\sigma + \xi'\xi/2.
\end{aligned}$$

Next, I compute

$$\begin{aligned}
&E(\delta_{t+\tau-2,t+\tau-1} E(\delta_{t+\tau-1,t+\tau}|Y_{t+\tau-1})|Y_{t+\tau-2}) = \\
&= \exp\left(\mathcal{A}_0(2) + \mathcal{A}_g(2)\log g_{t+\tau-3,t+\tau-2} + \mathcal{A}_\pi(2)\log\pi_{t+\tau-3,t+\tau-2} + \mathcal{A}_i(2) i^1_{t+\tau-2} + \mathcal{A}_\sigma(2)\sigma^2_{t+\tau-2}\right),
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{A}_0(2) &= \log\delta + \mathcal{A}_0(1) + [\mathcal{A}_g(1),\ \mathcal{A}_\pi(1),\ \mathcal{A}_i(1),\ \mathcal{A}_\sigma(1)]F \\
&= \log\delta + \mathcal{A}_0(1) + \mathcal{A}_g(1)F_1 + \mathcal{A}_\pi(1)F_2 + \mathcal{A}_i(1)F_3 + \mathcal{A}_\sigma(1)F_4, \\
\mathcal{A}_g(2) &= \mu_g + \mathcal{A}_g(1)G_{11} + \mathcal{A}_\pi(1)G_{21} + \mathcal{A}_i(1)G_{31} + \mathcal{A}_\sigma(1)G_{41}, \\
\mathcal{A}_\pi(2) &= \mu_\pi + \mathcal{A}_g(1)G_{12} + \mathcal{A}_\pi(1)G_{22} + \mathcal{A}_i(1)G_{32} + \mathcal{A}_\sigma(1)G_{42}, \\
\mathcal{A}_i(2) &= \mu_i + \mathcal{A}_g(1)G_{13} + \mathcal{A}_\pi(1)G_{23} + \mathcal{A}_i(1)G_{33} + \mathcal{A}_\sigma(1)G_{43}, \\
\mathcal{A}_\sigma(2) &= \mu_\sigma + \mathcal{A}_g(1)G_{14} + \mathcal{A}_\pi(1)G_{24} + \mathcal{A}_i(1)G_{34} + \mathcal{A}_\sigma(1)G_{44} \\
&+ 0.5([\mathcal{A}_g(1),\ \mathcal{A}_\pi(1),\ \mathcal{A}_i(1),\ \mathcal{A}_\sigma(1)]H + \xi')([\mathcal{A}_g(1),\ \mathcal{A}_\pi(1),\ \mathcal{A}_i(1),\ \mathcal{A}_\sigma(1)]H + \xi')'/2.
\end{aligned}$$

Finally, for a generic τ,

$$E(\delta_{t,t+\tau}|Y_t) = \exp\left(\mathcal{A}_0(\tau) + \mathcal{A}_g(\tau)\log g_{t-1,t} + \mathcal{A}_\pi(\tau)\log\pi_{t-1,t} + \mathcal{A}_i(\tau) i^1_t + \mathcal{A}_\sigma(\tau)\sigma^2_t\right),$$

where the parameters of the conditional expectation are determined by the system of difference equations:

$$\begin{aligned}
\mathcal{A}_0(\tau) &= \log\delta + \mathcal{A}_0(\tau-1) + [\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]F, \\
\mathcal{A}_g(\tau) &= \mu_g + \mathcal{A}_g(\tau-1)G_{11} + \mathcal{A}_\pi(\tau-1)G_{21} + \mathcal{A}_i(\tau-1)G_{31} + \mathcal{A}_\sigma(\tau-1)G_{41}, \\
\mathcal{A}_\pi(\tau) &= \mu_\pi + \mathcal{A}_g(\tau-1)G_{12} + \mathcal{A}_\pi(\tau-1)G_{22} + \mathcal{A}_i(\tau-1)G_{32} + \mathcal{A}_\sigma(\tau-1)G_{42}, \\
\mathcal{A}_i(\tau) &= \mu_i + \mathcal{A}_g(\tau-1)G_{13} + \mathcal{A}_\pi(\tau-1)G_{23} + \mathcal{A}_i(\tau-1)G_{33} + \mathcal{A}_\sigma(\tau-1)G_{43}, \\
\mathcal{A}_\sigma(\tau) &= \mu_\sigma + \mathcal{A}_g(\tau-1)G_{14} + \mathcal{A}_\pi(\tau-1)G_{24} + \mathcal{A}_i(\tau-1)G_{34} + \mathcal{A}_\sigma(\tau-1)G_{44} \\
&+ 0.5([\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]H + \xi') \\
&\quad ([\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]H + \xi')'/2.
\end{aligned}$$

In this case, the random variable associated with the change of measure is

$$L^{\delta}_{t,\tau} = \frac{\exp\left(\tilde{e}'_\delta(\tau-1, Y_t)\varepsilon_{t+1}\right)}{\exp\left(0.5(\tilde{e}'_\delta(\tau-1, Y_t)\tilde{e}_\delta(\tau-1, Y_t))'\right)},$$

where

$$\tilde{e}_\delta(\tau-1, Y_t) = ([\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]H + \xi')'\sigma_t.$$

The shock-exposure elasticity immediately follows

$$\begin{aligned}
\ell_\delta(Y_t, \tau) &= \alpha_h(Y_t) \cdot \tilde{e}_\delta(\tau-1, Y_t) \\
&= \alpha_h(Y_t) \cdot ([\mathcal{A}_g(\tau-1),\ \mathcal{A}_\pi(\tau-1),\ \mathcal{A}_i(\tau-1),\ \mathcal{A}_\sigma(\tau-1)]H + \xi')'\sigma_t.
\end{aligned}$$

Shock-price elasticity

To compute the shock-price elasticity (3.4.16), I need to evaluate the following object

$$\ell_v(Y_t, \tau) = \left.\frac{\mathrm{d}\log E[\tilde{\delta}_{t,t+\tau} m_{t,t+\tau}|Y_t]}{\mathrm{dv}}\right|_{\mathrm{v}=0}$$

which has a similar mathematical structure to the shock-exposure elasticity. Borovička and Hansen (2011) call this object the shock-value elasticity. The shock-price elasticity, $\ell_p(Y_t, \tau)$, follows by means of subtracting the shock-value elasticity from the shock-exposure elasticity:

$$\ell_p(Y_t,\tau) \;\; = \;\; \ell_\delta(Y_t,\tau) - \ell_v(Y_t,\tau).$$

The derivation of the shock-value elasticity mirrors one of the shock-exposure elasticity. Therefore, the solution has a similar mathematical representation:

$$\ell_v(Y_t,\tau) \;\; = \;\; \alpha_v(Y_t)\cdot([\mathcal{B}_g(\tau-1),\ \mathcal{B}_\pi(\tau-1),\ \mathcal{B}_i(\tau-1),\ \mathcal{B}_\sigma(\tau-1)]H + \xi' + q')'\sigma_t,$$

where $\mathcal{B}_g$, $\mathcal{B}_\pi$, $\mathcal{B}_i$, and $\mathcal{B}_\sigma$ solve the system of difference equations:

$$\begin{aligned}
\mathcal{B}_0(\tau) &= \log\delta + \log m + \mathcal{B}_0(\tau-1) + [\mathcal{B}_g(\tau-1),\ \mathcal{B}_\pi(\tau-1),\ \mathcal{B}_i(\tau-1),\ \mathcal{B}_\sigma(\tau-1)]F,\\
\mathcal{B}_g(\tau) &= \mu_g + \eta_g + \mathcal{B}_g(\tau-1)G_{11} + \mathcal{B}_\pi(\tau-1)G_{21} + \mathcal{B}_i(\tau-1)G_{31} + \mathcal{B}_\sigma(\tau-1)G_{41},\\
\mathcal{B}_\pi(\tau) &= \mu_\pi + \eta_\pi + \mathcal{B}_g(\tau-1)G_{12} + \mathcal{B}_\pi(\tau-1)G_{22} + \mathcal{B}_i(\tau-1)G_{32} + \mathcal{B}_\sigma(\tau-1)G_{42},\\
\mathcal{B}_i(\tau) &= \mu_i + \eta_i + \mathcal{B}_g(\tau-1)G_{13} + \mathcal{B}_\pi(\tau-1)G_{23} + \mathcal{B}_i(\tau-1)G_{33} + \mathcal{B}_\sigma(\tau-1)G_{43},\\
\mathcal{B}_\sigma(\tau) &= \mu_\sigma + \eta_\sigma + \mathcal{B}_g(\tau-1)G_{14} + \mathcal{B}_\pi(\tau-1)G_{24} + \mathcal{B}_i(\tau-1)G_{34} + \mathcal{B}_\sigma(\tau-1)G_{44}\\
&+ \ 0.5(q' + \xi' + [\mathcal{B}_g(\tau-1),\ \mathcal{B}_\pi(\tau-1),\ \mathcal{B}_i(\tau-1),\ \mathcal{B}_\sigma(t-1)]H)\\
&\quad\ (q' + \xi' + [\mathcal{B}_g(\tau-1),\ \mathcal{B}_\pi(\tau-1),\ \mathcal{B}_i(\tau-1),\ \mathcal{B}_\sigma(\tau-1)]H)'.
\end{aligned}$$

with the following initial conditions

$$\begin{aligned}
\mathcal{B}_0(1) &= \log m + \log\delta,\\
\mathcal{B}_g(1) &= \mu_g + \eta_g,\\
\mathcal{B}_\pi(1) &= \mu_\pi + \eta_\pi,\\
\mathcal{B}_i(1) &= \mu_i + \eta_i,\\
\mathcal{B}_\sigma(1) &= \mu_\sigma + \eta_\sigma + (\xi+q)'(\xi+q)/2
\end{aligned}$$

and

$$\ell_v(Y_t,1) \;\; = \;\; \alpha_h(Y_t)\cdot(\xi+q)\sigma_t.$$

A.2.7 Model diagnostics

Table A.9 provides diagnostics of the fitted residuals of the vector autoregression, Table A.10 and Table A.11 provide diagnostics of the fitted residuals of the FX cash flow process under the "Fast Inflation" and "Fast Consumption" identification schemes, respectively (here I consider the cross section of currency returns as defined in the main body of the paper). The diagnostics evaluate whether the shocks to consumption growth, inflation, nominal rate, stochastic variance, and FX cash flow are from the standard normal distribution. I construct the posterior distribution of these shocks and assess three summary statistics – skewness, kurtosis, and autocorrelation.

I find that the model fits data well. There are only slight signs of non-normalities in the fitted residuals of the FX cash flows: mild excess kurtosis for cash flows of "Low" and "Intermediate" baskets and mild excess skewness for the cash flow of "High" basket. Additionally, I find positive autocorrelation for the inflation innovations and signs of non-normalities for the innovations to the nominal rate. The primer can be attributed to the effect of cross-equation restrictions. One way to improve the fit is to estimate the preference parameters. The latter is the standard problem of fitting nominal interest rate data that exhibit dramatic behaviour during the Monetary Experiment of 1979–1982. I leave improvements to future research.

A.2.8 Simulation exercise. Bansal and Yaron (2004) economy

Economy

Consider the model of Bansal and Yaron (2004) with one modification, that is replace the autoregressive process for the stochastic variance with the discretized version of the square-root process.

A representative agent has recursive utility of Epstein and Zin (1989) and Weil (1989)

$$U_t = [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho} \tag{A.2.65}$$

with the certainty equivalent function

$$\mu_t(U_{t+1}) = [E_t(U_{t+1}^{\alpha})]^{1/\alpha}, \qquad \text{(A.2.66)}$$

where $(1-\alpha)$ is the coefficient of relative risk aversion, $1/(1-\rho)$ is the elasticity of intertemporal substitution (EIS) and β is the subjective discount factor. The consumption growth process is specified in terms of two latent states x_t and σ_t^2:

$$\begin{aligned} \log g_{t,t+1} &= \log g + x_t + \sigma_g \sigma_t \varepsilon_{g,t+1}, && \text{(A.2.67)} \\ x_{t+1} &= \rho_x x_t + \psi_x \sigma_t \varepsilon_{x,t+1}, && \text{(A.2.68)} \\ \sigma_{t+1}^2 &= (1-v) + v\sigma_t^2 + \sigma_w \sigma_t \varepsilon_{\sigma,t+1}. && \text{(A.2.69)} \end{aligned}$$

To solve the model, work with with an approximate Bellman equation

$$\log u_t \approx b_0 + b_1 \log \mu_t(u_{t+1} g_{t,t+1}),$$

where

$$\begin{aligned} b_1 &= \beta e^{\rho \log \mu} / [1 - \beta + \beta e^{\rho \log \mu}], && \text{(A.2.70)} \\ b_0 &= \rho^{-1} \log(1 - \beta + \beta e^{\rho \log \mu}) - b1 \log \mu. && \text{(A.2.71)} \end{aligned}$$

See Backus, Chernov, and Zin (2012) for the details of the solution method.

I guess that the value function is an affine function of the states

$$\log u_t = \log u + p_x x_t + p_\sigma \sigma_t^2$$

and subsequently verify my guess.

Firstly, I compute

$$\begin{aligned}
\log u_{t+1}g_{t,t+1} &= [\log ug + p_\sigma(1-v)] + (p_x\rho_x+1)x_t + p_\sigma v\sigma_t^2 + \sigma_g\sigma_t\varepsilon_{g,t+1} + p_x\psi_x\sigma_t\varepsilon_{x,t+1} \\
&+ p_\sigma\sigma_w\sigma_t\varepsilon_{\sigma,t+1} \\
\log\mu_t(u_{t+1}g_{t,t+1}) &= [\log ug + p_\sigma(1-v)] + (p_x\rho_x+1)x_t \\
&+ [p_\sigma v + \alpha\sigma_g^2/2 + \alpha p_x^2\psi_x^2/2 + \alpha p_\sigma^2\sigma_w^2/2]\sigma_t^2.
\end{aligned}$$

The parameters of the value function $\log u$, p_x, and p_σ are solutions to the following three equations:

$$\begin{aligned}
\log u &= b_0 + b_1(\log ug + p_\sigma(1-v)), \\
p_x &= b_1(p_x\rho_x + 1), \\
p_\sigma &= b_1(p_\sigma v + \alpha p_x^2\psi_x^2/2 + \alpha p_\sigma^2\sigma_w^2/2 + \alpha\sigma_g^2/2).
\end{aligned}$$

Hence,

$$\log u = [b_0 + b_1\log g + b_1 p_\sigma(1-v)]/(1-b_1), \tag{A.2.72}$$

$$p_x = b_1/[1-b_1\rho_x]. \tag{A.2.73}$$

Equation for p_σ is quadratic

$$\alpha b_1\sigma_w^2 p_\sigma^2/2 + p_\sigma(b_1 v - 1) + b_1(\alpha p_x^2\psi_x^2/2 + \alpha\sigma_g^2/2) = 0$$

and has a real solution if

$$D = (b_1 v - 1)^2 - \alpha^2 b_1^2(p_x^2\psi_x^2 + \sigma_g^2)\sigma_w^2 \geq 0.$$

If the discriminant is positive and $b_1 v - 1 < 0$, I choose the minus root to guarantee stochastic stability:

$$p_\sigma = [-b_1 v + 1 - D^{1/2}]/[\alpha b_1\sigma_w^2] \tag{A.2.74}$$

Expressions (A.2.70)–(A.2.74) form the fixed point problem.

Calibration

For numerical evaluation of the model, I use calibration of Borovička, Hansen, Hendricks, and Scheinkman (2011). On a monthly frequency, the consumption growth process is

$$\begin{aligned}\log g_{t+1} &= 0.0015 + x_t + 0.0078\sigma_t\varepsilon_{g,t+1},\\ x_{t+1} &= 0.979x_t + 0.00034\sigma_t\varepsilon_{x,t+1},\\ \sigma^2_{t+1} &= 0.013 + 0.987\sigma^2_t + 0.038\sigma_t\varepsilon_{\sigma,t+1}.\end{aligned}$$

I assume the following preference parameters: $\beta = 0.999$ (subjective discount factor), $\gamma = 1-\alpha = 10$ (risk aversion) and $\psi = 1/(1-\rho) = 3/2$ (EIS).

I solve the fixed point problem on a two-dimensional grid. I find: $b_0 = -10^{-5}$, $b_1 = 0.999$, $p_x = 45.5354$, $p_\sigma = -0.1017$, $\log u = 0.1749$.

Asset prices

The pricing kernel in this economy is

$$\begin{aligned}\log m_{t,t+1} &= \log\beta + (\rho-1)\log g_{t,t+1} + (\alpha-\rho)(\log g_{t,t+1}u_{t+1} - \log\mu_t(g_{t,t+1}u_{t+1}))\\ &= [\log\beta + (\rho-1)\log g] + (\rho-1)x_t - \alpha(\alpha-\rho)p_x^2\psi_x^2\sigma_t^2/2 - \alpha(\alpha-\rho)p_\sigma^2\sigma_w^2\sigma_t^2/2\\ &- \alpha(\alpha-\rho)\sigma_g^2\sigma_t^2/2 + (\alpha-1)\sigma_g\sigma_t\varepsilon_{g,t+1} + (\alpha-\rho)p_x\psi_x\sigma_t\varepsilon_{x,t+1}\\ &+ (\alpha-\rho)p_\sigma\sigma_w\sigma_t\varepsilon_{\sigma,t+1}.\end{aligned}$$

The equilibrium one-period real risk-free rate is

$$\begin{aligned}r^1_{t,f} &= [-\log\beta - (\rho-1)\log g] - (\rho-1)x_t + [\alpha(\alpha-\rho)/2 - (\alpha-1)^2/2]\sigma_g^2\sigma_t^2\\ &+ (\alpha-\rho)\rho[p_x^2\psi_x^2 + p_\sigma^2\sigma_w^2]\sigma_t^2/2 = [-\log\beta - (\rho-1)\log g] - (\rho-1)x_t\\ &+ (\alpha-\rho)\rho[p_x^2\psi_x^2 + p_\sigma^2\sigma_w^2]\sigma_t^2/2 + (2\alpha - \rho\alpha - 1)\sigma_g^2\sigma_t^2/2.\end{aligned}$$

Introduce new notations

$$
\begin{aligned}
B_0 &= -\log\beta - (\rho-1)\log g, && \text{(A.2.75)}\\
B_1 &= -(\rho-1), && \text{(A.2.76)}\\
B_2 &= [\alpha(\alpha-\rho)/2 - (\alpha-1)^2/2]\sigma_g^2 + \rho(\alpha-\rho)[p_x^2\psi_x^2 + p_\sigma^2\sigma_w^2]/2 && \text{(A.2.77)}
\end{aligned}
$$

and re-write subsequently the real risk-free rate as

$$
r_{t,f}^1 = B_0 + B_1 x_t + B_2\sigma_t^2, \qquad \text{(A.2.78)}
$$

For the calibration used: $B_0 = 0.002$, $B_1 = 0.6667$, and $B_2 = -8.828\cdot 10^{-4}$.

Solve equation (A.2.78) for x_t in terms of the real rate $r_{t,f}^1$ and stochastic variance σ_t^2:

$$
x_t = r_{t,f}^1/B_1 - B_0/B_1 - B_2\sigma_t^2/B_1.
$$

Equivalent model

Next, I re-write the model (A.2.67)-(A.2.69) in terms of the real risk-free rate and stochastic variance that play the role of the model's states:

$$
\begin{aligned}
\log g_{t,t+1} &= [\log g - B_0/B_1] + r_{t,f}^1/B_1 - B_2\sigma_t^2/B_1 + \sigma_g\sigma_t\varepsilon_{g,t+1}, && \text{(A.2.79)}\\
r_{t+1,f}^1 &= [B_0 - B_0\rho_x + B_2(1-v)] + \rho_x r_{t,f}^1 + B_2(v-\rho_x)\sigma_t^2 + B_1\psi_x\sigma_t\varepsilon_{x,t+1} \\
&+ B_2\sigma_w\sigma_t\varepsilon_{\sigma,t+1}, && \text{(A.2.80)}\\
\sigma_{t+1}^2 &= (1-v) + v\sigma_t^2 + \sigma_w\sigma_t\varepsilon_{\sigma,t+1}. && \text{(A.2.81)}
\end{aligned}
$$

I denote a vector of variables Y_t: $Y_t = (\log g_{t-1,t},\ r_{t,f}^1,\ \sigma_t^2)'$ and re-write the evolution (A.2.79-A.2.81) in a more compact form:

$$
Y_{t+1} = F + GY_t + H\sigma_t\varepsilon_{t+1}. \qquad \text{(A.2.82)}
$$

Mapping between the two models

Intuitively, there is a one-to-one mapping between the model specified in terms of latent states x_t and σ_t^2 and the model written down in terms of the real risk-free rate $r_{t,f}^1$ and stochastic variance σ_t^2. The equivalence is reflected in the functional form of the parameters of the matrices F, G, and H that are the functions of the structural parameters of the original model of consumption growth and preference parameters. Below I illustrate that if the real risk-free rate is priced internally consistently under the model determined by the preferences (A.2.65)-(A.2.66) and consumption growth process (A.2.82) then the equivalence of the two specifications holds.

I solve an approximate Bellman equation

$$\log u_t \approx b_0 + b_1 \log \mu_t(u_{t+1} g_{t,t+1}).$$

I guess that the value function is the affine function of the model's states:

$$\log u_t \quad = \quad \log u + P'Y_t = p_g \log g_{t-1,t} + p_r r_{f,t}^1 + p_\sigma \sigma_t^2,$$

where the vector P is $P = (p_g,\ p_r,\ p_\sigma)'$. Here I allow consumption growth to be a model's state. Later I will compute p_g and show that in fact consumption growth is not a state ($p_g = 0$).

Compute

$$\begin{aligned}
\log u_{t+1} g_{t,t+1} &= [e_1'F + \log u + P'F] + [P'G + e_1'G]Y_t + [P + e_1]'\sigma_t w_{t+1}. \\
\log \mu_t(u_{t+1} g_{t,t+1}) &= [e_1'F + \log u + P'F] + (P'G + e_1'G)Y_t + \alpha(P + e_1)'\Sigma(P + e_1)\sigma_t^2/2,
\end{aligned}$$

where $e_1 = (1,\ 0,\ 0)'$.

Parameters $\log u$, p_g, p_r, and p_σ are solutions to the following four equations:

$$\log u = b_0 + b_1(e_1'F + \log u + P'F), \quad (A.2.83)$$

$$p_g = b_1(p_g + 1)G_{11} + b_1 p_r G_{21} + b_1 p_\sigma G_{31}, \quad (A.2.84)$$

$$p_r = b_1(p_g G_{12} + p_r G_{22} + G_{12}) + b_1 p_\sigma G_{32}, \quad (A.2.85)$$

$$p_\sigma = b_1(p_g G_{13} + p_r G_{23} + p_\sigma G_{33} + G_{13} + \alpha(P + e_1)'\Sigma(P + e_1)/2). \quad (A.2.86)$$

Because the stochastic variance follows the discretized version of the square-root process, $G_{31} = G_{32} = 0$. Equations (A.2.84) and (A.2.85) are linear in p_g and p_r, and therefore, there are unique parameters p_g and p_r satisfying both. Equation (A.2.86) is quadratic in p_σ. If the discriminant is positive, I choose the root that guarantees stochastic stability. Finally, parameter $\log u$ is uniquely determined as follows

$$\log u = [b_0 + b_1(PF + F_{11})]/[1 - b_1]$$

The log pricing kernel is

$$\begin{aligned} \log m_{t,t+1} &= \log\beta + (\rho - 1)\log g_{t,t+1} + (\alpha - \rho)[\log g_{t,t+1}u_{t+1} - \log \mu_t(u_{t+1}g_{t,t+1})] \\ &= [\log\beta + (\rho - 1)e_1'F] + (\rho - 1)e_1'GY_t - \alpha(\alpha - \rho)(P + e_1)'\Sigma(P + e_1)'\sigma_t^2/2 \\ &+ [(\alpha - 1)e_1 + (\alpha - \rho)P]'\sigma_t w_{t+1}. \end{aligned}$$

The one-period real risk-free rate is

$$\begin{aligned} r_{t-1,t} &= -\log\beta - (\rho - 1)e_1'F - (\rho - 1)e_1'GY_t \\ &+ (\rho(\alpha - \rho)P'\Sigma P/2 + (2\alpha - \alpha\rho - 1)e_1'\Sigma e_1/2 + (\alpha - \rho)P'\Sigma e_1)\sigma_t^2. \quad (A.2.87) \end{aligned}$$

The real risk-free rate plays twofold role in the model: it is an equilibrium outcome of the model and one of the states of the model. To guarantee the internal consistency of the model, I have to make sure that the left-hand side and the right-hand side of the equation (A.2.87) coincide.

The latter holds if the following four parameter restrictions are satisfied:

$$\log\beta + (\rho-1)e_1'F = 0, \tag{A.2.88}$$

$$(\rho-1)e_1'Ge_1 = 0, \tag{A.2.89}$$

$$1 + (\rho-1)e_1'Ge_2 = 0, \tag{A.2.90}$$

$$\begin{aligned}&(\rho-1)e_1'Ge_3 - (\rho(\alpha-\rho)P'\Sigma P/2 + (2\alpha-\alpha\rho-1)e_1'\Sigma e_1/2\\ &+(\alpha-\rho)P'\Sigma e_1) = 0,\end{aligned} \tag{A.2.91}$$

where $e_3 = (0,\ 0,\ 1)'$.

Restrictions (A.2.88) and (A.2.90) deliver unique solutions B_0 given in (A.2.75) and B_1 given in $(A.2.76)$. Restriction (A.2.89) suggests that the consumption growth is not a state of the model. Restriction (A.2.91) delivers the expression for B_2 given in (A.2.77). Therefore, the pricing restrictions provide the unique mapping between the original model of consumption growth written down in terms of unobservable states and the model represented in terms of the real risk-free rate and stochastic variance.

Implied consumption growth process. Calibration

The implied consumption growth process in terms of the real risk-free rate and stochastic variance is

$$\begin{aligned}\log g_{t,t+1} &= -0.0015 + 1.5r^1_{t,f} + 0.001324\sigma^2_t + 0.0078\sigma_t\varepsilon_{g,t+1},\\ r^1_{t+1,f} &= 3.1\cdot 10^{-5} + 0.979r^1_{t,f} - 7.06\cdot 10^{-6}\sigma^2_t + 0.0227\sigma_t\varepsilon_{x,t+1} - 3.35\cdot 10^{-5}\sigma_t\varepsilon_{\sigma,t+1},\\ \sigma^2_{t+1} &= 0.013 + 0.987\sigma^2_t + 0.038\sigma_t\varepsilon_{\sigma,t+1}.\end{aligned}$$

I solve the fixed-point problem on a two-dimensional grid. I find $b_0 = 10^{-5}$, $b_1 = 0.999$, $\log u = 0.1958$, $p_g = 0$, $p_r = 68.3031$, and $p_\sigma = -0.0414$. Additionally, I compute the approximation error involved while solving the fixed point problem. I find that expression (A.2.91) is equal to $5.4\cdot 10^{-20}$. This is negligibly different from zero because the maximal in absolute terms deviation of the equilibrium interest rate from the interest rate simulated under the null of the model is $5.2\cdot 10^{-16}$.

Estimation

I use parameter values from Borovička, Hansen, Hendricks, and Scheinkman (2011) to simulate $N = 260$ observations of consumption growth and states x_t and σ_t^2 under the null of the model (A.2.67)-(A.2.69).[4] Next, I compute the time-series of the real risk-free rate as in (A.2.78). I use consumption growth and interest rate data to estimate the model (A.2.79)-(A.2.81) with pricing restrictions (A.2.88)-(A.2.91).

Table A.12 shows the parameter estimates for the free elements of the vector autoregression (A.2.82) with cross-equation restrictions (A.2.88)-(A.2.91), together with their 95% confidence intervals and true values. Figure A.5 displays the true stochastic variance and the 95% confidence interval for the estimated stochastic variance.

In the estimation exercise, my main interest lies into identifying parameters p_g, p_r, and p_σ. These parameters are loadings on the economic states in the value function and important ingredients of the prices of risk represented by the vector $-[(\alpha - 1)e_1 + (\alpha - \rho)P]'H$. Table A.13 contains the estimates of the parameters p_g, p_r, and p_σ together with their confidence intervals and true values. Evidently, the parameter p_σ is estimated with high uncertainty. Consequently, the estimate for the price of variance risk is associated with high uncertainty too.

[4] I choose $N = 260$ observations to have a comparable length of the dataset with one I use in Chapter 3.

A.3 Tables and figures

Table A.1: AUD, CHF, GBP, JPY events

AUD Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Dec 24, 1986	-1.41	-0.90 (0.59)		No significant news. Thin pre-Christmas trading day	
Jan 14, 1987	-2.68	-2.67 (1.00)		Listing of the Australian dollar futures for the first time on the Chicago futures exchange	JPY^u
Jan 20, 1987	2.62	2.36 (1.00)		Encouraging narrowing of the AU monthly deficit along with worst-ever current account figures	
Oct 29, 1987	-2.46	-1.56 (0.74)		Upheaval in world financial markets	
Feb 16, 1989	-4.06	-3.72 (1.00)		Positive US trade news. Poor balance of payment figures in AU	
Feb 27, 1989	-3.24	-2.67 (0.93)		Australian Treasurer says that figures for the trade balance would continue bad. No increase of interest rates	
Apr 5, 1989	-3.82	-3.50 (0.98)		Intervention by the Reserve Bank of Australia (RBA) to temper the rise of AUD	
Jan 23, 1990	-2.42	-2.21 (0.97)		Reserve Bank of Australia relaxes restrictive monetary policy. Interest rate cut	
Apr 26, 1990	-1.66	-1.02 (0.62)		Intervention by the Reserve Bank of Australia. Sell AUD for USD	
Aug 30, 1990	-2.32	-1.26 (0.63)		Third RBA intervention in four trading days. Sell AUD for USD	
Oct 15, 1990	-3.38	-3.19 (1.00)		The Reserve Bank of Australia cuts its official money market rate	
May 22, 1991	-1.72	-1.58 (0.93)		Intervention by the Reserve Bank of Australia. Announcement of the RBA's governor about intention to devalue AUD	
June 5, 1991	-1.91	-1.89 (0.99)		Jittery market in anticipation of the release of AU unemployment figures	
May 18, 1995	-2.19	-1.74 (0.86)		Goldman Sachs report shows negative prospects for AUD	
Nov 21, 1996	1.09	0.84 (0.58)		Australian and New Zealand banking group release – strong annual result	
Dec 3, 1996	-1.41	-0.96 (0.67)		Speculation of an interest rate cut following the meeting of the Reserve Bank of Australia	GBP^d
May 21, 1997	1.63	0.94 (0.59)		The US Federal Reserve decides not to raise interest rates	
Oct 22, 1997	-1.38		0.14 (0.79)	Release of the AU CPI data – annual inflation reaches 35-year low. Anticipation of a regular cut in AU interest rates. Uncertainty about the effect of the Asian crisis on the AU economy	
Oct 24, 1997	-2.48	-0.98 (0.53)		Hong Kong suffered the biggest equity market slump in its history on Oct 23, 1997	
Oct 27, 1997	-0.42		0.16 (0.95)	Impact of the South East Asian currency crisis. Price of gold sliding – Swiss government considers selling about a half of its gold reserves	
Oct 28, 1997	3.42	2.30 (0.85)		Huge losses in the US stock market – largest drop since Black Monday	CHF^v
Jan 6, 1998	-0.90		0.11 (0.87)	Sharp drop in the price of gold which recorded an 18-year low. Soaring US dollar. Concerns over the Asian currency crisis	CHF^v

Table A1: AUD, CHF, GBP, JPY events continued

AUD Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
June 18, 1998	-1.06		0.14 (0.66)	Speculation that the US may try to help stop the slide of the Japanese yen. Intervention by the Bank of Japan and the Fed to buy the yen for USD	
Oct 7, 1998	4.68	3.34 (0.89)		Greenspan: negative prospects for the US economy, credit crunch. Possible further cut of the US interest rates. Threat of Clinton impeachment. Strong AU employment report	JPY^u
Oct 8, 1998	-0.78		0.20 (1.00)	Large moves in the US dollar/yen rate	CHF^v, JPY^v
Jan 28, 2000	-2.97	-2.75 (1.00)	0.11 (1.00)	AU Release of the December quarter CPI – lower than expected	
Dec 22, 2000	3.22	1.75 (0.68)		Downward revision to US Q3 growth data. Concerns over a sharp economic US slowdown loom. Thin trading exaggerates FX moves	
Apr 18, 2001	-0.02		0.12 (0.59)	Westpac/Melbourne Institute releases an index showing that AU economy is in a weak state	
Sep 17, 2001	-2.41		0.12 (0.94)	Markets are turbulent ahead of Wall Street's open after September 11. Coordinated cut of interest rates by the Fed, the Bank of Canada, the ECB the Swiss National Bank, and the Swedish Riksbank	CHF^v, JPY^v
Jan 6, 2003	1.47		0.19 (0.51)	Robust AU economic data	
Feb 20, 2004	-2.61	-1.35 (0.64)		Japan is on terror alert. Japan dispatches troops on a humanitarian mission to Iraq	JPY^d
July 27, 2007	-2.24	-1.80 (0.89)	0.11 (0.98)	Flight to safety – sharp fall on Wall Street	
Aug 10, 2007	-0.61		0.11 (0.99)	Bad signs of credit crisis. BNP Paribas warns of credit problems	
Aug 16, 2007	-3.57	-3.14 (0.98)	0.25 (1.00)	Hedge fund liquidation. The Fed unexpectedly cuts the discount rate on its lending to banks	JPY^v
Oct 16, 2007	-1.19		0.12 (0.80)	Flight to safety away from high-yielding currencies. Talk that G7 would act to stop the US dollar falling any further	
Nov 12, 2007	-4.00	-1.59 (0.61)		Credit crunch continues to batter high-yielding currencies	
Jan 16, 2008	-0.12		0.11 (1.00)	Positive AU economic data but flight to quality effect dominates market sentiment	
Sep 5, 2008	-0.82		0.14 (0.85)	Downbeat US employment figures, disappointing retail sales data, growing speculation about troubles at major hedge funds	JPY^v
Sep 15, 2008	-2.08		0.13 (0.99)	Lehman Brothers collapse	CHF^v, GBP^v, JPY^u, JPY^v
Sep 16, 2008	-0.77		0.20 (0.85)	Sharp drop in commodity prices. Expectations of a big rate cut by the Reserve Bank of Australia	
Sep 29, 2008	-3.21		0.24 (0.97)	Unprecedented coordinated attempt to pour liquidity into the financial system through short-term loans by the Fed, the ECB, the Bank of Japan, the Reserve bank of Australia, and the Bank of England. The US House of Representatives rejects a proposed $700 billion financial bailout package	CHF^v, GBP^v, JPY^v
Sep 30, 2008	-1.51		0.14 (0.60)	Banking crisis deepens in Europe. Banking bailouts	GBP^v

Table A1: AUD, CHF, GBP, JPY events continued

AUD Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Oct 6, 2008	-6.92	-5.64 (0.99)	0.46 (1.00)	Expectations of easing in AU. European and US stocks are crashed	CHF^v, GBP^v, JPY^v
Oct 8, 2008	-6.06		0.45 (1.00)	Coordinated cut of interest rates by the FED, the ECB, the Bank of England, the Bank of Canada, the Swiss National Bank, and the Swedish Riksbank	
Oct 22, 2008	0.31		0.30 (0.95)	Investors deleverage	CHF^v, GBP^v, JPY^v
Oct 24, 2008	-7.28		0.14 (0.61)	The White House announces that the widespread recession is inevitable. British Prime Minister acknowledges that the UK is likely entering recession. Global stocks plumb multi-year lows	GBP^v, JPY^v
Nov 12, 2008	-2.56		0.17 (0.96)	Turmoil on equity markets, thin trading before the Veteran's day in the US	GBP^v, JPY^v
Dec 17, 2008	1.39		0.14 (0.89)	Fed decides to cut interest rates to a zero level	CHF^v, GBP^v
Jan 5, 2009	0.90		0.11 (0.58)	Hopes that fresh stimulus plans from the US and Germany would help the global economy recover. Rebounding stock market encourages investors to pick up higher-yielding currencies	GBP^v
Jan 13, 2009	-2.52		0.13 (0.95)	Negative US job figures. Deepening global slowdown. Decline in commodity prices	
Feb 10, 2009	-3.54		0.14 (0.88)	The National Bank's of Australia measure of business confidence dives to the historical lowest. The US Treasury Secretary announces a plan to rescue the banking system which disappoints the market. The US Senate passes a massive economic stimulus package	CHF^v
Mar 23, 2009	2.64		0.12 (0.65)	The US government fleshes out the plan to purge banks from toxic assets	
June 3, 2009	-2.47		0.11 (0.65)	Weaker-than expected US economic data. Comments from Asian monetary officials that Asian central banks would keep buying US Treasuries even if the US credit rating were to be cut	
May 6, 2010	-2.34		0.21 (1.00)	Weak AU retail report. Mounting fears over the Greek debt, Greek riots	GBP^v, JPY^u, JPY^v
May 17, 2010	-0.95		0.12 (0.99)	Meeting of the Reserve Bank of Australia. No interest rate rise	
May 19, 2010	-2.00		0.23 (1.00)	German ban on naked short sales of euro-zone government bonds and CDSs	GBP^v
June 29, 2010	-2.71		0.13 (0.67)	Dismal reading of the US consumer confidence. Fear about the pace of global growth	

Table A1: AUD, CHF, GBP, JPY events continued

CHF Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
June 2, 1987	2.25	1.29 (0.71)		Paul Volcker leaves the Fed	JPY^u
Aug 18, 1987	1.97	1.27 (0.75)		Disappointing US trade deficit report	JPY^u
Nov 5, 1987	3.08	1.30 (0.61)		Pessimism over the US budget deficit negotiations. Comments by the US Treasure Secretary – the US may be willing to let the dollar ease	
Jan 5, 1988	-3.08	-1.37 (0.64)		Coordinated intervention by the G7 on behalf of the Federal Reserve Board and the US Treasury	JPY^d
Apr 14, 1988	2.21	1.91 (0.94)		Disappointing US trade deficit report	JPY^u
Sep 18, 1989	2.45	1.03 (0.58)		Fear of central banks' intervention to depreciate strong dollar	JPY^u
Jan 2, 1990	-2.29	-1.12 (0.63)		Favorable US economic data: the US National Association of Purchasing Management announces about the growth in monthly index of economic activity	JPY^d
Jan 4, 1990	2.34	0.96 (0.56)		Intervention by the Bundesbank, the Bank of Japan, the Bank of England, and the Swiss National Bank. Sell US dollars	JPY^u
May 9, 1990	2.05	0.93 (0.58)		Japan buys CHF to redeem franc debt	
June 4, 1993	-2.45	-0.90 (0.51)		Stronger than expected US May jobs report	
Dec 28, 1994	1.93	0.52 (0.80)		Speculation that Mexico may have drawn on its multi-billion dollar lines of credit expanded by the US and Canada one week earlier to halt the peso's decline	
Jan 9, 1995	2.11	1.08 (0.64)		Concerns of a protectionist American trade policy. US Ambassador warns Japan that the Clinton administration might use Super 301 trade sanctions against Japan. Fed's surprise intervention to support the Mexican peso	
Mar 1, 1995	-0.13		0.29 (0.41)	Richmond Federal Reserve Bank President: Fed did not target foreign exchange rates	JPY^v
May 11, 1995	-3.85	-2.35 (0.81)		Optimistic US producer price figures	
May 25, 1995	3.63	2.56 (0.88)		Weak US economic figures. Fear that trade war with Japan will depreciate the US dollar. Vague rumors that Mexico might be forced to default on $20 billion US loan	
Aug 15, 1995	-3.30	-2.65 (0.94)		Unexpected intervention by the Fed, the Bundesbank, the Bank of Japan, and the Swiss National Bank. Buy US dollars	JPY^d
Sep 20, 1995	2.62	1.68 (0.80)		Unexpected widening of the US trade deficit	
Sep 21, 1995	2.70	1.87 (0.84)		Disappointment over the lack of intervention to support the US dollar by the Fed, the Bundesbank, and the Bank of Japan	JPY^u, GBP^u
July 16, 1996	2.49	2.31 (1.00)		US stock market plummets for the second consecutive day	
July 17, 1996	0.60		0.20 (1.00)	Markets anticipate Greenspan's speech on the economic outlook	GBP^v

Table A1: AUD, CHF, GBP, JPY events continued

CHF Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
May 20, 1997	3.01	2.56 (0.97)		Fed's announcement: no interest rate increase	JPY^u, JPY^v
July 15, 1997	-0.04		0.11 (0.99)	The US dollar is lifted by continued weakness in the Deutsche Mark	
Oct 28, 1997	1.10		0.16 (1.00)	Huge losses in the US equity market – largest drop since Black Monday	AUD^u
Jan 6, 1998	-0.27		0.10 (0.58)	Soaring US dollar. Concerns over the Asian currency crisis	AUD^v
Jan 26, 1998	-1.04		0.11 (0.87)	Clinton negates that he had an affair with a White House intern	GBP^v
Aug 28, 1998	2.82	2.62 (1.00)		Yeltsin dismisses the rumors he would quit over Russian's financial crises. Investors see Russia as a big risk to Latin America, and that's a big risk for the US	GBP^u
Aug 31, 1998	-0.15		0.19 (1.00)	Japan's Defence Agency reports that North Korea fires a ballistic missile into the Sea of Japan	GBP^v
Sep 11, 1998	-0.43		0.19 (1.00)	Markets are under influence of the political scandal around Clinton. Possibility of impeachment	GBP^v
Oct 2, 1998	0.23		0.10 (0.78)	Slow US jobs growth. Tumbling equity market in the US	
Oct 8, 1998	-0.57		0.09 (0.84)	Large moves in the US dollar/yen rate	AUD^v, JPY^v
Dec 11, 1998	0.58		0.11 (0.81)	Dollar is hurt by the Clinton impeachment proceedings	
Dec 29, 1998	0.01		0.09 (0.94)	Uncertainty about the January launch of the euro	JPY^v
Jan 14, 1999	0.28		0.10 (0.97)	Deepening of the Brazil's financial crises. Brazil's central bank president resigns. Official trading band for Brazil's real is widened	
Nov 24, 1999	-0.94		0.11 (0.57)	Weak euro-zone economic data	
Nov 29, 1999	-0.74		0.12 (0.55)	Intervention by the Bank of Japan to support the US dollar	
Jan 31, 2000	-0.42		0.11 (0.98)	Euro erodes further. Expectation of US strong economic data. Heightened expectation of the aggressive Fed credit tightening	
Feb 28, 2000	-0.35		0.21 (1.00)	Dramatic drop of the euro against the US dollar	GBP^v
Sep 7, 2000	0.02		0.13 (1.00)	The Euro keeps depreciating	
Oct 12, 2000	-0.06		0.11 (1.00)	Escalating conflict in the Middle East – Israel vs Palestina. Israel fires at targets near Yasser Arafat's headquarters, the US Navy destroyer is bombed in Yemen	
Oct 25, 2000	-1.30		0.09 (0.87)	US equities are recovering. Fading expectations of central bank intervention to support the Euro against the rampant US dollar	GBP^v
Sep 11, 2001	2.76	1.81 (0.82)		Terrorist attack on the US	GBP^v, JPY^v

Table A1: AUD, CHF, GBP, JPY events continued

CHF Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Sep 12, 2001	-1.05		0.14 (0.97)	Aftermath of the terrorist attacks on the US. The ECB, the Swiss National Bank, and the Bank of Japan add liquidity to the financial system	
Sep 17, 2001	1.10		0.11 (0.54)	Markets are turbulent ahead of Wall Street's open after September 11. Chairman of the Swiss National Bank claims that it is up to currency markets to set exchange rates; the mandate of the Swiss National Bank is price stability. Coordinated cut of the interest rates by the Fed, the Bank of Canada, the ECB, the Swiss National Bank, and the Swedish Riksbank	AUD^v, JPY^v
Dec 24, 2001	-2.50	-2.31 (0.99)		Argentina's massive debt constraints euro	
Dec 25, 2001	0.25		0.09 (0.98)	Argentina stops servicing most of its foreign debt	
Jan 2, 2002	1.12		0.14 (1.00)	Euro is boosted by launch of physical currency	
Jan 25, 2002	-1.83		0.09 (0.92)	Alan Greenspan's says that the US economy is coming out of its recession	
June 24, 2002	-0.01		0.10 (0.79)	Bush's speech on the Middle East boosts stock market. Intervention by the Bank of Japan to support the US dollar	
June 26, 2002	0.26		0.11 (0.52)	Accounting WorldCom scandal. US Securities and Exchange Comission launches investigation	GBP^v
Nov 18, 2003	2.52	0.89 (0.51)		Geopolitical jitters: weekend bombings in Turkey and reports that al-Qaeda could target Japan. The US reduces import quotas on selected Chinese textiles. Fear that protectionism would hurt the US economic recovery	
Nov 7, 2007	0.96		0.11 (1.00)	Ben Bernanke emphasizes bleak picture of the US economy	GBP^v, JPY^v
Mar 17, 2008	1.38		0.15 (0.97)	JP Morgan Chase offers to acquire Bear Sterns at a price of 2 US dollars Dramatic sell-off in global equity market	
June 9, 2008	-0.88		0.09 (0.60)	Better than expected US pending home sales data	
Aug 8, 2008	-1.85		0.10 (0.99)	President of the ECB predicts that eurozone economy would weaken substantially in the coming months	GBP^v
Sep 15, 2008	1.30		0.12 (0.99)	Lehman Brothers collapse	AUD^v, GBP^v, JPY^u, JPY^v
Sep 29, 2008	0.03		0.13 (0.95)	Unprecedented coordinated attempt to pour liquidity into the financial system through short-term loans by the Fed, the ECB, the Bank of Japan, the Reserve bank of Australia, and the Bank of England. The US House of Representatives rejects a proposed $700 billion financial bailout package	AUD^v, GBP^v, JPY^v
Oct 6, 2008	-1.74		0.12 (0.98)	European and US stocks are crashed	AUD^v, AUD^d, GBP^v, JPY^v

Table A1: AUD, CHF, GBP, JPY events continued

CHF Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Oct 22, 2008	-1.04		0.15 (0.72)	Investors deleverage	AUD^v, GBP^v, JPY^v
Nov 21, 2008	-0.06		0.11 (0.93)	Surprise interest rate cut by the Swiss National Bank	
Dec 15, 2008	1.56		0.23 (0.99)	Widely expected interest rate cut by the Fed. Concerns over the health of the US economy and the impact of the US government's rescue plan	GBP^v
Dec 17, 2008	4.69		0.23 (0.98)	Fed decides to cut interest rates to a zero level	AUD^v, GBP^v
Dec 29, 2008	0.73		0.19 (0.99)	Israeli air strikes in the Gaza Strip boost dollar-denominated oil prices	
Feb 10, 2009	0.63		0.10 (0.92)	The US Treasury Secretary announces a plan to rescue the banking system which disappoints the market. The US Senate passes a massive economic stimulus package	AUD^v
Mar 12, 2009	-2.74		0.09 (0.90)	The Swiss National Bank targets to decrease LIBOR	
May 22, 2009	0.74		0.09 (0.83)	Signs of higher inflation in the US. US Labor Department report: unemployment hits a record high	GBP^v
Aug 3, 2009	0.85		0.09 (0.91)	Signs of recovery from data on manufacturing surveys across the globe	
Feb 4, 2010	-0.69		0.11 (0.51)	Strong economic US data	
May 5, 2010	-1.37		0.12 (0.69)	Turbulent European markets, debt problems	

Table A1: AUD, CHF, GBP, JPY events continued

GBP Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Jan 20, 1986	0.02		0.29 (0.54)	Financial representatives of the US, the UK, France, West Germany, and Japan reject a Japanese proposal of interest rate cut	
Mar 7, 1988	2.46	1.38 (0.73)		The Bank of England abandons its defense of the 3.00-mark level. Expected negative release of the US trade figures	
Sep 25, 1989	2.87	1.26(0.64)		G-7 meeting stresses that strong USD contributes to a world trade imbalance. Coordinated intervention by the Bank of Japan, the Fed, the Bank of Canada, the Swiss National Bank, the Bank of France, the Bank of Italy, and the central bank of Denmark. Sell USD	JPY^u
Oct 26, 1989	-2.39	-1.23 (0.67)		Surprise news – Britain's finance minister Lawson has resigned. Greenspan says that the Fed is concerned about inflation	
Jan 9, 1992	-3.38	-1.34 (0.62)		Speculation about devaluation or an ERM realignment	
July 20, 1992	-2.25	-0.87 (0.54)		Two rounds of concerted central bank intervention to support the US dollar by the Bundesbank, the Fed, and Western-European central banks	
Sep 4, 1992	1.09		0.25 (0.39)	Bad unemployment US data. UK Treasury announces that it would borrow money in foreign currency to buy pounds	
Sep 8, 1992	0.63		0.25 (0.47)	The Bank of England announces that it temporally stops linking the Finnish markka to the Deutsche mark. Finland is expected to devalue. Investors buy the Deutsche mark; sterling is under pressure, US dollar suffers even more	
Dec 29, 1993	-2.03	-1.36 (0.79)		Positive US economic data	
Aug 26, 1994	-1.86	-0.98 (0.65)		Positive US economic data	
Sep 21, 1995	2.54	2.35 (1.00)		Growing concern about the monetary system in Europe. Disappointment over the lack of intervention to support the US dollar by the Fed, the Bundesbank, and the Bank of Japan.	CHF^u, JPY^u
May 29, 1996	1.14	0.87 (0.81)		Richmond Federal Reserve Bank President says that the Fed may need to move toward greater monetary restraint	
May 30, 1996	0.17		0.09 (0.90)	Surprise positive UK economic data. Germany decides to keep interest rates steady contrary to expectations to cut the rates	
July 17, 1996	-0.76		0.10 (0.82)	Markets anticipate Greenspan's speech on the US economic outlook	CHF^v
Oct 31, 1996	-0.33		0.10 (0.80)	Short-covering before the key US employment data release helps USD closes firmer	
Nov 5, 1996	0.16		0.11 (0.62)	US presidential elections	
Dec 3, 1996	-2.23	-2.03 (1.00)		British Chancellor of the Exchequer says that strong pound worries the UK business, UK is exempted from a proposed European stability act in the run-up to a single currency	AUD^d

Table A1: AUD, CHF, GBP, JPY events continued

GBP Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Dec 6, 1996	0.73		0.09 (0.89)	Surprisingly weak US payrolls report	
Jan 6, 1997	0.54		0.10 (0.52)	British Prime Minister refuses to rule out sterling's participation in European Economic and Monetary Union.	
Jan 23, 1997	-0.30		0.10 (0.93)	Strong economic data in the US. Signs of economic weakness in Japan and Germany. Remarks suggesting that all G7 countries want dollar to continue to rise	
Aug 7, 1997	-1.02		0.09 (1.00)	The Bank of England: no need to increase interest rates further	
Oct 29, 1997	0.23		0.18 (1.00)	Jittering US and Asian equity markets	
Jan 26, 1998	-0.92		0.10 (0.52)	Clinton negates that he had an affair with a White House intern	CHF^v
Aug 28, 1998	1.88	1.74 (0.98)		Yeltsin dismisses the rumors he would quit over Russian's financial crisis. Investors see Russia as a big risk to Latin America, and that's a big risk for the US	CHF^u
Aug 31, 1998	-0.04		0.10 (1.00)	Japan's Defence Agency reports that North Korea fires a ballistic missile into the Sea of Japan	CHF^v
Sep 11, 1998	-1.00		0.09 (0.80)	The Bank of England's Monetary Policy Committee: British inflation could fall below government's target. Fears that the US Congress may launch impeachment against Clinton are lifted	CHF^v
Oct 9, 1998	-0.17		0.16 (0.96)	Instability in Brazil. Expectations of interest rate cuts in the UK and the US	
Dec 4, 1998	-0.11		0.11 (0.93)	Positive employment data in the US: surprisingly strong job growth	
Jan 4, 1999	-0.00		0.10 (0.61)	The first day of Euro trade	
Nov 30, 1999	-0.52		0.16 (1.00)	Intervention by the Bank of Japan: sell yen for the US dollar. Speculation that the ECB will intervene to support the Euro	
Feb 28, 2000	0.03		0.10 (0.63)	Dramatic drop of the Euro against the US dollar	CHF^v
Apr 28, 2000	-1.33		0.09 (0.80)	Weak UK growth data dampens expectations of interest rate increase	
May 11, 2000	-0.55		0.08 (0.97)	Inflation report of the Bank of England: interest rates won't be raised for some time Fear of rasing the US interest rates	
Sep 8, 2000	-1.61		0.08 (0.98)	Speculation that the UK interest rates would stay on hold for the months to come amid the benign inflation	
Oct 17, 2000	0.18		0.10 (0.66)	Progress at an emergency Middle East summit. Firmer stock markets, easing oil prices. Sterling is weighed down by relentless euro weakness	
Oct 25, 2000	-1.25		0.09 (0.66)	US equities are recovering. Quarterly survey by the Confederation of the UK industry shows that the country's manufacturing sector suffers a profit squeeze	CHF^v

Table A1: AUD, CHF, GBP, JPY events continued

GBP Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Jan 6, 2001	-1.51		0.09 (1.00)	Expected cut of the UK interest rate. Fear that the UK is heading for a slowdown. Fear that US is sliding into recession	
Sep 11, 2001	1.34		0.13 (0.99)	Terrorist attacks on the US	CHF^u, JPY^v
June 26, 2002	1.18		0.13 (1.00)	Accounting WorldCom scandal. US securities and Exchange Comission launches investigation. British Prime Minister Brown says that he supports any decision the Bank of England might make to increase interest rates in order to prevent house prices rising too far and stop strong consumer demand from high inflation	CHF^v
Nov 24, 2006	0.87		0.07 (0.99)	The deputy governor of the People's Bank of China: dollar's recent decline increased risk for Asian reserve assets; possibility of selling the US dollar.	
Nov 7, 2007	0.76		0.08 (1.00)	Ben Bernanke: bleak picture of the US economy	CHF^v, JPY^v
Dec 17, 2007	0.16		0.10 (0.62)	US Trade deficit fell to its lowest level in two years. Falling global stock prices	
Mar 14, 2008	-0.64		0.09 (0.77)	The US government and JPMorgan Chase bail out Bear Sterns	JPY^v
Aug 8, 2008	-1.16		0.11 (1.00)	President of the ECB: eurozone economy would weaken substantially in the coming months	CHF^v
Sep 15, 2008	0.37		0.11 (1.00)	Lehman Brothers collapse	AUD^v, CHF^v, JPY^u, JPY^v
Sep 29, 2008	-1.96		0.13 (0.96)	Unprecedented coordinated attempt to pour liquidity into the financial system through short-term loans by the Fed, the ECB, the Bank of Japan, the Reserve bank of Australia, and the Bank of England. The US House of Representatives rejects a proposed $700 billion bailout package	AUD^v, CHF^v, JPY^v
Sep 30, 2008	-1.56		0.08 (0.82)	Bank crisis deepens in Europe. Banking bailouts	AUD^v
Oct 6, 2008	-1.55		0.13 (0.99)	European and US stocks are crashed	AUD^d, AUD^v, CHF^v, JPY^v
Oct 21, 2008	-2.63		0.17 (0.97)	Bernanke's speech about fiscal stimulus supports the view that the US will recover from a global economic slowdown earlier than other countries. Fear that European banks may be forced to pay default protection at a Lehman Brothers Holding CDS settlement	
Oct 22, 2008	-2.65		0.18 (0.97)	Investors deleverage	AUD^v, CHF^v, JPY^v
Oct 24, 2008	-2.06		0.31 (1.00)	The White House announces that the widespread recession is inevitable. British Prime Minister acknowledges that the UK is likely entering recession. Global stocks plumb multi-year lows	AUD^v, JPY^v
Oct 30, 2008	0.49		0.31 (1.00)	The Fed cuts interest rates and stresses downside economic risks	
Nov 12, 2008	-2.76		0.20 (1.00)	Turmoil on equity markets, thin trading before the Veteran's day in the US. The Bank of England considers a further cut of the interest rates as disinflation is forecasted	AUD^v, JPY^v
Dec 1, 2008	-3.25		0.09 (0.61)	Meltdown in the equity markets	

Table A1: AUD, CHF, GBP, JPY events continued

GBP Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Dec 15, 2008	2.40		0.08 (0.75)	Widely expected interest rate cut by the Fed. Concerns over the health of the US economy and the impact of the US government's rescue plan	CHF^v
Dec 17, 2008	-0.28		0.13 (0.97)	Fed decides to cut interest rates to a zero level. The Bank of England considers a larger interest rate cut than before	AUD^v, CHF^v
Jan 5, 2009	1.05		0.07 (0.62)	Hopes that fresh stimulus plans from the US and Germany would help the global economy recover. Rebounding stock market encourages investors to pick up higher-yielding currencies. Pound takes advantage from the pressure on Euro	AUD^v
Jan 19, 2009	-2.14		0.09 (0.67)	The UK expands bailout package for the banking system. The UK government decides to take a larger stake in RBS	
Jan 20, 2009	-3.47		0.19 (0.97)	Crisis in UK banking sector	JPY^v
Feb 11, 2009	-1.00		0.09 (0.87)	Governor of the Bank of England: perspectives of implementing quantitative easing	
May 22, 2009	0.57		0.09 (0.67)	Signs of higher inflation in the US. US Labor Department report – unemployment hits a record high	CHF^v
Sep 28, 2009	-0.43		0.08 (0.75)	Traders interpret comments by governor of the Bank of England as suggesting that British authorities would be comfortable with a weaker pound	
Feb 5, 2010	-0.72		0.08 (0.88)	Stronger than expected US service-sector data	
Mar 1, 2010	-1.63		0.11 (1.00)	Worries about the outcome of forthcoming UK general election and ability of the UK government to remedy the high fiscal deficit	
Mar 22, 2010	0.58		0.08 (0.77)	Director of currency research at Global Forex Trading: final approval of the health care bill has contributed to the US dollar weakness	
May 6, 2010	-1.80		0.17 (1.00)	General election in the UK. Mounting fears over the Greek debt, Greek riots	AUD^v, JPY^u, JPY^v
May 19, 2010	0.77		0.08 (0.73)	German ban on naked short sales of euro-zone government bonds and CDSs	AUD^v

Table A1: AUD, CHF, GBP, JPY events continued

JPY Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Oct 24, 1986	-2.02	-1.30 (0.77)		Positive US trade deficit report	
Jan 14, 1987	1.90	0.73 (0.50)		The Reagan administration was reported to support decline of the US dollar in order to maintain the trade deficit	AUD^d
June 2, 1987	2.21	1.36 (0.76)		Paul Volcker leaves the Fed	CHF^u
Aug 18, 1987	2.59	2.16 (0.95)		Disappointing US trade deficit report	CHF^u
Dec 10, 1987	2.28	1.04 (0.62)		Record US trade deficit report	
Dec 28, 1987	2.37	0.80 (0.50)		Persistent US budget and trade deficit	
Jan 5, 1988	-3.38	-2.47 (0.90)		Coordinated intervention by the G7 on behalf of the Federal Reserve Board and the US Treasury	CHF^d
Jan 15, 1988	-2.94	-1.85 (0.82)		Positive US trade deficit report	
Apr 14, 1988	2.13	2.11 (1.00)		Disappointing US trade deficit report	CHF^u
Oct 11, 1988	1.67	0.80 (0.58)		Disagreement among the US, Germany, and Japan how to keep the US dollar stable Failed intervention by the Bundesbank, the Bank of Tokyo, and the Bank of England	
Oct 12, 1988	1.61	0.68 (0.51)		Expectations of very high US trade deficit figures to be released on Oct, 13	
Sep 18, 1989	1.96	0.95 (0.61)		Intervention by the Bank of Japan to support the yen	CHF^u
Sep 25, 1989	2.00	1.09 (0.68)		G-7 meeting: strong US dollar contributes to a world trade imbalance. Fear of intervention	GBP^u
Jan 2, 1990	-1.70	-1.25 (0.83)		Rumors about a political scandal in Japan. Favorable US economic data: the US National Association of Purchasing Management announces about the growth in monthly index of economic activity	CHF^d
Jan 4, 1990	1.61	1.21 (0.83)		Intervention by the Bundesbank, the Bank of Japan, the Bank of England, and Swiss National Bank. Sell USD	CHF^u
Apr 19, 1990	1.64	0.73 (0.53)		Japanese stock market is recovering from the recent plunge. Persistent sentiment that the US dollar has been overbought. US Treasury Undersecretary said that yen should be stronger based on Japan's economic fundamentals	
May 11, 1990	2.75	2.46 (0.99)		Negative announcements of the US Labor Department and Department of Commerce.	
Jan 17, 1991	2.95	2.54 (0.97)		Gulf War: start of the Desert Storm	
Nov 27, 1991	-1.68	-0.73 (0.53)		False rumor of a second Soviet coup. Good news about the US economy	
Jan 20, 1992	2.77	2.40 (0.98)		Intervention by the Fed and the Bank of Japan. Buy the yen	
May 12, 1992	2.03	1.68 (0.93)		Treasury department official (Mulford): huge trade surplus of Japan, G7 should intervene	

Table A1: AUD, CHF, GBP, JPY events continued

JPY Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Sep 22, 1992	2.20	0.91 (0.57)		French approval of the Maastricht treaty	
Feb 9, 1993	2.09	1.42 (0.82)		European parliament - yen is undervalued, raised speculation that the Clinton administration wants to see a stronger yen	
June 11, 1993	-0.12		0.38 (0.59)	Expectation of the CPT release. No US interest rate tightening	
Aug 19, 1993	-4.06	-3.41 (0.97)		Intervention by the Fed and the Bank of Japan to support the US dollar after news on widest US trade gap since Oct 1987	
Feb 14, 1994	4.87	4.41 (1.00)		US failure to reach a trade pact with Japan	
Mar 1, 1995	-0.09		0.61 (0.88)	Richmond Federal Reserve Bank President: Fed did not target foreign exchange rates	CHF^v
Aug 2, 1995	-3.47	-1.65 (0.68)		Japan's Finance Minister' speech: promote Japanese investments overseas	
Aug 15, 1995	-3.80	-2.21 (0.78)		Unexpected intervention by the Fed, the Bundesbank, the Bank of Japan, and the Swiss National Bank. Buy US dollar	CHF^d
Sep 21, 1995	3.45	2.35 (0.86)		New economic stimulus program of Japan. Disappointment over the lack of intervention to support the US dollar by the Fed, the Bundesbank, and the Bank of Japan	CHF^u, GBP^u
Apr 30, 1996	-0.20		0.17 (0.86)	Strong US economic data. Increase of the US interest rate	
Jan 29, 1997	-0.88		0.14 (0.98)	Expectation of the US interest rate increase due to sharply higher inflation	
May 9, 1997	2.94	2.51 (1.00)		Japan's Finance Minister: probable intervention to limit the US dollar rise	
May 15, 1997	1.06		0.14 (0.86)	US economic reports generate uncertainty regarding the interest-rate policy	
May 20, 1997	2.66	0.91 (0.53)	0.17 (0.59)	Fed's announcement: no interest rate increase.	CHF^u
June 9, 1997	1.13		0.21 (1.00)	US Trade Representative: "the US won't tolerate a widening trade gap with Japan"	
Aug 8, 1997	3.18	2.82 (0.99)		Sterling's free fall after a Bundesbank warning that German interest rates could rise	
Jan 5, 1998	-0.85		0.12 (0.83)	Alan Greenspan's speech: global deflation	
June 12, 1998	0.17		0.20 (0.89)	Release of official Japanese figures: recession	
June 15, 1998	-1.36		0.16 (0.57)	No significant news. Yen is victim of the Asian crisis and weak state of the JP economy	
June 17, 1998	4.57	3.19 (0.93)		Coordinated intervention by the US and Japan. Buy the yen for the US dollar	
Sep 8, 1998	-0.18		0.12 (0.65)	A surprise cut in Japanese interest rate	
Oct 7, 1998	6.93	5.33 (1.00)		Greespan: negative prospects for the US economy, credit crunch. Possible further cut of the US interest rate. Threat of Clinton impeachment	AUD^u
Oct 8, 1998	2.02		0.49 (1.00)	Large moves in the US dollar/yen on Oct 7. Japanese shares collapse	AUD^v, CHF^v

Table A1: AUD, CHF, GBP, JPY events continued

JPY Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Nov 11, 1998	0.31		0.15 (0.57)	Japanese Liberal Democratic Party confirms to cut consumption tax	
Dec 3, 1998	1.67		0.18 (0.99)	Negative news from the US equity market	
Dec 10, 1998	0.69		0.13 (0.58)	Japanese Finance Minister suggests that the US would like to have a weaker dollar	
Dec 29, 1998	0.38		0.13 (0.53)	Uncertainty about the January launch of the Euro. Sharp increase of the Japanese interest rates and stock prices	CHF^v
Feb 2, 1999	2.61		0.15 (0.98)	Very large increase of long-term interest rates in Japan	
Feb 16, 1999	-2.46		0.13 (0.86)	Japan's Ministry of Finance: resume purchase of outstanding government bonds	
July 20, 1999	-0.60		0.17 (0.51)	Expected intervention by the Bank of Japan to buy US dollar	
Aug 18, 1999	1.84		0.17 (1.00)	Disappointing US trade deficit report	
Sep 15, 1999	1.12		0.22 (0.95)	Growing optimism about the strength of the Japanese economy. Record US current account deficit	
Sep 21, 1999	1.17		0.13 (0.67)	US trade deficit ballooned to a record $25.2 billion	
Nov 26, 1999	2.45	1.46 (0.76)	0.30 (1.00)	JP Trade Ministry: positive news about GDP	
Mar 31, 2000	2.65		0.18 (0.90)	Positive quarterly survey of the Bank of Japan	
Jan 15, 2001	-0.40		0.13 (0.79)	Rumors that two large JP banks were facing financial difficulty in morning trading	
Mar 2, 2001	-1.38		0.16 (0.57)	Record figures for Japanese unemployment and fall in Tokyo area consumer prices	
May 23, 2001	2.24	0.89 (0.59)		Negative economic news in Europe	
Sep 11, 2001	1.33		0.16 (1.00)	Terrorist attack on the US	CHF^u, GBP^v
Sep 17, 2001	-0.41		0.16 (0.61)	Intervention by the Bank of Japan to support the US dollar. Markets are turbulent ahead of Wall Street's open after September 11. Coordinated cut of the interest rates by the Fed, the Bank of Canada, the ECB, the Swiss National Bank, and the Swedish Riskbank	AUD^v, CHF^v
Mar 7, 2002	2.40	0.98 (0.60)		US imposes tariffs on steel imports	
Dec 2, 2002	-1.55	-1.20 (0.75)		Japanese Finance minister calls for yen weakening to support Japanese companies	
Sep 19, 2003	1.09		0.13 (0.97)	Speculation about the intention of G7 to object Japan's weakening intervention policies	
Feb 20, 2004	-1.88	-2.00 (0.98)		Japan is on terror alert. Japan dispatches troops on a humanitarian mission to Iraq	AUD^d
July 21, 2005	2.35	1.87 (0.98)		China revalues its currency	
Dec 14, 2005	2.15	1.58 (0.92)		Negative US trade deficit report	

Table A1: AUD, CHF, GBP, JPY events continued

JPY Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
Feb 27, 2007	2.27	1.79 (0.99)		Steepest single-session US stock market decline in more than five years	
July 10, 2007	1.34	1.04 (0.82)		Moody's downgrades subprime-backed bonds. Standard and Poor's warns that could cut ratings on $12 billion of bonds backed by US subprime mortgages	
Aug 16, 2007	2.34		0.22 (0.91)	The Fed unexpectedly cuts the discount rate on its lending to banks	AUD^v, AUD^d
Nov 7, 2007	1.83		0.14 (0.89)	Ben Bernanke: bleak picture of the US economy	CHF^v, GBP^v
Nov 9, 2007	1.70		0.17 (0.98)	Oil prices near record highs. Persistent fears of ongoing credit crisis	
Feb 29, 2008	1.55		0.15 (0.98)	Worsening US economic data. Fears of further aggressive US interest rate cut	
Mar 13, 2008	1.12		0.17 (0.65)	Negative US economic data and concerns that Japanese authorities may intervene	
Mar 14, 2008	1.55		0.16 (0.57)	The US government and JPMorgan chase bail out Bear Sterns	GBP^v
Sep 5, 2008	-0.61		0.14 (0.71)	Downbeat US employment figures, disappointing retail sales data, growing speculation about troubles at major hedge funds	AUD^v
Sep 15, 2008	3.08	1.68 (0.75)	0.28 (1.00)	Lehman Brothers collapse	AUD^v, CHF^v, GBP^v
Sep 29, 2008	1.73		0.13 (0.55)	Unprecedented coordinated attempt to pour liquidity into the financial system through short-term loans by the Fed, the ECB, the Bank of Japan, the Reserve bank of Australia, and the Bank of England. The US House of Representatives rejects a proposed $700 billion financial bailout package	AUD^v, CHF^v, GBP^v
Oct 6, 2008	3.37		0.34 (1.00)	European and US stocks are crashed	AUD^d, AUD^v, CHF^v, GBP^v
Oct 8, 2008	2.31		0.15 (0.63)	Coordinated interest rate cut: the Fed, the ECB, the Bank of England, the Bank of Canada, the Swiss National Bank, and the Swedish Riksbank	AUD^v
Oct 22, 2008	2.50		0.19 (0.50)	Investors deleverage	AUD^v, CHF^v, GBP^v
Oct 23, 2008	0.35		0.17 (0.67)	US stocks decline on crumbling global economic outlook	
Oct 24, 2008	3.11		0.19 (0.62)	The White House announces that the widespread recession in inevitable. British Prime Minister acknowledges that the UK is likely entering recession Global stocks plumb multi-year lows	AUD^v, GBP^v
Nov 12, 2008	2.74		0.20 (0.97)	Turmoil on equity markets, thin trading before the Veteran's day in the US	AUD^v, GBP^v
Dec 12, 2008	0.26		0.17 (0.96)	Negative US trade deficit news. Failure of a proposed US government plan to bail out US auto makers	
Jan 20, 2009	0.98		0.15 (0.54)	Crisis in UK banking sector	GBP^v
Jan 21, 2009	0.30		0.21 (0.90)	Crisis in UK banking sector	

Table A1: AUD, CHF, GBP, JPY events continued

JPY Date	Excess return	Jump in FX	Jump in Vol	Events/Sources of uncertainty	Impact on FX
May 12, 2009	1.06		0.15 (0.54)	Financial Times: does US deserves to keep triple A credit rating?	
July 8, 2009	2.14		0.14 (0.64)	Improvement in JP current account surplus	
Nov 27, 2009	0.07		0.15 (0.68)	Debt problems of Dubai World	
Dec 4, 2009	-2.57	-1.07 (0.57)		Positive report of the US Labor department	
Mar 24, 2010	-2.08	-1.46 (0.75)		Downgrading of the Portugal's credit rating by Fitch Ratings boosts the US dollar	
May 6, 2010	3.50	3.05 (0.99)	0.19 (1.00)	Mounting fears over the Greek debt, Greek riots	AUD^v, GBP^v
May 20, 2010	2.24		0.19 (0.98)	Concerns over policymakers' response to the euro zone debt crisis. Negative US economic data	
Sep 15, 2010	-3.21	-1.98 (0.83)		Intervention by the Bank of Japan	

Notes: This table summarizes information about events and sources of uncertainty which cause substantial movements in FX market qualified as jumps in prices or volatility. The second column is daily excess log currency returns in percent per day. The third and fourth columns provide estimates of the size of jumps and probability of jumps (in parentheses); size of jumps is in percent. The last two columns describe what news bring about dramatic movements in currency markets and which cross rates are affected. If we cannot attribute FX dynamics to specific news or events we indicate what type of uncertainty causes market turbulence. Source of news: Factiva.

Table A.2: Properties of real log excess returns. Sorting currencies by one-period yields

	Mean	Std Dev	Skewness	Kurtosis	Autocorrelation
Basket “Low”	-0.0056	0.0544	0.46	3.38	0.06
Basket “Intermediate”	-0.0025	0.0431	-0.33	3.33	0.13
Basket “High”	0.0046	0.0532	-0.03	3.44	0.14

Notes. Currency baskets are formed by sorting currencies by their corresponding short yields at a quarterly basis. Sample period: 1986 – 2011. Quarterly.

Table A.3: Composition of currency baskets. Sorting currencies by one-period yields

Currency	Basket “Low”	Basket “Intermediate”	Basket “High”
Australia	6	29	64
Canada	26	71	6
Denmark	8	70	15
Germany	34	8	10
Euro area	18	11	0
Japan	98	5	0
Norway	7	18	30
New Zealand	4	15	68
Sweden	28	34	14
Switzerland	87	8	0
UK	6	41	56
South Africa	0	0	58

Notes. At the end of each period (quarter), currencies are sorted by their respective short interest rates. Table entry shows the number of periods each currency belongs to each basket. Sample period: 1986 – 2011, at a quarterly frequency.

Table A.4: Estimated FX cash flow process (identification "Fast Inflation"). Sorting currencies by one-period yields

Parameter	Basket "Low"	Basket "Intermediate"	Basket "High"
$\log\delta$	-0.0028	-0.0131	-0.0063
	(-0.0183, 0.0154)	(-0.0290, -0.0003)	(-0.0279, 0.0100)
μ_g	1.0241	0.5541	-1.2649
	(0.3843, 1.7303)	(-0.0510, 1.1418)	(-2.0504, -0.4687)
μ_π	-0.7762	-2.8042	-1.7922
	(-1.2466, -0.3394)	(-3.1966, -2.3946)	(-2.3308, -1.2560)
μ_i	0.4351	2.2262	1.9015
	(0.0317, 0.8669)	(1.8530, 2.5869)	(1.4407, 2.3758)
μ_σ	-0.0082	0.0008	-0.0044
	(-0.0176, 0.0241)	(-0.0129, 0.0090)	(-0.0227, 0.0058)
ξ_g	0.0010	0.0042	0.0135
	(-0.0036, 0.0052)	(0.0005, 0.0076)	(0.0081, 0.0189)
ξ_π	-0.0040	0.0093	0.0142
	(-0.0099, 0.0013)	(0.0048, 0.0136)	(0.0090, 0.0197)
ξ_i	-0.0138	-0.0031	0.0027
	(-0.0172, -0.0104)	(-0.0052, -0.0010)	($7.86 \cdot 10^{-5}$, 0.0053)
ξ_σ	-0.0020	0.0029	0.0046
	(-0.0126, 0.0088)	(-0.0061, 0.0117)	(-0.0069, 0.0158)

Notes: At the end of each quarter, I sort currencies into three currency baskets based on their short interest rates. For each currency basket, I estimate the FX cash flow process:

$$\log\delta_{t,t+1} = \log\delta + \mu' Y_t + \sigma_t \xi' \varepsilon_{t+1} + \xi_v \sigma_t v_{t+1},$$

where $\mu = (\mu_g\ \mu_\pi\ \mu_i\ \mu_\sigma)'$ and $\xi = (\xi_g\ \xi_\pi\ \xi_i\ \xi_\sigma)'$. Quarterly. There are 95% confidence intervals in the brackets.

Table A.5: Estimated FX cash flow process (identification "Fast Consumption"). Sorting currencies by one-period yields

Parameter	Basket "Low"	Basket "Intermediate"	Basket "High"
$\log\delta$	-0.0029	-0.0130	-0.0065
	(-0.0196, 0.0139)	(-0.0288, -0.0003)	(-0.0288, 0.0098)
μ_g	1.0261	0.5603	-1.2489
	(0.2442, 1.7742)	(-0.0154, 1.1356)	(-2.0605, -0.4486)
μ_π	-0.7688	-2.8020	-1.7887
	(-1.2167, -0.3003)	(-3.2413, -2.3854)	(-2.3078, -1.2601)
μ_i	0.4296	2.2240	1.8913
	(-0.0292, 0.8529)	(1.8474, 2.6096)	(1.4129, 2.3742)
μ_σ	-0.0086	0.0009	-0.0045
	(-0.0187, 0.0231)	(-0.0121, 0.0090)	(-0.0219, 0.0060)
ξ_g	0.0022	0.0009	0.0084
	(-0.0026, 0.0071)	(-0.0028, 0.0045)	(0.0032, 0.0133)
ξ_π	-0.0037	0.0101	0.0180
	(-0.0091, 0.0020)	(0.0064, 0.0139)	(0.0126, 0.0233)
ξ_i	-0.0137	-0.0031	0.0027
	(-0.0175, -0.0103)	(-0.0053, -0.0009)	(0.0002, 0.0054)
ξ_σ	-0.0021	0.0031	0.0044
	(-0.0139, 0.0078)	(-0.0057, 0.0120)	(-0.0069, 0.0160)

Notes: At the end of each quarter, I sort currencies into three currency baskets based on their short interest rates. For each currency basket, I estimate the FX cash flow process:

$$\log\delta_{t,t+1} = \log\delta + \mu' Y_t + \sigma_t \xi' \varepsilon_{t+1} + \xi_v \sigma_t v_{t+1},$$

where $\mu = (\mu_g\ \mu_\pi\ \mu_i\ \mu_\sigma)'$ and $\xi = (\xi_g\ \xi_\pi\ \xi_i\ \xi_\sigma)'$. Quarterly. There are 95% confidence intervals in the brackets.

Table A.6: Data description

Country	Data availability	FX data/Datastream Mnemonics	Source of term-structure data/Mnemonics
Australia	1987:I – 2011:IV	AUSTDOL and USDOLLR	Wright [1987:I – 2009:I]
			Bloomberg [2009:II – 2011:IV]: ADSW1, ..., ADSW10
Canada	1986:I – 2011:IV	CNDOLLR and USDOLLR	Wright [1986:I – 2009:I]
			Bloomberg [2009:II – 2011:IV]: CDSW1, ..., CDSW10
Denmark	1988:III – 2011:IV	DANISHK and USDOLLR	Bloomberg [1988:III – 2011:IV]: DKSW1, ..., DKSW10
Euro Area	2004:III – 2011:IV	EURSTER and USDOLLR	Bloomberg [2004:III – 2011:IV]: EUSW1V3, ..., EUSW10V3
Germany	1986:I – 1998:IV	DMARKER and USDOLLR	Wright [1986:I – 1998:IV]
Japan	1986:I – 2011:IV	JAPAYEN and USDOLLR	Wright [1986:I – 2009:I]
			Bloomberg [2009:II – 2011:IV]: JYSW1, ..., JYSW10
New Zealand	1990:I – 2011:IV	NZDOLLR and USDOLLR	Wright [1990:I – 2009:I]
			Bloomberg [2009:II – 2011:IV]: NDSW1, ..., NDSW10
Norway	1998:I – 2011:IV	NORKRON and USDOLLR	Wright [1998:I – 2009:I]
			Bloomberg [2009:II – 2011:IV]: NKSW1, ..., NKSW10
South Africa	1997:II – 2011:IV	COMRAND and USDOLLR	Bloomberg [1997:II – 2011:IV]: SASW1, ..., SASW10
Sweden	1992:IV – 2011:IV	SWEKRON and USDOLLR	Wright [1992:IV – 2009:I]
			Bloomberg [2009:II – 2011:IV]: SKSW1, ..., SKSW10
Switzerland	1988:I – 2011:IV	SWISSFR and USDOLLR	Wright [1986:I – 2009:I]
			Bloomberg [2009:II – 2011:IV]: SFSW1, ..., SFSW10
UK	1986:I – 2011:IV	USDOLLR	Wright [1986:I – 2009:I]
			Bloomberg [2009:II – 2011:IV]: BPSW1, ..., BPSW10

Notes: Data availability and data sources.

Table A.7: Priors for the parameters of the vector-autoregression

Parameter	Prior
$\Theta_1 = F_1$	$\mathcal{N}(0.0010, 0.0015^2)$
$\Theta_2 = G_{11}$	$\mathcal{N}(0.30, 0.10^2)$
$\Theta_3 = G_{12}$	$\mathcal{N}(-0.20, 0.06^2)$
$\Theta_4 = G_{13}$	$\mathcal{N}(0.15, 0.15^2)$
$\Theta_5 = G_{14}$	$\mathcal{N}(0.00, 0.0010^2)$
$\Theta_6 = G_{24}$	$\mathcal{N}(0.00, 0.0010^2)$
$\Theta_7 = F_3$	$\mathcal{N}(0.00, 0.0005^2)$
$\Theta_8 = G_{31}$	$\mathcal{N}(0.00, 0.05^2)$
$\Theta_9 = G_{32}$	$\mathcal{N}(0.00, 0.05^2)$
$\Theta_{10} = G_{33}$	$\mathcal{N}(0.95, 0.015^2)$
$\Theta_{11} = G_{34}$	$\mathcal{N}(0.00, 0.0010^2)$
$\Theta_{12} = G_{44}$	$\mathcal{N}(0.95, 0.015^2)$
Σ	$\mathcal{IW}(b, B)$

Notes. Prior distributions for the twenty two parameters of the matrices F, G, and Σ. The hyperparameters of the inverse Wishart distribution for Σ are such that

$$b = \begin{pmatrix} 0.69 & -0.03 & 0.06 & -0.03 \\ -0.03 & 0.96 & 0.06 & -0.03 \\ 0.06 & 0.06 & 0.3 & -0.03 \\ -0.03 & -0.03 & -0.03 & 0.015 \end{pmatrix}$$

and $B = 8$.

Table A.8: Priors for the parameters of the FX cash flow process

Parameter	"Low"	"Intermediate"	"High"
$\log\delta$	$\mathcal{N}(-0.0027, 0.0110^2)$	$\mathcal{N}(-0.0205, 0.0114^2)$	$\mathcal{N}(-0.0114, 0.0135^2)$
μ_g	$\mathcal{N}(1.3585, 0.3560^2)$	$\mathcal{N}(-0.0856, 0.3582^2)$	$\mathcal{N}(-1.3681, 0.4174^2)$
μ_π	$\mathcal{N}(-0.8053, 0.2650^2)$	$\mathcal{N}(-2.7442, 0.2776^2)$	$\mathcal{N}(-1.8320, 0.3378^2)$
μ_i	$\mathcal{N}(0.7241, 0.3966^2)$	$\mathcal{N}(2.4383, 0.4052^2)$	$\mathcal{N}(-2.1729, 0.5271^2)$
μ_σ	$\mathcal{N}(0.0068, 0.0104^2)$	$\mathcal{N}(0.0164, 0.0109^2)$	$\mathcal{N}(0.0090, 0.0125^2)$
ξ_g	$\mathcal{N}(-0.0021, 0.0023^2)$	$\mathcal{N}(0.0053, 0.0021^2)$	$\mathcal{N}(0.0115, 0.0026^2)$
ξ_π	$\mathcal{N}(-0.0047, 0.0026^2)$	$\mathcal{N}(0.0081, 0.0025^2)$	$\mathcal{N}(0.0199, 0.0030^2)$
ξ_i	$\mathcal{N}(-0.0123, 0.0016^2)$	$\mathcal{N}(-0.0102, 0.0014^2)$	$\mathcal{N}(0.0063, 0.0014^2)$
ξ_σ	$\mathcal{N}(-0.0010, 0.0059^2)$	$\mathcal{N}(-0.0002, 0.0053^2)$	$\mathcal{N}(0.0052, 0.0068^2)$
ξ_v	$\mathcal{IG}(2, 0.0025)$	$\mathcal{IG}(2, 0.0025)$	$\mathcal{IG}(2, 0.0025)$

Notes. Prior distributions for the parameters of the FX cash flow process for the "Low", "Intermediate", and "High" baskets.

Table A.9: Model diagnostics for the macro VAR

Parameter	Skewness	Kurtosis	Autocorrelation
Fitted residuals of consumption growth			
Percentile 2.5%	-0.25	3.22	0.10
Mean	-0.04	3.66	0.25
Percentile 97.5%	0.18	4.32	0.38
Fitted residuals of inflation			
Percentile 2.5%	-0.44	3.04	0.71
Mean	-0.21	3.38	0.74
Percentile 97.5%	0.03	3.78	0.77
Fitted residuals of nominal rate			
Percentile 2.5%	-0.98	5.06	0.18
Mean	-0.71	5.96	0.22
Percentile 97.5%	-0.43	7.03	0.26
Fitted residuals of stochastic variance			
Percentile 2.5%	-0.24	2.50	-0.06
Mean	0.05	2.98	0.07
Percentile 97.5%	0.34	3.65	0.19

Notes. Posterior means and 2.5 and 97.5 percentiles are reported for the fitted residuals of the four equations of the vector autoregression (A.2.63).

Table A.10: Model diagnostics for the FX cash flow process. "Fast Inflation" identification

Parameter	Skewness	Kurtosis	Autocorrelation
"Low" basket			
Percentile 2.5%	-0.17	3.14	-0.04
Mean	0.23	3.97	0.04
Percentile 97.5%	0.63	5.44	0.12
"Intermediate" basket			
Percentile 2.5%	-0.29	3.15	0.01
Mean	0.05	3.98	0.07
Percentile 97.5%	0.42	5.40	0.14
"High" basket			
Percentile 2.5%	-0.82	2.94	-0.10
Mean	-0.36	3.84	-0.04
Percentile 97.5%	-0.04	5.77	0.03

Notes. Posterior means and 2.5 and 97.5 percentiles are reported for the fitted residuals of the FX cash flow for the three currency baskets. I use the identification strategy "Fast Inflation".

Table A.11: Model diagnostics for the FX cash flow process. "Fast Consumption" identification

Parameter	Skewness	Kurtosis	Autocorrelation
"Low" basket			
Percentile 2.5%	-0.15	3.15	-0.03
Mean	0.22	3.95	0.04
Percentile 97.5%	0.62	5.35	0.12
"Intermediate" basket			
Percentile 2.5%	-0.30	3.17	0.01
Mean	0.05	3.97	0.07
Percentile 97.5%	0.42	5.40	0.14
"High" basket			
Percentile 2.5%	-0.82	2.94	-0.10
Mean	-0.36	3.84	-0.04
Percentile 97.5%	-0.04	5.77	0.03

Notes. Posterior means and 2.5 and 97.5 percentiles are reported for the fitted residuals of the FX cash flow for the three currency baskets. I use the identification strategy "Fast Consumption".

Table A.12: Parameter estimates. Simulated economy

Parameter	Estimate	Confidence interval	True value
G_{13}	0.0021	(0.0014, 0.0029)	0.0013
F_2	$2.2 \cdot 10^{-5}$	$(-2.13 \cdot 10^{-5}, 6.2 \cdot 10^{-5})$	$3 \cdot 10^{-5}$
G_{21}	0.0014	(-0.0025, 0.0049)	0
G_{22}	0.9735	(0.9671, 0.9817)	0.979
G_{23}	$6.05 \cdot 10^{-6}$	$(-2.34 \cdot 10^{-5}, 3.81 \cdot 10^{-5})$	$-7.06 \cdot 10^{-6}$
G_{33}	0.9807	(0.9415, 0.9955)	0.987
Σ_{11}	$4.68 \cdot 10^{-5}$	$(2.83 \cdot 10^{-5}, 7.84 \cdot 10^{-5})$	$6.08 \cdot 10^{-5}$
Σ_{12}	$-8.26 \cdot 10^{-6}$	$(-2.6 \cdot 10^{-5}, 1.04 \cdot 10^{-5})$	0
Σ_{13}	$-4.23 \cdot 10^{-5}$	$(-1.82 \cdot 10^{-5}, 6.78 \cdot 10^{-5})$	0
Σ_{22}	$4.86 \cdot 10^{-4}$	$(3.04 \cdot 10^{-4}, 8.07 \cdot 10^{-4})$	$5.27 \cdot 10^{-4}$
Σ_{23}	$1.22 \cdot 10^{-6}$	$(-1.06 \cdot 10^{-4}, 4.20 \cdot 10^{-4})$	$-1.29 \cdot 10^{-4}$
Σ_{33}	0.0017	(0.0005, 0.0041)	0.0014

Notes. Estimates of the free parameters of the vector autoregression

$$Y_{t+1} = F + GY_t + \Sigma^{1/2}\sigma_t\varepsilon_{t+1}, \qquad \text{(A.23)}$$

with cross-equation restrictions

$$\begin{aligned}
&\log\beta + (\rho-1)e_1'F = 0, && \text{(A.29)}\\
&(\rho-1)e_1'Ge_1 = 0, && \text{(A.30)}\\
&1 + (\rho-1)e_1'Ge_2 = 0 && \text{(A.31)},\\
&(\rho-1)e_1'Ge_3 - (\rho(\alpha-\rho)P'\Sigma P/2 + (2\alpha - \alpha\rho - 1)e_1'\Sigma e_1/2 &&\\
&+(\alpha-\rho)P'\Sigma e_1) = 0, && \text{(A.32)}
\end{aligned}$$

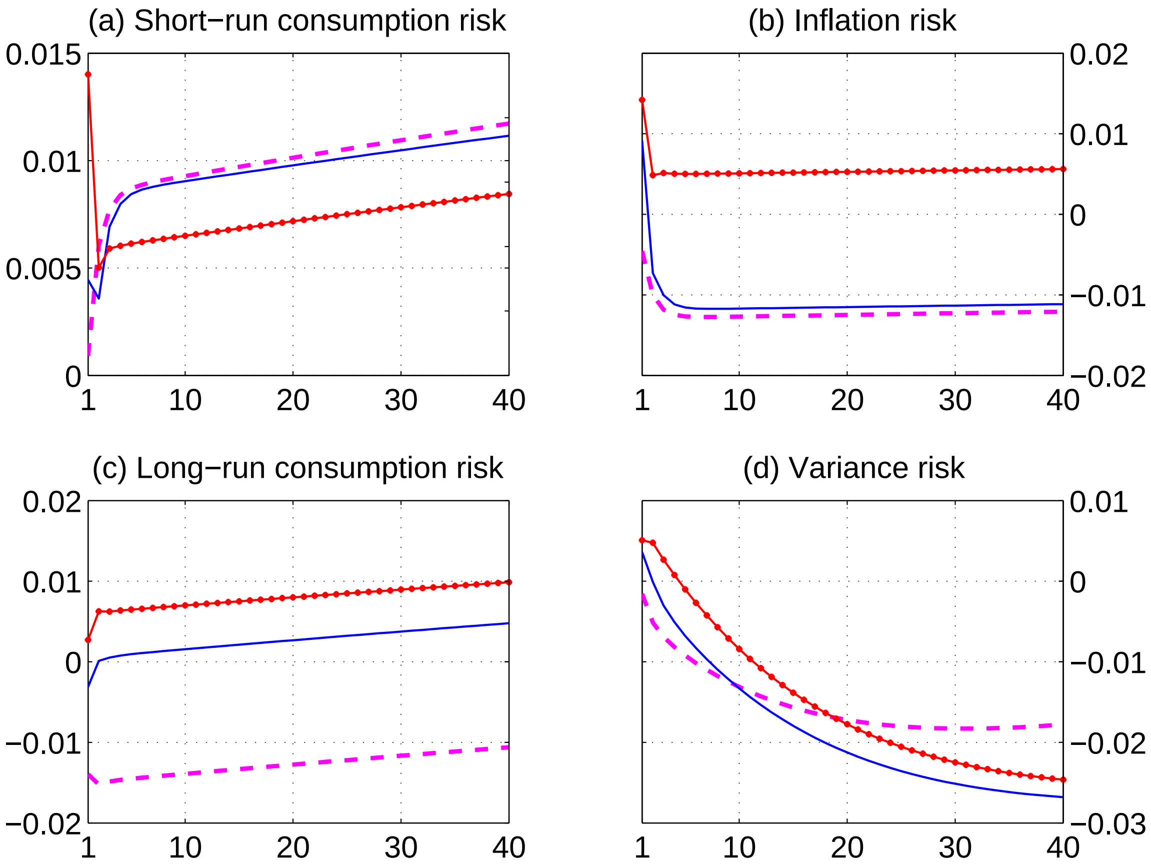
Table A.13: State loadings in the value function. Simulated economy

Parameter	Estimate	Confidence interval	True value
p_g	0.0938	(-0.0967, 0.2947)	0
p_r	60.4933	(48.9624, 76.8279)	68.3031
p_σ	0.0388	(-0.0476, 0.1718)	-0.0414

Notes. Loadings on the states $\log g_{t-1,t}$, $r^1_{t,f}$ and σ^2_t in the value function

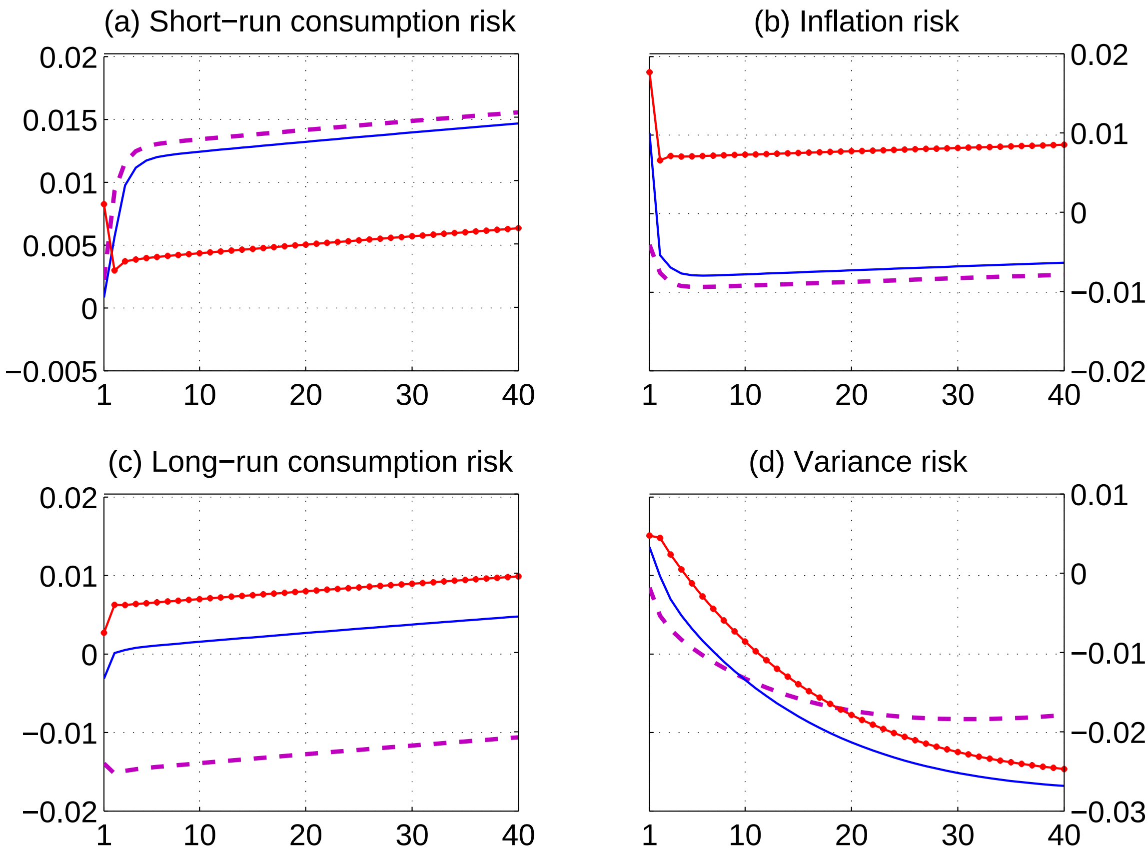
$$\log u_t = \log u + p_g \log g_{t-1,t} + p_r r^1_{t,f} + p_\sigma \sigma^2_t.$$

Figure A.1: Shock-exposure elasticity (identification “Fast Inflation”). Sorting currencies by one-period yields



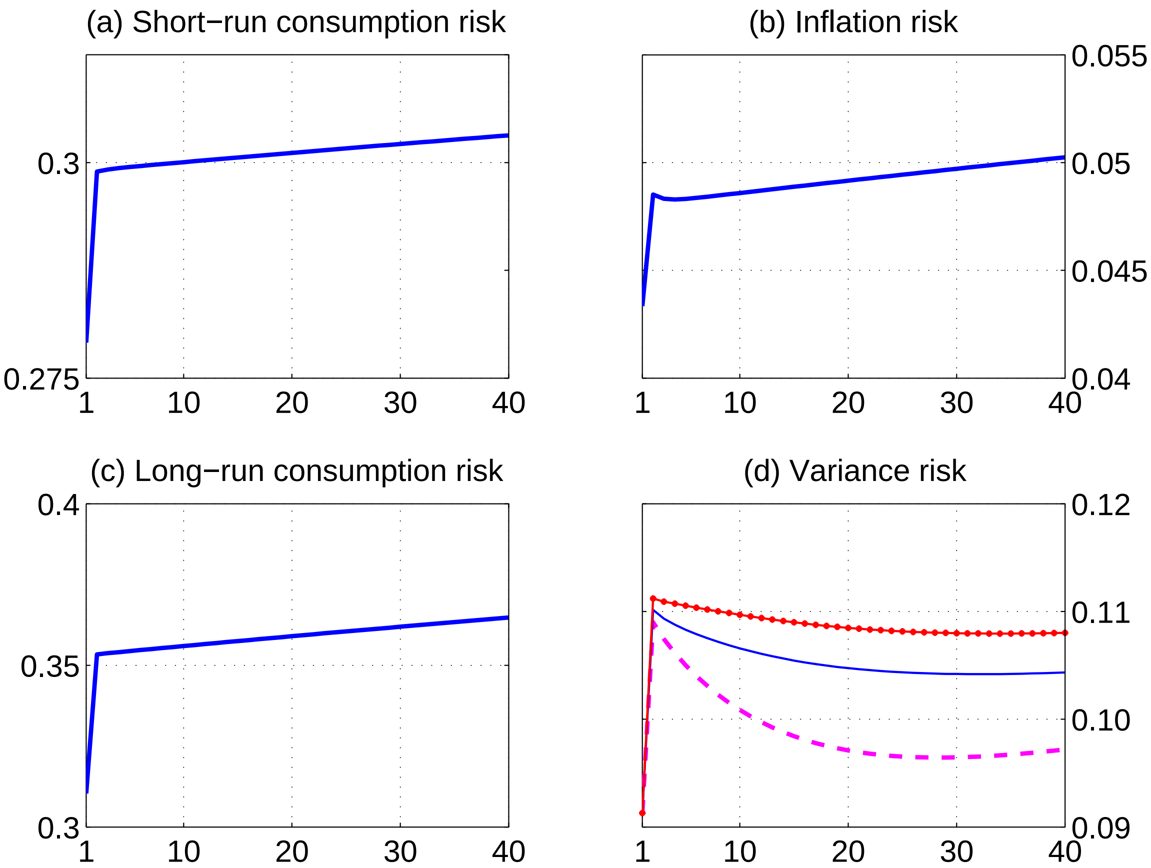
Panel (a) displays shock-exposure elasticity for the short-run risk. Panel (b) displays shock-exposure elasticity for the inflation risk. Panel (c) displays shock-exposure elasticity for the long-run risk. Panel (d) displays shock-exposure elasticity for the variance risk. Identification “Fast Inflation”. Quarterly. The magenta dashed line is for the basket “Low”, the blue solid line is for the basket “Intermediate”, the red marked line is for the basket “High”. The horizontal axes: from 1 quarter to 10 years.

Figure A.2: Shock-exposure elasticity (identification "Fast Consumption"). Sorting currencies by one-period yields



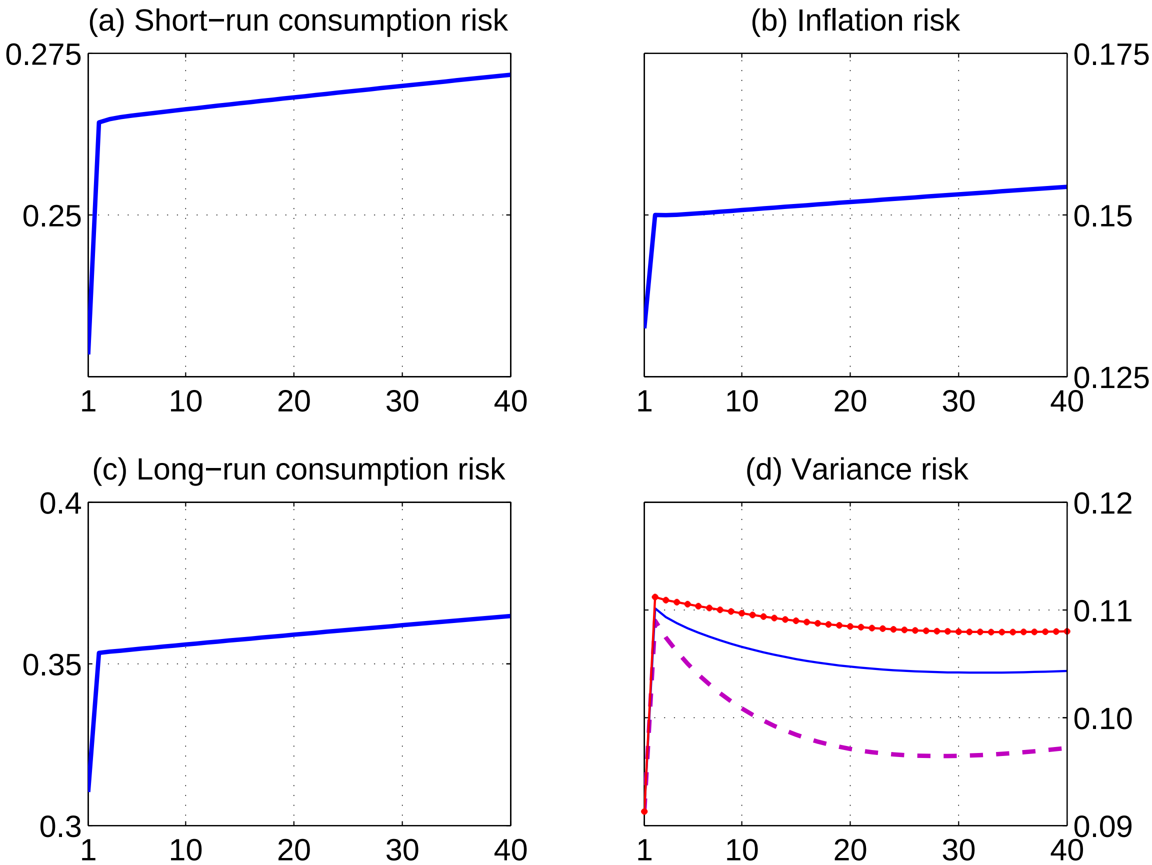
Panel (a) displays shock-exposure elasticity for the short-run risk. Panel (b) displays shock-exposure elasticity for the inflation risk. Panel (c) displays shock-exposure elasticity for the long-run risk. Panel (d) displays shock-exposure elasticity for the variance risk. Identification "Fast Consumption". Quarterly. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". The horizontal axes: from 1 quarter to 10 years.

Figure A.3: Shock-price elasticity (identification "Fast Inflation"). Sorting currencies by one-period yields



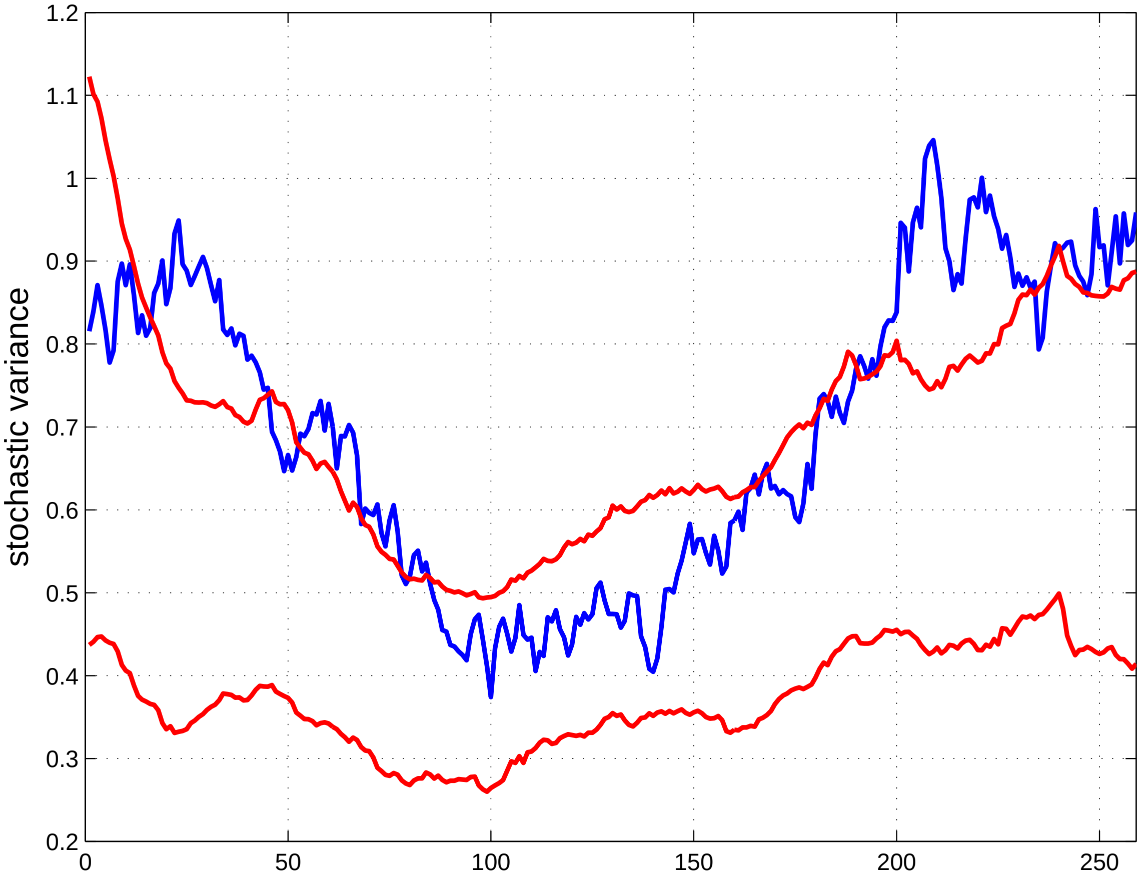
Panel (a) displays shock-price elasticity for the short-run risk. Panel (b) displays shock-price elasticity for the inflation risk. Panel (c) displays shock-price elasticity for the long-run risk. Panel (d) displays shock-price elasticity for the variance risk. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". The horizontal axes: from 1 quarter to 10 years. Identification "Fast Inflation". Quarterly.

Figure A.4: Shock-price elasticity (identification "Fast Consumption"). Sorting currencies by one-period yields



Panel (a) displays shock-price elasticity for the short-run risk. Panel (b) displays shock-price elasticity for the inflation risk. Panel (c) displays shock-price elasticity for the long-run risk. Panel (d) displays shock-price elasticity for the variance risk. Identification "Fast Consumption". Quarterly. The magenta dashed line is for the basket "Low", the blue solid line is for the basket "Intermediate", the red marked line is for the basket "High". The horizontal axes: from 1 quarter to 10 years.

Figure A.5: Stochastic variance of consumption growth. Simulated economy



The blue lines depict the simulated stochastic variance of consumption growth $\sigma_g^2 \sigma_t^2$. The red line delineates the 95% confidence interval for the estimated stochastic variance.