# TSO-DSO Operational Coordination Using a Look-Ahead Multi-Interval Framework

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***Abstract*—With the rise of distributed energy resources and the increasing activation of flexibility resources by Distribution Systems Operators (DSOs), the Transmission System Operators (TSOs) need to co-ordinate their actions with those of the DSOs. This research uses a look-ahead multi-interval (LA-MI) framework for analyzing this coordination and explores two formulations. Firstly in the exogenous DSO model, a mixed-integer linear program is developed to reflect the pragmatic approach in many real situations whereby the TSO can only anticipate statistically the actions of the DSO. In the embedded DSO model, as a comparator, we propose a new organizational setup for the TSO-DSO operational coordination mechanism. In the resulting bilevel decomposition, a new method to calculate Benders cuts is developed and tested on a modified IEEE 118-bus test system as a transmission network and two modified IEEE 33-bus test systems as distribution networks. The benefits of the LA-MI coordination framework are substantial in comparison with the current Look-Ahead Single-Interval (LA-SI) coordination framework widely used in Europe.**

***Index Terms*—Logic-based Benders decomposition, Look-ahead, Optimal power flow, TSO-DSO coordination.**

## NOMENCLATURE

**Indices**

| | |
|---|---|
| $b, c$ | Indices for transmission system buses. |
| $i, j$ | Indices for distribution system buses. |
| $d/u$ | Index for buses with down/up services in DSO. |
| $p$ | Index for buses with TSO-DSO connection. |
| $t/r$ | Index for rolling interval/LBBD iteration. |
| $n$ | Index for complicating variables in LBBD. |

**Sets**

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|---|---|
| $I^{TS}/K^{TS}$ | Set for buses/lines of the transmission system. |
| $I_b^{DS}/K_b^{DS}$ | Set for buses/lines of the distribution system at transmission bus $b$. |
| $N_{db}/N_{ub}$ | Sets of distribution system buses with down/up services at transmission system bus $b$. |
| $PCC$ | Set for points of common coupling. |
| $N_b^{mv}$ | Sub-sets of complicating variables at transmission bus $b$ in the LBBD method ($N_b^{mv} \subseteq N_{ub}$). |
| $RW/IT$ | Set of rolling windows/LBBD iteration number. |

**Parameters**

| | |
|---|---|
| $G_{bc}^{TS}/B_{bc}^{TS}$ | Conductance/Susceptance between buses $b$ and $c$ in transmission system. |
| $G_{ijb}^{DS}/B_{ijb}^{DS}$ | Conductance/Susceptance between buses $i$ and $j$ in distribution system at transmission bus $b$. |
| $\pi_{ubt}^{UP}/\pi_{dbt}^{DW}$ | Flexibility bid of turn-up/turn-down unit $u/d$ at transmission bus $b$ at time $t$ ($\$/MWh$). |
| $RU_b/RD_b$ | Ramp up/down capability of reserves at bus $b$ ($MW/h$). |
| $PV_{ibt}/WT_{ibt}$ | Scheduled photovoltaic/wind turbine generation in the forward market ($MW$) at distribution bus $i$ and transmission bus $b$ at time $t$ . |
| $\Phi_{ibt}^{DR}/\Phi_{ibt}^{BA}$ | Offered amount of flexibility services by DR aggregators/battery owners at bus $i$ of a distribution system which is connected to the bus $b$ of a transmission system at time $t$ ($MW$). |
| $\Phi_{ubt}^{PV}/\Phi_{ubt}^{WT}$ | Offered amount of turn-up flexibility services by PV/wind turbine owner at bus $u$ of a distribution system which is connected to the bus $b$ of a transmission system at time $t$ ($MW$). |
| $\tilde{P}_{g_{pt}}^{DS}/\tilde{Q}_{g_{pt}}^{DS}$ | Anticipated active/reactive consumption of the exogenous DSO model by TSO at point of common coupling at time $t$ ($MW/MVar$). |
| $\tilde{F}_{ipt}^{UPD/DWD}$ | Anticipated turn-up/-down service activation by the DSO at the point of common coupling for the exogenous model at time $t$ ($MW$). |
| $\overline{P}_{bc}^{TS}/\overline{Q}_{bc}^{TS}$ | Upper limit of active/reactive power flow of branch $bc$ in transmission system ($MW/MVar$). |
| $\underline{P}_{bc}^{TS}/\underline{Q}_{bc}^{TS}$ | Lower limit of active/reactive power flow of branch $bc$ in transmission system ($MW/MVar$). |
| $\overline{P}_{ijb}^{DS}/\overline{Q}_{ijb}^{DS}$ | Upper limit of active/reactive power flow of branch $ij$ in distribution system at transmission bus $b$ ($MW/MVar$). |
| $\underline{P}_{ijb}^{DS}/\underline{Q}_{ijb}^{DS}$ | Lower limit of active/reactive power flow of branch $ij$ in distribution system at transmission bus $b$ ($MW/MVar$). |
| $(\underline{v}^s/\overline{v}^s)_{i/b}$ | The limitation of square of the voltage magnitude at bus $i/b$ in distribution/transmission system. |
| $S_B/\tilde{a}_{1,b}, \tilde{a}_{0,b}$ | Base value in per unit system/The anticipated cost coefficients of generator units in transmission bus $b$. |

**Variables**

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|---|---|
| $F_{dbt}^{DWT/DWD}$ | Activated turn-down service by TSO/DSO from distribution bus ($d \in N_{db}$) and transmission bus $b$ at time $t$ ($MW$). |
| $F_{ubt}^{UPT/UPD}$ | Activated turn-up service by TSO/DSO from distribution bus ($u \in N_{ub}$) and transmission bus $b$ at time $t$ ($MW$). |
| $P_{g_{bt}}^{TS}/P_{g_{ibt}}^{DS}$ | Generated active power of generator $b^{th}/i^{th}$ in TSO/DSO region at time $t$ ($MW$). |
| $Q_{g_{bt}}^{TS}/Q_{g_{ibt}}^{DS}$ | Generated reactive power of generator $b^{th}/i^{th}$ in TSO/DSO region at time $t$ ($MVar$). |
| $P_{bt}^{TS}/Q_{bt}^{TS}$ | Active/Reactive power injection at transmission bus $b$ at time $t$ ($MW/MVar$). |
| $P_{ibt}^{DS}/Q_{ibt}^{DS}$ | Active/Reactive power injection at bus $i$ of a distribution system which is connected to bus $b$ of a transmission system at time $t$ ($MW/MVar$). |
| $P_{L_{bt}}^{TS}/Q_{L_{bt}}^{TS}$ | Active/Reactive power load connected to transmission bus $b$ at time $t$ ($MW/MVar$). |
| $P_{L_{ibt}}^{DS}/Q_{L_{ibt}}^{DS}$ | Active/Reactive power load connected to the bus $i$ of a distribution system which is connected to the bus $b$ of a |

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| | transmission system at time $t$ ($MW/MVar$). |
| $P^{TS}_{bct}/Q^{TS}_{bct}$ | Active/Reactive power flow from bus $b$ to bus $c$ in transmission system at time $t$ ($MW/MVar$). |
| $P^{DS}_{ijbt}/Q^{DS}_{ijbt}$ | Active/Reactive power flow from bus $i$ to bus $j$ of a distribution system which is connected to bus $b$ of a transmission system at time $t$ ($MW/MVar$). |
| $P^{DS}_{g_{pt}}/Q^{DS}_{g_{pt}}$ | Active/reactive consumption for the embedded DSO model at point of common coupling at time $t$ ($MW/MVar$). |
| $(\theta/v^s)_{(b/i)t}$ | Phase angle/Square of voltage magnitude at bus $b/i$ in transmission/distribution systems at time $t$. |
| $\beta^{UP}_{bt}/\beta^{DW}_{bt}$ | Binary variables that represent the state of up/down operating reserve activation at bus $b$ and time $t$ for the exogenous DSO activation model. It also represents the concept of implication in the inference dual theorem in the embedded DSO activation model. |
| $\alpha^{UP}_{ubt}/\alpha^{DW}_{dbt}$ | Binary variables that represent the state of up/down EERs activation at bus $(u/d) \in \{N_{ub}/N_{db}\}$ of distribution systems which is located at bus $b$ of a transmission system at time $t$ for the exogenous DSO activation model. It also represents the concept of implication in the inference dual theorem in the embedded DSO activation model. |
| $\lambda/\Pi/\mu/\delta$ | Dual variables of equality constraints (1b)-(1e). |
| $D/Y$ | Dual variables of complementary slackness conditions/Logical indicators. |
| $\Gamma$ | Boundary for the determined Benders cuts in the LBBD model. |
| $g$ | Binary variables that reflect combination of complicating variable. |
| $\hat{G}$ | Boolean variables for different combinations of complicating variables in LBBD. |

**Abbreviations**

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|---|---|
| $TSO/DSO$ | Transmission/Distribution system operator. |
| $TS/DS$ | Superscript for transmission/distribution system. |
| $DR/EER$ | Demand response/Embedded energy resource. |
| $GDP$ | Generalized disjunctive programming. |
| $LA-MI$ | Look-ahead multi-interval. |
| $LA-SI$ | Look-ahead single-interval. |
| $LBBD$ | Logic based Benders decomposition. |
| $SBD$ | Standard Benders decomposition. |
| $MILP$ | Mixed-Integer Linear Programming. |
| $MLLP$ | Mixed-Logical Linear programming. |

# I. Introduction

The rise of embedded energy resources (EERs) is partly due to the adoption of local renewable facilities, but also includes the increase in demand-side response, penetration of electric vehicles, and installations of battery storage facilities. In total, being "behind-the-meter", they create greater load uncertainty on the transmission system and may pose constraint management issues for the local distribution network operators. But they are also part of the solution as well. These resources can provide useful flexibility services that both the TSOs and DSOs can procure, perhaps through aggregators, and activate them if needed to ensure system quality. Whilst in reality the DSOs and TSOs are separate organizations, and they seek to optimize their own networks individually, their operational decisions inevitably interact and therefore require a coordination process [1]. Regulators are likely to require cooperation through the timely transparency of actions, but a fully integrated optimization is very unlikely to be mandated [2]. In particular, the TSO needs to react, and ideally anticipate, the activations of flexibility resources by the DSO, but in practice the TSOs have limited visibility of the state of the distribution networks [3]. In this context, anticipation of the activities of the DSO is an additional complication.

In general, three TSO-DSO coordination frameworks have been proposed in the existing literature specifically the TSO-managed model, the TSO-DSO hybrid managed model, and the DSO-managed model [3]. In the TSO-managed model, TSO is responsible for validating the submitted bids by the EERs and solving the TSO-DSO operational coordination problem. Typically, in this model, DSO does not activate any flexibility sources and they are obliged to share their confidential information with TSO. In the TSO-DSO hybrid-managed model, the TSO is responsible for solving the AC-OPF problem of the interconnected TSO-DSO systems, and DSO validates the submitted bids by the EERs. In the DSO-Managed model, DSO is responsible for solving the AC-OPF problem of the distribution system and validating the submitted bids by the EERs. In both TSO-DSO hybrid-managed model and the DSO-managed model, DSO activates its needed flexibilities prior to the TSO in a sequential market structure.

Two decentralized TSO-DSO coordination schemes are proposed based on the DSO-managed model and benchmarked against a centrally co-optimized TSO-DSO coordination scheme in [4]. In the first proposed decentralized scheme, the DSO validates the submitted bids by the EERs, whilst the second scheme proposes a sequential procurement process. A robust decentralized TSO-DSO coordination scheme based on the TSO-DSO hybrid-managed model is presented in [5], in which the TSO-DSO operational coordination is not considered in the ancillary flexibility service market with the EERs. Excluding EERs from the real-time operational coordination of TSO-DSO disregards the benefits of considering renewable energy sources as flexibility providers. In [6], based on a TSO-DSO hybrid-managed model, a TSO-DSO operational coordination framework is proposed in which TSO and DSO are responsible to create and solve the economic dispatch optimization problem of their networks. A distributed optimization approach is employed to find a solution to the OPF problem of a system including TSO and several DSOs in [7] which is based on the TSO-managed model. Although it co-optimizes the allocation of conventional resources considering TSO-DSO coordination, it is not related to the TSO-DSO coordination problem in the ancillary flexibility service market and it does not consider the impacts of EERs in the real-time operation of power systems.

A TSO-DSO coordination model is proposed in [2] which estimates the flexibility range of power exchange at the TSO-DSO injection area. In [8], a TSO-DSO coordination approach considering two trading markets including day-ahead and real-time is proposed in which DSO validates the bids of EERs in the day-ahead market. A dynamic economic dispatch model is proposed in [9] that considers the TSO-DSO operational coordination and automatic regulation schemes, but it does not evaluate how to find the solution to large-scale optimization problems from a decomposition viewpoint. Authors in the European SmartNet project [10] proposed four potential TSO-DSO coordination schemes in the ancillary service market all of which DSO validates the bids of EERs and provides them to the TSO in different sequential activation structures. A separated TSO-DSO coordination framework is presented in [11]–[13] which allows the design of the local flexibility market in the distribution level with the participation of EERs. In the TSO-DSO coordination scheme based on the local flexibility market, the DSO can activate its required flexibility from EERs. Then, the activated service by the DSO is eliminated from the available EERs for the TSO.

In summary, in the TSO-managed model, there is a possibility of inefficient facilitation of services since the DSO is not responsible for the bid validation process. In the TSO-DSO hybrid-managed model, since DSO is responsible for the bid validation process, DSO does not utilize the maximum capacity of the distribution system due to the lifetime issue. The DSO-managed model is based on a sequential process in which DSO activates its required service prior to the TSO. The drawback of the sequential process includes the inability to achieve efficiency in the overall service allotment, undesirable cross impacts between buyers, and the possibility of the free-rider strategy [1].

In the aforementioned papers, the DSO and TSO objectives regarding EER activations in the ancillary flexibility service market have not been considered in a coordinated mechanism and they mainly focus on different decentralized models. In some articles such as [1], [10]–[12], the objective functions of the TSO and DSO are modeled in the TDO-DSO coordination framework, but in a sequential procurement process.

Furthermore, other researchers such as [14] and [15] have noted, and this paper also argues, that the TSO needs to look beyond the immediate dispatching period to efficiently dispatch real-time resources. The look-ahead framework tries to hedge against different sources of uncertainties in the TSO-DSO coordination over different time intervals in the future. A Look-ahead Single Interval (LA-SI) framework, which is traditionally utilized in various electricity markets, is based on the forecasted uncertainties only for the next time step in the scheduling problems [15], [16]. However, in the TSO-DSO coordination problem, real-time dispatch may involve facilities that are energy limited (e.g. batteries), time-of-day limited (e.g. solar), and dynamically-limited (e.g. ramp rates) all of which motivate Look-Ahead Multi-Interval (LA-MI) optimization by the TSO. Moreover, considering inter-temporal constraints, the LA-MI framework improves the economy and security of the power system operation [17], and this in turn leads to higher utilization of renewable energy sources [18].

Therefore, one of the main contributions of this paper is implementing the LA-MI framework into the TSO-DSO operational coordination mechanism for the first time using the following two approaches. The first is a pragmatic reflection of the current state of imperfect information by which the TSO seeks to use empirical data on past activations by the DSO to have a statistical model of DSO responses in the TSO's look ahead model. We do include an approach in which the TSO does not have access to DSO data but just seeks to forecast their actions. Secondly, with the trend amongst regulators to deal with progressive unbundling and market complexities by creating new intermediary agents, it is quite likely that a regulator might seek to create a new independently regulated market operator to ensure impartiality and efficiency in the allocation of flexibility services. We refer to this as a Flexibility Market Operator (FMO). In particular, if the DSO and TSO also own flexibility assets, in addition to those of the various market participants, an FMO may be needed to avoid conflicts of interest. Therefore, to complete our developed TSO-DSO coordination mechanism and align it with today's practical settings in many markets, we propose a new organizational setup for TSO-DSO operational coordination mechanism which involves TSO, DSO, EER, and FMO. The proposed FMO is an independent non-profit and regulated organization that operates the flexibility market, validates the bids of service providers considering the constraints of the distribution system, forecasts the uncertainty sources, and most importantly has access to the required data to solve the TSO-DSO operational coordination problem as an interface optimizer. Accordingly, the FMO embeds the DSO's prospective optimization within the TSO's look ahead multi-interval real-time dispatch modeling and solves a bilevel mixed-logical linear programming problem (MLLP). Our framework is implemented into the proposed market mechanism in [1].

The LA-MI framework in this paper is applied to both approaches and it is benchmarked against the LA-SI framework. Using LA-MI and LA-SI in the second approach with the embedded DSO activations leads to a mixed-logical linear programming problem. In this regard, whilst the Karush-Kuhn-Tucker (KKT) conditions are widely used to solve bilevel optimization programming problems in one shot [19], these LA-MI and LA-SI frameworks create computational challenges which require special techniques.

To be more specific, the standard Benders decomposition was proposed for mixed-integer linear programming (MILP) in which all integer variables could be considered as complicating variables, and thus the standard duality theory could be applied to the sub-problem to determine the required Benders cuts [20], [21]. If there are numerous binary variables and complementary slackness conditions, as in our case, the sub-problem still contains some integer variables since not all variables can be considered as complicating variables. In this regard, standard duality theory cannot be utilized to determine the Benders cuts since the sub-problem is no longer convex with zero duality gap [22]. Consequently, a new modification of the decomposition technique is needed.

There are several modifications of the classical Benders decomposition method which can address integer sub-problems [23]–[25], logical propositions [22], [26], and non-linear constraints [27], [28]. However, these methods employ continuous relaxation in the sub-problem to calculate approximated Benders cuts using the duality theorem. Moreover, these methods are very slow, especially when the number of binary variables increases [24], [25]. We find that the logic-based Benders decomposition (LBBD) method is the most appropriate method since it allows the sub-problem to be any optimization problem rather than specifically a linear or convex nonlinear programming problem [20]. Moreover, our suggested LBBD is tractable for an expansive range of decomposable large-scale programming problems with integer decision variables [29]. We use the concept of ”inference dual” in the sub-problem to find the tightest bound on the objective function of the master problem as implied by its current solution. Then, the Benders cuts for the master problem are calculated based on these bounds [22]. Nonetheless, there is no general method to determine logic-based Benders cuts and each formulation requires a customized approach [20], [29], which means that a new method for generating Benders cuts should be developed for any specific optimization problem. Accordingly, one of the main contributions of this paper is to expand the LBBD method for the TSO-DSO co-ordination problem and develop a new formulation to calculate the associated logic-based Benders cuts.

Although a small scale co-ordination of TSO and DSO activities has been presented in [6], a large-scale network configuration is needed to show the effectiveness of the proposed optimization methodology. Consequently, in this paper, a combination of a modified IEEE 118-bus test system (as TSO network) and two modified IEEE 33-bus test systems (as DSO networks) is specified.

In summary, the contributions of this paper are as follows:

1) We propose a new organizational setup for the TSO-DSO operational coordination framework including an independent non-profit regulated organization called ”Flexibility Market Operator” (FMO). The proposed FMO embeds the DSO activation model into the TSO optimization problem. The proposed model leads to a bilevel programming problem in which the TSO and DSO objectives are optimized at the upper level and the lower level, respectively.
2) We develop the LBBD method for a bilevel TSO-DSO coordination problem with a scheduling MLLP problem at each level. Using LBBD, we decompose the problem into two mixed-integer linear logic-based problems as the master and sub-problem. Since parallel computing is applicable in the proposed method, we can mitigate the associated computational challenges.
3) We propose a new method to generate the logic-based Benders cuts for the proposed bilevel TSO-DSO coordination problem with an embedded DSO activation model.
4) Instead of utilizing the big-M method, we employ the generalized disjunctive programming along with the indicator constraints method to handle complementary slackness conditions and disjunctive equations.
5) We implement the LA-MI framework into the TSO-DSO operational coordination mechanism including both embedded and exogenous DSO activation models for the first time. Our results are benchmarked against the existing LA-SI framework. A comparison of our LA-MI coordination framework with the current LA-SI coordination framework, as widely used in Europe, reveals lower costs and more efficient system operation, as demonstrated through several numerical experiments, using a modified IEEE 118-bus test system and two modified IEEE 33-bus test systems.

The rest of the paper is organized as follows. Section II introduces the formulations for the look-ahead multi-interval TSO-DSO coordinations. Linearized models of optimal power flow of the coordinated networks are developed in Section III. Section IV presents the optimization procedures. The case studies are presented in Section V and Section VI summarizes the conclusions.

## II. The Look-Ahead Multi-Interval (LA-MI) Framework for TSO-DSO Coordination

The length of intervals in the rolling horizon depends on several factors. The more the number of intervals, the more the computational complexity of the optimization problem, and the less the accuracy of the forecasted parameters (e.g., solar radiation, wind speed, and bids of ERRs). Due to the ramping capabilities of reserve capacities and minimum up- or down-time constraints of generation units, we assume that the system operators are able to cope with the real-time operational uncertainties provided they are being prepared for the next four hours. Consequently, a LA-MI framework with four rolling windows is proposed for improving TSO-DSO coordination in the real-time operation of electricity market. The embedded energy resources in our set-up include demand response (DR), battery storage systems, Photovoltaic (PV) panels, and wind turbines. These uncertain energy resources are available to provide ancillary flexibility services to both the DSO and TSO.

The proposed framework is compared with LA-SI framework in the real-time operation of electricity market. All contracts related to the day-ahead and hour-ahead markets are taken into account as firm contracts. Consequently, the proposed LA-MI framework is considered for hedging against the uncertainty of Renewable Energy Source (RES) and load consumption in order to fulfil all signed contracts in the forward market platforms. We treat all generators and flexibility service providers as price takers with competitive offers. The LA-MI rolling horizon for TSO-DSO co-ordination is illustrated in Fig. 1. A series of TSO-DSO coordination problems are solved in a rolling implementation of the LA-MI framework. Each look-ahead rolling window contains four delivery periods but only the results of the AC-OPF for the upcoming interval are mandatory; the later ones are provisional. Likewise, only the prices for the upcoming interval are used for settlement, the forward prices for the subsequent three periods are used for the provisional look-ahead but are treated as indicative.

The objective function of the LA-MI framework is to minimize the total operation cost of the TSO-DSO coordination over the relevant time horizon of rolling windows by solving a properly linearized AC optimal power flow model for both transmission and distribution systems. The total cost of TSO-DSO coordination comprises the cost of generated power by the predetermined operating reserve generators and the cost of providing the flexibility services by service providers (aggregators).

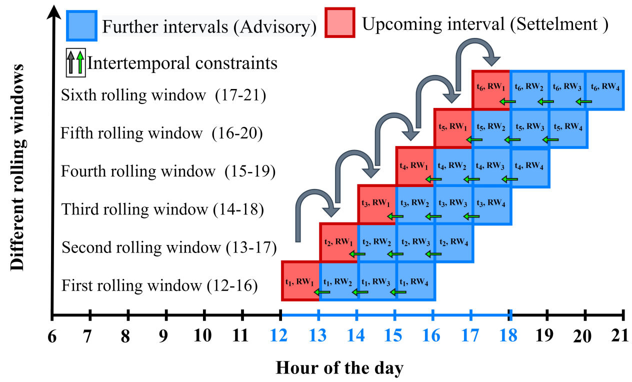


Fig. 1. Series of TSO-DSO coordination problems solved in a rolling implementation of the LA-MI framework.

*1) Market Mechanism and Organizational Setup:* The recently proposed mechanism in [1] is considered. The TSO and its inter-connected DSOs are possible service buyers. The EERs are competing with available operating reserves which include the spinning reserve as well as the non-spinning or supplemental reserve to provide flexibility at the TSO level. At the DSO level, EERs are the only flexibility providers. According to the trading platform of the market mechanism, flexibility is cleared in a one-shot auction. The complex utility functions of the buyers are reflected in the market-clearing problem to hedge against a possible free-rider strategy.

The TSO and DSO are independent non-profit flexibility service buyers whose flexibility interests are confidential and might conflict with each other. Moreover, neither TSO nor DSO is willing to reveal their network information. However, due to the free-rider strategy, the disadvantages of the sequential flexibility activation process, and to improve the real-time operation of power systems, the coordinated TSO-DSO operation is necessary. The coordinated operation needs a high level of information accessibility. Consequently, we propose a new independent non-profit regulated organization in the real-time flexibility market which is called Flexibility Market Operator (FMO). The FMO receives all the required information in a short period of time before power delivery in the power system (perhaps 15 minutes). The proposed FMO has different sub-organizations including anticipation, data collection, and interface optimizer. Consequently, our new proposed organizational structure for TSO-DSO operational coordination mechanism includes TSO, DSO, EER, and FMO. From the market participants' privacy viewpoint, since our proposed FMO is supposed to operate only during the ancillary flexibility service market and it is completely independent of TSO and DSO, privacy issues are mitigated. Furthermore, each sub-organization of FMO can be considered independent to ensure a high level of participants' privacy.

The FMO anticipates uncertainty sources based on the received information from the market participants, collects all required information to operate the proposed coordination model, forms and solves our bilevel TSO-DSO operational coordination problem, and finally sends all dispatch comments and market clearing outputs to the flexibility market participants. The FMO has no access to the whole interest functions of the market participants. The exchanged information is only the technical information of transmission and distribution systems as well as the offer functions of flexibility market participants in a short period of time before power delivery. Moreover, since TSO, DSO, and FMO are independent non-profit regulated organizations, it is assumed that TSO and DSOs would share the required information with FMO through an agreement.

The main objective function of the market-clearing problem is to maximize the total social welfare of all service buyers. The considered mechanism has the properties of truthfulness and efficiency which means that the FMO maximizes social welfare while participants are incentivized to submit true valuations.

In this paper, from the flexibility service term, we only refer to the "turn-up" and "turn-down" service activation. The "turn-down" service activation refers to demand decrease (or generation increase) and vice versa for "turn-up" service activation. We consider PV and wind generation units as the turn-up service providers and DR aggregators as the turn-down service providers. The battery storage systems can provide both turn-up and turn-down services which depend on the submitted offer functions by the battery system owner who includes all constraints and costs into their offer functions. Based on the battery storage constraints like SOC, battery system owners can act as turn-up or turn-down service providers. The amount of flexibility that they can offer depends on their SOC and other technical constraints which are confidential and FMO has no information on them. When the SOC of the battery storage system is too low, which means that the battery is fully discharged, the battery owners submit the maximum amount of their turn-up flexibility. On the other side, when the battery system is fully charged, the battery owner can offer the maximum amount of turn-down flexibility. Nevertheless, neither FMO nor any other organization has information about the actual capacity of the battery systems.

## III. Optimal Power Flows for TSO and DSO

The total cost of TSO-DSO coordination to be minimized comprises the cost of generation to meet demand and the cost of providing the flexibility services. The base AC-OPF model introduced in [30] is taken. The OPF for the DSOs is formulated as follows:

$$\underset{\{P_{g_i}^{DS}, v_i^{s^{DS}}, \theta_i^{DS}, P_{ij}^{DS}, Q_{ij}^{DS}, P_i^{DS}, Q_i^{DS}\}}{\text{Minimize}} \sum_{i \in I^{DS}} f_i(P_{g_i}^{DS}) \tag{1a}$$

subject to:

$$P_{ij}^{DS} \approx P_{ij,0}^{DS} + (\nabla P_{ij}^{DS}|_0)^T \begin{pmatrix} v_i^{s^{DS}} - v_{i,0}^{s^{DS}} \\ v_j^{s^{DS}} - v_{j,0}^{s^{DS}} \\ \theta_i^{DS} - \theta_{i,0}^{DS} \\ \theta_j^{DS} - \theta_{j,0}^{DS} \end{pmatrix}, \forall\, (ij) \in K^{DS} \tag{1b}$$

$$Q_{ij}^{DS} \approx Q_{ij,0}^{DS} + (\nabla Q_{ij}^{DS}|_0)^T \begin{pmatrix} v_i^{s^{DS}} - v_{i,0}^{s^{DS}} \\ v_j^{s^{DS}} - v_{j,0}^{s^{DS}} \\ \theta_i^{DS} - \theta_{i,0}^{DS} \\ \theta_j^{DS} - \theta_{j,0}^{DS} \end{pmatrix}, \forall\, (ij) \in K^{DS} \tag{1c}$$

$$P_i^{DS} = \sum_{(ij)\in K^{DS}} P_{ij}^{DS} + v_i^{s^{DS}} \sum_{j \in I^{DS}} G_{ij}^{DS},\ \forall\, i \in I^{DS} \tag{1d}$$

$$Q_i^{DS} = \sum_{(ij)\in K^{DS}} Q_{ij}^{DS} + v_i^{s^{DS}} \sum_{j \in I^{DS}} -B_{ij}^{DS},\ \forall\, i \in I^{DS} \tag{1e}$$

$$\underline{P}_i^{DS} \le P_i^{DS} \le \overline{P}_i^{DS},\ \forall\, i \in I^{DS} \tag{1f}$$

$$\underline{Q}_i^{DS} \le Q_i^{DS} \le \overline{Q}_i^{DS},\ \forall\, i \in I^{DS} \tag{1g}$$

$$\underline{v}_i^{s^{DS}} \le v_i^{s^{DS}} \le \overline{v}_i^{s^{DS}},\ \forall\, i \in I^{DS} \tag{1h}$$

$$\underline{P}_{ij}^{DS} \le P_{ij}^{DS} \le \overline{P}_{ij}^{DS},\ \forall\, (ij) \in K^{DS} \tag{1i}$$

$$\underline{Q}_{ij}^{DS} \le Q_{ij}^{DS} \le \overline{Q}_{ij}^{DS},\ \forall\, (ij) \in K^{DS} \tag{1j}$$

$$\{v_i^{s^{DS}}, \theta_i^{DS}, P_{ij}^{DS}, Q_{ij}^{DS}, P_i^{DS}, Q_i^{DS}\} \in \mathbb{R} \tag{1k}$$

where,

$$\nabla P_{ij}^{DS}|_0 = \begin{pmatrix} (1 - \frac{v_{j,0}^{DS} cos\theta_{ij,0}^{DS}}{2v_{i,0}^{DS}}) g_{ij}^{DS} - \frac{v_{j,0}^{DS} b_{ij}^{DS} sin\theta_{ij,0}^{DS}}{2v_{i,0}^{DS}} \\ \frac{-v_{i,0}^{DS} g_{ij}^{DS} cos\theta_{ij,0}^{DS}}{2v_{j,0}^{DS}} - \frac{v_{i,0}^{DS} b_{ij}^{DS} sin\theta_{ij,0}^{DS}}{2v_{j,0}^{DS}} \\ v_{i,0}^{DS} v_{j,0}^{DS} g_{ij}^{DS} sin\theta_{ij,0}^{DS} - v_{i,0}^{DS} v_{j,0}^{DS} b_{ij}^{DS} cos\theta_{ij,0}^{DS} \\ -v_{i,0}^{DS} v_{j,0}^{DS} g_{ij}^{DS} sin\theta_{ij,0}^{DS} + v_{i,0}^{DS} v_{j,0}^{DS} b_{ij}^{DS} cos\theta_{ij,0}^{DS} \end{pmatrix}$$

$$\nabla Q_{ij}^{DS}|_0 = \begin{pmatrix} -(1 - \frac{v_{j,0}^{DS} cos\theta_{ij,0}^{DS}}{2v_{i,0}^{DS}}) b_{ij}^{DS} - \frac{v_{j,0}^{DS} g_{ij}^{DS} sin\theta_{ij,0}^{DS}}{2v_{i,0}^{DS}} \\ \frac{v_{i,0}^{DS} b_{ij}^{DS} cos\theta_{ij,0}^{DS}}{2v_{j,0}^{DS}} - \frac{v_{i,0}^{DS} g_{ij}^{DS} sin\theta_{ij,0}^{DS}}{2v_{j,0}^{DS}} \\ -v_{i,0}^{DS} v_{j,0}^{DS} b_{ij}^{DS} sin\theta_{ij,0}^{DS} - v_{i,0}^{DS} v_{j,0}^{DS} g_{ij}^{DS} cos\theta_{ij,0}^{DS} \\ v_{i,0}^{DS} v_{j,0}^{DS} b_{ij}^{DS} sin\theta_{ij,0}^{DS} + v_{i,0}^{DS} v_{j,0}^{DS} g_{ij}^{DS} cos\theta_{ij,0}^{DS} \end{pmatrix}$$

$$\theta_{ij,0}^{DS} = \theta_{i,0}^{DS} - \theta_{j,0}^{DS}\ \forall (ij) \in K^{DS}$$

Equations (1b) and (1c) are the first-order Taylor series expansion for the active and reactive power flow on each branch (based out of the full AC-OPF equations [(3) and (4) of [30]] which preserve all the components of branch flow). Equations (1d) and (1e) represent the active and reactive power balance at each bus. The symbol $P_i^{DS}$ is the injected power at each distribution bus which equals generation minus load consumption. Since the load consumption is considered a parameter, the shown upper and lower boundaries for $P_i^{DS}$ represent the boundaries of generation units. Using $P_i^{DS}$ notation makes the problem formulation more concise. Constraints (1i) and (1j), represent the linearization of the quadratic apparent line flow limits $(P_{ij}^{DS})^2 + (Q_{ij}^{DS})^2 \le (S_{ij}^{DS})^2$ (see [30]–[32]).

The transmission systems have a very low R:X ratio. Due to this, we ignore the loss terms in the transmission network. The fast-decoupled load flow assumption is considered to decouple active power flow from $v^s$ terms and reactive power flow from $\theta_{ij}$ terms. These approximations (along with voltage angle approximations are discussed in [30]) lead to a simplified AC-OPF model for TSO as follows:

$$\underset{\{v_b^{s^{TS}}, \theta_b^{TS}, P_{bc}^{TS}, Q_{bc}^{TS}, P_b^{TS}, Q_b^{TS}\}}{\text{Minimize}} \sum_{b \in I^{TS}} f_b(P_{g_b}^{TS}) \tag{2a}$$

subject to:

$$P_{bc}^{TS} \approx -B_{bc}^{TS} \theta_{bc}^{TS},\ \forall (bc) \in K^{TS} \tag{2b}$$

$$Q_{bc}^{TS} \approx -B_{bc}^{TS} (\frac{v_b^{s^{TS}} - v_c^{s^{TS}}}{2}),\ \forall (bc) \in K^{TS} \tag{2c}$$

$$P_b^{TS} = \sum_{bc \in K^{TS}} P_{bc}^{TS} + v_b^{s^{TS}} \sum_{c \in I^{TS}} G_{bc}^{TS}, \forall b \in I^{TS} \tag{2d}$$

$$Q_b^{TS} = \sum_{bc \in K^{TS}} Q_{bc}^{TS} - v_b^{s^{TS}} \sum_{c \in I^{TS}} B_{bc}^{TS}, \forall b \in I^{TS} \tag{2e}$$

$$\underline{P}_b^{TS} \le P_b^{TS} \le \overline{P}_b^{TS},\ \forall b \in I^{TS} \tag{2f}$$

$$\underline{Q}_b^{TS} \le Q_b^{TS} \le \overline{Q}_b^{TS},\ \forall b \in I^{TS} \tag{2g}$$

$$\underline{v}_b^{s^{TS}} \le v_b^{s^{TS}} \le \overline{v}_b^{s^{TS}},\ \forall b \in I^{TS} \tag{2h}$$

$$\underline{P}_{bc}^{TS} \le P_{bc}^{TS} \le \overline{P}_{bc}^{TS},\ \forall (bc) \in K^{TS} \tag{2i}$$

$$\underline{Q}_{bc}^{TS} \le Q_{bc}^{TS} \le \overline{Q}_{bc}^{TS},\ \forall (bc) \in K^{TS} \tag{2j}$$

$$\{v_b^{s^{TS}}, \theta_{bc}^{TS}, P_{bc}^{TS}, Q_{bc}^{TS}, P_b^{TS}, Q_b^{TS}\} \in \mathbb{R} \tag{2k}$$

where, $\theta_{bc}^{TS} = \theta_b^{TS} - \theta_c^{TS},\ \forall (bc) \in K^{TS}$

The above OPF problems for TSO and DSO systems are generic formulations appropriate for each system operator individually. The objective functions (1a) and (2a) could be any convex cost function depending on the application. In next section, we develop these OPF formulations for our TSO-DSO coordination problem with our particular objective functions and additional constraints.

## IV. Optimization Procedure

We assume that PV panels, wind turbines, battery storage systems, and demand response resources are at the service providers' disposal which means that the service providers embed all corresponding constraints and cost functions into their submitted offers. We also assume that thermal units are the only flexibility service providers at the TSO level and they are allowed to send linear offer functions. Their submitted offer functions may or may not represent the actual cost function of the flexibility service providers. The TSO-DSO coordination problem has an inherent and natural asymmetrical and imperfect information property. We address this imperfect information issue using one of the following optimization models.

In the first approach, we assume DSOs and EERs publish enough data such that the TSO can properly anticipate their activities. Consequently, we propose the exogenous DSO modeling in the TSO optimization problem. The TSO predicts the required information for its optimization model from public published data of DSOs and EERs (and perhaps some agreements with them) and solves its optimization problem. In this approach, the burden of the coordination is put on TSO and TSO requires dealing with data collection and estimation, forecasting techniques, and perhaps employing stochastic programming techniques.

In the second approach, we assume limited published data such that the required information cannot be estimated by TSO. Consequently, we propose the embedded DSO model and an organizational setup based on the new FMO organization. The FMO is regulated and solves our TSO-DSO coordination model through its interface optimizer. It communicates with TSO, DSOs, and EERs to collect the required information and solves our optimization models. In our proposed model, FMO has access only to the submitted linear offer functions. The actual constraints, costs, and interests of flexibility service providers are confidential.

### A. *Optimization with Exogenous DSO Activations*

In this model, the DSO flexibility activation is a stochastic exogenous parameter and thus the amount of anticipated DSO flexibility activation is deducted from the total available for the TSO. The anticipation is based on previous knowledge of net energy transactions between the TSO and DSO. This knowledge includes renewable energy output as well as the requested energy from the TSO to DSO and vice versa. Consequently, the optimization problem is LA-MI mixed-integer linear programming as follows:

$$\underset{\Omega_1}{\text{Minimize}} \sum_{t\in RW}\sum_{b\in I^{TS}} f_b(P^{TS}_{g_{bt}})+ \sum_{t\in RW}\sum_{b\in I^{TS}}\sum_{d\in N_{db}} F^{DWT}_{dbt}\pi^{DW}_{dbt} + \sum_{t\in RW}\sum_{b\in I^{TS}}\sum_{u\in N_{ub}} F^{UPT}_{ubt}\pi^{UP}_{ubt} \tag{3a}$$

subject to (2b), (2c), (2e)-(2k), and:

$$P^{TS}_{g_{bt}} - P^{TS}_{L_{bt}} - \sum_{u\in N_{ub}} F^{UPT}_{ubt} + \sum_{d\in N_{db}} F^{DWT}_{dbt} = \sum_{(bc)\in K^{TS}} P^{TS}_{bct} + v^{s^{TS}}_{bt} \sum_{c\in I^{TS}} G^{TS}_{bc} \;, \quad \forall\, b \in I^{TS} \;, \;\; t \in RW \tag{3b}$$

$$P^{TS}_{L_{pt}} = \tilde{P}^{DS}_{g_{pt}} + \sum_{u\in N_{up}} \tilde{F}^{UPD}_{upt} - \sum_{d\in N_{dp}} \tilde{F}^{DWD}_{dpt} \;, \quad \forall\, p \in PCC \;, t \in RW \tag{3c}$$

$$Q^{TS}_{L_{pt}} = \tilde{Q}^{DS}_{g_{pt}} \;, \; \forall\, p \in PCC \;, \; t \in RW \tag{3d}$$

$$f_b(P^{TS}_{g_{bt}}) = \tilde{a}_{1,b} P^{TS}_{g_{bt}} + \tilde{a}_{0,b} \;, \; \forall\, b \in I^{TS} \;, \; t \in RW \tag{3e}$$

$$(F^{UPT}_{ubt} + \tilde{F}^{UPD}_{ubt}) \leq \alpha^{UP}_{ubt}(\Phi^{PV}_{ubt} + \Phi^{WT}_{ubt} + \Phi^{BA}_{ubt}) \;, \quad \forall\, b \in I^{TS} \;, \; u \in N_{ub}, \; t \in RW \tag{3f}$$

$$(F^{DWT}_{dbt} + \tilde{F}^{DWD}_{dbt}) \leq \alpha^{DW}_{dbt}(\Phi^{DR}_{dbt} + \Phi^{BA}_{dbt}) \;, \quad \forall\, b \in I^{TS} \;, \; d \in N_{db} \;, \; t \in RW \tag{3g}$$

$$P^{TS}_{g_{bt}} - P^{TS}_{g_{b(t-1)}} \leq \beta^{UP}_{bt} RU_b \;, \; \forall\, b \in I^{TS}, t \in RW \tag{3h}$$

$$-P^{TS}_{g_{bt}} + P^{TS}_{g_{b(t-1)}} \leq \beta^{DW}_{bt} RD_b, \forall b \in I^{TS}, t \in RW \tag{3i}$$

$$0 \leq \beta^{UP}_{bt} \perp \beta^{DW}_{bt} \geq 0 \;, \;\; \forall\, b \in I^{TS} \;, \; t \in RW \tag{3j}$$

$$0 \leq \alpha^{UP}_{ibt} \perp \alpha^{DW}_{ibt} \geq 0 \;, \quad \forall\, b \in I^{TS} \;, \; i \in I^{DS}_b \;, \; t \in RW \tag{3k}$$

$$\{\beta^{UP}_{bt}, \beta^{DW}_{bt}, \alpha^{UP}_{ubt}, \alpha^{DW}_{dbt}\} \in \{0,1\}, \quad \{F^{UPT}_{ubt}, F^{DWT}_{dbt}\} \in \mathbb{R}_{\geq 0} \tag{3l}$$

Set of variables $\Omega_1 = \{v^{s^{TS}}_{bt}, \theta^{TS}_{bt}, P^{TS}_{g_{bt}}, Q^{TS}_{g_{bt}}, P^{TS}_{bct}, Q^{TS}_{bct}, P^{TS}_{L_{pt}}, P^{TS}_{bt}, Q^{TS}_{bt}, \alpha^{UP}_{ubt}, \alpha^{DW}_{dbt}, \beta^{UP}_{bt}, \beta^{DW}_{bt}, F^{DWT}_{dbt}, F^{UPT}_{ubt}\}$. The objective function (3a) aims to minimize the system operation cost including generation cost and the cost of flexibility service activation. The active power balance at each node is expressed by (3b). The problem (3) is subject to constraint (2e) which represents the reactive power balance. The linking constraints between the TSO and DSO, constraints (3c) and (3d), represent that the load consumption at transmission buses connected to the distribution systems (point of common coupling) should be equal to the generated power at the slack bus of the distribution network. The estimated cost function of generators is shown by (3e). The ramping capabilities of generation units are described in (3h) and (3i). As mentioned earlier, PV and wind generation are turn-up service providers, DR aggregators are turn-down service providers, and battery storage systems provide both services. Binary variables $\alpha^{DW}_{dbt}$ and $\alpha^{UP}_{ubt}$ represent the state of the EERs activation at each distribution system bus with an EER flexibility service provider ("u/d") while the corresponding distribution system is located at the transmission bus "b". Variables $F^{DWT}_{dbt}$ and $F^{UPT}_{ubt}$ express the amount of the activated services by TSO. Accordingly, constraint (3f) represents the maximum available turn-up services at each distribution bus, which equals the summation of the validated submitted bids for providing turn-up service by the owner of PV, battery, and wind generation. Constraint (3g) expresses that the maximum available turn-down service is equal to the summation of the validated submitted bids of battery storage systems and DR aggregators. At the TSO level, the EERs compete with operating reserves to provide flexibility in the ancillary service market. There is a difference between spinning, non-spinning, and supplemental reserve generators from the cost function viewpoint. Since all types of operating reserves including the spinning reserve as well as the non-spinning or supplemental reserves, which have different start-up costs and minimum up/down time constraints, are assumed to be capable of providing flexibility services, the binary variables $\beta^{UP}_{bt}$ and $\beta^{DW}_{bt}$ are employed in constraints (3h) and (3i) to model the state of the reserve generation units activation. Constraints (3j) and (3k) express that turn-up and turn-down services, for both generation units and EERs, should not be activated at each bus simultaneously. Fig. 2 depicts the coupling constraints and the scheme of the proposed exogenous DSO activation model.

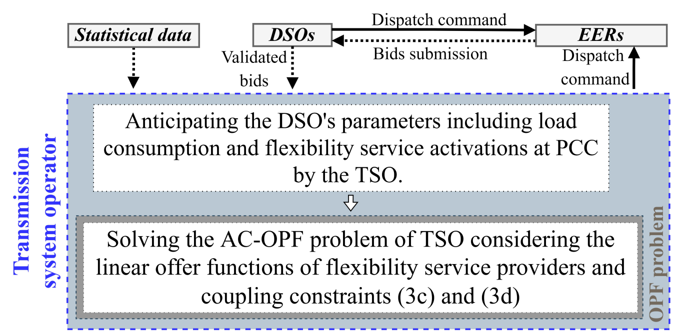


Fig. 2. The proposed TSO-DSO framework for the exogenous DSO activation model.

The Standard Benders Decomposition (SBD) is applicable to solve this formulation. In this version, the LA-MI optimization problem is decomposed into a master problem which selects the optimal TSO nodes for activating DSO flexibility and a sub-problem which decides on the optimal system operation levels. It implies that binary variables, $\alpha_{ubt}^{UP}$, $\alpha_{dbt}^{DW}$, $\beta_{bt}^{UP}$, and $\beta_{bt}^{DW}$ are complicating variables that are going to be optimized in the master problem whilst the rest of the variables are optimized through the sub-problem. Iterating between master problem and sub-problem through Benders cuts solves the original problem.

### B. Optimization with Embedded DSO Activations

From the optimization procedure viewpoint, it is assumed that the FMO has access to all the technical and historical data of the transmission and distribution networks. For such information, some monitoring devices (such as PMU and micro-PMU) are required in both transmission and distribution networks or alternatively the FMO can gain such information through agreements directly from TSO and DSOs. Besides, there might be some data which are needed to be directly anticipated by the FMO itself. With this perspective, the FMO embeds the DSO's prospective optimization within the TSO's real-time dispatch modeling. Consequently, this situation forms a bilevel LA-MI mixed-logical linear program as follows.

*1) Upper Level:* The upper level of this model is as follows.

$$\begin{aligned}
&\underset{\Omega_2}{\text{Minimize}} \sum_{t\in RW}\sum_{b\in I^{TS}} f_b(P_{g_{bt}}^{TS}) + \\
&\sum_{t\in RW}\sum_{b\in I^{TS}}\sum_{d\in N_{db}} F_{dbt}^{DWT}\pi_{dbt}^{DW} + \\
&\sum_{t\in RW}\sum_{b\in I^{TS}}\sum_{u\in N_{ub}} F_{ubt}^{UPT}\pi_{ubt}^{UP} \qquad (4a)
\end{aligned}$$

subject to (2b), (2c), (2e)-(2k), (3h)-(3l), and:

$$\begin{aligned}
&P_{g_{bt}}^{TS} - P_{L_{bt}}^{TS} - \sum_{u\in N_{ub}}(F_{ubt}^{UPT} + F_{ubt}^{UPD}) \\
&+ \sum_{d\in N_{db}}(F_{dbt}^{DWT} + F_{dbt}^{DWD}) = \sum_{(bc)\in K^{TS}} P_{bct}^{TS} \\
&+ v_{bt}^{s^{TS}} \sum_{c\in I^{TS}} G_{bc}^{TS} \;, \forall\, b \in I^{TS}, t \in RW \qquad (4b)
\end{aligned}$$

$$P_{L_{pt}}^{TS} = P_{g_{pt}}^{DS} \;, \forall\, p \in PCC,\ t \in RW \qquad (4c)$$

$$Q_{L_{pt}}^{TS} = Q_{g_{pt}}^{DS} \;, \forall\, p \in PCC\ ,\ t \in RW \qquad (4d)$$

$$v_{pt}^{s^{TS}} = v_{pt}^{s^{DS}} \;, \forall\, p \in PCC\ ,\ t \in RW \qquad (4e)$$

$$\theta_{pt}^{TS} = \theta_{pt}^{DS} \;, \forall\, p \in PCC\ ,\ t \in RW \qquad (4f)$$

$$\begin{aligned}
&(F_{ubt}^{UPT} + F_{ubt}^{UPD}) \le \alpha_{ubt}^{UP}(\Phi_{ubt}^{PV} + \Phi_{ubt}^{WT} + \Phi_{ubt}^{BA}) \;, \\
&\forall\, b \in I^{TS}\ ,\ u \in N_{ub},\ t \in RW \qquad (4g)
\end{aligned}$$

$$\begin{aligned}
&(F_{dbt}^{DWT} + F_{dbt}^{DWD}) \le \alpha_{dbt}^{DW}(\Phi_{dbt}^{DR} + \Phi_{dbt}^{BA}) \;, \\
&\forall\, b \in I^{TS}\ ,\ d \in N_{db}\ ,\ t \in RW \qquad (4h)
\end{aligned}$$

where, $\Omega_2$={$\Omega_1$, $Q_{L_{pt}}^{TS}$, $F_{dbt}^{DWD}$, $F_{ubt}^{UPD}$, $P_{g_{pt}}^{DS}$, $Q_{g_{pt}}^{DS}$, $\theta_{pt}^{DS}$, $v_{pt}^{s^{DS}}$}. The conditions of TSO-DSO connection are modeled by (4c)-(4f). Here, variables $P_{g_{pt}}^{DS}$, $Q_{g_{pt}}^{DS}$, $\theta_{pt}^{DS}$, $v_{pt}^{s^{DS}}$, $F_{dbt}^{DWD}$, and $F_{ubt}^{UPD}$ are determined through following lower-level optimization. Constraints (4g) and (4h) represent the maximum available turn-up and turn-down service activation, respectively. In the embedded DSO activation model, since it is assumed that reserve generators and the EERs embed all cost functions into the submitted offer functions, there is no need to consider binary variables $\beta$ for defining the operation state of reserve generators. However, due to the mathematical concept of implication in the inference dual theorem used in the logic-based Benders decomposition, we need to assign binary variables $\alpha$ and $\beta$ to the flexibility services again. The binary variables are necessary since we cannot use the concept of implication for continuous variables in the inference dual theorem which is based on Boolean logic.

*2) Lower Level:* The lower-level optimization problem is formulated as follows which represents the objective functions of all DSOs.[1]

$$\begin{aligned}
&\underset{\Omega_3}{\text{Minimize}} \sum_{t\in RW}\sum_{b\in I^{TS}}\sum_{u\in N_{ub}} F_{ubt}^{UPD}\pi_{ubt}^{UP} \\
&+ \sum_{t\in RW}\sum_{b\in I^{TS}}\sum_{d\in N_{db}} F_{dbt}^{DWD}\pi_{dbt}^{DW} \qquad (5a)
\end{aligned}$$

subject to (1b)*, (1c)*, (1e)*-(1k)*, and:

$$\begin{aligned}
&P_{g_{ibt}}^{DS} - P_{L_{ibt}}^{DS} + PV_{ibt} + WT_{ibt} + \\
&F_{ibt}^{DWT} - F_{ibt}^{UPT} + F_{ibt}^{DWD} - F_{ibt}^{UPD} = \\
&\sum_{(ij)\in K_b^{DS}} P_{ijbt}^{DS} + v_{ibt}^{s^{DS}} \sum_{j\in I_b^{DS}} G_{ijb}^{DS} \;, \\
&\forall\, b \in I^{TS}\ ,\ i \in I_b^{DS}\ ,\ t \in RW \qquad (5b)
\end{aligned}$$

$$\begin{aligned}
&F_{ubt}^{UPD} \le (\Phi_{ubt}^{PV} + \Phi_{ubt}^{WT} + \Phi_{ubt}^{BA}) \;, \\
&\forall\, b \in I^{TS}\ ,\ u \in N_{ub},\ t \in RW \qquad (5c)
\end{aligned}$$

$$\begin{aligned}
&F_{dbt}^{DWD} \le (\Phi_{dbt}^{DR} + \Phi_{dbt}^{BA}) \;, \\
&\forall\, b \in I^{TS}\ ,\ d \in N_{db}\ ,\ t \in RW \qquad (5d)
\end{aligned}$$

where, $\Omega_3$={$v_{ibt}^{s^{DS}}$, $\theta_{ibt}^{DS}$, $P_{ijbt}^{DS}$, $Q_{ijbt}^{DS}$, $P_{ibt}^{DS}$, $Q_{ibt}^{DS}$, $F_{ibt}^{UPD}$, $F_{ibt}^{DWD}$}. The objective function (5a) minimizes the total cost of flexibility service activation in each rolling window with four intervals for each DSO which is formulated based on the submitted linear offer function of flexibility service providers. A new power balance with the impact of TSO and DSO flexibility service activation is considered in constraint (5b). Constraints (5c) and (5d) represent the maximum available turn-up and turn-down service activation, respectively. For solving the optimization problem of the embedded DSO model, firstly, the lower level is replaced by its Karush-Kuhn-Tucker (KKT) optimality conditions as follows.

*3) Reformulated Mathematical Program with Complementarity Constraints:* This step reformulates the bilevel LA-MI optimization problem into a single-level LA-MI optimization problem:

$$\begin{aligned}
&\underset{\Omega_4}{\text{Minimize}} \sum_{t\in RW}\sum_{b\in I^{TS}} f_b(P_{g_{bt}}^{TS}) + \\
&\sum_{t\in RW}\sum_{b\in I^{TS}}\sum_{d\in N_{db}} F_{dbt}^{DWT}\pi_{dbt}^{DW} + \qquad (6a) \\
&\sum_{t\in RW}\sum_{b\in I^{TS}}\sum_{u\in N_{ub}} F_{ubt}^{UPT}\pi_{ubt}^{UP}
\end{aligned}$$

[1]Constraints (1b)*, (1c)*, (1e)*, and (1k)* are similar to (1b), (1c), (1e), and (1k), respectively, where in the * constraints, the time is added using index "$t$" and the location of each distribution system in the interconnected transmission-distribution system is considered using the index "$b$". For instance, $P_i^{DS}$ is replaced by $P_{ibt}^{DS}$.

subject to (1b)*, (1c)*, (1e)*, (1k)*, (2b)-(2c), (2e)-(2k), (4b), (3h)-(3l), (4c)-(4h), (5b)-(5d), and:

$$\{\lambda, \Pi, \mu, \delta\} \in \mathbb{R}, \ \{D\} \in \mathbb{R}_{\ge 0} \tag{6b}$$

$$\lambda_{ibt} + D_{3,ibt} - D_{4,ibt} = 0 \ , \quad \forall\, b \in I^{TS} \ , \ i \in I_b^{DS} \ , \ t \in RW \tag{6c}$$

$$\Pi_{ibt} + D_{5,ibt} - D_{6,ibt} = 0 \ , \quad \forall\, b \in I^{TS} \ , \ i \in I_b^{DS} \ , \ t \in RW \tag{6d}$$

$$\begin{aligned} &\Pi_{ibt}\sum\nolimits_{j\in I_b^{DS}} B_{ijb}^{DS} - \lambda_{it}\sum\nolimits_{j\in I_b^{DS}} G_{ijb}^{DS} \\ &+D_{1,ibt} - D_{2,ibt} + \sum\nolimits_{(ij)\in K_b^{DS}} \mu_{ijbt}\Big(-\frac{\partial P_{ijbt}^{DS}}{\partial v_{ibt}^{DS}}\Big) \\ &+\sum\nolimits_{(ij)\in K_b^{DS}} \delta_{ijbt}\Big(-\frac{\partial Q_{ijbt}^{DS}}{\partial v_{ibt}^{DS}}\Big) = 0 \ , \forall\, b \in I^{TS}, \\ &i \in I_b^{DS} \ , \ \ t \in RW \end{aligned} \tag{6e}$$

$$\begin{aligned} &\sum\nolimits_{(ij)\in K_b^{DS}} \mu_{ijbt}\Big(-\frac{\partial P_{ijbt}^{DS}}{\partial \theta_{ibt}^{DS}}\Big) \\ &+\sum\nolimits_{(ij)\in K_b^{DS}} \delta_{ijbt}\Big(-\frac{\partial Q_{ijbt}^{DS}}{\partial \theta_{ibt}^{DS}}\Big) = 0, \ \forall i \in I_b^{DS}, t \in RW \end{aligned} \tag{6f}$$

$$S_B \pi_{pt}^{UP} - \lambda_{upt} + D_{11,upt} = 0 \ , \quad \forall\, p \in PCC, \ u \in N_{up} \ , \ t \in RW \tag{6g}$$

$$S_B \pi_{pt}^{DW} + \lambda_{dpt} + D_{12,dpt} = 0 \ , \quad \forall\, p \in PCC \ , \ d \in N_{dp} \ , \ t \in RW \tag{6h}$$

$$0 \le D_{1,ibt} \perp (\overline{v}_{ib}^{s^{DS}} - v_{ibt}^{s^{DS}}) \ge 0, \quad \forall\, b \in I^{TS}, \ i \in I_b^{DS}, \ t \in RW \tag{6i}$$

$$0 \le D_{2,ibt} \perp (v_{ibt}^{s^{DS}} - \underline{v}_{ib}^{s^{DS}}) \ge 0, \quad \forall\, b \in I^{TS}, \ i \in I_b^{DS}, \ t \in RW \tag{6j}$$

$$0 \le D_{3,ibt} \perp (\overline{P}_{g_{ib}}^{DS} - P_{g_{ibt}^{DS}}) \ge 0, \quad \forall\, b \in I^{TS}, \ i \in I_b^{DS}, \ t \in RW \tag{6k}$$

$$0 \le D_{4,ibt} \perp (P_{g_{ibt}^{DS}} - \underline{P}_{g_{ib}}^{DS}) \ge 0, \quad \forall\, b \in I^{TS}, \ i \in I_b^{DS}, \ t \in RW \tag{6l}$$

$$0 \le D_{5,ibt} \perp (\overline{Q}_{ijb}^{DS} - Q_{ijbt}^{DS}) \ge 0, \quad \forall\, b \in I^{TS}, \ (ij) \in K_b^{DS}, \ t \in RW \tag{6m}$$

$$0 \le D_{6,ibt} \perp (Q_{ijbt}^{DS} - \underline{Q}_{ijb}^{DS}) \ge 0, \quad \forall\, b \in I^{TS}, \ (ij) \in K_b^{DS}, \ t \in RW \tag{6n}$$

$$0 \le D_{7,ijbt} \perp (\overline{P}_{ijb}^{DS} - P_{ijbt}^{DS}) \ge 0, \quad \forall\, b \in I^{TS}, \ (ij) \in K_b^{DS}, \ t \in RW \tag{6o}$$

$$0 \le D_{8,ijbt} \perp (P_{ijbt}^{DS} - \underline{P}_{ijb}^{DS}) \ge 0, \quad \forall\, b \in I^{TS}, \ (ij) \in K_b^{DS}, \ t \in RW \tag{6p}$$

$$0 \le D_{9,ijbt} \perp (\overline{Q}_{ijb}^{DS} - Q_{ijbt}^{DS}) \ge 0, \quad \forall\, b \in I^{TS}, \ (ij) \in K_b^{DS}, \ t \in RW \tag{6q}$$

$$0 \le D_{10,ijbt} \perp (Q_{ijbt}^{DS} - \underline{Q}_{ijb}^{DS}) \ge 0, \quad \forall\, b \in I^{TS}, \ (ij) \in K_b^{DS}, t \in RW \tag{6r}$$

$$0 \le D_{11,ubt} \perp (\Phi_{ubt}^{PV} + \Phi_{ubt}^{WT} + \Phi_{ubt}^{BA} - F_{ubt}^{UPD}) \ge 0 \ , \quad \forall\, b \in I^{TS}, \ u \in N_{ub} \ , \ t \in RW \tag{6s}$$

$$0 \le D_{12,dbt} \perp (\Phi_{dbt}^{DR} + \Phi_{dbt}^{BA} - F_{dbt}^{DWD}) \ge 0, \quad \forall\, b \in I^{TS}, \ d \in N_{db} \ , \ t \in RW \tag{6t}$$

$$0 \le F_{ubt}^{UPD} \perp F_{dbt}^{DWD} \ge 0, \quad \forall\{b \in I^{TS}, u \in N_{ub}, \ d \in N_{db}, \ t \in RW | u = d\} \tag{6u}$$

where, $\Omega_4$=$\{\Omega_1, \ \Omega_2, \ \Omega_3, \ D, \ \lambda, \ \Pi, \ \mu, \ \delta\}$. Constraints (6c)-(6h) represent the derivative of the Lagrangian function with respect to the optimization variables in the DSO model. Constraints (6i)-(6t) express the complementary slackness conditions for each inequality constraint in the lower level (DSO model). Constraints (6u) represents that turn-up and turn-down services should not be activated at one node of distribution network simultaneously. The whole structure of the proposed organizational setup involving the FMO organization is shown in Fig.3.

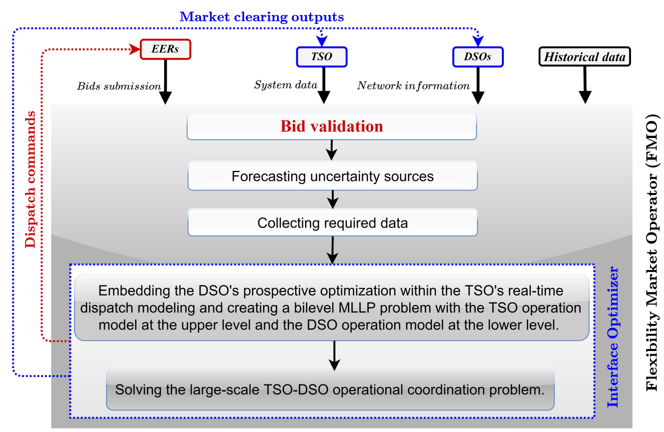


Fig. 3. The proposed organizational setup based on the FMO coordinating organization.

## C. Generalized Disjunctive Programming (GDP) Model

In this step, constraints including (3j), (3k), and (6i)-(6u) are reformulated as disjunctive inequalities.[2]

$$\underset{\Omega_4}{\text{Minimize}} \text{ Objective Function of (6a)} \tag{7a}$$

subject to (1b)*,(1c)*,(1e)*,(1k)*,(2b),(2c),(2e)-(2k),(4b),(3h)-(3l),(4c)-(4h),(5b)-(5d),(6b)-(6h), and the equivalent disjunctive form of constraints (6i)-(6u) as follows:

$$[D_{1,ibt} \le 0] \vee [\overline{v}_{ib}^{s^{DS}} - v_{ibt}^{s^{DS}} \le 0] \ , \quad \forall\, b \in I^{TS} \ , \ i \in I_b^{DS}, \ t \in RW \tag{7b}$$

$$[D_{2,ibt} \le 0] \vee [v_{ibt}^{s^{DS}} - \underline{v}_{ib}^{s^{DS}} \le 0], \quad \forall\, b \in I^{TS} \ , \ i \in I_b^{DS}, \ t \in RW \tag{7c}$$

Similarly for (6k)-(6u) (7d)

[2] $\vee$ : *Logical disjunction (OR)*

where, constraints (7b) and (7c) show the disjunctive equivalent of (6i) and (6j), respectively. Constraints (7d) express the same disjunctive equivalents for constraint (6k)-(6u).

One of the most common methods for finding the solution to disjunctive programming problems is to utilize the Big-M method to reformulate the problem as a mixed-integer linear/nonlinear programming model. In optimization problem (7), the Big-M method can be used as in (8) below to linearize the disjunctive inequality (7b):

$$D_{1,ibt} \leq M_{1,ibt}(1 - Y_{1,ibt}), \quad \forall\, b \in I^{TS},\, i \in I_b^{DS},\, t \in RW \tag{8a}$$

$$\overline{v}_{ib}^{s^{DS}} - v_{ibt}^{s^{DS}} \leq M_{2,ibt}(1 - Y_{2,ibt}), \quad \forall\, b \in I^{TS},\, i \in I_b^{DS},\, t \in RW \tag{8b}$$

$$\sum\nolimits_{k=1}^{2} Y_{k,ibt} = 1\,,\; Y_{k,ibt} \in \{1, 0\}\,, \quad \forall\, b \in I^{TS},\, i \in I_b^{DS},\, t \in RW \tag{8c}$$

Since solving optimization problems using the Big-M method with a large number of variables and complementary conditions is an NP-hard problem [33], these Big-M based models tend to lead to weak continuous relaxations and turn out to be unsolvable in practice [34]. Consequently, a logic-based model is proposed to reformulate the disjunctive inequalities. Boolean variables and propositional logic are considered in the optimization problem. The following represents the logical form of disjunctive inequalities (7b) and (7c).[3]

$$\begin{bmatrix} Y_{3,ibt} \\ D_{1,ibt} \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{3,ibt} \\ \overline{v}_{ib}^{s^{DS}} - v_{ibt}^{s^{DS}} \leq 0 \end{bmatrix}, \quad \forall\, b \in I^{TS},\, i \in I_b^{DS},\, t \in RW \tag{9a}$$

$$\begin{bmatrix} Y_{4,ibt} \\ D_{2,ibt} \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{4,ibt} \\ v_{ibt}^{s^{DS}} - \underline{v}_{ib}^{s^{DS}} \leq 0 \end{bmatrix}, \quad \forall\, b \in I^{TS},\, i \in I_b^{DS},\, t \in RW \tag{9b}$$

$$\neg Y_{3,ibt} \veebar \neg Y_{4,ibt},\; \forall\, b \in I^{TS},\, i \in I_b^{DS},\, t \in RW \tag{9c}$$

$$\begin{aligned} &\neg Y_{3,ibt} \implies Y_{4,ibt}\,, \\ &\neg Y_{4,ibt} \implies Y_{3,ibt}\,, \forall\, b \in I^{TS},\, i \in I_b^{DS}, t \in RW \end{aligned} \tag{9d}$$

Logical disjunction inequalities (9a) and (9b) represent that the equations corresponding to true Boolean variables are added to the optimization problem. Constraints (9c)-(9d) explain the logical propositions associated with constraints (9a) and (9b). For instance, in the logic proposition (9c), $\neg Y_{3,ibt}$ implies $\overline{v}_{ib}^{s^{DS}}$=$v_{ibt}^{s^{DS}}$. On the other side, $\neg Y_{4,ibt}$ implies $v_{ibt}^{s^{DS}}$=$\underline{v}_{ib}^{s^{DS}}$. Consequently, $\neg Y_{3,ibt}$ and $\neg Y_{4,ibt}$ cannot be true simultaneously. In the proposed logic-based model, all disjunctives are handled using the indicator constraints method and MIP solver CPLEX in GAMS platform [35].

[3] $\iff$ : *If and only if*, $\veebar$ : *Exclusive disjunction (XOR)*, $\implies$ : *Mathematical implication*

The following is the new form of optimization (7) which expresses and exploits the inherent logic structure of the optimization problem.

$$\begin{aligned} \underset{\Omega_5}{\text{Minimize}} &\sum\nolimits_{t\in RW}\sum\nolimits_{b\in I^{TS}} f_b(P_{g_{bt}}^{TS}) + \\ &\sum\nolimits_{t\in RW}\sum\nolimits_{b\in I^{TS}}\sum\nolimits_{d\in N_{db}} F_{dbt}^{DWT}\pi_{dbt}^{DW} + \\ &\sum\nolimits_{t\in RW}\sum\nolimits_{b\in I^{TS}}\sum\nolimits_{u\in N_{ub}} F_{ubt}^{UPT}\pi_{ubt}^{UP} \end{aligned} \tag{10a}$$

subject to (1b)*,(1c)*,(1e)*,(1k)*, (2b), (2c), (2e)-(2k), (4b), (3h)-(3l), (4c)-(4h), (5b)-(5d), (6b)-(6h), (9a)-(9d), and:

$$\begin{bmatrix} Y_{1,bt} \\ \beta_{bt}^{UP} \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{1,bt} \\ \beta_{bt}^{DW} \leq 0 \end{bmatrix}, \; \forall\, b \in I^{TS},\; t \in RW \tag{10b}$$

$$\begin{bmatrix} Y_{2,ibt} \\ \alpha_{ibt}^{UP} \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{2,ibt} \\ \alpha_{ibt}^{DW} \leq 0 \end{bmatrix}, \forall b \in I^{TS}, i \in I_b^{DS}, t \in RW \tag{10c}$$

$$\begin{bmatrix} Y_{5,ibt} \\ D_{3,ibt} \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{5,ibt} \\ \overline{P}_{g_{ib}}^{DS} - P_{g_{ibt}^{DS}} \leq 0 \end{bmatrix}, \quad \forall b \in I^{TS}, i \in I_b^{DS}, t \in RW \tag{10d}$$

$$\begin{bmatrix} Y_{6,ibt} \\ D_{4,ibt} \leq 0 \end{bmatrix} \vee \begin{bmatrix} \neg Y_{6,ibt} \\ P_{g_{ibt}^{DS}} - \underline{P}_{g_{ib}}^{DS} \leq 0 \end{bmatrix}, \quad \forall b \in I^{TS}, i \in I_b^{DS}, t \in RW \tag{10e}$$

$$\neg Y_{5,ibt} \veebar \neg Y_{6,ibt}, \quad \forall\, b \in I^{TS}, i \in I_b^{DS},\; t \in RW \tag{10f}$$

$$\begin{aligned} &\neg Y_{5,ibt} \implies Y_{6,ibt}\,, \\ &\neg Y_{6,ibt} \implies Y_{5,ibt}\,,\; \forall\, b \in I^{TS},\; i \in I_b^{DS},\; t \in RW \end{aligned} \tag{10g}$$

Similarly for (6m)-(6u) (10h)

$$Y_{k,ibt} \in \{False, True\}, \quad k = 1, ..., 15 \tag{10i}$$

where, the set of variables $\Omega_5$={$\Omega_4$,$Y_{k,ibt}$}. Due to the non-convexity of plausible sub-problems, obtaining the optimality cuts using the classical Benders decomposition method is challenging. As mentioned in the Introduction, the standard Benders decomposition method cannot solve this type of optimization programming with the non-convexity in the sub-problem. Accordingly, the Logic-Based Benders Decomposition (LBBD) method is introduced as follows.

### *D. Logic-Based Benders Decomposition Method*

In our proposed LBBD method, the master problem represents the relationship between the value of the binary complicating variables and the objective functions of the cut calculation process. The sub-problem optimizes the rest of the variables while the complicating variables are considered exogenous parameters. Then, at the cut calculation process, all possible combinations of binary complicating variables are enumerated while each combination implies particular values for sub-problem variables. An illustrative example is presented in the Appendix to make the LBBD process clearer.

Accordingly, the GDP problem (10) can be solved by our proposed LBBD method. Binary variables related to the TSO turn-up and turn-down service activations are considered as complicating variables (sub-set $N_b^{mv}$) to form the master problem. The sub-problem is formed to obtain the optimal nodes for activating DSO flexibility and dispatch levels. This process is shown in Fig.4. Accordingly, the sub-problem, master problem, and the cut calculation process are formulated as follows.

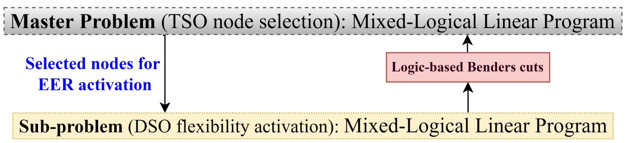


Fig. 4. The logic-based Benders decomposition for solving the embedded DSO model.

*1) Sub-Problem:* In the optimization problem (10), we define the complicating variables $\alpha_{ut}^{UP}$ and corresponding logical indicators $Y_{2,it}$ as exogenous parameters $\hat{\alpha}_{ut}^{UP}$ and $\hat{Y}_{2,it}$, respectively. These assumptions form the sub-problem as follows.

$$\underset{\Omega_5}{\text{Minimize}} \text{ Objective Function of (10a)} \tag{11a}$$

subject to (1b)*, (1c)*,(1e)*, (1k)*, (2b), (2c), (2e)-(2k), (4b), (3h)-(3l), (4c)-(4h), (5b)-(5d), (6b)-(6h), (9a)-(9d), (10b), (10d)-(10i), and:

$$\alpha_{ubt}^{UP} = \hat{\alpha}_{ubt}^{UP}, \quad \forall\, b \in I^{TS},\ u \in N_b^{mv},\ t \in RW \tag{11b}$$

$$Y_{2,it} = \hat{Y}_{2,it}, \quad \forall\, i \in I_b^{DS},\ t \in RW \tag{11c}$$

Solving the sub-problem (11) determines the upper bound of the original optimization problem (10). If the difference between the lower and upper bounds is less than a pre-defined tolerance, the process is finished.

*2) Master Problem:* The master problem determines the optimal value of complicating variables using the concept of inference dual. Before defining the inference dual concept, we introduce the notion of implication with respect to a domain $D$.

**Definition 1.** *We assume that $A$ and $B$ are two propositions about $x \in D$ where $D$ is the domain of $x$. We say, $A$ implies $B$ with respect to $D$ (noted $A \xrightarrow{D} B$), if $B$ is true for any $x \in D$ for which $A$ is true [26].*

Now we can define the concept of inference dual as follows:

**Definition 2.** *For general optimization problem $P_1 = \{Minimize\ f(x):\ x \in S,\ x \in D\}$ with feasibility set $S$ and domain set $D$, the inference dual is the problem of inferring the possible tightest lower bound on the optimal value of the objective function $f(x)$ from the constraints ($x \in S$). The inference dual of above optimization problem is as follows: $P_2 = \{Maximize\ \psi:\ x \in S \xrightarrow{D} f(x) \geq \psi\}$.*

Using above definitions we have the strong inference duality theorem as follows:

**Theorem 1.** *Strong inference duality: The optimization problem $P_1$ has the same optimal value as its inference dual problem $P_2$.*

*Proof.* Let $\psi^*$ be the optimal value of $P_1$. It is obvious that $x \in S$ implies $f(x) \geq \psi^*$, this in turn shows that the optimal value of the dual is at least $\psi^*$. The optimal value of dual cannot be greater than $\psi^*$, because this would mean that $f(x) = \psi^*$ cannot be achieved in $P_1$ for any feasible $x$. If $P_1$ is infeasible, then is $P_2$ unbounded with optimal value $\infty$. If $P_1$ is unbounded, then $P_2$ is infeasible with optimal value $-\infty$ [36]. □

Applying Theorem 1 to our problem leads to following optimization problem:

$$\underset{\Omega_6}{\text{Maximize}}\ W_t \tag{12a}$$

$$\text{subject to: } \Omega_5 \in f_S \xrightarrow{\text{D}} OF_{(11a)} \geq W_t, \forall\, t \in RW \tag{12b}$$

where, $f_S$ and $D$ are the feasibility and domain sets of sub-problem (11), respectively. Symbols $W_t$ and $RW$ are the tightest lower bound and set of rolling windows, respectively. $OF$ stands for objective function.

In this regard, the proposed master problem is presented as follows.[4]

$$\underset{g_{rnt}, \hat{G}_{ubrnt}, \alpha_{urnt}^{UP}, Z_t}{\text{Minimize}} \quad \sum\nolimits_{t \in RW} Z_t \tag{13a}$$

subject to:

$$Z_t \geq \Gamma_{rnt} g_{rnt}\ , \forall\, n \in \{1,..,2^{N_b^{mv}}\}, r \in IT\ ,\ t \in RW \tag{13b}$$

$$\begin{aligned} &g_{rnt} = 1 \iff \bigwedge_{b=1}^{I^{TS}} \bigwedge_{u=1}^{N_{ub}} \hat{G}_{ubrnt}\ , \\ &\quad \forall\, n \in \{1,..,2^{N_b^{mv}}\}, r \in IT\ ,\ t \in RW \end{aligned} \tag{13c}$$

$$\begin{aligned} &\hat{G}_{ubrnt} \iff \alpha_{ubrnt}^{UP} = 1\ ,\ \forall\, u \in N_b^{mv}\ , \\ &\quad b \in I^{TS}\ ,\ n \in \{1,..,2^{N_b^{mv}}\}, r \in IT\ ,\ t \in RW \end{aligned} \tag{13d}$$

$$\begin{aligned} &\neg\hat{G}_{ubrnt} \iff \alpha_{ubrnt}^{UP} = 0\ ,\ \forall\, u \in N_b^{mv}\ , \\ &\quad b \in I^{TS}\ ,\ n \in \{1,..,2^{N_b^{mv}}\}, r \in IT\ ,\ t \in RW \end{aligned} \tag{13e}$$

$$g \in \{0,1\}, \hat{G} \in \{False, True\} \tag{13f}$$

The calculated objective function in this step determines the lower bound of optimization problem. Constraint (13b) represents the Benders cuts which are calculated based on the achieved optimal value of the current sub-problem. Constraint (13c) explains the predefined logical proposition related to the different combinations of complicating variables which determines the selected nodes for EER activation at each transmission and distribution bus. At each iteration, only one combination is selected. Constraints (13d) and (13e) explain that each combination of the Boolean variables implies specific service activation.

[4] $\wedge$ : *Logical conjunction*

*3) Benders Cut Calculation:* The following optimization problem calculates the LBBD cut for flexibility service activation. The problem is solved for $2^{N_b^{mv}}$ iterations corresponding to all combinations of a subset of complicating variables ($N_b^{mv} \subseteq N_{ub}$). At each iteration, the optimal value of the objective function $\Gamma_{rnt}$ is the tightest bound to the master problem which is related to the specific combination of complicating variables ${\alpha_{ubrnt}^{UP}}^*$.

$$\underset{\Omega_7}{\text{Minimize}} \quad \Gamma_{rnt} = \sum\nolimits_{b\in I^{TS}} f_b(P_{g_{brnt}}^{TS}) + \sum\nolimits_{b\in I^{TS}} \sum\nolimits_{d\in N_{db}} F_{dbrnt}^{DWT} \pi_{dbt}^{DW} + \sum\nolimits_{b\in I^{TS}} \sum\nolimits_{u\in N_b^{mv}} F_{ubrnt}^{UPT} \pi_{ubt}^{UP} \ , \quad \forall\ n \in \{1,..,2^{N_b^{mv}}\},\ t \in RW,\ \ r \in IT \tag{14a}$$

subject to (1b)*, (1c)*, (1e)*, (1k)*, (2b), (2c), (2e)-(2k), (4b), (3h)-(3l), (4c)-(4h), (5b)-(5d), (6b)-(6h), (9a)-(9d), (10b), (10d)-(10i), and:

$$\alpha_{ubrnt}^{UP} = {\alpha_{ubrnt}^{UP}}^* \ , \forall\, b \in I^{TS}\ ,\ u \in N_b^{mv}, \quad t \in RW,\ n \in \{1,..,2^{N_b^{mv}}\}, r \in IT \tag{14b}$$

$$\alpha_{ibrnt}^{DW} = \{\hat{\alpha}_{ibrnt}^{DW} | {\alpha_{ibrnt}^{UP}}^* \implies [\hat{\alpha}_{ibrnt}^{DW}] \wedge [\hat{Y}_{2,ibrnt}]\}, \quad \forall\, b \in I^{TS}, i \in I_b^{DS}, t \in RW, n \in \{1,..,2^{N_b^{mv}}\},\ \ r \in IT \tag{14c}$$

$$Y_{ibrnt} = \{\hat{Y}_{ibrnt} | {\alpha_{ibrnt}^{UP}}^* \implies \hat{Y}_{ibrnt}\}\ , \quad \forall\, b \in I^{TS}, i \in I_b^{DS}, t \in RW, n \in \{1,..,2^{N_b^{mv}}\},\ \ r \in IT \tag{14d}$$

$$D_{ijbrnt} = \{\hat{D}_{ijbrnt} | {\alpha_{ibrnt}^{UP}}^* \implies \hat{D}_{ijbrnt}\}\ , \quad \forall\, b \in I^{TS}, i \in I_b^{DS}, (ij) \in K_b^{DS}, t \in RW, \quad n \in \{1,..,2^{N_b^{mv}}\},\ \ r \in IT \tag{14e}$$

$$F_{ibrnt}^{UPD} = \{\hat{F}_{ibrnt}^{UPD} | {\alpha_{ibrnt}^{UP}}^* \implies \hat{Y}_{15,ibrnt}\}\ , \quad \forall\, b \in I^{TS}, i \in I_b^{DS}, t \in RW, n \in \{1,..,2^{N_b^{mv}}\},\ \ r \in IT \tag{14f}$$

$$F_{ibrnt}^{DW} = \{\hat{F}_{ibrnt}^{DW} | {\alpha_{ibrnt}^{UP}}^* \implies [\hat{Y}_{14,ibrnt}] \wedge [\hat{Y}_{15,ibrnt}]\}\ , \quad \forall\, b \in I^{TS}, i \in I_b^{DS}, t \in RW, n \in \{1,..,2^{N_b^{mv}}\},\ \ r \in IT \tag{14g}$$

$$Q_{g_{ibrnt}} = \{\hat{Q}_{g_{ibrnt}} | {\alpha_{ibrnt}^{UP}}^* \implies [\hat{Y}_{7,ibrnt}] \wedge [\hat{Y}_{8,ibrnt}]\}, \quad \forall\, b \in I^{TS}, i \in I_b^{DS}, t \in RW, n \in \{1,..,2^{N_b^{mv}}\},\ \ r \in IT \tag{14h}$$

$$Q_{ijbrnt} = \{\hat{Q}_{ijbrnt} | {\alpha_{ibrnt}^{UP}}^* \implies [\hat{Y}_{11,ijbrnt}] \wedge [\hat{Y}_{12,ijbrnt}]\},\ \forall\, b \in I^{TS},\ i \in I_b^{DS}, \quad (ij) \in K_b^{DS}, t \in RW, n \in \{1,..,2^{N_b^{mv}}\},\ r \in IT \tag{14i}$$

$$\delta_{dbrnt} = \hat{\delta}_{dbrnt}, \forall\, b \in I^{TS},\ d \in N_d, \quad t \in RW, r \in IT, n \in \{1,..,2^{N_b^{mv}}\} \tag{14j}$$

$$\theta_{brnt}^{TS} = \hat{\theta}_{brnt}^{TS}\ , \quad \forall\, b \in I^{TS}\ ,\ t \in RW\ , \quad r \in IT\ ,\ n \in \{1,..,2^{N_b^{mv}}\} \tag{14k}$$

Where $\Omega_7 = \{\Gamma_{rnt},\ P_{bcrnt}^{TS},\ P_{brnt}^{TS},\ P_{ibrnt}^{DS},\ P_{ijbrnt}^{DS},\ v_{ibrnt}^{s^{DS}},\ F_{ubt}^{UPT}\}$. Constraints (14b) represents the selected combination of $\alpha_{ubrnt}^{UP}$. Constraints (14c)-(14k) express the exogenous parameters in the sub-problem (11). The whole process of the LBBD method is presented in **Algorithm 1** and Fig. 5.

**Algorithm 1** Logic-based Benders Decomposition

Take $\alpha_{ubrnt}^{UP}$ as complicating variables, r$\leftarrow$ 1
$\alpha_{ubrnt}^{UP} \leftarrow \alpha_{ubrnt}^{UP(0)}$(initial guess) , $LB \leftarrow$-$\infty$ , $UB \leftarrow \infty$
**while** $UB - LB \geq \epsilon$ **do**
  **Step1: Master problem (MLLP)**
    **if** $r \geq 2$ **then**
      Solve the master problem (13) considering $\Gamma_{rnt}$
      $(\hat{\alpha}_{ubrnt}^{UP} \leftarrow \alpha_{ubrnt}^{UP})$ , $(LB \leftarrow new\ LB)$
    **end if**
  **Step2: Sub-problem (MLLP)**
    Solve the sub-problem (11)
    $UB \leftarrow new\ UB$ , $\overline{\Omega_6} \leftarrow \Omega_6$ (save all optimal values)
  **Step3: Logic-based Benders cut calculation**
    **for** $n \leftarrow 1$ to $2^{N_b^{mv}}$ **do**
      **if** $n \geq 2$ **then**
        $g_{r(n-1)t} \leftarrow 0$
      **end if**
      $g_{rnt} \leftarrow 1 \iff \bigwedge_{b=1}^{I^{TS}} \bigwedge_{u=1}^{N_b^{mv}} \hat{G}_{ubrnt}$
      $(\hat{G}_{ubrnt} \iff \alpha_{ubrnt}^{UP} \leftarrow 1)$ , $({\alpha_{ubrnt}^{UP}}^* \leftarrow \alpha_{ubrnt}^{UP})$
      Solve the optimization problem (14)
      $\Gamma_{(r+1)nt} \leftarrow \Gamma_{rnt}$
    **end for** $r \leftarrow r + 1$
**end while**

In summary, our proposed LBBD has the following two strong properties compared to the SBD:

- The proposed LBBD can handle situations where the sub-problem has both logical propositions and binary variables. This is performed using the concept of inference dual we introduced in the current paper.
- Since, we do not reformulate the disjunctive constraints using the Big-M method, the disjunctive parameters (or big-Ms) do not appear in our LBBD. Hence, there is no need to tune these parameters (which is very hard task [33], if not impossible [34]).

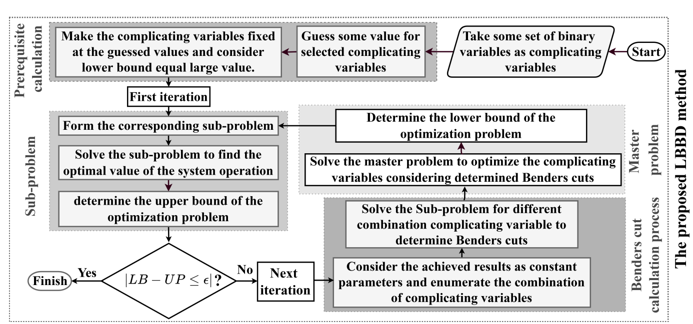


Fig. 5. The process of determining Benders cuts in our LBBD method.

## V. Case Studies

### A. Test System

A modified IEEE 118-bus test system is applied as the transmission system that is connected to two modified IEEE 33-bus test systems as distribution networks (at bus No.102 and bus No.109). Three different case studies assess the proposed framework, and with sensitivity variations on load consumption and renewable generation. In all case studies, it is assumed that forward market have cleared so that the objective of the TSO-DSO coordination framework is to manage the real-time uncertainties using the available flexibility services offered by market aggregators. Table I shows the expected amount of available flexibility sources in two different distribution systems for a six-hour interval. The battery storage systems are only considered at $DSO_B$. Hourly delivery periods are assumed so that the available sources for flexibility service activation are obtained by multiplying the values of Table I by the profiles in Fig. 6. The service activation bids by EERs in each DSO at each hour are shown in Fig. 7. The bids of EERs in $DSO_A$ and $DSO_B$ relate to the turn-up and turn-down services, respectively. The MIP solver CPLEX [35] in the GAMS platform was used, whilst the computer had Intel Core i7-8650U (2.11 GHz), and 16G RAM.

TABLE I
Expected Amount of Flexibility Services in the Case Studies

| *Distribution network* | *Node* | *Turn-Down* | *Turn-up* |
|---|---|---|---|
| $DSO_A$ | No.07 | $DR_1(0.2MW)$ | $WT_1(10.5MW)$ |
| | No.08 | $DR_2(0.2MW)$ | $PV_1(9MW)$ |
| | No.24 | - | $PV_2(9MW)$ |
| | No.30 | $DR_3(0.2MW)$ | $WT_2(9MW)$ |
| | No.32 | - | $PV_3(9MW)$ |
| $DSO_B$ | No.07 | $-$ | $PV_1(9MW)$ |
| | No.18 | $BA_1(5MW)$ | $BA_1(5MW)$ |
| | No.22 | $BA_2(7MW)$ | $BA_2(7MW)$ |
| | No.24 | $DR_1(1.52MW)$ | $PV_2(9MW)$ |
| | No.25 | $DR_2(1.42MW)$ | $PV_3(9MW)$ |
| | No.32 | $DR_3(0.8MW)$ | $WT1(13.5MW)$ |
| | No.33 | $BA_3(5MW)$ | $BA_3(5MW)$ |

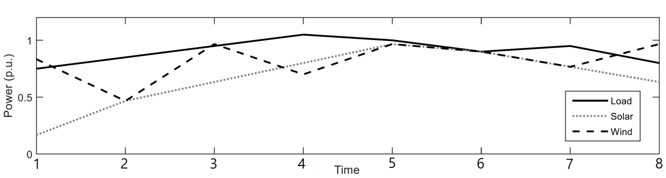


Fig. 6. Profile used in the case studies.

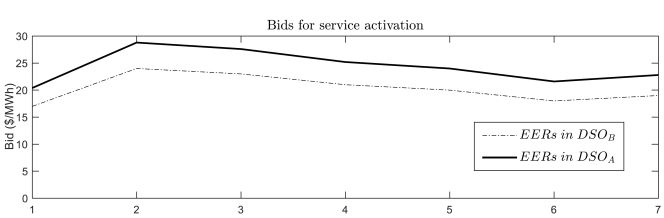


Fig. 7. Bids for service activation in different case studies.

Our proposed model takes 96, 95, and 100 seconds to find the solutions to the optimization problems of Case I, Case II, and Case III, respectively. Nevertheless, the convergence and the number of iterations of the LBBD method depends on the initial guesses relating to the complicating variables. Since each LBBD cut is calculated individually and independently, parallel computing techniques can be employed. Therefore, one of the main advantages of our developed LBBD method is that parallel computing can easily be employed in the computation process and this, in turn, decreases the computation time for large-scale optimization problems significantly.

*1) Case I (Unexpected Load Increment):* In this case, the average load consumption of $DSO_B$ is increased by 20 percent. This case is defined only to evaluate the proposed methodology. Evidently, the different percentages of load increments have different impacts on the results. Both TSO and DSO need to mitigate the resulting congestion and voltage problem associated with unexpected load increases. Therefore, our proposed framework mitigates these problems and fulfills the signed contracts in the forward electricity market by activating available EERs.

*2) Case II (Unexpected Surplus Renewable Energy Generation):* In this case, the scheduled renewable energy generation in the forward market is available but the actual load consumption becomes lower than the expected value due to the load decrease in the $DSO_A$. Therefore, there is an unexpected surplus of renewable energy generation. The EERs are capable of providing turn-up services to overcome the congestion, energy imbalance, and voltage problems in both TSO and DSO regions. In the exogenous DSO activation model, the maximum amount of available service equals the scheduled renewable generation in the forward market which is shown by constraints (3f) and (3g). Consequently, at each hour, the upper limit of available turn-up service is derived from the profile in Fig. 6 using the values in Table I.

*3) Case III (Conflict Among TSO and DSO Objectives):* This case deals with a situation in $DSO_B$ that the load consumption is unpredictably increased at buses No.31 and No.33, and it is decreased at buses No.24 and No.25. Therefore, there are surplus renewable energy generation and load increments simultaneously. The TSO and DSO procure flexibility services to overcome the congestion, energy imbalance, and voltage problems in their regions. Compared to Table I, the available DR at bus No.32 is increased to $2.8MW$. Moreover, we consider different offer functions for flexibility service provider in Case III to show the efficiency of our model. Accordingly, we multiply the presented bids at each hour in Fig. 7 by a random parameter, with uniformly distributed values between 0.9 and 1, to generate different offer functions for each individual flexibility service provider. Therefore, all results related to Case III are based on consideration of different offer functions for flexibility service providers.

In the TSO optimization with exogenous DSO activations, it is envisaged that the TSO would seek in practice to use empirical data on past activations by the DSO to have a statistical model of DSO responses in the TSO's look-ahead model. We can only make illustrative assumptions on this aspect. In Case I, it is anticipated that the $DSO_B$ activates 40 percent of available flexibility resources. In Case II, it is predicted that $DSO_A$ activates 90 percent of available flexibility resources. In Case III, it is forecasted that $DSO_B$ activates 75 percent of turn-up services and 40 percent of turn-down services. These are the exogenous DSO service activations in the optimization models of both the LA-MI and LA-SI frameworks. Evidently, these are arbitrary values simply to test the model formulations. How well TSOs can anticipate DSO activations and how much transparency they have on the distribution system will necessarily be situation specific.

### B. Results and Discussions

The LA-MI and LA-SI frameworks are applied to each case study. Two optimization models are implemented in each framework being the SBD (for the exogenous DSO model), and the LBBD (for the embedded DSO model). Consequently, this experimental testing results in four optimization problems comprising two bilevel mixed-logical linear programming problems related to the LA-MI and LA-SI frameworks in the embedded DSO model, and two single-level mixed-integer linear problems related to the exogenous DSO model. Fig. 8 depicts the optimization problems used in this paper. Six hours of system operations are taken into account to evaluate the proposed models and frameworks.

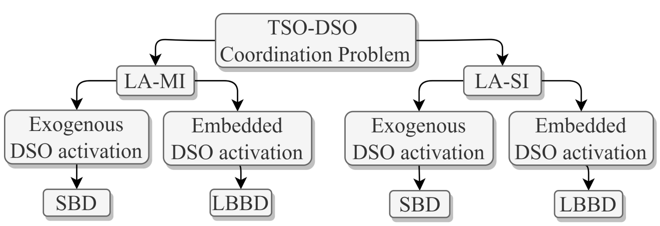


Fig. 8. The flowchart of all proposed optimization problems in the current paper.

Regarding the market mechanism in [1], the EERs compete with available operating reserves to provide flexibility services at the TSO level. Therefore, along with the available EERs in DSOs, there is one operating reserve unit at transmission bus No.1 that competes with EERs to provide flexibility services in the transmission system.

In the process of solving the mixed-logical linear programming problem using the LBBD method, the binary variables related to the turn-up flexibility service activation by TSO in the binding interval (from available turn-up services $\alpha_{ut}^{UP}$ in $DSO_B$) are considered as complicating variables. In the case studies, the number of complicating variables in the binding interval is equal to four, being related to the turn-up services by three PV generation units along with one wind farm in $DSO_B$. Therefore, there are 16 combinations of complicating variables, which implies 16 Benders cuts in our proposed LBBD methodology. Table II shows the logic propositions related to the combination of the considered complicating variables. These combinations are represented by logic propositions in the sub-problem. In each iteration of the LBBD method, after solving the sub-problem, sixteen constraints are added to the master problem based on the value of the optimized objective function in the sub-problem.

TABLE II
LOGIC PROPOSITION FOR DEFINING BENDERS CUTS IN THE PROPOSED LBBD METHOD

| ***Benders Cuts (Combination)*** | ***The location of turn up services at $DSO_B$*** | | | |
|---|---|---|---|---|
| | *Bus No.7* | *Bus No.23* | *Bus No.25* | *Bus No.32* |
| CUT 1 | NOT | NOT | NOT | NOT |
| CUT 2 | AND | NOT | NOT | NOT |
| CUT 3 | NOT | AND | NOT | NOT |
| CUT 4 | NOT | NOT | AND | NOT |
| CUT 5 | NOT | NOT | NOT | AND |
| CUT 6 | AND | AND | NOT | NOT |
| CUT 7 | AND | NOT | AND | NOT |
| CUT 8 | AND | NOT | NOT | AND |
| CUT 9 | NOT | AND | AND | NOT |
| CUT 10 | NOT | AND | NOT | AND |
| CUT 11 | NOT | NOT | AND | AND |
| CUT 12 | AND | AND | AND | NOT |
| CUT 13 | AND | AND | NOT | AND |
| CUT 14 | AND | NOT | AND | AND |
| CUT 15 | NOT | AND | AND | AND |
| CUT 16 | AND | AND | AND | AND |

*1) Results Regarding System Operation Cost:* Table III shows the system operation costs for the four optimization problems in Case I. Sign ”>” represents that the total operation cost is greater than the right-hand side value due to the fact that there are infeasible solutions in some hours. Evidently, the system operation cost in the LA-MI framework is lower than the LA-SI framework in both optimizations. Moreover, the cost of system operation in the second (embedded) DSO model is lower than the first (exogenous) DSO model in each framework. Evidently, the absolute value of this benefit is an artefact of the modelling assumptions and the results are only meant to be indicative of the feasibility of the methodology.

TABLE III
SYSTEM OPERATION COST FOR CASE I

| ***Binding Interval*** | ***Exogenous DSO model*** | | ***Embedded DSO model*** | |
|---|---|---|---|---|
| | *LA-SI* | *LA-MI* | *LA-SI* | *LA-MI* |
| $t_1$ | Infeasible | 72413 ($/h) | 74635 ($/h) | 72378 ($/h) |
| $t_2$ | 87554 ($/h) | 82200 ($/h) | 87373 ($/h) | 82134 ($/h) |
| $t_3$ | 100198 ($/h) | 92564 ($/h) | 99869 ($/h) | 92502 ($/h) |
| $t_4$ | Infeasible | 84989 ($/h) | Infeasible | 84964 ($/h) |
| $t_5$ | 76424 ($/h) | 76290 ($/h) | 76320 ($/h) | 76268 ($/h) |
| $t_6$ | 88098 ($/h) | 87426 ($/h) | 88006 ($/h) | 87386 ($/h) |
| **Total** | **>511873($)** | **495882($)** | **>511167($)** | **495632($)** |

When the LA-MI framework is implemented in both day-ahead and real-time markets, all forward contracts have been achieved with the available flexibility services. But, on the contrary, there are some hours when it is impossible to mitigate the congestion and fulfill all forward contracts with the available flexibility sources when the LA-SI framework is implemented. It implies that even when using all turn-down service activations, it is impossible to cope with the unexpected load increments. In practice, this would lead to a resource adequacy intervention by the TSO, possibly with disconnections.

Table IV depicts the system operation costs for the four optimization problems in Case II. In this case study, the

TSO hedges against the congestion and energy imbalance problems in the transmission network while DSO copes with voltage and congestion problems due to the unexpected energy surplus in the distribution network. Therefore, the TSO and DSO compete with each other to activate the available turn-up services to eliminate the surplus energy and relieve their network problems. Similar to Case I, the LA-MI framework has lower operational costs in both optimization models. The difference between the total operational cost of the proposed LA-MI and LA-SI frameworks are significant.

TABLE IV
SYSTEM OPERATION COST FOR CASE II

| *Binding Interval* | *Exogenous DSO model* | | *Embedded DSO model* | |
|---|---|---|---|---|
| | *LA-SI* | *LA-MI* | *LA-SI* | *LA-MI* |
| $t_1$ | 74893 ($/h) | 73194 ($/h) | 74865 ($/h) | 73153 ($/h) |
| $t_2$ | 87109 ($/h) | 84111 ($/h) | 86829 ($/h) | 83927 ($/h) |
| $t_3$ | 99833 ($/h) | 94472 ($/h) | 98740 ($/h) | 94320 ($/h) |
| $t_4$ | 89342 ($/h) | 86490 ($/h) | 88183 ($/h) | 86468 ($/h) |
| $t_5$ | 77820 ($/h) | 77306 ($/h) | 77786 ($/h) | 77272 ($/h) |
| $t_6$ | 89573 ($/h) | 88178 ($/h) | 89551 ($/h) | 88131 ($/h) |
| **Total** | **518570($)** | **503751($)** | **515954($)** | **503271($)** |

Table V depicts the system operation costs for the four optimization problems in Case III. From the operational cost point of view, Case III is similar to Case I and Case II. DSO activates both turn-up and turn-down services to mitigate the congestion, voltage, and energy imbalance problems for different parts of its network. TSO copes with the voltage and congestion problems in its network by activating turn-down services. Again these results are consistent with engineering intuition.

TABLE V
SYSTEM OPERATION COST FOR CASE III

| *Binding Interval* | *Exogenous DSO model* | | *Embedded DSO model* | |
|---|---|---|---|---|
| | *LA-SI* | *LA-MI* | *LA-SI* | *LA-MI* |
| $t_1$ | 73988 ($/h) | 74270 ($/h) | 73948 ($/h) | 74193 ($/h) |
| $t_2$ | Infeasible | 84539 ($/h) | Infeasible | 83992 ($/h) |
| $t_3$ | Infeasible | 94897 ($/h) | Infeasible | 94560 ($/h) |
| $t_4$ | Infeasible | 87331 ($/h) | Infeasible | 86793 ($/h) |
| $t_5$ | 78954 ($/h) | 78967 ($/h) | 79009 ($/h) | 77845 ($/h) |
| $t_6$ | 91660 ($/h) | 90320 ($/h) | 89976 ($/h) | 88942 ($/h) |
| **Total** | **$>$511369($)** | **510324($)** | **$>$508278($)** | **506325($)** |

The results in Table III, Table IV, and Table V show that the LA-MI framework in the exogenous DSO model is preferable to LA-SI in both models from both economic and feasibility viewpoints. Furthermore, as mentioned before, the exogenous DSO model is based on what actually occurs in the current flexibility markets. Therefore, although the LA-MI framework in the embedded DSO model has the minimum operation cost, the complexity of this model is much higher than the exogenous DSO model. The computational complexity of this framework is due to the existence of non-convex sub-problems. Nonetheless, using the proposed methods in this paper including GDP and LBBD reduces the computational complexity of the main optimization problem which in turn will make them more feasible. Therefore, we continue to analyze the TSO-DSO coordination with the LA-MI framework using LBBD.

*2) Congestion and Voltage Problems:* Table VI shows the congestion and voltage problems related to Case I before operating the flexibility market using our proposed model. As can be seen, there is no voltage problem in the transmission system. Nevertheless, due to the overloading problem in line no. 175 which connects bus no. 109 to bus no. 110, TSO needs to activate the flexibility sources in the $DSO_B$ to remove the overloading problem. There are both voltage and congestion problems in the distribution system $DSO_B$ due to the increased load consumption in Case I. Consequently, DSO needs to compete with the TSO to activate the available flexibility sources to mitigate its own problems.

TABLE VI
VOLTAGE AND CONGESTION PROBLEMS FOR CASE I

| ***Binding Interval*** | ***Transmission*** | | ***Distribution*** $(DSO_B)$ | |
|---|---|---|---|---|
| | *Voltage (Bus.No)* | *Congestion (Line.No)* | *Voltage (Bus.No)* | *Congestion (Line.No)* |
| $t_1$ | - | L175 | B15 | L22, L25 |
| $t_2$ | - | L175 | B15, B17 | L18, L22, L25 |
| $t_3$ | - | L175 | B15, B17 | L18, L22, L25 |
| $t_4$ | - | L175 | B15, B17, B31 | L18, L22, L25 |
| $t_5$ | - | L175 | B15, B17 | L18, L22, L25 |
| $t_6$ | - | L175 | B15, B17 | L22, L25 |

Table VII illustrates the voltage and congestion problems for Case II before activating any flexibility sources. As can be seen, there is no voltage problem in the transmission system, however, TSO is required to activate flexibility sources to overcome the overloading problem for line no. 162 which connects bus no. 102 to bus no. 92. The DSO has to deal with both bus-voltage and line-congestion problems in its own system for Case II. Therefore, DSO competes with TSO to activate its needed flexibility services.

TABLE VII
VOLTAGE AND CONGESTION PROBLEMS FOR CASE II

| ***Binding Interval*** | ***Transmission*** | | ***Distribution*** $(DSO_B)$ | |
|---|---|---|---|---|
| | *Voltage (Bus.No)* | *Congestion (Line.No)* | *Voltage (Bus.No)* | *Congestion (Line.No)* |
| $t_1$ | - | L161 | B24, B25 | L22, L23 |
| $t_2$ | - | L161 | B7, B24, B25 | L22, L23 |
| $t_3$ | - | L161 | B7, B23, B24, B25 | L22, L23 |
| $t_4$ | - | L161 | B7, B23, B24, B25, B32 | L2, L3, L22, L23 |
| $t_5$ | - | L161 | B7, B23, B24, B25, B32 | L2, L3, L22, L23 |
| $t_6$ | - | L161 | B7, B24, B25 | L3, L22, L23 |

Table VIII depicts the voltage and congestion problems for Case III. As can be seen, TSO has to manage the voltage problems at bus no. 107 and bus no. 112 and the congestion problems of line no. 175. Moreover, both DSO and TSO has voltage and congestion problems in their systems. Therefore, TSO and DSO compete with each other to activate the available flexibility sources to overcome their network problems.

TABLE VIII
VOLTAGE AND CONGESTION PROBLEMS FOR CASE III

| ***Binding Interval*** | ***Transmission*** | | ***Distribution*** $(DSO_B)$ | |
|---|---|---|---|---|
| | *Voltage (Bus.No)* | *Congestion (Line.No)* | *Voltage (Bus.No)* | *Congestion (Line.No)* |
| $t_1$ | B107, B112 | L175 | B23, B32 | L2, L22, L23 |
| $t_2$ | B107, B112 | L175 | B23, B24, B32 | L2, L22, L23 |
| $t_3$ | B107, B112 | L175 | B23, B24, B25, B32 | L2, L3, L18, L22, L23 |
| $t_4$ | B107, B112 | L175 | B23, B24, B25, B32 | L2, L3, L18, L22, L23 |
| $t_5$ | B107, B112 | L175 | B23, B24, B32 | L2, L3, L18,L22, L23 |
| $t_6$ | B107, B112 | L175 | B23, B24, B32 | L3, L22, L23 |

*3) Service Activation:* Table IX and Fig. 9 show the results of solving the logic-based problem related to the LA-MI framework using LBBD in Case I. As can be seen, however, the proposed TSO-DSO coordination has properly adapted to the unexpected variation in the load consumption. The TSO activates the turn-down services in $DSO_B$ to cope with overloading problems in the transmission system. At the same time, $DSO_B$ also activates down-services at its system to cope with the resulting congestion. In this case, all available turn-down services are utilized by the TSO and $DSO_B$. The rest of the unexpected load increment is compensated by the reserve capacity in the transmission system.

TABLE IX
FLEXIBILITY SERVICE ACTIVATION OF LA-MI MODEL FOR CASE I

| ***Binding Interval*** | ***TSO service activation*** | | ***DSO service activation*** | |
|---|---|---|---|---|
| | *Turn up* | *Turn down* | *Turn up* | *Turn down* |
| $t_1$ | - | 6.2 MW | - | 5.2 MW |
| $t_2$ | - | 5.6 MW | - | 8.6 MW |
| $t_3$ | - | 5.2 MW | - | 11.5 MW |
| $t_4$ | - | 5.7 MW | - | 9.5 MW |
| $t_5$ | - | 6.0 MW | - | 6.8 MW |
| $t_6$ | - | 5.4 MW | - | 9.1 MW |

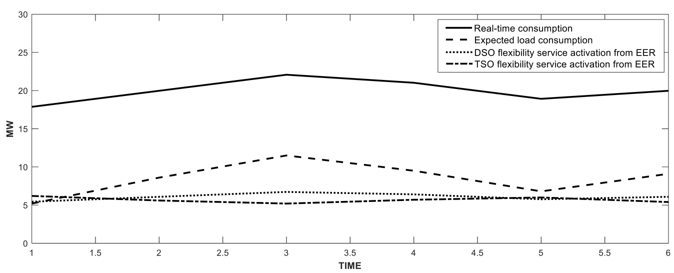


Fig. 9. Service Activation of LA-MI model for Case I.

Table X and Fig. 10 show the results of surplus renewable energy production in Case II. Evidently, the proposed TSO-DSO coordination with the LA-MI framework has properly coped with the unexpected surplus RES generation. TSO has activated available turn-up flexibility services to relieve the overloading problems. In the same way as the TSO, $DSO_A$ has also activated the available turn-up services to cope with the congestion problem inside its distribution network. Most of the available turn-up flexibility services has been activated by $DSO_A$ due to the impact of unexpected surplus RES generation on the distribution network.

TABLE X
FLEXIBILITY SERVICE ACTIVATION OF LA-MI FOR CASE II

| ***Binding Interval*** | ***TSO service activation*** | | ***DSO service activation*** | |
|---|---|---|---|---|
| | *Turn up* | *Turn down* | *Turn up* | *Turn down* |
| $t_1$ | 0.6 MW | - | 23.4 MW | - |
| $t_2$ | 0.9 MW | - | 30.6 MW | - |
| $t_3$ | 0.9 MW | - | 33.6 MW | - |
| $t_4$ | 0.6 MW | - | 42.9 MW | - |
| $t_5$ | 0.6 MW | - | 39.9 MW | - |
| $t_6$ | 1.1 MW | - | 33.4 MW | - |

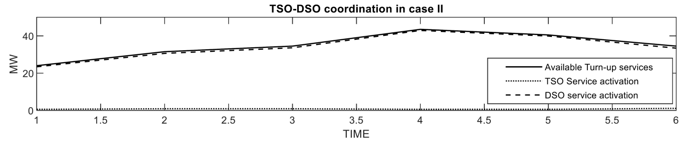


Fig. 10. Service Activation of LA-MI model for Case II.

Table XI and Fig. 11 show the results of both surplus renewable energy production and load increment occurring in Case III. This case demonstrates the ability of the proposed framework to handle the conflict among TSO and $DSO_B$ objectives to activate available EERs. In the third, fourth, and fifth hours of the system operation, $DSO_B$ needs to activate both turn-up and turn-down flexibility services to mitigate the problems in different parts of the $DSO_B$ region. On the other hand, the TSO also activates turn-down service to overcome the voltage and congestion problems in the transmission network. Consequently, the results show that our proposed LA-MI framework with embedded DSO activation can properly handle different simultaneous situations that might arise in the coordination between TSO and DSOs.

TABLE XI
FLEXIBILITY SERVICE ACTIVATION OF LA-MI FOR CASE III

| ***Binding Interval*** | ***TSO service activation*** | | ***DSO service activation*** | |
|---|---|---|---|---|
| | *Turn up* | *Turn down* | *Turn up* | *Turn down* |
| $t_1$ | - | 16.4 MW | 6.2 MW | - |
| $t_2$ | - | 16.6 MW | 6.1 MW | - |
| $t_3$ | - | 18.7 MW | 1.1 MW | 4.2 MW |
| $t_4$ | - | 19.4 MW | 1.5 MW | 2.4 MW |
| $t_5$ | - | 17.0 MW | 0.6 MW | 3.3 MW |
| $t_6$ | - | 22.8 MW | 1.0 MW | - |

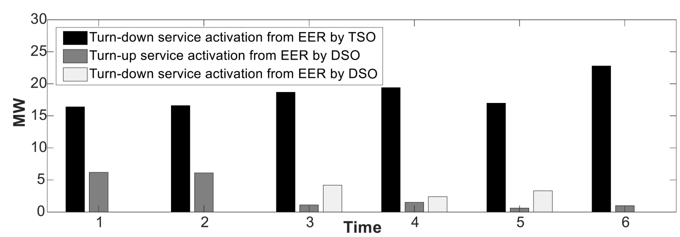


Fig. 11. Service Activation of LA-MI model for Case III.

In this research, the bilevel optimization problems for embedded DSO modeling case have binary variables in their subproblems and accordingly cannot be solved using the SBD. This is while, as reported in our numerical experiments, the proposed LBBD can efficiently solve these bilevel programs.

## VI. CONCLUSION

In this research, an LA-MI framework for coordinating TSO-DSO operational decisions is analyzed, in which, a new organizational setup based on the introduced FMO organization is proposed. Two linearized AC-OPF models are introduced for the transmission and distribution systems to evaluate the coordination and two optimization models are formulated.

A logic-based Benders decomposition method is advocated to solve the resulting bilevel mixed-logical linear programming problem and a new approach to determine the Benders cuts is proposed. The generalized disjunctive programming is used to reformulate the complementary slackness conditions. Instead of the Big-M method, the indicator constraint method is employed to represent all disjunctives in the proposed logic-based model. A comparison of our LA-MI coordination framework with the current LA-SI coordination framework widely used in Europe reveals lower costs and more efficient system operation, as demonstrated through several numerical experiments.

## VII. Future Work

Expanding the LBBD to stochastic bi-level optimization is a new challenge. Consequently, modeling the stochastic behavior of the influential uncertainty sources in a stochastic TSO-DSO coordination problem with the LBBD method is the scope of one important future study, which should improve the system operation and the resilience of both the TSO and DSO for coping with uncertainties.

The coordination approach can also be extended to include renewable energy sources with non-convex cost functions. For such an extension, new binary variables will be introduced into the transmission-level model to model the non-convex cost functions and then the same methodology as proposed in the paper should be applicable.

Finally, we are aware that the practical experiences in [22], [37] indicate that including a sub-problem relaxation in the master problem can significantly reduce the number of Benders iterations. However, determining such a sub-problem relaxation depends on the structure of different optimization problems. Consequently, developing an appropriate algorithm to determine a specific sub-problem relaxation would require a separate piece of research.

## VIII. Appendix: An illustrative example

For an illustration, we apply the LBBD method to a linear binary optimization problem, as follows.

$$\underset{X,Y,Z,I}{\text{Minimize}} \sum_{i=1}^{4} \alpha_i Y_i + \sum_{j=1}^{3} \beta_j X_j + \sum_{k=1}^{2} \gamma_k Z_k \tag{15a}$$

$$\text{subject to: } \sum_{i=1}^{4} Y_i = 1, \; \sum_{j=1}^{3} X_j = 1, \; \sum_{k=1}^{2} Z_k = 1 \tag{15b}$$

$$\begin{bmatrix} I_1 \\ Y_4 \le 0 \end{bmatrix} \vee \begin{bmatrix} \neg I_1 \\ X_1 \le 0 \\ X_2 \le 0 \\ X_3 \le 0 \\ Z_1 \le 0 \end{bmatrix}, \begin{bmatrix} I_2 \\ Z_2 \le 0 \end{bmatrix} \vee \begin{bmatrix} \neg I_2 \\ Y_1 \le 0 \\ Y_2 \le 0 \\ Y_3 \le 0 \end{bmatrix} \tag{15c}$$

$$\begin{bmatrix} I_3 \\ Y_1 \le 0 \end{bmatrix} \vee \begin{bmatrix} \neg I_3 \\ X_2 \le 0 \\ X_3 \le 0 \end{bmatrix}, \begin{bmatrix} I_4 \\ Y_2 \le 0 \end{bmatrix} \vee \begin{bmatrix} \neg I_4 \\ X_1 \le 0 \\ X_3 \le 0 \end{bmatrix} \tag{15d}$$

$$\begin{bmatrix} I_5 \\ Y_3 \le 0 \end{bmatrix} \vee \begin{bmatrix} \neg I_5 \\ X_1 \le 0 \\ X_2 \le 0 \end{bmatrix}, \begin{bmatrix} \{X, Y, Z\} \in \{1, 0\} \\ I \in \{False, True\} \end{bmatrix} \tag{15e}$$

We consider $Y$ as complicating variable to utilize the LBBD method. Consequently, the master problem is as follows.

$$\underset{Y,W}{\text{Minimize}}\ W, \quad \text{subject to: } W \ge \Gamma_{am} Y_i \ , \ \sum_{i=1}^{4} Y_i = 1 \tag{16}$$

There are four complicating variables. Symbol $\Gamma_{am}$ is the logic-based Benders cut coefficient where $m$ is an iteration counter and $a$ is an index for all possible combinations of $Y$. Since the summation of $Y$ should be equal to one, there are four logic-based Benders cuts. We fix the complicating variables $Y$ at $\overline{Y}$, which is found by solving (16), and form the sub-problem (17) below:

$$\underset{X,Y,Z,I}{\text{Minimize}} \text{ Objective Function of (15a)} \tag{17a}$$

$$\text{subject to (15b)-(15e), and } \; Y = \overline{Y}. \tag{17b}$$

At this stage for cut calculation, all possible combinations of the complicating variable $Y$ are considered. Then, for each combination of $Y$, $X$ and $Z$ are considered equal to the optimal values $\overline{X}$ and $\overline{Z}$ (calculated for $Y$ equals to $\overline{Y}$ from (17)) provided that specific combination of $Y$ implies $\overline{X}$ and $\overline{Z}$. Then, $\Gamma_{am}$ is calculated for all possible combinations of complicating variable $Y$ as follows:

$$\Gamma_{am} = \sum_{i=1}^{4} \alpha_i Y_i + \sum_{j=1}^{3} \beta_j X_j + \sum_{k=1}^{2} \gamma_k Z_k \quad , \forall\ a \in \{1, .., 4\}$$

subject to (15b)-(15e)

The master problem and sub-problem are solved iteratively until the difference between their solutions meet the predefined tolerance.

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