The Theory and Empirics of Cross Country Trade in Bonds and Equities

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Declaration

I certify that the work presented in this thesis is my own.

Ebrahim Rahbari

To my parents,

Fatemeh and Dr Yazdan Rahbari,

to whom I owe everything.

Abstract

This thesis extends the theory and empirics of cross country asset positions, paying particular attention to the fact that a large share of domestic equity is held by domestic investors ("equity home bias"). The thesis is built around three key insights. Firstly, we show that in a broad class of models equilibrium asset positions can be related to a small number of sources of risk reflecting fluctuations in consumption expenditure, nonfinancial income and government spending. Secondly, we show that it is important to account for the presence of all assets traded, as individual asset positions depend on the comovement of the portion of the returns of that asset and certain sources of risk that are orthogonal to the returns of the other assets traded. Thirdly, we show that the nature of the asset menu traded matters greatly, e.g. the maturity of bonds traded.

We focus on both theoretical and empirical results. We present a two country dynamic general equilibrium model with nominal rigidities, endogenous capital accumulation and trade in nominal bonds and equity that produces realistic equity home bias, while replicating many salient features of the international business cycle. Later, we introduce trade in long term nominal bonds into a simple two country model and show that the model generally predicts that domestic investors have a net short position in domestic currency, as found in data for the U.S..

On the empirical side, we use theory based sign restrictions to identify the impulse responses of asset returns and sources of risk to different shocks and find that the responses of all variables are similar for all shocks considered and similar to the nominal exchange rate, suggesting an important role for nominal exchange rate fluctuations. We use quarterly data to test the relative importance of three hedging based explanations of equity home bias and find a potentially important role for the motive to hedge human capital returns in the G7 countries. We also find that allowing a central bank to hedge sudden capital account reversals ("sudden stops") increases the optimal dollar in the central bank portfolio for 24 emerging market economies on average, but also find regional differences.

Acknowledgements

Many people have supported me in writing this doctoral thesis and I am indebted to all of them. Undoubtedly, I will miss some who have assisted me and to those I apologise and ask not to take my omission for ingratitude.

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In addition to my advisors, a number of other people proved to be a great source of help and advice during my studies. In particular, I would like to mention Roland Beck, Francisco Gomes, Emeric Henry, Martin Kliem, Albert Marcet, Anna Pavlova, Richard Portes, Paolo Surico, and my examiners, Gianluca Benigno and Philippe Martin, for their comments and suggestions, advice and encouragement. James Forder and David Vines introduced me to economics during my undergraduate studies in Oxford and many of the lessons I learnt from them have stayed with me and will continue to stay with me. The PhD Office at London Business School, in particular Judith Fry, Sarah Druce and Jenny Kovalaine Kwan, provided great support and resources over the years. I would also like to thank the Economic and Social Research Council (ESRC) and the Richard and Margaret Merrell Foundation for financial support and the European Central Bank and Universitat Pompeu Fabra for their hospitality during part of the period during which this thesis was completed.

Contrary to popular belief, a PhD is not all fun and games and it could well be unbearable without the company and support of great friends and colleagues. Thus, I would like to thank my "partners in crime" Sirio Aramonte, Fabian Eser, Francesca Monti, Hande Kucuk Tuger, Stefano Sacchetto, Kai Truempler and Bart Vanneste for insightful discussions, collegiality and some great times. I would also like to thank Alan Choi, Cecile Knai, Nico Neumann, Ceciel Perdikouli, Jonas Renz, Christian Reusch, Christoph Schmiedel and Clara Zverina for being great friends and for being there when I needed them. Special thanks are due to Georg Kaltenbrunner whose help and advice was essential in the early stages of my doctoral studies and who continues to be an outstanding friend.

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Chapter 1

Introduction

This thesis extends the theory and empirics of cross country asset positions. Cross border asset trade has grown substantially over the last few decades (see e.g. Lane and Milesi Feretti (2001, 2007)). The large size of these positions implies that relatively modest changes to valuations and/or capital flows can potentially have substantial macroeconomic effects. The complexity of cross border financial trade has also increased, now encompassing a dizzying variety of securities. The size and complexity of cross border asset trade and its potential welfare consequences imply that the study of the determinants and effects of cross border financial transactions is of high merit.

One particular issue that has sparked considerable work is the tendency of domestic investors to hold large shares of the domestic equity stock. This tendency has survived, despite the growing size and complexity of international financial markets (see French and Poterba (1991) and Table 1.1). The tendency is often referred to as "equity home bias", as basic models of international risk sharing would imply full diversification across countries (Lucas (1982)). Many explanations of this bias have been proposed (see Lewis (1999), Karolyi and Stulz (2003) and Sercu and Vanpée (2007) for surveys of the literature). These explanations can be divided into two broad categories. The first one includes explanations that ascribe equity home bias to frictions in cross border financial trade (e.g. financial transaction costs, informational asymmetries). The second category includes explanations that suggest that large equity holdings result from the fact that domestic equity could hedge against particular sources of risk. Among the sources of risk are labour income risk (Baxter and Jermann (1997)), real exchange rate risk (Obstfeld and Rogoff (2000)) and government spending risk (Berriel and Bhattarai (2009)).

Table 1.1: **Equity Home Bias in G7 countries**

Country	**Equity Market Cap as % of World Market Cap** (1)	**Share of domestic stocks in total equity portfolio (in %)** (2)	**Home Bias** (3)
Canada	2.4	84.0	81.6
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US	47.8	88.7	40.9

Home Bias in column (3) is computed as the difference between columns (2) and (1). Source: Coeurdacier et al (2007) based on CPIS data from 2001.

In this thesis, we present novel theoretical and empirical results on cross country asset trade. Our work is built around three key insights. Firstly, we note that financial markets are complex and we show that it is important to account for the presence of all assets traded. This is because individual asset positions in an equilibrium framework should depend on the comovement of the portion of the returns of that asset and certain sources of risk that are orthogonal to the returns of the other assets traded, and therefore the whole universe of assets would need to be taken into account when trying to account for the holdings of individual assets. Secondly, we show that in a broad class of equilibrium models the relevant sources of risk can be generically be related to fluctuations in consumption expenditure, nonfinancial income and government spending. This is an important result, as it builds the basis for parsimonious empirical work, as the relationship between asset positions and sources of risk should hold in many settings. Thirdly, we draw attention to the nature of the asset menu, e.g. by exploring the role of bond maturity in international portfolio choice.

We present a two country dynamic general equilibrium model with nominal rigidities, endogenous capital accumulation and trade in nominal bonds and equity that produces realistic equity home bias, while replicating many salient features of the international business cycle. This is noteworthy, as many models of international portfolio choice that can account for the pattern of cross country asset trade can only do so at the expense of generating counterfactual implications for macroeconomic time series, e.g. the cross country correlation in investment.

To our knowledge, we are the first to introduce trade in long term nominal bonds into a general equilibrium model of international portfolio choice. Long term nominal bonds are in fact the most common type of bonds traded internationally. What is more, we show that allowing for bonds of long maturity implies that many models predict that domestic investors have a net short position in domestic currency, as found in the data for the U.S., but contrary to the predictions of models with short term nominal bonds or with real bonds. Throughout this work, me make ample use of the methodological results and tools described in Devereux and Sutherland (2009).

We also present some empirical evidence that underscores the importance of nominal exchange rate fluctuations in cross border asset trade. We provide new evidence regarding the responses of different asset returns and sources of risk to different types of shocks using sign restrictions based on standard theory and find that the responses of all variables are similar for all shocks considered and similar to the nominal exchange rate. We also use quarterly data to test the relative importance of three hedging based explanations of equity home bias. We find a potentially important role for the motive to hedge human capital returns in the G7 countries, while the motives to hedge consumption expenditures and government spending usually go the other way. We also find that allowing a central bank to hedge sudden capital account reversals ("sudden stops") increases the optimal dollar in the central bank portfolio for 24 emerging market economies on average, but also find regional differences.

Chapter 2 — Portfolios and Business Cycles in an Open Economy DSGE Model

In this chapter, we present a two country, two good model with nominal rigidities, endogenous capital accumulation, endogenous monetary policy, trade in equities and nominal bonds, and shocks to neutral, and investment specific technology and the nominal interest rate. The key innovation in our work is the combination of a sophisticated endogenous portfolio decision with a realistic description of the real side of the economy.

We show that this model i) generates substantial international positions in nominal bonds and equity, ii) produces equity home bias of a realistic size, iii) matches the pattern of correlation between asset returns and sources of risk, and iv) matches many important features of the international business cycle. Agents use bonds and equity holdings to hedge fluctuations in labour income and consumption expenditures. Since bond positions hedge some of this risk, equity positions depend on the correlation of equity returns with the part of human capital and consumption expenditure risk that is orthogonal to bond returns.

Relative price movements drive a high degree of comovement of most variables and equity returns are unconditionally positively correlated with human capital returns, but the correlation with the part of human capital returns that is orthogonal to bond returns is negative, and this implies equity home bias. Consumption expenditure is mainly hedged by taking bond positions, and domestic investors in equilibrium take net positive positions in domestic bonds as a result. Monetary policy which is modelled as a Taylor type rule affects equilibrium portfolios even when prices are flexible, as markets are incomplete.

We confront the predictions of the model to data in two ways. Firstly, we calibrate the model and find that that it matches the stylised features of the international business cycle. We also compare the correlations of asset returns and sources of risk from the model with those in the data and find that the model is able to capture the qualitative and, to some extent, even the qualitative pattern of the correlations, both unconditionally and for those which are orthogonal to other asset returns.

Chapter 3 — International Portfolios and Hedging Demands — A Sign Restrictions Exploration

In this chapter, we use sign restrictions based on standard theory to provide evidence on the responses of dividends, nominal exchange rates, consumption expenditure and labour income to neutral and investment specific technology shocks and monetary shocks. The previous literature (including chapters 2 and 4 of this thesis) have shown that equilibrium asset positions should depend on the comovement of these variables, yet there is little evidence on the response of these variables to structural shocks.

We build on the insights of Faust (1998), Uhlig (2005) and the literature that followed them and choose an identification scheme that is based on sign restrictions on the impulse responses that are implied by a wide variety of macroeconomic models. We find that the response of dividends, nominal exchange rates, consumption expenditures and labour income to all of the shocks considered is quite similar. Thus, relative consumption expenditure, labour income and dividends rise in response to a relative productivity shock, while they fall in response to a contractionary monetary shock. The similarity of the impulse responses of labour income, dividends and consumption expenditures to the response of nominal exchange rates is particularly interesting, as it suggests that the dynamics of nominal exchange rates dominate the dynamic behaviour of the other cross country variables.

The response of all variables to investment specific technology shocks is often insignificant under our benchmark identification. However, for an extended identification scheme with more restrictions, we find that relative dividends fall in response to a positive investment specific technology shock, confirming the predictions of the models presented in Coeurdacier et al (2009) and the model presented in chapter 2. The response of relative consumption expenditures to relative productivity shocks is also noteworthy. We find that relative consumption expenditures fall in response to a positive relative shock to neutral productivity. This runs contrary to the predictions of most open economy macroeconomic models.

Chapter 4 — A Test of Hedging Based Explanations of Equity Home Bias

In chapter 4, we attempt to empirically investigate the relative contributions of different hedging motives to the phenomenon of equity home bias. Above, we noted that several hedging based explanations of equity home bias exist. Among these are that domestic equity holdings could hedge against fluctuations in the real exchange rate (Obstfeld and Rogoff (2000)), labour income (Baxter and Jermann (1997) and Heathcote and Perri (2009)) and government spending (Berriel and Bhattarai (2009)). We derive an expression for equilibrium equity holdings that makes the relationship to these three hedging sources explicit and that relies on a very limited set of assumptions and therefore holds in a very broad class of models, irrespective of the presence and nature of nominal rigidities, endogenous capital accumulation, or incomplete markets. Importantly, the formulation derived also accounts for other assets traded and highlights that in order to compute the moments that are relevant for equilibrium equity positions, only fluctuations that are orthogonal to the returns of the other assets traded need to be considered.

We use the methods presented in Campbell and Shiller (1988) and Campbell (1996) and quarterly data on G7 countries to estimate the relative contribution of the individual hedging motives. We find that the correlation of excess returns on domestic equity with human capital risk is negative in all G7 countries, conditional on the excess returns of domestic bonds, suggesting that the motive to hedge labour income can contribute to the equity home bias observed. On the contrary, the motives to hedge consumption expenditure and government spending risk appear to work in the opposite direction, as excess returns on domestic equity, conditional on bond returns, appear to be high when relative consumption expenditure and government spending are low.

Chapter 5 — Bond Maturity and Negative Net Debt Positions

In this chapter, we introduce long term nominal bonds into a simple open economy general equilibrium model. Long term nominal bonds are the most common class of bonds traded internationally, but previous work has only allowed for short term nominal bonds (Devereux and Sutherland (2007, 2008)) or long term real bonds (Coeurdacier et al (2009)).

We find that introducing long term nominal bonds has potentially important implications. The reason is that equilibrium bond positions depend on cross country bond return differentials which in turn depend on nominal exchange rate fluctuations. Relative short term bond returns only depend on exchange rate fluctuations in the immediate future, while long term bond differentials depend on changes in the expectation of nominal exchange rates far in the future.

We show that in many models, the short and the long run dynamics of the nominal exchange rate can be quite different, giving rise to different bond return differentials for short and long term bonds. Thus, in response to a relative productivity shock, nominal exchange rates initially depreciate giving rise to a negative bond return differential on short term bonds. Relative inflation falls, and since monetary policy stabilises inflation, but not the price level, so does the price level over time. For the real exchange rate to return to its steady state value, the nominal exchange rate therefore appreciates in the long run. For a wide range of parameter values, the long run appreciation dominates the short run depreciation of the nominal exchange rate, giving rise to a negative correlation between short and long run bond returns. As a result, these models predict an equilibrium net negative position in domestic bonds for domestic investors, as found in data for the U.S.. On the contrary, previous work with short term nominal bonds and long term real bonds implied positive net positions in domestic bonds in most settings.

Chapter 6 — Optimal Reserve Composition in the Presence of Sudden Stops: The Euro and the Dollar as Safe Haven Currencies

In this chapter, we introduce a transactions motive into the portfolio problem of a central bank in a country that is subject to sudden capital account reversals ("sudden stops"). We analytically derive optimal central bank portfolios in a minimum variance framework with two assets and "transaction demands" caused by sudden stops in capital inflows. In addition to standard portfolio terms, the optimal portfolio now reflects the extent to which the individual assets can hedge sudden stops. The transaction demands become less important relative to traditional portfolio objectives as debt to reserve ratios decrease.

We empirically estimate optimal dollar and euro shares for 24 emerging market countries and find that optimal reserve portfolios are dominated by anchor currencies and, at current debt to reserve ratios, introducing transactions demand has a relatively modest effect. We also find that euro and dollar bonds act as "safe haven currencies" during sudden stops. Dollars are better hedges for global sudden stops and for regional sudden stops in Asia and Latin America, while the euro is a better hedge for sudden stops in Emerging Europe. We reproduce qualitatively the recent decline in the share of the dollar in emerging market reserves and find that the denomination of foreign currency debt has very little importance for optimal reserve portfolios.

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Chapter 2

Portfolios and Business Cycles in an Open Economy DSGE Model

2.1 Introduction

This paper is motivated by four facts. Firstly, international trade in assets is large and it involves both debt and equity securities (see e.g. Lane and Milesi Feretti (2001, 2007)). In 2008, total U.S. holdings of foreign securities amounted to $4,291$ billion U.S. dollars, while foreign holdings of U.S. securities amounted to $10,322$ billion of dollars, or 31 and 72% of 2008 U.S. GDP, respectively (U.S. Treasury (2009a,b)). The large size of these positions implies that they can have significant effects on macroeconomic outcomes. For example, previous research has shown that fluctuations in asset prices and exchange rates can generate substantial wealth redistributions between countries.[1]

In addition to size, the composition of asset trade also matters. Gross U.S. portfolio equity positions (the sum of portfolio equity assets and liabilities) totalled $5,717$ billion U.S. dollars, while gross U.S. portfolio debt positions equalled $8,185$ billion U.S. dollars (U.S. Treasury (2009a,b)). And in 2007 and 2008, U.S. portfolio debt inflows fell much less, both in percentage and absolute terms, than equity inflows, and the valuation effects were much stronger for equity than for debt.[2] Understanding the drivers of both the size and the composition of international trade in assets is therefore of high interest, even for a country as large and

[1] Lane and Milesi-Ferretti (2001), Ghironi et al. (2005), Gourinchas and Rey (2007a), and Tille (2003, 2008) emphasize the quantitative importance of valuation effects on external assets and liabilities.

[2] based on an updated version of the dataset described in Gourinchas and Rey (2007a), kindly provided by Hélène Rey.

Table 2.1: **Equity Home Bias in G7 countries**

Country	**Equity Market Cap as % of World Market Cap** (1)	**Share of domestic stocks in total equity portfolio (in %)** (2)	**Home Bias** (3)
Canada	2.4	84.0	81.6
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Home Bias in column (3) is computed as the difference between columns (2) and (1). Source: Coeurdacier et al (2007) based on CPIS data from 2001.

developed as the U.S.

Secondly, despite the size and sophistication of international financial markets, countries around the world continue to display a marked tendency to invest disproportionately in domestic equity (see French and Poterba (1991) and Table 2.1). This tendency is often referred to as "equity home bias", as basic models of international risk sharing would imply full diversification across countries (Lucas (1982)). Many explanations have been put forward to account for the observed bias, including frictions in the trading of goods or assets or informational frictions (see Lewis (1999), Karolyi and Stulz (2003) and Sercu and Vanpée (2007) for surveys of the literature). One intuitive set of explanations has focused on the possibility that domestic investors may choose to hold domestic equity, because it insures them against a domestic source of risk, most notably labour income risk (see Heathcote and Perri (2009) amongst others). However, in the data relative equity and human capital returns appear to be positively correlated (Baxter and Jermann (1997)) and this is our third fact.

Fourthly, countries tend to exhibit significantly positive cross country correlations across a broad range of macroeconomic variables, including output, consumption, investment and employment (see e.g. Backus et al (1993)).

We propose a two country, two good model with nominal rigidities, endogenous capital accumulation and incomplete markets that i) implies substantial international equity and debt positions, ii) predicts equity home bias of a realistic size, iii) features a positive unconditional correlation between relative equity returns and relative human capital returns, and iv) matches important features of the international business cycle. Our model builds on the extensive literature on business cycles in open economies.[3] The key innovation in our work is the combination of a

[3]Backus et al (1992) is an early assessment of the performance of frictionless open economy

sophisticated endogenous portfolio decision with a realistic description of the real side of the economy. On the portfolio side, we allow for international trade of nominal bonds in addition to trade in equities, the two major types of portfolio assets traded globally, and markets are incomplete. On the real side, our model features nominal price and wage rigidities, endogenous capital accumulation, endogenous monetary policy, and shocks to productivity, investment specific technology, and the nominal interest rate.

Most previous work in international macroeconomics has featured only very limited portfolio decisions, with the most common settings involving trade in a noncontingent bond or country specific equities. This is because the standard approximation techniques that are used to solve these models only pin down portfolios for a number of restricted cases.[4] We use the recent approximation methods described in Devereux and Sutherland (2009a) to overcome these difficulties and solve for optimal portfolios in a setting with multiple assets and incomplete markets.[5] We show that under very general conditions, steady state positions in equity and bonds can be related to two sources of risk. Agents invest in equities and bonds to insure against fluctuations in the present value of their relative consumption expenditures ("consumption expenditure risk") and the present value of their relative labour income ("human capital risk"). Financial income is then more valuable in states of the world where the present value of consumption expenditures is high or when the value of human capital is low.

Crucially, however, positions in individual assets have to take into account returns on other assets also. Thus, in our framework, equilibrium equity positions depend on the covariance of equity returns with the parts of human capital and consumption expenditure risk that are orthogonal to bond returns. Following the recent literature, we will often refer to these covariances as "conditional on bond returns".[6] Similarly, bond holdings depend on the covariance of bond returns with

DSGE models in replicating international business cycles. Kollmann (2001a,b) and Chari et al (2002) are examples of models that feature nominal rigidities and endogenous capital accumulation.

[4]Examples of such specific cases include Coeurdacier et al (2009), Engel and Matsumoto (2009), Heathcote and Perri (2009) and Kollmann (2006).

[5]Tille and van Wincoop (2009) present a similar solution method that allows for incorporating portfolio choice in open economy DSGE models. Devereux and Saito (2005), Evans and Hnatkovska (2006) and Judd et al. (2002) describe alternative solution approaches. Other recent papers that featured trade in equities and bonds include Benigno and Nistico (2009), Coeurdacier et al (2007), Coeurdacier and Gourinchas (2009), Pavlova and Rigobon (2007, 2009), and Bui (2009).

[6]See e.g. Engel and Matsumoto (2009), Coeurdacier et al (2009), Coeurdacier and Gourinchas (2009).

the parts of human capital and consumption expenditure risk that are orthogonal to equity returns, and we will refer to these covariances as "conditional on equity returns".

The distinction between the unconditional comovement of asset returns and the sources of risk and the one conditioning on other asset returns turns out to be very important both in the model and in the data. Unconditionally, equity and bond returns are positively correlated with each other and both are positively correlated with human capital and consumption expenditure risk. Relatively low substitutability between Home and Foreign goods and home bias in consumption imply that increases in the supply of the Home good induce a large fall in its relative price, and, with nominal rigidities, nominal exchange rates. The behaviour of nominal exchange rates, in turn, has a strong effect on the comovement of many variables in our model, and, inter alia, implies that relative equity returns are positively correlated with relative human capital returns and relative bond returns.

Nevertheless, equity home bias is mainly driven by the motive to hedge human capital risk in our model. This is because once we condition on bond returns, equity returns are *negatively* correlated with human capital returns. In addition to the response of nominal exchange rates, the key forces are endogenous capital accumulation and nominal price rigidities. Relative equity returns are positively correlated with relative human capital returns in response to productivity and monetary shocks, but move in opposite directions in response to investment specific technology shocks. In response to an investment specific technology shock, firms increase their demand for investment, and, with home bias in investment, their demand for Home labour. Firms, increasing production, face rising marginal costs and would like to raise prices. Since nominal rigidities prevent some from doing so, the overall response in the price level is smaller and the ultimate increase in investment and labour demand is larger than in the case of price flexibility and large enough to imply a rise in relative human capital returns, despite a small nominal depreciation. The Home investment boom lowers payments to shareholders in order to finance investment, thereby generating negative comovement between equity and human capital returns, conditional on the investment shock. Since relative bond returns do not respond strongly to the investment efficiency shock, this also implies that relative equity returns are negatively correlated with relative human capital returns, conditioning on relative bond returns.

Previous work (e.g. Coeurdacier et al (2009) and Engel and Matsumoto (2009)) has noted that in the presence of trade in bonds, it is the covariance between human capital returns and equity returns, conditional on bond returns, that drives equity portfolios. There is also some evidence that, while equity returns are positively correlated with human capital returns unconditionally, they are negatively correlated, once we condition on relative bond returns (Coeurdacier and Gourinchas (2009) and below). However, to our knowledge, this is the first paper to present a model that replicates both the conditional and the unconditional empirical pattern of correlations between human capital and equity returns.

Consumption expenditure risk has a much smaller effect on equity positions, as it is mainly hedged using positions in one period nominal bonds. Relative consumption expenditures are strongly affected by movements in the nominal exchange rate, as fluctuations in relative consumption volume and relative inflation are relatively modest. Since relative bond returns are highly correlated with the nominal exchange rate, long positions in domestic bonds provide a very good hedge for consumption expenditure risk. While the (conditional and unconditional) positive correlation between relative bond returns and consumption expenditure risk induces Home agents to take long positions in Home bonds, the (conditional and unconditional) positive correlation between relative bond returns and human capital returns induces Home agents to go short in Home bonds. Overall, the former motive prevails quantitatively, implying long positions in Home bonds in our benchmark case.

Monetary policy is represented by a simple rule according to which the central bank reacts strongly to CPI inflation and less strongly to output growth. Engel and Matsumoto (2009) showed that nominal price stickiness can have strong effects on endogenous asset portfolios in a setting with exogenous money supply. We show that this is true still, once we allow for endogenous and realistically calibrated, but not optimal, monetary policy. Monetary policy does affect portfolios through two channels here. Firstly, due to price rigidities, monetary policy has real effects and therefore affects the dynamics of human capital and consumption expenditure risk. Secondly, monetary policy also affects the behaviour of asset returns, and in particular the nominal exchange rate. In our setting, markets are incomplete and risk sharing is imperfect. In this case, monetary policy has welfare effects beyond the conventional effects arising from price dispersion (Devereux and Sutherland (2007, 2008)). This is because by affecting the hedging properties of assets, it affects the degree of overall risk sharing. But since the degree of risk sharing is high under most reasonable specifications of monetary policy, these additional welfare effects are likely to be small.

We confront our model with the data in three ways. Firstly, we calibrate the model and compare its predictions for international asset positions with available evidence. Our benchmark calibration implies that domestic investors hold close to 90 percent of the domestic equity stock, which is virtually identical to the share reported for the US in 2008 (U.S. Treasury (2009a)). The model also predicts that domestic investors take an overall long position in domestic bonds of about 50 percent of GDP, and a corresponding short position in foreign bonds. This is consistent with the evidence presented in Lane and Shambaugh (2007, 2008) who show that advanced economies, on average, have negative net foreign currency debt positions. However, a notable exception is the U.S. which has a negative domestic currency debt position.

Secondly, we compare the business cycle implications of our benchmark calibration with the data and find that the model replicates many important features of the international business cycle. The presence of investment efficiency shocks lowers the cross country correlation in consumption, raises the volatility of the real exchange rate and generates countercyclical net exports, as in Coeurdacier et al (2009). However, our model is also able to generate positive cross country correlations in investment and strongly positive correlations between domestic consumption and output, due to the nominal rigidities in price setting and investment adjustment costs. Previous work has noted the difficulty of jointly generating realistic macroeconomic and asset pricing moments in a domestic context.[7] Our open economy model is no exception. We find that the equity premium in our benchmark economy is too low and an extension that allows for Epstein-Zin preferences, as in Rudebusch and Swanson (2009), can only generate a realistic level of the equity premium at the expense of excessive volatility in the risk free rate.

Finally, we use data on equity and bond returns, labour income and consumption expenditure for the G7 economies since 1970 to estimate the unconditional and conditional comovement of equity and bond returns with consumption and human capital risk for the US. We find that in the data, as in the model, relative equity returns are unconditionally moderately positively correlated with consumption expenditure and human capital risk, while relative bond returns are more strongly positively correlated with the two sources of risk. Once we condition on bond returns, relative equity returns are negatively correlated with relative human capital returns, while they remain positively correlated with consumption

[7]Mehra and Prescott (1985) document the difficulty of generating a realistic equity premium in standard general equilibrium endowment models. Rouwenhorst (1995), Lettau and Uhlig (2000), Uhlig (2004), and Cochrane (2005) show that the challenge is even bigger once production is endogenised and macroeconomic moments are considered at the same time.

expenditure risk. Our estimation therefore implies equity home bias driven by the motive to hedge human capital risk and, to a lesser extent, consumption expenditure risk. Quantitatively, the position implied by the estimation is very close to the position implied by our model calibration and both are close to data on actual U.S. holdings of U.S. equity. For bond positions, we find that, as in the model, the estimation predicts that the human capital motive should induce U.S. investors to go short in U.S. bonds, while the consumption expenditure motive would induce them to take long positions. Unlike in the model, the consumption expenditure motive dominates in the estimation which therefore predicts an overall short position in U.S. bonds, consistent with actual data on U.S. portfolio positions (e.g. U.S. Treasury (2009a)). However, the size of the bond position predicted by the estimation is too large in comparison with the available data.

This paper is related to a number of recent contributions. Engel and Matsumoto (2009) investigate international trade in equities and currency forward contracts in a model with nominal price stickiness and exogenous money supply, while Devereux and Sutherland (2007, 2008) study the effects of monetary policy on international portfolio choice in a model with nominal price stickiness and incomplete markets. Neither of the two feature endogenous capital accumulation or investigate the business cycle implications of their models. Heathcote and Perri (2009) were the first to show that endogenous capital accumulation can generate a negative correlation between dividends and labour income in a model where equity is the only asset traded and shocks to productivity are the only source of uncertainty. Coeurdacier et al (2009) extend the model of Heathcote and Perri (2009) by adding shocks to investment specific technology and trade in real bonds and investigate its predictions for international portfolio choice and business cycle properties. Unlike their framework, our model features nominal rigidities in price and wage setting, and endogenous monetary policy. What is more, the bonds traded in our setup are nominal and markets are incomplete, while the bonds traded in Coeurdacier et al (2009) are real and markets are complete, to a first order approximation.

This paper proceeds as follows. In the next section, we show that in a fairly general class of open economy models, steady state asset positions are related to the comovement of relative asset returns with consumption expenditure and human capital risk, conditional on the returns of other assets. In section 3, we describe our benchmark model. In section 4, we calibrate the model, present quantitative results for its equilibrium portfolios and business cycles predictions and discuss their drivers. Section 5 contains the empirical analysis and section 6 concludes.

2.2 Asset Positions and Hedging Demands

Below, we will present a two country, two good, production economy model that features endogenous capital accumulation, nominal price and wage rigidities and incomplete markets. A difficulty that arises from the incorporation of these features is that it is, in general, not possible to solve for portfolios in closed form. However, we show that, under a very limited set of assumptions, we can derive a reduced form expression that relates asset positions to fluctuations in (consumption) expenditure and nonfinancial income, at a first order of approximation, that can be used to understand the drivers of asset positions and that can be estimated. Assume that equities and short term bonds are traded between two symmetric countries, generically called "Home" and "Foreign". Equities are a claim to dividends, while nominal bonds pay one unit of local currency for one period only. The budget constraint of the Home agent is then:

$$\begin{aligned} & \underbrace{S_{H,t}^{H} P_{H,t}^{S} + S_{F,t}^{H} P_{F,t}^{S} Z_t + B_{H,t}^{H} P_{H,t}^{B} + B_{F,t}^{H} P_{H,t}^{B} Z_t}_{\text{Financial Investment}} + \underbrace{P_{C,t}^{H} C_{H,t}}_{\text{Consumption Expenditure}} \\ = \; & \underbrace{W_{H,t} L_{H,t}}_{\text{Nonfinancial Income}} + \underbrace{S_{H,t-1}^{H}\left(D_{H,t} + P_{H,t}^{S}\right) + S_{F,t-1}^{H}\left(D_{F,t} + P_{F,t}^{S}\right) Z_t + B_{H,t-1}^{H} + B_{F,t-1}^{H} Z_t}_{\text{Financial Income}}, \end{aligned} \quad (2.1)$$

where $S_{j,t}^{i}$ is the fraction of country j equity held by country i, $B_{j,t}^{i}$ are the number of country j bonds held by country i, $P_{i,t}^{S}$ are the prices of country i shares in local currency, $P_{C,t}^{i}$ is the consumption price index of country i, and $W_{i,t}, L_{i,t}, D_{i,t}$ are wages, labour demand, and dividends. Z_t is the nominal exchange rate, defined as the number of units of Home currency per unit of Foreign currency, implying that a rise in Z_t signifies a *depreciation* of the Home currency. This expression indicates that nonfinancial income and financial income are used to finance consumption expenditure today or (via financial investment) in the future. To make the dynamic nature of this relationship more precise and to translate it into an international context, it is useful to derive expressions for asset returns and our definitions of risk.[8] Due to the symmetry of the model, it is without loss of generality to focus on the relative value of all variables, including asset returns, and we will henceforth do so.

[8] The complete derivation is given in the appendix.

Realised relative returns in terms of Home currency between Home and Foreign nominal bonds, $\widehat{R}_t^B$, are, linearised around the nonstochastic steady state:

$$\widehat{R}_t^B \equiv \widehat{R}_{H,t}^B - \widehat{R}_{F,t}^B - \widehat{Z}_t + \widehat{Z}_{t-1} = \widetilde{E}_t\left[-\widehat{Z}_t\right], \tag{2.2}$$

where $R_{H,t}^B\left(R_{F,t}^B\right)$ are local currency bond returns. $E_t\left[X_{t+j}\right]$ is the conditional expectation of X_{t+j} at time t, $\widetilde{E}_t\left[X_t\right] \equiv E_t\left[X_t\right] - E_{t-1}\left[X_t\right]$ denote date t surprises in the value of X and $\widehat{X}_t$ is the linearised value of X_t, defined as $X_t \equiv \frac{X_t - X}{X}$, where X is the value of X_t in the nonstochastic steady state. Thus, relative returns of Home bonds equal the extent of the unexpected appreciation of the local currency in the same period. Relative linearised equity returns in Home currency, $\widehat{R}_t^S$, are determined by the present value of surprises to relative dividends:

$$\widehat{R}_t^S \equiv \widehat{R}_{F,t}^S - \widehat{R}_{H,t}^S - \widehat{Z}_t + \widehat{Z}_{t-1} = (1-\beta)\,\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j} - \widehat{D}_{H,t+j} + \widehat{Z}_{t+j}\right)\right], \tag{2.3}$$

where $R_{H,t}^S\left(R_{F,t}^S\right)$ are local currency equity returns, β is the subjective discount rate that we assume to be common between the two countries and we imposed the appropriate transversality conditions.

Above, we mentioned that agents use financial and nonfinancial income to finance their consumption expenditures. We therefore define two measures, R_t^{PC} and R_t^{WL}, that we will henceforth call “consumption expenditure risk” and “human capital risk”. In linearised form these are:

$$\widehat{R}_t^{PC} \equiv (1-\beta)\,\widetilde{E}_t\sum_{j=0}^{\infty}\beta^j\left(\widehat{P}_{C,t+j}^H + \widehat{C}_{H,t+j} - \widehat{P}_{C,t+j}^F - \widehat{C}_{F,t+j} - \widehat{Z}_{t+j}\right) \tag{2.4}$$

$$\widehat{R}_t^{WL} \equiv (1-\beta)\,\widetilde{E}_t\sum_{j=0}^{\infty}\beta^j\left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} - \widehat{W}_{F,t+j} - \widehat{L}_{F,t+j} - \widehat{Z}_{t+j}\right), \tag{2.5}$$

where we again imposed the appropriate transversality conditions. Consumption expenditure risk thus depends on the present value of surprises in relative consumption expenditure, while human capital risk, depends on the present value of surprises in relative labour income.[9]

[9]It is worth noting that the discounted sums in (2.3) – (2.5) are formed using the simple nonstochastic discount factor. With incomplete markets, the stochastic discount factors of the Home and Foreign investor are not necessarily equal. However, at the level of approximation that we consider, any time variation in the discount factors drops out and the discount factor is simply equal to β. Note also that, to a first order approximation, the *expected* stochastic discount factors are equal between Home and Foreign investors, as shown in the appendix.

Using a present value analogue of the period by period budget constraint, (2.1) , in linearised form, deducting the analogous expression for the foreign country, and using expressions (2.2) – (2.5) , we can then write the relative budget constraint as:[10]

$$\frac{C}{1-\beta}\widehat{R}_t^{PC} = \frac{WL}{1-\beta}\widehat{R}_t^{WL} + (2S-1)\frac{D}{1-\beta}\widehat{R}_t^{S} + 2B\widehat{R}_t^{B}, \tag{2.6}$$

where S and B are the steady share of the domestic equity stock and the number of domestic bonds held by domestic investors. C, D and WL are the steady state values of consumption, labour income and dividends, respectively. This expression states that innovations in the present value of relative consumption expenditures have to be met by innovations to the value of human capital or the value of equity and bond holdings. Projecting (2.6) on relative bond returns and deducting the resulting expression from (2.6) , we can then solve for equilibrium equity holdings in the steady state:

$$S = \underbrace{\frac{1}{2}}_{\substack{\text{Pure}\\ \text{Diversification}\\ \text{Motive}}} + \underbrace{\frac{1}{2}\frac{C}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{PC}, \widehat{R}_t^{S}\right)}{var_{R_t^B}\left(\widehat{R}_t^{S}\right)}}_{\substack{\text{Consumption}\\ \text{Expenditure}\\ \text{Motive}}} - \underbrace{\frac{1}{2}\frac{WL}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{WL}, \widehat{R}_t^{S}\right)}{var_{R_t^B}\left(\widehat{R}_t^{S}\right)}}_{\substack{\text{Human}\\ \text{Capital}\\ \text{Motive}}} \tag{2.7}$$

$cov_{R_t^B}(x, y)$ is the covariance between the parts of x and y that are orthogonal to relative bond returns and we will often refer to this measure of comovement as the covariance of x and y, conditional on relative bond returns.

Equation (2.7) is not structural, as equity positions are not expressed in terms of the exogenous shocks of the model, but it nevertheless provides useful intuition about the drivers of equity positions. According to (2.7), the equity position can be broken into three terms. The first term reflects the pure diversification motive. In our symmetric two country model, agents would hold exactly one half of the equity stock of each country if relative equity returns were uncorrelated with consumption expenditure and human capital risk, conditional on bond returns. The second term arises from a motive to hedge movements in relative consump-

[10] The market clearing conditions for assets in the steady state imply: $S_H^H = S_F^F = S = 1 - S_F^H = 1 - S_H^F$ and $B_H^H = B_F^F = B = -B_F^H = -B_H^F$. We also impose that the discounted sum of future net exports is equal to the initial net foreign asset position, a condition that is satisfied at our level of approximation. Full details of the derivation are given in the appendix.

tion expenditures. Domestic equity is more attractive, if relative equity returns are high, on average, when surprises in the present value of future consumption expenditures are high, conditional on relative bond returns. The third term in (2.7) reflects the motive to hedge human capital returns with equity holdings. This term captures the fact that equity returns are particularly valuable, if other sources of income, in this case human capital returns and relative bond returns, are low. Thus, the domestically held share of the equity stock increases the more negatively correlated relative equity return innovations are with relative human capital return innovations, conditional on relative bond returns.

The relative importance of the consumption expenditure and the human capital motive is indicated by the presence of the C, D and WL terms, reflecting the size of steady state consumption expenditure, labour income and dividends. Two features are worth highlighting. Firstly, it is the comovement of *relative* returns with *relative* consumption expenditure and labour income that matters. As mentioned before, the symmetry of the model implies that it is without loss of generality to focus on relative moments, but it is worth keeping in mind that relative returns may fall even when absolute returns rise, and vice versa. Secondly, note that the equity positions in (2.7) are functions of covariances and variances, conditional on relative bond returns. This reflects the fact that equity is not the only financial asset in this economy, and the other financial assets, in this case nominal bonds, are also used to hedge the two sources of risk. The conditional covariance of relative equity return with human capital risk can thus be interpreted as the residual comovement, once some part of this risk has been hedged using the bond positions. It is also worth noting that the expressions for bond and equity positions feature conditional covariance-variance ratios and thus look like multiple regression coefficients, a property which, following Coeurdacier and Gourinchas (2009), we will exploit below, both in the model and in the data.

Starting from (2.6), but projecting the equation on relative equity returns, we obtain the analogous expression for equilibrium bond holdings:

$$\widetilde{B} = \frac{1}{2}\frac{\beta}{(1-\beta)}\frac{C}{Y}\frac{cov_{R_t^S}\left(\widehat{R}_t^{PC}, \widehat{R}_t^{B}\right)}{var_{R_t^S}\left(\widehat{R}_t^{B}\right)} - \frac{1}{2}\frac{\beta}{(1-\beta)}\frac{WL}{Y}\frac{cov_{R_t^S}\left(\widehat{R}_t^{WL}, \widehat{R}_t^{B}\right)}{var_{R_t^S}\left(\widehat{R}_t^{B}\right)}, \tag{2.8}$$

where Y is annualised steady state output. $cov_{R_t^S}(x, y)$ is the covariance between the parts of x and y that are orthogonal to relative bond returns and we will refer to this measure as the covariance between x and y, conditional on relative equity returns. Since net bond positions are assumed to be zero in this framework, the pure diversification term is absent. Other than that, the bond position in (2.8) is driven by the same hedging motives as the equity position in (2.7). The first term reflects the desire to hedge relative consumption expenditure risk using the bond position, while the second term reflects the motive to hedge human capital risk. Thus, positions in domestic bonds are larger, the more positively correlated relative bond returns are with relative consumption expenditure risk and the more negatively correlated relative bond returns are with human capital risk, conditional on relative equity returns. As with equity positions, the fact that the relevant covariances and variances are conditional on relative equity returns reflects that equity returns are also used to hedge some of the consumption expenditure and human capital risk.[11]

Equations (2.7) and (2.8) will hold in the specific model studied below which features, inter alia, capital accumulation, nominal rigidities in price and wage setting, and incomplete markets. However, the expressions above are independent of the presence of these model features and in fact much more general than the models presented here.[12]

2.3 Model

There are two symmetric countries, Home (H) and Foreign (F), indexed by i. Each country is specialised in the production of a composite good using a continuum of country specific intermediate goods. Intermediate goods are produced using labour and capital and there are nominal rigidities in price and wage setting of the Calvo type. The factors of production and the intermediate goods are immobile between countries, but composite goods are traded. In addition to trade in composite goods, countries trade one period nominal bonds and equities and there are three aggregate sources of uncertainty per country: Neutral technology shocks, investment specific technology shocks and monetary shocks.

[11]The presence of the $\beta/(1-\beta)$ terms in the expression for bonds results from the fact that the maturity of bonds here is one period, while equity and the sources of risk do not mature.

[12]The expressions can easily be adapted to allow for asymmetry between countries, government spending, financial frictions or a more extensive menu of assets. Similar expressions have been derived in a static context by Coeurdacier and Gourinchas (2009) and in a dynamic context with complete markets by Coeurdacier et al (2009), among others.

2.3.1 Households

Country i is inhabited by a representative consumer with a utility function that is separable in consumption and labour:

$$U_i = \sum_{j=0}^{\infty} \beta_{t+j} \left(\frac{C_{i,t+j}^{1-\sigma}}{1-\sigma} - \frac{\widetilde{L}_{i,t+j}^{1+\omega}}{1+\omega} \right), \tag{2.9}$$

where $\widetilde{L}_{i,t}$ is labour supply and the distinction between labour demand and labour supply will be discussed in the context of optimal wage setting below. The intertemporal elasticity of substitution σ and inverse of the Frisch labour supply elasticity ω are common across countries. Following Schmitt-Grohe and Uribe (2003), the discount factor β is modelled as:

$$\beta_{t+j+1} = \beta_{t+j} \widetilde{\beta} C_{i,t}^{\eta}; \beta_0 = 1, \tag{2.10}$$

where $0 < \eta < \sigma$ and $\widetilde{\beta} C^{\eta} < 1$, and individual decision makers take η as exogenous. If $\eta = 0$, we obtain the standard case of an exogenous and constant discount factor. By allowing $\eta < 0$, we will ensure that net foreign asset positions are stationary in our approximations.[13] The consumption aggregator for country i is defined as:

$$C_{i,t} = \left[a^{1/\phi} \left(C_{i,t}^{i} \right)^{\frac{\phi-1}{\phi}} + (1-a)^{1/\phi} \left(C_{j,t}^{i} \right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \tag{2.11}$$

where $a > 1/2$ is the home bias parameter and ϕ is the elasticity of substitution between Home and Foreign goods. These preferences imply the following consumption price indices:

$$P_{C,t}^{i} = \left[a \left(P_{i,t}^{i} \right)^{1-\phi} + (1-a) \left(P_{j,t}^{i} \right)^{1-\phi} \right]^{\frac{1}{1-\phi}}, \tag{2.12}$$

where $P_{j,t}^{i}$ is the price in country i of the composite good produced in country j. All prices are quoted in terms of the local currency. The optimal allocation across consumption goods is then given by:

$$C_{i,t}^{i} = a \left(\frac{P_{i,t}^{i}}{P_{C,t}^{i}} \right)^{-\phi} C_{i,t} \qquad C_{j,t}^{i} = (1-a) \left(\frac{P_{j,t}^{i}}{P_{C,t}^{i}} \right)^{-\phi} C_{i,t} \tag{2.13}$$

[13]We solve our model using linear approximations around the nonstochastic steady state. It is well known that at this level of approximation, net foreign asset positions display a unit root due to market incompleteness. Setting η to a low, but positive value removes the unit root, while leaving our results virtually unaffected.

where $C^i_{j,t}$ denotes consumption of good j by agent i. Consumption of the Home and Foreign aggregate consumption goods are given by:

$$C^i_{j,t} = \left(\int_0^1 \left(C^i_{j,t}\left(k\right)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.14}$$

where $C^i_{j,t}\left(k\right)$ is consumption of the kth intermediate good in country j by the agent in country i and ε is the elasticity of substitution between varieties in consumption. Optimal consumption of Home and Foreign intermediate goods then implies:

$$C^i_{j,t}\left(k\right) = \left(\frac{P^i_{j,t}}{P^i_{j,t}\left(k\right)} \right)^{\varepsilon} C^i_{j,t} \qquad P^i_{j,t} = \left(\int_0^1 P^i_{j,t}\left(k\right)^{1-\varepsilon} dk \right)^{\frac{1}{1-\varepsilon}} \tag{2.15}$$

where $P^i_{j,t}$ is the price in country i of the composite good produced in country j. The real exchange rate Q is defined as:

$$Q_t = \frac{Z_t P^F_{C,t}}{P^H_{C,t}} \tag{2.16}$$

2.3.2 Capital Accumulation

At time t, each country possesses a capital stock $K_{i,t}$. Country specific capital stocks depreciate at rate δ and are augmented by country specific investment $I_{i,t}$:

$$K_{i,t+1} = \left(1-\delta\right) K_{i,t+1} + \chi_{i,t} \Upsilon \left(\frac{I_{i,t}}{I_{i,t-1}} \right) I_{i,t}, \tag{2.17}$$

where investment in country $i's$ capital stock is a CES aggregate of the Home and Foreign composite good

$$I_{i,t} = \left[a^{1/\phi_I} \left(I^i_{i,t}\right)^{\frac{\phi-1}{\phi}} + \left(1-a\right)^{1/\phi} \left(I^i_{j,t}\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}. \tag{2.18}$$

$\chi_{i,t}$ is a shock to investment specific technology, as in Greenwood et al (1997, 2000) and Fisher (2002, 2006), and the function $\Upsilon\left(\frac{I_{i,t}}{I_{i,t-1}}\right)$ reflects investment adjustment costs, as in Christiano et al (2005) or Smets and Wouters (2007).[14] Investment goods are modelled analogously to consumption goods, implying the

[14]The adjustment cost function is chosen to be $\Upsilon\left(\frac{I_t}{I_{t-1}}\right) = \frac{\alpha_1}{1-1/\upsilon}\left(\frac{I_t}{I_{t-1}}\right)^{1-1/\upsilon} + \alpha_2$ and the constants α_1 and α_2 are chosen to ensure that there are no adjustment costs in the steady state $\Upsilon\left(1\right) = \Upsilon'\left(1\right) = 1$.

following aggregators, investment allocations and price indices:

$$I_{j,t}^{i} = \left(\int_{0}^{1}\left(I_{j,t}^{i}(k)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad I_{j,t}^{i}(k) = \left(\frac{P_{j,t}^{i}}{P_{j,t}^{i}(k)}\right)^{\varepsilon} I_{j,t}^{i} \quad (2.19)$$

$$P_{I,t}^{i} = \left[a\left(P_{i,t}^{i}\right)^{1-\phi} + (1-a)\left(P_{j,t}^{i}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}}, \quad (2.20)$$

where we assume that substitution elasticities between individual varieties for investment and the price indices are equal to those for consumption.
At time t, the stock of capital in each country is rented out in a spot market at a rental rate $R_{i,t}^{K}$. Investment is then optimally chosen to satisfy the two following conditions:

$$-\lambda_{i,t}^{K} = \varpi_{i,t+1}R_{i,t+1}^{K} - (1-\delta)\lambda_{i,t+1}^{K} \quad (2.21)$$

$$P_{I,t}^{i} = -\lambda_{i,t}^{K}\chi_{i,t}\left(\frac{I_{t}^{i}}{I_{t-1}^{i}}\right)^{-\frac{1}{\upsilon}} - \varpi_{i,t+1}\chi_{i,t+1}\lambda_{i,t+1}^{K}\left(\frac{1}{\upsilon-1}\left(\frac{I_{t+1}^{i}}{I_{t}^{i}}\right)^{\frac{\upsilon-1}{\upsilon}} + \frac{1}{\upsilon-1}\right) \quad (2.22)$$

where υ is the elasticity of the investment adjustment cost function, $\varpi_{t,t+1}^{i}$ is the stochastic discount factor of the country i investor applied at time t to discount date $t+1$ profits and λ_{i}^{K} is the Lagrange multiplier on the law of motion for capital. In the case of no adjustment costs $(\upsilon \to \infty)$, the two expressions collapse into the familiar condition that investment is chosen to equate the expected discounted payoff from investment to its marginal cost at time t:

$$1 = \beta E_{t}\left[\varpi_{t,t+1}^{i}\frac{\chi_{i,t}}{P_{I,t}^{i}}\left(R_{i,t}^{K} + (1-\delta)\frac{P_{I,t+1}^{i}}{\chi_{i,t+1}}\right)\right]. \quad (2.23)$$

2.3.3 Firms

Each country contains a continuum of firms, each producing a differentiated intermediate good, indexed by k, using capital and labour. The production function is given by:

$$Y_{i,t}(k) = A_{i,t}\left(L_{i,t}(k)\right)^{\alpha}\left(K_{i,t}(k)\right)^{1-\alpha}, \quad (2.24)$$

where $A_{i,t}$ is the exogenous level of productivity in country i and α is the elasticity of output with respect to labour. Firms maximise the present discounted value of profits and choose labour and capital in aggregate country specific spot markets to minimise the cost of production. Firms face the same wages and rental rates of capital and, consequently, all firms in a given country choose the same labour to capital ratio :

$$\frac{K_{i,t}(k)}{L_{i,t}(k)} = \frac{K_{i,t}}{L_{i,t}} = \frac{1-\alpha}{\alpha}\frac{W_{i,t}}{R^K_{i,t}}. \tag{2.25}$$

Total demand for each intermediate good is composed of domestic and foreign demand for consumption and investment. Firms take this demand as given and choose prices in local currency in Home and Foreign to maximise profits. However, with probability θ, they cannot reset prices in the current period (Calvo (1983)). A firm reoptimising in period t will then choose a price $\widetilde{P}^j_{i,t}$ that maximises the current market value of profits generated while the price remains in effect, taking all other prices as given. Optimal prices for Home firms are then:

$$\widetilde{P}^H_{H,t}(k) = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi^H_{t,t+l}\varrho_{H,t}\left(P^H_{H,t+l}\right)^{\varepsilon}Y^H_{H,t+l}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi^H_{t,t+l}\left(P^H_{H,t+l}\right)^{\varepsilon}Y^H_{H,t+l}} \tag{2.26}$$

$$\widetilde{P}^F_{H,t}(k) = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{l=0}^{\infty}\theta^l\varpi^H_{t,t+l}\left(P^F_{H,t+l}\right)^{\varepsilon}\varrho_{H,t}Y^F_{H,t+l}}{E_t\sum_{l=0}^{\infty}\theta^l\varpi^H_{t,t+l}Z_{t+l}\left(P^F_{H,t+l}\right)^{\varepsilon}Y^F_{H,t+l}}, \tag{2.27}$$

where $Y^i_{i,t}(k) = C^i_{i,t}(k) + I^i_{i,t}(k), Y^j_{i,t}(k) = C^j_{i,t}(k) + I^j_{i,t}(k)$, $\varrho_{H,t}$ is the marginal cost function of a firm in country i, and foreign prices follow analogously. This expression implies that optimal prices are set as a markup over expected marginal costs over the expected lifetime of the price set, and collapses to the familiar static condition $\widetilde{P}^H_{H,t}(k) = \frac{\varepsilon}{\varepsilon-1}\varrho_{H,t}$, if prices are perfectly flexible. The nature of price rigidities implies that all firms will choose the same price when reoptimising. The price indices for Home and Foreign goods then evolve as:

$$P^i_{j,t} = \left(\theta\left(P^i_{j,t-1}\right)^{1-\varepsilon} + (1-\theta)\left(\widetilde{P}^i_{j,t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{2.28}$$

2.3.4 Wage Setting

As in Schmitt Grohe and Uribe (2007), we assume that households supply labour monopolistically to a continuum of labour markets, indexed by j. There is a single union for each labour market and firms, indexed by k, combine labour inputs from each of these different labour markets according to the following aggregator:

$$L_{i,t}(k) = \int_0^1 L_{i,t}(j,k)^{1-1/\varepsilon_w}\,dj \tag{2.29}$$

Optimal firm decisions imply that the demand by firm k for labour from sector j is given by:

$$L_{i,t}(j,k) = \left(\frac{W_{i,t}(j)}{W_{i,t}}\right)^{-\varepsilon_w} L_{i,t}(k), \tag{2.30}$$

where $L_{i,t}(k)$ is total labour demand of firm k and $W_{i,t}(j)$ are nominal wages in sector j. Aggregating across all sectors and assuming that unions satisfy demand at the going wage, we have that total labour supply of type j is:

$$\widetilde{L}_{i,t}(j) = \left(\frac{W_{i,t}(j)}{W_{i,t}}\right)^{-\varepsilon_w} \int_0^1 L_{i,t}(k)\,dk = \left(\frac{W_{i,t}(j)}{W_{i,t}}\right)^{-\varepsilon_w} L_{i,t}. \tag{2.31}$$

Aggregating across all labour markets, we arrive at total labour supply:

$$\widetilde{L}_{i,t} = \int_0^1 \widetilde{L}_{i,t}(j)\,dj = L_{i,t} \int_0^1 \left(\frac{W_{i,t}(j)}{W_{i,t}}\right)^{-\varepsilon_w} dj.^{15} \tag{2.32}$$

The nominal wage index $W_{i,t}$ is given by $W_{i,t} = \left(\int_0^1 \left(W_{i,t}(j)\right)^{1-\varepsilon_w}\right)^{\frac{1}{1-\varepsilon_w}}$ and has the property that it is the minimum cost of a bundle of intermediate labour inputs yielding one unit of composite labour. Similar to wage setting, we assume that each period the union cannot reset the wage optimally in a random fraction θ_w of labour markets. It can then be shown that unions that can reoptimise the wage at time t will choose the same wage, $\widetilde{W}_{i,t}(j) = \widetilde{W}_{i,t}$, given by:

$$\widetilde{W}_{i,t} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{\sum_{l=0}^{\infty} \beta_{i,t+l}\theta_w^l \left(-\frac{C_{i,t+l}^{-\sigma}}{P_{C,t+l}^i}\right) W_{t+l}^{\varepsilon_w} L_{t+l} \frac{-\widetilde{L}_{i,t}^{\omega}}{-\frac{C_{i,t+l}^{-\sigma}}{P_{C,t+l}^i}}}{\sum_{l=0}^{\infty} \beta_{i,t+l}\theta_w^l \left(-\frac{C_{i,t+l}^{-\sigma}}{P_{C,t+l}^i}\right) W_{t+l}^{\varepsilon_w} L_{t+l}}. \tag{2.33}$$

This expression states that the union chooses the wage as a markup of expected average marginal revenues over the expected average cost of supplying labour effort.[16] Aggregate wages then follow the law of motion:

$$W_{i,t}^{1-\varepsilon_w} = \theta_w W_{i,t-1}^{1-\varepsilon_w} + (1-\theta_w)\,\widetilde{W}_{i,t}^{1-\varepsilon_w}. \tag{2.34}$$

[16]Without nominal wage rigidities, this expression would collapse to: $W_{i,t} = \frac{\varepsilon_w}{\varepsilon_w - 1}\widetilde{L}_{i,t}^{\omega} C_{i,t}^{\sigma} P_{C,t}^i$, implying that wages are set at a markup over the marginal rate of substitution between consumption and labour.

2.3.5 Monetary Authority

We assume that the monetary authority responds to CPI inflation and output growth and sets the nominal interest rate, $R^N_{i,t}$, according to:

$$R^N_{i,t} = \left(R^N_{i,t}\right)^{\rho_r} \left(\frac{1}{\beta}\left(\frac{P^i_{C,t}}{P^i_{C,t-1}}\right)^{\gamma_\pi} \left(\frac{Y_{i,t}}{Y_{i,t-1}}\right)^{\gamma_Y} M_{i,t}\right)^{1-\rho_r}, \tag{2.35}$$

where ρ_r is the degree of interest rate smoothing and γ_π and γ_y parametrise the response of the monetary authority to fluctuations in CPI inflation and output, respectively. $M_{i,t}$ is a mean zero shock to interest rates. The monetary rule here is similar to the one used in Del Negro and Schorfheide (2009). It is a generalisation of the rule used in Devereux and Sutherland (2007, 2008) by allowing monetary policy to respond to output growth in addition to inflation and by allowing for interest rate smoothing.[17] It is worth noting that monetary policy is not optimal here, as it does not necessarily respond in an efficient manner to deviations of inflation and output from their natural levels.

2.3.6 Financial Markets

There is trade in Home and Foreign nominal bonds and Home and Foreign equity. Nominal bonds pay one unit of the domestic currency in the following period and are in net zero supply. Owners of equity of country i receive a claim to country i dividends $D_{i,t}$ which are composed of profits earned by the firm plus the receipts from renting out capital, minus investment spending:

$$D_{i,t} = \Pi_{i,t} + R^K_{i,t} K_{i,t} - P^i_{I,t} I_{i,t}. \tag{2.36}$$

Gross nominal returns in domestic currency for bonds and equity are:

$$R^S_{i,t+1} = \frac{D_{i,t} + P^S_{i,t+1}}{P^S_{i,t}} \qquad R^B_{i,t+1} = \frac{1}{P^B_{i,t}} \tag{2.37}$$

where $R^S_{i,t}$ $\left(R^B_{i,t}\right)$ is the return on holdings of country i equity (bonds) and $P^S_{i,t}$ $\left(P^B_{i,t}\right)$ are the prices of country i equity (bonds). The total supply of equity in each country is normalised to unity and it is assumed that in period 0 each household owns the stock of domestic equity. Asset prices are then determined by the usual

[17] Another difference is that monetary policy responds to PPI inflation in Devereux and Sutherland (2007, 2008), while it responds to CPI inflation here.

pricing equations. For the Home investor, these are:

$$E_t\left[\varpi_{H,t+1}\left(\frac{P^H_{C,t}}{P^H_{C,t+1}}R^S_{H,t+1}\right)\right] = 1 \quad E_t\left[\varpi_{H,t+1}\left(\frac{P^H_{C,t}}{P^H_{C,t+1}}R^S_{F,t+1}\frac{Z_{t+1}}{Z_t}\right)\right] = 1 \tag{2.38}$$

$$E_t\left[\varpi_{H,t+1}\left(\frac{P^H_{C,t}}{P^H_{C,t+1}}R^B_{H,t+1}\right)\right] = 1 \quad E_t\left[\varpi_{H,t+1}\left(\frac{P^H_{C,t}}{P^H_{C,t+1}}R^B_{F,t+1}\frac{Z_{t+1}}{Z_t}\right)\right] = 1 \tag{2.39}$$

and the pricing equations for the Foreign investor are analogous. The budget constraint for the Home investor in period t is then identical to (2.1) , the budget constraint used to derive our reduced form expression above:

$$\begin{aligned} & S^H_{H,t}P^S_{H,t} + S^H_{F,t}P^S_{F,t}Z_t + B^H_{H,t}P^B_{H,t} + B^H_{F,t}P^B_{H,t}Z_t \\ = \; & W_{H,t}L_{H,t} + S^H_{H,t-1}\left(D_{H,t} + P^S_{H,t}\right) + S^H_{F,t-1}\left(D_{F,t} + P^S_{F,t}\right)Z_t \\ & + B^H_{H,t-1} + B^H_{F,t-1}Z_t - P^H_{C,t}C_{H,t} \end{aligned} \tag{2.40}$$

where $S^i_{j,t}$ $\left(B^i_{j,t}\right)$ are holdings of country j equity (bonds) by country i in period t. The budget constraint of the Foreign agent can be written analogously.

2.3.7 Market Clearing and Exogenous Processes

The market clearing condition for goods are given by:

$$C^H_{H,t} + C^F_{H,t} + I^H_{H,t} + I^F_{H,t} = Y_{H,t} \qquad C^H_{F,t} + C^F_{F,t} + I^H_{F,t} + I^F_{F,t} = Y_{F,t} \tag{2.41}$$

For assets, we have:

$$S^H_{H,t} + S^F_{H,t} = 1 \qquad S^H_{F,t} + S^F_{F,t} = 1 \tag{2.42}$$

$$B^H_{H,t} + B^F_{H,t} = 0 \qquad B^H_{F,t} + B^F_{F,t} = 0. \tag{2.43}$$

There are three sources of uncertainty per country, neutral and investment specific technology shocks, and monetary shocks, and each is assumed to follow a first order autoregressive process in logarithms.

$$\log\left(A_{i,t+1}\right) = \rho_A \log\left(A_{i,t}\right) + \varepsilon_{A,i,t+1}, \tag{2.44}$$

$$\log\left(\chi_{i,t+1}\right) = \rho_\chi \log\left(\chi_{i,t}\right) + \varepsilon_{\chi,i,t+1} \tag{2.45}$$

$$\log\left(M_{i,t+1}\right) = \rho_M \log\left(M_{i,t}\right) + \varepsilon_{M,i,t+1}, \tag{2.46}$$

where $\varepsilon_{j,i,t+1}$ is a mean zero i.i.d shock and ρ_j governs the persistence of the autoregressive process.

2.3.8 Equilibrium and Solution Method

An equilibrium in this economy consists of a set of processes for prices, $P^H_{C,t}$, $P^F_{C,t}$, $P^H_{I,t}$, $P^F_{I,t}$, $P^H_{F,t}$, $P^F_{H,t}$, $P^F_{F,t}$, $P^H_{H,t}$, wages $W_{H,t}, W_{F,t}$, rental rates for capital $R^K_{H,t}, R^K_{F,t}$, nominal interest rates, $R^N_{H,t}, R^N_{F,t}$ and the exogenous processes $A_{H,t}$, $A_{F,t}$, $\chi_{H,t}$, $\chi_{F,t}$, $M_{H,t}$, $M_{F,t}$ such that, taking these as given, the following set of conditions are satisfied for all $t \geq 0$:

1. the first order conditions for intermediate goods purchases for consumption and investment, (2.13) and (2.19)
2. the first order conditions for capital accumulation, (2.21) and (2.22)
3. the optimal labour and capital hiring decisions of firms, (2.25)
4. the optimal price setting conditions for firms, (2.26) and (2.27)
5. the optimal wage setting conditions for unions, (2.33)
6. the first order conditions for stock purchases by households, (2.38) and (2.39)
7. the households' budget constraints, (2.1)
8. the market clearing conditions for goods, (2.41)
9. the market clearing conditions for assets, (2.42) and (2.43) .

We solve for a linear approximation to this equilibrium around the nonstochastic steady state, applying the methods developed by Devereux and Sutherland (2009a). One key insight of their work is that a first order accurate approximation of the dynamics of this class of models only depends on the non time varying part of portfolios ("steady state portfolios"). Standard linearisation approaches cannot solve for these portfolios, for two reasons. Firstly, optimal portfolios are not uniquely pinned down in a nonstochastic steady state, so there is no natural point around which to approximate. Secondly, optimal portfolios are also not uniquely defined in a first order approximation of a DSGE model, as it satisfies certainty equivalence and implies that all assets are perfect substitutes. The methods described in Devereux and Sutherland (2009a) show how to overcome these problems. Firstly, building on earlier work by Samuelson (1970), and Judd and Guu (2001), the authors demonstrate that the steady state portfolio corresponds to a bifurcation point in the set of nonstochastic equilibria. Secondly, they show that we need to combine a second order accurate approximation of the portfolio euler equations, (2.38) – (2.39), with a first order accurate approximation of the rest of the model in order to arrive at a solution for steady state portfolios.

2.4 Portfolios and Business Cycles in an Open Economy DSGE Model

In this section, we calibrate the model and present optimal portfolios and business cycle statistics. This section also discusses the intuition behind the portfolio positions and in particular, the role of nominal rigidities, endogenous capital accumulation, and monetary policy.[18] We find that optimal equity positions in the steady state feature home bias of a realistic size and that this home bias is driven mainly by the motive to hedge human capital risk, and less so by the motive to hedge consumption expenditure risk. Relative equity returns are unconditionally positively correlated with human capital returns, but negatively, once we condition on relative bond returns. Steady state positions in Home bonds are positive and equal to roughly 50% of GDP and the model generates realistic cross country correlations in output, consumption, investment and labour.

2.4.1 Calibration

We adopt a benchmark calibration that closely follows the literature on international business cycles and portfolios (e.g. Backus et al. (1994), Chari et al. (2002), Coeurdacier et al. (2007, 2009), Heathcote and Perri (2009)) and present our parameter choices in Table 2.2. The model is assumed to run at quarterly frequency. We choose $\widetilde{\beta}$ and η such that the steady state discount rate is equal to 0.99, implying an annual steady state real interest rate of 4.1%. The degrees of consumption and investment home bias are set to $a = 0.85$, implying a steady state import/GDP ratio of 15%. There is great variety in the estimates of the elasticity of substitution between Home and Foreign goods. Macroeconomic studies often estimate values close to, and sometimes below, one, while microeconomic estimates are much larger (see Harrigan (1993) and Hummels (2001) for micro studies and Heathcote and Perri (2002) and Backus and et al (1994) for macro estimates. See also Imbs and Mejean (2009) and the discussion in Coeurdacier

[18]The appendix also discusses the role of bond maturity and real bonds.

(2009)). We follow the macro literature and set the elasticity of substitution between Home and Foreign goods to 1.1. The elasticity of substitution between individual varieties of intermediate goods, ε, and individual labour varieties, ε_w, is set to 10. The former implies an average markup of 11% of prices over marginal costs, while the second implies an average markup of 11% of wages over the the marginal rate of substitution between consumption and labour. The intertemporal elasticity of substitution, σ, is set to 2, in line with much of the domestic and open economy business cycle literature (e.g. Backus et al (1992, 1994) or Coeurdacier et al (2009)). The nominal stickiness parameter for prices, θ, is set to 0.7, which implies an average duration for prices of 3.3 quarters. The analogous parameter for wages, θ_w, is set to 0.8, as the degree of wage rigidity is often found to be larger than the degree of price rigidity (see e.g. Christiano et al. (2005)). The rate of depreciation is set to $\delta = 0.025$ implying an annual rate of depreciation of around 10%. The elasticity of output with respect to labour, α, is set to 0.78 which, together with the other parameters, implies a steady state labour share of 0.70, the mean value for the US from $1970 - 2008$. ω is set to zero, implying that utility is linear in labour. The investment adjustment cost parameter, υ, is set to $\upsilon = 0.9$. For the monetary policy rule, we use the estimation results of Canzoneri et al. (2007) for the US which imply $\rho_r = 0.824$, $\gamma_\Pi = 2.02$, $\gamma_Y = 0.184$ and $\sigma_M = 0.00245$ and assume that monetary shocks across countries are not correlated. For the neutral technology process, we rely on Heathcote and Perri (2002) who estimate an AR(1) processes for TFP for US and rest of the world and obtain $\rho_A = 0.91$, $\sigma_A = 0.006$ and a cross country correlation of productivity shocks, $\rho_{A,A}$, of 0.25. For investment specific technology, we follow Fisher (2006) and Coeurdacier et al (2009) and use the ratio of CPI to the investment price deflator as an estimate of the investment shock. Estimating AR(1) processes for the US and rest of the world investment shock, we then obtain $\sigma_\chi = 0.0051$, $\rho_\chi = 0.83$ and $\rho_{\chi,\chi} = 0.21$, where the latter denotes the correlation of investment specific technology shocks between the two countries.[19]

2.4.2 International Bond and Equity Portfolios

In Table 2.3, we present equity and bond portfolios for our benchmark calibration, as well as the contributions of the two motives to hedge risks to these positions, and the unconditional correlations between asset return innovations and sources of risk.

[19] See the appendix for further details on data sources and the estimation of investment specific technology.

Table 2.2: **Calibration**

Parameter	**Interpretation**	**Value**
$\widetilde{\beta}$	Discount Rate	0.99
a	Consumption Home Bias	0.85
σ	1/Intertemporal Elasticity of Substitution	2
ϕ	Substitution Elasticity Home/Foreign	1.1
ε	Subst. Elasticity for Domestic Varieties	10
ε_w	Subst. Elasticity for Labour Varieties	10
θ	1/Average Duration of Prices	0.7
θ_w	1/Average Duration of Wages	0.8
δ	Depreciation Rate	0.025
α	Labour Share in Production	0.78
ω	1/Elasticity of Labour Supply	0
ρ_r	Persistence of Interest Rate	0.79
γ_π	Response to Inflation	2.02
γ_y	Response to Output Growth	0.184
η	Endogenous Discount Rate Elasticity	0.01
υ	Investment Adjustment Cost Elasticity	0.9
ρ_A	Persistence of Productivity Shock	0.91
ρ_M	Persistence of Monetary Shock	0
ρ_χ	Persistence of Investment Shock	0.83
σ_A	Std of Productivity Shock	0.0061
σ_χ	Std of Investment Shock	0.0051
σ_M	Std of Money Shock	0.0025
$\sigma_{A,A}$	Correlation between Productivity Shocks	0.25
$\sigma_{\chi,\chi}$	Correlation between Investment Shocks	0.19
$\sigma_{M,M}$	Correlation between Money Shocks	0

The fraction of U.S. equity held by foreign investors is almost perfectly matched by the model. According to the U.S. Treasury Report on Foreign Portfolio Holdings of U.S. Securities (2009a), foreign investors were holding 10.4 percent of the U.S. equity stock in 2008 and the report shows that this proportion has been stable in recent years. Our reduced form expressions, (2.7) and (2.8), allow us to make some further observations. The covariance variance ratios in these expressions can be recovered by simulating the model and running the following two regressions on the simulated data:

$$\widehat{R}_{t+1}^{WL} = k_1 + \beta_{wl,b}\widehat{R}_{t+1}^{B} + \beta_{wl,s}^{i}\widehat{R}_{t+1}^{S} + \varepsilon_t^{wl} \tag{2.47}$$

$$\widehat{R}_{t+1}^{PC} = k_2 + \beta_{pc,b}\widehat{R}_{t+1}^{B} + \beta_{pc,s}^{i}\widehat{r}_{t+1}^{S} + \varepsilon_t^{pc}, \tag{2.48}$$

where k_1, k_2 are constants, and $\varepsilon_t^{PC}, \varepsilon_t^{WL}$ are the error terms of the regression. The relevant covariance variance ratios are then given by the regression coefficients:

$$\beta_{pc,s} = \frac{cov_{R_t^B}\left(\widehat{R}_t^{PC}, \widehat{R}_t^{S}\right)}{var_{R_t^B}\left(\widehat{R}_t^{S}\right)} \quad \beta_{wl,s} = \frac{cov_{R_t^B}\left(\widehat{R}_t^{WL}, \widehat{R}_t^{S}\right)}{var_{R_t^B}\left(\widehat{R}_t^{S}\right)} \tag{2.49}$$

$$\beta_{pc,b} = \frac{cov_{R_t^S}\left(\widehat{R}_t^{PC}, \widehat{R}_t^{B}\right)}{var_{R_t^S}\left(\widehat{R}_t^{B}\right)} \quad \beta_{wl,b} = \frac{cov_{R_t^S}\left(\widehat{R}_t^{WL}, \widehat{R}_t^{B}\right)}{var_{R_t^S}\left(\widehat{R}_t^{B}\right)}. \tag{2.50}$$

Substituting the steady state values and regression coefficients into (2.7) and (2.8) , we can decompose the observed asset positions into the parts induced by the human capital and consumption expenditure motive, respectively. We observe that, in the model, equity home bias is driven mainly by the motive to hedge human capital risk, and to a much smaller degree the motive to hedge consumption expenditure risk. Interestingly, the motive to hedge human capital risk induces U.S. investors to hold U.S. equity despite the fact that the *unconditional* correlation between relative equity return innovations and human capital risk is *positive.* One argument against the hypothesis that equity home bias may arise from a motive to hedge human capital returns has been that the unconditional correlation between equity returns and human capital returns is positive in the data (Baxter and Jermann (1997)). However, as mentioned above, it is the correlation between the two, conditional on relative bond returns, that matters.[20] Previous work has shown that the conditional correlation between the two may well be negative in

[20] This point has also been emphasized by, amongst others, Coeurdacier et al (2009), Coeurdacier and Gourinchas (2009) and Engel and Matsumoto (2009).

the data and we provide some further evidence below. To our knowledge, however, ours is the first contribution that presents a model that can replicate the fact that the unconditional correlation between relative equity returns is positive, while the conditional one is negative.

Steady state holdings of Home bonds are 50% of GDP, with the equivalent short position in Foreign bonds. Available data indicates that, on aggregate, U.S. investors have a short position in U.S. bonds of between 35 and 45% of GDP in 2008.[21] Our benchmark model thus predicts a bond position of a realistic magnitude, but not of the right sign for the U.S. However, Lane and Shambaugh (2007, 2008) show that developed economies, on average, have negative net foreign-currency debt positions, in line with the prediction of our benchmark model. The U.S. is a notable exception and it can plausibly be argued that its outlier status is due to a number of features that are not captured by the model, such as the special role of the dollar as a reserve and vehicle currency. Comparison of the model implied bond positions with their real life counterpart is complicated by several other factors. Firstly, net positions in forward currency markets are equivalent to bond positions in the model and the correct measure of bond positions in the data should therefore net out positions in bond and currency forward markets, but data on the latter is scarce. Secondly, the model features net zero bond positions and firms are fully financed by equity. Coeurdacier et al (2009) show that when firms are partly financed by debt and the Modigliani-Miller theorem applies, agents should hold the same proportion of foreign corporate debt as of foreign equity. With positive foreign equity exposures, this would imply higher gross holdings of foreign debt and thereby reduced net debt positions in Home bonds. Table also indicates the drivers of bond positions. Relative bond returns are positively correlated with human capital and consumption expenditure risk, both unconditionally and conditioning on relative equity returns. Thus, the two motives are offsetting, with the consumption expenditure motive inducing agents to take a long position in Home bonds, while the human capital motive induces Home investors to go short in Home bonds. The consumption expenditure motive prevails, leading to an overall short position in Home bonds. Below, we will provide evidence that the conditional and unconditional correlations between bond returns

[21] According to the U.S. Treasury Report on U.S. Holdings of Foreign Securities (2009b), U.S. investors held debt securities worth $1,543$ billion US dollars in 2008, while the U.S. Treasury Report on Foreign Holdings of U.S. Securities (2009a) estimates foreign holdings of U.S. debt securities at 6642 billion U.S. dollars, implying a net short position of 5099 billion U.S. dollars, or 36% of GDP. An updated version of the Gourinchas and Rey (2007a), kindly provided by Hélène Rey, estimates the U.S. short position at 6550 billion U.S. dollars or 46% of GDP.

Table 2.3: **International Equity and Bond Portfolios and Correlations**

a) Portfolios	**Share of Home Equity**	**Home Bonds/ GDP**
Total	**0.90**	**0.49**
due to:		
Pure Diversification	0.50	0.00
Human Capital	0.34	−0.76
Consumption Expenditure	0.06	1.24
b) Correlations	$\widehat{R}^S$	$\widehat{R}^B$
$\widehat{R}^{WL}$	0.25	0.96
$\widehat{R}^{PC}$	0.49	0.99

Contributions of hedging motives and correlations were computed relying on 500 simulations of the benchmark model for 500 periods. Contributions of hedging motives were calculated according to the reduced form expressions (2.7) and (2.8) and using the coefficients obtained from running the regressions (2.47) and (2.48) on the simulated data. Relative bond and equity returns and human capital and consumption expenditure risk were Hodrick Prescott filtered with a smoothing parameter of 1600 before computing correlations.

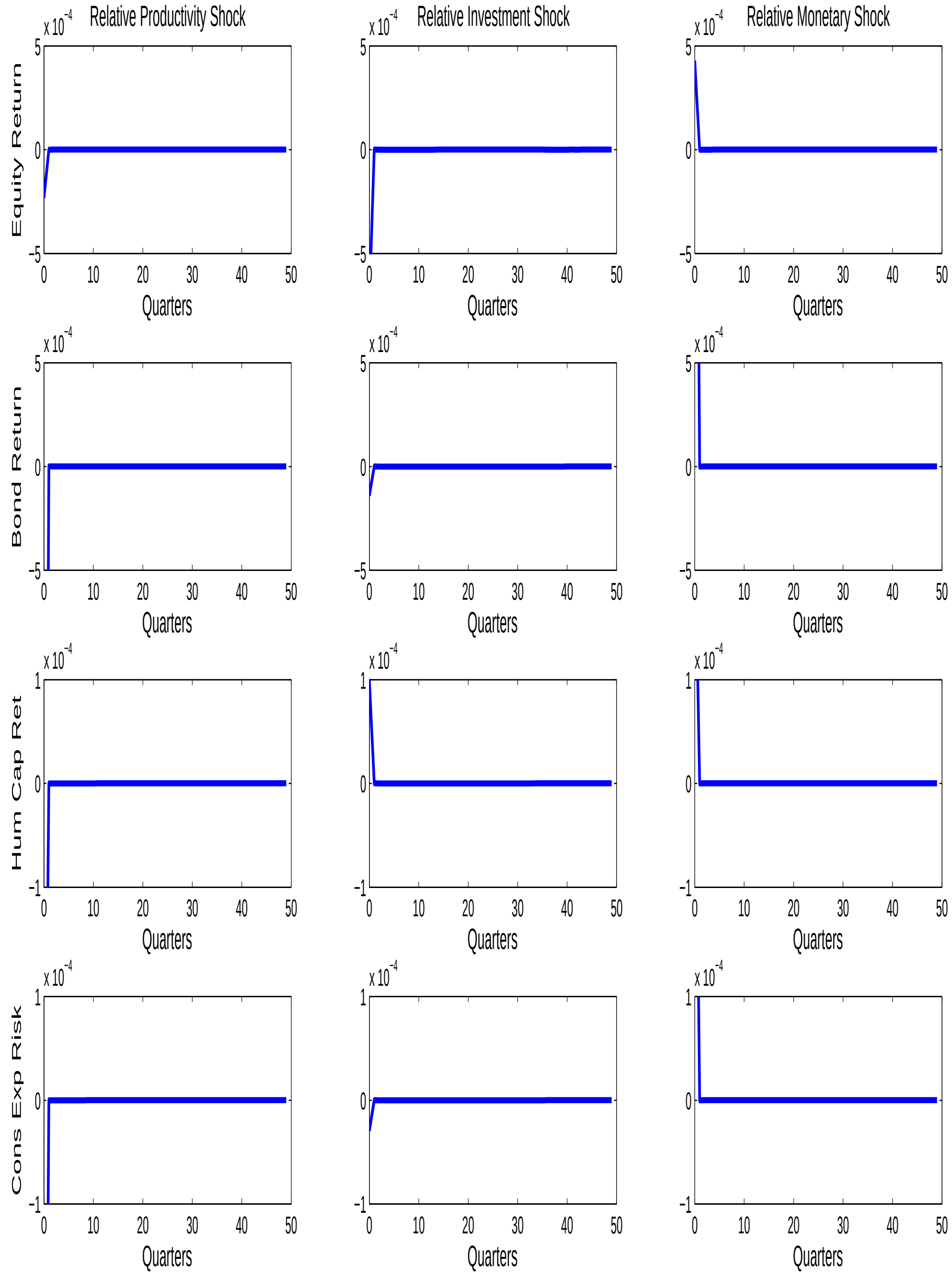
and human capital and consumption expenditure risk in the model qualitatively match those in the data for the U.S.. However, the consumption expenditure motive is stronger in the model, thereby implying the overall long position in U.S. bonds, while the data indicates that the human capital motive dominates and thus leads to an overall short position in U.S. bonds for U.S. investors.

2.4.3 Discussion and Robustness

In our framework, relative bond and equity returns, and consumption and human capital risk are positively correlated. This is because a relatively low elasticity of substitution and home bias in consumption imply that nominal exchanges respond quite strongly to shifts in supply and demand. Nominal exchange rates in turn contribute to the comovement of asset returns and sources of risk. In Figure **??**, we present the impulse responses of relative equity and bond returns and relative human capital and consumption expenditure risk to positive relative shocks

Figure 2.1: **Impulse Responses of Asset Returns and Sources of Risk**

This figure presents the response of relative equity returns, relative bond returns, relative human capital returns and consumption expenditure risk to relative shocks to neutral and investment specific technology and the interest rate. Relative shocks are defined as a positive shock to the Home variable of one standard deviation, with a negative shock to the foreign variable of the same size. Note that a relative monetary shock here is a positive shock to nominal interest rates and is therefore contractionary



to neutral and investment specific technology and the interest rate.[22,23] The first column of the figure indicates that equity and bond returns, as well as human capital and consumption expenditure risk fall in response to a relative productivity shock, while the third columns shows that all four variables increase in response to a contractionary monetary shock. Figure 2.2, which plots the responses of relative dividends, nominal exchange rates, relative labour income and relative consumption expenditures, indicates why.[24] Comparing the responses in the first column of Figure **??** with the second column of Figure 2.2, we can see that the asset returns and sources of risk move in the same direction as the nominal exchange rate which depreciates for about three years after a relative productivity shock.[25] Comparing the third column of the two figures, we see that the response of asset returns and sources of risk after a relative shock to investment efficiency again follows the nominal exchange rate, which in this case appreciates, further illustrating its role in driving comovement between asset returns and sources of risk.

While these figures mainly illustrate why many variables are unconditionally positively correlated, they also already include a hint for why equity home bias is driven by the motive to hedge human capital risk in our model. As we can see, the one exception to the general tendency of comovement occurs in response to relative shocks to investment efficiency. Relative equity returns fall strongly, while relative bond returns and consumption expenditure risk decrease only slightly, and human capital returns increase strongly. The responses of relative equity and human capital returns are driven by the combination of endogenous capital accumulation and nominal price rigidities. In response to shocks to investment specific technology, the demand for investment increases which, with home bias in investment, implies an increased demand for Home labour. Firms would like to react to the increase in demand by raising prices, but due to nominal rigidities,

[22]As noted before, the symmetry of the model implies that we can focus on the response of relative variables without loss of generality. Similarly, we can focus on the response of relative shocks whereby, e.g. a relative Home productivity shock is defined as a positive one standard deviation shock to Home productivity, with a negative shock to Foreign productivity of equal size. Note that a positive shock to the interest rate implies a contractionary monetary policy shock.

[23]Note that the fact that all of the variables here are defined as innovations implies that their impulse responses will be zero after one period.

[24]Equations (2.2) – (2.5) indicated that relative equity returns depend on the present value of relative dividend innovations, relative bond returns depend on the nominal appreciation in the following period, and human capital and consumption expenditure risk depend on the present value of relative labour income and consumption expenditure innovations, respectively. In the appendix, we also provide the impulse responses for output, consumption, investment, labour demand, the real exchange rate and net exports.

[25]Note that a rise in the nominal exchange rate here denotes a depreciation.

Figure 2.2: **Impulse Responses of Portfolio Relevant Variables**

This figure presents the response of relative dividends, nominal exchange rates, relative labour income and relative consumption expenditures to relative shocks to neutral and investment specific technology and the interest rate. Relative shocks are defined as a positive shock to the Home variable of one standard deviation, with a negative shock to the foreign variable of the same size. Note that a rise in the nominal exchange rate signifies a depreciation of the Home nominal currency.

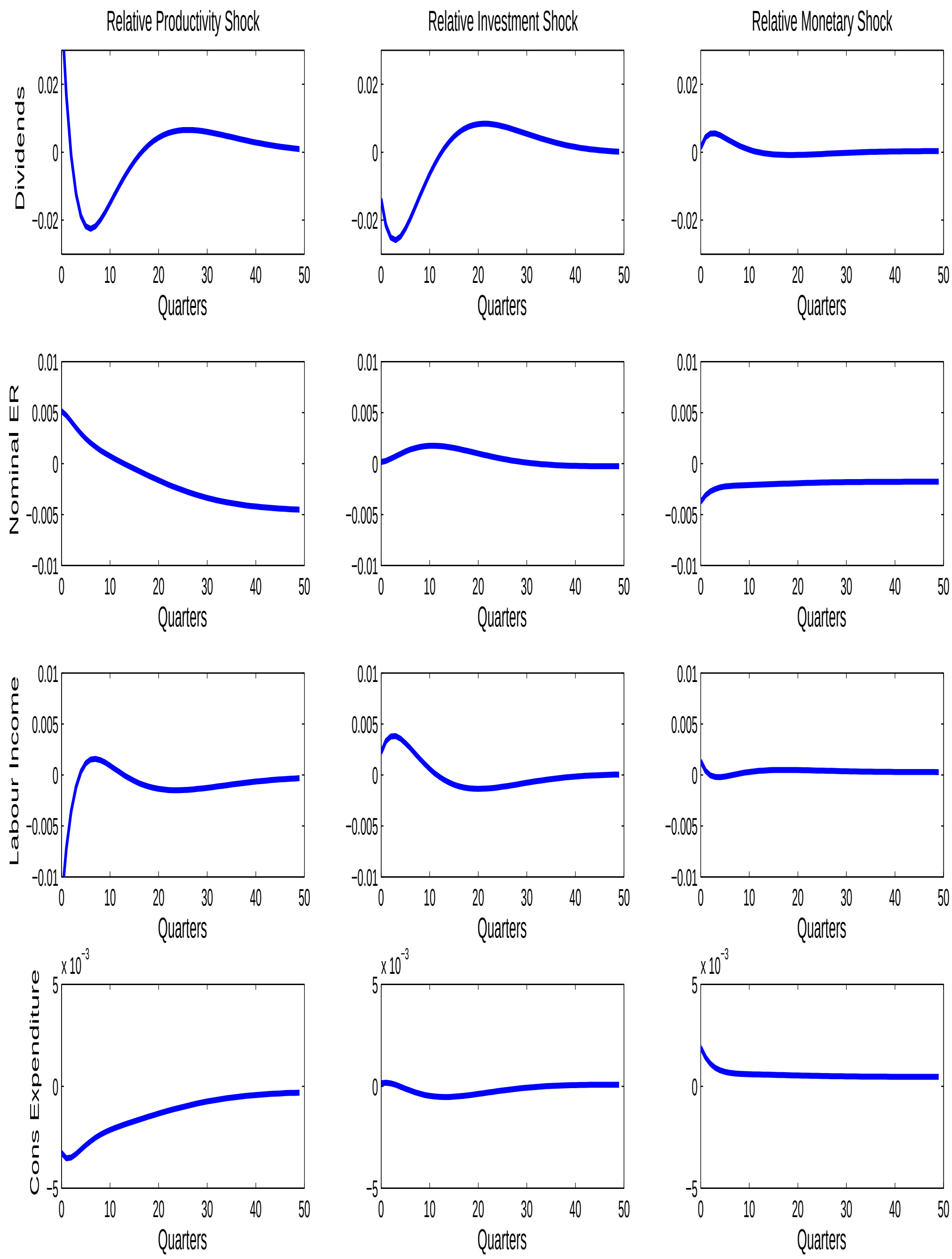
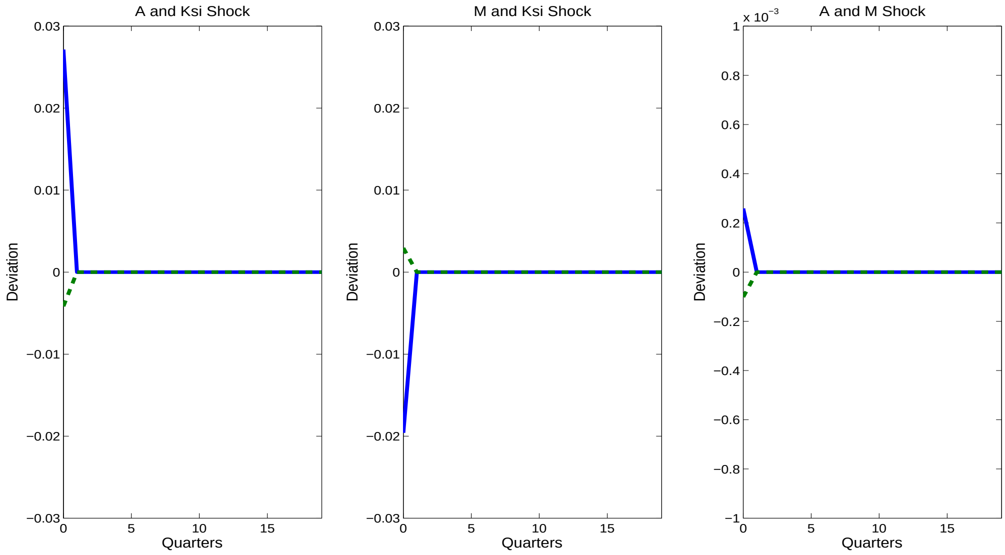


Figure 2.3: **Impulse Responses of Relative Equity Returns and Relative Human Capital Returns, conditional on Relative Bond Returns**

This figure presents the response of relative equity returns and relative human capital returns for combinations of shocks that leave relative bond returns unchanged. The first graph presents responses to a combination of neutral and investment specific technology shocks, the second to a combination of monetary and investment specific technology shocks and the third to a combination of neutral technology and monetary shocks. Responses of relative equity returns are given by the solid line, while responses of relative human capital returns are given by the dashed line.



not all firms can do so. The limited rise in prices implies that the increase in the demand for investment is higher than it would have been if prices were flexible and the corresponding increase in labour demand raises relative human capital returns. Payouts to shareholders fall, as the rise in investment is paid for by retained earnings, and the fact that the investment boom is even larger with price rigidities implies that relative equity returns fall strongly. Thus, conditional on the investment efficiency shock, relative equity and human capital returns are negatively correlated. The positive comovement in response to productivity and monetary shocks implies an overall positive correlation between equity and human capital returns. But since bond returns only fall slightly in response to investment specific shock, the correlation of human capital returns with bond returns is more strongly positive overall than with equity returns.

Above we stressed several times that it is the comovement of relative equity returns with human capital returns, conditional on relative bond returns, that drives equity positions. One way to investigate this relationship more directly is to consider several combinations of shocks that imply that relative nominal bond returns are unchanged. Since nominal bond returns are by construction zero in these scenarios, there is no distinction between the unconditional comovement of equity and human capital returns and the correlation, conditional on relative bond returns. Since markets are incomplete in our setup, the combination of shocks that implies a zero response of relative bond returns is not unique. However, for each pair of shocks, the combination is unique up to a scalar multiplication, and we present the three combinations in Figure 2.3. For example, Figure **??** indicates that relative bond returns fall by 0.5 percent in response to a one standard deviation shock to relative productivity, while they fall by 0.014 percent in response to an investment specific technology shock. A combination of a one standard deviation positive relative shock to productivity and a negative relative shock to investment specific technology of $0.5/0.014 = 36.79$ standard deviations therefore leaves relative bond returns unchanged. The response of relative equity returns and relative human capital returns to such a combination of shocks is depicted in the first column of Figure 2.3. It clearly shows that the two are negatively correlated. The second and third column present the responses to combinations of investment efficiency and interest rate shocks, and productivity and interest rate shocks, respectively. Both of them also show a negative comovement of relative equity returns with relative human capital returns. Thus, the conditional correlation of human capital returns and equity returns, conditional on relative bond returns, is negative for all three shocks, and therefore overall. Figure 2.3 only focuses on the conditional comovement of relative equity returns with relative human capital returns. The reason is that for the comovement of relative equity returns with consumption expenditure risk, and for the comovement of relative bond returns with both human capital and consumption expenditure risk, the distinction between the conditional and unconditional moments is not too substantial. The correlation between relative equity returns and consumption expenditure risk is positive, and induces Home investors to take long positions in Home equity beyond those induced by the human capital motive, but the effect is small. The reason is that relative bond returns are highly positively correlated with consumption expenditure risk and a long position in Home bonds therefore hedges consumption expenditure risk effectively. The correlation between relative bond returns and relative human capital returns is also positive, and this motive therefore induces the Home investor to take short positions in Home bonds, and thereby partly offsets the effects of the consumption expenditure motive on bond positions.

The Role of Nominal Rigidities

In order to highlight the effects of nominal rigidities in our framework, the second column of Table 2.4 presents a version of the model without nominal price rigidities ("Flex Price").[26] Nominal price rigidities affect portfolios through two mechanisms here. Firstly, the bonds traded are nominal and, as mentioned above, relative bond returns are related to fluctuations in the nominal exchange rate. Higher degrees of nominal rigidity imply that fluctuations in nominal exchange rates are more closely related with fluctuations in real exchange rates which are in turn closely associated with fluctuations in human capital and consumption expenditure risk. The Flex Price model therefore features a lower correlation between relative bond returns and human capital and consumption expenditure risk and lower nominal bond positions.

Secondly, both the human capital motive and the consumption expenditure motive for holding domestic equity are affected. The previous section discussed how nominal price rigidities imply that investment and labour demand respond strongly to shocks to investment specific technology and that the strong investment and labour response induces relative equity and human capital returns to comove negatively in response to this shock. Without price rigidity, the investment and labour response is correspondingly smaller and, relative human capital returns fall as a result, and relative equity returns also fall. Both the conditional and the unconditional correlation of equity returns with human capital returns are then positive which implies that the human capital motive induces Home agents to take a *short* position in Home equity in this case. It is worth pointing out here that the effect of nominal rigidities on equity portfolios is different from the one in Engel and Matsumoto (2009). In their model, there are nominal price rigidities, but there is no endogenous capital accumulation. As in our model, a sufficient degree of price rigidities implies that relative human capital returns decrease in response to a productivity shock. This is because a productivity shock implies that less workers are needed to produce a given level of output. Output increases, but nominal price rigidities prevent some firms from lowering prices in order to stimulate demand. If price rigidities are large enough, relative labour income falls persistently, and therefore relative human capital returns fall. But unlike in our

[26] We set $\theta = 0$. Note that despite the absence of nominal price rigidities, monetary shocks are not neutral, as there are still nominal rigidities in wages.

model, relative equity returns rise, both due to the increase in productivity, as well as a fall in the wage bill, and relative equity and human capital returns are therefore negatively correlated. In our model, relative equity returns fall, because the increase in endogenous investment implies a lower payout to shareholders. In our setting, equity and human capital returns are negatively correlated in response to *investment specific* technology shocks. Another difference to Engel and Matsumoto (2009) is that monetary policy responds endogenously in our model, whereas money supply was exogenous in their setting and therefore may magnify the importance of nominal rigidities. We show here that price rigidities remain effective, even in an environment where monetary policy is endogenous and responds actively, if not optimally, to inflation.

Table 2.4 shows that the equilibrium equity portfolio with flexible prices is nevertheless strongly home biased and that is because of the effect of price flexibility on the consumption expenditure motive for holding domestic equity. Since with price flexibility nominal bonds are no longer as effective in hedging consumption expenditure risk, Home agents now take a large long position in Home equity in order to hedge this source of risk.

Nominal wage rigidities have less effects on portfolios than nominal price rigidities. The reason is that the presence of nominal wage rigidities does not translate into markedly different behaviour of labour income. The limited movement in wages due to wage rigidity is counteracted by larger variation in the demand for labour, leaving overall labour income relatively little affected.[27]

The Role of Capital Accumulation

In the third column of Table 2.4 we also present portfolios for a model without capital accumulation ("Fixed Capital").[28] In Heathcote and Perri (2009) and Coeurdacier et al (2009), endogenous capital accumulation drives the negative comovement between equity returns and human capital returns. As we can see in the table, this is not the case in our benchmark model. Equity returns are negatively correlated with human capital returns, both conditionally and unconditionally in the Fixed Capital model, despite the absence of endogenous capital accumulation. The reason is that now equity and human capital returns are negatively correlated in response to a productivity shock. The mechanism is essentially the

[27]Portfolios for a model in which nominal wages are perfectly flexible $(\theta_w = 0)$, but that is otherwise identical to the benchmark model are presented in the appendix.

[28]In this model, we fix the capital stock by setting investment and the depreciation rate to zero at all times.

Table 2.4: **The Role of Nominal Rigidities, Capital Accumulation and Monetary Policy for Portfolios**

		Bench-mark	**Flex Price**	**Fixed Capital**	**Hawk**	**Dove**	**High** σ_M	**Low**σ_M	**Currency Union**
		(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	$(VIII)$
Share of Home Equity		**0.90**	**1.30**	**−0.35**	**0.98**	**0.74**	**0.76**	**1.00**	**0.49**
	due to:								
	Human Capital	0.36	−0.21	2.45	0.25	0.61	0.57	0.18	0.99
	Consumption Exp	0.04	1.02	−3.30	0.23	−0.37	−0.31	0.32	−0.98
Correlations:									
	$Corr\left(\widehat{R}^S, \widehat{R}^{WL}\right)$	0.24	0.91	−0.91	0.35	−0.02	0.16	0.32	−0.97
	$Corr\left(\widehat{R}^S, \widehat{R}^{PC}\right)$	0.49	0.97	−0.89	0.56	0.31	0.48	0.53	−0.97
Home Bonds/ GDP		**0.49**	**0.30**	**0.28**	**0.47**	**0.46**	**0.32**	**0.61**	**0.00**
	due to:								
	Human Capital	−0.76	−0.29	−0.73	−0.73	−0.71	−0.48	−0.95	0.00
	Consumption Exp	1.24	0.59	0.45	1.21	1.17	0.80	1.57	0.00
Correlations:									
	$Corr\left(\widehat{R}^B, \widehat{R}^{WL}\right)$	0.96	0.96	0.83	0.99	0.87	0.88	0.99	0.00
	$Corr\left(\widehat{R}^B, \widehat{R}^{PC}\right)$	0.99	0.91	0.86	1.00	0.98	0.99	0.99	0.00

(I) follows the benchmark calibration described in the text, (II) sets $\theta = 0$, (III) fixes the capital stock by setting both investment and the rate of depreciation, δ, to zero, (IV) sets $\gamma = 1.5$, (V) sets $\gamma = 2.5$, (VI) sets $\sigma_M = 0.012$, (VII) sets $\sigma_M = 0.049$, $(VIII)$ replaces the two monetary rules by one that responds to inflation and output in both countries symmetrically. Contributions of hedging motives and correlations were computed relying on 500 simulations of the benchmark model for 500 periods. Contributions of hedging motives were calculated according to the reduced form expressions (2.7) and (2.8) and using the coefficients obtained from running the regressions (2.47) and (2.48) on the simulated data. Relative bond and equity returns and human capital and consumption expenditure risk were Hodrick Prescott filtered with a smoothing parameter of 1600 before computing correlations.

same as in Engel and Matsumoto (2009) which was discussed in the last section. Importantly, this model implies that equity and human capital returns are both conditionally and unconditionally negatively correlated, unlike in the data. The role of endogenous capital accumulation is therefore to generate an unconditionally positive correlation between relative equity and human capital returns, while nominal rigidities generate a negative correlation between relative human capital returns and equity returns, conditional on relative bond returns.

Despite the negative correlation between equity and human capital returns, the overall equity position here exhibits strongly negative equity home bias. This is because equity returns are now strongly negatively correlated with consumption expenditure risk, unless price stickiness is large, and this induces the Home investor to take a large short position in Home equity.[29]

Relaxing investment adjustment costs has only small effects on portfolios here. The main effect of adjustment costs on portfolios here is to drive a wedge between equity returns and consumption expenditure returns. Removing investment adjustment costs thus increases the correlation between equity and consumption expenditure risk, but the effects are modest on bond and equity portfolios.[30]

The Role of Monetary Policy

We model monetary policy using a Taylor type rule, as is standard in the New Keynesian literature (see Gali (2007) and Woodford (2003) for excellent introductions). We already noted that monetary policy is endogenous in our benchmark calibration, and it responds actively, if not optimally, to inflation and output growth. While it does not undo the effects of nominal rigidities entirely, monetary policy does affect equilibrium portfolios, for two reasons. Firstly, monetary policy has real effects in this model, due to nominal rigidities in price and wage setting.

[29] At high degrees of nominal rigidity, we obtain equity home bias of a realistic size in this model also. This is because the correlation of consumption expenditure falls with regard to equity returns and rises with regard to bond returns, therefore reducing the effect of the consumption expenditure motive on equity holdings. Engel and Matsumoto (2009) obtain equity home bias with more modest levels of price rigidity in a model without endogenous capital accumulation. Our results are not directly comparable to theirs, since they assume a different form of price stickiness, but most likely the fact that money supply is exogenous in their framework implies that lower degrees of price rigidity are necessary to generate equity home bias.

[30] In the appendix, we also present a model calibration that features neither nominal rigidities in wage and price setting, nor investment adjustment costs ($\theta = \theta_w = 0$ and $\upsilon \to \infty$). This calibration can qualitatively produce positive unconditional and negative conditional correlations between equity returns and human capital returns. However, it cannot produce home bias of a reasonable size, as the effect of the human capital motive is too small, while equity returns are highly correlated with consumption expenditure risk. What is more, the lack of investment adjustment costs implies negative cross country correlations in investment.

Monetary policy will therefore affect the dynamics of labour income and consumption expenditures and therefore our sources of risk. Secondly, monetary policy also affects the dynamics of asset returns. For example, relative nominal bond returns solely depend on nominal exchange rate movements and these are strongly affected by monetary policy. The effects of more or less responsive monetary policy on portfolios are documented in the fourth and fifth column of Table 2.4. There, we present portfolios for monetary policy rules that are more responsive to inflation ("Hawk") or less responsive ("Dove") than in our benchmark calibration. More responsive monetary policy lowers the correlation between relative equity returns and human capital risk, and raises its correlation with consumption expenditure risk. As a result, the motive to hedge human capital risk increases equity home bias by less, while the consumption expenditure motive increases it by more than in the benchmark. Less activist monetary policy has the opposite effect. In terms of portfolios, the effects of changes in monetary policy responsiveness are therefore similar to changes in nominal price rigidities and, for example, increases in monetary policy responsiveness have similar effects as reductions in nominal price stickiness on equity portfolios. Increases in the volatility of the monetary shock ("High σ_M") have similar effects on equity portfolios as reductions in monetary policy responsiveness. However, their effect on bond positions is larger, with larger values of the monetary shock associated with lower absolute levels of bond holdings. This is because the larger level of volatility of monetary shocks reduces the efficacy of nominal bonds as a risk hedging instrument.[31]

In standard New Keynesian models, monetary policy has an effect on welfare by modifying the degree of inefficient price dispersion. As noted by Devereux and Sutherland (2007, 2008), in models with endogenous portfolio choice and incomplete markets, there is an additional channel through which monetary policy affects welfare. As monetary policy alters asset returns, it thereby changes the hedging properties of different assets. By changing the hedging properties of the asset menu, monetary policy affects the degree of risk sharing and thereby welfare. In our framework, markets are incomplete, even at the first order of approximation, as the number of shocks per country exceeds the number of assets and monetary policy therefore affects welfare beyond its effects on price dispersion. However, the degree of risk sharing in the model is high under most specifications of monetary policy, for two reasons. Firstly, two assets per country are enough to generate high

[31] Reductions in the persistence of the interest rate, ρ_r, also have similar effects to increases in monetary policy responsiveness, while changes in the responsiveness to output growth, γ_Y, have only small effects on portfolios.

risk sharing in many open economy settings (see also Benigno and Kucuk-Tuger (2009)), including ours. What is more, the response of relative prices does much to share risk between countries here (Cole and Obstfeld (1991)). The welfare effects of monetary policy through the additional channel of affecting the hedging properties of assets are therefore likely to be small.

In the last column of Table 2.4, we present portfolios for the case of a currency union between the two countries ("Currency Union"). In that case, the two monetary policy rules in (5.27) are replaced by a single rule:

$$R_t^N = \left(R_t^N\right)^{\rho_r} \left(\frac{1}{\beta}\left(\frac{P_{C,t}}{P_{C,t-1}}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\gamma_Y} M_t\right)^{1-\rho_r}, \tag{2.51}$$

where R_t^N is the single nominal interest rate for the currency union, $P_{C,t}/P_{C,t-1}$ is union wide inflation, Y_t/Y_{t-1} is union wide output growth and M_t is a union wide interest rate shock.[32] Thus, we now assume that the monetary policy authority responds to union wide changes in output and inflation. The currency union has the same effect as permanently fixing the nominal exchange here.[33] Several implications for portfolios result. The two nominal bonds are now equivalent from the viewpoint of both investors, as they offer the same nominal payout. Bond holdings in the steady state are therefore indeterminate and can be set to zero without loss of generality.[34] As a result, only equity portfolios are used to hedge both human capital and consumption expenditure risk. Neumeyer (1998) discusses the existence of a trade off for the effects of a monetary union on risk sharing. On the one hand, the formation of the currency union eliminates nominal exchange rate risk. On the other hand, it reduces the menu of assets available and thereby reduces hedging opportunities and both of these effects are present here. Under our benchmark calibration, a currency union would imply that investors would hold almost perfectly diversified portfolios. The human capital motive still induces domestic investors to take long positions in domestic equity. But the consumption expenditure motive now induces Home agents to take short positions and the two motives offset each other almost perfectly. For our benchmark calibration, currency union thus implies a large degree of diversification of equity holdings,

[32]Union wide inflation and output growth are given by: $\frac{Y_t}{Y_{t-1}} = \frac{1}{2}\left(\frac{Y_{H,t}}{Y_{H,t-1}} + \frac{Y_{F,t}}{Y_{F,t-1}}\right)$ and $\frac{P_{C,t}}{P_{C,t-1}} = \frac{1}{2}\left(\frac{P_{C,t}^H}{P_{C,t-1}^H} + \frac{P_{C,t}^F}{P_{C,t-1}^F}\right)$.

[33]Note, however, that real exchange rates are not constant, due to differences in preferences in the two countries.

[34]Relative bond returns are in any case zero and any bond holding would therefore not provide any hedging benefits for human capital or consumption expenditure risk.

but it is mainly driven by a change in the set of assets traded and not the removal of exchange rate risk.

2.4.4 Business Cycle Moments

In the previous subsection, we illustrated that the benchmark model implies equity home bias and bond holdings of a realistic size. Here, we show that, in addition to matching international asset portfolios, the model matches important features of the international business cycle. In Table 2.5, we present business cycle statistics for our benchmark model, several alternative model configurations, and the data. The table also shows the predictions of two other recent papers that investigate the business cycle implications of international financial asset trade in open economy models. Heathcote and Perri (2002) compare the business cycle implications of models with complete markets, trade in riskless bonds, and financial autarky. They argue that the model with financial autarky matches business cycle statistics most closely and its predictions are presented in the third column of Table 2.5 ("HP"). In Coeurdacier et al (2009), agents trade real consols and equities, but there are neither nominal rigidities, nor investment adjustment costs, and the business cycle predictions of their model are shown in the fourth column of Table 2.5 ("CKM").[35]

Our benchmark calibration is able to match many features of the international business cycle. In particular, it generates positive cross country correlations in output, consumption, investment, and output. Net exports are countercyclical and the correlation between domestic output and consumption and investment is highly positive, as in the data. As in most open economy models, the cross country correlation of consumption is higher than the one with output, while the reverse is true in the data. However, in our model the difference is relatively small, and substantially smaller than in the two other papers. Coeurdacier et al (2009) note that the presence of investment specific technology shocks lowers the cross country correlation of consumption and the correlation between output and the real exchange rate. This is true in our model also, but the table shows that it does not come at the expense of counterfactual behaviour of investment.[36] The fifth and sixth column illustrate that we can trace the difference in predictions to the

[35] The other two papers that are closely related to our study and that we discussed extensively above, Engel and Matsumoto (2009) and Heathcote and Perri (2009) do not present business cycle statistics.

[36] Heathcote and Perri (2002) also report business cycle statistics for models in which countries trade a riskless bond, or where markets are complete. In those cases, the model also predicts negative cross country correlations in investment and also employment.

Table 2.5: **Business Cycle Statistics**

Statistic	**Data** (I)	**Bench-mark** (II)	**HP** (III)	**CKM** (IV)	**Flex Price** (V)	**Free K** (VI)	**Fixed K** (VII)
Cross Country Correlation							
Output	0.57	0.43	0.24	0.17	0.27	0.34	0.19
Consumption	0.26	0.58	0.85	0.58	0.66	0.56	0.63
Investment	0.31	0.36	0.35	−0.37	0.37	−0.02	0.00
Hours worked	0.10	0.32	0.14	0.18	0.39	0.31	0.12
Correlation with GDP							
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.85	0.82	0.92	0.38	0.85	0.39	0.96
Investment	0.94	0.92	0.99	0.71	0.85	0.82	0.00
Hours worked	0.88	0.46	0.99	0.61	0.79	0.98	−0.71
Net exports	−0.35	−0.18	0.00	−0.07	0.10	−0.56	0.62
Real exchange rate	0.07	−0.44	−0.65	−0.22	−0.57	−0.18	−0.61
Standard Deviation[1]							
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.82	0.44	0.43	0.43	0.50	0.12	0.86
Investment	2.88	5.17	1.73	4.42	4.39	12.00	0.00
Hours worked	0.84	0.77	0.24	0.70	0.48	1.21	0.96
Net exports	1.50	0.11	0	0.57	0.10	0.24	0.14
Real exchange rate	6.35	0.77	1.00	0.74	0.82	0.23	1.48

(I) presents business cycle statistics for U.S. data from 1970 to 2008, (II) presents the statistics implied by the benchmark calibration, (III) is from Heathcote and Perri (2002), (IV) is from Coeurdacier et al (2009), (V) sets $\theta = 0$, (VI) sets $\upsilon \to \infty$, (VII) sets both investment and the rate of depreciation to zero. Standard deviations and correlations in model and data are for Hodrick Prescott filtered variables with a smoothing parameter of 1600. All variables with the exception of net exports were logged before applying the filter. Model moments result from 500 simulations of 500 periods.

[1]Standard deviations are expressed relative to the standard deviation of GDP

presence of nominal rigidities and investment adjustment costs. Without investment adjustment costs ("Flex K"), investment responds too strongly to shocks in investment specific technology which implies that the cross country correlation of investment becomes negative, and the correlation between domestic and output low, as in Coeurdacier et al (2009). As we noted above, nominal price rigidities imply that the response of investment and labour to investment efficiency shocks is quite strong. With perfect price flexibility, investment responds less strongly, implying procyclical net exports. The last column illustrates the effect of capital accumulation per se on the cyclicality of net exports. In models without endogenous capital accumulation, such as Engel and Matsumoto (2009), net exports are procyclical, due to the agents' motive to smooth consumption over time. During booms, agents would like to save in order to consume more in the future also. Without capital accumulation, the only way to do so is by increasing net foreign assets which implies a rise in net exports.

Like many other open economy models, our benchmark model nevertheless has difficulty to match some business cycle properties in the data. Thus, the real exchange rate is still not volatile enough and too highly correlated with GDP. The volatility of net exports is too low, while the volatility of investment is too high. Previous work has noted the difficulty of jointly generating realistic macroeconomic and asset pricing moments in a domestic context.[37] Our model is no exception. We find that the equity premium in our benchmark economy is too low. An extension of the model that allows for Epstein-Zin preferences, as in Rudebusch and Swanson (2009), and a more persistent productivity process can generate a realistic level of the equity premium, but at the expense of implying a volatility of the risk free rate that is too high.[38]

2.5 Estimated Moments and Portfolios

In this section, we test the empirical predictions of our model in an additional way. We construct asset return innovations and our measures of risk, (2.2) – (2.5), for the US, using quarterly data on G7 countries. Our estimation is closely related to Coeurdacier and Gourinchas (2009) who study international portfolio choice in a

[37]Mehra and Prescott (1985) document the difficulty of generating a realistic equity premium in standard general equilibrium endowment models. See Rouwenhorst (1995), Lettau and Uhlig (2000), Uhlig (2004), and Cochrane (2005) for a closed economy discussion of why allowing for endogenous production and further complicates the task of matching both asset pricing and macroeconomic moments.

[38]Results are available upon request.

very general static setting. The key difference in our estimation is that we focus on a dynamic measure of consumption expenditure risk as the second hedging motive in addition to human capital risk, while their second source of risk is a static measure of real exchange rate risk.[39] We find that, as in the model, the unconditional correlation between relative equity returns and relative human capital returns is moderately positive in the data. More importantly, we find that this correlation, conditional on relative bond returns, is negative, implying equity home bias. Relative equity returns are moderately positively correlated with relative consumption expenditure risk, both conditionally and unconditionally, again, as in the model. Relative bond are positively correlated with human capital and consumption expenditure risk, both conditionally and unconditionally. We then use our estimated moments and an adapted version of (2.7) and (2.8) to construct predicted equity and bond portfolios. The equity portfolio resulting from our estimation is virtually identical to the one implied by the benchmark model and available data on U.S. equity holdings. The bond portfolio implied by the estimation suggests a short position in U.S. bonds for U.S. investors, as observed in actual data on bond holdings. This is because the human capital motive outweighs the consumption expenditure motive, unlike in the model. However, the size of the predicted bond position is larger than observed bond positions for the U.S..

2.5.1 Constructing Innovations and Risk Measures

Our methodology follows Campbell and Shiller (1988) and Campbell (1996).[40] We are interested in estimating relative equity and bond return innovations, $(2.2) - (2.3)$, and human capital and consumption expenditure risk, $(2.4) - (2.5)$, for the U.S.. We therefore face two tasks. Firstly, the present value of future relative labour income and relative consumption expenditures are not observable - only current labour income and consumption expenditures are - and we therefore need to construct them using observable data. Secondly, we need to construct expectations of returns and risk in order to retrieve innovations in these variables.

For human capital, denote by $r^{WL}_{US,t+1}$ the log of the gross return on human capital in the US between t and $t+1$ and by $WL_{US,t}$ the level of labour income. Following Campbell (1996), under the assumption that the dividend price ratio

[39]In open economy models with complete markets, relative consumption expenditures are perfectly correlated with the real exchange rate. With incomplete markets, that is no longer the case, but relative consumption expenditures can be expressed as the sum of relative consumption volume and the real exchange rate, at the first order of approximation.

[40]Baxter and Jermann (1997), Coeurdacier and Gourinchas (2009) and Juillard (2003) have used similar methods.

on human wealth is stationary, we can write:

$$\begin{aligned} r^{WL}_{US,t+1} &= \log\left(WL_{US,t+1} + V^{WL}_{US,t+1}\right) - \log\left(V^{WL}_{US,t}\right) \\ &= k + \zeta_{US,t} - \rho\zeta_{US,t+1} + \Delta \log WL_{US,t+1}, \end{aligned} \tag{2.52}$$

where $V^{WL}_{US,t}$ measures human capital wealth, $\zeta_{US,t} = \log\left(WL_{US,t}/V^{WL}_{US,t}\right)$ is the log-dividend price ratio for human capital, and $\rho^{-1} = 1+\exp\left(\zeta_{US}\right) = \left(WL_{US} + V^{WL}_{US}\right)/V^{WL}_{US}$. As in Coeurdacier and Gourinchas (2009), we will use $\rho = 0.98$, while k is an unimportant constant. Solving this equation forward and imposing that $\lim_{t\to\infty} \rho^t \left(r^{WL}_{US,t} - \Delta \log WL_{US,t}\right) = 0$, we obtain (up to a constant):

$$\zeta_{US,t} = E_t \sum_{j=0}^{\infty} \rho^j \left(r^{WL}_{US,t+j} - \Delta \log WL_{US,t+j+1}\right). \tag{2.53}$$

This expression states that the ratio of labour income to the value of human capital is high today either when future human capital returns are high, or when future nonfinancial income growth is low. Following Campbell (1996), if we assume that the conditional expected return on financial wealth equals the conditional expected return on human wealth $\left(E_t\left[r^{WL}_{US,t+j}\right] = E_t\left[r^{S}_{US,t+j}\right], \forall j\right)$ and substitute (2.53) into (2.52), we obtain:[41]

$$\widetilde{E}_{t+1}\left[r^{WL}_{US,t+1}\right] = \widetilde{E}_{t+1} \sum_{j=0}^{\infty} \rho^j \left(\Delta \log WL_{US,t+1+j}\right) - \widetilde{E}_{t+1} \sum_{j=1}^{\infty} \rho^j r^{WL}_{US,t+j+1}. \tag{2.54}$$

This expression states that the innovation to the return on human capital depends upon innovations to the path of future expected labour income growth, as well as innovations to the path of future expected equity returns proxying for future expected human capital returns. Thus, human capital return innovations today are high, if innovations to expected labour income growth are high, or if innovations to expected human capital returns are low. We then obtain innovations to the relative expected human capital return by subtracting the analogous expression for the rest of the world, converted into dollars, from (2.53), assuming that ρ is

[41] For open economy applications, Coeurdacier and Gourinchas (2009) make the same assumption, while Baxter and Jermann (1997) and Benigno and Nistico (2009) assume that discount rates are constant and thereby omit the second term. See also the discussion in Benigno and Nistico (2009) and Coeurdacier and Gourinchas (2009) and recent empirical work by Lustig and Van Nieuwerburgh (2006) and Lustig et al (2009).

the same for all countries:

$$\widetilde{E}_{t+1} r_{t+1}^{WL} = \widetilde{E}_{t+1} \sum_{j=0}^{\infty} \rho^j \left(\Delta \log WL_{t+1+j}\right) - \widetilde{E}_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+j+1}^S, \tag{2.55}$$

where $\log WL_{t+1+j}$ and r_{t+j+1}^S are relative labour income and relative equity returns, expressed in US dollars. The relative return on human capital thus depends on innovations to expected future relative labour income, as well as innovations to future relative discount rates.

We also need to obtain a measure for relative consumption expenditure risk. Following the same steps as above, we obtain the following expression:

$$\widetilde{E}_{t+1} \left[r_{t+1}^{PC}\right] = \widetilde{E}_{t+1} \sum_{j=0}^{\infty} \rho^j \left(\Delta \log PC_{t+1+j}\right) - \widetilde{E}_{t+1} \sum_{j=1}^{\infty} \rho^j r_{US,t+j+1}^{PC}, \tag{2.56}$$

where r^{PC} is the relevant measure of real exchange rate risk and PC_t are relative consumption expenditures in US dollars. Since we are not aware of any work that suggests the predictability of future returns to the stream of relative consumption expenditures, we set the second term to zero and therefore estimate consumption expenditure risk solely as a function of expected growth in relative consumption expenditures. Equity and bond returns are observable and relative equity and bond return innovations are given by:

$$\widetilde{E}_{t+1} \left[r_{t+1}^S\right] = \widetilde{E}_{t+1} \left[r_{US,t+1}^S - r_{ROW,t+1}^S\right] \qquad \widetilde{E}_{t+1} \left[r_{t+1}^B\right] = \widetilde{E}_{t+1} \left[r_{US,t+1}^B - r_{ROW,t+1}^B\right], \tag{2.57}$$

where $r_{US,t+1}^S$ $\left(r_{US,t+1}^B\right)$ are log equity (bond) returns in the US, $r_{ROW,t+1}^S$ $\left(r_{ROW,t+1}^B\right)$ are log equity (bond) returns in the rest of the world, in dollars.

In order to estimate $\widetilde{E}_{t+1} \left[r_{t+1}^S\right], \widetilde{E}_{t+1} \left[r_{t+1}^B\right], \widetilde{E}_{t+1} \left[r_{t+1}^{WL}\right], \widetilde{E}_{t+1} \left[r_{t+1}^{PC}\right]$, we then run the following first order vector autoregression:

$$x_{t+1} = Ax_t + \varepsilon_{t+1}, \tag{2.58}$$

where $x_t' = \begin{bmatrix} r_t^S & \Delta li_t & r_t^B & \Delta pc_t & b_t \end{bmatrix}$. b_t represents other controls that help to predict the (relative) growth rate of labour income, bond and equity returns, and consumption expenditures. Here, we use the *nxa* variable from Gourinchas and Rey (2007b) and the term premium.[42] Having obtained the estimates $\widehat{A}$ and $\widehat{\varepsilon}$

[42] *nxa* is the ratio of net exports to net foreign assets and was found by Gourinchas and Rey (2007b) to predict relative equity returns for the US. We rely on an updated version of the original *nxa* series that runs until the first quarter of 2008, kindly supplied by Hélène Rey.

from the VAR, human capital and consumption expenditure risk are created as:

$$\widetilde{E}_{t+1}\left[r^{WL}_{i,t+1}\right] = \left(e_2^{'} - \rho e_1^{'}\widehat{A}\right)\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1} \tag{2.59}$$

$$\widetilde{E}_{t+1}\left[r^{PC}_{i,t+1}\right] = e_4^{'}\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1} \tag{2.60}$$

while equity and bond return innovations are:

$$\widetilde{E}_{t+1}\left[r^{S}_{i,t+1}\right] = e_1^{'}\widehat{\varepsilon}_{t+1} \qquad \widetilde{E}_{t+1}\left[r^{B}_{i,t+1}\right] = e_3^{'}\widehat{\varepsilon}_{t+1}. \tag{2.61}$$

e_i is a unit vector whose ith element is equal to one, while all other elements are equal to zero.

The data is quarterly and runs from the first quarter of 1970 until the first quarter of 2008. The rest of the world is constructed using the G7 economies (Canada, France, Germany, Italy, Japan, UK, US). Further details about data sources and construction are in the appendix. The VAR is estimated in levels, including a constant and one lag.[43] In Table 2.6, we present the unconditional correlations of the asset returns and sources of risk and contrast them with the ones implied by the benchmark model. As we can see, the model does a good job of qualitatively matching the correlations in the data. The correlation of equity return innovations is positive, but quite low, with human capital risk, and somewhat larger, with consumption expenditure risk, as in the model. The correlations of relative bond return innovations with human capital and consumption expenditure risk is positive and quite high in the data, while not quite as high as in our benchmark model.

Conditional Measures of Comovement

We are now in a position to estimate the relevant covariance variance ratios. We run the following two regressions:

$$\widetilde{E}_{t+1}\left[r^{WL}_{t+1}\right] = k_1 + \beta_{wl,b}\widetilde{E}_{t+1}\left[r^{B}_{t+1}\right] + \beta^{i}_{wl,s}\widetilde{E}_{t+1}\left[r^{S}_{t+1}\right] + \varepsilon^{wl}_t \tag{2.62}$$

$$\widetilde{E}_{t+1}\left[r^{PC}_{t+1}\right] = k_2 + \beta_{pc,b}\widetilde{E}_{t+1}\left[r^{B}_{t+1}\right] + \beta^{i}_{pc,s}\widetilde{E}_{t+1}\left[r^{S}_{t+1}\right] + \varepsilon^{pc}_t, \tag{2.63}$$

where the error terms ε^{wl} and ε^{pc} are attributed both to measurement error in the construction of the sources of risk as well as to fluctuations not spanned by relative bond and equity returns. As noted above, the coefficients obtained from

[43] Akaike and Bayesian information criteria suggest that including one lag is the preferred specification.

Table 2.6: **Unconditional Correlations between Asset Returns and Sources of Risk**

Correlations	**Data**	**Model**
Equity Return Innovations with:		
Human Capital Risk	0.21	0.24
Consumption Expenditure Risk	0.34	0.49
Bond Return Innovations with:		
Human Capital Risk	0.77	0.96
Consumption Expenditure Risk	0.64	0.99

Correlations in the data are based on quarterly data from Q1:1970 to Q3:2008. Model correlations were computed using 500 simulations of 500 periods of 500 periods each.

this regression are equal to the loadings that enter in our expression for equilibrium asset portfolios:

$$\beta_{pc,s} = \frac{cov_{R_t^B}\left(\widehat{R}_t^{PC}, \widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)} \quad \beta_{wl,s} = \frac{cov_{R_t^B}\left(\widehat{R}_t^{WL}, \widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)} \tag{2.64}$$

$$\beta_{pc,b} = \frac{cov_{R_t^S}\left(\widehat{R}_t^{PC}, \widehat{R}_t^B\right)}{var_{R_t^S}\left(\widehat{R}_t^B\right)} \quad \beta_{wl,b} = \frac{cov_{R_t^S}\left(\widehat{R}_t^{WL}, \widehat{R}_t^B\right)}{var_{R_t^S}\left(\widehat{R}_t^B\right)} \tag{2.65}$$

The results of the regressions are presented in table 2.7. The first row indicates that the correlation between relative equity return innovations and human capital risk, conditional on relative bond returns, is negative and the regression coefficient is in fact the same as in our benchmark model. The coefficient for consumption expenditure risk is positive and quite low in both model and data, but it is not significant in the data. For bond positions, our model can also qualitatively, if not quantitatively match the regression coefficients. Thus, we find that both the regression coefficients for human capital and consumption expenditure risk are positive and quite large. While they are also positive in the model, they are much smaller.

2.5.2 Implied Bond and Equity Positions

In our model, we assumed that the two countries are of the same size. Here, we relax that assumption and slightly adapt our reduced form expressions to allow

Table 2.7: **Regression Results**

	Human Capital Risk		**Consumption Expenditure Risk**	
	Data	Model	Data	Model
Equity Return Innovations	-0.16^{**}	-0.16	0.07	0.02
	(0.06)		(0.07)	
Bond Return Innovations	1.29^{**}	0.09	0.93^{**}	0.12
	(0.09)		(0.11)	
R^2	0.61		0.41	
No. of Observations	152		152	

Data and model regression coefficients were obtained from (4.37) and (4.38) . Data is from the first quarter of 1970 to the third quarter of 2007. Model regressions were run on data obtained from 500 simulations of 500 periods. ** implies that coefficients are significant at the 95% confidence level. Coefficients for the constants are suppressed.

for countries of different size:

$$S = \widetilde{\lambda} + \left(1-\widetilde{\lambda}\right)\left(\frac{C}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{PC}, \widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)} - \frac{1}{2}\frac{WL}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{WL}, \widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)}\right) \tag{2.66}$$

$$\widetilde{B} = \frac{1}{2}\frac{\beta}{(1-\beta)}\frac{C}{Y}\frac{cov_{R_t^S}\left(\widehat{R}_t^{PC}, \widehat{R}_t^B\right)}{var_{R_t^S}\left(\widehat{R}_t^B\right)} - \frac{1}{2}\frac{\beta}{(1-\beta)}\frac{WL}{Y}\frac{cov_{R_t^S}\left(\widehat{R}_t^{WL}, \widehat{R}_t^B\right)}{var_{R_t^S}\left(\widehat{R}_t^B\right)} \tag{2.67}$$

where $\widetilde{\lambda} = \frac{C_{US}}{C_{US}+C_{ROW}} = \frac{D_{US}}{D_{US}+D_{ROW}} = \frac{Y_{US}}{Y_{US}+Y_{ROW}}$ is the steady state ratio of consumption, dividends and output between the US and the rest of the world. The average share of US in aggregate GDP in our sample is 45% and we correspondingly set $\widetilde{\lambda} = 0.45$. The mean ratios of consumption over dividends, $\frac{C}{Y}$, and labour income over dividends, $\frac{WL}{Y}$, are 0.73 and 0.71, respectively. Substituting the regression coefficients obtained in the previous subsection and the steady state coefficients, we obtain the implied portfolios in Table 2.8. There, we see that the estimated moments imply substantial equity home bias for the US, as the benchmark model does and both are qualitatively and quantitatively similar to the available data on actual equity holdings. The correlation of relative equity returns with relative human capital returns, conditional on relative bond returns, is negative in the data, and it therefore contributes to equity home bias. The consumption expenditure motive also implies long positions in domestic equity for U.S. investors, as the correlation of relative equity returns with consumption expenditure risk, conditional on relative bond returns, is positive, but as in the benchmark model, the contribution of the human capital motive to equity home bias is larger. The

Table 2.8: **Estimated Equity and Bond Positions**

	Share of Home Equity	Home Bonds/ GDP
Total	**0.88**	**−1.73**
due to:		
Pure Diversification	0.50	–
Human Capital	0.30	−5.78
Consumption Expenditure	0.12	4.05
Home Bias	0.43	–

Equilibrium Equity and Bond Positions are calculated using (2.66) and (2.67) , the regression coefficients estimated in the previous subsection and $\beta = 0.99$, $\widetilde{\lambda} = 0.45$, $\frac{C}{Y} = 0.73$ and $\frac{WL}{Y} = 0.71$.

model and the data also agree on the directions of the drivers of bond portfolios. Both in the model and in the data, the human capital motive induces Home agents to take short positions in domestic bonds, while the consumption expenditure motive induces them to take a long position. In our estimation, the former motive dominates, implying overall short positions of U.S. investors in U.S. bonds, while in our benchmark model the consumption expenditure motive is larger and leads to overall long positions in domestic bonds. Finally, while the sign of the bond position is in accord with data on available U.S. holdings, the size is somewhat too large.

2.6 Conclusion

There is ample evidence on international trade in equities and nominal bonds, equity home bias, and positive cross country correlations in output, consumption, investment, and output. There is also strong evidence that the unconditional correlation between equity returns and human capital returns is positive. We present a two country, two good model with nominal rigidities, endogenous capital accumulation and incomplete markets that matches these facts. Agents trade equity and nominal bonds to hedge fluctuations in labour income and consumption expenditures. Importantly, in the presence of trade in multiple assets, equity holdings depend on the comovement with these sources of risk, conditional on the returns of other assets, as emphasized by, amongst others, Engel and Matsumoto (2009) and Coeurdacier and Gourinchas (2009). We show that in the model the correlation between equity returns and human capital returns is positive, but that once we condition on bond returns, this correlation turns negative. These relationships are confirmed using data on G7 countries and imply equity home bias. Long positions in domestic nominal bonds hedge most of consumption expenditure risk and we show that the model matches the conditional and unconditional correlations of bond returns with the sources of risk also, at least qualitatively. The model is able to match many features of the international business cycle, and in particular, generates positive cross country correlations in investment. We emphasize that both nominal rigidities and endogenous capital accumulation contribute to matching the patterns of international portfolios and cross country correlations observed in the data.

However, the model shares some of the weaknesses of previous models of international risk sharing. Thus, it predicts a correlation of relative consumption with the real exchange rate that is too high compared to the data (Backus and Smith (1993) and Kollmann (1995)). What is more, it is not able to simultaneously produce a realistic equity premium and risk free rate volatility. While the former raises questions about the appropriate way to model market incompleteness in open economy models, the latter gives reason to translate the lessons learned in the recent asset pricing literature into an open economy context. We regard both of these as worthy endeavours for future research. In addition, the integration of portfolio choice and macroeconomics allows the rigorous analysis of many issues that are at the center of current policy debates in the international policy sphere, but could not be examined systematically before. Among these topics are the origins and effects of global imbalances, the effects of policy decisions on capital flows or the interaction between real and financial globalisation.

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2.7 Appendix

2.7.1 Stochastic Discount Factors

The Euler equations for investment in Home equity are:

$$\beta E_t\left[\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma}\left(\frac{P^H_{C,t}}{P^H_{C,t+1}}\right)\left(\frac{D_{H,t+1}+P^S_{H,t+1}}{P^S_{H,t}}\right)\right] = 1 \quad (2.68)$$

$$\beta E_t\left[\left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\sigma}\left(\frac{P^F_{C,t}}{P^F_{C,t+1}}\right)\left(\frac{D_{H,t+1}+P^S_{H,t+1}}{P^S_{H,t}}\frac{Z_t}{Z_{t+1}}\right)\right] = 1 \quad (2.69)$$

Linearising:

$$\begin{aligned} 0 &= E_t\left[-\sigma\left(\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right)+\left(\widehat{P}^H_{C,t}-\widehat{P}^H_{C,t+1}\right)\right] \\ &\quad +E_t\left[(1-\beta)\,\widehat{D}_{H,t+1}+\beta\widehat{P}^S_{H,t+1}-\widehat{P}^S_{H,t}\right] \quad (2.70) \\ 0 &= E_t\left[-\sigma\left(\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right)+\left(\widehat{P}^F_{C,t}-\widehat{P}^F_{C,t+1}\right)\right] \\ &\quad +E_t\left[(1-\beta)\,\widehat{D}_{H,t+1}+\beta\widehat{P}^S_{H,t+1}-\widehat{P}^S_{H,t}+\widehat{Z}_t-\widehat{Z}_{t+1}\right] \quad (2.71) \end{aligned}$$

which imply:

$$\begin{aligned} & E_t\left[-\sigma\left(\widehat{C}_{H,t+1}-\widehat{C}_{H,t}\right)-\left(\widehat{P}^H_{C,t+1}-\widehat{P}^H_{C,t}\right)\right] \quad (2.72) \\ =\; & E_t\left[-\sigma\left(\widehat{C}_{F,t+1}-\widehat{C}_{F,t}\right)-\left(\widehat{P}^F_{C,t+1}-\widehat{P}^F_{C,t}\right)\right]+\widehat{Z}_t-\widehat{Z}_{t+1} \quad (2.73) \end{aligned}$$

Thus, in a first order approximation, the Home and Foreign stochastic discount factors are the same in expectation, once they are expressed in the same units.

2.7.2 Budget Constraint

The budget constraint for the Home agent is:

$$\begin{aligned} & S^H_{H,t}P^S_{H,t}+S^H_{F,t}P^S_{F,t}Z_t+B^H_{H,t}P^B_{H,t}+B^H_{F,t}P^B_{H,t}Z_t+P^H_{C,t}C_{H,t} \quad (2.74) \\ =\; & W_{H,t}L_{H,t}+S^H_{H,t-1}\left(D_{H,t}+P^S_{H,t}\right)+S^H_{F,t-1}\left(D_{F,t}+P^S_{F,t}\right)Z_t+B^H_{H,t-1}+B^H_{F,t-1}Z_t, \end{aligned}$$

The net foreign asset position is then given by assets held abroad minus domestic assets held by foreign agents:

$$NFA_{H,t}=S^H_{F,t}P^S_{F,t}Z_t+B^H_{F,t}P^B_{F,t}Z_t-S^F_{H,t}P^S_{H,t}-B^F_{H,t}P^B_{H,t}, \quad (2.75)$$

which using the asset market clearing conditions $\left(S_{H,t}^{F}=1-S_{H,t}^{H}, B_{H,t}^{F}=-B_{H,t}^{H}\right)$ becomes:

$$NFA_{H,t}=\left(S_{H,t}^{H}-1\right)P_{H,t}^{S}+S_{F,t}^{H}Z_{t}P_{F,t}^{S}+B_{F,t}^{H}P_{F,t}^{B}Z_{t}+B_{H,t}^{H}P_{H,t}^{B} \tag{2.76}$$

Define the portfolio excess return as:

$$\begin{aligned}\xi_{H,t} &= \left(S_{H,t-1}^{H}-1\right)\left(P_{H,t}^{S}+D_{H,t}\right)+S_{F,t-1}^{H}\left(P_{F,t}^{S}+D_{F,t}\right)Z_{t} \\ &\quad +B_{H,t-1}^{H}+B_{F,t-1}^{H}Z_{t}-NFA_{H,t-1}R_{H,t}^{S},\end{aligned} \tag{2.77}$$

i.e. the difference between actual net foreign assets at the beginning of period t and net foreign assets at period t had all wealth been invested in the home equity. Substituting for net foreign asset and writing in terms of returns, we have:

$$\begin{aligned}\xi_{H,t} &= S_{F,t-1}^{H}P_{F,t-1}^{S}Z_{t-1}\left(R_{F,t}^{S}\frac{Z_{t}}{Z_{t-1}}-R_{H,t}^{S}\right)+B_{H,t-1}^{H}P_{H,t-1}^{B}\left(R_{H,t}^{B}-R_{H,t}^{S}\right) \\ &\quad +B_{F,t-1}^{H}Z_{t-1}P_{F,t-1}^{B}\left(R_{F,t}^{B}\frac{Z_{t}}{Z_{t-1}}-R_{H,t}^{S}\right)\end{aligned} \tag{2.78}$$

Note that the left hand side of (2.74) is equal to $NFA_{H,t}+P_{H,t}^{S}-C_{H,t}$. Using the expressions for the portfolio excess returns and net foreign assets, we can then write the budget constraint as:

$$NFA_{H,t}=NFA_{H,t-1}R_{H,t}^{S}+P_{H,t-1}^{S}R_{H,t}^{S}+\xi_{H,t}+L_{H,t}W_{H,t}-P_{C,t}^{H}C_{H,t}-P_{H,t}^{S}. \tag{2.79}$$

Now, remembering that next exports were defined as: $NX_{H,t}=Y_{H,t}^{H}P_{H,t}^{H}+Y_{H,t}^{F}P_{H,t}^{F}Z_{t}-I_{H,t}P_{I,t}^{H}-P_{C,t}^{H}C_{H,t}$, we have:

$$NFA_{H,t}=NFA_{H,t-1}R_{H,t}^{S}+\xi_{H,t}+NX_{H,t} \tag{2.80}$$

or, in linear form:

$$\widehat{NFA}_{H,t}=\widehat{NFA}_{H,t-1}\frac{1}{\beta}+\widehat{\xi}_{H,t}+\widehat{NX}_{H,t}, \tag{2.81}$$

Linearising the expression for the portfolio excess return and using (2.81), we have:

$$\widehat{\xi}_{H,t}=\left(S_{H}^{H}-1\right)\frac{P^{S}}{\beta}\left(\widehat{R}_{H,t}^{S}-\widehat{R}_{F,t}^{S}-\widehat{Z}_{t}+\widehat{Z}_{t-1}\right)+\frac{P^{B}}{\beta}B\left(\widehat{R}_{H,t}^{B}-\widehat{R}_{F,t}^{B}-\widehat{Z}_{t}+\widehat{Z}_{t-1}\right), \tag{2.82}$$

We can then write the budget constraint in linearised form as:

$$\begin{aligned}
\widehat{NFA}_{H,t} &= \widehat{NFA}_{H,t-1}\frac{1}{\beta} + (S-1)\frac{P^S}{\beta}\left(\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right) \\
&\quad + \frac{P^B}{\beta}B\left(\widehat{R}^B_{H,t} - \widehat{R}^B_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right) + \widehat{NX}_{H,t} \qquad (2.83)
\end{aligned}$$

Taking expectations at time t and rewriting:

$$\begin{aligned}
\frac{1}{\beta}\widehat{NFA}_{H,t-1} &= E_t\left[\widehat{NFA}_{H,t} - \widehat{NX}_{H,t} - \frac{P^S}{\beta}(S-1)\left(\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right)\right] \\
&\quad - \frac{P^B}{\beta}BE_t\left[\widehat{R}^B_{F,t} - \widehat{R}^B_{H,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \qquad (2.84)
\end{aligned}$$

Iterating forward, imposing $T \to \infty$,$lim_{T\to\infty}E_t\left[NFA_{t+T}\right] = 0$ and using $E_t\left[\widehat{R}^S_{H,t+\tau} - \widehat{R}^S_{F,t+\tau} + \widehat{Z}_{t+\tau} - \widehat{Z}_{t+\tau-1}\right] = 0, \tau > 1$, we get:

$$\begin{aligned}
\frac{1}{\beta}\widehat{NFA}_{H,t-1} &= \sum_{j=0}^{T} -\beta^j E_t\left[\widehat{NX}_{H,t+j}\right] - \frac{P^S}{\beta}(S-1)E_t\left[\widehat{R}^S_{H,t} - \widehat{R}^S_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \\
&\quad - \frac{P^B}{\beta}BE_t\left[\widehat{R}^B_{F,t} - \widehat{R}^B_{H,t} - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \qquad (2.85)
\end{aligned}$$

Rearranging and using the expressions for linearised relative bond and equity returns, we have:

$$\begin{aligned}
\sum_{j=0}^{T} -\beta^j E_t\left[NX_{H,t+j}\right] &= \frac{1}{\beta}NFA_{H,t-1} + D(S-1)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{H,t+j} - \widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right] \\
&\quad + B\widetilde{E}_t\left[-\widehat{Z}_{t+j}\right], \qquad (2.86)
\end{aligned}$$

where $\widetilde{E}_t\left[X_t\right] = E_t\left[X_t\right] - E_{t-1}\left[X_t\right]$.
This budget constraint holds if and only if:

$$\sum_{j=0}^{T} -\beta^j \widetilde{E}_t\left[NX_{H,t+j}\right] = D(S-1)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{H,t+j} - \widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right] + B\widetilde{E}_t\left[-\widehat{Z}_{t+j}\right] \qquad (2.87)$$

and

$$\sum_{j=0}^{T} -\beta^j E_{t-1}\left[NX_{H,t+j}\right] = \frac{1}{\beta}NFA_{H,t-1} \qquad (2.88)$$

Using, $\widehat{NX}_{H,t} = \widehat{D}_{H,t}D + \left(\widehat{W}_{H,t} + \widehat{L}_{H,t}\right)WL - C\left(\widehat{P}^H_{C,t} + \widehat{C}_{H,t}\right)$ and rewriting, we then have:

$$\begin{aligned} & C\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\widehat{P}^H_{C,t} + \widehat{C}_{H,t}\right] \\ = \; & WL\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j}\right] + SD\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\widehat{D}_{H,t+j}\right] \\ & + (1-S)\,D\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right] + B\widetilde{E}_t\left[-\widehat{Z}_{t+j}\right] \end{aligned} \tag{2.89}$$

The analogous expression for the foreign country is, in terms of Home currency:

$$\begin{aligned} & C\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\widehat{P}^F_{C,t+j} + \widehat{C}_{F,t+j} + \widehat{Z}_{t+j}\right] \\ = \; & WL\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} + \widehat{Z}_{t+j}\right] + SD\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j} + \widehat{Z}_{t+j}\right)\right] \\ & + (1-S)\,D\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\widehat{D}_{H,t+j}\right] - B\widetilde{E}_t\left[-\widehat{Z}_{t+j}\right] \end{aligned} \tag{2.90}$$

Deducting the foreign budget constraint from the Home one, we obtain:

$$\begin{aligned} & C\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\left(\widehat{P}^H_{C,t} + \widehat{C}_{H,t}\right) - \left(\widehat{P}^F_{C,t+j} + \widehat{C}_{F,t+j} + \widehat{Z}_{t+j}\right)\right] \\ = \; & WL\sum_{j=0}^{T}\beta^j\widetilde{E}_t\left[\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} - \left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} + \widehat{Z}_{t+j}\right)\right] \\ & + (2S-1)\,D\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{H,t+j} - \left(\widehat{D}_{F,t+j} - \widehat{Z}_{t+j}\right)\right)\right] \\ & + 2B\widetilde{E}_t\left[-\widehat{Z}_{t+j}\right] \end{aligned} \tag{2.91}$$

Now define the return on human capital in country i as

$$R^{WL}_{H,t} = \frac{W_{H,t}L_{H,t} + P^W_{H,t}}{P^W_{H,t-1}}, \tag{2.92}$$

where $P^{W}_{H,t} = E_t \sum_{j=1}^{\infty} \beta^j \left(\frac{C_{H,t+j}}{C_{H,t}}\right)^{-\sigma} \frac{P^H_{C,t}}{P^H_{C,t+j}} W_{H,t+j} L_{H,t+j}$ is the present value of labour income in Home. Linearising, the expression for returns, we have:

$$R^{WL}_{H,t} = (1-\beta)\,\widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(-\sigma\left(\widehat{C}_{H,t+j} - \widehat{C}_{H,t}\right) + \widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+j} + \widehat{W}_{H,t+j} + \widehat{L}_{H,t+j}\right) \tag{2.93}$$

Deducting the analogous foreign expression in Home currency terms, we have:

$$\widehat{R}^{WL}_{H,t} - \widehat{R}^{WL}_{F,t} - Z_t + Z_{t-1} = (1-\beta)\,\widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} - \widehat{W}_{F,t+j} - \widehat{L}_{F,t+j} - \widehat{Z}_{t+j}\right) \tag{2.94}$$

Now define the return on consumption expenditure in country i as

$$R^{PC}_{H,t} = \frac{P^H_{C,t} C_{H,t} + P^{PC}_{H,t}}{P^{PC}_{H,t-1},}, \tag{2.95}$$

where $P^{PC}_{H,t} = E_t \sum_{j=1}^{\infty} \beta^j \left(\frac{C_{H,t+j}}{C_{H,t}}\right)^{-\sigma} \frac{P^H_{C,t}}{P^H_{C,t+j}} P^H_{C,t+j} C_{H,t+j}$ is the present value of consumption expenditure in the Home country. Linearising, the expression for returns, we have:

$$R^{PC}_{H,t} = (1-\beta)\,\widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(-\sigma\left(\widehat{C}_{H,t+j} - \widehat{C}_{H,t}\right) + \widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+j} + \widehat{P}^H_{C,t+j} + \widehat{C}_{H,t+j}\right) \tag{2.96}$$

Deducting the analogous foreign expression in Home currency terms, we have:

$$\widehat{R}^{PC}_{H,t} - \widehat{R}^{PC}_{F,t} - Z_t + Z_{t-1} = (1-\beta)\,\widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(\widehat{P}^H_{C,t+j} + \widehat{C}_{H,t+j} - \widehat{P}^H_{C,t+j} - \widehat{C}_{F,t+j} - \widehat{Z}_{t+j}\right) \tag{2.97}$$

Using the expressions, we just defined, we can then write the relative budget constraints as:

$$\frac{C}{1-\beta}\widehat{R}^{PC}_t = \frac{WL}{1-\beta}\widehat{R}^{WL}_t + (2S-1)\frac{D}{1-\beta}\widehat{R}^S_t + 2B\widehat{R}^B_t, \tag{2.98}$$

where $R^{WL}_t = \widehat{R}^{WL}_{H,t} - \widehat{\widehat{R}}^{WL}_{F,t}$, $R^S_t = \widehat{R}^S_{H,t} - \widehat{\widehat{R}}^S_{F,t}$, $\widehat{R}^B_t = \widehat{R}^B_{H,t} - \widehat{\widehat{R}}^B_{F,t}$ and $R^{PC}_t = \widehat{R}^{PC}_{H,t} - \widehat{\widehat{R}}^{PC}_{F,t}$.

Now project this equation on relative bond returns $\widehat{R}_t^B$:

$$\frac{C}{1-\beta}P\left[\widehat{R}_t^{PC}|\widehat{R}_t^B\right]=\frac{WL}{1-\beta}P\left[\widehat{R}_t^{WL}|\widehat{R}_t^B\right]+\frac{D}{1-\beta}\left(2S-1\right)P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]+2B\widehat{R}_t^B, \tag{2.99}$$

where $P\left[\widehat{X}_t|\widehat{Y}_t\right]$ is the projection of X_t on Y_t. Subtracting this equation from the one before, we have:

$$\begin{aligned}&\frac{C}{1-\beta}\left(\widehat{R}_t^{PC}-P\left[\widehat{R}_t^{PC}|\widehat{R}_t^B\right]\right)\\ =\ &\frac{WL}{1-\beta}\left(\widehat{R}_t^{WL}-P\left[\widehat{R}_t^{WL}|\widehat{R}_t^B\right]\right)+\frac{D}{1-\beta}\left(2S-1\right)\left(\widehat{R}_t^S-P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)\end{aligned} \tag{2.100}$$

Rearranging, we have:

$$S=\frac{1}{2}+\frac{1}{2}\frac{C}{D}\frac{\left(\widehat{R}_t^{PC}-P\left[\widehat{R}_t^{PC}|\widehat{R}_t^B\right]\right)}{\left(\widehat{R}_t^S-P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)}-\frac{1}{2}\frac{WL}{D}\frac{\left(\widehat{R}_t^{WL}-P\left[\widehat{R}_t^{WL}|\widehat{R}_t^B\right]\right)}{\left(\widehat{R}_t^S-P\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)} \tag{2.101}$$

Multiplying the numerator and the denominator by $\left(\widehat{R}_t^S-\left[\widehat{R}_t^S|\widehat{R}_t^B\right]\right)$, and taking unconditional expectations, we have:

$$S=\frac{1}{2}+\frac{1}{2}\frac{C}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{PC},\widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)}-\frac{1}{2}\frac{WL}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{W},\widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)},$$

where $cov_{R_t^B}\left(X_t,Y_t\right)$ is the covariance of X_t and Y_t conditional on relative bond returns R_t^B.

Projecting (2.98) onto to relative equity returns instead of bond returns and otherwise following the same steps, we can derive a closed form expression for bond positions.

$$\widetilde{B}=\frac{1}{2}\frac{\beta}{(1-\beta)}\frac{C}{Y}\frac{cov_{R_t^S}\left(\widehat{R}_t^{PC},\widehat{R}_t^B\right)}{var_{R_t^S}\left(\widehat{R}_t^B\right)}-\frac{1}{2}\frac{\beta}{(1-\beta)}\frac{WL}{Y}\frac{cov_{R_t^S}\left(\widehat{R}_t^{WL},\widehat{R}_t^B\right)}{var_{R_t^S}\left(\widehat{R}_t^B\right)}. \tag{2.102}$$

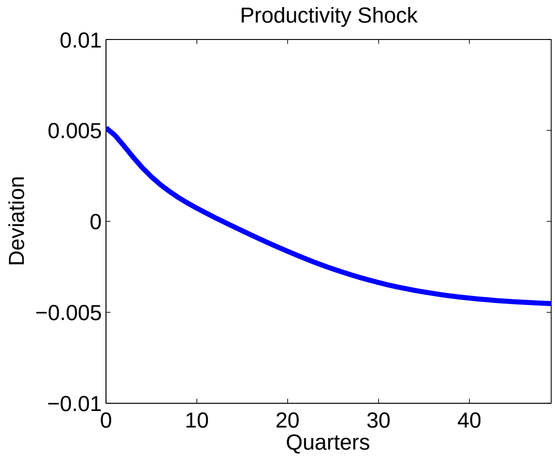
2.7.3 Bond Maturity and Real Bonds

In the benchmark model, agents trade equities and one period nominal bonds, in line with recent work on international with nominal rigidities (Devereux and Sutherland (2007, 2008), Engel and Matsumoto (2009)). In Table 2.9, we present portfolios for two additional asset menus. In the first case, nominal consols are traded in addition to equity. Cross border trade in longer maturity debt securities is in fact quite large, but, to our knowledge, it has not previously been investigated in the context of open economy models of portfolio choice. Nominal consols pay one unit of local currency per period and relative bond returns are then given by $\widehat{R}_t^{B,c} = (1-\beta)\,\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(-\widehat{Z}_{t+j}\right)\right]$, and thus depend on surprises to the present value of appreciations in the future. The correlations and equilibrium portfolios are quite different in this case. Figure 2.4 tells us why. In response to productivity shocks, the nominal exchange rate first depreciates, due to the rise in the supply of the Home good. In the long run, however, it appreciates.[44] Overall, the effect of the long run appreciation dominates and relative returns of long term bonds rise, while returns of short term bonds fall. As a result, nominal bond returns are now strongly negatively correlated with human capital risk and consumption expenditure risk, both conditionally and unconditionally. The human capital motive then induces Home investors to take long positions in Home bonds, while the consumption expenditure motive calls for short positions. Since the latter prevails, the sign of the bond position is thus more in line with the available U.S. evidence, though the size is too large. Equity positions are also quite different. The relationship between equity returns and human capital is qualitatively unchanged, but the size of the contribution of the human capital motive is much lower. Equity home bias is much larger overall, however, due to the consumption expenditure motive. This is mainly due to the fact that consumption expenditure risk is now less correlated with bond returns, increasing the conditional correlation between equity returns and consumption expenditure risk, conditional on bond returns.

Secondly, we consider the asset menu of Coeurdacier et al (2009). There, agents trade equities and real consols. Real consols have a payoff equal to the local price level each period. Relative returns are then given by $\widehat{R}_t^{B,real} = (1-\beta)\,\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta_{t+j}\left(-\widehat{Q}_{t+j}\right)\right]$, implying that relative bond returns now depend on the path of *real,* not nominal exchange rates. In our model, markets are incomplete, and relative consumption expenditures are not perfectly correlated with the real exchange rate, even at first order approximation (see also Backus and Smith (1993) and Kollmann (1995)). However, the correlation between the two is still very high. As a result, long posi-

[44] The long run appreciation of the nominal exchange rate is based on our assumptions about the monetary policy rule. Monetary policy here stabilises inflation, not the price level. In response to productivity shocks, relative inflation falls, leading over time to a fall in the relative price level. Since real exchange rates are stationary, this implies that the nominal exchange rate has to appreciate in the long run in order for the real exchange rate to return to its steady state value (of one).

Figure 2.4: **Impulse Response of Nominal Exchange Rates to a Relative Productivity Shock.**



tions in real bonds provide an almost perfect hedge for consumption expenditure risk and Home agents consequently take a large long position in them. The fact that real bonds can provide an almost perfect hedge for relative consumption expenditure risk then also explains why the effect of the consumption expenditure motive on equity positions is virtually zero in that case.[45]

2.7.4 Data Sources

The data is quarterly from the first quarter of 1970 to the third quarter of 2008. We take the U.S. as the Home country and a GDP weighted aggregate of the G7 countries with the exception of the U.S. (Canada, France, Germany, Italy, Japan, UK) as the Foreign country. National account data is from the OECD quarterly national accounts (QNA), Economic Outlook (EO) and Main Economic Indicators (MEI) databases, while data on equity and bond returns and nominal exchange rates is from the Global Financial Data (GFD) database.

Output: Gross domestic product, volume, market prices, EO

Consumption: Private final consumption expenditure, volume, EO

Investment: Gross capital formation, total, volume, EO

[45]The fact that relative consumption expenditure risk is perfectly hedged through bond bond positions is also at the heart of the result in Coeurdacier et al (2009) that equity positions do not depend on preference parameters, and in particular, the elasticity of substitution between Home and Foreign goods. In our benchmark case, bond positions do not provide a perfect hedge for consumption expenditure risk and as a result, preference parameters do have an effect on the equilibrium portfolios. In general, relative equity returns are a good hedge for relative human capital returns only for a sufficiently low value of ϕ.

Table 2.9: **Portfolios for additional model variants**

		Bench-mark (I)	**Consols** (II)	**Real Consols** (III)	**Flex Wage** (IV)	**No Inv Adj Cost** (V)	**No Frictions** (VI)
Share of Home Equity		**0.90**	**1.52**	**0.88**	**0.89**	**0.98**	**1.12**
	due to:						
	Human Capital	0.36	0.02	0.38	0.36	0.22	−0.01
	Consumption Exp	0.04	0.99	0.00	0.03	0.26	0.63
Correlations:							
	$Corr\left(\widehat{R}^S, \widehat{R}^{WL}\right)$	0.24	0.04	0.37	0.37	0.13	0.64
	$Corr\left(\widehat{R}^S, \widehat{R}^{PC}\right)$	(0.49)	0.41	0.56	0.63	0.85	0.94
Home Bonds/ GDP		**0.49**	**−1.99**	**2.76**	**0.50**	**0.45**	**0.39**
	due to:						
	Human Capital	−0.76	1.04	−4.32	−0.72	−0.68	−0.41
	Consumption Exp	1.24	−3.02	7.07	1.22	1.13	0.82
Correlations:							
	$Corr\left(\widehat{R}^B, \widehat{R}^{WL}\right)$	0.96	−0.96	0.98	0.95	0.78	0.98
	$Corr\left(\widehat{R}^B, \widehat{R}^{PC}\right)$	0.99	−0.94	1.00	0.99	0.97	0.86

(I) follows the benchmark calibration described in the text, (II) features trade in nominal consols and equities, (III) has trade in equities and real consols, (IV) sets $\theta_w = 0$, (V) sets $\upsilon \rightarrow \infty$, (VI) sets $\theta = \theta_w = 0$ and $\upsilon \rightarrow \infty$. Unconditional correlations between the asset return innovation and source of risk are shown in brackets. Contributions of hedging motives and correlations were computed using 500 simulations of 500 periods. Contributions of hedging motives were calculated from regressions of the sources of risk on relative bond and equity return innovations and expressions (2.7) and (2.8). Asset returns and sources of risk were Hodrick-Prescott filtered with a smoothing parameter of 1600 before computing correlations

Labour: Total employment times Hours worked per employee, Total Economy, EO

Real exchange rate: Ratio of Consumer Prices, all items, MEI, converted into U.S. dollars

Net exports: Net exports of goods and services, USD, EO

Labour income: Compensation of employees (QNA) plus share of mixed surplus (QNA), as described in Coeurdacier and Gourinchas (2009) and based on Gollin (2002)

Consumption expenditures: Private Final Consumption Expenditures, QNA

Equity returns: S&P/TSX-300 Total Return Index (Canada), SBF-250 Total Return Index (France), CDAX Total Return Index (Germany), BCI Global Return Index (Italy), Topix Total Return Index (Japan), FTSE All-Share Return Index (UK), S&P 500 Total Return Index (US), all from GFD

Bond returns: 3 month bill rates, GFD

Estimation of the Process for Investment Efficiency Shocks

We follow the literature on investment specific technology shocks and used real investment prices as a measure of the investment specific technology shock (e.g. Fisher (2002, 2006)). The real investment price index is computed as the total capital formation deflator divided by the CPI deflator, both from the OECD Economic Outlook database. We estimate an AR(1) process for real investment prices for the U.S. and the Foreign country, where we aggregate the Foreign country, as above. The series are logged and Hodrick-Prescott filltered with a smoothing parameter of 1600 before estimating. Fisher (2006) notes that prior to 1982, investment deflators for the U.S. included little adjustments for quality changes. Consequently, the sample period chosen for our regressions is from the first quarter of 1982 until the third quarter of 2008.

2.7.5 Impulse Responses of Macro Variables

Figure 2.5: **Impulse Responses of Other Macro Variables I**

This figure presents the response of relative output, consumption, investment and labour demand to relative shocks to neutral and investment specific technology and the interest rate. Relative shocks are defined as a positive shock to the Home variable of one standard deviation, with a negative shock to the foreign variable of the same size. Note that a relative monetary shock here is a positive shock to nominal interest rates and is therefore contractionary.

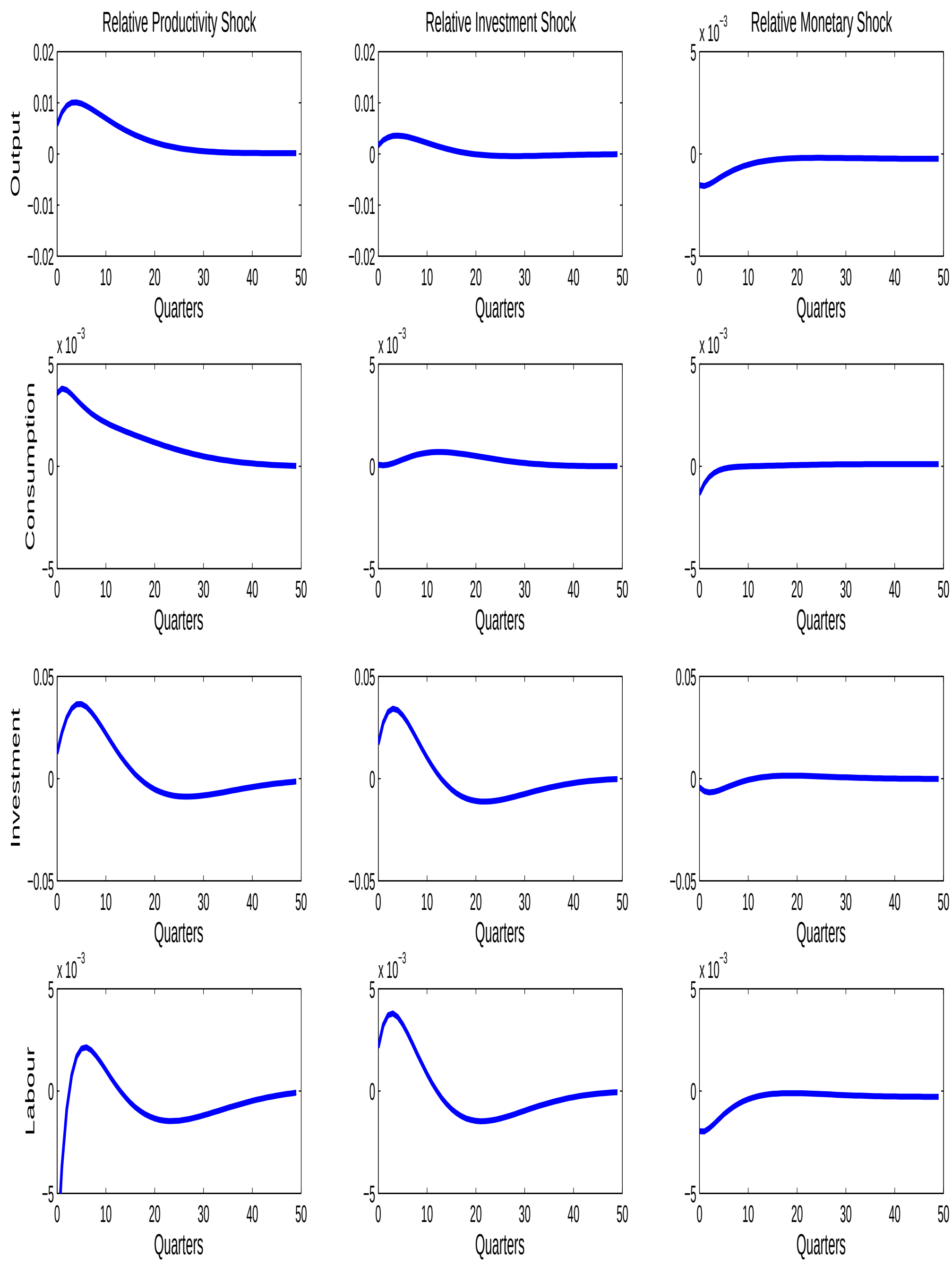
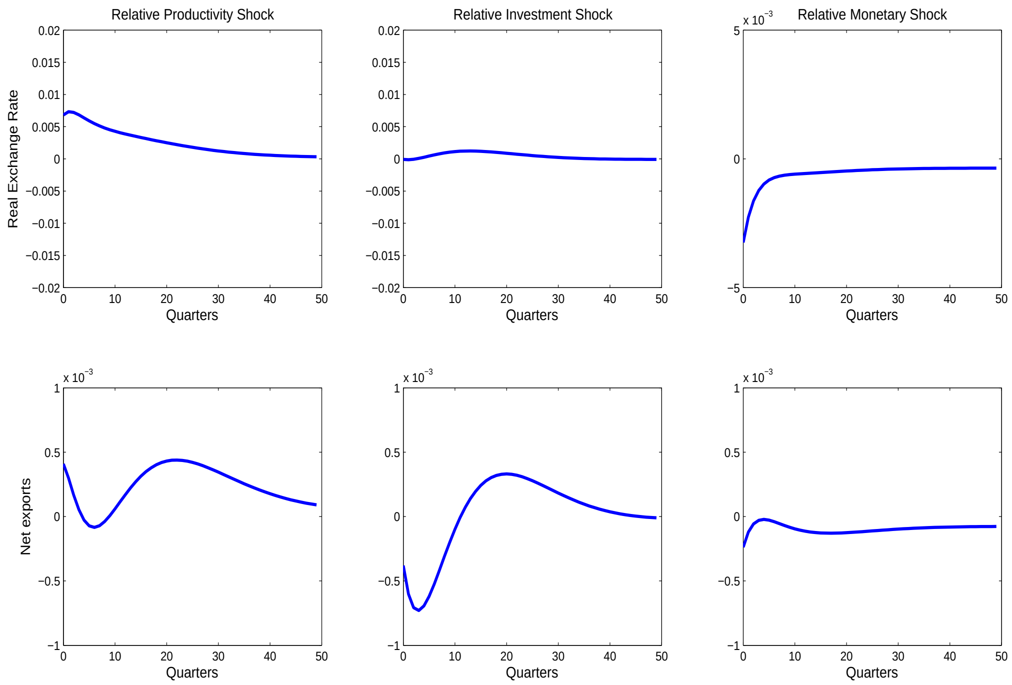


Figure 2.6: **Impulse Responses of Other Macro Variables II**

This figure presents the response of the real exchange rate and net exports to relative shocks to neutral and investment specific technology and the interest rate. Relative shocks are defined as a positive shock to the Home variable of one standard deviation, with a negative shock to the foreign variable of the same size. Note that a relative monetary shock here is a positive shock to nominal interest rates and is therefore contractionary.



Chapter 3

International Portfolios and Hedging Demands: A Sign Restriction Exploration

3.1 Introduction

The growth of cross border asset trade and methodological breakthroughs have recently led to a strong increase in work on international macroeconomic models that allow for more sophisticated asset markets.[1] This work has already led to a strong increase of our understanding of financial interactions between countries and the interaction between asset markets and the real economy. However, much of this work has been theoretical, gradually extending the baseline open economy macroeconomic model to incorporate a variety of features and shocks. Much of modern macroeconomic analysis involves comparing the predictions of models with those of the data. One common approach consists of comparing the impulse responses derived from models with impulse responses derived from data in order to assess the performance of the model and judge their merits relative to previous contributions. The purpose of this paper is to derive empirical impulse responses to various structural shocks based on a robust identification scheme that facilitates a more rigorous assessment of recent contributions in the literature on open economy macroeconomics with endogenous cross country portfolios.

Our work is motivated by recent work that shows that equilibrium asset posi-

[1] A very selective list includes Benigno and Kucuk Tuger (2009), Benigno and Nistico (2009), Coeurdacier et al (2007), Coeurdacier and Gourinchas (2009), Coeurdacier et al (2009), Devereux and Saito (2005), Evans and Hnatkovska (2006), Engel and Matsumoto (2009), Heathcote and Perri (2009), Kollmann (2006), Pavlova and Rigobon (2007, 2009), Tille and van Wincoop (2009).

tions in a wide class of models can be related to a small numbers of sources of risk.[2] In particular, this literature shows that equilibrium asset positions should depend on the comovement of dividends and nominal exchange rates - the determinants of relative equity and bond returns - with consumption expenditures and labour income. Coeurdacier and Gourinchas (2009), Coeurdacier et al (2009) and Rahbari (2010a,b) estimate the relevant moments, which account for the asset returns of other assets, and compare the results with predictions of certain equilibrium models and with observed cross country asset holdings. Here, we take a different route and follow the spirit of much recent macroeconomic work and investigate the impulse responses of these variables to various shocks. We focus here on four variables, consumption expenditures, labour income, dividends and nominal exchange rates, as their dynamics play a key role for equilibrium asset positions. To our knowledge, the impulse responses, at least for the first three, have not been the subject of much empirical work. We base our choice of the shocks to be identified on the recent literature on cross border portfolios which in turn is based on business cycle work in a domestic and international context. In Coeurdacier et al (2009), there are shocks to neutral and investment specific technology, while Engel and Matsumoto (2009) focus on monetary and neutral productivity shocks. Here, we therefore examine the response of the different variables to neutral and investment specific technology shocks and monetary shocks.

We pursue a sign restriction based identification strategy, i.e. we identify the structural shocks by imposing theory based predictions on the impulse responses of a number of variables. These methods were initiated by Faust (1998) and Uhlig (2005) and have recently been applied by, among others, Canova and de Nicolo (2002), Corsetti et al (2009) and Mountford and Uhlig (2009). These methods have the advantage that, while they use theory to derive identification restrictions, these restrictions are picked such that they are consistent with a large class of models. We use restrictions on consumer price inflation, output, investment, nominal interest rates and real investment prices to identify the three types of shocks and our identification assumptions were picked to be consistent with previous work in the sign restriction literature in the domestic and open economy context.

Our results indicate that there is a large degree of comovement between relative consumption expenditures, relative labour income, relative dividends, and the nominal exchange rate. "Relative" variables are constructed as cross coun-

[2] See e.g. Coeurdacier and Gourinchas (2009), Coeurdacier et al (2009) and Rahbari (2010a,b).

try differences between the response of U.S. variables relative to an aggregate of the G7. We find that the first three variables respond positively in a persistent manner to a relative productivity shock, while the nominal exchange rate appreciates. In response to a monetary loosening, the first three variables fall, while the nominal exchange rate depreciates. The responses to an investment specific technology shock are mostly nonsignificant. The construction of relative consumption expenditures, relative labour income and relative dividends includes nominal exchange rates. The high comovement, qualitatively, but also quantitatively, of all variables (with each other, and in particular with the nominal exchange rate) suggests that fluctuations in nominal exchange rates dominate the fluctuations of the other nominal cross country variables.

Two other responses deserve special comment. Firstly, the response of consumption expenditures to productivity shocks is not consistent with the predictions of many macroeconomic models. This puzzle is strongly related to the behaviour of real exchange rates in response to productivity shocks, discussed in, inter alia, Corsetti et al (2008) and Enders and Mueller (2009). This is because, under certain assumptions that are very common in the open economy macro literature, namely CRRA utility and complete markets, real exchange rates and relative consumption expenditures are perfectly correlated to a first order approximation, and the correlation is often still very high once one allows for incomplete markets or different specifications of the utility function. As Corsetti et al (2008) and Enders and Mueller (2009) discuss, many standard open economy macro models imply that the real exchange rate depreciates in response to productivity shocks, mainly due to the response of relative prices. The close association with consumption expenditures implies that relative consumption expenditures also fall, while our results indicate that they do in fact increase initially in the data. Corsetti et al (2008) and Enders and Mueller (2008) present models that do allow for real exchange rate appreciations in response to productivity shocks. However, since their models also imply substantially incomplete markets, it is not clear what the implications are for relative consumption expenditures.

The second impulse response of note is the response of relative dividends to investment specific technology shocks. This type of shock is often interpreted as technological improvements in the investment goods producing sector, and identified using fluctuations in the real price of investment goods, following the work of Greenwood et al (1997, 2000), and Fisher (2002, 2006). In our benchmark calibration, the responses of all variables, including relative dividends, are mostly insignificant. However, under our alternative identification that imposes more restrictions on the investment specific shock, we find that relative dividends fall, as predicted by open economy models that allow for these types of shocks. In the models, the behaviour of relative dividends is due to the fact that improvements in investment specific technology induce an increase in investment, and that this increase in investment is financed by a reduction in dividends.

The rest of the paper is structured as follows. Section two presents some results that show the relation between equilibrium asset positions, asset returns and sources of risk in a wide class of open economy macro models, and introduces the theoretical motivation for the restrictions imposed to identify neutral and investment specific technology shocks, and monetary shocks. Section three presents the empirical strategy in some detail, while the resulting impulse responses are presented and discussed in section four. Section five concludes.

3.2 Equilibrium Asset Positions, Hedging Demands, and Structural Shocks

Recent work has shown that equilibrium asset positions in a large class of models can be related to fluctuations in a small number of sources of risk (see e.g. Coeurdacier and Gourinchas (2009), Coeurdacier et al (2009) and Rahbari (2010a,b)). For example, Rahbari (2010a) shows that in a wide class of new open economy macroeconomic models with trade in equities and nominal bonds, equilibrium equity and bond positions can be shown to be given by:

$$S = \lambda + (1-\lambda)\left(\frac{C}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{PC}, \widehat{R}_t^{S}\right)}{var_{R_t^B}\left(\widehat{R}_t^{S}\right)} - \frac{WL}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{WL}, \widehat{R}_t^{S}\right)}{var_{R_t^B}\left(\widehat{R}_t^{S}\right)}\right) \tag{3.1}$$

$$\widetilde{B} = (1-\lambda)\left(\frac{\beta}{(1-\beta)}\frac{C}{Y}\frac{cov_{R_t^S}\left(\widehat{R}_t^{PC}, \widehat{R}_t^{B}\right)}{var_{R_t^S}\left(\widehat{R}_t^{B}\right)} - \frac{\beta}{(1-\beta)}\frac{WL}{Y}\frac{cov_{R_t^S}\left(\widehat{R}_t^{WL}, \widehat{R}_t^{B}\right)}{var_{R_t^S}\left(\widehat{R}_t^{B}\right)}\right) \tag{3.2}$$

where S and $\widetilde{B}$ are the share of domestic equity and the ratio of domestic debt to GDP held by domestic investors in the steady state, $\widehat{R}_t^S$ $\left(\widehat{R}_t^B\right)$ are relative equity (bond) returns, $\widehat{R}_t^{PC}$ is consumption expenditure risk and $\widehat{R}_t^{WL}$ is human capital risk. λ is the share of the domestic economy in world output and C, D, Y and WL are the steady state ratios of consumption, dividends, output and labour income to output. $cov_{R_t^B}(x_t, y_t)$ and $cov_{R_t^S}(x_t, y_t)$ are the covariance between the components of x_t and y_t that are orthogonal to relative bond and relative equity returns, respectively, while β is the discount factor.

The term "relative" in all of the above variables is meant to imply cross country differences. Specifically, relative asset returns and sources of risk are given to a first order approximation by:

$$\begin{aligned}
\widehat{R}_t^S &= \widehat{R}_{F,t}^S - \widehat{R}_{H,t}^S - \widehat{Z}_t + \widehat{Z}_{t-1} \\
&= (1-\beta)\left(E_t - E_{t-1}\right)\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j} - \widehat{D}_{H,t+j} + \widehat{Z}_{t+j}\right)\right] \qquad (3.3) \\
\widehat{R}_t^B &= \widehat{R}_{H,t}^B - \widehat{R}_{F,t}^B - \widehat{Z}_t + \widehat{Z}_{t-1} = (1-\beta)\left(E_t - E_{t-1}\right)\left[\sum_{j=0}^{\infty}\beta^j\left(-\widehat{Z}_{t+j}\right)\right] \qquad (3.4) \\
\widehat{R}_t^{PC} &= (1-\beta)\,\widetilde{E}_t\sum_{j=0}^{\infty}\beta_{t+j}\left(\widehat{P}_{C,t+j}^H + \widehat{C}_{H,t+j} - \widehat{P}_{C,t+j}^F - \widehat{C}_{F,t+j} - \widehat{Z}_{t+j}\right) \qquad (3.5) \\
\widehat{R}_t^{WL} &= (1-\beta)\,\widetilde{E}_t\sum_{j=0}^{\infty}\beta_{t+j}\left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} - \widehat{W}_{F,t+j} - \widehat{L}_{F,t+j} - \widehat{Z}_{t+j}\right), \qquad (3.6)
\end{aligned}$$

where $\widehat{X}_t = \frac{X_t - X}{X}$ is the proportional deviation of X_t from its steady state value, $\widetilde{E}_t = E_t - E_{t-1}$ is the difference between time t and time $t-1$ expectations, Z_t is the nominal exchange rate defined as the number of units of the Home currency per unit of the Foreign currency. $D_{i,t}$ are dividends in country i, $\widehat{P}_{C,t}^i$ is the consumption price index, $C_{i,t}$ is real consumption, $W_{i,t}$ is the (nominal) wage, and $L_{i,t}$ denotes labour supply. As equations (3.3) to (2.52) show, equilibrium asset positions only depend on cross country differences or relative values. Thus, relative equity returns, $\widehat{R}_t^S$, depend on the present discounted value of future relative dividends, $\widehat{D}_{F,t} - \widehat{D}_{H,t} + \widehat{Z}_t$, while relative bond returns, $\widehat{R}_t^B$, depend on the present discounted value of nominal exchange rate changes, $\widehat{Z}_t$. Consumption expenditure risk, $\widehat{R}_t^{PC}$, depends on the present discounted value of future relative consumption expenditures, $\widehat{P}_{C,t}^H + \widehat{C}_{H,t} - \widehat{P}_{C,t}^F - \widehat{C}_{F,t} - \widehat{Z}_t$, while human capital risk depends on the present discounted value of future relative labour income, $\widehat{W}_{H,t} + \widehat{L}_{H,t} - \widehat{W}_{F,t} - \widehat{L}_{F,t} - \widehat{Z}_t$.

Equations (3.3) to (2.52) show that equilibrium asset positions depend on the comovement of relative asset returns with the risk arising from fluctuations in consumption expenditure and labour income. For example, domestic investors would hold on to a large share of their domestic equity stock, if the relative payoff of domestic equity is high when relative consumption expenditures, in present value terms, are high, or when relative labour income, again in present value terms, is low. As Coeurdacier and Gourinchas (2009), Coeurdacier et al (2009) and Engel and Matsumoto (2009) emphasize, the relevant moments for equilibrium asset positions need to take into account the returns on other assets traded also, as some of the risk is already hedged by the other asset positions taken. Coeurdacier and Gourinchas (2009), Coeurdacier et al (2009) and Rahbari (2010a,b) estimate the relevant moments which account for other assets traded directly.

Here, we follow a different route and instead identify the response of the components of the relative asset returns and sources of risk, namely dividends, nominal exchange rates, consumption expenditure and labour income, conditional on different types of shocks. While equilibrium asset positions depend on the moments which reflect the returns of other assets, the models that we construct in order to investigate and explain cross border portfolio decisions need to be consistent with the empirical responses of the portfolio relevant variables to various types of shocks also. To date, there is little work on the response of dividends, consumption expenditure and labour income to different shocks and this paper therefore tries to fill that gap. In order to do so, we first need to decide on the types of shocks to be investigated, and then on the identification scheme. Both choices reflect our desire for parsimony. The first choice is mainly based on the recent literature on international portfolio choice which in turn reflects previous work on business cycles. In Coeurdacier et al (2009), there are shocks to neutral and investment specific technology, while Engel and Matsumoto (2009) focus on monetary and neutral productivity shocks. Here, we therefore examine the response of the different variables to neutral and investment specific technology shocks and monetary shocks, and fiscal shocks. We choose an identification scheme that is based on sign restrictions that are satisfied in a wide range of models to identify the structural shocks.

3.2.1 Productivity Shocks

The first shock we identify are neutral shocks to productivity. These shocks are assumed to be present in the overwhelming majority of modern macroeconomic models, going back at least to the work of Kydland and Prescott (1982) and Long and Plosser (1983) and have also been the subject of much empirical work. Increases in productivity imply that the same amount of goods can be produced with fewer inputs. The cost of production therefore decreases which in general implies that the price level falls or, more generally, the rate of increase of prices slows down. The rise in productivity also generally implies an increase in total output produced. A further implication of productivity shocks is an increase in investment, for two reasons. Firstly, the increase in productivity increases the marginal product of capital and, if at all persistent, generates an incentive to increase the capital stock. Secondly, a rise in productivity increases the level of wealth of agents in the economy. Forward looking agents then want to smooth their consumption by saving which in turn fuels investment, at least part of which will be domestic. Our identification assumptions for neutral relative productivity shocks are then i) a fall in relative CPI inflation, ii) a rise in relative output, iii) a rise in relative investment.[3]

A number of papers have used a sign restriction based identification scheme to identify technology shocks. Peersman and Straub (2009) show that technology shocks have a positive effect on output, a negative effect on prices and a positive effect on real wages under a wide range parameter values for a standard medium scale DSGE model and use these restrictions to investigate the response of hours worked to technology shocks for the euro area. Corsetti et al (2009) focus on the response of the real exchange rate to supply shocks and identify positive supply shocks as increases in manufacturing output relative to aggregate output in the US, decreases in the domestic price of US manufacturing relative to US nontradables, increases in US manufacturing output relative to Foreign manufacturing output; and increases in US labor productivity relative to Foreign labor productivity in manufacturing. Enders et al (2010) investigate the response of the real exchange rate to fiscal and productivity shocks using a sign restriction based identification scheme. The authors derive a long list of restrictions from simulations of a two country DSGE model that randomises the value of some parameters, and the list includes increases in output and investment and a fall in (consumer price) inflation.

3.2.2 Investment Specific Technology Shocks

A positive shock to investment specific technology reduces the cost of an efficiency unit of investment. We follow the recent literature on investment specific technology shocks (see e.g. Fisher (2002, 2006)) and identify positive shocks to effiency in the investment goods sector with a decrease in the relative price of investment, measured as a fall in the ratio of the total capital formation deflator and the consumption price index. If technological improvements in the investment

[3]Our narrative justification of the identification restrictions neglects the open economy dimension of the investigation, while we highlighted above that many implications of open economy models depend on cross country differences. However, it can be shown that the same mechanisms apply in a cross country context, both here and throughout the paper when no explicit reference is made to the cross country dimension.

specific technology sector involve increases in quality as well as the efficiency of production, the identification strategy presupposes that increases in the quality of investment goods are reflected in the investment price index. Fisher (2007) discusses this issue at length and uses a series for the US for the total capital formation deflator that includes some additional changes in quality. Since these adjustments are not available for the sample of countries that we use to construct the relative variables, we use the original total capital formation deflators provided by the OECD (details on data sources and construction are presented in the appendix). In addition to differences in the data sources, our identification strategy also differs from Fisher (2007). Fisher (2007) identifies investment specific technology shocks using a long run restriction identification scheme that assumes that increases in investment specific technology imply reductions in the ratio of the investment price and the consumption price index in the long run, that total labour productivity rises in response to an investment specific technology shock in the long run, and a further assumption that is more model specific. Fisher (2007) shows that the assumption that the long run relative price of investment is reduced by (permanent) investment specific technology shocks is satisfied in a standard model with investment specific technology shocks. What is more, he shows that, in the model, this price rises also in the short term. Our identification strategy assumes that a positive relative shock to investment specific technology reduces the relative price of investment in the short run and is thus consistent with the model and the empirical results in Fisher (2007). We impose two other restrictions to identify investment specific technology shocks, namely that the investment shock increases both investment and the interest rate. The only paper that we are aware of that employs a sign restriction based identification scheme to identify investment specific technology shocks is Peersman and Straub (2006). There, the authors derive the implications of investment specific shocks from simulations of a New Keynesian DSGE model and arrive at a number of restrictions that include, but go beyond, those employed in our identification scheme.

3.2.3 Monetary Shocks

The third shock we identify is a monetary shock. Monetary shocks have been quite prominent in the recent literature on the use of sign restriction in macroeconomics. We assume that positive monetary shocks: i) decrease the relative nominal interest rate, and ii) increase relative inflation. Faust (1998), Uhlig (2005) and Peersman and Straub (2006, 2009) also impose both of these restrictions (while also including others), while Canova and de Nicolo (2002) impose restrictions on the comovement of different variables in the VAR in order to identify a monetary shock.

3.3 Empirical Strategy

As noted above, we build on the work of Faust (1998), Canova and De Nicoló [2002] and Uhlig (2005) and use sign restrictions to identify the underlying shocks. Other recent applications of sign restrictions methods have been Dedola and Neri (2007), Mountford and Uhlig (2005) and Peersman and Straub (2009) in a domestic context, and Corsetti et al (2009), Enders et al (2009) and Scholl and Uhlig (2008) in an open economy context. Our discussion of the empirical strategy largely follows Corsetti et al (2009).

Firstly, note that we take advantage of the fact that many open economy macroeconomic models are formulated as symmetric two country models. The inbuilt symmetry of these models implies that it is without loss of generality to focus on the relative response of variables to relative shocks, where the relative response is given by the cross country differential and relative shocks are defined as a positive shock in one country, with a negative shock to the other country of equal size. In this, we follow Clarida and Gali (1994) and Glick and Rogoff (1995), and Corsetti et al (2009) and this approach has the advantage that it reduces the number of variables that appear in the VAR, as only the cross country differentials of variables enter the VAR. Nevertheless, the assumption of symmetry that underlies this approach can raise potential issues in cases, where asymmetries across countries are assumed to be large, or when country size is small.[4] In our analysis, we are interested in the response of variables for the US vis-a-vis the rest of the world. To that end, we will compute relative values for the variables of interest, as the cross country differential between values for the US and values for an aggregate of G7 economies (Canada, France, Germany, Italy, Japan, UK and US) and refer to the latter as the rest of the world (ROW). The choice of the latter countries was mainly dictated by data availability, but they also account for a very large proportion of asset holdings, particularly in the early part of our sample. Since the US is a large country and the rest of the world in our application is of similar size, the assumption of symmetry may be somewhat less problematic here.

[4]The alternative is to include all country specific variables in the VAR. However, this approach quickly becomes infeasible, as the number of parameters to be estimated quickly runs up against constraints imposed by data availability. However, see Scholl and Uhlig (2008) for an application.

The previous section outlined the theoretical results that equilibrium equity and bond positions should be related to relative dividends, relative labour income, relative consumption expenditure and nominal exchange rates and it is the responses of these variables to the three types of shocks that we are interested in here. In order to identify the response of these variables, we will estimate a vector autoregression and impose the identification restrictions discussed in the previous section. We estimate several specifications of the following reduced from VAR model (omitting the constant):

$$J_t = A(L)\, J_{t-1} + u_t, \tag{3.7}$$

where the vector J_t includes several variables of interest, $A(L)$ is a polynomial in the lag operator and u_t are the reduced form residuals, with covariance matrix Σ. The VARs considered here will involve six variables and our identification strategy will imply imposing restrictions on the impulse responses of the first five variables, but leaving the sixth variable unrestricted. As is common in the open economy literature, we keep the first five variables and thus equations fixed, and interchange the sixth variable to consider the responses of different variables, to deal with the curse of dimensionality and the fact that sample sizes are relatively small.

The empirical implementation closely follows Uhlig (2005) and Mountford and Uhlig (2009). We estimate (3.7) using ordinary least squares (OLS) which is equivalent to maximum likelihood (ML) estimation under the assumption of Gaussian innovations. We are interested in investigating the impulse responses of several variables to shocks. The residuals in (3.7) , however, are correlated with each other and impulse responses calculated on the basis of the reduced form coefficients will therefore be a linear combination of the true impulse responses. Identification then implies solving for the matrix F that maps the orthogonal residuals ε_t in to the reduced form residuals, such that $u_t = F\varepsilon_t$. The matrix F can then be represented as a solution to the matrix equation $\Sigma = FF'$. With n variables in the VAR, the covariance matrix is of dimension $n \times n$ and the orthogonality of the shocks and the symmetry of Σ then imply that $n(n-1)$ additional restrictions need to be imposed on F in order to be able to uniquely (up to an orthonormal transformation) solve for it. There are several different ways of imposing these restrictions, including imposing restrictions on the impact response of variables (Sims (1980)) or restrictions on the long run response of variables (Blanchard and Quah (1989)). Criticisms of these approaches have noted that the short run restrictions imposed are often not satisfied in structural models, while long run restrictions are often problematic in the light of the short samples used to estimate the reduced form VAR. Using sign restrictions on the impulse responses of variables is then seen as preferable, as these restrictions are chosen so as to be explicitly compatible with a fairly general class of models.

Following Uhlig (1998) and Corsetti et al (2009), we call the jth column of F an "impulse vector" in R^n. The impulse vector maps the innovation to the jth structural shock ε_j into the contemporaneous impact responses of all the n variables, $\Psi_{0,j}$. We can then calculate the structural impulse responses of the n variables recursively, using the identification matrix F and the estimated coefficient matrices $A(L)$ of the reduced form VAR:

$$\Psi_{s,j} = \sum_{h=0}^{s} A_{s-h}\Psi_{h,j}, s \geq 1, A_{s-h} = 0, s-h \geq p \tag{3.8}$$

$$\Psi_{0,j} = F_j, \tag{3.9}$$

where p is the lag length of the VAR. Uhlig (2005) shows that any structural impulse vector F_j from a given identification matrix F can be represented as Pq for an appropriate vector q belonging to the hypersphere of unitary radius $S^n \subset R^n$, and an arbitrary matrix P such that $PP' = \Sigma$. Common choices for the orthogonal decomposition P are then, for example, the eigenvalue-eigenvector or the Cholesky decomposition of Σ. The sign restriction approach uses theoretical insights to constrain the behaviour of impulse responses directly. Since the decompositions are not unique, the resulting impulse vectors and impulse matrices are not, either. The sign restrictions approach then uses the theoretical insights to attribute a zero probability to impulse vectors which are not consistent with the restrictions implied by theory. The restrictions on the sign of impulse responses are chosen so as to fit a general class of models and should thus be more immune to changes to model specification and parameterisation. Impulse vectors that satisfy the assumed set of restrictions are kept and attributed equal probability. In a Bayesian manner, this approach thus attributes zero weight to impulse responses which do not satisfy the assumed restrictions, while it attributes equal weight to those which satisfy them.

We follow Uhlig (2005) and Corsetti et al (2009) and simulate the posterior distribution of impulse responses consistent with our sign restrictions by jointly drawing from the Normal-Wishart posterior for Σ, $B(L)$ and the uniform for q over S^n, discarding the impulse responses that violate the restrictions, using the following algorithm: for a given estimate of the VAR reduced-form matrices and

$B(L)$, and the associated decomposition P, we draw a large number of candidate q vectors from a uniform distribution over S^n, and compute the associated impulse vector F_j and impulse response matrix Ψ, discarding those that do not satisfy the assumed sign restrictions. In practice, the q vectors are drawn from a multivariate standard normal and normalized with their Euclidean norm to make sure they have unitary length. The coefficients of the reduced-form VAR and the impulse vector are therefore estimated simultaneously.

3.4 Impulse Responses to Neutral and Investment Specific Technology Shocks, and Monetary Shocks

In the previous two sections, we described the nature of the sign restrictions we imposed as well as their motivation. As noted above, we estimate several specifications of the following reduced from VAR model (omitting the constant):

$$J_t = A(L)\, J_{t-1} + u_t. \tag{3.10}$$

In our benchmark specification, the vector J_t is given by:

$$J_t' = \begin{bmatrix} \pi_t & y_t & i_t & r_t^n & p_{I,t} - p_{C,t} & x_t \end{bmatrix},$$

where π_t is log relative CPI inflation, y_t, i_t are log relative output and investment, r_t^n are log nominal interest rates, $p_{I,t} - p_{C,t}$ is the relative ratio of the CPI and the investment price index, and x_t is a final variable of interest. The VAR is estimated in levels, with four lags and without a constant. The sample period is quarterly and extends from the first quarter of 1970 to the third quarter 2008, the longest sample for which data is available. Our identification strategy will imply imposing restrictions on the impulse responses of the first five variables, but leaving the sixth variable unrestricted. As is common in the open economy literature, we keep the first five variables and thus equations fixed, and interchange the sixth variable to consider the responses of different variables, to deal with the curse of dimensionality and the fact that sample sizes are relatively small. The four variables of primary interest are i) (log) relative consumption expenditure, pc, ii) (log) relative labour income, wl, iii) (log) relative dividends, d, and iv) the (log) nominal exchange rate, z. Details on data sources and construction are presented in the

Table 3.1: **Sign Restrictions**

	Productivity	**Investment**	**Monetary**
CPI inflation	< 0		> 0
Output	> 0		
Investment		> 0	
Nominal Interest Rate		> 0	< 0
Real Investment Price		< 0	

appendix. Table 3.1 summarises our identification restrictions. All restrictions are imposed for five quarters and all restrictions are imposed simultaneously. In our experiments, we consider 500 draws from the posterior for each shock and 1000 rotations each.

3.4.1 Productivity Shocks

Figure 3.1 presents the responses of the five variables used for identification to a relative productivity shock. Each figure reports confidence bands around the median impulse response, with the upper dashed line showing the 16th percentile, while the lower dashed line shows the 84th percentile response. Relative inflation is significantly negative for about seven quarters and therefore only marginally longer than our restriction horizon. It remains close to zero thereafter, even though it is on the margin of becoming significantly positive around 15 quarters after the shock . Relative output increases more persistently and remains positive for about 20 quarters. Relative investment, relative interest rates and relative investment prices were unrestricted in this identification scheme. Relative investment rises initially and falls thereafter, but the response is never significant. Relative interest rates fall after about one quarter, but the fall is only briefly significant after six quarters. The relative price of investment goods increases and the increase is significant after about 12 quarters.

Turning to the unrestricted variables of interest for portfolios, Figure 3.2 presents the impulse responses for the relative consumption expenditure, relative labour income, relative dividends and the nominal exchange rate to the relative productity shock in the first column. Relative consumption expenditures rise strongly on impact, and the response only slowly fades out over time, being significant for around ten quarters. It is worth pointing out that this response of consumption expenditures is not consistent with many open economy macro models. This is because in many of these models, consumption expenditures and the real exchange rate are perfectly correlated, and the real exchange rate depreciates in response to a productivity shock.[5]

Figure 3.2 shows that relative labour income exhibits a very persistent increase, with the response being significant for more than 50 quarters. Relative dividends show the largest increase, almost five percent on impact, but the response fades over time and ceases to be significant after around 10 periods. Finally, the nominal exchange rate appreciates on impact, but the appreciation is only significant for around 8 quarters.

As we pointed out above, equilibrium asset positions depend on the comovement of asset returns with sources of risk, conditional on the response of other assets traded. This implies that we cannot derive direct implications for equilibrium asset positions from these graphs. However, one feature is noteworthy and will return in the responses to other shocks below: At least qualitatively, the response of all of the variables shown in Figure 3.2 is similar, as they all show a significant increase on impact. One interpretation for this observation is that the nominal exchange drives much of this comovement.

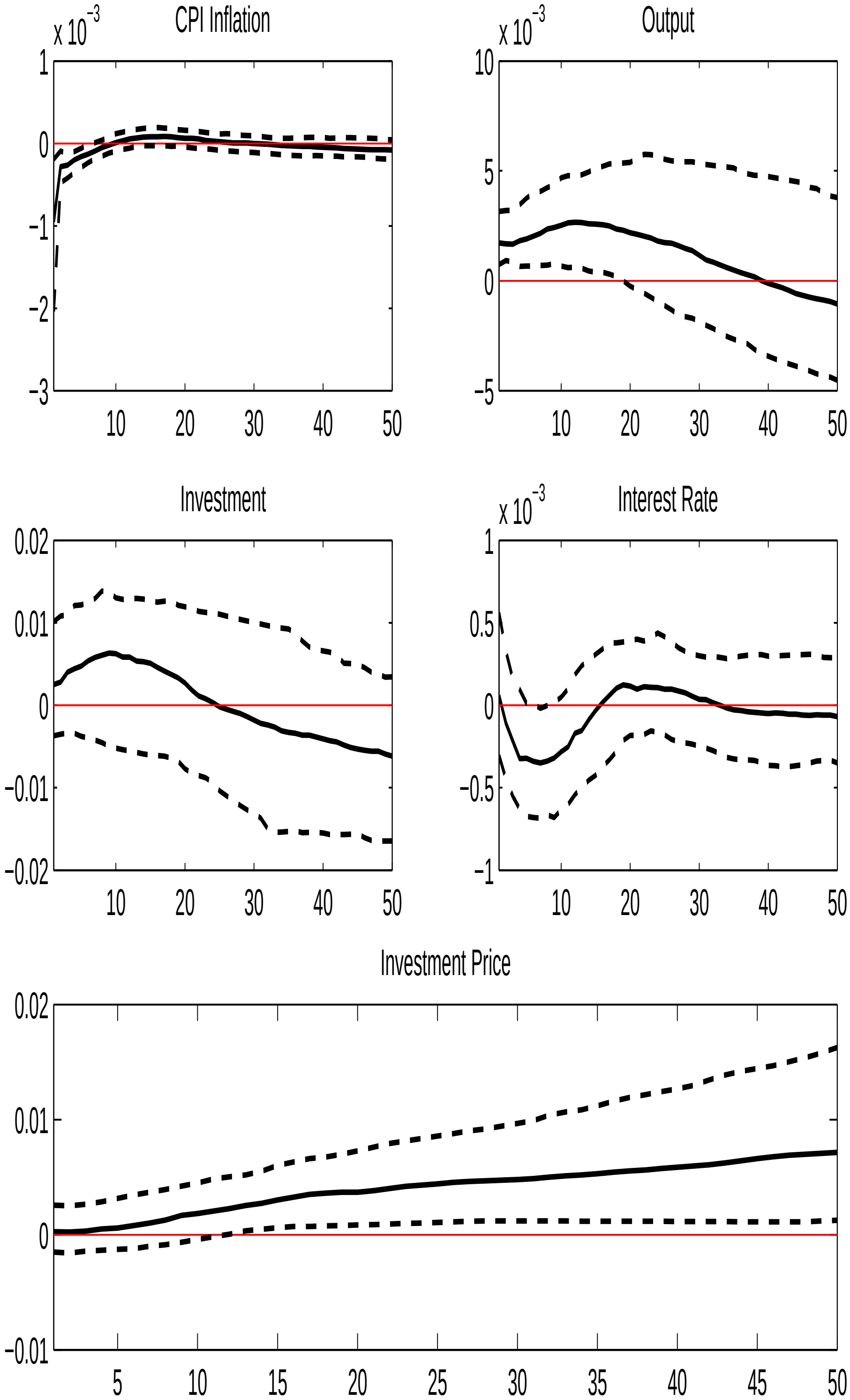
3.4.2 Investment Specific Technology Shocks

The responses of the variables used for identication to relative shocks to investment specific technology are shown in Figure 3.3. As noted above, the response of investment, the interest rate and the real price of investment are used for the identification of this shock. Investment rises on impact, whereafter the response falls and becomes insignificant after about 10 quarters. Interest rates show a similar response, but the decline is steeper and turns insignificant after eight quarters. The fall in the investment price is very persistent and remains significant for around 25 quarters. The response of output is similar to the response of investment, but it falls less quickly, remaining significant for around 40 quarters, while the response of consumer prices is insignificant throughout.

The second column of Figure 3.2 shows the responses of consumption expenditures, labour income, dividends and nominal exchange rates. All four impulse

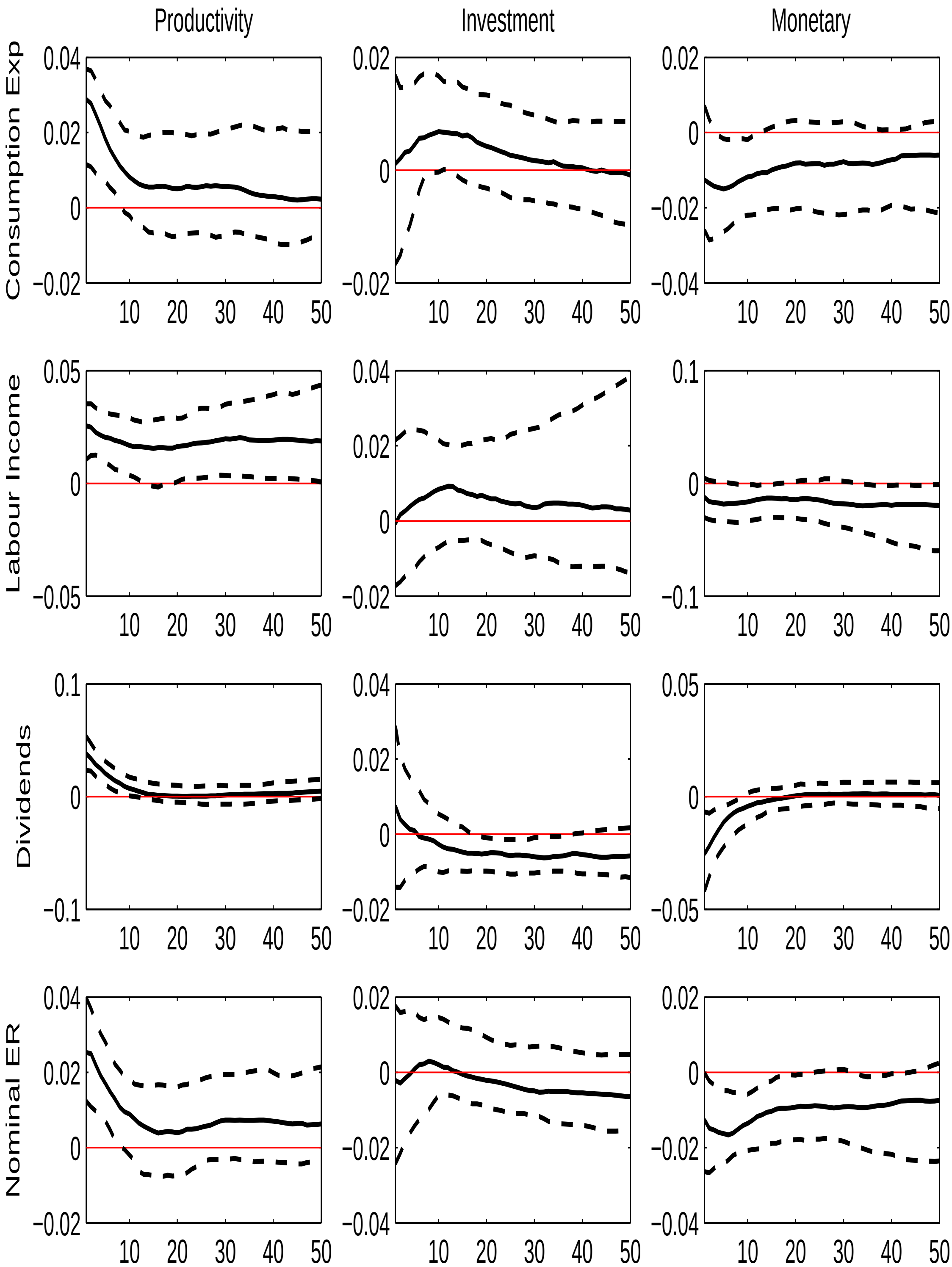
[5]Corsetti et al (2008) and Enders and Mueller (2009) are important exceptions. In these models, the real exchange rate depreciates in response to positive productivity shocks. Neither paper highlights the response of consumption expenditures which can potentially be different, as markets are incomplete.

Figure 3.1: **Impulse Responses to a Relative Productivity Shock**



The solid lines represent the median impulse responses. Dashed lines are 16th and 84th percentile responses

Figure 3.2: **Impulse Responses of Hedging Demands and Sources of Risk: Benchmark Identification**



The solid lines represent the median impulse responses. Dashed lines are 16th and 84th percentile responses

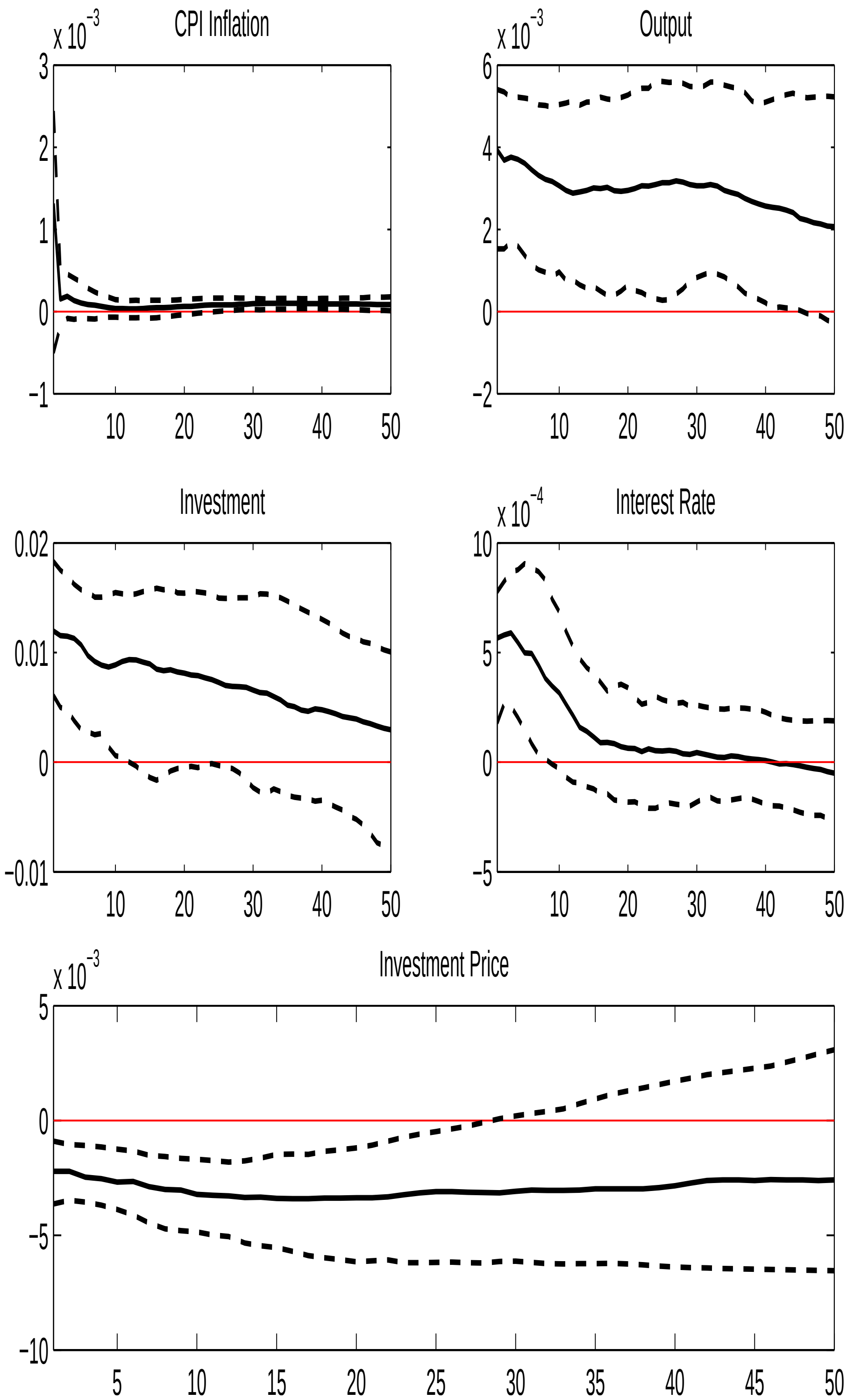
responses are insignificant for most of the horizon. Consumption expenditures increase, but the increase is gradual and only significant only very briefly, around 11 quarters after the shock, and even then only marginally so. The response of labour income is somewhat similar, but never exceeds our thresholds of significance. Relative dividends increase on impact and then fall and the fall is marginally significant after around 20 quarters, while the response of the nominal exchange rate is never significant and remains close to zero throughout.

3.4.3 Monetary Shocks

The responses of relative CPI inflation, output, investment, the relative interest rate and the relative price of investment to monetary shocks are shown in Figure 3.4. CPI inflation and interest rates are used for identification. Consumer price inflation rises on impact and then slowly returns to zero, with the increase significant for around 10 quarters. The fall in the nominal interest rate is only significant for around 6 quarters, but remains negative for the rest of the sample horizon. The other three variables in this graph, output, investment and the real price of investment do not respond significantly to this shock.

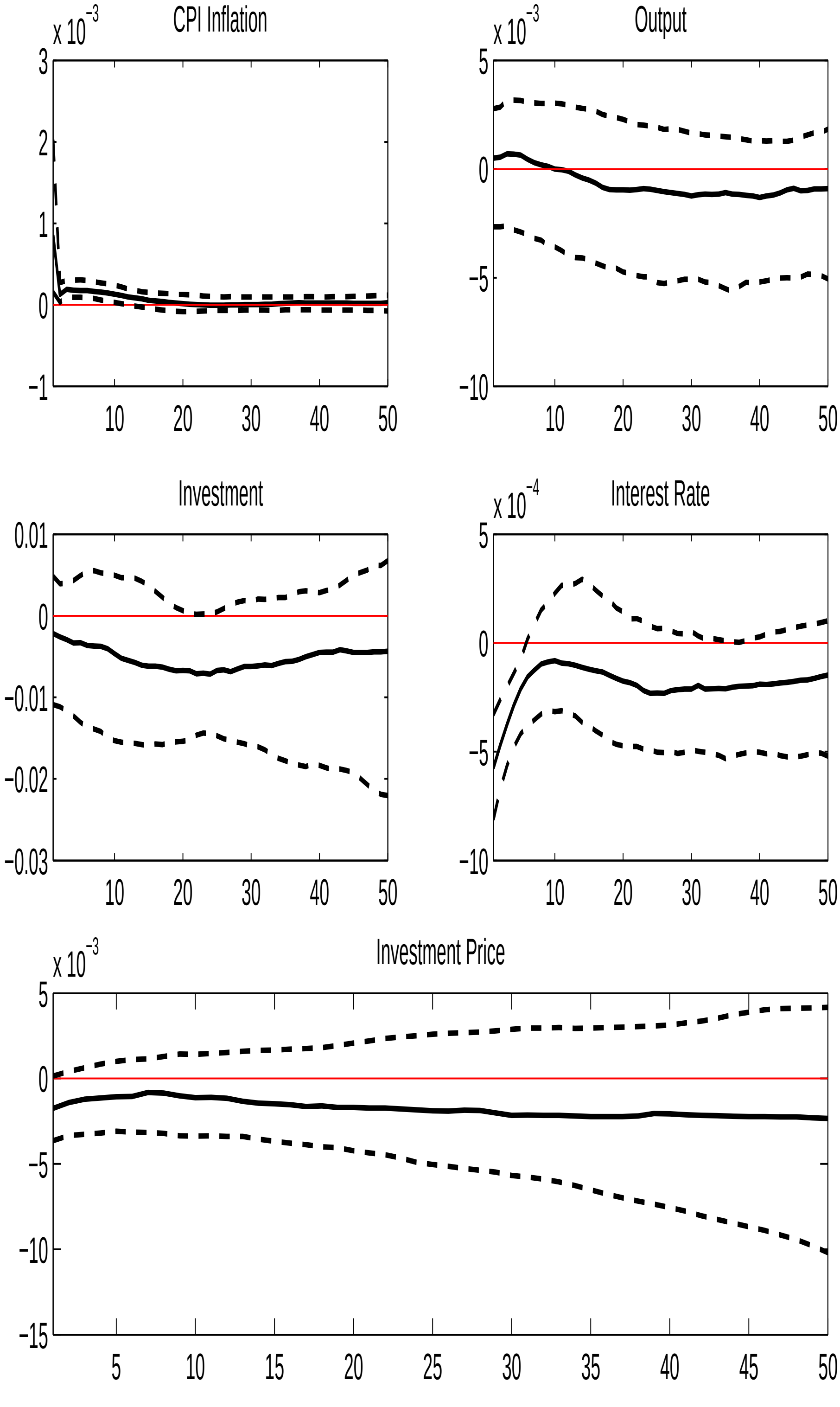
Turning to the third column of Figure 3.2, we observe that the response of consumption expenditures, labour income, dividends and the nominal exchange rate is again qualitatively similar, as all four variables show a decrease. The fall in relative consumption expenditures is persistent, but is close to our significance bounds, from either side, for the entire 50 quarter period shown. Similarly, labour income falls, but is only marginally significant for most of the period. Relative dividends also show a fall on impact, but the response becomes insignificant after around 10 quarters and is close to zero thereafter. Finally, nominal exchange rates show a persistent depreciation, but, again, the response is only marginally signficant. It is noteworthy that not only do all of the responses show a decline on impact, but it is also of comparable size, slightly below a two percent fall in most cases. We again interpret this finding as evidence of the importance of nominal exchange rate fluctuations for the dynamics of the other variables.

Figure 3.3: **Impulse Responses to a Relative Investment Shock**



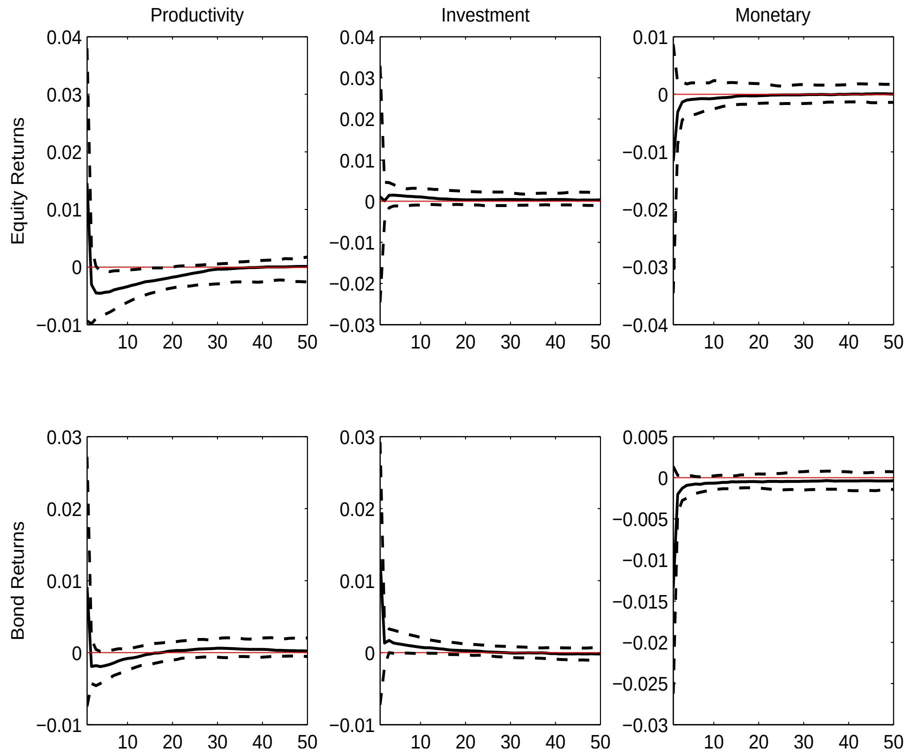
The solid lines represent the median impulse responses. Dashed lines are 16th and 84th percentile responses

Figure 3.4: **Impulse Responses to a Relative Monetary Shock**



The solid lines represent the median impulse responses. Dashed lines are 16th and 84th percentile responses

Figure 3.5: **Impulse Responses of Asset Returns**



The solid lines represent the median impulse responses. Dashed lines are 16th and 84th percentile responses

3.5 Alternative Configurations

3.5.1 Using Asset Returns Directly

In Figure 3.5, we present impulse responses from VARs in which asset returns are used directly, rather than using dividends or nominal exchange rates. Thus, the first row of Figure 3.5 shows the results from a VAR in which (log) relative equity returns are used, while the second row presents the responses resulting from a VAR that includes relative bond returns as the sixth variable. The responses of equity and bond returns are similar to the responses of dividends and nominal exchange rates. Thus, relative equity and bond returns increase in response to a positive productivity shock, while they decrease in response to a monetary loosening. However, once we use actual asset returns, the responses are no longer significant.

3.5.2 Other Assumptions for Investment Shocks

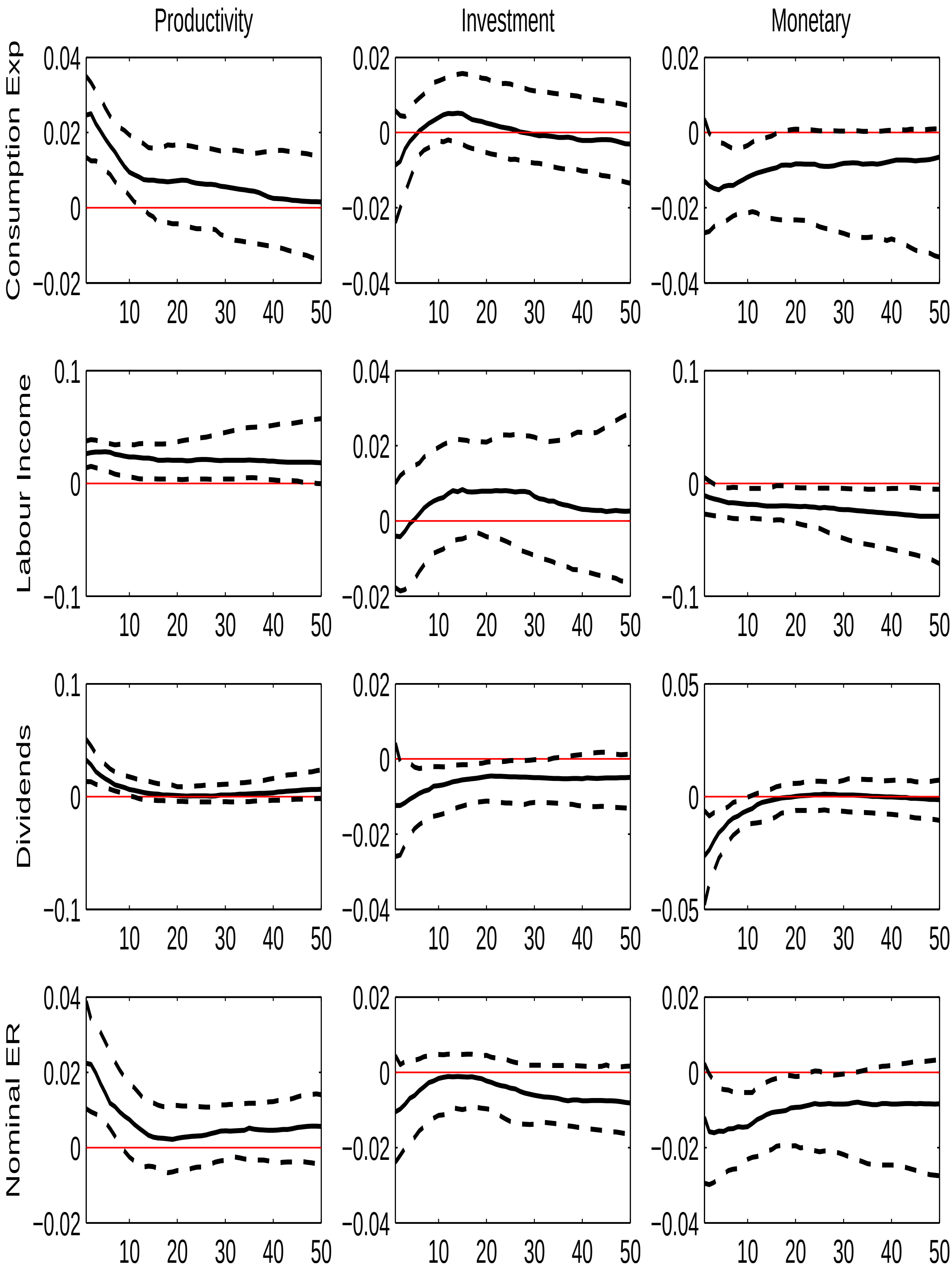
Above we noted that the responses to investment efficiency shocks are mostly non-significant. Here, we impose additional restrictions on the impulse vector to see whether we can generate more significant responses. In particular, we now impose that consumer price inflation falls, while output rises for the first five quarters, conditions which are satisfied in most models with investment specific technology shocks. The resulting impulse responses are presented in Figure 3.6. Comparing Figure 3.2 with Figure 3.6, we note that the responses to investment specific technology shocks are very similar.[6] The only exception is the initial response of dividends. While our benchmark identification scheme found that relative dividends initially rose and then fell, but that the response was insignificant until about 20 quarters after the shock, the modified identification scheme implies that relative relative dividends fall, with the fall significant after about one quarter. It is worth noting that most open economy models with investment specific technology shocks do imply a *fall* in relative dividends in response to this shock. The reason is that such a shock induces higher investment and investment is paid for by reducing dividends.

3.6 Conclusion

The response of macroeconomic variables to different types shocks has long been used to compare the predictions of macroeconomic models with actual data. This paper makes a first step to apply such methods to the recent work on cross border portfolios. Recent work has shown that we can relate equilibrium asset positions in a large class of models to the comovement of relative dividends, relative labour income, relative consumption expenditures and nominal exchange rates. These variables therefore carry special importance for open economy models with endogenous cross border asset trade in equities and bonds, but they have not been subject to much empirical research. We use sign restriction based identification methods to investigate the empirical responses of these variables to the three most commonly used shocks in this recent literature, namely neutral and investment specific technology shocks, and monetary shocks. The sign restrictions imposed are robust in the sense that they should be expected to hold in a fairly large class of models.

[6] Reassuringly, we find that the responses to the other two types of shocks - the identification of which we did not change - are virtually identical to those found in Figure 3.2.

Figure 3.6: **Impulse Responses of Asset Returns and Sources of Risk: Alternative Investment Identification**



The solid lines represent the median impulse responses. Dashed lines are 16th and 84th percentile responses

We find that the most striking feature of the responses is the large degree of comovement between all of the variables. Thus, all four variables increase, at least initially, in response to a productivity shock, while they fall in response to a monetary contraction. What is more, even quantitatively, the response is rather similar. We argue that these results can be interpreted as evidence that the dynamics of the nominal exchange rate, which is included in the computation of the other cross country nominal variables, accounts for a large fraction of the volatility of the other variables. It could then plausibly be argued that nominal exchange rates also potentially have a large effect on cross border portfolio decisions, and, at the very least, suggests that due attention should be given to the modelling of nominal variables, and in particular nominal exchange rates in open economy models, including further empirical testing. Our results also resonate with recent findings that pointed out that standard macroeconomic models have difficulty replicating the behaviour of real exchanges, in particular their dynamic responses to productivity shocks (see Corsetti et al (2009) and Enders and Mueller (2009)). In these models, the real exchange rate and relative consumption expenditures are highly correlated and we find that relative consumption expenditures increase in response to productivity shocks, while they fall in most standard open economy models.

Our analysis, however, also offers some more positive evidence for existing macro models. While in our benchmark calibration, the response of relative dividends to investment specific technology shocks is not significant, an extended identification scheme finds a significant negative response, in line with the predictions of recent models that allow for these types of shocks (e.g. Coeurdacier et al (2009), Rahbari (2010a)).

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3.7 Appendix - Data Sources and Variable Construction

The data is quarterly from the first quarter of 1970 to the third quarter of 2008. We take the U.S. as the Home country and a GDP weighted aggregate of the G7 countries with the exception of the U.S. (Canada, France, Germany, Italy, Japan, UK) as the Foreign country. As mentioned above, all the variables are constructed as cross country differences in logarithms. Also, all variables are expressed in current U.S. dollars. In most cases, the data needed to construct the relevant cross country differences is supplied directly by the databases mentioned below. Exceptions are dividends and the real investment price. Dividends D are constructed as $D = GDP - WL - T$, where WL is nonfinancial income and T are taxes. Nonfinancial income WL is computed as the $WL = LI + \phi\pi$, where LI is compensation of employees, π is mixed income and $\phi = \frac{LI}{\pi + LI}$ is the share of labour income in the sume of labour income and mixed income.[7] The real investment price is taken as the ratio of total capital formation deflator to the consumer price index.

National account data is from the OECD quarterly national accounts (QNA), Economic Outlook (EO) and Main Economic Indicators (MEI) databases, while data on equity and bond returns and nominal exchange rates is from the Global Financial Data (GFD) database. The basic data series are:

Output: Gross domestic product, volume, market prices, EO

Investment: Gross capital formation, total, volume, EO

Labour income: Compensation of employees (QNA) plus share of mixed surplus (QNA), as described in Coeurdacier and Gourinchas (2009) and based on Gollin (2001).

Consumption expenditures: Private Final Consumption Expenditures, QNA

Interest Rates: Short term interest rate (EO)

Nominal Exchange Rate: Nominal Exchange Rate, GFD

Investment Price: Total Capital Formation Deflator (EO)/ Consumer Price Inflation (QNA)

Equity returns: S&P/TSX-300 Total Return Index (Canada), SBF-250 Total Return Index (France), CDAX Total Return Index (Germany), BCI Global Return Index (Italy), Topix Total Return Index (Japan), FTSE All-Share Return Index (UK), S&P 500 Total Return Index (US), all from GFD

[7] See Gollin (1992) and Coeurdacier and Gourinchas (2009) for a discussion.

Bond returns: 3 month bill rates, GFD

Chapter 4

A Test of Hedging Based Explanations of Equity Home Bias*

4.1 Introduction

Despite the strong increase in cross border financial transactions, a large share of the domestic equity stock continues to be held by domestic investors (see Table 4.1). On the contrary, an international version of the frictionless capital asset pricing model would suggest full international diversification which implies that the share of domestic equity held should equal the share of the country in the world equity market. A large literature has developed that tries to explain the observed tendency for domestic investors to hold on to large shares of the domestic equity stock (see Lewis (1999), Karolyi and Stulz (2003) and Sercu and Vanpee (2007) for surveys of the literature). These explanations can broadly be classified into two classes. The first class of explanations emphasizes frictions in cross border asset trade, including transaction costs and informational asymmetries. A second class of explanations focusses on the possibility that holding a large share of domestic equities may be optimal for domestic investors, as it hedges against particular sources of risk.

The literature has pointed out three possible sources of risk that may imply equity home bias. The first possibility is that the structure of optimal portfolios should reflect the hedging properties of relative equity returns against real

*This chapter is joint with Professor Nicolas Coeurdacier of London Business School

Table 4.1: **Equity Home Bias in G7 countries**

Country	**Equity Market Cap as % of World Market Cap** (1)	**Share of domestic stocks in total equity portfolio (in %)** (2)	**Home Bias** (3)
Canada	2.4	84.0	81.6
France	4.3	79.8	75.5
Germany	4.0	61.3	57.3
Italy	2.2	67.3	65.1
Japan	11.3	89.5	78.2
UK	8.1	77.0	68.9
US	47.8	88.7	40.9

Home Bias in column (3) is computed as the difference between columns (2) and (1). Source: Coeurdacier et al (2007) based on CPIS data from 2001.

exchange rate fluctuations.[1] These contributions note that, under certain assumptions, home equity bias arises when excess domestic equity returns are positively correlated with an appreciation of the real exchange rate. The reason is that, with locally complete markets, efficient risk sharing implies that relative consumption expenditures increase when the real exchange rate appreciates, and if domestic excess returns on equity are high precisely at that time, domestic equity provides the appropriate hedge against real exchange rate risk. Rahbari (2010a) notes that when markets are not locally complete, the same reasoning goes through, but without taking the intermediate step of going through the real exchange rate: If excess returns on domestic equity are high in the states of nature when relative domestic consumption expenditures are high, then the optimal portfolio will be tilted towards domestic equity. We call this influence the motive to hedge "consumption expenditure risk".

It has also been argued that fluctuations in nonfinancial income, and in particular, labour income, could plausibly imply that domestic investors' equity portfolio will optimally feature equity home bias (see Baxter and Jermann (1997), Heathcote and Perri (2009), Coeurdacier and Gourinchas (2009), Coeurdacier et al (2009) and Rahbari (2010a)). This work emphasises that domestic investors are subject to sizable fluctuations in their nonfinancial income. If excess returns on domestic equity are high in the states of nature when the returns on nonfinancial capital are low, then a large share of domestic equity holdings can plausibly feature in the optimal investor portfolio and Heathcote and Perri (2009), Coeurdacier et al (2009) and Rahbari (2010a) provide examples of structural macroeconomic models that have this property. We refer to this motive as the motive to hedge "human

[1] This point was emphasised by Obstfeld and Rogoff (2000) and Coeurdacier (2009), but also already in an earlier, partial equilibrium, literature (see Adler and Dumas (1983)).

capital risk".

Finally, Berriel and Bhattaraj (2009) note that equity holdings could in principle also hedge fluctuations in taxation. If excess returns to domestic equity are high in states of nature when the excess burden of taxation rises, then again it would be optimal to tilt the structure of the investor portfolio to include a high share of domestic equities. This influence will be referred to as "government spending risk" in the rest of the paper.

The early contributions such as Baxter and Jermann (1997), Coeurdacier (2009) and Obstfeld and Rogoff (2009) derived these results and intuitions in the context of models that only featured trade in equities. A more recent literature has studied optimal portfolios in environments when multiple classes of assets are traded. While it remains true that the optimal structure of the portfolio would be chosen to reflect the three different influences described above, there is then a distinction between the payoff of the overall portfolio and the payoff of the equity portfolio. Put differently, the portfolio should be chosen to deliver high payoffs in states of nature when consumption expenditures are high, nonfinancial income is low or government spending is high. However, the (unconditional) covariance of domestic excess returns on equity with these sources of risk is no longer a relevant statistic to determine the share of domestic equity held. Rather, it has to be taken into account to what extent these sources of risk are already hedged by other assets and the relevant statistic for equity holdings becomes the covariance of the component of domestic excess returns on equity and the relevant sources of risk that is orthogonal to the excess returns on the other assets traded. This point has been discussed, amongst others, by Engel and Matsumoto (2009), Coeurdacier et al (2007, 2009), Coeurdacier and Gourinchas (2009) and Rahbari (2010a). These contributions also show that in general the distinction between the unconditional correlation between domestic equity returns and the sources of risk an the correlation, once the returns on other assets are taken into account, is key both in the data and in the context of certain macroeconomic models. We will henceforth call the covariance of the component of relative equity returns and some source of risk that is orthogonal to relative bond returns the covariance of relative equity returns and the respective source of risk, *conditional on bond returns.*

The contribution of this paper is to assess empirically the relative importance of the different hedging motives on optimal equity portolios. We use a standard period budget constraint to derive a formulation that relates equilibrium equity holdings to fluctuations in nonfinancial income, consumption expenditures and government spending. We stress that this formulation holds in a wide class of equilibrium models, irrespective of the presence and nature of nominal rigidities, capital accumulation and market completeness. It also naturally encompasses the three different hedging based motivations for equity home bias. What is more, the formulation we derive can explicitly allow for the impact of trade in other assets, is explicitly dynamic and can be estimated, relying on the methods described in Campbell and Shiller (1988) and Campbell (1996). We then use quarterly national account data from 1970 to 2008 for the G7 countries to assess the relative contribution of the motives to hedge human capital risk, consumption expenditure risk and government spending risk to equity home bias.

We find that the covariance of excess returns on domestic equity with relative human capital returns, conditional on bond returns, is negative in all countries and significant in all countries and significant in all countries but the U.S.. This implies that, conditional on bond returns, domestic equity has a high payoff when the returns on human capital are low and the motive to hedge human capital risk therefore contributes to equity home bias. The covariance of excess returns on domestic equity with consumption expenditure and government spending risk, conditional on bond returns, is also negative in all countries and significant in all but Italy and the U.S.. This implies that the motives to hedge consumption expenditure and government spending risk work *against* equity home bias, as it implies that, conditional on bond returns, domestic equity has a high payoff when consumption expenditure and the burden from government spending are low.

Our paper builds on previous empirical work that has attempted to study the comovement of domestic equity returns with some identified sources of risk. Van Wincoop and Warnock (2006) show that the correlation between excess returns on domestic equity and the contemporaneous real exchange rate is very low and unable to explain equity home bias. Baxter and Jermann (1997) argue that the correlation between the returns to financial capital and the returns to human capital are positively correlated, implying that allowing for human capital would make the international diversification puzzle worse. The papers closest to ours are Coeurdacier and Gourinchas (2009) and Rahbari (2010a). Coeurdacier and Gourinchas (2009) examine the correlation between domestic excess equity returns and the real exchange rate, conditional on bond returns, for the G7 countries and find that, while the conditional correlation between equity returns and the real exchange rate is very low and often insignificant, the correlation between equity returns and nonfinancial income is in general negative and able to imply a substantial degree of equity home bias. Rahbari (2010a) examines the correlation between domestic excess returns to equity and a dynamic measure of consumption expenditure and human capital risk, conditional on bond returns, for the U.S. and finds that the correlation is negative and significant for human capital risk, while it is insignificant for consumption expenditure risk. In this paper, we study all G7 countries, as Coeurdacier and Gourinchas (2009) do, and can therefore also establish certain patterns of cross country evidence. Like in Rahbari (2010a), we use an explicitly dynamic setup - reflecting the dynamic nature of portfolio decisions - and examine fluctuations in consumption expenditure rather than real exchange rate fluctuations. What is more, we allow for a third source of risk - government spending risk - that previous work has not captured.

This paper proceeds as follows. In the next section, we derive a relationship that relates equilibrium equity positions with fluctuations in consumption expenditures, nonfinancial income, and government spending that holds in a large class of macroeconomic models. The third section presents the data construction and empirical methodology, based on the work of Campbell and Shiller (1988) and Campbell (1996). Section four presents our results on the loadings of equity on the three sources of risk and the implied equity portfolios. Section five concludes.

4.2 Equity Positions and Hedging Demands

In this section, we derive an expression that relates equity positions to consumption expenditure risk, human capital risk, and government spending risk in a large class of models. Assume that there are two countries, generically called "Home" and "Foreign", symmetric in all respects, except that they are potentially of unequal size, whereby λ is the relative share of the Home country in world output in the steady state. Assume also that there is trade in equities and one period nominal bonds and that taxes are levied as lump sums. Equities are a claim to dividends, while nominal bonds pay one unit of local currency for one period only. Assume also at this stage that net foreign assets and government debt are zero at time 0. The budget constraint of the Home agent can be written as:

$$\begin{aligned}
&\underbrace{S^H_{H,t}P^S_{H,t} + S^H_{F,t}P^S_{F,t}Z_t + B^H_{H,t}P^B_{H,t} + B^H_{F,t}P^B_{H,t}Z_t}_{\text{Financial Investment}} + \underbrace{P^H_{C,t}C_{H,t}}_{\text{Consumption Expenditure}} + \underbrace{T_{H,t}}_{\text{Taxes}} \\
= \quad &\underbrace{W_{H,t}L_{H,t}}_{\text{Nonfinancial Income}} + \underbrace{S^H_{H,t-1}\left(D_{H,t} + P^S_{H,t}\right) + S^H_{F,t-1}\left(D_{F,t} + P^S_{F,t}\right)Z_t + B^H_{H,t-1} + B^H_{F,t-1}Z_t}_{\text{Financial Income}}, \qquad (4.1)
\end{aligned}$$

where $S^i_{j,t}$ is the fraction of country j equity held by country i, $B^i_{j,t}$ are the number of country j bonds held by country i, $P^S_{i,t}\left(P^B_{i,t}\right)$ are the prices of country i shares (bonds) in local currency, $P^i_{C,t}$ is the consumption price index of country i, and $W_{i,t}, L_{i,t}, D_{i,t}, T_{i,t}$ are wages, labour demand, dividends and taxes. Z_t is the nominal exchange rate, defined as the number of units of Home currency per unit of Foreign currency, implying that a rise in Z_t signifies a *depreciation* of the Home currency. This expression indicates that nonfinancial income and financial income are used to finance consumption expenditure or taxes today or (via financial investment) in the future. To make the dynamic nature of this relationship more precise and to translate it into an international context, it is useful to derive expressions for asset returns and our definitions of risk. Due to the symmetry of the model, it is without loss of generality to focus on the relative value of all variables, including asset returns, and we will henceforth do so.

Realised relative returns in terms of Home currency between Home and Foreign nominal bonds, $\widehat{R}^B_t$, are, linearised around the nonstochastic steady state:

$$\widehat{R}^B_t \equiv \widehat{R}^B_{H,t} - \widehat{R}^B_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1} = \widetilde{E}_t\left[-\widehat{Z}_t\right], \tag{4.2}$$

where $R^B_{H,t}\left(R^B_{F,t}\right)$ are local currency bond returns. $E_t\left[X_{t+j}\right]$ is the conditional expectation of X_{t+j} at time t, $\widetilde{E}_t\left[X_t\right] \equiv E_t\left[X_t\right] - E_{t-1}\left[X_t\right]$ denote date t surprises in the value of X and $\widehat{X}_t$ is the linearised value of X_t, defined as $X_t \equiv \frac{X_t - X}{X}$, where X is the value of X_t in the nonstochastic steady state. Thus, relative returns of Home bonds equal the extent of the unexpected appreciation of the local currency in the same period. Relative linearised equity returns in Home currency, $\widehat{R}^S_t$,are determined by the present value of surprises to relative dividends:

$$\widehat{R}^S_t \equiv \widehat{R}^S_{F,t} - \widehat{R}^S_{H,t} - \widehat{Z}_t + \widehat{Z}_{t-1} = (1-\beta)\,\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j} - \widehat{D}_{H,t+j} + \widehat{Z}_{t+j}\right)\right], \tag{4.3}$$

where $R^S_{H,t}\left(R^S_{F,t}\right)$ are local currency equity returns, β is the common discount rate and we imposed the appropriate transversality conditions.

Above, we mentioned that agents use financial and nonfinancial income to finance their consumption expenditures and taxes. We therefore define three measures, R^{PC}_t, R^{WL}_t and R^G that we will henceforth call "consumption expenditure risk", "human capital risk", and "government spending risk". In linearised form these are:

$$\widehat{R}_t^{PC} \equiv (1-\beta)\widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(\widehat{P}_{C,t+j}^H + \widehat{C}_{H,t+j} - \widehat{P}_{C,t+j}^F - \widehat{C}_{F,t+j} - \widehat{Z}_{t+j} \right) \quad (4.4)$$

$$\widehat{R}_t^{WL} \equiv (1-\beta)\widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} - \widehat{W}_{F,t+j} - \widehat{L}_{F,t+j} - \widehat{Z}_{t+j} \right) \quad (4.5)$$

$$\widehat{R}_t^{G} \equiv (1-\beta)\widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(\widehat{P}_{G,t+j}^H + \widehat{G}_{H,t+j} - \widehat{P}_{G,t+j}^F - \widehat{G}_{F,t+j} - \widehat{Z}_{t+j} \right) \quad (4.6)$$

where $\widehat{P}_{G,t+j}^i, \widehat{G}_{i,t+j}$ are the price index for government expenditures and real government spending respectively, and we again imposed the appropriate transversality conditions. Consumption expenditure risk thus depends on the present value of surprises in relative consumption expenditure, human capital risk depends on the present value of surprises in relative labour income and government spending risk depends on surprises in the present value of relative government spending.[2]

Using a present value equivalent of the period by period budget constraint, (4.1), in linearised form, deducting the analogous expression for the foreign country, and using expressions (4.2) − (4.6), we can then write the relative budget constraint as:[3]

$$\begin{aligned} & \frac{C}{Y}\widehat{R}_t^{PC} + \frac{G}{Y}\widehat{R}_t^{G} \\ = \; & \frac{WL}{Y}\widehat{R}_t^{WL} + \left(S - \frac{\lambda}{1-\lambda}(1-S) \right)\frac{D}{Y}\widehat{R}_t^{S} - \frac{B}{Y}\frac{1-\beta}{1-\lambda}\widehat{R}_t^{B} \end{aligned} \quad (4.7)$$

where S and B/Y are the steady share of the Home equity stock and the value of Home bonds divided by Home output held by Home investors. C, G, D and WL are the steady state values of consumption, government spending, labour income and dividends in the Home country, respectively. This expression states that innovations in the present value of relative consumption and government expenditures have to be met by innovations to the value of human capital or the value of equity and bond holdings. Projecting (4.7) on relative bond returns and deducting the resulting expression from (4.7), we can then solve for equilibrium

[2] It is worth noting that the discounted sums in (4.2) − (4.6) are formed using the simple discount factor β and not the stochastic discount factor, due to the order of approximation.

[3] The market clearing conditions for assets in the steady state imply: $S_H^H = S_F^F = S = 1 - S_F^H = 1 - S_H^F$ and $B_H^H = B_F^F = B = -B_F^H = -B_H^F$. We also impose that the discounted sum of future net exports is equal to the initial net foreign asset position, a condition that is satisfied at our level of approximation. Full details of the derivation are given in the appendix.

equity holdings in the steady state:

$$\begin{aligned}
S &= \lambda + (1-\lambda)\frac{C}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{PC}, \widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)} - (1-\lambda)\frac{WL}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{WL}, \widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)} \\
&\quad + (1-\lambda)\frac{G}{D}\frac{cov_{R_t^B}\left(\widehat{R}_t^{G}, \widehat{R}_t^S\right)}{var_{R_t^B}\left(\widehat{R}_t^S\right)} \\
&= \lambda + (1-\lambda)\left(\frac{C}{D}\beta_{pc,s} - \frac{WL}{D}\beta_{wl,s} + \frac{G}{D}\beta_{g,s}\right), \qquad (4.8)
\end{aligned}$$

where $cov_{R_t^B}(x,y)$ is the covariance between x and y, conditional on relative bond returns, and $\beta_{x,y} = cov_{R_t^B}(X,Y)/var_{R_t^B}(X)$.

Equation (4.8) is not structural, as equity positions are not expressed in terms of the exogenous shocks of the model, but it nevertheless provides useful intuition about the drivers of equity positions. According to (4.8), the equity position can be broken into four terms. The first term reflects the pure diversification motive. In the absence of any other sources of risk, the two countries would each hold the world equity market portfolio, implying that the Home investor will hold a share of the Home equity stock that is equal to the share of Home in world equity. The second term arises from a motive to hedge movements in relative consumption expenditures. Domestic equity is more attractive, if relative equity returns are high, on average, when surprises in the present value of future consumption expenditures are high, conditional on relative bond returns. The third term in (4.8) reflects the motive to hedge human capital returns with equity holdings. This term captures the fact that equity returns are particularly valuable, if other sources of income, in this case human capital returns and relative bond returns, are low. Thus, the domestically held share of the equity stock increases the more negatively correlated relative equity return innovations are with relative human capital return innovations, conditional on relative bond returns. Finally, the fourth term indicates the motive to hedge government spending risk and implies that the share of domestically held equity rises the more positively correlated relative equity returns are with positive surprises in the present value of relative government spending.
The relative importance of the consumption expenditure, human capital and government spending motive is indicated by the presence of the C, D, WL, and G terms, reflecting the size of steady state consumption expenditure, labour income, dividends and government spending. Two features are worth highlighting. Firstly, it is the comovement of *relative* returns with *relative* consumption expenditure and

labour income that matters. As mentioned before, the symmetry of the model implies that it is without loss of generality to focus on relative moments, but it is worth keeping in mind that relative returns may fall even when absolute returns rise, and vice versa. Secondly, note that the equity positions in (4.8) are functions of covariances and variances, conditional on relative bond returns. This reflects the fact that equity is not the only financial asset in this economy, and the other financial assets, in this case nominal bonds, are also used to hedge the two sources of risk. The conditional covariance of relative equity return with human capital risk can thus be interpreted as the residual comovement, once some part of this risk has been hedged using the bond positions. It is also worth noting that the expressions for bond and equity positions feature conditional covariance-variance ratios and thus look like multiple regression coefficients, a property which, following Coeurdacier and Gourinchas (2009), we will exploit below.

The formulation we present here has three important advantages. Firstly, as we pointed out above, equation (4.8) holds in a very large class of models, i.e. all those for which we can write the budget constraint in the form of (4.1) . In particular, equation (4.8) holds irrespective of our assumptions regarding, inter alia, the presence and nature of capital accumulation, nominal rigidities in price and wage setting, and incomplete markets. Secondly, equation (4.8) encompasses the main hedging based explanations for equity home bias previously discussed in the literature and above, namely equity home bias arising from motives to hedge fluctuations in consumption expenditure/ real exchange rates, labour income and government spending. Thirdly, the relevant terms in equation (4.8) are covariance variance ratios that can be estimated through regressions.

4.3 Empirical Strategy

In this section, we discuss our strategy for obtaining estimates for equation (4.8) using evidence from the G7 countries. The exercise consists of four steps. Firstly, we construct a number of relative variables, such as relative nonfinancial income, relative consumption spencing, etc, using observable country level information. Secondly, we obtain estimates for the asset returns and sources of risk we defined above and for innovations in these series, noting that as these depend on expectations of future variables, we can only estimate but not observe them. Thirdly, we compute the loadings of the different sources of risk on equity and bond returns using a regression based approach. Finally, we insert the obtained loadings into equation (4.8) in order to arrive at the equity holdings predicted by our estimates and the relative contributions of the consumption expenditure, labour income and government spending hedging motive.

While our approach builds on a large body of earlier work, this exercise is most closely related to Coeurdacier and Gourinchas (2009) and Rahbari(2010a).[4] Coeurdacier and Gourinchas (2009) obtain the loadings of equity and bond returns on nonfinancial income and the contemporaneous real exchange rate for the G7 countries. Rahbari (2010a) obtain estimates for the US for the loadings of equity and bond returns on nonfinancial income and consumption expenditure. The key differences with respect to Coeurdacier and Gourinchas (2009) are the use of consumption expenditures instead of the real exchange rate, the use of explicitly dynamic definition of asset returns and sources of risk, and the inclusion of government spending risk as a third source of risk.[5] With respect to Rahbari (2010a), the key differences are the addition of government spending risk as the third source of risk, the use of national income data to construct returns to financial income and that estimates are provided for all G7 countries and not just for the US.

4.3.1 Data description and construction

We use quarterly data for G-7 countries from the first quarter of 1970 to the third quarter of 2008. The data is from Coeurdacier and Gourinchas (2009) with the exception of government spending which we collect. To estimate the empirical counterpart of equation (4.8), we treat in turn each member of the G-7 as the Home country and aggregate the remaining members of the G-7 as the Foreign country. For each country, we build a measure of non-financial income, financial income, government spending, bond returns and consumption expenditure. Details of the data sources for each series are given in the appendix. All measures are expressed in US dollars.

[4]See also Campbell and Shiller (1988), Campbell (1996), Baxter and Jermann (1997), Juillard (2003), and Warnock and van Wincoop (2008)

[5]In open economy models with complete markets, relative consumption expenditures are perfectly correlated with the real exchange rate. With incomplete markets, that is no longer the case, but relative consumption expenditures can be expressed as the sum of relative consumption volume and the real exchange rate, at the first order of approximation.

Non-financial income

Following Coeurdacier and Gourinchas (2009) and Gollin (2002), we use the GDP accounts by income to construct the empirical counterpart of nonfinancial income and of the average share of nonfinancial income in total income. Denote by $Y_{i,t}$ the GDP of country i at time t in US dollars. It can be decomposed in the following way:

$$Y_{it} = LI_{it} + MI_{i,t} + \Pi_{i,t} + DFC_{i,t} + T_{i,t}, \tag{4.9}$$

where $LI_{i,t}$ denotes the aggregate compensation of employees, $MI_{i,t}$ denotes mixed income, $\Pi_{i,t}$ represents net operating surplus, $DFC_{i,t}$ is the consumption of fixed capital, and $T_{i,t}$ represents taxes minus subsidies on production and imports. Mixed income represents income from self-employment as well as proprietary income. Since this income also includes compensation for labour, we follow Gollin (2002) and construct our estimate of labour income, WL_{it},as the sum of the compensation of employees, $LI_{i,t}$, plus a share $\gamma_{i,t}$ of mixed income:

$$WL_{i,t} = LI_{i,t} + \gamma_{i,t} MI_{i,t}, \tag{4.10}$$

where $\gamma_{i,t}$ is set to the share of compensation of employees in the sum of compensation of employees and the net operating surplus:

$$\gamma_{i,t} = \frac{LI_{i,t}}{LI_{i,t} + WL_{i,t}}. \tag{4.11}$$

We calculate an equivalent measure for the rest of the world $W_{-i,t}$ by summing non-financial incomes (in USD) for the 6 remaining countries:

$$WL_{-i,t} = \sum_{j \neq i} LI_{j,t} + \gamma_{j,t} MI_{j,t}. \tag{4.12}$$

Finally, we compute relative nonfinancial income in US dollars as the log difference between nonfinancial income in country i and in the rest of the world:

$$rwl_{i,t} = \log WL_{i,t} - \log W_{-i,t}. \tag{4.13}$$

Consumption Expenditure and Government Spending

As with nonfinancial income, relative consumption expenditure is computed as the difference between consumption expenditure in country i and consumption expenditure in the rest of the world:

$$rpc_{i,t} = \log PC_{i,t} - \log PC_{-i,t}, \tag{4.14}$$

where consumption expenditure in the rest of the world is arrived at by summing the values for the other six G7 countries:

$$PC_{-i,t} = \sum_{j \neq i} PC_{j,t}. \tag{4.15}$$

Since quarterly data on total government spending is not available, we use quarterly data on government consumption. Relative government spending is then computed as the difference between government consumption in country i and the remaining six countries:

$$rpg_{i,t} = \log PG_{i,t} - \log PG_{-i,t} \tag{4.16}$$

$$PC_{-i,t} = \sum_{j \neq i} PC_{j,t}. \tag{4.17}$$

Financial Income

We also rely on national account data to construct a measure of financial income. Financial income $D_{i,t}$ is constructed as:

$$D_{i,t} = Y_{i,t} - WL_{i,t} - T_{i,t} - \left(I_{i,t} - I^R_{i,t}\right), \tag{4.18}$$

where $I_{i,t}$ is gross fixed capital formation in country i and $I^R_{i,t}$ is residential investment. Relative financial income is then computed as:

$$rd_{i,t} = \log D_{i,t} - \log D_{-i,t}, \tag{4.19}$$

where

$$D_{-i,t} = \sum_{i \neq j} D_{j,t}. \tag{4.20}$$

Bond Returns

We take $R^B_{i,t}$ to be the local currency gross return on 3-month Treasury bills in country i at time t and construct the corresponding dollar return as $R^B_{i,t}\frac{Z_{i,t}}{Z_{i,t-1}}$, where $Z_{i,t}$ is the value of the exchange rate of country i per US dollar. The equivalent measure for the rest of the world is computed using GDP weights α_i :

$$R^B_{-i,t}\frac{Z_{-i,t}}{Z_{-i,t-1}} = \sum_{j\neq i} \alpha_{j,t} R^B_{j,t}\frac{Z_{j,t}}{Z_{j,t-1}}. \tag{4.21}$$

The relative log-bond return is then:

$$rrb_{i,t} = \log\left(R^B_{i,t}\frac{Z_{i,t}}{Z_{i,t-1}}\right) - \log\left(R^B_{-i,t}\frac{Z_{-i,t}}{Z_{-i,t-1}}\right). \tag{4.22}$$

4.3.2 Innovations and Risk Measures

In the previous subsection, we described how we construct measures for relative nonfinancial income, consumption expenditure, government spending, financial income and bond returns. Now, we use these measures to construct the measures for relative equity and bond return innovations, $(4.2)-(4.3)$, and human capital, consumption expenditure and government spending risk, $(4.4)-(4.6)$, following the work of Campbell and Shiller (1988) and Campbell (1996).[6] We therefore face two tasks. Firstly, the present value of future relative labour income, relative financial income, relative consumption expenditure and relative government spending are not observable - only current labour and financial income and consumption expenditures and government spending are - and we therefore need to construct them using observable data. Secondly, we need to construct expectations of returns and risk in order to retrieve innovations in these variables.

For human capital, denote by $r^{WL}_{i,t+1}$ the log of the gross return on human capital in country i between t and $t+1$. Following Campbell (1996), under the assumption that the dividend price ratio on human wealth is stationary, we can write:

$$\begin{aligned} r^{WL}_{i,t+1} &= \log\left(WL_{i,t+1} + V^{WL}_{i,t+1}\right) - \log\left(V^{WL}_{i,t}\right) \\ &= k + \zeta_{i,t} - \rho\zeta_{i,t+1} + \Delta \log WL_{i,t+1}, \end{aligned} \tag{4.23}$$

where $V^{WL}_{i,t}$ measures human capital wealth, $\zeta_{i,t} = \log\left(WL_{i,t}/V^{WL}_{i,t}\right)$ is the log-dividend price ratio for human capital, $\rho^{-1} = 1 + \exp(\zeta_i) = \left(WL_i + V^{WL}_i\right)/V^{WL}_i$ and Δ is the difference operator ($\Delta x_t = x_t - x_{t-1}$). As in Coeurdacier and Gourinchas (2009), we will use $\rho = 0.98$, while k is an unimportant constant. Solving this equation forward and imposing that $\lim_{t\to\infty} \rho^t \left(r^{WL}_{i,t} - \Delta \log WL_{i,t}\right) = 0$, we

[6] Baxter and Jermann (1997), Coeurdacier and Gourinchas (2009) and Juillard (2003) use similar approaches.

obtain (up to a constant):

$$\zeta_{i,t} = E_t \sum_{j=0}^{\infty} \rho^j \left(r_{i,t+j}^{WL} - \Delta \log WL_{i,t+j+1} \right). \tag{4.24}$$

This expression states that the ratio of labour income to the value of human capital is high today either when future human capital returns are high, or when future nonfinancial income growth is low. Following Campbell (1996), if we assume that the conditional expected return on financial wealth equals the conditional expected return on human wealth $\left(E_t \left[r_{i,t+j}^{WL}\right] = E_t \left[r_{i,t+j}^{S}\right], \forall j\right)$ and substitute (4.24) into (4.23), we obtain:[7]

$$\widetilde{E}_{t+1}\left[r_{i,t+1}^{WL}\right] = \widetilde{E}_{t+1} \sum_{j=0}^{\infty} \rho^j \left(\Delta \log WL_{i,t+1+j}\right) - \widetilde{E}_{t+1} \sum_{j=1}^{\infty} \rho^j r_{i,t+j+1}^{WL}. \tag{4.25}$$

This expression states that the innovation to the return on human capital depends upon innovations to the path of future expected labour income growth, as well as innovations to the path of future expected equity returns proxying for future expected human capital returns. Thus, human capital return innovations today are high, if innovations to expected labour income growth are high, or if innovations to expected human capital returns are low. We then obtain innovations to the relative expected human capital return by subtracting the analogous expression for the rest of the world, converted into dollars, from (4.25), assuming that ρ is the same for all countries:

$$\widetilde{E}_{t+1} rr_{i,t+1}^{WL} = \widetilde{E}_{t+1} \sum_{j=0}^{\infty} \rho^j \left(\Delta rwl_{i,t+1+j}\right) - \widetilde{E}_{t+1} \sum_{j=1}^{\infty} \rho^j rr_{i,t+j+1}^{E}, \tag{4.26}$$

where $rr_{i,t+1}^{WL}$, $rwl_{i,t+1+j}$ and $rr_{i,t+j+1}^{S}$ are relative returns on human capital, log relative labour income and log relative equity returns, expressed in US dollars. The relative return on human capital thus depends on innovations to expected future relative labour income, as well as innovations to future relative discount rates.

We also need to obtain a measure for relative consumption expenditure and government spending risk and for returns to financial income. Following the same

[7]For open economy applications, Coeurdacier and Gourinchas (2009) make the same assumption, while Baxter and Jermann (1997) and Benigno and Nistico (2009) assume that discount rates are constant and thereby omit the second term. See also the discussion in Benigno and Nistico (2009) and Coeurdacier and Gourinchas (2009) and recent empirical work by Lustig and Van Nieuwerburgh (2006) and Lustig et al (2009).

steps as above, we obtain the following expression:

$$\widetilde{E}_{t+1}\left[rr^{PC}_{i,t+1}\right] = \widetilde{E}_{t+1}\sum_{j=0}^{\infty}\rho^{j}\left(\Delta rpc_{t+1+j}\right) - \widetilde{E}_{t+1}\sum_{j=1}^{\infty}\rho^{j} rr^{E}_{US,t+j+1} \tag{4.27}$$

$$\widetilde{E}_{t+1}\left[rr^{G}_{i,t+1}\right] = \widetilde{E}_{t+1}\sum_{j=0}^{\infty}\rho^{j}\left(\Delta rpg_{t+1+j}\right) - \widetilde{E}_{t+1}\sum_{j=1}^{\infty}\rho^{j} rr^{E}_{US,t+j+1} \tag{4.28}$$

$$\widetilde{E}_{t+1}\left[rr^{D}_{i,t+1}\right] = \widetilde{E}_{t+1}\sum_{j=0}^{\infty}\rho^{j}\left(\Delta rd_{t+1+j}\right) - \widetilde{E}_{t+1}\sum_{j=1}^{\infty}\rho^{j} rr^{E}_{US,t+j+1} \tag{4.29}$$

where $rr^{PC}_{i,t}, rr^{G}_{i,t}, rr^{D}_{i,t}$ are relative consumption expenditure risk, relative government spending risk and relative returns to financial capital, respectively. rpc are log relative consumption expenditures in US dollars, rpg is log relative government spending, and rd is log relative financial income. Equity and bond returns are observable and relative equity and bond return innovations are given by:

$$\widetilde{E}_{t+1}\left[rr^{E}_{i,t+1}\right] = \widetilde{E}_{t+1}\left[r^{E}_{i,t+1} - r^{E}_{-i,t+1}\right] \qquad \widetilde{E}_{t+1}\left[rr^{B}_{i,t+1}\right] = \widetilde{E}_{t+1}\left[r^{B}_{i,t+1} - r^{B}_{-i,t+1}\right], \tag{4.30}$$

where $r^{S}_{i,t+1}\left(r^{B}_{i,t+1}\right)$ are log equity (bond) returns in country i and $r^{S}_{-i,t+1}\left(r^{B}_{-i,t+1}\right)$ are log equity (bond) returns in the rest of the world, in dollars.

In order to estimate $\widetilde{E}_{t+1}\left[rr^{D}_{t+1}\right], \widetilde{E}_{t+1}\left[rr^{B}_{t+1}\right], \widetilde{E}_{t+1}\left[rr^{WL}_{t+1}\right]$, $\widetilde{E}_{t+1}\left[rr^{PC}_{t+1}\right]$, and $\widetilde{E}_{t+1}\left[rr^{G}_{t+1}\right]$ we then run the following first order vector autoregression:

$$x_{t+1} = Ax_t + \varepsilon_{t+1}, \tag{4.31}$$

where $x'_t = \left[\begin{array}{cccccc} rr^{E}_{t} & \Delta rli_t & rr^{B}_{t} & \Delta rpc_t & \Delta rpg_t & \Delta rd_t \end{array}\right]$. The VAR is estimated in levels, including a constant and one lag.[8] Having obtained the estimates $\widehat{A}$ and $\widehat{\varepsilon}$ from the VAR, human capital, consumption expenditure and government spending risk and financial return innovations are created as:

$$\widetilde{E}_{t+1}\left[rr^{WL}_{i,t+1}\right] = \left(e'_2 - \rho e'_1\widehat{A}\right)\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1} \tag{4.32}$$

$$\widetilde{E}_{t+1}\left[rr^{PC}_{i,t+1}\right] = \left(e'_4 - \rho e'_1\widehat{A}\right)\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1} \tag{4.33}$$

$$\widetilde{E}_{t+1}\left[rr^{G}_{i,t+1}\right] = \left(e'_5 - \rho e'_1\widehat{A}\right)\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1} \tag{4.34}$$

$$\widetilde{E}_{t+1}\left[rr^{D}_{i,t+1}\right] = \left(e'_6 - \rho e'_1\widehat{A}\right)\left(I - \rho\widehat{A}\right)^{-1}\widehat{\varepsilon}_{t+1} \tag{4.35}$$

[8] Akaike and Bayesian information criteria suggest that including one lag is the preferred specification.

while equity and bond return innovations are:

$$\widetilde{E}_{t+1}\left[rr^{S}_{i,t+1}\right] = e_1^{'}\widehat{\varepsilon}_{t+1} \qquad \widetilde{E}_{t+1}\left[rr^{B}_{i,t+1}\right] = e_3^{'}\widehat{\varepsilon}_{t+1}. \tag{4.36}$$

e_i is a unit vector whose ith element is equal to one, while all other elements are equal to zero. The final step then consists of obtaining the loadings by means of the following regressions separately for each country:

$$\widetilde{E}_{t+1}\left[rr^{WL}_{t+1}\right] = k_{wl} + \beta_{wl,b}\widetilde{E}_{t+1}\left[rr^{B}_{t+1}\right] + \beta_{wl,s}\widetilde{E}_{t+1}\left[rr^{S}_{t+1}\right] + \varepsilon^{wl}_{t} \tag{4.37}$$

$$\widetilde{E}_{t+1}\left[rr^{PC}_{t+1}\right] = k_{pc} + \beta_{pc,b}\widetilde{E}_{t+1}\left[rr^{B}_{t+1}\right] + \beta_{pc,s}\widetilde{E}_{t+1}\left[rr^{S}_{t+1}\right] + \varepsilon^{pc}_{t} \tag{4.38}$$

$$\widetilde{E}_{t+1}\left[rr^{G}_{t+1}\right] = k_{g} + \beta_{g,b}\widetilde{E}_{t+1}\left[rr^{B}_{t+1}\right] + \beta_{g,s}\widetilde{E}_{t+1}\left[rr^{S}_{t+1}\right] + \varepsilon^{g}_{t}, \tag{4.39}$$

where k_{wl}, k_{pc}, k_g are constants and ε^{wl}, ε^{pc} and ε^{g} are error terms of the regressions.

4.4 Results

In this section, we present the results from the procedure described in the previous section. First, we present the results from the regressions of the three sources of risk on equity and bond returns. We find that the correlation of relative returns to financial income with human capital, consumption expenditure and government spending risk is negative for all countries and significant in all countries but the US and Italy.

4.4.1 Loadings

Nonfinancial Income

Table 4.2 presents the results from the regression of human capital risk on relative equity and bond returns: $\widetilde{E}_{t+1}\left[rr^{WL}_{t+1}\right] = k_{wl} + \beta_{wl,b}\widetilde{E}_{t+1}\left[rr^{B}_{t+1}\right] + \beta_{wl,s}\widetilde{E}_{t+1}\left[rr^{S}_{t+1}\right] + \varepsilon^{wl}_{t}$. As the table shows, the coefficients are negative for all countries and significant for all countries but the U.S.. This implies that, conditional on relative bond returns, excess returns on domestic equity are high when excess returns on human capital are low, suggesting that hedging against human capital risk can drive equity home bias. These results are consistent with those presented in Coeurdacier and Gourinchas (2009), Heathcote and Perri (2009), Coeurdacier et al (2009) and Rahbari (2010a) who also argue that human capital returns can drive equity home bias, using very similar methods.

Table 4.2: **Loadings of Human Capital Risk**

	Canada	France	Germany	Italy	Japan	UK	US
a) equity and bond returns as regressors							
Equity	-0.10^{**}	-0.22^{**}	-0.21^{**}	-0.14^{**}	-0.14^{**}	-0.11^{**}	-0.05
	(0.03)	(0.03)	(0.02)	(0.07)	(0.02)	(0.04)	(0.04)
Bonds	1.19^{**}	1.15^{**}	1.51^{**}	1.32^{**}	0.90^{**}	1.21^{**}	1.09^{**}
	(0.06)	(0.05)	(0.03)	(0.09)	(0.03)	(0.07)	(0.06)
R2	0.75	0.81	0.95	0.70	0.86	0.73	0.73
Obs.	153	153	153	153	153	153	153
b) equity returns as regressors							
Equity	0.14^{**}	0.11^{**}	0.33^{**}	0.57^{**}	0.15^{**}	0.30^{**}	0.31^{**}
	(0.04)	(0.05)	(0.05)	(0.07)	(0.04)	(0.05)	(0.06)
R2	0.07	0.03	0.22	0.29	0.08	0.20	0.16
Obs.	153	153	153	153	153	153	153

Results for panel a) are from the regression of $rr_{t+1}^{WL} = k_{wl} + \beta_{wl,b} rr_{t+1}^{B} + \beta_{wl,s} rr_{t+1}^{S}$ $+\varepsilon_t^{wl}$. Results for panel b) are from the regression $rr_{t+1}^{WL} = k_{wl} + \beta_{wl,s} rr_{t+1}^{S} + \varepsilon_t^{wl}$. (**) and (*) denote significance at the 5% and 1% level, respectively. Standard erros are provided in parentheses, while the coefficients for the constant terms are. not shown.

Table 4.2 also shows the coefficients on bonds which are positive and large and highly statistically significant for all countries. This already suggests that it may be important to allow for trade in bonds which is further reaffirmed by the lower panel of the panel which presents the results from the regression of human capital risk on relative equity returns (and a constant) only. We can see that in the case where we do not allow for trade in bonds, the coefficient between equity returns and human capital returns is positive suggesting that allowing for human capital may make the international diversification puzzle worse (Baxter and Jermann (1997)).

Consumption Expenditure

In Table 4.3, we then present the results from the regression of consumption expenditure risk on relative equity and bond returns: $\widetilde{E}_{t+1}\left[rr_{t+1}^{PC}\right] = k_{wl} + \beta_{pc,b}\widetilde{E}_{t+1}\left[rr_{t+1}^{B}\right] + \beta_{pc,s}\widetilde{E}_{t+1}\left[rr_{t+1}^{S}\right] + \varepsilon_t^{pc}$. The coefficients for equity returns are negative for all countries and they are significant for all countries but the U.S. and Italy. Relative returns on domestic equity are then high, conditional on bond returns, in states of nature when relative consumption expenditures are low, implying that the motive for consumption expenditure risk cannot help to explain the home equity bias puzzle.

It is worthwhile to compare our results with some previous work. Most of this work has been done on movements in the real exchange rate. When markets are complete, efficient risk sharing implies that the real exchange rate and relative consumption expenditure are often perfectly correlated, at the first order of approximation. Van Wincoop and Warnock (2009) find that the correlation between the contemporaneous real exchange rate and relative equity returns is low and insignificant in many specifications. Benigno and Nistico (2010) point out that once a dynamic measure of real exchange rates is considered, the correlation often becomes significant for the US, while Rahbari (2010a) finds that the correlation of (a dynamic measure of) consumption expenditure risk with relative equity returns is insignificant for the U.S.. We also find an insignificant correlation for the U.S., but the correlation, conditional on bond returns, for the other G7 countries is significantly negative.

The coefficients on bond returns are positive and significant for all countries. As in the case of human capital, we find that allowing for trade in bonds changes the sign of the correlation between equity returns and consumption expenditures. When we regress consumption expenditure risk on equity returns only, the coefficients are positive and significant in all countries.

Government Spending

Table 4.4 presents the results from the regression of government spending risk on relative equity and bond returns: $\widetilde{E}_{t+1}\left[rr^{WL}_{t+1}\right] = k_{wl} + \beta_{wl,b}\widetilde{E}_{t+1}\left[rr^{B}_{t+1}\right] + \beta_{wl,s}\widetilde{E}_{t+1}\left[rr^{S}_{t+1}\right] + \varepsilon^{wl}_{t}$. As the table shows, the coefficients are negative for all countries and significant for all countries but the U.S. and Italy. This implies that, conditional on relative bond returns, excess returns on domestic equity are high when the present value of relative government spending and the associated burden of taxation is low, suggesting that hedging against government spending risk cannot drive equity home bias. We are not aware of any prior work that has examined this relationship which is why we consider it an important test of the hypothesis presented in Berriel and Bhattaraj (2009) who suggest that fluctuations in government spending and taxation could drive home bias in equities as well as bonds.

Table 4.4 also shows the coefficients on bonds which are positive and large and highly statistically significant for all countries. Once again, this highlights the

Table 4.3: **Loadings of Consumption Expenditure Risk**

	Canada	France	Germany	Italy	Japan	UK	US
a) equity and bond returns as regressors							
Equity	-0.06^{**}	-0.17^{**}	-0.18^{**}	-0.07	-0.07^{**}	-0.08^{**}	0.00
	(0.02)	(0.03)	(0.02)	(0.05)	(0.02)	(0.02)	(0.03)
Bonds	1.12^{**}	1.08^{**}	1.42^{**}	1.22^{**}	0.86^{**}	1.18^{**}	1.02^{**}
	(0.05)	(0.05)	(0.03)	(0.07)	(0.03)	(0.04)	(0.05)
R2	0.77	0.77	0.94	0.78	0.9	0.88	0.81
Obs.	153	153	153	153	153	153	153
b) equity returns as regressors							
Equity	0.17^{**}	0.13^{**}	0.32^{**}	0.58^{**}	0.21^{**}	0.32^{**}	0.34^{**}
	(0.04)	(0.05)	(0.05)	(0.06)	(0.04)	(0.04)	(0.05)
R2	0.1	0.04	0.23	0.36	0.15	0.27	0.22
Obs.	153	153	153	153	153	153	153

Results for panel a) are from the regression of $rr_{t+1}^{PC} = k_{pc} + \beta_{pc,b} rr_{t+1}^{B} + \beta_{pc,s} rr_{t+1}^{S}$ $+\varepsilon_t^{pc}$. Results for panel b) are from the regression $rr_{t+1}^{PC} = k_{pc} + \beta_{pc,s} rr_{t+1}^{S} + \varepsilon_t^{pc}$. (**) and (*) denote significance at the 5% and 1% level, respectively. Standard erros are provided in parentheses, while the coefficients for the constant terms are. not shown.

importance for allowing for trade in bonds and is again also confirmed by the fact that the sign of the correlation between equity returns and government spending risk changes if we only regress on equity returns.

4.4.2 Equity Positions

Table 4.5 uses equation (4.8) as well as the coefficients of the regressions in the previous subsections to compute the implied equity portfolios for the G7 countries as well as the contributions of the individual hedging motives. The table indicates that the regression coefficients imply equity home bias as the result of the three hedging motives in all countries, apart from Germany. The table therefore shows that without allowing for financial or informational frictions in cross border asset trade, the correlation properties of equity returns with the three identified sources of risk would suggest that investors should tilt their portfolios optimally towards domestic equity. But while the aggregate of the three hedging motives induces domestic investors to hold a larger share of the domestic equity stock, this effect is entirely due to the motive to hedge human capital risk. The motive to consumption expenditure and government spending risk work in the opposite direction, driving

Table 4.4: **Loadings of Government Spending Risk**

	Canada	France	Germany	Italy	Japan	UK	US
a) equity and bond returns as regressors							
Equity	-0.08^{**}	-0.18^{**}	-0.18^{**}	-0.27	-0.10^{**}	-0.09^{**}	-0.05
	(0.02)	(0.02)	(0.02)	(0.05)	(0.02)	(0.03)	(0.03)
Bonds	1.15^{**}	0.93^{**}	1.31^{**}	1.42^{**}	0.65^{**}	1.05^{**}	0.59^{**}
	(0.04)	(0.04)	(0.05)	(0.07)	(0.03)	(0.03)	(0.06)
R2	0.85	0.76	0.88	0.79	0.73	0.73	0.50
Obs.	153	153	153	153	153	153	153
b) equity returns as regressors							
Equity	0.16^{**}	0.08^{**}	0.29^{**}	0.49^{**}	0.11^{**}	0.27^{**}	0.15^{**}
	(0.04)	(0.04)	(0.05)	(0.07)	(0.03)	(0.04)	(0.04)
R2	0.09	0.02	0.20	0.24	0.07	0.21	0.09
Obs.	153	153	153	153	153	153	153

Results for panel a) are from the regression of $rr^G_{t+1} = k_{pg} + \beta_{pg,b} rr^B_{t+1} + \beta_{pg,s} rr^S_{t+1} + \varepsilon^{pg}_t$. Results for panel b) are from the regression $rr^G_{t+1} = k_{pg} + \beta_{pg,s} rr^S_{t+1} + \varepsilon^{pg}_t$. (**) and (*) denote significance at the 5% and 1% level, respectively. Standard erros are provided in parentheses, while the coefficients for the constant terms are. not shown.

investors to hold smaller shares of the domestic equity stock.

4.5 Conclusion

Domestic investors continue to hold a large share of the domestic equity stock, despite substantial increases in cross border financial asset trade. Many explanations have been proposed to account for this phenomenon. In this paper, we investigate the empirical relevance of the explanation that investors optimally choose to hold large shares of domestic equity, as such holdings hedge certain sources of risk. We derive a simple and intuitive expression that links equilibrium equity positions to the three sources of risk discussed in the literature. These three explanations are that equity returns may hedge fluctuations in consumption expenditures/ real exchange rates, labour income or government spending. Our formulation is simple, but can be shown to hold in a very wide class of models and can be estimated using relatively simple methods and building on earlier work by Campbell and Shiller (1988) and Campbell (1996). Importantly, our approach allows to account for trade in other assets, notably bonds, a point highlighted recently in many contributions.

Table 4.5: **Implied Equity Portfolios**

in %	Canada	France	Germany	Italy	Japan	UK	US
Output Share	4.1	7.9	10.9	7.6	16.3	7.7	45.4
Equity Share	20.0	11.1	4.6	11.5	44.5	16.1	59.9
Home Bias	15.5	3.3	−6.3	3.9	28.2	8.4	14.4
due to:							
Human Capital	61.7	132.5	121.3	83.0	78.5	66.8	18.5
Consumption Expenditure	−35.1	−103.3	−103.0	−40.3	−37.8	−45.9	0.0
Government Spending	−11.1	−25.9	−24.6	−38.9	−12.5	−12.5	−4.0

Contributions of different hedging motives were calculated according to the reduced form equation (4.8) and using the coefficients obtained from running the regressions (4.37) to (4.39).

We find that the correlation between domestic excess returns on equity and human capital risk is negative once we condition on relative returns on bonds, implying that the motive to hedge human capital risk contributes to equity home bias in the G7 countries. On the contrary, we show that the correlation between equity returns and government spending and consumption expenditures in general suggests that it would be optimal for investors to hold smaller shares of the domestic equity stock.

We build on earlier work that investigates the comovement of equity returns with different sources of risk in order to assess its hedging properties. Our work extends earlier approaches by allowing for all three main hedging based explanations of equity home bias. However, throughout we have noted that a second important class of explanations exists, namely that equity home bias is driven by financial or informational frictions in cross border asset trade. While our work suggests that hedging based explanations can account for some degree of equity home bias, an approach that encompasses both hedging based and frictional explanations would be of high merit.

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4.6 Appendix

4.6.1 Derivation of the Relationship between Equity Holdings and Sources of Risk

The budget constraint for the Home agent is:

$$\begin{aligned} & S^H_{H,t}P^S_{H,t} + S^H_{F,t}P^S_{F,t}Z_t + B^H_{H,t}P^B_{H,t} + B^H_{F,t}P^B_{F,t}Z_t + P^H_{C,t}C_{H,t} \\ = \; & S^H_{H,t-1}\left(P^S_{H,t} + D_{H,t}\right) + S^H_{F,t-1}\left(P^S_{F,t} + D_{F,t}\right)Z_t + B^H_{H,t-1} \\ & + B^H_{F,t-1}Z_t + L_{H,t}W_{H,t} - T_t \end{aligned} \tag{4.40}$$

The net foreign asset position is then given by assets held abroad minus domestic assets held by foreign agents:

$$NFA_{H,t} = S^H_{F,t}P^S_{F,t}Z_t + B^H_{F,t}P^B_{F,t}Z_t - S^F_{H,t}P^S_{H,t} - B^F_{H,t}P^B_{H,t} \tag{4.41}$$

From the asset market clearing conditions:

$$S^F_{H,t} = 1 - S^H_{H,t} \qquad B^F_{H,t} = B_{H,t} - B^H_{H,t} \tag{4.42}$$

which gives:

$$NFA_{H,t} = \left(S^H_{H,t} - 1\right)P^S_{H,t} + S^H_{F,t}P^S_{F,t}Z_t + B^H_{F,t}P^B_{F,t}Z_t + \left(B^H_{H,t} - B_{H,t}\right)P^B_{H,t} \tag{4.43}$$

Define the portfolio excess return as:

$$\begin{aligned} \xi_{H,t} \; = \; & \left(S^H_{H,t-1} - 1\right)\left(P^S_{H,t} + D_{H,t}\right) + S^H_{F,t-1}\left(P^S_{F,t} + D_{F,t}\right)Z_t \\ & + \left(B^H_{H,t-1} - B_{H,t-1}\right) + B^H_{F,t-1}Z_t - NFA_{H,t-1}R^B_{H,t}, \end{aligned} \tag{4.44}$$

i.e. the difference between actual net foreign assets at the beginning of period t and net foreign assets at period t had all wealth been invested in Home bonds. Now note that $R^S_H = \frac{P^S_{H,t}+D_{H,t}}{P^S_{H,t-1}}$ and $R^S_F\frac{Z_t}{Z_{t-1}} = \frac{P^S_{F,t}+D_{F,t}}{P^S_{F,t-1}}\frac{Z_t}{Z_{t-1}}$, $R^B_H = \frac{1}{P^B_{H,t-1}}, \frac{1}{P^S_{H,t-1}}\frac{Z_t}{Z_{t-1}}$ denote the Home currency returns of Home and Foreign stocks and bonds. Then,

$$\begin{aligned} \xi_{H,t} \; = \; & \left(S^H_{H,t-1} - 1\right)P^S_{H,t-1}R^S_{H,t} + S^H_{F,t-1}P^S_{F,t-1}Z_{t-1}R^S_{F,t}\frac{Z_t}{Z_{t-1}} \\ & + \left(B^H_{H,t-1} - B_{H,t-1}\right)P^B_{H,t-1}R^B_{H,t} + B^H_{F,t-1}Z_{t-1}P^B_{F,t-1}R^B_{F,t}\frac{Z_t}{Z_{t-1}} \\ & - NFA_{H,t-1}R^B_{H,t}. \end{aligned} \tag{4.45}$$

From $NFA_{H,t} = \left(S^H_{H,t} - 1\right) P^S_{H,t} + S^H_{F,t} P^S_{F,t} Z_t + B^H_{F,t} P^B_{F,t} Z_t + \left(B^H_{H,t} - B_{H,t}\right) P^B_{H,t}$

$$\begin{aligned} \xi_{H,t} \quad = \quad & \left(S^H_{H,t-1} - 1\right) P^S_{H,t-1} \left(R^S_{H,t} - R^B_{H,t}\right) + \\ & S^H_{F,t-1} P^S_{F,t-1} Z_{t-1} \left(R^S_{F,t} \frac{Z_t}{Z_{t-1}} - R^B_{H,t}\right) \\ & + B^H_{F,t-1} Z_{t-1} P^B_{F,t-1} \left(R^B_{F,t} \frac{Z_t}{Z_{t-1}} - R^B_{H,t}\right) \end{aligned} \tag{4.46}$$

Now rewrite the original budget constraint:

$$\begin{aligned} NFA_{H,t} + P^S_{H,t} + B_{H,t} P^B_{H,t} \quad = \quad & S^H_{H,t-1} \left(P^S_{H,t} + D_{H,t}\right) + S^H_{F,t-1} Z_t \left(P^S_{F,t} + D_{F,t}\right) \\ & + B^H_{H,t-1} + B^H_{F,t-1} Z_t + L_{H,t} W_{H,t} - T_t - P^H_{C,t} C_{H,t} \\ = \quad & S^H_{H,t-1} P^S_{H,t-1} R^S_{H,t} + S^H_{F,t-1} Z_{t-1} P^S_{F,t-1} R^S_{F,t} \frac{Z_t}{Z_{t-1}} \\ & + B^H_{H,t-1} P^B_{H,t-1} R^B_{H,t} + B^H_{F,t-1} P^B_{F,t-1} Z_{t-1} R^B_{F,t} \frac{Z_t}{Z_{t-1}} \\ & + L_{H,t} W_{H,t} - T_t - P^H_{C,t} C_{H,t} \end{aligned} \tag{4.47}$$

From $\xi_{H,t} = \left(S^H_{H,t-1} - 1\right) P^S_{H,t-1} \left(R^S_{H,t} - R^B_{H,t}\right) + S^H_{F,t-1} P^S_{F,t-1} Z_{t-1} \left(R^S_{F,t} \frac{Z_t}{Z_{t-1}} - R^B_{H,t}\right) + B^H_{F,t-1} P^B_{F,t-1} Z_{t-1} \left(R^B_{F,t} \frac{Z_t}{Z_{t-1}} - R^B_{H,t}\right)$:

$$\begin{aligned} NFA_{H,t} \quad = \quad & S^H_{H,t-1} P^S_{H,t-1} R^B_{H,t} + P^S_{H,t-1} \left(R^S_{H,t} - R^B_{H,t}\right) + \xi_{H,t} + S^H_{F,t-1} P^S_{F,t-1} Z_{t-1} R^B_{H,t} \\ & + B^H_{H,t-1} P^B_{H,t-1} R^B_{H,t} + B^H_{F,t-1} P^B_{F,t-1} Z_{t-1} R^B_{H,t} \\ & + L_{H,t} W_{H,t} - T_t - P^H_{C,t} C_{H,t} - P^S_{H,t} - B_{H,t} P^B_{H,t} \end{aligned} \tag{4.48}$$

From $NFA_{H,t} = \left(S^H_{H,t} - 1\right) P^S_{H,t} + S^H_{F,t} P^S_{F,t} Z_t + B^H_{F,t} P^B_{F,t} Z_t + \left(B^H_{H,t} - B_{H,t}\right) P^B_{H,t}$:

$$\begin{aligned} NFA_{H,t} \quad = \quad & NFA_{H,t-1} R^B_{H,t} + P^S_{H,t-1} R^B_{H,t} + P^S_{H,t-1} \left(R^S_{H,t} - R^B_{H,t}\right) + \xi_{H,t} \\ & + B_{H,t-1} P^B_{H,t-1} R^B_{H,t} + L_{H,t} W_{H,t} - T_t - P^H_{C,t} C_{H,t} - P^S_{H,t} \\ & - B_{H,t} P^B_{H,t} \\ = \quad & NFA_{H,t-1} R^B_{H,t} + P^S_{H,t-1} R^S_{H,t} + \xi_{H,t} + B_{H,t-1} + L_{H,t} W_{H,t} - T_t \\ & - P^H_{C,t} C_{H,t} - P^S_{H,t} - B_{H,t} P^B_{H,t} \end{aligned} \tag{4.49}$$

From $R^S_{H,t} = \frac{D_{H,t}+P^S_{H,t}}{P^S_{H,t-1}}$, $D_{H,t} = \Pi_{H,t} + R^K_{H,t}K_{H,t} - I_{H,t}P^H_{I,t}$ and $\Pi_{H,t} = Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t - R^K_{H,t}K_{H,t} - W_{H,t}L_{H,t}$ and $P^B_{H,t}B_{H,t} = B_{H,t-1} + P_{G,t}G_t - T_t$

$$\begin{aligned}
NFA_{H,t} &= NFA_{H,t-1}R^B_{H,t} + D_{H,t} + P^S_{H,t} + \xi_{H,t} + B_{H,t-1} - P^H_{C,t}C_{H,t} - P^S_{H,t} \\
&\quad +W_{H,t}L_{H,t} - T_t - B_{H,t}P^B_{H,t} \\
&= NFA_{H,t-1}R^B_{H,t} + \xi_{H,t} + B_{H,t-1} + Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t - I_{H,t}P^H_{I,t} \\
&\quad -P^H_{C,t}C_{H,t} - T_t - B_{H,t}P^B_{H,t} \\
&= NFA_{H,t-1}R^B_{H,t} + \xi_{H,t} + Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t - I_{H,t}P^H_{I,t} - P^H_{C,t}C_{H,t} \\
&\quad -P^H_{G,t}G_{H,t} \qquad (4.50)
\end{aligned}$$

Net exports were defined as: $NX_{H,t} = Y^H_{H,t}P^H_{H,t} + Y^F_{H,t}P^F_{H,t}Z_t - I_{H,t}P^H_{I,t} - P^H_{C,t}C_{H,t} - P^H_{G,t}G_{H,t}$, we have:

$$NFA_{H,t} = NFA_{H,t-1}R^B_{H,t} + \xi_{H,t} + NX_{H,t} \qquad (4.51)$$

or, in linear form:

$$\widehat{NFA}_{H,t} = \widehat{NFA}_{H,t-1}\frac{1}{\beta} + \widehat{\xi}_{H,t} + \widehat{NX}_{H,t}, \qquad (4.52)$$

where $\widehat{NFA}_{H,t} = \frac{NFA_{H,t}}{Y}$, $\widehat{\xi}_{H,t} = \frac{\xi_{H,t}}{Y}$, $\widehat{NX}_{H,t} = \frac{NX_{H,t}}{Y}$.

Now realise that the Home country accounts for a share λ of would output. We then have:

$$\begin{aligned}
P^S_H &= \frac{\lambda}{1-\lambda}P^S_F = P^S && (4.53) \\
Y_H &= \frac{\lambda}{1-\lambda}Y_F && (4.54) \\
S^H_F &= \frac{\lambda}{1-\lambda}S^F_H && (4.55) \\
S^H_H &= S = 1 - S^F_H && (4.56) \\
S^F_F &= S^* = 1 - S^H_F && (4.57)
\end{aligned}$$

Linearising the expression for the portfolio excess return and using (4.52), we have:

$$\widehat{\xi}_{H,t}Y = (S-1)\frac{P_H^S}{\beta}\left(\widehat{R}_{H,t}^S - \widehat{R}_{H,t}^B\right) + (1-S^*)\frac{P_F^S}{\beta}\left(\widehat{R}_{F,t}^S + \widehat{Z}_t - \widehat{Z}_{t-1} - \widehat{R}_{H,t}^B\right)$$
$$+B\frac{P^B}{\beta}\left(\widehat{R}_{F,t}^B + \widehat{Z}_t - \widehat{Z}_{t-1} - \widehat{R}_{H,t}^B\right); B_H^H + B_H^F = B_H; B_H^F = B \quad (4.58)$$
$$= (S-1)\frac{P^S}{\beta}\left(\widehat{R}_{H,t}^S - \widehat{R}_{F,t}^S - \widehat{Z}_t + \widehat{Z}_{t-1}\right) + B\frac{P^B}{\beta}\left(\widehat{R}_{F,t}^B + \widehat{Z}_t - \widehat{Z}_{t-1} - \widehat{R}_{H,t}^B\right) \quad (4.59)$$

We can then write the budget constraint in linearised form as:

$$\widehat{NFA}_{H,t} = \widehat{NFA}_{H,t-1}\frac{1}{\beta} + (S-1)\frac{P^S}{\beta Y}\left(\widehat{R}_{H,t}^S - \widehat{R}_{F,t}^S - \widehat{Z}_t + \widehat{Z}_{t-1}\right)$$
$$-B\frac{P^B}{\beta Y}\left(\widehat{R}_{H,t}^B - \widehat{R}_{F,t}^B - \widehat{Z}_t + \widehat{Z}_{t-1}\right) + \widehat{NX}_{H,t} \quad (4.60)$$

Taking expectations at time t and rewriting:

$$\frac{1}{\beta}\widehat{NFA}_{H,t-1} = E_t\left[\widehat{NFA}_{H,t} - \widehat{NX}_{H,t} - \frac{P^S}{\beta Y}(S-1)\left(\widehat{R}_{H,t}^S - \widehat{R}_{F,t}^S - \widehat{Z}_t + \widehat{Z}_{t-1}\right)\right]$$
$$-B\frac{P^B}{\beta Y}E_t\left[\widehat{R}_{F,t}^B - \widehat{R}_{H,t}^B - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \quad (4.61)$$

Iterating forward, imposing $T \to \infty$, $lim_{T\to\infty}E_t\left[NFA_{t+T}\right] = 0$ and using $E_t\left[\widehat{R}_{H,t+\tau}^S - \widehat{R}_{F,t+\tau}^S + \widehat{Z}_{t+\tau} - \widehat{Z}_{t+\tau-1}\right] = 0, \tau > 1$, we get:

$$\frac{1}{\beta}\widehat{NFA}_{H,t-1} = \sum_{j=0}^{T} -\beta^j E_t\left[\widehat{NX}_{H,t+j}\right] - \frac{P^S}{\beta Y}(S-1)E_t\left[\widehat{R}_{H,t}^S - \widehat{R}_{F,t}^S - \widehat{Z}_t + \widehat{Z}_{t-1}\right]$$
$$-B\frac{P^B}{\beta Y}E_t\left[\widehat{R}_{F,t}^B - \widehat{R}_{H,t}^B - \widehat{Z}_t + \widehat{Z}_{t-1}\right] \quad (4.62)$$

Rearranging and using the expressions for linearised relative bond and equity returns, we have:

$$\sum_{j=0}^{T} -\beta^j E_t\left[NX_{H,t+j}\right] = \frac{1}{\beta}NFA_{H,t-1} + \frac{D}{Y}(S-1)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j \widehat{D}_{H,t+j}\right]$$
$$+\frac{D}{Y}\frac{1-\lambda}{\lambda}\left(1-S_F^F\right)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j} + \widehat{Z}_{t+j}\right)\right]$$
$$-B\frac{P^B}{\beta Y}\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(-\widehat{Z}_{t+j}\right)\right], \quad (4.63)$$

where $\widetilde{E}_t [X_t] = E_t [X_t] - E_{t-1} [X_t]$.
This budget constraint holds if and only if:

$$\begin{aligned}\sum_{j=0}^{T} -\beta^j \widetilde{E}_t [NX_{H,t+j}] &= \frac{D}{Y}(S-1)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j \widehat{D}_{H,t+j}\right] \\ &+\frac{D}{Y}\frac{1-\lambda}{\lambda}\left(1-S_F^F\right)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j}+\widehat{Z}_{t+j}\right)\right] \\ &-B\frac{P^B}{\beta Y}\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(-\widehat{Z}_{t+j}\right)\right] \qquad (4.64)\end{aligned}$$

and

$$\sum_{j=0}^{T} -\beta^j E_{t-1} [NX_{H,t+j}] = \frac{1}{\beta} NFA_{H,t-1} \qquad (4.65)$$

From $NX_{H,t} = Y_{H,t}^H P_{H,t}^H + Y_{H,t}^F P_{H,t}^F Z_t - I_{H,t} P_{I,t}^H - P_{C,t}^H C_{H,t} - P_{G,t}^H G_{H,t}, D_{H,t} = \Pi_{H,t} + R_{H,t}^K K_{H,t} - I_{H,t} P_{H,t}^I$, $\Pi_{H,t} = Y_{H,t}^H P_{H,t}^H + Y_{H,t}^F P_{H,t}^F Z_t - R_{H,t}^K K_{H,t} - W_{H,t} L_{H,t}$ we have:

$$NX_{H,t} = D_{H,t} + W_{H,t} L_{H,t} - P_{C,t}^H C_{H,t} - P_{G,t}^H G_{H,t}$$

Linearising, we have:

$$\begin{aligned}&\widehat{NX}_{H,t} = \widehat{D}_{H,t}\frac{D}{Y} + \left(\widehat{W}_{H,t} + \widehat{L}_{H,t}\right)\frac{WL}{Y} - \frac{C}{Y}\left(\widehat{P}_{C,t}^H + \widehat{C}_{H,t}\right) \\ &-\frac{G}{Y}\left(\widehat{P}_{G,t}^H + \widehat{G}_{H,t}\right) \qquad (4.66)\end{aligned}$$

and substituting above, we then have:

$$\begin{aligned}&\sum_{j=0}^{T} -\beta^j \widetilde{E}_t \left[\widehat{D}_{H,t}\frac{D}{Y} + \left(\widehat{W}_{H,t} + \widehat{L}_{H,t}\right)\frac{WL}{Y} - \frac{C}{Y}\left(\widehat{P}_{C,t}^H + \widehat{C}_{H,t}\right) - \frac{G}{Y}\left(\widehat{P}_{G,t}^H + \widehat{G}_{H,t}\right)\right] \\ = \; &\frac{D}{Y}(S-1)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j \widehat{D}_{H,t+j}\right] + \frac{D}{Y}\frac{1-\lambda}{\lambda}\left(1-S_F^F\right)\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(\widehat{D}_{F,t+j}+\widehat{Z}_{t+j}\right)\right] \\ &-B\frac{P^B}{\beta Y}\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(-\widehat{Z}_{t+j}\right)\right] \qquad (4.67)\end{aligned}$$

The analogous expression for the foreign country is, in terms of Home currency:

$$
\begin{aligned}
& \frac{1-\lambda}{\lambda}\frac{C}{Y}\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{P}^{F}_{C,t+j}+\widehat{C}_{F,t+j}+\widehat{Z}_{t+j}\right] \\
= \;& \frac{1-\lambda}{\lambda}\frac{WL}{Y}\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}+\widehat{Z}_{t+j}\right] \\
& +S^{*}\frac{1-\lambda}{\lambda}\frac{D}{Y}\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\left(\widehat{D}_{F,t+j}+\widehat{Z}_{t+j}\right)\right] \\
& +\left(1-S^{*}\right)\frac{1-\lambda}{\lambda}\frac{D}{Y}\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\widehat{D}_{H,t+j}\right] \\
& -\frac{B}{Y}\frac{P^{B}}{\beta}\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\left(-\widehat{Z}_{t+j}\right)\right] \\
& -\frac{1-\lambda}{\lambda}\frac{G}{Y}\left[\sum_{j=0}^{\infty}\beta^{j}\left(\widehat{P}^{F}_{G,t}+\widehat{G}_{F,t}\right)\right],
\end{aligned}
\tag{4.68}
$$

where we have divided by Y from the normalisation, but where all the steady state terms are given by $C_F = \frac{1-\lambda}{\lambda}C$ and so on.

Deducting the foreign budget constraint from the Home one, we obtain:

$$
\begin{aligned}
& \frac{C}{Y}\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\left(\widehat{P}^{H}_{C,t}+\widehat{C}_{H,t}\right)-\left(\widehat{P}^{F}_{C,t+j}+\widehat{C}_{F,t+j}+\widehat{Z}_{t+j}\right)\right] \\
= \;& \frac{WL}{Y}\sum_{j=0}^{T}\beta^{j}\widetilde{E}_{t}\left[\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}-\left(\widehat{W}_{H,t+j}+\widehat{L}_{H,t+j}+\widehat{Z}_{t+j}\right)\right] \\
& +\left(S+S^{*}-1\right)\frac{D}{Y}\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\left(\widehat{D}_{H,t+j}-\left(\widehat{D}_{F,t+j}-\widehat{Z}_{t+j}\right)\right)\right] \\
& -B\frac{P^{B}}{\beta}\left(\frac{1}{Y_H}+\frac{1}{Y_F}\right)\widetilde{E}_{t}\left[\sum_{j=0}^{\infty}\beta^{j}\left(-\widehat{Z}_{t+j}\right)\right] \\
& -\frac{G}{Y}\left[\sum_{j=0}^{\infty}\beta^{j}\left(\left(\widehat{P}^{H}_{G,t}+\widehat{G}_{H,t}\right)-\left(\widehat{P}^{F}_{G,t}+\widehat{G}_{F,t}+\widehat{Z}_{t}\right)\right)\right]
\end{aligned}
\tag{4.69}
$$

Now define the return on human capital in country i as

$$
R^{WL}_{H,t}=\frac{W_{H,t}L_{H,t}+P^{WL}_{H,t}}{P^{WL}_{H,t-1},}, \tag{4.70}
$$

where $P^{WL}_{H,t} = E_t \sum_{j=1}^{\infty} \beta^j \left(\frac{C_{H,t+j}}{C_{H,t}}\right)^{-\sigma} \frac{P^H_{C,t}}{P^H_{C,t+j}} W_{H,t+j} L_{H,t+j}$ is the present value of labour income in Home. Linearising, the expression for returns, we have:

$$\widehat{R}^{WL}_{H,t} = (1-\beta) \sum_{j=0}^{\infty} \beta^j \left(-\sigma \left(\widehat{C}_{H,t+j} - \widehat{C}_{H,t}\right) + \widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+j} + \widehat{W}_{H,t+j} + \widehat{L}_{H,t+j}\right) \tag{4.71}$$

Deducting the analogous foreign expression in Home currency terms, we have:

$$\begin{aligned} \widehat{R}^{WL}_t &= \widehat{R}^{WL}_{H,t} - \widehat{R}^{WL}_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1} \\ &= (1-\beta)\, \widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(\widehat{W}_{H,t+j} + \widehat{L}_{H,t+j} - \widehat{W}_{F,t+j} - \widehat{L}_{F,t+j} - \widehat{Z}_{t+j}\right) \end{aligned} \tag{4.72}$$

Similarly, define the capitalised cost of government expenditure as:

$$P^G_{H,t} = \sum_{j=0}^{\infty} \beta^j \left(\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma} \frac{P^H_{C,t}}{P^H_{C,t+1}} \left(P^H_{G,t} G_{H,t}\right)\right) \tag{4.73}$$

Defining:

$$R^G_{H,t} = \frac{P^H_{G,t} G_{H,t} + P^G_{H,t}}{G^K_{H,t-1}}, \tag{4.74}$$

We then have:

$$\widehat{R}^G_{H,t} = (1-\beta) \sum_{j=0}^{\infty} \beta^j \left(-\sigma \left(\widehat{C}_{H,t+j} - \widehat{C}_{H,t}\right) + \widehat{P}^H_{C,t} - \widehat{P}^H_{C,t+j} + \widehat{G}_{H,t+j} + \widehat{P}^H_{G,t+j}\right) \tag{4.75}$$

Deducting the analogous foreign expression in Home currency terms, we have:

$$\begin{aligned} \widehat{R}^G_t &= \widehat{R}^G_{H,t} - \widehat{R}^G_{F,t} - \widehat{Z}_t + \widehat{Z}_{t-1} \\ &= (1-\beta)\, \widetilde{E}_t \sum_{j=0}^{\infty} \beta^j \left(\widehat{P}^G_{H,t+j} + \widehat{G}_{H,t+j} - \widehat{P}^G_{F,t+j} - \widehat{G}_{F,t+j} - \widehat{Z}_{t+j}\right) \end{aligned} \tag{4.76}$$

Similarly, define capitalised consumption expenditure as:

$$P^{PC}_{H,t} = \sum_{j=0}^{\infty} \beta^j \left(\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma} \frac{P^H_{C,t}}{P^H_{C,t+1}} \left(P^H_{C,t} C_{H,t}\right)\right) \tag{4.77}$$

Defining:

$$R^{PC}_{H,t} = \frac{P^H_{C,t} C_{H,t} + P^{PC}_{H,t}}{P^{PC}_{H,t-1}}, \tag{4.78}$$

We then have:

$$\widehat{R}_{H,t}^{PC} = (1-\beta)\sum_{j=0}^{\infty}\beta^{j}\left(-\sigma\left(\widehat{C}_{H,t+j}-\widehat{C}_{H,t}\right)+\widehat{P}_{C,t}^{H}-\widehat{P}_{C,t+j}^{H}+\widehat{C}_{H,t+j}+\widehat{P}_{C,t+j}^{H}\right) \tag{4.79}$$

Deducting the analogous foreign expression in Home currency terms, we have:

$$\begin{aligned}\widehat{R}_{t}^{PC} &= \widehat{R}_{H,t}^{PC}-\widehat{R}_{F,t}^{PC}-\widehat{Z}_{t}+\widehat{Z}_{t-1}\\ &= (1-\beta)\,\widetilde{E}_{t}\sum_{j=0}^{\infty}\beta^{j}\left(\widehat{P}_{C,t+j}^{H}+\widehat{C}_{H,t+j}-\widehat{P}_{C,t+j}^{F}-\widehat{C}_{F,t+j}-\widehat{Z}_{t+j}\right)\end{aligned} \tag{4.80}$$

Now we can write the budget constraint as:

$$\begin{aligned}&\frac{C}{(1-\beta)\,Y}\widehat{R}_{t}^{PC}+\frac{G}{(1-\beta)\,Y}\widehat{R}_{t}^{G}\\ =\ &\frac{WL}{(1-\beta)\,Y}\widehat{R}_{t}^{WL}+(S+S^{*}-1)\,\frac{D}{(1-\beta)\,Y}\widehat{R}_{t}^{S}-B\frac{P_{B}}{\beta}\frac{1+\lambda}{Y}\widehat{R}_{t}^{B}\end{aligned} \tag{4.81}$$

Now project this equation on relative bond returns $\widehat{R}_{t}^{B}$:

$$\begin{aligned}\frac{C}{Y}P\left[\widehat{R}_{t}^{PC}|\widehat{R}_{t}^{B}\right] &= \frac{WL}{Y}P\left[\widehat{R}_{t}^{W}|\widehat{R}_{t}^{B}\right]+\frac{D}{Y}(S+S^{*}-1)\,P\left[\widehat{R}_{t}^{S}|\widehat{R}_{t}^{B}\right]\\ &\quad -(1-\beta)\,\frac{B}{Y}(1+\lambda)\,\widehat{R}_{t}^{B}-2\frac{G}{Y}P\left[\widehat{R}_{t}^{G}|\widehat{R}_{t}^{B}\right],\end{aligned} \tag{4.82}$$

where $P\left[\widehat{X}_{t}|\widehat{Y}_{t}\right]$ is the projection of X_t on Y_t. Subtracting this equation from the one before, we have:

$$\begin{aligned}&\frac{C}{Y}\left(\widehat{R}_{t}^{PC}-P\left[\widehat{R}_{t}^{PC}|\widehat{R}_{t}^{B}\right]\right)\\ =\ &\frac{WL}{Y}\left(\widehat{R}_{t}^{W}-P\left[\widehat{R}_{t}^{W}|\widehat{R}_{t}^{B}\right]\right)+\frac{D}{Y}(S+S^{*}-1)\left(\widehat{R}_{t}^{S}-P\left[\widehat{R}_{t}^{S}|\widehat{R}_{t}^{B}\right]\right)\\ &-\frac{G}{Y}\left(\widehat{R}_{t}^{G}-P\left[\widehat{R}_{t}^{G}|\widehat{R}_{t}^{B}\right]\right)\end{aligned} \tag{4.83}$$

We have:

$$\begin{aligned}&\frac{C}{Y}\left(\widehat{R}_{t}^{PC}-P\left[\widehat{R}_{t}^{PC}|\widehat{R}_{t}^{B}\right]\right)\\ =\ &\frac{WL}{Y}\left(\widehat{R}_{t}^{W}-P\left[\widehat{R}_{t}^{W}|\widehat{R}_{t}^{B}\right]\right)+\frac{D}{Y}\left(S-\frac{\lambda}{1-\lambda}(1-S)\right)\left(\widehat{R}_{t}^{S}-P\left[\widehat{R}_{t}^{S}|\widehat{R}_{t}^{B}\right]\right)\\ &-\frac{G}{Y}\left(\widehat{R}_{t}^{G}-P\left[\widehat{R}_{t}^{G}|\widehat{R}_{t}^{B}\right]\right)\end{aligned} \tag{4.84}$$

Rearranging, we have:

$$\begin{aligned}
\left(S - \frac{\lambda}{1-\lambda}(1-S)\right) &= \frac{\frac{C}{Y}\left(\widehat{R}_t^{PC} - P\left[\widehat{R}_t^{PC}|\widehat{R}_t^{B}\right]\right)}{\frac{D}{Y}\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)} \\
&\quad - \frac{\frac{WL}{Y}\left(\widehat{R}_t^{W} - P\left[\widehat{R}_t^{W}|\widehat{R}_t^{B}\right]\right)}{\frac{D}{Y}\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)} + \frac{\frac{G}{Y}\left(\widehat{R}_t^{G} - P\left[\widehat{R}_t^{G}|\widehat{R}_t^{B}\right]\right)}{\frac{D}{Y}\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)} \\
\frac{1}{(1-\lambda)}S &= \frac{\lambda}{1-\lambda} + \frac{\frac{C}{Y}\left(\widehat{R}_t^{PC} - P\left[\widehat{R}_t^{PC}|\widehat{R}_t^{B}\right]\right)}{\frac{D}{Y}\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)} \\
&\quad - \frac{\frac{WL}{Y}\left(\widehat{R}_t^{W} - P\left[\widehat{R}_t^{W}|\widehat{R}_t^{B}\right]\right)}{\frac{D}{Y}\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)} + \frac{\frac{G}{Y}\left(\widehat{R}_t^{G} - P\left[\widehat{R}_t^{G}|\widehat{R}_t^{B}\right]\right)}{\frac{D}{Y}\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)} \\
S &= \lambda + (1-\lambda)\frac{\frac{C}{Y}\left(\widehat{R}_t^{PC} - P\left[\widehat{R}_t^{PC}|\widehat{R}_t^{B}\right]\right)}{\frac{D}{Y}\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)} \\
&\quad - (1-\lambda)\frac{\frac{WL}{Y}\left(\widehat{R}_t^{W} - P\left[\widehat{R}_t^{W}|\widehat{R}_t^{B}\right]\right)}{\frac{D}{Y}\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)} \\
&\quad + (1-\lambda)\frac{\frac{G}{Y}\left(\widehat{R}_t^{G} - P\left[\widehat{R}_t^{G}|\widehat{R}_t^{B}\right]\right)}{\frac{D}{Y}\left(\widehat{R}_t^{S} - P\left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)} \qquad (4.85)
\end{aligned}$$

Multiplying the numerator and the denominator by $\left(\widehat{R}_t^{S} - \left[\widehat{R}_t^{S}|\widehat{R}_t^{B}\right]\right)$, and taking unconditional expectations, we have:

$$\begin{aligned}
S &= \lambda + (1-\lambda)\frac{\frac{C}{Y}cov_{R_t^B}\left(\widehat{R}_t^{PC}, \widehat{R}_t^{S}\right)}{\frac{D}{Y}var_{R_t^B}\left(\widehat{R}_t^{S}\right)} \\
&\quad - (1-\lambda)\frac{\frac{WL}{Y}cov_{R_t^B}\left(\widehat{R}_t^{W}, \widehat{R}_t^{S}\right)}{\frac{D}{Y}var_{R_t^B}\left(\widehat{R}_t^{S}\right)} \\
&\quad + (1-\lambda)\frac{\frac{G}{Y}cov_{R_t^B}\left(\widehat{R}_t^{G}, \widehat{R}_t^{S}\right)}{\frac{D}{Y}var_{R_t^B}\left(\widehat{R}_t^{S}\right)}, \qquad (4.86)
\end{aligned}$$

where $cov_{R_t^B}(X_t, Y_t)$ is the covariance of X_t and Y_t conditional on relative bond returns R_t^B.

4.6.2 Data

The data is quarterly from the first quarter of 1970 to the third quarter of 2008 for the G7 economies (Canada, France, Germany, Italy, Japan, UK). We take each country in turn as the Home country and an aggregate of the other G7 countries as the Foreign country. National account data is from the OECD quarterly national accounts (QNA), Economic Outlook(EO) and Main Economic Indicators (MEI) databases, while data on equity and bond returns and nominal exchange rates is from the Global Financial Data (GFD) database.

Labour income: Compensation of employees (QNA) plus share of mixed surplus (QNA), as described in Coeurdacier and Gourinchas (2009) and based on Gollin (2002)

Consumption expenditures: Private Final Consumption Expenditures, QNA

Government spending: Final government consumption expenditure, EO

Equity returns: S&P/TSX-300 Total Return Index (Canada), SBF-250 Total Return Index (France), CDAX Total Return Index (Germany), BCI Global Return Index (Italy), Topix Total Return Index (Japan), FTSE All-Share Return Index (UK), S&P 500 Total Return Index (US), all from GFD

Bond returns: 3 month bill rates, GFD

Chapter 5

Bond Maturity and Negative Net Debt Positions

5.1 Introduction

Cross border trade in bonds is large. In 2008, foreign investors held $7,353$ billions of dollars of U.S. debt, while U.S. investors held $1,543$ billion U.S. dollars of foreign debt, implying that U.S. investors had a net negative position in domestic debt of $5,810$ billion U.S. dollars for U.S. based investors (U.S. Treasury (2009a,b)). The wide majority of this debt is dollar denominated and offers a nominal payoff that does not compensate for changes in inflation or the price level. What is more, the majority of this debt is of a long term nature. Thus, U.S. Treasury (2009a,b) indicate that 82% of foreign bonds held by U.S. investors and 88% of U.S. bonds held by foreign investors were classified as "long term debt", where long term is defined as securities with a maturity of more than a year. This paper studies the implications of allowing for trade in long term nominal bonds for equilibrium cross border bond positions.

We study a simple two country, two good model with nominal rigidities and endogenous monetary policy and show that allowing for trade in long term bonds usually switches the sign of equilibrium bond positions. In particular, we find that when countries trade equities and long term nominal bonds, the model predicts that domestic investors choose to take a net negative position in domestic bonds, as found in the data for the U.S. By contrast, previous work generally found a net positive position in domestic bonds of domestic investors, when agents trade short term nominal bonds or long term real bonds.[1]

[1]This is generally true as long as the elasticity of substitution between domestic and foreign

Agents choose bond positions such that they offset the risk from their equity positions and other sources of income, in this case labour income. In booms, firm profits and labour income rise and agents optimally choose a bond portfolio that delivers a negative payoff in these states of nature. Since nominal exchange rates initially depreciate, relative returns of short term bonds fall and a long position in domestic bonds will generate the needed wealth transfer. The returns of long term bonds, however, often rise. The reason is that, while the nominal exchange rate depreciates initially, it *appreciates* in the long run. This is because the rise in productivity subdues inflation in the Home country. Since monetary policy stabilises inflation, but not the price level, the Home price level falls relative to the Foreign price level over time. For the real exchange rate to be unaffected, the nominal exchange rate therefore needs to appreciate in the long run. While the return of one period nominal bonds only depend on the response of the nominal exchange rate in the current period, returns of long term bonds depend on the discounted value of current and future exchange rate fluctuations. We show that under reasonable parameterisations of the monetary policy rule, the long run appreciation dominates the short run depreciation, and returns of long term bonds respond negatively to the productivity shock. This implies that in order to deliver a negative wealth effect in booms, domestic investors need to take a *short* position in domestic bonds. We deliver our finding in the context of a simple setup. However, in related work, Rahbari (2010a) we show that our results are robust to the introduction of local currency pricing, endogenous capital accumulation, consumption home bias, and incomplete markets.

Recent work has emphasised the implications of trade in bonds for, inter alia, international risk sharing and equity home bias.[2] However, despite the prominence given to bonds in recent work, the empirically most relevant type of bond - long term nominal bonds - have not received any attention. Thus, Engel and Matsumoto (2009a,b) and Devereux and Sutherland (2007, 2008) allow for trade in one period nominal bonds, while Coeurdacier et al (2009) feature trade in real consols, i.e. infinitely lived real bonds. Coeurdacier and Gourinchas (2009) study a static framework that does not allow a role for different maturities. This paper shows that this is potentially an important omission. We show that allowing for trade in long term nominal bonds often reverses the sign of the equilibrium bond position and brings them in line with observed data for the U.S.

goods is larger than one, the empirically relevant range.

[2]Examples include Engel and Matsumoto (2009), Coeurdacier and Gourinchas (2009), Coeurdacier et al (2009), and Devereux and Sutherland (2007, 2008).

The rest of the paper proceeds as follows. The next section presents a simple two country model with nominal price rigidities, a Taylor type monetary policy rule, and cross country trade in equities and nominal bonds. The third section presents the implications of this model for equilibrium bond positions, using a standard calibration, and the fourth section concludes.

5.2 Model

The benchmark model is virtually identical to the model presented in Devereux and Sutherland (2007, 2008), with one key exception. There are two symmetric countries, Home (H) and Foreign (F), indexed by i. Each country is specialised in the production of a composite good using a continuum of country specific intermediate goods. Intermediate goods are produced solely using labour. Factors of production and intermediate goods are immobile between countries, but composite goods are traded. The key difference to Devereux and Sutherland (2007, 2008) is that we assume that countries trade infinitively lived nominal bonds ("nominal consols"), rather than one period nominal bonds.

5.2.1 Households

Country i is inhabited by a representative consumer with a utility function that is separable in consumption and labour:

$$U_i = \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{i,t+j}^{1-\sigma}}{1-\sigma} - \iota L_{i,t+j} \right), \tag{5.1}$$

where $C_{i,t}$ is the consumption aggregator of country i,, $L_{i,t}$ labour supply and $P_{C,t}^i$ is the consumption price index. The discount rate β, the intertemporal elasticity of substitution σ and the parameter ι governing labour supply in the steady state are common across countries. The consumption aggregator for country i is defined as:

$$C_{i,t} = \left[\frac{1}{2}^{1/\phi} \left(C_{i,t}^i\right)^{\frac{\phi-1}{\phi}} + \frac{1}{2}^{1/\phi} \left(C_{j,t}^i\right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \tag{5.2}$$

where ϕ is the elasticity of substitution between Home and Foreign goods. These preferences imply the following consumption price indices for country i :

$$P_{C,t}^i = \left[\frac{1}{2} \left(P_{i,t}^i\right)^{1-\phi} + \frac{1}{2} \left(P_{j,t}^i\right)^{1-\phi} \right]^{\frac{1}{1-\phi}}, \tag{5.3}$$

where $P^i_{j,t}$ is the price in country i of the composite good produced in country j. All prices are quoted in terms of the local currency. The optimal allocation across consumption goods is then given by:

$$C^i_{i,t} = \frac{1}{2}\left(\frac{P^i_{i,t}}{P^i_{C,t}}\right)^{-\phi} C_{i,t} \qquad\qquad C^i_{j,t} = \frac{1}{2}\left(\frac{P^i_{j,t}}{P^i_{C,t}}\right)^{-\phi} C_{i,t} \tag{5.4}$$

where $C^i_{j,t}$ denotes consumption of good j by agent i. The first order conditions for labour supply are given by:

$$\iota = \left(\frac{W_{i,t}}{P^i_{C,t}}\right) C^{-\sigma}_{i,t}, \tag{5.5}$$

which describe the standard condition that the marginal disutility of labour today has to equal the marginal utility of consumption times the real wage.
Consumption of the Home and Foreign aggregate consumption goods are given by

$$C^i_{j,t} = \left(\int_0^1 \left(C^i_{j,t}(k)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{5.6}$$

where $C^i_{j,t}(k)$ is consumption of the kth intermediate good in country j by the agent in country i and ε is the elasticity of substitution between varieties in consumption. Optimal consumption of Home and Foreign intermediate goods then implies:

$$C^i_{j,t}(k) = \left(\frac{P^i_{j,t}}{P^i_{j,t}(k)}\right)^{\varepsilon} C^i_{j,t} \tag{5.7}$$

where $P^i_{j,t}$ is the price in country i of the good produced in country j. The price indices for Home and Foreign composite goods are then:

$$P^i_{j,t} = \left(\int_0^1 \left(P^i_{j,t}(k)\right)^{1-\varepsilon} dk\right)^{\frac{1}{1-\varepsilon}} \tag{5.8}$$

and the real exchange rate Q is defined as:

$$Q_t = \frac{P^H_{C,t}}{Z_t P^F_{C,t}}, \tag{5.9}$$

where Z_t is the nominal exchange rate defined as number of units of Home currency per unit of Foreign currency.

5.2.2 Firms

Each country contains a continuum of firms, each producing a differentiated intermediate good, indexed by k, labour. The production function is given by:

$$Y_{i,t}(k) = A_{i,t} L_{i,t}(k), \tag{5.10}$$

where $A_{i,t}$ is the exogenous level of productivity in country i and α is the elasticity of production with respect to labour. Labour is traded in an aggregate country specific spot market and is immobile between countries. The total and marginal cost functions, $\Xi(Y)$ and $\varrho(Y)$ are then given by:

$$\Xi_{i,t}(Y_{i,t}(k)) = W_{i,t} L_{i,t}(k) = \frac{Y_{i,t}(k)}{A_{i,t}} W_{i,t} \tag{5.11}$$

$$\varrho_{i,t}(Y_{i,t}(k)) = \frac{\partial TC_{i,t}}{\partial Y_{i,t}} = \frac{W_{i,t}}{A_{i,t}}. \tag{5.12}$$

Total demand for each intermediate good is composed of domestic demand for consumption and foreign demand for consumption. Equating production with total demand then implies:

$$Y_{i,t}(k) = C^{i}_{i,t}(k) + C^{j}_{i,t}(k), \tag{5.13}$$

where the demand functions are as given above. Firms take these demand functions as given and choose prices in their own currency to maximise profits. Producer currency pricing implies:

$$P^F_{H,t} = \frac{P^H_{H,t}}{Z_t} \qquad P^H_{F,t} = P^F_{F,t} Z_t. \tag{5.14}$$

Per period profits of Home firms are given by:

$$\Pi_{H,t}(k) = P^H_{H,t}(k)\left(C^H_{H,t}(k) + C^F_{H,t}(k)\right) - \Xi_H(Y_{H,t}(k)) \tag{5.15}$$

$$= \left(P^H_{H,t}(k) - \frac{W_{i,t}}{A_{i,t}}\right) Y_{H,t}(k). \tag{5.16}$$

While firms rent labour on a spot market, they can only reset prices with a probability of $1-\theta$ every period. A firm reoptimising in period t will choose a price $\widetilde{P}^{i}_{i,t}$ that maximises the current market value of profits generated while the price remains in effective, taking all other prices as given. Thus, it solves:

$$\max_{\widetilde{P}^i_{i,t},\widetilde{P}^i_{i,t}} \sum_{l=0}^{\infty} \theta^l E_t \left[\varpi^i_{t,t+l} \left(\Pi_{i,t} \left(k, \widetilde{P}^i_{i,t} \right) \right) \right], \tag{5.17}$$

where $\varpi^i_{t,t+l}$ is the stochastic discount factor used to discount profits of a firm in country i at time $t+l$ back to time t. Optimal prices for the Home country are then given by:

$$\widetilde{P}^H_{H,t}(k) = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{l=0}^{\infty} \theta^l \varpi^H_{t,t+l} \frac{W_{H,t+l}}{A_{H,t+l}} \left(P^H_{H,t+l} \right)^{\varepsilon} Y_{H,t+l}}{E_t \sum_{l=0}^{\infty} \theta^l \varpi^H_{t,t+l} \left(P^H_{H,t+l} \right)^{\varepsilon} Y_{H,t+l}}, \tag{5.18}$$

with an analogous condition holding for the Foreign country. As usual, optimal prices thus depend on discounted marginal costs over the expected lifetime of the price set. The nature of price rigidities implies that Home and Foreign goods prices evolve according to:

$$P^i_{j,t} = \left(\theta \left(P^i_{j,t-1} \right)^{1-\varepsilon} + (1-\theta) \left(\widetilde{P}^i_{j,t} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{5.19}$$

5.2.3 Financial Markets

The innovation this paper lies in the structure of asset markets. We assume that the two countries trade equity and long term nominal bonds. Owners of equity of country i receive a claim to country i dividends $D_{i,t}$ which equal firm profits $\Pi_{i,t}$. The total supply of equity in each country is normalised to unity. Nominal bonds pay one unit of local currency in each period and are in zero net supply. As noted in the introduction, nominal long term bonds are the most commonly traded asset class in international asset markets, but previous work has focussed on trade in one period nominal bonds or in infinitively lived real bonds. For comparison, we also show the results that arise in the case of trade in short term nominal bonds. In that case, the model and results are similar to the "NBE" case in Devereux and Sutherland (2007, 2008).[3] It is assumed that in period 0 each household owns the stock of domestic equity and that bond positions are zero. The gross nominal returns in domestic currency for bonds and equity are:

$$R^S_{i,t+1} = \frac{D_{i,t} + P^S_{i,t+1}}{P^S_{i,t}} \qquad R^B_{i,t+1} = \frac{1 + P^B_{i,t+1}}{P^B_{i,t}} \qquad R^{B,s}_{i,t+1} = \frac{1}{P^{B,s}_{i,t}}, \tag{5.20}$$

[3] We disregard the results for real bonds. This is because, with no consumption home bias and producer currency pricing, the real exchange rate is constant. Relative returns on real bonds, which are determined by fluctuations in the real exchange rate, are therefore zero and bonds cannot be used to hedge any cross country risk.

where $R^S_{i,t}$ $\left(R^B_{i,t}\right)$ is the return on holdings of country i equity (long term bonds) in local currency, and $P^S_{i,t}$ $\left(P^B_{i,t}\right)$ are the prices of country i equity (long term bonds). $R^{B,s}_{i,t}$ is the return to short term bonds which pay one unit of local currency in one period, and $P^{B,s}_{i,t}$ is the corresponding price.
The budget constraint for the Home household in the case of trade in long term bonds is then given by:

$$\begin{aligned} & S^H_{H,t}P^S_{H,t} + S^H_{F,t}P^S_{F,t}S_t + B^H_{H,t}P^B_{H,t} + B^H_{F,t}P^B_{H,t}S_t \\ = \; & W_{H,t}L_{H,t} + S^H_{H,t-1}\left(D_{H,t} + P^S_{H,t}\right) + S^H_{F,t-1}\left(D_{F,t} + P^S_{F,t}\right)S_t \\ & + B^H_{H,t-1}\left(1 + P^B_{H,t}\right) + B^H_{F,t-1}S_t\left(1 + P^B_{F,t}\right) - P^H_{C,t}C_{H,t}, \end{aligned} \tag{5.21}$$

where $S^i_{j,t}$ $\left(B^i_{j,t}\right)$ are holdings of country j equity (bonds) by country i in period t. The asset Euler equations for the Home investor are then given by:

$$\beta E_t\left[\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma}\frac{P^H_{C,t}}{P^H_{C,t+1}}\frac{D_{H,t+1} + P^S_{H,t+1}}{P^S_{H,t}}\right] = 1 \tag{5.22}$$

$$\beta E_t\left[\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma}\frac{P^H_{C,t}}{P^H_{C,t+1}}\frac{1 + P^B_{H,t+1}}{P^B_{H,t}}\right] = 1, \tag{5.23}$$

with those for foreign assets and the foreign agent given analogously.

5.2.4 Market Clearing

The market clearing condition for goods are given by:

$$C^H_{H,t} + C^F_{H,t} = Y_{H,t} \qquad C^H_{F,t} + C^F_{F,t} = Y_{F,t} \tag{5.24}$$

For assets, we have:

$$S^H_{H,t} + S^F_{H,t} = 1 \qquad S^H_{F,t} + S^F_{F,t} = 1 \tag{5.25}$$

$$B^H_{H,t} + B^F_{H,t} = 0 \qquad B^H_{F,t} + B^F_{F,t} = 0 \tag{5.26}$$

5.2.5 Monetary Authorities and Exogenous Processes

In order to close the model, we need to specify the monetary policy rule and the exogenous processes. As in Devereux and Sutherland (2007,2008), we assume that the monetary authority sets the nominal interest rate according to a simple rule that is, however, subject to shocks. The central bank here stabilises producer price inflation according to:

$$R_{i,t}^{N}=\frac{1}{\beta}\left(\frac{P_{i,t}^{i}}{P_{i,t-1}^{i}}\right)^{\gamma}\exp\left(m_{i,t}\right), \tag{5.27}$$

where $R_{i,t}^{N}$ is the nominal interest rate, γ determines the monetary policy responsiveness to inflation and m_t is a mean zero shock to interest rates.

There are two sources of uncertainty per country. Productivity shocks in each country evolve according to:

$$\log\left(A_{i,t+1}\right)=\rho_A\log\left(A_{i,t}\right)+\varepsilon_{A,i,t+1}, \tag{5.28}$$

where $\varepsilon_{A,i,t+1}$ is a mean zero shock to productivity, and ρ_A governs the persistence of productivity. The interest rate shock is represented by:

$$m_{i,t+1}=\rho_M m_{i,t}+\varepsilon_{M,i,t+1} \tag{5.29}$$

where $\varepsilon_{M,i,t+1}$ is a mean zero shock to interest rates, and ρ_M governs the persistence of interest rates.

5.2.6 Equilibrium and Solution Method

An equilibrium in this economy is a set of quantities $NFA_{H,t}$, $Y_{F,t}$, $C_{H,t}$, $C_{F,t}$, $C_{H,t}^{H}$, $C_{F,t}^{H}$, $C_{F,t}^{F}$, $C_{H,t}^{F}$, $D_{H,t}$, $D_{F,t}$, $W_{H,t}$, $W_{F,t}$, $S_{H,t}^{H}$, $S_{F,t}^{H}$, $S_{H,t}^{F}$, $S_{F,t}^{F}$, $B_{H,t}^{H}$, $B_{F,t}^{H}$, $B_{H,t}^{F}$, $B_{F,t}^{F}$, $L_{H,t}L_{F,t}$, prices $P_{H,t}^{H}$, $P_{F,t}^{H}$, $P_{F,t}^{F}$, $P_{H,t}^{F}$, $P_{C,t}^{H}, P_{C,t}^{F}$, $P_{I,t}^{H}$, $P_{I,t}^{F}$, Q_t, Z_t and exogenous processes $A_{H,t}$, $A_{F,t}$, $M_{H,t}$, $M_{F,t}$ which satisfy the following conditions:

1. the consumption allocations (equations 5.4 & 5.7)
2. the first order conditions for labour supply (equation 5.5)
3. the Home household's budget constraint (equation 5.21)
4. the household's first order conditions for asset purchases (equations 5.22 - 5.23)
5. the firms' pricing decisions (equation 5.18)
6. the market clearing conditions for goods (equations 5.24)
7. the market clearing conditions for assets (equations 5.25 - 5.26)

8. the laws of motion for the exogenous processes (equations 5.28 - 5.29)

9. the monetary policy rule (equations 5.27)

However, we do not solve for this equilibrium. Instead, we solve for a linear approximation to this equilibrium around the nonstochastic steady state, applying the methods developed by Devereux and Sutherland (2009). The authors show that in order to solve for a first order accurate solution in a class of models that include the one studied here, we only need to solve for the non time varying part of cross country asset portfolios, termed "steady state portfolios". The steady state portfolios in turn can be derived, utilising a second order accurate version of the portfolio Euler equations as well as a linear approximation to the other equations of the model. For details on the technique, see Devereux and Sutherland (2009). Despite the simplicity of the model, it cannot be solved analytically even for the first order approximation.[4]

5.3 Exchange Rates, Portfolios and Asset Returns

It can be shown that equilibrium portfolios are solely a function of cross country differences. Relative returns on nominal bonds are a function of nominal exchange rate fluctuations only, to a first order of approximation. Relative returns on long term bonds, $\widehat{R}_t^B$, depend on the present discounted value of future exchange rate appreciations, while relative returns on one period bonds, $\widehat{R}_t^{B,s}$, depend on the innovation in the exchange rate in the next period only:

$$\widehat{R}_t^B = \widehat{R}_{F,t}^S - \widehat{R}_{H,t}^S - \widehat{Z}_t + \widehat{Z}_{t-1} = (1-\beta)\,\widetilde{E}_t\left[\sum_{j=0}^{\infty}\beta^j\left(-\widehat{Z}_{t+j}\right)\right] \tag{5.30}$$

$$\widehat{R}_t^{B,s} = \widehat{R}_{F,t}^B - \widehat{R}_{H,t}^B - \widehat{Z}_t + \widehat{Z}_{t-1} = \widetilde{E}_t\left[-\widehat{Z}_t\right], \tag{5.31}$$

where $\widetilde{E}_t = (E_t - E_{t-1})$. Thus, the returns of short and long term bonds can only be different if the short run dynamics of the nominal exchange rate differ from its long run dynamics. To illustrate that this is in general the case, we derive equilibrium portfolios for an illustrative calibration of the parameters of the model

[4] Devereux and Sutherland (2007, 2008) manage to solve for equilibrium portfolios analytically in a very similar model. The reason is that they impose that productivity shocks are perfectly persistent, i.e. $\rho_A = 1$, a condition we do not impose.

that is in line with the literature in international macroeconomics. There are ten parameters to be calibrated. The discount factor β, the elasticity of substitution between Home and Foreign goods, ϕ, the price stickiness parameter, θ, the elasticity of substitution between individual varieties, ε, and the persistence of the productivity and interest rate shocks, ρ_A and ρ_M are set to 0.99, 1.5, $0.75, 10, 0.9$, and 0, in line with previous literature. a, i, ω, σ and are set to $0.5, 1, 0$ and 1, respectively, as in Devereux and Sutherland (2007, 2008).[5]

Figure 5.1 plots the impulse response of nominal exchange rates to a positive shock to the difference between Home and Foreign productivity.[6] We can see that the short run and the long run response of the nominal exchange rate are quite different. Initially, the nominal exchange rate depreciates in response to the productivity shock. In the long run, however, the nominal exchange rate appreciates. The reason is that the positive productivity shock subdues inflation in the Home country. Monetary policy stabilises inflation, and not the price level. The persistent fall in relative inflation therefore implies that the Home price level falls relative to the Foreign price level. For the *real* exchange rate to be unaffected in the long run, the nominal exchange therefore needs to appreciate. As noted above, relative returns to long term nominal bonds depend on the present discounted value of current and future nominal exchange rate movements. In our benchmark case, as well as for a large class of parameter values, the long run appreciation dominates the short run depreciation and the relative returns to long term nominal bonds rise. The productivity shock also implies an increase in the value of the dividend and labour income streams of domestic investors. An efficient response for Home investors would then be to have a short exposure to Home bonds, as the negative wealth effect from the bond holdings would offset the gain from the additional income induced by the productivity shock, as can be seen in Table 5.1. The initial depreciation of the Home currency in response to the productivity shock imply that relative returns to Home bonds respond negatively. In that case, a *long* position in domestic bonds would deliver the appropriate change in wealth, as in Devereux and Sutherland (2007, 2008) or Engel and Matsumoto (2009a,b).

We illustrated the effect of bond maturity on equilibrium bond positions in the context of a very simple model and a particular combination of parameters. In

[5]Note that, with two shocks per country and two assets, markets are complete at the first order of approximation. Equilibrium portfolios are therefore invariant to changes in the covariance matrix of shocks, as long as the shocks are not perfectly correlated. We therefore do not need to specify values for the elements of the covariance matrix.

[6]Note that the nominal exchange rate is defined as the number of units of Home currency per unit of Foreign currency. A rise in Z therefore implies a *depreciation* of the Home currency.

Figure 5.1: **Impulse Response of the Nominal Exchange Rate**

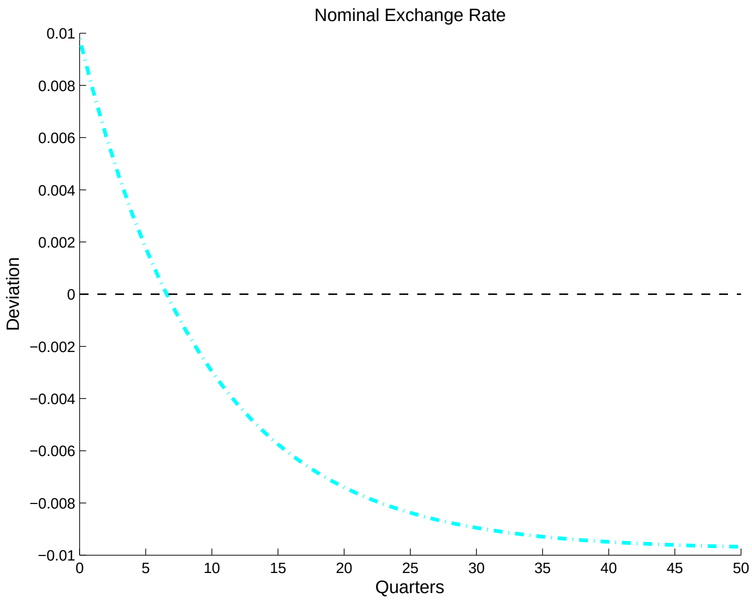


Table 5.1: **Cross Border Bond Portfolios**

	Long Term Bonds	Short Term Bonds
Domestic Bond Position (in % of GDP)	-1.38	0.35

In both cases, equities are traded in addition to nominal bonds. In the long term bond economy, there is also trade in nominal bonds that pay one unit of the local currency in each period. In the short term bond economy, bonds pay off one unit of currency in the following period only.

Table 5.2: **Bond Positions and Monetary Policy**

	$\gamma = 1.1$	$\gamma = 1.5$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 10$
Domestic Bond Position (in % of GDP)	-0.26	-0.61	-1.15	-2.69	-12.04	10.95

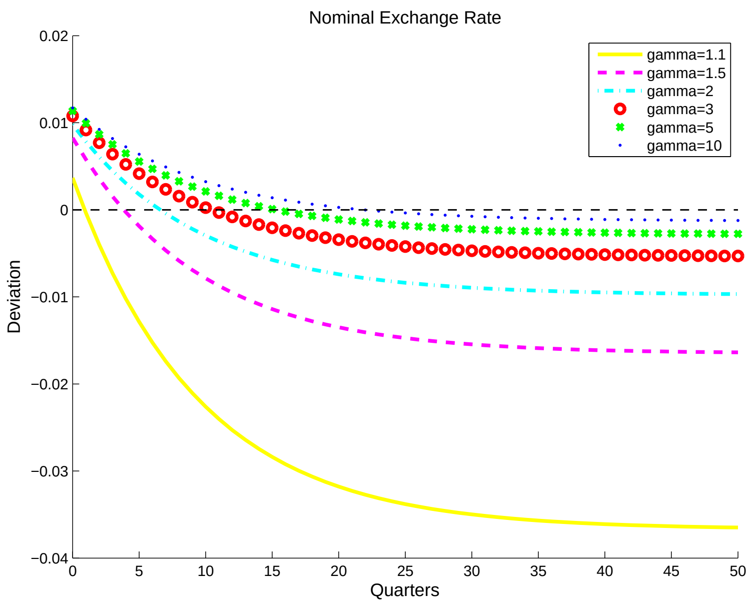
In both cases, equities are traded in addition to nominal bonds. In the long term bond economy, there is also trade in nominal bonds that pay one unit of the local currency in each period. In the short term bond economy, bonds pay off one unit of currency in the following period only.

particular, our benchmark model does not feature consumption home bias, local currency pricing, endogenous capital accumulation or nominal rigidities in wage setting. However, our results are robust to the introduction of these features, as shown in Rahbari (2010a), but also hold for a wide range of parameter values. This is because the result depends on the dynamics of the nominal exchange rate, and all of these extensions do not change the fact that the short run and the long run responses of the nominal exchange rate can be very different. One particular ingredient of the model, the specification of monetary policy, does have an important effect, as illustrated in Figure 5.2 and Table 5.2. In Figure 5.2, we reproduce the impulse response of nominal exchange rates to a relative shock to Home productivity, for various values of monetary policy responsiveness, γ. We can clearly see that, as monetary policy becomes more responsive to producer price inflation, the initial depreciation becomes larger and the ultimate appreciation smaller. Table 5.2 shows the resulting equilibrium bond portfolios. As γ rises, relative bond returns respond less positively to productivity shocks and investors therefore need to take larger positions. For very high values of γ, the initial depreciation quantitatively dominates the ultimate appreciation and relative bond returns fall in response to a relative productivity shock, implying that a *long* position in domestic bonds now becomes preferable. However, it is worth noting that this only occurs for values of γ that are much higher than the estimates usually found in the literature which cluster around 2.

5.4 Conclusion

Recent work on international portfolios has stressed the importance of allowing for cross country trade in bonds when studying home bias in equity holdings or international risk sharing. However, this small literature has neglected the empirically most relevant class of bonds traded internationally - long term nominal bonds. We show that introducing long term bonds into these models can address

Figure 5.2: **Effect of Monetary Policy Responsiveness**



the recent puzzle that most models predicted that domestic agents take a long position in domestic bonds, while the U.S. has a net negative position in dollar denominated bonds. Unlike short term nominal bonds or real bonds, the relative returns of long term nominal bonds rise in response to relative productivity shocks, implying that it is efficient for investors to take a short position in domestic bonds.

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Chapter 6

Optimal Reserve Composition in the Presence of Sudden Stops: The Euro and the Dollar as Safe Haven Currencies*

6.1 Introduction

Emerging market central banks have recently accumulated very large amounts of foreign exchange reserves. Since 1999, foreign exchange reserves held by developing countries have more than quadrupled and now amount to more than four trillion dollars (around 75% of global foreign exchange reserves, see Figure 6.1). Until now, a large share of these foreign exchange reserves of emerging market central banks has been invested in US government securities. However, recently a debate has emerged whether more diversification was to be expected, in particular in the light of high absolute reserve levels for many countries. This debate is all the more topical, as emerging market central banks have financed an increasing part of the US current account deficit and may have contributed in recent years to low US interest rates (Warnock and Warnock (2006)). In addition, some have argued that the euro has become a "credible competitor" (Chinn and Frankel (2005)) for the dollar as reserve currency. In fact, euro-denominated bond markets have become very liquid and caught up with the dollar-denominated markets in terms of size.[1]

*This chapter is joint with Dr Roland Beck of the European Central Bank

[1] Bid-ask spreads of euro-denominated government bonds declined from about 0.08% in 2003 to 0.05% in 2006 (ECB (2007b, p. 61)). For a comparison of the microstructure of euro debt markets with those of the US and the UK, see Dunne et al (2006). According to the BIS, the

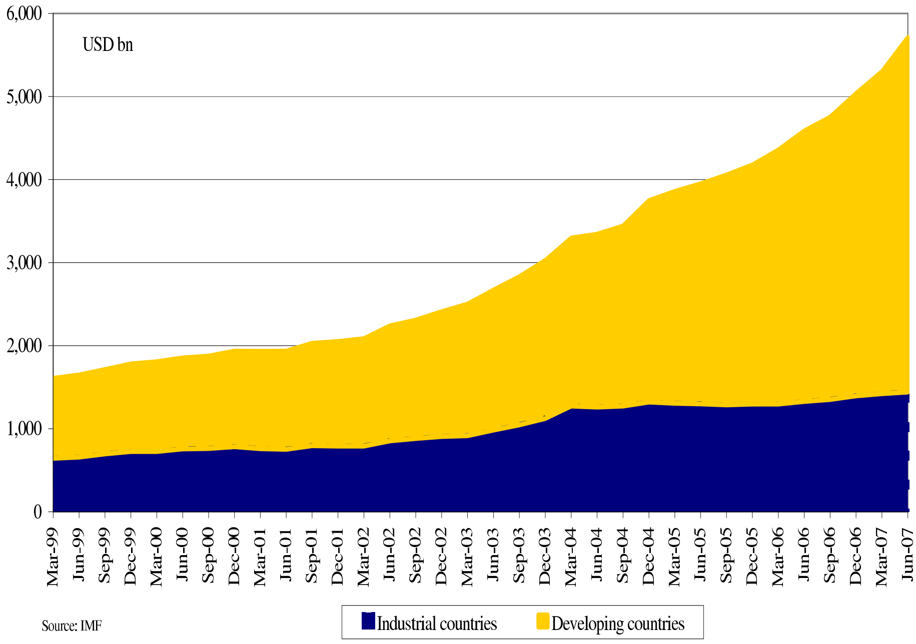

Figure 6.1: **Global Foreign Exchange Reserves**

But despite the strong policy interest, thorough analysis of official reserve choices has been hampered by poor data availability on the empirical side and the lack of a convincing model for central bank behaviour on the theoretical side. Only around one half of emerging market reserves are included in the International Monetary Fund(IMF)'s "Currency Composition of Official Foreign Exchange Reserves" (COFER) database. In addition, many countries have transferred official foreign exchange reserves from their central bank to sovereign wealth funds (see e.g. ECB (2007a)). For those countries which report the currency composition of their "traditional" reserves to the Fund, the IMF publishes figures for the emerging market aggregate which suggests that the share of the dollar in emerging market reserves has decreased from 71% in 1999 to 61% as of June 2007. During the same time, the share of the euro has increased from 18% to 29%. However, the bulk of these developments have taken place until end-2004. Since then, aggregate currency shares have remained relatively stable (see Figure 6.2).

We offer an explanation of these stylised facts which is based on a new framework for optimal reserve portfolios in emerging markets. Central bank behaviour is thought to be motivated by "transaction needs" and "wealth diversification"

share of the euro in global bond markets has gradually increased since 1999 from around 24% to around 30%. During the same time period, the share of the dollar has decreased from around 47% to 42%.

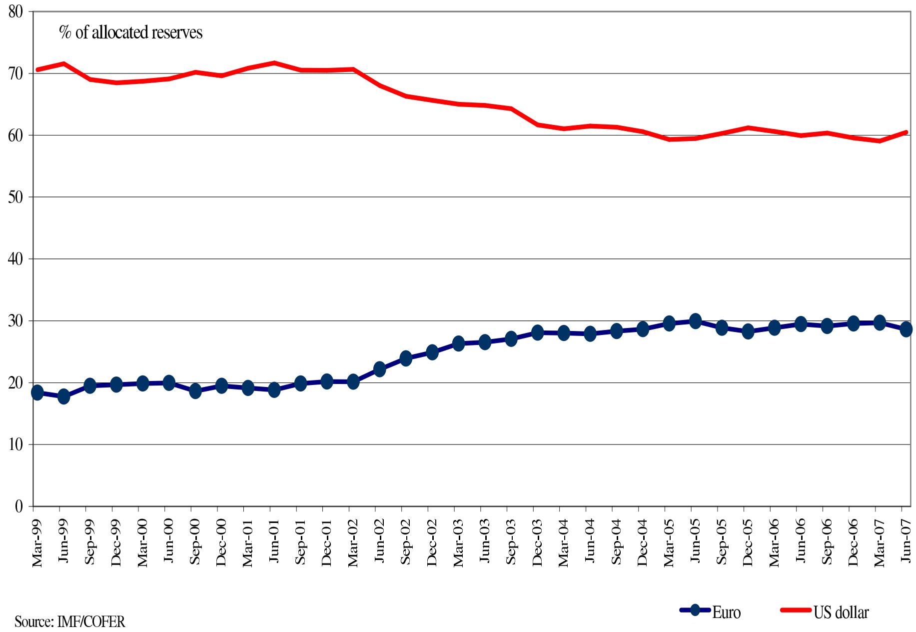

Figure 6.2: **Currency Composition of Foreign Exchange Reserves**

(Roger (1993), ECB (2004)). The wealth diversification motive for central banks is supposed to be working in the same way as for normal investors, motivating them to pursue the maximum return for a given amount of risk, with the only potential difference being a smaller appetite for risk for central banks as opposed to normal investors. Transaction needs are somewhat unique to central banks and encompass temporary import financing, foreign exchange interventions or the balancing of capital outflows. While the need for reserves for import financing purposes is rather small even for most emerging market countries, the financial crises in Asia and Latin America have reaffirmed the importance of the other transaction motives for holding foreign exchange reserves. But while empirical studies using confidential data provided by the IMF (Heller and Knight (1978), Dooley et al (1988) and Eichengreen and Mathieson (2000)) or the publicly available aggregates (Chinn and Frankel (2005)) have generally found transaction motives to be important for central bank currency choices, quantitative theoretical work has largely ignored these and focussed solely on portfolio considerations (see e.g. Ben Bassat (1980)).

In this paper, we attempt to combine both transaction needs and wealth diversification in a single framework and compute optimal portfolios for a number of emerging market economies and regional aggregates. Here, the central banks can invest in dollar or euro-denominated bonds and minimise the portfolio variance in real local currency terms. In our framework, transaction needs arise because countries are subject to sudden reversals in capital flows ("sudden stops") and the central bank uses its reserves to repay the short term foreign denominated debt that is not rolled over in these events (see Calvo et al (2004), Jeanne and Ranciére (2007), Rothenberg and Warnock (2006) for evidence on sudden stops). We focus on emerging market economies, as these economies have accumulated a large amount of reserves in recent years and are more likely to be subject to high volatility in capital inflows.

In our framework, optimal asset shares depend, in addition to their variances and the covariances of the assets with each other, on the extent to which these assets can be used to hedge against sudden stops. This sudden stops risk consists of both exchange rate risk – as the liabilities are denominated in foreign currency – and risk associated with the occurrence of a sudden stop. While the former provides a rationale for matching the currency composition of debt with the composition of liabilities, the latter may or may not imply such a pattern depending on the covariances of currency returns with the sudden stops. We use our simple model to arrive at an analytical solution for optimal currency shares and show that a rise (decline) in reserves (short-tern debt) leads to a decline of the importance of the transactions demand.

In our empirical application, we first document that a standard minimum variance portfolio in local currency (abstracting from transactions demand considerations) is dominated by the "anchor currency" if the country operates de facto an exchange rate peg or tightly managed float, as already indicated by Papaioannou, Portes and Siourounis (2006) and others. Therefore, countries in Asia and Latin America tend to have high optimal dollar shares, whereas the euro dominates the reserve portfolios of countries in Emerging Europe. Countries with more flexible exchange rates tend to have more diversified portfolios.

We then compute optimal central bank portfolios using three different definitions of transactions demand taking into account global, regional and country-specific sudden stops. We find that introducing transactions demand in general has a modest effect on optimal portfolios, with optimal dollar shares changing by a few percentage points. Using our global measure of sudden stops, we find that optimal dollar shares tend to be higher when we include transactions demand. When we use our regional measure of transactions demand, we obtain a regionally varying pattern, with the optimal dollar share rising in Asia and Latin America, while countries in Emerging Europe would hold more euro denominated assets. We tentatively interpret these findings as reflecting the status of the dollar and the euro as "safe haven currencies" which tend to appreciate during sudden stops in which investors redirect capital to mature markets. While the dollar appears to retain his traditional role as the currency of choice in such circumstances, the euro appears to have assumed a similar role in Emerging Europe.

Our results are also consistent with the observed trends in aggregate data on the currency composition of reserves in emerging market economies. According to our model, the decline of the share of the dollar could reflect the notion that transaction motives have become less important as a consequence of rising reserve levels. Our results also suggest, however, that the optimal dollar share is often still very high when transactions considerations are neglected, suggesting that we should not expect a further rise in reserve levels to lead to strong diversification away from the dollar. If anything, we find that the optimal euro share is somewhat higher in the available data than our model would suggest.

A further significant decline of dollar-denominated assets in the reserve portfolios of emerging economies is only likely if currency arrangements in these countries were to change, i.e. with countries moving towards more flexible exchange rate arrangements or increasing the weight attributed to the euro in the case of pegged or tightly managed exchange rate regimes. In fact, this has happened, for example, in the case of Russia which introduced a dollar-euro currency basket in February 2005 and subsequently raised the share of the euro in its foreign exchange reserves to 45%. Likewise, smaller reserve holders which recently become new EU members or EU candidates appear to have increased the share of the euro in their reserve portfolios, following a stronger exchange rate orientation towards the single European currency. More recently a debate on the appropriateness of the dollar pegs in the Gulf Co-operation Countries has emerged. However, a change to a currency basket which includes the euro has so far only occurred to a small extent in the case of Kuwait. Beyond these cases, a more prominent role for the euro as currency anchor with a possible diversification into euro-denominated reserves has been either politically motivated or very gradual, suggesting substantial inertia both in exchange rate arrangements and reserve management decisions. In our framework, the inertia in exchange rate arrangements will also imply very gradual changes in reserve portfolios.

6.2 Related Literature

The academic literature on the currency composition of official foreign exchange reserves can be traced back at least to Heller and Knight (1978) and can broadly be classified into two general categories: an empirical literature trying to relate the reserve portfolio of central banks to observable country or reserve currency characteristics, and a literature that uses portfolio theory to derive the optimal currency composition of reserves.[2]

Empirical work on the determinants of the currency composition of reserves has been hampered by a lack of publicly available data on the reserve portfolio of individual countries. The International Monetary Fund (IMF) collects data from a number of individual countries, but only publishes aggregate figures in its Currency Composition of Official Foreign Exchange Reserves (COFER) database. In addition to the data made available through the COFER database, the IMF grants researchers access to the confidential data set once every decade or so. Using for the first time a confidential data set on the currency composition of reserves of 76 countries, Heller and Knight (1978) find that a country's exchange rate regime and its trade patterns are significantly related with the currency composition of its reserves. These findings led Heller and Knight to conclude that transaction needs play a major role in determining the currency composition of reserves. Dooley et al (1988) use an updated version of this dataset and find further evidence that exchange rate regimes and trade flows are empirical determinants of the currency composition of reserves and interpret these findings as suggesting that countries alter the currency composition of their net foreign asset position through the composition of assets and liabilities other than reserve assets. Using even more recent confidential country level data, Eichengreen and Mathieson (2000) document for the period 1979-1996 that exchange rate pegs, trade flows and financial flows (i.e. the currency composition of external debt) determine the currency composition of reserves in a sample of 84 emerging and transition countries.

Chinn and Frankel (2005), using the aggregate data for the currency composition of reserves published in the COFER database, regress the currency shares of the main reserve currencies on various characteristics of the corresponding reserve currencies and find evidence that the size of the home country, the inflation rate (or the lagged depreciation trend) of the reserve currency, exchange rate volatility

[2]Prior to Heller and Knight (1978) the literature focused on the broader choice between gold, foreign exchange reserves and IMF assets since little information on the currency composition was available.

and the size of the home financial market centre are significant determinants of the currency shares in central bank reserve portfolios.

While the empirical literature in general finds evidence for a strong role of transaction motives as a determinant of reserve composition, the existing theoretical literature has for the most part ignored transaction motives and derived the currency composition of optimal reserves as the solution to an international version of a Markowitz type portfolio problem. The discussion then mainly revolved around the right method of applying optimal portfolio theory in an international context (i.e. the choice of deflator to calculate real from nominal returns, derivation of exchange rate return expectations, etc), rather than explicitly taking into account that central banks pursue objectives different from a normal investor. The resulting optimal portfolio was then compared to actual portfolios and a small difference between the two interpreted as support for the hypothesis that central banks pursue portfolio objectives. Thus, Ben-Bassat (1980) suggests applying mean-variance optimization in terms of a basket of import currencies. When comparing optimal to actual reserve portfolios using data for 1976 and 1980, he finds some evidence for portfolio objectives as a determinant of the currency composition of reserves of the emerging markets but not for industrialised countries.

Dellas and Yoo (1991) use data on the currency denomination of imports and the reserve composition for South Korea to test both a mean variance optimisation model and an import based version of the consumption capital asset pricing model (CCAPM). They show that the actual central bank portfolio was quite close to the efficient frontier computed and that the restrictions implied by the CCAPM could not be rejected, but admit that the power of these tests are low.

In rare attempts to take account of the transaction motives of central banks, Dooley (1983) and Dooley, Lizondo and Mathieson (1988) use a very simple model to show that in the presence of both foreign currency assets and liabilities as well as transaction costs, the composition of gross assets would depend on the structure of transaction costs, and the composition of net assets on expected returns and covariances, in the case of a mean variance optimising central bank. Papaioannou, Portes and Siourounis (2006) investigate the mean variance optimal portfolio at the world level using a variety of methods to estimate covariance matrices and return expectations and different reference currencies. They also experiment with imposing different ad hoc constraints that reflect transaction considerations. The authors find that the reference currency is quantitatively very important and that the computed optimal euro at the world level share is lower than the actual aggregate share published in the COFER database.

On the empirical side, recent papers like Wong (2007) and Lim (2007) examine the impact of past exchange rate changes on aggregate currency shares of foreign exchange reserves and document that currency diversification in response to exchange rate changes have thus far tended to be rather stabilizing for foreign exchange markets, i.e. central banks have tended to pursue "portfolio rebalancing" à la Perold and Sharpe (1995) in which they buy (sell) falling (rising) currencies rather than market trend strategies in which one would buy (sell) rising (falling) currencies. Lim (2007) concludes that these findings are consistent with relatively stable currency shares in the COFER database. He also suggests that these findings may support the view that optimal reserve portfolios have hardly changed over time. Alternatively, Lim (p. 18) suggests that his findings may also support the view that optimal reserve portfolios have changed over time, but reserve managers have on average implemented the change very gradually.

Lately, there has also been renewed interest in explicitly modelling optimal levels of foreign exchange reserves, in part in response to the financial crises of the 1990s and, more recently, the rapid accumulation of reserves in many developing countries since 2002.[3] In the academic literature, this massive reserve build-up has been explained by "insurance" and "mercantilist" motives. The insurance motive suggests that "hoarding international reserves can be viewed as a precautionary adjustment, reflecting the desire for self-insurance against exposure to future sudden stops" (Aizenman and Lee (2007, p. 192)). The mercantilist motive, on the other hand, views the recent accumulation of foreign reserves as a response to concerns about export competitiveness – in particular in the case of China. Aizenman and Lee (2007) document empirically that the insurance motive dominates in the hoarding of international reserves by developing countries. Rodrik (2006) stresses that similar objectives in terms of improving external liquidity positions could have been achieved at lower costs by reducing more forcefully short-term debt. Jeanne and Ranciere (2006) and Jeanne (2007) assume that central banks use reserves to smooth a fall in domestic absorption in the case of sudden capital account reversals and find that optimal reserve levels in their framework are quite close to actual levels in many developing countries, even after the recent rise.

We add to the literature by explicitly introducing transaction motives into the optimal portfolio problem of the central bank, similar to recent efforts in the literature on optimal reserve levels. In our framework, the central bank uses reserves to smooth adjustment in the case of sudden capital account reversals and takes

[3]For an overview of the reserve accumulation phenomenon, see e.g. ECB (2006).

these reversals into account when choosing the optimal reserve portfolio. Since we are interested in the global implications of introducing transactions demand objectives, we also deviate from most of the literature by first obtaining optimal portfolios at the country level and combine the country results to arrive at regional and global aggregates.

6.3 The Model

6.3.1 Minimum Variance Analysis for Central Banks

We consider the problem of a benign central bank that chooses between investing in dollar- and euro-denominated bonds and takes into account transaction needs for foreign exchange. These needs can arise because the central bank would like to intervene in the foreign exchange market in order to support the domestic currency, because the central bank would like to cushion the impact of a sudden reversal of capital flows on domestic output or in order to temporarily finance an amount of imports. In the empirical application, we focus on sudden capital account reversals. In the related literature these events are often referred to as "sudden stops" (Calvo (1998)) and we will sometimes use this term below. We assume that the economy is subject to an exogenous risk of sudden stops which have the effect that any maturing foreign held short term debt is not rolled over and therefore has to be repaid. The restriction to two assets is made for tractability and because we leave many aspects that differentiate the dollar and the euro from other possible reserve currencies unspecified in our model (liquidity, capital market size, etc).[4]

In our analysis, we assume the investor is minimizing the variance of end of period wealth in domestic real terms taking the level of reserves, the level of foreign debt and the level of the transaction need as given.[5] In addition, we impose a short selling constraint. We believe that the right model would be dynamic, let the central bank choose the level of reserves and foreign debt endogenously and simultaneously with the choice of currency composition, include a richer menu of assets and liabilities and a more general form of preferences. Nevertheless we believe that our approach is meritful for various reasons. Firstly, we would like to study the question of currency composition separately from the question of optimal reserve levels, as the former question has received relatively less attention

[4] As argued in the introduction, euro and dollar-denominated assets have become reasonably similar in these respects.

[5] In the appendix we consider also the case of mean-variance optimisation.

than the latter. Nevertheless, we do allow the (exogenous) level of reserves to have an impact on the optimal currency composition. Secondly, in the real world the choice of the level and composition of debt in the economy and the choice of the level of foreign exchange reserves of central banks are usually independent of the choice of currency composition of foreign exchange reserves. Thirdly, central banks are very conservative investors and have until recently invested the bulk of their assets in short-term sovereign debt securities. Fourthly, the finance literature has documented that the impact of estimation error on optimal portfolio weights is particularly severe for expected returns but less so for estimation of the variances and co-variances. In response, Chopra and Ziemba (1993) have suggested to simply set all expected returns in an asset allocation problem equal to each other and focus on minimising the portfolio variance which is the approach we take. Finally, these assumptions allow us to arrive at analytical expressions for the drivers of the optimal portfolio decision and these drivers would also feature in a more comprehensive framework.

Formally, the central bank solves the following problem:

$$\begin{aligned}
&\min_{\alpha} Var\left[W\right] \\
&s.t. \\
&W = \alpha A R_{US} + (1-\alpha) A R_E - S_{US} b B - S_E (1-b) B \\
&0 \leq \alpha \leq 1,
\end{aligned} \tag{6.1}$$

where W is the real end of period level of wealth, A, α, B, b are the level of foreign exchange reserves, the share of dollar-denominated assets in reserves, the level of foreign debt and the share of dollar-denominated debt at the beginning of the period. R_{US} is the real return on dollar bonds, while $R_E$is the real return on euro bonds and the two are defined as:

$$R_{US} = \frac{1+i_{US}}{1+\pi}\left(1+e_{US}\right) \qquad R_E = \frac{1+i_E}{1+\pi}\left(1+e_E\right), \tag{6.2}$$

where $i_{us}\,(i_e)$ is the nominal dollar (euro) interest rate, $e_{us}\,(e_e)$ is the appreciation of the dollar (euro) against the reference currency, and π is a measure of the change in purchasing power (i.e. domestic inflation). S_{US} and S_E are defined as:

$$S_{US} = \frac{1+e_{US}}{1+\pi} S \quad S_E = \frac{1+e_E}{1+\pi} S, \tag{6.3}$$

where S is a random variable the realization of which lies between zero and one and indicates the "extent" of the sudden stop. The transactions demand in our model thus arises because in the case of a sudden stop a certain amount of foreign denominated short-term debt is not rolled over and has to be repaid. We assume that the central bank uses its reserves in these cases (see Jeanne and Ranciere (2006) for some evidence that during sudden capital account reversals central banks cushion the impact on domestic absorption). Below we will construct several different variables corresponding to S and we therefore postpone a more detailed discussion until then.

Under our assumptions (and for the moment disregarding the short selling constraint), we can arrive at an analytical solution for the optimal dollar share in the central bank portfolio:

$$\begin{aligned}\alpha = \quad & \frac{A\left(var\left[R_E\right]-cov\left[R_{US},R_E\right]\right)}{A\left(var\left[R_{US}\right]+var\left[R_E\right]-2cov\left[R_{US},R_E\right]\right)} \\ & +\frac{bB\left(cov\left[R_{US},S_{US}\right]-cov\left[R_E,S_{US}\right]\right)}{A\left(var\left[R_{US}\right]+var\left[R_E\right]-2cov\left[R_{US},R_E\right]\right)} \\ & +\frac{\left(1-b\right)B\left(cov\left[R_{US},S_E\right]-cov\left[R_E,S_E\right]\right)}{A\left(var\left[R_{US}\right]+var\left[R_E\right]-2cov\left[R_{US},R_E\right]\right)}\end{aligned} \tag{6.4}$$

The corresponding euro share is:

$$\begin{aligned}1-\alpha = \quad & \frac{A\left(var\left[R_{US}\right]-cov\left[R_{US},R_E\right]\right)}{A\left(var\left[R_{US}\right]+var\left[R_E\right]-2cov\left[R_{US},R_E\right]\right)} \\ & -\frac{bB\left(cov\left[R_{US},S_{US}\right]-cov\left[R_E,S_{US}\right]\right)}{A\left(var\left[R_{US}\right]+var\left[R_E\right]-2cov\left[R_{US},R_E\right]\right)} \\ & -\frac{\left(1-b\right)B\left(cov\left[R_{US},S_E\right]-cov\left[R_E,S_E\right]\right)}{A\left(var\left[R_{US}\right]+var\left[R_E\right]-2cov\left[R_{US},R_E\right]\right)}\end{aligned} \tag{6.5}$$

These equations have an intuitive interpretation, with the first term reflecting conventional minimum variance considerations and the final two terms reflecting the transactions motive we introduced. In the first term, the euro variance enters positively, while the covariance between euro and dollar assets enters negatively. The latter reflects the strength of diversification benefits between investing in euro and dollars and has the effect of making dollar and euro shares more unequal, if the correlation between dollar and euro returns is positive, while it pushes the shares towards equality, if the correlation is negative. While this term is completely standard, it is worthwhile pointing out that in the context of optimal reserve management two important issues discussed in the literature can be directly linked to this term. The first is the importance of the exchange rate arrangement, and in particular of an exchange rate peg or tightly managed float. A peg or tightly managed float to a particular "anchor" currency implies a low volatility of returns of that currency and will tend to increase the optimal share of that currency in our framework. In general, however, there will also be a benefit to diversification, i.e. investing in more than one asset or currency. But the size of these benefits will depend crucially on the correlation between the different currencies.

The second and third term reflect transaction demand concerns. The second term is pre-multiplied by the amount of dollar debt, and quantifies the extent to which dollar assets are a relatively better hedge than euro assets against a sudden stop in dollar-denominated debt. The optimal dollar share in reserves increases if the covariance between dollar returns and transaction needs arising from dollar debt is higher than the covariance of euro returns with these transaction needs. The third term reflects an analogous consideration with respect to euro-denominated debt and is pre-multiplied with the amount of euro debt. Note that there are two factors that determine the effect of the currency composition of debt on the optimal composition of reserves. Firstly, the second term is weighted by b, the share of dollar debt, while the third term is weighed by $(1 - b)$. This will only have a substantial effect, however, if the second factor, i.e. the relative covariances with dollar and euro sudden stops are very different between the two currencies. In addition, note that it is not necessarily true that the second term is positive (or the third term negative), i.e. it is not necessarily true that dollar assets are a better hedge for dollar sudden stops than euro assets. We revisit this point below.

It is worth noting that the above expression collapses to a standard minimum variance portfolio for two assets if the level of debt, B, is equal to zero. Note also that the second and third term are multiplied by the ratio of debt to reserves, B/A, which captures that the importance of the transactions demand relative to conventional portfolio objectives increases with increases in the debt to reserve ratio. In this regard, our framework generates a simple link between reserve accumulation (or a decline of short-term debt) and the optimal currency composition of reserves.

While we regard the simplicity of our approach to be a virtue, it necessarily implies that we cannot address a number of issues that have been raised in the academic or policy debate on developments in the international financial system. Firstly, note that the transactions demand is taken to be an exogenous event. Therefore reserves have no role in preventing the possibility of a crisis, and neither does reserve composition. What is more, the exogeneity of sudden stops precludes analysis of moral hazard considerations associated with excessive lending or overborrowing in foreign currency. Since the currency composition of reserves is not a state variable in our framework, we do not include transaction costs, and take returns to be exogenous, we also cannot use our model to analyse the merits or possible effects of diversification of large reserve holders which could have an impact on currency returns through price pressure (e.g. in the case of China).

6.3.2 The Effect of Changes in Reserve Levels, Debt Levels and the Currency Composition of Foreign Debt

We can partially differentiate the analytical expression for the optimal dollar share with respect to the level of reserves A, the level of debt B, the debt/reserves ratio B/A and the fraction of debt denominated in dollars b to make some predictions about the effect of changes in reserve or debt levels and the currency composition of foreign-denominated debt on central bank reserve portfolio decisions.

Changes in Reserve Levels

Since the level of reserves only appears in the denominator of the second and third term of the optimal dollar share equation, we can see that, as the level of reserves tends towards infinity, the second and third term go to zero and the optimal dollar share converges to the optimal share in a standard minimum variance portfolio. Until that point, we have:

$$\begin{aligned}\frac{\partial \alpha}{\partial A} &= -\frac{B}{A^2}\frac{1}{\left(var\left[R_{US}\right]+var\left[R_E\right]-2cov\left[R_{US},R_E\right]\right)}\times \\ &\quad \left(b\left(cov\left[R_{US},S_{US}\right]-cov\left[R_E,S_{US}\right]\right)+\left(1-b\right)\left(cov\left[R_{US},S_E\right]-cov\left[R_E,S_E\right]\right)\right)\end{aligned} \tag{6.6}$$

The effect of an increase in the level of reserves on the optimal dollar share thus depends crucially on whether the dollar is a better hedge for sudden stops than the euro. If the dollar is a better hedge, then the above expression is negative and the optimal dollar share falls with an increase in the level of reserves.

Changes in Debt Levels

$$\frac{\partial \alpha}{\partial B} = \frac{1}{A}\frac{1}{(var[R_{US}] + var[R_E] - 2cov[R_{US}, R_E])} \times \\ (b(cov[R_{US}, S_{US}] - cov[R_E, S_{US}]) + (1-b)(cov[R_{US}, S_E] - cov[R_E, S_E])) \tag{6.7}$$

An increase in the level of debt changes the optimal currency composition of reserves in the opposite direction of an increase in reserve levels. If dollars are a better hedge for sudden stops, then an increase in the level of debt will increase the size of the optimal dollar share.

We can also differentiate the optimal share with respect to the ratio of debt to reserves, B/A:

$$\frac{\partial \alpha}{\partial (B/A)} = \frac{1}{(var[R_{US}] + var[R_E] - 2cov[R_{US}, R_E])} \times \\ (b(cov[R_{US}, S_{US}] - cov[R_E, S_{US}]) + (1-b)(cov[R_{US}, S_E] - cov[R_E, S_E])) \tag{6.8}$$

Again, we can see that an increase in the debt/reserve ratio will only reduce the dollar share if the dollar is a better hedge for the sudden stops.

Change in the Currency Denomination of Debt

Eichengreen and Mathieson (2000), among others, have empirically documented that the currency composition of debt is a significant determinant of the currency composition of foreign exchange reserves. In our framework, the effect of an increase in the fraction of debt denominated in dollars is given by:

$$\frac{\partial \alpha}{\partial b} = \frac{B}{A}\frac{1}{(var[R_{US}] + var[R_E] - 2cov[R_{US}, R_E])} \times \\ ((cov[R_{US}, S_{US}] - cov[R_E, S_{US}]) - (cov[R_{US}, S_E] - cov[R_E, S_E])) \tag{6.9}$$

This expression implies that an increase in the dollar share in debt will increase the optimal dollar share in reserves if the dollar is a relatively better hedge for dollar debt sudden stops than for euro debt sudden stops. Note that this is not trivially true. This is because the risk the central bank would like to hedge against is, loosely speaking, composed of exchange risk caused by the foreign denomination of the debt, as well as the possibility of a sudden stop which would in principle be there even if debt was denominated in domestic currency, but held by domestic agents. While the former is strongly linked to debt denomination and implies that risk can be minimized by holding assets in the currency of the liability, the sudden stop risk would best be insured by investing in assets that have a high payoff during capital account crises. In the absence of such contingent assets (so far, there are no "macro markets" for insuring against sudden stops), the co-movement of dollar and euro returns with occurrences of sudden stops will determine their relative hedging demands. A priori, there is no strong theoretical presumption that any one currency is more strongly linked with sudden stops. What is more, it is, a priori, unclear whether the exchange rate risk or the sudden stop risk is quantitatively more important. Finally, since both terms are multiplied by the debt to reserves ratio, B/A, it is worth noting that the impact of debt denomination is limited by the strength of the transaction motive for reserves: If the transactions motives quantitatively not important, then debt denomination cannot be, either.

6.4 Empirical Implementation

6.4.1 Reference Currency and Choice of Deflator

The previous literature (see, for example, Papaiouannou et al (2006)) has noted the importance of the reference currency in international portfolio models. What is usually meant by reference currency is the currency of account. It is then often assumed that an investment in the reference currency is risk-free. This is only strictly true if one abstracts from inflation and interest rate risk which is sometimes justified by an assumption of perfect foresight for the former and buy-and-hold investing for the latter. In these models the existence of a risk-free asset would make the portfolio problem trivial when the objective is to minimize the portfolio variance. In our model this is not the case, as we include inflation and interest rate risk as well as another source of risk, the sudden stops. However, the reference currency is still quantitatively very important, as the variability of the returns of assets we consider is greatly reduced once we strip them of exchange rate risk. It is worth noting that the introduction of the transactions demand will in general reduce the importance of the reference currency, as we add another source of risk. This does not mean that the optimal share of the reference currency is necessarily smaller once we introduce transactions demand, but rather that in cases where the optimal share is higher, it is because the currency returns comove positively with sudden stops, not because of its reference currency status.

Overall, there is no single theoretically convincing choice of reference currency. Previous papers have mostly either assumed that returns are measured in local currency (Dellas and Yoo (1991)) or in dollars. Papaiouannou et al (2006) also present results using euros and the SDR as the currency of account. When the local currency is used as the unit of account, it is justified on the grounds that the central bank maximizes domestic consumption which would in general be measured using real local currency units. Using the dollar as the reference currency is sometimes motivated by the fact that central banks in reality often use the dollar as their unit of account, both for internal and external accounting purposes. We agree with the former view, and will report results using the domestic currency as the reference currency.

It is worth noting that there is a second use of the term “reference currency” in the context of official reserve decisions, namely the currency whose exchange rate with the local currency is managed, e.g. in the case of pegged exchange rates or tightly managed floats. We suggest using the term “anchor currency” for this purpose, as for example in ECB (2007a). As noted above, exchange rate pegs or tightly managed floats will lead to high optimal shares in the anchor currency, exactly because the exchange rate risk associated with these investments is very small and exchange rate risk is a large part of the total risk for the assets we consider.

A second, related choice is the choice of deflator to convert nominal into real returns. Again, there is no consensus in the existing literature. Different alternatives that have been considered are to use a domestic consumption or production price index, to use an import price index, or to abstract from inflation altogether. The choice of one over the other would ideally reflect strong priors on the eventual use of the currency. If the eventual aim is to purchase imports using foreign exchange reserves, then an import price index would appear to be most appropriate, while the use of a domestic consumption price index is preferred if reserves are used to finance domestic consumption. In practice, data is only available for many countries for consumer price indices, so we rely on these in our computations and, where available, provide some robustness checks with regard to using import price indices.

6.4.2 Transactions Demand

In order to arrive at optimal portfolio shares, we need to construct an empirical counterpart to our transaction demands. This presents several challenges. Firstly, we need to specify the use of reserves we have in mind in order to measure it. In the existing literature several motives for holding reserves have been noted, ranging from financing imports, smoothing adjustment during capital account reversals or repaying short term debt of the private or the public sector (see e.g. Roger (1993), Aizenman and Marion (2004) or Jeanne (2007)). While we believe that all of them are potentially important, we focus here on abrupt changes in capital flows. Since we need to estimate the covariance with this measure of transactions demand, we need to construct a whole series for our sudden stop variables. Since, in general, sudden stops are relatively rare events, we face two difficulties: Many countries have not experienced a sudden stop during our sample and any estimation results that are based on the sample will therefore not attribute any importance to this transactions demand. More generally, sudden stops are relatively rare events which makes estimation problematic. Furthermore, we do not have bilateral capital flow data, thus it is not possible for us to distinguish between a sudden stop in dollar capital flows and a sudden stop in euro capital flows. In light of these difficulties, we make the following choices: Firstly, we do not distinguish between dollar and euro sudden stops and define a sudden stop in capital inflows using the methodology described in Rothenberg and Warnock (2006).[6] This method follows Calvo et al (2004) in generating monthly data on capital account reversals based on capital account data, but corrects for capital outflows of local investors. With annual data, the number of observations is relatively small and due to the often short lived nature of crises the number of crises observations is greatly reduced. We construct three different measures for sudden stops based on this method. Firstly, we construct a series relying solely on country specific observations for country i. For our second sudden stop measure, we divide our sample into four regions (Emerging Europe, Latin America, Middle East & Africa, Asia) and calculate regional sudden stops as the sum of sudden stops in any country in the region divided by the number of countries in that region. Thirdly, we calculate a global sudden stop measure S as the sum of all sudden stops divided by the number of countries in the sample. All measures are monthly and lie between zero and one (see Figure 6.3 for a graphical illustration of all sudden stop dates at country, regional and global level). The second and the third measure are based on some understanding that patterns in other countries do carry some information that the central bank takes into account when estimating its future transaction demands.

The second substantial choice pertains to the size of the intervention in the event of a sudden stop. We assume that in the event of a sudden stop, the whole

[6] Note that despite no distinction between dollar and euro sudden stops, the effect of sudden stops will still be different for dollar and euro debt, respectively, because of changes in exchange rates.

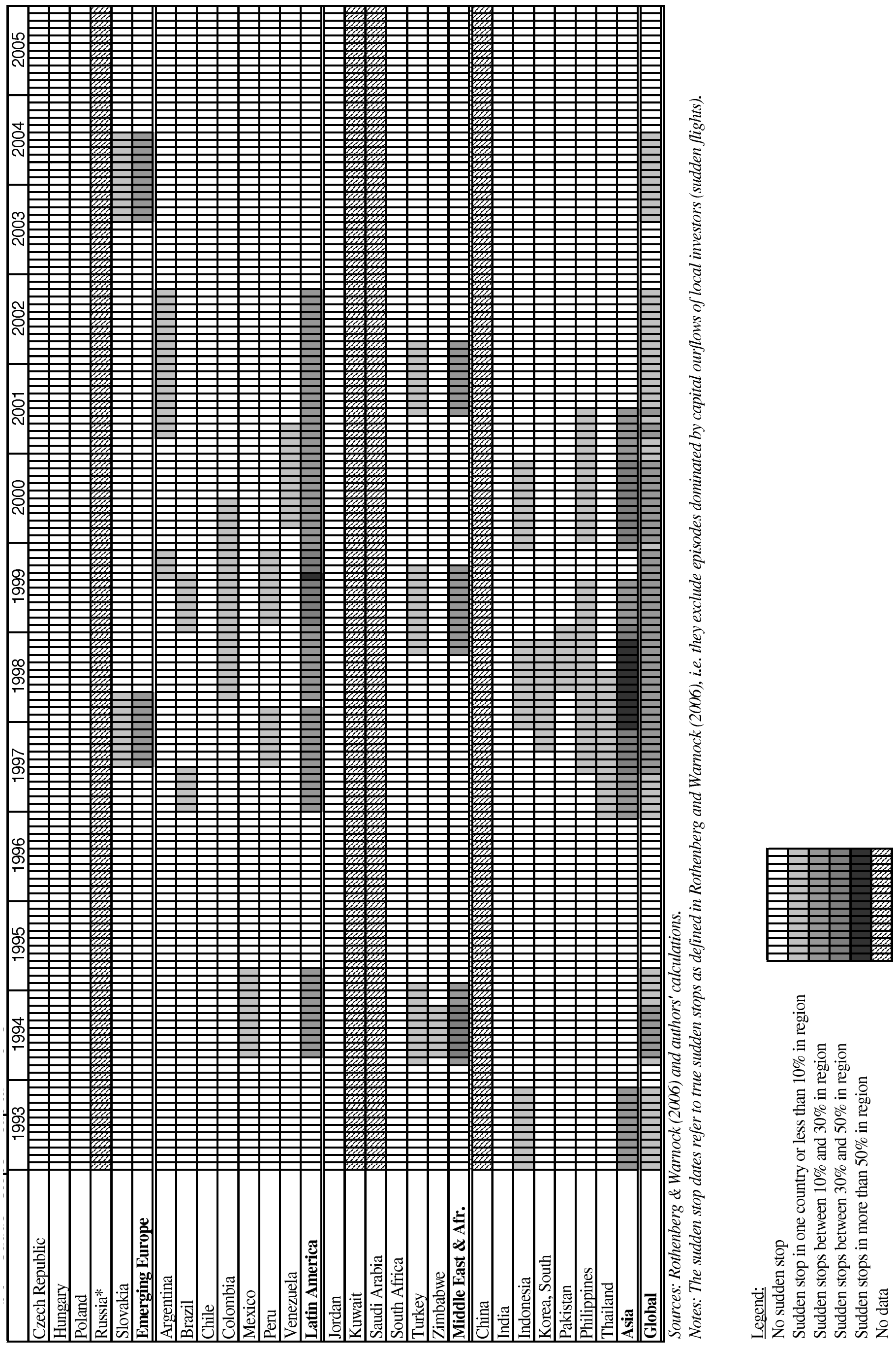

Figure 6.3: **Sudden Stops in Capital Inflows**

stock of foreign short-term debt that is maturing at the end of the period is not rolled over and that the central bank uses its reserves to make this repayment. Since we do not have data on the maturity composition of short term debt (defined as debt with maturity of less than a year), we simply assume that average maturity is six months, and that one sixth of the outstanding amount of short term debt is maturing at the end of each month. Similarly, geographical ownership data is not available for foreign currency, so we assume that most of the foreign currency debt is held by investors abroad.[7]

6.4.3 Estimation of Moments

It has been noted before (see e.g. Jorion (1992) or De Santis et al (2003)) that in portfolio models similar to ours, the estimation results for variances and co-variances will depend quite strongly on the sample period and the estimation method used. Since we do not see our contribution in adding to this literature and are somewhat constrained in terms of data availability, we simply pick the simplest estimation method and estimate the population moments by their sample analogues.[8]

6.4.4 Data

We compute optimal reserve portfolios for 24 emerging market economies which are Argentina, Brazil, Chile, China, Colombia, Czech Republic, Hungary, India, Indonesia, Jordan, Korea (South), Kuwait, Mexico, Pakistan, Peru, Philippines, Poland, Russia, Saudi Arabia, Slovakia, South Africa, Thailand, Turkey and Venezuela.

Our sample runs for most countries from January 1993 to December 2005, the longest possible sample we can construct for the variables used.[9] For the monthly euro and dollar real return series, we deflate 3-month LIBOR and EURIBOR rates

[7]For example, according to the ECB (2007a), the stock of outstanding of euro-denominated debt securities which has been issued by non-residents amounted at end-2005 to USD 1,933 bn. Comparing this amount to euro area iip data (which differs somewhat in terms of valuation methodology) suggests that almost half of this amount was held by euro area residents. Lane and Shambaugh (2007) state that “from the Bank of Japan data, it is clear that Japanese investors purchase (virtually) all of the yen-denominated debt issued by other countries.

[8]More sophisticated techniques for the estimation of the variance-covariance matrix (e.g. GARCH estimation or realised volatility methods) require the availability of higher frequency data which is not an option in our set-up with macroeconomic variables which are available at most at monthly frequency.

[9]In the case of Saudi Arabia, data is only available since January 2002. In the case of Jordan, data are only available since January 1998. In the case of Kuwait, data start in January 1995.

with domestic consumer price indices. Since the currency composition for short-term debt is not available, we use the currency composition for all foreign debt as a proxy for it. In four cases (Czech Republic, Kuwait, Saudi Arabia and South Korea) the composition of external debt was not available. In these cases we used figures for similar countries as a proxy. Prior to 1999, the euro is proxied by a "synthetic" euro computed by Eurostat and based on the ECU. Likewise, for pre-1999 euro interest rates, we use a synthetic Eurostat series for the EURIBOR rate which is a GDP-weighted average of legacy currency interest rates. Sudden stops are taken from Rothenberg and Warnock (2006)'s monthly series of true sudden stops. We supplement their country sample with four major reserve accumulators for which we have no sudden stop data (China, Kuwait, Russia and Saudi Arabia). In these cases, we cannot compute the optimal dollar share with country-specific sudden stops and proxy the respective regional sudden stops and S using the country sample for which such data is available.

All data except the sudden stops have been retrieved using the Haver Analytics database. The original data sources are the World Bank's debt tables (for short-term debt and the currency composition of external debt), the IMF's International Financial Statistics (for the level of foreign exchange reserves and local exchange rates against the dollar), Eurostat (for the euro-dollar exchange rate and the EURIBOR rate), the Financial Times (for LIBOR rates) and national sources (for domestic consumer price and import price indices).

6.5 Results

6.5.1 Benchmark Case: Minimum Variance Portfolio without Transactions Demand

We present our results in terms of the optimal dollar share α - in our two-asset model, the respective optimal euro share is always given by $1 - \alpha$. In Figure 6.4, we report in column 1 traditional minimum variance portfolios (i.e. the case $B = 0$) as a benchmark. As described above, these computations are based on the longest possible time span (January 1993 – December 2005 except for Saudi Arabia and Russia). The stock data for the level of reserves, short-term debt and the denomination of external debt are as of end-2005.

The results exhibit a strong regional pattern which is mainly a reflection of (de facto) exchange rate regimes in the respective regions. Whereas countries in Asia and Latin America have a high optimal dollar share, countries in Emerging Europe

	Without transaction motive	With transaction motive			Memo:
	-	Country-specific S	Regional S	Global S	Actual dollar share**
Country	(1)	(2)	(3)	(4)	(5)
Czech Republic	0.0%	0.0%	0.0%	0.0%	na
Hungary	20.1%	20.1%	17.0%	23.3%	na
Poland	42.3%	42.3%	40.2%	44.4%	na
Russia*	62.9%	na	63.3%	63.0%	65%
Slovakia	3.0%	0.0%	0.0%	8.0%	22%
Emerging Europe	**47.3%**	**na**	46.9%	48.1%	
Argentina	96.0%	84.0%	100.0%	99.5%	na
Brazil	71.3%	82.5%	75.2%	73.5%	na
Chile	89.6%	89.6%	93.7%	91.8%	72%
Colombia	95.6%	100.0%	99.6%	97.6%	87%
Mexico	100.0%	100.0%	100.0%	100.0%	na
Peru	94.8%	99.4%	97.0%	95.9%	81%
Venezuela	100.0%	100.0%	100.0%	100.0%	na
Latin America	**91.1%**	93.2%	93.2%	92.4%	
Jordan	94.9%	94.9%	95.5%	95.7%	na
Kuwait	89.4%	na	91.0%	94.3%	na
Saudi Arabia	99.1%	na	97.2%	97.2%	na
South Africa	60.6%	60.6%	58.9%	63.3%	na
Turkey	76.8%	71.5%	74.0%	80.8%	na
Middle East & Afr.	**80.9%**	**na**	79.0%	83.3%	
China	97.0%	na	99.1%	97.9%	na
India	97.5%	97.5%	98.3%	97.8%	na
Indonesia	30.8%	47.6%	43.6%	35.7%	na
Korea, South	80.4%	79.6%	83.9%	81.8%	na
Pakistan	92.4%	92.6%	93.8%	93.0%	na
Philippines	89.5%	100.0%	94.4%	91.5%	na
Thailand	58.7%	66.9%	63.2%	60.5%	na
Asia Total	**90.9%**	**na**	93.5%	92.0%	
Asia ex China	**79.9%**	**na**	83.4%	81.3%	
All EMEs	**83.9%**	**na**	85.7%	85.0%	
All EME's ex China	**73.7%**	**na**	75.3%	75.0%	**68%*** **
EU Neighbouring Regions†	**52.3%**	**na**	51.5%	53.7%	
EU Neighbouring Regions ex Russia	**41.4%**	**na**	39.3%	44.1%	**35%*** **
Dollar area‡	**84.2%**	**na**	87.0%	85.5%	**81.2%*** **

Sources: *Authors'calculations for optimal dollar shares. National central banks, IMF and Lim (2006) for actual dollar shares.*
Notes: *Grey-shaded areas indicate cases in which the optimal dollar share is higher when transactions motives are included, disregarding short-sale constraints. Short-sale constraints are applied to the currency share numbers. Real returns are computed with the CPI as deflator. Sudden stops for individual countries from Rothenberg and Warnock (2006). Regional and global sudden stops are computed as sum over the respective countries divided by the number of countries. Regional aggregates are weigted by reserve levels as of end-2005. * Results for Russia based on Jan.2004 to Dec. 2005 due to change in currency basket. The actual dollar share in Russia's reserves is an estimate which is based on public statements made during 2005, stating that the share of the euro is about one third. ** Re-scaled to 2-asset case. *** Broadly corresponding aggregates from the IMF's COFER database and Lim (2006) as described in section 5.1. † All countries in Emerging Europe plus South Africa and Turkey. ‡ All countries in Latin America and Asia (excluding China) plus Jordan, Kuwait and Saudi Arabia.*

Figure 6.4: **Optimal dollar share for the 2-asset minimum variance portfolio, end 2005**

tend to have a high optimal euro share.[10] In the Middle East and Africa, Middle Eastern countries have high optimal dollar shares whereas the respective figures for South Africa and Turkey are somewhat lower. Generally, countries operating de facto a relatively flexible exchange rate regime (e.g. South Africa, Poland) have more diversified optimal portfolios than countries with explicit or implicit exchange rate targets. The reason that the exchange rate regime has such a strong effect on optimal reserve portfolios is that it has a strong effect on the exchange rate risk that the assets have and this exchange rate risk accounts for a large portion of the total risk of the assets we consider. In fact, countries which operate a de facto fixed exchange rate regime or manage their currency against the dollar or the euro have the opportunity to invest their foreign exchange reserves in an asset with very low variance. As the correlation between dollar and euro returns is usually quite high, the diversification benefits are limited and portfolios are dominated by the low variance currency asset. It is worth pointing out here that most emerging market economies have explicit or implicit exchange rate targets so that these findings would apply to the majority of emerging market central banks.[11] In a number of countries, the short sale constraint is hit, again reflecting the strong effect of the exchange rate regime and limited diversification benefits. In Mexico and Venezuela, the optimal portfolio consists of only dollar assets, while in the Czech Republic the optimal portfolio only contains euro denominated assets.

It should be stressed in this context that the results for the optimal dollar share can be very sensitive to the choice of sample period. In addition to the instability of covariance matrix estimation noted before in the literature, there are two aspects that aggravate this problem in our context. Firstly, many emerging market economies exhibit few, but very large depreciations (which often coincide with capital account crises) and these crises periods have a strong effect on measured variances and covariances. Secondly, during our sample period some countries underwent a change in their exchange rate regime (see the appendix for a graphical illustration of real dollar and euro returns for all countries in the sample). Russia is a case in point. It had an implicit dollar peg until 2004, but changed to an operational currency basket which included the euro in February 2005.[12] Since we

[10]Note that the fact that Hungary has a larger optimal dollar share that the Czech Republic is likely to be a reflection of Hungary's currency basket which included prior to 2000 a sizable proportion of dollars. In fact, the share of the dollar was changed from 50% in 1993 to 30% during 1994-1999. Only since 2000 does Hungary's reference basket only contain the euro.

[11]According to the latest list on "De Facto Exchange Rate Arrangements and Anchors of Monetary Policy" compiled by the IMF as of July 2006, only 25 out of 146 countries with own legal tender are classified as "independently floating".

[12]The share of the euro in this currency basket which is used by the Central Bank of Russia

identify variances and covariances with their sample analogues and do not allow for structural breaks, the optimal dollar share using data for the whole sample period is very high (in fact it is 100%). Since Russian authorities have announced that this change in exchange rate policy has been accompanied by a rising share of the euro in foreign exchange reserves, we believe that the Central Bank of Russia could have implicitly decreased the weight of past observations which still refer to the old exchange rate regime. Therefore, we recomputed the optimal dollar share using only data from January 2004 to February 2005 and arrive at a lower dollar share. This number is reported in the table.

Our results are broadly in line with previous findings in the literature and actual reserve levels. As noted above, the "anchor" currency tends to be dominant in reserve portfolios, similar to the roles played by the "reference currency" (meaning currency of account) in Papaionnaou et al (2006). Our results are also in line with recent papers on home bias which document that home bias is more pronounced for bonds than for equities (see Fidora, Fratzscher and Thimann (2007)). This is because exchange rate risk accounts for a major portion of total risk for bonds, while it accounts for a smaller portion of the total risk of (the much riskier) equities. In quantitative terms, our country-specific findings for the minimum variance benchmark portfolio are broadly comparable to those in Papaioannou et al. (2006) for the BRIC countries (Brazil, Russia, India and China). Using a different methodology and data from January 1995 until December 2004, the authors find optimal dollar allocations (re-scaled to the 2-asset case) for Russia (75%), India (92%), China (93%), and Brazil (88%).[13]

A comparison of our model results with actual reserve portfolios at the country level is difficult since country data on the currency composition of foreign exchange reserves is only available for a few of the countries in our sample. Rescaling the available actual currency shares to our two-asset case, we find that our model results are generally quite close to the actual central bank portfolios (Figure 6.4, column 1 and 5). In the case of Russia, the optimal dollar share is close to the actual dollar share in 2005 (which is roughly known from public statements by central bank officials). Optimal dollar shares are somewhat higher than actual

for the management of daily volatility has been gradually increased from 10% to 20% (March 2005), 35% (August 2005), 40% (December 2005) and 45% (February 2007).

[13] Papaioannou et al (2006) compute optimal portfolio as solutions to a mean variance optimisation problem with transaction costs in terms of dollars and abstracting from inflation and interest rate risk. They estimate the variance covariance matrix using multivariate GARCH methods that allow for changing correlations. The authors also impose an ad hoc constraint which requires that the central bank holds reserves in the currencies of the country's main trading partners at levels equal to 50% of the respective trade share.

shares in Peru, Columbia and Chile. In the case of Slovakia, the optimal dollar share is lower than the actual dollar share.

Aggregating the countries in our sample, weighted by reserve levels, we find that the optimal dollar share lies at around 84% (Figure 6.4, column 1, bottom). One useful benchmark to compare this aggregate figure with is the IMF's COFER data which reports a dollar share of 61% for developing countries at the end of 2005 which would correspond to 68% in a portfolio that includes only dollar and euro-denominated assets (Figure 6.4, column 5, bottom). However, it is worth noting that, while our sample includes China, the COFER data does not, and, given China's large level of reserves and high optimal dollar share, including China increases the optimal dollar share quite significantly. Recomputing the emerging market total of our sample without China, we arrive at an optimal dollar share of around 74% (Figure 6.4, column 1, bottom).[14]

Given the strong regional character of optimal portfolios, we also compute various regional aggregates. We thus arrive at an optimal dollar share of 47% in Emerging Europe, 91% in Latin America, 81% in the Middle East & Africa and 91% in Asia, including China (Figure 6.4, column 1). While no regional breakdown of the IMF data is regularly available, Lim (2006) presents figures for certain subgroups that can be compared to our regional aggregates. The author reports that the rescaled optimal dollar share for "all European countries surrounding the euro area and all countries worldwide that largely peg their currencies to the euro" (EU neighbouring regions) stood at 35% which is somewhat lower than our estimate (excluding Russia which is likely to have been classified by Lim as belonging to the dollar area) of 47%. The optimal dollar share for "Asia, the Western Hemisphere and all countries that largely peg their currencies to the US dollar" (dollar area) is equal to 81% which compares to a weighted average optimal dollar share from our model of 84% for Asia and Latin America. As regards countries in the Middle East and Africa, we believe that the dollar peggers in our sample have also been included by Lim in the dollar area. Therefore, we have also included these countries into our aggregate for the dollar area.

We thus see that a simple model of variance minimisation in local currency can rationalise the strongly regional pattern of reserve composition displayed in the disaggregated COFER data presented by Lim, mainly as a result of the prevailing

[14] According to the IMF (2005, p. 110), reporting compliance on the currency composition of reserves is particularly low in Asia. Given the (publicly known) size of China's foreign exchange reserves, it is highly plausible that China's reserves are not part of the IMF's "allocated reserves" on which aggregate currency shares are based.

exchange rate arrangements. In the following section, we analyse the effect of explicitly introducing transactions demands for foreign exchange.

6.5.2 Optimal Portfolios with Transactions Demand

As described above, we consider three different definitions for the transactions demand, based on country-specific, regional and global sudden stops.

Hedging against Country-specific Sudden Stops

First, we consider only country-specific sudden stops to predict future transactions needs and present the results in Figure 6.4, column 2. As stated above, we cannot compute optimal dollar shares for Russia, Kuwait, Saudi Arabia and China since no country-specific sudden stop dates are available for these countries. Note that in countries that did not suffer a sudden stop during our sample period, the optimal share is unchanged from the benchmark portfolio presented in the previous subsection (Czech Republic, Hungary, Poland, Chile, Jordan, South Africa, and India). For countries that have suffered a sudden stop during our sample period, the optimal dollar share often increases with the introduction of the transactions demand. This is the case for Brazil, Colombia, Peru, Indonesia, Pakistan, the Philippines and Thailand, while the euro share increases in Slovakia, Argentina, and Turkey. In some countries (Mexico, Venezuela) the short-sale constraint is hit with and without transactions demand. In Mexico, the optimal dollar share would be lower with hedging against country-specific sudden stops whereas in Venezuela it would be higher. Note also that in countries that have suffered a sudden stop during our sample period, the size of the change in the optimal reserve portfolio caused by the introduction of the transactions varies quite widely between the different countries, but is in general quite small. The smallest change is observed in the case of Pakistan (92.6% dollar share with transactions demand, 92.4% without), while the biggest change is observed in Indonesia (47.6% vs 30.8%). The reason that the effect of the transactions demands are quite small is that, in general, sudden stops are rare events, and reserve levels are quite high, in particular compared to current levels of short-term foreign debt.

Hedging against Regional Sudden Stops

It is widely believed that crises in emerging market economies have effects on other emerging market economies that go beyond their direct trade links. This phenomenon is usually called "contagion" (for a recent survey of the contagion literature related to emerging market economies, see Forbes (2007)). While there is no consensus on the underlying mechanism leading to contagion, we regard it as highly plausible that central banks will use information from other emerging market economies with similar characteristics to forecast the possibility of swift capital account reversals. We therefore calculate optimal shares using our regional measure of sudden stop and present the results in column 3 of Figure 6.4. With very few exceptions, the effect of introducing a transactions demand on the optimal dollar share now follows a regional pattern. Whereas in Emerging European countries, the euro is a better hedge against regional sudden stops and introducing sudden stops now reduces the optimal dollar share, the respective optimal dollar shares in Latin American and Asian countries are higher than in our benchmark case. Within the Middle East and Africa, countries in the Middle East tend to have higher optimal dollar shares (with the exception of Saudi Arabia) whereas in Turkey and South Africa, the optimal dollar share is lower than in our benchmark portfolio. The magnitude of changes in optimal reserve portfolios compared to our minimum variance benchmark is again relatively modest, ranging from 0.4% in the case of Russia to 12.8% for Indonesia. It should be noted though that our results for Emerging Europe are based on only one sudden stop observation (i.e. that in Slovakia) and should therefore be treated with caution.

Hedging against Global Sudden Stops

In column 4 of Figure 6.4, we present optimal portfolios that are calculated using the global measure of sudden stops described above. We can see that in this case the optimal dollar share increases in virtually all cases.[15] We therefore see that introducing transactions demands based on sudden stops has different effects on optimal portfolios depending on the definition of sudden stops. For global sudden stop measures, the optimal dollar share increases, reflecting high dollar returns during periods of crises. For regional sudden stop measures, the effects depend on the region, with the optimal dollar share increasing in Latin America and Asia, and the euro share increasing in Emerging Europe. For country-specific sudden stops, the optimal dollar share increased in most countries.

Our interpretation of these results is based on the idea that during periods of heightened uncertainty or crises in emerging markets, capital is redirected to

[15]Only in the case of Saudi Arabia is the euro share slightly higher reflecting the different sample period.

mature economies and financial markets bidding up the prices of assets in those countries, and therefore also their currencies. In Latin America and Asia the role of the "safe haven currency" is played by the US dollar, while investment into Emerging Europe and some parts of the Middle East and Africa is dominated by euro area investors, making euro-denominated assets the destination of choice during crisis periods. In fact, according to the BIS' consolidated banking statistics, as of end-2005 more than 70% of foreign bank claims on countries in EU neighbouring regions were held by euro area banks.

A Rise in Reserves or Decline in Short-term Debt

Irrespective of how we measure sudden stops, Figure 6.4 suggests that, quantitatively, the impact of introducing transactions demands is relatively small for most countries. As demonstrated in section 3, a rise in reserves leads in our model to a decline of the transactions motive. Likewise, a decline in short-term debt leads to smaller hedging terms in our equation for the optimal dollar share. Since foreign exchange reserves have increased considerably over the past few years (recall Figure **??**) and levels of foreign debt have decreased, one might interpret the small impact of transactions demand as reflecting very high reserve levels (or low levels of short-term debt) as of end-2005. It is worth noting in this context that in our sample all countries had reserves to short-term debt ratios of more than 100%, sometimes many times more, thereby comfortably exceeding the Greenspan-Guidotti rule. To quantitatively investigate the impact of changing reserve levels, we therefore recalculate optimal portfolios with all parameters kept equal to their values in Figure 6.4, but with reserve levels which are 50% lower than at the end of 2005. The results are summarised in 6.9. As expected, the effect of introducing transactions demands is now larger, and, given that, on balance, the dollar is a better hedge for sudden stops for most measures of sudden stops, the aggregate optimal dollar share also increases. For individual countries, the impact can be quite large (in the extreme in the order of magnitude of 20 to 30 percentage points). At the regional level, the largest impact lies in the order of five to seven percentage points. These results are therefore also consistent with the evidence that dollar shares in actual central bank reserves have decreased somewhat during the period of strong reserve accumulation, though in our case the difference in the aggregate dollar share is very small. Note that the results would be very similar if we increased short term debt instead.

In Figure 6.6, we present results based on reserve levels which are twice as

	Without transaction motive	With transaction motive		
	-	Country-specific S	Regional S	Global S
Country	**(1)**	**(2)**	**(3)**	**(4)**
Czech Republic	0.0%	0.0%	0.0%	2.7%
Hungary	20.1%	20.1%	13.9%	26.5%
Poland	42.3%	42.3%	38.1%	46.4%
Russia*	62.9%	na	63.7%	63.0%
Slovakia	3.0%	0.0%	0.0%	13.1%
Emerging Europe	**47.3%**	**na**	46.6%	49.2%
Argentina	96.0%	72.1%	100.0%	100.0%
Brazil	71.3%	93.7%	79.0%	75.7%
Chile	89.6%	89.6%	97.8%	94.0%
Colombia	95.6%	100.0%	100.0%	99.5%
Mexico	100.0%	100.0%	100.0%	100.0%
Peru	94.8%	100.0%	99.3%	97.1%
Venezuela	100.0%	100.0%	100.0%	100.0%
Latin America	**91.1%**	94.7%	94.6%	93.3%
Jordan	94.8%	94.9%	96.0%	96.5%
Kuwait	89.4%	na	92.6%	99.2%
Saudi Arabia	99.1%	na	95.2%	95.3%
South Africa	60.6%	60.6%	57.2%	66.1%
Turkey	76.8%	66.2%	71.2%	84.8%
Middle East & Afr.	**80.9%**		77.1%	85.7%
China	97.0%	na	100.0%	98.8%
India	97.5%	97.5%	99.1%	98.2%
Indonesia	30.8%	64.4%	56.3%	40.6%
Korea, South	80.4%	78.9%	87.3%	83.2%
Pakistan	92.4%	92.9%	95.2%	93.5%
Philippines	89.5%	100.0%	99.3%	93.5%
Thailand	58.7%	75.0%	67.6%	62.3%
Asia	**90.9%**	**na**	95.3%	93.1%
Asia ex China	**79.9%**	**na**	86.9%	82.7%
All EME's	**83.9%**	**na**	87.0%	86.1%
All EME's ex China	**73.7%**	**na**	76.8%	76.3%

Source: Authors'calculations.
*Notes: Grey-shaded areas indicate cases in which the optimal dollar share is higher when transactions motives are included, disregarding short-sale constraints. Short-sale constraints are applied to the currency share numbers. Real returns are computed with the CPI as deflator. Sudden stops for individual countries from Rothenberg and Warnock (2006). Regional and global sudden stops are computed as sum over the respective countries divided by the number of countries. Regional aggregates are weigted by reserve levels as of end-2005. * Results for Russia based on Jan.2004 to Dec. 2005 due to change in currency basket.*

Figure 6.5: **Optimal dollar share for the 2-asset minimum variance portfolio, end 2005, half reserve levels**

high as at the end of 2005. As can be seen, the effect of the transactions demand is now very small in most cases and the optimal shares become very close to the minimum variance benchmark. Note that, as in our minimum variance benchmark the optimal dollar share remains very high, no widespread diversification out of dollar-denominated assets should be expected as reserve levels increase to even higher levels.

A Change in the Denomination of External Debt

Finally, we reconsider the impact of the denomination of external debt on optimal portfolios. As explained in section 6.3, the impact of debt denomination depends on the extent to which the relative hedging properties of dollar and euro vary between dollar and euro sudden stops. In practice, it turns out that this difference is quite small, i.e. the dollar and the euro have similar hedging properties for dollar sudden stops as for euro sudden stops. To a certain extent this reflects our modelling assumptions. In our framework, the central bank repays short term debt only in the event of a sudden stop. This has the effect that exchange rate risk is quite limited relative to the risk associated with the occurrence of a sudden stop. Put differently, in our framework, the central bank is concerned with holding an asset that has a high payoff when the sudden stop occurs, but does not care as much about the exchange rate risk of its liabilities. Of course, the preceding section indicated that the effect of introducing transactions demands is quite small in many cases which implies that in those cases, debt denomination cannot have strong quantitative effects, even if the hedging properties for euro and dollar sudden stops were very different. In order to demonstrate this in quantitative terms, we consider two extreme scenarios in Figures 6.7 and 6.8. In Figure 6.7, we present optimal portfolios assuming that all external debt is euro-denominated ($b = 0$) and in Figure 6.8, we present assuming that all external debt is dollar-denominated ($b = 1$). Comparing Figures 6.7 and 6.8, it can be seen that the difference between optimal portfolios is very limited even in the extreme examples we consider in this table.

Robustness to Alternative Deflators

Since countries may also care about real purchasing power in import terms, we also compute optimal currency shares using import price indices for deflating nominal returns (Figure **??**). As such data is only available for a few countries in our sample (Czech, Republic, Hungary, Poland, Brazil, Mexico, Peru, Jordan, Turkey

	Without transaction motive	With transaction motive		
	-	Country-specific S	Regional S	Global S
Country	**(1)**	**(2)**	**(3)**	**(4)**
Czech Republic	0.0%	0.0%	0.0%	0.0%
Hungary	20.1%	20.1%	18.5%	21.7%
Poland	42.3%	42.3%	41.3%	43.3%
Russia*	62.9%	na	63.1%	62.9%
Slovakia	3.0%	0.0%	0.5%	5.5%
Emerging Europe	**47.3%**	na	47.0%	47.7%
Argentina	96.0%	90.0%	99.6%	97.8%
Brazil	71.3%	76.9%	73.2%	72.4%
Chile	89.6%	89.6%	91.6%	90.7%
Colombia	95.6%	100.0%	97.6%	96.6%
Mexico	100.0%	100.0%	100.0%	100.0%
Peru	94.8%	97.1%	95.9%	95.4%
Venezuela	100.0%	100.0%	100.0%	100.0%
Latin America	**91.1%**	92.3%	92.3%	91.7%
Jordan	94.9%	94.9%	95.2%	95.3%
Kuwait	89.3%	na	90.2%	91.8%
Saudi Arabia	99.1%	na	98.2%	98.2%
South Africa	60.6%	60.6%	59.8%	62.0%
Turkey	76.8%	74.1%	75.4%	78.8%
Middle East & Afr.	**80.9%**	na	80.0%	82.1%
China	97.0%	na	98.1%	97.4%
India	97.5%	97.5%	97.9%	97.7%
Indonesia	30.8%	39.2%	37.2%	33.3%
Korea, South	80.4%	80.0%	82.1%	81.1%
Pakistan	92.4%	92.5%	93.1%	92.7%
Philippines	89.5%	97.0%	91.9%	90.5%
Thailand	58.7%	62.8%	60.9%	59.6%
Asia Total	**90.9%**	na	92.2%	91.5%
Asia ex China	**79.9%**	na	81.6%	80.6%
All EME's	**83.9%**	na	84.8%	84.5%
All EME's ex China	**73.7%**	na	74.5%	74.3%

Source: Authors'calculations.

*Notes: Grey-shaded areas indicate cases in which the optimal dollar share is higher when transactions motives are included, disregarding short-sale constraints. Short-sale constraints are applied to the currency share numbers. Real returns are computed with the CPI as deflator. Sudden stops for individual countries from Rothenberg and Warnock (2006). Regional and global sudden stops are computed as sum over the respective countries divided by the number of countries. Regional aggregates are weigted by reserve levels as of end-2005. * Results for Russia based on Jan.2004 to Dec. 2005 due to change in currency basket.*

Figure 6.6: **Optimal dollar share for the 2-asset minimum variance portfolio, end 2005, double reserve levels**

	Without transaction motive	With transaction motive		
	-	Country-specific S	Regional S	Global S
Country	**(1)**	**(2)**	**(3)**	**(4)**
Czech Republic	0.0%	0.0%	0.0%	0.0%
Hungary	20.1%	20.1%	16.9%	23.1%
Poland	42.3%	42.3%	40.2%	44.2%
Russia	62.9%	na	63.2%	63.0%
Slovakia	3.0%	0.0%	0.0%	7.9%
Emerging Europe	**47.3%**	**na**	**46.8%**	**48.1%**
Argentina	96.0%	82.8%	100.0%	98.8%
Brazil	71.3%	82.1%	74.6%	73.0%
Chile	89.6%	89.6%	93.1%	91.2%
Colombia	95.6%	100.0%	98.9%	97.0%
Mexico	100.0%	100.0%	100.0%	100.0%
Peru	94.8%	99.1%	96.7%	95.6%
Venezuela	100.0%	100.0%	100.0%	100.0%
Latin America	**91.1%**	**92.9%**	**92.9%**	**92.1%**
Jordan	94.9%	94.9%	95.4%	95.6%
Kuwait	89.4%	na	90.6%	93.3%
Saudi Arabia	99.1%	na	98.1%	98.1%
South Africa	60.6%	60.6%	58.6%	62.8%
Turkey	76.8%	70.2%	73.6%	80.1%
Middle East & Afr.	**80.9%**		**79.0%**	**83.0%**
China	97.0%	na	97.9%	97.3%
India	97.5%	97.5%	98.1%	97.7%
Indonesia	30.8%	45.4%	42.5%	35.0%
Korea, South	80.4%	79.4%	83.3%	81.5%
Pakistan	92.4%	92.5%	93.6%	92.8%
Philippines	89.5%	100.0%	93.6%	91.0%
Thailand	58.7%	66.4%	62.5%	60.1%
Asia	**90.9%**	**na**	92.6%	91.5%
Asia ex China	**79.9%**	**na**	82.9%	81.0%
All EME's	**83.9%**	**na**	**85.0%**	**84.6%**
All EME's ex China	**73.7%**	**na**	**75.0%**	**74.8%**

Source: Authors'calculations.

*Notes: Real returns are computed with the CPI as deflator. Sudden stops for individual countries from Rothenberg and Warnock (2006). Regional and global sudden stops are computed as sum over the respective countries divided by the number of countries. No short-sales are allowed. Regional aggregates are weigted by reserve levels as of end-2005. * Results for Russia based on Jan.2004 to Dec. 2005 due to change in currency basket. ** Re-scaled to 2-asset case.*

Figure 6.7: **Optimal dollar share for the 2-asset minimum variance portfolio, end 2005, only Euro debt**

	Without transaction motive	With transaction motive		
	-	Country-specific S	Regional S	Global S
Country	**(1)**	**(2)**	**(3)**	**(4)**
Czech Republic	0.0%	0.0%	0.0%	0.3%
Hungary	20.1%	20.1%	17.2%	24.1%
Poland	42.3%	42.3%	40.4%	44.8%
Russia	62.9%	na	63.3%	63.0%
Slovakia	3.0%	0.0%	0.0%	9.3%
Emerging Europe Total	**47.3%**	**na**	**46.9%**	**48.4%**
Argentina	96.0%	84.8%	100.0%	99.9%
Brazil	71.3%	82.5%	75.2%	73.6%
Chile	89.6%	89.6%	93.7%	91.8%
Colombia	95.6%	100.0%	99.6%	97.6%
Mexico	100.0%	100.0%	100.0%	100.0%
Peru	94.8%	99.4%	97.1%	96.0%
Venezuela	100.0%	100.0%	100.0%	100.0%
Latin America Total	**91.1%**	**93.3%**	**93.2%**	**92.4%**
Jordan	94.9%	94.9%	95.5%	95.8%
Kuwait	89.4%	na	91.0%	94.4%
Saudi Arabia	99.1%	na	98.2%	98.2%
South Africa	60.6%	60.6%	59.0%	63.5%
Turkey	76.8%	71.9%	74.1%	81.0%
Middle East and Africa Total	**80.9%**		**79.3%**	**83.7%**
China	97.0%	na	98.1%	97.5%
India	97.5%	97.5%	98.3%	97.8%
Indonesia	30.8%	48.3%	43.9%	35.9%
Korea, South	80.4%	79.7%	84.0%	81.9%
Pakistan	92.4%	92.6%	93.9%	93.0%
Philippines	89.5%	100.0%	94.5%	91.5%
Thailand	58.7%	66.9%	63.2%	60.5%
Asia Total	**90.9%**	**na**	92.9%	91.7%
Asia ex China	**79.9%**	**na**	83.5%	81.3%
All Emerging Markets in sample	**83.9%**	**na**	**85.3%**	**84.9%**
All Emerging Markets ex China	**73.7%**	**na**	**75.4%**	**75.1%**

Source: Authors'calculations.

*Notes: Real returns are computed with the CPI as deflator. Sudden stops for individual countries from Rothenberg and Warnock (2006). Regional and global sudden stops are computed as sum over the respective countries divided by the number of countries. No short-sales are allowed. Regional aggregates are weigted by reserve levels as of end-2005. * Results for Russia based on Jan.2004 to Dec. 2005 due to change in currency basket. ** Re-scaled to 2-asset case.*

Figure 6.8: **Optimal dollar share for the 2-asset minimum variance portfolio, end 2005, only Dollar debt**

and South Korea) and in some cases only for shorter samples (Czech Republic, Hungary, Poland, Jordan), we can only draw limited conclusions from this exercise. In theory, we would expect import price indices to reflect the location of trading partners, the currency of invoicing, as well as other determinants of pricing decisions. Thus, countries that trade a lot with euro area economies and invoice in euros would be expected to have a lower variance of euro returns and thus a higher optimal euro share. In practice, we find that, with some exceptions, optimal portfolios have the same order of magnitude as in the case of CPI-deflated returns (6.4). In the case of Turkey, the considerably higher optimal dollar shares obtained when deflating with the import price deflator are mainly a result of a higher relative variance of real dollar returns. This can be rationalised with a high trade share with euro area economies.[16] In addition, we observe notable differences to the CPI-case for Hungary (lower optimal dollar share than in the CPI-case) and Brazil (higher optimal dollar share than in the CPI case). In the case of Hungary, this difference mostly reflects a difference in the sample period, as the import price sample starts only in May 2003. In fact, we obtain very similar optimal portfolios using the CPI for the same period.[17] In the case of Brazil, the differences appear to stem from varying behaviour of consumer and import prices during the period of hyperinflation in 1993. When we remove the hyperinflation period from the sample, we get similar optimal dollar shares for the CPI and the import price case.

The inclusion of transactions demand tends to again increase optimal dollar shares when we consider hedging against global sudden stops. Also, the regional pattern in the case of hedging against regional sudden stops is robust to this change of deflators. When considering hedging against country-specific sudden stops, we find similar results to the CPI-case as well, i.e. unchanged optimal currency shares in countries with no sudden stop during the sample period (Czech Republic, Hungary, Poland), a higher optimal dollar share in Brazil, Peru, Jordan (ignoring the short-sale constraint) and a higher optimal euro share in Mexico (ignoring the short-sale constraint), Turkey and South Korea.

[16]In June 2007, imports from euro area economies accounted for 30.0% of total Turkish imports, while imports from the US accounted for 5.0%.

[17]As stated above Hungary's reference currency basket contains only the euro since 2000. Using a sample starting in May 2003, we thus obtain much lower optimal dollar shares (which are in fact zero) both using the CPI and the IPI as the deflator on account of the low relative euro variance reflecting the changed de facto exchange rate regime.

	Without transaction motive	With transaction motive		
	-	Country-specific S	Regional S	Global S
Country	**(1)**	**(2)**	**(3)**	**(4)**
Czech Republic	0.0%	0.0%	0.0%	2.7%
Hungary	20.1%	20.1%	13.9%	26.5%
Poland	42.3%	42.3%	38.1%	46.4%
Russia*	62.9%	na	63.7%	63.0%
Slovakia	3.0%	0.0%	0.0%	13.1%
Emerging Europe	**47.3%**	**na**	46.6%	49.2%
Argentina	96.0%	72.1%	100.0%	100.0%
Brazil	71.3%	93.7%	79.0%	75.7%
Chile	89.6%	89.6%	97.8%	94.0%
Colombia	95.6%	100.0%	100.0%	99.5%
Mexico	100.0%	100.0%	100.0%	100.0%
Peru	94.8%	100.0%	99.3%	97.1%
Venezuela	100.0%	100.0%	100.0%	100.0%
Latin America	**91.1%**	94.7%	94.6%	93.3%
Jordan	94.8%	94.9%	96.0%	96.5%
Kuwait	89.4%	na	92.6%	99.2%
Saudi Arabia	99.1%	na	95.2%	95.3%
South Africa	60.6%	60.6%	57.2%	66.1%
Turkey	76.8%	66.2%	71.2%	84.8%
Middle East & Afr.	**80.9%**		77.1%	85.7%
China	97.0%	na	100.0%	98.8%
India	97.5%	97.5%	99.1%	98.2%
Indonesia	30.8%	64.4%	56.3%	40.6%
Korea, South	80.4%	78.9%	87.3%	83.2%
Pakistan	92.4%	92.9%	95.2%	93.5%
Philippines	89.5%	100.0%	99.3%	93.5%
Thailand	58.7%	75.0%	67.6%	62.3%
Asia	**90.9%**	**na**	95.3%	93.1%
Asia ex China	**79.9%**	**na**	86.9%	82.7%
All EME's	**83.9%**	**na**	87.0%	86.1%
All EME's ex China	**73.7%**	**na**	76.8%	76.3%

Source: Authors'calculations.

*Notes: Grey-shaded areas indicate cases in which the optimal dollar share is higher when transactions motives are included, disregarding short-sale constraints. Short-sale constraints are applied to the currency share numbers. Real returns are computed with the CPI as deflator. Sudden stops for individual countries from Rothenberg and Warnock (2006). Regional and global sudden stops are computed as sum over the respective countries divided by the number of countries. Regional aggregates are weigted by reserve levels as of end-2005. * Results for Russia based on Jan.2004 to Dec. 2005 due to change in currency basket.*

Figure 6.9: **Optimal dollar share for the 2-asset minimum variance portfolio with import deflators used for real returns, end 2005**

6.6 Conclusions

We derive optimal central bank portfolios in cases where the country is subject to sudden reversals in capital flows and the central bank uses its reserves to smooth decreases in absorption in the case of a reversal. We show that in our two asset minimum variance approach with transaction demand, the optimal shares can be decomposed analytically into asset demand derived from traditional portfolio objectives and hedging demands related to sudden stops. We further show that the hedging demands become less important relative to traditional portfolio objectives as debt to reserve ratios decrease. Whether the introduction of transactions demand increases or decreases the optimal dollar share depends on whether the dollar is a better hedge for sudden stops than the euro. In our empirical section, we find that dollars are a better hedge for global sudden stops, and for country specific sudden stops in many cases, but that there is a regional pattern if we consider regional sudden stops. In that case, the dollar appears to be a better hedge for sudden stops in Latin America and Asia, while the euro is a better hedge for sudden stops in Emerging Europe, which we interpret to reflect safe haven properties stemming from the geographical origin of investment flows in these regions. We also find that as reserve levels increase the global share of the dollar falls, as countries become less concerned with the risk of sudden stops, but our model predicts smaller changes in portfolios then seen in the data.

In any case, optimal dollar shares remain quite high in many cases, both with and without transactions demand, in particular in Asia, Latin America and those countries of the Middle East with a traditionally high dollar orientation. This finding to a large extent reflects current exchange rate arrangements in many emerging market economies (as well as our assumption of variance minimisation as the central bank objective). In many emerging market economies, exchange rates vis-à-vis the dollar are closely managed which implies a low volatility of dollar returns. Since the correlation of euro and dollar returns is often quite high, diversification benefits are limited and the low variance of dollar assets dominates the portfolio choice. Our framework therefore suggests that strong diversification out of dollar-denominated assets is unlikely as long as many emerging market economies continue to manage their exchange rate mainly against the dollar. However, we have recently seen some cases in which central banks have increased the weight of the euro in the exchange rate basket they are managing. In those cases which were mainly to be found among the EU Neighbouring countries, our model suggests that the share of the euro in the reserves should also rise. This is in line with the swift diversification into euro-denominated assets by the Central Bank of Russia, following the introduction of a dollar-euro currency basket.

We also find that the effect of the denomination of foreign currency debt on central bank portfolios is quite low, both because debt-to-reserve ratios are low, and because the exchange rate risk associated with sudden stops accounts for a small portion of its total risk.

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6.7 Appendix

6.7.1 Mean Variance Optimisation

Formally, the central bank solves the following problem:

$$\begin{aligned}
&\max_{\alpha} E[W] - \frac{\gamma}{2} Var[W] \\
&s.t. \\
&W = \alpha A R_{US} + (1-\alpha) A R_E - S_{US} b B - S_E (1-b) B \\
&0 \leq \alpha \leq 1.
\end{aligned} \tag{6.10}$$

where γ is a coefficient reflecting the risk aversion of the central bank. The optimal dollar share is then given by:

$$\begin{aligned}
\alpha \quad = \quad & \frac{E[R_{US}] - E[R_E]}{\gamma A (var[R_{US}] + var[R_E] - 2cov[R_{US}, R_E])} \\
& + \frac{A (var[R_E] - cov[R_{US}, R_E])}{A (var[R_{US}] + var[R_E] - 2cov[R_{US}, R_E])} \\
& + \frac{bB (cov[R_{US}, S_{US}] - cov[R_E, S_{US}])}{A (var[R_{US}] + var[R_E] - 2cov[R_{US}, R_E])} \\
& + \frac{(1-b) B (cov[R_{US}, S_E] - cov[R_E, S_E])}{A (var[R_{US}] + var[R_E] - 2cov[R_{US}, R_E])}
\end{aligned} \tag{6.11}$$

Compared to the version presented in the main part of the paper, there are two differences. Firstly, there exists now an additional term that reflects differences in expected returns between the two assets. The higher the expected return of dollar assets relative to euro assets, the higher is the optimal dollar share. The second difference is the presence of the risk aversion parameter, γ. The higher the value of γ, the smaller is the difference between the minimum variance and the mean variance optimization results, ceteris paribus. Also note that, for the case of no difference in expected returns, the first term drops out and we are back to the minimum variance case. In general, expected return differentials are small and do not materially affect our results, even for low values of the risk aversion parameter.

6.7.2 Dollar and Euro Returns

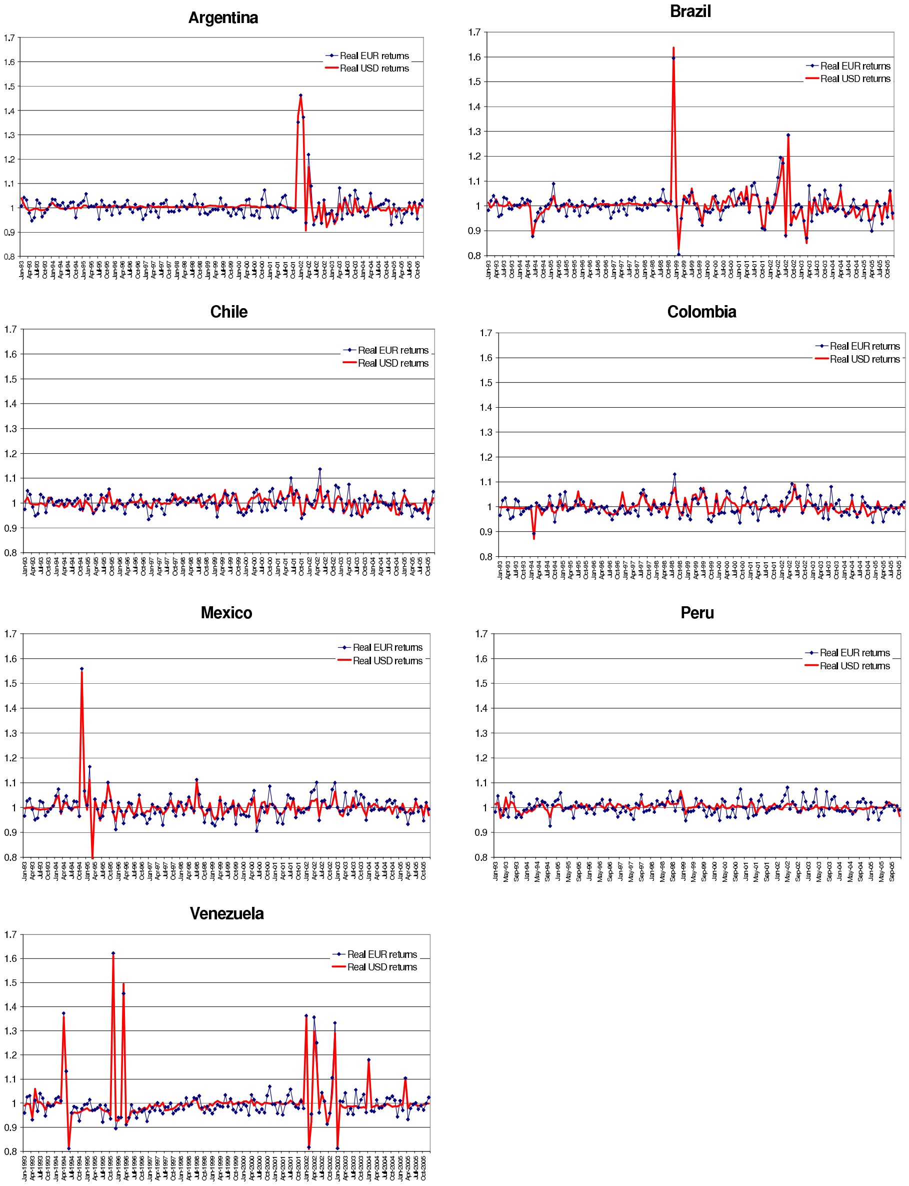

Figure 6.10: **Dollar and Euro Returns by Region: South America**
Monthly returns in real CPI deflated local currency units

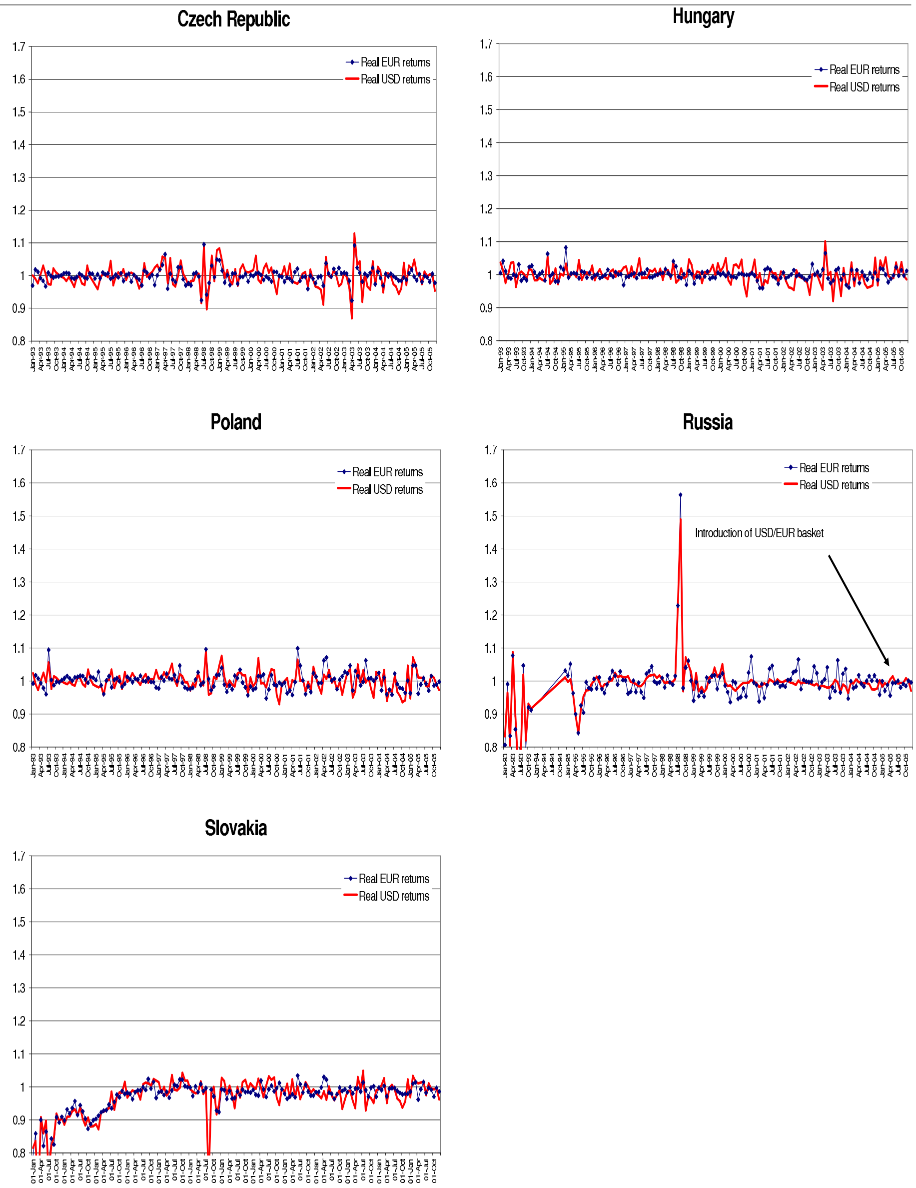

Figure 6.11: **Dollar and Euro Returns by Region: Emerging Europe**
Monthly returns in real CPI deflated local currency units

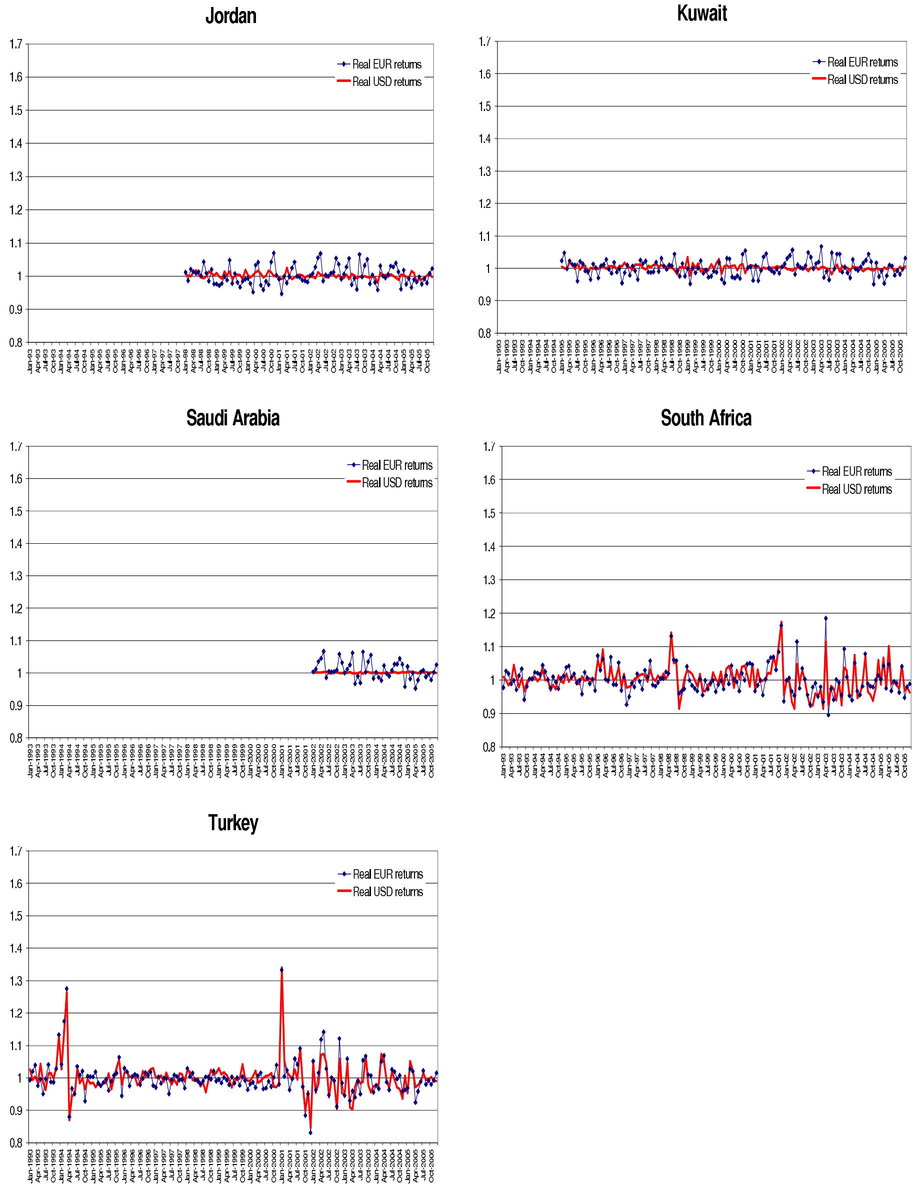

Figure 6.12: **Dollar and Euro Returns by Region: Middle East and Africa**
Monthly returns in real CPI deflated local currency units

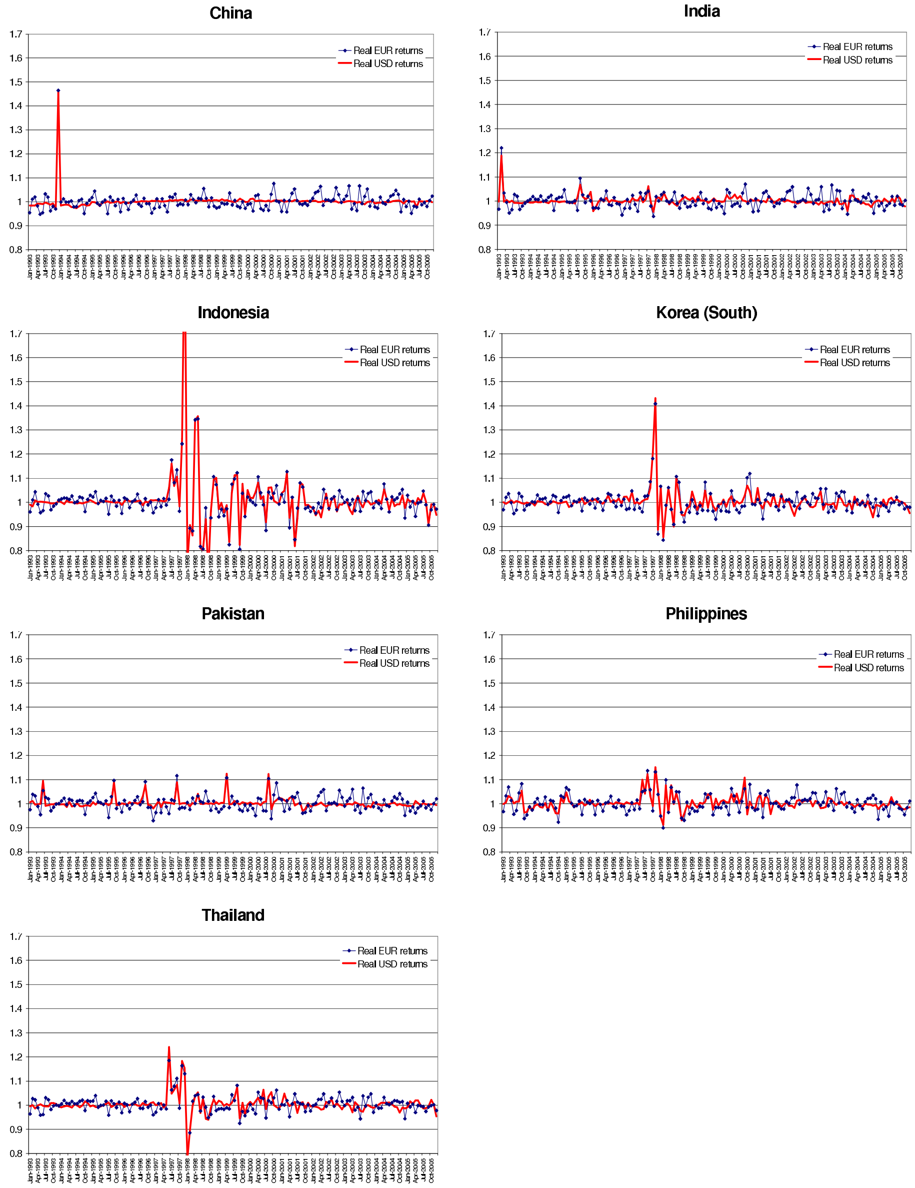

Figure 6.13: **Dollar and Euro Returns by Region: Asia**
Monthly returns in real CPI deflated local currency units