DOI: 10.1111/eufm.12405

ORIGINAL ARTICLE

EUROPEAN FINANCIAL MANAGEMENT WILEY

Optimal equity valuation using multiples: The number of comparable firms

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Abstract

We examine how the accuracy of a multiples-based valuation changes as the number of comparable firms used to estimate the valuation multiple increases. Our research is motivated by a contrast between the approach followed by practitioners, who typically use a small number of closely comparable firms, and the academic literature which often uses all firms in an industry. Using a simple selection rule based on growth rates, we find that using 10 closely comparable firms is as accurate on average as using the entire cross-section of firms in an industry. The loss of accuracy from using five comparable firms rather than 10 firms or the entire industry is not great.

KEYWORDS

equity valuation, multiples valuation, valuation

JEL CLASSIFICATION

G11, G24, D81

We are grateful to an anonymous referee and the editor for helpful comments and suggestions. We would also like to thank Leonardo Cordeiro for his valuable research assistance. All errors remain our responsibility.

1 | INTRODUCTION

In this study we examine how the accuracy of a multiples-based valuation changes when the number of comparable firms used to estimate the valuation multiple increases. Our research is motivated by a contrast between the approach commonly followed by practitioners and that generally used in the academic literature. In multiples-based valuation, practitioners generally use a small number of closely comparable firms to estimate the multiple.[1] The academic literature generally uses all firms in an industry as the comparable group. We seek to investigate which of these is optimal. We also examine a number of related issues, such as the method of selecting the small sample, and the variation of accuracy across industries and over time.

Using an entire industry as the comparable group has two main advantages. It does not require the choice of a procedure to select a smaller sample and it uses all the information contained in the multiples of the firms in the industry. However, we show that the use of more information is not necessarily better. If multiples that contain little incremental information about a valuation are given weight at the expense of those that contain the most information, increasing the number of comparable firms can decrease accuracy.

Using a smaller sample has the advantage that it uses only the most relevant information. However, this requires a procedure to select the comparable firms whose share prices are likely to contain the most information about the value of the target firm being valued. A small sample also allows the valuer to exercise judgement about which are the most comparable firms and what weights their multiples should be given. The disadvantage of a small sample is that it may ignore the information contained in the multiples of those firms not contained in the sample.

We show theoretically that for a larger sample to improve a valuation the relative weights given to the multiples of comparable firms must satisfy a particular criterion. For any given valuation procedure, we show that it is not possible to guarantee that this condition will be fulfilled. Therefore, it is not clear theoretically whether a large or small sample will give a more accurate valuation. The issue is, essentially, an empirical one.

To carry out our empirical test we use the method of valuation using multiples that has been found to be optimal in the academic literature. This uses forward earnings as the value driver and the harmonic mean to average the multiples of comparable firms. We vary the number of comparable firms used in the valuation, and measure the accuracy of the resulting value estimate by comparing it with the market price. We select comparable firms based on the absolute difference between the growth rate of the comparable firm and the growth rate of the target firm.

We measure accuracy by the bias, mean absolute deviation (MAD), and mean squared error (MSE) of the value estimates. By all these criteria, we find that using about 10 closely comparable firms is as accurate as using the entire cross-section of firms in an industry. Using five comparable firms is slightly less accurate. However, the loss of accuracy from using five comparable firms rather than 10 firms or the entire industry is not great.

We also examine whether the relative accuracy of a small sample relative to a large sample varies in a systematic way between industries or over time. We find that industry characteristics help only marginally in explaining the relative accuracy of large and small numbers of comparables. What is far more important is the closeness of the growth rates of the comparable

[1]Examination of brokers' reports shows that a typical sample size is 4–6 firms.

firms to that of the target firm. We show that a small number of comparables performs very well when the comparables have a growth rate that is close to the target firm. This suggests that more sophisticated selection and weighting rules may be able to improve the performance of valuations using a small number of comparables even further.

2 | PRIOR RESEARCH

Although using multiples is the most common equity valuation technique among practitioners, it has received limited academic interest. Empirical studies have shown that multiple-based valuation can give similar accuracy to discounted cash flow (DCF) valuation. Kaplan and Ruback (1995) show, for a sample of highly leveraged transactions, that valuation using Earnings before Interest, Tax, Depreciation, and Amortization multiples is as accurate as DCF. Other developments have also demonstrated that there is a strong theoretical basis for using multiples (Feltham & Ohlson, 1995).

Much of the empirical research on valuation using multiples has focused on the optimal value driver and statistical procedure used to estimate the multiple of the target firm. In a comprehensive study of these issues, Liu et al. (2002) (LNT) conclude that the most accurate valuation procedure uses forward earnings as the value driver (see also Kenton, 2004). Kim and Ritter (1999) find a similar result in their investigation of how IPO prices are set. LNT find that the best statistic is the harmonic mean of the price–earnings multiples of comparable firms, as suggested by Baker and Ruback (1999). They also find that using the entire sample of firms in an industry is better than using the entire cross-section of firms in all industries. However, they do not examine the properties of estimates based on samples smaller than all the firms in an industry.

In an early study of the selection of the comparable group, Alford (1992) finds that using all firms in a 3-digit Standard Industrial Classification industry is as good as other selection procedures. The alternatives he examines select the comparable group based on leverage, return on equity, assets, and growth. Of these variables, he finds that the analysts' long-term growth forecast is the fundamental measure that contributes most to valuation accuracy. However, he finds that selecting the comparable group on the basis of forecast growth rates adds little or no predictive accuracy to portfolios of comparable firms formed on the basis of industry.

Using a different approach, Bhojraj and Lee (2002) (BL) show that accuracy in forecasting future multiples can be improved by using a closely matched set of comparable firms. They suggest that their method of selection offers a significant improvement in accuracy over comparable firms selected on the basis of industry. Their approach involves two components: (1) regression analysis of multiples using standard value-drivers as independent variables and (2) the selection of closely comparable firms based on the estimated relationships. The evidence in LNT suggests that regression analysis alone does not improve the accuracy of valuations. Therefore, it seems likely that the extra accuracy in BL's procedure is coming from the information contained in the multiples of a small number of closely comparable firms.

However, because BL's tests involve forecasting future multiples rather than simply examining the accuracy of current valuations, they are not directly comparable with the results in LNT. Our study adds to this literature in three ways. First, we examine theoretically the way that valuation accuracy changes as more comparable firms are added. We show the trade-off involved in increasing the number of comparable firms.

Adopting a different valuation perspective, Bartram and Grinblatt (2018) propose a simple approach to fundamental analysis to approximate a firm's fair equity value as a linear function of almost all of its most recently reported information in financial statements. Similar to our theoretical model, this statistician's approach to valuation is also based on the law of one price and the values obtained are the market values of synthetic stocks or replicating portfolios (as each of the portfolios' fundamental characteristics is identical to those of the firm being valued). They find that the estimation of fair market values in this way leads to trading strategies that can earn abnormal profits.

Gao et al. (2019) develop a hybrid valuation model by combining the advantage principles of multiples-valuation approach and standard discount models. They show that their hybrid model outperforms the price-to-earnings multiple which Liu et al. (2002) find performs remarkably well. Another hybrid approach is given in Cornell and Gokhale (2016), where they extend the usual multiples approach by including the information in the entire term structure of earnings forecasts and find that it improves accuracy.

Second, we show empirically how the number of comparable firms affects valuation accuracy. Across our entire sample we obtain a result that explains the preference of practitioners for relatively small samples of comparable firms. Using the 10 firms in the industry with forecast growth rates most similar to the target firm is as accurate as using the entire industry. Using only five firms selected on the basis of growth rates results in only a small decrease in accuracy. This suggests that the largest gain to be made in the accuracy of valuation using multiples may be in selecting and weighting the evidence in a small number of closely comparable firms, rather than statistical procedures for processing the information in a large sample.

We also show that the incremental accuracy from using an entire industry rather than a small number of comparable firms varies between industries. This suggests that the search for a "one-size-fits-all" procedure for valuation using multiples may not work unless it makes the sample of comparable firms depend on industry characteristics.

3 | METHODOLOGY

3.1 | Estimation of the target value and measurement of pricing errors

We use the procedure that gives the most accurate valuation, according to LNT. Given a target firm i, a date t, and a sample of n comparable firms, $j = 1, \ldots, n$ we use the share prices of the comparable firms on the same date, p_{jt}, and their 2-year out forward earnings, x_{jt}. From these we form the harmonic mean of their price–earnings multiples[2]:

$$\widehat{\beta}_{it}(n) = \frac{n}{\sum_{j=1}^{n} \frac{x_{jt}}{p_{jt}}}. \tag{1}$$

The estimated value of the target firm is given by the estimated price–earnings multiple applied to the forward earnings of the target firm:

[2]This formulation implicitly assumes that the expected pricing error is zero.

$$\widehat{p}_{it}(n) = \widehat{\beta}_{it}(n)x_{it}. \tag{2}$$

The harmonic mean is a nonlinear function of the price–earnings ratio, but a simpler interpretation of the procedure can be seen by rearranging (1) and (2) to give

$$\frac{x_{it}}{\widehat{p}_{it}(n)} = \frac{1}{n}\sum_{j=1}^{n}\frac{x_{jt}}{p_{jt}}. \tag{3}$$

Thus the result of the procedure is that the price of the target firm is set so that its earnings-to-price ratio is equal to the average of the comparable firms' earnings-to-price ratios.

We calculate the pricing error by comparing the estimated value of the target firm with its actual price:

$$\widehat{\varepsilon}_{it}(n) = \frac{\widehat{\beta}_{it}(n)x_{it} - p_{it}}{p_{it}}. \tag{4}$$

Substituting the definition of $\widehat{\beta}_{it}(n)$ from (1) gives

$$\widehat{\varepsilon}_{it}(n) = \frac{M_{it}(n) - \widehat{M}_{it}(n)}{\widehat{M}_{it}(n)}, \tag{5}$$

where M_{it} is the actual earnings-to-price ratio of the target firm, and $\widehat{M}_{it}$ is its estimated earnings-to-price ratio based on the comparable firms using (1). Thus an alternative interpretation of the pricing error is that it is the proportional error between the actual and estimated earnings-to-price multiples of the target firm.

To evaluate the performance of different methods, we examine the Bias, MAD, and MSE of the pricing error. We examine how these vary across our entire sample as a function of the number of comparable firms used n. Given a particular value of n, we calculate accuracy measures for the entire sample as follows:

$$Bias(n) = \frac{1}{N}\sum_{t}\sum_{i}\widehat{\varepsilon}_{it}(n), \tag{6}$$

$$MAD(n) = \frac{1}{N}\sum_{t}\sum_{i}|\widehat{\varepsilon}_{it}(n)|, \tag{7}$$

$$MSE(n) = \frac{1}{N}\sum_{t}\sum_{i}\widehat{\varepsilon}_{it}(n)^2, \tag{8}$$

where N is the total number of target firm-years.

We also calculate these accuracy measures using only observations within each year, industry, and industry–year pairs, for example, for the Bias indicator we have: $Bias_t(n)$, $Bias_m(n)$, $Bias_{mt}(n)$, for every year t, industry m, and industry–year pair mt, respectively. We construct these measures for MAD and MSE in a similar fashion.

We also examine how the relative accuracy of small versus large sample valuation varies between industries, over time, and between every year–industry combination. We define a relative accuracy measure for a particular industry–year as

$$RelBias_{mt}(n) = \frac{Bias_{mt}(n)}{Bias_{mt}(N_{mt})}, \tag{9}$$

where N_{mt} is the total number of firms in industry m in year t and $Bias_{mt}(n)$ is the bias for each industry–year pair resulting from using n comparable firms. The relative accuracy measures $RelMAD_{mt}$ and $RelMSE_{mt}$ are defined in similar ways.

3.2 | Trade-off between many and few comparable firms

To interpret our empirical analysis, we first show theoretically the trade-off between using a few closely comparable firms or all comparable firms in an industry. A small sample guarantees that the information used is the most relevant, but ignores any information contained in the other firms in the industry. A large sample uses more information.

In Appendix 1 we derive the condition under which the use of more comparable firms increases or decreases the accuracy of a valuation. This depends on the trade-off between the extra information contained in the prices of the additional firms and the weight given to this information in the averaging procedure. When an extra firm is added to the sample of comparable firms, the incremental information in its price replaces some of the information in the smaller sample. If the weight given to the incremental firm is too high relative to its incremental information, its addition will decrease the accuracy of the valuation. So the use of a larger sample can decrease the accuracy of a valuation.

As an example, LNT finds that using all firms in an industry is better than using all firms in all industries. The multiples of firms in other industries contain little relevant information, so giving them any weight in a valuation displaces the more relevant information contained in the multiples of firms from the same industry.

This might suggest that the procedure could be improved by assigning the “correct” weight to the incremental information in the larger sample. That is true, if the correct weight is known. Appendix 2 derives the optimal weighting scheme and shows that the correct weight depends on unobservable parameters. The correct weight to give is uncertain. Because of this, the addition of an incremental comparable firm may decrease the accuracy of the valuation.

It might also appear that sophisticated estimation schemes, such as regression methods of estimating multiples, would not suffer from this problem. Appendix 3 shows that regression-based methods of estimating multiples are essentially just different weighting schemes. As such, they suffer from the same trade-offs discussed above. The empirical evidence in LNT is that regression estimates perform worse than the harmonic mean when using all firms in the industry.

3.3 | Selection of comparable firms

Given a target firm, we select our sample of comparable firms from the same industry as the target. When we do not use all the firms in the industry we need a selection criterion to determine the sample we use for a valuation. Our sample selection criterion is to use those firms with long-term growth rate forecasts closest to that of the target. Growth is the measurable characteristic that has been found to be most useful in using price–earnings multiples for valuation, so this procedure is designed to select the comparables whose multiples

TABLE 1 Summary statistics

This table reports summary statistics of N, the number of firms in the industry–year pair; g, the arithmetic average of the long-term growth rate estimates; size, the arithmetic average of market capitalization; σ_g, the standard deviation of g; σ_{size}, the standard deviation of *size*.

	Mean	*Median*	*SD*	75%–25%	90%–10%	95%–5%	Observations (target)
N	337	133	498	410	981	1304	23,614
g	0.14	0.14	0.07	0.08	0.16	0.22	23,614
size ($1000 s)	5909	1174	20,710	3276	11,159	23,095	23,614
σ_g	0.15	0.15	0.04	0.04	0.08	0.11	23,614
σ_{size}	4270	2332	5219	4123	10,534	14,790	23,614

should contain the most relevant information.[3] Both intuitively, and also for the reasons discussed above, these are the firms whose multiples should be given the most weight in any multiples-based valuation.

The sample selection procedure for target firm i, year t, and sample size n, is to select the n comparable firms with the smallest absolute difference from the target in their long-term expected growth rate g_{jt}, that is, the n firms with the smallest values of $|g_{jt} - g_{it}|$.[4] This selection procedure gives a ranking of all comparable firms in terms of their closeness to the target firm. As we increase the number of comparable firms used, we add the firm with the next smallest distance measure at each stage.

3.4 | Sample and data

We use Institutional Brokers' Estimate System (IBES) data to construct our sample. We select firms using a procedure similar to LNT.[5] As of April of each year (labelled year t) we select firms that satisfy the following criteria: (1) price (P), forecasted earnings per share (EPS) for year $t + 1$ (EPS1), and long-term growth forecast are available in the IBES summary file; (2) nonnegative earnings and share price greater than or equal $2; (3) all price to forward earnings ratios lie within the 1st and 99th percentiles of the pooled distribution; and (4) all industry–year pairs contain at least 21 firms and selected industries have at least 10 years of data. The resulting sample contains 23,614 firms (i.e., 1,245,456 firm-years observations) in the period from 1982 to 2014. Descriptive statistics for the sample are given in Table 1.

For our industry definition, we use the IBES "Industry" classification, the intermediate level from the IBES Sector/Industry/Group classification. IBES's industry classification is loosely based on the S&P 500 industry groupings.

[3]See Alford (1992) and Zarowin (1990).
[4]In instances where the long-term expected growth was the same for two different firms we used additional ranking criteria: the absolute difference from the target in their PE ratios: $|PE_{jt} - PE_{it}|$.
[5]Our sample is less constrained than theirs, because they require nonmissing data on COMPUSTAT and CRSP for some of the tests they run.

TABLE 2 Ratio of value driver to price

This table reports descriptive statistics of the value driver used to estimate valuation multiples and pricing errors: EPS1 is the 1-year out EPS forecast and P is the stock price as of April of each year in the sample. Sample includes all firms from the IBES Summary file in the period from 1982 to 2014 with nonmissing information for price and forecasts of 1-year ahead EPS and long-term growth rate. Firms with negative earnings and share price smaller than $2 were excluded. Observations with resulting P/E rations outside the 1st and 99th percentiles of the pooled distribution were also excluded as well as firms from industries with less than 25 firms in any given year and with less than 10 years of data. The resulting sample contains 23,614 firms (1,245,456 firm-year observations).

	Mean	*Median*	*SD*	*75%–25%*	*90%–10%*	*95%–5%*	**Observations**
P/EPS1	18.4128	16.0153	10.4072	8.6916	19.0442	27.9718	23,614

Abbreviations: EPS, earnings per share; IBES, Institutional Brokers' Estimate System.

Table 2 gives descriptive statistics of the value driver (1-year out EPS forecast scaled by price) used to estimate valuation multiples. The distribution of scaled value drivers is skewed to the right, with an implied average and median P/EPS1 ratio of 18.41 and 16.01, respectively.

4 | RESULTS

4.1 | Pricing errors for different number of comparable firms

Table 3 reports descriptive information on the distribution of pricing errors across the entire sample for different numbers of comparable firms used in the estimation of the valuation multiples.

For n equal to 5 or greater, the mean errors are close to zero, indicating little bias. The absolute bias with 50 comparables is similar to that with 10, suggesting that increasing the number of comparables beyond 10 does not improve the bias. However, with $n < 10$ there is a significant bias, suggesting that very small samples may be less accurate. The other measures of accuracy show similar patterns. $n = 10$ is as good or better than using all firms in an industry for all measures. However $n = 5$ or less is generally slightly worse. Figure 1 shows the dependence of accuracy on sample size graphically. In the figure the horizontal line represents the measure calculated using an increasing number n of comparable firms. All accuracy measures exhibit a decrease for less than 10 comparable firms (i.e., Bias, MAD, and MSE increase). Also, all accuracy measures show a little detectable decrease as the number of comparables increases, particularly when n is higher than 15–20 firms.

The cause of the inaccuracy of small samples is the higher frequency of extreme errors. Figure 2 shows the distribution of pricing errors for different levels of n. As n increases, the distribution becomes less leptokurtic. The frequency of very small errors increases, but so does the frequency of very large ones. Figure 3 shows the tail of the error distribution for different levels of n. The four charts in Figure 3 illustrate this. As n increases, the number of extreme errors decreases. For $n = 2, 5, 10$, and *All*, the number of extreme observations is 185, 60, 30, and 16, respectively.[6]

[6]We define an extreme observation when the pricing error is greater than 200%.

TABLE 3 Distribution of pricing errors for the P/EPS1 multiple

This table reports summary statistics of pricing errors derived from estimated P/EPS1 multiples using 2, 5, 10, 10, 20, 40, 50, and all comparable firms in a given industry/year. p_{it} is the stock price and x_{it} is the 2-year our EPS forecast from IBES. Prices are calculated by $\hat{p}_{it}(n) = \hat{\beta}_{it}(n)x_{it}$, where $\hat{\beta}_{it}(n)$ is the estimated multiple using information from n comparable firms $\hat{\beta}_{it}(n) = \frac{n}{\sum_{j=1}^{n} \frac{x_{jt}}{p_{jt}}}$. Pricing errors are given by $\hat{\varepsilon}_{it}(n) = \frac{\hat{\beta}_{it}(n)x_{it} - p_{it}}{p_{it}}$.

n	*Mean* (%)	*Median* (%)	*SD* (%)	*MAD* (%)	*MSE* (%)	25%–75% (%)	10%–90% (%)	5%–95% (%)	Observations
2	5.8	−0.8	45.5	30.5	21.1	43.3	95.2	136.0	1,245,256
5	2.3	−2.0	37.8	26.5	14.3	38.3	83.7	116.3	1,245,256
10	0.5	−3.0	35.0	25.1	12.2	36.6	80.0	111.3	1,245,256
20	−0.6	−3.3	33.8	24.6	11.4	36.2	78.8	108.7	1,245,256
30	−0.8	−3.3	33.8	24.7	11.4	36.7	79.1	108.8	1,245,256
40	−0.8	−3.4	34.5	25.4	11.9	37.9	81.2	111.9	1,245,256
50	−0.8	−3.3	34.2	25.2	11.7	37.6	81.4	112.0	1,245,256
All	0.3	−2.3	35.1	26.0	12.3	39.2	83.6	114.3	1,245,256

Abbreviations: EPS, earnings per share; IBES, Institutional Brokers' Estimate System; MAD, mean absolute deviation; MSE, mean squared error; SD, standard deviation.

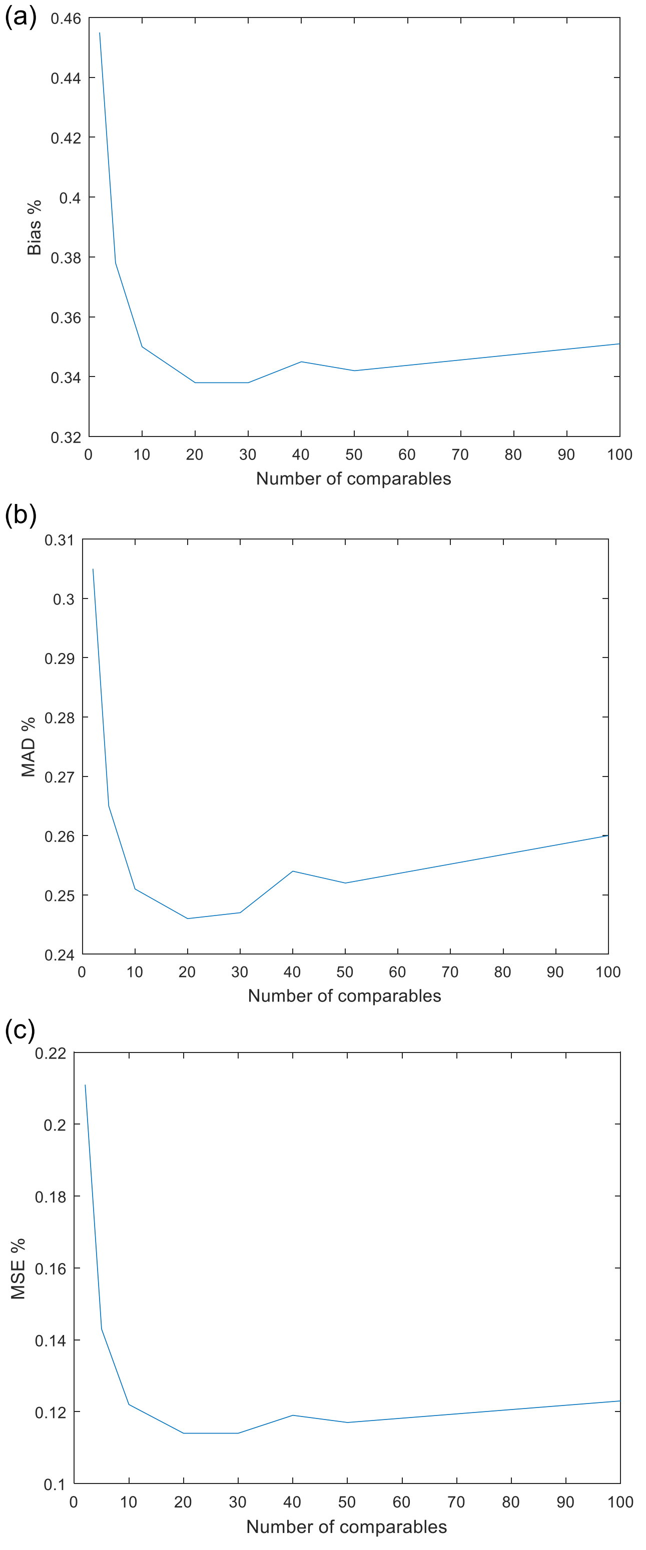

FIGURE 1 Accuracy measures for the whole sample by the number of comparable firms. (a) Bias, (b) mean absolute deviation (MAD), and (c) mean squared error (MSE).

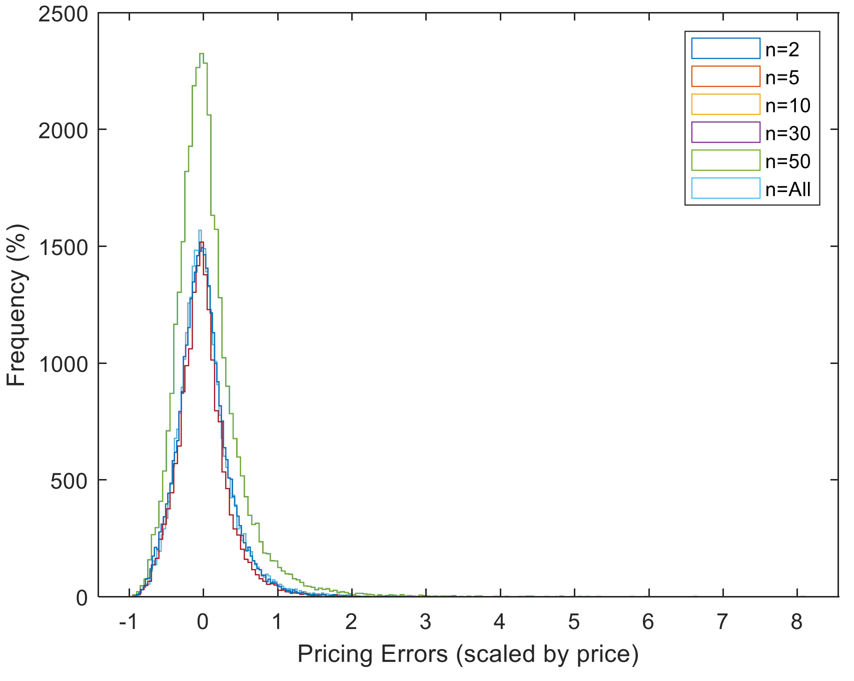

FIGURE 2 Distribution of pricing errors $\hat{\varepsilon}_{it}(n)$ for different values of n, the number of comparables

This result suggests that the main cause of higher error in forecasts made with small numbers of comparables is the greater frequency of very large errors when n is small. We show later how it is possible to predict when a valuation will suffer from this problem and, therefore, use small samples of comparables with confidence.

We also examine the evolution of the accuracy measures through time. Table 4 and Figure 4 show the absolute and relative levels of the average valuation errors by year. None of the measures shows a strong trend over time. However, in the adversity periods 1999–2003 and 2008–2009—which correspond to the Internet bubble and the US financial crises—there is a large increase in the average pricing error, both in absolute terms and also in the relative error from using a small number of comparables.

4.2 | The effect of industry

Table 5 reports the accuracy measures across industries. We focus on measures of the relative accuracy of small sample estimators versus the entire industry, to examine whether industry gives a reliable basis on which to decide to use a small number of comparables. From Table 5 small sample valuations appear to have greater merit in some industries rather than others. For instance, using *RelBias*(5) as the measure of relative accuracy, "ELECTRICAL" produces a higher relative bias than "INDUSTRIAL SERVICES", which in turn is higher than the relative bias of "AUTO PART MFG". However, the pattern is not stable for different accuracy measures. For instance, "INDUSTRIAL SERVICES" has the lowest relative MSE and the highest relative MAD of these three industries. Also, these relative rankings also change for different values of

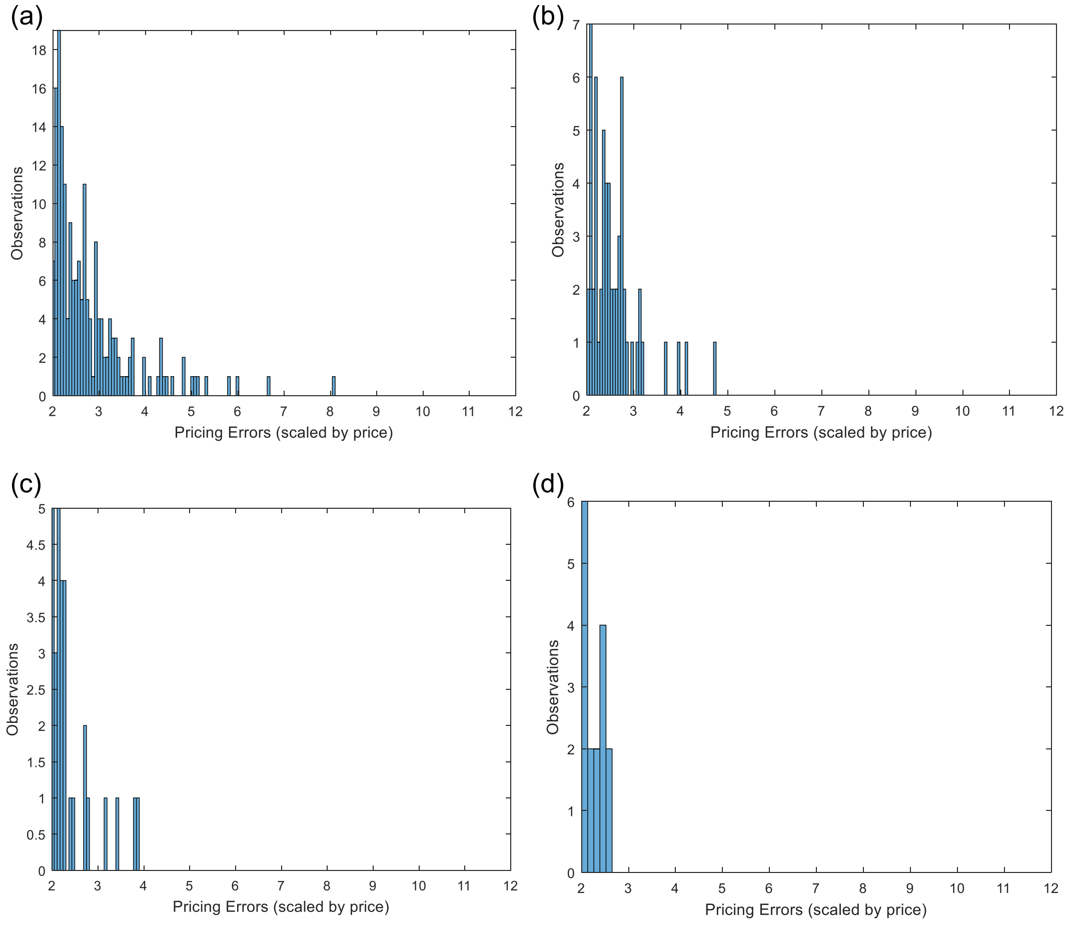

FIGURE 3 Detail of the tails of the $\hat{\varepsilon}_{it}(n)$ distributions for different values of n, the number of comparables. (a) $n = 2$, (b) $n = 5$, (c) $n = 10$, and (d) $n = All$.

n. In the case of these three industries they differ between $n = 5$ and $n = 10$. Hence it is not clear that industry is a reliable guide to the relative accuracy of small sample valuations.

To investigate the effect of industry in a more rigorous way, Tables 7–9 report the results of regressing the relative accuracy of small sample valuations on industry characteristics. We run a series of panel regressions of the measures of the relative accuracy of a small sample valuation on industry characteristics for each industry–year combination. All variables are as averages for each industry–year pair. The dependent variables are those shown in Table 6: $RelBias_m(n)$, $RelMAD_m(n)$, and $RelMSE_m(n)$ for $n \in \{5, 10, 24, 50\}$. The independent variables are: n, number of firms used; g, average of the long-term growth rate estimates; *size*, average of market capitalization; σ_g, standard deviation of g; σ_{size}, standard deviation of *size*; and their respective logs. The distributions of *size* and σ_g are highly skewed, so their logs were used in the regressions. Table 1 gives statistics for these independent variables.

There are 625 industry–year combinations for $n \leq 20$ and this number decreases as n increases. This is a consequence of our sampling procedure in which we imposed a minimum of 21 firms in each industry–year pair. For $n = 50$ the number of observations falls to 139.

TABLE 4 Bias, mean absolute deviation (MAD), and mean squared error (MSE) of pricing errors across years

This table reports Bias, MAD, and MSE statistics across years calculated with pricing errors $\hat{\varepsilon}_{it}(n) = \frac{\hat{\beta}_{it}(n)x_{it} - p_{it}}{p_{it}}$ in each year t: $Bias(t, n) = \frac{1}{N_t}\sum_i \varepsilon_{it}(n)$, $MAD(t, n) = \frac{1}{N_t}\sum_i |\varepsilon_{it}(n)|$, $MSE(t, n) = \frac{1}{N_t}\sum_i \varepsilon_{it}(n)^2$, where N_t is the total number of observations in a given year and n is the number of comparable firms used. $RelBias(t, n) = Bias(t, n)/Bias(t, All)$. Other relative measures are defined analogously.

	n = 5			n = 10			n = all			n = 5			n = 10		
Year	***Bias***	***MAD***	***MSE***	***Bias***	***MAD***	***MSE***	***Bias***	***MAD***	***MSE***	***RelBias***	***RelMAD***	***RelMSE***	***RelBias***	***RelMAD***	***RelMSE***
1982	−0.0007	0.1888	0.0684	−0.0048	0.1810	0.0588	0.0040	0.2034	0.0712	−0.1767	0.9280	0.9604	−1.1921	0.8899	0.8255
1983	0.0161	0.2103	0.0772	0.0066	0.1982	0.0672	0.0041	0.2121	0.0747	3.8871	0.9911	1.0334	1.5875	0.9341	0.8993
1984	0.0057	0.1804	0.0587	−0.0022	0.1747	0.0532	0.0037	0.1929	0.0657	1.5261	0.9351	0.8941	−0.5928	0.9059	0.8096
1985	−0.0038	0.1815	0.0615	−0.0010	0.1704	0.0547	0.0033	0.1885	0.0602	−1.1546	0.9630	1.0214	−0.3087	0.9040	0.9089
1986	0.0023	0.1851	0.0608	−0.0057	0.1742	0.0535	0.0029	0.1834	0.0589	0.7820	1.0094	1.0317	−1.9633	0.9496	0.9083
1987	0.0098	0.2124	0.0886	0.0088	0.1989	0.0814	0.0044	0.2085	0.0824	2.2432	1.0187	1.0743	2.0175	0.9541	0.9874
1988	−0.0001	0.1951	0.0738	−0.0043	0.1803	0.0651	0.0039	0.2008	0.0725	−0.0327	0.9718	1.0172	−1.1187	0.8982	0.8975
1989	0.0133	0.2162	0.0889	−0.0013	0.1984	0.0737	0.0045	0.2205	0.0842	2.9546	0.9807	1.0560	−0.2827	0.8997	0.8753
1990	0.0176	0.2272	0.0983	0.0089	0.2118	0.0840	0.0043	0.2250	0.0883	4.0663	1.0100	1.1130	2.0539	0.9414	0.9518
1991	0.0051	0.2225	0.0927	−0.0055	0.2110	0.0827	0.0048	0.2235	0.0921	1.0602	0.9957	1.0065	−1.1345	0.9442	0.8972
1992	0.0108	0.2234	0.0922	−0.0008	0.2129	0.0825	0.0043	0.2205	0.0867	2.5176	1.0133	1.0640	−0.1883	0.9655	0.9512
1993	−0.0024	0.2134	0.0830	−0.0063	0.2091	0.0786	0.0034	0.2127	0.0833	−0.6886	1.0031	0.9964	−1.8459	0.9832	0.9442
1994	0.0238	0.2297	0.1046	0.0077	0.2228	0.0909	0.0040	0.2280	0.0941	5.9706	1.0074	1.1122	1.9424	0.9772	0.9664
1995	0.0097	0.2501	0.1171	−0.0040	0.2411	0.1062	0.0044	0.2534	0.1127	2.2183	0.9870	1.0389	−0.9067	0.9515	0.9425
1996	0.0166	0.2345	0.0964	0.0010	0.2284	0.0900	0.0032	0.2347	0.0922	5.1396	0.9993	1.0452	0.2960	0.9732	0.9762
1997	0.0272	0.2351	0.0993	0.0052	0.2240	0.0881	0.0030	0.2301	0.0885	9.1282	1.0216	1.1227	1.7469	0.9732	0.9963

(Continues)

TABLE 4 (Continued)

	$n = 5$			$n = 10$			$n = $ all			$n = 5$			$n = 10$		
Year	***Bias***	***MAD***	***MSE***	***Bias***	***MAD***	***MSE***	***Bias***	***MAD***	***MSE***	***RelBias***	***RelMAD***	***RelMSE***	***RelBias***	***RelMAD***	***RelMSE***
1998	0.0281	0.3709	0.2622	0.0107	0.3481	0.2177	0.0074	0.3482	0.2116	3.7854	1.0653	1.2393	1.4453	0.9998	1.0288
1999	0.0688	0.4167	0.3404	0.0134	0.3827	0.2654	0.0085	0.3932	0.2637	8.0944	1.0597	1.2910	1.5730	0.9734	1.0067
2000	0.0526	0.3584	0.2618	0.0163	0.3423	0.2320	0.0087	0.3590	0.2246	6.0360	0.9982	1.1656	1.8779	0.9533	1.0328
2001	0.0424	0.3099	0.1869	0.0143	0.2963	0.1626	0.0061	0.2996	0.1543	6.9199	1.0344	1.2119	2.3393	0.9890	1.0541
2002	0.0478	0.3343	0.2209	0.0226	0.3177	0.1872	0.0062	0.3129	0.1715	7.6611	1.0682	1.2879	3.6272	1.0151	1.0915
2003	0.0135	0.2481	0.1093	0.0051	0.2359	0.1029	0.0036	0.2469	0.1042	3.7581	1.0051	1.0486	1.4205	0.9554	0.9870
2004	0.0157	0.2352	0.1032	0.0033	0.2213	0.0912	0.0031	0.2305	0.0923	5.0568	1.0205	1.1173	1.0494	0.9602	0.9876
2005	0.0029	0.2417	0.1070	−0.0077	0.2339	0.0970	0.0037	0.2532	0.1086	0.7783	0.9544	0.9851	−2.0742	0.9236	0.8929
2006	0.0003	0.2367	0.0977	−0.0090	0.2229	0.0855	0.0038	0.2386	0.0959	0.0696	0.9921	1.0188	−2.4046	0.9344	0.8914
2007	0.0030	0.2669	0.1264	−0.0060	0.2563	0.1129	0.0047	0.2630	0.1188	0.6429	1.0148	1.0644	−1.2678	0.9743	0.9506
2008	0.0326	0.3303	0.2309	0.0142	0.3093	0.1891	0.0072	0.3137	0.1882	4.5189	1.0527	1.2270	1.9726	0.9860	1.0051
2009	0.0060	0.2848	0.1420	−0.0039	0.2743	0.1297	0.0051	0.2847	0.1330	1.1650	1.0004	1.0678	−0.7525	0.9638	0.9753
2010	0.0189	0.3016	0.1685	0.0010	0.2833	0.1442	0.0063	0.2957	0.1508	2.9962	1.0199	1.1169	0.1617	0.9579	0.9563
2011	0.0292	0.3064	0.1823	0.0001	0.2852	0.1493	0.0063	0.3019	0.1653	4.6147	1.0147	1.1030	0.0083	0.9444	0.9033
2012	0.0189	0.2890	0.1564	0.0072	0.2805	0.1410	0.0062	0.2801	0.1408	3.0315	1.0315	1.1107	1.1542	1.0012	1.0018
2013	0.0180	0.2683	0.1253	−0.0003	0.2477	0.1042	0.0050	0.2558	0.1109	3.5934	1.0491	1.1304	−0.0548	0.9685	0.9399
2014	0.0118	0.2724	0.1602	−0.0106	0.2562	0.1324	0.0059	0.2698	0.1293	1.9972	1.0098	1.2392	−1.7949	0.9499	1.0241
1999–2003	0.0450	0.3335	0.2239	0.0144	0.3150	0.1900	0.0066	0.3223	0.1837	6.4939	1.0331	1.2010	2.1676	0.9772	1.0344
2008–2009	0.0193	0.3075	0.1864	0.0052	0.2918	0.1594	0.0062	0.2992	0.1606	2.8420	1.0266	1.1474	0.6100	0.9749	0.9902
Excluded: 1999–2003 and 2008–2009	0.0115	0.2383	0.1096	−0.0003	0.2259	0.0956	0.0044	0.2375	0.1012	2.5387	1.0003	1.0659	−0.1410	0.9484	0.9352

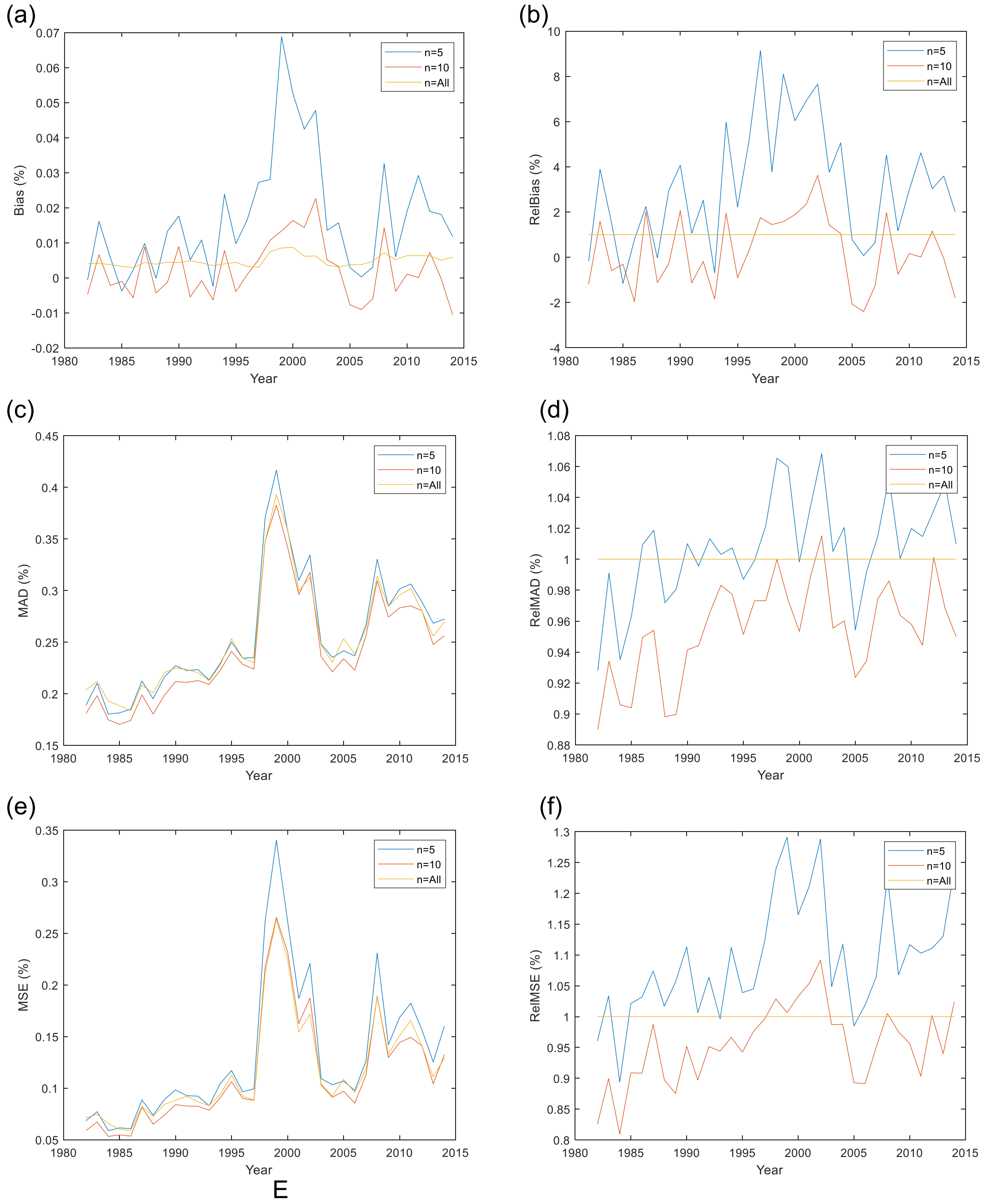

FIGURE 4 Accuracy measures across time by the number of comparable firms. (a) Bias, (b) RelBias, (c) MAD, (d) RelMAD, (e) MSE, and (f) RelMSE. MAD, mean absolute deviation; MSE, mean squared error; RelBias, relative bias; RelMAD, relative mean absolute deviation; RelMSE, relative mean squared error.

Regression results for *RelBias*, presented in Table 7, indicate that none of the industry characteristics hold any consistent explanatory power. Only the number of firms in each industry–year pair (N) is a significant regressor. N is positive for $n = 5$, 10, and 20 but negative for $n = 50$.

TABLE 5 Bias, mean absolute deviation (MAD), and mean squared error (MSE) of pricing errors across industries

This table reports Bias, MAD, and MSE statistics across industries calculated with pricing errors $\hat{\varepsilon}_{it}(n) = \frac{\hat{\beta}_{it}(n)x_{it} - p_{it}}{p_{it}}$ in each industry m: $Bias(m, n) = \frac{1}{N_m}\sum_{i\in m} \varepsilon_{it}(n)$, $MAD(m, n) = \frac{1}{N_m}\sum_{i\in m}|\varepsilon_{it}(n)|$, $MSE(m, n) = \frac{1}{N_m}\sum_{i\in m} \varepsilon_{it}(n)^2$, where N_m is the total number of observations in a given industry and n is the number of comparable firms used. $RelBias(m, n) = Bias(m, n)/Bias(m, All)$. Other relative measures are defined analogously.

	n = 5			*n* = 10			*n* = all			*n* = 5			*n* = 10		
Industry	***Bias***	***MAD***	***MSE***	***Bias***	***MAD***	***MSE***	***Bias***	***MAD***	***MSE***	***RelBias***	***RelMAD***	***RelMSE***	***RelBias***	***RelMAD***	***RelMSE***
AIRLINES	0.0190	0.2987	0.1749	0.0030	0.2866	0.1448	0.0115	0.2933	0.1490	1.6517	1.0182	1.1744	0.2610	0.9769	0.9722
AUTO PART MFG	0.0105	0.2456	0.1285	−0.0026	0.2179	0.0799	0.0068	0.2251	0.0973	1.5569	1.0910	1.3197	−0.3875	0.9679	0.8210
BEVERAGES	−0.0002	0.2387	0.1034	−0.0124	0.2561	0.1092	0.0083	0.2440	0.1009	−0.0254	0.9785	1.0244	−1.4989	1.0500	1.0817
BIOTECHNOLOGY	−0.0125	0.3072	0.1746	−0.0401	0.3022	0.1530	0.0122	0.3471	0.1972	−1.0309	0.8853	0.8857	−3.2951	0.8707	0.7758
BUILDING & RELATED	0.0066	0.2522	0.1131	0.0079	0.2602	0.1216	0.0106	0.2566	0.1167	0.6216	0.9828	0.9700	0.7456	1.0138	1.0424
BUILDING MATERIALS	0.0378	0.2860	0.1477	0.0167	0.2693	0.1297	0.0061	0.2642	0.1214	6.1852	1.0824	1.2164	2.7272	1.0194	1.0685
CHEMICALS	0.0110	0.2281	0.0920	0.0018	0.2147	0.0820	0.0021	0.2180	0.0842	5.1853	1.0464	1.0916	0.8235	0.9850	0.9731
CLOTHING	0.0127	0.2445	0.1184	0.0017	0.2282	0.0976	0.0037	0.2296	0.0941	3.3982	1.0648	1.2590	0.4427	0.9940	1.0372
COMMUNICATIONS	0.0142	0.2952	0.1783	0.0004	0.2823	0.1610	0.0039	0.2950	0.1607	3.6859	1.0007	1.1098	0.0957	0.9568	1.0020
COMPUTER MFRS	0.0101	0.2611	0.1159	0.0171	0.2472	0.0959	0.0099	0.2799	0.1268	1.0163	0.9329	0.9142	1.7195	0.8832	0.7565
CONTAINERS	0.0010	0.2051	0.0868	0.0270	0.3409	0.2267	0.0082	0.2027	0.0821	0.1236	1.0116	1.0578	3.2724	1.6815	2.7625
DEFENCE	−0.0045	0.2327	0.0965	−0.0213	0.2285	0.0874	0.0047	0.2489	0.1000	−0.9592	0.9349	0.9652	−4.4973	0.9181	0.8739
DRUGS	0.0006	0.2102	0.0841	−0.0094	0.2086	0.0808	0.0039	0.2273	0.0910	0.1538	0.9249	0.9237	−2.4526	0.9178	0.8884
ELECTRICAL	0.0352	0.2691	0.1581	0.0150	0.2580	0.1349	0.0053	0.2566	0.1212	6.6609	1.0490	1.3047	2.8391	1.0055	1.1132
ELECTRICAL UTILITIES	0.0038	0.1262	0.0329	0.0018	0.1212	0.0306	0.0005	0.1206	0.0290	8.0757	1.0465	1.1347	3.7730	1.0051	1.0532
ELECTR SYST/DEVICES	0.0139	0.3109	0.1527	0.0031	0.3013	0.1514	0.0078	0.3025	0.1510	1.7740	1.0279	1.0114	0.3928	0.9961	1.0030

TABLE 5 (Continued)

	$n = 5$			$n = 10$			$n = $ all			$n = 5$			$n = 10$		
Industry	***Bias***	***MAD***	***MSE***	***Bias***	***MAD***	***MSE***	***Bias***	***MAD***	***MSE***	***RelBias***	***RelMAD***	***RelMSE***	***RelBias***	***RelMAD***	***RelMSE***
FINANCIAL SERVICES	0.0107	0.2731	0.1482	0.0069	0.2623	0.1330	0.0096	0.2647	0.1311	1.1182	1.0320	1.1303	0.7160	0.9913	1.0145
FOOD PROCESSORS	0.0059	0.2144	0.0946	0.0012	0.2060	0.0886	0.0029	0.2073	0.0857	2.0232	1.0343	1.1039	0.4059	0.9934	1.0345
FOREST PRODUCTS	0.0236	0.2819	0.1588	0.0276	0.2687	0.1482	0.0093	0.2693	0.1452	2.5315	1.0465	1.0934	2.9641	0.9977	1.0201
GAS UTILITIES	0.0006	0.1612	0.0504	−0.0102	0.1517	0.0442	0.0017	0.1593	0.0459	0.3739	1.0117	1.0971	−5.9738	0.9518	0.9629
HOME FURNISHINGS	0.0082	0.2519	0.1165	−0.0048	0.2359	0.0986	0.0068	0.2416	0.1032	1.2049	1.0424	1.1289	−0.7005	0.9761	0.9557
HOME HEALTH CARE	−0.0108	0.2614	0.1144	−0.0095	0.2828	0.1332	0.0116	0.2560	0.1088	−0.9312	1.0209	1.0511	−0.8161	1.1048	1.2245
HOME PRODUCTS	0.0217	0.2362	0.0933	0.0094	0.2879	0.1658	0.0096	0.2251	0.0870	2.2727	1.0493	1.0726	0.9830	1.2788	1.9047
HOSPITALS	−0.0043	0.2659	0.1212	−0.0296	0.2562	0.1126	0.0109	0.2698	0.1271	−0.3957	0.9857	0.9533	−2.7144	0.9498	0.8860
INDUSTRIAL SERVICES	0.0208	0.3423	0.2010	0.0036	0.3290	0.1771	0.0087	0.3129	0.1590	2.3958	1.0941	1.2644	0.4105	1.0515	1.1139
LEISURE	0.0041	0.2919	0.1467	−0.0053	0.2854	0.1405	0.0074	0.2828	0.1357	0.5603	1.0321	1.0808	−0.7236	1.0093	1.0352
LEISURE PRODUCTS	0.0049	0.2755	0.1147	0.0104	0.2720	0.1166	0.0242	0.3047	0.1693	0.2034	0.9043	0.6777	0.4306	0.8928	0.6886
LEISURE TIMES	0.0207	0.3224	0.1820	0.0005	0.3481	0.1874	0.0193	0.3015	0.1532	1.0737	1.0695	1.1883	0.0260	1.1547	1.2236
MACHINERY	0.0080	0.2495	0.1147	−0.0051	0.2338	0.0980	0.0021	0.2484	0.1050	3.8197	1.0042	1.0920	−2.4265	0.9412	0.9330
MARITIME	0.0467	0.3438	0.1950	0.0337	0.3386	0.1784	0.0202	0.3358	0.1854	2.3137	1.0239	1.0523	1.6667	1.0083	0.9624
MEDICAL SUPPLIES	0.0001	0.2335	0.0995	−0.0061	0.2273	0.0934	0.0024	0.2408	0.0952	0.0321	0.9699	1.0445	−2.5021	0.9439	0.9813
METAL FABRI & DIST	0.0334	0.2553	0.1207	0.0205	0.2368	0.0939	0.0107	0.2485	0.1054	3.1270	1.0275	1.1444	1.9206	0.9531	0.8906
MULT-IND CAP GOOD	0.0232	0.2303	0.1017	0.0205	0.2389	0.1091	0.0089	0.2356	0.1006	2.5954	0.9773	1.0116	2.2981	1.0139	1.0846
OFFICE PRODUCTS	−0.0146	0.2139	0.0842	−0.0229	0.1792	0.0642	0.0096	0.2136	0.0890	−1.5145	1.0015	0.9465	−2.3791	0.8388	0.7215
OFFICE/COMM EQUIP	0.0188	0.3481	0.2247	−0.0007	0.3292	0.1859	0.0082	0.3272	0.1824	2.2771	1.0638	1.2323	−0.0817	1.0062	1.0191
OIL	0.0451	0.3396	0.2150	0.0191	0.3217	0.1817	0.0027	0.3333	0.1729	16.9605	1.0190	1.2431	7.1855	0.9654	1.0507

(Continues)

TABLE 5 (Continued)

Industry	n = 5			n = 10			n = all			n = 5			n = 10		
	Bias	*MAD*	*MSE*	*Bias*	*MAD*	*MSE*	*Bias*	*MAD*	*MSE*	*RelBias*	*RelMAD*	*RelMSE*	*RelBias*	*RelMAD*	*RelMSE*
OTHER COMPUTERS	0.0275	0.3298	0.1885	0.0013	0.3107	0.1597	0.0046	0.3180	0.1653	5.9379	1.0370	1.1403	0.2837	0.9768	0.9659
PRECIOUS METALS	0.0043	0.3309	0.1750	0.0283	0.3470	0.1905	0.0206	0.3307	0.1783	0.2098	1.0005	0.9818	1.3717	1.0493	1.0688
RETAILING—FOODS	0.0064	0.2356	0.1030	−0.0158	0.2265	0.0945	0.0028	0.2514	0.1112	2.2899	0.9369	0.9262	−5.6145	0.9011	0.8499
RETAILING—GOODS	0.0449	0.2729	0.1435	0.0153	0.2559	0.1198	0.0013	0.2625	0.1172	35.6141	1.0398	1.2245	12.1626	0.9751	1.0222
SEMICOND/COMP.	0.0413	0.3551	0.2406	0.0236	0.3349	0.2060	0.0046	0.3316	0.1924	8.9938	1.0709	1.2505	5.1457	1.0100	1.0703
SERVS TO MED PROF	−0.0137	0.2537	0.1203	−0.0288	0.2662	0.1211	0.0086	0.3195	0.1653	−1.5943	0.7942	0.7277	−3.3446	0.8331	0.7322
SOFTW & EDP SERVS	0.0574	0.3266	0.2444	0.0184	0.3052	0.1901	0.0038	0.3368	0.2071	14.9199	0.9697	1.1798	4.7920	0.9061	0.9177
STEEL	0.0232	0.2757	0.1272	0.0135	0.2398	0.0931	0.0128	0.2716	0.1228	1.8137	1.0151	1.0355	1.0590	0.8827	0.7576
TELEPHONE UTILITIES	0.0098	0.2591	0.1451	−0.0143	0.2449	0.1204	0.0049	0.2547	0.1243	1.9845	1.0174	1.1666	−2.9018	0.9618	0.9683
TRUCKING	0.0109	0.2025	0.0722	−0.0073	0.2098	0.0735	0.0047	0.2013	0.0710	2.3069	1.0061	1.0165	−1.5501	1.0423	1.0356
UNDESIGN CONR SVC	0.0317	0.3170	0.1876	0.0016	0.3095	0.1656	0.0043	0.3092	0.1574	7.3732	1.0251	1.1918	0.3632	1.0009	1.0524

TABLE 6 *RelBias*, *RelMAD*, and *RelMSE* panel regressions—Summary statistics

The table reports summary statistics of the dependent variables used in the panel regression of relative accuracy measures. Dependent variables are the three relative accuracy measures $RelBias_m(n)$, $RelMAD_m(n)$, and $RelMSE_m(n)$, with $n = 5, 10, 20, 50$.

	Mean	*Median*	*SD*	75%–25% (%)	90%–10% (%)	95%–5% (%)	Observations
*RelBias*5	2.80	1.00	12.78	9.22	22.46	33.63	625
*RelBias*10	−0.05	0.16	10.84	9.65	22.89	33.52	625
*RelBias*20	−2.55	−2.22	12.44	10.28	24.89	36.50	625
*RelBias*50	−2.41	−2.15	21.91	13.27	42.60	67.74	139
*RelMAD*5	1.01	1.01	0.12	0.13	0.29	0.40	625
*RelMAD*10	0.97	0.99	0.09	0.10	0.22	0.28	625
*RelMAD*20	0.96	0.98	0.06	0.07	0.16	0.20	625
*RelMAD*50	0.97	0.98	0.05	0.05	0.12	0.14	139
*RelMES*5	1.06	1.03	0.25	0.27	0.60	0.80	625
*RelMES*10	0.97	1.00	0.16	0.19	0.39	0.52	625
*RelMES*20	0.94	0.95	0.11	0.12	0.27	0.36	625
*RelMES*50	0.94	0.97	0.08	0.09	0.21	0.27	139

Abbreviations: RelBias, relative bias; RelMAD, relative mean absolute deviation; RelMSE, relative mean squared error; SD, standard deviation.

When relative mean absolute deviations *RelMAD* are considered in Table 8 the predictive power of the number of firms in each industry–year pair N is significant positive for a small number of comparables ($n = 5$ and 10) and negative for a large number of comparables ($n = 50$), This finding implies that for larger industries is needed higher number of comparables to achieve valuation accuracy. For example, for specification (12), if we multiply the coefficient of N, −0.001, by the interquartile range of N, 43.3, we get a difference of *RelMAD*50 equal to −0.043. Thus the relative accuracy of a valuation based on $n = 50$ improves by 4% in large industries. This result is similar using the relative MSE measure.[7] The result indicates that large industries perform relatively better using a larger number of comparables. The results on *RelMAD* in Table 8 also show a negative significant coefficient for the standard deviation of the growth rate (σ_g) for a small number of firms ($n \leq 20$). Again, this result suggests that smaller volatility of growth rate leads to higher accuracy. However, this result is not confirmed in Table 9 based on *RelMSE*.

Overall, these results suggest that it is not possible reliably to choose to use a small number of comparables based on the characteristics of an industry. The only reliable effect is that the valuation for large industries works better by using a higher number of comparable firms.

[7]On the other hand, the positive and significant N for a small number of comparables ($n = 5$ and 10) is meaningless and likely statistical artefact due to the small sample of firms in both groups (i.e., $n \approx N$); that is, this derives more from the inadequacy of using the entire set of comparable firms in such industries.

TABLE 7 Relative accuracy measures and industry characteristics—*Bias*

This table reports panel regression of relative accuracy measures and industry characteristics for all industry–year combinations. Dependent variables are the relative accuracy measures $RelBias_{mt}(n) = Bias_{mt}(n)/Bias_{mt}(All)$ for each year t, industry m and number of comparables n = 5, 10, 20, 50. Independent variables were calculated for each industry–year pair: N, number of firms in each industry–year pair; g, arithmetic average of the long-term growth rate estimates; *size*, arithmetic average of market capitalization; σ_g, standard deviation of g; σ_{size}, standard deviation of *size*; $\log(\sigma_{size})$, log of the standard deviation of *size*; $\log(size)$, log of the arithmetic average of market capitalization. The absolute value of robust t statistics is reported in parenthesis, accompanied by the standard indication of statistical significance.

	(1) *RelBias5*	(2) *RelBias10*	(3) *RelBias20*	(4) *RelBias50*	(5) *RelBias5*	(6) *RelBias10*	(7) *RelBias20*	(8) *RelBias50*	(9) *RelBias5*	(10) *RelBias10*	(11) *RelBias20*	(12) *RelBias50*
g	27.910	16.423	−16.60	−162.881	24.38	15.29	−28.37	−123.19	34.79	4.303	−24.069	−54.519
	(0.00)***	(0.10)	(0.22)	(0.00)***	(0.01)**	(0.16)	(0.06)*	(0.01)**	(0.02)**	(0.81)	(0.34)	(0.55)
σ_g	−37.993	−30.958	−37.703	1.123	−38.05	−29.92	−32.55	−31.702	−34.841	−24.86	−38.48	−142.74
	(0.00)***	(0.03)**	(0.12)	(0.99)	(0.00)***	(0.05)**	(0.21)	(0.71)	(0.01)**	(0.12)	(0.16)	(0.15)
$\log(size)$	1.184	−0.243	−3.034	−16.51	0.134	−1.378	−4.966	−4.019	−0.769	−1.906	−5.732	−13.75
	(0.34)	(0.87)	(0.19)	(0.06)*	(0.93)	(0.46)	(0.11)	(0.79)	(0.55)	(0.21)	(0.01)**	(0.14)
$\log(\sigma_{size})$	−0.702	0.231	2.147	7.574	−0.315	0.756	2.684	2.210	0.977	1.440	3.813	5.148
	(0.48)	(0.84)	(0.24)	(0.29)	(0.77)	(0.57)	(0.21)	(0.82)	(0.34)	(0.23)	(0.04)**	(0.49)
N	0.269	0.120	0.103	−0.237	0.269	0.122	0.107	−0.185	0.272	0.136	0.181	−0.101
	(0.00)***	(0.00)***	(0.00)***	(0.01)**	(0.00)***	(0.00)***	(0.00)***	(0.08)*	(0.00)***	(0.00)***	(0.00)***	(0.40)
Intercept	−9.044	−4.408	3.941	109.80	0.001	0.001	0.001	0.015	−1.898	−2.342	6.807	91.62
	(0.01)***	(0.27)	(0.53)	(0.00)***	(0.82)	(0.76)	(0.60)	(0.03)**	(0.75)	(0.67)	(0.40)	(0.01)**
Observations	625	625	625	139	625	625	625	139	625	625	625	139
Industry FE	N	N	N	N	Y	Y	Y	Y	N	N	N	N
Year FE	N	N	N	N	N	N	N	N	Y	Y	Y	Y

Abbreviations: FE, fixed effects; N, no; RelBias, relative bias; Y, yes.

TABLE 8 Relative accuracy measures and industry characteristics—*MAD*

This table reports panel regression of relative accuracy measures and industry characteristics for all industry–year combinations. Dependent variables are the relative accuracy measures $RelMADm_t(n) = MAD_{mt}(n)/MAD_{mt}(All)$ for each year t, industry m, and a number of comparables $n = 5, 10, 20, 50$. Independent variables were calculated for each industry–year pair: N, number of firms in each industry–year pair; g, arithmetic average of the long-term growth rate estimates; *size*, arithmetic average of market capitalization; σ_g, standard deviation of g; σ_{size}, standard deviation of *size*; $\log(\sigma_{size})$, log of the standard deviation of *size*; $\log(size)$, log of the arithmetic average of market capitalization. The absolute value of robust t statistics is reported in parenthesis, accompanied by the standard indication of statistical significance.

	(1) RelMA-D5	(2) RelMA-D10	(3) RelMA-D20	(4) RelMA-D50	(5) RelMA-D5	(6) RelMA-D10	(7) RelMA-D20	(8) RelMA-D50	(9) RelMA-D5	(10) RelMA-D10	(11) RelMA-D20	(12) RelMA-D50
g	−0.066	−0.160	−0.109	−0.122	0.219	0.141	0.139	0.233	−0.103	−0.223	−0.178	−0.135
	(0.49)	(0.06)*	(0.12)	(0.15)	(0.14)	(0.32)	(0.28)	(0.23)	(0.30)	(0.01)**	(0.02)**	(0.17)
σ_g	−0.353	−0.356	−0.274	−0.132	−0.326	−0.329	−0.237	−0.387	−0.375	−0.300	−0.270	−0.114
	(0.00)***	(0.00)***	(0.03)**	(0.40)	(0.02)**	(0.01)**	(0.09)*	(0.06)*	(0.00)***	(0.02)**	(0.04)**	(0.51)
$\log(size)$	0.041	0.038	0.013	−0.013	0.043	0.037	0.009	−0.005	0.028	0.041	0.005	0.011
	(0.00)***	(0.00)***	(0.27)	(0.49)	(0.00)***	(0.00)***	(0.47)	(0.81)	(0.07)*	(0.01)**	(0.76)	(0.70)
$\log(\sigma_{size})$	−0.024	−0.022	−0.001	0.009	−0.027	−0.023	0.001	0.006	−0.019	−0.025	−0.001	−0.004
	(0.02)**	(0.03)**	(0.88)	(0.56)	(0.01)**	(0.02)**	(0.94)	(0.72)	(0.08)*	(0.02)**	(0.91)	(0.84)
N	0.000	0.000	−0.001	−0.001	0.001	0.000	0.000	−0.001	0.000	0.000	−0.001	−0.001
	(0.01)**	(0.59)	(0.00)***	(0.00)***	(0.01)**	(0.40)	(0.22)	(0.00)***	(0.03)**	(0.64)	(0.00)***	(0.00)***
Intercept	0.892	0.892	0.912	1.100	0.855	0.853	0.875	1.018	0.973	0.884	0.975	0.972
	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***
Observations	625	625	625	139	625	625	625	139	625	625	625	139
Industry FE	N	N	N	N	Y	Y	Y	Y	N	N	N	N
Year FE	N	N	N	N	N	N	N	N	Y	Y	Y	Y

Abbreviations: FE, fixed effects; MAD, mean absolute deviation; N, no; RelMAD, relative mean absolute deviation; Y, yes.

TABLE 9 Relative accuracy measures and industry characteristics—*MSE*

This table reports panel regression of relative accuracy measures and industry characteristics for all industry–year combinations. Dependent variables are the relative accuracy measures $RelMSE_{mt}(n) = MSE_{mt}(n)/MSE_{mt}(All)$ for each year t, industry m, and a number of comparables n = 5, 10, 20, 50. Independent variables were calculated for each industry–year pair: N, number of firms in each industry–year pair; g, arithmetic average of the long-term growth rate estimates; *size*, arithmetic average of market capitalization; σ_g, standard deviation of g; σ_{size}, standard deviation of *size*; $\log(\sigma_{size})$, log of the standard deviation of *size*; $\log(size)$, log of the arithmetic average of market capitalization. The absolute value of robust t statistics is reported in parenthesis, accompanied by the standard indication of statistical significance.

	(1) *RelMSE5*	(2) *RelMSE10*	(3) *RelMSE20*	(4) *RelMSE50*	(5) *RelMSE5*	(6) *RelMSE10*	(7) *RelMSE20*	(8) *RelMSE50*	(9) *RelMSE5*	(10) *RelMSE10*	(11) *RelMSE20*	(12) *RelMSE50*
g	0.089	−0.362	−0.307	−0.378	0.752	0.314	0.237	0.271	−0.043	−0.487	−0.467	−0.366
	(0.65)	(0.02)**	(0.01)**	(0.01)**	(0.02)**	(0.22)	(0.30)	(0.42)	(0.83)	(0.00)***	(0.00)***	(0.04)**
σ_g	−0.407	−0.475	−0.345	−0.268	−0.537	−0.615	−0.440	−0.943	−0.442	−0.391	−0.302	−0.183
	(0.11)	(0.03)**	(0.12)	(0.33)	(0.06)*	(0.01)**	(0.08)*	(0.01)**	(0.10)	(0.09)*	(0.19)	(0.55)
$\log(size)$	0.065	0.047	0.020	−0.030	0.075	0.052	0.013	−0.015	0.035	0.042	0.007	0.024
	(0.01)**	(0.04)**	(0.35)	(0.36)	(0.00)***	(0.02)**	(0.53)	(0.66)	(0.25)	(0.14)	(0.81)	(0.64)
$\log(\sigma_{size})$	−0.030	−0.022	0.000	0.018	−0.040	−0.027	0.002	0.009	−0.022	−0.022	−0.002	−0.011
	(0.14)	(0.22)	(1.00)	(0.50)	(0.06)*	(0.13)	(0.89)	(0.74)	(0.33)	(0.26)	(0.90)	(0.76)
N	0.002	0.000	0.000	−0.002	0.002	0.001	0.000	−0.002	0.002	0.001	0.000	−0.002
	(0.00)***	(0.07)*	(0.05)**	(0.00)***	(0.00)***	(0.20)	(0.62)	(0.00)***	(0.00)***	(0.06)*	(0.07)*	(0.00)***
Intercept	0.740	0.832	0.849	1.239	0.710	0.731	0.765	1.132	0.963	0.871	0.985	0.980
	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***	(0.00)***
Observations	625	625	625	139	625	625	625	139	625	625	625	139
Industry FE	N	N	N	N	Y	Y	Y	Y	N	N	N	N
Year FE	N	N	N	N	N	N	N	N	Y	Y	Y	Y

Abbreviations: FE, fixed effects; MSE, mean squared error; N, no; RelMSE, relative mean squared error; Y, yes.

TABLE 10 Valuation rule—Small differences in growth rates

This table reports statistics of a subsample of pricing errors. Pricing errors resulting from observations whose absolute value of the difference between the target firm's growth rate and the average growth rate of comparable firms fell below the 80% percentile of the growth rate difference distribution when $n = 5$. Statistics are calculated for a different number of comparable firms—$n = 2, 5, 10, 20, 30, 40, 50$, and *All*—and averaged across all observations of the subsample.

n	*Mean* (%)	*Median* (%)	*SD* (%)	*MAD* (%)	*MSE* (%)	25%–75% (%)	10%–90% (%)	5%–95% (%)	Observations
2	6.0	−0.5	44.7	30.4	20.4	43.7	94.4	134.1	996,204
5	2.6	−1.6	36.9	26.3	13.7	38.4	82.9	113.9	996,204
10	1.2	−2.5	34.3	24.8	11.8	36.5	78.6	109.6	996,204
20	0.5	−2.7	33.2	24.3	11.0	36.4	76.7	106.7	996,204
30	0.7	−2.5	33.4	24.5	11.1	37.0	77.6	108.1	996,204
40	0.9	−2.4	34.4	25.4	11.8	38.3	80.2	111.2	996,204
50	0.8	−2.5	34.4	25.5	11.8	38.4	81.5	112.8	996,204
All	2.6	−0.7	34.6	25.6	12.0	38.9	81.3	111.7	996,204

Abbreviations: MAD, mean absolute deviation; MSE, mean squared error; SD, standard deviation.

TABLE 11 Valuation rule—Large differences in growth rates

This table reports statistics of a subsample of pricing errors. Pricing errors resulting from observations whose absolute value of the difference between the target firm's growth rate and the average growth rate of comparable firms fell above the 80% percentile of the growth rate difference distribution when $n = 5$. Statistics are calculated for a different number of comparable firms—$n = 2, 5, 10, 20, 30, 40, 50$, and *All*—and averaged across all observations of the subsample.

n	*Mean* (%)	*Median* (%)	*SD* (%)	*MAD* (%)	*MSE* (%)	25%–75% (%)	10%–90% (%)	5%–95% (%)	Observations
2	4.9	−1.7	48.5	31.2	23.8	41.8	99.1	143.3	249,051
5	1.2	−3.1	41.0	27.5	16.8	37.7	86.7	123.7	249,051
10	−2.4	−4.6	37.4	26.2	14.0	36.5	82.6	115.9	249,051
20	−5.2	−5.6	35.8	26.0	13.1	38.3	83.2	111.6	249,051
30	−6.3	−5.8	34.7	25.5	12.4	37.6	80.8	109.0	249,051
40	−6.7	−6.2	34.5	25.5	12.3	37.1	81.8	110.6	249,051
50	−5.6	−5.1	33.3	24.5	11.4	34.3	81.0	108.3	249,051
All	−8.9	−8.4	35.6	27.7	13.5	41.8	85.7	115.0	249,051

Abbreviations: MAD, mean absolute deviation; MSE, mean squared error; SD, standard deviation.

4.3 | Improving small sample valuations

Our analysis suggests that problems with sample valuations are caused when there is a likelihood of extreme valuation errors. Motivated by this, we subdivide the full sample in two subsamples and undertake additional analysis. We create the subsamples based on the difference between the growth rate of the target firm, g, and the average growth rate of the comparable firms, using $n = 5$ to measure this difference. The main subsample consists of that 80% of the valuations with the smallest absolute difference in these growth rates. In other words, we segment the sample at the 80th percentile of this difference in growth rates. This corresponds to an absolute difference in growth rates of 0.92%. Thus the test is essentially the same as segmenting the sample into those valuations where the average forecast growth rate of the five comparable firms is within 1% of the growth rate of the target firm, and those where this difference is greater. The former case has a frequency of 80% in our sample and the latter case a frequency of 20%.

Tables 10 and 11 show the accuracy measures for these two samples. Table 10 is for the sample with close growth rates and Table 11 for the rest of the sample. The tables show that the problem with small samples lies primarily in the cases when there is a large difference between the growth rates of the comparable group and the company being valued. In particular, in Table 11, where there is a large growth difference, the MSE for small sample valuations is much higher than the MSE for large sample valuations in those cases (16.8% for $n = 5$ vs. 13.5% for $n = All$). In contrast, when the growth rates are similar, Table 10 shows that a valuation with $n = 5$ is essentially as accurate as one which uses all firms in an industry.

Thus valuations using five comparable firms are as accurate, on average, as those using a large number of firms if the firms used are from the same industry, if they are chosen to be the firms with expected growth rates closest to the target firm, and if their average growth rate is within 1% of the target firm's growth rate. This, unsurprisingly, is a close approximation to the rule practitioners use when carrying out valuations based on multiples. However, because this rule is very simple and automatic, it is likely that the rule can be improved by incorporating other considerations, whether based on an extended automatic rule or judgement.

5 | CONCLUSIONS

Typical practitioner valuations use small numbers of comparable firms. Our study provides evidence that using about five comparables is optimal when the comparable firms used are those from the same industry with expected growth rates closest to the target firm, and if their average growth rate is within 1% of the target firm's growth rate. Adding more comparables to the valuation has the benefit of adding more information, but the cost of adding more noise. We show theoretically that this trade-off can go either way. We show empirically that beyond $n = 5$ there is, on average, a net benefit of adding more comparables only in the relatively small number of cases where it is not possible to find five comparables with an average growth rate close to the target firm's growth rate.

Our conclusions are based on simple automatic rules for selection of comparables and weighting of the evidence they contain. If more complex automatic rules or judgemental rules can be used to select comparable firms, the weighting scheme, or even the value drivers used, it may be possible to improve the relative performance of small sample valuations even further. We leave this for further research.

Given that there is no "one-size-fits-all" procedure for choosing comparable firms, in practice judgement beyond standard criteria is generally used in selecting comparable companies. For example, how should one choose the right set of companies for valuing Google or Amazon? What industry is Google even in? Similarly, whether adding more firms is beneficial or not is highly dependent on the nature of the firm being valued. If research could find a solution to this problem that can be described by complex automated rules it would be a major next step in the use of multiples.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the authors with the permission of a third party (i.e., all relevant Databases used in the study).

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How to cite this article: Cooper, I. & Lambertides, N. (2022). Optimal equity valuation using multiples: The number of comparable firms. *European Financial Management*, 1–29. https://doi.org/10.1111/eufm.12405

APPENDIX 1: THE EFFECT OF INCREASING THE NUMBER OF COMPARABLE FIRMS

Consider a target firm, i, being valued using multiples. We show how the error in a valuation varies as we increase the number of comparable firms used. We demonstrate the effect using a generalized procedure based on a weighted average of the multiples of the comparable firms j. For sample size n, the estimated multiple for firm i is a weighted average of the multiples with weights w_j:

$$\widehat{M}_n = \sum_{j=1}^{n} w_j M_j, \quad \text{with} \quad \sum_{j=1}^{n} w_j = 1, \tag{A1}$$

where M_j is the multiple for comparable firm j. We measure the error in the estimated value as[8]

$$\hat{\varepsilon}_n = \widehat{M}_n - M_i. \tag{A2}$$

This has variance $V_n = VAR(\hat{\varepsilon}_n)$. We are interested in how the accuracy of the valuation, measured by V_n, varies as we increase n.

Suppose that we add to the sample the $(n + 1)$th closest comparable firm, with weight w_{n+1}. We preserve the relative weights of the other firms.[9] The new estimate of the multiple is

$$\begin{aligned} \widehat{M}_{n+1} &= (1 - w_{n+1})\widehat{M}_n + w_{n+1}M_{n+1} \\ &= \widehat{M}_n - w_{n+1}(\widehat{M}_n - M_{n+1}) \end{aligned} \tag{A3}$$

and the new error

$$\begin{aligned} \hat{\varepsilon}_{n+1} &= \widehat{M}_{n+1} - M_i \\ &= \widehat{M}_n - w_{n+1}(\widehat{M}_n - M_{n+1}) - M_i. \end{aligned} \tag{A4}$$

The new error variance V_{n+1} is

$$\begin{aligned} V_{n+1} = VAR(\widehat{M}_n - M_i) + w_{n+1}^2 VAR(\widehat{M}_n - M_{n+1}) \\ - 2w_{n+1}COV(\widehat{M}_n - M_{n+1}, \widehat{M}_n - M_i). \end{aligned} \tag{A5}$$

The first term on the right-hand side is equal to V_n, so

$$\begin{aligned} V_{n+1} - V_n = w_{n+1}^2 VAR(\widehat{M}_n - M_{n+1}) \\ - 2w_{n+1}COV(\widehat{M}_n - M_{n+1}, \widehat{M}_n - M_i). \end{aligned} \tag{A6}$$

The condition for the addition of the extra comparable firm to reduce the error variance is

$$V_{n+1} - V_n < 0. \tag{A7}$$

Rearranging and assuming a nonzero weight w_{n+1}, this condition is

$$w_{n+1} < 2\frac{COV(\widehat{M}_n - M_{n+1}, \widehat{M}_n - M_i)}{VAR(\widehat{M}_n - M_{n+1})}. \tag{A8}$$

[8]Alternatively, we could measure the proportional error. This will vary in the same way.

[9]For example, if $n = 3$ we would have: $\widehat{M}_3 = \sum_{j=1}^{3} w_j M_j = w_1M_1 + w_2M_2 + w_3M_3$. Let $\overline{w}_4$ be the weight of the fourth multiple to be included, M_4, and $\overline{w}_\iota$, $\iota \in \{1, 2, 3\}$, the new weights of the previously included multiples. We arrive at (A3) by rewriting $\overline{w}_\iota = (1 - \overline{w}_4)w_i$, $\iota \in \{1, 2, 3\}$, and rearranging terms.

The right-hand side of this expression is a measure of the incremental information about M_i that is contained in M_{n+1}. The condition says that the weight given to M_{n+1} must not be too great relative to the incremental information it contains.

The trade-off involved in adding an extra comparable is this. If the extra firm contains little incremental information about the valuation, it should be assigned a low weight. If the weighting scheme gives it too high a weight, it will add mainly noise to the valuation and reduce the weight given to more informative multiples. The correct weight is uncertain. Therefore, if the incremental information in the extra firm is too low, it is best simply to give it a zero weight. The point where this happens is the optimal sample size. If this occurs for a sample less than the entire industry, it will be better not to use all the firms in the industry.

The above analysis is based on using the error variance as the accuracy criterion and a weighted average of comparable multiples as the valuation method. However, the same basic trade-off applies to any accuracy criterion and weighting method.

APPENDIX 2: THE OPTIMAL WEIGHTING SCHEME

Let b_{n+1} be the slope coefficient from a regression of $\widehat{M}_n - M_{n+1}$, the incremental contribution from the $(n+1)$th firm, on $\widehat{M}_n - M_i$, the estimation error from using n comparable firms. Then the condition (A8) in Appendix 1 is

$$w_{n+1} < 2b_{n+1}. \tag{A9}$$

This condition states that adding an extra comparable reduces the error if w_{n+1} is not too large relative to b_{n+1}. Even if there is extra information in M_{n+1}, it must not be given too much weight, because doing so will crowd out the information in the other multiples.

The regression coefficient b_{n+1} is high if the extra information in M_{n+1} is highly correlated with the error $\widehat{M}_n - M_i$, if the variance of the error $\widehat{M}_n - M_i$ is high, and the extra information in M_{+1} has low variance.

We can use the formula for the incremental variance (A5) to derive the optimal value for w_{n+1}:

$$\frac{\partial(\widehat{V}_{n+1} - \widehat{V}_n)}{\partial w_{n+1}} = 2w_{n+1}VAR(\widehat{M}_n - M_{n+1}) - 2w_{n+1}COV(\widehat{M}_n - M_{n+1}, \widehat{M}_n - M_i). \tag{A10}$$

Setting this equal to zero yields the optimal weight:

$$w_{n+1}^{*} = b_{n+1}. \tag{A11}$$

Thus the optimal weight to give the $(n+1)$th multiple can be estimated by running a regression of the variable on $\widehat{M}_n - M_{n+1}$ on the variable on $\widehat{M}_n - M_i$. The slope coefficient of this regression is an estimate of the optimal weight for M_{n+1}.

This estimated value also illustrates whether other weighting schemes are placing too much weight on M_{n+1}. The addition of an extra comparable reduces the error if condition (A8) is fulfilled:

$$w_{n+1} < 2w_{n+1}^{*}. \tag{A12}$$

If the actual weight is less than twice the optimal weight, the addition of the extra comparable reduces the error. If the actual weight is greater than twice the optimal weight, the addition of the extra comparable increases the error.

Note that if the amount of additional information contained in an incremental multiple is known, as measured by w^*, extra multiples should always be added. As long as the weight is correct it will always improve accuracy. However, the optimal weight cannot be observed, but only estimated. Therefore, there will always be error in this procedure. As a result, if there is uncertainty about the optimal weight the multiple may receive too high a weight and reduce accuracy. Also, the optimal weight could be negative. In that case, any positive weight will reduce accuracy.

APPENDIX 3: LINEAR REGRESSION AS A WEIGHTING SCHEME

Consider a characteristic, y_i, that linearly affects a given multiple. This could be a vector, but here it is a scalar. Consider the regression of M_i on y_i:

$$M_i = \hat{a} + \widehat{\beta} y_i + e_i, \tag{A13}$$

where $\hat{a} = \overline{M} - \widehat{\beta}\overline{y}$.

The regression estimate of the target multiple involves inserting the target characteristic, y_i, in (A13):

$$\widehat{M_i} = \hat{a} + \widehat{\beta} y_i = \overline{M} + \widehat{\beta}(y_i - \overline{y}). \tag{A14}$$

Using the OLS definition of $\widehat{\beta}$, the estimated multiple can be shown to be given by

$$\widehat{M_i} = \sum_{j=1}^{n} w_j M_j, \tag{A15}$$

where

$$w_j = \left[\frac{1}{N} + \frac{(y_j - \overline{y})(y_i - \overline{y})}{\sum_{j=1}^{n}(y_j - \overline{y})^2}\right]. \tag{A16}$$

Thus the regression method is a weighted average, with the weights given by (A16).