Higher Moments in the Fundamental Specification of Electricity Forward Prices

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Abstract

An extended specification for estimating the risk premia necessary for the forward pricing of wholesale electricity is developed in order to respond to the increasing need for more precise risk management of hedging positions in practice. Using Taylor expansions, we provide new specifications for the electricity forward premium including its dependency on all four moments of the expected wholesale price density as well as the higher moments of the demand density including skewness and kurtosis. Overall we argue that previous models have been underspecified and that the extended formulation proposed in this analysis is robust and worthwhile.

Keywords: Electricity Prices, Electricity Demand, Monte Carlo Simulations, Forward Risk Premium, Student-t Distribution, Skew-t Distribution, Kurtosis.

1 Introduction

The forward curve for wholesale electricity is conventionally analysed in terms of the underlying expectations for future spot prices with adjustments for forward risk premia.

Whilst the means of producing electricity can be stored (fuels, water, batteries, etc), electricity itself is essentially an instantaneous nonstorable commodity that is produced to meet immediate demand. As such, there is no physical cost-of-carry arbitrage between spot and forward prices. Rather, analysts following the influential work of Bessembinder and Lemmon (2002), view the forward prices as the sum of expected spot prices and forward premia, in which the premia reflect the balance of risk aversion in the market for hedges. Thus, Bessembinder and Lemmon (2002) (henceforth, BL) formulated a general economic equilibrium for risk averse electricity producers and retailers' trading in spot and forward markets. Their model resulted in the forward risk premium being expressed in terms of the first three moments of the expected spot price densities (using a Taylor approximation) and demand densities (via simulation). This switched the forward price analysis to that of the determinants of the risk premia and a large body of work has consequently followed on that theme.

Thus, electricity forward risk premia have been extensively investigated in many international markets, sometimes with considerations of exogenous and fundamental factors, and sometimes with operational and technological constraints: eg Longstaff and Wang (2004), Karakatsani and Bunn (2005), Diko et al. (2006), Hadsell and Shawky (2007), Wilkens and Wimschulte, 2007, Álvaro Cartea and Villaplana (2008), Douglas and Popova (2008), Ronn and Wimschulte (2009), Redl et al. (2009), Botterud et al. (2010), Furió and Meneu (2010), Viehmann (2011), Lucia and Torró (2011), Haugom and Ullrich (2012), Huisman and Kilic (2012), Redl and Bunn (2013), Weron and Zator (2014), Fleten et al. (2015), Xiao et al. (2015), van Koten (2020), van Koten (2021), Peura and Bunn (2021), Koolen et al. (2021), Huisman et al. (2021) and Jacobs et al. (2022) among many others. However, the empirical evidence that has emerged with respect to the BL model has been mixed.

One cause of the contradictory implications for how the BL premia depend upon the price and demand densities could be an underspecification in the model. That motivates consideration of a more enhanced specification in this research. With this objective, we test the goodness of the Taylor approximation and confirm that an *extension* to include the fourth moment is worthwhile. We also derive a more explicit expression for the premium in terms of the anticipated demand distribution and investigate the dependence of the premium on the third and fourth demand moments by simulation. As part of this, we relax the normality assumption for both the price and demand densities to consider student-t and skew-t distributions. This allows us to investigate more properly the dependence of the premia on the anticipated price and demand skewness and kurtosis. Whilst this work is about the usefulness of higher moments in determining electricity forward risk premia, it can be viewed alongside the increasing interest in using higher moments (including kurtosis) more generally in financial engineering models for asset pricing (eg Dittmar, 2002), portfolio analysis (Harvey et al., 2010) and value-at-risk (Bali et al., 2008).

The structure of the paper is organized as follows: Section 2 presents the original BL model extended to include price kurtosis and the more explicit formulation relating to demand. Section 3 provides the details on the simulation setting, together with parameters used, and simulation process. The results are presented in Section 4, whereas Section 5 concludes.

2 An Extended Specification of the BL Model

The BL model considers the electricity trading to be organized through a day-ahead (or spot) market and a forward market, within which three types of agents act. These are *Producers*, of whom there are N_P, and formally represented by P_i with $i = 1, 2, \ldots, N_P$, who produce power and sell it (either spot or forward) to retailers; *Retailers*, indicated with R_j with $j = 1, 2, \ldots, N_R$, who buy power from producers and resell it to final consumers; and *Consumers* who cannot access the spot/forward market and can only buy electricity at a fixed price from retailers.

In this BL setting, the following prices are considered: (i) the *spot* or *day-ahead* price, P_W, at which (unit) power is traded on the spot market at time T; (ii) the *forward* price, P_F, at which the contract is agreed upon at time 0 for delivery of (unit) power at the maturity T; and finally, (iii) the *retail* price, P_R, at which (unit) power is sold to consumers at time T; and it is reasonably higher than the spot price.

Each *producer* P_i at time 0 *sells* the quantity Q^F_{Pi} at a forward price P_F in the forward market with delivery T. Whereas, at the maturity T, the producer generates the quantity $Q_{Pi} \geqslant 0$, which can be seen as a combination of two parts: the first one being Q^F_{Pi}, delivered to honour the forward contracts, granting a revenue given by $P_F\, Q^F_{Pi}$; and, the second one being the residual $Q^W_{Pi} = Q_{Pi} - Q^F_{Pi}$ which is instead sold on the spot market, with a revenue given by $P_W\, Q^W_{Pi}$. Note that Q^F_{Pi} or Q^W_{Pi} may be negative, which means that the producer *buys* $|Q^F_{Pi}|$ or $|Q^W_{Pi}|$ on the forward/spot market. However, the sum $Q_{Pi} = Q^F_{Pi} + Q^W_{Pi}$ cannot be negative. At time T, producer i incurs into the *total production costs* to generate the quantity Q_{Pi}, that is $TC(Q_{Pi})$. The (total production) cost function is assumed to be the same for all producers, and defined as $TC(Q) = C_0 + aQ^c/c$, with $a > 0$ and $c \geqslant 2$ and where C_0 are fixed costs. Marginal costs are increasing and convex in production Q. The (ex-post) profit for producer i at time T is the same (concave) function of (Q^F_{Pi}, Q^W_{Pi}) for all producers.

On the other hand, each *retailer*, R_j, at time 0 *buys* the quantity Y^F_{Rj} in the forward market for delivery in T at the forward price P_F, facing at time T the *aggregated demand* D_j requested from all consumers, which is covered by the quantity bought at time 0 on forward markets, Y^F_{Rj} (with an outflow at time T equal to $P_F Y^F_{Rj}$) and the remaining part, $Y^W_{Rj} = D_j - Y^F_{Rj}$, is bought on the spot market (on T, at a cost of $P_W Y^W_{Rj}$). Note that Y^F_{Rj} or Y^W_{Rj} may be negative, but the sum (D_j) cannot be negative. In addition, at time T, retailer R_j sells the total purchased quantity to consumers at the retail price P_R, receiving a revenue of $P_R D_j$. Hence, the (ex-post) profit at time T is a linear function of (Y^F_{Rj}, Y^W_{Rj}), and again is equal for all retailers. To summarize, at the initial time 0, the forward and retail prices (P_F and P_R) are known together with the quantities traded forward by producers and retailers, whereas, P_W, Q^W_{Pi}, Q_{Pi}, π_{Pi}, Y^W_{Rj}, D_j, π_{Rj} will become known only at the maturity T.

The optimal choice is determined at the maturity, when spot prices and demand are known. Then, to determine the optimal choice at the initial time 0, P_W and D are unknown and considered as random variables.

Each producer/retailer has the same mean-variance utility function for random profits at time T to be maximized, where the common risk aversion coefficient is $A > 0$. The equilibrium forward price at time 0 is then determined by matching the total quantity sold by producers with the total quantity purchased by retailers. Finally, the expression of the forward premium $\mathrm{FP} = P_F - \mu_W$, where $\mu_W = \mathbb{E}[P_W]$, is obtained as

$$\mathrm{FP} = \omega \cdot \mathrm{cov}(P_W,\, P_W^{b+1} - c\, P_R\, P_W^b) \tag{1}$$

where $b = 1/(c-1)$ and

$$\omega = \frac{A N_P}{(N_P + N_R)\, c\, a^b}.$$

Note that $0 < b \leqslant 1$ and $\omega > 0$. Moreover, FP is finite provided that P_W has a finite moment of order $b+2$, which is between 2 and 3. Equation (1) shows that the forward premium FP depends on the distribution of the wholesale prices P_W, in particular on its moments.

Using a Taylor expansion around μ_W, BL derived the premium in terms of price moments. Indeed, from

$$\mathrm{FP} = \omega \cdot \mathrm{cov}(P_W,\, P_W^{b+1} - cP_R\, P_W^b) = \omega \cdot \mathbb{E}\left[(P_W - \mu_W) \cdot (P_W^{b+1} - cP_R\, P_W^b)\right]$$

we see that the premium can be written in the form $\mathrm{FP} = \mathbb{E}[h(P_W)]$ in terms of the function

$$h(x) = (x - \mu_W)(x^{b+1} - cP_R x^b). \tag{2}$$

The function h is infinitely differentiable for $x > 0$, so we can consider Taylor expansions of any order. In particular, since $h(\mu_W) = 0$, the *quartic* Taylor approximation of h around μ is[1]

$$h(x) \approx \beta_1(x-\mu_W) + \beta_2(x-\mu_W)^2 + \beta_3(x-\mu_W)^3 + \beta_4(x-\mu_W)^4 \tag{3}$$

with $\beta_k = h^{(k)}(\mu_W)/k!$ for $k = 1,\ldots,4$, where $h^{(k)}$ denotes the k-th derivative of h. Some algebra shows that $\beta_1 = \mu_W^b(\mu_W - cP_R)$ and

[1] We can arrive at (3), just by taking the cubic Taylor approximation of a simpler function $g(x) = (x^{b+1} - cP_R x^b)$ and multiplying it by $(x - \mu_W)$.

$$\beta_2 = \mu_W^{b-1}(b+1)(\mu_W - P_R),$$

$$\beta_3 = \mu_W^{b-2}\frac{b+1}{2}\left(\mu_W b - P_R(b-1)\right),$$

$$\beta_4 = \mu_W^{b-3}\frac{1-b^2}{6}\left(P_R(b-2) - \mu_W b\right).$$

According to (3), since $\mathbb{E}\left[P_W - \mu_W\right] = 0$, it is possible to write

$$\text{FP} = \mathbb{E}\left[h(P_W)\right] \approx \beta_2\mathbb{E}\left[(P_W - \mu_W)^2\right] + \beta_3\mathbb{E}\left[(P_W - \mu_W)^3\right] + \beta_4\mathbb{E}\left[(P_W - \mu_W)^4\right].$$

Denoting with σ_W^2, ξ_W and κ_W the variance, skewness and kurtosis of wholesale prices P_W respectively, and recalling that $\xi_W = \sigma_W^{-3}\,\mathbb{E}\left[(P_W - \mu_W)^3\right]$ and $\kappa_W = \sigma_W^{-4}\,\mathbb{E}\left[(P_W - \mu_W)^4\right]$, we can then write the approximation of the forward premium as

$$\text{FP} \approx \omega \cdot (\beta_2\sigma_W^2 + \beta_3\sigma_W^3\xi_W + \beta_4\sigma_W^4\kappa_W), \tag{4}$$

where β_2, β_3 and β_4 have been derived above.

Note that BL just consider the *cubic* Taylor expansion of h, thus obtaining the approximation of the forward premium up to the third moment, that is $\text{FP} \approx \omega \cdot (\beta_2\sigma_W^2 + \beta_3\sigma_W^3\xi_W)$. Whereas, we expand their derivation by including the term with kurtosis, that is $\omega\beta_4\sigma_W^4\kappa_W$ (and we call it the *extended approximation* to distinguish from the BL one).

Let us now look at the sign of the β_k coefficients. Given the obvious assumption that expected wholesale prices are strictly positive (that is $\mu_W > 0$), and recalling that $0 < b \leqslant 1$ (then $b + 1 > 0$), the following conclusions can be drawn. First, we see that β_2 is *negative* whenever $P_R > \mu_W$, which is reasonable given that the retail price contains also taxes and levies in addition to the cost of energy. Second, β_3 is always *positive*, since $\mu_W b > 0$ and $P_R(b-1) \leqslant 0$. Finally, β_4 is always *negative* if $c > 2$ since $1 - b^2 > 0$ and $P_R(b-2) < 0$; when $c = 2$, then $b = 1$ and $\beta_4 = 0$.

As $\omega > 0$, we can conclude that the forward premium FP is, ceteris paribus, *decreasing* in the standard deviation of wholesale prices and *increasing* in the skewness. These are, respectively, hypotheses (H1) and (H2) as stated by BL. Thanks to our approximation, we can add a new hypothesis (H2′), that is *the forward premium is decreasing in the kurtosis of prices* when $c > 2$.

Looking now at the approximation in (4), it is worth emphasizing that its accuracy can be an issue, since it has been derived by using the Taylor approximation (3) inside an expectation. In other words, if p is some Taylor polynomial for the function h, from $h(x) \approx p(x)$ around $x = \mu$ we have inferred that $\mathbb{E}[h(X)] \approx \mathbb{E}[p(X)]$ for a given random variable X. This is obviously not always the case[2], even when both expectations are finite.

Therefore, we should be extremely careful when using (1). To emphasize this point, we have tested numerically the goodness of this approximation, when using the reduced approximation in BL (that is neglecting the kurtosis term) or the *extended* one that we propose. Based on these considerations and accounting for the empirical evidence on time-varying shapes of electricity prices provided by Gianfreda and Bunn (2018)[3], we also investigate the dependence of the risk premium on price moments under different distributional assumptions for wholesale prices P_W. And we additionally investigate the sensitivity to different parameter choices via simulations, hence providing new computational evidence.

Moving forward and considering the electricity demand, BL derive the equilibrium relation between demand (D) and wholesale prices (P_W) simply as

$$P_W = a \left(\frac{D}{N_P} \right)^{c-1} . \tag{5}$$

Whereas, and as an additional contribution, we derive a more explicit expression for

[2]For instance, consider $h(x) = \mathrm{e}^x$ and the corresponding quartic Taylor polynomial (around $x = 0$): $p_4(x) = 1 + x + x^2/2 + x^3/6 + x^4/24$. If $X \sim N(0,1)$ we can compute $\mathbb{E}[\mathrm{e}^X] = \mathrm{e}^{1/2} \approx 1.649$ and $\mathbb{E}[p_4(X)] = 13/8 = 1.625$ and we see that the quartic Taylor approximation holds reasonably well also for the expectations. However, if X has a Laplace distribution with parameter $\lambda \in (0,1)$, that is $X = \lambda(Y_1 - Y_2)$ where Y_1 and Y_2 are independent and exponentially distributed with mean 1, then we can compute $\mathbb{E}[\mathrm{e}^X] = (1-\lambda^2)^{-1}$. Next, we have $\mathbb{E}[X] = \mathbb{E}[X^3] = 0$, $\mathbb{E}[X^2] = 2\lambda^2$ and $\mathbb{E}[X^4] = 24\lambda^4$, so that $\mathbb{E}[p_4(X)] = 1 + \lambda^2 + \lambda^4$. We see that $\mathbb{E}[p_4(X)]$ can be far away from $\mathbb{E}[\mathrm{e}^X]$ if λ is close to 1. For instance, if $\lambda = 0.9$, then the former expectation is 5.26, while the latter one is just 2.47.

[3]They show from a detailed analysis of German prices that these prices, under a wide range of market conditions, can be characterized by two-, three- and four-moment distributions.

the forward premium in terms of demand. Precisely, we substitute the expression (5) into equation (1) obtaining, after some algebra, the forward premium as

$$\mathrm{FP} = \omega' \cdot \operatorname{cov}(D^{c-1},\, D^c - \omega'' D), \tag{6}$$

where

$$\omega' = \omega \cdot \frac{a^{b+2}}{N_P^{2c-1}} \qquad \text{and} \qquad \omega'' = \frac{c\, P_R\, N_P^{c-1}}{a}.$$

Note that ω' and ω'' are both positive. Moreover, the covariance in (6) is finite provided that the demand D has a finite moment of order $2c-1$, where $2c-1 \geqslant 3$.

Expression (6) allows us to investigate the dependence of the forward premium directly on the distribution of demand or on its moments. In this regard, BL propose two additional hypotheses: (H3) FP is convex in σ_D, that is the premium is first decreasing in the demand standard deviation, and then it is increasing; (H4) FP is increasing in μ_D, that is the premium is increasing with the mean of demand. No hypotheses about the dependence of the premium on the demand skewness (ξ_D) and kurtosis (κ_D) were formulated by BL. Therefore, we aim at exploring this issue by using expression (6) in Monte Carlo simulations.

Note that we do not attempt to find an approximation for the forward premium in terms of the four moments of demand, as we did in (4) when studying the dependence on the moments of wholesale prices. And the reason for this choice is explained in what follows. Rewriting (6) as

$$\mathrm{FP} = \omega' \cdot \Big(\mathbb{E}\left[D^{2c-1} - \omega'' D^c\right] - \mathbb{E}\left[D^c\right] \cdot \mathbb{E}\left[D^c - \omega'' D\right]\Big), \tag{7}$$

shows that there is the need of approximating three expectations of non-linear functions of demand D. Trying to approximate these functions by a quartic polynomial of $(D - \mu_D)$ is certainly doomed to fail. Indeed, the function h in (2) is a sum of (fractional) powers of x with degrees at most $b + 2 \leqslant 3$, so that a Taylor polynomial of degree 4 is able to give a good approximation even for values of x moderately far from μ_W. On the contrary, in the function $v(x) = x^{2c-1} - \omega'' x^c$, appearing in the first expectation in (7), there is a power with exponent $(2c-1)$, which is above 4 if $c > 2.5$. In this latter case, the quartic Taylor polynomial p_4 gives an approximation of v which worsens quickly as we depart from μ_D, thus making the statement $\mathbb{E}[v(D)] \approx \mathbb{E}[p_4(D)]$ hard to justify. For this reason, we do not pursue the route of Taylor approximations for (6). Instead, we study the dependence on the distribution of D only by simulations.

3 Simulation Setting

In what follows we carefully consider the key variables and parameters, and then describe precisely the simulation process.

3.1 Simulation Parameters

Let us recall that in this system only generation costs are considered. No other costs for transmission, distribution, taxes and levies are included; however, this assumption will be relaxed in the determination of retail prices as discussed below. We set the parameters values as follows:

- the number of producers and retailers is left unchanged with respect to the seminal BL paper: $N_P = N_R = 20$; because even if these numbers approach infinity the forward price is expected to converge to the expected spot prices;
- the production convex cost parameter c is left to vary in the range between 2 and 5, that is $c \in [2,5]$ as in BL. Let us recall that it incorporates issues related to extreme high demand covered by inefficient peakload plants as well as limited capacity in production or transmission;
- retail prices are assumed to be higher than expected wholesale prices to incentive risk-averse retailers to enter the market; thus $P_R = \lambda \cdot \mathbb{E}[P_W]$ with $\lambda > 1$, originally set to 1.2 as in BL. However, for arguments analyzed in Appendix A, we have decided to relax this assumption to include system costs and consequently consider lower (net) margins for retailers with $\lambda = 1.02, 1.05$;

- wholesale prices and demand are initially assumed to be normally distributed, then other distributional assumptions are considered to include asymmetries and fat tails.

Following Bessembinder and Lemmon (2002), the variable cost parameter is set as $a = 30(N_P/100)^{c-1}$, and the risk relevance parameter is set as $A = 0.8/c^2$.

3.2 Simulation Process

Once a distribution for wholesale prices P_W or demand D has been selected and the parameters of the model have been set, we can compute the corresponding forward premium via simulation in a natural way following these steps:

1. produce N independent and identically distributed (iid) realizations of $X = P_W$ or D, say X_i, with $i = 1, \ldots, N$ replicates;
2. compute the forward premium FP as a sample covariance, using (1) when $X = P_W$ and (6) when $X = D$. More explicitly, in the former case we compute

$$\widehat{\mathrm{FP}} = \omega \cdot \left(\frac{1}{N} \sum_{i=1}^{N} X_i (X_i^{b+1} - cP_R X_i^b) - \frac{1}{N} \sum_{i=1}^{N} X_i \cdot \frac{1}{N} \sum_{i=1}^{N} (X_i^{b+1} - cP_R X_i^b) \right) \tag{8}$$

   whereas in the latter case we compute

$$\widehat{\mathrm{FP}} = \omega' \cdot \left(\frac{1}{N} \sum_{i=1}^{N} X_i^{c-1} (X_i^c - \omega'' X_i) - \frac{1}{N} \sum_{i=1}^{N} X_i^{c-1} \cdot \frac{1}{N} \sum_{i=1}^{N} (X_i^c - \omega'' X_i) \right). \tag{9}$$

In both cases, the numerical results are clearly prone to simulation variability. However, we find that using $N = 10^5$ gives sufficiently stable results across all our investigations. In order to further improve the quality, we increase the value of N to 10^6 or we keep $N = 10^5$ and perform the computations for 100 batches of N iid realizations, and then take the median of the 100 numerical results. In few cases, we increase the number of batches to 400.

For all selected distributions, X (P_W or D) can take negative values with positive probability. Therefore, we discard all those values of X at step 1.[4] Doing so, we actually compute $\widehat{FP}$ with less than N replicates for X. However, the fraction of discarded realizations is less than 10% in all our investigations (and, in most cases, less than 1%).

In order to study the dependence of the premium FP on a particular moment of X (P_W or D), we fix a parametric class of distributions and, by changing the parameters in a suitable way, we let that moment vary across a specified range, while keeping the other moments constant. For each such choice of the parameters, we go through steps 1. and 2. above, thus obtaining the corresponding value for the premium.

3.3 Selected Distributions

To uncover the characteristics of the premium, we consider the following parametric classes of distributions of X (P_W or D):

- **Normal**. This is a location-scale family of symmetric distributions with no shape parameters. In particular, if $X \sim N(m,s^2)$, then $\mu_X = m$, $\sigma_X = s$, $\xi_X = 0$ and $\kappa_X = 3$. By keeping m fixed while varying s, we can study the dependence of FP on σ_W or σ_D. On the other hand, by keeping s fixed while varying m, we can study the dependence of FP on μ_D. Simulations from $N(m,s^2)$ is straighforward in Matlab, through `randn`.

- **Student-T**. This is a location-scale family of symmetric distributions with one shape parameter: $\nu > 0$ (*degrees-of-freedom*, *dof*), controlling for the kurtosis. In particular, if $X \sim T(m,s,\nu)$, i.e. $(X - m)/s \sim T(0,1,\nu)$, with $\nu > 4$, then

  $$\mu_X = m \qquad \sigma_X = s \cdot \sqrt{\frac{\nu}{\nu - 2}} \qquad \xi_X = 0 \qquad \kappa_X = 3 \cdot \frac{\nu - 2}{\nu - 4}$$

  By keeping m and ν fixed while varying s, we can again study the dependence of FP on σ_W or σ_D. By keeping s and ν fixed while varying m, we can study the dependence of FP on μ_D. Finally, by keeping s and m fixed while varying ν, we can study the dependence of FP on κ_W and κ_D (note that as ν ranges in $(4,+\infty)$, κ_X

[4] In other words we consider the distributions left-truncated at 0, which is in line with van Koten (2020). However, differently from him, we do not discard replicates outside the interval $\mu \pm 5\sigma$.

ranges in $(3, +\infty)$). Simulations from this distribution are also straightforward in Matlab, through the function `trnd`.

- **Skew-T**. This is a location-scale family of distributions with two additional parameters, that allow for both asymmetry and fat tails. It has been proposed and extensively discussed in Azzalini (2013) and nests the Student-T class as a particular case. It turns out that if $X \sim ST(m, s, \alpha, \nu)$, i.e. X is Skew-T distributed with location $m \in \mathbb{R}$, scale $s > 0$, shape $\alpha \in \mathbb{R}$ and degrees-of-freedom (dof) $\nu > 0$, then the first two moments μ_X and σ_X depend on all four parameters, while ξ_X and κ_X depend on α and ν only. In particular, for all values of $\mu_X \in \mathbb{R}$, $\sigma_X > 0$ and (ξ_X, κ_X) in a suitable region in $\mathbb{R}^2$ it is possible to find (unique) values of the four parameters that yield the chosen moments. Thanks to this fact, it is possible to keep three out of the four moments fixed and vary the fourth moment in an admissible range. In this way, we can study the dependence of FP separately on each moment. We will use the Skew-T class to investigate the dependence on ξ_W, κ_W and ξ_D, but also to study the dependence on lower moments (μ and σ) when the distribution is asymmetric. Simulations from this distribution are not directly provided in Matlab, but can be easily implemented via a stochastic representation provided in Azzalini (2013). We refer to Appendix B for further details about this distribution, in particular regarding its moments and the simulation algorithm.

4 Results via Simulations

4.1 Testing the Accuracy of the Approximation for the Forward Premium

Given that the first two hypotheses H1 and H2 in BL are based on the approximate expression for the forward premium provided in (4), and considering that no investigations were performed so far to test the accuracy of that approximation, we aim at filling this gap. Then, through simulation, we compute the "exact" forward premium FP using eq. (1) and compare it with the *BL approximation* (i.e using (4) without the kurtosis term) and with our *extended* one (i.e using (4) with the kurtosis term).

Table 1 shows the results for selected combinations of the mean μ_W and standard deviation σ_W of wholesale prices, with the constant c determining from quadratic ($c = 2$) to quintic ($c = 5$) cost functions. Retail prices are set as $\lambda \cdot \mu_W$ with three choices for λ (set to 1.02, 1.05, and also to 1.2 as in BL). Here, wholesale prices P_W are assumed to be normally distributed. Similar results are obtained using a Skew-T distribution for P_W, with kurtosis levels higher than 3, at least when the skewness is not too large (say, between -1 and 1). Numerical results are available upon request.

We observe that the Taylor approximation works reasonably well when σ_W is small and when c is low. However, the accuracy worsens when c and σ_W increase, with a general underestimation of the premium (in absolute value). It is interesting to observe that, as expected, the *extended* approximation works better than the BL approximation especially when costs increase, consistently across all three λs. These results are robust for mean prices set at 35 or 40 €/MWh. Additionally, since extreme prices can occur, we inspect the case of $\mu_W = 100$, as prices observed in recent years 2021 and 2022; with the range of volatility enlarged consequently to include higher levels (20 and 30). Results in Table 2 confirm previous conclusions about the goodness of the *extended* approximation, consistently across all parameters.

Overall, these results suggest that the quartic Taylor approximation that we propose can provide better estimations of electricity risk premia than the cubic one in BL, also in extreme market conditions affecting mean and volatility prices. Therefore, particular attention should be paid in modelling premia by including all four moments of the anticipated distribution of wholesale prices.

4.2 Testing the BL Hypotheses (H1) and (H2), and inspecting Premia Dependence on Price Kurtosis (H2')

In order to verify the validity of BL (H1) hypothesis, that the forward premium is decreasing in the standard deviation of wholesale prices σ_W, we consider a normal distribution for prices P_W with fixed mean $\mu_W = 100$, and let the standard

Table 1: Exact and Approximated Forward Premium for given μ_W, σ_W and c, with $\lambda = 1.02, 1.05, 1.2$. Prices P_W are normally distributed. BL stands for the BL approximation, whereas BGS stands for our approximation.

			$\lambda = 1.02$			$\lambda = 1.05$			$\lambda = 1.2$		
μ_W	σ_W	c	Exact	BL	BGS	Exact	BL	BGS	Exact	BL	BGS
35	3	2	-0.105	-0.105	-0.105	-0.262	-0.262	-0.262	-1.051	-1.050	-1.050
35	5	2	-0.289	-0.290	-0.290	-0.727	-0.727	-0.727	-2.919	-2.915	-2.915
35	10	2	-1.044	-1.072	-1.072	-2.792	-2.822	-2.822	-11.534	-11.572	-11.572
35	12	2	-0.830	-1.024	-1.024	-3.313	-3.544	-3.544	-15.730	-16.144	-16.144
35	3	3	-0.026	-0.022	-0.026	-0.058	-0.054	-0.058	-0.221	-0.216	-0.221
35	5	3	-0.092	-0.060	-0.091	-0.183	-0.150	-0.181	-0.637	-0.600	-0.635
35	10	3	-0.812	-0.220	-0.713	-1.185	-0.580	-1.084	-3.050	-2.379	-2.937
35	12	3	-1.403	-0.209	-1.208	-1.940	-0.726	-1.747	-4.625	-3.308	-4.440
35	3	4	-0.010	-0.008	-0.010	-0.022	-0.020	-0.022	-0.081	-0.079	-0.081
35	5	4	-0.038	-0.022	-0.037	-0.071	-0.055	-0.070	-0.238	-0.219	-0.237
35	10	4	-0.384	-0.081	-0.321	-0.523	-0.212	-0.459	-1.217	-0.869	-1.145
35	12	4	-0.703	-0.076	-0.564	-0.904	-0.265	-0.764	-1.913	-1.207	-1.767
35	3	5	-0.005	-0.004	-0.005	-0.010	-0.009	-0.010	-0.039	-0.037	-0.039
35	5	5	-0.019	-0.010	-0.018	-0.035	-0.026	-0.034	-0.114	-0.104	-0.113
35	10	5	-0.204	-0.038	-0.167	-0.271	-0.100	-0.232	-0.604	-0.412	-0.560
35	12	5	-0.383	-0.036	-0.296	-0.481	-0.125	-0.392	-0.967	-0.572	-0.872
40	3	2	-0.120	-0.120	-0.120	-0.300	-0.300	-0.300	-1.202	-1.200	-1.200
40	5	2	-0.331	-0.332	-0.332	-0.832	-0.832	-0.832	-3.337	-3.332	-3.332
40	10	2	-1.284	-1.299	-1.299	-3.287	-3.299	-3.299	-13.300	-13.299	-13.299
40	12	2	-1.603	-1.674	-1.674	-4.474	-4.554	-4.554	-18.831	-18.954	-18.954
40	3	3	-0.026	-0.023	-0.026	-0.061	-0.058	-0.061	-0.235	-0.231	-0.235
40	5	3	-0.090	-0.064	-0.089	-0.187	-0.160	-0.186	-0.672	-0.641	-0.670
40	10	3	-0.718	-0.250	-0.656	-1.114	-0.635	-1.050	-3.094	-2.559	-3.019
40	12	3	-1.330	-0.322	-1.154	-1.905	-0.875	-1.727	-4.781	-3.643	-4.587
40	3	4	-0.010	-0.008	-0.010	-0.022	-0.021	-0.022	-0.085	-0.083	-0.084
40	5	4	-0.035	-0.023	-0.035	-0.070	-0.057	-0.070	-0.244	-0.229	-0.243
40	10	4	-0.320	-0.089	-0.283	-0.464	-0.227	-0.426	-1.181	-0.915	-1.137
40	12	4	-0.623	-0.115	-0.513	-0.833	-0.313	-0.720	-1.883	-1.302	-1.758
40	3	5	-0.005	-0.004	-0.005	-0.011	-0.010	-0.011	-0.040	-0.039	-0.040
40	5	5	-0.017	-0.011	-0.017	-0.034	-0.027	-0.033	-0.115	-0.107	-0.115
40	10	5	-0.166	-0.042	-0.144	-0.234	-0.106	-0.211	-0.573	-0.429	-0.547
40	12	5	-0.330	-0.054	-0.264	-0.429	-0.147	-0.362	-0.928	-0.610	-0.852

Table 2: Exact and Approximated Forward Premium for $\mu_W = 100$ and given σ_W and c, with $\lambda = 1.02, 1.05, 1.2$. Prices P_W are normally distributed. BL stands for the BL approximation, whereas BGS stands for our approximation.

			$\lambda = 1.02$			$\lambda = 1.05$			$\lambda = 1.2$		
μ_W	σ_W	c	Exact	BL	BGS	Exact	BL	BGS	Exact	BL	BGS
100	3	2	-0.300	-0.300	-0.300	-0.751	-0.750	-0.750	-3.005	-3.000	-3.000
100	5	2	-0.832	-0.832	-0.832	-2.084	-2.082	-2.082	-8.347	-8.332	-8.332
100	10	2	-3.312	-3.319	-3.319	-8.322	-8.319	-8.319	-33.372	-33.319	-33.319
100	12	2	-4.761	-4.775	-4.775	-11.976	-11.975	-11.975	-48.048	-47.975	-47.975
100	20	2	-13.136	-13.219	-13.219	-33.176	-33.219	-33.219	-133.377	-133.219	-133.219
100	30	2	-25.045	-26.159	-26.159	-69.910	-71.159	-71.159	-294.236	-296.159	-296.159
100	3	3	-0.037	-0.037	-0.037	-0.092	-0.091	-0.092	-0.367	-0.365	-0.366
100	5	3	-0.108	-0.101	-0.108	-0.260	-0.253	-0.260	-1.023	-1.014	-1.021
100	10	3	-0.509	-0.404	-0.507	-1.121	-1.013	-1.118	-4.181	-4.055	-4.172
100	12	3	-0.800	-0.581	-0.795	-1.683	-1.458	-1.676	-6.098	-5.839	-6.081
100	20	3	-3.404	-1.609	-3.257	-5.884	-4.043	-5.728	-18.281	-16.214	-18.081
100	30	3	-13.142	-3.179	-11.408	-18.826	-8.650	-17.061	-47.247	-36.001	-45.324
100	3	4	-0.012	-0.011	-0.012	-0.028	-0.028	-0.028	-0.113	-0.112	-0.112
100	5	4	-0.034	-0.031	-0.034	-0.081	-0.078	-0.080	-0.315	-0.311	-0.314
100	10	4	-0.167	-0.124	-0.166	-0.355	-0.311	-0.354	-1.296	-1.244	-1.293
100	12	4	-0.269	-0.178	-0.266	-0.540	-0.447	-0.537	-1.898	-1.792	-1.892
100	20	4	-1.243	-0.494	-1.169	-2.010	-1.240	-1.933	-5.845	-4.975	-5.750
100	30	4	-5.283	-0.975	-4.347	-7.064	-2.653	-6.108	-15.973	-11.041	-14.911
100	3	5	-0.005	-0.005	-0.005	-0.012	-0.012	-0.012	-0.049	-0.049	-0.049
100	5	5	-0.015	-0.014	-0.015	-0.035	-0.034	-0.035	-0.137	-0.135	-0.137
100	10	5	-0.075	-0.054	-0.074	-0.157	-0.135	-0.156	-0.566	-0.540	-0.564
100	12	5	-0.122	-0.077	-0.120	-0.240	-0.194	-0.238	-0.830	-0.778	-0.827
100	20	5	-0.585	-0.214	-0.545	-0.919	-0.539	-0.877	-2.592	-2.160	-2.541
100	30	5	-2.592	-0.423	-2.072	-3.375	-1.152	-2.842	-7.292	-4.793	-6.696

deviation σ_W vary in the range $(1,30)$. Then, we repeat with $\mu_W = 35$ and a reduced range $(1,12)$ for σ_W. For cost parameters $c = 2,3,4,5$ and two values for λ (1.2 and the more realistic 1.02) we go through the steps of the procedure described before, obtaining the forward premium FP as a function of price standard deviation, σ_W. Results are depicted in Figure 1. We can see that in all cases the premium is clearly decreasing in σ_W, thus confirming the first BL hypothesis (H1). Similar evidence is produced when prices are skew-t distributed with skewness $\xi_W = 0.5$ and $\kappa_W = 4$ (see bottom row of the same Figure 1).

For testing the BL (H2) hypothesis, that the premium is increasing in the skewness of prices, contrarily to what assumed by BL, we consider the Skew-T distribution for P_W (see Appendix for details). We fix $\mu_W = 35$, $\sigma_W = 5$ and $\kappa_W = 4$ (kurtosis) and let ξ_W (skewness) vary in the range $(-0.5,1)$. Results for $c = 3,4,5$ and $\lambda = 1.02$ and 1.2 are reported in Figure 2. A similar behaviour can be observed for other values of μ_W, σ_W, κ_W and λ, and results are available on request. We can conclude that the BL (H2) hypothesis is confirmed when wholesale prices are skew-t distributed.

Moving forward, the dependence of the forward premium on price kurtosis κ_W is inspected. Let us recall that our derivation lead us to formulate an additional hypothesis (H2') that the forward premium is decreasing in price kurtosis when $c > 2$. We investigate this fact in two contrasting cases: the first one using a symmetric distribution and the second one using an asymmetric distribution.

Assuming symmetry (that is $\xi_W = 0$), we consider a Skew-T distribution for wholesale prices P_W and fixed price mean $\mu_W = 35$, standard deviation $\sigma_W = 5$ and skewness $\xi_W = 0$, letting price kurtosis κ_W vary in the range $(3,6)$. Results across c and for $\lambda = 1.02$ are presented in Figure 3. Results for $\lambda = 1.2$ are similar and have been omitted. As per the mathematical derivation in (4), results for $c \geqslant 3$ show indeed that the premium is decreasing with price kurtosis. When $c = 2$ instead $\beta_4 = 0$, that is the kurtosis term vanishes, but this relation is unveiled via

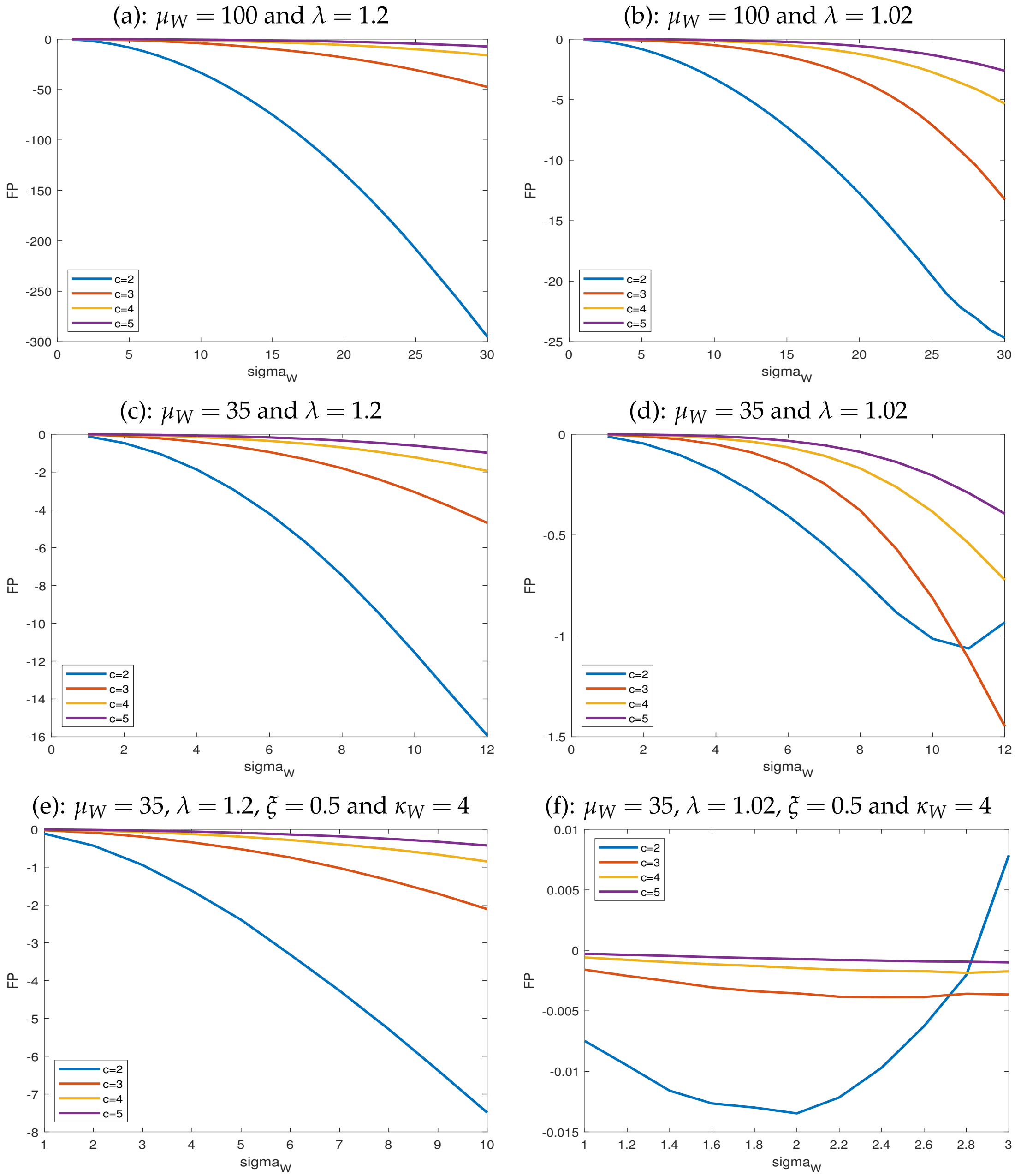

Figure 1: Behaviour of the Forward Premium FP vs Price Standard Deviation σ_W across c and for selected values of μ_W and λ. Wholesale prices P_W are Normally distributed in the first two rows, and Skew-T distributed in the bottom row. Testing (H1): FP decreasing in σ_W.

simulations. Only in this case, we can observe that the effect of fat tails increases the premium. Consistent results are obtained when asymmetry is assumed (that is $\xi_W = 1$), and using the same values as before for μ_W, σ_W, c and λ; results are in Figure 4. Those for $\lambda = 1.2$ are similar hence omitted. Therefore, we can conclude that our hypothesis of forward premium FP *decreasing* in price kurtosis

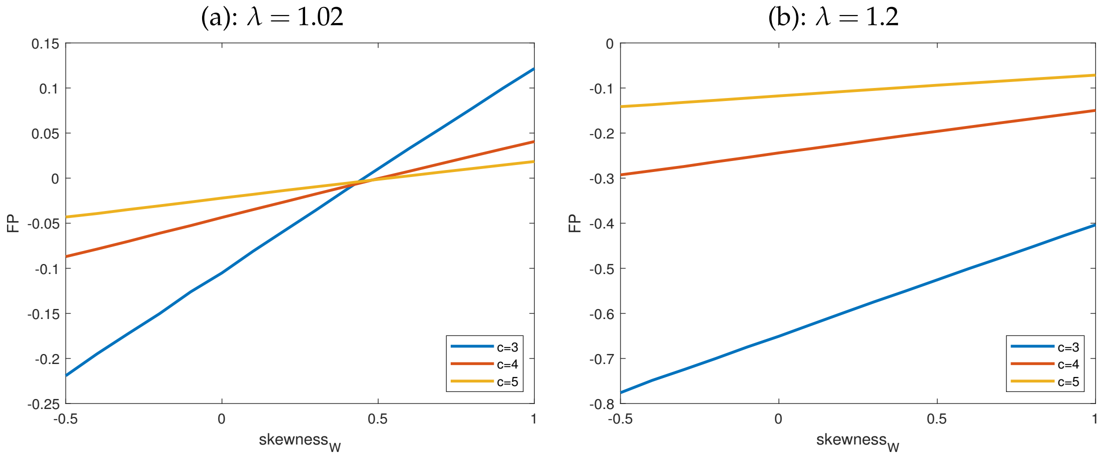

Figure 2: Behaviour of the Forward Premium FP vs Price Skewness ξ_W for prices P_W Skew-T distributed, with $\mu_W = 35$, $\sigma_W = 5$, $\kappa_W = 4$, and selected values of c and λ. Consistent results for $c = 2$ have been omitted since in a larger scale. Testing (H2): FP increasing in ξ_W.

κ_W is supported when $c > 2$ for both symmetric and asymmetric distributions, consistently with (4). The same conclusion does not hold when $c = 2$ as shown by simulation results.

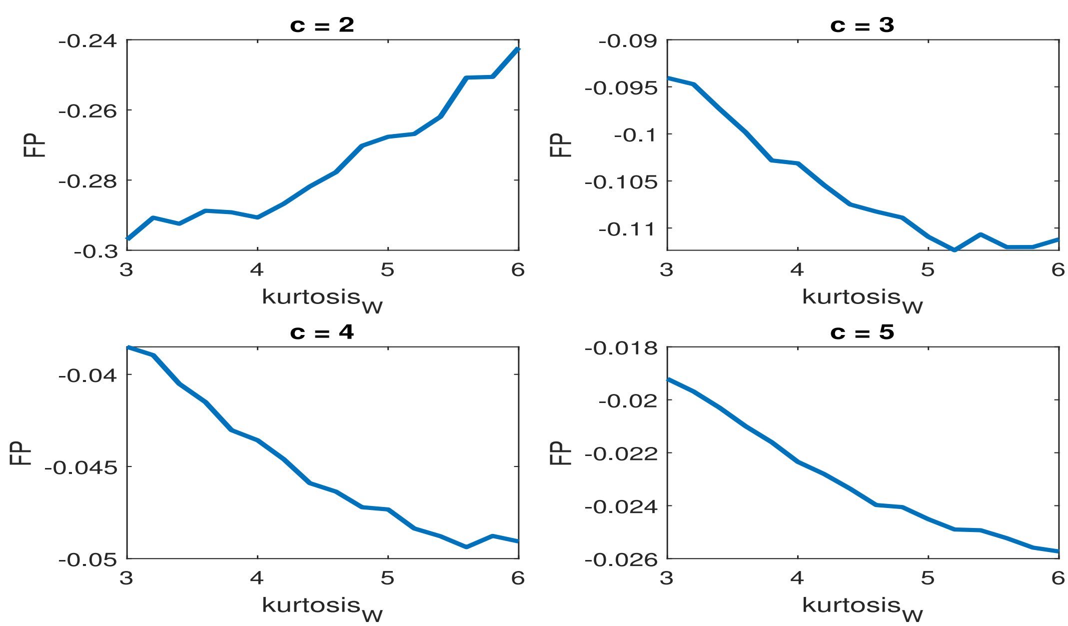

Figure 3: Behaviour of the Forward Premium FP vs Price Kurtosis κ_W across c, for Wholesale Prices P_W symmetrically Skew-T distributed with $\mu_W = 35$, $\sigma_W = 5$, $\xi_W = 0$, and $\lambda = 1.02$. Testing (H2'): FP decreasing in κ_W.

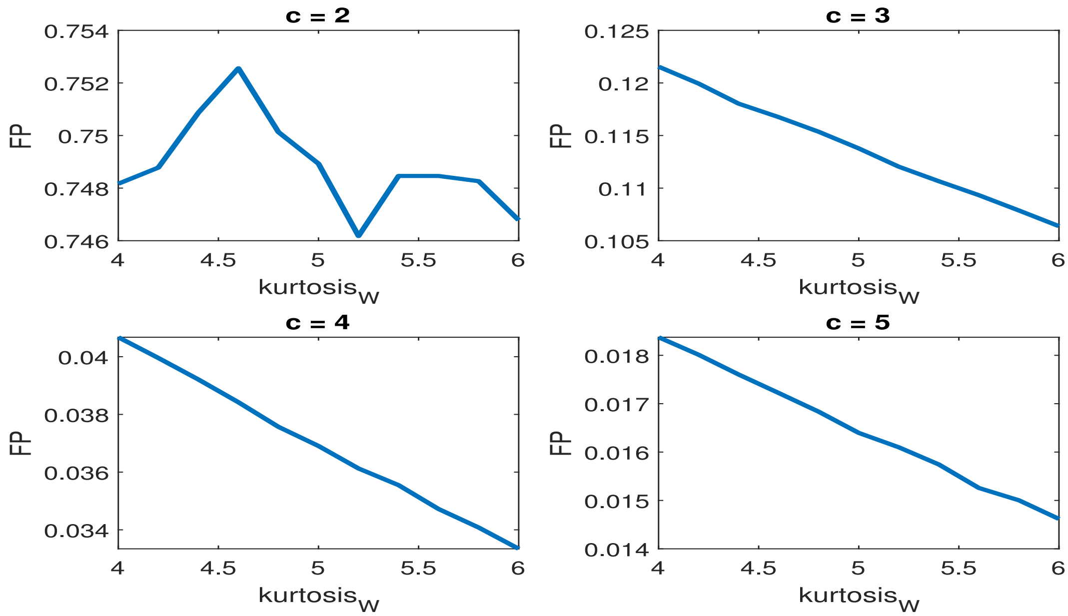

Figure 4: Behaviour of the Forward Premium FP vs Price Kurtosis κ_W across c, for Wholesale Prices P_W asymmetrically Skew-T distributed with $\mu_W = 35$, $\sigma_W = 5$, $\xi_W = 1$, and $\lambda = 1.02$. Testing (H2'): FP decreasing in κ_W.

4.3 Testing the BL Hypotheses (H3) and (H4), and inspecting for Premia Dependence on Demand Skewness and Kurtosis

Given the development of distributed generation of solar panels and micro-wind plants, it is reasonable to expect changes and variations in aggregated demand, see for instance recent contributions in Koolen et al. (2021) and Huisman et al. (2021). Thus, we investigate the behaviour of the forward premium FP with respect to the demand mean μ_D and its standard deviation σ_D as in Bessembinder and Lemmon (2002) and van Koten (2020), but considering in addition to the normal assumption the possibility that demand could follow asymmetric or fat-tailed distributions. This allows us to inspect the premium also with respect to demand skewness ξ_D and kurtosis κ_D, using the exact formula that we derived in eq. (6).

Let us start with BL (H3) hypothesis according to which the forward premium FP is convex in the standard deviation of demand σ_D; that is, first decreasing, and then increasing. Initially, we consider a normal distribution for electricity demand D with fixed μ_D (set to 50, 75, 100, 125 and 150) and let σ_D varying in the range $(1,40)$; as in van Koten (2020). Results across c and for $\lambda = 1.2$ are depicted in Figure 5; which coincide with those in Figure 2 (top row) in van Koten (2020). Results for $\lambda = 1.02$ are reported in Figure 6, with μ_D set to 100 and 150 and with different ranges for σ_D, in order to better observe convexity.

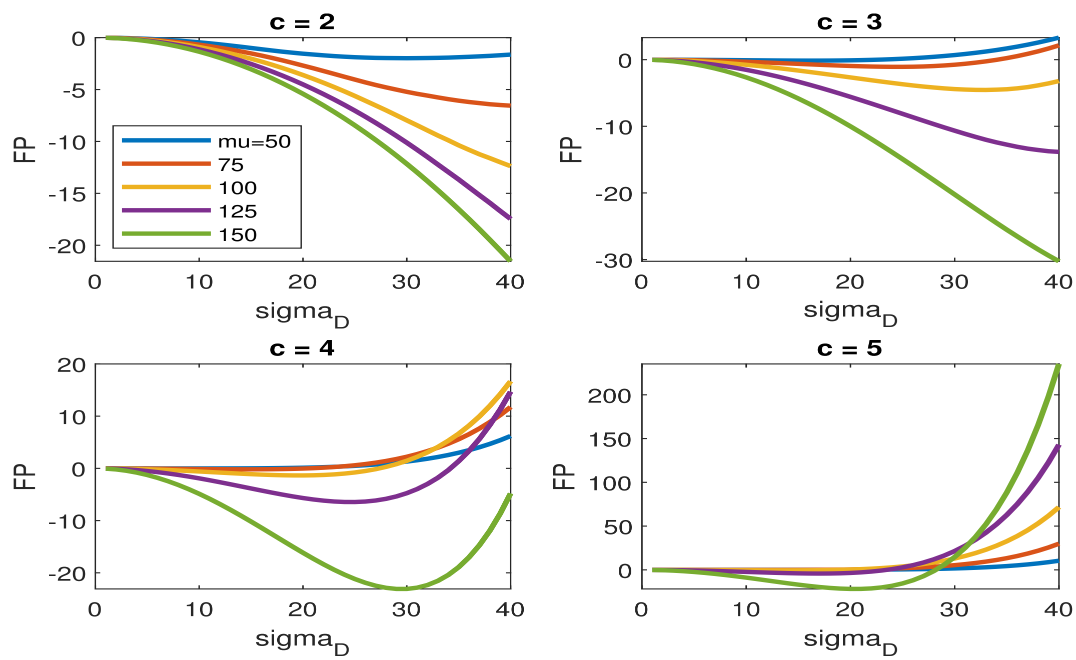

Figure 5: Behaviour of the Forward Premium FP vs Demand Standard Deviation σ_D for Demand D Normally distributed for some fixed values of μ_D and $\lambda = 1.2$. Testing (H3): FP convex in σ_D.

Moving forward, we assume now that the demand D follows a Student-T distribution with $\mu_D = 100$, *dof* ν set to $(5.1, 5.5, 8)$ and let σ_D vary in suitable ranges. Results across c and for $\lambda = (1.02, 1.2)$ are provided in Figure 7. Overall, we see that the forward premium is indeed convex,[5] thus confirming BL (H3) hypothesis not only under the hypothesis of normal demand, but also and interestingly under different distributional assumptions run in our simulations; hence, providing new further support in favour of this hypothesis.

Let us now move to the BL hypothesis (H4), stating that the forward premium FP is increasing in the expected mean demand μ_D. As before, we consider first

[5]Convexity can be observed also in the case $c = 2,3$ and $\lambda = 1.2$ by widening suitably the range for σ_D. In Figure 5 we kept the range $(1,40)$ for σ_D in order to ease the comparison with results in van Koten (2020).

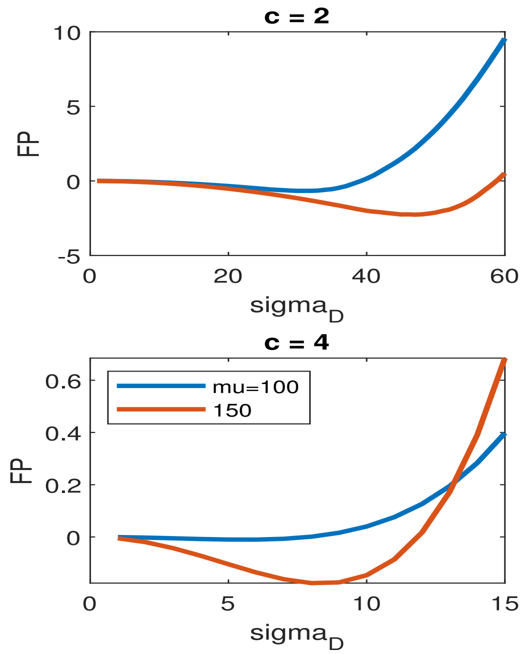

Figure 6: Behaviour of the Forward Premium FP vs Demand Standard Deviation σ_D for Demand D Normally distributed with for two fixed values of μ_D and $\lambda = 1.02$. Testing (H3): FP convex in σ_D.

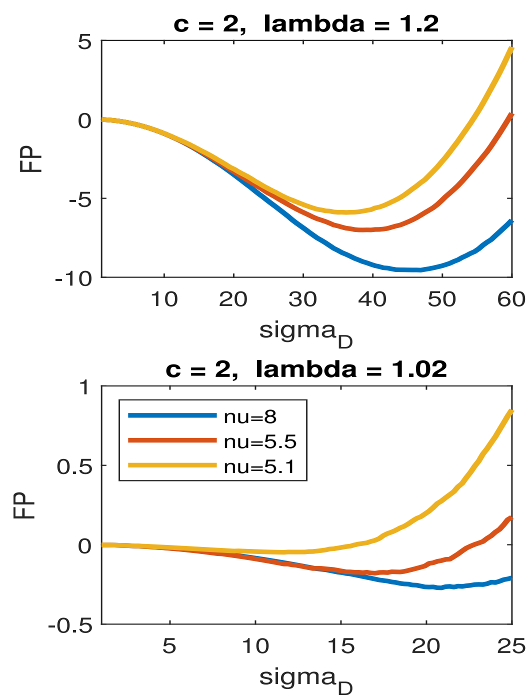

Figure 7: Behaviour of the Forward Premium FP vs Demand Standard Deviation σ_D across c and λ, for Demand D Student-T distributed with $\mu_D = 100$ and 3 values for $dof\ \nu$. Testing (H3): FP convex in σ_D.

a normal distribution for demand D with fixed σ_D (set to 5, 15, 25, and 35 as in van Koten (2020)) and let μ_D range in $(50, 150)$. Numerical results across c and for $\lambda = (1.2, 1.02, 1.05)$ are provided in Figures 8, 9 and 10, respectively.

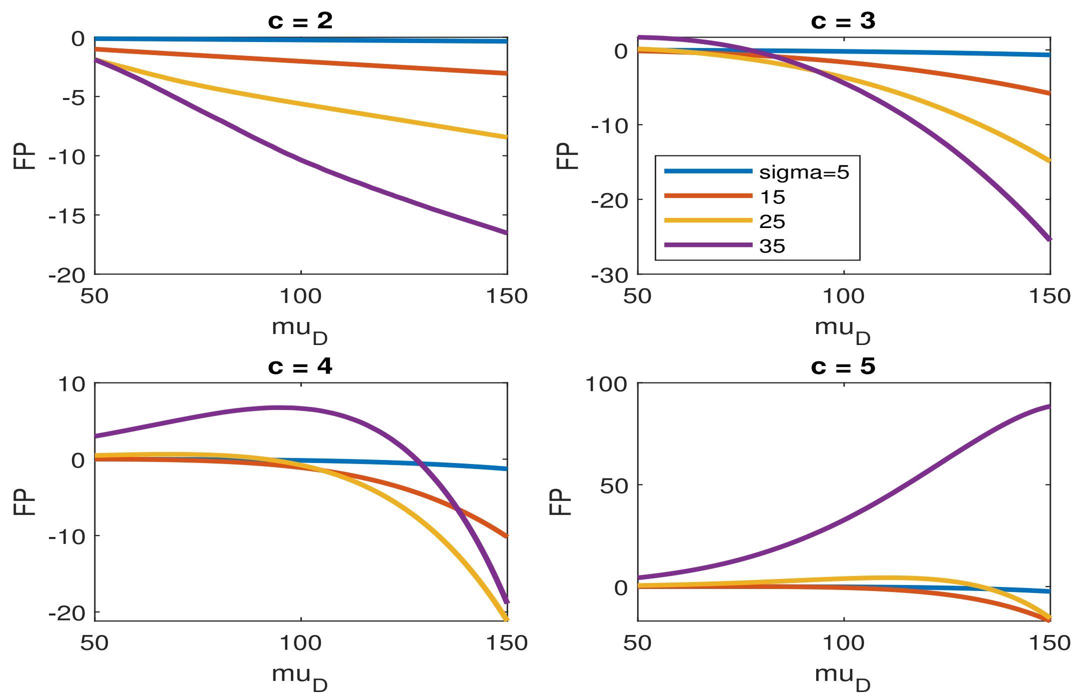

Figure 8: Behaviour of the Forward Premium FP vs Mean Demand μ_D across c and for $\lambda = 1.2$, with Demand D Normally distributed with some fixed values for σ_D. Testing (H4): FP increasing in μ_D.

Then, we assume that the expected demand follows a Student-T distribution with $\sigma_D = 20$ and *dof* ν set to 5.1,5.5 and 8, as before, and let μ_D vary in the range $(50, 150)$. Results across c and for $\lambda = 1.02$ are provided in Figure 11.

As far as (H4) is concerned, it must be recalled that previous evidence was mixed; indeed van Koten (2020) did not find support in favour of this hypothesis. Our simulations instead show that when more reasonable risk aversion are considered (for low values of $\lambda = 1.02, 1.05$) and cost functions are high ($c = 4, 5$), forward premia clearly increase in the expected mean demand μ_D especially when demand risk increases. However, when there is an higher risk aversion ($\lambda = 1.2$ as in BL and van Koten (2020)) and the cost function is moderately high ($c = 4$), the premium first increases but then quickly decreases. It also decreases for low cost

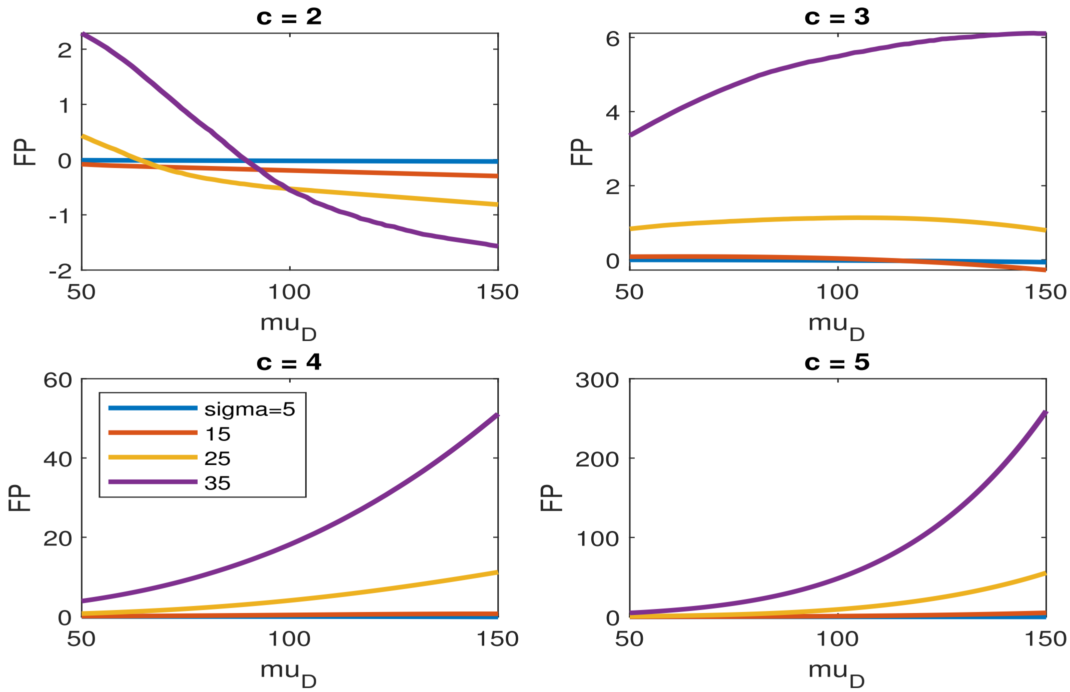

Figure 9: Behaviour of the Forward Premium FP vs Mean Demand μ_D across c for $\lambda = 1.02$, with Demand D Normally distributed with some fixed values for σ_D. Testing (H4): FP increasing in μ_D.

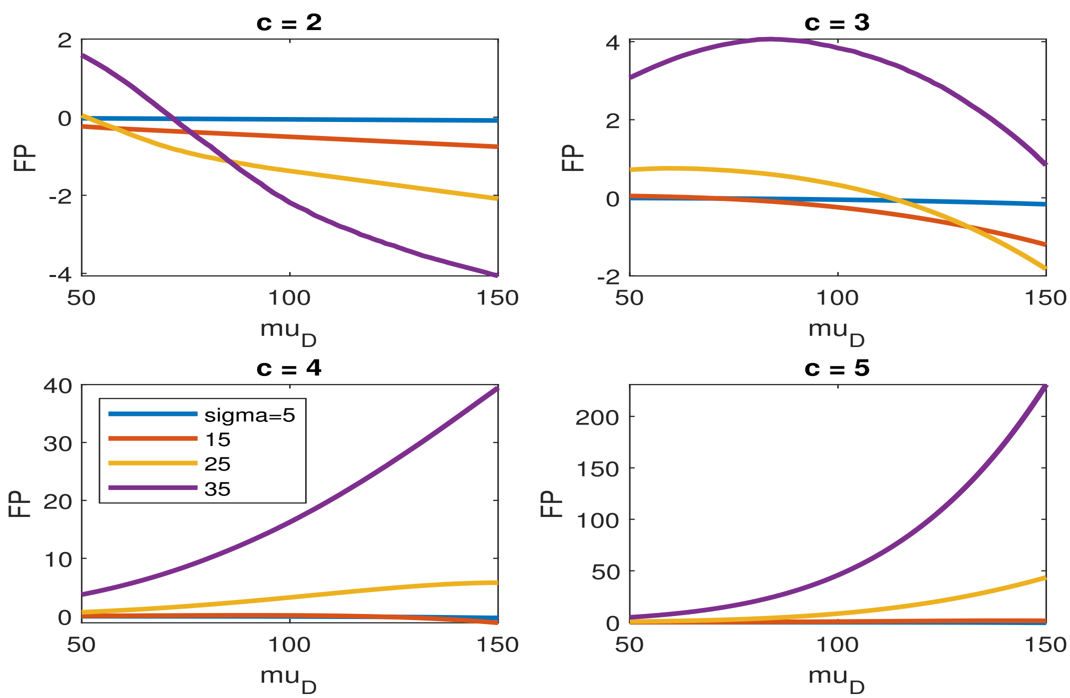

Figure 10: Behaviour of the Forward Premium FP vs Mean Demand μ_D across c for $\lambda = 1.05$, with Demand D Normally distributed with some fixed values for σ_D. Testing (H4): FP increasing in μ_D.

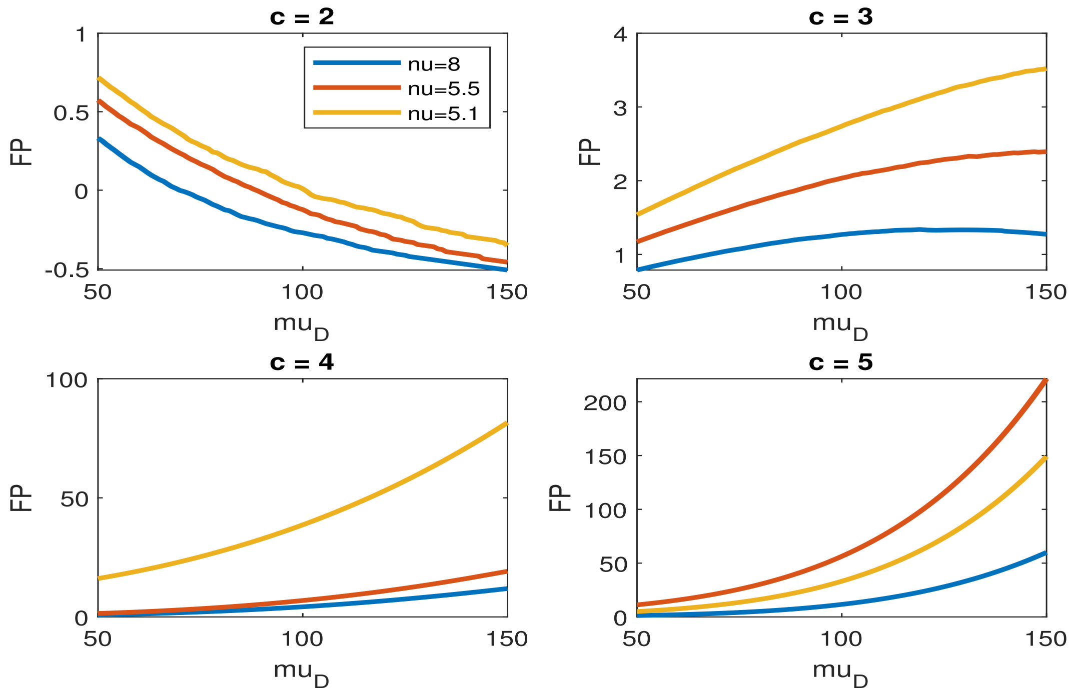

Figure 11: Behaviour of the Forward Premium FP vs Demand Mean μ_D across c and for $\lambda = 1.02$, with Demand D Student-T distributed with $\sigma_D = 20$, and 3 values for ν. Testing (H4): FP increasing in μ_D.

functions ($c = 2$) and all inspected volatility levels and risk aversion profiles. According to these results, it can be argued that under low production costs, generators do not look for higher compensation, but that is required as soon as producing electricity becomes more costly. This is particularly evident and supported by the behaviour of the premium for both risk aversions $\lambda = 1.02, 1.05$ when moving across cost parameters. When normality of demand is relaxed towards more realistic dynamics of asymmetric demand especially in view of a higher RES penetration, our previous conclusions are even more evident (see Figure 11). Thus, this hypothesis which has received controversial support elsewhere is shown to depend upon specific market conditions: while it can be justified for high cost functions, it does not seem to hold for low costs.

In trying to clarify the dependence of the forward premium on higher demand moments, we first focus on demand skewness ξ_D considering the Skew-T distribution for demand D, and then we look at its kurtosis κ_D using the Student-t distribution.

As far as demand skewness in concerned, we fix $\mu_D = (50,100)$, $\sigma_D = (10,20)$, $\kappa_D = 4$, and we let ξ_D vary in the range $(-0.5,1)$. Results across c and for $\lambda = 1.02$ are presented in Figure 12. We can observe that the premium is clearly increasing with respect to the skewness of demand, and this behaviour is consistent across different cost levels c and the expected mean demand values considered. Results for other (fixed) values of $\kappa_D > 3$ an/or for $\lambda = 1.2$ are similar hence omitted.

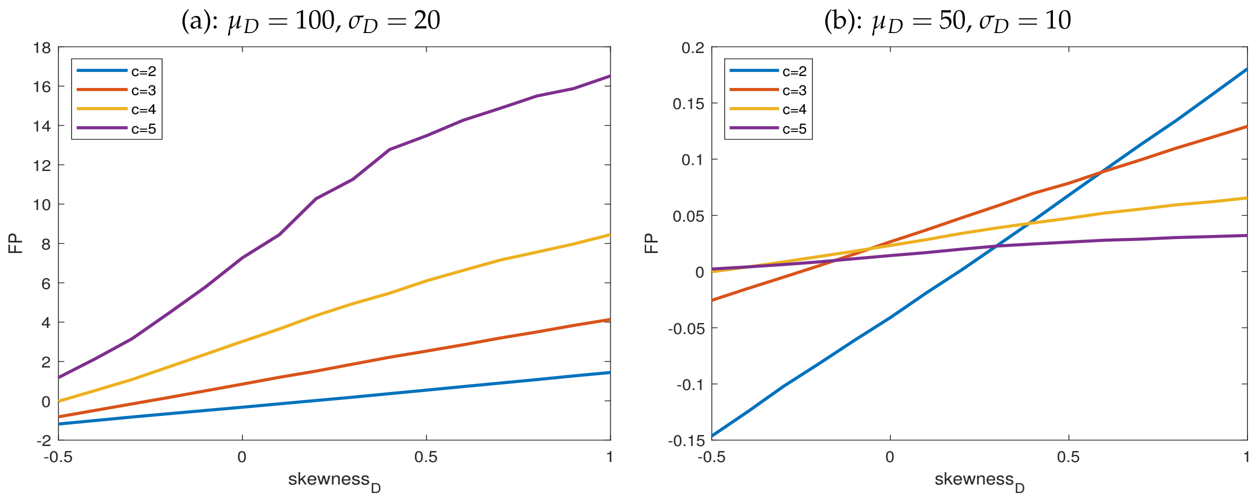

Figure 12: Behaviour of the Forward Premium FP vs Demand Skewness ξ_D across c and for $\lambda = 1.02$, with Demand D Skew-T distributed with $\kappa_D = 4$ and selected values of μ_D and σ_D.

As far as demand kurtosis in concerned, we assume demand D to be Student-T distributed with fixed μ_D and σ_D, while leaving *dof* ν_D to vary in such a way that the kurtosis ranges in $(3,6)$. Simulations show that the premium is increasing in the expected demand kurtosis, independently from the choice of parameters; see Figure 13.

5 Conclusions

With the financial performance of participants in competitive electricity markets becoming more dependent upon adequate hedging in the forward markets, we considered how this could be improved with more precise modelling of the for-

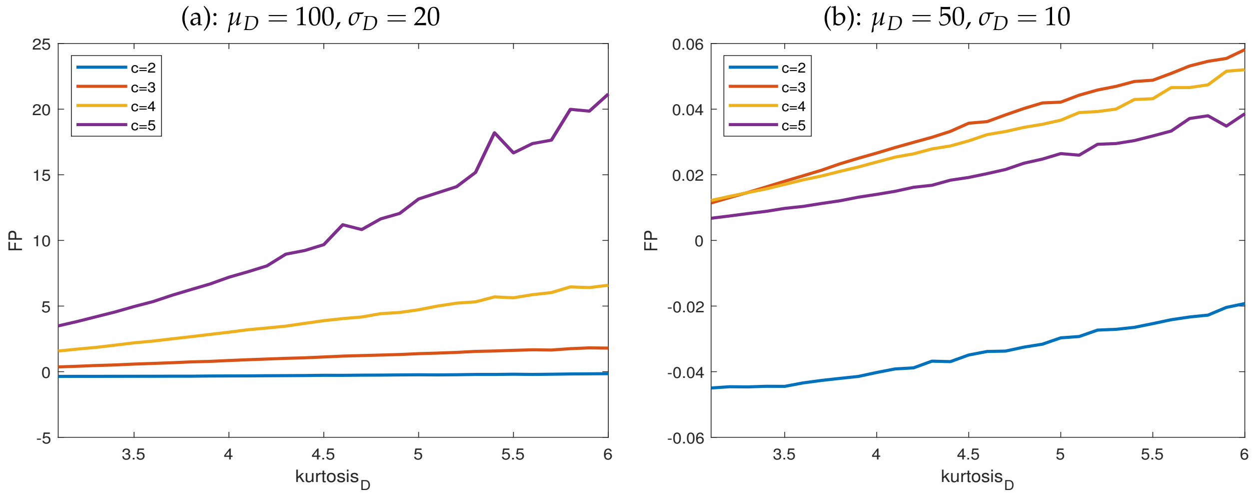

Figure 13: Behaviour of the Forward Premium FP vs Demand Kurtosis κ_D across c and for $\lambda = 1.02$, with Demand D Student-T distributed with selected values of μ_D and σ_D.

ward curve. In particular we considered the widely applied model proposed by Bessembinder and Lemmon (2002) could be reformulated with an extended specification to include higher moments. Thus we developed a more thorough specification and contributed in the following ways.

Firstly, using a quartic Taylor expansion, we provided an extended approximation for the electricity forward premium including its dependency from all the four moments of the anticipated wholesale price distribution. This aligns the forward risk premium specification for electricity with the increasing research that is incorporating kurtosis into financial risk and asset pricing models.

Secondly, we tested the goodness of the Taylor approximations and confirm that the one we propose (*extended* to include the fourth moment) is more precise in estimating the electricity risk premia, also in extreme market conditions affecting price mean and volatility.

Thirdly, we derived a more explicit expression for the premium in terms of the expected demand density and investigated the dependence of the premium on the third and fourth demand moments by simulations, explaining why the above approximation used for price is not feasible in this (demand) case.

Fourthly, our investigations have been carried relaxing the normality assumption for both wholesale prices and demand, considering student-t and skew-t distributions.

All of the above has allowed us to investigate more thoroughly the dependence of the premium on the expected price skewness and kurtosis. Reassuringly, we were able to draw the same conclusions supporting the original BL hypothesis of premia increasing in price skewness even when prices are skew-t distributed. Moreover, we provided new insights that the premia decrease with price kurtosis consistently across the specifications of symmetric or asymmetric distributions when marginal cost are high.

As far as the dependence of the premium with respect to demand is concerned, following the same simulation style of analysis as in BL, we confirmed the original BL hypothesis of the premium being convex in the expected demand standard deviation also for asymmetric distributed demand. Secondly, we showed that the BL hypothesis of the premium increasing with the expected mean demand holds for high costs and demand risk, for both symmetric and asymmetric demand distributional assumptions.

Finally, by simulations and using skew-t and student-t distributions, we reveal the dependence of the forward premium on the anticipated demand skewness and kurtosis, showing that the premium increases in both cases, independently of considered cost functions and levels of expected demand mean and risk.

Overall, this research has sought to clarify some of the controversial results in the various applications of the original BL model for electricity premia and, by extending the specification, we argue that new insights and clarity have been developed. More generally, the forward premia can be specified with increasing complexity with additional exogenous variables and/or different specifications of risk aversion, but such elaborations would be outside the scope of this work. We focussed upon the fundamental relationship to price and demand that has been central to the BL theme of research which still remains at the core of the forward premia modelling in relation to price and demand.

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Appendix A. Inspection of Retail and Wholesale (Gross) Price Levels

Considering the relationships between wholesale and retail prices in the modelling, we inspect the retail prices observed in Germany, France, Italy, The Netherlands, Austria and Spain for a sample of years from 2008 to 2020, collected from DeStatis[6]. These are annual average prices (in cent€/kWh) to supply electricity to households with annual consumption from 2.5 kWh to under 5 kWh, and include taxes, levies and VAT. Comparing these (gross) retail prices with wholesale prices (in the same scale), we observe that BL expected computation rule of *retail prices determined as 1.2 times the conditionally expected wholesale prices* is not reflected in the gross data, see Table 3. Hence, we propose to consider a more reasonable ratio of 1.02 or 1.05 to determine the *net* retail prices, that is $P_R = 1.02P_W$ or $P_R = 1.05P_W$.

[6] Data is from the website www.destatis.com accessed in February 2022.

Table 3: Yearly Ratios of Annual Gross Retail over Annual Average Wholesale Prices in c€/kWh

	DE	FR	IT	NL	AT	ES
2008	3.03	1.61	2.37	2.37	2.68	2.25
2009	5.44	2.59	3.22	4.64	4.91	4.37
2010	5.07	2.59	3.08	3.73	4.34	4.73
2011	4.68	2.68	2.82	3.31	3.82	4.01
2012	5.72	2.83	2.93	3.69	4.63	4.58
2013	6.98	3.28	3.66	3.54	5.47	4.81
2014	8.30	4.32	4.39	4.26	6.09	5.04
2015	8.54	4.04	4.51	4.64	6.30	4.44
2016	9.52	4.24	5.29	4.74	6.94	5.36
2017	8.09	3.60	3.84	3.81	5.71	4.14
2018	6.33	3.33	3.38	3.15	4.29	4.15
2019	7.33	4.38	3.85	4.79	5.11	4.85
2020	8.91	5.51	5.38	4.07	6.44	6.36

Appendix B. The Skew-T Distributions

In this section, we recall definitions and basic facts about the Skew-T distribution taken from Azzalini (2013) to which we refer for further details and proofs.

The (*normalized*) *skew-T* distribution $ST(0,1,\alpha,\nu)$, with *shape* parameter $\alpha \in \mathbb{R}$ and *degrees-of-freedom* (dof) $\nu > 0$, is defined by the density

$$f(x) = 2\,g_\nu(x)\,G_{\nu+1}\left(\alpha x\sqrt{\frac{\nu+1}{\nu+x^2}}\right), \qquad x \in \mathbb{R}, \tag{10}$$

where g_ν and G_ν are the density and the cdf of the $T(0,1,\nu)$ distribution (normalized *Student-T* with ν dof), i.e.[7]

$$g_\nu(x) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\,\Gamma(\nu/2)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}} \qquad G_\nu(x) = \int_{-\infty}^{x} g_\nu(y)\,dy$$

If $\alpha = 0$, then $f(x) = 2\,g_\nu(x)\,G_{\nu+1}(0) = g_\nu(x)$, so that $ST(0,1,0,\nu)$ coincides with $T(0,1,\nu)$. Also, in the limit $\nu \to \infty$ we recover the *Skew-Normal* distribution, characterized by the density $2\varphi(x)\Phi(\alpha x)$ (φ and Φ are the density and the cdf of a standard normal).

More generally, the *skew-T* distribution $ST(m,s,\alpha,\nu)$ with *location* parameter $m \in \mathbb{R}$, *scale* parameter $s > 0$, shape parameter $\alpha \in \mathbb{R}$ and dof $\nu > 0$, is the distribution of $X = m + s\,X'$ where $X' \sim ST(0,1,\alpha,\nu)$.

The Skew-T distribution admits the following stochastic representation: if $Z_1, Z_2 \sim N(0,1)$ and $V \sim \chi^2_\nu$ (*chi-square* distribution with $\nu > 0$ dof), with Z_1, Z_2, V independent, then

$$X = m + s \cdot \frac{Z_1 \cdot [1 - 2 \cdot I(Z_2 > \alpha Z_1)]}{\sqrt{V/\nu}}$$

has distribution $ST(m,s,\alpha,\nu)$. Here, $I(A)$ denotes the indicator function of the event A. The representation above allows us to easily simulate from the Skew-T

[7] Γ denotes the Gamma function.

distribution, using the Matlab function `chi2rnd` for simulating from a χ^2_ν distribution.

Assuming $\nu > 4$, the mean, standard deviation, skewness and kurtosis of $ST(m,s,\alpha,\nu)$ are given by

$$\mu = m + s \cdot \mu_0 \qquad \sigma = s \cdot \sigma_0$$

$$\xi = \frac{\mu_0}{\sigma_0^3}\left(\frac{\nu(3-\delta_\alpha^2)}{\nu-3} - \frac{3\nu}{\nu-2} + 2\mu_0^2\right)$$

$$\kappa = \frac{1}{\sigma_0^4}\left(\frac{3\nu^2}{(\nu-2)(\nu-4)} - \frac{4\mu_0^2\nu(3-\delta_\alpha^2)}{\nu-3} + \frac{6\mu_0^2\nu}{\nu-2} - 3\mu_0^4\right)$$

where

$$\mu_0 = \frac{\sqrt{\nu}\,\Gamma((\nu-1)/2)}{\sqrt{\pi}\Gamma(\nu/2)} \cdot \delta_\alpha \qquad \sigma_0^2 = \frac{\nu}{\nu-2} - \mu_0^2 \qquad \delta_\alpha = \frac{\alpha}{\sqrt{1+\alpha^2}}$$

We can see that the choice of the parameters α and ν affects all four moments and, as a consequence, that m and s are *not* in general the mean and standard deviation of $ST(m,s,\alpha,\nu)$. Also, as m and s are location and scale parameters, they have no influence on ξ and κ. It can also be shown that as α ranges in $\mathbb{R}$ and ν ranges in $(4,+\infty)$, the couple (ξ,κ) spans the region R depicted in grey in Figure 14. We can see that, for instance, if $\kappa = 10$ then all (ξ,κ) with (roughly) $|\xi| \leqslant 1.8$ are in R.

For each $\mu \in \mathbb{R}$, $\sigma > 0$ and $(\xi,\kappa) \in R$, there is, unique, a set of parameters (m,s,α,ν) that yield the chosen moments. We can numerically derive these parameters, by solving a system of four non-linear equations. We perform this task in Matlab using the function `fsolve`.

Figure 14: The joint *range* skewness-kurtosis. Levels for skewness are on the x-axis, whereas those for kurtosis on the y-axis.