Online Appendix

A. Proofs

Proof of Proposition 1.

A consumer at x purchases a product of quality q and price p from a monopolist if $u = q - p - \tau|x - \frac{1}{2}| \geq 0$. Hence, consumers at $\frac{1}{2} - \frac{q-p}{\tau} \leq x \leq \frac{1}{2} + \frac{q-p}{\tau}$ purchase and the remainder do not, leading to demand $d = \min\{2\left(\frac{q-p}{\tau}\right), 1\}$.

The firm, in each period, maximizes its (expected) profit given by:

$$\begin{cases} \pi_t^M(k) - \theta\Delta = d \cdot p - \theta(\delta \cdot d \cdot p + \rho) = (1-\theta\delta)d \cdot p - \theta\rho & \text{if } k = in \text{ and } t = 1, \\ \pi_t^M(k) = d \cdot p & \text{otherwise.} \end{cases}$$

If the firm launches its innovation at $t = 1$, it must bear the pecuniary costs of a recall Δ, which occurs with probability θ. This cost, however, does not directly affect the firm's equilibrium price, since δ enters the profit function proportionally.[1]

For an interior solution (i.e., the market is not entirely served, $d = 2\left(\frac{q-p}{\tau}\right)$), the first-order condition (FOC) yields $\frac{-p}{\tau} + \frac{q-p}{\tau} = 0 \Leftrightarrow p = \frac{q}{2}$. The condition for an interior solution is $q < \tau$.[2] For a corner solution (i.e., the market is fully covered, $d = 1$), the consumer farthest from the monopolist must be indifferent between purchasing or not. This yields $q - p - \frac{\tau}{2} = 0 \Leftrightarrow p = q - \frac{\tau}{2}$. For the remainder of the proof, we focus on the case wherein the market is fully covered (i.e., $q \geq \tau \; \forall \, q(k)$).

The firm's test decision $QA \in \{0, 1\}$ governs the subsequent product offering $k \in \{ma, in, rc\}$ in periods $t \in \{1, 2\}$. Given the timeline of the game, there are four pricing scenarios to analyze. The monopolist's price p and profits π under each case are as follows:

- Sell mature product at $t = 1$ ($k_1^M = ma$): Product quality $q(ma) = V$ and thus $p_1^M(ma) = \pi_1^M(ma) = V - \frac{\tau}{2}$.
- Sell innovation at $t = 1$ ($k_1^M = in$): Product quality $q(in) = V + B$ and thus $p_1^M(in) = \pi_1^M(in) = V + B - \frac{\tau}{2}$. In addition, the firm incurs a probabilistic cost of a recall, $\theta\Delta = \theta(\rho + \delta \cdot \pi_1(in)) = \theta\left(\rho + \delta \cdot (V + B - \frac{\tau}{2})\right)$.
- Sell innovation at $t = 2$ ($k_2^M = in$): Product quality $q(in) = V + B$ and thus $p_2^M(in) = \pi_2^M(in) = V + B - \frac{\tau}{2}$. Since the product uncertainty is resolved in $t = 2$, the firm does not incur any additional costs.
- Sell recalled innovation at $t = 2$ ($k_2^M = rc$): Product quality $q(rc) = (V + B)(1 - \gamma)$ and thus $p_2^M(rc) = \pi_2^M(rc) = (V + B)(1 - \gamma) - \frac{\tau}{2}$.

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[1] See also Section B.5 of the Online Appendix in which the marginal cost δ depends on per-unit demanded and thus affects firm's pricing decisions.

[2] For an interior solution to exist, it must be the case that $2\left(\frac{q-p}{\tau}\right) < 1 \Leftrightarrow p > q - \frac{\tau}{2}$. Under such case, the monopolist sets price $p = \frac{q}{2}$, and thus $\frac{q}{2} > q - \frac{\tau}{2}$ if and only if $q < \tau$.

If the monopolist conducts testing ($QA=1$), it launches the mature product in the first period ($k_1^M=ma$) and the (fail-proof) innovation in the second period ($k_2^M=in$). Hence, the overall payoff

$$\Pi^{QA=1}=\pi_1^M(ma)+\pi_2^M(in)=2V+B-\tau.$$

Conversely, if the monopolist does not conduct testing ($QA=0$), it launches its innovative product immediately in the first period ($k_1^M=in$). With probability θ, the product is recalled, and (1) if $\rho\leq\rho^M$, the monopolist covers the associated costs and markets the recalled product in the second period ($k_2^M=rc$); (2) if $\rho>\rho^M\equiv(1-\delta)(V+B-\frac{\tau}{2})$, the costs exceed $t=1$ profits and the firm exits the market, leading to zero profits in both periods ($\pi_1^M+\pi_2^M=0$). With probability $1-\theta$, the product does not fail and the monopolist markets the innovation once again in the second period ($k_2^M=in$). Hence, the overall payoff

$$\Pi^{QA=0}=\begin{cases}\pi_1^M(in)+\theta(-\Delta+\pi_2^M(rc))+(1-\theta)\cdot\pi_2^M(in)=\frac{(V+B)(2-\theta(\delta+\gamma))}{4}-\theta\rho & \text{if } \rho\leq\rho^M,\\ \theta\cdot 0+(1-\theta)(\pi_1^M(in)+\pi_2^M(in))=(1-\theta)(2V+2B-\tau) & \text{if } \rho>\rho^M.\end{cases}$$

The monopolist engages in testing ($QA=1$) iff

$$\Pi^{QA=1}\geq\Pi^{QA=0}\Leftrightarrow\theta\geq\theta^M,$$

where

$$\theta^M\equiv\begin{cases}\frac{2B}{2((V+B)(\gamma+\delta)+\rho)-\delta\tau} & \text{if } \rho\leq\rho^M,\\ \frac{B}{2(V+B)-\tau} & \text{if } \rho>\rho^M.\end{cases}$$

We note that qualitatively similar results hold if a monopolist is located at either endpoint of the unit segment; or if it is located at both endpoints, jointly maximizing the profits of two products.[3]

Proof of Corollary 1.

The following holds from the derivatives of the cutoff θ^M if $\rho\leq\rho^M$:

$$\begin{aligned}
\frac{\partial\theta^M}{\partial V}&=-\left(\frac{4B(\gamma+\delta)}{(2((V+B)(\gamma+\delta)+\rho)-\delta\tau)^2}\right)<0,\\
\frac{\partial\theta^M}{\partial B}&=\frac{4V(\gamma+\delta)+4\rho-2\delta\tau}{(2((V+B)(\gamma+\delta)+\rho)-\delta\tau)^2}>0,\\
\frac{\partial\theta^M}{\partial\rho}&=-\left(\frac{4B}{(2((V+B)(\gamma+\delta)+\rho)-\delta\tau)^2}\right)<0,\\
\frac{\partial\theta^M}{\partial\delta}&=-\left(\frac{2B(2(V+B)-\tau)}{(2((V+B)(\gamma+\delta)+\rho)-\delta\tau)^2}\right)<0,\\
\frac{\partial\theta^M}{\partial\gamma}&=-\left(\frac{4B(V+B)}{(2((V+B)(\gamma+\delta)+\rho)-\delta\tau)^2}\right)<0,\\
\frac{\partial\theta^M}{\partial\tau}&=\frac{2B\delta}{(2((V+B)(\gamma+\delta)+\rho)-\delta\tau)^2}>0.
\end{aligned}$$

[3] When the monopolist is located at either endpoint, we have $p=\pi=q-\tau$, implying that the impact of τ on firm's profits becomes stronger by a factor of 2. Nonetheless, this yields qualitatively similar firm decision and comparative statics. Alternatively, when the monopolist is situated at both endpoints, a consumer at x purchases a product if $u=q-p-\tau x\geq 0$, when buying from $x=0$, or if $u=q-p-\tau(1-x)\geq 0$, when buying from $x=1$. Hence, consumers at $x\leq\frac{q-p}{\tau}$ and $1-\frac{q-p}{\tau}\leq x$ purchase, leading to identical demand $d=\min\{2\left(\frac{q-p}{\tau}\right),1\}$ and thus profits.

For $\frac{\partial \theta^M}{\partial B}$ and $\frac{\partial \theta^M}{\partial \delta}$, the inequality holds since $\tau < \min\{V, (V+B)(1-\gamma)\}$.

If $\rho > \rho^M$, the following holds:[4]

$$\frac{\partial \theta^M}{\partial V} = \frac{-2B}{(2(V+B)-\tau)^2} < 0,$$
$$\frac{\partial \theta^M}{\partial B} = \frac{2V-\tau}{(2(V+B)-\tau)^2} > 0,$$
$$\frac{\partial \theta^M}{\partial \rho} = 0,$$
$$\frac{\partial \theta^M}{\partial \delta} = 0,$$
$$\frac{\partial \theta^M}{\partial \gamma} = 0,$$
$$\frac{\partial \theta^M}{\partial \tau} = \frac{B}{(2(V+B)-\tau)^2} > 0.$$

Proof of Lemma 1.

Let firm i be situated at location 0 and i' at 1. The respective utility a consumer at x derives is $u^i = q^i - p^i - \tau x$ if purchasing from firm i and $u^{i'} = q^{i'} - p^{i'} - \tau(1-x)$ if purchasing from firm i'. The location of the indifferent consumer (between purchasing from firm i and i') along the horizontal dimension is given by $\tilde{x} = \frac{q^i - q^{i'} - p^i + p^{i'} + \tau}{2\tau}$.

Given the timeline in Figure 1, there are three scenarios in which competition may arise:

- The products are of identical quality ($k_t^i = k_t^{i'} \in \{in, ma, rc\}$): Since $q^i = q^{i'}$, the indifferent consumer is located at $\tilde{x} = \frac{-p^i + p^{i'} + \tau}{2\tau}$. This leads to

$$p_t^i(k,k) = p_t^{i'}(k,k) = \tau;\ d_t^i(k,k) = d_t^{i'}(k,k) = \frac{1}{2};\ \text{and}\ \pi_t^i(k,k) = \pi_t^{i'}(k,k) = \frac{\tau}{2}.$$

- Mature product versus innovation ($k_1^i = ma$; $k_1^{i'} = in$) at $t = 1$: Since $q^i(ma) = V$ and $q^{i'}(in) = V + B$, the indifferent consumer is located at $\tilde{x} = \frac{-B - p^i + p^{i'} + \tau}{2\tau}$. This leads to

$$p_1^i(ma,in) = \frac{-B+3\tau}{3};\ d_1^i(ma,in) = \frac{-B+3\tau}{6\tau};\ \text{and}\ \pi_1^i(ma,in) = \frac{(-B+3\tau)^2}{18\tau};$$
$$p_1^{i'}(in,ma) = \frac{B+3\tau}{3};\ d_1^{i'}(in,ma) = \frac{B+3\tau}{6\tau};\ \text{and}\ \pi_1^{i'}(in,ma) = \frac{(B+3\tau)^2}{18\tau}.$$

- Recalled innovation versus fail-proof innovation ($k_2^i = rc$; $k_2^{i'} = in$) at $t = 2$: Since $q^i(rc) = (V+B)(1-\gamma)$ and $q^{i'}(in) = V + B$, the indifferent consumer is located at $\tilde{x} = \frac{-(V+B)\gamma - p^i + p^{i'} + \tau}{2\tau}$. This leads to

$$p_2^i(rc,in) = \frac{-(V+B)\gamma+3\tau}{3};\ d_2^i(rc,in) = \frac{-(V+B)\gamma+3\tau}{6\tau};\ \text{and}\ \pi_2^i(rc,in) = \frac{(-(V+B)\gamma+3\tau)^2}{18\tau};$$
$$p_2^{i'}(in,rc) = \frac{(V+B)\gamma+3\tau}{3};\ d_2^{i'}(in,rc) = \frac{(V+B)\gamma+3\tau}{6\tau};\ \text{and}\ \pi_2^{i'}(in,rc) = \frac{((V+B)\gamma+3\tau)^2}{18\tau}.$$

[4] We note further that, because ρ^M decreases in τ, the cutoff θ^M may exhibit a discontinuous decrease as τ increases to the point of triggering the monopolist's bankruptcy.

In deriving above, the following constraint is imposed: $\frac{B}{3} < \tau < \frac{2V+B}{3}$ if $0 < \gamma < \frac{B}{V+B}$ or $\frac{(V+B)\gamma}{3} < \tau < \frac{(V+B)(2-\gamma)}{3}$ if $\frac{B}{V+B} < \gamma < 1$. The upper bound ensures that the market is fully covered by the two competing firms; the lower bound ensures that there is some competition in the market (and no single firm dominates).

Proof of Proposition 2.

Using the equilibrium profits of the pricing subgame per Lemma 1, we now characterize the equilibrium of the testing strategy $QA^i = \{0, 1\}$. In equilibrium, each firm's strategy must form a best response to its rival's strategy, $QA^{i'}$. Firm i's total expected profits $\Pi^{QA^i, QA^{i'}}$ for each subgame $(QA^i, QA^{i'})$ are given by:

- Rival strategy is to test, $QA^{i'} = 1$. Thus, $k_1^{i'} = ma$ and $k_2^{i'} = in$.
  (a) If the firm conducts testing ($QA^i = 1$), $k_1^i = ma$ and $k_2^i = in$ and competing firms market products of similar quality in both periods. Hence,

$$\Pi^{1,1} = \pi_1(ma, ma) + \pi_2(in, in) = \tau. \tag{A.1}$$

  (b) If the firm does not conduct testing ($QA^i = 0$), it earns $\pi_1^i(in, ma)$ in the first period. At $t = 2$, with probability θ, the firm's product is recalled: (1) If $\rho < \rho^{\bar{B}}$, the firm bears the cost of a recall $\Delta = (\rho + \delta \cdot \pi_1^i(in, ma))$ and earns $\pi_2^i(rc, in)$; (2) If $\rho > \rho^{\bar{B}}$, the costs exceed $t = 1$ profits and the firm exits the market, leading to zero profits in both periods ($\pi_1 + \pi_2 = 0$). With probability $1 - \theta$, the firm's product does not fail and competing firms market products of similar quality. Hence,

$$\Pi^{0,1} = \begin{cases} \pi_1(in, ma) + \theta(-\delta \cdot \pi_1(in, ma) - \rho + \pi_2(rc, in)) + (1-\theta)\pi_2(in, in) & \\ \quad = (1-\theta\delta) \cdot \frac{(B+3\tau)^2}{18\tau} - \theta\rho + \theta \cdot \frac{(-(V+B)\gamma + 3\tau)^2}{18\tau} + (1-\theta) \cdot \frac{\tau}{2} & \text{if } \rho \leq \rho^{\bar{B}}, \\ \theta \cdot 0 + (1-\theta)\left(\pi_1(in, ma) + \pi_2(in, in)\right) & \\ \quad = (1-\theta)\left(\frac{(B+3\tau)^2}{18\tau} + \frac{\tau}{2}\right) & \text{if } \rho > \rho^{\bar{B}}. \end{cases} \tag{A.2}$$

  The firm, once a recall takes place, exits the market if $\Delta > \pi_1(in, ma) \Leftrightarrow \rho > (1-\delta)\frac{(B+3\tau)^2}{18\tau} \equiv \rho^{\bar{B}}$.

- Rival strategy is not to test, $QA^{i'} = 0$. Thus, $k_1^{i'} = in$ and (1) with probability θ, either $k_2^{i'} = rc$ or $k_2^{i'} = \emptyset$ depending on the rival's bankruptcy (where $\emptyset$ denotes bankruptcy); (2) with probability $(1-\theta)$, $k_2^{i'} = in$.
  (a) If the firm conducts testing ($QA^i = 1$), it earns $\pi_1^i(ma, in)$ in the first period. At $t = 2$, with probability θ, the rival's product is recalled: (1) If $\rho < \rho^{\bar{B}}$, the rival withstands the recall and the firm earns $\pi_2^i(in, rc)$; (2) If $\rho > \rho^{\bar{B}}$, the rival exits the market and the firm earns monopoly-level profit from selling the innovation—i.e., $\pi_2^i(in, \emptyset) = V + B - \frac{\tau}{2}$.[5] With probability $1 - \theta$, the rival's product does not fail and the competing

[5] The profit level follows from Proposition 1. The results are robust if the remaining monopolist firm (1) keeps its initial location, i.e., either $x = 0$ or $x = 1$, or (2) moves to the center of the unit segment.

firms market products of similar quality. Hence,

$$\Pi^{1,0} = \begin{cases} \pi_1(ma, in) + \theta \cdot \pi_2(in, rc) + (1-\theta) \cdot \pi_2(in, in) & \\ \quad = \frac{(-B+3\tau)^2}{18\tau} + \theta \cdot \frac{((V+B)\gamma+3\tau)^2}{18\tau} + (1-\theta) \cdot \frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ \pi_1(ma, in) + \theta \cdot \pi_2^i(in, \emptyset) + (1-\theta) \cdot \pi_2(in, in) & \\ \quad = \frac{(-B+3\tau)^2}{18\tau} + \theta \cdot (V + B - \frac{\tau}{2}) + (1-\theta) \cdot \frac{\tau}{2} & \text{if } \rho > \rho^{\bar{B}}. \end{cases} \tag{A.3}$$

(b) If the firm does not conduct testing ($QA^i = 0$), $k_1 = in$ and the competing firms market products of similar quality at $t = 1$, leading to $\pi_1^i(in, in)$. At $t = 2$ (1) if $\rho < \rho^{\underline{B}}$, neither firm exits the market following a recall: (1a) With probability $\theta(1-\theta)$, the firm's own product experiences a recall and the rival's does not, earning $\pi_2^i(rc, in)$; (1b) With probability $(1-\theta)\theta$, the rival's product experiences a recall and the firm's own does not, earning $\pi_2^i(in, rc)$; (1c) With probabilities θ^2 and $(1-\theta)^2$, the competing firms market similar quality products. In addition, with probability θ (regardless of the rival's failure), the firm's own product fails and it incurs the fixed cost of a recall $\Delta = (\rho + \delta \cdot \pi_1^i(in, in))$.

In contrast, (2) if $\rho > \rho^{\underline{B}}$, a firm is (or both firms are) withdrawn from the market in the event of a recall: (2a) With probability θ, the firm exits the market, leading to zero profits for both periods ($\pi_1 + \pi_2 = 0$); (2b) With probability $(1-\theta)\theta$, the rival exits the market and the firm enjoys monopoly-level profits from selling the innovation ($\pi_2^i(in, \emptyset) = V + B - \frac{\tau}{2}$); (2c) With probability $(1-\theta)^2$, the competing firms sell similar quality products. Hence,

$$\Pi^{0,0} = \begin{cases} \pi_1(in, in) + \theta(-\delta \cdot \pi_1(in, in) - \rho + \theta\pi_2(rc, rc) + (1-\theta)\pi_2(rc, in)) & \\ \quad +(1-\theta)(\theta\pi_2(in, rc) + (1-\theta)\pi_2(in, in)) & \\ \quad = (2 - \theta\delta) \cdot \frac{\tau}{2} - \theta\rho + 2\theta(1-\theta) \cdot \frac{(V+B)^2\gamma^2}{18\tau} & \text{if } \rho \le \rho^{\underline{B}}, \\ \theta \cdot 0 + (1-\theta)\left(\pi_1(in, in) + \theta \cdot \pi_2(in, \emptyset) + (1-\theta) \cdot \pi_2(in, in)\right) & \\ \quad = (1-\theta)\left(\theta(V + B - \tau) + \tau\right) & \text{if } \rho > \rho^{\underline{B}}. \end{cases} \tag{A.4}$$

The cutoff for firm's bankruptcy $\rho^{\underline{B}}$ is $\Delta > \pi_1(in, in) \Leftrightarrow \rho > (1-\delta)\frac{\tau}{2} \equiv \rho^{\underline{B}}$. Note that because we have $\rho^{\underline{B}} < \rho^{\bar{B}}$, there always exists a region in which $\rho^{\underline{B}} < \rho < \rho^{\bar{B}}$ holds.

Using the above profit levels, we can specify the conditions required for each equilibrium to hold.

No-test equilibrium (Symmetric). The condition of this equilibrium is derived by establishing when the profits for not testing $QA^i = 0$ is greater than that of testing $QA^i = 1$, given the rival's decision not to test $QA^{i'} = 0$. That is,

$$\Pi^{0,0} \ge \Pi^{1,0} \Leftrightarrow \theta \le \theta^{NT},$$

where

$$\theta^{NT} = \begin{cases} \frac{(V+B)^2\gamma^2 - 6((V+B)\gamma+3\rho)\tau - 9\delta\tau^2 + \sqrt{8B(V+B)^2\gamma^2(-B+6\tau) + ((V+B)^2\gamma^2 - 6((V+B)\gamma+3\rho)\tau - 9\delta\tau^2)^2}}{4(V+B)^2\gamma^2} & \text{if } \rho \le \rho^{\underline{B}}, \\ \frac{-(V+B)^2\gamma^2 + 6(V+B)(3-\gamma)\tau - 36\tau^2 + \sqrt{72B(-B+6\tau)(V+B-\tau)\tau + ((V+B)^2\gamma^2 - 6(V+B)(3-\gamma)\tau + 36\tau^2)^2}}{36(V+B-\tau)\tau} & \text{if } \rho^{\underline{B}} < \rho \le \rho^{\bar{B}}, \\ \frac{-\tau + \sqrt{2B(-B+6\tau)(V+B-\tau)/9\tau + \tau^2}}{2(V+B-\tau)} & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$

Dual-test equilibrium (Symmetric). The condition of this equilibrium is derived by establishing when profits for testing $QA^i = 1$ is greater than that of not testing $QA^i = 0$, given the rival's decision to test $QA^{i'} = 1$. That is,

$$\Pi^{1,1} \geq \Pi^{0,1} \Leftrightarrow \theta \geq \theta^{DT},$$

where

$$\theta^{DT} = \begin{cases} \frac{B(B+6\tau)}{-(V+B)^2\gamma^2+6((V+B)\gamma+3\rho)\tau+\delta(B+3\tau)^2} & \text{if } \rho \leq \rho^{\bar{B}}, \\ \frac{B(B+6\tau)}{(B+3\tau)^2+9\tau^2} & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$

Single-test equilibrium (Asymmetric). The condition of this equilibrium is derived by establishing

(1) when the profits for $QA^i = 0$ is greater than that of $QA^i = 1$, given $QA^{i'} = 1 \Rightarrow \theta < \min(\theta^{DT}, 1)$, and

(2) when the profits for $QA^i = 1$ is greater than that of $QA^i = 0$, given $QA^{i'} = 0 \Rightarrow \theta > \max(0, \theta^{NT})$.

Proof of Result 1.

The result can be derived by comparing the value of θ^{NT} as $\rho \to \rho^{\underline{B}(-)}$ vs. as $\rho \to \rho^{\underline{B}(+)}$. To show the existence of the result, let $V \to 1$, $B \to 0.5$ and $\gamma \to 1/3$. Then, $\lim_{\rho \to \rho^{\underline{B}(-)}} \theta^{NT} < \lim_{\rho \to \rho^{\underline{B}(+)}} \theta^{NT}$ holds if $0.083 < \tau < 0.880$, whose region is a superset of the constraint $\tau \in (0.167, 0.833)$ in Lemma 1.

Proof of Result 2.

Rearranging the cutoff of the no-test equilibrium (we illustrate for the case $\rho \leq \rho^{\bar{B}}$) in terms of ρ yields

$$\rho < \frac{-B^2(1+\gamma^2(2\theta^2-\theta)) + B(2V\gamma^2(1-2\theta)\theta + 6(1-\gamma\theta)\tau) + \theta(V^2\gamma^2(1-2\theta) - 6V\gamma\tau - 9\delta\tau^2)}{18\theta\tau} \equiv \rho^{R2}.$$

Taking the derivative with respect to τ yields

$$\frac{\partial \rho^{R2}}{\partial \tau} = \frac{B^2 + (V+B)^2\gamma^2\theta(2\theta-1) - 9\delta\theta\tau^2}{18\theta\tau^2}.$$

The value is positive (i.e., the no-test equilibrium region is expanding in τ) if

$$\tau < \sqrt{\frac{B^2 + (V+B)^2\gamma^2\theta(2\theta-1)}{9\delta\theta}} \equiv \tau^{R2}$$

and negative otherwise. In solving for the conditions under which τ^{R2} satisfies the constraint (specified in Lemma 1), we illustrate for the case $0 < \gamma < \frac{B}{V+B}$. We have that $\frac{B}{3} < \tau^{R2} < \frac{2V+B}{3}$ holds if $\frac{B^2+(V+B)^2\gamma^2\theta(2\theta-1)}{(2V+B)^2\theta} < \delta < \frac{B^2+(V+B)^2\gamma^2\theta(2\theta-1)}{B^2\theta}$. Deriving the specific conditions under which the expression satisfies $0 < \delta < 1$ involves tedious algebra and the details are available from the authors upon request. Note that as $V \to 1$, $B \to 0.5$, $\gamma \to \frac{1}{3}$, and $\theta \to 0.1$, the former expression becomes $0.176 < \delta < 4.4$ (and thus $0.176 < \delta < 1$ is binding). Therefore, there is a nonempty range of parameter values for which the result is sustained.

Proof of Corollary 2.

The following holds from the derivatives of the cutoff θ^{DT} if $\rho \le \rho^{\bar{B}}$:

$$\begin{aligned}
\frac{\partial\theta^{DT}}{\partial V} &= -\left(\frac{2B\gamma(3\tau-(V+B)\gamma)(B+6\tau)}{(2VB\gamma^2+V^2\gamma^2+B^2(\gamma^2-\delta)-6V\gamma\tau-6B(\gamma+\delta)\tau-9\tau(2\rho+\delta\tau))^2}\right)<0,\\
\frac{\partial\theta^{DT}}{\partial B} &= \frac{-2VB(V+B)\gamma^2+6((2VB-V^2\gamma+B^2(1+\gamma))\gamma+6B\rho)\tau+18(2V\gamma+B\delta+6\rho)\tau^2+54\delta\tau^3}{(2VB\gamma^2+V^2\gamma^2+B^2(\gamma^2-\delta)-6V\gamma\tau-6B(\gamma+\delta)\tau-9\tau(2\rho+\delta\tau))^2}>0,\\
\frac{\partial\theta^{DT}}{\partial \rho} &= -\left(\frac{18B\tau(B+6\tau)}{(2VB\gamma^2+V^2\gamma^2+B^2(\gamma^2-\delta)-6V\gamma\tau-6B(\gamma+\delta)\tau-9\tau(2\rho+\delta\tau))^2}\right)<0,\\
\frac{\partial\theta^{DT}}{\partial \delta} &= -\left(\frac{B(B+3\tau)^2(B+6\tau)}{(2VB\gamma^2+V^2\gamma^2+B^2(\gamma^2-\delta)-6V\gamma\tau-6B(\gamma+\delta)\tau-9\tau(2\rho+\delta\tau))^2}\right)<0,\\
\frac{\partial\theta^{DT}}{\partial \gamma} &= -\left(\frac{2B(V+B)(3\tau-(V+B)\gamma)(B+6\tau)}{(2VB\gamma^2+V^2\gamma^2+B^2(\gamma^2-\delta)-6V\gamma\tau-6B(\gamma+\delta)\tau-9\tau(2\rho+\delta\tau))^2}\right)<0,\\
\frac{\partial\theta^{DT}}{\partial \tau} &= -\left(\frac{6B((V+B)\gamma(B+(V+B)\gamma)+9\delta\tau^2+3B(\rho+\delta\tau)}{(2VB\gamma^2+V^2\gamma^2+B^2(\gamma^2-\delta)-6V\gamma\tau-6B(\gamma+\delta)\tau-9\tau(2\rho+\delta\tau))^2}\right)<0.
\end{aligned}$$

The signs for $\frac{\partial\theta^{DT}}{\partial\rho}$, $\frac{\partial\theta^{DT}}{\partial\delta}$ and $\frac{\partial\theta^{DT}}{\partial\tau}$ are straightforward. The inequalities for $\frac{\partial\theta^{DT}}{\partial V}$ and $\frac{\partial\theta^{DT}}{\partial\gamma}$ hold since $\tau > \frac{(V+B)\gamma}{3}$ from the constraint per Lemma 1. Lastly, because $\frac{\partial\theta^{DT}}{\partial B}$ is increasing in τ, the inequality for $\frac{\partial\theta^{DT}}{\partial B}$ is obtained by plugging in the minimum constraint value for τ (i.e., $\frac{B}{3}$ if $0<\gamma<\frac{B}{V+B}$ or $\frac{(V+B)\gamma}{3}$ if $\frac{B}{V+B}<\gamma<1$ per Lemma 1).

If $\rho > \rho^{\bar{B}}$, the following holds:

$$\begin{aligned}
\frac{\partial\theta^{DT}}{\partial V} &= 0,\\
\frac{\partial\theta^{DT}}{\partial B} &= \frac{36\tau^2(B+3\tau)}{(B+3\tau)^2+9\tau^2}>0,\\
\frac{\partial\theta^{DT}}{\partial \rho} &= 0,\\
\frac{\partial\theta^{DT}}{\partial \delta} &= 0,\\
\frac{\partial\theta^{DT}}{\partial \gamma} &= 0,\\
\frac{\partial\theta^{DT}}{\partial \tau} &= -\left(\frac{36\tau^2(B+3\tau)}{(B+3\tau)^2+9\tau^2}\right)<0.
\end{aligned}$$

Proof of Result 3.

Using the profit expressions from Proposition 2, $\Pi^{0,0} > \Pi^{1,1}$ holds if

$$\theta > \theta^{R3} \equiv \begin{cases} 1-\frac{9\tau(2\rho+\delta\tau)}{2(V+B)^2\gamma^2} & \text{if } \rho \le \rho^{B},\\ \frac{V+B-2\tau}{V+B-\tau} & \text{if } \rho > \rho^{B}.\end{cases}$$

We see that $\theta^{R3} < \theta^{NT}$ holds (i.e., the parameter lies within the region of the no-test equilibrium) if $\frac{B^2+(V+B)^2\gamma^2}{9\tau} - \frac{\delta\tau}{2} - \frac{2B(B+(V+B)\gamma)}{3(V+B)\gamma+18\tau} < \rho < \rho^{B}$ or, by rearranging the terms, if $\frac{3V+2B+\sqrt{9V^2+12VB+8B^2}}{12} < \tau < \tilde{\tau}$ and $\rho > \rho^{B}$, where $\tilde{\tau}$ is the upper bound of the constraint in Lemma 1. Note that as $V\to 1$, $B\to 0.5$, and $\gamma\in(0,\frac{B}{V+B})$, the expression yields $0.677 < \tau < \tilde{\tau} = 0.833$ and thus there is a nonempty range of parameter values for which the result is sustained.

Proof of Proposition 3.

First, the outcomes of the product testing/recalls subgame are established; afterwards, firms' equilibrium R&D investment decisions are discussed. For below, let $\Pi^{QA^i,QA^{i'}}_{RD^i,RD^{i'}}$ denote firm i's profit in the subgame $(QA^i, QA^{i'})$ conditional on the two firms' R&D investment decisions $(RD^i, RD^{i'})$. For instance, $\Pi^{1,0}_{1,0}$ denotes the total profits for firm i when it invests in R&D and conducts testing, whereas the rival i' decides not to invest in R&D.

Testing/Recall Subgame. In the product testing/recall subgame, the expected profits are characterized by the following three scenarios:

(1) **Both firms invest in R&D:** The profit structure carries over from the duopoly case of the main model discussed in Proposition 2. The specific payoffs depend on firms' testing decisions following their R&D investments. If the resulting subgame lies in the region of the dual-test equilibrium or the no-test equilibrium, the profit for either firm follows Equation (A.1) or Equation (A.4), respectively. If the resulting subgame is the single-test (asymmetric) equilibrium, however, a coordination issue of which specific firm will be the one to conduct testing arise. To resolve this issue, we make the following additional assumption regarding the sequence of moves. When both firms engage in R&D, their respective innovations are materialized sequentially during $t = 0$. The first-mover (the firm that completes its innovation first) decides whether to conduct quality assurance testing. Upon observing this decision, the second-mover makes the test decision.[6] The probability of being the first-mover is assumed to be equal across firms. Given the assumption, the expected profit (for either firm) in the asymmetric case is given by taking the average of Equations (A.2) and (A.3).

(2) **Only one firm invests in R&D:** In this case, the sole innovator makes the testing decision, whereas the non-innovating firm markets the mature product through the remainder of the game. To determine the sole innovator's test decision, we first need to consider an additional scenario of product competition: At $t = 2$, a recalled innovation ($k^i_2 = rc$), from the sole innovator if it did not conduct testing and its product failed, faces a mature product ($k^{i'}_2 = ma$), from the non-innovating firm. The following characterizes the outcomes of this pricing subgame:

$$p^i_2(rc, ma) = \frac{B-(V+B)\gamma+3\tau}{3};\ d^i_2(rc, ma) = \frac{B-(V+B)\gamma+3\tau}{6\tau};\ \text{and}\ \pi^i_2(rc, ma) = \frac{(B-(V+B)\gamma+3\tau)^2}{18\tau};$$
$$p^{i'}_2(ma, rc) = \frac{(V+B)\gamma-B+3\tau}{3};\ d^{i'}_2(ma, rc) = \frac{(V+B)\gamma-B+3\tau}{6\tau};\ \text{and}\ \pi^{i'}_2(ma, rc) = \frac{((V+B)\gamma-B+3\tau)^2}{18\tau}.$$

In deriving above, the following constraint is imposed: $\frac{B-(V+B)\gamma}{3} < \tau < \frac{V+(V+B)(1-\gamma)}{3}$ if $0 < \gamma < \frac{B}{V+B}$ (i.e., reputation damage is modest) or $\frac{(V+B)\gamma-B}{3} < \tau < \frac{V+(V+B)(1-\gamma)}{3}$ if $\frac{B}{V+B} < \gamma < 1$ (i.e., reputation damage is

[6] The second-mover takes an asymmetric action relative to the first-mover under the relevant parameter space for the single-test equilibrium (see Proof of Proposition 2).

severe). Here, the lower bound is naturally satisfied per Lemma 1. Note also that the reputation damage γ determines the relative profitability of the two firms. When the reputation damage is modest (i.e., $\gamma \leq \frac{B}{V+B} \Leftrightarrow (V+B)(1-\gamma) \geq V$), the value of (and hence the resulting profit from selling) the recalled product is higher than the mature product, and the opposite holds when the reputation damage is severe (i.e., $\gamma > \frac{B}{V+B} \Leftrightarrow (V+B)(1-\gamma) < V$).

Given above profits and Lemma 1, the overall expected payoff for firms can be calculated.

- If the sole innovator conducts testing ($QA^i = 1$), $k_1^i = ma$ and $k_2^i = in$. Hence,

$$\Pi_{1,0}^{1,0} = \pi_1(ma, ma) + \pi_2(in, ma) = \frac{\tau}{2} + \frac{(B+3\tau)^2}{18\tau}$$

  and conversely for the non-innovator,

$$\Pi_{0,1}^{0,1} = \pi_1(ma, ma) + \pi_2(ma, in) = \frac{\tau}{2} + \frac{(-B+3\tau)^2}{18\tau}$$

- If the sole innovator does not conduct testing ($QA^i = 0$), it earns $\pi_1^i(in, ma)$ in the first period. At $t = 2$, with probability θ, the firm's product is recalled: (1) If $\rho < \rho^{SB}$, the firm bears the cost of a recall $\Delta = (\rho + \delta \cdot \pi_1^i(in, ma))$ and earns $\pi_2^i(rc, ma)$; (2) If $\rho > \rho^{SB}$, the costs exceed $t = 1$ profits and the sole innovator exits the market, earning zero profits in both periods ($\pi_1 + \pi_2 = 0$). With probability $1-\theta$, the firm's product does not fail and the sole innovator again earns $\pi_1^i(in, ma)$. Hence,

$$\Pi_{1,0}^{0,0} = \begin{cases} \pi_1(in, ma) + \theta(-\delta \cdot \pi_1(in, ma) - \rho + \pi_2(rc, ma)) + (1-\theta)\pi_2(in, ma) & \\ \quad = (2-\theta-\theta\delta) \cdot \frac{(B+3\tau)^2}{18\tau} - \theta\rho + \theta \cdot \frac{(B-(V+B)\gamma+3\tau)^2}{18\tau} & \text{if } \rho \leq \rho^{SB}, \\ \theta \cdot 0 + (1-\theta)\left(\pi_1(in, ma) + \pi_2(in, ma)\right) & \\ \quad = (1-\theta)\left(2 \cdot \frac{(B+3\tau)^2}{18\tau}\right) & \text{if } \rho > \rho^{SB}. \end{cases}$$

  The sole innovator exits the market following a product recall if $\Delta > \pi_1(in, ma) \Leftrightarrow \rho > (1-\delta)\frac{(B+3\tau)^2}{18\tau} \equiv \rho^{SB}$. Note that $\rho^{SB} = \rho^{\bar{B}}$ holds.

  Conversely for the non-innovator,

$$\Pi_{0,1}^{0,0} = \begin{cases} \pi_1(ma, in) + \theta \cdot \pi_2(ma, rc) + (1-\theta)\pi_2(ma, in) & \\ \quad = (2-\theta) \cdot \frac{(B-3\tau)^2}{18\tau} + \theta \cdot \frac{((V+B)\gamma-B+3\tau)^2}{18\tau} & \text{if } \rho \leq \rho^{SB}, \\ \pi_1(ma, in) + \theta \cdot \pi_2^i(ma, \emptyset) + (1-\theta)\pi_2(ma, in) & \\ \quad = (2-\theta) \cdot \frac{(-B+3\tau)^2}{18\tau} + \theta \cdot (V - \frac{\tau}{2}) & \text{if } \rho > \rho^{SB}. \end{cases}$$

The sole innovator conducts testing ($QA = 1$) if

$$\theta > \frac{18\theta\rho\tau + 9\tau^2 - (1-\theta)(B+3\tau)^2 + \delta\theta(B+3\tau)^2 - \theta(B-(V+B)\gamma+3\tau)^2}{18\tau} \equiv \theta^{SI},$$

and does not otherwise.

(3) **No firm invests in R&D:** For either firm, $\Pi^{RD^i=0, RD^{i'}=0} = \tau$.

Firms' R&D Investment Decisions. The above testing/recall subgame outcomes illustrate how the parameter values associated with a recall, ρ and θ, lead to various subgame outcomes, which vary along two dimensions: the competing firms' testing decisions (dual-test, single-test and no-test following both firms' innovation; the sole innovator's test and no-test following one firm's innovation) and resulting bankruptcy outcomes (whether the firm stays in the market vs. goes bankrupt). To establish the equilibrium of the firms' R&D investment strategy, we illustrate using two representative cases (which are also used for proving Result 4). The conditions for other cases can be derived in an analogous manner.

(1) *Case* 1. *If both firms innovate, the single-test equilibrium emerges; if only one firm innovates, the sole-innovator conducts testing. The fixed cost of a recall is modest such that firms stay in the market following a recall. (i.e.,* $\max\{\theta^{NT},\theta^{SI}\}<\theta<\theta^{DT}$ *and* $\rho<\rho^{\underline{B}}$*).*

The subgame profits are given by

$$\Pi^{RD^i=1,RD^{i'}=1}=\Pi_{1,1}^{1/2,1/2}=-\eta+\frac{1}{2}\left((1-\theta\delta)\cdot\frac{(B+3\tau)^2}{18\tau}-\theta\rho+\theta\cdot\frac{(-(V+B)\gamma+3\tau)^2}{18\tau}+(1-\theta)\cdot\frac{\tau}{2}\right.$$
$$\left.\frac{(-B+3\tau)^2}{18\tau}+\theta\cdot\frac{((V+B)\gamma+3\tau)^2}{18\tau}+(1-\theta)\cdot\frac{\tau}{2}\right)$$
$$\Pi^{RD^i=0,RD^{i'}=1}=\Pi_{0,1}^{0,1}=\frac{(-B+3\tau)^2}{18\tau}+\frac{\tau}{2}$$
$$\Pi^{RD^i=1,RD^{i'}=0}=\Pi_{1,0}^{1,0}=-\eta+\frac{(B+3\tau)^2}{18\tau}+\frac{\tau}{2}$$
$$\Pi^{RD^i=0,RD^{i'}=0}=\Pi_{0,0}^{0,0}=\tau$$

Using above profit levels, we can specify the conditions required for each equilibrium to hold.

No-innovation equilibrium (Symmetric). The condition for this equilibrium is derived by establishing when profits of not investing ($RD^i=0$) are greater than those of investing ($RD^i=1$); given the rival's decision to not invest ($RD^{i'}=0$). That is,

$$\Pi^{RD^i=0,RD^{i'}=0}\geq\Pi^{RD^i=1,RD^{i'}=0}$$
$$\Leftrightarrow\eta\geq\frac{B(B+6\tau)}{18\tau}\equiv\eta^{NI}.$$

Dual-innovation equilibrium (Symmetric). The condition for this equilibrium is derived by establishing when profits of investing ($RD^i=1$) are greater than those of not investing ($RD^i=0$); given the rival's decision to invest ($RD^{i'}=1$). That is,

$$\Pi^{RD^i=1,RD^{i'}=1}\geq\Pi^{RD^i=0,RD^{i'}=1}$$
$$\Leftrightarrow\eta\leq\frac{(2(V+B)^2\gamma^2-B^2\delta)\theta-6(B(-2+\delta\theta)+3\theta\rho)\tau-9\delta\theta\tau^2}{36\tau}\equiv\eta^{DI}.\tag{A.5}$$

Single-innovation equilibrium (Asymmetric). The condition for this equilibrium is derived by establishing

(i) when the profits of investing ($RD^i = 1$) are greater than those of not investing ($RD^i = 0$); given the rival's decision to not invest ($RD^{i'} = 0$): $\eta < \eta^{NI}$, *and*

(ii) when the profits of not investing ($RD^i = 0$) are greater than those of investing ($RD^i = 1$); given the rival's decision to invest ($RD^{i'} = 1$): $\eta^{DI} < \eta$.

(2) *Case 2. If both firms innovate, the no-test equilibrium emerges; if only one firm innovates, the sole-innovator does not conduct testing. The fixed cost of a recall is intermediate. (i.e.,* $0 < \theta < \min\{\theta^{NT}, \theta^{SI}\}$ *and* $\rho^{\underline{B}} < \rho < \rho^{\bar{B}} = \rho^{SB}$*).*

The subgame profit levels are given by

$$\begin{aligned}
\Pi^{RD^i=1,RD^{i'}=1} &= \Pi^{0,0}_{1,1} = -\eta + (1-\theta)(\theta(V+B-\tau)+\tau) \\
\Pi^{RD^i=0,RD^{i'}=1} &= \Pi^{0,0}_{0,1} = (2-\theta)\cdot\frac{(B-3\tau)^2}{18\tau} + \theta\cdot\frac{((V+B)\gamma - B + 3\tau)^2}{18\tau} \\
\Pi^{RD^i=1,RD^{i'}=0} &= \Pi^{0,0}_{1,0} = -\eta + (2-\theta-\theta\delta)\cdot\frac{(B+3\tau)^2}{18\tau} - \theta\rho + \theta\cdot\frac{(B-(V+B)\gamma+3\tau)^2}{18\tau} \\
\Pi^{RD^i=0,RD^{i'}=0} &= \Pi^{0,0}_{0,0} = \tau
\end{aligned}$$

Using above profit levels, we can specify the conditions required for each equilibrium to hold.

No-innovation equilibrium (Symmetric). The condition for this equilibrium is derived by establishing when profits of not investing ($RD^i = 0$) are greater than those of investing ($RD^i = 1$); given the rival's decision to not invest ($RD^{i'} = 0$). That is,

$$\begin{aligned}
&\Pi^{RD^i=0,RD^{i'}=0} \geq \Pi^{RD^i=1,RD^{i'}=0} \\
\Leftrightarrow\ &\eta \geq \frac{-2VB(1-\gamma)\gamma\theta + B^2(2-(2-\gamma)\gamma\theta-\delta\theta) - 6V\gamma\theta\tau + 6B(2-(\gamma+\delta)\theta)\tau - 9\tau(2\eta+2\theta\rho+\delta\theta\tau)}{18\tau} \equiv \eta^{NI}.
\end{aligned}$$

Dual-innovation equilibrium (Symmetric). The condition for this equilibrium is derived by establishing when profits of investing ($RD^i = 1$) are greater than those of not investing ($RD^i = 0$); given the rival's decision to invest ($RD^{i'} = 1$). That is,

$$\begin{aligned}
&\Pi^{RD^i=1,RD^{i'}=1} \geq \Pi^{RD^i=0,RD^{i'}=1} \\
\Leftrightarrow\ &\eta \leq \frac{2VB(1-\gamma)\gamma\theta - V^2\gamma^2\theta + B^2(2\gamma\theta-2-\gamma^2\theta) + 6V\theta(3-\gamma-3\theta)\tau + 6B(2+(3-\gamma)\theta-3\theta^2)\tau - 18(2-\theta)\theta\tau^2}{18\tau} \equiv \eta^{DI}. \qquad \text{(A.6)}
\end{aligned}$$

Single-innovation equilibrium (Asymmetric). The condition for this equilibrium is derived by establishing

(i) when the profits of investing ($RD^i = 1$) are greater than those of not investing ($RD^i = 0$); given the rival's decision to not invest ($RD^{i'} = 0$): $\eta < \eta^{NI}$, *and*

(ii) when the profits of not investing ($RD^i = 0$) are greater than those of investing ($RD^i = 1$); given the rival's decision to invest ($RD^{i'} = 1$): $\eta^{DI} < \eta$.

Proof of Result 4.

(1) *Case* 1. From Equation (A.5), $\frac{\partial \eta^{DI}}{\partial \theta} = \frac{2(V+B)^2\gamma^2 - \delta(B+3\tau)^2 - 18\rho\tau}{36\tau} > 0$ holds if $0 \leq \rho < \frac{2(V+B)^2\gamma^2 - \delta(B+3\tau)^2}{18\tau}$. Because $\frac{2(V+B)^2\gamma^2 - \delta(B+3\tau)^2}{18\tau} > 0$ holds as $\delta \to 0$, there is nonempty range of parameter values for which the result is sustained.

(2) *Case* 2. From Equation (A.6), $\frac{\partial \eta^{DI}}{\partial \theta} = \frac{(V+B)(3-\gamma-6\theta)}{3} + \frac{(V+B)\gamma(2B-(V+B)\gamma)}{18\tau} - 2(1-\theta)\tau > 0$ holds if $0 \leq \theta < \frac{6(V+B)(3-\gamma)\tau + (V+B)\gamma(2B-(V+B)\gamma) - 36\tau^2}{36(V+B-\tau)\tau} \equiv \theta^{R4}$. Note that as $V \to 1$, $B \to 0.5$, $\gamma \to \frac{1}{3}$, and $\tau \to 0.5$ (here, the values jointly satisfy the constraints in Lemma 1 and Proposition 3), we have $\theta^{R4} = 0.181$ and thus there is a nonempty range of parameter values for which the result is sustained.

B. Extensions and Robustness Checks

This section provides details of the analyses discussed in Section 6 of the main paper. We refer to the model under horizontal demand structure (provided in Section 3) as the "basic model" throughout the section. The notation remains the same as in the main paper unless otherwise defined.

B.1. Consumer Heterogeneity in Willingness-To-Pay (WTP) for Innovation

Under the *vertical demand structure*, consumers are heterogeneous in how much value they place on quality. As will become clear, such demand structure amplifies the firms' differentiation incentive. Therefore, the single-test region expands over a wide range of parameter values as compared to the basic model.

The discussion begins by illustrating the model of vertical demand structure. We then analyze the equilibria under the alternative demand structure for both unlimited- and limited-liability settings. The analysis then proceeds to consider the firms' product selection decisions.

Vertical Demand Structure. Suppose that the market is composed of a unit mass of consumers who are heterogeneous in their WTP for product quality. Each consumer is characterized by a marginal valuation for product quality, φ, distributed uniformly on $[0, 1]$. For brevity, we exclude consumers' horizontal preference for each brand by setting $\tau = 0$. All other settings of the basic model continue to hold.

Under the alternative setting, the utility a consumer with valuation for quality φ derives from purchasing firm i's product type k at price p is given by

$$u^i = \varphi \cdot q(k^i) - p^i,$$

where $q(k^i) \in \{V, V+B, (V+B)(1-\gamma)\}$ as defined in Equation (1) of the main paper. The coverage of the market depends on the competitive environment. When competing firms offer differentiated products, the equilibrium prices are positive and some consumers whose valuations are close to zero will not buy (i.e., the market will not be fully covered). In contrast, when competing firms offer products of equal quality, the resulting head-to-head competition drives prices down to zero and the market becomes fully covered.

Firms Have Unlimited Liability. The analysis begins by solving for the firms' optimal actions given unlimited liability, which allows us to better characterize the equilibrium conditions that pertain to the alternative demand structure, and then proceeds by incorporating firms' limited liability, where we illustrate how the qualitative results of the basic model also apply to the vertical demand structure.

In a duopoly under vertical models of consumer preferences, if both firms offer products of equal quality level, Bertrand price competition (Tirole 1988) ensues, driving prices and profits to zero. This implies that firm i prices above zero only when the product it offers is differentiated from its rival's. The following lemma characterizes the outcomes of the pricing subgame depending on the competitive products offered.

Lemma B1. *In the pricing subgame, firm i's equilibrium prices and profits are as follows:*

- *If firm i markets a mature product ($k_1^i = ma$) while rival i′ markets its innovation ($k_1^{i'} = in$),*

$$p_1^i(ma,in) = \frac{VB}{3V+4B},\ \pi_1^i(ma,in) = \frac{VB(V+B)}{(3V+4B)^2};\ p_1^{i'}(in,ma) = \frac{2B(V+B)}{3V+4B},\ \pi_1^{i'}(in,ma) = \frac{4B(V+B)^2}{(3V+4B)^2}.$$

- *If firm i markets its innovation ($k_2^i = in$) while rival i′ markets its recalled innovation ($k_2^{i'} = rc$),*

$$p_2^i(in,rc) = \frac{2(V+B)\gamma}{3+\gamma},\ \pi_2^i(in,rc) = \frac{4(V+B)\gamma}{(3+\gamma)^2};\ p_2^{i'}(rc,in) = \frac{(V+B)(1-\gamma)\gamma}{3+\gamma},\ \pi_2^{i'}(rc,in) = \frac{(V+B)(1-\gamma)\gamma}{(3+\gamma)^2}.$$

- *Otherwise, $p_t^i = p_t^{i'} = 0$, $\pi_t^i = \pi_t^{i'} = 0$.*

Proof. If both firms introduce nondifferentiated products, the equilibrium price p_t^i and profit π_t^i are zero for $i \in \{1,2\}$. In the following, let H (L) denote a high (low) quality product. The marginal consumers are $\varphi_H = \frac{p_H - p_L}{q_H - q_L}$ and $\varphi_L = \frac{p_L}{q_L}$. The competing firms each maximize the (expected) profit with respect to demand $d_H = 1 - \varphi_H$, for the firm with a high-quality product; and $d_L = \varphi_H - \varphi_L$, for the firm with a low-quality product. There are two cases in which product differentiation arise:

- Mature versus innovative product ($k_1^i = ma$; $k_1^{i'} = in$) at $t = 1$: The low quality product is $q_L = q(ma) = V$; and the high quality product is $q_H = q(in) = V + B$. The marginal consumers are $\varphi_L = \frac{p_L}{V}$ and $\varphi_H = \frac{p_H - p_L}{B}$. This leads to $p_L = p_1^i(ma,in) = \frac{VB}{3V+4B}$ and $\pi_L = \pi_1^i(ma,in) = \frac{VB(V+B)}{(3V+4B)^2}$; $p_H = p_1^{i'}(in,ma) = \frac{2B(V+B)}{3V+4B}$ and $\pi_H = \pi_1^{i'}(in,ma) = \frac{4B(V+B)^2}{(3V+4B)^2}$.
- Innovative versus recalled product ($k_2^i = in$; $k_2^{i'} = rc$) at $t = 2$: The high quality product is $q_H = q(in) = V + B$; and the low quality product is $q_L = q(rc) = (V + B)(1-\gamma)$. The marginal consumers are $\varphi_H = \frac{p_H - p_L}{(V+B)\gamma}$ and $\varphi_L = \frac{p_L}{(V+B)(1-\gamma)}$. This leads to $p_H = p_2^i(in,rc) = \frac{2(V+B)\gamma}{3+\gamma}$ and $\pi_H = \pi_2^i(in,rc) = \frac{4(V+B)\gamma}{(3+\gamma)^2}$; $p_L = p_2^{i'}(rc,in) = \frac{(V+B)(1-\gamma)\gamma}{3+\gamma}$ and $\pi_L = \pi_2^{i'}(rc,in) = \frac{(V+B)(1-\gamma)\gamma}{(3+\gamma)^2}$. □

Given firms' equilibrium prices and profits in the pricing subgame, we now characterize the equilibria of the entire game in terms of the decision to conduct quality assurance testing.

Proposition B1. *In a duopoly featuring two identical firms, quality assurance testing decisions are determined by the following mutually exclusive regions categorized by θ^{DT} and $\theta^{NT} \equiv [\theta^{\underline{NT}}, \theta^{\overline{NT}}]$:*

- *No-test equilibrium (symmetric): If $\theta \in \theta^{NT}$, neither firm conducts testing and both rush to the market with their respective innovations.*
- *Dual-test equilibrium (symmetric): If $\theta \geq \theta^{DT}$, both firms conduct testing and delay launching their innovations.*
- *Single-test equilibrium (asymmetric): If $\theta \leq \theta^{DT}$ and $\theta \notin \theta^{NT}$, only one firm conducts testing and the other rushes to the market with its innovation.*

Proof. In equilibrium, each firm's strategy, QA^i forms a best response to its rival's strategy, $QA^{i'}$. Firm i's total expected profits $\Pi^{QA^i,QA^{i'}}$ for each subgame $(QA^i,QA^{i'})$ are given by:

- The rival's strategy is to test ($QA^{i'}=1$).
  (1) If the firm conducts testing ($QA^i=1$): $\Pi^{1,1}=0$.
  (2) If the firm does not conduct testing ($QA^i=0$): $\Pi^{0,1}=\frac{4B(V+B)^2}{(3V+4B)^2}+\theta\left(-\delta\cdot\frac{4B(V+B)^2}{(3V+4B)^2}-\rho+\frac{(V+B)(1-\gamma)\gamma}{(3+\gamma)^2}\right)$.
- Rival strategy is not to test ($QA^{i'}=0$).
  (1) If the firm conducts testing ($QA^i=1$): $\Pi^{1,0}=\frac{VB(V+B)}{(3V+4B)^2}+\theta\cdot\frac{4(V+B)\gamma}{(3+\gamma)^2}$.
  (2) If the firm does not conduct testing ($QA^i=0$): $\Pi^{0,0}=\theta(1-\theta)\cdot\frac{(V+B)(1-\gamma)\gamma}{(3+\gamma)^2}+(1-\theta)\theta\cdot\frac{4(V+B)\gamma}{(3+\gamma)^2}-\theta\rho$.

Using the above profit levels, we specify the conditions required for each equilibrium to hold.

- The no-test equilibrium holds if

$$\Pi^{0,0}=(1-\theta)\theta\cdot\frac{(V+B)(5-\gamma)\gamma}{(3+\gamma)^2}-\theta\rho\geq\frac{VB(V+B)}{(3V+4B)^2}+\theta\cdot\frac{4(V+B)\gamma}{(3+\gamma)^2}=\Pi^{1,0}$$
$$\Leftrightarrow\theta\in\theta^{NT}\equiv[\theta^{\underline{NT}},\theta^{\overline{NT}}],$$

  where

$$\theta^{\underline{NT}}=\frac{(V+B)(1-\gamma)\gamma-(3+\gamma)^2\rho-\sqrt{((V+B)(1-\gamma)\gamma-(3+\gamma)^2\rho)^2-\frac{4VB(V+B)^2(5-\gamma)\gamma(3+\gamma)^2}{(3V+4B)^2}}}{2(V+B)(5-\gamma)\gamma}\quad\text{and}$$
$$\theta^{\overline{NT}}=\frac{(V+B)(1-\gamma)\gamma-(3+\gamma)^2\rho+\sqrt{((V+B)(1-\gamma)\gamma-(3+\gamma)^2\rho)^2-\frac{4VB(V+B)^2(5-\gamma)\gamma(3+\gamma)^2}{(3V+4B)^2}}}{2(V+B)(5-\gamma)\gamma}.$$

  For θ^{NT} to exist ($0<\theta^{NT}<1$), the following conditions are required: $B<\frac{335+29\sqrt{145}-\sqrt{234026+19430\sqrt{145}}}{16}V$, $\rho<\frac{(V+B)(3V+4B)(1-\gamma)\gamma-2(V+B)(3+\gamma)\sqrt{VB(5-\gamma)\gamma}}{(3V+4B)(3+\gamma)^2}$, and $\gamma^{\underline{NT}}<\gamma<\gamma^{\overline{NT}}$. Solving for $\gamma^{\underline{NT}}$ and $\gamma^{\overline{NT}}$ involves tedious algebra and the details are available from the authors. Note that when $B\to0$ and $\rho\to0$, we have that $\theta^{\underline{NT}}\to0$ and $\theta^{\overline{NT}}\to\frac{1-\gamma}{5-\gamma}$. Therefore, there is a nonempty range of γ values for which θ^{NT} exists.

- The dual-test equilibrium holds if

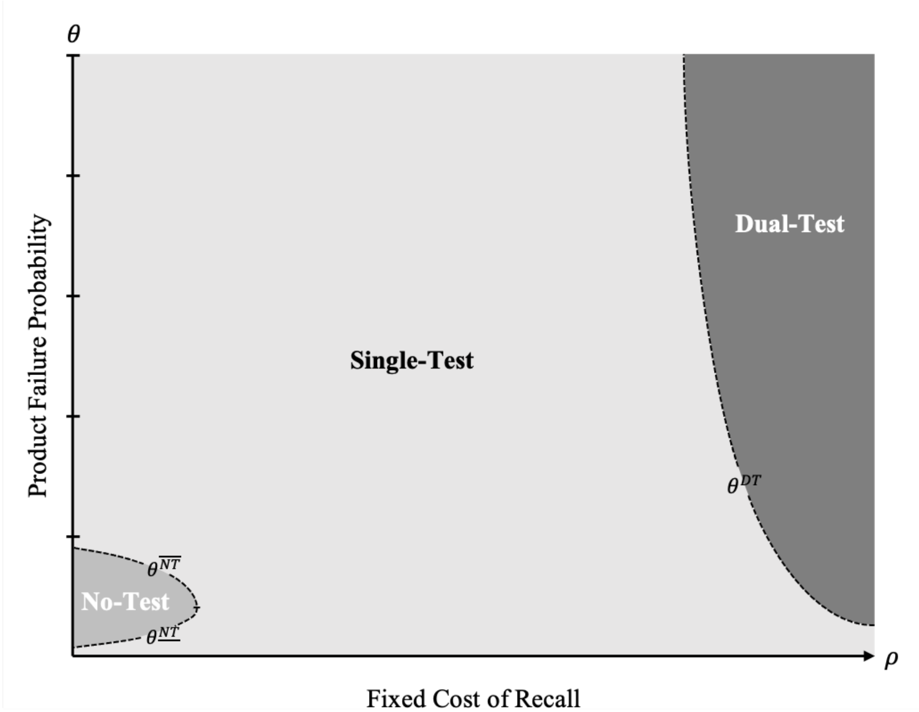
$$\Pi^{1,1}=0\geq\frac{4B(V+B)^2}{(3V+4B)^2}+\theta\left(-\delta\cdot\frac{4B(V+B)^2}{(3V+4B)^2}-\rho+\frac{(V+B)(1-\gamma)\gamma}{(3+\gamma)^2}\right)=\Pi^{0,1}$$
$$\Leftrightarrow\theta\geq\frac{4B(V+B)^2(3+\gamma)^2}{(V+B)(4B(V+B)(3+\gamma)^2\delta-(3V+4B)^2(1-\gamma)\gamma)+(3V+4B)^2(3+\gamma)^2\rho}\equiv\theta^{DT}.$$

  Because the failure probability satisfies $\theta\in(0,1)$, θ^{DT} exists ($0<\theta^{DT}<1$) if $\rho>\frac{(V+B)(9V^2(1-\gamma)\gamma+4B^2(9+10\gamma-3\gamma^2-(3+\gamma)^2\delta)+4VB(9+12\gamma-5\gamma^2-(3+\gamma)^2\delta))}{(3V+4B)^2(3+\gamma)^2}$.

- The single-test equilibrium holds if $\theta<\min(\theta^{DT},1)$ and $\theta\in\{\theta>0,\theta\notin\theta^{NT}\}$. □

Figure B1 provides a graphical illustration of the equilibrium regions described in Proposition B1 in the $\theta\leftrightarrow\rho$ parameter space. We find that all three equilibria of the basic model continue to hold, despite in a slightly different form: (1) the no-test equilibrium arises when the cost of a recall is relatively small and the failure probability is low

Figure B1 Equilibrium Regions: Duopoly Under Vertical Demand Structure



Notes. The dotted lines depict cutoff values θ^{DT} and θ^{NT}. The y-axis shows product failure probability θ and the x-axis shows the fixed cost of a recall ρ.

but not too low, (2) the dual-test equilibrium arises when the cost of a recall and the failure probability are high, and (3) the single-test equilibrium arises otherwise.

We note the following characteristics of the equilibrium under the vertical setting as compared to the basic model. First, the single-test equilibrium region exists over a wider region, which is driven by the firms' differentiation incentive. Unlike the basic model, the vertical demand structure implies that the firm, conditional on the rival offering its innovation, gains more by offering an inferior differentiated product than by offering its innovation as well (as illustrated in Lemma B1). Hence, the firms' differentiation incentive grows significantly under the vertical setting. This incentive drives firms to vary the timing of their innovation launch, thereby expanding the single-test equilibrium region.

Second, the no-test equilibrium region *can arise* in the vertical setting, despite the two firms taking identical actions and thus their products will likely be undifferentiated. The intuition for the emergence of this equilibrium is the probabilistic product recalls. That is, if one of the firms' innovation experiences a recall, the two products become differentiated in the second period, leading to positive profits per Lemma B1. Thus, both firms benefit from product differentiation due to a recall by allowing them to avoid head-to-head competition. Further supporting this equilibrium is the intense competition that drives down the price of the (untested) innovations at $t = 1$, which reduces the cost burden in case of a product recall. Note that without product recall considerations (i.e., under conventional launch-timing games), this equilibrium region would not exist because a firm is always better of deviating and differentiating its product offering over time.

In addition to its existence, the no-test equilibrium region exhibits an inverse-U shaped form with respect to failure probability θ. The mechanism behind this equilibrium is that the benefits of a probabilistic recall arises if only one of the firms' innovation fails, but not both. Thus, with probability $\theta(1-\theta)$, the firms benefit from positive profit at $t = 2$ due to product differentiation. This quadratic relationship between the recall probability and the expected second-period profits gives rise to the the upper and lower bounds, which creates an inverse-U shape of the no-test equilibrium region.

Lastly, the dual test equilibrium exists only if the fixed cost of a recall, ρ, is sufficiently large, such that the firms would need to forfeit more than or equal to the revenue they could generate, in order to deal with a recall (e.g., to settle legal issues and incur marketing expenses). In other words, for both firms to conduct testing, the pecuniary costs of a product recall must equal or exceed the potential gains form being the sole firm with an innovation in the market. This has implications for firms' behavior under limited liability, which is explained below.

Incorporating Limited Liability. We now incorporate firms' limited liability assumption into the vertical setting and show how the qualitative results of the basic model continue to carry over. The analysis considers Results 1 and 3 in order. Note that Result 2 discusses about consumers' sensitivity to horizontal mismatch and therefore does not apply to the vertical setting.

We first state the following result, which mirrors Result 1 of the basic model.

Result B1. *In the intermediate range of product failure probability θ, an increase in the fixed cost of a recall ρ above the threshold $\rho^{\underline{B}}$ can increase the likelihood of two competing firms releasing their innovations immediately under the vertical demand structure.*

Proof. Suppose $\rho^{\underline{B}} < \rho < \rho^{\bar{B}}$. Given rival strategy to not test, if the firm conducts testing, the profits follow the unlimited liability case (since the rival stays in the market after a recall). Hence, $\Pi^{1,0} = \frac{VB(V+B)}{(3V+4B)^2} + \theta \cdot \frac{4(V+B)\gamma}{(3+\gamma)^2}$. Conversely, if the firm does not conduct testing, it benefits from monopoly-level profits in case the rival incurs a product recall and exits the market, which occurs with probability $(1-\theta)\theta$. The monopoly-level profits from selling the innovation is given by $\pi_2^i(in, \emptyset) = (V + B)/4$. Hence, $\Pi^{0,0} = (1-\theta)\theta \cdot (V + B)/4$. Using these profit levels, the condition required for the no-test equilibrium to hold (if $\rho^{\underline{B}} < \rho < \rho^{\bar{B}}$) is:

$$\Pi^{0,0} = (1-\theta)\theta \cdot \frac{V+B}{4} \geq \frac{VB(V+B)}{(3V+4B)^2} + \theta \cdot \frac{4(V+B)\gamma}{(3+\gamma)^2} = \Pi^{1,0}$$
$$\Leftrightarrow \theta \in \theta^{NT'} \equiv [\theta^{\underline{NT'}}, \theta^{\overline{NT'}}],$$

where

$$\theta^{\underline{NT'}} = \frac{(9-\gamma)(1-\gamma)}{2(3+\gamma)^2} - \frac{\sqrt{(V+B)^2((9-\gamma)^2(1-\gamma)^2/(3+\gamma)^4 - 16VB/(3V+4B)^2)}}{2(V+B)} \text{ and}$$
$$\theta^{\overline{NT'}} = \frac{(9-\gamma)(1-\gamma)}{2(3+\gamma)^2} + \frac{\sqrt{(V+B)^2((9-\gamma)^2(1-\gamma)^2/(3+\gamma)^4 - 16VB/(3V+4B)^2)}}{2(V+B)}.$$

To conclude the proof, we compare the values of θ^{NT} and $\theta^{NT'}$, which determine the size of the no-test region for $0 \leq \rho \leq \rho^{\underline{B}}$ and $\rho^{\underline{B}} < \rho < \rho^{\bar{B}}$, respectively. To show the existence of the result, assume $B \to 0$ and $\rho \to 0$. Then, $\theta^{\underline{NT}} = \theta^{\underline{NT'}} = 0$, $\theta^{\overline{NT}} = \frac{1-\gamma}{5-\gamma}$ and $\theta^{\overline{NT'}} = \frac{(9-\gamma)(1-\gamma)}{(3+\gamma)^2}$. Because $\frac{1-\gamma}{5-\gamma} < \frac{(9-\gamma)(1-\gamma)}{(3+\gamma)^2}$ for any $0 < \gamma < 1$, the no-test equilibrium region expands at $\rho^{\underline{B}}$. $\square$

The result illustrates that the finding in which a stronger penalty for experiencing a product recall prompting both firms to rush to the market is applicable for the vertical setting. We further note that the threshold $\rho^{\underline{B}}$ under the vertical setting is lower than the horizontal setting—in fact, $\rho^{\underline{B}} = 0$ under the vertical setting. This arises from the nature of vertical models under which a head-to-head competition drives price and profits to zero. Since $\rho^{\underline{B}}$ is determined by the profit level at which both firms sell their innovations in the first period, in the vertical setting, *any* cost of a recall triggers a firm's bankruptcy, thereby facilitating both firms to immediately launch their innovations.

Thus far, we replicated the main result of the basic model under the vertical demand structure. We note, however, that Result 3, which discusses how the competitive forces may drive both firms to fall into a sub-ideal situation (i.e., both firms rush to the market and take on the risk of a recall), does not hold under the vertical setting. As discussed in the unlimited liability case, the dual-test equilibrium exists only if the costs of a recall are sufficiently large. Hence, under the limited liability assumption, the dual-test equilibrium region only weakly holds.[7] Because any product differentiation leads to positive profits under the vertical setting, the firms' expected payoffs are higher under the no-test equilibrium (as compared to had they both been able to commit to testing), which offers a chance to benefit from product differentiation due to a recall and thus avoid head-to-head competition.

Product Selection and Shelving an Innovation. This subsection allows firms to offer either product variant in their portfolio. Specifically, at the beginning of $t = 2$, let each firm choose the product to launch. For simplicity, we maintain the assumption that a firm markets only one product per period. Given this modified setup, the following result identifies the conditions for firms' product selection and shelving an innovation.

Result B2. *Consider the single-test equilibrium in which at $t = 2$ firm i has conducted testing and possesses a product portfolio $k_2^i \in \{ma, in\}$ and firm i' has rushed to the market and experienced a recall so that it possesses a product portfolio $k_2^{i'} \in \{ma, rc\}$. Under the vertical demand structure, there exist cutoff values $0 < \gamma^{NS} < \gamma^{\underline{RS}} \leq \gamma^{\overline{RS}} < 1$ for the reputation-damage parameter γ such that the following equilibria can be sustained:*

[7] The dual-test equilibrium holds as $\rho \to (1-\delta)4B(V+B)^2/(3V+4B)^2$, $\theta \to 1$, and either $\gamma \to 0$ or $\gamma \to 1$.

- *If $\gamma \in (0, \gamma^{NS})$, firm i, when faced with a recalled product from rival i', shelves its fail-proof innovation and markets the mature product ($k_2^i = ma$, $k_2^{i'} = rc$).*
- *If $\gamma \in (0, \gamma^{\underline{RS}})$ or $\gamma \in (\gamma^{\overline{RS}}, 1)$, firm i', when faced with an innovation from rival i, withdraws its recalled product and markets the mature product ($k_2^i = in$, $k_2^{i'} = ma$).*
- *If $\gamma \in [\gamma^{\underline{RS}}, \gamma^{\overline{RS}}]$, firm i', when faced with an innovation from rival i, continues to market its recalled product ($k_2^i = in$, $k_2^{i'} = rc$).*

In the range $\gamma \in (0, \gamma^{NS})$, multiple equilibria, ($k_2^i = ma$, $k_2^{i'} = rc$) and ($k_2^i = in$, $k_2^{i'} = ma$), may coexist.

Proof. Using the profit levels under the vertical demand structure, the best response for each firm, given the rival's action, is derived as follows:

- When reputation damage is small: $\gamma \leq \frac{B}{V+B} \Leftrightarrow (V+B)(1-\gamma) \geq V$

  For firm i that conducted testing, facing rival i' that encountered a recall:

  (1) If the rival's action is to launch the mature product ($k^{i'} = ma$), we have

  $$\pi^i(in, ma) - \pi^i(ma, ma) = \frac{4B(V+B)^2}{(3V+4B)^2} > 0.$$

  Hence, the firm is always better off launching the innovation.

  (2) If the rival's action is to launch the recalled product ($k^{i'} = rc$), firm i's best response is to launch the innovation if

  $$\pi^i(in, rc) - \pi^i(ma, rc) = \frac{4(V+B)\gamma}{(3+\gamma)^2} - \frac{V(V+B)(1-\gamma)((V+B)(1-\gamma)-V)}{(4(V+B)(1-\gamma)-V)^2} > 0,$$

  and launch the mature product otherwise.

  It turns out that deriving a closed form solution for the quartic inequality is cumbersome. Instead, we illustrate that there exists a unique value $\gamma^{NS} \in \left(0, \frac{B}{V+B}\right)$ such that $\pi^i(in, rc) - \pi^i(ma, rc) = 0$ holds, and that for $0 < \gamma < \gamma^{NS}$ ($\gamma^{NS} < \gamma < \frac{B}{V+B}$), the firm is better off launching its mature (innovative) product. Let us denote $\hat{\pi}^{NM} \equiv \pi^i(in, rc) - \pi^i(ma, rc)$. We have (1) $\hat{\pi}^{NM} = -\frac{VB(V+B)}{(3V+4B)^2} < 0$ as $\gamma \to 0$; and (2) $\hat{\pi}^{NM} = \frac{4B}{(3+\frac{B}{V+B})^2} > 0$ as $\gamma \to \frac{B}{V+B}$. Further, the function $\hat{\pi}^{NM}$ is continuous in the domain $\gamma \in \left(0, \frac{B}{V+B}\right)$; and there exists no value of γ such that $\frac{\partial \hat{\pi}^{NM}}{\partial \gamma} = 0$ holds. Hence, $\hat{\pi}^{NM}$ is a monotone increasing function in $\gamma \in \left(0, \frac{B}{V+B}\right)$, and, thus, there exists some $\gamma^{NS} \in \left(0, \frac{B}{V+B}\right)$ such that $\hat{\pi}^{NM} = 0$ as $\gamma \to \gamma^{NS}$.

  For firm i' that encountered a recall, facing rival i that conducted testing:

  (1) If the rival's action is to launch the mature product ($k^i = ma$), we have

  $$\pi^{i'}(rc, ma) - \pi^{i'}(ma, ma) = \frac{4(V+B)^2(1-\gamma)^2((V+B)(1-\gamma)-V)}{(4(V+B)(1-\gamma)-V)^2} > 0.$$

  Hence, the firm is always better off launching the recalled product.

(2) If the rival's action is to launch the innovation ($k^i = in$), the best response of firm i' is to launch the recalled product if

$$\pi^{i'}(rc, in) - \pi^{i'}(ma, in) = \frac{(V+B)(1-\gamma)\gamma}{(3+\gamma)^2} - \frac{VB(V+B)}{(3V+4B)^2} > 0,$$

and launch the mature product otherwise. We have that $\pi^{i'}(rc, in) - \pi^{i'}(ma, in) > 0$ holds if $V > B > \frac{3V}{4}$ and $\frac{B}{V+B} > \gamma > \frac{9V}{9V+16B} \equiv \hat{\gamma}^{\underline{RM}}$.

In determining the order of cutoffs γ^{NS} and $\hat{\gamma}^{\underline{RM}}$, note that we have

$$\hat{\pi}^{NM}(\hat{\gamma}^{\underline{RM}}) = \frac{V(V+B)(9V+16B)}{4(3V+4B)^2} + \frac{16VB(V+B)(9V^2-16B^2)}{(9V^2-48VB-64B^2)^2} > 0.$$

for any $V > B > \frac{3V}{4}$. Hence, $\gamma^{NS} < \hat{\gamma}^{\underline{RM}}$ holds.

- When reputation damage is large: $\gamma > \frac{B}{V+B} \Leftrightarrow (V+B)(1-\gamma) < V$

For firm i that conducted testing, facing rival firm i' that encountered a recall:

(1) if the rival's action is to launch the mature product ($k^{i'} = ma$), we have

$$\pi^i(in, ma) - \pi^i(ma, ma) = \frac{4B(V+B)^2}{(3V+4B)^2} > 0.$$

Hence, the firm is always better off launching the innovation.

(2) if the rival's action is to launch the recalled product ($k^{i'} = rc$), we have

$$\pi^i(in, rc) - \pi^i(ma, rc) = \frac{4(V+B)\gamma}{(3+\gamma)^2} - \frac{4V^2(V-(V+B)(1-\gamma))}{(4V-(V+B)(1-\gamma))^2} > 0.$$

Hence, the firm is always better off launching the innovation.

For firm i' that encountered a recall, facing rival firm i that conducted testing:

(1) if the rival's action is to launch the mature product ($k^i = ma$), we have

$$\pi^{i'}(rc, ma) - \pi^{i'}(ma, ma) = \frac{V(V+B)(1-\gamma)(V-(V+B)(1-\gamma))}{(4V-(V+B)(1-\gamma))^2} > 0.$$

Hence, the firm is always better off launching the recalled product.

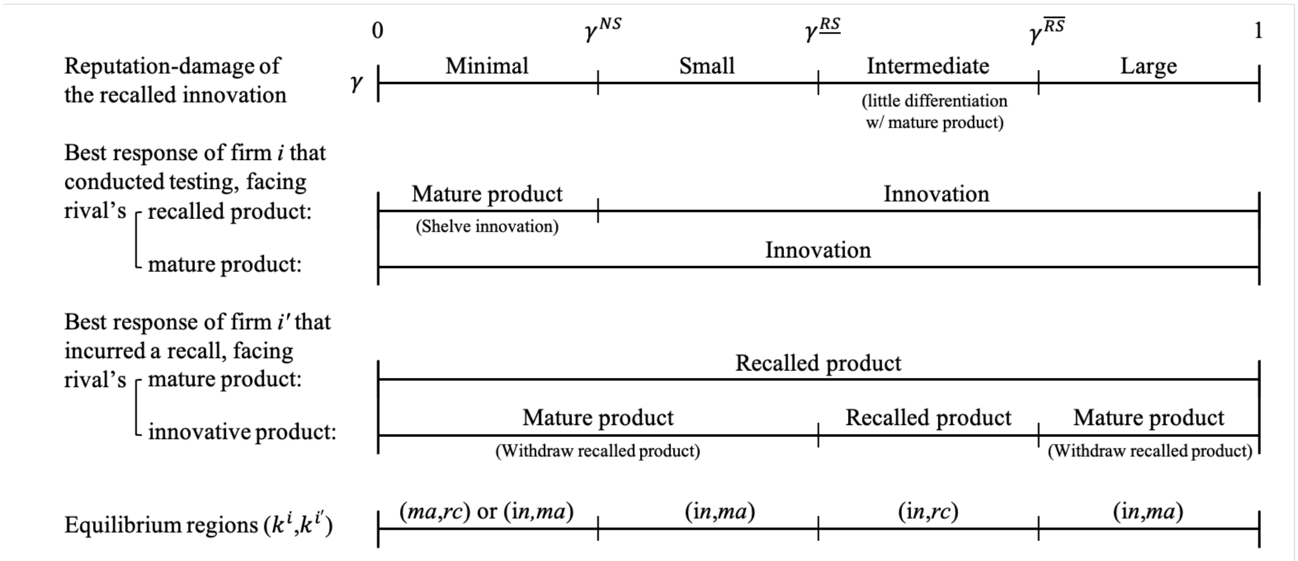
(2) if the rival's action is to launch the innovation ($k^i = in$), the best response of firm i' is to launch the recalled product if

$$\pi^{i'}(rc, in) - \pi^{i'}(ma, in) = \frac{(V+B)(1-\gamma)\gamma}{(3+\gamma)^2} - \frac{VB(V+B)}{(3V+4B)^2} > 0,$$

and launch the mature product otherwise. We have that $\pi^{i'}(rc, in) - \pi^{i'}(ma, in) > 0$ holds if $\frac{3V}{4} > B > 0$ and $\frac{B}{V+B} < \gamma < \frac{9V}{9V+16B} \equiv \hat{\gamma}^{\overline{RM}}$.

To conclude the proof, (1) if $0 < B < \frac{3V}{4}$, let $\gamma^{\underline{RS}} = \frac{B}{V+B}$ and $\gamma^{\overline{RS}} = \hat{\gamma}^{\overline{RM}}$; and (2) otherwise, let $\gamma^{\underline{RS}} = \hat{\gamma}^{\underline{RM}}$ and $\gamma^{\overline{RS}} = \frac{B}{V+B}$. $\square$

Figure B2 Firms' Best Responses and Resulting Product Selection Equilibrium Regions



The key to understanding the intuition behind Result B2 is examining the role of reputation damage on the relative value of competing products at $t = 2$. A firm's decision regarding which product from its portfolio to offer is governed in large part by the desire to avoid head-to-head competition. Since consumers are heterogeneous in their WTP for product quality, the firm can capture demand while charging a positive price (and, thus, generating a profit) even if the product it markets is of lower quality than the rival's. Figure B2 graphically summarizes this intuition by illustrating the best responses by the firms based on the degree of reputation damage γ that a recalled product faces.

Let us first consider the best response of firm i, which conducted testing (choosing between products $k^i \in \{ma, in\}$), when facing rival i', which offers its recalled innovation ($k^{i'} = rc$). If the reputation damage is significant ($\gamma \in [\gamma^{NS}, 1)$), the firm that conducted testing can take advantage of the situation by launching its innovation and targeting consumers who value high quality. In contrast, if the rival's recalled product experiences only minimal reputational damage ($\gamma \in (0, \gamma^{NS})$), the firm that tested is better off shelving its innovation and marketing the mature product to consumers whose valuation for quality is low, thereby avoiding intense price competition with its rival's relatively high-quality product.

A similar rationale holds for firm i', which incurred a recall (choosing between products $k^{i'} \in \{ma, rc\}$), when facing rival i, which conducted testing and offers its innovation ($k^i = in$). When the reputation damage from a recall is either limited or severe, firm i' is better off offering its legacy, mature product since offering the recalled innovation leads to a lower profit: (i) with limited reputation damage, the recalled product is not much differentiated from the innovation, and (ii) when the reputation damage is severe, the value of the recalled product in the eyes of consumers is less than that of the mature product. However, when the reputation damage is intermediate, the firm gains a greater profit from continuing with the recalled product since it is sufficiently differentiated from the rival's innovation and of greater value than the mature product.

Result B2 focuses on the single-test equilibrium region. For completeness, we also describe the product selection outcomes under the dual-test and no-test equilibria. If the rival firm also conducted testing or if its innovation did not fail despite forgoing testing, both firms are endowed with a mature product and a fail-proof innovation ($k^i \in \{ma, in\}$ for $i \in \{1,2\}$). Hence, at $t=2$ in equilibrium, one firm will offer its innovation and the other firm's best response is to offer its mature product, once again shelving the innovation. A similar asymmetric outcome holds if both firms skipped testing and experienced a recall (i.e., $k^i \in \{ma, rc\}$ for $i \in \{1,2\}$).[8]

B.2. Consumers Consider the Recall Probability of an Innovation Marketed Early

We allow firms' testing decisions and product failure probability to be of common knowledge to consumers. That is, consumers strategically consider the possibility of a product recall (and potential bankruptcy of the firm) when they observe an innovation being offered in the first period. Specifically, let consumer utility $u^i = \mathrm{E}_{\theta,\delta}[q(k_t^i) - p_t^i] - \tau(i-1+(3-2i)x)$ for $i \in \{1,2\}$, where

$$\mathrm{E}_{\theta,\delta}[q(k_t^i) - p_t^i] = \begin{cases} (1-\theta \cdot \mathbb{I}_{k_1^i=in})q(k_t^i) - (1-\delta\theta \cdot \mathbb{I}_{k_1^i=in})p_t^i & \text{if } \rho \le \rho^i, \\ (1-\theta \cdot \mathbb{I}_{k_1^i=in})q(k_t^i) - p_t^i & \text{if } \rho > \rho^i. \end{cases}$$

Here, $\mathbb{I}_{k_1^i=in}$ denotes an indicator function equal to 1 if $k_1^i = in$ and 0 otherwise, and ρ^i is firm i's threshold for bankruptcy following a recall. Hence, upon observing an early launch of the innovation, consumers consider both (1) the possible loss in product value they might need to bear ($= -\theta \cdot q(k^i)$) and (2) the monetary reimbursement they can retrieve from the recall ($= \theta \cdot \delta \cdot p^i$, if the firm stays in the market, and $=0$, if it goes bankrupt).

By solving for the new equilibrium price p and profits π, the following holds:

- Mature product versus innovation ($k_1^i = ma$; $k_1^{i'} = in$) at $t=1$: Product quality $q^i(ma) = V$ and $q^{i'}(in) = V+B$, the indifferent consumer is located at $\tilde{x} = \frac{-B+(V+B)\theta - p^i + p^{i'}(1-\delta\theta)+\tau}{2\tau}$. This leads to

$$p_1^i(ma,in) = \frac{-B+(V+B)\theta+3\tau}{3};\ d_1^i(ma,in) = \frac{-B+(V+B)\theta+3\tau}{6\tau};\ \text{and } \pi_1^i(ma,in) = \frac{(-B+(V+B)\theta+3\tau)^2}{18\tau};$$
$$p_1^{i'}(in,ma) = \frac{B-(V+B)\theta+3\tau}{3(1-\theta\delta)};\ d_1^{i'}(in,ma) = \frac{B-(V+B)\theta+3\tau}{6\tau};\ \text{and } \pi_1^{i'}(in,ma) = \frac{(B-(V+B)\theta+3\tau)^2}{18(1-\theta\delta)\tau},$$

if $\rho \le \rho^{\bar{B}} = (1-\delta)(B-(V+B)\theta+3\tau)^2/(18(1-\delta\theta)\tau)$, and

$$p_1^i(ma,in) = \frac{-B+(V+B)\theta+3\tau}{3};\ d_1^i(ma,in) = \frac{-B+(V+B)\theta+3\tau}{6\tau};\ \text{and } \pi_1^i(ma,in) = \frac{(-B+(V+B)\theta+3\tau)^2}{18\tau};$$
$$p_1^{i'}(in,ma) = \frac{B-(V+B)\theta+3\tau}{3};\ d_1^{i'}(in,ma) = \frac{B-(V+B)\theta+3\tau}{6\tau};\ \text{and } \pi_1^{i'}(in,ma) = \frac{(B-(V+B)\theta+3\tau)^2}{18\tau},$$

otherwise.

[8] In the no-test equilibrium, if one firm faces a recall while the other does not, the outcome is analogous to Result B2.

- Innovation versus innovation ($k_1^i = in$; $k_1^{i'} = in$) at $t=1$: Product quality $q^i(in) = q^{i'}(in) = V + B$, and the indifferent consumer is located at $\tilde{x} = \frac{(-p^i + p^{i'})(1-\delta\theta)+\tau}{2\tau}$. This leads to

$$p_t^i(in,in) = p_t^{i'}(in,in) = \frac{\tau}{1-\theta\delta};\ d_t^i(in,in) = d_t^{i'}(in,in) = \frac{1}{2};\ \text{and}\ \pi_t^i(in,in) = \pi_t^{i'}(in,in) = \frac{\tau}{2(1-\theta\delta)},$$

if $\rho \le \rho^{\underline{B}} = (1-\delta)\tau/(2(1-\delta\theta))$, and

$$p_t^i(in,in) = p_t^{i'}(in,in) = \tau;\ d_t^i(in,in) = d_t^{i'}(in,in) = \frac{1}{2};\ \text{and}\ \pi_t^i(in,in) = \pi_t^{i'}(in,in) = \frac{\tau}{2},$$

otherwise.

- For all other cases, firms' price p and profits π follow Lemma 1.

Three observations are noteworthy. First, if $(1-\theta)(V+B) > V$, consumers' expected utility from the innovation, despite the risk of its failure, is greater than their expected utility from the mature product, and thus the price and profit of the innovation are kept higher than the mature product. Conversely, if $(1-\theta)(V+B) < V$, the risk of product failure renders the expected utility lower than that of the mature product, and thus the price and profit of the mature product are higher in this case. Second, when the fixed cost of a recall is modest, consumers consider the amount they will be reimbursed (in proportion to δ) in the event of a recall. This behavior is factored in the pricing of the firm that launches its innovation early—represented by $(1-\theta\delta)$ in the denominator. The reimbursement, by reducing consumers' potential loss from a product failure, increases the price and profit from selling the innovation at $t=1$. Such increase cancels out the variable cost of a recall ($-\theta \cdot \delta \cdot \pi_1$ at $t=2$) and thus, in a sense, the firm effectively transfers a part of the risk to consumers. Third, when the fixed cost of a recall is high and consumers expect the firm(s) to go bankrupt, such risk transfer does not occur because consumers no longer expect to be reimbursed in the event of a recall.

We now illustrate how the three equilibria presented in the basic model continue to hold under the alternative setting. Using the equilibrium profit levels derived above, the following holds.

$$\Pi^{1,1} = \tau.$$

$$\Pi^{0,1} = \begin{cases} \frac{(B-(V+B)\theta+3\tau)^2}{18\tau} - \theta\rho + \theta \cdot \frac{(-(V+B)\gamma+3\tau)^2}{18\tau} + (1-\theta)\cdot\frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ (1-\theta)\left(\frac{(B-(V+B)\theta+3\tau)^2}{18\tau} + \frac{\tau}{2}\right) & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$

$$\Pi^{1,0} = \begin{cases} \frac{(-B+(V+B)\theta+3\tau)^2}{18\tau} + \theta \cdot \frac{((V+B)\gamma+3\tau)^2}{18\tau} + (1-\theta)\cdot\frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ \frac{(-B+(V+B)\theta+3\tau)^2}{18\tau} + \theta\cdot(V+B-\frac{\tau}{2}) + (1-\theta)\cdot\frac{\tau}{2} & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$

$$\Pi^{0,0} = \begin{cases} \tau - \theta\rho + 2\theta(1-\theta)\cdot\frac{(V+B)^2\gamma^2}{18\tau} & \text{if } \rho \le \rho^{\underline{B}}, \\ (1-\theta)\left(\theta(V+B-\tau)+\tau\right) & \text{if } \rho > \rho^{\underline{B}}. \end{cases}$$

The remainder of the analysis is straightforward and analogous to Proposition 2 in the main paper: the no-test equilibrium holds when $\Pi^{0,0} \ge \Pi^{1,0}$; the dual-test equilibrium holds when $\Pi^{1,1} \ge \Pi^{0,1}$; and the single-test equilibrium complements the other two conditions. Deriving the specific conditions in terms of failure probability θ involves tedious algebra and the details are available from the authors. Note that as $V \to 1$, $B \to 0.5$, $\gamma \to 0.5$, $\delta \to 1$, $\tau \to 0.2$, and $\rho \to 0$, we have that $\theta^{DT} = 0.397$ and $\theta^{NT} = 0.241$. Therefore, there is a nonempty range of parameter values for which all three equilibria are sustained.

B.3. A Segment of Consumers Avoids Purchase of a Risky Innovation

We consider a segment of consumers who avoid the purchase of a risky innovation and let the market size vary accordingly across periods. More specifically, define a risk-averse segment of size α as those consumers who desire to possess an innovation, but make their purchase only when is no risk of a product recall (i.e., at $t=2$). Further, let the total market size to be $1-\alpha$ at $t=1$ and $1+\alpha$ at $t=2$. Hence, as the risk-averse segment grows in size, more consumers delay their purchase until the second period, when product uncertainty is fully resolved. Note that the basic model is a special case in which $\alpha = 0$ and thus the key results continue to hold under the alternative setting.

The alternative setting leads to subgame profits given by:

$$\Pi^{1,1} = \tau.$$
$$\Pi^{0,1} = \begin{cases} (1-\theta\delta)\cdot\frac{(1-\alpha)(B+3\tau)^2}{18\tau} - \theta\rho + \theta\cdot\frac{(1+\alpha)(-(V+B)\gamma+3\tau)^2}{18\tau} + (1+\alpha)(1-\theta)\cdot\frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ (1-\theta)\left(\frac{(1-\alpha)(B+3\tau)^2}{18\tau} + (1+\alpha)\cdot\frac{\tau}{2}\right) & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$
$$\Pi^{1,0} = \begin{cases} \frac{(1-\alpha)(-B+3\tau)^2}{18\tau} + \theta\cdot\frac{(1+\alpha)((V+B)\gamma+3\tau)^2}{18\tau} + (1+\alpha)(1-\theta)\cdot\frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ \frac{(1-\alpha)(-B+3\tau)^2}{18\tau} + (1+\alpha)\cdot\theta(V+B-\frac{\tau}{2}) + (1+\alpha)(1-\theta)\cdot\frac{\tau}{2} & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$
$$\Pi^{0,0} = \begin{cases} (2-\theta\delta(1-\alpha))\cdot\frac{\tau}{2} - \theta\rho + 2\theta(1-\theta)\cdot\frac{(1+\alpha)(V+B)^2\gamma^2}{18\tau} & \text{if } \rho \le \rho^{\underline{B}}, \\ (1-\theta)\left((1+\alpha)\cdot\theta(V+B-\tau)+\tau\right) & \text{if } \rho > \rho^{\underline{B}}, \end{cases}$$

where $\rho^{\bar{B}} = \frac{(1-\alpha)(1-\delta)(B+3\tau)^2}{18\tau}$ and $\rho^{\underline{B}} = (1-\alpha)(1-\delta)\cdot\frac{\tau}{2}$.

The above illustrates how firms' profits shrink at $t=1$ and grow at $t=2$ in proportion to the risk-averse segment α. The likelihood of firms' testing, in general, increases with α as it lessens the advantages of the innovation at $t=1$ and grows the benefits of a robust innovation at $t=2$. Considering the bankruptcy concerns, however, the likelihood of both firms rushing to the market may increase with α, since a firm's prospect for monopoly profits (at $t=2$) grows.

B.4. Asymmetric Recall Probabilities

We examine a modified setting in which competing firms possess different product recall probabilities. Specifically, let firm i possess product failure probability $\theta_i = \theta$ and the rival firm i' possess $\theta_{i'} = \beta\cdot\theta$. Here, firm i's likelihood of a recall is higher if $0 < \beta < 1$ and lower if $1 < \beta < 1/\theta$. Note that the basic model is a special case in which $\beta = 1$ and thus the key results continue to hold under the alternative setting. The difference in two competing firms' recall probabilities lead to asymmetric subgame profits. Let $\Pi_i^{QA^i, QA^{i'}}$ denote firm i's total expected profits for subgame $(QA^i, QA^{i'})$. The subgame profits for firm i are given by:

$$\Pi_i^{1,1} = \tau.$$

$$\Pi_i^{0,1} = \begin{cases} (1-\theta\delta)\cdot\frac{(B+3\tau)^2}{18\tau} - \theta\rho + \theta\cdot\frac{(-(V+B)\gamma+3\tau)^2}{18\tau} + (1-\theta)\cdot\frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ (1-\theta)\left(\frac{(B+3\tau)^2}{18\tau} + \frac{\tau}{2}\right) & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$

$$\Pi_i^{1,0} = \begin{cases} \frac{(-B+3\tau)^2}{18\tau} + \beta\theta\cdot\frac{((V+B)\gamma+3\tau)^2}{18\tau} + (1-\beta\theta)\cdot\frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ \frac{(-B+3\tau)^2}{18\tau} + \beta\theta\cdot(V+B-\frac{\tau}{2}) + (1-\beta\theta)\cdot\frac{\tau}{2} & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$

$$\Pi_i^{0,0} = \begin{cases} (1-\theta\delta)\cdot\frac{\tau}{2} - \theta\rho + \theta\left(\beta\theta\cdot\frac{\tau}{2} + (1-\beta\theta)\cdot\frac{(-(V+B)\gamma+3\tau)^2}{18\tau}\right) & \\ \quad +(1-\theta)\left(\beta\theta\cdot\frac{((V+B)\gamma+3\tau)^2}{18\tau} + (1-\beta\theta)\cdot\frac{\tau}{2}\right) & \text{if } \rho \le \rho^{\underline{B}}, \\ (1-\theta)\left(\beta\theta(V+B-\tau)+\tau\right) & \text{if } \rho > \rho^{\underline{B}}, \end{cases}$$

and those of firm i' are given by:

$$\Pi_{i'}^{1,1} = \tau.$$

$$\Pi_{i'}^{0,1} = \begin{cases} (1-\beta\theta\delta)\cdot\frac{(B+3\tau)^2}{18\tau} - \beta\theta\rho + \beta\theta\cdot\frac{(-(V+B)\gamma+3\tau)^2}{18\tau} + (1-\beta\theta)\cdot\frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ (1-\beta\theta)\left(\frac{(B+3\tau)^2}{18\tau} + \frac{\tau}{2}\right) & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$

$$\Pi_{i'}^{1,0} = \begin{cases} \frac{(-B+3\tau)^2}{18\tau} + \theta\cdot\frac{((V+B)\gamma+3\tau)^2}{18\tau} + (1-\theta)\cdot\frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ \frac{(-B+3\tau)^2}{18\tau} + \theta\cdot(V+B-\frac{\tau}{2}) + (1-\theta)\cdot\frac{\tau}{2} & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$

$$\Pi_{i'}^{0,0} = \begin{cases} (1-\beta\theta\delta)\cdot\frac{\tau}{2} - \beta\theta\rho + \beta\theta\left(\theta\cdot\frac{\tau}{2} + (1-\theta)\cdot\frac{(-(V+B)\gamma+3\tau)^2}{18\tau}\right) & \\ \quad +(1-\beta\theta)\left(\theta\cdot\frac{((V+B)\gamma+3\tau)^2}{18\tau} + (1-\theta)\cdot\frac{\tau}{2}\right) & \text{if } \rho \le \rho^{\underline{B}}, \\ (1-\beta\theta)\left(\theta(V+B-\tau)+\tau\right) & \text{if } \rho > \rho^{\underline{B}}, \end{cases}$$

where $\rho^{\bar{B}} = \frac{(1-\delta)(B+3\tau)^2}{18\tau}$ and $\rho^{\underline{B}} = \frac{(1-\delta)\tau}{2}$.

Given the subgame profit levels, we can specify the conditions required for each equilibrium to hold: the no-test equilibrium holds when $\Pi^{0,0} \ge \Pi^{1,0}$ for both i and i'; the dual-test equilibrium holds when $\Pi^{1,1} \ge \Pi^{0,1}$ for both i and i'; and the single-test equilibrium complements the other two conditions. For brevity in examining the role of asymmetric recall probabilities—represented by parameter β—on firms' decisions to test, we illustrate the case in which both firms remain in the market following a recall and under the condition $\gamma \to 0$.

The condition of the no-test equilibrium is derived by establishing when the profits for not testing is greater than that of testing, given the rival's decision to not test, for *both* firms i and i'. That is,

$$\Pi_i^{0,0} \ge \Pi_i^{1,0} \Leftrightarrow \theta \le \frac{B(6\tau - B)}{9\tau(2\rho+\delta\tau)}, \text{ and}$$
$$\Pi_{i'}^{0,0} \ge \Pi_{i'}^{1,0} \Leftrightarrow \theta \le \frac{1}{\beta}\cdot\frac{B(6\tau - B)}{9\tau(2\rho+\delta\tau)}.$$

Since $\frac{B(6\tau-B)}{9\tau(2\rho+\delta\tau)} < \frac{1}{\beta}\cdot\frac{B(6\tau-B)}{9\tau(2\rho+\delta\tau)}$ if $0 < \beta < 1$ and, conversely, $\frac{B(6\tau-B)}{9\tau(2\rho+\delta\tau)} > \frac{1}{\beta}\cdot\frac{B(6\tau-B)}{9\tau(2\rho+\delta\tau)}$ if $1 < \beta < 1/\theta$, we have

$$\theta^{NT} = \begin{cases} \frac{B(6\tau-B)}{9\tau(2\rho+\delta\tau)} & \text{if } 0 < \beta < 1, \\ \frac{1}{\beta}\cdot\frac{B(6\tau-B)}{9\tau(2\rho+\delta\tau)} & \text{if } 1 < \beta < \frac{1}{\theta}. \end{cases}$$

Note here that the decision for both firms to forgo testing is governed by the firm with a higher probability of a recall: firm i (with failure probability θ) if $0<\beta<1$ and firm i' (with $\beta\cdot\theta$) if $1<\beta<1/\theta$. If the benefits of early launch are large enough for the firm with a higher failure rate to forgo testing, the rival with a lower failure rate would certainly rush to the market as well. In sum, the conditions imply that if $0<\beta<1$, the no-test equilibrium region is unaffected by β, whereas if $1<\beta<1/\theta$, the region shrinks in β.

Similarly, The condition of the dual-test equilibrium is derived by establishing when the profits for testing is greater than that of not testing, given the rival's decision to test, for *both* firms i and i'. That is,

$$\Pi_i^{1,1}\geq \Pi_i^{0,1}\Leftrightarrow \theta\geq \frac{B(6\tau+B)}{\delta(B+3\tau)^2+18\tau\rho},\ \textit{and}$$
$$\Pi_i^{1,1}\geq \Pi_i^{0,1}\Leftrightarrow \theta\geq \frac{1}{\beta}\cdot\frac{B(6\tau+B)}{\delta(B+3\tau)^2+18\tau\rho}.$$

Since $\frac{B(6\tau+B)}{\delta(B+3\tau)^2+18\tau\rho}<\frac{1}{\beta}\cdot\frac{B(6\tau+B)}{\delta(B+3\tau)^2+18\tau\rho}$ if $0<\beta<1$ and, conversely, $\frac{B(6\tau+B)}{\delta(B+3\tau)^2+18\tau\rho}>\frac{1}{\beta}\cdot\frac{B(6\tau+B)}{\delta(B+3\tau)^2+18\tau\rho}$, we have

$$\theta^{DT}=\begin{cases}\frac{1}{\beta}\cdot\frac{B(6\tau+B)}{\delta(B+3\tau)^2+18\tau\rho} & \text{if } 0<\beta<1,\\ \frac{B(6\tau+B)}{\delta(B+3\tau)^2+18\tau\rho} & \text{if } 1<\beta<\frac{1}{\theta}.\end{cases}$$

Here, the decision for both firms to conduct testing is governed by the firm with a lower probability of a recall: firm i' (with failure probability $\beta\cdot\theta$) if $0<\beta<1$ and firm i (with θ) if $1<\beta<1/\theta$. If the consequences of a recall are severe enough to trigger it to test, the rival with a higher failure rate would certainly do the same. In sum, the conditions imply that, if $0<\beta<1$, the dual-test region expands in β, whereas if $1<\beta<1/\theta$, the region is invariant to β.

B.5. The Cost of a Recall Depends on the Number of Units Demanded

We consider an alternative scenario in which the recall cost is a function of "units" demanded (d_1), rather than the "revenue" generated (π_1). Let us redefine the direct cost of a recall by $\hat{\Delta}=\rho+\hat{\delta}\cdot d_1$, where $\hat{\delta}>0$ expresses the degree of penalty per-unit demanded at $t=1$. Given this modified setup, the firms' subgame (expected) profit maximization problem becomes

$$\begin{cases}\pi_t(k)-\theta\hat{\Delta}=\pi_t(k)-\theta(\rho+\hat{\delta}\cdot d)=d\cdot p-\theta(\rho+\hat{\delta}\cdot d) & \text{if } k=in \text{ and } t=1,\\ \pi_t(k)=d\cdot p & \text{otherwise.}\end{cases}$$

The change affects firms' pricing and thus profits when the innovative product is offered at $t=1$. By solving for the new equilibrium price p and profits π,[9] the following holds:

- Mature product versus innovation ($k_1^i=ma$; $k_1^{i'}=in$) at $t=1$: Since $q^i(ma)=V$ and $q^{i'}(in)=V+B$, the indifferent consumer is located at $\tilde{x}=\frac{-B-p^i+p^{i'}+\tau}{2\tau}$. This leads to

[9] The equilibrium price and profits are derived for the case in which the firms remain in the market following a recall. Alternatively, if the firms were to consider the bankruptcy concerns, their price and profits under the parameter region of bankruptcy follow Lemma 1 in the main paper, since the variable cost $\hat{\delta}$ no longer affects a firm's payoff in case of a recall.

(1) If $\hat{\delta}\theta < B + 3\tau$,

$$p_1^i(ma, in) = \frac{-B + \hat{\delta}\theta + 3\tau}{3};\ d_1^i(ma, in) = \frac{-B + \hat{\delta}\theta + 3\tau}{6\tau};\ \text{and } \pi_1^i(ma, in) = \frac{(-B + \hat{\delta}\theta + 3\tau)^2}{18\tau};$$
$$p_1^{i'}(in, ma) = \frac{B + 2\hat{\delta}\theta + 3\tau}{3};\ d_1^{i'}(in, ma) = \frac{B - \hat{\delta}\theta + 3\tau}{6\tau};\ \text{and } \pi_1^{i'}(in, ma) = \frac{(B + 2\hat{\delta}\theta + 3\tau)(B - \hat{\delta}\theta + 3\tau)}{18\tau}.$$

(2) Otherwise,

$$p_1^i(ma, in) = \frac{-B + \hat{\delta}\theta + 3\tau}{3};\ d_1^i(ma, in) = 1;\ \text{and } \pi_1^i(ma, in) = \frac{-B + \hat{\delta}\theta + 3\tau}{3};$$
$$p_1^{i'}(in, ma) = \frac{B + 2\hat{\delta}\theta + 3\tau}{3};\ d_1^{i'}(in, ma) = 0;\ \text{and } \pi_1^{i'}(in, ma) = 0.$$

- Innovation versus innovation ($k_1^i = in$; $k_1^{i'} = in$) at $t = 1$: Since $q^i(in) = q^{i'}(in) = V + B$, the indifferent consumer is located at $\tilde{x} = \frac{-p^i + p^{i'} + \tau}{2\tau}$. This leads to

$$p_t^i(in, in) = p_t^{i'}(in, in) = \hat{\delta}\theta + \tau;\ d_t^i(in, in) = d_t^{i'}(in, in) = \frac{1}{2};\ \text{and } \pi_t^i(in, in) = \pi_t^{i'}(in, in) = \frac{\hat{\delta}\theta + \tau}{2}.$$

- For all other cases, firms' price p and profits π follow Lemma 1.

Two observations are noteworthy. First, both the per-unit penalty $\hat{\delta}$ and the recall probability θ jointly affect *both* of the firms' pricing. As the direct cost of a recall increases in the units demanded at $t = 1$, d_1, the firm rushing to the market with the (untested) innovation now has an incentive to price higher so to reduce this demand as the negative consequences of a recall becomes severe. In so doing, the firm factors the probability θ and the severity $\hat{\delta}$ of a recall into pricing its innovation, and, due to competition, the rival's pricing of the mature product also increases in proportion. Second, when the risk associated with a recall is too high, the firm may be better off not selling its innovation at all during the first period. Note that in the above derivation, the demand for the innovative product, $d_1^{i'}(in, ma) = \frac{B - \hat{\delta}\theta + 3\tau}{6\tau}$, is positive only if $\hat{\delta}\theta < B + 3\tau$. Otherwise, the consequences of a recall renders the firm to price its innovation too high, leading to zero demand and profits. In such case, the firm selling its mature product covers the market as shown in case (2).

We now illustrate how the three equilibria presented in the basic model continue to hold under the alternative setting. For brevity, we illustrate under the condition $\hat{\delta}\theta < B + 3\tau$. Using the equilibrium profit levels derived above, the following holds.

$$\Pi^{1,1} = \tau.$$
$$\Pi^{0,1} = \begin{cases} \frac{(B - \hat{\delta}\theta + 3\tau)^2}{18\tau} - \theta\rho + \theta \cdot \frac{(-(V+B)\gamma + 3\tau)^2}{18\tau} + (1-\theta) \cdot \frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ (1-\theta)\left(\frac{(B + 2\hat{\delta}\theta + 3\tau)(B - \hat{\delta}\theta + 3\tau)}{18\tau} + \frac{\tau}{2}\right) & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$
$$\Pi^{1,0} = \begin{cases} \frac{(-B + \hat{\delta}\theta + 3\tau)^2}{18\tau} + \theta \cdot \frac{((V+B)\gamma + 3\tau)^2}{18\tau} + (1-\theta) \cdot \frac{\tau}{2} & \text{if } \rho \le \rho^{\bar{B}}, \\ \frac{(-B + \hat{\delta}\theta + 3\tau)^2}{18\tau} + \theta \cdot (V + B - \frac{\tau}{2}) + (1-\theta) \cdot \frac{\tau}{2} & \text{if } \rho > \rho^{\bar{B}}. \end{cases}$$
$$\Pi^{0,0} = \begin{cases} \tau - \theta\rho + 2\theta(1-\theta) \cdot \frac{(V+B)^2\gamma^2}{18\tau} & \text{if } \rho \le \rho^{\underline{B}}, \\ (1-\theta)\left(\theta(V + B - \tau) + \frac{\hat{\delta}\theta + 2\tau}{2}\right) & \text{if } \rho > \rho^{\underline{B}}. \end{cases}$$

where $\rho^{\bar{B}} = \frac{(B-\hat{\delta}\theta+3\tau)(B+2\hat{\delta}\theta+3\tau-3\hat{\delta}\theta)}{18\tau}$ and $\rho^{\underline{B}} = \frac{\tau-\hat{\delta}(1-\theta)}{2}$.

The remainder of the analysis is straightforward and analogous to Proposition 2 in the main paper: the no-test equilibrium holds when $\Pi^{0,0} \geq \Pi^{1,0}$; the dual-test equilibrium holds when $\Pi^{1,1} \geq \Pi^{0,1}$; and the single-test equilibrium complements the other two conditions. Deriving the specific conditions in terms of failure probability θ involves tedious algebra and the details are available from the authors. Note that as $V \to 1$, $B \to 0.5$, $\gamma \to 0.5$, $\delta \to 1$, $\tau \to 0.2$, and $\rho \to 0$, we have that $\theta^{DT} = 0.397$ and $\theta^{NT} = 0.299$. Therefore, there is a nonempty range of parameter values for which all three equilibria are sustained.

B.6. Cost of Addressing the Product Issues

This section considers the cost of addressing a quality issue of the innovation. We illustrate how the basic model is robust to varying assumptions about this cost. The first assumption considers a generic case whereby a firm incurs a cost to *fix* a product defect, which increases in proportion to failure probability θ. As discussed in footnote 7 of the main paper, this cost applies whether firms conduct testing or not—because we assume that if a firm discovers a flaw, it must “fix” the product in order to sell it at $t = 2$, regardless of whether the defect was discovered through quality assurance (if the firm conducted testing) or through consumer experience (if the firm skipped testing). Hence, the two costs cancel out during a firm’s decision making (between testing and skipping the test) and including such an expense in the model would not change the results.

An alternative assumption on the cost of addressing the product issues is to view it as the cost to *discover* the product flaw as part of testing. This cost would be proportional to the product recall probability θ, but only incurred by a firm that conducts testing (because if a flaw is revealed post-launch due to customer usage, the discovery is, in a sense, free to the firm). Let us denote this cost by ε. We have that if the firm tests, it incurs a cost of discovering a product flaw of $\varepsilon\theta$; whereas if it firm skips testing, it bears no discovery costs but incurs the direct cost of a recall, $\rho\theta$. Hence, mathematically, by reparametrizing $\tilde{\rho}\theta = \rho\theta - \varepsilon\theta$, the basic model accommodates this scenario as well. The conceptual difference is that the new parameter $\tilde{\rho}$ represents the net cost (fixed cost − discovery cost) associated with a product recall.

B.7. The Role of Reputation Damage to a Recalled Innovation in the Model

This section discusses the implications of reputation damage to a recalled innovation, represented by the parameter γ in the model. Reputation damage, in our context, captures the situation in which a firm struggles to market a previously recalled innovation. This is because the negative impact on consumers’ perceptions and attitudes toward the innovation, once done, cannot be fully recovered from, even if the firm resolves the issues associated with the recall and the product delivers the promised benefits.[10] The setting helps represent a realistic market environment by suppressing the expected profit levels for a firm that forgoes testing its innovation (and possibly incurs a recall).

From a model analysis standpoint, by indirectly penalizing an innovation that is recalled, reputation damage limits the gains of rushing to the market (and skipping the test) and thus restricts the no-test equilibrium from outweighing the other equilibria in certain regions of the parameter space. For instance, as the parameter $\gamma \to 0$, the no-test equilibrium region expands and, in cases where the fixed cost of a recall is very small (i.e., $\rho \to 0$) and the recall probability θ is moderate, the regions of no-test and dual-test equilibria may overlap, leading to multiple equilibria.

Though reputation damage affords a more realistic environment, the study's key propositions and results are largely unaffected under the setting $\gamma = 0$. The only exception is part of Result 4, which covers two scenarios whereby an increase in the failure probability (θ) can result in both firms innovating. More specifically, the first scenario, which relates to the case of the single-test equilibrium following firms' R&D investment decisions, no longer holds. This is because the premise therein relies on the rival incurring reputation damage in the event of a recall and, without a such consequence, the likelihood of both firms innovating decreases in the failure probability in this case. Nonetheless, the second part of Result 4 continues to hold because its premise relies on the prospect of the rival's bankruptcy if a recall is incurred.

B.8. Additional Observation Regarding the Dual-Innovation Cutoff η^{DI}

This section discusses an additional observation on how the dual-innovation cutoff η^{DI} might change as a function of ρ. Under a narrow circumstance in which γ is significantly large and τ is small, the likelihood of both firms innovating may exhibit a further increase at ρ^{SI} and a subsequent decrease at ρ^{DT}. In this region of parameters, the non-innovating firm will greatly benefit if the rival's product is recalled. The rival's switch to testing (at ρ^{SI}), however, removes this opportunity and thus reduces the benefits of not innovating.

[10] For example, GoPro, Inc. recalled its newly launched Karma Drone in November 2016, following reports of losing electrical power during operation and falling from the sky (https://gopro.com/en/us/2016karmareturn). The company, upon fixing the issue, relaunched the drone in February 2017. However, consumers were reluctant to purchase the product: GoPro suffered an $80 million hit in revenues due to heavily discounting its drones and, eventually, exited the drone market after a year of sluggish sales (https://investor.gopro.com/press-releases/press-release-details/2018/GoPro-Announces-Preliminary-Fourth-Quarter-2017-Results/). Notwithstanding, the company's main product line, the HERO action camera series, had minimal impact on its reputation and was still highly regarded.