LONDON BUSINESS SCHOOL

Essays in Asset Pricing with Market Imperfections

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A thesis submitted to the London Business School for the degree of Doctor of Philosophy

April 2012

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Declaration

I certify that the thesis I have presented for examination for the Ph.D. degree of the London Business School is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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London, April 2012

Andrea M. Buffa

Abstract

This thesis is structured around two main chapters, which analyze the impact of market imperfections on financial markets in the presence of strategic behavior.

An economic relevant friction I concentrate my research on regards default externalities and systemic risk. In particular, I study the strategic risk taking of highly-levered financial institutions within a structural model of credit risk, where I consider a context in which systemic default induces externalities that amplify the cost of financial distress. This represents a source of strategic interaction and mandates an analysis of financial institutions' asset allocations in coalescence. I derive a unique strategic equilibrium in which two heterogenous institutions adopt polarized and stochastic risk exposures, without sacrificing full diversification. In the presence of systemic externalities, both financial firms are concerned with maintaining sufficient wealth in adverse states. To this purpose, the conservative institution reduces its risk exposure, whereas the aggressive institution optimally gambles on positive and negative outcomes by taking long and short positions in risky securities over time. This equilibrium mechanism increases the likelihood of a systemic crisis.

In the second part of the thesis I explore the role of disclosure regulation in the presence of asymmetric information, as an institutional way to improve the efficiency and liquidity of the market. Is a more transparent market also more efficient and liquid? I address this question by analyzing the impact of mandatory ex-post disclosure of corporate insider trades (as in Section 16(a) of the U.S. Securities Exchange Act) in a dynamic model of strategic risk averse insider trading. I show that trade disclosure reduces informational efficiency of prices, may cause the market to be less liquid, and may increase insider's expected utility. In my model, the informed trader uses a less aggressive trading strategy in a more transparent market (i.e. with trade disclosure) in order to prevent the market maker from inferring perfectly the private information from public records, and to maintain her informational advantage over time. My result then questions the effectiveness of such securities regulation.

Acknowledgements

This thesis owes its existence to the help and inspiration of several people who have supported me along this long but fulfilling road.

First and foremost I am deeply indebted to my advisor, Süleyman Başak. He has been a tremendous mentor, teacher and friend, providing me with invaluable insights about research, and always encouraging me to have passion and fun in doing what we do. I feel exceedingly privileged to have had his guidance and I owe him a great many heartfelt thanks.

I am truly grateful to Dimitri Vayanos for his precious help and direction; working with him in the last years has been extremely inspiring. I would also like to express my deepest gratitude to Francisco Gomes and Christopher Hennessy for their invaluable support and fruitful discussions.

And last, but not least, I am forever indebted to my parents, Enrica and my close friends for their understanding, endless patience and encouragement when it was most needed. To all of you, thank you!

To my beloved family

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Chapter 1

Introduction

This thesis is structured around two main chapters, which analyze the impact of market imperfections on financial markets in the presence of strategic behavior. In the second chapter, entitled "Strategic Risk Taking with Systemic Externalities", I study the investment decisions of highly-levered financial institutions within a structural model of credit risk with default externalities. In the third chapter, entitled "Insider Trade Disclosure", I explore the impact of corporate insider's trade behavior when an ex-post trade disclosure regulation is enforced. A brief overview of the two next chapters is provided in what follows.

Chapter 2: Strategic Risk Taking with Systemic Externalities. In this chapter I study the optimal risk taking behavior of levered financial institutions in a scenario in which the cost of default of one institution depends on whether or not other institutions are defaulting as well. This *systemic externality* makes the institutions behave strategically. Hence, I consider a strategic game between two financial firms in a continuous-time structural model of credit risk. The financial institutions have some debt in place and choose portfolio allocations as the optimal combination of risky and riskless securities that will constitute the asset side of their balance sheet. This implies that the asset dynamics of these firms are endogenously determined by the outcome of the game. If the debt promises can not be repaid at maturity, default occurs and the associated costs get amplified if the two institutions are defaulting together.

In the strategic equilibrium the two institutions are characterized by a very polarized risk taking behavior; indeed, in order to reduce the cost of joint default they hold different portfolios. However, a reduced portfolio correlation is not obtained by tilting the composition of risky securities they are holding, but rather by taking different exposures to the same tangency portfolio. So, "diversity" between these institutions is in the relative weights of the tangency portfolio and the risk-free asset, and this is true regardless of whether the firms are ex-ante identical or characterized by some degree of heterogeneity. For the case of heterogenous institutions, we always find two types: a conservative institution, with a low risk profile, and an aggressive one, with a high risk profile. A reduced risk correlation is obtained in equilibrium because the institutions' exposures to the same portfolio are stochastic and optimally set to have a lower correlation. When one invests a lot, the other invests a little and vice versa. This highlights how in our model there is no trade-off between diversity and diversification.

Furthermore, I show how the most aggressive of the two institutions holds extreme positions which can switch between long and short over time. This trading strategy effectively generates an insurance contract against joint default. However, as any insurance policy, it requires a premium which in this case is financed by higher shortfall if an idiosyncratic default occurs. The short position generates the payout of the insurance, the long position the premium.

Despite the attempt to avoid the higher cost of joint default, in equilibrium the probability of joint default may actually increase. This somehow surprising result is due to a substitution and an income effect, generated by the amplified cost of joint default. The substitution effect means that institutions want to the transfer wealth from good to bad states. And this *per se* would reduce the probability of joint default. However, an income effect is also present: the higher cost of joint default hits the budget constraints of the two institutions, and hence increases the probability of joint default. In the paper I show that when leverage is high the income effect dominates.

Finally, financial institutions' debt inherits the endogenously determined risk-profile of their assets. Losses given joint default are reduced in equilibrium because of the payout of the implicit insurance contract. This makes the debt in bad states of the world less risky. On the other hand, the higher shortfall associated with idiosyncratic default makes the debt in good states of the world more risky.

Overall, my results unveil some of the underlying mechanisms that drive financial institutions' strategic risk taking when credit risk matters, Moreover, since any recent proposal for macro-prudential regulation effectively tries to render joint default more costly, the results in this chapter shed some light on how financial institutions would change their investment behavior in the presence of such regulation.

Chapter 3: Insider Trade Disclosure. In this chapter I study whether a more transparent market is also more efficient and liquid. Specifically, I analyze this question by studying the impact of mandatory ex-post disclosure of corporate insider trades in a dynamic model of insider trading. In an opaque market, that is without any disclosure regulation, a risk-averse insider trader would trade on his own information very aggressively because of future price risk induced by noise traders. In other words, he is afraid that noise traders will move the price against his own trade. By being so aggressive, most of the private information is revealed in early trading rounds, thus making the market very efficient in the sense that the price would reflect the fundamentals quickly. Along the same lines, once most of the information is revealed in the market, the adverse selection is severely reduced and hence market liquidity becomes very high.

The introduction of an ex-post disclosure regulation forces the insider to behave less aggressive. In order to exploit his information over time, he adopts a mixed strategy whereby he adds a noise component to his trade. The equilibrium reveals that the private information is incorporated into prices at a slower pace, which implies a lower market efficiency and liquidity. In the continuous time limit the risk averse insider behaves as if he was risk-neutral, thus minimizing the leakage of private information through prices. In essence, this regulation has both a positive-direct and a negative-indirect effect. The positive effect reduces the asymmetric information in the market, because of the information disclosed by the insider trader. The negative effect increases the asymmetric information because of the change in the insider trading strategy. In this paper, I show that this indirect exists and dominates the direct one. The introduction of such securities regulation can be considered as an institutional friction that prevents prices to convey relevant information to market participants.

Chapter 2

Strategic Risk Taking with Systemic Externalities

Over the past two decades, the financial system has evolved to be dominated by a small number of highly levered financial institutions.[1] Given the degree of interconnectedness between them and the sheer scale at which they operate, the decision making of each institution individually affects, and is affected by, the decisions of the others. Hence, their behavior should not be studied in isolation. This paper analyzes the risk taking of financial institutions in the presence of costly default and systemic externalities. We consider a context in which systemic default induces externalities that amplify the cost of default. This represents a source of strategic interaction.

The recent financial crisis has highlighted the costliness of systemic default. When Lehman Brothers filed for Chapter 11 protection in September 2008, a few months after the collapse of Bear Sterns, not only were its assets subjected to an emergency liquidation at discount prices, it also set in motion what is described in the final report of the Financial Crisis Inquiry Commission as "[...] one of the largest, most complex, multi-faceted and far-reaching bankruptcy procedures ever filed in the United States. The costs of the bankruptcy administration are approaching one billion USD" (Financial Crisis Inquiry Commission, 2011). There is a vast literature detailing such direct costs of

[1] In 2008, US commercial banks had leverage ratios (Total Assets/Total Equity) of roughly 15, while those of US investment banks were in the range of 20-30 (see `www.bis.org`).

default, with an emphasis on the costs of legal settlement. The case of Lehman Brothers illustrates that when default is systemic, the increased complexity of the settlement process between different claimholders only amplifies these costs. The same applies for indirect costs of default, encompassing reputational damage, loss of trading opportunities and the liquidation of assets at fire-sales prices. The systemic nature of losses due to fire-sales is equally well-documented (Schleifer and Vishny, 1992; Acharya, Bharath and Srinivasan, 2007). However, the strategic behavior of financial institutions in the presence of such systemic externalities of default requires more understanding. Hence, we believe a comprehensive analysis of financial institutions' strategic risk taking in a dynamic asset allocation framework is needed.

The challenge to better understand the strategic interactions at play, has recently been invigorated by policymakers' efforts to design and reinforce macro-prudential regulation. By virtue of modeling the systemic externalities in a reduced form, our set-up easily extends to an analysis of macro-prudential policies. Several proposals have been circulated, ranging from Pigouvian taxes over systemic capital requirements and risk-surcharges to capital insurance (Acharya, Pedersen, Philippon and Richardson, 2010; Hart and Zingales, 2010; Hansen, Kayshap and Stein, 2011; Webber and Willison, 2011). All these initiatives share the objective of making the most global and systemically important institutions internalize the negative externalities they impose on the non-financial sector. While choosing among the proposed regulations is beyond the scope of this paper, we do contribute to the debate with a positive analysis of banks' risk taking under incentives to avoid a systemic crisis.

We consider two highly levered financial institutions (the *banks*) within a structural model of credit risk. As in Merton (1974), Black and Cox (1976) and Longstaff and Schwartz (1995), we take the capital structure of the two institutions as given, with debt already in place. To maintain as simple a setting as possible, and following Carlson and Lazrak (2010) among others, we assert that default may occur only upon maturity of the debt contracts, when the banks fail to repay the debt obligations. Each bank is run by a manager whose incentives are aligned with those of the equityholders. Both managers have access to a complete financial market and their task consists of selecting a portfolio of risky and riskless securities which endogenously determines the asset side of the banks' balance sheet.

Default induces pecuniary costs affecting the banks' budget constraint. Specifically, we adopt a cost function that is linear in the shortfall of the debt repayments, where the slope parameter identifies the cost per unit of default. In order to capture the presence of default externalities beyond the costs of idiosyncratic failure, we attribute a higher slope parameter to a systemic crisis, which is defined by the joint default of the institutions under consideration. In characterizing the banks' optimal asset allocations, we appeal to the pure-strategy Nash equilibrium concept, in which each bank strategically accounts for the dynamic investment policies of the other bank, and the equilibrium policies of the two institutions are mutually consistent. As in Basak and Makarov (2011), by virtue of dynamically complete markets, the horizon equity is enough to characterize each bank's strategy. This means that both managers optimally select an equity profile, which prescribes the value of their bank's equity for any state of the world at maturity, given any possible equity profile of the other bank. Thus, we pin down the best response strategies.

For the case of heterogenous banks, we derive a unique equilibrium. The strategic interaction between financial institutions, captured by the interplay of their best response strategies, produces two distinct equilibrium equity profiles at maturity. The equilibrium has the following properties. First, in good states of the world both banks have optimal equity levels at least as high as their default boundaries, hence neither of them defaults. Second, in intermediate states wealth becomes expensive and only one bank can afford to maintain the level of the equity value higher or equal to the default boundary. This implies that intermediate states give rise to an idiosyncratic default regime. Finally, in the worst states of the world resisting default is too costly for both institutions, thus triggering a systemic crisis. Under these circumstances banks' equilibrium equities are strictly below their respective default boundaries. For expositional convenience, we label the bank prone to idiosyncratic default as "early-defaulter" and the other one as "late-defaulter".[2]

When banks are sufficiently homogenous, we show that multiple equilibria arise. This means that there are states of the world in which more than one pair of (mutually consistent) strategies can be part of an equilibrium. Specifically, this is the case when both banks agree on the fact that only one bank should default but they can not agree on which one. While selecting among the multiple equilibria is beyond the scope of this paper, we do establish the result that even in the extreme case of perfectly homogenous banks the equilibrium investment strategies are indeed heterogeneous. This confirms that in the case of the unique equilibrium, our results are effectively driven by the presence of the strategic interaction, and not by the ex-ante heterogeneity between the banks. We acknowledge that the existence of multiple equilibria creates scope for regulatory intervention.

When the equilibrium is unique, we provide a full characterization of the equilibrium investment policies under an isoelastic objective function and lognormal security prices. Our paper presents a tractable framework to study the dynamics of banks' strategic ex-

[2]Note that *early* and *late* refer to the state space dimension, not the time dimension, as default may only occur at maturity. In Section 3 we show that our results are qualitatively robust to varying the source of heterogeneity. Any unique equilibrium features an *early-* and a *late-defaulter*.

posure to risky securities. In particular, we focus on the level of each bank's risk exposure (*risk taking*) and on their correlation (*diversity*) in the presence of strategic interactions. We evaluate our findings against a benchmark of no systemic default externalities on the one hand, and one of entirely costless default on the other hand.

While the banks' efforts to internalize the systemic externalities induce them to implement less correlated asset allocations vis-à-vis the benchmarks (higher diversity), both banks attain full diversification by investing in the (same) mean-variance tangency portfolio. Thus, reduced correlation is not achieved by altering the composition of their portfolios, but rather by stochastically varying their exposure to the tangency portfolio. This means that at any point in time, conditional on the realization of a state of the world, the correlation between the banks' portfolios is indeed equal to one. Unconditionally however, this correlation is less than one because the banks select optimal stochastic exposures that are not perfectly correlated. This set of results is at odds with the findings in Wagner (2011), where systemic liquidation costs lead agents to sacrifice diversification for diversity. In our model, full diversification is not compromised in the presence of negative systemic externalities because both banks can move along the efficient frontier by altering the fraction invested in the riskless bond. Furthermore, because of the stochastic nature of the banks' investment policies, we ascertain that their correlation exhibits some interesting patterns. Diversity tends to increase upon the realization of intermediate states of the world, especially when time approaches maturity. These are precisely the circumstances under which banks are most concerned with systemic default and hence value diversity most.

Regarding the risk taking dynamics of the two financial institutions, we establish the novel result that strategic interaction drives a wedge in the equilibrium level of risk desired by the *early*- and the *late-defaulter*. While both banks aim at transferring horizon equity from good/intermediate states of the world to those characterized by (very costly) systemic default, in equilibrium they choose polarized strategies. The *late-defaulter* adopts a conservative strategy. By implementing a low-risk investment policy, it generates sufficiently high wealth to finance an optimal equity profile where wealth is maintained above and at the default boundary in good and intermediate states, respectively. In contrast, the *early-defaulter* displays a more aggressive strategy. Close to maturity, in states of the world where both idiosyncratic and systemic default are likely to occur (intermediate states), the *early-defaulter*'s risk taking exhibits two radically opposite behaviors. Although defaulting at maturity is very likely, it either invests a high fraction of the assets in risky securities, or, at the other extreme, takes a short position in the tangency portfolio. Effectively, the only way the *early-defaulter* can allocate more wealth to systemic states at maturity is by accepting idiosyncratic default in intermediate states. Thus, in these states the bank allows its asset value to be very sensitive and positively correlated to economic fluctuations by investing heavily in the market for risky securities close to maturity. This high risk taking allows the bank to take away wealth from the states with idiosyncratic default and finance an asset value that is less sensitive to the severity of the systemic crisis. In the event of such a crisis, the early-defaulter also wants to increase the asset value in order to minimize the shortfall in the debt repayments. The only way it can deliver an asset profile that will jump upwards in case of joint default, is by investing in a portfolio that is negatively correlated with the economic fluctuations. This rationalizes the desire to short the market close to maturity.

Summarizing, we find that in the presence of systemic externalities, both banks are concerned with maintaining sufficient wealth in adverse states. However, while the conservative bank reduces its risk exposure, the radical bank optimally gambles on positive and negative outcomes, by taking either a large long or a short position in risky securities. We believe these results on banks' strategic risk taking are new.

Given that the equilibrium equity profiles of the two financial institutions endogenously determine in which states idiosyncratic and systemic default occurs, they also carry implications for the probabilities of default. To appreciate the effect of negative systemic externalities (e.g., macro-prudential regulation) on the occurrence and the magnitude of systemic crises, we compare the equilibrium default probabilities and expected shortfalls in the strategic model to those in the benchmarks. Most notable, we find that in the presence of negative systemic externalities, both banks are more likely to default. This immediately implies that a systemic crisis becomes more likely.

This finding does not easily reconcile with the ex-ante objective of macro-prudential regulation to reduce the likelihood of a systemic event. We rationalize this outcome by recognizing that the systemic externalities implicitly generate what we term as a *substitution* and an *income* effect. The former captures the wealth transfer from idiosyncratic to systemic states which decreases the probability of a systemic crisis. The latter, instead, increases the probability of joint default since it reflects the asset value reduction caused by the negative externalities. The equilibrium outcome, being the net of these two forces, shows that the income effect dominates for financial institutions with realistic leverage ratios. We complete this analysis by examining the expected shortfalls. We ascertain that, under extremely adverse economic conditions, the expected losses given default are lower in the presence of strategic interactions, by virtue of the banks' wealth transfers into these states. In all, we believe these results are indicative of a friction between systemic default probabilities on the one hand and systemic losses on the other hand. Hence, this should not be overlooked in the design of a macro-prudential framework.

A last set of implications of our paper examines the pricing of credit spreads and credit default swap premiums for the two financial institutions. Since debt is a straightforward claim on the value of the assets, its equilibrium price is also endogenously determined by the strategic risk taking of the banks. Indeed, we find that the credit spreads inherit the distinctive features of banks' risk exposures. This is especially clear for the *early-defaulter* whose radical risk taking behavior, translates into non-monotonic credit spreads. This means that lower credit spreads can be associated with worse states when time to maturity is low. More generally, we document a decrease in credit spreads for the most adverse states and an increase of the spreads for the more favorable ones. Once more, this reflects the banks' wealth transfers from good to systemic states, in a bid to internalize the negative externalities associated with joint default. With respect to the CDS spreads, which are positively related to both the probability of default and the credit spread, we document that the price to pay for protection against default is higher in the strategic model than in the benchmarks. This is also true for the systemic states, where the credit spreads of the banks are markedly lower than for the benchmark models. Hence, we acknowledge the dominance of default probabilities on equilibrium CDS prices.

This paper relates to several strands of literature. We build on the literature of structural credit risk models. We employ the modeling approach presented by Basak and Shapiro (2005), allowing the asset-value dynamics to be endogenously determined. In this paper, we deviate from the standard structural contingent-claim approach, with exogenous asset-value dynamics, as first employed by Merton (1974) and extended, among others by Leland (1994), Longstaff and Schwarz (1995) and Anderson and Sundaresan (1996). By virtue of endogenizing the asset-values, this paper allows for an analysis of portfolio choice effects under strategic interactions. In this regard, our work is most closely related to Basak and Makarov (2011) who analyze dynamic portfolio strategies of money managers, in the presence of strategic interactions arising from relative performance concerns.

Notwithstanding a literature on banks' portfolio choice in the presence of systemic externalities (Gorton and Huang, 2004; Wagner, 2011), our paper contributes by explicitly considering such externalities as a source of strategic interaction. This most clearly differentiates our paper from Wagner (2011). In a set-up with systemic liquidation costs, he considers atomistic agents' investment in risky securities only and establishes that agents optimally sacrifice diversification for the sake of avoiding systemic failure. In our model, because both banks can move along the efficient frontier by altering the fraction invested in the riskless bond, full diversification is not compromised.

In relating our paper to a more general literature on strategic behavior among banks, we acknowledge recent contributions by Perotti and Suarez (2002), Acharya (2009). Both works consider banks' strategic portfolio decisions, in related yet different contexts of systemic default. While these papers consider potential gains from the acquisition of failed bank assets as a source of strategic interaction among financial institutions, we restrict our attention to amplified costs of joint default. With this, we touch upon the vast literature on costly default (Altman, 1984; Weiss, 1990; Andrade and Kaplan, 1998) and fire-sales (Schleifer and Vischny, 1992; Acharya, Bharath and Srinivasan, 2007). These papers extensively document how a systemic crisis exacerbates the costs of default, emphasizing the economic relevance of the systemic externalities considered in our model. A set of recent papers analyzes the optimal resolution of bank failures (Acharya and Yorulmazer, 2007, 2008; Farhi and Tirole, 2011). These articles show how systemic bailouts distort the incentives for banks to correlate the risk in their investment choices. Our results on risk correlation in a context of negative systemic externalities complement their findings, by showing how banks' preference for *diversity* evolves dynamically. Moreover, our full characterization of strategic risk taking adds a further layer of analysis to this literature.

Finally, and perhaps most importantly, we also add to the rapidly growing literature on systemic risk. Seminal contributions on the measurement of systemic risk include Adrian and Brunnermeier (2011) and Acharya, Pedersen, Philippon and Richardson (2010) among others. Much work in this field is motivated by the view that regulation should be designed in a way that financial institutions are penalized based on their contribution to systemic risk. By interpreting the systemic externalities in our model as any generic systemic policy, we are able to draw conclusions on financial institutions' responsiveness to potential regulatory changes of this nature. We believe that our analysis of systemic crises, both in terms of likelihood and expected shortfall, within a workhorse dynamic asset allocation framework is new and delivers a rich set of implications regarding financial institutions' strategic risk taking.

The remainder of the paper is organized as follows. Section 2 presents the economic set-up and lays out the micro-foundations for the strategic game in the presence of systemic externalities of default. Section 3 solves for the best-response strategies and characterizes the unique equilibrium for the case of heterogenous banks. Section 4 investigates the properties of the unique equilibrium. We analyze optimal risk taking, default probabilities and shortfalls, and debt pricing. Section 5 concludes. Proofs and minor results are derived in the Appendix A.

2.1 The Model

2.1.1 The Economic Setting

We consider a continuous-time, finite horizon economy, $t \in [0, T]$, in which the uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$ on which an N-dimensional standard Brownian motion, $w_t = (w_{1t}, \cdots, w_{Nt})'$, is defined. We assume that all stochastic processes are adapted to the augmented filtration $\{\mathcal{F}_t\}$ generated by w, and that regularity conditions that make the processes well-defined (Karatzas and Shreve, 1998) are satisfied.

Financial market. Financial investment opportunities are given by $N + 1$ assets: an instantaneously riskless bond and N risky securities, whose prices evolve according to the following dynamics

$$dB_t = B_t r dt \tag{2.1}$$

$$dS_t = S_t \mu dt + S_t \sigma dw_t. \tag{2.2}$$

The bond provides a continuously compounded constant interest rate of r, whereas μ and σ represent the N-dimensional vector of mean returns and the $N \times N$ non-degenerate volatility matrix of the risky securities, respectively. Markets are dynamically complete, implying the existence of a unique state price density process ξ such that

$$d\xi_t = -\xi_t r dt - \xi_t \kappa' dw_t \tag{2.3}$$

where $\kappa \equiv \sigma^{-1}[\mu - r\mathbf{1}]$ is the N-dimensional market price of risk, $\mathbf{1}$ is the N-dimensional vector $(1, \cdots, 1)'$, and ξ_0 is set to 1.

Agents. Our economy is populated by two financial institutions, which for simplicity we will refer to as *banks* hereafter. If we let V_{it} denote the value of the assets of bank i at time t, and W_{it} and D_{it} the value of the equity and of the debt, respectively, then the following accounting identity must hold,

$$V_{it} = W_{it} + D_{it}. \tag{2.4}$$

Each bank $i \in \{1, 2\}$ is run by a manager whose incentives are aligned with equityholders' interests. The manager is guided by an isoelastic objective function defined over the terminal value of the equity (the bank's net wealth),

$$u_i(W_{iT}) = \frac{W_{iT}^{1-\gamma_i} - 1}{1-\gamma_i}, \qquad \gamma_i > 0. \tag{2.5}$$

It is important to stress that the interpretation of the aforementioned objective function is broader than a standard concave utility function. Indeed, it could capture managerial self interest, compensation structures, concave non-stochastic investment opportunities beyond the terminal date, and, more in general, market frictions (Allen and Santomero, 1998; John and John, 1993; John, Saunders and Senbet, 2000; Froot, Scharfstein and Stein, 1993; Froot and Stein, 1998).

The manager of bank i maximizes the expected value of (2.5) by dynamically choosing an investment policy π_{it}, which denotes the (N-dimensional) vector of fractions of bank i's assets invested in each risky security, given an initial capital of W_{i0}. We refer to π_{it} as the *risk taking* of bank i. Clearly, $(1 - \pi_{it}'\mathbf{1})$ pins down the fraction of bank i's assets invested in the riskless bond. Hence, the optimization problem faced by the manager of bank i is subject to the dynamics of the value of the assets:

$$dV_{it} = V_{it}[r + \pi_{it}'(\mu - r\mathbf{1})]dt + V_{it}\pi_{it}'\sigma dw_t. \tag{2.6}$$

2.1.2 Leverage, Default and Externalities

As in many structural models of credit risk (Merton, 1974; Leland, 1994; Longstaff and Schwarz, 1995), we do not derive the optimal capital structure of the two financial institutions, but rather we take it as given and study the implications of their strategic interaction on optimal investment decisions and systemic risk.[3] Therefore, we assume that both banks are levered; specifically, they are bound by a zero-coupon debt contract with face value of F_i and price D_{i0} at the initial date, where the latter will be determined endogenously by the optimal asset choice of bank i. The presence of debt in the banks' balance sheets captures the possibility of these institutions defaulting.

Debt contract. Financial distress may occur at the terminal date T and it is triggered if bank i fails to repay its debt obligation, equal to the face value F_i. In such a case, debtholders can force liquidation or reorganization and can seize only a fraction of the bank's total assets, $(1-\beta_i)V_{iT}$, while the remaining $\beta_i V_{iT}$ is retained by the equityholders. By assuming β_i greater than zero, we capture violations of the Absolute Priority Rule (APR), a fact which is extensively documented in the empirical literature (Franks and

[3] Optimal capital structure within a credit risk model is studied in Leland (1994), Leland and Toft (1996). This papers assume exogenous asset value processes.

Torous, 1989, 1994; Eberhart, Moore, and Roenfeldt, 1990; Weiss, 1990; Betker, 1995). Departures from APR can be rationalized as the optimal outcome of a bargaining game among corporate claimants after time T (Anderson and Sundaresan, 1996; Mella-Barral and Perraudin, 1997, Fan Sundaresan, 2000; Acharya, Huang, Subrahmanyam and Sundaram, 2006; Garlappi, Shu and Yan, 2008).[4] Hence, the payoff of the debt contract is given by:

$$D_{iT} = \min\{(1-\beta_i)V_{iT}, F_i\} \tag{2.7}$$

where $\beta_i \in [0,1]$. Bank i enters financial distress when $V_{iT} < F_i/(1-\beta_i)$. It is worth clarifying that in this model there is no formal distinction between default and distress: what really matters is that in both cases the bank can not repay the face value of the debt. In what follows, we make use of both terms interchangeably.

Cost of default and systemic externalities. Default is costly. An extensive literature has documented the nature and severity of (direct and indirect) costs of financial distress. Guided by these insights, and following Basak and Shapiro (2005), we model cost of default by adopting the following reduced form:

$$C_{iT} = \begin{cases} 0 & \textit{if} \quad D_{iT} = F_i \\ \phi + \lambda(F_i - D_{iT}) & \textit{if} \quad D_{iT} < F_i \quad \wedge \quad D_{jT} = F_j \\ \phi + (\lambda + \eta_i)(F_i - D_{iT}) & \textit{if} \quad D_{iT} < F_i \quad \wedge \quad D_{jT} < F_j, \end{cases} \tag{2.8}$$

for any $i \in \{1,2\}$ and $j \neq i$. Upon default, $D_{iT} < F_i$, bank i incurs a fixed and a proportional costs, $(\phi, \lambda) \geqslant 0$, where the latter is proportional to the extent of default $(F_i - D_{iT})$. When financial distress is systemic, that is when both banks default on their debt, the proportional cost become equal to $(\lambda + \eta_i)$. If $\eta_i > 0$ we have a *negative systemic externality*; if $-\lambda < \eta_i < 0$ we have a *positive systemic externality*. Hence, systemic defaults can be more or less costly than idiosyncratic defaults depending on whether the systemic externality is negative or positive. We allow the systemic components of the cost to be heterogeneous among banks.

Besides the advantage of tractability, this parsimonious yet flexible specification allows us to capture, within the same theoretical framework, different economic scenarios:

- *Direct financial distress costs.* Direct costs of default encompass the costs of lawyers, accountants, and other professionals involved in the bankruptcy filling, including the value of managerial time spent to this purpose (Warner, 1977; Weiss, 1990; Andrade and Kaplan, 1998). These costs are both fixed and proportional,

[4]As a complementary interpretation, we can view $\beta_i V_{iT}$ as bank i's intangible assets, which can not be collateralized and hence transferred to the debtholders.

and it is reasonable to assume that in the event of a systemic crisis they would rise because of the increased complexity of negotiating the disputes between claimholders.

- *Indirect financial distress costs.* Impaired business reputation, loss of trading opportunities (Dow and Rossiensky, 2001), loss of market share (Opler and Titman, 1994), liquidation of assets at fire-sales prices (Allen and Gale, 1994, 1998; Acharya and Yorulmazer, 2007; Wagner, 2011) represent the large part of indirect costs of default. ÊSince such opportunity costs depend on the market setting, they become more severe during a systemic crisis. For instance, it is more likely that some of the assets of the distressed institutions are acquired by investors who are not the most efficient user of these assets, thus valuing them below their fundamental value (Williamson, 1988; Shleifer and Vishny, 1992).

- *Macro-prudential/systemic regulation.* Inspired by the events of the recent financial crisis, several proposals to contain systemic risk have been advocated (Acharya, Pedersen, Philippon and Richardson, 2010; Hart and Zingales, 2010; Hansen, Kayshap and Stein, 2011; Webber and Willison, 2011). A general consensus seems to highlight the inability of micro-prudential regulations to prevent the collapse of the financial system as a whole, and the urgent need for macro-prudential regulations. Of these kind, among others, are Pigouvian taxes, systemic capital requirements, systemic-based risk constraints, capital insurance, systemic risk surcharges, all tailored to induce systemic financial institutions to internalize their externalities on the entire economy. In other words, they are designed to induce the financial sector to bear some of the social costs that they would generate in the event of a crisis. Discussing different aspects of the Dodd-Frank Act, Acharya, Cooley, Richardson and Walter (2010) concisely summarizes this view:

  > "*The basic idea is that, to the extent these stricter standards impose costs on financial firms, these firms will have an incentive to avoid them and therefore be less systemically risky.*"

  Therefore, we can deem our postulated cost function as capturing (in reduced form) the implementation of such systemic regulations through the (positive) coefficient η_i.[5]

[5] As an example, consider the optimal (Pigouvian) tax system obtained in Acharya, Pedersen, Philippon and Richardson (2010): each financial institution is taxed based on: (i) the expected shortfall in case of default, and (ii) the expected shortfall in case of a systemic crisis. Therefore, the expected default cost that the banks face in our model can be easily interpreted as a systemic tax:

$$\begin{aligned}\mathbb{E}[\xi_T C_T] = &+ \lambda \mathbb{E}[\xi_T (F_i - D_{iT}) | \textit{single distress}] \mathbb{P}(\textit{single distress}) \\ &+ \eta_i \mathbb{E}[\xi_T (F_i - D_{iT}) | \textit{systemic distress}] \mathbb{P}(\textit{systemic distress}) + K\end{aligned}$$

where the event $\{\textit{systemic distress}\} \subset \{\textit{single distress}\}$ and K captures the expected fixed costs which do not depend on the banks' shortfall.

- *Systemic bail-out.* While the previous scenarios represent *negative systemic externalities*, our framework can also accommodate *positive systemic externalities* such as explicit and implicit promises and transfers from the rest of the economy to the financial sector. Moral hazard problems related to the resolution of bank failures (Acharya and Yorulmazer, 2007, 2008; Panageas, 2010; Farhi and Tirole, 2011) can make the cost of distress in the event of a financial crisis lower than the one in the event of a single default. In our model, this corresponds to negative values of η_i.

Therefore, the coefficients in our cost specification are meant to capture the net effect of all the potential determinants of the aforementioned distress costs. Note that when η_i is equal to zero, systemic externalities are absent and the two banks are not connected with each other, thus behaving as the single borrower in Basak and Shapiro (2005). This implies that their optimal investment problems can be solved independently. In contrast, when externalities are present, bank i acts strategically by taking into account the effect that bank j has on bank i's cost function, knowing that bank j takes into account the effect that bank i has on bank j's cost function, and so on. Hence, the optimal investment policies of the two banks are the equilibrium outcome of a strategic game.

2.1.3 Default Regions

Before formally defining the strategic game between the two financial institutions, let us determine the banks' equity and total asset value in all the possible regions of default. At the terminal date T, taking into account of the default cost, the accounting identity in (2.4) becomes equal to

$$V_{iT} - C_{iT} = W_{iT} + D_{iT} \tag{2.9}$$

so that the value of the equity of bank i at time T is given generically by

$$\begin{aligned} W_{iT} = V_{iT} - \min\{(1-\beta_i)V_{iT}, F_i\} - \Big(\phi + \lambda(F_i - \min\{(1-\beta_i)V_{iT}, F_i\}) \\ + \eta_i(F_i - \min\{(1-\beta_i)V_{iT}, F_i\})\mathbb{1}_{\{(1-\beta_j)V_{jT}<F_j\}}\Big)\mathbb{1}_{\{(1-\beta_i)V_{iT}<F_i\}}. \end{aligned} \tag{2.10}$$

Based on this we can consider the following sub-cases:

No-default region. None of the two banks are in financial distress: $(1-\beta_1)V_{1T} \geqslant F_1$ and $(1-\beta_2)V_{2T} \geqslant F_2$.

$$W_{iT} = V_{iT} - F_i \quad \Rightarrow \quad V_{iT} = W_{iT} + F_i \tag{2.11}$$

for $i \in \{1,2\}$.

Single default region. Only bank i is in financial distress: $(1-\beta_i)V_{iT} < F_i$ and $(1-\beta_j)V_{jT} \geqslant F_j$.

$$W_{iT} = V_{iT}[\beta_i + \lambda_i(1-\beta_i)] - [\phi + \lambda F_i] \quad \Rightarrow \quad V_{iT} = \frac{W_{iT} + [\phi + \lambda F_i]}{\beta_i + \lambda(1-\beta_i)} \tag{2.12}$$

$$W_{jT} = V_{jT} - F_j \quad \Rightarrow \quad V_{jT} = W_{jT} + F_j \tag{2.13}$$

Systemic default region. Both banks are in financial distress: $(1-\beta_1)V_{1T} < F_1$ and $(1-\beta_2)V_{2T} < F_2$.

$$W_{iT} = V_{iT}[\beta_i + (\lambda+\eta_i)(1-\beta_i)] - [\phi + (\lambda+\eta_i)F_i] \quad \Rightarrow \quad V_{iT} = \frac{W_{iT} + [\phi + (\lambda+\eta_i)F_i]}{\beta_i + (\lambda+\eta_i)(1-\beta_i)} \tag{2.14}$$

for $i \in \{1,2\}$.

In order to express the default boundary in terms of the value of the equity, let $\underline{W}_i$ denote the lower-bound value of W_{iT} in the no-default region:

$$\underline{W}_i : \quad (1-\beta_i)(\underline{W}_i + F_i) = F_i \quad \Rightarrow \quad \underline{W}_i \equiv \frac{\beta_i F_i}{1-\beta_i}, \tag{2.15}$$

for $i \in \{1,2\}$. When bank i's equity at maturity is greater than or equal to the threshold $\underline{W}_i$, the bank does not default. Note, however, that the upper-bound value of W_{iT} in the default regions is given by $\underline{W}_i - \phi$. Therefore, the fixed cost creates a discontinuity such that the value of the equity at maturity can not take values in the interval $[\underline{W}_i - \phi,\ \underline{W}_i)$. In the next section, we verify that the optimal policies of the two banks satisfy this condition. To avoid abuse of notation, throughout this paper we refer to the event $\{W_{iT} < \underline{W}_i\}$ as default. Since there is a one-to-one mapping between equity and asset values in all the default regions, we can express all the relevant quantities as a function of the horizon equities (W_{iT}, W_{jT}). In the following Lemma we do this for the cost function.

Lemma 1. *Bank i's cost of default can be written as*

$$C_{iT}(W_{iT}, W_{jT}) = \begin{cases} 0 & \text{if} \quad W_{iT} \geqslant \underline{W}_i \\ x_i\phi + (1-x_i)(\underline{W}_i - W_{iT}) & \text{if} \quad W_{iT} < \underline{W}_i \quad \wedge \quad W_{jT} \geqslant \underline{W}_j \\ z_i\phi + (1-z_i)(\underline{W}_i - W_{iT}) & \text{if} \quad W_{iT} < \underline{W}_i \quad \wedge \quad W_{jT} < \underline{W}_j, \end{cases} \tag{2.16}$$

for $j \neq i$, where

$$x_i \equiv \frac{1}{1+\hat{\lambda}_i}, \qquad z_i \equiv \frac{1}{1+(\hat{\lambda}_i+\hat{\eta}_i)}, \qquad \hat{\lambda}_i \equiv \lambda\left(\frac{1-\beta_i}{\beta_i}\right), \qquad \hat{\eta}_i \equiv \eta\left(\frac{1-\beta_i}{\beta_i}\right). \tag{2.17}$$

If the systemic externality is negative (positive), then $x_i > z_i$ ($x_i < z_i$).

Proof. See the Appendix A. □

2.1.4 Martingale Representation and The Strategic Game

The manager of bank i maximizes the expected value of the objective function over the value of the final horizon equity, subject to the dynamic budget constraint in (2.6), and the default cost in (2.8). Under the assumption of dynamically complete markets, we can solve the dynamic optimization problem of bank i using the martingale representation approach (Karatzas, Lehoczky and Shreve, 1987; Cox and Huang, 1989). This entails solving the following static problem:

$$\max_{W_{iT}} \quad \mathbb{E}[u_i(W_{iT})] \qquad s.t. \quad \mathbb{E}[\xi_T V_{iT}] \leqslant V_{i0} \tag{2.18}$$

where $V_{i0} = W_{i0} + D_{i0}$ and $V_{iT} = W_{iT} + D_{iT} + C_{iT}$. Since markets are dynamically complete, the debt contract is fairly priced, $D_{i0} = \mathbb{E}[\xi_T D_{iT}]$, and the bank's problem can be restated as

$$\max_{W_{iT}} \quad \mathbb{E}[u_i(W_{iT})] \qquad s.t. \quad \mathbb{E}[\xi_T(W_{iT} + C_{iT})] \leqslant W_{i0} \tag{2.19}$$

where, by Lemma 1, $C_{iT} = C_{iT}(W_{iT}, W_{jT})$.

The two banks are interconnected through the (systemic) cost of default: the choice of one bank to default affects and is affected by the choice of the other. Hence, they play a strategic dynamic game. The strategy of each bank consists of a non-negative horizon equity W_{iT} and a sequence of fractions of bank's assets to invest in the risky financial securities $\{\pi_{it}\}_{t\in[0,T]}$. By virtue of dynamically complete markets, (i) the horizon equity W_{iT} is enough to characterize each bank's strategy, and (ii) the dynamic game can be viewed as a game played only at time 0, where each bank decides its strategic actions for the whole time period $[0, T]$. Note that this is equivalent to solving for an open-loop equilibrium.[6] The two banks play such game under the assumption of complete

[6]We refer to Basar and Olsder (1982, 1995) for an exhaustive discussion on open and closed-loop equilibria of dynamic games. As highlighted by Back and Paulsen (2009), there are some issues in defining closed-loop equilibria for dynamic games in continuous-time and continuous-action; for this reason we leave the investigation of such equilibria for future research.

information, given that the set $\{W_{i0}, F_i, u_i(\cdot), \beta_i, \phi, \lambda, \eta_i\}$, for any $i \in \{1,2\}$, is common knowledge.

The game. Let $\langle \mathcal{I}, \Omega, \mathbb{P}, (\mathcal{S}_i), (v_i) \rangle$ denote the strategic game, where $\mathcal{I}$ refers to the set of players; $\mathbb{P}$ denotes a probability measure defined over the set of states Ω; $\mathcal{S}_i$ represents the nonempty set of strategies s_i available to bank i, and v_i its payoff function. Specifically,

- $\mathcal{I} = \{1, 2\}$;
- $\Omega = \mathbf{R}^{++}$;
- $\mathbb{P} : \log(\xi_T) = -\left(r + \kappa^2/2\right) T - \kappa w_T$;
- $\mathcal{S}_i = \{W_{iT}(\xi_T) : \Omega \to \mathbf{R}^{++} : \ \mathbb{E}[\xi_T(W_{iT} + C_{iT})] \leqslant W_{i0}\}$
- $v_i : (\mathcal{S}_i \times \mathcal{S}_j) \to \mathbf{R}$ is such that $v_i(W_{iT}, W_{jT}) = \max_{W_{iT} \in \mathcal{S}_i} \ \mathbb{E}[u_i(W_{iT})]$

Definition 1 (**Best Response Strategies**). *Consider the strategic game $\langle \mathcal{I}, \Omega, \mathbb{P}, (\mathcal{S}_i), (v_i) \rangle$. For any $W_{jT}(\xi_T) \in \mathcal{S}_j$, let $\mathcal{B}_i(W_{jT})$ define the set of bank i's best response strategies given W_{jT}:*

$$\mathcal{B}_i(W_{jT}) = \left\{ W_{iT}(\xi_T) \in \mathcal{S}_i : \ v_i(W_{iT}, W_{jT}) > v_i(W'_{iT}, W_{jT}) \ \ \textit{for all} \ \ W'_{iT}(\xi_T) \in \mathcal{S}_i \right\}. \tag{2.20}$$

Then, $\hat{W}_{iT}(W_{jT})$ denotes an element of $\mathcal{B}_i(W_{jT})$.

Definition 2 (**Pure-Strategy Nash Equilibrium**). *A pure-strategy Nash equilibrium of the strategic game $\langle \mathcal{I}, \Omega, \mathbb{P}, (\mathcal{S}_i), (v_i) \rangle$ is a profile of strategies $(W^*_{iT}, W^*_{jT}) \in (\mathcal{S}_i, \mathcal{S}_j)$ for which*

$$W^*_{iT} \in \mathcal{B}_i(W^*_{jT}) \quad \textit{for all} \quad i \in \mathcal{I}. \tag{2.21}$$

2.2 Strategic Equilibrium with Negative Externalities

In the current section we solve for the strategic game played by the two levered financial institutions subject to default costs, as described in the earlier section. First we derive each bank's best response strategy, then we characterize the strategic equilibrium by selecting those strategies that are mutual consistent.

2.2.1 Best Response Strategies

The manager of bank i faces the optimization problem in (2.19): he maximizes the expected value of the objective function defined over the equity value at the terminal date, subject to the static budget constraint. The budget constraint states that that expected discounted value of the sum of the horizon equity plus potential default costs (discounted with the pricing kernel of the economy, ξ_T) can not be higher than the value of the initial capital. In simpler terms, the implemented policy must be affordable.

As equation (2.16) highlights, the cost of default of bank i is affected by the policy adopted by bank j. Depending on whether bank j is solvent or not at maturity, bank i's cost function would exhibit *low* or *high* proportional costs, respectively. The best response strategy of bank i prescribes the optimal level of equity at time T for any possible realization of the uncertainty, $W_{iT}(\xi_T)$, conditional on the strategy of bank j. In other words, bank i takes W_{jT} as given and solves the optimization problem in (2.19) for all possible values of W_{jT}. However, since the cost function of bank i is not affected by the level of W_{jT} *per se*, but rather by whether W_{jT} is *above* or *below* the default boundary $\underline{W}_j$, the bank needs to solve only two optimization problems. Each problem is conditional to one of the two collectively exhaustive and mutually exclusive events,

$$\{W_{jT} \geqslant \underline{W}_j\} \qquad \text{and} \qquad \{W_{jT} < \underline{W}_j\}.$$

Because of the nonlinearity and discontinuity in the default cost functions, the banks' optimization problems are non-standard as they are not globally concave. In fact, they exhibit local convexity around the default boundaries $\underline{W}_i$, for $i \in \{1, 2\}$. To tackle this issue, we adapt the common convex-duality approach (e.g., Karatzas and Shreve, 1998) to incorporate kinks and discontinuities in the objective and in the budget constraint.[7] In the following Proposition we characterize the banks' best response strategies explicitly in closed-form.

Proposition 1. *The best response function of bank i with respect to bank j, for any $i \neq j$, is given by*

$$\hat{W}_{iT}(W_{jT}) = \begin{cases} (y_i\xi_T)^{-\frac{1}{\gamma_i}} & \text{if} \quad \xi_T \leqslant \underline{\xi}_i \\ \underline{W}_i & \text{if} \quad \underline{\xi}_i < \xi_T \leqslant \bar{\xi}_i \\ (y_i x_i \xi_T)^{-\frac{1}{\gamma_i}} + \left[\underline{W}_i - (y_i x_i \xi_T)^{-\frac{1}{\gamma_i}}\right] \mathbb{1}_{\{W_{jT} < \underline{W}_j\}} & \text{if} \quad \bar{\xi}_i < \xi_T \leqslant \bar{\bar{\xi}}_i \\ (y_i x_i \xi_T)^{-\frac{1}{\gamma_i}} + \left[(y_i z_i \xi_T)^{-\frac{1}{\gamma_i}} - (y_i x_i \xi_T)^{-\frac{1}{\gamma_i}}\right] \mathbb{1}_{\{W_{jT} < \underline{W}_j\}} & \text{if} \quad \xi_T > \bar{\bar{\xi}}_i. \end{cases} \tag{2.22}$$

[7] Examples of non-standard optimization problems include Carpenter (2000); Basak and Shapiro (2001, 2005); Basak, Pavlova and Shapiro (2007); Carlson and Lazrak (2010); Basak and Makarov (2011).

The thresholds $(\underline{\xi}_i, \bar{\xi}_i, \bar{\bar{\xi}}_i)$ *are given by*

$$\underline{\xi}_i \equiv \frac{(\underline{W}_i)^{-\gamma_i}}{y_i}, \qquad \bar{\xi}_i \equiv \frac{\alpha_i}{y_i x_i}, \qquad \bar{\bar{\xi}}_i \equiv \frac{\alpha_i}{y_i z_i}, \tag{2.23}$$

where $\alpha_i > (\underline{W}_i)^{-\gamma_i}$ *is the solution to the following equation*

$$\underline{W}_i^{1-\gamma_i} - \gamma_i \alpha_i^{1-\frac{1}{\gamma_i}} - \alpha_i \left(\underline{W}_i - \phi\right)(1-\gamma_i) = 0, \tag{2.24}$$

The Lagrange multiplier y_i *is set such that*

$$\mathbb{E}\left[\xi_T \hat{W}_{iT}(W_{jT}, \xi_T; y_i) + \xi_T C_{iT}(\hat{W}_{iT}(W_{jT}, \xi_T; y_i))\right] - W_{i0} = 0. \tag{2.25}$$

If the systemic externality is negative, $\eta_i > 0$*, then the following ordering holds:* $\underline{\xi}_i < \bar{\xi}_i < \bar{\bar{\xi}}_i$.

Proof. See the Appendix A. □

The best response strategy of bank i in equation (2.22) is characterized by three different thresholds that originate four distinct regions spanning the entire state space at time T. Before describing the best response behavior adopted in each of these regions, recall that the objective of the manager of the bank is to choose the optimal equity profile to implement at maturity. Such profile, which must be affordable, prescribes the how much net wealth (equity) to have at maturity for all possible economic scenarios.

For a graphical representation of the best response strategy, Panel A in Figure 2.1 plots the optimal equity profile of bank i at time T as a function of the realizations of the economic uncertainty (the state price density ξ_T). In Panel A1 the equity profile is conditional on bank j not defaulting; in Panel A2 it is conditional on bank j being in distress; the graph in Panel A3 combines the previous two. Panel B, on the contrary, provides a more commonly used representation of the best response strategy by plotting the optimal equity of bank i at time T as a function of the equity of bank j. Panel B1-B4 correspond to different economic scenarios.

The economic intuition underlying the optimal equity profile is as follows. When economic conditions are good ($\xi_T \leqslant \underline{\xi}_i$), the price (per unit of probability $\mathbb{P}$) of one unit of wealth (that is ξ_T) is low. Since wealth in these states of the world is "cheap" it is optimal for the manager of bank i to have high equity. High equity means that the firms do not default. Moreover, this holds whether or not bank j is in distress. When economic prospects deteriorate ($\underline{\xi}_i < \xi_T \leqslant \bar{\xi}_i$), the price of wealth become more expensive and bank i would default if there were no costs associated to this choice. However, to avoid distress costs, it is optimal to "buy" the minimum amount of wealth that allows the bank to be

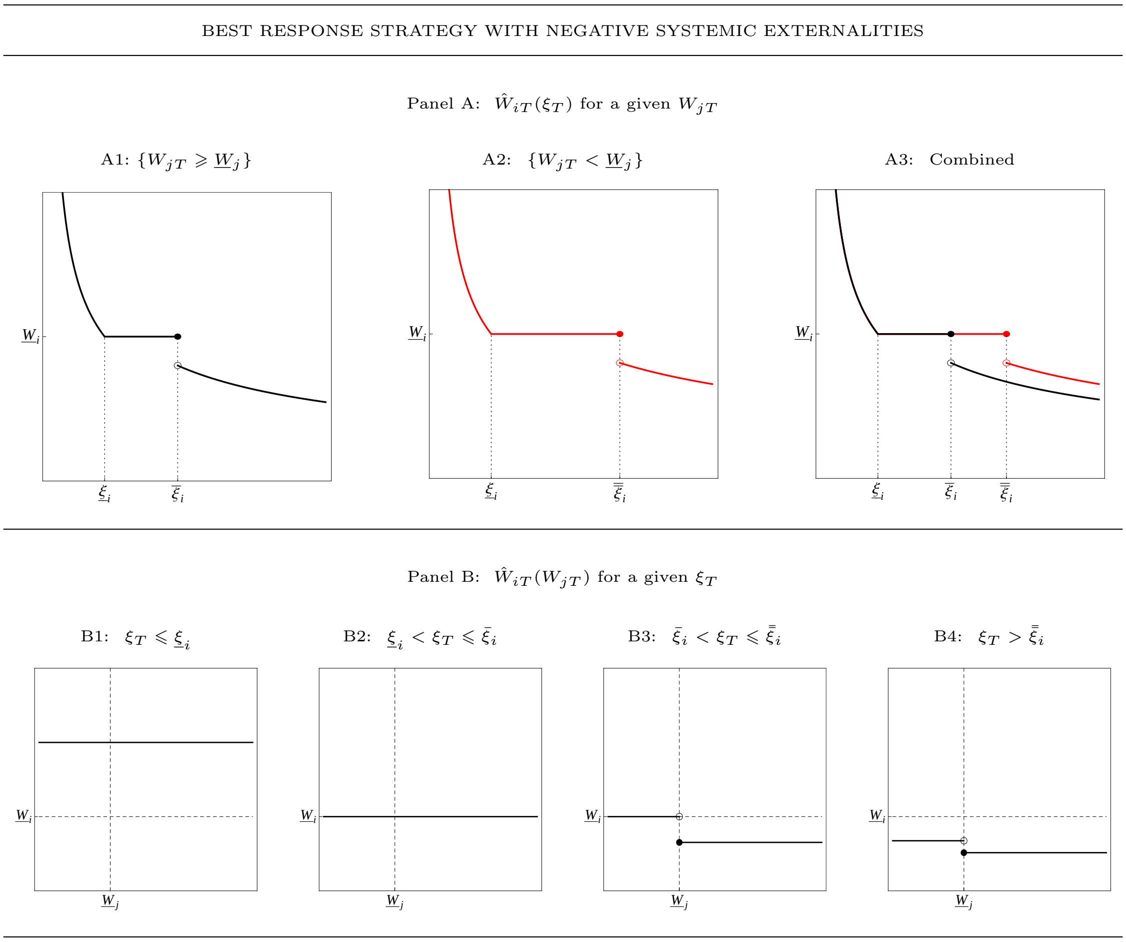

Figure 2.1 Bank i's best response strategy

solvent. This corresponds to an equity value equal to the (constant) default boundary $\underline{W}_i$, as shown by the black flat line in Panel A3. Even in this case the financial soundness of bank j does not affect the decision of bank i to *resist* default.

If economic conditions get worse ($\bar{\xi}_i < \xi_T \leqslant \bar{\bar{\xi}}_i$), then the trade-off between paying default costs if defaulting and paying expensive wealth (at the expenses of wealth in other states of the world) if resisting default becomes dependent on the equity choice of bank j. Indeed, if bank j does not default, it is optimal for bank i to do so because distress costs (per unit of default) are, in relative terms, low. In contrast, if bank j is in distress, it is optimal for bank i to resist default in order to avoid high systemic costs. Finally, in the very bad states of the world ($\xi_T > \bar{\bar{\xi}}_i$), the price of wealth is so high that makes default the optimal choice for bank i. Note that, in these states, the financial condition of bank j does not affect the bank i's decision to default but rather the level of default. In fact, if bank j is in distress, the loss given default of bank i is lower, thus implying a higher equity value. This is explained by the attempt of bank i to at least reduce the unavoidable systemic costs, as captured by the gap between the red and the black lines in Panel A3, or by the jump in correspondence to $\underline{W}_j$ in Panel B4.

Note that in an economic environment without externalities ($\eta_i = 0$, in our model), the optimal policy of bank i would coincide with its best response strategy since conditioning on any policy adopted by bank j has no impact on bank i's decision. However, in a strategic setting ($\eta_i > 0$), the optimal policies of the two institutions are given by the interaction of their best response strategies. Such "interaction" pins down those strategies that are mutually consistent with each other, and it is formally characterized in the next section.

2.2.2 Equilibrium Strategies

In the current section, we show that the strategic interaction between financial institutions can generate both unique and multiple equilibria. We establish the conditions for these two possible outcomes, and we provide the economic intuition of the underlying mechanism.

Since each bank's best response strategy is characterized by three different thresholds, the entire state space can be generically divided into seven partitions. Definition 2 implies that an equilibrium of the strategic game exists provided that: (i) a Nash equilibrium exists for any ξ_T in the seven partitions; (ii) each bank's budget constraint is satisfied. In particular, following the characterization in Basak and Makarov (2011), a Nash equilibrium is unique if for any ξ_T there is one and only one strategy-pair (W_{1T}^*, W_{2T}^*) both banks agree on and have no incentive to deviate from:

$$W_{1T}^* = \hat{W}_{1T}(W_{2T}^*, \xi_T) \qquad \text{and} \qquad W_{2T}^* = \hat{W}_{2T}(W_{1T}^*, \xi_T) \qquad \forall\ \xi_T.$$

Multiple equilibria, instead, occur if for each ξ_T the banks agree on at least one strategy-pair (W_{1T}^*, W_{2T}^*) and for some states ξ_T they agree on more than one. The following Proposition provides the condition for uniqueness of a pure-strategy equilibrium, and, in such case, characterizes the banks' equilibrium strategies.

Proposition 2. *Consider two heterogeneous banks. If the initial capital of one, and only one, of the two banks is below some threshold,* $W_{j0} \leqslant \bar{W}_{j0}$*, then the Nash Equilibrium is unique, otherwise multiple equilibria occur. When unique, the equilibrium is characterized* *by*

$$W_{iT}^* = \begin{cases} (y_i^* \xi_T)^{-\frac{1}{\gamma_i}} & \text{if} \quad \xi_T \leqslant \underline{\xi}_i \\ \underline{W}_i & \text{if} \quad \underline{\xi}_i < \xi_T \leqslant \bar{\bar{\xi}}_i \\ (y_i^* z_i \xi_T)^{-\frac{1}{\gamma_i}} & \text{if} \quad \xi_T > \bar{\bar{\xi}}_i, \end{cases} \qquad W_{jT}^* = \begin{cases} (y_j^* \xi_T)^{-\frac{1}{\gamma_j}} & \text{if} \quad \xi_T \leqslant \underline{\xi}_j \\ \underline{W}_j & \text{if} \quad \underline{\xi}_j < \xi_T \leqslant \bar{\xi}_j \\ (y_j^* x_j \xi_T)^{-\frac{1}{\gamma_j}} & \text{if} \quad \bar{\xi}_j < \xi_T \leqslant \bar{\bar{\xi}}_i \\ (y_j^* z_j \xi_T)^{-\frac{1}{\gamma_j}} & \text{if} \quad \xi_T > \bar{\bar{\xi}}_i, \end{cases} \tag{2.26}$$

where y_i^* and y_j^* are such that

$$\mathbb{E}\left[\xi_T(W_{iT}^*(y_i^*) + C_{iT}(W_{iT}^*(y_i^*)))\right] = W_{i0} \tag{2.27}$$

$$\mathbb{E}\left[\xi_T(W_{jT}^*(y_j^*, y_i^*) + C_{jT}(W_{jT}^*(y_j^*, y_i^*)))\right] = W_{j0}. \tag{2.28}$$

The thresholds $\bar{W}_{j0}$ for $j \in \{1, 2\}$ are defined in the Appendix A.

Proof. See the Appendix A. □

Remark 1. *To highlight the generality of our result, the equilibrium is solved in the Appendix A for a generic objective function $u_i(\cdot)$ that satisfies the usual assumptions: it is strictly increasing, strictly concave, twice continuously differentiable, and it satisfies the Inada conditions.*

Proposition 2 reveals that, if there is some degree of heterogeneity among banks, the equilibrium of the strategic game is unique. The intuition is that sufficient heterogeneity guarantees that in equilibrium there are no states of the world (at time T) in which both banks want to default but only one can. This would be the case when the two regions

$$\bar{\xi}_1 < \xi_T \leqslant \bar{\bar{\xi}}_1 \qquad \text{and} \qquad \bar{\xi}_2 < \xi_T \leqslant \bar{\bar{\xi}}_2 \tag{2.29}$$

overlap. The existence of such set of states,

$$\bar{\xi}_i < \xi_T \leqslant \bar{\bar{\xi}}_j, \tag{2.30}$$

creates multiple equilibria since there is no rule/mechanism to select which bank should default for each state in that set. The banks agree on the fact that only one bank should default but they can not agree on which one. For instance, suppose that $\bar{\xi}_1 < \xi_T \leqslant \bar{\bar{\xi}}_2$. According to (2.22), bank 1 wants to default if bank 2 is solvent, and resist default otherwise. At the same time, bank 2 wants to default only if bank 1 is solvent. Therefore, for any state of the world in that interval, two possible strategy-pairs can be part of an equilibrium:

$$(W_{1T}^* < \underline{W}_1, W_{2T}^* = \underline{W}_2) \qquad \text{or} \qquad (W_{1T}^* = \underline{W}_1, W_{2T}^* < \underline{W}_2). \tag{2.31}$$

We can conclude that the condition for uniqueness requires that the two regions defined in (2.29) do not overlap:

$$\bar{\bar{\xi}}_2 < \bar{\xi}_1 \qquad \text{or} \qquad \bar{\bar{\xi}}_1 < \bar{\xi}_2. \tag{2.32}$$

In the Appendix A, we show that (2.32) translates into a simple threshold condition on the initial capital of each of two banks, $W_{i0} \leqslant \bar{W}_{i0}$ for $i \in \{1, 2\}$. When one, and at most one, of these conditions is satisfied, the equilibrium is unique. While selecting among multiple equilibria is beyond the scope of this paper, we do establish the result that also homogenous banks adopt heterogenous investment strategies, in an attempt to minimize the systemic externality.[8]

Corollary 1. *Consider two homogeneous banks. Then:*

(i) There are Multiple Nash Equilibra.

(ii) A symmetric equilibrium, $W_{1T}^* = W_{2T}^*$, does not exists.

Proof. See the Appendix A. □

Let us now focus on the unique equilibrium. Proposition 2 unveils the distinct equilibrium strategies of the two institutions. In particular, the unique equilibrium is such that the two banks can be classified based on their default behavior. If $W_{j0} \leqslant \bar{W}_{j0}$, we label bank j as *early-defaulter*, and bank i as *late-defaulter*, because in equilibrium the former will default "earlier" in the state-space dimension (and not in the time-dimension since default can happen only at the final horizon T). Indeed, the *early-defaulter* bank will enter financial distress if $\xi_T > \bar{\xi}_j$, whereas the *late-defaulter* if $\xi_T > \bar{\bar{\xi}}_i$. Given (2.32), it is straightforward to show that $\bar{\xi}_j < \bar{\bar{\xi}}_i$.

We now describe the unique equilibrium strategies. Without loss of generality, let bank 1 be the *late-defaulter*. Equation (2.26) highlights how in equilibrium bank 1 can afford resisting default in a larger set of states, and hence its equity profile exhibits a wider region in which the optimal equity levels at the default boundary $\underline{W}_1$. When extremely poor economic conditions materialize ($\xi_T > \bar{\bar{\xi}}_1$), default occurs. Despite the discontinuity at $\bar{\bar{\xi}}_1$ caused by the fixed cost of default, the equity profile of bank 1 is monotonic in the state price density.

[8]Appendix A.3 presents an example of multiple equilibria with homogeneous banks.

In contrast, bank 2, the *early-defaulter*, finds it very expensive to maintain its equity value to a level ($\underline{W}_2$) that is insensitive to economic fluctuation, especially considering that bank 1 does not default for any $\xi_T < \bar{\xi}_1$, as established in the previous section. Therefore, since $W_{20} \leqslant \bar{W}_{20}$ implies that $\bar{\bar{\xi}}_2 < \bar{\xi}_1$, it is optimal for bank 2 to enter financial distress for any realizations of the state price density in the interval $\bar{\xi}_2 < \xi_T \leqslant \bar{\xi}_1$. When $\bar{\xi}_1 < \xi_T \leqslant \bar{\bar{\xi}}_1$, bank 1 finds it optimal to *resist* default since bank 2 has no incentive to do so. This explains the extended region in which the equity of bank 2 sharply decreases below the default threshold (*idiosyncratic default*). Effectively, this sharp decrease, allows bank 2 to finance an increase in the default level once also bank 1 becomes insolvent (*systemic default*). The transfer of wealth from the idiosyncratic default states to the systemic default states makes the equity profile of bank 2 non-monotonic in the state price density. Indeed, at $\bar{\bar{\xi}}_1$, the value of the equity of bank 2 exhibits a upward jump equal to $(y_2^* z_2 \bar{\bar{\xi}}_1)^{-1/\gamma_2} - (y_2^* x_2 \bar{\bar{\xi}}_1)^{-1/\gamma_2}$. Moreover, in the systemic default region ($\xi_T > \bar{\bar{\xi}}_1$), the curvature of the equity profile becomes flatter, highlighting bank 2's desire to reduce its exposure to economic fluctuations. These unique features are brought about by the strategic interaction between the financial institutions. They would be absent in an economy without systemic externalities.

For completeness, in the following Corollary we show how to recover the equilibrium value of the assets and the debt at maturity from the equilibrium value of the equity.

Corollary 2. *The optimal value of the assets and the optimal value of the debt of bank i at time T are given by*

$$V_{iT}^* = \frac{1}{\beta_i} \left[(W_{iT}^* + C_{iT}^*) + (1 - \beta_i) \left(\underline{W} - W_{iT}^* \right) \mathbb{1}_{\{\xi_T \leqslant \underline{\xi}_i\}} \right] \tag{2.33}$$

$$D_{iT}^* = \frac{1 - \beta_i}{\beta_i} \left[(W_{iT}^* + C_{iT}^*) + \left(\underline{W} - W_{iT}^* \right) \mathbb{1}_{\{\xi_T \leqslant \underline{\xi}_i\}} \right] \tag{2.34}$$

respectively, where $C_{iT}^* \equiv C_{iT}(W_{iT}^*)$.

Proof. See the Appendix A. □

We conclude this section by presenting some economically relevant examples of sources of heterogeneity among banks that lead to a unique equilibrium.

Example 1: Heterogeneity in leverage. Consider bank 2 as more levered in the sense that $(F_2/W_{20}) - (F_1/W_{10}) > 0$. Bank 2 is the *early-defaulter* because its default boundary is higher. This case is illustrated in Panel A and Panel B, Figure 2.2.

Example 2: Heterogeneity in the objective function. Consider bank 2 as the one with a higher curvature in the objective function (more risk averse if $u_i(\cdot)$ represents

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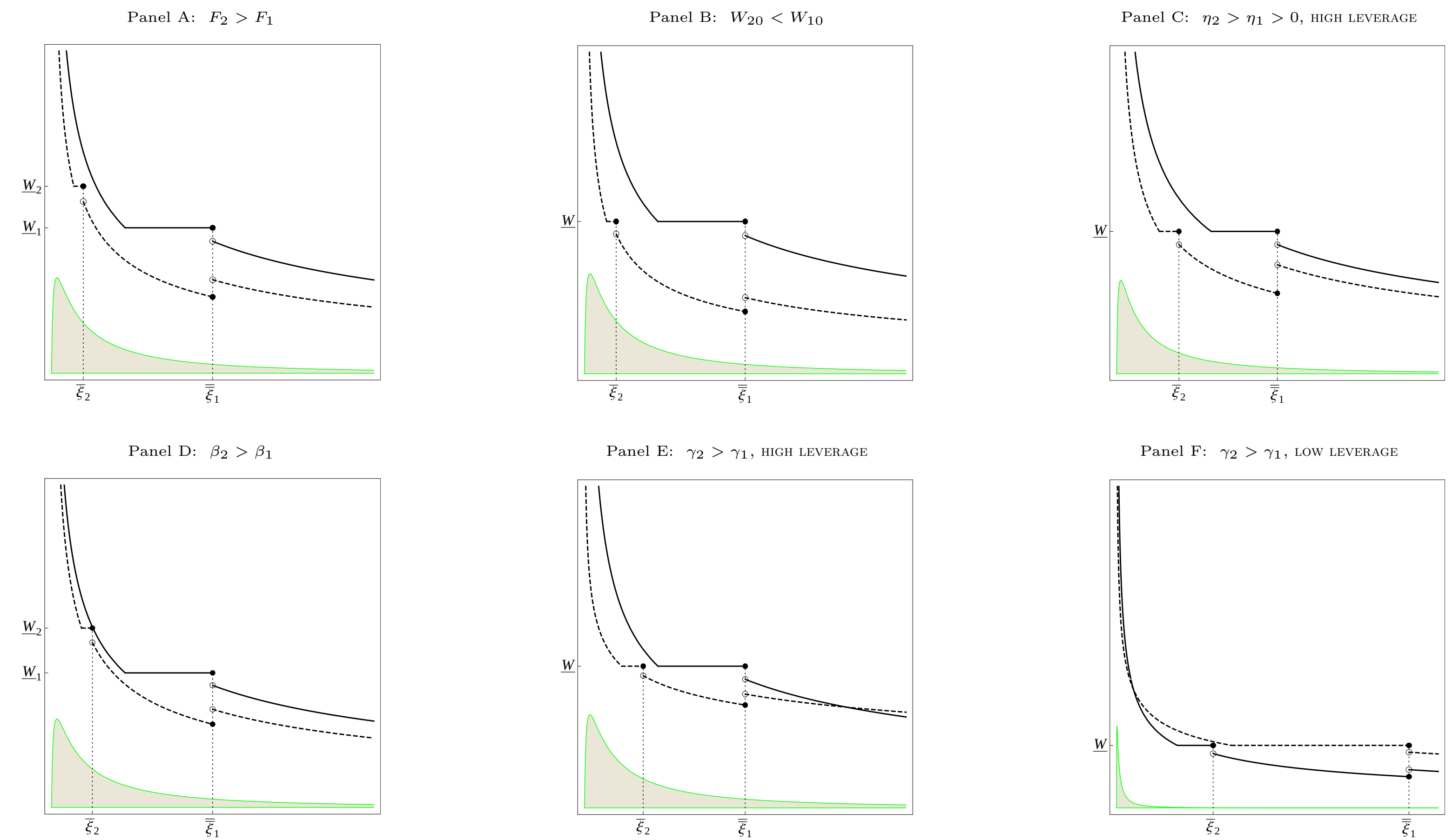
solid line: $W^*_{1T}(\xi_T)$; *dashed line*: $W^*_{2T}(\xi_T)$; *green line*: $pdf(\xi_T|\mathcal{F}_0)$



Parameter values. Financial market (monthly): $r = 0.005$, $||\kappa|| = 0.2$. Horizon (years): $T = 5$. Banks: $W_{10} = W_{20} = 1$, $F_1 = F_2 = 7$, $\beta_1 = \beta_2 = 0.2$, $\gamma_1 = \gamma_2 = 2$, $\phi = 1.5\% W_{i0}$, $\lambda = 5\%$, $\eta_1 = \eta_2 = 15\%$. Each panel contains a specific source of heterogeneity. Panel A: $F_2 = 9$. Panel B: $W_{20} = 0.778$, Panel C: $\eta_1 = 5\%$, $\eta_2 = 25\%$. Panel D: $\beta_2 = 0.25$. Panel E: $\gamma_2 = 4$. Panel F: $\gamma_2 = 4$, $F_1 = F_2 = 3.5$.

Figure 2.2 Equilibrium horizon equity with heterogeneous banks

a utility function), $\gamma_2 > \gamma_1$. Then, bank 2 is the *early-defaulter* if the level of leverage is high, whereas it becomes the *late-defaulter* if leverage is low. The intuition is the following: being more averse to the bad states of the world, bank 2 wants to transfer wealth from the good states to the bad states. If leverage is high, it means that default can occur in relatively good states; hence, by removing wealth from those states, bank 2 will increase the probability of default thus becoming the *early-defaulter*. If instead, leverage is low, default occurs only in very bad states of the world, exactly those states that bank 2 has "insured" by transferring wealth to. Therefore, it will decrease the probability of default, thus becoming the *late-defaulter*. These two case are illustrated in Panel E and Panel F, Figure 2.2.

Example 3: Heterogeneity in systemic costs. Consider bank 2 as to be more affected by systemic externalities, $\eta_2 > \eta_1$. If leverage is high, systemic costs become relevant and induce bank 2 to transfer wealth from the good states to the bad states. When leverage is high, bank 2 increases the probability of default because it transfers wealth away from those states in which default gets triggered. Hence, bank 2 is the *early-defaulter*. This case is illustrated in Panel C, Figure 2.2.

Example 4: Heterogeneity in intangible assets/bargaining power (APR violations). Consider the case in which bank 2 has a higher fraction of intangible assets or its equityholders have a higher bargaining power, $\beta_2 > \beta_1$. Since debtholders of bank 2 are going to seize a lower fraction of the assets in case of default, then the default boundary must increase, making bank 2 the *early-defaulter*. This case is illustrated in Panel D, Figure 2.2.

2.3 Unique Equilibrium Properties

In the current section we analyze properties and implications of the unique strategic equilibrium derived in Section 2.2. Since different sources of heterogeneity lead to very similar (unique) equilibrium profiles, as shown in Figure 2.2, we concentrate, for the sake of clarity, on only one. Specifically, we consider heterogeneity in leverage in the form of different face values of debt ($F_j > F_i$).[9]

Moreover, we highlight the relevance of these results by comparing them with those derived from two benchmark models: (a) *no cost of default*; (b) *no systemic cost of default*. Both benchmarks are special cases of our model where banks behave non-strategically. Benchmark (a) represents a frictionless economy where there are no costs associated to financial distress, $\phi = \lambda = \chi_i = \eta_i = 0$, for any i. In benchmark (b), instead, default

[9] Equilibrium properties relevant to other source of heterogeneity are very similar to the one presented in this section, and are available from the author upon request.

is costly but systemic externalities are absent, $\chi_i = \eta_i = 0$, for any i. We refer to Appendix A.2 for the solution of the two benchmark models. In what follows, we adopt the upscripts $*$, a and b as notational convention for any equilibrium quantity related to the strategic model, benchmark (a) and benchmark (b), respectively.

2.3.1 Risk Taking Behavior: Optimal Asset Allocation

How are the banks' risk exposures affected by the attempt to internalize (negative) systemic externalities? We provide an answer to this question by analyzing the optimal investment of the banks' assets in risky securities. The next Proposition formally states the closed-form solution for the optimal asset allocation of the two financial institutions.

Proposition 3. *The fractions of bank i's assets invested in risky securities at time t is given by*

$$\pi^*_{it} = \hat{\pi}^*_{it} \cdot (\sigma')^{-1}\kappa \qquad \textit{where} \qquad \hat{\pi}^*_{it} = -\frac{\xi_t}{V_{it}} \frac{\partial V_{it}}{\partial \xi_t}. \tag{2.35}$$

W.l.o.g., let bank 1 be the late-defaulter and bank 2 the early-defaulter. Then,

$$\begin{aligned}\hat{\pi}^*_{1t} = \frac{1}{\gamma_1} + \frac{1}{V^*_{1t}} \Bigg[& \frac{e^{-A_1(T-t)}(y^*_1\xi_t)^{-\frac{1}{\gamma_1}}}{\beta_1 ||\kappa|| \sqrt{T-t}} \left(\beta_1 n(-\hat{d}_{1t}(\underline{\xi}_1)) - z_1^{1-\frac{1}{\gamma_1}} n(-\hat{d}_{1t}(\bar{\bar{\xi}}_1)) \right) \\ & - \frac{e^{-r(T-t)}}{\beta_1\gamma_1} \left(\underline{W}_1 \left[1 - \beta_1 \mathcal{N}(-\bar{d}_t(\underline{\xi}_1)) \right] - z_1(\underline{W}_1 - \phi) \left[1 - \mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1)) \right] \right) \\ & - \frac{e^{-r(T-t)}}{\beta_1 ||\kappa|| \sqrt{T-t}} \left(\beta_1 \underline{W}_1 n(-\bar{d}_t(\underline{\xi}_1)) + z_1(\underline{W}_1 - \phi) n(-\bar{d}_t(\bar{\bar{\xi}}_1)) \right) \Bigg],\end{aligned} \tag{2.36}$$

$$\begin{aligned}\hat{\pi}^*_{2t} = \frac{1}{\gamma_2} + \frac{1}{V^*_{2t}} \Bigg[& \frac{e^{-A_2(T-t)}(y^*_2\xi_t)^{-\frac{1}{\gamma_2}}}{\beta_2 ||\kappa|| \sqrt{T-t}} \left(\beta_2 n(-\hat{d}_{2t}(\underline{\xi}_2)) + x_2^{1-\frac{1}{\gamma_2}} \left[n(-\hat{d}_{2t}(\bar{\bar{\xi}}_1)) - n(-\hat{d}_{2t}(\bar{\xi}_2)) \right] - z_2^{1-\frac{1}{\gamma_2}} n(-\hat{d}_{2t}(\bar{\bar{\xi}}_1)) \right) \\ & - \frac{e^{-r(T-t)}}{\beta_2\gamma_2} \left(\underline{W}_2 \left[1 - \beta_2 \mathcal{N}(-\bar{d}_t(\underline{\xi}_2)) \right] - x_2(\underline{W}_2 - \phi) \left[\mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1)) - \mathcal{N}(-\bar{d}_t(\bar{\xi}_2)) \right] - z_2(\underline{W}_2 - \phi) \left[1 - \mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1)) \right] \right) \\ & - \frac{e^{-r(T-t)}}{\beta_2 ||\kappa|| \sqrt{T-t}} \left(\beta_2 \underline{W}_2 n(-\bar{d}_t(\underline{\xi}_2)) + x_2(\underline{W}_2 - \phi) \left[n(-\bar{d}_t(\bar{\bar{\xi}}_1)) - n(-\bar{d}_t(\bar{\xi}_2)) \right] - z_2(\underline{W}_2 - \phi) n(-\bar{d}_t(\bar{\bar{\xi}}_1)) \right) \Bigg],\end{aligned} \tag{2.37}$$

where $\mathcal{N}(\cdot)$ and $n(\cdot)$ are the cumulative distribution function and the probability density function of a standard-normal distribution, respectively. V^*_{it}, A_i, $\hat{d}_{it}(\cdot)$, $\bar{d}_t(\cdot)$ are reported in the Appendix A.

Proof. See the Appendix A. □

Table 2.1 Unconditional portfolio moments

Parameter values. Financial market (monthly): $r = 0.005$, $||\kappa|| = 0.2$. Horizon (years): $T = 5$. Banks: $W_{10} = W_{20} = 1$, $\beta_1 = \beta_2 = 0.2$, $\gamma_1 = \gamma_2 = 2$, $\phi = 1.5\% W_{i0}$, $\lambda = 5\%$, $\eta_1 = \eta_2 = 15\%$. Source of heterogeneity: $F_1 = 7$, $F_2 = 9$.

	Fraction of time: t/T				
	0.05	0.25	0.5	0.75	0.95
Panel A: Correlation					
$\rho_0(\hat{\pi}^*_{1t}, \hat{\pi}^*_{2t})$	**0.794**	**-0.044**	**-0.394**	**-0.463**	**-0.480**
$\rho_0(\hat{\pi}^a_{1t}, \hat{\pi}^a_{2t})$	0.999	0.990	0.970	0.930	0.845
$\rho_0(\hat{\pi}^b_{1t}, \hat{\pi}^b_{2t})$	0.903	0.887	0.845	0.757	0.575
Panel B: Variance					
$\mathbb{V}\text{ar}_0(\hat{\pi}^*_{1t})/\mathbb{V}\text{ar}_0(\hat{\pi}^a_{1t})$	0.013	0.012	0.034	0.118	0.276
$\mathbb{V}\text{ar}_0(\hat{\pi}^*_{2t})/\mathbb{V}\text{ar}_0(\hat{\pi}^a_{2t})$	0.135	0.187	0.253	0.381	1.147
$\mathbb{V}\text{ar}_0(\hat{\pi}^*_{1t})/\mathbb{V}\text{ar}_0(\hat{\pi}^b_{1t})$	0.079	0.051	0.116	0.301	0.521
$\mathbb{V}\text{ar}_0(\hat{\pi}^*_{2t})/\mathbb{V}\text{ar}_0(\hat{\pi}^b_{2t})$	**3.013**	**1.338**	**1.050**	**1.100**	**2.388**
Panel C: Mean					
$\mathbb{E}_0(\hat{\pi}^*_{1t})/\mathbb{E}_0(\hat{\pi}^a_{1t})$	0.252	0.301	0.382	0.481	0.567
$\mathbb{E}_0(\hat{\pi}^*_{2t})/\mathbb{E}_0(\hat{\pi}^a_{2t})$	0.439	0.476	0.520	0.563	0.598
$\mathbb{E}_0(\hat{\pi}^*_{1t})/\mathbb{E}_0(\hat{\pi}^b_{1t})$	0.344	0.395	0.482	0.588	0.680
$\mathbb{E}_0(\hat{\pi}^*_{2t})/\mathbb{E}_0(\hat{\pi}^b_{2t})$	0.573	0.605	0.643	0.678	0.705

Proposition 3 immediately reveals that the optimal asset allocation of the two banks satisfies the two-fund separation theorem: both banks in equilibrium attain full diversification. Indeed, they invest in the same portfolio of risky assets, the *mean-variance tangency portfolio* (MVTP hereafter), with an exposure to it proportional to the elasticity of the equilibrium value of the assets with respect to economic fluctuations (represented by realizations of the state price density). This result highlights how the banks' attempt to minimize systemic default costs does not distort the optimality of full diversification. This is, for instance, in contrast to the findings in Wagner (2011), where joint liquidation costs make (atomistic) agents prefer diversity at the expense of diversification. In our model full diversification is not compromised because both banks can move along the efficient frontier by altering (in a state-contingent manner) the fraction invested in the riskless bond, $(1 - \hat{\pi}_{it}((\sigma')^{-1}\kappa)'\mathbf{1})$. By moving in different directions along the frontier, they attain diversity and preserve diversification.[10]

[10] We conjecture that in the absence of a riskless asset (hence incomplete markets) banks would choose two distinct risky portfolios. The analysis of such case is beyond the scope of this paper and it is left for future research.

The fact that both institutions invest in the (same) MVTP entails that at any point in time, conditional on the realization of a state of the world, the correlation between their portfolios is indeed perfect,

$$\rho_t(\pi^*_{1t}, \pi^*_{2t}) = \pm 1 \tag{2.38}$$

for $t \in (0, T)$. Unconditionally however, this correlation is less than one because the banks select optimal stochastic exposures that are not perfectly correlated. Equations (2.36) and (2.37) unveil the stochastic nature of the banks' exposures as the source of heterogeneity in their trading strategies. Table 2.1 helps us to address the question of whether systemic costs of default reduce the (unconditional) correlation between these strategies. The answer is yes. Panel A presents the correlation coefficient as of time 0 between banks' optimal time t risk exposure, for our strategic model and for the two benchmarks. A clear ranking appears:

$$\rho_0(\hat{\pi}^*_{1t}, \hat{\pi}^*_{2t}) < \rho_0(\hat{\pi}^b_{1t}, \hat{\pi}^b_{2t}) < \rho_0(\hat{\pi}^a_{1t}, \hat{\pi}^a_{2t}) \tag{2.39}$$

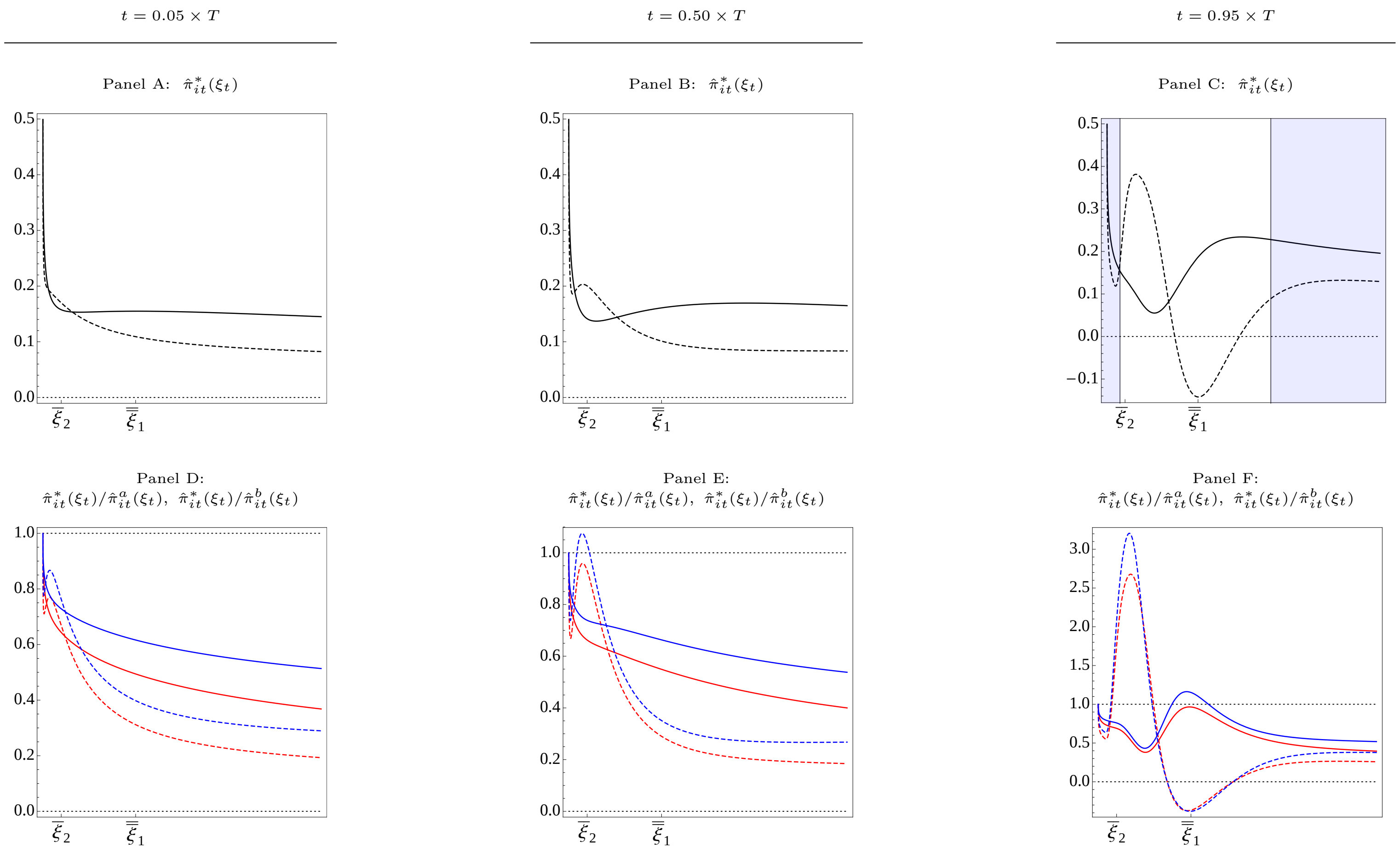
for $t \in (0, T)$. The attempt to internalize systemic externalities, and hence to reduce systemic costs of joint default, induces the two banks to become more diverse. Table 2.1 also shows that the unconditional correlation decreases in all three models when maturity gets closer, becoming even negative in the strategic one. These results are qualitatively robust to different parameter values.

Figure 2.3 provides a graphical illustration of the investment behavior of the two financial institutions. In the top panels, exposures $\hat{\pi}^*_{it}$ to the MVTP are plotted against realizations of the state price density for different points in time. In the bottom panels, instead, we plot the ratio between time t exposures coming from the strategic model and the two benchmarks (red line for benchmark (a), blue line for benchmark (b)). The following results emerge.

First, as shown in Table 2.1, the banks' investment strategies become more uncorrelated as time to maturity decreases, and this is especially true for realizations of the state price density around the default thresholds (Panel A-C, Figure 2.3). The intuition is the following. Banks find it optimal to adopt diverse investment strategies if they reduce the likelihood of a systemic default. When t is close to T and economic conditions are very good (low values of ξ_t), it is very likely that at maturity such conditions will remain good; therefore, there is no need for diversity since any default is very unlikely. Analogously, when economic conditions are extremely bad (high values of ξ_t), it is very likely that at maturity bad conditions will persist: in this case, it would be too costly for both banks to avoid joint default, thus making diversity unnecessary. If instead, economic conditions are not so extreme (values of ξ_t around the default thresholds $\bar{\xi}_2$ and $\bar{\bar{\xi}}_1$), then diversity pays off. Trying to avoid joint default is not too costly and can be done by adopting

RISK TAKING BEHAVIOR

solid line: Bank 1; *dashed line*: Bank 2; *black line*: Strategic Model; *red line*: Benchmark Model (a); *blue line*: Benchmark Model (b)

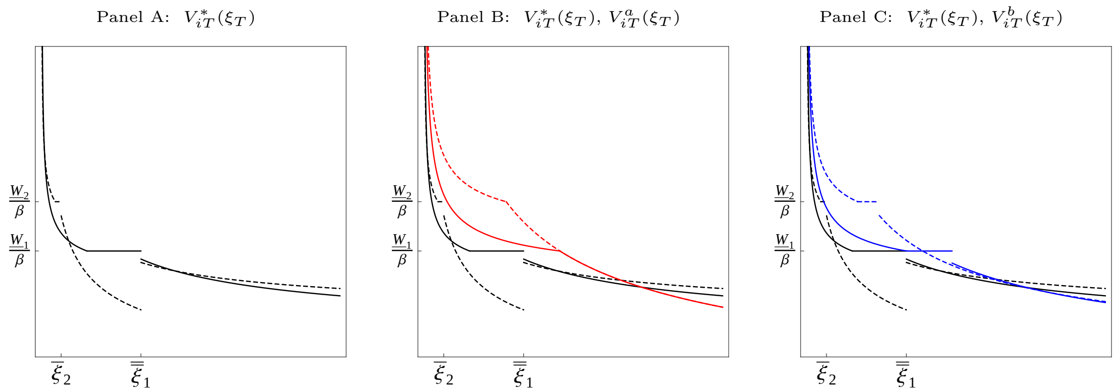


Parameter values. Financial market (monthly): $r = 0.005$, $||\kappa|| = 0.2$. Horizon (years): $T = 5$. Banks: $W_{10} = W_{20} = 1$, $\beta_1 = \beta_2 = 0.2$, $\gamma_1 = \gamma_2 = 2$, $\phi = 1.5\% W_{i0}$, $\lambda = 5\%$, $\eta_1 = \eta_2 = 15\%$. Source of heterogeneity: $F_1 = 7$, $F_2 = 9$.

Figure 2.3 Banks' optimal asset allocation

UNIQUE EQUILIBRIUM: VALUE OF BANKS' ASSETS

solid line: Bank 1; *dashed line*: Bank 2;
black line: Strategic Model; *red line*: Benchmark Model (a); *blue line*: Benchmark Model (b)



Parameter values. Financial market (monthly): $r = 0.005$, $||\kappa|| = 0.2$. Horizon (years): $T = 5$. Banks: $W_{10} = W_{20} = 1$, $\beta_1 = \beta_2 = 0.2$, $\gamma_1 = \gamma_2 = 2$, $\phi = 1.5\% W_{i0}$, $\lambda = 5\%$, $\eta_1 = \eta_2 = 15\%$. Source of heterogeneity: $F_1 = 7$, $F_2 = 9$.

Figure 2.4 Horizon value of banks' assets

negatively correlated investment strategies. Panel C in Figure 2.3 clearly highlights how diversity (correlation between investment strategies) changes with economic fluctuations, where the shaded areas correspond to extremely good (left area) and extremely bad (right area) economic conditions. Instead, when time to maturity is high (low t as in Panel A), current economic conditions are not representative of the economic outlook at time T; therefore, it is optimal to wait before becoming diverse.

Second, conditional on becoming diverse ("medium" economic conditions and t close to maturity), we observe interesting patterns in the risk taking decision. Consider bank 2, the *early-defaulter*. Panel C in Figure 2.3 shows that when both idiosyncratic default and systemic default are likely to occur (i.e., the intermediate unshaded area), bank 2's risk taking at time t exhibits two radically opposite behaviors. Indeed, it can be either very high, or negative. So, although defaulting is very likely, bank 2 could either invest a high fraction of the assets in risky securities, or, at the other extreme, take a short position in the MVTP.

To help explain the intuition behind this somehow surprising result, let us consider the equilibrium value of banks' assets at time T plotted in Figure 2.4, and recall that the optimal trading strategies are implemented to "deliver" those equilibrium asset values in each possible state of the world. Panel A of Figure 2.4 shows how the time T value of the assets inherits all the properties of the equilibrium equity, described in details in Section 2.2. Because joint default is more costly (the negative externality), bank 2 wants to transfer wealth to the states that correspond to a systemic crisis ($\xi_T > \bar{\bar{\xi}}_1$), in order to reduce the shortfall and hence the associated costs. This is attained by removing wealth from the most valuable states before $\bar{\bar{\xi}}_1$. Therefore, this underlying mechanism produces two key properties of the model: (i) the sharp decrease in asset value in the idiosyncratic default region ($\bar{\xi}_2 < \xi_T \leqslant \bar{\bar{\xi}}_1$), and (ii) the positive jump in correspondence to the threshold $\bar{\bar{\xi}}_1$. Property (i) implies that the asset value is very sensitive and positively correlated to economic fluctuations, whereas property (ii) implies that it is negatively correlated.

Returning to the risk taking behavior at time t (close to maturity), suppose T is 5 years and we are 3 months away from maturity (as in Panel C, Figure 2.3). If economic prospects are not very good today, say ξ_t just above $\bar{\xi}_2$, then, most likely (since we are close to maturity) such prospects will not change much in 3 months and bank 2 will optimally enter (idiosyncratic) financial distress. The only way it can "deliver" an asset profile very correlated with the underlying uncertainty is by investing extensively in the market for risky securities today. To attain full diversification, a positive exposure to the MVTP is the optimal strategy. This explains the high risk taking. If, instead, economic conditions are worse today, say ξ_t just below $\bar{\bar{\xi}}_1$, then the economy will face a systemic crisis with both banks defaulting if economic prospects slightly deteriorates. The only way bank 2 can "deliver" an asset profile that will jump upwards in such a case is by investing in financial securities that are negatively correlated with the economic fluctuations. To attain full diversification, a negative exposure to the MVTP is the optimal strategy. This explains the shorting behavior. The investment decisions of bank 1 can be explained in a similar fashion. However, high risk taking and short positions are absent since both idiosyncratic default and non-monotonic behavior (upward jump) are not part of the equilibrium value of the assets of bank 1. In summary, the banks adopt polarized and stochastic risk exposure. The *early-defaulter* is the radical bank. The *late-defaulter* is the conservative one.

Moreover, banks' risk exposures when time to maturity is high are less volatile and some of the properties discussed above may not be present. For instance, the short position of bank 2 is absent in Panel B (Figure 2.3) because it would be too costly to short the MVTP two years and half before maturity. In other words, Panel B reveals that it is optimal to wait before taking extreme positions.

Third, compared to the non-strategic benchmarks, the unconditional mean of both banks' risk exposure decreases, as reported in Panel C, Table 2.1. This is not surprising since the systemic externality considered in this paper imposes an extra burden to financial institutions. However, once we condition on time to maturity and economic conditions, we obtain the following. While bank 1's risk taking is lower in the strategic model in (almost) all periods and states, bank 2's behavior exhibits significantly more risk taking when time to maturity is low.[11] Not surprisingly, the need to finance the steep reduction in asset value in the idiosyncratic default region, absent in the two benchmark models, is the cause of the high exposure to the MVTP. Indeed, Panel F in Figure 2.3 shows how that the risk exposures in the strategic model can be around 2.5 and 3 times larger than in benchmark (a) and (b), respectively. Another feature that differentiates the strategic model from the benchmarks is the aforementioned shorting behavior. Since in both non-strategic models the optimal value of the assets at maturity is monotonic in the state price density (see Panel B and C in Figure 2.4), there is no need to take a (costly) short position in the MVTP. This explains the difference in the variability of the asset allocation of bank 2, between the strategic model and the two benchmark models. The unconditional variance reported in Panel B, Table 2.1 confirms these findings.

2.3.2 Default Probabilities and Expected Shortfalls

In this section we study another set of important implications of the model. We analyze how internalizing systemic externalities affects the likelihood of idiosyncratic defaults and systemic crises. We then complement these findings with results on the expected shortfalls.

The equilibrium equity profiles of the two financial institutions, derived in Section 2.2, endogenously determine the banks' optimal default thresholds in the state-space. For $\xi_T \leqslant \bar{\xi}_2$ neither of the banks default; for $\bar{\xi}_2 < \xi_T \leqslant \bar{\bar{\xi}}_1$ bank 2 (the *early-defaulter*) is insolvent; for $\xi_T > \bar{\bar{\xi}}_1$ both banks default. The equilibrium default probabilities are defined only by the (absolute) positioning of the default thresholds in the state-space domain, as presented in the next Proposition.

Proposition 4. *Let $\mathcal{D}_i$ denote the financial distress event of bank i, and $\bar{\mathcal{D}}_i$ its complement. W.l.o.g., let bank 1 be the late-defaulter and bank 2 the early-defaulter. The strategic model produces the following time-t probabilities:*

(i) Marginal probability of default:

$$\mathbb{P}_t(\mathcal{D}_1) = \mathcal{N}(d_t(\bar{\bar{\xi}}_1)) \qquad \textit{and} \qquad \mathbb{P}_t(\mathcal{D}_2) = \mathcal{N}(d_t(\bar{\xi}_2)) \tag{2.40}$$

(ii) Probability of systemic default:

$$\mathbb{P}_t(\mathcal{D}_1 \cap \mathcal{D}_2) = \mathcal{N}(d_t(\bar{\bar{\xi}}_1)) \tag{2.41}$$

[11]Panel F in Figure 2.3 shows that when t is close to T and ξ_t is close to $\bar{\bar{\xi}}_1$, bank 1 can in fact take slightly more risk than in benchmark (b). This is driven by the need to finance a slightly bigger fall in asset value, should a systemic distress occur. This effect, though, is quantitatively small, if compared to the risk taking of bank 2.

(iii) Probability of idiosyncratic default:

$$\mathbb{P}_t(\cup_{j\neq i\in\{1,2\}}\mathcal{D}_i \cap \bar{\mathcal{D}}_j) = \mathcal{N}(d_t(\bar{\xi}_2)) - \mathcal{N}(d_t(\bar{\bar{\xi}}_1)) \tag{2.42}$$

(iv) Conditional probability of default:

$$\mathbb{P}_t(\mathcal{D}_1|\mathcal{D}_2) = \mathcal{N}(d_t(\bar{\bar{\xi}}_1))/\mathcal{N}(d_t(\bar{\xi}_2)) \qquad \textit{and} \qquad \mathbb{P}_t(\mathcal{D}_2|\mathcal{D}_1) = 1 \tag{2.43}$$

where $\mathcal{N}(\cdot)$ represents the standard-normal cumulative distribution function. The function $d_t(\cdot)$ and the thresholds $\bar{\xi}_2$ and $\bar{\bar{\xi}}_1$ are reported in the Appendix A.

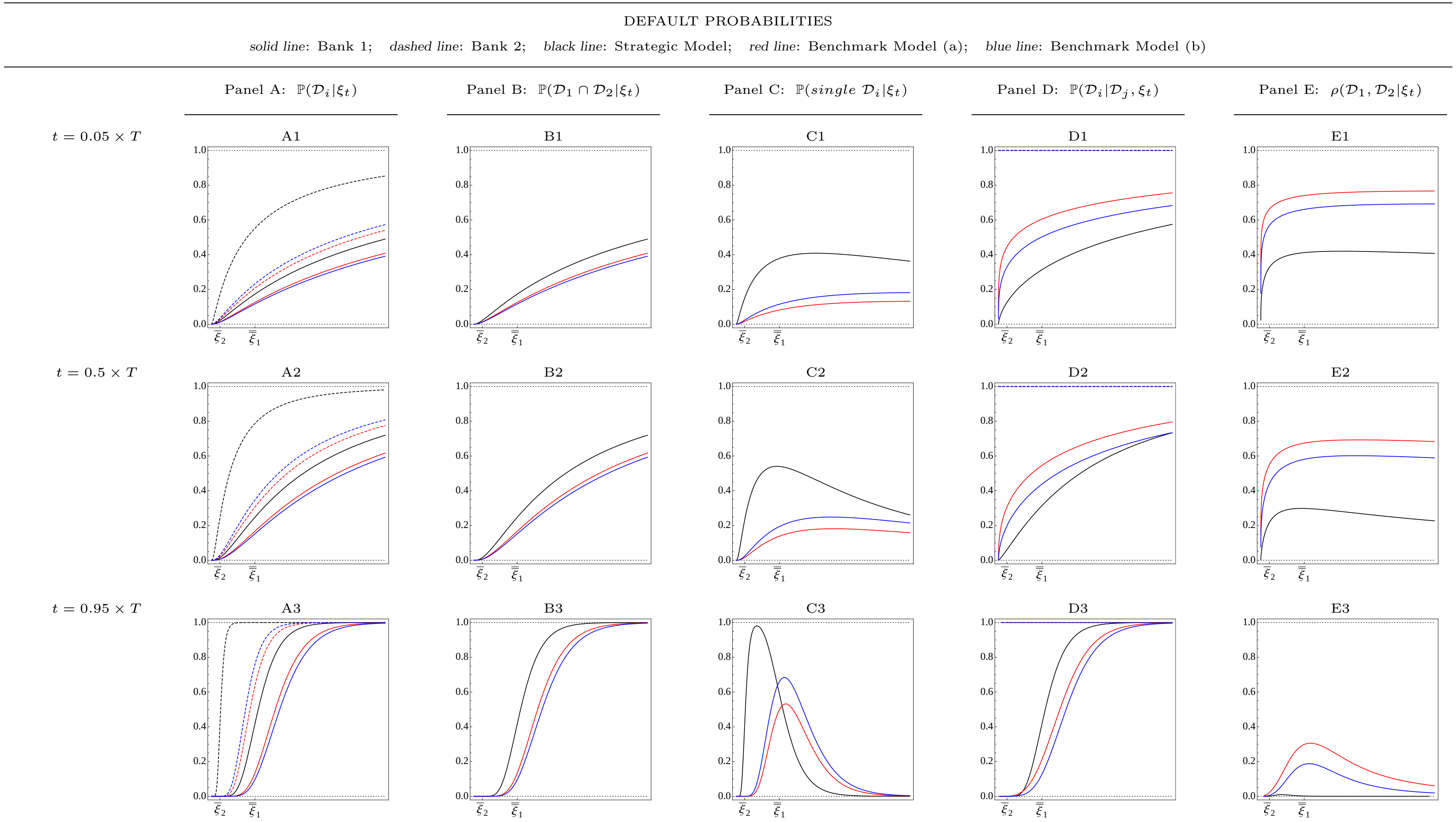
Proof. See the Appendix A. □

To appreciate the effect of the strategic interactions between the two financial institutions on the idiosyncratic and systemic default probabilities, we compare those of the strategic model with those of the two benchmarks. Similarly, we compare the expected shortfalls across the three models. A key question is: Are incentives to internalize systemic externalities effective in reducing the occurrence and the magnitude of systemic crises? The answer to this question is particularly relevant from a regulator's viewpoint. Thanks to our flexible specification, the systemic costs of default can be readily interpreted as a macro-prudential regulation, allowing us to provide an answer.

Regarding default probabilities, we exploit the graphs in Figure 2.5 to highlight our findings. The following results emerge. Relative to the benchmarks, both banks are more likely to default, as illustrated in Panel A, where the black lines (the strategic model) are substantially higher than the corresponding colored lines (the benchmarks). This is true regardless of the time t and state ξ_t in which we evaluate these probabilities. An immediate implication of this result is that systemic default becomes more likely, since the event of a systemic crisis coincides with the default of bank 1 (Panel B).

Panel C reveals that, compared to the benchmarks, also the probability of an idiosyncratic default rises substantially. When close to maturity (Panel C3), however, this probability may become lower if the current economic conditions are particularly poor. If this is the case, an idiosyncratic default is less likely because the probability of systemic default is particularly high. Hence, the positioning of the black line to the left of benchmarks' curves and its steeper profile confirms how the idiosyncratic default interval shifts to the left in the presence of negative systemic externalities.

Panel D considers the (non-degenerate) conditional probability of default defined in (2.43). Such a probability can be interpreted as the relative importance (in probabilistic



Parameter values. Financial market (monthly): $r = 0.005$, $||\kappa|| = 0.2$. Horizon (years): $T = 5$. Banks: $W_{10} = W_{20} = 1$, $\beta_1 = \beta_2 = 0.2$, $\gamma_1 = \gamma_2 = 2$, $\phi = 1.5\% W_{i0}$, $\lambda = 5\%$, $\eta_1 = \eta_2 = 15\%$. Source of heterogeneity: $F_1 = 7$, $F_2 = 9$.

Figure 2.5 Banks' default probabilities

terms) of a systemic crisis with respect to an idiosyncratic default:

$$\mathbb{P}_t(\mathcal{D}_1|\mathcal{D}_2) = \frac{\mathbb{P}_t(systemic\ \mathcal{D}efault)}{\mathbb{P}_t(idiosyncratic\ \mathcal{D}efault) + \mathbb{P}_t(systemic\ \mathcal{D}efault)}. \tag{2.44}$$

Panels D1-D2 show that, away from maturity, systemic externalities and strategic interaction lower the relative importance of a systemic crisis. However, close to maturity, the opposite holds.

Finally, Panel E considers the *default correlation*, defined as

$$\rho_t(\mathcal{D}_1, \mathcal{D}_2) \equiv \frac{\mathbb{P}_t(\mathcal{D}_1 \cap \mathcal{D}_2) - \mathbb{P}_t(\mathcal{D}_1)\mathbb{P}_t(\mathcal{D}_2)}{\sqrt{\mathbb{P}_t(\mathcal{D}_1)[1-\mathbb{P}_t(\mathcal{D}_1)]}\sqrt{\mathbb{P}_t(\mathcal{D}_2)[1-\mathbb{P}_t(\mathcal{D}_2)]}}. \tag{2.45}$$

In line with the results on the optimal asset allocation, the default correlation in the strategic model lower for any time to maturity and current state of the economy.

This set of findings does not easily reconcile with the ex-ante objective of macro-prudential regulation to reduce the likelihood of a systemic event. We provide next a simple explanation for these results. We argue that, compared to the benchmarks, default thresholds and hence default probabilities, are affected by the net result of a substitution and an income effect. The substitution effect captures the banks' desire to transfer wealth from the good states to the bad states in order to reduce the potential exposure to high cost of systemic default. Therefore, the substitution effect translates into a lower probability of joint default, since banks attempt to internalize systemic externalities. However, although it incentivizes a transfer of wealth across states, a high cost of systemic default also represents a burden to the banks' budget constraint. Equivalent to an overall drop in the banks' capital, the income effect causes a higher probability of joint default. The equilibrium outcome is thus the net of these two opposing effects. The two competing effects are "extracted" from the ratio of systemic default thresholds across models:

$$\bar{\bar{\xi}}_1/\underline{\xi}_1^a = \underbrace{\left(\frac{\alpha_1}{\underline{W}_1^{-\gamma_1} z_1}\right)}_{\text{substitution effect}} \Big/ \underbrace{\left(\frac{y_1^*}{y_1^a}\right)}_{\text{income effect}}, \tag{2.46}$$

$$\bar{\bar{\xi}}_1/\bar{\xi}_1^b = \underbrace{\left(\frac{x_1}{z_1}\right)}_{\text{substitution effect}} \Big/ \underbrace{\left(\frac{y_1^*}{y_1^b}\right)}_{\text{income effect}}. \tag{2.47}$$

Hence, we define the substitution and income effects as follows,

$$\varsigma^a \equiv \alpha_1/(\underline{W}_1^{-\gamma_1} z_1), \qquad \iota^a \equiv y_1^*/y_1^a, \qquad \varsigma^b \equiv x_1/z_1, \qquad \iota^b \equiv y_1^*/y_1^b, \tag{2.48}$$

where α_1 solves equation (2.24). The substitution effect, with respect to benchmark k, dominates if $\varsigma^k > \iota^k$ for $k \in \{a, b\}$. The income effect dominates otherwise.

Our model predicts that for financial institutions with moderate and high leverage ratios, the income effect always dominates, regardless of the magnitude of the default externalities. These dynamics lead to the documented increase in probabilities of idiosyncratic and systemic default. We assess this effect to be economically significant, considering the prevalence of extremely high leverage ratios among financial institutions. Figure 2.6 illustrates this result. Each bar-chart plots the pair of substitution (bright bar) and income (dark bar) effect for different leverage ratios. In Panel B and Panel C default costs are lower and higher than in Panel A, respectively. Compared to both benchmarks (top panels refers to benchmark (a), bottom panels to benchmark (b)), we confirm that: (i) the income effect dominates the substitution effect for medium-high leverage ratios; (ii) this relationship is not affected by the magnitude of default costs.

Default probabilities describe the likelihood of a default, but they are not informative on the extent of default. For this reason, we complete our analysis by examining expected shortfalls. Specifically, we consider those arising from an idiosyncratic and a systemic default.

Proposition 5. *W.l.o.g., let bank 1 be the late-defaulter and bank 2 the early-defaulter. The idiosyncratic and systemic expected shortfalls at time t are given by*

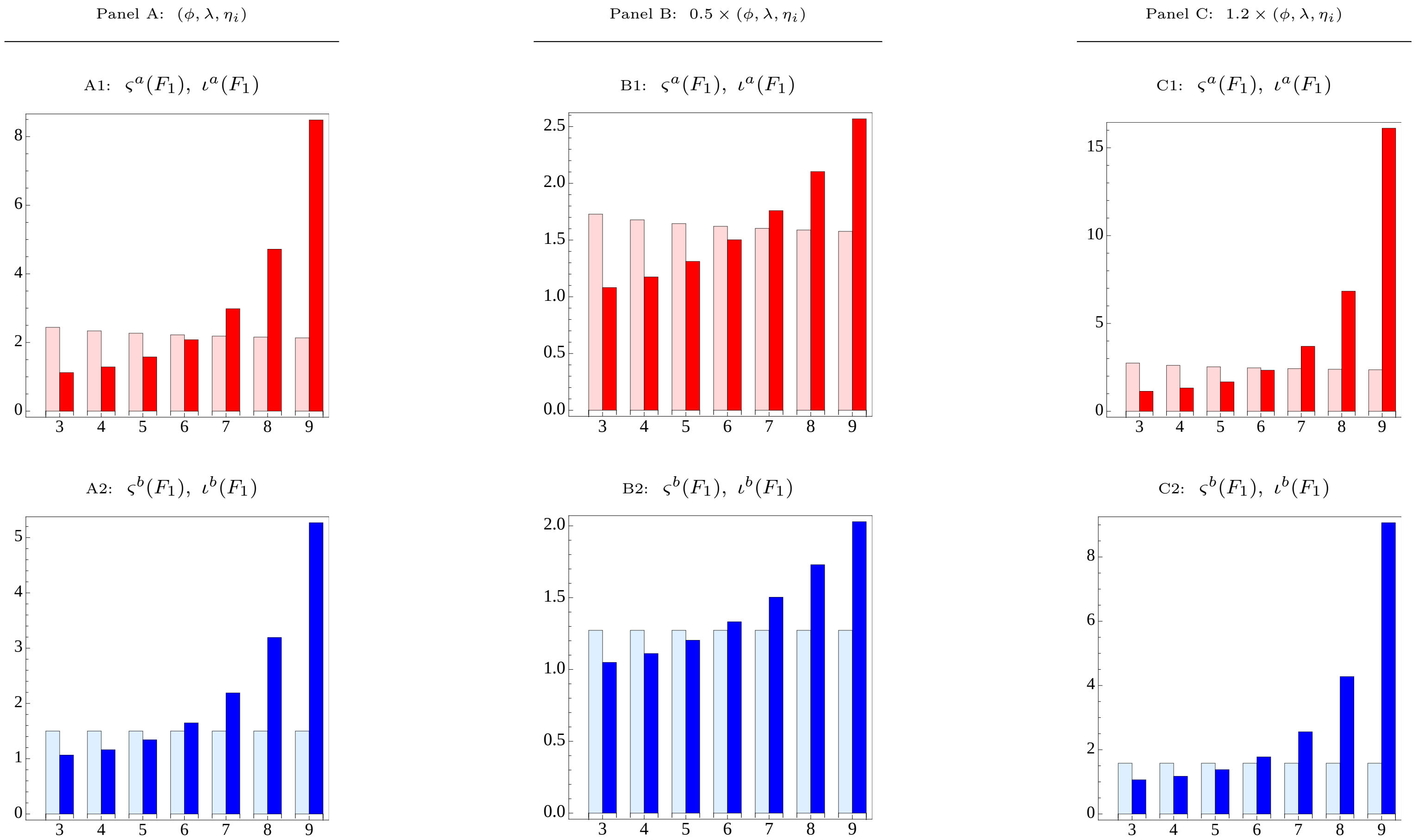
$$\begin{aligned}
IES_t &= \mathbb{E}_t\left[\frac{\xi_T}{\xi_t}(F_2 - D_{2T})\mathbb{1}_{\{\bar{\xi}_2 < \xi_T \leqslant \bar{\bar{\xi}}_1\}}\right] \\
&= \left(\frac{1-\beta_2}{\beta_2}\right)\Big[x_2(\underline{W}_2 - \phi)e^{-r(T-t)}\Big\{\mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1)) - \mathcal{N}(\bar{d}_t(\bar{\xi}_2))\Big\} \\
&\quad - x_2(y_2^* x_2 \xi_t)^{-\frac{1}{\gamma_2}} e^{-A_2(T-t)}\Big\{\mathcal{N}(-\hat{d}_{2t}(\bar{\bar{\xi}}_1)) - \mathcal{N}(\hat{d}_{2t}(\bar{\xi}_2))\Big\}\Big], && (2.49)\\
SES_t &= \mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\Big\{(F_1 - D_{1T}) + (F_2 - D_{2T})\Big\}\mathbb{1}_{\{\xi_T > \bar{\bar{\xi}}_1\}}\right] \\
&= \sum_{i=1,2}\Big(F_i e^{-r(T-t)} - D_{it}\Big) - IES_t, && (2.50)
\end{aligned}$$

respectively, where

$$\begin{aligned}
D_{1t} = &\left(\frac{1-\beta_1}{\beta_1}\right)\left[e^{-A_1(T-t)}(y_1^*\xi_t)^{-\frac{1}{\gamma_1}}\left(z_1^{1-\frac{1}{\gamma_1}}\Big[1 - \mathcal{N}(-\hat{d}_{1t}(\bar{\bar{\xi}}_1))\Big]\right)\right. \\
&\left. + e^{-r(T-t)}\left(\underline{W}_1 - z_1(\underline{W}_1 - \phi)\Big[1 - \mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1))\Big]\right)\right], && (2.51)\\
D_{2t} = &\left(\frac{1-\beta_2}{\beta_2}\right)\left[e^{-A_2(T-t)}(y_2^*\xi_t)^{-\frac{1}{\gamma_2}}\left(x_2^{1-\frac{1}{\gamma_2}}\big[\mathcal{N}(-\hat{d}_{2t}(\bar{\bar{\xi}}_1)) - \mathcal{N}(-\hat{d}_{2t}(\bar{\xi}_2))\big] + z_2^{1-\frac{1}{\gamma_2}}\big[1 - \mathcal{N}(-\hat{d}_{2t}(\bar{\bar{\xi}}_1))\big]\right)\right. \\
&\left. + e^{-r(T-t)}\left(\underline{W}_2 - x_2(\underline{W}_2 - \phi)\big[\mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1)) - \mathcal{N}(-\bar{d}_t(\bar{\xi}_2))\big] - z_2(\underline{W}_2 - \phi)\big[1 - \mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1))\big]\right)\right]. && (2.52)
\end{aligned}$$

SYSTEMIC DEFAULT THRESHOLD: SUBSTITUTION AND INCOME EFFECTS

light bars: Substitution Effect; *dark bars*: Income Effect; *top panels (red)*: Strategic Model vs Benchmark (a); *bottom panels (blue)*: Strategic Model vs Benchmark (b)



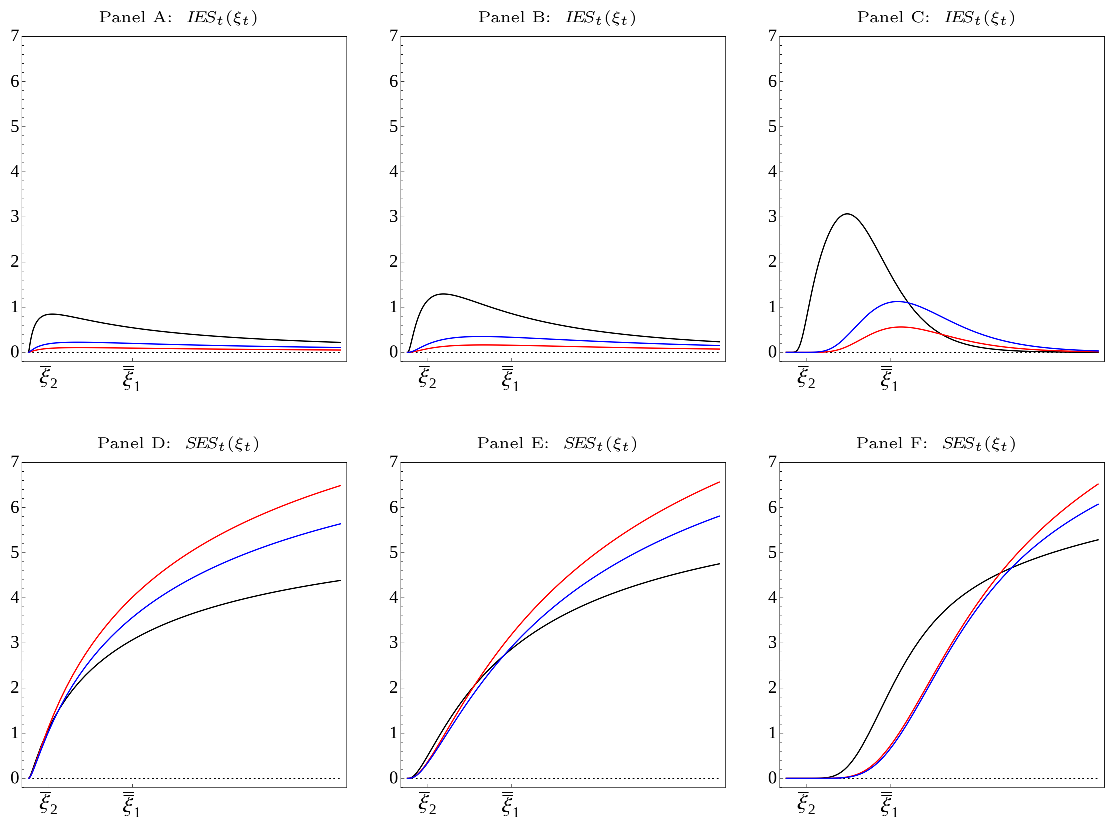
Parameter values. Financial market (monthly): $r = 0.005$, $||\kappa|| = 0.2$. Horizon (years): $T = 5$. Banks: $W_{10} = W_{20} = 1$, $\beta_1 = \beta_2 = 0.2$, $\gamma_1 = \gamma_2 = 2$, $\phi = 1.5\% W_{i0}$, $\lambda = 5\%$, $\eta_1 = \eta_2 = 15\%$. Source of heterogeneity: $F_2 = 9/7 \times F_1$.

Figure 2.6 Substitution and income effects

IDIOSYNCRATIC and SYSTEMIC EXPECTED SHORTFALL

black line: Strategic Model; *red line*: Benchmark Model (a); *blue line*: Benchmark Model (b)

$t = 0.05 \times T$	$t = 0.5 \times T$	$t = 0.95 \times T$



Parameter values. Financial market (monthly): $r = 0.005$, $||\kappa|| = 0.2$. Horizon (years): $T = 5$. Banks: $W_{10} = W_{20} = 1$, $\beta_1 = \beta_2 = 0.2$, $\gamma_1 = \gamma_2 = 2$, $\phi = 1.5\% W_{i0}$, $\lambda = 5\%$, $\eta_1 = \eta_2 = 15\%$. Source of heterogeneity: $F_1 = 7$, $F_2 = 9$.

Figure 2.7 Expected shortfall

$\mathcal{N}(\cdot)$ *represents the standard-normal cumulative distribution function.* A_i, $\hat{d}_{it}(\cdot)$, $\bar{d}_t(\cdot)$ *are reported in the Appendix A.*

Proof. See the Appendix A. □

By means of Figure 2.7, we highlight the following results. Idiosyncratic expected shortfall are significantly higher in the strategic model than in the benchmarks. This precisely reflects the transfer of wealth that the *early-defaulter* engages in. However, when time to maturity is low (Panel C), the expected shortfall may become lower if the current economic conditions are particularly poor. This is justified by the fact that, as reported in Figure 2.5 (Panel C3), the probability of an idiosyncratic default becomes very low. While we have shown that the probabilities of systemic default are unambiguously higher for the strategic model, the same does not apply to the systemic expected shortfall. Indeed, as shown in Panel D-F, by internalizing the externalities of default, the expected shortfall is always below the expected shortfall of the benchmarks in the most adverse states. Thus, we can conclude that also the expected losses *given* default must be lower under extremely adverse economic conditions, considering that systemic default probabilities are higher overall. We still find though that around the default thresholds, the expected systemic shortfall may become higher then in the benchmark cases.

We believe the results presented in this section are indicative of a friction between systemic default probabilities on one hand and systemic losses on the other hand, important to a regulator concerned with the design of a macro-prudential framework.

2.3.3 Debt Pricing: Credit Spreads and CDS

Since Merton (1974), structural models of credit risk have been developed with the scope of deriving the value of debt issued by a firm, by means of a contingent claim (no-arbitrage) analysis. In this section, we discuss the strategic equilibrium implications on the value of credit spreads and credit default swap premiums for the two banks and relate them to the results presented in the previous sections. Proposition 6 presents the closed-form equilibrium values.

Proposition 6. *W.l.o.g., let bank 1 be the late-defaulter and bank 2 the early-defaulter. The credit spreads and the credit default swap premium written on the bond of bank i are given by*

$$CS_{it} = \frac{1}{T-t}\ln\left(\frac{F_i}{D_{it}}\right) - r \tag{2.53}$$

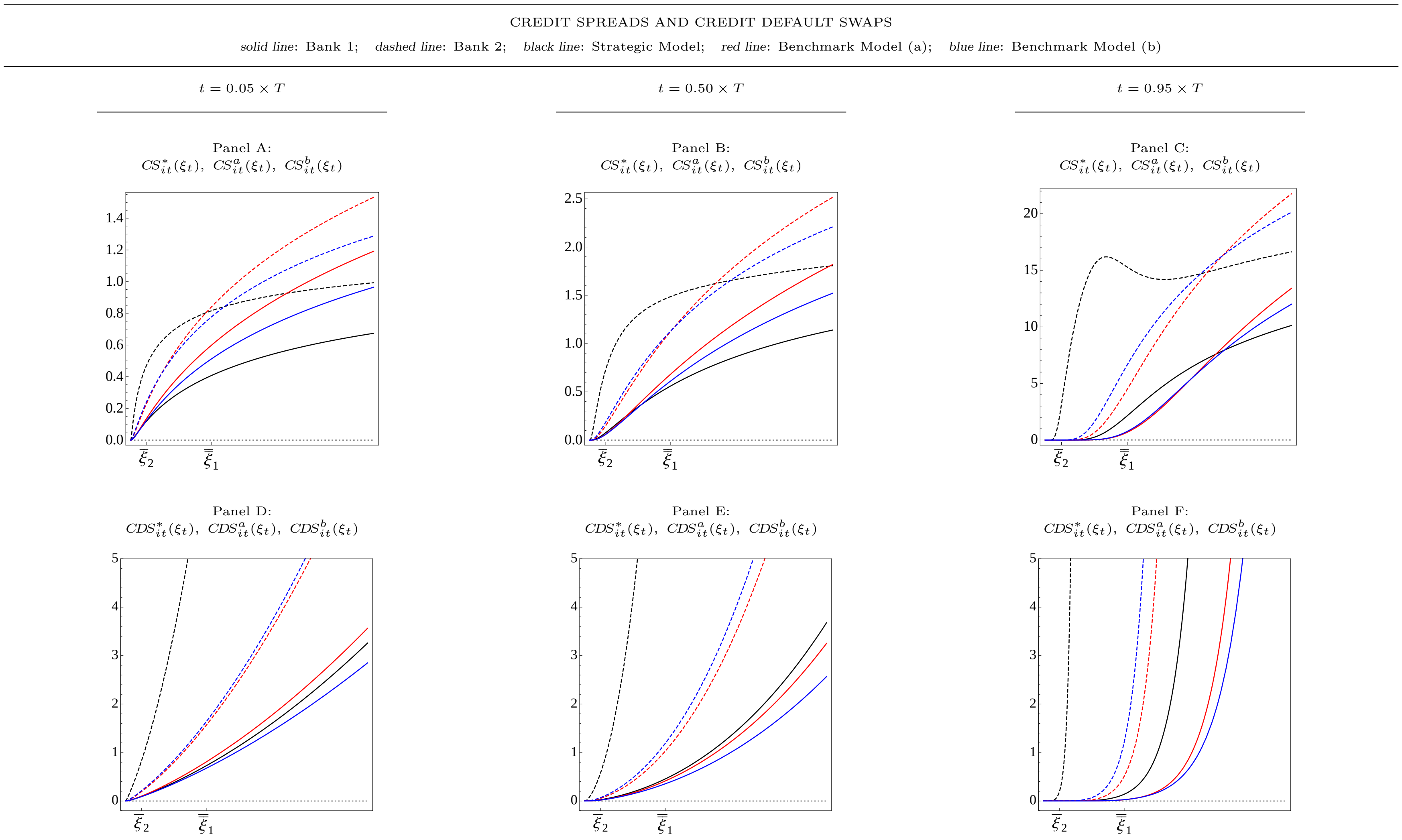
$$CDS_{it} = \frac{\mathbb{E}_t[(\xi_T/\xi_t)(F_i - D_{iT})\mathbb{1}_{\{\mathcal{D}_i\}}]}{\mathbb{E}_t[(\xi_T/\xi_t)\mathbb{1}_{\{\bar{\mathcal{D}}_i\}}]} \tag{2.54}$$

respectively, where D_{it} is provided in equations (2.51) and (2.52). Hence,

$$\frac{CDS_{1t}}{F_1} = \frac{1-e^{-CS_{1t}(T-t)}}{\mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1))}, \qquad \frac{CDS_{2t}}{F_2} = \frac{1-e^{-CS_{2t}(T-t)}}{\mathcal{N}(-\bar{d}_t(\bar{\xi}_2))}. \tag{2.55}$$

Proof. See the Appendix A. □

Debt is a claim on the value of the assets, which in the equilibrium of our model are determined endogenously by the strategic risk taking of the banks. Therefore the value of the debt is directly linked to the primitives of the strategic interactions, as is clear from (2.51) and (2.52). This facilitates the interpretation of the credit spread dynamics.



Parameter values. Financial market (monthly): $r = 0.005$, $||\kappa|| = 0.2$. Horizon (years): $T = 5$. Banks: $W_{10} = W_{20} = 1$, $\beta_1 = \beta_2 = 0.2$, $\gamma_1 = \gamma_2 = 2$, $\phi = 1.5\% W_{i0}$, $\lambda = 5\%$, $\eta_1 = \eta_2 = 15\%$. Source of heterogeneity: $F_1 = 7$, $F_2 = 9$.

Figure 2.8 Banks' debt: Credit spreads and CDS

For both the *early-* and the *late-defaulter*, we find that the value of the credit spreads is lower than in the benchmark models for adverse states of the world, at any time to maturity (Panel A-C, Figure 2.8). This is driven by the wealth transfers that the banks implement, from good states to systemic states, in order to minimize the burden of joint default. These wealth transfers have a trade-off: by removing wealth from the good states, the banks' bonds become riskier in spite of favorable economic conditions. The patterns in Panel A-C of Figure 2.8 highlight the close connection between the bonds' prices and the banks' strategic risk taking. In particular, we want to emphasize the risk-profile of bank 2's credit spreads close to maturity (Panel C, Figure 2.8). Indeed, these credit spreads inherit the distinctive features of bank 2's risk exposure (Panel C, Figure 2.3). In the vicinity of threshold $\bar{\xi}_2$, it is very likely that the idiosyncratic default of bank 2 occurs, implying a large shortfall on the debt repayment. This is naturally reflected in the credit spreads. In the vicinity of threshold $\bar{\bar{\xi}}_1$ the credit spreads behave non-monotonically: lower credit spreads are associated with worse states. The shorting position, that guarantees the financing of a higher wealth level in case of a systemic default, effectively makes the banks' debt less risky.

The credit default swap spreads in our model reflect the price one needs to pay for protection against the possibility of time T default of the reference bank. This price is positively related to both the probability of default and the credit spread, as revealed in (2.55). We document in the lower panels of Figure 2.8 that the price to pay for protection against default is higher in the strategic model than in the benchmarks. This is also true for the systemic states where the credit spreads of the banks are markedly lower than for the benchmark models. This indicates that the effect of the increased default probabilities dominates on the equilibrium CDS prices.

2.4 Concluding Remarks

Arguably, the financial sector is dominated by a small set of highly levered financial institutions with strong interlinkages, giving rise to strategic interactions. This paper analyzes the strategic risk taking of two such financial institutions, when systemic default induces externalities that amplify the cost of financial distress. We develop a structural model of credit risk in which, for a given capital structure, the asset value dynamics are endogenously determined by the optimal portfolio allocation.

We derive a unique strategic equilibrium in which heterogenous banks adopt polarized and stochastic risk exposure, without sacrificing full diversification. In the presence of systemic externalities, both banks care about financing a sufficiently high level of wealth in adverse states. To this purpose, the conservative bank reduces its risk exposure, whereas the radical bank optimally gambles on positive and negative outcomes, by taking either a large long or a short position in risky securities. The underlying economic mechanism increases the likelihood of systemic default. We believe that our analysis of systemic crises, both in terms of likelihood and expected shortfall, within a workhorse dynamic asset allocation framework is new and delivers a rich set of implications regarding financial institutions' strategic risk taking.

The tractable framework developed in this paper provides a platform to investigate other relevant questions. For instance, evaluating the effectiveness of different proposals to regulate systemic risk in a context of externalities represents one possible extension. Moreover, in this paper we show that a multiplicity of equilibria arises when financial institutions are very homogeneous. Thus, selecting amongst these equilibria and understanding their interplay with macro-prudential policies represents another promising direction for future research. Finally, one could bring the analysis to a general equilibrium level in order to study the impact of strategic interaction and systemic externalities on asset prices.

Chapter 3

Insider Trade Disclosure

Market transparency has been for many years at the heart of policy debates concerning the design of securities markets. However, there is no agreement among financial market regulators on what should be the optimal degree of transparency. In the United States, the Securities and Exchange Commission's (SEC) view is straightforward: market transparency "plays a fundamental role in the fairness and efficiency of the secondary market, [...] improves the price discovery, fairness, competitiveness and attractiveness of U.S. market" (SEC, 1995). Similarly, the International Organization of Securities Commissions (IOSCO) states that "market transparency – in essence, the widespread availability of information relating to current opportunities to trade and recently completed trades – is generally regarded as central to both the fairness and efficiency of a market, and in particular to its liquidity and quality of price-formation" (IOSCO, 2001). In contrast, in the United Kingdom, the Securities and Investment Board (SIM) has argued that there is "a tradeoff between liquidity and trade transparency" and consequently "transparency should be restricted if this is necessary to assure adequate liquidity" (SIB, 1994).

Differently from previous works on this issue, this paper analyzes ex-post disclosure of corporate insider trades, as a less explored but relevant aspect of market transparency. According to Section 16(a) of the Securities Exchange Act of 1934, insider traders (officers and directors) must report to the SEC transactions in equity securities directly with the related issuer within ten days following the end of the month in which the trade had occurred. The rationale for this securities regulation, as recognized by the authority, is to make private information available to all market participants more rapidly, thus increasing price efficiency and market liquidity:

> *Section 16(a) is likely to provide significant benefits by making information concerning insiders' transactions in issuer equity securities publicly available substantially sooner than it was before. Making this information available to all investors on a more timely basis should increase market transparency, which will likely enhance market efficiency and liquidity.*
>
> — U.S. Securities and Exchange Commission, File No. S7-31-02.

Recently, after the introduction of the Sarbanes-Oxley Act (August 2002), the US financial market regulator tightened up this regulation by requiring insiders to report their trades not later than two business days following the transaction. This drastic change is a clear sign of the intention to reduce the degree of asymmetric information in the market.

A similar rule is enforced in the U.K., where corporate insiders must inform their company as soon as possible and no later than the fifth business day after a transaction for their own account or on behalf of their spouses and children. In turn, a company must inform the London Stock Exchange (LSE) without delay and no later than the end of the business day following receipt of the information.[1] Therefore, an improvement in market transparency through the disclosure of insider trades should translate into a higher liquidity and efficiency of the market. In this paper we present a theoretical model to study whether and to which extent this is true.

The natural choice to analyze dynamic strategic insider trading behavior is the rational expectation trading model pioneered by Kyle (1985). We present here an extended version of it in which we introduce risk aversion and mandatory ex-post trade disclosure in a continuous-time framework. We model the regulation by requiring the informed agent to disclose in each trading period the trade she has made in the previous one. Then, the comparison between equilibrium outcomes for the same economy with and without regulation (hereafter referred as transparent and opaque market respectively) allows us to determine the impact on market components.

Our main result is that transparency reduces informational efficiency of prices and may cause the market to be less liquid. The analysis of the following two scenarios should clarify the intuition behind this somewhat surprising result. In an opaque market

[1] This implies that information about an insider transaction can reach the market as late as 6 days after the transaction. However, in practice, this information is disclosed faster: as documented by Fidrmuc et al. (2006) the announcement day for most of the directors' dealings (85% in their sample) coincides with the transaction day or is the following day.

(i.e. without disclosure regulation), as showed by Holden and Subrahmanyam (1994) and Baruch (2002), the risk averse informed trader chooses to exploit her informational advantage very rapidly to protect herself against future price risk imposed by liquidity traders. This aggressive trading behavior affects positively both market efficiency and liquidity. Indeed, almost all the private information is revealed to the market at the very early stages of the trading rounds, thus strongly reducing the adverse selection problem faced by the market maker. This implies a very liquid market. In a transparent market (i.e. with disclosure regulation), instead, we find an equilibrium in which the aggressiveness of the informed trader is severely reduced. The introduction of the disclosure requirement creates a tradeoff in the informed agent's strategy between future price risk and the revelation of private information through disclosure. In equilibrium the concern for the latter prevails.

There are indeed two opposite effects associated with the enforcement of such securities regulation. The positive direct effect is represented by the flow of information disclosed by the insider at the end of any trading round that clearly decreases the uncertainty caused by liquidity traders' order flow. This by itself reduces the adverse selection problem of the market maker. However, the indirect and negative effect, due to the change in the insider's trading strategy, intensifies the degree of asymmetric information. In this paper we show that when the informed agent is risk averse the indirect effect exist, and most importantly it dominates.

Consistently with Huddart et al. (2001) the insider plays a mixed strategy by adding a random component to her order flow. This prevents the market maker from inferring perfectly the private information from public records, and allows the insider to maintain an informational advantage over time. A lower market efficiency is caused by the fact that private information is now slowly incorporated into prices. A possible interpretation of this result is that corporate trade disclosure can be seen as the institutional means that forces the risk averse insider to behave as if she were risk neutral. As a matter of fact, to sustain an equilibrium in mixed strategy the trading costs set by the market maker must be constant over time, which directly implies a risk neutral trading behavior. An interesting feature of the model is that the transparent market equilibrium we derive is independent of the level of risk aversion.

We also study the profitability of insider trading and show that when the private information owned by the corporate insider is sufficiently unexpected – that is when the degree of asymmetric information is sufficiently high – the introduction of a trade disclosure regulation increases the insider's expected utility. In particular, we show that her expected utility can be easily interpreted a function of the positive direct and negative indirect effects associated with the enforcement of the regulation, where the relative weight of the negative effect (and hence positive for the insider profitability) is given by how unexpected is her private information.

More in details, our results can be summarized as follows: when ex-post insider trade disclosure is imposed (i) the insider maintains an informational advantage over the entire trading period despite the disclosure; (ii) market is less efficient at any trading round; (iii) the difference in market inefficiency, in terms of how much of the private information is not incorporated into prices, even widens if risk aversion/the variance of the liquidation value of the risky asset/the volatility of liquidity trading increase; (iv) trading prices have constant volatility over time and information is incorporated into prices at a constant rate; (v) even when the insider is risk neutral market efficiency does not improve if trading is continuous; (vi) market liquidity is constant over time; (vii) aggregate execution cost increases if the insider is sufficiently risk averse; (viii) the insider's ex-ante expected utility, conditional on her private information, increases if this information is sufficiently unexpected. These results thus question the effectiveness of such securities regulation.

This paper relates to the literature on market transparency, in which the key issue is the tradeoff between (pre- and post-trade) transparency and liquidity. Analytical results (Madhavan, 1995; Pagano and Röell, 1996; Naik et al., 1999; Frutos and Manzano, 2002), laboratory experiments (Bloomfield and O'Hara, 1999; Flood et al., 1999) and natural experiments (Madhavan et al., 2005) have shown that in a dealer market transparency improves informational efficiency but causes opening spreads to widen.[2]

This paper also relates to the literature on strategic insider trading with mandated disclosure. Fishman and Hagerty (1995) consider a two period model in which an insider may become informed with a certain probability. When the insider becomes informed, she never manipulates the market, while she might do it (imitating an informed insider with good news) if uninformed. John and Narayanan (1997) extend their model by introducing asymmetry in the probability of receiving a good or a bad signal. In this setting also an informed insider may manipulate the market. In both models manipulation is driven by an uninformed insider's attempt to pool with an informed insider and never occurs in equilibrium with good and bad news simultaneously. The most related article to this paper is Huddart et al. (2001). They extend the discrete time model by Kyle (1985) with risk neutral insider trading by adding the trade disclosure constraint. They find that an equilibrium in which the insider adds a noise component to her trading strategy (*dissimulation strategy*) exists and that the regulation accelerates price discovery and increases market depth. In what follows we show that this result is driven by the restrictive assumption of risk neutrality. Cao and Ma (1999) introduce imperfect competition among insiders. In their model market efficiency is unambiguously higher with disclosure, while market liquidity may be lower when insiders' signal are negatively

[2] Flood et al. (1999), instead, find that quote transparency reduces both opening spreads and market efficiency.

correlated.[3]

We contribute to the existing literature on market transparency and strategic insider trading by (i) examining a different aspect of transparency (the requirement for corporate insiders to ex-post disclose their trades) which is of primary importance for the design of securities markets; (ii) obtaining close-form solution for the transparent and opaque market equilibrium with risk aversion; (iii) formally showing the detrimental effect that transparent markets have on market efficiency and liquidity.

Consistent with our *mixed strategy equilibrium*, empirical studies on legal insider trading show that insiders place both informed and uninformed trades, and that on average the information content is small. Lakonishok and Lee (2001) provides event-study evidence of statistically but not economically significant market reaction around US legal insider purchases. Fidrmuc et al. (2006) reports abnormal returns of higher magnitude for the UK. However, abnormal returns could be a noisy proxy for insider information and the possible endogenous relation between abnormal returns and insider trading may lead to inconsistent results. To overcome these problems, Aktas et al. (2008) measure the contribution of insider trades to market efficiency by estimating the correlation between returns and the relative order imbalance, a methodology recently introduced by Chordia et al. (2005). Within this setting they find that insiders contribute significantly to faster price discovery. To our knowledge, Degryse et al. (2009) is the only work that analyze the information content of insider trades across different regulatory regimes. Using de-aggregated data for Dutch listed firms, the authors find that the implementation of the Market Abuse Directive (European Union Directive 2003/6/EC), which makes the reporting requirements harsher, reduces the information content of sales by top executive. This finding seems in line with our result.

The remainder of the paper is organized as follow. Section 3.1 introduces the features of the model. An opaque market equilibrium is derived in Section 3.2, whereas in Section 3.3 we solve for a transparent market equilibrium. In Section 3.4 we compare and discuss the properties of the equilibria, and assess the effectiveness of the disclosure regulation. In Section 3.5 we describe some findings concerning the insider ex-ante expected profits. Section 3.6 outlines the policy implications of our findings and concludes. In the Appendix B we relegate all the proofs and the derivation of a transparent market equilibrium in discrete-time.

[3] Cao and Ma (1999) adopt the same signals structure as in Foster and Viswanathan (1996) and use as a benchmark to their continuous time extension the model without trade disclosure by Back et al. (2000).

3.1 The Model

Our model is based on Kyle (1985). Consider a single risky asset traded in continuous auctions by an informed trader (*the corporate insider*) and a number of liquidity traders. We assume that the sequence of auction dates $\langle t \rangle$ partitions the time interval $[0, 1]$. At any auction a marker maker observes the total order flow and fixes the price at which all orders are filled. All agents are risk averse but the market maker who is risk neutral.[4]

Let us denote with v the *ex-post* liquidation value of the risky asset, which is assumed to be normally distributed, $v \sim \mathcal{N}(p_0, \Sigma_0)$. The informed trader has perfect information in the sense she observes at $t = 0$ the liquidation value v.[5] Let $\mathrm{d}x(t)$ be the order placed by the monopolistic insider trader at time t. The aggregate order by liquidity traders, who trade for liquidity reasons, is denoted by $\mathrm{d}u(t)$, and we assume it follows an arithmetic Brownian motion,

$$\mathrm{d}u(t) = \sigma_u \mathrm{d}B^u(t) \tag{3.1}$$

for some constant σ_u, independent of v.[6] Furthermore, let $\mathrm{d}y(t)$ be the total order flow given by the sum of the order flows place by the insider trader and the liquidity traders: $\mathrm{d}y(t) = \mathrm{d}x(t) + \mathrm{d}u(t)$.

We denote with $W(0)$ the initial wealth of the informed trader and with $W(t)$ the wealth at time t. The informed trader has negative exponential utility, with risk aversion coefficient A, for terminal wealth (denoted by $W(1)$):

$$U(W(1)) = -\exp\{-AW(1)\} \tag{3.2}$$

At time t the market maker sets the market price $p(t)$ by a competitive process, such that the price equals the expected value of v conditional on all public information available at that auction.[7] The monopolistic insider instead chooses a trading strategy that maximizes her expected utility conditional on all public and private information. Equilibrium is therefore defined as follows:[8]

[4] As argued by Holden and Subrahmanyam (1994), market making is typically performed by large financial institutions, which have large capacity to bear risk. Therefore, their behavior can be modeled by assuming risk neutrality.

[5] The assumption of perfectly informed monopolistic insider seems the most reasonable given the specific type of regulation we analyze: corporate insiders subject to this regulation are very likely to have perfect information and to be able to collude with each other, thus behaving as single informed agent.

[6] Back (1992), Back and Pedersen (1998), and Baruch (2002) consider a model in which the instantaneous variance changes over time. Since this feature would not add new insights to our result, we prefer to consider the standard case of constant variance.

[7] This is equivalent to assuming Bertrand competition among several market makers, which implies that the profits of the market maker(s) are driven to zero.

[8] Since the aim of this paper is far from (technically) generalizing the Kyle (1985) model, we consider throughout the paper a locally linear choice space (space in which linear pricing rules and linear trading

Definition 3. *A linear equilibrium of the continuous-time trading game is defined by the following conditions:*

1. UTILITY MAXIMIZATION. *Given the linear pricing rule $p(t)$, the linear trading strategy $\mathrm{d}x(t)$ maximizes*
$$\mathbb{E}[-\exp\{-AW(1)\}] \tag{3.3}$$

2. MARKET EFFICIENCY. *Given the linear trading strategy $\mathrm{d}x(t)$, the pricing rule is competitive:*
$$p(t) = \mathbb{E}_t^m(v) \tag{3.4}$$
where the expectation is taken conditional on the marker maker information set at time t.

3.2 The Benchmark: Opaque Market Equilibrium

In this Section we characterize the market equilibrium in a setting with continuous trading and no mandatory disclosure: this is the benchmark that allows us to assess the effectiveness of the securities regulation. A more general version of this model, in which different elastic liquidity demand functions are considered, is studied in Baruch (2002). The next theorem show how to construct a linear equilibrium:

Theorem 1. *There exists a recursive linear equilibrium in which the constants $\tilde{\lambda}(t)$, $\tilde{\beta}(t)$, $\tilde{\alpha}(t)$, $\tilde{\delta}(t)$, and $\tilde{\Sigma}(t)$ characterize the following:*

$$\begin{aligned}
\mathrm{d}x(t) &= \tilde{\beta}(t)M(t)\mathrm{d}t && (3.5)\\
M(t) &\equiv v - p(t) && (3.6)\\
\mathrm{d}p(t) &= \tilde{\lambda}(t)\mathrm{d}y(t) && (3.7)\\
\tilde{\Sigma}(t) &= \mathbb{V}\mathrm{ar}[v|\mathcal{F}(t)] && (3.8)\\
J(M,t) &= -\exp\{-A\left[W(t) + V(M,t)\right]\} && (3.9)\\
V(M,t) &= \tilde{\alpha}(t)[M(t)]^2 + \tilde{\delta}(t) && (3.10)\\
W(t) &= \int_0^t (v - p(s))\mathrm{d}x(s) && (3.11)
\end{aligned}$$

strategies are defined) in which doubling strategies are ruled out. We refer to Back (1992) for further details.

for every $t \in [0,1]$. *The constants* $\tilde{\beta}(t)$, $\tilde{\lambda}(t)$, $\tilde{\alpha}(t)$, $\tilde{\delta}(t)$, $\tilde{\Sigma}(t)$ *are given by*

$$\tilde{\Sigma}(t) = \frac{\left[2+\left(A^2\Sigma(0)\sigma_u^2-\sqrt{A^2\Sigma(0)\sigma_u^2(4+A^2\Sigma(0)\sigma_u^2)}\right)t\right]\Sigma(0)(1-t)}{2+2A^2\Sigma(0)\sigma_u^2(1-t)t} \tag{3.12}$$

$$\tilde{\lambda}(t) = \left[\frac{\sigma_u\sqrt{4+A^2\Sigma(0)\sigma_u^2}}{2\sqrt{\Sigma(0)}}+A\sigma_u^2\left(t-\frac{1}{2}\right)\right]^{-1} \tag{3.13}$$

$$\tilde{\beta}(t) = \frac{2\sigma_u}{\left[\sqrt{\Sigma(0)(4+A^2\Sigma(0)\sigma_u^2)}-A\Sigma(0)\sigma_u\right](1-t)} \tag{3.14}$$

$$\tilde{\alpha}(t) = \frac{1}{2\tilde{\lambda}(t)} \tag{3.15}$$

$$\tilde{\delta}(t) = \frac{\sigma_u^2}{2}\int_t^1 \tilde{\lambda}(s)\mathrm{d}s \tag{3.16}$$

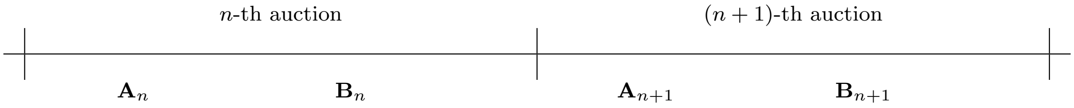
Proof. See the Appendix B. □

The linear strategy of the corporate insider is characterized by $\tilde{\beta}(t)$, the one of the market maker by $\tilde{\lambda}(t)$. $\tilde{\Sigma}(t)$ denotes the conditional variance of prices, while $\tilde{\alpha}(t)$ and $\tilde{\delta}(t)$ characterize the value function of the insider.

A risk averse informed agent faces the following trade-off: on the one hand she would like to use all the private information immediately because concerned about future price risk induced by liquidity traders, on the other hand she would like to concentrate all her trade at the time in which the trading cost, $\tilde{\lambda}(t)$ is minimized. In equilibrium the optimal balance of these two components is determined by the decreasing dynamic of the trading cost. At the early stages of trading the insider behaves more aggressively, yet maintaining an informational advantage over time and hence trading till the last auction, in which all the private information is reveled, $\tilde{\Sigma}(1) = 0$. When the corporate insider is risk neutral the concern for future price risk disappears, and only a constant trading cost can be part of an equilibrium. We refer to Baruch (2002) for further discussions and comparative statics.

3.3 Transparent Market Equilibrium

In this Section we characterize the market equilibrium when mandatory trade disclosure is enforced. To easily communicate the intuition of how such regulation works in a dynamic model of strategic trading, we prefer to introduce and describe it in the context of a more intuitive discrete-time setting – in which we can make use of the time between two subsequent auctions – before moving to a more elegant continuous-time framework.



Events (for all n)

$\mathbf{A}_n$: (i) insider places the order Δx_n; (ii) liquidity traders place the order Δu_n; (iii) market maket observes Δy_n and sets market price p_n.

$\mathbf{B}_n$: (i) insider discloses her trade Δx_n; (ii) market maket updates his beliefs on v and compute the *updating* price p_n^*.

Figure 3.1 Disclosure regulation in discrete-time: The sequence of events

Disclosure regulation in discrete-time. Mandatory trade disclosure is modeled by imposing the informed trader to reveal at the end of any auction the amount of trade she placed at the beginning of the auction. Hence the insider's past trades become public information. Therefore, the market maker, after having set the price at the n-th auction, but before setting the market price of the subsequent one, observes the order placed by the informed trader at the n-th auction. Thus, he adjusts his belief of the asset value using this new piece of information reported by the insider. We define p^* as the *updating* price, which is not a market price but rather the price that the market maker would have set for the n-th auction if he had known the insider's order at the beginning of the period. This *updating* price is computed by the market maker in order to determine the market price p of the next auction. The sequence of events in Figure 3.1 makes this clear. We leave in the Appendix B the derivation of the transparent market equilibrium in the discrete-time setting, and we show at the end of this Section that as the interval between auctions in the discrete-time model becomes uniformly small, the sequential auction equilibrium with mandatory trade disclosure converges to the equilibrium in continuous-time.

Disclosure regulation in continuous-time. The same intuition holds for the continuos-time setting, in which, instead, the insider simultaneously reveals at each instant t the trade made a $\mathrm{d}t$ before and places the new order (i.e., the events $\mathbf{B}_n$ and $\mathbf{A}_{n+1}$ in Figure 3.1 happen at the same time). The main feature of the disclosure regulation is that the insider maintains an informational advantage between the trading and the disclosing periods. We keep this essential features in our continuous-time framework by assuming that the market maker, although continuously observing the insider's disclosed trades, uses the total order flow $\mathrm{d}y(t)$ to determine the market price $p(t)$. This can be considered as the limit of the discrete-time framework. We define the *updating* price, $p^*(t)$, which is not a market price but rather the price that the market maker would have set in the previous auction (a dt before) if he had been able to observe the insider's order. The sequence of updating prices will be used by the market maker as a "base" to determine the sequence of market prices:

$$\begin{aligned} p^*(t) &= f\Big(p^*(t-\mathrm{d}t), \mathrm{d}x(t-\mathrm{d}t)\Big) \\ p(t) &= h\Big(p^*(t), \mathrm{d}y(t)\Big) \end{aligned}$$

As argued by Huddart et al. (2001), trading and pricing strategies characterizing a linear equilibrium in an opaque market can not be part of an equilibrium in a transparent market. To see this, suppose the contrary holds. Suppose the market maker conjectures that insider's demand is just a linear function of the liquidation value, v, of the risky asset. Then, after the first disclosure, the market maker would be able to invert the insider's trading strategy, thus perfectly inferring the liquidation value. So, no private information would be left to the insider. As a consequence, the market maker would set the price in the next auction equal to v by choosing zero sensitivity to the order flow in his pricing function (infinite market depth). Rationally behaving, the insider would anticipate this and have an incentive to deviate from her trading strategy in order to induce mispricing and make unbounded profits. Therefore, no invertible trading strategy can be part of an equilibrium with trade disclosure (*no equilibrium in pure strategy*).

However, an equilibrium in mixed strategy does exist. At any auction the insider's trading strategy is determined by two components: (i) a private information-based linear component, and (ii) a plain noise component. The next theorem presents the closed-form solution for the linear equilibrium of the trading game in the continuous-time framework.

Theorem 2. *There exists a recursive linear equilibrium in which the constants $\lambda(t)$, $\gamma(t)$, $\beta(t)$, $\alpha(t)$, $\delta(t)$, $\sigma_z(t)$, and $\Sigma(t)$ characterize the following:*

$$\mathrm{d}x(t) = \beta(t)M(t)\mathrm{d}t + \mathrm{d}z(t) \tag{3.17}$$

$$M(t) \equiv v - p^*(t) \tag{3.18}$$

$$\mathrm{d}z(t) = \sigma_z(t)\mathrm{d}B^z(t) \quad \textit{with} \quad \mathrm{d}\langle B^z, B^u\rangle_t = 0 \tag{3.19}$$

$$p(t) = p^*(t) + \lambda(t)\mathrm{d}y(t) \tag{3.20}$$

$$p^*(t+\mathrm{d}t) = p^*(t) + \gamma(t)\mathrm{d}x(t) \tag{3.21}$$

$$\Sigma(t) = \mathbb{V}\mathrm{ar}[v|\mathcal{F}'(t)] \tag{3.22}$$

$$J(M,t) = -\exp\{-A\,[W(t) + V(M,t)]\} \tag{3.23}$$

$$V(M,t) = \alpha(t)[M(t)]^2 + \delta(t) \tag{3.24}$$

$$W(t) = \int_0^t (v - p(s))\mathrm{d}x(s) \tag{3.25}$$

for every $t \in [0,1]$. *Given the initial condition* $p^*(0) = p(0)$, *the constants* $\beta(t)$, $\lambda(t)$, $\gamma(t)$, $\alpha(t)$, $\delta(t)$, $\sigma_z(t)$, $\Sigma(t)$ *are given by*

$$\Sigma(t) = (1-t)\Sigma(0) \tag{3.26}$$

$$\gamma(t) = \hat{\gamma} = \left(\Sigma(0)/\sigma_u^2\right)^{\frac{1}{2}} \tag{3.27}$$

$$\lambda(t) = \frac{1}{2}\hat{\gamma} = \frac{1}{2}\left(\Sigma(0)/\sigma_u^2\right)^{\frac{1}{2}} \tag{3.28}$$

$$\beta(t) = \hat{\gamma}^{-1}/(1-t) = \sigma_u \Sigma(0)^{-\frac{1}{2}}/(1-t) \tag{3.29}$$

$$\alpha(t) = \frac{1}{2}\hat{\gamma}^{-1} = \frac{1}{2}\left(\sigma_u^2/\Sigma(0)\right)^{\frac{1}{2}} \tag{3.30}$$

$$\delta(t) = 0 \tag{3.31}$$

$$\sigma_z(t) = \sigma_u \ , \quad t \in (0,1) \tag{3.32}$$

with the boundary condition $\sigma_z(1) = 0$.

Proof. See the Appendix B. □

The linear strategy of the corporate insider is now characterized by $\beta(t)$ and $\sigma_z(t)$, the one of the market maker by $\lambda(t)$. $\gamma(t)$ represents the updating trading cost that the market maker can compute after any trade disclosure. $\Sigma(t)$ denotes the conditional variance of prices, and $\alpha(t)$ and $\delta(t)$ characterize the value function of the insider.

Since no invertible trading strategy can be part of an equilibrium, the informed agent adds a random component to her market order: at each trading round she chooses the optimal variance of such noise, $\sigma_z(t)$. Equation (3.32) states that in equilibrium this variance must be equal to the instantaneous volatility of liquidity trading, σ_u, except for the last trading round in which no *dissimulation component* is needed. As a consequence, trading costs $\lambda(t)$ must be constant over time in order to make the insider indifferent among all the possible values of his order flow. Private information is incorporated into prices gradually and market prices have constant volatility over time.

Finally, a somehow surprising feature of this equilibrium is that it is independent of the level of risk aversion. This and other properties are discussed in detail in the next Section.

A convergence result. In the Appendix B we derive a transparency market equilibrium in discrete-time. We study now how the equilibrium properties of the sequential trading model are related to the properties of the continuous trading model when the interval between auctions become smaller and smaller. A convergence result shows that as the interval between auctions in the discrete-time model becomes uniformly small, the sequential auction equilibrium with mandatory trade disclosure converges to our continuous auction equilibrium.

Proposition 7. *Holding* $\Sigma(0) = \Sigma_0$ *and* σ_u^2 *constant, and given the convention* $x(t) = x_n$ *for all* $t \in [t_{n-1}, t_n)$*, consider a sequence of sequential equilibria such that* $\Delta t \to 0$*. Then the values* $\Sigma(t), \gamma(t), \lambda(t), \beta(t), \alpha(t), \delta(t)$*, and* $\sigma_z(t)$ *characterized in Theorem 3 converge to the corresponding value in the continuous auction equilibrium obtained in Theorem 2.*

Proof. See the Appendix B. □

3.4 The Effectiveness of the Transparency Regulation

In the introduction of this paper we argue that the aim of a corporate insider disclosure regulation, as recognized by the authority, is to make private information available to all market participants more rapidly in order to promote price efficiency and market liquidity. In this Section we formally show that within the theoretical framework presented, the regulation fails at achieving these goals.

Our main result is that transparency reduces informational efficiency of prices and may cause the market to be less liquid. Before formalizing this in the next propositions, we first discuss the intuition behind. When the (risk averse) corporate insider is not required to disclose her trade, she will adopt a very aggressive trading strategy in order to exploit her informational advantage rapidly. The reason why a risk averse insider would behave more aggressively than the informed agent in Kyle (1985), is that the former wants to protect herself against future price risk imposed by liquidity traders. In other words, she is concerned about the possibility that profitable trading opportunities will be lost as the liquidity traders move market prices against her. By trading intensively at the early trading rounds, most of the private information is incorporated into prices, thus reducing the adverse selection problem faced by the market maker. Since the most of the private information is now in the market, the market maker reduces the sensitivity of prices with respect to the order flows. This means a liquid market. Therefore, as stated by Holden and Subrahamanyam (1994), "from a regulatory perspective, [...] insider trading may be much less of a potential problem than the analysis of Kyle (1985) indicates". It is the risk attitude of the informed agent that makes the market more efficient and liquid. In our benchmark model (the opaque market) the higher the risk aversion of the corporate insider, the faster private information get disseminated to the market, and the faster the market becomes liquid.

Surprisingly, the introduction of this transparency regulation prevents this effective "mechanism" based on risk attitude to succeed. Indeed, in equilibrium market prices convey less private information and liquidity does not decline over time. This represents a failure of the securities regulation. How can a transparency regulation worsen financial markets? Quoting Pagano and Röell (1996), this is possible "[...] because informed traders adapt their strategy to the market mechanism they face [...]". In order to maintain an informational advantage over time, the corporate insider can not adopt the same trading strategy she would if no regulation was enforced, otherwise the market maker would be able to back out the private information from public records, thus eliminating any future profits. For this reason, we show in the model that an optimal strategy for the insider is to add a random component to her market order. On the one hand this allows her to maintain an informational advantage over time, but on the other hand it induces her to behave less aggressively. It is exactly this low trading intensity that reduces the dissemination of private information: trading prices have constant volatility over time, meaning that information is incorporated into prices at a constant rate. Moreover, since the insider must be indifferent to all possible values of her market order (otherwise she would deviate from that strategy), the adverse selection problem and hence market depth do not improve over time. Therefore, if a more realistic assumption on the risk preference of the informed agent increases informativeness of prices and reduces the adverse selection problem (Holden and Subrahamanyam, 1994; Baruch, 2002), the introduction of a trade disclosure regulation makes market prices less informative and the adverse selection problem more severe.

So, why do we observe mandatory trade disclosure regulation, if it can be so detrimental to financial markets? Our argument is that there are two opposite effects associated with the enforcement of such securities regulation – a positive *direct* effect and a negative *indirect* effect – and that regulators may have not considered or underestimated the latter. The positive direct effect is represented by the flow of information disclosed by the insider at the end of any trading round that clearly decreases the uncertainty caused by liquidity traders' order flow. This by itself reduces the adverse selection problem of the market maker. Hence, a thoughtless argument may be: more information available to all market participants, less asymmetric information, more efficiency, more liquidity. However, the indirect and negative effect, due to the change in the insider's trading strategy, intensifies the degree of asymmetric information. In this paper we show not only that there is also an indirect effect, but most importantly that it dominates the direct one.

Finally, another interesting interpretation of the effect of this insider trading regulation is that mandatory trade disclosure represents the institutional means that makes the informed agent behave as if she were risk neutral. To have a transparent market equilibrium in which the corporate insider maintains an informational advantage over time a non-invertible trading strategy is needed: adding a plain noise component to the market order of each auction is a tractable way to obtain one, as proposed by Huddart et al. (2001). However, to sustain this mixed strategy in equilibrium the trading costs must be constant over time, otherwise any disparity in such costs would induce a deviation from this strategy to exploit the lower cost. As a consequence insider trader's concern for future price risk is completely disregarded in equilibrium. Hence, how risk averse the insider is does not affect her trading behavior: the transparent market equilibrium turns out to be independent of the level of risk aversion. Therefore, the introduction of the disclosure regulation "forces" any risk averse insider to behave as a risk neutral one. Needless to say, the monopolistic risk neutral insider represents the most harmful case in terms of market efficiency and liquidity.[9]

As in Kyle (1985), the parameter $\Sigma(t)$, which gives the error variance of prices at time t, is an inverse measure of the informational efficiency of prices, which indicates how much of the insider's private information is not yet incorporated into prices. The parameter $\lambda(t)$, which characterizes the market maker pricing function, is an inverse measure of market liquidity (or more precisely of market depth). The parameter $\beta(t)$, which characterizes the insider's trading strategy, represents the intensity (aggressiveness) of insider trading, that is the sensitivity of the informed order flow to private information.

Panel A of Table 3.1 shows the effect of the disclosure regulation when the insider trader is assumed risk neutral. The three graphs contrast respectively $\Sigma(t)$, $\lambda(t)$, and $\beta(t)$, (i) in a transparent market (solid line), and (ii) in an opaque market (dashed line). This means plotting Huddart et al. (2001) versus Kyle (1985) in a continuous-time setting, in which exogenous parameters are normalized by setting $\Sigma(0) = \sigma_u^2 = 1$. In contrast to the results in a discrete-time setting (Huddart et al., 2001), market efficiency does not improve with the enforcement of the regulation. Insider trading intensity and conditional variance of prices coincides in the two market equilibria. The only difference relies on market liquidity, which is higher in a transparent market. The risk neutral case is very instructive because it clearly highlights the positive direct effect of the regulation aforementioned. Indeed, since insider aggressiveness remains unchanged, the negative indirect effect is null. This is what we believe being the reason why financial market regulators consider such regulation as beneficial for market participants.

However, Panel B of Table 3.1 shows that the results are reversed once we relax the restrictive assumption on the risk attitude. Specifically, we consider an informed agent characterized by a CARA utility function with coefficient of risk aversion of 4 and the same normalization on the exogenous parameters. The first graph shows that in an opaque market $\Sigma(t)$ declines very rapidly through time, whereas in a transparent market

[9]In the case of a monopolistic risk neutral insider the trading costs are constant over time to eliminate profitable destabilization schemes that can generate unbounded profits, and hence not compatible with a rational expectation equilibrium.

the instantaneous change in the conditional variance, $d\Sigma(t)$, is constant. Concerning the level, market efficiency is always higher in an opaque market. The third graph shows that in a transparent market insider trading aggressiveness is reduced at any auction. Finally, the second graph shows that with disclosure market depth is constant, while without disclosure it declines sharply over time, becoming lower in a final trading sub-period. In an opaque market the adverse selection problem is severe at the beginning of the trading period because the total order flow is very informative, and it becomes almost irrelevant once most of the information is incorporated into prices.

3.4.1 Market Efficiency

The normalized difference in conditional variance of prices between transparent and opaque market is a measure of the efficiency loss caused by the regulation, that is how much more information remains private and it is not incorporated into prices at each point in time if the disclosure regulation is introduced. An aggregate measure of efficiency loss is given by the sum of the efficiency losses at each point in time. Table 3.2 presents comparative statics for the instantaneous and the aggregate measure of efficiency loss with respect to the exogenous parameters of the model: the coefficient of risk aversion, A, the prior variance of the liquidation value of the risky asset, $\Sigma(0)$, and the variance of liquidity trading, σ_u^2.

The three graphs in Panel A show that the instantaneous measure of efficiency loss is increasing in the level of risk aversion, and in both variances. For instance, according to the first graph, when the coefficient of risk aversion is equal to 4, the enforcement of the disclosure regulation prevents 57% of all private information to be incorporated into prices after just one-tenth of the trading rounds. When the coefficient of risk aversion is equal to 8, the instantaneous efficiency loss becomes 78%. Moreover, the higher the risk aversion the faster the private information reaches the market: this is confirmed by the right-skewness of the curves. Similar conclusions can be drawn with respect to the other two graphs.

Panel B, instead, considers the aggregate measure of efficiency loss and shows specular comparative statics. The aggregate efficiency loss is represented by the shaded area in the first graph (that is the sum of all the instantaneous losses) scaled by 1/2 (that is the area below the solid line). Not surprisingly the second and the third graphs show that also the aggregate measure is increasing in all the three exogenous parameters. The following proposition formalizes these results.

Proposition 8. *For any level of risk aversion and at each trading round, insider trading* *is more aggressive and the market is more efficient in an opaque market:*

$$\tilde{\beta}(t) - \beta(t) \geqslant 0 \quad \forall\, t \in [0,1] \tag{3.33}$$

$$\tilde{\Sigma}(t) - \Sigma(t) \geqslant 0 \quad \forall\, t \in [0,1] \tag{3.34}$$

If we define the "instantaneous efficiency loss" as the difference between the conditional variances of prices in a transparent and in an opaque market, normalized by the prior variance of the liquidation value of the risky asset, for every $t \in [0,1]$,

$$\Omega(t) = \frac{\Sigma(t) - \tilde{\Sigma}(t)}{\Sigma(0)} \tag{3.35}$$

and the "aggregate efficiency loss" as the sum of all the instantaneous losses scaled by 1/2,

$$\Omega^a = 2\int_0^1 \Omega(t)\mathrm{d}t \tag{3.36}$$

then the following cross-sectional results hold:

i. $\Omega(t)$ *increases with the level of the coefficient of risk aversion* A*;*

ii. $\Omega(t)$ *increases with the level of the prior variance of the liquidation value* $\Sigma(0)$*;*

iii. $\Omega(t)$ *increases with the level of the instantaneous variance of liquidity trading* σ_u^2*;*

iv. the same comparative statics hold for Ω^a*.*

Proof. See the Appendix B. □

3.4.2 Market Liquidity

In a similar fashion we define a measure of liquidity loss as the difference in the execution cost of liquidity traders between transparent and opaque market. Table 3.3 presents comparative statics with respect to the exogenous parameters of the model. All the three graphs in Panel A show that instantaneous liquidity loss is increasing over time, reflecting the change in the adverse selection problem. The aggregate liquidity loss is represented by the shaded area in the first graph of Panel B and it coincides with the difference in the aggregate execution cost, as defined by Back and Pedersen (1998). The second and the third graphs show that if the corporate insider is sufficiently risk averse, then the aggregate execution cost is higher in a transparent market. The following proposition formalizes the results on market liquidity.

Proposition 9. *If we define the "instantaneous liquidity loss" as the difference between the execution costs of liquidity traders in a transparent and in an opaque market for every* $t \in [0,1]$,

$$\Psi(t) = (\lambda(t) - \tilde{\lambda}(t))\sigma_u^2 \tag{3.37}$$

and the "aggregate liquidity loss" as the sum of all the instantaneous losses,

$$\Psi^a = \int_0^1 \Psi(t)\mathrm{d}t \tag{3.38}$$

then the following cross-sectional results hold:

i. if the informed agent is sufficiently risk averse $(A > A^*)$*, then there exists trading sub-period in which market liquidity is higher in an opaque market, and the length of this sub-period increases with the level of the coefficient of risk aversion* A*;*

ii. if the informed agent is sufficiently risk averse $(A > A^{**} > A^*)$*, then* Ψ^a *increases with the level of the prior variance of the liquidation value,* $\Sigma(0)$*, and the level of the instantaneous variance of liquidity trading,* σ_u^2*;*

iii. if the informed agent is sufficiently risk averse $(A > A^{***} > A^{**})$*, then* $\Psi^a > 0$*.*

Proof. See the Appendix B. □

3.5 Insider Trading Profitability

To this point we have analyzed the implications of insider trade disclosure regulation for market efficiency and liquidity, since identified by the regulator as being the two main objective of such regulation. In the current section we describe some findings concerning the insider ex-ante expected profits.[10]

In order to have a more flexible specification for the utility function, which encompasses the risk neutral case when the coefficient of risk aversion goes to zero, let us consider the following functional form,

$$U(W(1)) = \frac{1 - \exp\{-AW(1)\}}{A} \tag{3.39}$$

Note that the equilibrium results are the same regardless of the use of the utility functions specified in Equations (3.2) and (3.39) because we can take a monotonic transformation

[10] I thank Eric Hughson for inspiring the development of this section.

of utility and still represent the same preferences.[11] Now, let π_0 denote the insider's unconditional expected utility (at time 0),

$$\pi_0 = \mathbb{E}_0[U(W(1))] \tag{3.40}$$

and Π_0 the insider's expected utility conditional on her private information (still at time 0, that is before the trading takes place),

$$\Pi_0 = \mathbb{E}_0[U(W(1))|v] \tag{3.41}$$

where clearly $\pi_0 = \mathbb{E}_0[\Pi_0]$ by the Law of Iterated Expectation.

The following proposition examines how the insider's (un)conditional expected utility changes across the two regulatory regimes, the *opaque* and the *transparent* market, focusing in particular on the role played by her risk attitude.

Proposition 10. *In an opaque market equilibrium the insider's unconditional and conditional expected utility are given by*

$$\tilde{\pi}_0 = \frac{1}{A}\left[1 - \left(1 + A\left(A\Sigma(0)\sigma_u^2 + \sqrt{\Sigma(0)\sigma_u^2(4 + A^2\Sigma(0)\sigma_u^2)}\right)\right)^{-1/2} \exp\Big\{-AW(0)\Big\}\right] \tag{3.42}$$

$$\tilde{\Pi}_0 = \frac{1}{A}\left[1 - \Xi \exp\left\{-A\left[W(0) + \frac{1}{4}\left(\sqrt{\frac{\sigma_u^2(4 + A^2\Sigma(0)\sigma_u^2)}{\Sigma(0)}} - A\sigma_u^2\right)\Big(v - \mathbb{E}_0(v)\Big)^2\right]\right\}\right] \tag{3.43}$$

respectively, where

$$\Xi \equiv \left(\frac{2}{2 + A\left(A\Sigma(0)\sigma_u^2 + \sqrt{\Sigma(0)\sigma_u^2(4 + A^2\Sigma(0)\sigma_u^2)}\right)}\right)^{1/2} \tag{3.44}$$

while in a transparent market equilibrium they are given by

$$\pi_0 = \frac{1}{A}\left[1 - \left(1 + A\sqrt{\Sigma(0)\sigma_u^2}\right)^{-1/2} \exp\Big\{-AW(0)\Big\}\right] \tag{3.45}$$

$$\Pi_0 = \frac{1}{A}\left[1 - \exp\left\{-A\left[W(0) + \left(\frac{1}{2}\sqrt{\frac{\sigma_u^2}{\Sigma(0)}}\right)\Big(v - \mathbb{E}_0(v)\Big)^2\right]\right\}\right] \tag{3.46}$$

Then the following results hold:

i. $\tilde{\pi}_0 > \pi_0$ *and* $\partial\left(\tilde{\pi}_0 - \pi_0\right)/\partial A \leqslant 0$*;*

[11]With the utility function specified in Equation (3.39), $\lim_{A\to 0} U(W(1)) = W(1)$.

ii. if we define the insider's initial informational advantage as $\Gamma \equiv |v - \mathbb{E}_0(v)|$*, then the set*

$$\mathcal{G} = \{\Gamma : \ \Pi_0(\Gamma) \geqslant \tilde{\Pi}_0(\Gamma)\}$$

is non-empty if the insider is risk averse ($A > 0$*), and takes the form of open intervals* $[\Gamma^*, \infty)$ *on* $\mathbf{R}^+$*;* Γ^* *is increasing in* $\Sigma(0)$*, decreasing in* σ_u^2*, and decreasing in* A*; in the limit, as* A *approaches 0,* Γ^* *goes to infinity.*

Proof. See the Appendix B. □

The main finding can be summarized as follows: in a transparent market, compared to an opaque market, the insider's *unconditional* expected utility is reduced; however, if she is risk averse, once the private information is realized, her *conditional* expected utility may be higher. This is the case when the unexpected component of the private information is sufficiently large.

The first part of the proposition states that the introduction of a transparency regulation reduces the ex-ante expected utility of the corporate insider. However, the difference in utility across the two regulatory regimes becomes smaller and smaller the more averse to the risk the insider is. We describe these *unconditional* results in Panel A of Table 3.4. The three graphs contrast the unconditional expected utility of the insider over the prior variance of the private information (the first two graphs) and over the level of risk aversion (third graph). The dashed line, which represents an opaque market, is always above the solid line, a transparent market, while the solid bold line, which captures the difference between the two is clearly decreasing in A.

The second part of the proposition highlights a more interesting feature of the model, that further differentiates the general case of risk aversion from the specific case of risk neutrality. With risk neutrality no matter what type of information the insider receives, she is worse off if she has to disclose her trade; with risk aversion, instead, if the private information is particularly unexpected, the insider can indeed be better off. This result is important because it shows that there are states of the world in which the insider can benefit from the introduction of such regulation. Therefore, trade disclosure not only fails at improving market efficiency and liquidity, but it may also allow the informed agent to exploit her private information in a more profitable way. To understand why this is possible, let us consider the value function at time 0, given zero initial wealth:

$$\Pi_0 = \frac{1}{A}\Big[1 - \exp\Big\{ - A\Big(\alpha(0)[M(0)]^2 + \delta(0)\Big)\Big\}\Big] \tag{3.47}$$

where $M(0) = v - p(0)$ represents the component of the private information which is not expected by the market.[12]

The coefficient $\alpha(0)$ captures the initial liquidity of the market, which in equilibrium depends on the aggressiveness of the insider. In a transparent market the insider adopts a mixed strategy which makes her trade less aggressive, thus inducing a more liquid market. In a opaque market, instead, the market maker anticipates the willingness of the insider to exploit the private information in a timely manner, and hence sets a price which is very sensitive to the order flow: this makes the market illiquid. Indeed, straightforward algebra shows that $\alpha(0) \geqslant \tilde{\alpha}(0)$, where the equality holds only for the case of risk neutrality.

The coefficient $\delta(0)$ reflects the possibility for the insider to profit by hiding her trades behind the ones placed by liquidity traders for the entire trading period. It is the "aggregate value" of noise trading activities for the insider. This is positive in an opaque market since the insider's future informational advantages $|v - p(t)|$ depend on $\mathrm{d}u(t)$. In particular $\tilde{\delta}(0)$ is proportional to the "quantity" of noise trading activities, measured by the variance σ_u^2, times the "price" of such activities, measured by the coefficients $\tilde{\lambda}(t)$, for all t, which capture impact of noise trades on market prices. Not surprisingly, in a transparent market the coefficient $\delta(0)$ is equal to zero. As a matter of fact the insider's future informational advantages $|v - p^*(t)|$ do not depend on $\mathrm{d}u(t)$ because the disclosure of her trades eliminates the noise trading uncertainty faced by the market maker. It follows that $\delta(0) < \tilde{\delta}(0)$.

Qualitatively, we can interpret $[\tilde{\delta}(0) - \delta(0)]$ as a measure of the positive *direct* effect, and $[\tilde{\alpha}(0) - \alpha(0)]$ as a measure as the negative *indirect* effect brought by the introduction of the trade disclosure regulation. With risk neutrality, $\tilde{\alpha}(0) = \alpha(0)$, and only the positive effect remains.

Having examined the role played by the two components that characterize the value function at time 0, now it should be clear that the relative weight of these components determines whether the insider's conditional expected utility is higher in a transparent rather than in an opaque market. As appears in Equation (3.47), the relative weight is given by the (square of the) insider's informational advantage. The lower Γ, the lower the importance of the negative effect, and the lower the insider's expected utility. This justifies the existence of a threshold level Γ^*, such that the insider's expected utility is higher in a transparent market for any level of the informational advantage greater than Γ^*. By continuity, when the informational advantage equal to Γ^*, the expected utilities associated to the two regulatory regimes coincide. Panel B in Table 3.4 highlights these results. The first two graphs describe how the conditional expected utility changes with Γ for the case of risk neutrality and risk aversion respectively. When A approaches zero,

[12] Note that the insider's initial informational advantage Γ is equal to $|M(0)|$.

$\tilde{\alpha}(0)$ approaches $\alpha(0)$, and $\tilde{\delta}(0)$ approaches its maximum: therefore, the threshold level Γ^* must be very high. In the limit, Γ^* goes to infinity, and this is the reason why it does not appear in the first graph. With risk aversion, Γ^* is finite, as pictured in the second graph. Finally, the third graph presents comparative statics of Γ^* for the exogenous parameters A, $\Sigma(0)$, and σ_u^2.

The current section provides interesting findings concerning the profitability of the informed trading activity for corporate insiders. A more rigorous welfare analysis should also take into account the impact of the disclosure regulation on the welfare of liquidity traders. However, in the framework here presented, the preferences of agents who trade for liquidity purposes are un-modeled and their aggregate trade is exogenously assumed. Departures from this assumption are analyzed in Admati and Pfleiderer (1988), Bernhardt and Hughson (1997), and Mendelson and Tunca (2004) among others. An extension in this direction, which goes beyond the scope of this paper, is left for further research.

3.6 Conclusions and Policy Implications

Securities markets worldwide have different degrees of transparency with implications that are not well understood. In this article we focus on mandatory ex-post trade disclosure by corporate insiders as a particular aspect of market transparency. In a continuous-time model of risk averse strategic insider trading we show that informational efficiency and market liquidity are significantly lower in a transparent market (i.e. with disclosure regulation) than in an opaque market (i.e. without disclosure regulation). The reason for this detrimental effect is that a risk averse insider optimally chooses a less aggressive trading strategy in order to prevent the market maker from inferring perfectly the private information from public records, and to maintain her informational advantage over time. Moreover, if the initial informational advantage of the insider is sufficiently high, then her expected utility from trading is higher in a transparent market.

Our result adds an interesting theoretical evidence to the existing debate on the relation between the degree of transparency and the optimal design and regulation of securities markets. A number of implications for regulatory policy can be drawn from our analysis. First, a mandatory insider trade disclosure does not eliminate the presence of insider trading once the informed agent reveals her trade: this constitutes a relevant empirical implication to test. Second, if the main goal of market design is to sustain informational efficiency and liquidity, as explicitly stated by several financial regulations such as the Section 16(a) of the U.S. Securities Exchange Act, then an opaque market should be preferred. Strategic corporate insiders would exploit their informational advantage more rapidly, inducing private information to reach the market (and hence to be available to all investors) on a more timely basis. Under this view a trade disclosure regulation would represent a friction in the system of efficient prices. Such regulation may also not be able to sufficiently reduce the adverse selection problem in the market, and consequently to enhance market liquidity. Finally, when the degree of asymmetric information is high – that is when an effective policy intervention is more needed – the introduction of a trade disclosure regulation increases the insider trading profitability. Testable implications on insiders' trading behavior and on market components then follow.

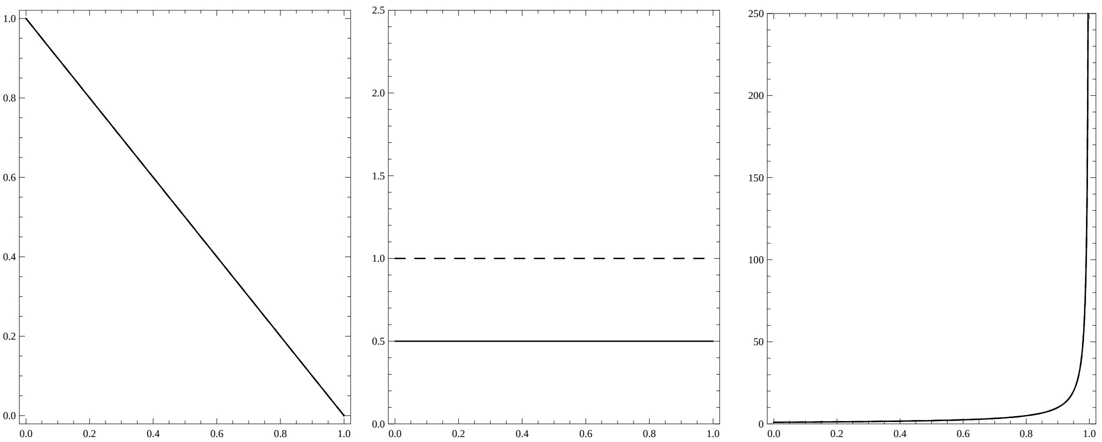
The change in the U.S. regulation on insider trade disclosure with the introduction of the Sarbanes-Oxley Act in 2002 offers the opportunity to carry out a natural experiment to test the implications derived in this article. Moreover, having shown the failure of the existing regulation, a further step would be to study and endogenously derive what should be the optimal one. These interesting and challenging projects are left for further research.

Table 3.1 Equilibria

The three graphs in each Panel contrast over time the conditional variance of prices (an inverse measure of market efficiency), $\Sigma(t)$, the sensitivity of the pricing function to the total order flow (an inverse measure of market liquidity), $\lambda(t)$, and the sensitivity of the insider's trading strategy to private information (a direct measure of insider aggressiveness) $\beta(t)$, respectively for the continuous-time equilibrium (i) in a transparent market (solid line), and (ii) in an opaque market (dashed line). Panel A represents the case of risk neutrality ($A = 0$); Panel B the case of risk aversion ($A = 4$). Exogenous parameters are normalized by setting $\Sigma(0) = 1$, $\sigma_u^2 = 1$.

MARKET INEFFICIENCY: $\Sigma(t)$ — MARKET ILLIQUIDITY: $\lambda(t)$ — INSIDER AGGRESSIVENESS: $\beta(t)$

Panel A: Risk Neutrality ($A = 0$)



Panel B: Risk Aversion ($A = 4$)

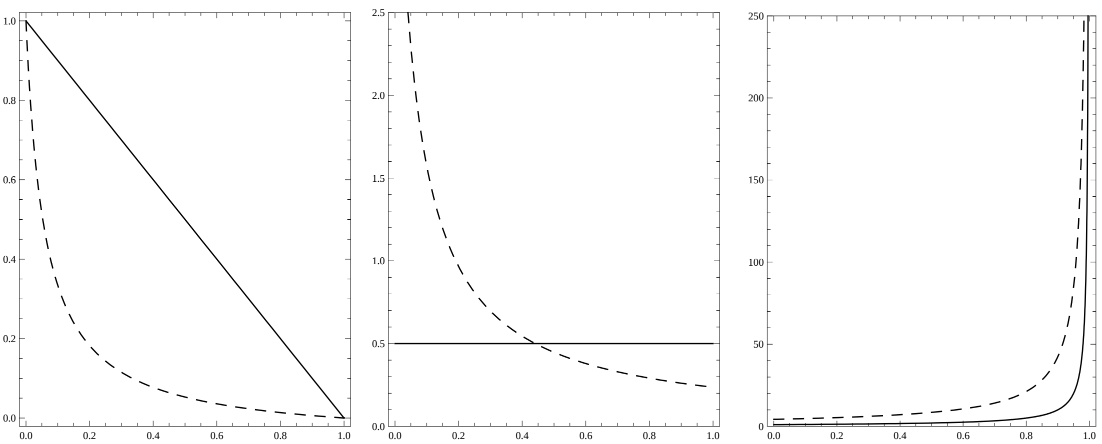
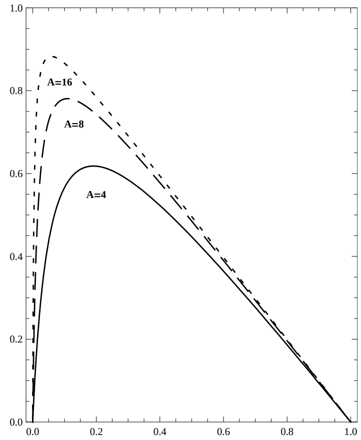


Table 3.2 Efficiency loss

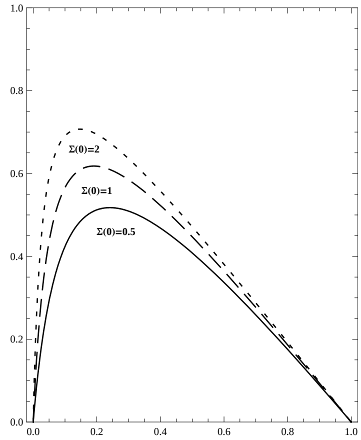
The three graphs in Panel A contrast over time the instantaneous efficiency loss and present comparative statics for the exogenous parameters $A, \Sigma(0), \sigma_u^2$. The shaded area in the first graph in Panel B identifies the aggregate measure of efficiency loss (to be scaled by 1/2). The second and the third graphs in Panel B contrast over the level of risk aversion the aggregate efficiency loss and present comparative statics for the exogenous parameters $\Sigma(0), \sigma_u^2$. Unless otherwise stated, exogenous parameters are normalized by setting $A = 4$, $\Sigma(0) = 1$, and $\sigma_u^2 = 1$.

Panel A: INSTANTANEOUS EFFICIENCY LOSS, $\Omega(t) = \big(\Sigma(t) - \tilde{\Sigma}(t)\big)/\Sigma(0)$

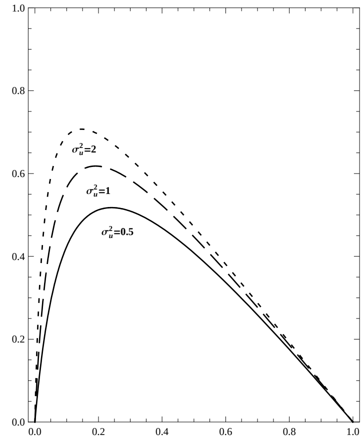
$\Omega(t)$ vs A



$\Omega(t)$ vs $\Sigma(0)$

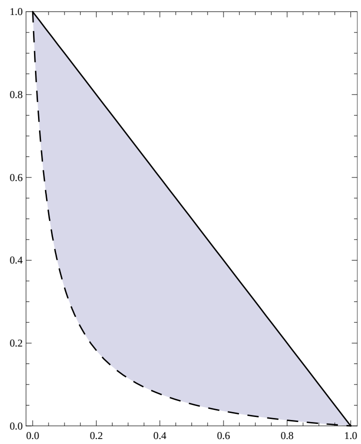


$\Omega(t)$ vs σ_u^2

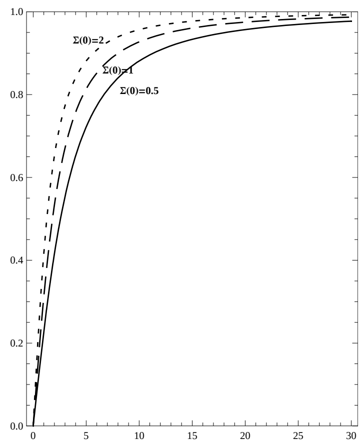


Panel B: AGGREGATE EFFICIENCY LOSS, $\Omega^a = 2\int_0^1 \Omega(t)\mathrm{d}t$

Ω^a



$\Omega^a(A)$ vs $\Sigma(0)$



$\Omega^a(A)$ vs σ_u^2

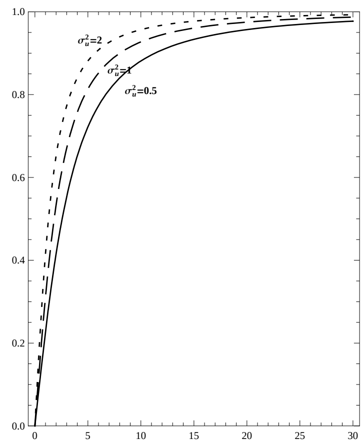
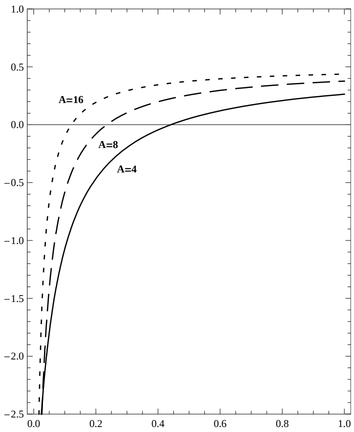


Table 3.3 Liquidity loss

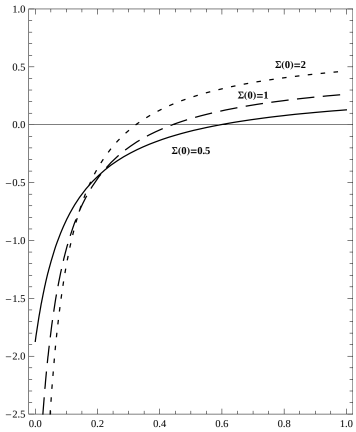
The three graphs in Panel A contrast over time the instantaneous liquidity loss and present comparative statics for the exogenous parameters $A, \Sigma(0), \sigma_u^2$. The shaded area in the first graph in Panel B identifies the aggregate measure of liquidity loss. The second and the third graphs in Panel B contrast over the level of risk aversion the aggregate liquidity loss and present comparative statics for the exogenous parameters $\Sigma(0), \sigma_u^2$. Unless otherwise stated, exogenous parameters are normalized by setting $A = 4$, $\Sigma(0) = 1$, and $\sigma_u^2 = 1$.

Panel A: INSTANTANEOUS LIQUIDITY LOSS, $\Psi(t) = \big(\lambda(t) - \tilde{\lambda}(t)\big)\sigma_u^2$

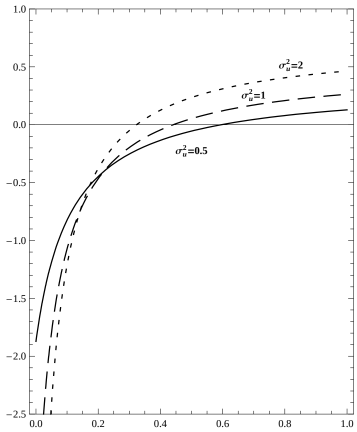
$\Psi(t)$ vs A



$\Psi(t)$ vs $\Sigma(0)$

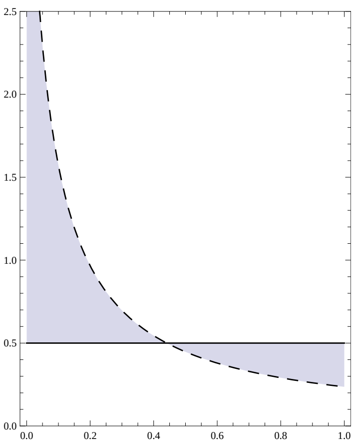


$\Psi(t)$ vs σ_u^2

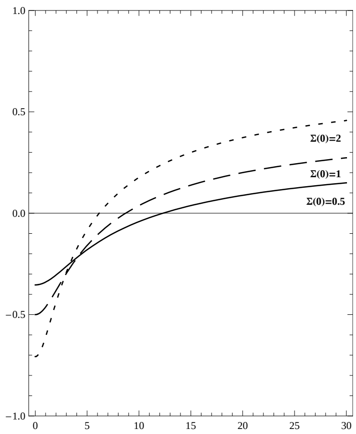


Panel B: AGGREGATE LIQUIDITY LOSS, $\Psi^a = \int_0^1 \Psi(t)\mathrm{d}t$

Ψ^a



$\Psi^a(A)$ vs $\Sigma(0)$



$\Psi^a(A)$ vs σ_u^2

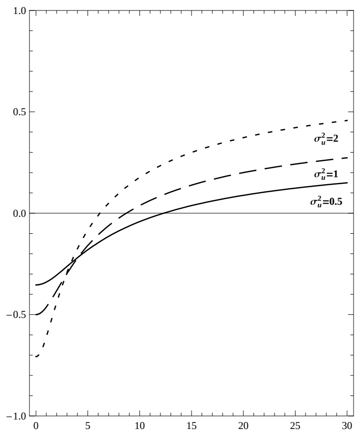
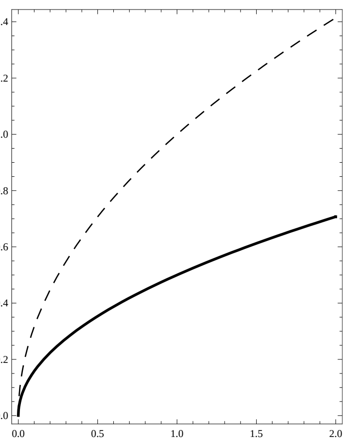


Table 3.4 Insider trading profitability

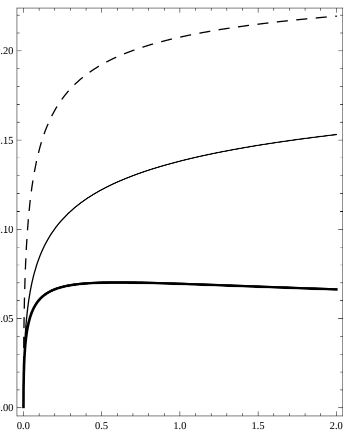
The three graphs in Panel A contrast the unconditional expected utility of the insider over the prior variance of the private information (the first two graphs) and the level of risk aversion (third graph). The first two graphs in Panel B contrast the conditional expected utility of the insider over her informational advantage, $\Gamma \equiv |v - \mathbb{E}_0(v)|$, while the third graph contrast the minimum level of the insider's informational advantage, Γ^* such that for any $\Gamma > \Gamma^*$ her conditional expected utility is higher in a transparent market. The dashed line represents an opaque market, the solid line a transparent market, and the solid bold line the difference between the two. In the 3-dimensional graph the darker surface represents the threshold level Γ^* as a function of $\Sigma(0)$ and A; the brighter surface as a function of σ_u^2 and A. Unless otherwise stated, exogenous parameters are normalized by setting $W(0) = 0$, $A = 4$, $\Sigma(0) = 1$, and $\sigma_u^2 = 1$.

Panel A: UNCONDITIONAL EXPECTED UTILITY, $\pi_0 = \mathbb{E}_0[U(W(1))]$

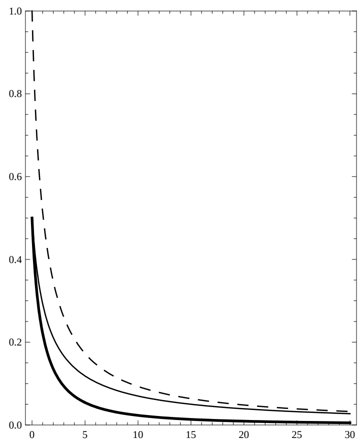
$\pi_0(\Sigma(0))$ when $A = 0$



$\pi_0(\Sigma(0))$ when $A = 4$

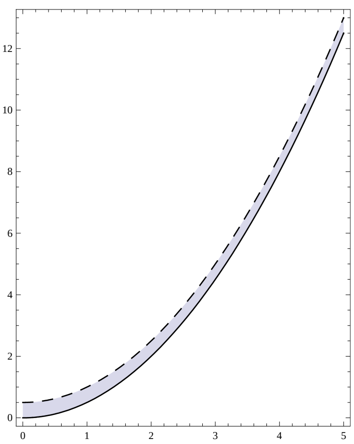


$\pi_0(A)$

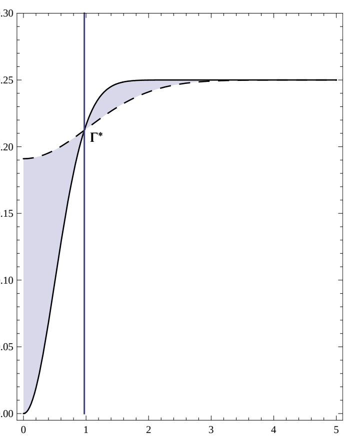


Panel B: CONDITIONAL EXPECTED UTILITY, $\Pi_0 = \mathbb{E}_0[U(W(1))|v]$

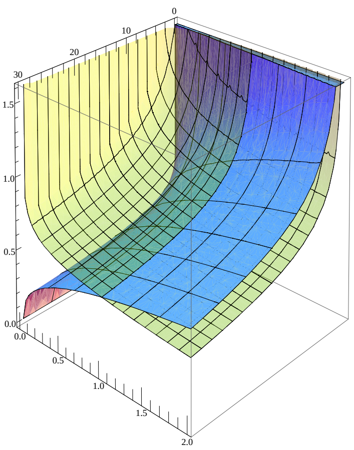
$\Pi_0(\Gamma)$ when $A = 0$



$\Pi_0(\Gamma)$ when $A = 4$



$\Gamma^*(\Sigma(0), A)$ and $\Gamma^*(\sigma_u^2, A)$



Appendix A

Strategic Risk Taking with Systemic Externalities

A.1 Proofs

Proof of Lemma 1. For each default region, substitute in (2.8) the relevant value of the assets as a function of (W_{iT}, W_{jT}). Straightforward algebra leads to (2.16). □

Proof of Proposition 1. We solve for bank i's best response function considering a generic objective function $u_i(\cdot)$ that satisfies the properties in Remark 1. The problem faced by bank i can be restated using Lagrangian as

$$\max_{W_{iT}, y_i} \mathbb{E}\Big[u_i(W_{iT}) + y_i[W_{i0} - \xi_T(W_{iT} + C_{iT})]\Big] \tag{A.1}$$

For each state ξ_T:

$$\begin{aligned}\hat{W}_{iT} &= \arg\max\, u_i(W_{iT}) + y_i W_{i0} - y_i \xi_T W_{iT} - y_i \xi_T C_{iT} \\ &= \arg\max\, u_i(W_{iT}) - y_i \xi_T W_{iT} - y_i \xi_T \Big\{[x_i\phi + (1 - x_i)(\underline{W}_i - W_{iT})]\mathbb{1}_{\{W_{iT}<\underline{W}_i \,\wedge\, W_{jT}\geqslant\underline{W}_j\}} \\ &\qquad + [z_i\phi + (1 - z_i)(\underline{W}_i - W_{iT})]\mathbb{1}_{\{W_{iT}<\underline{W}_i \,\wedge\, W_{jT}<\underline{W}_j\}}\Big\}\end{aligned} \tag{A.2}$$

Let $h_i(W_{iT})$ denote the objective function on the RHS of (A.2). $h_i(W_{iT})$ is not concave in W_{iT}

because of the discontinuity in $\underline{W}_i$, and it exhibits possible maxima at:

$$I_i(y_i\xi_T) \quad \text{if} \quad I_i(y_i\xi_T) \geqslant \underline{W}_i \tag{A.3}$$
$$\underline{W}_i \tag{A.4}$$
$$I_i(y_i x_i \xi_T) \quad \text{if} \quad I_i(y_i x_i \xi_T) < \underline{W}_i \ \wedge \ W_{jT} \geqslant \underline{W}_j \tag{A.5}$$
$$I_i(y_i z_i \xi_T) \quad \text{if} \quad I_i(y_i z_i \xi_T) < \underline{W}_i \ \wedge \ W_{jT} < \underline{W}_j \tag{A.6}$$

where $I_i(\cdot)$ is the inverse function of $u_i'(\cdot)$. In what follows, we show for which regions of the domain of ξ_T the above candidates are indeed a global maximum. Note that, since $I_i(\alpha)$ is decreasing in α and $(x_i, z_i) \in (0, 1)$:

- when $I_i(y_i\xi_T) \geqslant \underline{W}_i$, then the maxima at $I_i(y_i x_i \xi_T)$ and $I_i(y_i z_i \xi_T)$ are not feasible since $I_i(y_i z_i \xi_T) > I_i(y_i x_i \xi_T) > \underline{W}_i$: hence the possible candidates are $I_i(y_i\xi_T)$ and $\underline{W}_i$;
- when $I_i(y_i x_i \xi_T) < \underline{W}_i$, then the maximum at $I_i(y_i\xi_T)$ is not feasible sicne $I_i(y_i\xi_T) < \underline{W}_i$: hence the possible candidates are $I_i(y_i x_i \xi_T)$ and $\underline{W}_i$;
- when $I_i(y_i z_i \xi_T) < \underline{W}_i$, then the maximum at $I_i(y_i\xi_T)$ is not feasible sicne $I_i(y_i\xi_T) < \underline{W}_i$: hence the possible candidates are $I_i(y_i z_i \xi_T)$ and $\underline{W}_i$.

Therefore, we can distinguish among the following cases:

(i) $h_i(I_i(y_i\xi_T))$ is a global maximum if the following two conditions are satisfied:

$$I_i(y_i\xi_T) - \underline{W}_i \geqslant 0 \tag{A.7}$$
$$h_i(I_i(y_i\xi_T)) - h_i(\underline{W}_i) \geqslant 0 \tag{A.8}$$

Since for any $W_{iT} \geqslant \underline{W}_i$ bank i does not default, (A.7) implies (A.8). Hence, $\hat{W}_{iT} = I_i(y_i\xi_T)$ if $\xi_T \leqslant \underline{\xi}_i$ where $\underline{\xi}_i \equiv u_i'(\underline{W}_i)/y_i$.

(ii) When $W_{jT} \geqslant \underline{W}_j$, $h_i(I_i(y_i x_i \xi_T))$ is a global maximum if the following two conditions are satisfied:

$$I_i(y_i x_i \xi_T) - \underline{W}_i < 0 \tag{A.9}$$
$$h_i(I_i(y_i x_i \xi_T)) - h_i(\underline{W}_i) \geqslant 0 \tag{A.10}$$

Equation (A.9) implies that $\xi_T > \underline{\xi}_i/x_i$, and (A.10) can be written as $f_i(y_i x_i \xi_T) \leqslant 0$ where

$$f_i(\alpha) \equiv \Big[u_i(\underline{W}_i) - u_i(I_i(\alpha))\Big]\frac{1}{\alpha} + I_i(\alpha) - \underline{W}_i + \phi \tag{A.11}$$
$$f_i(y_i\underline{\xi}_i) = \phi \tag{A.12}$$
$$\frac{\partial f_i(\alpha)}{\partial \alpha} < 0 \quad \text{if} \quad I_i(\alpha) < \underline{W}_i \tag{A.13}$$

Let $\bar{\xi}_i$ be such that $f_i(y_i x_i \bar{\xi}_i) = 0$ and $\bar{\xi}_i > \underline{\xi}_i/x_i$. From (A.12) and (A.13) we can conclude that $\bar{\xi}_i$ exists and it is unique. Hence, $\hat{W}_{iT} = I_i(y_i x_i \xi_T)$ if $\xi_T > \bar{\xi}_i$. Since $f_i(u_i'(\underline{W}_i - \phi)) > 0$, in case of default, $\hat{W}_{iT} = I_i(y_i x_i \xi_T) < \underline{W}_i - \phi$ for any $\xi_T > \bar{\xi}_i$. This proves that the optimal equity of bank i does not take values in the interval $[\underline{W}_i - \phi, \underline{W}_i)$.

(iii) Symmetrically, when $W_{jT} < \underline{W}_j$, $h_i(I_i(y_i z_i \xi_T))$ is a global maximum if the following two conditions are satisfied:

$$I_i(y_i z_i \xi_T) - \underline{W}_i < 0 \tag{A.14}$$
$$h_i(I_i(y_i z_i \xi_T)) - h_i(\underline{W}_i) \geqslant 0 \tag{A.15}$$

Equation (A.14) implies that $\xi_T > \underline{\xi}_i/z_i$, and (A.15) can be written as $f_i(y_i z_i \xi_T) \leqslant 0$ where $f_i(\cdot)$ is defined above in (A.11). Let $\bar{\bar{\xi}}_i$ be such that $f_i(y_i z_i \bar{\bar{\xi}}_i) = 0$ and $\bar{\bar{\xi}}_i > \underline{\xi}_i/z_i$. From (A.12) and (A.13) we can conclude that $\bar{\bar{\xi}}_i$ exists and it is unique. Hence, $\hat{W}_{iT} = I_i(y_i z_i \xi_T)$ if $\xi_T > \bar{\bar{\xi}}_i$.

(iv) When $W_{jT} \geqslant \underline{W}_j$, $h_i(\underline{W}_i)$ is a global maximum if either

$$I_i(y_i \xi_T) - \underline{W}_i < 0 \tag{A.16}$$
$$I_i(y_i x_i \xi_T) - \underline{W}_i \geqslant 0 \tag{A.17}$$

that is if $\underline{\xi}_i < \xi_T \leqslant \underline{\xi}_i/x_i$, or

$$I_i(y_i x_i \xi_T) - \underline{W}_i < 0 \tag{A.18}$$
$$h_i(I_i(y_i x_i \xi_T)) - h_i(\underline{W}_i) > 0 \tag{A.19}$$

that is if $\underline{\xi}_i/x_i < \xi_T \leqslant \bar{\xi}_i$. Hence, $\hat{W}_{iT} = \underline{W}_i$ if $\underline{\xi}_i < \xi_T \leqslant \bar{\xi}_i$.

(v) When $W_{jT} < \underline{W}_j$, $h_i(\underline{W}_i)$ is a global maximum if either

$$I_i(y_i \xi_T) - \underline{W}_i < 0 \tag{A.20}$$
$$I_i(y_i z_i \xi_T) - \underline{W}_i \geqslant 0 \tag{A.21}$$

that is if $\underline{\xi}_i < \xi_T \leqslant \underline{\xi}_i/z_i$, or

$$I_i(y_i z_i \xi_T) - \underline{W}_i < 0 \tag{A.22}$$
$$h_i(I_i(y_i z_i \xi_T)) - h_i(\underline{W}_i) > 0 \tag{A.23}$$

that is if $\underline{\xi}_i/z_i < \xi_T \leqslant \bar{\bar{\xi}}_i$. Hence, $\underline{W}_i$ is a global maximizer if $\underline{\xi}_i < \xi_T \leqslant \bar{\bar{\xi}}_i$.

Putting together all the five cases and considering an isoelastic objective function as in (2.5), where

$$I_i(x) = x^{-\frac{1}{\gamma_i}}, \tag{A.24}$$

we obtain (2.22). We have already shown that $\underline{\xi}_i < \bar{\xi}_i$, and it is straightforward to see that $\bar{\xi}_i < \bar{\bar{\xi}}_i$, since $x_i > z_i$.

Given the optimal solution $\hat{W}_{iT}(W_{jT}; y_i)$, y_i is set such that the static budget constraint in (2.19) holds with equality. Let $\tilde{W}_{iT}$ be any feasible solution (i.e., that satisfies the static budget constraint), then we can show that

$$\begin{aligned} \mathbb{E}[u_i(\hat{W}_{iT})] - \mathbb{E}[u_i(\tilde{W}_{iT})] &= \mathbb{E}[u_i(\hat{W}_{iT}) - y_i W_{i0}] - \mathbb{E}[u_i(\tilde{W}_{iT}) - y_i W_{i0}] \\ &\geqslant \mathbb{E}[u_i(\hat{W}_{iT}) - y_i \xi_T(\hat{W}_{iT} + C_{iT}(\hat{W}_{iT}))] - \mathbb{E}[u_i(\tilde{W}_{iT}) - y_i \xi_T(\tilde{W}_{iT} + C_{iT}(\tilde{W}_{iT}))] \\ &\geqslant 0 \end{aligned} \tag{A.25}$$

The first inequality follows from (2.19) holding with equality for $\hat{W}_{iT}$. The second inequality follows from (A.2). $\square$

Lemma 2. *If bank i defaults, then*

$$W_{iT} + C_{iT} < \underline{W}_i \tag{A.26}$$

Proof. Let us consider first the case $W_{jT} \geqslant \underline{W}_j$:

$$\begin{aligned} W_{iT} + C_{iT} &= I_i(y_i x_i \xi_T) + [x_i \phi + (1 - x_i)(\underline{W}_i - I_i(y_i x_i \xi_T))] &\text{(A.27)}\\ &= \underline{W}_i + x_i [\phi - \underline{W}_i + I_i(y_i x_i \xi_T)] &\text{(A.28)} \end{aligned}$$

Since $I_i(\alpha)$ is decreasing in α, by using the definition of $\bar{\xi}_i$ in (2.23), we can conclude that

$$\phi - \underline{W}_i + I_i(y_i x_i \xi_T) < \phi - \underline{W}_i + I_i(y_i x_i \bar{\xi}_i) = -\Big[u_i(\underline{W}_i) - u_i(I_i(y_i x_i \bar{\xi}_i))\Big] \frac{1}{y_i x_i \bar{\xi}_i} < 0 \tag{A.29}$$

Hence, $W_{iT} + C_{iT} < \underline{W}_i$. A symmetric proof, which makes use of the definition of $\bar{\bar{\xi}}_i$ in (2.23), holds for the case $W_{jT} < \underline{W}_j$. $\square$

Proof of Proposition 2. As for the proof of Proposition 1, we consider a generic objective function $u_i(\cdot)$. Consider any realization of ξ_T in the set $(\bar{\xi}_i, \bar{\bar{\xi}}_i] \cap (\bar{\xi}_j, \bar{\bar{\xi}}_j]$, where the thresholds $\bar{\xi}_j, \bar{\bar{\xi}}_i$ are defined in (2.23). From the best response functions in (2.22), it follows that the Nash equilibrium of the game for a fixed level of ξ_T (which we refer as the ξ_T-game hereafter) is multiple. Hence, a unique pure-strategy equilibrium of the strategic game requires that the set $(\bar{\xi}_i, \bar{\bar{\xi}}_i] \cap (\bar{\xi}_j, \bar{\bar{\xi}}_j]$ is empty: $\bar{\bar{\xi}}_j < \bar{\xi}_i$. In turn, this requires that

$$y_j > \bar{y}_j \qquad \text{where} \qquad \bar{y}_j \equiv y_i \left(\frac{\alpha_j x_i}{\alpha_i z_j} \right). \tag{A.30}$$

Let assume that $y_j > \bar{y}_j$. then the Nash equilibrium of each ξ_T-game, for any ξ_T, is unique. This can be written as

$$W^*_{iT}(y_i) = \begin{cases} I_i(y_i \xi_T) & \text{if} \quad \xi_T \leqslant \underline{\underline{\xi}}_i \\ \underline{W}_i & \text{if} \quad \underline{\xi}_i < \xi_T \leqslant \bar{\bar{\xi}}_i \\ I_i(y_i z_i \xi_T) & \text{if} \quad \xi_T > \bar{\bar{\xi}}_i, \end{cases} \qquad W^*_{jT}(y_j, y_i) = \begin{cases} I_j(y_j \xi_T) & \text{if} \quad \xi_T \leqslant \underline{\xi}_j \\ \underline{W}_j & \text{if} \quad \underline{\xi}_j < \xi_T \leqslant \bar{\bar{\xi}}_j \\ I_j(y_j x_j \xi_T) & \text{if} \quad \bar{\bar{\xi}}_j < \xi_T \leqslant \bar{\bar{\xi}}_i \\ I_j(y_j z_j \xi_T) & \text{if} \quad \xi_T > \bar{\bar{\xi}}_i, \end{cases} \tag{A.31}$$

This corresponds to a unique pure-strategy equilibrium of the strategic game if and only if the Lagrange multipliers (y^*_i, y^*_j) that make the budget constraints binding

$$\begin{aligned} &\mathbb{E}\left[\xi_T(W^*_{iT}(y^*_i) + C_{iT}(W^*_{iT}(y^*_i)))\right] = W_{i0} &\text{(A.32)}\\ &\mathbb{E}\left[\xi_T(W^*_{jT}(y^*_j, y^*_i) + C_{jT}(W^*_{jT}(y^*_j, y^*_i)))\right] = W_{j0}, &\text{(A.33)} \end{aligned}$$

are such that the assumed condition is indeed satisfied: $y^*_j > y^*_i \left(\frac{\alpha_j x_i}{\alpha_i z_j}\right)$. To verify this we proceed as follows. First notice that the budget constraint in (A.32) depends only on y_i and not on y_j. This allows us to get y^*_i independently of y_j. Then, by means of the following *budget-contraint* operator

$$BC_j(y_j) \equiv \mathbb{E}\left[\xi_T(W^*_{jT}(y_j, y^*_i) + C_{jT}(W^*_{jT}(y_j, y^*_i)))\right] \tag{A.34}$$

we define

$$\bar{W}_{j0} \equiv BC_j\left(y^*_i \frac{\alpha_j x_i}{\alpha_i z_j}\right) \tag{A.35}$$

By monotonicity of the operator in (A.34), for which we omit the proof, we can conclude that

$$W_{j0} < \bar{W}_{j0} \quad \Leftrightarrow \quad y^*_j > y^*_i \left(\frac{\alpha_j x_i}{\alpha_i z_j}\right) \tag{A.36}$$

When this condition holds for only one of the two banks, the equilibrium is unique. □

Proof of Corollary 1. Part (i) follows from Proposition 2. Define $\bar{y}_j \equiv y_i(x/z) > y_i$. It follows from (A.31) that $W^*_{iT}(y_i) > W^*_{jT}(\bar{y}_j, y_i)$ for any ξ_T. Hence it must be that $W_0 > \underline{W}_j 0$. This clearly holds for both banks. To prove part (ii), suppose by contradiction that a symmetric equilibrium exists: $(W^*_{1T}, y^*_1) = (W^*_{2T}, y^*_2)$. Then, since $y^*_1 = y^*_2$,

$$\underline{\xi} \equiv \underline{\xi}_1 = \underline{\xi}_2, \qquad \bar{\xi} \equiv \bar{\xi}_1 = \bar{\xi}_2, \qquad \bar{\bar{\xi}} \equiv \bar{\bar{\xi}}_1 = \bar{\bar{\xi}}_2 \tag{A.37}$$

To conclude that this can not be part of an equilibrium of the strategic game played by the two banks, it is enough to show that there exists a realization of the state of nature (the SPD) for which the Nash equilibrium is not symmetric. So, consider any realization ξ_T in the interval $\bar{\xi} < \xi_T \leqslant \bar{\bar{\xi}}$; according to Equation (2.22) in Proposition 1, if $W_{jT} \geqslant \underline{W}$, the optimal response of bank i is to default, $W_{iT} < \underline{W}$, whereas if $W_{jT} < \underline{W}$, the optimal response of bank i is not to default, $W_{iT} \geqslant \underline{W}$. This contradicts the initial assumption that $W_{1T} = W_{2T}$ can be part of an equilibrium. □

Proof of Corollary 2. The optimal value of the assets follows from (2.11), (2.12), (2.14), and Lemma 1. The optimal value of the debt follows from: $D^*_{iT} = V^*_{iT} - W^*_{iT} - C^*_{iT}$. □

Lemma 3. *The state price density follows a Geometric Brownian Motion:*

$$\frac{d\xi_t}{\xi_t} = -rdt - \kappa dw_t$$

Then, the following results hold:

$$\mathbb{E}_t\left[\mathbb{1}_{\{\xi_T \leqslant H\}}\right] = \mathcal{N}(-d_t(H)) \tag{A.38}$$

$$\mathbb{E}_t\left[\xi_T \mathbb{1}_{\{\xi_T \leqslant H\}}\right] = \xi_t e^{-r(T-t)}\mathcal{N}(-\bar{d}_t(H)) \tag{A.39}$$

$$\mathbb{E}_t\left[\xi_T^{(\gamma_i-1)/\gamma_i} \mathbb{1}_{\{\xi_T \leqslant H\}}\right] = \xi_t^{(\gamma_i-1)/\gamma_i} e^{-A_i(T-t)}\mathcal{N}(-\hat{d}_{it}(H)) \tag{A.40}$$

where $\mathcal{N}(\cdot)$ *is the standard-normal cumulative distribution function and*

$$d_t(H) = \frac{\ln(\xi_t/H) - \left(r - (||\kappa||^2/2)\right)(T-t)}{||\kappa||\sqrt{T-t}} \tag{A.41}$$

$$\bar{d}_t(H) = d_t(H) + ||\kappa||\sqrt{T-t} \tag{A.42}$$

$$\hat{d}_{it}(H) = \bar{d}_t(H) - \frac{||\kappa||}{\gamma_i}\sqrt{T-t} \tag{A.43}$$

$$A_i = \left(\frac{\gamma_i - 1}{\gamma_i}\right)\left[r + \frac{||\kappa||^2}{2\gamma_i}\right] \tag{A.44}$$

Proof. The results follow from the conditional expectation of a log-normal random variable. □

Proof of Proposition 3. To derive the optimal asset allocation we first compute the time t value of the equity and of the assets:

$$W_{it}^* = \mathbb{E}_t\left[\frac{\xi_T}{\xi_t}(W_{iT}^* + C_{iT}^*)\right] \tag{A.45}$$

$$V_{it}^* = \frac{1}{\beta_i}\mathbb{E}_t\left[\frac{\xi_T}{\xi_t}(W_{iT}^* + C_{iT}^*) + (1-\beta_i)\frac{\xi_T}{\xi_t}(\underline{W} - W_{iT}^*)\,\mathbb{1}_{\{\xi_T \leqslant \underline{\xi}_i\}}\right]. \tag{A.46}$$

Specifically,

$$\begin{aligned} W_{1t}^* = e^{-r(T-t)}&\left\{\underline{W}_1\left[1 - \mathcal{N}(-\bar{d}_t(\underline{\xi}_1))\right] - z_1(\underline{W}_1 - \phi)\left[1 - \mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1))\right]\right\} \\ &+ e^{-A_1(T-t)}(y_1^*\xi_t)^{-\frac{1}{\gamma_1}}\left\{\mathcal{N}(-\hat{d}_{1t}(\underline{\xi}_1)) + z_1^{1-\frac{1}{\gamma_1}}\left[1 - \mathcal{N}(\hat{d}_{1t}(\bar{\bar{\xi}}_1))\right]\right\}, \end{aligned} \tag{A.47}$$

$$\begin{aligned} W_{2t}^* = e^{-r(T-t)}&\left\{\underline{W}_2\left[1-\mathcal{N}(-\bar{d}_t(\underline{\xi}_2))\right] - x_2(\underline{W}_2-\phi)\left[\mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1)) - \mathcal{N}(-\bar{d}_t(\underline{\xi}_2))\right] - z_2(\underline{W}_2-\phi)\left[1-\mathcal{N}(-\bar{d}_t(\bar{\bar{\xi}}_1))\right]\right\} \\ &+ e^{-A_2(T-t)}(y_2^*\xi_t)^{-\frac{1}{\gamma_2}}\left\{\mathcal{N}(-\hat{d}_{2t}(\underline{\xi}_2)) + x_2^{1-\frac{1}{\gamma_2}}\left[\mathcal{N}(-\hat{d}_{2t}(\bar{\bar{\xi}}_1)) - \mathcal{N}(-\hat{d}_{2t}(\underline{\xi}_2))\right] + z_2^{1-\frac{1}{\gamma_2}}\left[1-\mathcal{N}(\hat{d}_{2t}(\bar{\bar{\xi}}_1))\right]\right\} \end{aligned} \tag{A.48}$$

and

$$V_{it}^* = \frac{W_{it}^*}{\beta_i} - \frac{1-\beta_i}{\beta_i}\left\{e^{-A_i(T-t)}(y_i^*\xi_t)^{-\frac{1}{\gamma_i}}\mathcal{N}(-\hat{d}_{it}(\underline{\xi}_i)) - e^{-r(T-t)}\underline{W}_i\mathcal{N}(-\bar{d}_t(\underline{\xi}_i))\right\} \tag{A.49}$$

Applying Ito's Lemma on $V_{it}^* = V_i^*(\xi_t, t)$, we obtain:

$$dV_{it}^* = (\cdot)\,dt + \left(-\xi_t\frac{\partial V_{it}}{\partial \xi_t}\kappa\right)dw_t. \tag{A.50}$$

Equating the diffusion terms in (A.50) and (2.6), we obtain (2.35). □

Proof of Proposition 4. Given the equilibrium default thresholds in (2.26), default probabilities follow from Lemma 3. □

Proof of Proposition 5. Results follow from Lemma 3. □

Proof of Proposition 6. Results follow from Lemma 3. □

A.2 Benchmarks

A.2.1 Benchmark (a): No Cost of Default

$$C_{iT} = 0 \quad \forall\, i \in 1, 2 \tag{A.51}$$

Bank i's problem. The representative equityholder of Bank i faces the following optimization problem:

$$\max_{W_{iT}} \ \mathbb{E}[u_i(W_{iT})] \qquad s.t. \qquad \mathbb{E}[\xi_T W_{iT}] \leqslant W_{i0} \tag{A.52}$$

where the static budget constraint obtains by substituting $V_{i0} = W_{i0} + D_{i0}$, $V_{iT} = W_{iT} + D_{iT}$ and $D_{i0} = \mathbb{E}[\xi_T D_{iT}]$ into $\mathbb{E}[\xi_T V_{iT}] \leqslant V_{i0}$.

Proposition 11. *The optimal value of the equity is not affected by the leverage in place:*

$$W_{iT}^a = I_i(y_i^a \xi_T) \qquad where \qquad y_i^a = \left(\frac{e^{-A_i T}}{W_{i0}}\right)^{\gamma_i} \tag{A.53}$$

Bank i defaults when $W_{iT}^a < \underline{W}_i$:

$$I_i(y_i^a \xi_T) < \underline{W}_i \qquad \Rightarrow \qquad \xi_T > u_i'(\underline{W}_i)/y_i^a \equiv \underline{\xi}_i^a \tag{A.54}$$

Proof. Special case of Proposition 2 where $\phi = \lambda = \eta_1 = \eta_2 = 0$. □

A.2.2 Benchmark (b): No Systemic Cost of Default

$$C_{iT} = \begin{cases} 0 & if \quad D_{iT} = F_i \\ \phi + \lambda(F_i - D_{iT}) & if \quad D_{iT} < F_i \end{cases} \qquad \forall\, i \in 1, 2 \tag{A.55}$$

Bank i's problem. The representative equityholder of Bank i faces the following optimization problem:

$$\max_{W_{iT}} \ \mathbb{E}[u_i(W_{iT})] \qquad s.t. \qquad \mathbb{E}[\xi_T(W_{iT} + C_{iT}(W_{iT}))] \leqslant W_{i0} \tag{A.56}$$

where the static budget constraint obtains by substituting $V_{i0} = W_{i0} + D_{i0}$, $V_{iT} = W_{iT} + D_{iT} + C_{iT}$ and $D_{i0} = \mathbb{E}[\xi_T D_{iT}]$ into $\mathbb{E}[\xi_T V_{iT}] \leqslant V_{i0}$.

Proposition 12. *The optimal value of the equity is not affected by the leverage in place:*

$$W^b_{iT} = \begin{cases} I_i(y^b_i \xi_T) & \text{if} \quad \xi_T \leqslant \underline{\xi}^b_i \\ \underline{W}_i & \text{if} \quad \underline{\xi}^b_i < \xi_T \leqslant \bar{\xi}^b_i \\ I_i(y^b_i x_i \xi_T) & \text{if} \quad \xi_T > \bar{\xi}^b_i, \end{cases} \tag{A.57}$$

where y^b_i solves

$$\mathbb{E}\left[\xi_T(W^b_{iT}(y^b_i) + C_{iT}(W^b_{iT}(y^b_i)))\right] = W_{i0} \tag{A.58}$$

Bank i defaults when $\xi_T > \bar{\xi}^b_i$.

Proof. Special case of Proposition 2 where $\eta_1 = \eta_2 = 0$. □

A.3 Example of Multiple Equilibria

This section provides an example of multiple equilibria for the case of homogenous banks. As highlighted by Corollary 1, all multiple equilibria are characterized by asymmetric optimal policies. Even though *ex-ante* identical, the two banks optimally choose different equilibrium asset allocations. Proposition 13 characterizes the equilibrium. Figure A.1 illustrates.

Proposition 13. *Consider two homogeneous banks. One of the asymmetric (multiple) equilibria can be constructed as follows:*

$$W^*_{iT} = \begin{cases} I(y^*_i \xi_T) & \text{if} \quad \xi_T \leqslant \underline{\xi}_i \\ \underline{W} & \text{if} \quad \underline{\xi}_i < \xi_T \leqslant \bar{\xi}_i \\ I(y^*_i x \xi_T) & \text{if} \quad \bar{\xi}_i < \xi_T \leqslant \bar{\bar{\xi}}_j \\ \underline{W} & \text{if} \quad \bar{\bar{\xi}}_j < \xi_T \leqslant \bar{\bar{\xi}}_i \\ I(y^*_i z \xi_T) & \text{if} \quad \xi_T > \bar{\bar{\xi}}_i, \end{cases} \qquad W^*_{jT} = \begin{cases} I(y^*_j \xi_T) & \text{if} \quad \xi_T \leqslant \underline{\xi}_j \\ \underline{W} & \text{if} \quad \underline{\xi}_j < \xi_T \leqslant \bar{\xi}_j \\ I(y^*_j x \xi_T) & \text{if} \quad \bar{\xi}_j < \xi_T \leqslant \bar{\xi}_i \\ \underline{W} & \text{if} \quad \bar{\xi}_i < \xi_T \leqslant \bar{\bar{\xi}}_j \\ I(y^*_j x \xi_T) & \text{if} \quad \bar{\bar{\xi}}_j < \xi_T \leqslant \bar{\bar{\xi}}_i \\ I(y^*_j z \xi_T) & \text{if} \quad \xi_T > \bar{\bar{\xi}}_i, \end{cases} \tag{A.59}$$

where y^*_i and y^*_j are such that

$$y^*_i < y^*_j < y^*_j \left(\frac{x}{z}\right) \tag{A.60}$$

$$\mathbb{E}\left[\xi_T(W^*_{iT}(y^*_i, y^*_j) + C_T(W^*_{iT}(y^*_i, y^*_j)))\right] = W_0 \tag{A.61}$$

$$\mathbb{E}\left[\xi_T(W^*_{jT}(y^*_j, y^*_i) + C_T(W^*_{jT}(y^*_j, y^*_i)))\right] = W_0 \tag{A.62}$$

Proof. We present the steps to construct the asymmetric equilibrium posited in the Proposition.

Step 1. From part (i) we deduce that y_i must be different from y_j. If this is not the case, the thresholds $(\underline{\xi}_i, \bar{\xi}_i, \bar{\bar{\xi}}_i)$ would coincides across banks, and W_{iT} would be equal to W_{jT} for any realization of the SPD at time T except for the interval $\bar{\xi} < \xi_T \leqslant \bar{\bar{\xi}}$, where either $W_{iT} > W_{jT}$ or $W_{iT} < W_{jT}$. This implies that the horizon equity of one bank would always be higher than the one of the other. Since the two banks are endowed with the same initial wealth, one of the two budget constraints can not be satisfied. Therefore, it must be that in equilibrium y_i differs from

y_j, and w.l.o.g. we can assume that $y_2 > y_1$. Then, it follows that

$$\underline{\xi}_2 < \underline{\xi}_1, \qquad \bar{\xi}_2 < \bar{\xi}_1, \qquad \bar{\bar{\xi}}_2 < \bar{\bar{\xi}}_1 \tag{A.63}$$

Step 2. We impose restrictions on the thresholds $(\underline{\xi}_i, \bar{\xi}_i, \bar{\bar{\xi}}_i)$ for $i \in \{1,2\}$ to ensure that the final horizon equity profile of one bank does not dominate the one of the other. Specifically, we impose that $\bar{\bar{\xi}}_2 > \bar{\xi}_1$, thus inducing to the following ordering:

$$\underline{\xi}_2 < \min\{\underline{\xi}_1, \bar{\xi}_2\} < \max\{\underline{\xi}_1, \bar{\xi}_2\} < \bar{\xi}_1 < \bar{\bar{\xi}}_2 < \bar{\bar{\xi}}_1 \tag{A.64}$$

Without such restriction, it is straightforward to show that the final horizon equity of bank 1 would be higher than the final horizon equity of bank 2 in any possible state of the world.

Step 3. Finding the Nash equilibrium of the strategic game entails solving for the Nash equilibrium for a given realization of ξ_T in the seven partitions of the state space. By combining the banks' best response functions in (2.22), we obtain that

- for the states $\xi_T \leqslant \bar{\xi}_1$ and $\xi_T > \bar{\bar{\xi}}_2$, the Nash equilibrium (in pure strategies) is unique and such that $W^*_{1T} \geqslant W^*_{2T}$;
- for the states $\bar{\xi}_1 < \xi_T \leqslant \bar{\bar{\xi}}_2$, the Nash equilibrium (in pure strategies) is multiple and such that either $W^*_{1T} > W^*_{2T}$ or $W^*_{1T} < W^*_{2T}$. However, only $W^*_{1T} < W^*_{2T}$ can be part of the Nash equilibrium of the strategic game, otherwise the final horizon equity of bank 1 would be higher than the final horizon equity of bank 2 in all the state of the world, thus preventing the banks' budget constraints to be simultaneously satisfied.

Therefore, for each of the following seven partitions the equilibrium final horizon equities are equal to:

	W^*_{1T}		W^*_{2T}
$\xi_T \leqslant \underline{\xi}_2$	$I(y_1\xi_T)$	$>$	$I(y_2\xi_T)$
$\underline{\xi}_2 < \xi_T \leqslant \min\{\underline{\xi}_1, \bar{\xi}_2\}$	$I(y_1\xi_T)$	$>$	$\underline{W}$
$\min\{\underline{\xi}_1, \bar{\xi}_2\} < \xi_T \leqslant \max\{\underline{\xi}_1, \bar{\xi}_2\}$	$\underline{W} + [I(y_1\xi_T) - \underline{W}]\mathbb{1}_{\{\underline{\xi}_1 > \bar{\xi}_2\}}$	$\geqslant$	$\underline{W} + [I(y_2 x\xi_T) - \underline{W}]\mathbb{1}_{\{\underline{\xi}_1 > \bar{\xi}_2\}}$
$\max\{\underline{\xi}_1, \bar{\xi}_2\} < \xi_T \leqslant \bar{\xi}_1$	$\underline{W}$	$>$	$I(y_2 x\xi_T)$
$\bar{\xi}_1 < \xi_T \leqslant \bar{\bar{\xi}}_2$	$I(y_1 x\xi_T)$	$<$	$\underline{W}$
$\bar{\bar{\xi}}_2 < \xi_T \leqslant \bar{\bar{\xi}}_1$	$\underline{W}$	$>$	$I(y_2 x\xi_T)$
$\xi_T > \bar{\bar{\xi}}_1$	$I(y_1 z\xi_T)$	$>$	$I(y_2 z\xi_T)$

Step 4. The equilibrium constructed above exists providing that a consistent set of Lagrange multipliers (y_1, y_2) exists. Consistent multipliers means that they have to satisfy the restrictions imposed in *Step 1* and *Step 2*. Note that the restriction in *Step 2*, $\bar{\bar{\xi}}_2 > \bar{\xi}_1$, implies that

$$\frac{\alpha}{y_2 z} > \frac{\alpha}{y_1 x} \quad \Rightarrow \quad y_2 < y_1\left(\frac{x}{z}\right) \tag{A.65}$$

where α solves the following equation

$$\Big[u(\underline{W}) - u(I(\alpha))\Big]\frac{1}{\alpha} + I(\alpha) - \underline{W} + \phi = 0. \tag{A.66}$$

Hence, the optimal Lagrange multipliers (y_1^*, y_2^*) must satisfy the restriction

$$y_1^* < y_2^* < y_1^* \left(\frac{x}{z}\right) \tag{A.67}$$

and make the budget constraints of the two banks binding

$$\mathbb{E}\left[\xi_T(W_{1T}^*(y_1^*, y_2^*) + C_T(W_{1T}^*(y_1^*, y_2^*)))\right] = W_0 \tag{A.68}$$

$$\mathbb{E}\left[\xi_T(W_{2T}^*(y_2^*, y_1^*) + C_T(W_{2T}^*(y_2^*, y_1^*)))\right] = W_0 \tag{A.69}$$

□

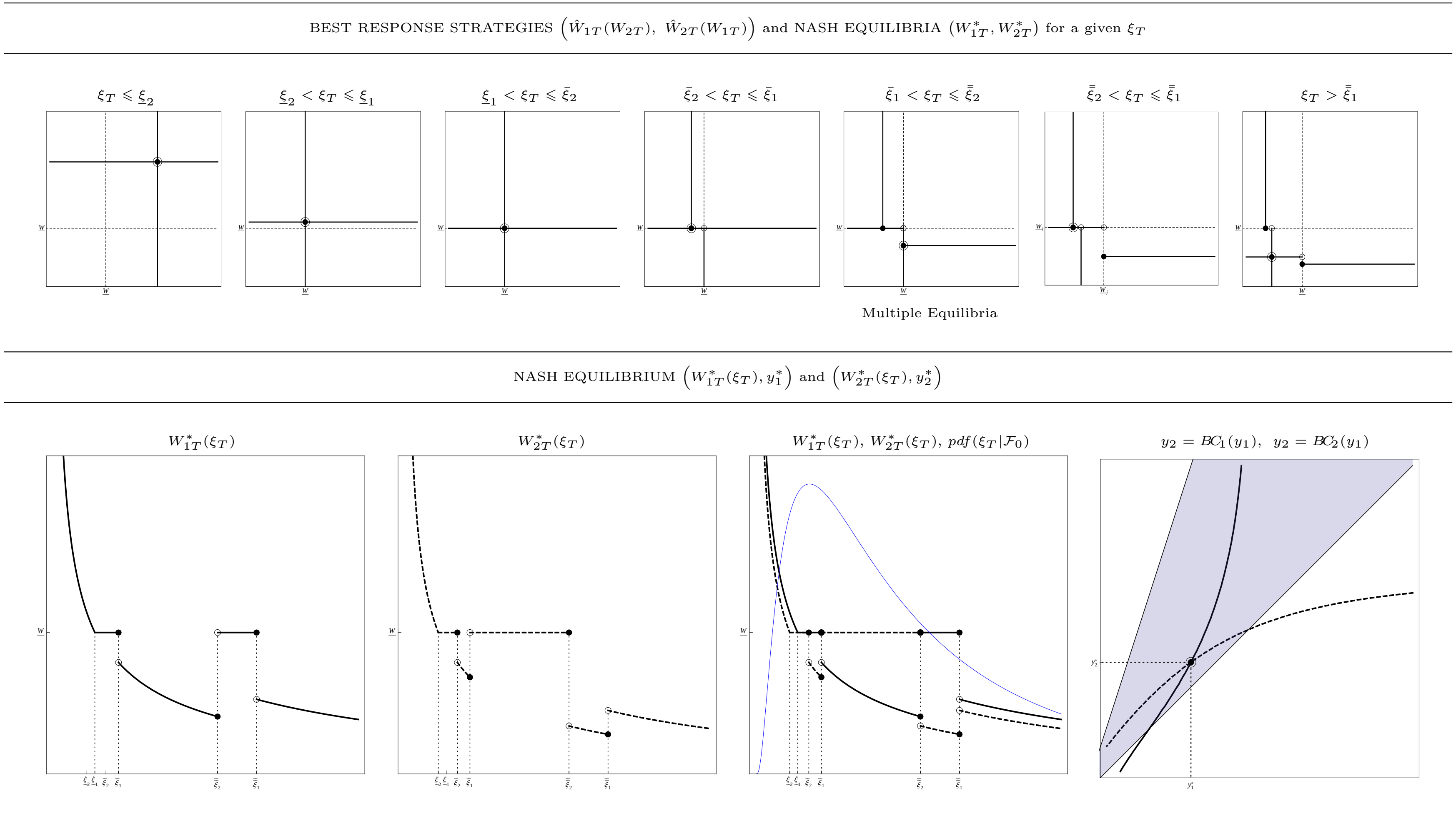


Figure A.1 Example of a multiple Nash equilibrium with homogeneous banks

Appendix B

Insider Trade Disclosure

B.1 Proofs

Proof of Theorem 1. The following filtrations allows us to define a linear equilibrium, in which both the trading strategy and the pricing rule are linear functions:

$$\mathbb{F} = \{\mathcal{F}(t)\}_{t\geqslant 0} \quad \text{where} \quad \mathcal{F}(t) = \sigma\big(y(s) : 0 \leqslant s \leqslant t\big)$$
$$\mathbb{G} = \{\mathcal{G}(t)\}_{t\geqslant 0} \quad \text{where} \quad \mathcal{G}(t) = \sigma\big(v\big) \vee \sigma\big(p(s) : 0 \leqslant s \leqslant t\big)$$

The objective of the insider trader is to maximize

$$\max_{\{\mathrm{d}x(t)\}_t} \mathbb{E}[-\exp\{-AW(1)\}] = \max_{\{\mathrm{d}x(t)\}_t} \mathbb{E}\left[-\exp\left\{-A\int_0^1 (v-p(s))\mathrm{d}x(s)\right\}\right] \tag{B.1}$$

given the dynamic of the conjectured pricing rule $p(t)$ defined in Equations (3.7). This is a Markovian stochastic control problem:[1]

$$J(M,t) = \max_{\mathrm{d}x} \mathbb{E}_t[J(M,t+\mathrm{d}t)] \tag{B.2}$$
$$\Updownarrow$$
$$-\exp\{-A\,[W(t)+V(M,t)]\} = \max_{\mathrm{d}x} \mathbb{E}_t[-\exp\{-A[W(t+\mathrm{d}t)+V(M,t+\mathrm{d}t)]\}] \tag{B.3}$$
$$\Updownarrow$$
$$1 = \max_{\mathrm{d}x} \mathbb{E}_t[\exp\{-A[\mathrm{d}W(t)+\mathrm{d}V(M,t)]\}] \tag{B.4}$$

[1]Note that, because of the exponential function, Equation (B.4) is equivalent to the standard formulation of the HJB equation,

$$0 = \max_{\mathrm{d}x} \mathbb{E}_t[\mathrm{d}J(M,t)]$$

Application of Ito's Lemma allows us to identify the following dynamics:[2]

$$\begin{aligned}
\mathrm{d}W(t) &= (v - p(t))\mathrm{d}x(t) && \text{(B.5)}\\
\mathrm{d}M(t) &= -\tilde{\lambda}(t)[\mathrm{d}x(t) + \mathrm{d}u(t)] && \text{(B.6)}\\
\mathrm{d}\langle M, M\rangle_t &= \tilde{\lambda}(t)^2\sigma_u^2\mathrm{d}t && \text{(B.7)}\\
\mathrm{d}V(M,t) &= V_M\mathrm{d}M(t) + \frac{1}{2}V_{MM}\mathrm{d}\langle M, M\rangle_t + V_t\mathrm{d}t && \text{(B.8)}\\
&= [2\tilde{\alpha}(t)M(t)]\mathrm{d}M(t) + [\tilde{\alpha}(t)]\mathrm{d}\langle M, M\rangle_t + M(t)^2\mathrm{d}\tilde{\alpha}(t) + \mathrm{d}\tilde{\delta}(t) && \text{(B.9)}
\end{aligned}$$

Note that $[\mathrm{d}W(t)+\mathrm{d}V(M,t)]$ is random because of $\mathrm{d}u(t)$, and hence normally distributed. Using the first moment for log-normal distributions, we can rewrite Equation (B.4) as

$$\begin{aligned}
1 &= \max_{\mathrm{d}x}\ \exp\left\{-A\left[\mathbb{E}_t[\mathrm{d}W(t)+\mathrm{d}V(M,t)] - \frac{A}{2}\mathbb{V}\mathrm{ar}_t[\mathrm{d}W(t)+\mathrm{d}V(M,t)]\right]\right\} && \text{(B.10)}\\
&= \max_{\mathrm{d}x}\ \exp\left\{-A\left[\mathrm{d}x\left[1-2\tilde{\alpha}(t)\tilde{\lambda}(t)\right]M(t)+\tilde{\alpha}(t)\tilde{\lambda}(t)^2\sigma_u^2\mathrm{d}t+\mathrm{d}\tilde{\delta}(t)+\left[\mathrm{d}\tilde{\alpha}(t)-\frac{A}{2}4\tilde{\alpha}(t)^2\tilde{\lambda}(t)^2\sigma_u^2\mathrm{d}t\right]M(t)^2\right]\right\} && \text{(B.11)}
\end{aligned}$$

with first-order condition (FOC) equal to

$$\exp\{\cdot\}\left[-A\left(1 - 2\tilde{\alpha}(t)\tilde{\lambda}(t)\right)M(t)\right] = 0 \tag{B.12}$$

which implies that for any $t \in [0,1]$

$$\tilde{\alpha}(t)\tilde{\lambda}(t) = \frac{1}{2} \tag{B.13}$$

Evaluating Equation (B.11) with the above optimal condition we get

$$1 = \exp\left\{-A\left[\frac{1}{2}\tilde{\lambda}(t)\sigma_u^2\mathrm{d}t + \mathrm{d}\tilde{\delta}(t) + \left[\mathrm{d}\tilde{\alpha}(t) - \frac{A}{2}\sigma_u^2\mathrm{d}t\right]M(t)^2\right]\right\} \tag{B.14}$$

which holds for every level of $M(t)$ and A if and only if:

$$\begin{aligned}
\mathrm{d}\tilde{\alpha}(t) &= \frac{A}{2}\sigma_u^2\mathrm{d}t && \text{(B.15)}\\
\mathrm{d}\tilde{\delta}(t) &= -\frac{1}{2}\tilde{\lambda}(t)\sigma_u^2\mathrm{d}t && \text{(B.16)}
\end{aligned}$$

Given the equilibrium condition in Equation (B.13) and the dynamic of $\tilde{\alpha}(t)$, it follows that

$$\mathrm{d}\tilde{\lambda}(t) = -A\tilde{\lambda}(t)^2\sigma_u^2\mathrm{d}t \tag{B.17}$$

Solving this differential equation:

$$\tilde{\lambda}(t) = (A\sigma_u^2 t + k)^{-1} \tag{B.18}$$

[2]The differential $\mathrm{d}\langle H, K\rangle_t$ simply coincide with $\mathrm{d}H(t)\mathrm{d}K(t)$ and represents the differential of the quadratic variation process. Note that $\mathrm{d}\langle x, x\rangle_t = 0$ because of order $(\mathrm{d}t)^2$.

for some constant k. Moreover, since no utility can be gained after trading is complete, the boundary condition $\tilde{\delta}(1) = 0$ must hold. Therefore,

$$\tilde{\delta}(t) = \frac{\sigma_u^2}{2} \int_t^1 \tilde{\lambda}(s)\mathrm{d}s \tag{B.19}$$

Following closely Back et al. (2000) we now define in the following Lemma the filtering problem of the market maker.

Lemma 4. *Assume the insider trader follow the linear strategy as in Equation (3.5) and let us define* $V(t) = \mathbb{E}[v|\mathcal{F}(t)]$ *and* $\Sigma(t) = \mathbb{V}\mathrm{ar}[v|\mathcal{F}(t)]$. *Then the following process*

$$Q(t) = \int_0^t \tilde{\beta}(s)[v - V(s)]\mathrm{d}s + u(t) \tag{B.20}$$

is a Wiener process on the market maker's information structure $\mathbb{F}$ *and it is called "innovation" process for the market maker's estimation problem. The differential*

$$\mathrm{d}Q(t) = \tilde{\beta}(t)[v - V(t)]\mathrm{d}t + \mathrm{d}u(t) \tag{B.21}$$

is the unpredictable part of the total order flow. Moreover,

$$V(t) = \int_0^t \frac{\tilde{\beta}(s)\tilde{\Sigma}(s)}{\sigma_u^2}\mathrm{d}Q(s) \tag{B.22}$$

The market maker's estimate of v *is revised according to*

$$\mathrm{d}V(t) = \frac{\tilde{\beta}(t)\tilde{\Sigma}(t)}{\sigma_u^2}\mathrm{d}Q(t) \tag{B.23}$$

Finally,

$$\mathrm{d}\tilde{\Sigma}(t) = -\frac{[\tilde{\beta}(t)\tilde{\Sigma}(t)]^2}{\sigma_u^2}\mathrm{d}t \tag{B.24}$$

Proof. This is an application of the Kalman-Bucy filter. See Kallianpur (1980, Sec. 10.3). □

Comparing Equation (B.23) with Equation (3.7) we can easily conclude that

$$\tilde{\lambda}(t) = \frac{\tilde{\beta}(t)\tilde{\Sigma}(t)}{\sigma_u^2} \tag{B.25}$$

Given our linear choice space $\lim_{s\to 1} p(s) = v$, meaning that all the private information is incorporated into prices at the end of the trading game, $\Sigma(1) = 0$. Hence, considering the conditional variance process specified in Equation (B.24), it follows that for every $t \in [0,1]$

$$\tilde{\Sigma}(t) = \int_t^1 \tilde{\lambda}(s)^2 \sigma_u^2 \mathrm{d}s \tag{B.26}$$

In order to have a closed-form solution for this equilibrium – in which all the equilibrium parameters are expressed in terms of the exogenous ones, $\{\Sigma(0), \sigma_u^2, A\}$ – let us substitute Equation (B.18) in the expression for $\tilde{\Sigma}(t)$ evaluated at $t = 0$, and solve for (the positive root of) k:

$$\begin{aligned} \Sigma(0) &= \int_0^1 \tilde{\lambda}(s)^2 \sigma_u^2 \mathrm{d}s & \text{(B.27)} \\ &= \int_0^1 (A\sigma_u^2 s + k)^{-2} \sigma_u^2 \mathrm{d}s & \text{(B.28)} \\ &= \frac{\sigma_u^2}{k(k + A\sigma_u^2)} & \text{(B.29)} \\ &\Downarrow & \\ k &= \frac{\sqrt{\Sigma(0)\sigma_u^2(4 + A^2\Sigma(0)\sigma_u^2)} - A\Sigma(0)\sigma_u^2}{2\Sigma(0)} & \text{(B.30)} \end{aligned}$$

Substituting this last expression back into Equation (B.18), we obtain through simple algebra Equation (3.13).[3] Similarly, evaluating the integral in Equation (B.26) and considering that $\tilde{\beta}(t) = \tilde{\lambda}(t)\sigma_u^2/\tilde{\Sigma}(t)$, we get Equation (3.12) and (3.14) respectively. This complete the proof of the theorem. □

Proof of Theorem 2. We follow closely the Proof of Theorem 1. The information structure is then represented by the following filtrations:

$$\begin{aligned} \mathbb{F} = \{\mathcal{F}(t)\}_{t \geqslant 0} &\quad \text{where} \quad \mathcal{F}(t) = \sigma\big(y(s) : 0 \leqslant s \leqslant t\big) \vee \sigma\big(x(s) : 0 \leqslant s < t\big) \\ \mathbb{F}' = \{\mathcal{F}'(t)\}_{t \geqslant 0} &\quad \text{where} \quad \mathcal{F}'(t) = \mathcal{F}(t) \vee \mathrm{d}x(t) \\ \mathbb{G} = \{\mathcal{G}(t)\}_{t \geqslant 0} &\quad \text{where} \quad \mathcal{G}(t) = \sigma\big(v\big) \vee \sigma\big(p^*(s), p(s) : 0 \leqslant s \leqslant t\big) \end{aligned}$$

The objective of the insider trader is to maximize her expected utility over terminal wealth. The

[3] An alternative way to determine the equilibrium $\tilde{\lambda}(t)$ is to express it as a function of $\tilde{\lambda}(1)$ (instead of k), and then solve the following system of non linear equations for the positive roots $\{\tilde{\lambda}(t), \tilde{\lambda}(1)\}$:

$$\begin{cases} \tilde{\lambda}(t) &= \frac{\tilde{\lambda}(1)}{1 - A\sigma_u^2 \tilde{\lambda}(1)(1-t)} \\ \Sigma(0) &= \int_0^1 \left(\frac{\tilde{\lambda}(1)\sigma_u}{1 - A\sigma_u^2 \tilde{\lambda}(1)(1-s)} \right)^2 \mathrm{d}s \end{cases} \tag{B.31}$$

following dynamics allow us to characterize the stochastic control problem faced by the insider:[4]

$$\begin{aligned}
\mathrm{d}W(t) &= (v - p(t))\mathrm{d}x(t) && \text{(B.32)}\\
&= M(t)\mathrm{d}x(t) - \lambda(t)\mathrm{d}\langle x,x\rangle_t - \lambda(t)\mathrm{d}\langle u,x\rangle_t && \text{(B.33)}\\
&= M(t)\mathrm{d}x(t) - \lambda(t)\mathrm{d}\langle x,x\rangle_t && \text{(B.34)}\\
\mathrm{d}M(t) &= -\gamma(t)\mathrm{d}x(t) && \text{(B.35)}\\
\mathrm{d}\langle M,M\rangle_t &= \gamma(t)^2\mathrm{d}\langle x,x\rangle_t && \text{(B.36)}\\
\mathrm{d}V(M,t) &= V_M\mathrm{d}M(t) + \frac{1}{2}V_{MM}\mathrm{d}\langle M,M\rangle_t + V_t\mathrm{d}t && \text{(B.37)}\\
&= [2\alpha(t)M(t)]\mathrm{d}M(t) + [\alpha(t)]\mathrm{d}\langle M,M\rangle_t + M(t)^2\mathrm{d}\alpha(t) + \mathrm{d}\delta(t) && \text{(B.38)}
\end{aligned}$$

Note that, given $\mathrm{d}x$, $[\mathrm{d}W(t) + \mathrm{d}V(M,t)]$ is not random. Therefore,

$$1 = \max_{\mathrm{d}x} \exp\Big\{ - A\Big[\mathrm{d}x\Big[1 - 2\alpha(t)\gamma(t)\Big]M(t) + (\mathrm{d}x)^2\Big[\alpha(t)\gamma(t)^2 - \lambda(t)\Big] + M(t)^2\mathrm{d}\alpha(t) + \mathrm{d}\delta(t)\Big]\Big\} \tag{B.39}$$

The first-order condition (FOC) is

$$\exp\{\cdot\}\Big[\Big(1 - 2\alpha(t)\gamma(t)\Big)M(t) + 2\mathrm{d}x(t)\Big(\alpha(t)\gamma(t)^2 - \lambda(t)\Big)\Big] = 0 \tag{B.40}$$

Because the conjectured trading strategy of insider incorporates a random component, in equilibrium she must be indifferent across all possible values of her trade, that is the FOC must hold for any $\mathrm{d}x(t)$. The following equations characterize the two *dissimulation conditions* for any $t \in [0,1]$:[5]

$$\begin{aligned}
1 - 2\alpha(t)\gamma(t) &= 0 && \text{(B.41)}\\
\alpha(t)\gamma(t)^2 - \lambda(t) &= 0 && \text{(B.42)}
\end{aligned}$$

Evaluating Equation (B.39) with the above optimal conditions we get

$$1 = \exp\Big\{ - A\Big[M(t)^2\mathrm{d}\alpha(t) + \mathrm{d}\delta(t)\Big]\Big\} \tag{B.43}$$

which holds for every level of $M(t)$ and A if and only if $\alpha(t)$ and $\delta(t)$ are constant:

$$\begin{aligned}
\mathrm{d}\alpha(t) &= 0 && \text{(B.44)}\\
\mathrm{d}\delta(t) &= 0 && \text{(B.45)}
\end{aligned}$$

Given the boundary condition $\tilde{\delta}(1) = 0$ (no utility can be gained after trading is complete),

[4]Note that $\mathrm{d}\langle u,x\rangle_t = 0$ since $\mathrm{d}\langle B^u, B^z\rangle_t = 0$ (the two Brownian motion are assumed independent) and $\mathrm{d}B^u\mathrm{d}t$ is of order higher than $\mathrm{d}t$, and that $\mathrm{d}\langle x,x\rangle_t \neq 0$ because of the dissimulation component.

[5]An alternative way to solve for the equilibrium is to explicitly specify the conjectured trading strategy, and maximize over $\beta(t)$ and $\sigma_z(t)$. Although with this formulation $[\mathrm{d}W(t)+\mathrm{d}V(M,t)]$ becomes stochastic because of the noise added to the informative component of the trading strategy, the two approaches in terms of equilibrium conditions are equivalent. We prefer to leave the trading strategy unspecified because it offers a more intuitive (and comparable to the discrete-time) explanation of the *dissimulation conditions*.

$\delta(t) = 0$ for all $t \in [0,1]$. Moreover, since $\alpha(t)$ is constant, by Equation (B.41) it must be the case that also $\gamma(t)$ is constant. The same holds true for $\lambda(t)$ once considered Equation (B.42). Therefore,

$$\hat{\lambda} = \frac{1}{2}\hat{\gamma} \tag{B.46}$$

$$\hat{\alpha} = \frac{1}{2}\hat{\gamma}^{-1} \tag{B.47}$$

where $\{\hat{\gamma}, \hat{\lambda}, \hat{\alpha}\}$ denote the equilibrium values for $\{\gamma(t), \lambda(t), \alpha(t)\}$ respectively for all $t \in [0,1]$.

A simple application of Lemma 4 on both the filtration $\mathbb{F}$ and $\mathbb{F}'$ allows us to characterize the filtering problems faced by the market maker:

$$\gamma(t) = \frac{\beta(t)\Sigma(t)}{\sigma_z^2(t)} \tag{B.48}$$

$$\lambda(t) = \frac{\beta(t)\Sigma(t)}{\sigma_z^2(t) + \sigma_u^2} \tag{B.49}$$

$$\mathrm{d}\Sigma(t) = -\frac{[\beta(t)\Sigma(t)]^2}{\sigma_z^2(t)}\mathrm{d}t \tag{B.50}$$

The optimal dissimulation component follows from $\lambda(t) = (1/2)\gamma(t)$:

$$\frac{\beta(t)\Sigma(t)}{\sigma_z^2(t) + \sigma_u^2} = \frac{1}{2}\frac{\beta(t)\Sigma(t)}{\sigma_z^2(t)} \tag{B.51}$$

$$\Downarrow$$

$$\sigma_z^2(t) = \sigma_u^2 \tag{B.52}$$

Given the constant drift for the conditional variance process,

$$\Sigma(t) = \Sigma(0) - \int_0^t \gamma(s)^2 \sigma_z(s)^2 \mathrm{d}s \tag{B.53}$$

$$= \Sigma(0) - (\hat{\gamma}^2\sigma_u^2)t \tag{B.54}$$

for every $t \in [0,1]$, and given our linear choice space ($\lim_{s\to 1} p^*(s) = \lim_{s\to 1} p(s) = v$), all the private information is incorporated into prices at the end of the trading game: $\Sigma(1) = 0$. Then, it follows that

$$\hat{\gamma} = \left(\Sigma(0)/\sigma_u^2\right)^{\frac{1}{2}} \tag{B.55}$$

All the other constants ($\beta(t)$, $\lambda(t)$, $\alpha(t)$) are obtained through straightforward algebra. The boundary condition $\sigma_z(1) = 0$ states that no dissimulation component is added to the trading strategy at the last trading instant. This complete the proof of the theorem. □

Proof of Proposition 7. Once the positive root of the polynomial in Equation (B.90) is determined, the sequential auction equilibrium is fully characterized for any number of auctions N. It is straightforward to see that for $\Delta t \to 0$ the polynomial becomes of order two and the only positive root λ is equal to $\sqrt{\Sigma(0)/4\sigma_u^2}$. Since $t \in [0,1]$, then $n = tN$ and the following limits (for

$\Delta t \to 0$) hold true:

$$\gamma_n \to \left(\Sigma(0)/\sigma_u^2\right)^{\frac{1}{2}} \tag{B.56}$$

$$\Lambda \to \left(\Sigma(0)/\sigma_u^2\right)^{\frac{1}{2}} \tag{B.57}$$

$$\beta_n \to \sigma_u \Sigma(0)^{-\frac{1}{2}}/(1-t) \tag{B.58}$$

$$\sigma_{z_n}^2 \to \sigma_u^2 \tag{B.59}$$

$$\alpha_n \to \frac{1}{2}\left(\sigma_u^2/\Sigma(0)\right)^{\frac{1}{2}} \tag{B.60}$$

$$\Sigma_n \to (1-t)\Sigma(0) \tag{B.61}$$

$$\delta_n \equiv \ln(\eta_n) \to 0 \tag{B.62}$$

□

Proof of Proposition 8: $\Sigma(t) - \tilde{\Sigma}(t) \geqslant 0$ and $\tilde{\beta}(t) - \beta(t) \geqslant 0$ imply respectively

$$\frac{A[\Sigma(0)]^{\frac{3}{2}}\sigma_u\left[\sqrt{4 + A^2\Sigma(0)\sigma_u^2} + \sqrt{A^2\Sigma(0)\sigma_u^2}(1-2t)\right](1-t)t}{[2 + 2A^2\Sigma(0)\sigma_u^2(1-t)t]} \geqslant 0 \tag{B.63}$$

$$\frac{A\Sigma(0)\sigma_u^2 + \sqrt{\Sigma(0)}\sigma_u\left[\sqrt{4 + A^2\Sigma(0)\sigma_u^2} - 2\right]}{2\Sigma(0)(1-t)} \geqslant 0 \tag{B.64}$$

Straightforward algebra can show that these two inequalities hold true for any $t \in [0,1]$. Using the first result we can derive $\Omega(t)$ simply by dividing the l.h.s of Equation (B.63). The partial derivative of $\Omega(t)$ with respect to the coefficient of risk aversion is equal to

$$\frac{\partial\Omega(t)}{\partial A} = \frac{\sqrt{\Sigma(0)\sigma_u^2}\left[2 + A\sqrt{\Sigma(0)\sigma_u^2}\sqrt{4 + A^2\Sigma(0)\sigma_u^2}(1-t) + A^2\Sigma(0)\sigma_u^2(1 + 2(t-1)t)\right](1-t)t}{\sqrt{4 + A^2\Sigma(0)\sigma_u^2}\left[1 + A^2\Sigma(0)\sigma_u^2(1-t)t\right]^2} \tag{B.65}$$

Straightforward algebra can show that this partial derivative is non-negative for any $t \in [0,1]$. This proves part (i). Part (ii) and (iii) follow since

$$\frac{\partial\Omega(t)}{\partial\Sigma(0)} = \frac{\partial\Omega(t)}{\partial A}\frac{A}{2}\frac{1}{\Sigma(0)} \tag{B.66}$$

$$\frac{\partial\Omega(t)}{\partial\sigma_u^2} = \frac{\partial\Omega(t)}{\partial A}\frac{A}{2}\frac{1}{\sigma_u^2} \tag{B.67}$$

Finally, a simple application of Leibniz's rule proves part (iv). □

Proof of Proposition 9: Since $\tilde{\lambda}(t)$ is decreasing over time,

$$\frac{\partial\tilde{\lambda}(t)}{\partial t} = -\frac{4A\Sigma(0)}{\left(\sqrt{4 + A^2\Sigma(0)\sigma_u^2} + A\sigma_u\sqrt{\Sigma(0)}(2t-1)\right)^2} \tag{B.68}$$

this means that $\tilde{\lambda}(t)$ and $\lambda(t)$ cross at most once in $t \in [0,1]$. Moreover, since $\tilde{\lambda}(1)$ is decreasing in the level of risk aversion,

$$\left.\frac{\partial \tilde{\lambda}(t)}{\partial A}\right|_{t=1} = -\frac{\Sigma(0)}{2}\left(1 - \sqrt{\frac{A^2\Sigma(0)\sigma_u^2}{4 + A^2\Sigma(0)\sigma_u^2}}\right) \tag{B.69}$$

then for any $A > A^*$, with $A^* = 3\left[2\sqrt{\Sigma(0)\sigma_u^2}\right]^{-1}$, market liquidity is higher in an opaque market for the trading sub-period $[t^*, 1]$, with

$$t^* = \frac{A\Sigma(0)\sigma_u + \sqrt{\Sigma(0)}(4 - \sqrt{4 + A^2\Sigma(0)\sigma_u^2})}{2A\Sigma(0)\sigma_u} \tag{B.70}$$

Moreover $\partial t^*/\partial A < 0$: this proves part (i). The aggregate liquidity loss is equal to

$$\Psi^a = \frac{\sqrt{\Sigma(0)\sigma_u^2}}{2} + \frac{1}{A}\ln\left(\frac{\sqrt{4 + A^2\Sigma(0)\sigma_u^2} - A\sqrt{\Sigma(0)\sigma_u^2}}{\sqrt{4 + A^2\Sigma(0)\sigma_u^2} + A\sqrt{\Sigma(0)\sigma_u^2}}\right) \tag{B.71}$$

Straightforward algebra can show that for any $A > A^{**} > A^*$, with $A^{**} = 2\sqrt{3}\left[\sqrt{\Sigma(0)\sigma_u^2}\right]^{-1}$, the partial derivatives $\partial\Psi^a/\partial\Sigma(0)$ and $\partial\Psi^a/\partial\sigma_u^2$ are both positive. This proves part (ii). Finally, since Ψ^a is monotone increasing in A ($\partial\Psi^a/\partial A > 0$ for any level of A) and since $\Psi^a(A^{**}) < 0$ and $\lim_{A\to\infty}\Psi^a = \sqrt{\Sigma(0)\sigma_u^2}/2$, then there must exist a level $A^{***} > A^{**}$ such that $\Psi^a > 0$ for any $A > A^{***}$. This concludes the proof of the proposition. □

Proof of Proposition 10. Given the monotonic transformation, $T(x) = (1+x)/A$, of the utility function, as in Equation (3.39), the insider's conditional utility at time zero (conditional on the private information v) is given by

$$\Pi_0 = \frac{1}{A}\left[1 + J(M, 0)\right] = \frac{1}{A}\left[1 - \exp\{-A[W(0) + \alpha(0)M(0)^2 + \delta(0)]\}\right] \tag{B.72}$$

Substituting the equilibrium values of $\alpha(0)$ and $\delta(0)$ (Equations (3.30) and (3.31) respectively) into Equation (B.72), we obtain Equation (3.46). Similarly, if we substitute $\tilde{\alpha}(0)$ and $\tilde{\delta}(0)$ (Equations (3.15) and (3.16) respectively) into Equation (B.72), we obtain the expression in Equation (3.43). Taking expectation with respect to the random variable $M(0) \sim \mathcal{N}(0, \Sigma(0))$ of Equations (3.43) and (3.46), we obtain Equations (3.42) and (3.45) respectively:[6]

$$\tilde{\pi}_0 = \frac{1}{A}\left[1 - \tilde{\Upsilon}\exp\{-AW(0)\}\right] \tag{B.73}$$

$$\pi_0 = \frac{1}{A}\left[1 - \Upsilon\exp\{-AW(0)\}\right] \tag{B.74}$$

where

$$\tilde{\Upsilon} = \left(1 + A\left(A\Sigma(0)\sigma_u^2 + \sqrt{\Sigma(0)\sigma_u^2(4 + A^2\Sigma(0)\sigma_u^2)}\right)\right)^{-1/2} \tag{B.75}$$

$$\Upsilon = \left(1 + A\sqrt{\Sigma(0)\sigma_u^2}\right)^{-1/2} \tag{B.76}$$

[6]If $w \sim \mathcal{N}(0, \Sigma)$, then

$$\mathbb{E}[\exp(w'Aw + b'w + c)] = |I - 2\Sigma A|^{\frac{1}{2}}\exp\left[\frac{1}{2}b'(I - 2\Sigma A)^{-1}\Sigma b + c\right]$$

It is easy to see that for any finite and positive A, $\tilde{\Upsilon} < \Upsilon$, and hence that $\tilde{\pi}_0 > \pi_0$. Moreover, we can conclude that $\partial\left(\tilde{\pi}_0 - \pi_0\right)/\partial A \leqslant 0$ by the following factors: (i) $\nexists$ a finite and positive value of A such that $\partial\left(\tilde{\pi}_0 - \pi_0\right)/\partial A = 0$; and (ii) $\lim_{A\to 0}\partial\left(\tilde{\pi}_0 - \pi_0\right)/\partial A = -(5\Sigma(0)\sigma_u^2 + 4\sqrt{\Sigma(0)\sigma_u^2}W_0)/8$. Straightforward but computationally intense algebra confirms the result.

Solving the equation $(\tilde{\Pi}_0 - \Pi_0) = 0$ for $M(0)$, we find:

$$M^*(0) = \pm\frac{\sqrt{\left(A\sqrt{\Sigma(0)(\sigma_u^2)^3} + \sigma_u^2\left(2 + \sqrt{4 + A^2\Sigma(0)\sigma_u^2}\right)\right)\log\left(\frac{\sqrt{2 + A^2\Sigma(0)\sigma_u^2 + A\sqrt{\Sigma(0)\sigma_u^2(4 + A^2\Sigma(0)\sigma_u^2)}}}{\sqrt{2}}\right)}}{A\sigma_u^2} \tag{B.77}$$

hence $\Gamma^* = |M^*(0)|$. Since

$$\left.\frac{\partial(\tilde{\Pi}_0 - \Pi_0)}{\partial M(0)}\right|_{M(0)=\Gamma^*} < 0$$

then we can conclude that for any $\Gamma > \Gamma^*$, $\Pi_0 > \tilde{\Pi}_0$. This and the sign of the partial derivatives of Γ^* with respect to $(\Sigma(0), \sigma_u^2, A)$ are again obtained through straightforward but computationally intense algebra, that we prefer to omit. □

B.2 Transparent Market Equilibrium in Discrete-time

Let $v \sim \mathcal{N}(p_0, \Sigma_0)$. The informed trader has perfect information in the sense she observes at t_0 the liquidation value v. Let Δx_n be the order placed by the monopolistic insider trader at the n-th auction. The aggregate order at the n-th auction by liquidity traders, who trade for liquidity reasons, is denoted by Δu_n. We assume $\Delta u_n \sim \mathcal{N}(0, \sigma_u^2\Delta t_n)\ \ \forall n$, serially uncorrelated and independent of v. Furthermore, let Δy_n be the total order flow observed by the market maker in period n: $\Delta y_n = \Delta x_n + \Delta u_n$. As in Holden and Subrahmanyam (1994) we denote with W_0 the initial wealth of the informed trader and with W_n the wealth at auction n. Moreover, the informed trader has negative exponential utility, with risk aversion coefficient A, for terminal wealth (denoted by W_{N+1}):

$$U(W_{N+1}) = -\exp\{-AW_{N+1}\} \tag{B.78}$$

The information of the market maker is represented by a *prior* (pre-disclosure) and an *updating* (post-disclosure) filtrations, denoted respectively by $\mathbb{F}$ and $\mathbb{F}'$, whereas the information of the informed agent is represented by a single filtration $\mathbb{G}$:

$$\begin{aligned}
\mathbb{F} &= \{\mathcal{F}_n\}_{n\geqslant 0} \quad \text{where} \quad \mathcal{F}_n = \sigma\big(y_m : 0 \leqslant m \leqslant n\big) \vee \sigma\big(x_m : 0 \leqslant m < n\big)\\
\mathbb{F}' &= \{\mathcal{F}'_n\}_{n\geqslant 0} \quad \text{where} \quad \mathcal{F}'_n = \sigma\big(y_m, x_m : 0 \leqslant m \leqslant n\big)\\
\mathbb{G} &= \{\mathcal{G}_n\}_{n\geqslant 0} \quad \text{where} \quad \mathcal{G}_n = \sigma\big(v\big) \vee \sigma\big(p_m, p_m^* : 0 \leqslant m < n\big)
\end{aligned}$$

The monopolistic insider behaves strategically by choosing a $\mathcal{G}$-mesurable function as optimal trading strategy that maximizes her expected utility conditional on all public and private information. Let $J(W_n)$ denote the indirect utility from W_n.

Let $\Delta x_n = X_n(\mathcal{G}_n)$ represent the optimal strategy of the insider trader at the n-th auction, and $p_n = P_n(\mathcal{F}_n)$, $p_n^* = P_n^*(\mathcal{F}'_n)$ the optimal strategies of the market maker before and after disclosure at that auction respectively. Then, let us define $X = \langle X_1, ..., X_N \rangle$, $P = \langle P_1, ..., P_N \rangle$ and $P^* = \langle P_1^*, ..., P_N^* \rangle$ as the vectors of strategy functions.

Definition 4. *An equilibrium of the discrete-time trading game is defined as a triple* (X, P, P^*) *such that the following conditions hold:*

1. Utility Maximization. *For any* $n \in \{1, ...N\}$ *and for any alternative trading strategy* $X' = \langle X_1', ..., X_N' \rangle$*:*

$$\mathbb{E}\Big[J\Big(W_{n+1}(X, P, P^*)\Big)\Big|\mathcal{G}_n\Big] \geqslant \mathbb{E}\Big[J\Big(W_{n+1}(X', P, P^*)\Big)\Big|\mathcal{G}_n\Big] \tag{B.79}$$

2. Market Efficiency. *For any* $n \in \{1, ...N\}$ *the two prices are competitive:*

$$p_n = \mathbb{E}[v|\mathcal{F}_n] \tag{B.80}$$
$$p_n^* = \mathbb{E}[v|\mathcal{F}_n'] \tag{B.81}$$

Given the definition of the equilibrium we restrict our attention on the class of linear equilibria. In particular, we search for an equilibrium in which both the insider trader and the market maker's strategies are linear functions. Moreover, without loss of generality we consider auctions occurring at equally spaced intervals $\Delta t_n = \Delta t = 1/N$ for all n.

Theorem 3. *There exists a recursive linear equilibrium in which the constants* β_n, λ_n, γ_n, α_n, η_n, $\sigma_{z_n}^2$, *and* Σ_n *characterize the following:*

$$\Delta x_n = \beta_n M_{n-1} \Delta t + \Delta z_n \tag{B.82}$$
$$M_{n-1} \equiv v - p_{n-1}^* \tag{B.83}$$
$$\Delta z_n \sim \mathcal{N}(0, \sigma_{z_n}^2 \Delta t) \tag{B.84}$$
$$p_n = p_{n-1}^* + \lambda_n \Delta y_n \tag{B.85}$$
$$p_n^* = p_{n-1}^* + \gamma_n \Delta x_n \tag{B.86}$$
$$\Sigma_n = \mathbb{V}\mathrm{ar}[v|\mathcal{F}_n'] \tag{B.87}$$
$$J(W_n) = -\eta_{n-1} \exp\Big\{-A\left(W_n + \alpha_{n-1}(v - p_{n-1}^*)^2\right)\Big\} \tag{B.88}$$
$$W_n = W_{n-1} + \Delta x_{n-1}(v - p_{n-1}) \tag{B.89}$$

for all auctions $n \in \{1, ..., N\}$*. Given the initial condition* $p_0^* = p_0$*, and denoting with* λ *the positive root of the following polynomial, that satisfy the SOC of the insider maximization problem at the last trading round,*

$$\lambda^4\left[A^2\sigma_u^2(\sigma_u^2\Delta t)^2\right] + \lambda^3\Big[4A\sigma_u^2(\sigma_u^2\Delta t)\Big] + \lambda^2\Big[4\sigma_u^2\Big] - \lambda\Big[A\sigma_u^2\Delta t\Sigma_0\Big] - \Sigma_0 = 0 \tag{B.90}$$

the constants λ_n, γ_n, α_n, η_n, β_n, $\sigma^2_{z_n}$, Σ_n are given by

$$\lambda_n = \lambda \tag{B.91}$$

$$\gamma_n = \gamma = 2\lambda + A\lambda^2\sigma_u^2\Delta t \tag{B.92}$$

$$\eta_n = 1 \tag{B.93}$$

$$\beta_n = \left(\frac{\Lambda\sigma_u^2}{\Sigma_0}\right)\frac{N}{N-n+1} \tag{B.94}$$

$$\sigma^2_{z_n} = \sigma_u^2\left(\frac{\Lambda}{\gamma}\right)\frac{N-n}{N-n+1} \tag{B.95}$$

$$\Sigma_n = \Sigma_0\frac{N-n}{N} \tag{B.96}$$

for all auctions $n \in \{1, ..., N\}$, and by

$$\alpha_n = (2\gamma)^{-1} \tag{B.97}$$

$$\alpha_N = 0 \tag{B.98}$$

for all auctions $n \in \{1, ..., N-1\}$, where the constant Λ is defined as follows:

$$\Lambda = \gamma(1 + A\lambda\sigma_u^2\Delta t)^{-1} \tag{B.99}$$

Proof. We proceed by backward induction. Suppose that for constants α_n and η_n

$$J(W_{n+1}) = -\eta_n \exp\bigl[-A\left(W_{n+1} + \alpha_n M_n^2\right)\bigr] \tag{B.100}$$

We have then

$$\begin{aligned} J(W_n) &= \max_{\Delta x}\ \mathbb{E}[J(W_{n+1})|\mathcal{G}_n] \\ &= \max_{\Delta x}\ \mathbb{E}_n\bigl[-\eta_n \exp\bigl[-A\left(W_n + \Delta x(v - p_n) + \alpha_n M_n^2\right)\bigr]\bigr] \end{aligned} \tag{B.101}$$

$$\text{s.t.} \begin{cases} p_n = p^*_{n-1} + \lambda_n(\Delta x + \Delta u_n) \\ p^*_n = p^*_{n-1} + \gamma_n \Delta x \end{cases} \tag{B.102}$$

where Δx denotes the control quantity of the insider trader at the n-th auction, and the two constraints highlight the linear functional form of the conjectured pricing functions. Substituting Equations (B.102) in (B.101), which clearly shows the strategic behavior of the insider, and evaluating the conditional expectation for log-normal distributions, we obtain

$$J(W_n) = \max_{\Delta x} -\eta_n \exp\biggl\{-A\biggl[W_n + \Delta x[1-2\alpha_n\gamma_n]M_{n-1} + (\Delta x)^2\biggl[\alpha_n\gamma_n^2 - \lambda_n - \frac{1}{2}A\lambda_n^2\sigma_u^2\Delta t\biggr] + \alpha_n M_{n-1}\biggr]\biggr\} \tag{B.103}$$

The first-order condition (FOC), where Δx_n denote the optimized value of Δx for the above expression, turns out to be

$$M_{n-1}[1 - 2\alpha_n\gamma_n] + \Delta x_n[2\alpha_n\gamma_n^2 - 2\lambda_n - A\lambda_n^2\sigma_u^2\Delta t] = 0 \tag{B.104}$$

Since the conjectured trading strategy of the informed agent incorporates a random component (the dissimulation part of the trade), it means that in equilibrium the insider must be indifferent across all possible values of her trade, that is the FOC must hold for any Δx_n. This implies that the following conditions (hereafter called *dissimulation conditions*) must hold for any $n \in$

$\{1, ..., N-1\}$:

$$\begin{aligned} 1 - 2\alpha_n \gamma_n &= 0 \qquad \text{(B.105)} \\ \alpha_n \gamma_n^2 - \lambda_n - \frac{1}{2} A \lambda_n^2 \sigma_u^2 \Delta t_n &= 0 \qquad \text{(B.106)} \end{aligned}$$

Note that, if these conditions are satisfied, both FOC and SOC hold.

In equilibrium the conjectured indirect utility must be correct. This means that α_{n-1} and η_{n-1} must be such that Equation (B.100) at time t_n is equal to Equation (B.101):

$$-\eta_{n-1} \exp\left[-A\left(W_n + \alpha_{n-1} M_{n-1}^2\right)\right] = \max_{\Delta x} \mathbb{E}[J(W_{n+1})|\mathcal{G}_n] \qquad \text{(B.107)}$$

When computing the expectation in the r.h.s. of the above expression we must take into account that dissimulation of insider trading implies randomness in the insider trading strategy. In particular, $J(W_{n+1})$ contains linear and quadratic functions of two independent (normally distributed) random components: Δu_n and Δz_n. For this purpose let us consider the following lemma.

Lemma 5. *Let X and Z be two independent normally distributed random variables: $X \sim \mathcal{N}(0, \sigma_X^2)$, $Z \sim \mathcal{N}(0, \sigma_Z^2)$. Then, if $\sigma_X^2(2a + c^2\sigma_Z^2) < 1$:*

$$\mathbb{E}\left[\exp\left(aX^2 + bX + dZ + cXZ\right)\right] = \exp\left(\frac{\Xi}{2\Phi}\right)\frac{1}{\sqrt{\Phi}} \qquad \text{(B.108)}$$

where

$$\begin{aligned} \Xi &\equiv d^2\sigma_Z^2 + \sigma_X^2(b^2 + 2d\sigma_Z^2(bc - ad)) \qquad \text{(B.109)} \\ \Phi &\equiv 1 - \sigma_X^2(2a + c^2\sigma_Z^2) \qquad \text{(B.110)} \end{aligned}$$

If $\sigma_X^2(2a + c^2\sigma_Z^2) > 1$, the above expectation is not well-defined.

Proof. See Brunnermeier (2001, pag. 64). □

Simple algebra leads to the following expression for the indirect utility at t_n:

$$J(W_n) = -\eta_n \left[\exp(-AQ_n)\mathbb{E}_n\left[\exp\left(a_n(\Delta z_n)^2 + b_n\Delta z_n + d_n\Delta u_n + c_n\Delta z_n\Delta u_n\right)\right]\right] \qquad \text{(B.111)}$$

where

$$\begin{cases} Q_n \equiv W_n + [\beta_n(1 - 2\alpha_n\gamma_n) + \beta_n^2(\alpha_n\gamma_n^2 - \lambda_n) + \alpha_n]M_{n-1}^2 \\ a_n \equiv -A(\alpha_n\gamma_n^2 - \lambda_n) \\ b_n \equiv -A[(1 - 2\alpha_n\gamma_n) + 2\beta_n(\alpha_n\gamma_n^2 - \lambda_n)]M_{n-1} \\ d_n \equiv A\lambda_n\beta_n M_{n-1} \\ c_n \equiv A\lambda_n \end{cases} \qquad \text{(B.112)}$$

Using Lemma 5 and the two *dissimulation conditions* in Equations (B.105) and (B.106), we find that α and η are constant over time until the second last auction. Therefore, since

$$\alpha_n = [2\lambda_n(2 + A\lambda_n\sigma_u^2\Delta t)]^{-1} \quad \text{and} \quad \alpha_{n-1} = \alpha_n \qquad \forall\, n \in \{1, ..., N-1\} \qquad \text{(B.113)}$$

it follows that

$$\gamma_{n-1} = \gamma_n \qquad \forall\, n \in \{1, ..., N-1\} \quad \text{(B.114)}$$
$$2(\lambda_n - \lambda_{n-1}) + A\sigma_u^2 \Delta t(\lambda_n^2 - \lambda_{n-1}^2) = 0 \quad \text{(B.115)}$$

which implies

$$\lambda_{n-1} = \lambda_n \qquad \forall\, n \in \{1, ..., N-1\} \quad \text{(B.116)}$$

Solving the maximization problem that the insider faces at the last trading round, in which it is clearly optimal not to add any noise component to the order flow, we get as in Holden and Subrahmanyam (1994) the following equations:

$$\beta_N \Delta t = [\lambda_N(2 + A\lambda_N \sigma_u^2 \Delta t)]^{-1} \quad \text{(B.117)}$$
$$\alpha_{N-1} = [2\lambda_N(2 + A\lambda_N \sigma_u^2 \Delta t)]^{-1} \quad \text{(B.118)}$$
$$\eta_{N-1} = 1 \quad \text{(B.119)}$$

Finally, Equation (B.91) follows from Equation (B.113) evaluated at second last auction and from Equation (B.118). As in Huddart et al. (2001) market depth is constant over all trading rounds. Moreover, since no utility can be gained after trading is complete, $\alpha_N = 0$ and $\eta_N = 1$.

Market efficiency conditions requires that $p_n = \mathbb{E}[v|\mathcal{F}_n]$ and $p_n^* = \mathbb{E}[v|\mathcal{F}_n']$. Simple application of the projection theorem for normally distributed random variables confirms the linear pricing rules specified in Equations (B.85) and (B.86), where the slope coefficients are respectively given

$$\lambda_n = \frac{\beta_n \Sigma_{n-1}}{\beta_n^2 \Delta t \Sigma_{n-1} + \sigma_{z_n}^2 + \sigma_u^2} \quad \text{(B.120)}$$
$$\gamma_n = \frac{\beta_n \Sigma_{n-1}}{\beta_n^2 \Delta t \Sigma_{n-1} + \sigma_{z_n}^2} \quad \text{(B.121)}$$

By the same theorem and using Equation (B.121) we find the expression for the conditional variance:

$$\Sigma_n = \Sigma_{n-1}(1 - \gamma_n \beta_n \Delta t) \quad \text{(B.122)}$$

Furthermore, simple algebra allows us to conclude that also γ_n is constant for any auction $n \in \{1, ..., N\}$.

Now, combining Equations (B.120) and (B.121) we can derive a convenient expression for β_n:

$$\beta_n = \Lambda \sigma_u^2 (\Sigma_{n-1})^{-1} \quad \text{(B.123)}$$

where Λ is a constant defined in Equation (B.99). Substituting β_n in Equation (B.122) we clearly see that the conditional variance decreases at a constant rate:

$$\Sigma_n = \Sigma_{n-1} - (\gamma \Lambda \sigma_u^2 \Delta t) \quad \text{(B.124)}$$
$$= \Sigma_0 - (\gamma \Lambda \sigma_u^2 \Delta t)n \qquad \forall\, n \in \{1, ..., N\} \quad \text{(B.125)}$$

Since all the private information is incorporated at the end the trading game, $\Sigma_N = 0$. Therefore,

$$0 = \Sigma_0 - (\gamma \Lambda \sigma_u^2 \Delta t) N \tag{B.126}$$

$$\Sigma_n = \Sigma_0 (N - n)/N \qquad \forall\, n \in \{1, ..., N\} \tag{B.127}$$

Equations (B.94) and (B.95) are obtained through straightforward algebra.

Finally, in order to obtain the optimal constant λ we need to solve the polynomial of degree four in Equation (B.90), obtained by combining Equations (B.117), (B.118) and (B.127). Note that any positive root satisfies the SOC of the insider maximization problem at the last trading round: $-2\lambda - A\lambda^2\sigma_u^2\Delta t \leqslant 0$. This complete the proof of the theorem. Note that if we set $A = 0$ we obtain the same equilibrium as in Huddart et al. (2001), in which case the positive root of the polynomial in Equation (B.90) is equal to $\sqrt{\Sigma_0/4\sigma_u^2}$. $\square$

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