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Dynamic Trade Finance in the Presence of Information Frictions and FinTech

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Problem Definition: The paper focuses on an innovative bank-intermediated trade finance contract, which we call *dynamic trade finance* (DTF, under which banks dynamically adjust loan interest rates as an order passes through different steps in the trade process). We examine the value of DTF, the impact of process uncertainties and the associated information frictions on this value, and the strategic interaction between DTF and FinTech. **Academic/Practical Relevance**: As more than 30% of global trade involves bank-intermediated trade finance (Bank for International Settlements 2014), examining contract innovation in trade finance (DTF) and its strategic interaction with FinTech is of practical importance. Also, analyzing trade finance in the presence of process dynamics and information frictions complements the existing academic literature. **Methodology**: We construct a parsimonious model of a supply chain process consisting of two steps. The duration of each step is uncertain, and the process may fail at either step. *Information delay* may also occur when verifying the process passing a step. The seller borrows from a bank to finance this 2-step process either through *uniform financing* (the interest rate remains constant throughout the process) or *DTF* (the interest rates are adjusted according to a pre-committed schedule as the process passes each step). While lending, the bank faces regulatory capital requirement (the bank is required to hold capital reserve when issuing risky loans) or information asymmetry (the seller/borrower possesses more accurate information about the trade process than the bank). **Results**: The value of DTF lies in its ability of reducing transactional deadweight loss (under regulatory capital requirement) and screening (separate high-quality borrowers from the low-quality ones under information asymmetry). This value is greater for more reliable or lengthier trade processes, yet DTF's ability of screening is stronger when the process is less reliable. The severity of information delay hurts the value of DTF convexly. FinTech that expedites information transmission and verification and enables automatic execution *complements* DTF, and those that segment customers more efficiently could *substitute* DTF. **Managerial Implications**: Our results shed light on how the underlying trade process dynamics and the type of information frictions involved affect the optimal deployment of contract innovations (DTF) and FinTech in trade finance.

Key words: Trade finance, supply chain finance, structured trade finance, information friction, information asymmetry, FinTech, blockchain, smart contracts

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1. Introduction

Efficient global supply chain operations require the facilitation by trade finance, which funds the entire trade cycle from order issuance until sales proceedings received. Trade finance can be broadly divided into three categories: (a) seller finance (trade credit), (b) buyer finance (cash in advance), and (c) bank-intermediated trade finance, e.g., letter of credit (Schmidt-Eisenlohr 2013). Unlike trade credit or cash in advance, which is offered by a supply chain partner, bank-intermediated trade finance is provided by a bank (or other types of financial intermediaries). This form of financing is indispensable to global trade when sellers and buyers do not have sufficient internal resources to fund the entire trade cycle, especially when the process is lengthy. According to the Bank for International Settlements (2014), trade finance in cross-border trade amounted to more than $6.5 trillion, approximately one third of the total global trade volume.

When providing trade finance, the bank, as an "outsider" of the supply chain, requires a certain level of knowledge of the trade process. The journey for an order to move from a seller to a buyer consists of a series of steps, such as export/import clearance, transportation, and product inspection. As the order passes through a step, the associated risks (e.g., product quality, passing custom control, and buyer payment) are realized. If the order fails to pass a step, the products from the failed order are liquidated and the bank collects the liquidation value; otherwise, the process moves to the next step and the bank's risk exposure is updated. However, traditional trade finance products, which apply a constant interest rate over the entire lending period, fail to take such process dynamics into consideration. Recognizing this opportunity, banks and other trade finance providers have developed more sophisticated contracts. For example, the international bank Wells Fargo has developed a set of "variable trade financing" products, providing either "levels of incremental funding" or "an interest rate that is reduced as the trade transaction progresses" (Dowling et al. 2018). Trade finance products of similar nature have also been referred to as "structured trade finance" (Jones 2018). To capture the fact that lending terms in these trade finance contracts are in general adjusted dynamically according to the trade process, we refer to such contracts as *dynamic trade finance* ("DTF"). We shall focus on contracts where the interest rates are dynamically adjusted as the order passes a step. Intuitively, as the collateral value of the order normally increases as the trade process passes a step successfully,[1] DTF adjusts the terms of the loan dynamically as the order passes a step, and hence lowering the overall financing cost borne by the borrower during the trade cycle.

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[1] For example, when the products are designed under the specification of the importing country, the liquidation value of the order often increases significantly after it clears the import custom (Lee et al. 2008).

Despite its potential, DTF could be arduous because it requires accurate and prompt information flow over the trade cycle, which is complicated by the uncertainties embedded in the process and the associated information frictions. Specifically, each step in the trade process involves two types of *process uncertainties*: passing uncertainty and duration uncertainty. For the former, the process may fail to pass a certain step for various reasons, ranging from product quality issues to geopolitical tensions.[2] For the latter, the time for the process to pass through each step is also uncertain.[3] Moreover, both forms of process uncertainties are amplified by large-scale disruptions. Indeed, during the COVID-19 pandemic, government-imposed health and safety measures caused extra delays in customs clearance, and some ports even refused certain ships to dock (International Chamber of Shipping 2020).

These process uncertainties also give rise to two forms of *information frictions* that the bank may experience. First, it is common that the bank has visibility into the material flow during the process only after some time delay. Such *information delay* could be due to the operational inefficiencies or regulatory requirements. For example, according to Cognizant (2017), in 80% of the related transactions, sellers presented to banks with incomplete documents, causing delay for banks to verify the authentication of the information. Even when the documentation is complete, it usually takes 7-10 days to authenticate the information to meet compliance requirement. In addition, some countries require detailed item-level documentations at customs (Lee and Silverman 2008), further increasing the verification burden carried by the bank and causing additional information delay. Second, it is also prevalent that *information asymmetry* exists between the lender and the borrower before the two parties enter into the lending contract: the seller is likely to possess more accurate information than the lender regarding the expected duration of each step and the probability of passing each step. The impact of these frictions are further compounded by other transactional frictions banks face in practice. Indeed, the Basel regulation requires banks to hold a certain amount of core capital (Basel Committee 2006). Such capital requirement is considered as a major factor that influences the price of trade finance. For example, a 2009 IMF survey reports that 58% respondents cite increased capital requirements as the reasons for recent increases in pricing during the Global Financial Crisis in 2008 (IMF 2009). These observations have motivated our first research question: *what is the value of DTF, and how would this value be affected by the information frictions associated with process uncertainties?*

[2] For example, since the September 11 attacks, international shipment has been under heightened scrutiny at ports. The US-China trade war that started in 2018 has also made customs clearance increasingly uncertain (Leonard 2018).

[3] According to Arvis et al. (2018), the average import lead time for the orders in the bottom quintile is approximately three times as those in the top quintile (Figure 2.4). Moreover, the time for an ocean freight shipment to clear U.S. customs could range from 3-5 days under normal circumstances to up to 15 days under “intensive examination” with a probability of 1–2% (Rogers Worldwide 2020).

Beyond contract innovations such as DTF, practitioners view Financial Technology (FinTech) as a disruptive force in trade finance, which traditionally has been plagued with labor and paper-intensive processes. One such technology is blockchain, or more generally, distributed ledger technology (DLT). By enhancing transparency through track and trace and digitalization of trade, blockchain facilitates sharing credible and timely information among various parties who are involved in the trade process. This feature has been exploited not only by traditional banks such as Barclays, Citibanks and HSBC, but also new business ventures and government programs, including eTradeConnect, Komgo, skuchain, and the People's Bank of China Blockchain Trade Finance Platform (CitiBank 2018, Kelly 2016, HSBC 2018, Patel and Ganna 2020). Chod et al. (2020) provide the details of the blockchain protocol b_verify, and how it, combined with Internet of Things (IoT), is used to improve supply chain finance through enhanced transparency. Another attractive function associated with blockchain is "smart contracts", which are computer programs stored on blockchain that are executed automatically when certain conditions are met. For instance, Jensen et al. (2019) document that on TradeLens, a blockchain-based platform jointly founded by Maersk and IBM, "import clearance could be pre-programmed into a smart contract distributed to the blockchain network, thus preventing other global supply chain participants from changing the business logic and making automatic self-execution possible based on events." In addition to blockchain, big data analytics have also become a valuable tool for banks to better understand operational characteristics in the trade process as well as the risk profiles of the borrowers, enabling customer segmentation at a more granular level. For example, MyBank, a Chinese FinTech provider, leverages on alternative data and proprietary algorithms to make credit granting decisions on small businesses (Chen et al. 2021). Seeing such potential, banks, technology and logistics providers are investing heavily in FinTech with the aim to improve trade finance (Gronholt-Pedersen 2018). These observations motivate our second research question: *what is the strategic interaction between FinTech (as technology innovation) and DTF (as contract innovation)?*

To answer these two aforementioned research questions, we develop a parsimonious model of a two-step supply chain process. Each step in the process faces two sources of uncertainties: the time to complete the step, and the probability of passing the step.[4] The seller finances this process through a bank, who faces an *information delay* when verifying that the process has successfully passed the intermediate step.[5] We focus on two forms of trade finance contracts: (a) *uniform*

[4] The passing uncertainty captures that the process may fail to pass to the next step for a variety of reasons including product quality, failing to meet regulatory requirement, or buyer default. Further, we assume such risks are exogenous and not influenced by the trade finance contract forms.

[5] In the paper, we treat the supply chain seller as the borrower, which fits the scenario that when the buyer only pays after the order is successfully delivered, the seller needs to seek for bank-intermediated trade finance to fund the process. That said, our results readily apply to the scenario where the buyer pays the seller in advance and borrows from a bank to finance the process.

financing (UF, the bank offers a uniform interest rate until the order either fails at any step or successfully completed both steps); and (b) *dynamic trade finance* (DTF, the bank adjusts the interest rate once receiving the verified information that the order has passed the first step). Based on this setting, we consider two separate models, each with a specific form of financial market imperfection: (a) regulatory capital requirement (banks are required to hold capital reserve at a high cost of capital based on the riskiness of the loan); (b) information asymmetry (the borrower possesses private information about the passing probability and duration of each step).

Analyzing these two models reveals that the value of DTF hinges upon the specific forms of financial market imperfection and the operational features of the trade processes. First, when the focal financial friction is caused by regulatory capital requirement, the value of DTF lies upon its capability of reducing deadweight loss, and it is more pronounced when (1) the trade process is more reliable, (2) the trade process is lengthier, (3) the liquidation value of the order increases significantly once passing a step, and (4) the lender's cost of (equity) capital is higher. Further, information delay lowers the value of DTF convexly. Numerical results based on calibrated parameters allude that DTF could substantially lower the seller's financing cost relative to UF.

On the other hand, when information asymmetry is the focal source of financial market imperfection, DTF creates value by separating the high-quality borrowers (with shorter duration and higher passing probability) from the low-quality ones more efficiently, thus lowering the financing cost high-quality firms borne under UF. The impact of operational characteristics of the trade process on the efficacy of DTF is more complex. On the one hand, when separation is feasible, the financing cost saved by DTF is higher for more reliable trade processes (higher passing probability and liquidation value). On the other hand, the DTF's capability of separation is greater when the process is less reliable or the profit margin of the product is smaller.

Based on the above results, we further investigate the interaction between DTF (as a form of contract innovation) and FinTech (as a form of technology innovation). We find they could be complements or substitutes through three channels. First, through technologies such as smart contracts, FinTech allows the bank to commit to more sophisticated loan terms structure, thus complements DTF. Second, as a technology that expedites information transmission and verification (e.g,. blockchain or other forms of digitalization), FinTech enhances the value of DTF. Finally, when allowing the bank to better profile borrowers (e.g., through big data analytics), FinTech offers an alternative mechanism that separate different types of customers, thus partly substituting the screening role of DTF.

As an initial attempt to examine DTF and its interaction with emerging FinTech, the goal of this paper is threefold. First, we develop an analytical framework that incorporates process uncertainties and the related financial and informational frictions in trade finance. Doing so allows us to identify two channels through which DTF creates value. Second, we identify how the value of DTF is related to the operational features in the trade process. Finally, the paper finds that FinTech could complement or substitute DTF, offering insights on how businesses should coordinate investments in contract and technology innovations.

2. Literature

This paper is closely related to three streams of literature: trade processes, trade and supply chain finance, and the interaction of FinTech and business operations. International trade is a complex process involving various parties. We refer the readers to Hausman et al. (2010) for a comprehensive description of the complete cross-border trade process. Various studies focus on certain steps during the entire trade process, such as international logistics (Limão and Venables 2001, Hausman et al. 2013, Hummels and Schaur 2013), inland transportation (Djankov et al. 2010), and export/import customs clearance (Fernandes et al. 2015). Based on the above literature, we have summarized two types of operational uncertainties during the international trade process (passing uncertainty and duration uncertainty) and incorporated them in our model.

Given its vast importance in the global economy, trade and supply chain finance has received attentions from the international trade, finance, and OM communities. Trade scholars have examined how firms' financing choices are affected by factors such as financial market characteristics and contracting environment (Schmidt-Eisenlohr 2013), repeated interaction (Olsen 2016), and contractual enforcement (Antras and Foley 2015). Our study complements the above studies by focusing on operational risks in the trade process and how they affect contract terms and FinTech adoption. In the finance literature, the focus has been mainly on trade credit, including both theoretical and empirical investigation. See Chod et al. (2019a) for a review of related works. Differently, we focus on bank-intermediated trade finance, which is particularly relevant when the seller (exporter) is financially constrained. In the area of OM, our work is most related to the literature on OM-Finance interface (Babich and Sobel 2004, Boyabatlı and Toktay 2011, Yang et al. 2015, Iancu et al. 2017, Lai and Xiao 2018, Ning and Babich 2018, Luo and Shang 2019, Tanrisever et al. 2021, Zhang et al. 2022). This stream of literature has examined various forms of trade and supply chain finance, including trade credit (Babich and Tang 2012, Kouvelis and Zhao 2012, Peura et al. 2017, Yang and Birge 2018, Chod et al. 2019b, Chen et al. 2020), receivables financing (Tunca and Zhu 2018, Hu et al. 2018, Kouvelis and Xu 2021), purchase order financing (Tang et al. 2018, Reindorp et al. 2018), and logistics financing (Chen et al. 2018). Our work extends the above literature in two dimensions. First, we explicitly model a *dynamic* trade process consisting of multiple steps, the associated process uncertainties and information frictions. Second, we focus on the value of DTF as a contract innovation, and its strategic interaction with FinTech in the presence of the aforementioned dynamic process. In addition, the DTF contract we consider shares some similarities with stage financing, which is observed in venture capital financing (Cornelli and Yosha 2003, Tian 2011). The central theme of this strand of research is on how the lender could use flexible contracts to mitigate the borrower's moral hazard. In contrast, the DTF contracts in our setting focus on committed contractual terms and different financial frictions, that is, bank capital regulation and information asymmetry.

Finally, the recent emergence of FinTech has motivated a growing literature on the interaction of FinTech and business operations. Fuster et al. (2019) empirically document that digitalization significantly reduces process time and switching cost in mortgage lending. Babich et al. (2020) and Chakraborty and Swinney (2020) examine the operational implications of crowd-funding as enabled by digital technologies. Babich and Hilary (2020) provide a comprehensive overview and outlook on blockchain in OM. More recent studies focus on the application of blockchain in specific OM settings such as inventory management (Gan et al. 2021), supply chain traceability (Dong et al. 2019, Cui et al. 2020), and platform operations (Chod et al. 2021). Most relevant to ours is Chod et al. (2020), who also consider the use of blockchain technology in signaling firms' market potential and thus facilitating finance. Our work complements the above literature by focusing on process dynamic and showing that transparency into the firm's operations through FinTech could either complement or substitute more sophisticated trade finance contracts in the presence of different forms of information frictions.

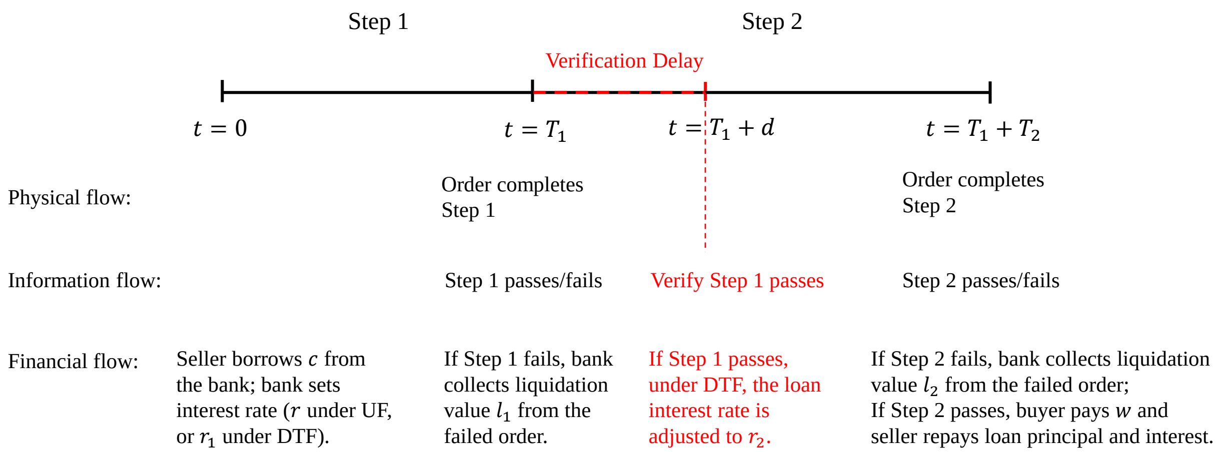
3. Model

By considering a typical process of trade finance as described in the literature (Willsher 1995, Hausman et al. 2010, Jones 2018), we present a parsimonious model of a two-level global supply chain in which a buyer (importer) orders a fixed amount of products (normalized to one) from a seller (exporter). The seller has no initial wealth and needs to borrow c from the bank through a trade finance arrangement. The buyer pays the wholesale price w to the seller when the order completes the process successfully.[6] All parties are risk-neutral, and the risk-free interest rate is normalized to zero. Figure 1 illustrates the sequence of events with separate physical, information, and financial flows.

At $t = 0$, the seller borrows c from the bank to initiate the transaction (trade process). For parsimony, we assume the trade process involves two serial steps: Steps 1 and 2 (e.g., export clearance and import clearance, respectively). Each step entails two types of uncertainties.

[6] Focusing on how incorporating process dynamics could improve bank-intermediated trade finance, we do not consider the possibility that the buyer finances the transaction (e.g., cash in advance).

Figure 1 **Sequence of Events**



1. *Passing uncertainty.* Due to various risks in the trade process (product defect, customs clearance, etc.), the process will proceed successfully to the next stage with a certain probability, which we assume to be exogenous to the trade finance contract. We use p_1 to denote the probability that the order will pass Step 1, and use p_2 to represent the probability that the order will pass Step 2 conditional on passing Step 1. Should the order fails to pass Step j, the order is liquidated, and the bank receives the liquidation value l_j. We assume that $l_1 < l_2 < c < w$ to capture the reality that financing this order is risky before it is received by the buyer, yet the product can be salvaged at a higher value as it getting closer to successful completion of the 2-step process.[7]
2. *Duration uncertainty.* Due to the inherent nature of the trade process, the completion time of each step is uncertain. Let $\tilde{T}_1$ and $\tilde{T}_2$ be the completion time of Steps 1 and 2 respectively, where $\tilde{T}_j \sim Exp(\mu_j)$ for $j = 1, 2$. The exponential duration assumption captures the empirical evidence that the completion time of each step in a trade process can be highly variable.[8] For ease of exposition, we normalize μ_1 to 1, and denote μ_2 as μ. Finally, duration uncertainty and passing uncertainty are assumed to be independent for tractability.

[7] For example, according to Lee et al. (2008), when the Chinese government took away the export license of Lee Der, a contract manufacturer for Mattel, due to the product adulteration scandal, Lee Der could not export the product out of China. When that happened, the lead-tainted toys already produced were worth much less, especially considering that Lee Der faced significant risk that the products were not allowed to be sold in China due to Mattel's contract clause.

[8] The exponential processing time assumption is also commonly made in the project management literature for tractability (Gaukler et al. 2008, Kwon et al. 2010). For robustness, in Appendix C, we relax the exponential assumption and show that the qualitative results of the paper remain unchanged when we allow the duration of Step 1 $\tilde{T}_1$ consists of a fixed component K (e.g., seller's production time for the order) and an exponentially distributed random component $\tilde{X}_1$ (e.g., variable customs clearance time).

By accounting for different values of the order at different stages (l_1, l_2, w), the passing probabilities at different steps (p_1, p_2), and the production cost c, the expected net value of the order, which we define as the seller's payoff without any financial friction (Π_0), is:

$$\Pi_0 = p_1 p_2 w + (1 - p_1) l_1 + p_1 (1 - p_2) l_2 - c, \tag{1}$$

where the first term, $p_1 p_2 w$, is the seller's expected revenue, and the second and third terms are the respective liquidation value when the process fails at Step 1 and Step 2. The wholesale price w is assumed to be sufficiently high such that the seller can repay the principal and interest rate of the loan once the order is received by the buyer. Note that as the risk-free rate is normalized to 0, the items in Eq. (1) do not need to be discounted. Finally, we assume that the seller's outside option is π_0. To avoid trivial cases, we assume $\pi_0 < \Pi_0$ to ensure that, without any financial friction, it is economically efficient for the seller to accept the order and trade.[9]

Regarding the information available to the bank, upon failing Step 1 or Step 2, the information is reported immediately to the bank for liquidation. Similarly, the information that the order has passed Step 2 and arrived at the buyer's also becomes immediately available to the bank, who then receives the payment w from the buyer, deducts the principal and interest of the loan, and passes the remaining amount to the seller. Further, we assume that the reported information about the passing of Step 1 successfully can only be verified by the bank after a random delay $\tilde{d}$ (even though the order has already proceeded to Step 2 physically).[10] For tractability, we assume that $\tilde{d} \sim Exp(\lambda)$, where $\lambda > \mu$. That is, the expected information delay is shorter than the average processing time of Step 2.[11] Further, information delay is assumed to be independent of processing time. We refer to this friction as *information delay.*

Facing the above trade process, the bank sets the interest rates. The total interest is thus continuously compounded at the corresponding interest rates, and repaid (together with the principal) as the trade process completes. We consider two types of trade finance contracts:

1. *Uniform financing ("UF").* The bank offers a single interest rate r over the entire process. The interest is continuously compounded at rate r. For example, suppose the seller borrows an amount of c at time 0 and pays back the loan at $T_1 + T_2$, where T_j is the realized passing time for Step $j = 1, 2$. the total repayment (principal plus interest) is $ce^{r(T_1+T_2)}$, where c is the principal and $c[e^{r(T_1+T_2)} - 1]$ is the cumulated interest.

[9] In Section 5, we extend the model to include two types of sellers with potentially different outside options.

[10] Even if the bank receives reports about the passing of Step 1 immediately at T_1, it needs to verify the authenticity of this information for compliance reasons, thus incurring a time delay. For the failure of passing Step 1, as the seller has no incentive to lie, the bank can trust the information and collect the liquidation value from the failed order immediately.

[11] This assumption is to ensure that refinancing under DTF is likely to happen.

2. *Dynamic trade finance ("DTF").* At the time when the two parties enter the lending contract, the bank offers an interest rate schedule (r_1, r_2) with $r_1 > r_2$. The initial interest rate is r_1 from $t = 0$ at the outset, and if the order passes Step 1 successfully and the information is verified by the bank at time $T_1 + d$, where d is the realized information delay, then the interest rate is changed to r_2, until the loan is repaid.[12]

Under the assumption that the bank operates in a competitive market, the interest rates are set such that the expected repayment from the loan equals to the expected cost of offering the loan.

Finally, to assess the value of DTF, we examine two scenarios, each capturing one form of financial frictions.[13] The first form of financial friction is a transactional one. In particular, we model that the bank is required to hold regulatory capital reserve based on the risk of the loan as specified in BASEL II (Basel Committee 2006). We focus on this scenario in Section 4. The second form of financial friction we focus on is information asymmetry. In this case, the expected processing time and the passing probability of each step are private information that is known to the seller but not to the bank. We discuss the details in Section 5.

4. Trade Finance in the Presence of Regulatory Capital Requirement

In this section, we focus on bank regulatory capital requirement as the source of financial friction. We first describe how the bank prices the trade loan in the presence of bank capital regulation, then we analyze uniform finance (UF) in §4.1, followed by the analysis of dynamic trade finance (DTF) in §4.2. We conclude this section with a set of numerical results (§4.3).

To incorporate bank capital regulation in trade loan pricing, we follow the internal rating based method specified in BASEL II, and assume that, when issuing a risky loan, the bank is required to set aside an amount of regulatory capital according to the Value at Risk (VaR) of the loan at a specific confidence level (Zhang et al. 2022). The rationale behind this requirement is to limit the extent to which loan defaults at one bank will create a negative spillover effect to the rest of the financial system and harm depositors. Such capital reserve is held at the cost of the bank's core equity capital. A bank's cost of equity capital is significantly higher than the cost of deposit, which in turn is close to risk-free rate (normalized to zero in the paper).[14] In the model, let the bank's

[12] In the main body of the paper, we focus on the case that the bank could pre-commit an interest schedule (r_1, r_2). As shown later (see Section 6.1 for details), this is to ensure the value of DTF can be fully unleashed.

[13] As shown in Section 4, without any form of financial frictions, the value of DTF is zero despite of the process uncertainties. This result is also consistent with the seminal Modigliani-Miller Theorem that financial market imperfections are necessary for operational features to have an impact on financing terms (Modigliani and Miller 1958).

[14] According to the data collected by Aswath Damodaran (`http://people.stern.nyu.edu/adamodar/`), the average annualized cost of equity capital among U.S. banks is approximately 7% between 1998 and 2018. The difference between this cost of equity and the cost of deposit reflects the fact that the bank's equity investors demand a premium for the risk the bank takes.

instantaneous cost of core capital be $\delta > 0$. To focus on the more interesting and realistic cases, we further assume $\delta \le \min(\mu, 1)$.[15] In this case, a loan is priced according to the following equation:

$$\text{Expected Repayment} = \text{Loan Principal} + \text{Net Cost of Holding Regulatory Capital Reserve.} \quad (2)$$

For example, if the process requires the bank to hold 1 dollar of regulatory capital reserve at cost δ from time 0 to T, the net cost of holding this regulatory capital reserve is $(e^{\delta T} - 1)$. Intuitively, the riskier the loan, the greater the amount of regulatory capital the bank needs to reserve. Thus, this friction is equivalent to the bank charging an additional premium on risky loans.

4.1. Uniform Financing (UF)

Under UF, the bank charges a single interest rate r over the entire process. This benchmark captures the case where the bank has limited capability in offering more sophisticated contracts or verifying the reported information about the passing of a step. To evaluate the bank's loan pricing decision and the seller's payoff under UF, we consider the following three cases depending on whether the order passes each step successfully.

(i) **With probability $(1 - p_1)$, the product fails to pass Step 1**. In this case, upon notification that the order failed to pass Step 1, the bank collects the liquidation value l_1 from the failed order. As the banks' confidence levels for calculating VaR are usually high,[16] we assume that p_1 and p_2 are lower than the required confidence level so that the bank needs to hold $(c - l_1)$ amount of capital to meet the regulatory capital requirement, where c is the loan principal. Under continuous compounding and the assumption of zero cost of deposit, the net cost of holding regulatory capital $(c - l_1)$ from 0 to T_1 is $(c - l_1)(e^{\delta T_1} - 1)$. On the other hand, as the loan defaults, the seller's payoff is 0.

(ii) **With probability $p_1(1 - p_2)$, the product passes Step 1 but fails to pass Step 2 .** The bank collects liquidation value l_2 from the failed order at $t = T_1 + T_2$. However, because the verified information regarding the passing of Step 1 is not available, the bank needs to hold $(c - l_1)$ so that its net cost of holding $(c - l_1)$ from 0 to $T_1 + T_2$ is $(c - l_1)\left[e^{\delta(T_1 + T_2)} - 1\right]$. The seller's payoff remains at 0.

(iii) **With probability $p_1 p_2$, the product successfully passes Step 2.** In this case, the seller receives payment w from the buyer at $T_1 + T_2$ and repays the loan's principal and interest. Thus, its payoff is $\left(w - ce^{r(T_1 + T_2)}\right)$. The bank's net cost of capital is the same as case (ii).

[15] This assumption ensures that the total expected financing cost is finite under uncertain completion time. Please refer to Proposition 1 and in particular, Eq. (6) for the technical reasons behind. As shown in numerical example (Section 4.3), this assumption is satisfied under the calibrated parameters. Further, in the case where different banks have different costs of core capital, δ represents the cost of equity from the second-lowest cost provider.

[16] Basel Committee (2006) requires the confidence level to be no less than 99.9%.

Combining all three cases, the seller's payoff under interest rate r and continuous compounding is:

$$\Pi_U = p_1 p_2 \left(w - \mathbb{E}\left[c e^{r(\tilde{T}_1 + \tilde{T}_2)} \right] \right). \tag{3}$$

On the bank side, the expected repayment of the loan (EP) and the associated net cost of capital (NCC) are:

$$\mathrm{EP} = (1-p_1)l_1 + p_1(1-p_2)l_2 + p_1 p_2 \mathbb{E}[\, c e^{r(\tilde{T}_1+\tilde{T}_2)}\,], \tag{4}$$

$$\mathrm{NCC} = (c-l_1)\left[(1-p_1)(\mathbb{E}[e^{\delta \tilde{T}_1}]-1) + p_1(\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}]-1) \right]. \tag{5}$$

As the banking market is assumed to be competitive, the bank can determine the equilibrium interest rate r^* according to (2) so that $EP = c + NCC$, where c is the loan principal. Substituting this equilibrium r^* into (3), we get the seller's payoff as follows.

Proposition 1. *Under Uniform Financing, the equilibrium interest rate is:*

$$r^* = \frac{1}{2}\left(1 + \mu - \sqrt{1 + \mu^2 - \mu(2 - \frac{4cp_1p_2}{c - (1-p_1)l_1 - p_1(1-p_2)l_2 + \frac{(\mu+p_1-\delta)(c-l_1)\delta}{(1-\delta)(\mu-\delta)}})} \right). \tag{6}$$

The seller's corresponding payoff is:

$$\Pi_U = \Pi_0 - (c-l_1)\left(\frac{\delta}{1-\delta}\right)\left(1 + \frac{p_1}{\mu-\delta}\right), \tag{7}$$

where Π_0 *is given in Eq.* (1)*.*

Proposition 1 reveals that under competitive loan pricing, the uniform interest rate r^* decreases when the trade process is more reliable (higher passing probability p_1 or p_2), the order's liquidation value is higher (greater l_1 or l_2), or the bank's cost of equity (δ) decreases. Facing this interest rate, the seller's payoff (Π_U) is strictly lower than the benchmark without any financial friction Π_0 given in (1), where the difference $\Pi_0 - \Pi_U$ quantifies the inefficiency caused by the financial friction (i.e., the bank needs to hold a certain amount of regulatory capital). This inefficiency decreases in the liquidation value l_1, as the bank is required to hold less capital over the entire process, and increases in the bank's cost of capital δ. Further, this inefficiency is more severe as it takes longer for the order to complete Step 2 (smaller μ) as it increases bank's cost of holding regulatory capital. For the same reason, this gap also increases in the order's probability of passing Step 1 (p_1). Finally, we note that this inefficiency is independent of the liquidation value of the order after it passes Step 1 (l_2), because UF fails to incorporate this information into the bank's regulatory capital holding decision.

4.2. Value of DTF

Recognizing the inefficiency under UF, we now examine the value of DTF, under which the bank offers r_1 at the outset at time 0, and adjust the interest rate to r_2 after the bank received the verified information that the order has successfully passed Step 1 at time (T_1+d). We now determine the bank's interest rates (r_1, r_2) and the seller's payoff under DTF by considering the following three scenarios in the same manner as presented in §4.1.

(i) **With probability $(1-p_1)$, the product fails to pass Step 1.** This case is identical to case (i) presented in §4.1. Hence, the bank's net cost of holding capital $(c-l_1)$ over $[0, T_1]$ is $(c-l_1)[e^{\delta T_1}-1]$, and the seller's payoff is 0.

(ii) **With probability $p_1(1-p_2)$, the product passes Step 1 but fails Step 2.** Upon failing Step 2, the seller's payoff is 0 and the bank receives liquidation value l_2 at $t = T_1+T_2$. To determine the bank's net capital regulation cost, consider scenarios: (a) when $d \geq T_2$, and (b) when $d < T_2$.

   (a) If the delay $d \geq T_2$, then the bank holds $(c-l_1)$ amount of capital from time 0 to T_1+T_2 so that the bank's net cost of capital is: $(c-l_1)[e^{\delta(T_1+T_2)}-1]$.

   (b) If the delay $d < T_2$, then the bank holds $(c-l_1)$ amount of capital from time 0 to T_1+d and then $(c-l_2)$ amount of capital from time T_1+d to T_1+T_2 so that the bank's net cost of capital can be simplified as: $(c-l_2)[e^{\delta(T_1+T_2)}-1]+(l_2-l_1)[e^{\delta(T_1+d)}-1]$.

(iii) **With probability p_1p_2, the product passes Step 2.** The seller is able to repay the loan's principal and interest at time T_1+T_2. Similar to case (ii), we consider two scenarios:

   (a) If $d \geq T_2$, there is no refinancing, and the interest rate is r_1 throughout the entire process so that the seller repays $ce^{r_1(T_1+T_2)}$.

   (b) If $d < T_2$, the interest rate changes from r_1 to the refinanced interest rate r_2 at time T_1+d, that is, when the verification of the passing of Step 1 is complete. In this case, the seller repays $ce^{r_1(T_1+d)+r_2(T_2-d)}$.

   The bank's cost of capital is the same as case (ii) as stated above.

Combining the above three scenarios along with the payment from the buyer w, the seller's expected profit Π can be expressed as follows:

$$\Pi = p_1p_2\left\{w - \Pr(\tilde{d}<\tilde{T}_2)\mathbb{E}[c\cdot e^{r_1(\tilde{T}_1+\tilde{d})+r_2(\tilde{T}_2-\tilde{d})}|\tilde{d}<\tilde{T}_2] - \Pr(\tilde{d}\geq\tilde{T}_2)\mathbb{E}[c\cdot e^{r_1(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}\geq\tilde{T}_2]\right\}. \quad (8)$$

On the bank side, by considering the repayment, the loan's principal c and the bank's net cost of capital across these three cases in the same manner as presented in §4.1, we can apply (2) to determine the constraint that the bank's competitive interest rates (r_1, r_2) schedule has to satisfy.[17] Substituting the resulting interest rate schedule (r_1, r_2) into (8), we can determine the corresponding seller's payoff Π_D in equilibrium.

PROPOSITION 2. *Under the DTF contract, there exists a threshold $\bar{r}_2^*$ such that the bank could offer any interest rate schedule (r_1^*, r_2^*) where $r_2^* \leq \bar{r}_2^*$ and (r_1^*, r_2^*) satisfy the bank's competitive loan pricing equation* (2). *The seller's payoff under such an interest rate schedule is:*

$$\Pi_D = \Pi_0 - \frac{\delta}{1-\delta}\left[(c-l_1)\left(1+\frac{p_1}{\lambda+\mu-\delta}\right)+\frac{(c-l_2)\lambda p_1}{(\lambda+\mu-\delta)(\mu-\delta)}\right]. \tag{9}$$

We note that the optimal DTF interest rate schedules are not unique. In fact, as shown in the proof of the proposition, when (r_1, r_2) satisfy Eq. (2), for sufficiently small r_2, we have $r_1 > r_2$, which encourages the seller to refinance upon passing Step 1.[18] Relative to the UF benchmark Π_U given in (7), the improvement in the seller's payoff is a consequence of the adoption of DTF. The next result characterizes how the value of DTF ($V = \Pi_D - \Pi_U$, the difference between the seller's payoffs under DTF and UF) is affected by process uncertainties, information delay, and the cost of regulatory capital.

PROPOSITION 3. *In the presence of regulatory capital requirement, the value of DTF (V) is:*

$$V = p_1(l_2 - l_1)\frac{\delta}{(1-\delta)(\mu-\delta)}\cdot\frac{\lambda}{(\lambda+\mu-\delta)} \tag{10}$$

1. *V increases in the passing probability of Step 1 (p_1), the expected processing time of Step 2 ($1/\mu$), the bank's cost of capital (δ), and the difference in liquidation value ($l_2 - l_1$).*
2. *V decreases in the expected information delay $\bar{d} := \frac{1}{\lambda}$. Specifically, reducing information delay is more beneficial when $\bar{d}$ is small, p_1 is high, $(l_2 - l_1)$ is high, μ is small, and δ is high.*

Proposition 3 asserts that the value of DTF is affected by various operational characteristics of the trade process. Observe from (10) that the value of DTF is created when the bank can successfully verify the order passing Step 1 so that it can hold a smaller amount of capital, that is, $(c - l_2)$ instead of $(c - l_1)$, and refinance the loan at time $T_1 + d$. Therefore, the value of DTF is higher when Step 1's passing probability p_1 is higher. This highlights that DTF is valuable for intrinsically solid suppliers making quality products. This lends support to the fact that DTF-type innovation is strongly advocated by companies such as PCH International and Li and Fung, who have carefully vetted suppliers working with them and are confident of the quality (Lee and Tung 2008).

In addition, the value of DTF also increases in the order's incremental liquidation value as passing Step 1 $(l_2 - l_1)$. This value can be substantial during international trade, especially those

[17] Note that under only one break-even constraint, the interest rates r_1 and r_2 are not uniquely determined. We shall examine this issue in detail in Section 6.1.

[18] In Section 6.1, we further discuss the implication of this condition.

where developing countries export to developed countries. In this case, if the products fail to pass customs and have to be sold in the domestic market in the developing countries, which tends to be less developed, the liquidation value of the products (l_1) is much lower than that when the order passes customs and can be sold in the importing countries. In addition, this value also tends to be high when the products are designed under the specification of the importing country. For instance, in the case of Mattel (Lee et al. 2008), the toy is specifically designed to satisfy U.S. requirements and cannot be easily sold elsewhere. Thus, the passing of U.S. custom will significantly increase the toy's liquidation value. Such circumstances deem DTF more valuable.

Furthermore, the value of DTF is higher when the trade process corresponds to a longer mean processing time of Step 2 ($1/\mu$) because DTF allows the bank to hold a smaller amount of capital over a longer period time. This is particularly relevant as buyers impose extended payment terms, which can be seen as a component of T_2. Relatedly, Proposition 3 also suggests when companies/governments are building a technology platform for international trade finance with many milestones, they should first focus on the more critical milestones with longer processing times.

Proposition 3 also reveals that the value of DTF is also affected by the severity of information delay, as captured in the average time it takes for the verified information to arrive at the bank after the order passes Step 1. Intuitively, reducing information delay increases the value of DTF. Further, it is worth noting that such information delay reduction effort is most beneficial when the mean information delay is already short. Put differently, the investment in reducing information delay exhibit increasing marginal return. Finally, we note that the directional impact of other modeling parameters, including passing probability (p_1), bank cost of capital (δ), mean processing time of Step 2 ($1/\mu$), and difference in liquidation value between the two steps ($l_2 - l_1$), on the benefit of delay reduction is the same as theirs on the value of DTF. This suggests that under the circumstances when adopting DTF is valuable, reducing information delay is also more beneficial.

4.3. Numerical Study

To further assess the value of DTF, we present a set of numerical study based on calibrated model parameters. The parameters and the data sources used for calibration are summarized in Table 1. By considering all combinations of the parameters, we evaluate more than 20 million scenarios. To make the cases more comparable between different sets of parameters, we measure the value of DTF as $\frac{V}{\Pi_0 - \Pi_U} = \frac{\Pi_D - \Pi_U}{\Pi_0 - \Pi_U}$, that is, the fraction of financial costs under UF that can be eliminated by DTF.

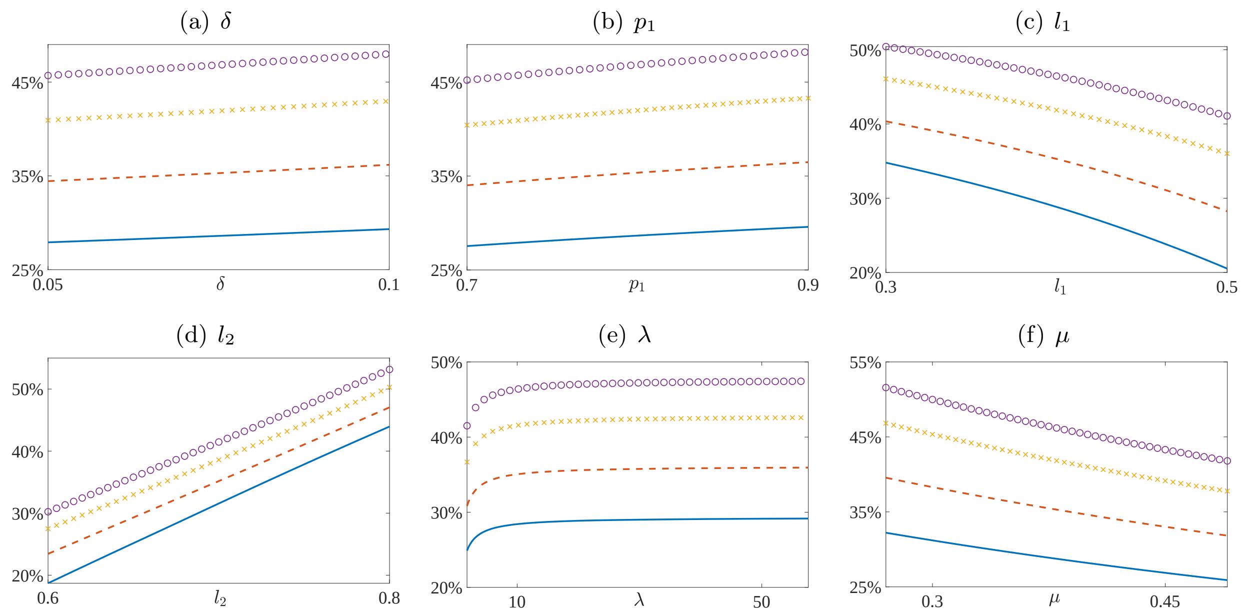
The results are presented in Figure 2. As we consider many parameter combinations, it is impossible to plot all instances. Thus, we present the impact of each parameter in one sub-figure, with four lines corresponding to the 25th-, 50th-, 75th-, and 90th-percentiles when fixing the focal parameter

Table 1 Data Sources and Parameter Calibration

Parameter Range	Step	Number of Values	Data Source
$p_1 \in [0.7, 0.9]$ $p_2 \in [0.7, 0.9]$	0.02 0.02	11 11	World Bank Logistics Performance Index (LPI) Database 2014: percentage of shipments meeting quality criteria by country (Question 26). We use the 25th (0.7) and 75th (0.9) percentile across 100 economies as the parameter range.
profit margin $\in [0.46, 0.67]$	0.03	8	We use the 50th (0.46) and 75th (0.67) percentile of profit margins for low variability goods in Zhang et al. (2022). We normalize $c = 1$ and profit margin $= w/c - 1$. We choose the higher percentiles to avoid $\Pi_U < 0$ cases.
$\mu \in [0.27, 0.49]$	0.02	12	World Bank Doing Business Database 2014 collects total export time (days) and total import time (days) for 211 economies. The 25th, 50th, and 75th percentile for export time is: 12, 18, and 25. The 25th, 50th, and 75th percentile for import time is: 13, 19.4, and 30. According to a case study in Lee and Whang (2005), the transit time from Malaysia to Seattle for a US electronic manufacturer is 30 days, with a standard deviation of 6 days. We let Step 1 represent exporting, and Step 2 for international shipment and importing. The duration range for Step 2 is between 37 days $(= 13 + 30 - 6)$ and 66 days $(= 30 + 30 + 6)$. We use the median of Step 1 (18 days) as the benchmark and normalize Step 1's rate to 1. Thus, the range for μ is calculated as $18/37 = 0.49$ and $18/66 = 0.27$.
$\lambda \in [1.8, 57.6]$	3.1	19	Without blockchain, the bank's information delay is about 10 days (Cognizant 2017). With blockchain, the delay is reduced to 2.5 hours (World Economic Forum 2018). Thus, the range for λ is calculated as $18/10 = 1.8$ and $18/(2.5/8) = 57.6$ assuming 8-hours working days.
$\delta \in [0.05, 0.1]$	0.005	11	According to Damodaran (2018), for US, the banks' cost of equity capital across 1998 to 2018 is between 5% and 10%.
$l_1 \in [0.3, 0.5]$ $l_2 \in [0.6, 0.8]$	0.02 0.02	11 11	We consider a wide range of l_1, l_2 values while satisfying $0 \le l_1 < l_2 < c$.

but varying all other parameters. Directionally, we observe that the impacts of the characteristics of the trade process (p_1, l_1, l_2, μ), the information delay (λ) and the cost of regulatory capital (δ) on the relative value of DTF are all consistent to those in Proposition 3. Quantitatively, we note that the impact of l_2 on the relative value of DTF is the most pronounced. Further, while the cost of regulatory capital (δ) captures the severity of financial friction, we note that the relative value of DTF is not very sensitive to δ: this is because δ affects the financial costs under UF and DTF in the same direction, although its exposure on DTF is marginally greater. Thus regardless of the value of δ, the relative value of DTF remains at around 25% and 45% at 25th- and 90th-percentile respectively.

Figure 2 **Impact of Various Parameters on the Relative Value of DTF ($V/(\Pi_0 - \Pi_U)$)**



Notes. The y-axis for the above figures is $V/(\Pi_0 - \Pi_U)$. Specifically, in Figure (a), for every δ, we run 303,468 ($= 11 \times 11 \times 11 \times 19 \times 12$) cases for combinations of parameter p_1, l_1, l_2, λ, and μ based on Table 1, and then plot its 25th (blue solid line), 50th (red dashed line), 75th (yellow line with crosses), and 90th percentile (purple line with circles). Similar graphs are plotted against other parameters in Figures (b)–(f).

5. Trade Finance in the Presence of Information Asymmetry

In this section, we focus on a different source of financial market imperfection: information asymmetry. Specifically, before the bank and the seller enters into the lending contract, the seller has superior information about the uncertainties of the supply chain process (the passing probability and the expected processing time of each step) than the bank. This situation commonly arises as the seller has more domain knowledge of the order, and is particularly pronounced when the order is specialized and/or the seller lacks of track record.

To capture such information asymmetry, we augment the basic model in §3 by considering the case when there are two types of sellers: efficient ones (high-quality, "H") and inefficient ones (low-quality, "L"). The proportion of efficient sellers in the market is $\theta \in (0,1)$, and the proportion of inefficient ones is $(1-\theta)$. The seller knows its own type, but the bank only knows the distribution (θ). The two types of firms differ in both their passing probability of each step, and the expected processing time of each step. Specifically, the type-i seller's probability of passing Step j is p_j^i for $i = H, L$ and $j = 1, 2$, and the processing time is $\tilde{T}_j^i \sim \text{Exp}(\mu_j^i)$, where $\tilde{T}_1^i$ and $\tilde{T}_2^i$ are independent.

The above parameters for the high-type firms are the same as in Section 4, that is, $\mu_1^H = 1$, $\mu_2^H = \mu$, $p_1^H = p_1$, and $p_2^H = p_2$. For low-type firms, we assume $\mu_1^L = \alpha$ ($\alpha < 1$), $\mu_2^L = \alpha\mu$, $p_1^L = \beta p_1$ ($\beta < 1$), and $p_2^L = \beta p_2$. α and β capture the differences between the two types of firms along the dimension of duration and passing probability respectively, thus measuring the severity of information asymmetry.

To isolate the effect of information asymmetry, we set the bank's cost of capital $\delta = 0$. Note that under this simplification, the value of DTF in the absence of information asymmetry (V in Proposition 3) is zero. In this case, the high-type firm's payoff without any information friction (Π_0^H), is:

$$\Pi_0^H = p_1 p_2 w + (1 - p_1) l_1 + p_1(1 - p_2) l_2 - c. \tag{11}$$

Similarly, the low-type firm's payoff without any information friction (Π_0^L), is:

$$\Pi_0^L = \beta^2 p_1 p_2 w + (1 - \beta p_1) l_1 + \beta p_1 (1 - \beta p_2) l_2 - c. \tag{12}$$

To rule out the trite case, we assume that a high-type (or an efficient) firm's outside option π_0^H follows $\pi_0^H < \Pi_0^H$ so that the high-type firm will always accept the order when there is no information asymmetry. Further, for expositional brevity, we shall focus on the case when $\pi_0^L \geq \Pi_0^L$ ("Attractive outside option for the low-type") so that a low-type (or an inefficient) firm will resort to attractive outside options when there is no information asymmetry.[19]

5.1. Inefficiencies under Uniform Financing

As in Section 4, we first examine the uniform financing case. That is, the bank offers a single interest rate r over the entire trade process.[20] Note that facing a single r, the incentive compatibility constraint is irrelevant. Thus, the analysis focus on the firm's individual rationality (IR) constraint. We conduct the analysis by backward induction, starting from the seller's participation decision when facing r, and followed by the bank's decision on r.

Facing r, a high-type firm accepts the order if and only if its IR constraint is satisfied, that is,

$$p_1 p_2 \left(w - \mathbb{E}\left[c e^{r(\tilde{T}_1^H + \tilde{T}_2^H)} \right] \right) > \pi_0^H. \tag{13}$$

[19] In Appendix D, we extend our analysis to the case when $\pi_0^L < \Pi_0^L$ ("Unattractive outside option for the low-type") so that the low-type (or inefficient) firm will also accept the order when there is no information asymmetry. We show that the main insights regarding the value of DTF remain unchanged.

[20] To highlight the value of DTF, this section focuses on interest-rate-only contracts as a screening device. However, there exists other mechanisms such as screening contracts with other levers (e.g., loan size) and signaling (Tang et al. 2018, Chod et al. 2019b) that could also mitigate the issue of information asymmetry. Given the scope of this paper, we leave these contracts to future study.

By using the fact that $\tilde{T}_1^H$ follows Exp(1) and $\tilde{T}_2^H$ follows Exp(μ), the IR constraint can be simplified as:

$$r < \bar{r}^H(\pi_0^H) := \frac{1+\mu}{2} - \sqrt{\frac{p_1 p_2 c\mu}{p_1 p_2 w - \pi_0^H} + \left(\frac{1-\mu}{2}\right)^2}. \tag{14}$$

Similarly, a low-type firm accepts the order if and only if:

$$r < \bar{r}^L(\pi_0^L) := \frac{\alpha(1+\mu)}{2} - \sqrt{\frac{\beta^2\alpha^2 p_1 p_2 c\mu}{\beta^2 p_1 p_2 w - \pi_0^L} + \left(\frac{\alpha(1-\mu)}{2}\right)^2}. \tag{15}$$

Intuitively, a seller only accepts the order when the interest rate is sufficiently low.[21] The highest acceptable interest rate ($\bar{r}^H$ and $\bar{r}^L$) decreases in the seller's outside option (π_0^H and π_0^L respectively) and increases in the reliability of the process (p^H and p^L respectively). In addition, for the inefficient firm, $\bar{r}^L$ decreases in the expected process duration ($1/\mu$).

On the bank side, the lowest interest rate the bank is willing to offer depends on its belief of the seller it faces. Specifically, when the bank believes the firm is of high-type, the bank's interest rate r^H should satisfy:

$$(1-p_1)l_1 + p_1(1-p_2)l_2 + p_1 p_2 \mathbb{E}[\,ce^{r^H(\tilde{T}_1^H+\tilde{T}_2^H)}\,] = c. \tag{16}$$

By substituting $\mathbb{E}[e^{r^H\tilde{T}_1^H}] = \frac{1}{1-r^H}$ and $\mathbb{E}[e^{r^H\tilde{T}_2^H}] = \frac{\mu}{\mu-r^H}$ into the above equation, we have:

$$r^H = \frac{1}{2}\left(1+\mu-\sqrt{1+\mu^2-\mu(2-\frac{4cp_1p_2}{c-(1-p_1)l_1-p_1(1-p_2)l_2})}\right). \tag{17}$$

Correspondingly, the high-type firm's payoff under interest rate r^H and continuous compounding is:

$$\Pi_U^H = p_1 p_2\left(w - \mathbb{E}\left[ce^{r^H(\tilde{T}_1^H+\tilde{T}_2^H)}\right]\right) = \Pi_0^H, \tag{18}$$

where the two equalities in the equation follow directly from (16) and (11). Similarly, when the bank believes the firm is of low-type, the bank's interest rate r^L should satisfy:

$$(1-\beta p_1)l_1 + \beta p_1(1-\beta p_2)l_2 + \beta^2 p_1 p_2 \mathbb{E}[\,ce^{r^L(\tilde{T}_1^L+\tilde{T}_2^L)}\,] = c. \tag{19}$$

By substituting $\mathbb{E}[e^{r^L\tilde{T}_1^L}] = \frac{\alpha}{\alpha-r^L}$ and $\mathbb{E}[e^{r^L\tilde{T}_2^L}] = \frac{\alpha\mu}{\alpha\mu-r^L}$ into the above equation, we have:

$$r^L = \frac{\alpha}{2}\left(1+\mu-\sqrt{1+\mu^2-\mu(2-\frac{4c\beta^2p_1p_2}{c-(1-\beta p_1)l_1-\beta p_1(1-\beta p_2)l_2})}\right). \tag{20}$$

[21] For ease of exposition, we assume that if the low-type firm is indifferent between trade and its outside option, it chooses its outside option. The preference can be made strictly by increasing the interest rate by an arbitrarily small amount.

The low-type firm's payoff under interest rate r^L and continuous compounding is:

$$\Pi_U^L = \beta^2 p_1 p_2 \left(w - \mathbb{E}\left[ce^{r^L(\tilde{T}_1^L+\tilde{T}_2^L)}\right]\right) = \Pi_0^L. \tag{21}$$

Similar to the uniform interest rate in Proposition 1, r^H and r^L also decrease when the trade process is more reliable (higher p_1, p_2, β), the order's liquidation value is higher (greater l_1, l_2), or processing duration increases (smaller μ, α).[22] Finally, note that given the assumption on π_0^H and π_0^L, we have: $\bar{r}^L(\pi_0^L) \le r^L$ and $\bar{r}^H(\pi_0^H) > r^H$.

Combining the sellers' participation constraints, (14) and (15), and the bank's willingness to lend, we obtain the equilibrium interest rate and sellers' decision as follows.

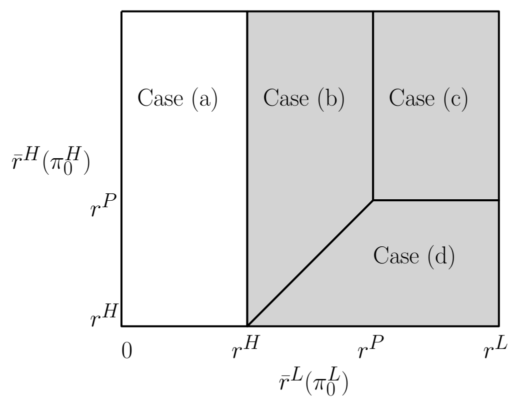
Proposition 4. *In the presence of information asymmetry, under UF, the interest rate, the high- and low-type firms' trade decisions, and their respective payoff (Π_U^H and Π_U^L) satisfy:*

1. *When $\beta \ge \bar{\beta}(\alpha)$, the bank offers interest rate r^H and only the high-type firms trade. $\Pi_U^H = \Pi_0^H$ and $\Pi_U^L = \pi_0^L$.*
2. *When $\beta < \bar{\beta}(\alpha)$,*
   - *(a) if $\bar{r}^L(\pi_0^L) \le r^H$, the bank offers interest rate r^H and only the high-type firms trade, $\Pi_U^H = \Pi_0^H$ and $\Pi_U^L = \pi_0^L$;*
   - *(b) if $\bar{r}^H(\pi_0^H) > \bar{r}^L(\pi_0^L) \in (r^H,\, r^P)$, the bank sets the interest rate at $\bar{r}^L(\pi_0^L)$; only the high-type firms trade; $\Pi_U^H = p_1 p_2 \left[w - \frac{c\mu}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))}\right]$ and $\Pi_U^L = \pi_0^L$;*
   - *(c) if $\min(\bar{r}^H(\pi_0^H),\, \bar{r}^L(\pi_0^L)) > r^P$, the bank offers interest rate r^P, and both types of firm trade; $\Pi_U^H = p_1 p_2 \left[w - \frac{c\mu}{(1-r^P)(\mu-r^P)}\right]$ and $\Pi_U^L = \beta^2 p_1 p_2 \left[w - \frac{\alpha^2 c\mu}{(\alpha-r^P)(\alpha\mu-r^P)}\right]$, where r^P is the bank's break-even interest rate when financing both types of firms;*
   - *(d) if $\bar{r}^H(\pi_0^H) < \min(\bar{r}^L(\pi_0^L),\, r^P)$, the bank does not lend and no firms trade; $\Pi_U^i = \pi_0^i$ for $i = H, L$.*

For expositional brevity, the expressions of the thresholds and interest rates are provided in the proof. The result is illustrated in Figure 3. Consider Statement 1. when $\beta \ge \bar{\beta}(\alpha)$, that is, when the passing probability of the inefficient (low-quality) firm is not sufficiently lower than that of the efficient (high-quality) firm, uniform pricing is sufficient to separate the two types of firms. Under this condition, we have $r^L \le r^H$. That is, by setting the interest rate at r^H, which allows the bank to break even when only facing efficient sellers, the inefficient sellers are deterred from trading. Similarly, when the low-quality firm's outside option is sufficiently attractive ($\bar{r}^L(\pi_0^L) \le r^H$), as in Statement 2(a), setting the interest rate at r^H is also sufficient to prevent the inefficient sellers from accepting the order. In these cases, information asymmetry does not have any impact on the sellers' decision as well as their payoffs, leaving no room of improvement for DTF or FinTech.

[22] Please see Technical Lemma 1 in Appendix F for detailed proof.

Figure 3 Equilibrium Outcomes under Uniform Financing when $\beta < \bar{\beta}(\alpha)$.



Notes. Cases (a) – (d) correspond respectively to Statements 2(a) – 2(d) in Proposition 4. Gray areas indicate uniform financing leads to inefficient outcomes.

As the difference in passing probabilities between the two types of firms becomes larger ($\beta < \bar{\beta}(\alpha)$) and the low-quality firm's outside option is not sufficiently attractive ($\bar{r}^L(\pi_0^L) > r^H$), uniform financing could lead to three types of inefficiencies. First, when the inefficient firm's outside option is relatively valuable ($\bar{r}^L(\pi_0^L) \in (r^H,\, r^P)$) and the efficient firm's outside option is less attractive, that is, $\bar{r}^H(\pi_0^H) > \bar{r}^L(\pi_0^L)$, the bank sets the interest rate just high enough to deter inefficient firms, while still acceptable to efficient firms. As this interest rate is greater than the competitive one under symmetric information (r^H), the high-type sellers essentially pay an information rent, although the trading decision remains efficient. This corresponds to Statement 2(b) in the Proposition.

Second, as the firms' outside options become even less attractive, that is, $\min\left(\bar{r}^H(\pi_0^H),\, \bar{r}^L(\pi_0^L)\right) > r^P$ (Statement 2(c)), it is more economical for the high-type firms to subsidize the low-type firms through the pooling interest rate r^P. Consequently, both types of firms accept the order, resulting in not only inefficient trade decisions, but also lower payoff for the efficient firms.

Finally, when the efficient firms' outside option is attractive such that they will not accept the pooling interest rate r^P, nor any interest rate that is sufficient high to deter the low-type firms, these firms will not participate in the trade even though it is efficient to do that. In this case, the potential cost of subsidizing the low-type firm pushes the high-type firm out of the market, resulting in a complete market collapse (Statement 2(d)).

Combining Cases 2(b) – 2(d), we can see that inefficiencies occur when $\bar{r}^L(\pi_0^L) > r^H$. In other words, in the presence of information asymmetry, UF does not cause inefficiencies when $\bar{r}^L(\pi_0^L) \le r^H$, or equivalently,

$$\beta \le \bar{\beta}_U := \min\left(\sqrt{\frac{\pi_0^L}{p_1 p_2\left[w - \frac{\alpha^2 c\mu}{(\alpha - r^H)(\alpha\mu - r^H)}\right]}}\,,\ 1\right), \tag{22}$$

where r^H satisfies Eq. (17). We refer to the region $\beta \le \bar{\beta}_U$ as the screening region. In the following analysis, we will compare this region with the corresponding threshold under DTF.

5.2. The Value of DTF

The above results reveal that using UF to screen borrowers is associated with different forms of inefficiency. In this section, we investigate whether DTF could separate borrowers more efficiently. Under DTF, the interest rate is r_1 during Step 1. Upon passing Step 1 and verifying the information at time $T_1 + d$, the interest rate is changed to r_2 during Step 2. For this DTF contract to separate the two types of firms, we must have:

$$\Pi_D^H = p_1 p_2 \left[w - \left\{ \Pr(\tilde{d} < \tilde{T}_2^H) \mathbb{E}\left[c e^{r_1(\tilde{T}_1^H + \tilde{d}) + r_2(\tilde{T}_2^H - \tilde{d})} | \tilde{d} < \tilde{T}_2^H \right] + \Pr(\tilde{d} \ge \tilde{T}_2^H) \mathbb{E}\left[c e^{r_1(\tilde{T}_1^H + \tilde{T}_2^H)} | \tilde{d} \ge \tilde{T}_2^H \right] \right\} \right] > \pi_0^H; \tag{23}$$

$$\Pi_D^L = \beta^2 p_1 p_2 \left[w - \left\{ \Pr(\tilde{d} < \tilde{T}_2^L) \mathbb{E}\left[c e^{r_1(\tilde{T}_1^L + \tilde{d}) + r_2(\tilde{T}_2^L - \tilde{d})} | \tilde{d} < \tilde{T}_2^L \right] + \Pr(\tilde{d} \ge \tilde{T}_2^L) \mathbb{E}\left[c e^{r_1(\tilde{T}_1^L + \tilde{T}_2^L)} | \tilde{d} \ge \tilde{T}_2^L \right] \right\} \right] \le \pi_0^L. \tag{24}$$

Specifically, Eq. (23) guarantees that the DTF contract satisfies the high-type firm's IR constraint. Similarly, Eq. (24) ensures that the low-type firm's IR constraint will not be satisfied under the DTF contract. Combining these IR constraints with the bank's competitive loan pricing constraint for the high-type firm:

$$(1-p_1)l_1 + p_1(1-p_2)l_2 + p_1 p_2 \left\{ \Pr(\tilde{d} < \tilde{T}_2^H) \mathbb{E}\left[c e^{r_1(\tilde{T}_1^H + \tilde{d}) + r_2(\tilde{T}_2^H - \tilde{d})} | \tilde{d} < \tilde{T}_2^H \right] + \Pr(\tilde{d} \ge \tilde{T}_2^H) \mathbb{E}\left[c e^{r_1(\tilde{T}_1^H + \tilde{T}_2^H)} | \tilde{d} \ge \tilde{T}_2^H \right] \right\} = c, \tag{25}$$

the impact of DTF in mitigating information asymmetry is summarized in the following result.

PROPOSITION 5. *Among all DTF contracts that make the bank break even, the maximum screening region is achieved when*

$$r_1^* = \frac{1}{2}\left(1 + \mu + \lambda - \sqrt{(\lambda + \mu - 1)^2 + \frac{4c(\lambda + \mu) p_1 p_2}{c - (1-p_1)l_1 - p_1(1-p_2)l_2}} \right), \;\; r_2^* = 0. \tag{26}$$

Under this interest rate schedule, the DTF could separate the two types of firms if and only if $\beta \le \bar{\beta}_D^*$, *where*

$$\bar{\beta}_D^* = min\left(\sqrt{\frac{\pi_0^L}{p_1 p_2 \left[w - \frac{\alpha c(\lambda + \alpha\mu)}{(\alpha - r_1^*)(\lambda + \alpha\mu - r_1^*)} \right]}} \;,\; 1 \right) \ge \bar{\beta}_U; \tag{27}$$

In this case ($\beta \le \bar{\beta}_D^*$), *the high-type firm trades and obtains a payoff* $\Pi_D^H = \Pi_0^H$, *whereas the low-type firm does not trade with a payoff* $\Pi_D^L = \pi_0^L$.

Proposition 5 formalizes that, relative to the UF contract, a DTF contract (r_1^*, r_2^*) enlarges the screening region from $\beta \le \bar{\beta}_U$ to $\beta \le \bar{\beta}_D^*$. The rationale behind separation mechanism is as follows.

Compared to the uniform interest rate under UF as presented in §5.1, differentiated interest rates before and after passing Step 1 under DTF impose a heavier financial burden on the low-type firms (who need to spend more time in each processing step and have lower passing probabilities than the high-type firms). The bigger the difference between the two interest rates, the larger the separation region will be.[23] Thus, by setting $r_2^* = 0$, the bank maximizes the possible difference between r_1^* and r_2^* conditionally on r_1^* and r_2^* satisfying the competitive loan pricing condition, allowing the two types of firms to be separated via the IR constraints when $\beta \le \bar{\beta}_D^*$.

PROPOSITION 6. *The screening region boundary $\bar{\beta}_D^*$ has the following properties:*

1. *$\bar{\beta}_D^*$ decreases in the passing probability of Step 1 and Step 2 (p_1, p_2), the liquidation value of Step 1 and Step 2 (l_1, l_2), the profit margin (w), and the relative difference in duration between high- and low-type firms (α);*
2. *When α is sufficiently small, $\bar{\beta}_D^*$ decreases in the average information delay ($\bar{d} := \frac{1}{\lambda}$), and the expected processing time of Step 2 ($1/\mu$).*

Proposition 6 indicates that it is easier to separate the two types of firms when the DTF contract $(r_1^*, 0)$ exerts a heavier financial burden for low-type firms compared with high-type ones. This is achieved by either extending the expected processing duration of low-type firms (smaller α), or raising the initial interest rate r_1^*. Based on the proof of Proposition 6, we know that r_1^* decreases in p_1, p_2, l_1, and l_2. Therefore, lower passing probabilities or smaller liquidation values increase the bank's initial interest rate, and as a result, strengthens the screening capability of the DTF contract. Further, Proposition 6 reveals that $\bar{\beta}_D^*$ decreases in the average information delay $\bar{d}$, which suggests the DTF contract's screening capability strengthens as the average information delay reduces. This is because when the bank's average information delay decreases, the bank will charge a higher initial interest rate r_1^* to break even ($\partial r_1^* / \partial \bar{d} < 0$), and this puts a heavier financial burden on the low-type firm as its passing duration is longer than that of the high-type firm, therefore, making the separation easier.

Finally, we note from Proposition 5 that when the DTF contract is able to separate the two types of firms (i.e., when $\beta \le \bar{\beta}_D^*$), such a DTF contract can completely eliminate the inefficiencies caused by uniform financing as summarized in Proposition 4. The following result formally quantifies the value of DTF under information asymmetry (denoted by V^A), which is defined as the difference in the weighted average payoffs across the two types of firms between DTF and uniform financing. More formally,

$$V^A = \theta \left(\Pi_D^H - \Pi_U^H \right) + (1-\theta) \left(\Pi_D^L - \Pi_U^L \right). \tag{28}$$

[23] Please see Technical Lemma 3 in Appendix F for detailed proof.

Proposition 7. *In the presence of information asymmetry, the value of DTF (V^A) follows:*

$$V^A = \theta\left[c\left(\frac{p_1 p_2 \mu}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))}-1\right)+(1-p_1)l_1+p_1(1-p_2)l_2\right]^+ \geq 0. \tag{29}$$

$V_A > 0$ *if and only if* $\beta \in (\bar{\beta}_U, \bar{\beta}_D^*]$. *In this region,*

1. V^A *increases in the passing probability of Step 1 and Step 2* (p_1, p_2)*, the liquidation value of Step 1 and Step 2* (l_1, l_2)*, and the relative difference in duration and passing probability between high- and low-type firms* (α, β)*;*
2. V^A *decreases in the average information delay* $\bar{d}$*. Specifically, reducing information delay is more beneficial when* $\bar{d}$ *is small,* p_1*,* p_2*,* l_1*,* l_2*,* α *or* β *is high.*

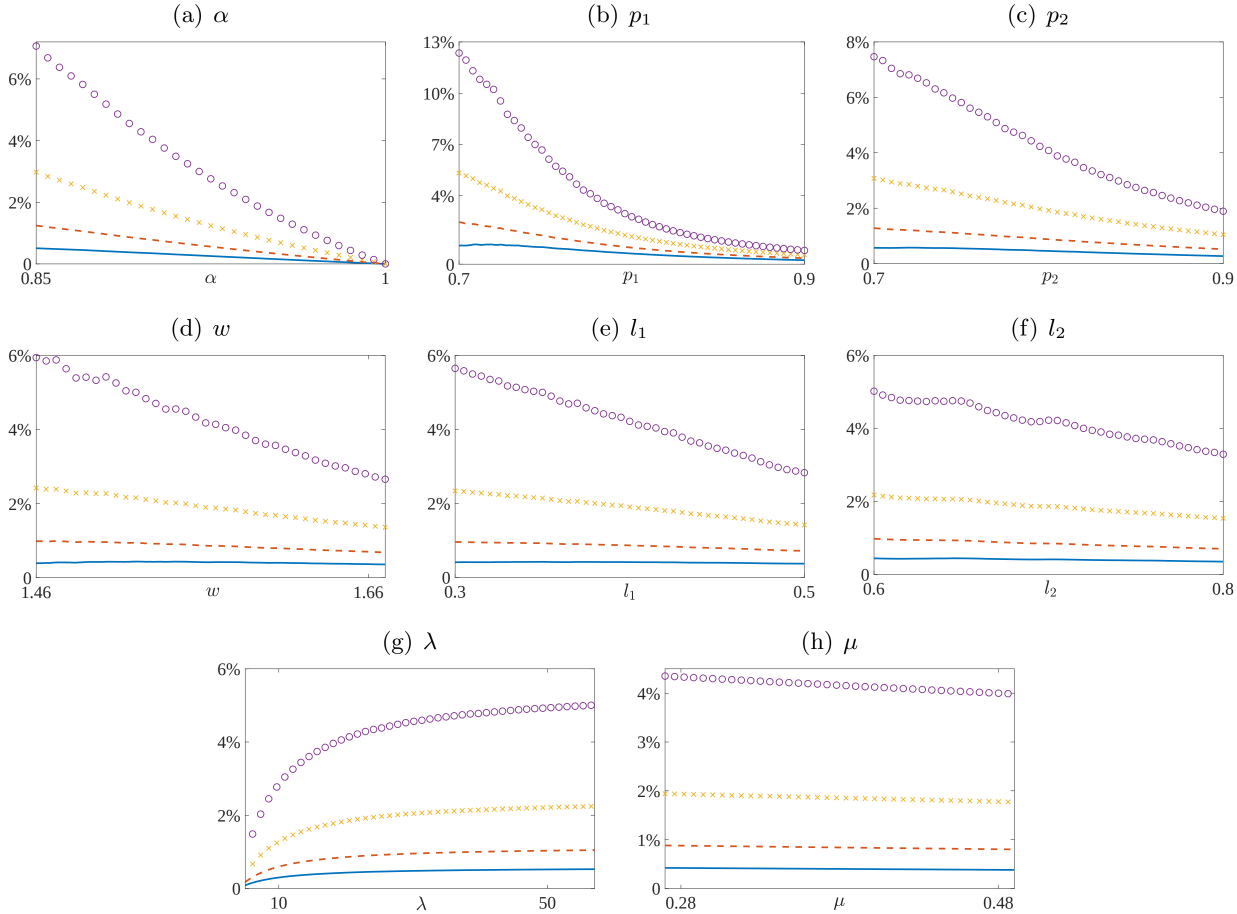
Proposition 7 reveals how DTF creates value in the presence of information asymmetry. When uniform financing could induce separating but results in a higher uniform interest rate ($\bar{r}^L(\pi_0^L)$) faced by the high-type firm (Case 2(b) in Proposition 4), the value of DTF is based on its capability of lowering the overall financing cost faced by the high-type firm by charging differentiated interest rates before and after passing Step 1, while still deterring the low-type firm from trading. When the low-type firm has higher passing probabilities (higher p_1, p_2 or β) or shorter expected processing duration (higher α), the profit from trading is greater, thus increasing the maximum uniform interest rate that a low-type firm is willing to take ($\bar{r}^L(\pi_0^L)$). This makes it more costly to separate under uniform financing. Under these conditions, adopting DTF generates greater value. In the other two scenarios (Case 2(c) and 2(d) in Proposition 4), as the two types of firm are relatively similar (β not too small), DTF is also incapable of separating the two types of firms. Finally, similar to Proposition 3 in Section 4, Proposition 7 reveals that in the presence of information asymmetry, reducing information delay also enhances the value of DTF, and such enhancement is more pronounced when the value of DTF is already high.

5.3. Numerical Study

Similar to Section 4.3, in this section, we present numerical results to illustrate the efficacy of DTF as a screening tool. Specifically, we continue to use the parameters in Table 1. In addition, we note from Proposition 5 that when DTF is able to separate the two types of firms, it could eliminate financial inefficiencies. Thus, instead of comparing payoff differences, we measure the efficacy of DTF by how much it could expand the region of separation beyond UF, that is, $\frac{\bar{\beta}_D^* - \bar{\beta}_U}{\bar{\beta}_U}$.

The results are presented in Figure 4. Similar to Figure 2, we also plot the relative increase of $\bar{\beta}_D^*$ at its 25th-, 50th-, 75th-, and 90th-percentiles. Directionally, the results are mostly consistent with Proposition 6. The only exception is the impact of μ: while $\bar{\beta}_D^*$ increases in μ, so does $\bar{\beta}_U$. Thus, when $\bar{\beta}_D^*$ is normalized by $\bar{\beta}_U$, the trend becomes ambiguous. Quantitatively, we note that the expansion of separation region is relatively stable, with the 90th-percentile at high single digits. The only exception is p_1, where the separation region could expand by up to 13%.

Figure 4 **Impact of Various Parameters on the Screening Capability of DTF (relative to UF, $\frac{\bar{\beta}_D^* - \bar{\beta}_U}{\bar{\beta}_U}$)**



Notes. The y-axis for Figures (a)–(h) represent $\frac{\bar{\beta}_D^* - \bar{\beta}_U}{\bar{\beta}_U}$. Specifically, in Figure (a), for every α, we run 26,705,184 $(= 11 \times 11 \times 8 \times 11 \times 11 \times 19 \times 12)$ cases for combinations of parameter p_1, p_2, w, l_1, l_2, λ, and μ based on Table 1, and then plot its 25th (blue solid line), 50th (red dashed line), 75th (yellow line with crosses), and 90th percentile (purple line with circles). For Figures (b)–(h), we fix $\alpha = 0.9$ and $\pi_0^L = 0.005$, and plot similar graphs against other parameters.

6. Interaction between DTF and FinTech

Thus far, we have focused on the value of DTF, and in particular, how this value is determined by the characteristics of process uncertainties and financial frictions. Based on these results, in this section, we focus on the interaction between DTF and FinTech, which has been touted as a force that could disrupt the trade finance industry by better facilitating information transmission among different stakeholders.

6.1. Contract execution

Despite its value, DTF is undoubtedly more complicated to execute than a simple Uniform Financing contract. For example, in previous sections, we observe that for DTF to achieve the best screening performance, the bank may need to subsidize the loan during the second step of the trade process (e.g., $r_2^* = 0$ in Proposition 5). This could potentially create a disincentive for the bank to share the results of the verification step with the borrower, dampening the value of DTF.[24] Put differently, in order to ensure both the bank and the borrower to have incentive to participate in DTF, it is desirable for the bank to be able to commit to an interest rate schedule and report the verification result promptly. On the other hand, there is a rich literature in economics arguing that the power to make commitment should not be taken for granted (Grossman and Hart 1986, Hart and Moore 1994). Fortunately, emerging FinTech has shown the promise of making commitment easier. One such technology is smart contracts. A smart contract is a computer protocol that is designed to automatically facilitate, verify, or enforce the negotiation or execution of a contract. Various trade finance platforms (such as Populous) have used smart contracts to allow for immediate and automatic payments when certain contractual conditions are met (Babich and Hilary 2020). Thus, such FinTechs could be seen as an enabler of DTF.

6.2. Information delay reduction

Another channel through which FinTech could complement the value of DTF is by reducing information delay. From Propositions 3 and 7, we observe that information delay has a dampening effect on the value of DTF. Thus, FinTechs that reduce such delay could enhance the efficacy of DTF. For example, Cognizant (2017) suggests that a blockchain-based solution can reduce the time it takes to verify information from an average 7–10 days to a few hours. Further, recall from Proposition 3 that reducing information delay exhibits increasing marginal return. Managerially, this suggests that any improvement that reduces the delay marginally (e.g., streamlining internal processes) should generate more value for those information verification processes that are relatively efficient. For those processes that suffer from severe delay (e.g., international shipping), a more drastic technological shift is needed to result in meaningful value improvement.

6.3. Information asymmetry mitigation

As shown in Section 5, DTF plays the role of mitigating information asymmetry through screening. Arguably, some FinTechs could play a similar role. For example, big data analytics can allow banks better collect and analyze the trade process and the profile of the sellers through various data sources before entering the lending contract, enabling more granular customer profiling and segmentation. Thus, by offering the option for the borrowers to participate in such programs (e.g., by providing more information), the banks could also induce the borrowers to reveal their risk profiles. This logic is formalized in Proposition B.2 in the Appendix. The proposition states that as

[24] Conversely, if the bank sets the set the second-stage interest rate (r_2) competitively at the time of refinancing, this could result in the undesirable situation that $r_2 > r_1$, especially when it takes the process more time than expected to pass the first step, thus discouraging the borrower to participate in DTF. We refer the readers to Appendix E, in particular, Proposition E.1, where we formalize this argument.

long as the cost of participating in such programs is lower than a threshold,[25] the efficient firm has the incentive to participate in such programs to reveal its type in exchange for a lower financing cost. In this case, those FinTech that segment customers more efficiently could act as a partial substitute and partial complement to DTF when information asymmetry is the focal financial friction. Specifically, when the difference between the two types of firms are sufficiently small and DTF could not separate them ($\beta > \bar{\beta}_D^*$), FinTech can then act as a valuable alternative. On the other hand, for smaller β such that separation is achievable with DTF (i.e., $\beta \leq \bar{\beta}_D^*$), FinTech is more of a substitute.

In addition, we note that FinTech is not the only tool that could help mitigate information asymmetry. More traditional tools, such as intermediation, could play a similar role. For example, when the transaction was done through intermediaries such as Alibaba and Li & Fung, the bank can make use of the intermediaries' expertise, knowledge and performance record of the supplier, and thus alleviate information asymmetry.

In summary, the interaction between DTF and FinTech depends crucially on the form of frictions that FinTech aims to alleviate. On the one hand, when FinTech acts as a commitment mechanism, or expediting data verification, DTF and FinTech are complementary. On the other hand, the two also can be substitutes when FinTech could alleviate information asymmetry and assess borrowers' risk. Managerially, this suggests for financial institutions adopting or planning to adopt DTF as a contract innovation, they should focus on those complementary FinTechs, such as blockchain. On the other hand, for financiers who already have invested technologies that could better segment customers, they may find DTF only valuable as a mechanism to lower deadweight loss (e.g., regulatory capital requirement).

7. Conclusion

Bank-intermediated trade finance is an important component of trade finance that fuels global and domestic trade. Taking into consideration of the dynamic structure in a trade process, practitioners have developed dynamic trade finance contracts that adjust loan terms as the order passes different steps. In this paper, we find that the value of DTF relies crucially on the characteristics of the underlying trade process, as well as the various underlying information frictions. The interaction between DTF and FinTech depends on the form of information frictions: DTF and FinTech are complementary in the presence of information delay, but they can be substitutes in the presence of information asymmetry.

[25] The cost of participation may be directly incurred by customers (e.g., by providing additional information), or passed through to customers by banks (e.g., FinTech investments)

As an initial attempt to study bank-intermediated trade finance in the presence of dynamic supply chain process uncertainties and information frictions, this paper can be extended in different aspects. First, for parsimony, we focus on a stylized supply chain process. Generalizing the process with calibrated parameters could potentially better quantify the impact of DTF and FinTech, yet possibly at the expense of losing analytical tractability. In addition, to focus on the impact of trade process on financial terms, we simplify the operational decision to a binary one, namely to pass the step and accept the order or not. One extension could be to enrich the operational decisions (e.g., order quantity or negotiating discounts for quality deviations). While our results imply that reducing financial friction through contract innovation and FinTech shall lead to larger order quantities, and thus greater supply chain profit and consumer surplus, more quantitative studies on different settings (e.g., demand uncertainty, competition) could be a promising direction for future study. Our framework could also be extended into the production process, where the product quality, and thus success probability in the process, is endogenously determined by the manufacturer (the borrower in the financing contract). Finally, while our paper is mainly motivated by practice in bank-intermediated trade finance, our results highlight that in general, incorporating the operational dynamics in a transaction when making financing decisions could be valuable, and thus have the potential to inform other forms of supply chain and trade finance, as well as insurance.

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Appendix A: List of Notations

Table 2 summarizes the notation used in the paper.

Table 2 Notation.

symbol	explanation
c	production cost
w	wholesale price
l_j	the liquidation value of the order before passing Step j $(j=1,2)$
p_j	the probability that the order can successfully pass Step j conditional on it has passed Step $j-1$
δ	bank's cost of regulatory capital
$\tilde{T}_j$	the processing time of Step $j=1,2$, $\tilde{T}_j \sim Exp(\mu_j)$; $\mu_1=1$ and $\mu_2=\mu$
$\tilde{d}$	information verification delay, $\tilde{d} \sim Exp(\lambda)$
θ	proportion of high-type firms in the market
p_j^i	type-i $(i=H,L)$ firm's probability of passing Step j $(j=1,2)$, $p_1^H=p_1$, $p_2^H=p_2$, $p_1^L=\beta p_1$ $(\beta<1)$, $p_2^L=\beta p_2$
$\tilde{T}_j^i$	type-i $(i=H,L)$ firm's processing time of Step j $(j=1,2)$, $\tilde{T}_j^i \sim Exp(\mu_j^i)$, $\mu_1^H=1$, $\mu_2^H=\mu$, $\mu_1^L=\alpha$ $(\alpha<1)$, $\mu_2^L=\alpha\mu$
π_0^i	outside option payoff for a type-i firm, $i=H,L$
Π_0	the value of the order without any financial friction
Π_U	the seller's payoff under uniform financing
Π_D	the seller's payoff under dynamic trade finance
V (or V^A)	the value of dynamic trade finance under regulatory capital requirement (or information asymmetry)

Appendix B: Supplemental Results

Proposition B.1 *With regulatory capital δ, a committed interest rate schedule (r_1, r_2) encourages the seller to refinance if and only if $r_2 \le \min\left(r_2^{C,0}, \bar{r}_2\right)$, where*

$$r_2^{C,0} = \frac{\mu(c-l_2)[\mu(1-p_2)+\delta p_2]}{\mu(c-l_2)+l_2p_2(\mu-\delta)}; \tag{30}$$

$$\bar{r}_2 = \frac{1+\mu-\sqrt{(1-\mu)^2+4\mu p_1p_2c\left(c+\frac{\delta}{1-\delta}\left[(c-l_1)\left(1+\frac{p_1}{\lambda+\mu-\delta}\right)+\frac{(c-l_2)\lambda p_1}{(\lambda+\mu-\delta)(\mu-\delta)}\right]-(1-p_1)l_1-p_1(1-p_2)l_2\right)^{-1}}}{2}. \tag{31}$$

Proposition B.2 *In the case under information asymmetry, suppose the bank adopts uniform financing for the case when $\beta > \bar{\beta}_D^*$. Then, if a firm can reveal its true type by incurring a fixed cost F, we get:*

1. *When $F < c\left[\frac{p_1p_2\mu}{(1-\min(\bar{r}^H(\pi_0^H),\bar{r}^L(\pi_0^L),r^P))(\mu-\min(\bar{r}^H(\pi_0^H),\bar{r}^L(\pi_0^L),r^P))}-1\right]+(1-p_1)l_1+p_1(1-p_2)l_2$, the high type firm incurs cost F and reveals its true type, the bank offers interest rate r^H and only the high-type firm trades so that $\Pi_U^H=\Pi_0^H-F$ and $\Pi_U^L=\pi_0^L$.*
2. *When $F \ge c\left[\frac{p_1p_2\mu}{(1-\min(\bar{r}^H(\pi_0^H),\bar{r}^L(\pi_0^L),r^P))(\mu-\min(\bar{r}^H(\pi_0^H),\bar{r}^L(\pi_0^L),r^P))}-1\right]+(1-p_1)l_1+p_1(1-p_2)l_2$, no firm reveals its type, and the equilibrium outcomes are as stated in Proposition 4.*

Appendix C: Trade Finance with Information Delay when Step 1 Completion Time Includes a Fixed Component

In this section, we use a more generalized processing time structure. We assume the completion time for Step 1 $\tilde{T}_1$ consists a constant component K_1 and a stochastic component $\tilde{X}_1$, which is exponentially distributed with rate 1. The constant component represents the fixed production time that is included in Step 1. The completion time for Step 2, $\tilde{T}_2$, remains exponentially distributed with rate μ as in the main body of the paper. $\tilde{X}_1$ and $\tilde{T}_2$ are assumed to be independently distributed. In terms of information friction, we focus on the case with regulatory capital requirement (Section 4).

C.1. Uniform Financing

Under uniform financing (UF), following the same analysis from Section 4.1 and Proposition 1, the equilibrium interest rate is:

$$r^* = \frac{1}{2}\left(1+\mu-\sqrt{1+\mu^2-\mu(2-\frac{4cp_1p_2e^{\delta K_1}}{c-(1-p_1)l_1-p_1(1-p_2)l_2+(c-l_1)\left(\frac{e^{\delta K_1}(\mu-(1-p_1)\delta)}{(1-\delta)(\mu-\delta)}-1\right)})}\right). \tag{32}$$

And the seller's corresponding payoff is:

$$\Pi_U = \Pi_0 - (c-l_1)\left[\frac{e^{\delta K_1}(\mu-(1-p_1)\delta)}{(1-\delta)(\mu-\delta)}-1\right]. \tag{33}$$

C.2. The Value of DTF

Similarly, under DTF, following the same three scenarios under Section 4.2, the bank's net capital regulation cost (NCC) under UF and DTF are as follows:

$$\mathrm{NCC}_U = (c-l_1)\left[(1-p_1)(\mathbb{E}[e^{\delta\tilde{T}_1}]-1)+p_1(\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}]-1)\right]; \tag{34}$$

$$\begin{aligned}\mathrm{NCC}_D =& p_1\Pr(\tilde{d}<\tilde{T}_2)\left[(c-l_2)(\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}<\tilde{T}_2]-1)+(l_2-l_1)(\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{d})}|\tilde{d}<\tilde{T}_2]-1)\right]\\ &+p_1\Pr(\tilde{d}\geq\tilde{T}_2)\left[(c-l_1)(\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}\geq\tilde{T}_2]-1)\right]+(1-p_1)(c-l_1)(\mathbb{E}[e^{\delta\tilde{T}_1}]-1),\end{aligned} \tag{35}$$

where

$$\Pr(\tilde{d}<\tilde{T}_2) = 1-\frac{\mu}{\lambda+\mu}; \tag{36}$$

$$\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}<\tilde{T}_2] = \frac{e^{\delta K_1}\mu(\lambda+\mu)}{(1-\delta)(\mu-\delta)(\lambda+\mu-\delta)}; \tag{37}$$

$$\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{d})}|\tilde{d}<\tilde{T}_2] = \frac{e^{\delta K_1}(\lambda+\mu)}{(1-\delta)(\mu+\lambda-\delta)}; \tag{38}$$

$$\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}\geq\tilde{T}_2] = \frac{e^{\delta K_1}(\lambda+\mu)}{(1-\delta)(\lambda+\mu-\delta)}. \tag{39}$$

Therefore, the value of DTF is:

$$\begin{aligned}V :=& \Pi_D-\Pi_U = \mathrm{NCC}_U-\mathrm{NCC}_D\\ =& e^{\delta K_1}p_1(l_2-l_1)\frac{\delta}{(1-\delta)(\mu-\delta)}\cdot\frac{\lambda}{(\lambda+\mu-\delta)}.\end{aligned} \tag{40}$$

Compared to Proposition 3, the value of DTF with the constant time component K_1 is amplified by the constant term $e^{\delta K_1}$. Thus, the constant component K_1 enhances the value of DTF, while the impact of other parameters remain the same as in Proposition 3.

Appendix D: Trade Finance with Information Asymmetry when the Inefficient Firms' Outside Option is Unattractive

This appendix supplements Section 5 by examining the case where the inefficient firms' outside option is unattractive, that is, $\pi_0^L < \Pi_0^L$. Thus, without information asymmetry, it is efficient for the low-type firm also to accept the order. In this section, we follow the same structure as in Section 5, first examining the inefficiency under uniform financing, followed by when and how DTF creates value. Finally, to focus on the more interesting case, we assume $\beta < \bar{\beta}(\alpha)$, that is, under symmetric information, the uniform interest rate faced by the high-type firm is lower than that of the low-type firm, that is, $r^H < r^L$. For expositional brevity, we also assume $\tilde{d} = 0$ and $l_1 = l_2 = 0$.

D.1. Inefficiency under Uniform Financing

Similar to Proposition 4 in Section 5.1, the following result summarizes the inefficiency caused by information asymmetry when the bank adopts uniform financing.

Proposition D.1 *When the bank adopts uniform financing, in the presence of information asymmetry, the interest rate, the high- and low-type firms' trade decisions, and their respective payoff (Π_U^H and Π_U^L) satisfy:*

1. *if $\bar{r}^H(\pi_0^H) \le r^P$, the bank offers interest rate r^L, and only low-type firms trade; $\Pi_U^H = \pi_0^H$ and $\Pi_U^L = \Pi_0^L$.*
2. *if $\bar{r}^H(\pi_0^H) > r^P$, the bank offers interest rate r^P, and both types of firm trade; $\Pi_U^H = p_1 p_2 \left[w - \frac{c\mu}{(1-r^P)(\mu-r^P)}\right]$ and $\Pi_U^L = \beta^2 p_1 p_2 \left[w - \frac{\alpha^2 c\mu}{(\alpha-r^P)(\alpha\mu-r^P)}\right]$. There is no efficiency loss, but creates unfairness.*

By the definition of $\bar{r}^H(\pi_0^H)$ in Eq. (14), we know that $\bar{r}^H(\pi_0^H)$ decreases with π_0^H. Thus, Proposition D.1 reveals that when π_0^H is high, the high-type firm prefers its outside option π_0^H to accepting the order under the pooling interest rate r^P. In this case, the overall efficiency loss, which is defined as the weighted average difference in profit between the case with information asymmetry and the one without, is $\theta\,[\Pi_0^H - \pi_0^H]$. On the other hand, when π_0^H is low, both types of firms accept the order, so there is no efficiency loss. That said, as both types of firms face the pooling interest rate r^P, the low-type firms are effectively subsidized by the high-type ones, creating a concern of fairness.

D.2. The Value of DTF

Similar to Proposition 5 in Section 5.2, the following result summarizes when and how DTF contracts can mitigate the inefficiency of information asymmetry.

Proposition D.2 *When adopting DTF, the bank can separate the two types of firms using a menu of one DTF contract with interest rates schedule (r_1^H, r_2^H) and one uniform financing contract with interest rate r^L if and only if $\beta \in [\bar{\bar{\beta}}, \bar{\beta}(\alpha))$. Under this condition, the high-type firm chooses the DTF contract and the low-type firm chooses the UF contract. One such menu of contracts is $r_1^H = 1 - p_1 p_2$, $r_2^H = 0$, and $r^L = \frac{\alpha}{2}\left(1 + \mu - \sqrt{(\mu-1)^2 + 4\mu\beta^2 p_1 p_2}\right)$. Under this contract menu, the payoff of type-i firm is $\Pi_D^i = \Pi_0^i$.*

Proposition D.2 reveals that, when the two types of firms are moderately different ($\beta \in [\bar{\bar{\beta}}, \bar{\beta}(\alpha))$), using a menu of one DTF contract and one uniform contract as a screening mechanism can help to either restore efficiency or alleviate unfairness.

Appendix E: Interest rate schedule under DTF and competitive refinancing.

In this Appendix, we illustrate the possible inefficiencies when the bank does not adopt a committed interest rate path, but instead uses competitive market rate to determine r_2. Specifically, we assume that the bank sets the original interest rate at r_1 at time 0. Upon receiving the verified information regarding the order successfully passing Step 1 at time T_1+d, the bank adjusts the interest rate to r_2 such that it breaks even when it looks forward, that is, from T_1+d to $T_1+\tilde{T}_2$ (conditional on $\tilde{T}_2>d$). Under this setting, the outstanding loan at T_1+d, including the cumulated interest up to T_1+d, is $c\cdot e^{r_1(T_1+d)}$ at time T_1+d. Thus, after taking into consideration of the updated liquidation value of the order l_2, the amount of regulatory capital the bank is required to hold is $(ce^{r_1(T_1+d)}-l_2)$ from T_1+d to $T_1+\tilde{T}_2$, which corresponds to the net cost of regulatory capital $(ce^{r_1(T_1+d)}-l_2)(\mathbb{E}[e^{\delta(\tilde{T}_2-d)}]-1)$.

Next, conditional that the order has passed Step 1, we can use the passing probability of Step 2 to determine the expected repayment of the loan refinanced at time T_1+d:

$$\begin{aligned}&\mathbb{E}[\,\text{Loan Repayment}\,|\,\text{Verified passing of Step 1 at } T_1+d \text{ and } \tilde{T}_2>d\,]\\&=p_2\cdot ce^{r_1(T_1+d)}\mathbb{E}[e^{r_2(\tilde{T}_2-d)}|\tilde{T}_2>d]+(1-p_2)l_2.\end{aligned}\tag{41}$$

From (2), the competitive interest rate r_2 for refinancing the outstanding loan at time T_1+d (conditional on $\tilde{T}_2>d$) satisfies:

$$p_2\cdot ce^{r_1(T_1+d)}\mathbb{E}[e^{r_2(\tilde{T}_2-d)}|\tilde{T}_2>d]+(1-p_2)\cdot l_2=c\cdot e^{r_1(T_1+d)}+(c\cdot e^{r_1(T_1+d)}-l_2)(\mathbb{E}[e^{\delta(\tilde{T}_2-d)}|\tilde{T}_2>d]-1). \tag{42}$$

By jointly considering (42) and the bank's overall break-even constraint from $t=0$ to $t=\tilde{T}_1+\tilde{T}_2$, we can uniquely determine the equilibrium interest rate (r_1^C,r_2^C) as follows.

Proposition E.1 *When the refinance rate r_2 is determined such that the bank breaks even upon verifying the order passing Step 1, the equilibrium interest rate schedule (r_1^C,r_2^C) is:*

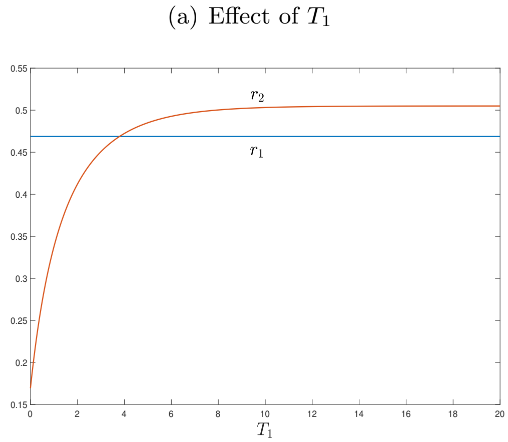
$$r_1^C=\frac{1}{2}\left(1+\lambda+\mu-\sqrt{(\lambda+\mu-1)^2+\frac{4c\mu p_1p_2(\lambda+\mu-r_2^C)}{(\mu-r_2^C)\left\{c-(1-p_1)l_1-p_1(1-p_2)l_2+\frac{\delta}{1-\delta}\left[(c-l_1)\left(1+\frac{p_1}{\lambda+\mu-\delta}\right)+\frac{(c-l_2)\lambda p_1}{(\lambda+\mu-\delta)(\mu-\delta)}\right]\right\}}}\right); \tag{43}$$

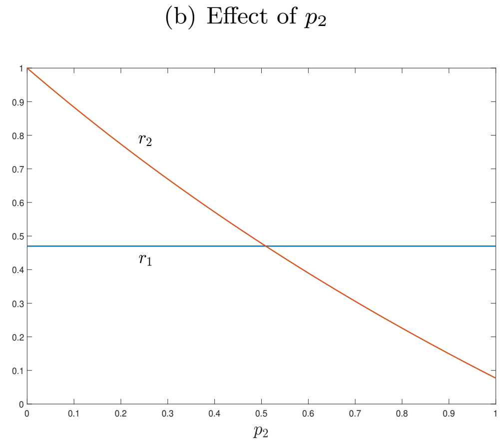
$$r_2^C=\frac{\mu[\mu(1-p_2)+\delta p_2](ce^{r_1^C(T_1+d)}-l_2)}{\mu(ce^{r_1^C(T_1+d)}-l_2)+p_2l_2(\mu-\delta)}. \tag{44}$$

The refinance rate r_2^C decreases in p_2; increases in T_1 and d.

Among others, Proposition E.1 reveals that the rate after refinance (r_2) increases in T_1, the realized passing time for Step 1. The reason is as follows. As the realized T_1 increases, the size of the loan at the time of refinancing, $ce^{r_1(T_1+d)}$, also increases. However, the liquidation value of the order remains unchanged at l_2. Thus, the required regulatory capital held by the bank at T_1+d, $(ce^{r_1(T_1+d)}-l_2)$, increases in T_1, which increases the bank's cost of capital, and thus resulting in a higher interest rate r_2. For the same reason, even when the liquidation value of the order increases from l_1 to l_2 upon passing Step 1, for the realization of T_1 that is sufficiently large, the refinanced rate r_2 could be higher than the original rate r_1, which is independent

Figure 5 Effect of T_1 and p_2 on the Relative Magnitude of r_1 and r_2 under Competitive Refinance





Notes. Parameter values for figure (a): $p_1 = 0.5$, $p_2 = 0.55$, $l_1 = 0.2$, $l_2 = 0.8$, $\delta = 0.1$, $c = 1$, $\mu = 1$, $d = 0$; parameter values for figure (b): $p_1 = 0.5$, $l_1 = 0.2$, $l_2 = 0.4$, $\delta = 0.1$, $c = 1$, $\mu = 1$, $T_1 = 1$, $d = 0$.

of the realized T_1. This phenomenon is illustrated in Figure 5(a). Further, Figure 5(b) reveals that r_2 is more likely to be higher than r_1 for smaller p_2, as the risk associated with Step 2 is high.

We note that the possibility that refinancing can result in a higher interest rate $(r_2 > r_1)$ is certainly undesirable as it could discourage the seller from refinancing, especially when the seller has the discretion to decide whether to do so. Hence, setting interest rates in this way can diminish the value of DTF.

Appendix F: Technical Lemma

Technical Lemma 1 *Under UF, r^H and r^L decrease in p_1, p_2, l_1 and l_2; increase in μ; r^L increases in α and decreases in β.*

Technical Lemma 2 *In the presence of information asymmetry, under UF, there exist thresholds $\bar{\beta}_U$, $\hat{\beta}$, $\tilde{\beta}$, and $\tilde{\pi}_0^H$, such that, the interest rate, the high- and low-type firms' trade decisions, and their respective payoff (Π_U^H and Π_U^L) satisfy:*

1. *When $\beta \le \bar{\beta}_U$, the bank offers interest rate r^H and only the high-type firms trade. $\Pi_U^H = \Pi_0^H$ and $\Pi_U^L = \pi_0^L$.*
2. *When $\beta > \bar{\beta}_U$,*
   - *(a) (Case 2(b) in Proposition 4) if $\bar{\beta}_U < \beta \le \hat{\beta}$, the bank sets the interest rate at $\bar{r}^L(\pi_0^L)$; only the high-type firms trade; $\Pi_U^H = p_1 p_2 \left[w - \frac{c\mu}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))}\right]$ and $\Pi_U^L = \pi_0^L$;*
   - *(b) (Case 2(c) in Proposition 4) if $\hat{\beta} < \beta \le \tilde{\beta}$ and $\pi_0^H < \tilde{\pi}_0^H$ or $\tilde{\beta} < \beta \le \bar{\beta}$, the bank offers interest rate r^P, and both types of firm trade; $\Pi_U^H = p_1 p_2 \left[w - \frac{c\mu}{(1-r^P)(\mu-r^P)}\right]$ and $\Pi_U^L = \beta^2 p_1 p_2 \left[w - \frac{\alpha^2 c\mu}{(\alpha-r^P)(\alpha\mu-r^P)}\right]$, where r^P is the bank's break-even interest rate when financing both types of firms;*
   - *(c) (Case 2(d) in Proposition 4) if $\hat{\beta} < \beta \le \tilde{\beta}$ and $\pi_0^H > \tilde{\pi}_0^H$, the bank does not lend and no firms trade; $\Pi_U^i = \pi_0^i$ for $i = H, L$.*

Technical Lemma 3 *There exists a threshold $\bar{\beta}_D(r_1, r_2)$, which decreases in r_2, such that when $\beta \le \bar{\beta}_D$, the bank can separate the two types of firms by offering a DTF contract (r_1, r_2) that makes the bank break even.*

Appendix G: Proofs

Proof of Proposition 1. By applying (2) and the loan principal c, the equilibrium interest rate r^* solves:

$$(1-p_1)l_1+p_1(1-p_2)l_2+p_1p_2\mathbb{E}[ce^{r^*(\tilde{T}_1+\tilde{T}_2)}]=c+(c-l_1)\left[(1-p_1)(\mathbb{E}[e^{\delta\tilde{T}_1}]-1)+p_1(\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}]-1)\right].\quad(45)$$

Because $\tilde{T}_1\sim Exp(1)$ and $\tilde{T}_2\sim Exp(\mu)$, we have: $\mathbb{E}[e^{\delta\tilde{T}_1}]=\frac{1}{1-\delta}$ and $\mathbb{E}[\,ce^{r^*(\tilde{T}_1+\tilde{T}_2)}\,]=\frac{c}{1-r^*}\cdot\frac{\mu}{\mu-r^*}$. Substituting these into (45) leads to:

$$(1-p_1)l_1+p_1(1-p_2)l_2+p_1p_2\left(\frac{c\mu}{(1-r^*)(\mu-r^*)}\right)=c+(c-l_1)\left[\frac{(1-p_1)\delta}{1-\delta}+p_1\left(\frac{1}{1-\delta}\frac{\mu}{\mu-\delta}-1\right)\right].\quad(46)$$

$$(1-p_1)l_1+p_1(1-p_2)l_2+p_1p_2\left(\frac{c\mu}{(1-r^*)(\mu-r^*)}\right)=c+(c-l_1)\frac{\delta}{1-\delta}\left[1-p_1+p_1\left(\frac{1+\mu-\delta}{(\mu-\delta)}\right)\right].\quad(47)$$

$$(1-p_1)l_1+p_1(1-p_2)l_2+p_1p_2\left(\frac{c\mu}{(1-r^*)(\mu-r^*)}\right)=c+(c-l_1)\left(\frac{\delta}{1-\delta}\right)\left(1+\frac{p_1}{\mu-\delta}\right).\quad(48)$$

There exists two possible r^* based on Eq. (48). By choosing the smaller one (due to market competition), the equilibrium interest rate r^* follows:

$$r^*=\frac{1}{2}\left(1+\mu-\sqrt{1+\mu^2-\mu(2-\frac{4cp_1p_2}{c-(1-p_1)l_1-p_1(1-p_2)l_2+\frac{(\mu+p_1-\delta)(c-l_1)\delta}{(1-\delta)(\mu-\delta)}})}\right).\quad(49)$$

Since $p_1,p_2<1$, $\delta<1$, and $l_1<l_2<c$, taking derivative of r^* with respect to p_1, p_2, l_1, l_2, and δ, we have:

$$\frac{\partial r^*}{\partial p_1}=\frac{-(c-l_1)(\frac{1}{p_1}+\frac{\delta}{1-\delta})(1-r^*)^2(\mu-r^*)^2}{c\mu p_1p_2(1+\mu-2r^*)}<0;\quad(50)$$

$$\frac{\partial r^*}{\partial p_2}=\frac{(l_2-\frac{c\mu}{(1-r^*)(\mu-r^*)})(1-r^*)^2(\mu-r^*)^2}{c\mu p_2(1+\mu-2r^*)}<0;\quad(51)$$

$$\frac{\partial r^*}{\partial l_1}=\frac{\left[(1-\frac{\delta}{(1-\delta)(\mu-\delta)})p_1-\frac{1}{1-\delta}\right](1-r^*)^2(\mu-r^*)^2}{c\mu p_1p_2(1+\mu-2r^*)}<0;\quad(52)$$

$$\frac{\partial r^*}{\partial l_2}=\frac{-(1-p_2)(1-r^*)^2(\mu-r^*)^2}{c\mu p_2(1+\mu-2r^*)}<0;\quad(53)$$

$$\frac{\partial r^*}{\partial \delta}=\frac{(c-l_1)(1-r^*)(\mu-r^*)\left[(\mu-\delta)^2+p_1(\mu-\delta^2)\right]}{c\mu p_1p_2(\frac{1}{\mu-r^*}+\frac{1}{1-r^*})(1-\delta)^2(\mu-\delta)^2}>0.\quad(54)$$

As for the seller's payoff, based on Eq. (3), we have:

$$\Pi_U=p_1p_2\left(w-\mathbb{E}\left[ce^{r^*(\tilde{T}_1+\tilde{T}_2)}\right]\right)=p_1p_2w-p_1p_2\left(\frac{c\mu}{(1-r^*)(\mu-r^*)}\right).\quad(55)$$

Substituting Eq. (48) into the above equation, we have:

$$\Pi_U=p_1p_2w+(1-p_1)l_1+p_1(1-p_2)l_2-c-(c-l_1)\left(\frac{\delta}{1-\delta}\right)\left(1+\frac{p_1}{\mu-\delta}\right);\quad(56)$$

$$=\Pi_0-(c-l_1)\left(\frac{\delta}{1-\delta}\right)\left(1+\frac{p_1}{\mu-\delta}\right),\quad(57)$$

as desired. □

Proof of Proposition 2. By applying (2) and considering the three scenarios, r_1^* and r_2^* under the DTF contract should satisfy:

$$(1-p_1)l_1 + p_1(1-p_2)l_2 + p_1p_2\left\{\Pr(\tilde{d}<\tilde{T}_2)\mathbb{E}\left[ce^{r_1^*(\tilde{T}_1+\tilde{d})+r_2^*(\tilde{T}_2-\tilde{d})}|\tilde{d}<\tilde{T}_2\right] + \Pr(\tilde{d}\geq\tilde{T}_2)\mathbb{E}\left[ce^{r_1^*(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}\geq\tilde{T}_2\right]\right\} =$$
$$c+(1-p_1)(c-l_1)(\mathbb{E}[e^{\delta\tilde{T}_1}]-1) + p_1\Pr(\tilde{d}<\tilde{T}_2)\left[(c-l_2)(\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}<\tilde{T}_2]-1) + (l_2-l_1)(\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{d})}|\tilde{d}<\tilde{T}_2]-1)\right] +$$
$$p_1\Pr(\tilde{d}\geq\tilde{T}_2)\left[(c-l_1)(\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}\geq\tilde{T}_2]-1)\right]. \tag{58}$$

The terms on the left hand side of the equation correspond to the bank's repayments, which is calculated as the bank's expected payoff with the product passing two steps (hence receiving principal and interest) and not passing (and hence recovering liquidation value). Also, the terms on the right hand side are the loan principal c and the net cost of holding bank capital that correspond to those three cases as described in §4.2. Based on Eq. (58), the interest rate schedule (r_1^*, r_2^*) satisfies:

$$\frac{c\mu p_1p_2(\lambda+\mu-r_2)}{(\lambda+\mu-r_1)(1-r_1)(\mu-r_2)} = c-(1-p_1)l_1 - p_1(1-p_2)l_2 + \frac{\delta}{1-\delta}\left[(c-l_1)\left(1+\frac{p_1}{\lambda+\mu-\delta}\right) + \frac{(c-l_2)\lambda p_1}{(\lambda+\mu-\delta)(\mu-\delta)}\right]. \tag{59}$$

Based on Proposition B.1, a committed interest rate schedule (r_1^*, r_2^*) that encourages the seller to refinance should also satisfy $r_2^* \leq \bar{r}_2^* := \min\left(r_2^{C,0}, \bar{r}_2\right)$. Therefore, the DTF contract that the bank offers should satisfy the bank's competitive loan pricing equation and has $r_2^* \leq \bar{r}_2^*$.

Given the interest rates (r_1^*, r_2^*), we now determine the seller's expected profit with time delay in information verification as defined in Eq. (8). By using the fact that $\Pr(\tilde{d}<\tilde{T}_2) = \frac{\lambda}{\lambda+\mu}$, $\mathbb{E}[e^{\delta(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}<\tilde{T}_2] = \frac{1}{1-\delta}\cdot\frac{\mu}{\mu-\delta}\cdot\frac{\lambda+\mu}{\lambda+\mu-\delta}$, and by taking Eq. (58) into Eq. (8), we can show that the seller's expected profit with delayed information verification $\tilde{d}$ can be simplified as:

$$\begin{aligned}\Pi_D &= p_1p_2\left[w - \Pr(\tilde{d}<\tilde{T}_2)\mathbb{E}[\,c\cdot e^{r_1(\tilde{T}_1+\tilde{d})+r_2(\tilde{T}_2-\tilde{d})}|\tilde{d}<\tilde{T}_2\,] - \Pr(\tilde{d}\geq\tilde{T}_2)\mathbb{E}[\,c\cdot e^{r_1(\tilde{T}_1+\tilde{T}_2)}|\tilde{d}\geq\tilde{T}_2\,]\right] \\ &= \Pi_0 - \frac{\delta}{1-\delta}\left[(c-l_1)\left(1+\frac{p_1}{\lambda+\mu-\delta}\right) + \frac{(c-l_2)\lambda p_1}{(\lambda+\mu-\delta)(\mu-\delta)}\right].\end{aligned} \tag{60}$$

where Π_0 is given in Eq. (1). □

Proof of Proposition E.1. Based on Eq. (42), we can solve for r_2^C as in Eq. (44). Taking r_2^C into the bank's competitive loan pricing equation (59), we have r_1^C as in Eq. (43). When r_1^C has multiple values, banking competition drives r_1^C to the smallest one as in Eq. (43).

Taking derivatives of r_2^C with respect to p_2 and T_1, we have:

$$\frac{\partial r_2^C}{\partial p_2} = -\frac{c\mu^2(\mu-\delta)e^{r_1^CT_1}\left(ce^{r_1^CT_1}-l_2\right)}{\left[c\mu e^{r_1^CT_1}-\delta l_2p_2+l_2\mu(p_2-1)\right]^2} < 0, \tag{61}$$

$$\frac{\partial r_2^C}{\partial T_1} = \frac{cl_2\mu p_2(\mu-\delta)[\mu(1-p_2)+\delta p_2]r_1^Ce^{r_1^CT_1}}{\left[c\mu e^{r_1^CT_1}-\delta l_2p_2+l_2\mu(p_2-1)\right]^2} > 0, \tag{62}$$

as $\frac{\partial r_2^C}{\partial d} = \frac{\partial r_2^C}{\partial T_1} > 0$, we have the results as desired. □

Proof of Proposition B.1. Motivated by the phenomenon in Appendix E that competitive refinancing could lead to a higher interest rate; i.e., $r_2 > r_1$, we now focus our attention on the interest rate schedules that encourage the seller to refinance. To do so, we impose two constraints. First, refinancing only results in interest rate reduction, that is, $r_1 \geq r_2$. Second, r_2 is no larger than the competitive refinanced interest rate that satisfies Eq. (44). This rules out the possibility of having the seller to refinance the loan with another bank at $T_1 + d$.

We examine the two imposed constraints separately. First, for the constraint that $r_2 \leq r_1$, according to the bank's competitive loan pricing equation, (r_1, r_2) needs to satisfy Eq. (59), which is:

$$r_1(r_2) = \frac{1}{2}\left(1+\lambda+\mu-\sqrt{(\lambda+\mu-1)^2 + \frac{4c\mu p_1 p_2(\lambda+\mu-r_2)}{(\mu-r_2)\left\{c-(1-p_1)l_1-p_1(1-p_2)l_2+\frac{\delta}{1-\delta}\left[(c-l_1)\left(1+\frac{p_1}{\lambda+\mu-\delta}\right)+\frac{(c-l_2)\lambda p_1}{(\lambda+\mu-\delta)(\mu-\delta)}\right]\right\}}}\right); \tag{63}$$

Observe from the above equation that r_1 is decreasing in r_2. Hence, the constraint $r_2 \leq r_1$ will hold as long as $r_2 \leq \bar{r}_2$, where $\bar{r}_2$ satisfies $r_1(\bar{r}_2) = \bar{r}_2$, which is

$$(1-\bar{r}_2)(\mu-\bar{r}_2) = \frac{\mu p_1 p_2 c}{c+\frac{\delta}{1-\delta}\left[(c-l_1)\left(1+\frac{p_1}{\lambda+\mu-\delta}\right)+\frac{(c-l_2)\lambda p_1}{(\lambda+\mu-\delta)(\mu-\delta)}\right]-(1-p_1)l_1-p_1(1-p_2)l_2}. \tag{64}$$

Or equivalently,

$$\bar{r}_2 = \frac{1+\mu-\sqrt{(1-\mu)^2+4\mu p_1 p_2 c\left(c+\frac{\delta}{1-\delta}\left[(c-l_1)\left(1+\frac{p_1}{\lambda+\mu-\delta}\right)+\frac{(c-l_2)\lambda p_1}{(\lambda+\mu-\delta)(\mu-\delta)}\right]-(1-p_1)l_1-p_1(1-p_2)l_2\right)^{-1}}}{2}. \tag{65}$$

For the second constraint, that is, r_2 is no larger than the competitive refinanced interest rate that satisfies (42), we note from Proposition E.1, r_2^C increases in T_1 and d. Thus, for r_2 to be smaller than r_2^C for any realization of T_1 and d, we need (and only need) that r_2 is smaller than r_2^C at $T_1 = 0$ and $d = 0$, or equivalently, $r_2^{C,0}$, which follows:

$$r_2^{C,0} = \frac{\mu(c-l_2)[\mu(1-p_2)+\delta p_2]}{\mu(c-l_2)+l_2 p_2(\mu-\delta)}. \tag{66}$$

Combining these two scenarios lead to the constraint on r_2. And r_1 follows directly from Eq. (63). □

Proof of Proposition 3. By taking Π_D from Eq. (9) and Π_U from Eq. (7), we have:

$$V = \Pi_D - \Pi_U = p_1(l_2-l_1)\frac{\delta}{(1-\delta)(\mu-\delta)}\cdot\frac{\lambda}{(\lambda+\mu-\delta)} \tag{67}$$

The monotonicity in the first statement follow directly from Eq. (67).

Regarding the monotonicity in the second statement, under $d = \frac{1}{\lambda}$, we re-write Eq. (67) as:

$$V = p_1(l_2-l_1)\frac{\delta}{(1-\delta)(\mu-\delta)}\cdot\frac{1}{1+(\mu-\delta)d} \tag{68}$$

Thus,

$$\frac{\partial V}{\partial d} = -p_1(l_2-l_1)\frac{\delta}{(1-\delta)[1+(\mu-\delta)d]^2} < 0, \tag{69}$$

and

$$\frac{\partial^2 V}{\partial d^2} = p_1(l_2 - l_1)\frac{2\delta}{(1-\delta)[1+(\mu-\delta)d]^3} > 0. \tag{70}$$

For cross derivatives:

$$\frac{\partial^2 V}{\partial d \partial p_1} = -(l_2 - l_1)\frac{\delta}{(1-\delta)[1+(\mu-\delta)d]^2} < 0; \tag{71}$$

$$\frac{\partial^2 V}{\partial d \partial (l_2 - l_1)} = -p_1\frac{\delta}{(1-\delta)[1+(\mu-\delta)d]^2} < 0; \tag{72}$$

$$\frac{\partial^2 V}{\partial d \partial \mu} = p_1(l_1 - l_2)\frac{2\delta d}{(1-\delta)[1+(\mu-\delta)d]^3} > 0; \tag{73}$$

$$\frac{\partial^2 V}{\partial d \partial \delta} = -p_1(l_2 - l_1)\frac{(1-\delta)[1+(\mu-\delta)d]^2 - \delta[-[1+(\mu-\delta)d]^2 - 2d(1-\delta)[1+(\mu-\delta)d]}{(1-\delta)^2[1+(\mu-\delta)d]^4} \tag{74}$$

$$= -p_1(l_2 - l_1)\frac{(1-\delta)[1+(\mu-\delta)d] + \delta[1+(\mu-\delta)d + 2d(1-\delta)]}{(1-\delta)^2[1+(\mu-\delta)d]^3} < 0; \tag{75}$$

as desired. □

Proof of Technical Lemma 1. Taking derivative of r^H and r^L with respect to p_1, p_2, l_1, l_2 and μ, we have:

$$\frac{\partial r^H}{\partial p_1} = -\frac{(1-r^H)^2(\mu - r^H)^2\left(l_2 - l_1 - l_2 p_2 + \frac{c\mu p_2}{(\mu - r^H)(1-r^H)}\right)}{c\mu p_1 p_2(1+\mu-2r^H)} < 0; \tag{76}$$

$$\frac{\partial r^H}{\partial p_2} = -\frac{(1-r^H)^2(\mu - r^H)^2\left(-l_2 p_1 + \frac{c\mu p_1}{(\mu - r^H)(1-r^H)}\right)}{c\mu p_1 p_2(1+\mu-2r^H)} < 0; \tag{77}$$

$$\frac{\partial r^H}{\partial l_1} = -\frac{(1-r^H)^2(\mu - r^H)^2(1-p_1)}{c\mu p_1 p_2(1+\mu-2r^H)} < 0; \tag{78}$$

$$\frac{\partial r^H}{\partial l_2} = -\frac{(1-r^H)^2(\mu - r^H)^2(1-p_2)}{c\mu p_2(1+\mu-2r^H)} < 0; \tag{79}$$

$$\frac{\partial r^H}{\partial \mu} = -\frac{r^H(1-r^H)}{\mu\left[-(1-r^H)-(\mu-r^H)\right]} > 0; \tag{80}$$

$$\frac{\partial r^L}{\partial p_1} = -\frac{(\alpha - r^L)^2(\alpha\mu - r^L)^2\left(\beta l_2 - \beta l_1 - \beta^2 l_2 p_2 + \frac{\alpha^2\beta^2 c\mu p_2}{(\alpha\mu - r^L)(\alpha - r^L)}\right)}{\alpha^2\beta^2 c\mu p_1 p_2(\alpha+\alpha\mu-2r^L)} < 0; \tag{81}$$

$$\frac{\partial r^L}{\partial p_2} = -\frac{(\alpha - r^L)^2(\alpha\mu - r^L)^2\left(-\beta^2 l_2 p_1 + \frac{\alpha^2\beta^2 c\mu p_1}{(\alpha\mu - r^L)(\alpha - r^L)}\right)}{\alpha^2\beta^2 c\mu p_1 p_2(\alpha+\alpha\mu-2r^L)} < 0; \tag{82}$$

$$\frac{\partial r^L}{\partial l_1} = -\frac{(\alpha - r^L)^2(\alpha\mu - r^L)^2(1-\beta p_1)}{\alpha^2\beta^2 c\mu p_1 p_2(\alpha+\alpha\mu-2r^L)} < 0; \tag{83}$$

$$\frac{\partial r^L}{\partial l_2} = -\frac{(\alpha - r^L)^2(\alpha\mu - r^L)^2(1-\beta p_2)}{\alpha^2\beta c\mu p_2(\alpha+\alpha\mu-2r^L)} < 0; \tag{84}$$

$$\frac{\partial r^L}{\partial \mu} = -\frac{r^L(\alpha - r^L)}{\mu\left[-(\alpha - r^L)-(\alpha\mu - r^L)\right]} > 0. \tag{85}$$

Taking derivative of r^L wrt. β, we have:

$$\frac{\partial r^L}{\partial \beta} = -\frac{\alpha\mu c p_1 p_2}{\sqrt{1+\mu^2-\mu(2-\frac{4c\beta^2 p_1 p_2}{c-(1-\beta p_1)l_1-\beta p_1(1-\beta p_2)l_2})}} \cdot \frac{\beta\left[2(c-l_1)+\beta p_1 l_1 - \beta p_1 l_2\right]}{\left(c-(1-\beta p_1)l_1 - \beta p_1(1-\beta p_2)l_2\right)^2} < 0, \tag{86}$$

as $c-(1-\beta p_1)l_1 - \beta p_1(1-\beta p_2)l_2 > 0$ also holds when $p_2 = 0$, we have $2(c-l_1)+\beta p_1 l_1 - \beta p_1 l_2 > c - l_1 + \beta p_1 l_1 - \beta p_1 l_2 > 0$, therefore, $\frac{\partial r^L}{\partial \beta} < 0$. Based on Eq. (20), it is obvious that r^L increases in α. □

Proof of Proposition 4.

We define $\bar{\beta}(\alpha)$ such that $r^L(\bar{\beta}(\alpha)) = r^H$. Based on Technical Lemma 1, we know that when $\beta \geq \bar{\beta}(\alpha)$, we have $r^L \leq r^H$. The bank charges r^H and only the high-type firms trade.

When $\beta < \bar{\beta}(\alpha)$, we first prove that if both types of firm accept the order, there exists $r^P \in (r^H, r^L)$ that makes the bank break-even when financing both types of firms. As the fraction of high-type firms in the market is θ, the pooling interest rate r^P should satisfy the following constraint:

$$\begin{aligned}
&\theta\left((1-p_1)l_1 + p_1(1-p_2)l_2 + p_1p_2\mathbb{E}\left[ce^{r^P(\tilde{T}_1^H+\tilde{T}_2^H)}\right]\right)\\
&+(1-\theta)\left((1-\beta p_1)l_1 + \beta p_1(1-\beta p_2)l_2 + \beta^2 p_1p_2\mathbb{E}\left[ce^{r^P(\tilde{T}_1^L+\tilde{T}_2^L)}\right]\right) = c. \qquad (87)
\end{aligned}$$

By using the fact that $\tilde{T}_1^H \sim Exp(1)$, $\tilde{T}_2^H \sim Exp(\mu)$, $\tilde{T}_1^L \sim Exp(\alpha)$ and $\tilde{T}_2^L \sim Exp(\alpha\mu)$, Eq. (87) can be simplified to:

$$\begin{aligned}
&\theta \cdot \frac{p_1p_2c\mu}{(\mu - r^P)(1-r^P)} + (1-\theta)\cdot\frac{\alpha^2\beta^2p_1p_2c\mu}{(\alpha\mu - r^P)(\alpha - r^P)}\\
&= c - \theta[(1-p_1)l_1 + p_1(1-p_2)l_2] - (1-\theta)[(1-\beta p_1)l_1 + \beta p_1(1-\beta p_2)l_2]. \qquad (88)
\end{aligned}$$

Based on the definition of r^H (Eq. (16)) and r^L (Eq. (19)), we know that:

$$\begin{aligned}
&\theta \cdot \frac{p_1p_2c\mu}{(\mu - r^H)(1-r^H)} + (1-\theta)\cdot\frac{\alpha^2\beta^2p_1p_2c\mu}{(\alpha\mu - r^L)(\alpha - r^L)}\\
&= c - \theta[(1-p_1)l_1 + p_1(1-p_2)l_2] - (1-\theta)[(1-\beta p_1)l_1 + \beta p_1(1-\beta p_2)l_2]. \qquad (89)
\end{aligned}$$

Comparing Eq. (88) and Eq. (89), it is easy to check that the left hand side (LHS) of Eq. (88) is smaller than the right hand side (RHS) when $r^P = r^H$ and vice versa when $r^P = r^L$. As Eq. (88) can be re-arranged as a quartic equation of r^P, $\exists r^P \in (r^H, r^L)$ that satisfies Eq. (88). In case of multiple roots, the market competition pulls r^P to be the smallest one satisfying Eq. (88).

After showing the existence of r^P, we can then analyze the four specific cases when $\beta < \bar{\beta}(\alpha)$. In Case (a), if $\bar{r}^L(\pi_0^L) \leq r^H$, which suggests that the low-type firm's outside option is so attractive that it will not accept interest rate r^H. In this case, the bank offers interest rate r^H and only the high-type firms trade. In Case (b), when $\bar{r}^H(\pi_0^H) > \bar{r}^L(\pi_0^L) \in (r^H,\, r^P)$, the lowest interest rate that prevents low-type firms from trading is $\bar{r}^L(\pi_0^L) + \epsilon$, where ϵ is positive and sufficiently small. As $\bar{r}^H(\pi_0^H) > \bar{r}^L(\pi_0^L)$, high-type firms still trade under this interest rate. In Case (c), if $\min(\bar{r}^H(\pi_0^H),\, \bar{r}^L(\pi_0^L)) > r^P$, both types of firm trade under the pooling interest rate r^P that satisfies Eq. (88). In Case (d), if $\bar{r}^H(\pi_0^H) < \bar{r}^L(\pi_0^L) < r^P$, the lowest interest rate that prevents low-type firms from trading is $\bar{r}^L(\pi_0^L) + \epsilon$, which is higher than $\bar{r}^H(\pi_0^H)$, therefore, both types of firm do not trade; if $\bar{r}^H(\pi_0^H) < r^P < \bar{r}^L(\pi_0^L)$, as $\bar{r}^L(\pi_0^L) \leq r^L$, there does not exist a uniform interest rate that makes the bank break-even and motivates either type of firm to trade, thus the bank does not lend and no firms trade. □

Proof of Technical Lemma 2 We intend to transform the interest rate boundaries in Proposition 4 into β boundaries. We define β_0 that satisfies $\bar{r}^L(\beta_0) = r^L(\beta_0)$, as $\bar{r}^L$ increases in β and r^L decreases in β, we have when $\beta \leq \beta_0$, $\bar{r}^L \leq r^L$. We define β_1 that satisfies $\bar{r}^L(\beta_1) = \bar{r}^H$, as $\bar{\beta}_U$ satisfies $\bar{r}^L(\bar{\beta}_U) = r^H$, we have $\bar{r}^L(\bar{\beta}_U) = r^H < \bar{r}^H = \bar{r}^L(\beta_1)$, thus $\bar{\beta}_U < \beta_1$. We define β_2 that satisfies $\bar{r}^L(\beta_2) = r^P(\beta_2)$, as r^P decreases in β, if $\beta_2 > \beta_0$, we have $r^H < \bar{r}^H = \bar{r}^L(\bar{\beta}_U) < r^L(\beta_0) = \bar{r}^L(\beta_0) < \bar{r}^L(\beta_2) = r^P(\beta_2) < r^L(\beta_2)$, the last inequality holds as r^P lies between r^H and r^L, as $r^L(\beta_0) < r^L(\beta_2)$ when $\beta_2 > \beta_0$ contradicts with the decreasing relationship between r^L and β, thus we have $\beta_2 < \beta_0$. Next, we prove that $\bar{\beta}_U < \beta_2$. If $\bar{\beta}_U > \beta_2$, we have $r^L(\beta_2) < r^P(\beta_2) = \bar{r}^L(\beta_2) < \bar{r}^L(\bar{\beta}_U) = r^H$, as $r^L(\beta_2) < \bar{r}^L(\beta_2)$ indicates $\beta_2 > \beta_0$, which contradicts with $\beta_2 < \beta_0$, thus we have $\bar{\beta}_U < \beta_2$. To summarize, $\forall \pi_0^H$, we have $\bar{\beta}_U < \beta_2 < \beta_0$ and $\bar{\beta}_U < \beta_1$.

We define β_3 that satisfies $r^P(\beta_3) = \bar{r}^H$. If $\beta_1 < \beta_3$, as $\bar{r}^L$ increases in β and r^P decreases in β, we have $\beta_1 < \beta_2 < \beta_3$. Similarly, if $\beta_1 > \beta_3$, we have $\beta_1 > \beta_2 > \beta_3$. We define $\tilde{\pi}_0^H$ that satisfies $\bar{r}^H(\tilde{\pi}_0^H) = r^P(\beta_2) = \bar{r}^L(\beta_2)$, as $\bar{r}^H$ decreases in π_0^H, we have when $\pi_0^H > \tilde{\pi}_0^H$, $\beta_1 < \beta_3$ holds.

Next, we transform boundaries in r in Proposition 4 to boundaries in β based on the above definitions and properties. When $\pi_0^H > \tilde{\pi}_0^H$, we have $\beta_1 < \beta_2 < \beta_3$. When $\beta_3 > \beta_0$, we have $\bar{r}^H = r^P(\beta_3) < r^P(\beta_0)$, we further define $\bar{\pi}_0^H$ that satisfies $\bar{r}^H(\bar{\pi}_0^H) = r^P(\beta_0)$, such that when $\pi_0^H > \bar{\pi}_0^H$, we have $\beta_3 > \beta_0$. As $\bar{r}^H(\tilde{\pi}_0^H) = r^P(\beta_2) > r^P(\beta_0) = \bar{r}^H(\bar{\pi}_0^H)$, we have $\tilde{\pi}_0^H < \bar{\pi}_0^H$. Under the case of $\beta_1 < \beta_2 < \beta_3$ (which is $\pi_0^H > \tilde{\pi}_0^H$), the interest rate regions can be transformed into the following:

1. region $\bar{r}^H(\pi_0^H) > \bar{r}^L(\pi_0^L) \in (r^H,\, r^P)$ can be expressed as $\bar{\beta}_U < \beta < \beta_1$;
2. when $\tilde{\pi}_0^H < \pi_0^H < \bar{\pi}_0^H$, region $\bar{r}^H(\pi_0^H) < \min(\bar{r}^L(\pi_0^L),\, r^P)$ can be expressed as $\beta_1 < \beta < \beta_3$; when $\pi_0^H > \bar{\pi}_0^H$, the regions becomes $\beta_1 < \beta < \beta_0$;
3. when $\tilde{\pi}_0^H < \pi_0^H < \bar{\pi}_0^H$, region $\min(\bar{r}^H(\pi_0^H),\, \bar{r}^L(\pi_0^L)) > r^P$ and $\bar{r}^L > \bar{r}^H$ can be expressed as $\beta_3 < \beta < \beta_0$;
4. region $\min(\bar{r}^H(\pi_0^H),\, \bar{r}^L(\pi_0^L)) > r^P$ and $\bar{r}^L < \bar{r}^H$ does not exist.

Summarizing the above results, we have the equilibrium outcomes as in Figure 6 and 7. Costly separation refers to Case 2(b) in Proposition 4, pooling refers to Case 2(c) in Proposition 4, and market collapse refers to Case 2(d) in Proposition 4. Costless separation indicates uniform financing does not create inefficiencies.

When $\pi_0^H < \tilde{\pi}_0^H$, we have $\beta_1 > \beta_2 > \beta_3$. When $\beta_1 < \beta_0$, we have $\bar{r}^H = \bar{r}^L(\beta_1) < \bar{r}^L(\beta_0)$. We further denote $\hat{\pi}_0^H$ that satisfies $\bar{r}^H(\hat{\pi}_0^H) = \bar{r}^L(\beta_0)$, such that when $\pi_0^H > \hat{\pi}_0^H$, we have $\beta_1 < \beta_0$. As $\bar{r}^H(\hat{\pi}_0^H) = \bar{r}^L(\beta_0) > \bar{r}^L(\beta_2) = \bar{r}^H(\tilde{\pi}_0^H)$, we have $\hat{\pi}_0^H < \tilde{\pi}_0^H$. Under the case of $\beta_1 > \beta_2 > \beta_3$ (which is $\pi_0^H < \tilde{\pi}_0^H$), the interest rate regions can be transformed into the following:

1. region $\bar{r}^H(\pi_0^H) > \bar{r}^L(\pi_0^L) \in (r^H,\, r^P)$ can be expressed as $\bar{\beta}_U < \beta < \beta_2$;
2. region $\bar{r}^H(\pi_0^H) < \min(\bar{r}^L(\pi_0^L),\, r^P)$ does not exist;
3. when $\hat{\pi}_0^H < \pi_0^H < \tilde{\pi}_0^H$, region $\min(\bar{r}^H(\pi_0^H),\, \bar{r}^L(\pi_0^L)) > r^P$ and $\bar{r}^L < \bar{r}^H$ can be expressed as $\beta_2 < \beta < \beta_1$; when $\pi_0^H < \hat{\pi}_0^H$, the region becomes $\beta_2 < \beta < \beta_0$;
4. when $\hat{\pi}_0^H < \pi_0^H < \tilde{\pi}_0^H$, region $\min(\bar{r}^H(\pi_0^H),\, \bar{r}^L(\pi_0^L)) > r^P$ and $\bar{r}^L > \bar{r}^H$ can be expressed as $\beta_1 < \beta < \beta_0$.

Summarizing the above results, we have the equilibrium outcomes as in Figure 8 and 9.

Summarizing all the four cases (or figures), if we define $\hat{\beta} := \min(\beta_1, \beta_2)$, $\tilde{\beta} := \min(\beta_0, 1, \max(\beta_1, \beta_3))$, and $\bar{\beta} := \min(\beta_0, 1)$, the above results can be summarized as follows:

1. When $\bar{\beta}_U < \beta \leq \hat{\beta}$, the equilibrium is costly separation;
2. When $\hat{\beta} < \beta \leq \tilde{\beta}$ and $\pi_0^H > \tilde{\pi}_0^H$, the equilibrium is market collapse;
3. When $\hat{\beta} < \beta \leq \tilde{\beta}$ and $\pi_0^H < \tilde{\pi}_0^H$, the equilibrium is pooling;

Figure 6 **Equilibrium Outcomes under Uniform Financing when $\pi_0^H > \bar{\pi}_0^H$.**

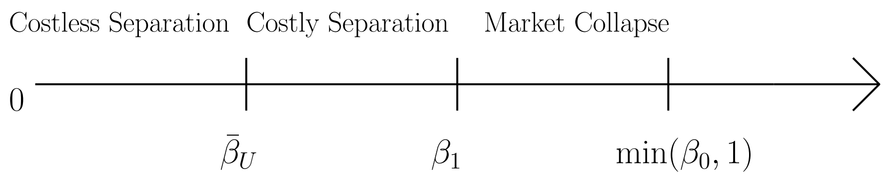


Figure 7 **Equilibrium Outcomes under Uniform Financing when $\tilde{\pi}_0^H < \pi_0^H < \bar{\pi}_0^H$.**

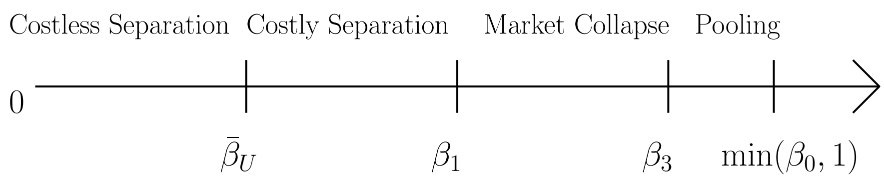


Figure 8 **Equilibrium Outcomes under Uniform Financing when $\hat{\pi}_0^H < \pi_0^H < \tilde{\pi}_0^H$.**

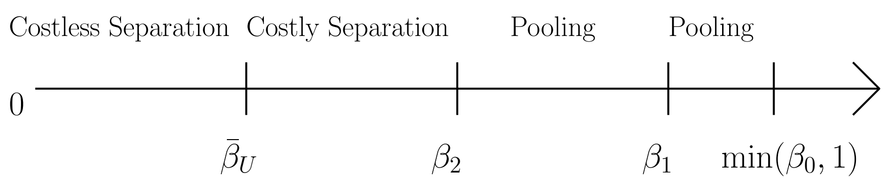
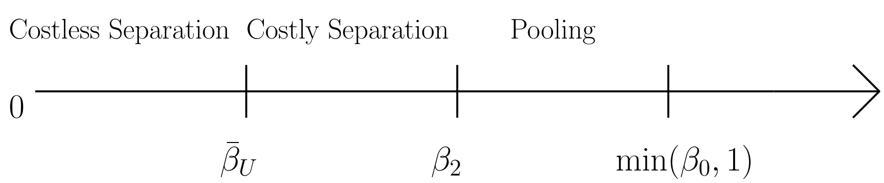


Figure 9 **Equilibrium Outcomes under Uniform Financing when $\pi_0^H < \hat{\pi}_0^H$.**



4. When $\tilde{\beta} < \beta \leq \bar{\beta}$, the equilibrium is pooling.

Summarizing the above results, we have Technical Lemma 2. □

Proof of Technical Lemma 3. By using the fact that $\tilde{T}_1^H \sim Exp(1)$, $\tilde{T}_2^H \sim Exp(\mu)$, $\tilde{T}_1^L \sim Exp(\alpha)$ and $\tilde{T}_2^L \sim Exp(\alpha\mu)$, Eq. (23) becomes:

$$\Pi_D^H = p_1 p_2 \left[w - \frac{c\mu(\lambda+\mu-r_2)}{(\lambda+\mu-r_1)(1-r_1)(\mu-r_2)} \right] > \pi_0^H; \tag{90}$$

Eq. (24) becomes:

$$\Pi_D^L = \beta^2 p_1 p_2 \left[w - \frac{\alpha^2 c\mu(\lambda+\alpha\mu-r_2)}{(\alpha-r_1)(\lambda+\alpha\mu-r_1)(\alpha\mu-r_2)} \right] \leq \pi_0^L, \tag{91}$$

or equivalently,

$$\beta \leq \sqrt{\frac{\pi_0^L}{p_1 p_2 \left[w - \frac{\alpha^2 c\mu(\lambda+\alpha\mu-r_2)}{(\alpha-r_1)(\lambda+\alpha\mu-r_1)(\alpha\mu-r_2)} \right]}}\ ; \tag{92}$$

and Eq. (25) becomes:

$$\frac{c\mu p_1 p_2(\lambda+\mu-r_2)}{(\lambda+\mu-r_1)(1-r_1)(\mu-r_2)} = c - (1-p_1)l_1 - p_1(1-p_2)l_2. \tag{93}$$

Taking Eq. (93) into Eq. (90), we have $\Pi_D^H = p_1p_2w + (1-p_1)l_1 + p_1(1-p_2)l_2 - c = \Pi_0^H > \pi_0^H$. This suggests that under bank's competitive loan pricing, the high-type firm's IR constraint is always satisfied.

We define:

$$\bar{\beta}_D := \min\left(\sqrt{\frac{\pi_0^L}{p_1p_2\left[w - \frac{\alpha^2 c\mu(\lambda+\alpha\mu-r_2)}{(\alpha-r_1)(\lambda+\alpha\mu-r_1)(\alpha\mu-r_2)}\right]}}\ ,\ 1\right), \tag{94}$$

where (r_1, r_2) satisfies Eq. (93). Next, we will prove that $\bar{\beta}_D$ decreases in r_2. Re-arrange Eq. (93), we have:

$$r_1(r_2) = \frac{1}{2}\left(1+\mu+\lambda - \sqrt{(\lambda+\mu-1)^2 + \frac{4c\mu(\lambda+\mu-r_2)p_1p_2}{(c-(1-p_1)l_1 - p_1(1-p_2)l_2)(\mu-r_2)}}\right), \tag{95}$$

taking derivative wrt. r_2, we have:

$$\frac{\partial r_1(r_2)}{\partial r_2} = -\frac{1}{4}\left((\lambda+\mu-1)^2 + \frac{4c\mu(\lambda+\mu-r_2)p_1p_2}{(c-(1-p_1)l_1-p_1(1-p_2)l_2)(\mu-r_2)}\right)^{-1/2} \cdot \frac{4c\mu\lambda p_1p_2}{(c-(1-p_1)l_1-p_1(1-p_2)l_2)(\mu-r_2)^2} < 0. \tag{96}$$

Based on the definition of $\bar{\beta}_D$, we know that when $\bar{\beta}_D < 1$, we have:

$$(\bar{\beta}_D)^2 p_1p_2\left[w - \frac{\alpha^2 c\mu(\lambda+\alpha\mu-r_2)}{(\alpha-r_1)(\lambda+\alpha\mu-r_1)(\alpha\mu-r_2)}\right] = \pi_0^L, \tag{97}$$

taking derivative wrt. r_2 on both sides of the above equation, we have:

$$\begin{aligned}&\frac{(\bar{\beta}_D)^2\alpha^2 c\mu p_1p_2\left[(\alpha-r_1)(\lambda+\alpha\mu-r_1)(\alpha\mu-r_2) + (\lambda+\alpha\mu-r_2)\left((\alpha\mu-r_2)(2r_1-\alpha-\lambda-\alpha\mu)\frac{\partial r_1}{\partial r_2}\right)\right]}{(\alpha-r_1)^2(\lambda+\alpha\mu-r_1)^2(\alpha\mu-r_2)^2}\\ &+2\bar{\beta}_D p_1p_2\left[w - \frac{\alpha^2 c\mu(\lambda+\alpha\mu-r_2)}{(\alpha-r_1)(\lambda+\alpha\mu-r_1)(\alpha\mu-r_2)}\right]\frac{\partial\bar{\beta}_D}{\partial r_2} = 0,\end{aligned} \tag{98}$$

as $2r_1 - \alpha - \lambda - \alpha\mu < 0$ and $\frac{\partial r_1}{\partial r_2} < 0$, we have $\frac{\partial\bar{\beta}_D}{\partial r_2} < 0$ in order to satisfy the above equation. Therefore, $\bar{\beta}_D$ decreases in r_2, and when $\beta \le \bar{\beta}_D$, the bank can separate the two types of firms by offering a competitively priced DTF contract. □

Proof of Proposition 5.

According to Technical Lemma 3, $\bar{\beta}_D$ decreases in r_2. Therefore, the maximum $\bar{\beta}_D$ is achieved when $r_2 = 0$. Taking $r_2 = 0$ into Eq. (95), we have (r_1^*, r_2^*) as in Eq. (26). Taking (r_1^*, r_2^*) into Eq. (94), we have $\bar{\beta}_D^*$ as in Eq. (27). As this DTF contract can separate the two types of firms, the high-type firm trades with a payoff $\Pi_D^H = \Pi_0^H$, whereas the low-type firm does not trade with a payoff $\Pi_D^L = \pi_0^L$.

Next, we will prove that $\bar{\beta}_D^*$ increases in λ when $\alpha < \bar{\alpha}$. Based on the expression of r_1^*, taking derivative of r_1^* wrt. λ, we have:

$$\frac{\partial r_1^*}{\partial\lambda} = \frac{r_1^*(1-r_1^*)}{(\lambda+\mu)(\lambda+\mu+1-2r_1^*)} > 0. \tag{99}$$

Taking derivative of $\bar{\beta}_D^*$ wrt. λ, we have:

$$\frac{\partial\bar{\beta}_D^*}{\partial\lambda} = \frac{\pi_0^L\alpha c\left[-(\alpha-r_1^*)r_1^* + (\lambda+\alpha\mu)(\lambda+\alpha(1+\mu)-2r_1^*)\frac{\partial r_1^*}{\partial\lambda}\right]}{2p_1p_2(\alpha-r_1^*)^2(\lambda+\alpha\mu-r_1^*)^2\left[w - \frac{\alpha c(\lambda+\alpha\mu)}{(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)}\right]^2\sqrt{\frac{\pi_0^L}{p_1p_2\left[w-\frac{\alpha c(\lambda+\alpha\mu)}{(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)}\right]}}}. \tag{100}$$

We denote

$$\begin{aligned}K(\alpha) &= -(\alpha-r_1^*)r_1^* + (\lambda+\alpha\mu)(\lambda+\alpha(1+\mu)-2r_1^*)\frac{\partial r_1^*}{\partial\lambda}\\ &= \mu(1+\mu)\frac{\partial r_1^*}{\partial\lambda}\alpha^2 + \left[-r_1^* + \lambda(1+\mu)\frac{\partial r_1^*}{\partial\lambda} + \mu(\lambda-2r_1^*)\frac{\partial r_1^*}{\partial\lambda}\right]\alpha + \lambda(\lambda-2r_1^*)\frac{\partial r_1^*}{\partial\lambda} + (r_1^*)^2.\end{aligned} \tag{101}$$

Taking $\frac{\partial r_1^*}{\partial \lambda}$ into the above equation, we have $K(\alpha=1)=0$ and $K(\alpha=r_1^*)>0$. As $\frac{\partial r_1^*}{\partial \lambda}>0$, there exists $\bar{\bar{\alpha}}>r_1^*$ that satisfies $K(\bar{\bar{\alpha}})=0$. We define $\bar{\alpha}:=\min(\bar{\bar{\alpha}},1)$, such that when $\alpha<\bar{\alpha}$, we have $K(\alpha)>0$ and $\frac{\partial \bar{\beta}_D^*}{\partial \lambda}>0$. As $\bar{\beta}_U$ corresponds to the case when $\lambda \to 0$, we have $\bar{\beta}_D^* \geq \bar{\beta}_U$. □

Proof of Proposition 6.

When $\bar{\beta}_D^*<1$, taking derivative of $\bar{\beta}_D^*$ wrt. α, we have:

$$\frac{\partial \bar{\beta}_D^*}{\partial \alpha} = -\frac{c\pi_0^L r_1^*\left[(\lambda-r_1^*)(\lambda+2\alpha\mu)+2\alpha^2\mu^2\right]}{2p_1p_2\left[\alpha c(\lambda+\alpha\mu)-(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)w\right]^2\sqrt{\frac{\pi_0^L}{p_1p_2\left[w-\frac{\alpha c(\lambda+\alpha\mu)}{(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)}\right]}}}. \tag{102}$$

Based on the definition of r_1^*, we have $r_1^*<\frac{1}{2}\left(1+\lambda+\mu-|\lambda+\mu-1|\right)$. If $\lambda+\mu>1$, we have $r_1^*<\frac{1}{2}<\lambda+\mu$; if $\lambda+\mu<1$, we have $r_1^*<\lambda+\mu$. As the above condition satisfies for any $\mu>0$, we have $r_1^*<\lambda$. Therefore, $\frac{\partial \bar{\beta}_D^*}{\partial \alpha}<0$.

Taking derivative of r_1^* wrt. p_1, p_2, l_1 and l_2, we have:

$$\frac{\partial r_1^*}{\partial p_1} = -\frac{c(\lambda+\mu)p_2(c-l_1)}{\left[c-(1-p_1)l_1-p_1(1-p_2)l_2\right]^2\sqrt{(\lambda+\mu-1)^2+\frac{4c(\lambda+\mu)p_1p_2}{c-(1-p_1)l_1-p_1(1-p_2)l_2}}}<0; \tag{103}$$

$$\frac{\partial r_1^*}{\partial p_2} = -\frac{c(\lambda+\mu)p_1(c-(1-p_1)l_1-p_1l_2)}{\left[c-(1-p_1)l_1-p_1(1-p_2)l_2\right]^2\sqrt{(\lambda+\mu-1)^2+\frac{4c(\lambda+\mu)p_1p_2}{c-(1-p_1)l_1-p_1(1-p_2)l_2}}}<0; \tag{104}$$

$$\frac{\partial r_1^*}{\partial l_1} = -\frac{c(\lambda+\mu)(1-p_1)p_1p_2}{\left[c-(1-p_1)l_1-p_1(1-p_2)l_2\right]^2\sqrt{(\lambda+\mu-1)^2+\frac{4c(\lambda+\mu)p_1p_2}{c-(1-p_1)l_1-p_1(1-p_2)l_2}}}<0; \tag{105}$$

$$\frac{\partial r_1^*}{\partial l_2} = -\frac{c(\lambda+\mu)(1-p_2)p_1^2p_2}{\left[c-(1-p_1)l_1-p_1(1-p_2)l_2\right]^2\sqrt{(\lambda+\mu-1)^2+\frac{4c(\lambda+\mu)p_1p_2}{c-(1-p_1)l_1-p_1(1-p_2)l_2}}}<0. \tag{106}$$

When $\bar{\beta}_D^*<1$, taking derivative of $\bar{\beta}_D^*$ wrt. p_1 and p_2, we have:

$$\frac{\partial \bar{\beta}_D^*}{\partial p_1} = -\frac{p_2}{2\pi_0^L}\left(\frac{\pi_0^L}{p_1p_2\left[w-\frac{\alpha c(\lambda+\alpha\mu)}{(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)}\right]}\right)^{3/2}\times \tag{107}$$

$$\left[\left(w-\frac{\alpha c(\lambda+\alpha\mu)}{(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)}\right)-\frac{p_1\alpha c(\lambda+\alpha\mu)\left[\lambda+(1+\mu)\alpha-2r_1^*\right]\frac{\partial r_1^*}{\partial p_2}}{(\alpha-r_1^*)^2(\lambda+\alpha\mu-r_1^*)^2}\right]<0; \tag{108}$$

$$\frac{\partial \bar{\beta}_D^*}{\partial p_2} = -\frac{p_1}{2\pi_0^L}\left(\frac{\pi_0^L}{p_1p_2\left[w-\frac{\alpha c(\lambda+\alpha\mu)}{(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)}\right]}\right)^{3/2}\times \tag{109}$$

$$\left[\left(w-\frac{\alpha c(\lambda+\alpha\mu)}{(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)}\right)-\frac{p_2\alpha c(\lambda+\alpha\mu)\left[\lambda+(1+\mu)\alpha-2r_1^*\right]\frac{\partial r_1^*}{\partial p_2}}{(\alpha-r_1^*)^2(\lambda+\alpha\mu-r_1^*)^2}\right]<0. \tag{110}$$

Based on the proof of Proposition 5, we know that when $\alpha<\bar{\alpha}$, $\bar{\beta}_D^*$ increases in λ, thus decreases in the average information delay $\bar{d}=\frac{1}{\lambda}$. Taking derivative of $\bar{\beta}_D^*$ wrt. μ, we have:

$$\frac{\partial \bar{\beta}_D^*}{\partial \mu} = \frac{\pi_0^L\alpha c\left[-\alpha(\alpha-r_1^*)r_1^*+(\lambda+\alpha\mu)(\lambda+\alpha(1+\mu)-2r_1^*)\frac{\partial r_1^*}{\partial \lambda}\right]}{2p_1p_2(\alpha-r_1^*)^2(\lambda+\alpha\mu-r_1^*)^2\left[w-\frac{\alpha c(\lambda+\alpha\mu)}{(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)}\right]^2\sqrt{\frac{\pi_0^L}{p_1p_2\left[w-\frac{\alpha c(\lambda+\alpha\mu)}{(\alpha-r_1^*)(\lambda+\alpha\mu-r_1^*)}\right]}}}. \tag{111}$$

Comparing Eq. (111) with Eq. (100), we can suggest that when $\alpha<\bar{\alpha}$, we have $\frac{\partial \bar{\beta}_D^*}{\partial \mu}>\frac{\partial \bar{\beta}_D^*}{\partial \lambda}>0$. As $\frac{\partial r_1^*}{\partial l_1}<0$ and $\frac{\partial r_1^*}{\partial l_2}<0$, based on the expression of $\bar{\beta}_D^*$ in Eq. (27), we know that $\bar{\beta}_D^*$ decreases in l_1, l_2 and w. □

Proof of Proposition 7. First, we would like to prove $\bar{\beta}_D^* < \hat{\beta} = \min(\beta_1, \beta_2)$ satisfies for any $\lambda > 0$, where $\hat{\beta}, \beta_1, \beta_2$ are given in Technical Lemma 2 and its proof. As $\bar{\beta}_D^*$ increases in λ according to the proof of Proposition 5, the maximum of $\bar{\beta}_D^*$ obtains when $\lambda \to \infty$. In this case, there is no information delay, the DTF contract $(r_1^*(\lambda \to \infty), 0)$ and $\bar{\beta}_D^*(\lambda \to \infty)$ satisfy:

$$\left(\bar{\beta}_D^*(\lambda \to \infty)\right)^2 p_1 p_2 \left[w - \frac{\alpha c}{\alpha - r_1^*(\lambda \to \infty)}\right] = \pi_0^L; \tag{112}$$

$$(1-p_1)l_1 + p_1(1-p_2)l_2 + \frac{c p_1 p_2}{1 - r_1^*(\lambda \to \infty)} = c. \tag{113}$$

As β_1 satisfies $\bar{r}^L(\beta_1) = \bar{r}^H$, we have:

$$\beta_1^2 p_1 p_2 \left[w - \frac{\alpha c}{\alpha - \bar{r}^H} \cdot \frac{\alpha \mu}{\alpha \mu - \bar{r}^H}\right] = \pi_0^L; \tag{114}$$

as

$$p_1 p_2 \left[w - \frac{c\mu}{(1-\bar{r}^H)(\mu - \bar{r}^H)}\right] = \pi_0^H < \Pi_0^H = (1-p_1)l_1 + p_1(1-p_2)l_2 + p_1 p_2 w - c, \tag{115}$$

we have

$$(1-p_1)l_1 + p_1(1-p_2)l_2 + \frac{c\mu p_1 p_2}{(1-\bar{r}^H)(\mu - \bar{r}^H)} > c. \tag{116}$$

Compare Eq. (113) and Eq. (116), we have

$$\frac{\mu}{(1-\bar{r}^H)(\mu - \bar{r}^H)} > \frac{1}{1 - r_1^*(\lambda \to \infty)}. \tag{117}$$

Next, we prove that as Eq. (117) satisfies for any $\mu > 0$, we have the following inequality:

$$\frac{\alpha}{\alpha - \bar{r}^H} \cdot \frac{\alpha \mu}{\alpha \mu - \bar{r}^H} > \frac{\alpha}{\alpha - r_1^*(\lambda \to \infty)}. \tag{118}$$

Taking derivative of $\frac{\mu}{\mu - \bar{r}^H}$ wrt. μ, we have:

$$\frac{\partial \left(\frac{\mu}{\mu - \bar{r}^H}\right)}{\partial \mu} = \frac{-\bar{r}^H + \mu \frac{\partial \bar{r}^H}{\partial \mu}}{(\mu - \bar{r}^H)^2}. \tag{119}$$

As $\bar{r}^H$ satisfies

$$p_1 p_2 \left[w - \frac{c\mu}{(1-\bar{r}^H)(\mu - \bar{r}^H)}\right] = \pi_0^H, \tag{120}$$

if $\frac{\partial \bar{r}^H}{\partial \mu} < 0$, then based on Eq. (120) we have $\partial \left(\frac{\mu}{\mu - \bar{r}^H}\right) / \partial \mu > 0$, which contradicts with Eq. (119). Therefore, we have $\frac{\partial \bar{r}^H}{\partial \mu} > 0$, in this case, based on Eq. (120) we have $\partial \left(\frac{\mu}{\mu - \bar{r}^H}\right) / \partial \mu < 0$. The minimum of $\frac{\mu}{\mu - \bar{r}^H}$ obtains when $\mu \to \infty$, as Eq. (117) satisfies for any μ, we have $\frac{1}{1-\bar{r}^H} > \frac{1}{1 - r_1^*(\lambda \to \infty)}$, which is $\bar{r}^H > r_1^*(\lambda \to \infty)$. Under this condition, it is easy to see that Eq. (118) holds. Comparing Eq. (112) and Eq. (114), we have $\beta_1 > \bar{\beta}_D^*(\lambda \to \infty)$.

As β_2 satisfies $\bar{r}^L(\beta_2) = r^P(\beta_2) \in (r^H, r^L)$, we have:

$$\beta_2^2 p_1 p_2 \left[w - \frac{\alpha c}{\alpha - r^P} \cdot \frac{\alpha \mu}{\alpha \mu - r^P}\right] = \pi_0^L; \tag{121}$$

$$(1-p_1)l_1 + p_1(1-p_2)l_2 + \frac{c\mu p_1 p_2}{(1-r^P)(\mu - r^P)} > c. \tag{122}$$

Following the same reasoning as β_1, we have $\beta_2 > \bar{\beta}_D^*(\lambda \to \infty)$. Therefore, $\bar{\beta}_D^* < \hat{\beta} = \min(\beta_1, \beta_2)$ satisfies for any $\lambda > 0$.

Based on Technical Lemma 2, we know that when $\bar{\beta}_U < \beta \leq \hat{\beta}$, the type of inefficiencies under UF is characterized by Case 2(b) in Proposition 4. Therefore, the value of DTF under this case is:

$$V^A = \theta\left(\Pi_0^H - p_1p_2\left[w - \frac{c\mu}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))}\right]\right)^+ \tag{123}$$

$$= \theta\left[c\left(\frac{p_1p_2\mu}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))} - 1\right) + (1-p_1)l_1 + p_1(1-p_2)l_2\right]^+. \tag{124}$$

Next, we prove the sensitivities of V^A wrt. various modeling parameters. Based on the definition of $\bar{r}^L(\pi_0^L)$, we have:

$$\beta^2 p_1p_2\left[w - \frac{\alpha^2 c\mu}{(\alpha-\bar{r}^L(\pi_0^L))(\alpha\mu-\bar{r}^L(\pi_0^L))}\right] = \pi_0^L. \tag{125}$$

It is easy to check that $\bar{r}^L(\pi_0^L)$ increases with β, p_1 and p_2. Taking derivative wrt. α on both sides of the above equation, we have:

$$\frac{\alpha\beta^2 c\mu p_1p_2\left[\alpha+\alpha\mu-2\bar{r}^L(\pi_0^L)\right]\left[-\bar{r}^L(\pi_0^L)+\alpha\frac{\partial\bar{r}^L(\pi_0^L)}{\partial\alpha}\right]}{(\alpha-\bar{r}^L(\pi_0^L))^2(\alpha\mu-\bar{r}^L(\pi_0^L))^2} = 0, \tag{126}$$

as $\alpha+\alpha\mu-2\bar{r}^L(\pi_0^L) > 0$, we have $\frac{\partial\bar{r}^L(\pi_0^L)}{\partial\alpha} = \frac{\bar{r}^L(\pi_0^L)}{\alpha} > 0$. Therefore, V^A increases with α. As $\bar{r}^L(\pi_0^L)$ increases with β, we have V^A increases with β. As $\bar{r}^L(\pi_0^L)$ does not depend on l_1 and l_2, based on the expression of V^A, we know that V^A increases in l_1 and l_2. Taking derivative of V^A wrt. p_1, we have:

$$\frac{\partial V^A}{\partial p_1} = \theta\left[-l_1 + (1-p_2)l_2 + \frac{c\mu p_2}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))} + \frac{c\mu p_1p_2(1+\mu-2\bar{r}^L(\pi_0^L))\frac{\partial\bar{r}^L(\pi_0^L)}{\partial p_1}}{(1-\bar{r}^L(\pi_0^L))^2(\mu-\bar{r}^L(\pi_0^L))^2}\right], \tag{127}$$

Based on the definition of r^H, we have:

$$(1-p_1)l_1 + p_1(1-p_2)l_2 + p_1p_2\mathbb{E}[ce^{r^H(\tilde{T}_1^H+\tilde{T}_2^H)}] = c \tag{128}$$

$$(1-p_1)l_1 + p_1(1-p_2)l_2 + \frac{c\mu p_1p_2}{(1-r^H)(\mu-r^H)} = c \tag{129}$$

$$p_1\left(-l_1 + (1-p_2)l_2 + \frac{c\mu p_2}{(1-r^H)(\mu-r^H)}\right) = \frac{c-l_1}{p_1} \tag{130}$$

As V^A obtains when $\bar{r}^L(\pi_0^L) > r^H$, we have:

$$p_1\left(-l_1 + (1-p_2)l_2 + \frac{c\mu p_2}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))}\right) > p_1\left(-l_1 + (1-p_2)l_2 + \frac{c\mu p_2}{(1-r^H)(\mu-r^H)}\right) > 0. \tag{131}$$

As $\frac{\partial\bar{r}^L(\pi_0^L)}{\partial p_1} > 0$ and $1+\mu-2\bar{r}^L(\pi_0^L) > 0$, we have $\frac{\partial V^A}{\partial p_1} > 0$. Similarly, taking derivative of V^A wrt. p_2, we have:

$$\frac{\partial V^A}{\partial p_2} = \theta\left[-l_2p_1 + c\left(\frac{\mu p_1}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))} + \frac{\mu p_1p_2\frac{\partial\bar{r}^L(\pi_0^L)}{\partial p_2}}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))^2} + \frac{\mu p_1p_2\frac{\partial\bar{r}^L(\pi_0^L)}{\partial p_2}}{(1-\bar{r}^L(\pi_0^L))^2(\mu-\bar{r}^L(\pi_0^L))}\right)\right] \tag{132}$$

Based on the definition of r^H and $\bar{r}^L(\pi_0^L) > r^H$, we have:

$$-l_2p_1 + \frac{c\mu p_1}{(1-\bar{r}^L(\pi_0^L))(\mu-\bar{r}^L(\pi_0^L))} > -l_2p_1 + \frac{c\mu p_1}{(1-r^H)(\mu-r^H)} = \frac{c-(1-p_1)l_1-p_1l_2}{p_2} > 0. \tag{133}$$

As $\frac{\partial\bar{r}^L(\pi_0^L)}{\partial p_2} > 0$, we have $\frac{\partial V^A}{\partial p_2} > 0$. To summarize, we have proved that V^A increases in α, β, p_1, p_2, l_1 and l_2.

Based on the expression of V^A, we can suggest that as $\bar{r}^L(\pi_0^L)$ does not depend on $\bar{d}$, the magnitude of V^A does not depend on $\bar{d}$. However, as $\bar{d}$ decreases, $\bar{\beta}_D^*$ increases, the value of DTF (V^A) either increase from zero to a positive value or remain unchanged, thus proving the decreasing relationship between V^A and $\bar{d}$. For cross derivatives, it is the same with the sensitivities of V^A wrt. various modeling parameters. □

Proof of Proposition B.2. When $\beta > \bar{\beta}^*_D$, there are three types of inefficiencies under uniform financing (Case 2(b), 2(c) and 2(d) in Proposition 4). For the three inefficient cases, the firm may choose to reveal its type when the fixed cost F is small as follows:

1. For Case 2(b) in Proposition 4, if the high-type firm chooses to signal, its profit is $\Pi^H_U = \Pi^H_0 - F$. This profit is higher than the high-type firm's profit without signaling (which is $p_1p_2\left[w - \frac{c\mu}{(1-\bar{r}^L(\pi^L_0))(\mu-\bar{r}^L(\pi^L_0))}\right]$) when $F < c\left[\frac{p_1p_2\mu}{(1-\bar{r}^L(\pi^L_0))(\mu-\bar{r}^L(\pi^L_0))} - 1\right] + (1-p_1)l_1 + p_1(1-p_2)l_2$.
2. For Case 2(c) in Proposition 4, if the high-type firm chooses to signal, its profit is $\Pi^H_U = \Pi^H_0 - F$. This profit is higher than the high-type firm's profit without signaling (which is $p_1p_2\left[w - \frac{c\mu}{(1-r^P)(\mu-r^P)}\right]$) when $F < c\left[\frac{p_1p_2\mu}{(1-r^P)(\mu-r^P)} - 1\right] + (1-p_1)l_1 + p_1(1-p_2)l_2$.
3. For Case 2(d) in Proposition 4, if the high-type firm chooses to signal, its profit is $\Pi^H_U = \Pi^H_0 - F$. This profit is higher than the high-type firm's profit without signaling (which is π^H_0) when $F < c\left[\frac{p_1p_2\mu}{(1-\bar{r}^H(\pi^H_0))(\mu-\bar{r}^H(\pi^H_0))} - 1\right] + (1-p_1)l_1 + p_1(1-p_2)l_2$.

Summarizing the above three cases, when $F < c\left[\frac{p_1p_2\mu}{(1-\min(\bar{r}^H(\pi^H_0),\bar{r}^L(\pi^L_0),r^P))(\mu-\min(\bar{r}^H(\pi^H_0),\bar{r}^L(\pi^L_0),r^P))} - 1\right] + (1-p_1)l_1 + p_1(1-p_2)l_2$, the high type firm reveals its type, the bank offers interest rate r^H and only the high-type firm trades. Therefore, $\Pi^H_U = \Pi^H_0 - F$ and $\Pi^L_U = \pi^L_0$. □

Proof of Proposition D.1. The pooling interest rate r^P is determined according to Eq. (87). As $r^H < r^L$, we have $r^H < r^P < r^L$ when $\theta \in (0,1)$. If $\bar{r}^H(\pi^H_0) \le r^P$, the high-type firms will not trade under r^P. Therefore, the bank offers r^L and only the low-type firms trade. If $\bar{r}^H(\pi^H_0) > r^P$, the high-type firms will trade under r^P. As both types of firm trade, there is no efficiency loss. However, as $r^H < r^P < r^L$, the high-type firms subsidize the low-type firms, creating unfairness. □

Proof of Proposition D.2. In order to use (r^H_1, r^H_2) and r^L to screen two types of firms, the interest rates must satisfy the following Incentive Compatibility (IC) constraints:

$$p_1p_2\left(w - \mathbb{E}\left[ce^{r^H_1\tilde{T}^H_1 + r^H_2\tilde{T}^H_2}\right]\right) \ge p_1p_2\left(w - \mathbb{E}\left[ce^{r^L(\tilde{T}^H_1+\tilde{T}^H_2)}\right]\right); \tag{134}$$

$$\beta^2p_1p_2\left(w - \mathbb{E}\left[ce^{r^L(\tilde{T}^L_1+\tilde{T}^L_2)}\right]\right) \ge \beta^2p_1p_2\left(w - \mathbb{E}\left[ce^{r^H_1\tilde{T}^L_1 + r^H_2\tilde{T}^L_2}\right]\right). \tag{135}$$

The above constraints are based on the firm's expected profit stated in (8) for the case when $\tilde{d} = 0$ and when the seller chooses different contracts. Specifically, observe that (134) guarantees that the high-type firm's expected profit under the DTF contract is higher than that of under the uniform contract. Similarly, (135) ensures that the low-type firm's expected profit under the uniform contract is higher than that of under the DTF contract. By using the fact that $\tilde{T}^H_1 \sim Exp(1)$, $\tilde{T}^H_2 \sim Exp(\mu)$, $\tilde{T}^L_1 \sim Exp(\alpha)$, and $\tilde{T}^L_2 \sim Exp(\alpha\mu)$, we can simplify the IC constraints as:

$$\frac{1}{(1-r^L)(\mu-r^L)} \ge \frac{1}{(1-r^H_1)(\mu-r^H_2)}; \tag{136}$$

$$\frac{1}{(\alpha-r^H_1)(\alpha\mu-r^H_2)} \ge \frac{1}{(\alpha-r^L)(\alpha\mu-r^L)}. \tag{137}$$

Under the separating equilibrium, the bank applies (2) to determine the interest rates competitively under the DTF contract (the uniform contract) for the high-type (low-type) firm. By using the assumptions that

$l_1 = l_2 = 0$, $\delta = 0$ and the time delay $\tilde{d} = 0$, we can use the same approach (i.e., those three cases) presented in §4.1 and §4.2 and apply (2) to show that the uniform interest rate r^L and the DTF interest rates (r_1^H, r_2^H) satisfy:

$$p_1 p_2 \mathbb{E}\left[c e^{r_1^H \tilde{T}_1^H + r_2^H \tilde{T}_2^H} \right] = c; \tag{138}$$

$$\beta^2 p_1 p_2 \mathbb{E}\left[c e^{r^L(\tilde{T}_1^L + \tilde{T}_2^L)} \right] = c. \tag{139}$$

We can solve the above equations and get:

$$r^L = \frac{\alpha}{2}\left(1 + \mu - \sqrt{(\mu-1)^2 + 4\mu\beta^2 p_1 p_2}\right); \tag{140}$$

$$r_2^H = \mu - \frac{p_1 p_2 \mu}{1 - r_1^H}. \tag{141}$$

Substituting r^L and r_2^H into (136), we get:

$$p_1 p_2 \mu > (1 - r^L)(\mu - r^L), \tag{142}$$

as r^L decreases in β according to Technical Lemma 1, we define $\hat{\hat{\beta}}$ that satisfies:

$$p_1 p_2 \mu = (1 - r^L(\hat{\hat{\beta}}))(\mu - r^L(\hat{\hat{\beta}})), \tag{143}$$

such that when $\beta < \hat{\hat{\beta}}$, Eq. (136) holds. We further note that according to the bank's competitive loan pricing when facing the high-type firm, we have:

$$p_1 p_2 \mu = (1 - r^H)(\mu - r^H), \tag{144}$$

therefore, we have $\hat{\hat{\beta}} = \bar{\beta}(\alpha)$.

Similarly, substituting r^L and r_2^H into (137), it is easy to check that r_1^H satisfies:

$$(1-\alpha)(r_1^H)^2 + \left[p_1 p_2(1-\alpha^2\beta^2) - (1-\alpha^2)\right] r_1^H + \alpha\left[1 - \alpha + (\alpha\beta^2 - 1)p_1 p_2\right] \geq 0. \tag{145}$$

We denote $A = 1 - \alpha > 0$, $B = p_1 p_2(1-\alpha^2\beta^2) - (1-\alpha^2)$, and $C = \alpha\left[1 - \alpha + (\alpha\beta^2 - 1)p_1 p_2\right]$.

1. When $C < 0$, which is:

$$\beta^2 < \frac{p_1 p_2 + \alpha - 1}{\alpha p_1 p_2}, \tag{146}$$

   we have $\Delta = B^2 - 4AC > 0$. There exists positive r_1^H that satisfies Eq. (145). When $r_1^H = 1$, Eq. (145) reduces to: $p_1 p_2(1-\alpha) > 0$. Therefore, there exists $r_1^H \in [0,1)$ that satisfies Eq. (145).

   To ensure there also exists $r_2^H \in [0,1)$, according to Eq. (141), we further require $r_1^H \leq 1 - p_1 p_2$. When $r_1^H = 1 - p_1 p_2$, Eq. (145) becomes:

$$\alpha p_1 p_2 \left(1 - \alpha + (\alpha\beta^2 - 1)p_1 p_2\right) < 0, \tag{147}$$

   which is negative given $C < 0$. Therefore, in this case, there does not exist $r_1^H, r_2^H \in [0,1)$ that satisfy Eq. (137).

2. When $C \geq 0$, together with $\beta < \bar{\beta}(\alpha)$, we have:

$$\sqrt{\frac{p_1p_2+\alpha-1}{\alpha p_1p_2}} \leq \beta < \bar{\beta}(\alpha). \tag{148}$$

Next, we prove that the above inequality holds. Based on Proposition 4, $\bar{\beta}(\alpha)$ satisfies $r^L(\bar{\beta}(\alpha)) = r^H$, which is:

$$(\bar{\beta}(\alpha))^2 = \frac{(\mu-1)^2+4\mu p_1p_2+(1+\mu)^2(1-\alpha)^2-(\mu-1)^2\alpha^2-2(1+\mu)(1-\alpha)\sqrt{(\mu-1)^2+4\mu p_1p_2}}{4\alpha^2\mu p_1p_2}. \tag{149}$$

We further have:

$$(\bar{\beta}(\alpha))^2 - \frac{p_1p_2+\alpha-1}{\alpha p_1p_2} = \frac{(1-\alpha)\left[1+\mu(\mu+2p_1p_2)-(1+\mu)\sqrt{(\mu-1)^2+4\mu p_1p_2}\right]}{2\alpha^2\mu p_1p_2}, \tag{150}$$

as $1+\mu(\mu+2p_1p_2)-(1+\mu)\sqrt{(\mu-1)^2+4\mu p_1p_2}$ decreases in p_1p_2, as $p_1p_2 \in (0,1)$, we have:

$$1+\mu(\mu+2p_1p_2)-(1+\mu)\sqrt{(\mu-1)^2+4\mu p_1p_2} > 1+\mu^2+2\mu-(1+\mu)^2 = 0. \tag{151}$$

Therefore, we have $\sqrt{\frac{p_1p_2+\alpha-1}{\alpha p_1p_2}} < \bar{\beta}(\alpha)$.

When $r_1^H = 1-p_1p_2$ and $\bar{\bar{\beta}} := \sqrt{\frac{p_1p_2+\alpha-1}{\alpha p_1p_2}} \leq \beta < \bar{\beta}(\alpha)$, Eq. (145) is non-negative. Therefore, there exists $r_1^H, r_2^H \in [0,1)$ that satisfy Eq. (137). One feasible interest rate schedule under DTF is $r_1^H = 1-p_1p_2$ and $r_2^H = 0$. □