Appendix 1: Proofs of Propositions

Proof of Proposition 1: If there is no upcoding or cherry picking the profit of provider i is given by

$$\pi_i(c_i,\delta_i,0,0)=\lambda q\left[h(p_{Mi}-\delta_i-c_i)+(1-h)(p_{mi}-c_i)\right]-R_c(c_i)-R_\delta(\delta_i)+T_i$$

(see Equation (1)). Since the provider's choice of c_i and δ_i does not affect the reimbursement received, and this is true irrespective of the number of DRGs used by the HO, the profit-maximizing choice of the provider is given by

$$-\frac{d}{dc}R_c(c_i)=\lambda q,\ -\frac{d}{d\delta}R_\delta(\delta_i)=\lambda qh.$$

Any values of c_i and δ_i that satisfy these $2\times N$ conditions are equilibria. Naturally, if all providers choose $c_i=c^*$ and $\delta_i=\delta^*$ the conditions above are identical for all providers, and in fact reduce to the first order conditions of the welfare-maximization problem. In addition, the transfer payment ensures that all providers break even. Therefore, the first-best investment decisions constitute a symmetric Nash equilibrium that achieves first-best investment in cost reduction. Furthermore, since $R_c''>0$ and $R_\delta''>0$, the symmetric equilibrium is unique and no asymmetric equilibrium exists.□

Proof of Proposition 2: In the absence of upcoding ($\bar{\alpha}=0$), under the yardstick competition scheme with a single DRG, the profit of provider i is given by

$$\pi_i(c_i,\delta_i,0,\gamma_i)=\lambda q\left[(1-h\gamma_i)\bar{c}_i-\delta_i h(1-\gamma_i)-c(1-h\gamma_i)\right]-R_c(c_i)-R_\delta(\delta_i)+\bar{R}_i,$$

where $\bar{c}_i=\frac{1}{N-1}\sum_{j\neq i}\left[c_j+\delta_j\frac{h(1-\gamma_j)}{1-h\gamma_j}\right]$ and $\bar{R}_i:=\frac{1}{N-1}\sum_{j\neq i}\left[R_c(c_j)+R_\delta(\delta_j)\right]$ as defined in §4. The derivatives of the profit function of provider i are given by:

$$\frac{\partial}{\partial\gamma_i}\pi_i = \lambda qh(c_i+\delta_i-\bar{c}_i), \tag{9}$$

$$\frac{\partial}{\partial c_i}\pi_i = -\frac{d}{dc}R_c(c_i)-\lambda q(1-h\gamma_i), \tag{10}$$

$$\frac{\partial}{\partial\delta_i}\pi_i = -\frac{d}{d\delta}R_\delta(\delta_i)-\lambda qh(1-\gamma_i). \tag{11}$$

In any equilibrium outcome the last two conditions will be equal to zero for all providers. Otherwise the provider for whom one of these conditions is not zero could increase their profit by changing c_i or δ_i. Furthermore, the conditions above imply that any two providers with $\gamma_i=\gamma_j$ will have the same costs $c_i=c_j$ and $\delta_i=\delta_j$. Furthermore, since $R_c''>0$ and $R_\delta''>0$, if a provider has $\gamma_i>\gamma_j$ then $c_i>c_j$, $\delta_i>\delta_j$ and the converse is also true – if a provider has costs such that $\delta_i>\delta_j$ (or $c_i>c_j$) then $\gamma_i>\gamma_j$. Furthermore, since $0\leq\gamma_i\leq\bar{\gamma}$, from (10) and (11), provider costs must satisfy $c^*\leq c_i\leq c^{e1},\delta^*\leq\delta_i\leq\delta^{e1}$.

Consider a symmetric equilibrium where all providers choose (γ, c, δ). In such a symmetric equilibrium then $\frac{\partial}{\partial\gamma_i}\pi_i = \lambda q h \delta \frac{1-h}{1-h\gamma} > 0$. Therefore, $\gamma = \bar{\gamma}$ which also implies that $c = c^{e1}, \delta = \delta^{e1}$ would constitute a candidate for a symmetric equilibrium. Furthermore, since $R_c'' > 0$ and $R_\delta'' > 0$ the symmetric equilibrium candidate is unique. We note that in this symmetric equilibrium candidate, the transfer payment ensures that all providers break even (i.e., make a profit of zero). For this to be an equilibrium outcome no provider must find it profitable to unilaterally deviate to a different strategy. Consider the payoff of one provider (labeled j) that chooses to deviate to a different strategy $(\gamma_j, c_j, \delta_j)$ when all other providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$. The derivative of the profit function of provider j with respect to γ_j is given by $\frac{\partial}{\partial\gamma_i}\pi_i = \lambda q h (c_j + \delta_j - c^{e1} - \delta^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}})$. Since $c_j + \delta_j \geq c^* + \delta^*$ then if $c^* + \delta^* > c^{e1} + \delta^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$ (i.e., if cherry-picking-best costs are not too extreme) which we have assumed to be the case, then choosing $\gamma_j < \bar{\gamma}$ cannot be a profitable deviation. Therefore, $(\bar{\gamma}, c^{e1}, \delta^{e1})$ is the unique symmetric equilibrium.

We will next investigate the existence of asymmetric equilibria. If an asymmetric equilibrium exists, then at least one provider (labeled j) would have the highest $(\gamma_j, c_j, \delta_j)$ (i.e., $\gamma_j \geq \gamma_i$ for all i and the inequality is strict for at least one i, and similarly for c_j, δ_j). Therefore, $c_j + \delta_j > \bar{c}_j$ (recall that $\bar{c}_j$ is the average cost of all other providers and at least some of these providers will have lower costs). From (9), this implies that $\gamma_j = \bar{\gamma}$, which also implies that the costs $c_j = c^{e1}, \delta_j = \delta^{e1}$. Therefore, in any asymmetric equilibrium, some providers (at least one) will choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$. The rest of the providers will have $\gamma_k < \bar{\gamma}, c_k < c^{e1}, \delta_k < \delta^{e1}$. For this to be an equilibrium outcome, from (9) it must be the case that $c_k + \delta_k - \bar{c}_k \leq 0$. If it was not the case then $\frac{\partial}{\partial\gamma_k}\pi_k > 0$, implying that the provider's profit could increase by increasing γ_k which is a contradiction. Consider a provider with $c_k + \delta_k - \bar{c}_k = 0$. The profit of this provider can be written as $[\lambda q[-\delta_k h - c_k] - R_c(c_k) - R_\delta(\delta_k)] - \lambda q \gamma_k h(\bar{c}_k - \delta_k - c_k) + C$, where C is an exogenous constant. Note that the first term is independent of γ_i and is maximized at c^* and δ^*. Consider a deviation from $(\gamma_k, c_k, \delta_k)$ to $(0, c^*, \delta^*)$. This deviation does not affect the second term (it is zero under both strategies) and increases the first term (the first term is maximized at c^*, δ^*). Therefore this deviation is profitable. This suggests than no provider with costs $c_k + \delta_k - \bar{c}_k = 0$ can exist, which implies that any provider with cost other that c^{e1}, δ^{e1} must satisfy $c_k + \delta_k - \bar{c}_k < 0$, which implies $\frac{\partial}{\partial\gamma_k}\pi_k < 0$. Therefore, this provider must choose $(0, c^*, \delta^*)$ (any other choice of $\gamma_k > 0$ cannot be an equilibrium outcome as provider k can increase their profit by reducing γ). Therefore the condition $c_k + \delta_k - \bar{c}_k < 0$ becomes $c^* + \delta^* < \bar{c}_k$, and note that $\bar{c}_k > c^{e1} + \delta^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$. In words, in any asymmetric equilibrium, providers will divide in two groups, θ_0 providers will not drop any patients and choose to operate at a cost as low as first best $(0, c^*, \delta^*)$, and $N - \theta_0$ providers will drop the maximum number of patients and operate at a higher cost compared to first best $(\bar{\gamma}, c^{e1}, \delta^{e1})$.

Consider one of the θ_0 low-cost providers. The fee per patient treated by this provider will be given by $\bar{c}_k = \frac{\theta_0-1}{N-1}(c^* + h\delta^*) + \frac{N-\theta_0}{N-1}(c^{e1} + h\frac{1-\bar{\gamma}}{1-\bar{\gamma}h}\delta^{e1})$. The condition $c_k + \delta_k - \bar{c}_k < 0$ implies that $c^* + \delta^* < \nu(c^* + h\delta^*) + (1-\nu)(c^{e1} + h\frac{1-\bar{\gamma}}{1-\bar{\gamma}h}\delta^{e1})$, for $\nu = \frac{\theta_0-1}{N-1}$. This is a contradiction as $c^* + \delta^* > (c^* + h\delta^*)$ and since we have assume that cherry-picking-best costs are not extreme, $c^* + \delta^* > c^{e1} + h\frac{1-\bar{\gamma}}{1-\bar{\gamma}h}\delta^{e1}$. Therefore, an asymmetric equilibrium cannot exist. □

Proof of Proposition 3: In the absence of upcoding ($\bar{\alpha} = 0$), under the yardstick competition scheme with two DRGs, the profit of provider i is given by

$$\pi_i(c_i, \delta_i, 0, \gamma_i) = \lambda q\left[h(1-\gamma_i)(\bar{c}_{Mi} - \delta_i - c_i) + (1-h)(\bar{c}_{mi} - c_i)\right] - R_c(c_i) - R_\delta(\delta_i) + \bar{R}_i,$$

where $\bar{c}_{Mi} := \frac{1}{N-1}\sum_{j\neq i}[c_j + \delta_j]$, $\bar{c}_{mi} := \frac{1}{N-1}\sum_{j\neq i} c_j$, and $\bar{R}_i := \frac{1}{N-1}\sum_{j\neq i}[R_c(c_j) + R_\delta(\delta_j)]$ as defined in §4. The derivatives of the profit function of provider i are given by:

$$\frac{\partial}{\partial\gamma_i}\pi_i = \lambda qh(c_i + \delta_i - \bar{c}_{Mi}), \tag{12}$$

$$\frac{\partial}{\partial c_i}\pi_i = -\frac{d}{dc}R_c(c_i) - \lambda q(1 - h\gamma_i), \tag{13}$$

$$\frac{\partial}{\partial\delta_i}\pi_i = -\frac{d}{d\delta}R_\delta(\delta_i) - \lambda qh(1-\gamma_i). \tag{14}$$

In any equilibrium outcome, the last two conditions will be equal to zero for all providers. Otherwise the provider for whom one of these conditions is not zero could increase their profit by changing c_i or δ_i. Furthermore, the conditions above imply that any two providers with $\gamma_i = \gamma_j$ will have the same costs $c_i = c_j$ and $\delta_i = \delta_j$. Since $R_c'' > 0$ and $R_\delta'' > 0$, if a provider has $\gamma_i > \gamma_j$ then $c_i > c_j$, $\delta_i > \delta_j$ and the converse is also true – if a provider has costs such that $\delta_i > \delta_j$ (or $c_i > c_j$) then $\gamma_i > \gamma_j$. Furthermore, since $0 \le \gamma_i \le \bar{\gamma}$, from (13) and (14), provider costs satisfy $c^* \le c_i \le c^{e1}, \delta^* \le \delta_i \le \delta^{e1}$.

Clearly, in any symmetric equilibrium $\frac{\partial}{\partial\gamma_i}\pi_i = 0$. Therefore, any $\gamma_i = \gamma$ where $0 \le \gamma \le \bar{\gamma}$ along with $c_i = c$ and $\delta_i = \delta$ such that $-\frac{d}{dc}R_c(c) = \lambda q(1-h\gamma)$, $-\frac{d}{d\delta}R_\delta(\delta) = \lambda qh(1-\gamma)$ would be a candidate for a symmetric equilibrium outcome. For any such γ, the values of c and δ are unique and are increasing in γ (since $R_c'' > 0$ and $R_\delta'' > 0$). Now consider any such symmetric equilibrium candidate where $\gamma > 0$ and consider the profit of provider i which will be given by $\pi_i = [-\lambda q(h\delta_i + c_i) - R_c(c_i) - R_\delta(\delta_i)] - \lambda qh\gamma_i(\bar{c}_{Mi} - \delta_i - c_i) + C$, where C is an exogenous constant. Note that the first term is independent of γ_i and is maximized at c^* and δ^*. Consider a deviation from the symmetric equilibrium candidate (γ, c, δ) to $(0, c^*, \delta^*)$. This deviation will be profitable for provider i as it would increase the first term and leave the second term unaffected (it is zero under both strategies). Therefore, $\gamma > 0$ cannot be a symmetric equilibrium outcome. Therefore, the only symmetric equilibrium candidate that survives is $(0, c^*, \delta^*)$. For this to be an equilibrium outcome, no provider must find it profitable to unilaterally deviate to a different strategy. Consider the payoff of one provider (labeled j) that chooses to deviate to a different strategy $(\gamma_j, c_j, \delta_j)$ when all other providers choose $(0, c^*, \delta^*)$. The derivative of the profit function of provider j with respect to γ_j is given by $\frac{\partial}{\partial \gamma_j}\pi_j = \lambda q h(c_j + \delta_j - c^* - \delta^*) > 0$. Therefore, this provider can improve their profit by deviating to a strategy where they drop some patients (i.e., $\gamma_j > 0$) and invest less in cost reduction, (i.e., $c_j > c^*, \delta_j > \delta^*$). Therefore no symmetric equilibrium outcome can exist.

We will now turn to asymmetric equilibria. In any asymmetric equilibrium, at least one provider (labeled j) would have the highest $(\gamma_j, c_j, \delta_j)$ (i.e., $\gamma_j \geq \gamma_i$ for all i and the inequality is strict for at least one i, and similarly for c_j, δ_j). Therefore, $c_j + \delta_j > \bar{c}_{Mj}$ (recall that $\bar{c}_{Mj}$ is the average cost of all other providers and at least some of these providers will have lower costs). From (12), this implies that $\gamma_j = \bar{\gamma}$, which also implies that the costs $c_j = c^{e1}, \delta_j = \delta^{e1}$. Conversely, at least one provider (labeled k) will have the lowest c_k, δ_k, γ_k (i.e., $c_k \leq c_i$ for all i and the inequality is strict for at least one i, and similarly for δ_k, γ_k). Therefore, $c_k + \delta_k < \bar{c}_{Mk}$ (recall that $\bar{c}_{Mk}$ is the average cost of all other providers and at least some of these providers will have higher costs). This implies that this provider will choose $\gamma_k = 0$ and $c_k = c^*, \delta_k = \delta^*$. Furthermore, consider a provider with costs other than c^* or c^{e1}, which we label as provider s. This provider must have costs c_s and δ_s such that $c_s + \delta_s = \bar{c}_{Ms}$ and a corresponding γ_s. Consider the profit of this provider, which can be written as $\pi_s = [-\lambda q(h\delta_s + c_s) - R_c(c_s) - R_\delta(\delta_s)] - \lambda q h \gamma_s(\bar{c}_{Ms} - \delta_s - c_s) + C$, where C is an exogenous constant. Note that the first term is independent of γ_s and is maximized at $c_s = c^*$ and $\delta_s = \delta^*$. Consider a deviation from $(\gamma_s, c_s, \delta_s)$ to $(0, c^*, \delta^*)$. This deviation does not affect the second term (it is zero under both strategies) and increases the first term. Therefore this deviation is profitable. This suggests than no provider with costs γ_s, c_s, δ_s can exist. In words, in any asymmetric equilibrium, providers will divide in two groups: θ_1 providers will not drop any patients and choose to operate at a cost as low as first best $(0, c^*, \delta^*)$ and $N - \theta_1$ providers will drop the maximum number of patients and operate at a higher cost compared to first best $(\bar{\gamma}, c^{e1}, \delta^{e1})$.

For such an asymmetric equilibrium to exist, the profit of the θ_1 low-cost providers and the profit of the $N - \theta_1$ high-cost providers need to be non-negative. Consider one of the θ_1 low-cost providers. The fee for providing the major treatment is given by $\bar{c}_{Mk} = \frac{N-\theta_1}{N-1}(\delta^{e1} + c^{e1}) + \frac{\theta_1 - 1}{N-1}(\delta^* + c^*)$, the minor treatment is given by $\bar{c}_{mk} = \frac{N-\theta_1}{N-1}c^{e1} + \frac{\theta_1-1}{N-1}(c^*)$, and the transfer payment they will receive is given by $\bar{T}_k = \frac{N-\theta_1}{N-1}(R_\delta(\delta^{e1}) + R_c(c^{e1})) + \frac{\theta_1-1}{N-1}(R_\delta(\delta^*) + R_c(c^*))$. After some algebra, the profit of the efficient provider can be written as $\frac{N-\theta_1}{N-1}v_1$, where

$$v_1 := \lambda q(h(\delta^{e1} + c^{e1} - \delta^* - c^*) + (1-h)(c^{e1} - c^*)) + R_c(c^{e1}) + R_\delta(\delta^{e1}) - R_c(c^*) - R_\delta(\delta^*).$$

Note that the expression $-\lambda q(h(\delta + c) + (1-h)c) - R_c(c) - R_\delta(\delta))$ is maximized at $c = c^*$ and $\delta = \delta^*$, therefore $v_1 > 0$. Similarly, the profit of one of the $N - \theta_1$ high cost providers can be written as $\frac{\theta_1}{N-1}u_1$, where

$$u_1 := \lambda q(h(1-\bar{\gamma})(\delta^* + c^* - \delta^{e1} - c^{e1}) + (1-h)(c^* - c^{e1})) + R_c(c^*) + R_\delta(\delta^*) - R_c(c^{e1}) - R_\delta(\delta^{e1}).$$

Note that the expression $-\lambda q(h(1-\bar{\gamma})(\delta+c)+(1-h)c)-R_c(c)-R_\delta(\delta))$ is maximized at $c=c^{e1}$ and $\delta=\delta^{e1}$, therefore $u_1>0$.

We will next determine the value of θ_1. For this to be an equilibrium outcome, it must be the case that the profit one of the low-cost providers makes by being low cost is greater than the profit they would make if they deviated to being a high-cost provider. After some algebra, this condition can be written as

$$\frac{N-\theta_1}{N-1}v_1 \geq \frac{\theta_1-1}{N-1}u_1.$$

Conversely, the profit of one of the high-cost providers must be greater than the payoff they would make if they deviated to being an low-cost provider. After some algebra, this condition reduces to

$$\frac{\theta_1}{N-1}u_1 \geq \frac{N-\theta_1-1}{N-1}v_1.$$

Together the last two inequalities imply that the number of efficient providers must satisfy

$$\frac{(N-1)v_1}{v_1+u_1} \leq \theta_1 \leq \frac{Nv_1+u_1}{v_1+u_1}.$$

Note that this interval contains exactly 1 integer as the difference $\frac{Nv_1+u_1}{v_1+u_1}-\frac{(N-1)v_1}{v_1+u_1}=1$. Furthermore, since $\frac{Nv_1+u_1}{v_1+u_1}<N$, this integer is always less than N. □

Proof of Proposition 4: Under the yardstick competition scheme with two DRGs, the profit of provider i is given by

$$\begin{aligned}\pi_i(c_i,\delta_i,\alpha_i,\gamma_i) &= \lambda q\{[h(1-\gamma_i)+(1-h)\alpha_i]\bar{c}_{Mi}+(1-h)(1-\alpha_i)\bar{c}_{mi} \\ &\quad - [(h(1-\gamma_i)+(1-h)\alpha_i\beta)\delta_i+(1-h\gamma_i)c_i]\}-R_c(c_i)-R_\delta(\delta_i)+\bar{R}_i,\end{aligned}$$

where $\bar{c}_{Mi} := \frac{1}{N-1}\sum_{j\neq i}\left[c_j+\delta_j\frac{h(1-\gamma_j)+(1-h)\alpha_j\beta}{h(1-\gamma_j)+(1-h)\alpha_j}\right]$, $\bar{c}_{mi} := \frac{1}{N-1}\sum_{j\neq i}c_j$, and $\bar{R}_i := \frac{1}{N-1}\sum_{j\neq i}[R_c(c_j)+R_\delta(\delta_j)]$ as defined in §4. The derivatives of the profit function of provider i are given by:

$$\frac{\partial}{\partial\alpha_i}\pi_i = \lambda q(1-h)(\bar{c}_{Mi}-\bar{c}_{mi}-\beta\delta_i), \tag{15}$$

$$\frac{\partial}{\partial\gamma_i}\pi_i = \lambda qh(c_i+\delta_i-\bar{c}_{Mi}), \tag{16}$$

$$\frac{\partial}{\partial c_i}\pi_i = -\frac{d}{dc}R_c(c_i)-\lambda q(1-h\gamma_i), \tag{17}$$

$$\frac{\partial}{\partial\delta_i}\pi_i = -\frac{d}{d\delta}R_\delta(\delta_i)-\lambda q[h(1-\gamma_i)+(1-h)\alpha_i\beta]. \tag{18}$$

In any equilibrium outcome, the last two conditions will be equal to zero for all providers. Otherwise the provider for whom one of these conditions is not zero could increase their profit by changing c_i or δ_i. Furthermore, since $0\leq\gamma_i\leq\bar{\gamma}$, $0\leq\alpha_i\leq\bar{\alpha}$, and $R_c''>0$ and $R_\delta''>0$, from (17) and (18) provider costs satisfy $c^* \leq c_i \leq c^{e1}, \delta^{e3} \leq \delta_i \leq \delta^{e1}$, where δ^{e3} is the unique solution to $-\frac{d}{d\delta}R_\delta(\delta_i) = \lambda q[h + (1-h)\bar{\alpha}\beta]$.

Turning to (15), this condition will be positive for provider i if $\frac{1}{N-1}\sum_{j\neq i}\left[\delta_j \frac{h(1-\gamma_j)+(1-h)\alpha_j\beta}{h(1-\gamma_j)+(1-h)\alpha_j}\right] > \beta\delta_i$. Note that the LHS is minimized when all providers other than i choose $\gamma_j = 0, \alpha_j = \bar{\alpha}, \delta_j = \delta^{e3}$. The RHS is maximized if provider i chooses $\delta_i = \delta^{e1}$. Therefore, a sufficient condition for (15) to be positive for all providers i is

$$\beta < \frac{h\delta^{e3}}{h\delta^{e1} + (\delta^{e1} - \delta^{e3})(1-h)\bar{\alpha}}, \tag{19}$$

which we have assumed holds. Therefore, in any equilibrium outcome, all providers will choose $\alpha_i = \bar{\alpha}$. Furthermore, (17) and (18) imply that any two providers with $\gamma_i = \gamma_j$ will have the same costs $c_i = c_j$ and $\delta_i = \delta_j$. Since $R''_c > 0$ and $R''_\delta > 0$, if a provider has $\gamma_i > \gamma_j$ then $c_i > c_j$, $\delta_i > \delta_j$ and the converse is also true – if a provider has costs such that $\delta_i > \delta_j$ (or $c_i > c_j$) then $\gamma_i > \gamma_j$.

Consider a symmetric equilibrium such that $\alpha_i = \alpha$, $\gamma_i = \gamma$, $c_i = c$ and $\delta_i = \delta$ for all i. In any symmetric equilibrium $\bar{c}_{Mi} - \bar{c}_{mi} - \beta\delta = \delta\frac{h(1-\gamma)(1-\beta)}{h(1-\gamma)+(1-h)\alpha} > 0$, which implies $\frac{\partial}{\partial\alpha_i}\pi_i > 0$. Therefore, in any symmetric equilibrium $\alpha_i = \bar{\alpha}$ for all i. This, implies that $c + \delta - \bar{c}_{Mi} = \frac{(1-h)(1-\beta)\delta\bar{\alpha}}{h(1-\gamma)+(1-h)\bar{\alpha}} > 0$, which also implies that $\frac{\partial}{\partial\gamma_i}\pi_i > 0$. Therefore in any symmetric equilibrium $\gamma = \bar{\gamma}$ for all i. The values of $c = c^{e1}$ and $\delta = \delta^{e2}$ are the solution to $-\frac{d}{dc}R_c(c) = \lambda q(1-h\bar{\gamma})$, $-\frac{d}{d\delta}R_\delta(\delta) = \lambda q[h(1-\bar{\gamma}) + (1-h)\bar{\alpha}\beta]$), and they are unique (since $R''_c > 0$ and $R''_\delta > 0$). In addition, the transfer payment ensures that all providers break even. Furthermore, since $(1-h\bar{\gamma}) < 1$, this implies that $c^{e1} > c^*$. If $h(1-\bar{\gamma}) + (1-h)\bar{\alpha}\beta < h$ then $\delta^{e2} > \delta^*$, otherwise the opposite holds. For this to be an equilibrium outcome, no provider must find it profitable to unilaterally deviate to a different strategy. Consider the payoff of one provider (labeled j) that chooses to deviate to a different strategy $(\alpha_j, \gamma_j, c_j, \delta_j)$ when all other providers choose $(\bar{\alpha}, \bar{\gamma}, c^{e1}, \delta^{e2})$. Due to condition (7), it is not profitable to choose any $\alpha_j < \bar{\alpha}$. The derivative of the profit function of provider j with respect to γ_j is given by $\frac{\partial}{\partial\gamma_j}\pi_j = \lambda qh(c_j + \delta_j - c^{e1} - \delta^{e2}\frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}})$. Since $c_j + \delta_j \geq c^* + \delta^{e3}$ then if $c^* + \delta^{e3} \geq c^{e1} + \delta^{e2}\frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$ then choosing $\gamma_j < \bar{\gamma}$ cannot be a profitable deviation. Therefore, $(\bar{\alpha}, \bar{\gamma}, c^{e1}, \delta^{e2})$ will constitute a symmetric equilibrium. If however, $c^* + \delta^{e3} \leq c^{e1} + \delta^{e2}\frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$ then it may be profitable to deviate to $(\bar{\alpha}, 0, c^*, \delta^{e3})$. For this to be the case, it must be the case that the profit of the provider who deviates to $(\bar{\alpha}, 0, c^*, \delta^{e3})$ when all other providers choose $(\bar{\alpha}, \bar{\gamma}, c^{e1}, \delta^{e2})$ is non-negative (as the profit associated with not deviating is zero). This condition can be written as $v_2 \geq 0$, where

$$\begin{aligned} v_2 := \lambda q &\left((h + (1-h)\bar{\alpha})\frac{(h(1-\bar{\gamma}) + (1-h)\bar{\alpha}\beta)}{h(1-\bar{\gamma}) + (1-h)\bar{\alpha}}\delta^{e2} - (h + (1-h)\bar{\alpha}\beta)\delta^{e3} + c^{e1} - c^*\right) \\ &+ R_c(c^{e1}) + R_\delta(\delta^{e2}) - R_c(c^*) - R_\delta(\delta^{e3}). \end{aligned}$$

We will then consider asymmetric equilibria. Due to condition (7), in any asymmetric equilibrium, all providers will choose $\alpha_j = \bar{\alpha}$. At least one provider (labeled j) would have the highest $(\gamma_j, c_j, \delta_j)$

(i.e., $\gamma_j \geq \gamma_i$ for all i and the inequality is strict for at least one i, and similarly for c_j, δ_j). Therefore, $c_j + \delta_j > \bar{c}_{Mj}$ (recall that $\bar{c}_{Mj}$ is the average cost of all other providers and at least some of these providers will have lower costs). From (16), this implies that $\gamma_j = \bar{\gamma}$, which also implies that the costs $c_j = c^{e1}, \delta_j = \delta^{e2}$. Conversely, at least one provider (labeled k) will have the lowest $(\gamma_k, c_k, \delta_k)$ (i.e., $\gamma_k \leq \gamma_i$ for all i and the inequality is strict for at least one i, and similarly for c_k, δ_k). Therefore, $c_k + \delta_k < \bar{c}_{Mk}$ (recall that $\bar{c}_{Mk}$ is the average cost of all other providers and at least some of these providers will have higher costs). From (16), this implies that $\gamma_k = 0$ and $c_k = c^*, \delta_k = \delta^{e3}$. For this to be possible, it must be the case that $c^* + \delta^{e3} < c^{e1} + \delta^{e2} \frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$.

Furthermore, consider a provider with γ_s other than 0 or $\bar{\gamma}$, which we label as provider s. Due to condition (7), this provider will still have $\alpha_s = \bar{\alpha}$, and from (16) must have costs c_s and δ_s such that $c_s + \delta_s = \bar{c}_{Ms}$. Consider the profit of this provider, which can be written as $\pi_s = [-\lambda q(h\delta_s + c_s) - R_c(c_s) - R_\delta(\delta_s)] - \lambda q h \gamma_s(\bar{c}_{Ms} - \delta_s - c_s) + C$, where C is a constant that does not depend on $(\gamma_s, c_s, \delta_s)$. Note that the first term is independent of γ_s and is maximized at $c_s = c^*$ and $\delta_s = \delta^*$. Consider a deviation from $(\gamma_s, c_s, \delta_s)$ to $(0, c^*, \delta^*)$. This deviation does not affect the second term (it is zero under both strategies) and increases the first term. Therefore this deviation is profitable. This suggests than no provider with costs γ_s, c_s, δ_s can exist. In words, in any asymmetric equilibrium, providers will divide in two groups: θ_2 providers will upcode the maximum number of patients, will not drop any patients and choose to operate at relatively low costs $(\bar{\alpha}, 0, c^*, \delta^{e3})$ and $N - \theta_2$ providers that will upcode the maximum number of patients, will drop the maximum number of patients and operate at a higher cost $(\bar{\alpha}, \bar{\gamma}, c^{e1}, \delta^{e2})$.

For such an asymmetric equilibrium to exist, the profit of the θ_2 low-cost providers and the profit of the $N - \theta_2$ high-cost providers need to be non-negative. Consider one of the θ_2 low-cost providers. The fee they are paid for providing the major treatment is given by $\bar{c}_{Mk} = \frac{N-\theta_2}{N-1}(\delta^{e2} \frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}} + c^{e1}) + \frac{\theta_2-1}{N-1}(\delta^{e3} \frac{h+(1-h)\bar{\alpha}\beta}{h+(1-h)\bar{\alpha}} + c^*)$, the fee for the minor treatment is given by $\bar{c}_{mk} = \frac{N-\theta_2}{N-1} c^{e1} + \frac{\theta_2-1}{N-1} c^*$, and the transfer payment they will receive is given by $\bar{T}_k = \frac{N-\theta_2}{N-1}(R_\delta(\delta^{e2}) + R_c(c^{e1})) + \frac{\theta_2-1}{N-1}(R_\delta(\delta^{e3}) + R_c(c^*))$. The profit of this provider will be given by $\frac{N-\theta_2}{N-1} v_2$. Similar algebra shows that the profit of one of the high-cost providers will be given by $\frac{\theta_2}{N-\theta_2} u_2$, where

$$
\begin{aligned}
u_2 := &\lambda q(\delta^{e3}(h + (1-h)\bar{\alpha}\beta) \frac{h(1-\bar{\gamma}) + (1-h)\bar{\alpha}}{h + (1-h)\bar{\alpha}} - (h(1-\bar{\gamma}) + (1-h)\bar{\alpha}\beta)\delta^{e2} + (1-\bar{\gamma}h)(c^* - c^{e1})) \\
&+ R_c(c^*) + R_\delta(\delta^{e3}) - R_c(c^{e1}) - R_\delta(\delta^{e2}).
\end{aligned}
$$

Therefore, for the asymmetric equilibrium to exist, it must be the case that $v_2 \geq 0$ and $u_2 \geq 0$. Note that $u_2 > 0$. To see this, note that u_2 can be written as

$$
\begin{aligned}
u_2 = &\left[-\lambda q \left((1-\bar{\gamma}h)c^{e1} + ((1-\bar{\gamma})h + (1-h)\bar{\alpha}\beta)\delta^{e2}\right) - R_c(c^{e1}) - R_\delta(\delta^{e2})\right] \\
&+ \left[\lambda q \left((1-\bar{\gamma}h)c^* + ((1-\bar{\gamma})h + (1-h)\bar{\alpha}\beta)\delta^{e3}\right) - R_c(c^*) - R_\delta(\delta^{e3})\right]
\end{aligned}
$$

$$+ h(1-h)\frac{\bar{\alpha}\bar{\gamma}(1-\beta)}{h+(1-h)\bar{\alpha}}\delta^*.$$

Note that the expression $[-\lambda q\left[(1-\bar{\gamma}h)c+((1-\bar{\gamma})h+(1-h)\bar{\alpha}\beta)\delta\right]-R_c(c)-R_\delta(\delta)]$ is maximized at c^{e1},δ^{e2}, therefore the sum of the terms in the first two brackets is positive. The third term is also positive, which implies that $u_2>0$. The sign of v_2 will depend on model parameters.

We next determine the value of θ_2. If one of the θ_2 low-cost providers was to deviate and become a high-cost provider then their profit would be given by $\frac{\theta_2-1}{N-1}u_2$ and if one of the $N-\theta_2$ high-cost providers was to deviate and become a low-cost provider it would be $\frac{N-\theta_2-1}{N-1}v_2$. In the symmetric equilibrium it must be the case that these deviations are not profitable. Therefore, $\frac{N-\theta_2}{N-1}v_2>\frac{\theta_2-1}{N-1}u_2$ and $\frac{\theta_2}{N-1}u_2\geq\frac{N-\theta_2-1}{N-1}v_2$. These conditions imply that θ_2 satisfies $\frac{(N-1)v_2}{u_2+v_2}\leq\theta_2\leq\frac{Nv_2+u_2}{u_2+v_2}$. If $v_2\geq 0$, this interval contains exactly 1 integer as the difference between the RHS and the LHS of the inequalities is $\frac{Nv_2+u_2}{u_2+v_2}-\frac{(N-1)v_2}{u_2+v_2}=1$ and this integer is always less than N. □

Proof of Proposition 5: Under one DRG, if upcoding is possible (i.e., $\bar{\alpha}>0$) the derivative of the profit of any provider with respect to α_i is given by $\frac{\partial}{\partial\alpha_i}\pi_i=-\lambda q(1-h)\beta\delta_i<0$. Therefore, in equilibrium no provider would choose to upcode and the equilibrium outcome is identical to that presented in Proposition 3 – namely, given the condition $c^{e1}+\delta^{e1}h\frac{1-\bar{\gamma}}{1-h\bar{\gamma}}<c^*+\delta^*$ the symmetric equilibrium is characterized by all providers choosing $(0,\bar{\gamma},c^{e1},\delta^{e1})$. Note that this is equivalent to the solution that maximizes total welfare (i.e., maximizes the objective of the HO as defined in (2)) under the constraint $\gamma=\bar{\gamma}$. Under two DRGs, the equilibrium outcome is given by Proposition 5. Namely, given that CPU-best costs are comparable to upcoding best costs, the equilibrium is symmetric and characterized by all providers choosing $(\bar{\alpha},\bar{\gamma},c^{e1},\delta^{e2})$. Note that this is the solution that maximizes total welfare (i.e., maximizes the objective of the HO as defined in (2)) under the constraints $\gamma=\bar{\gamma}$ and $\alpha=\bar{\alpha}$. Note that the feasible region of this welfare-maximization problem is a subset of the feasible region of the pervious welfare-maximization problem. Therefore welfare under two-DRG symmetric equilibrium cannot be greater than the welfare under the one-DRG equilibrium.□

Proof of Proposition 6: Under this yardstick competition scheme, the profit of provider i is given by

$$\begin{aligned}\pi_i(c_i,\delta_i,\alpha_i,\gamma_i) &= \lambda q\left([h(1-\gamma_i)+(1-h)\alpha_i]\bar{c}_{Mi}+(1-h)(1-\alpha_i)\bar{c}_{mi}\right.\\ &\quad - \left.[(h(1-\gamma_i)+(1-h)\alpha_i\beta)\delta_i+(1-h\gamma_i)c_i]\right)\\ &\quad - R_c(c)-R_\delta(\delta)+\bar{R}_i+\kappa(M_i-\bar{M}_i)+\phi_i(m_i-\bar{m}_i),\end{aligned}$$

where $M_i=\lambda q(h(1-\gamma_i)+(1-h)\alpha_i)$, $m_i=\lambda q(1-h)(1-\alpha_i)$, $\kappa\geq 0$, $\phi_i-\kappa>\bar{\delta}_i-\beta\delta^{e3}$, $\bar{\delta}_i:=\bar{c}_{Mi}-\bar{c}_{mi}$, $\bar{c}_{Mi}=\frac{1}{N-1}\sum_{j\neq i}\left[c_j+\delta_j\frac{h(1-\gamma_j)+(1-h)\alpha_j\beta}{h(1-\gamma_j)+(1-h)\alpha_j}\right]$, $\bar{c}_{mi}=\frac{1}{N-1}\sum_{j\neq i}c_j$, and $\bar{R}_i=\frac{1}{N-1}\sum_{j\neq i}[R_c(c_j)+R_\delta(\delta_j)]$.

The derivatives of the profit function of provider i are given by:

$$\frac{\partial}{\partial \alpha_i}\pi_i = \lambda q(1-h)(-\beta\delta_i + \bar{\delta}_i + \kappa - \phi_i), \tag{20}$$

$$\frac{\partial}{\partial \gamma_i}\pi_i = \lambda q h(c_i + \delta_i - \bar{c}_{Mi} - \kappa), \tag{21}$$

$$\frac{\partial}{\partial c_i}\pi_i = -\frac{d}{dc}R_c(c_i) - \lambda q(1-h\gamma_i), \tag{22}$$

$$\frac{\partial}{\partial \delta_i}\pi_i = -\frac{d}{d\delta}R_\delta(\delta_i) - \lambda q[h(1-\gamma_i) + (1-h)\alpha_i\beta]. \tag{23}$$

In any equilibrium outcome, the expressions (22) and (23) have to be equal to zero for all providers. Otherwise the provider for whom one of these is not zero could increase their profit by changing c_i or δ_i. Furthermore, since $0 \le \gamma_i \le \bar{\gamma}$ and $0 \le \alpha_i \le \bar{\alpha}$, from (22) and (23) provider costs satisfy $c^* \le c_i \le c^{e1}, \delta^{e3} \le \delta_i \le \delta^{e1}$. Turning to (20), since $\phi_i - \kappa > \bar{\delta}_i - \beta\delta^{e3}$, in any equilibrium $\frac{\partial}{\partial \alpha_i}\pi_i < 0$ which implies that $\alpha_i = 0$ for all i. This implies that any two providers with $\gamma_i = \gamma_j$ will have the same costs $c_i = c_j$ and $\delta_i = \delta_j$ and since $R_c'' > 0$ and $R_\delta'' > 0$, if a provider has $\gamma_i > \gamma_j$ then $c_i > c_j$, $\delta_i > \delta_j$ and the converse is also true – if a provider has costs such that $\delta_i > \delta_j$ (or $c_i > c_j$) then $\gamma_i > \gamma_j$. Furthermore, since $\alpha_i = 0$ for all i we can use (23) to narrow down the range of possible costs δ_i to $\delta^* \le \delta_i \le \delta^{e1}$.

In any symmetric equilibrium $\frac{\partial}{\partial \gamma_i}\pi_i = -\kappa < 0$, therefore $\gamma_i = 0$, and $\delta_i = \delta^*$ and $c_i = c^*$ for all i. Therefore, the strategy $(0, 0, c^*, \delta^*)$ is the only candidate for a symmetric equilibrium outcome. For this to be an equilibrium outcome no provider must find it profitable to unilaterally deviate to a different strategy. Consider the payoff of one provider (labeled j) that chooses to deviate to a different strategy $(0, \gamma_j, c_j, \delta_j)$ when all other providers choose $(0, 0, c^*, \delta^*)$. The derivative of the profit function of provider j with respect to γ_j is given by $\frac{\partial}{\partial \gamma_j}\pi_j = \lambda q h(c_j + \delta_j - c^* - \delta^* - \kappa)$. If $\kappa > c^{e1} + \delta^{e1} - c^* - \delta^*$ then no profitable deviation can exist, therefore the strategy $(0, 0, c^*, \delta^*)$ is the unique symmetric equilibrium outcome. Otherwise, provider j may find it profitable to deviate to $(0, \bar{\gamma}, c^{e1}, \delta^{e1})$. In this case, the profit of provider j needs to be non-negative

$$\begin{aligned} u_4 &:= \lambda q(h(1-\bar{\gamma})(\delta^* + c^* - \delta^{e1} - c^{e1}) + (1-h)(c^* - c^{e1})) + R_c(c^*) + R_\delta(\delta^*) - R_c(c^{e1}) - R_\delta(\delta^{e1}) - \lambda q h\bar{\gamma}\kappa \\ &= u_1 - \lambda q h \bar{\gamma}\kappa. \end{aligned}$$

Note that $u_1 > 0$ (see Proof of Proposition 4). Therefore, if $\kappa < \min\{c^{e1} + \delta^{e1} - c^* - \delta^*, \frac{u_1}{\lambda q h \bar{\gamma}}\}$ then a profitable deviation will exist and no symmetric equilibrium outcome can exist.

We now turn to asymmetric equilibria. As shown above, in any asymmetric equilibrium $\alpha_i = 0$ for all i. Therefore, if $\kappa = 0$ for all i, then the problem of finding asymmetric equilibria reduces to that of finding equilibria in the case where there is cherry picking but not upcoding (see Proposition 4). We will consider the case where $\kappa > 0$. In this case, the provider with the highest γ_j will also be the provider with the highest costs c_j and δ_j (see similar argument given in the Proof of Proposition 4). Consider one such provider. From (21), $\frac{\partial}{\partial \gamma_j}\pi_j = \lambda q h(c_j + \delta_j - \bar{c}_{Mj} - \kappa_j)$. Note that $c_j + \delta_j > \bar{c}_{Mj}$, nevertheless, if $\kappa \geq c^{e1} + \delta^{e1} - c^* - \delta^*$ then $\frac{\partial}{\partial \gamma_j}\pi_j < 0$, suggesting that lowering γ_j would increase the provider's profit. Therefore, for sufficiently high κ there cannot exist a provider with higher rate γ_i or higher costs than other providers, suggesting that an asymmetric equilibrium does not exist. If $\kappa < c^{e1} + \delta^{e1} - c^* - \delta^*$, then $\gamma_j = \bar{\gamma}$, which also implies that the costs $c_j = c^{e1}, \delta_j = \delta^{e1}$. Conversely, at least one provider (labeled k) will have the lowest c_k, δ_k, γ_k (i.e., $c_k \leq c_i$ for all i and the inequality is strict for at least one i, and similarly for δ_k, γ_k). Therefore, $c_k + \delta_k < \bar{c}_{Mk}$ (recall that $\bar{c}_{Mk}$ is the average cost of all other providers and at least some of these providers will have higher costs). This implies that this provider will choose $\gamma_k = 0$ and $c_k = c^*, \delta_k = \delta^*$. Furthermore, consider a provider with costs other than c^* or c^{e1}, which we label as provider s. This provider must have costs c_s and δ_s such that $c_s + \delta_s = \bar{c}_{Ms} + \kappa$ and a corresponding γ_s. Consider the profit of this provider, which can be written as $\pi_s = [-\lambda q(h\delta_s + c_s) - R_c(c_s) - R_\delta(\delta_s)] - \lambda q h \gamma_s(\bar{c}_{Ms} - \delta_s - c_s + \kappa) + C$, where C is an exogenous constant. Note that the first term is independent of γ_s and is maximized at $c_s = c^*$ and $\delta_s = \delta^*$. Consider a deviation from $(0, \gamma_s, c_s, \delta_s)$ to $(0, 0, c^*, \delta^*)$. This deviation does not affect the second term (it is zero under both strategies) and increases the first term. Therefore this deviation is profitable. This suggests than no provider with costs γ_s, c_s, δ_s can exist. In words, in any asymmetric equilibrium providers will divide in two groups: θ_3 providers will not drop any patients and choose to operate at a cost as low as first best $(0, 0, c^*, \delta^*)$ and $N - \theta_3$ providers will drop the maximum number of patients and operate at a higher cost compared to first best $(0, \bar{\gamma}, c^{e1}, \delta^{e1})$.

For such an asymmetric equilibrium to exist, the profit of the θ_3 low-cost providers and the profit of the $N - \theta_3$ high-cost providers need to be non-negative. Consider one of the θ_3 low-cost providers. The fee for providing the major treatment is given by $\bar{c}_{Mk} = \frac{N-\theta_1}{N-1}(\delta^{e1} + c^{e1}) + \frac{\theta_1 - 1}{N-1}(\delta^* + c^*)$, the minor treatment is given by $\bar{c}_{mk} = \frac{N-\theta_1}{N-1}c^{e1} + \frac{\theta_1-1}{N-1}(c^*)$, the number of major treatments provided by others $\bar{M}_i = \lambda q\left(\frac{N-\theta_1}{N-1}h(1-\bar{\gamma}) + \frac{\theta_1 - 1}{N-1}h\right)$, the number of minor treatments provided by others $\bar{m}_i = \lambda q\left(\frac{N-\theta_1}{N-1}(1-h\bar{\gamma}) + \frac{\theta_1-1}{N-1}\right)$, and the transfer payment they will receive is given by $\bar{T}_k = \frac{N-\theta_1}{N-1}(R_\delta(\delta^{e1}) + R_c(c^{e1})) + \frac{\theta_1-1}{N-1}(R_\delta(\delta^*) + R_c(c^*))$. After some algebra, the profit of the low-cost provider can be written as $\frac{N-\theta_1}{N-1}v_4$, where

$$
\begin{aligned}
v_4 &:= \lambda q(h(\delta^{e1} + c^{e1} - \delta^* - c^*) + (1-h)(c^{e1} - c^*) + h\bar{\gamma}\kappa) + R_c(c^{e1}) + R_\delta(\delta^{e1}) - R_c(c^*) - R_\delta(\delta^*) \\
&= v_1 + \lambda q h \bar{\gamma}\kappa.
\end{aligned}
$$

Note that $v_1 > 0$ (see Proof of Proposition 4), therefore, $v_4 > 0$. Similarly, the profit of one of the $N - \theta_3$ high-cost providers can be written as $\frac{\theta_3}{N-1}u_4$, where $u_4 := u_1 - \lambda q h \bar{\gamma}\kappa$. Note that $u_1 > 0$ (see

Proof of Proposition 4). Therefore, for the asymmetric equilibrium to exist, it must be the case that $\kappa < \frac{u_1}{\lambda q h \bar{\gamma}}$.

We will next determine the value of θ_3. For this to be an equilibrium outcome the profit one of the low-cost providers makes by being low cost must be greater than the profit they would make if they deviated to being a high-cost provider. After some algebra this condition can be written as

$$\frac{N-\theta_3}{N-1} v_4 \geq \frac{\theta_3 - 1}{N-1} u_4.$$

Conversely, the profit of one of the high-cost providers must be greater than the payoff they would make if they deviated to being a low-cost provider. After some algebra this condition reduces to

$$\frac{\theta_3}{N-1} u_4 \geq \frac{N-\theta_3-1}{N-1} v_4.$$

Together the last two inequalities imply that the number of low-cost providers must satisfy

$$\frac{(N-1)(v_1 + \lambda q h \bar{\gamma} \kappa)}{v_1 + u_1} \leq \theta_3 \leq \frac{N v_1 + u_1 + (N-1)\lambda q h \bar{\gamma} \kappa}{v_1 + u_1}.$$

Note that this interval contains exactly 1 integer as the difference between the RHS and the LHS of the inequalities is 1. Furthermore, θ_3 is non-decreasing in κ. Since $\theta_3 = \theta_1$ when $\kappa = 0$, it follows that $\theta_3 \geq \theta_1$. □

Appendix 2: Cherry picking and Upcoding under Alternative Assumptions

In this Appendix we examine how patient cherry picking and upcoding affect the DRG design problem under cost-of-service regulation and under alternative assumptions. The numbering of equations continues from that of the main paper.

A2.0. Cost of service regulation

Under cost-of-service regulation, the HO observes the chosen costs of the providers and sets reimbursement on the basis of these observed costs. This is similar to how hospitals used to be reimbursed by Medicare until 1983 (Dranove 1987). For this section will assume that the status quo, where costs are c_0, δ_0 and there is no upcoding or lemon dropping ($\alpha = \gamma = 0$), are going to be the chosen equilibrium outcome unless there is an incentive to deviate – in other words, there is an implicit cost of managerial effort in changing the cost structure and/or finding ways to lemon drop some patients and upcode others that breaks ties in favour of the status quo. We need to distinguish two possibilities. In the first, the HO covers the provider's costs by paying a single fee per patient episode (p) irrespective of the treatment provided (i.e., $p_m = p_M = p$) – this is equivalent to assuming that there is only one coarse DRG associated with the condition. The fee per patient episode will be set equal to the ex post observed average cost of treating patients reduces to:

$$p = c + \delta \frac{h(1-\gamma) + (1-h)\alpha\beta}{1 - h\gamma}. \tag{24}$$

Alternately, the HO could break the condition into two distinct DRGs – one for the minor and another one for the major condition – and cover the provider's costs by paying two distinct fees, one for each DRG. This type of cost-of-service payment is more akin to fee-for-service, where the payment depends on the volume and intensity of services the provider offers to patients. In this case, the fee for the minor and the major condition will be set equal to the ex post average cost of treating patients, which are given by

$$p_m = c \text{ and } p_M = c + \delta \frac{h(1-\gamma) + (1-h)\alpha\beta}{h(1-\gamma) + (1-h)\alpha}, \tag{25}$$

respectively. In order to cover any investment cost incurred by the provider, in addition to the fees described above, the provider also receives a transfer payment equal to the observed investment costs $T = R_c(c) + R_\delta(\delta)$. All payments depend on the cost levels c, δ and/or the degree of upcoding and lemon dropping α, γ, all of which are all chosen by the provider. Furthermore, although the HO can observe the realized costs and the number of patients treated, they are not able to observe if the cost investments made by the providers are optimal (because they do not know the cost functions $R_c(.)$ and $R_\delta(.)$) and are not in a position to ascertain whether patient selection or upcoding are taking place. We characterize the providers' optimal actions with the proposition below.

Proposition 7 *Under cost-of-service regulation, irrespective of the number of DRGs used, i) the provider does not upcode or lemon drop (i.e., chooses $\alpha = \gamma = 0$); ii) the provider makes no investment in cost reduction (i.e., chooses costs c_0, δ_0).*

On the downside, the proposition shows that, because payments are linked to costs, providers have no incentive to invest in any cost reduction as any such cost reduction will result in a lower fee per treatment. This result is well known in the literature (see Shleifer 1985, Ma 1994). On the upside, the proposition shows that cost-of-service regulation is effective in curtailing patient upcoding and cherry picking practices. That it does so irrespective of the number of DRGs specified is a little surprising. For example, if the HO uses one DRG then the payment per patient will, by definition, be greater than the cost of treating low-complexity patients (c) but lower than the cost of treating high-complexity patients ($c+\delta$) – see Equation (24). Nevertheless, the provider has no incentive to cherry pick low-complexity patients or lemon drop high-complexity patients – if they did that, the average cost of treating patients and, therefore, the payment received per patient would be reduced accordingly. Similarly, if the HO were to use two DRGs, then the payment for the major treatment would be greater than the payment for the minor treatment (see Equation (25)). Nevertheless, the provider has no incentive to upcode low-complexity patients because such upcoding would reduce the average cost of providing the major treatment and, as a result, the payment for the major treatment would be reduced accordingly.

At a high level, the proposition suggests that in situations where the main concern is not to incentivize investment in cost reduction (e.g., because costs are largely fixed and exogenous to the providers' efforts) but instead to incentivize providers to prescribe the right treatment (i.e., solve the credence-goods problem), cost-of-service regulation, where providers are reimbursed for their costs of providing the service, performs well.

A2.1. Provider downcoding

In the main paper we had assume that providers were not able to undertreat patients (because of ethical and liability considerations). We had also assumed that providers do not downcode patients. That is, providers never code a high-complexity patient that was given the major treatment as having received the minor treatment for reimbursement purposes. In this section we relax this assumption and allow providers to downcode a proportion $1-\zeta$ of the non-dropped high complexity patients. We assume that the rate of downcoding is bounded from above by $1-\bar{\zeta} \geq 0$. The updated model is shown in Figure 5. We will focus on the case where the regulator is using two DRGs as downcoding is not possible/meaningful otherwise.

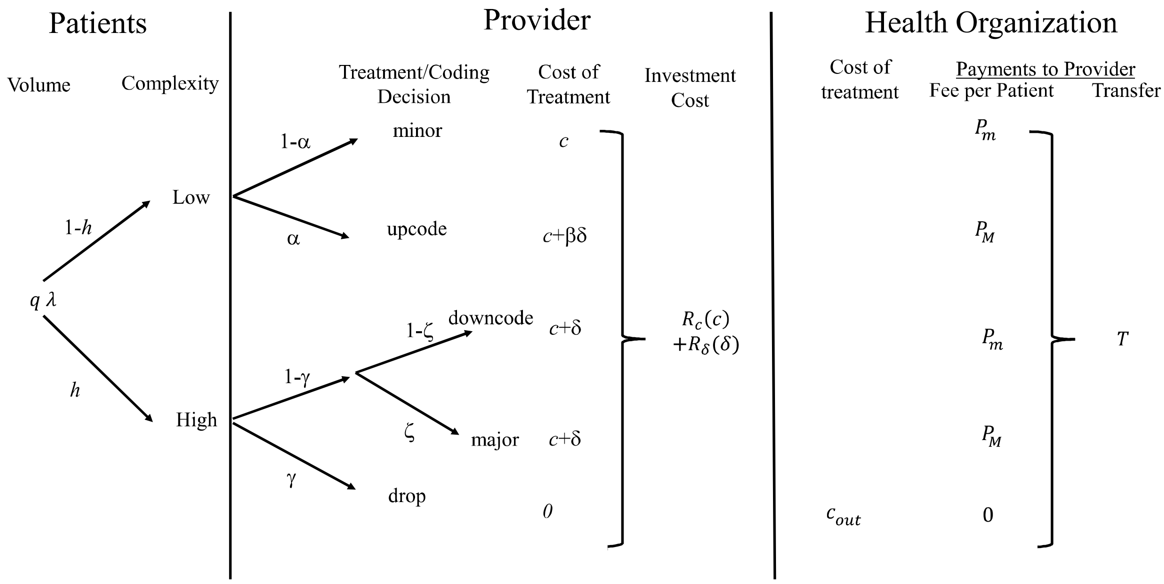

Figure 5 In this Appendix in addition to the assumptions made in the main paper (see Figure 1), providers may decide that a proportion $1-\zeta$ of the high-complexity patients that they do not lemon-drop will be downcoded. These patients still cost $c+\delta$ to treat (as they receive the major treatment) but will be reimbursed at the minor rate P_m.

We first examine the case of cost-of-service regulation. In this case, provider reimbursement for the minor and the major condition are given by the average cost of providing treatment:

$$p_m = c+\delta\frac{h(1-\gamma)(1-\zeta)}{h(1-\gamma)(1-\zeta)+(1-h)(1-\alpha)}, \text{ and } p_M = c+\delta\frac{h(1-\gamma)\zeta+(1-h)\alpha\beta}{h(1-\gamma)\zeta+(1-h)\alpha}. \tag{26}$$

Note that if $\zeta=1$ (i.e., providers do not downcode) these equations reduce to those of (25).

Proposition 7d: *Under cost-of-service regulation providers never find it optimal to downcode ($\zeta=1$), and the optimal outcome is given by Proposition 1.*

Turning to the case where the regulator uses yardstick competition to reimburse providers, the realized average costs for major and minor treatments at all other providers besides provider i, are given by:

$$\bar{c}_{Mi} := \frac{1}{N-1}\sum_{j\neq i}\left[c_j+\delta_j\frac{h(1-\gamma_j)\zeta_j+(1-h)\alpha_j\beta}{h(1-\gamma_j)\zeta_j+(1-h)\alpha_j}\right],$$
$$\bar{c}_{mi} := \frac{1}{N-1}\sum_{j\neq i}c_j+\delta_j\frac{h(1-\gamma_j)(1-\zeta_j)}{h(1-\gamma_j)(1-\zeta_j)+(1-h)(1-\alpha_j)},$$

respectively. Again, note that if $\zeta=1$ (i.e., providers do not downcode) these equations reduce to those of (4)).

We characterize the equilibrium outcome when providers can upcode, downcode, and lemon drop patients with the proposition below, which extends the results of Proposition 4 of the main paper.

Proposition 4d: *If the HO implements yardstick competion based on two DRGs, then in any equilibrium providers do not engage in downcoding if $\bar{c}_{Mi}-\bar{c}_{mi}>0$ for all i. A sufficient condition* *for this to be true is $\bar{\zeta} > \bar{\alpha}$. If this condition holds the equilibrium outcome is given by Proposition 4.*

The proposition suggests that under very mild conditions (i.e., as long as the reimbursement for the major treatment is higher than the reimbursement for the minor treatment) then providers find it optimal not to engage in downcoding. A sufficient condition based on model primitives for this to hold is that for every provider the proportion of high complexity patients that are coded correctly (ζ_i) is not lower than the proportion of low complexity patients that are upcoded (α_i).

Next we turn our attention to the case where the HO uses input statistics as described in §6. The following proposition extends the results of Proposition 6 of the main paper to the case where providers can downcode.

Proposition 6d: *Under the two-DRG payment scheme with input statistics, in any equilibrium providers do not engage in downcoding if $\bar{c}_{Mi} - \bar{c}_{mi} > 0$ for all i. A sufficient condition for this to be true is $\bar{\zeta} > \bar{\alpha}$. If this condition holds the equilibrium outcome is given by Proposition 6.*

A2.2. Extreme Cherry-Picking-Best costs

In this section we extend the results of Proposition 2 to the case where cherry-picking-best costs are extreme (i.e., $c^* + \delta^* \leq c^{e1} + \delta^{e1} \frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$). We will define the following two quantities:

$$v_0 := \lambda q \left[c^{e1} - c^* + h\left(\frac{1-\bar{\gamma}}{1-\bar{\gamma}h}\delta^{e1} - \delta^*\right) \right] + R_c(c^{e1}) + R_\delta(\delta^{e1}) - R_c(c^*) - R_\delta(\delta^*),$$
$$u_0 := \lambda q \left[(1-\bar{\gamma}h)(c^* - c^{e1}) + h(\delta^* - \delta^{e1}) + \bar{\gamma}h(\delta^{e1} - h\delta^*) \right] + R_c(c^*) + R_\delta(\delta^*) - R_c(c^{e1}) - R_\delta(\delta^{e1}).$$

The quantity v_0 is the profit of a provider who is paid according to yardstick competition and chooses $(0, c^*, \delta^*)$ when all other providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$, and vice versa for u_0.

The following proposition describes equilibrium outcomes when cherry-picking-best costs are extreme and complements the results presented in Proposition 2.

Proposition 8 *In the absence of upcoding ($\bar{\alpha} = 0$) if cherry-picking-best costs are extreme (i.e., $c^* + \delta^* \leq c^{e1} + \delta^{e1} \frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$), then if the HO implements yardstick competition based on a single DRG, then there exists a unique Nash equilibrium:*

- *If $v_0 < 0$ the equilibrium is symmetric and is given by Proposition 2.*
- *Otherwise, the equilibrium is asymmetric, where $N - \theta_0$ providers drop as many patients as possible and choose cherry-picking-best costs (i.e., these providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$, and $c^{e1} > c^*$, $\delta^{e1} > \delta^*$) and θ_0 providers do not engage in cherry picking and invest in cost reduction as much as in first best (i.e., these providers choose $(0, c^*, \delta^*)$). The number of efficient providers θ_0 is the only integer in the interval $\left[\frac{(N-1)v_0}{u_0+v_0}, \frac{Nv_0+u_0}{u_0+v_0}\right]$ and $\theta_1 \geq \theta_0$. All providers receive a positive rent.*

The proposition shows that even if costs are extreme (i.e., $c^* + \delta^* \leq c^{e1} + \delta^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$) the equilibrium might still be the same as that described in Proposition 2 (this is the case if $v_0 < 0$) where all providers drop as many high-complexity patients as possible and underinvest in cost reduction compared to first best. Otherwise, if $v_0 \geq 0$ the equilibrium is asymmetric. Some providers choose to drop high-complexity patients and underinvest in cost reduction while other providers treat all patients and invest optimally in cost reduction. This result is similar to the case where the provider uses two DRGs (see Proposition 3). Nevertheless, the number of efficient providers is lower under one DRG compared to two DRGs, suggesting that, in all cases, the equilibrium outcome with two DRGs dominates in terms of welfare compared to that with one DRG.

A2.3. Continuous increasing cost of patient upcoding and cherry picking

For the main paper we assumed that if providers choose to upcode or drop patients, they can do so without any cost up to the predetermined maximum amount of upcoding rate $\bar{\alpha}$ and patient lemon-dropping rate $\bar{\gamma}$, after which we have implicitly assumed that the cost of additional upcoding or lemon dropping becomes infinite. In this Appendix we investigate the case of continuously increasing cost in engaging with patient upcoding and cherry picking. More specifically, we assume that when the provider decides to upcode (or drop) a proportion of patients α (γ), the provider incurs a cost $E_\alpha(\alpha)$ ($E_\gamma(\gamma)$). Furthermore, we assume that the cost functions $E_\gamma(.)$ and $E_\alpha(.)$ are positive, increasing, convex, and 'sufficiently' well-behaved (see conditions A and B imposed below) with $E_\alpha(0) = E_\gamma(0) = 0$ and $\lim_{x\to 1} E_\alpha(x) = \lim_{x\to 1} E_\gamma(x) = \infty$. These assumptions reflect the situation where it is relatively cheap for the provider to upcode (or drop) a small number of patients, but as they engage more in this practice, the costs start to increase (in a convex manner). They also ensure that the chosen rate of upcoding/dropping is in the $[0,1)$ interval. (Alternately, one could view the cost as the expected penalty that the provider may incur if caught engaging in this practice, and that the size of the penalty and/or the probability of getting caught are increasing in proportion to the amount of upcoding/lemon dropping that takes place.)

In this case, the profit of the provider is as defined in §3.2 minus the cost of engaging in upcoding and overtreatment $E_\alpha(\alpha_i) + E_\gamma(\gamma_i)$. We note that, under these alternative assumptions, the first-best solution remains unchanged – the HO would still find it optimal not to engage in any lemon dropping or upcoding. Similarly, cost-of-service regulation would continue to provide the right incentives for providers not to engage in lemon dropping or upcoding but at the expense of eliminating incentives to invest in cost reduction.

Under yardstick competition, providers would be reimbursed with per-patient fees given by (3) or (4), if the HO uses one or two DRGs, respectively. We also assume that the transfer payment is now going to be equal to $T_i = \bar{R}_i + \overline{E_{\alpha i}} + \overline{E_{\gamma}}_i$, where the last two terms are the average cost of upcoding and cherry picking at other providers (i.e., $\overline{E_{\alpha i}} = \sum_{j\neq i} \frac{E_\alpha(\alpha_j)}{N-1}$ and similarly for $\overline{E_{\gamma_i}}$). This assumption is consistent with the case where the HO can only measure total investment cost and is not able to distinguish whether the investment was made to reduce costs or to upcode/cherry pick patients. (This assumption is not critical for any of our results, as the transfer payment does not affect incentives other than participation.)

We analyze the case where the HO uses one DRG and upcoding is not possible ($\alpha_i = 0$ for all i). This is the equivalent of Proposition 2.

Proposition 2c: *In the absence of upcoding ($\alpha_i = 0$ for all i), if the HO implements yardstick competition based on a single DRG and if Conditions A hold, then there exists a unique symmetric Nash equilibrium where providers choose $(\gamma^c, c^c, \delta^c)$ with $\gamma^c > 0, c^c > c^*$, $\delta^c > \delta^*$.*

Conditions A: Let $c(\gamma)$ and $\delta(\gamma)$ be the implicit functions defined by the unique solutions to $-R_c'(c) - \lambda q(1-h\gamma) = 0$, $R_\delta'(\delta) - \lambda qh(1-\gamma) = 0$, respectively.

- The following inequality holds for all $0 \le \gamma < 1$: $(\lambda qh)^2 \left(\frac{1}{R_\delta''(\delta(\gamma))} + \frac{1}{R_c''(c(\gamma))}\right) - E_\gamma''(\gamma) < 0$.
- If γ^c is the unique solution to $\lambda qh\delta(\gamma^c)\frac{1-h}{1-\gamma^c h} - E_\gamma'(\gamma^c) = 0$ then $\lambda qh(c^* + \delta^* - c(\gamma^c) - h\delta(\gamma^c)\frac{1-\gamma^c}{1-\gamma^c h}) - E_\gamma'(0) > 0$.

Clearly, using one DRG continues to be an issue – providers will choose to drop patients and investment in cost reduction will be reduced accordingly. We now turn our attention to the case where the HO uses two DRGs.

Proposition 3c: *In the absence of upcoding ($\alpha_i = 0$ for all i), if the HO implements yardstick competition based on two DRGs and if Conditions A hold, then there exists a unique symmetric Nash equilibrium where providers choose $(0, c^*, \delta^*)$.*

The results confirm that expanding the number of DRGs is helpful, at least in the absence of upcoding. If cherry picking costs are increasing at a sufficiently high rate, there exists a unique symmetric equilibrium where providers choose not to upcode.

Turning to the case where upcoding is possible, the following proposition summarizes the main results.

Proposition 4c: *If the HO implements yardstick competition based on two DRGs and if Conditions B hold, then there exists a unique symmetric Nash equilibrium where providers choose $(\alpha^d, \gamma^d, c^d, \delta^d)$ with $\alpha^d > 0, \gamma^d > 0, c^d > c^*$. If $\beta < \frac{h}{1-h}\gamma^d\alpha^d$ then there is underinvestment for the major treatment ($\delta^d > \delta^*$), otherwise there is overinvestment ($\delta^d < \delta^*$).*

Conditions B: Let $c(\gamma), \delta(\alpha, \gamma)$ be the implicit functions defined by the unique solutions to $-R_c'(c) - \lambda q(1-h\gamma) = 0$, $R_\delta'(\delta) - \lambda q(h(1-\gamma) + (1-h)\alpha) = 0$, respectively. Let

$$\pi(\alpha, \gamma) = -\lambda q((h(1-\gamma) + (1-h)\alpha\beta)\delta(\alpha, \gamma) + (1-h\gamma)c(\gamma))$$
$$- R_c(c(\gamma)) - R_\delta(\delta(\alpha, \gamma)) - E_\gamma(\gamma) - E_\alpha(\alpha).$$

Also define α^d and γ^d as the unique solutions to the system of two equations: $\lambda qh\delta(\alpha^d,\gamma^d)\frac{h(1-\beta)(1-\gamma^d)}{h(1-\gamma^d)+(1-h)\alpha^d} - E'_\alpha(\alpha^d) = 0$, $\lambda qh\delta(\alpha^d,\gamma^d)\frac{(1-h)(1-\beta)\alpha^d}{h(1-\gamma^d)+(1-h)\alpha^d} - E'_\gamma(\gamma^d) = 0$. The following relationships hold:

- The Hessian of $\pi(\alpha,\gamma)$ is negative definite,
- $\lambda qh(\delta(\alpha^d,\gamma^d)\frac{h(1-\gamma^d)+(1-h)\alpha^d\beta}{h(1-\gamma^d)+(1-h)\alpha^d} - \beta\delta^*) - E'_\alpha(0) > 0$,
- $\lambda qh(c^* + \delta^* - c^d - \delta(\alpha^d,\gamma^d)\frac{h(1-\gamma^d)+(1-h)\alpha^d\beta}{h(1-\gamma^d)+(1-h)\alpha^d}) - E'_\gamma(0) > 0$.

The result described above mirrors the results of Proposition 4 – the presence of upcoding reintroduces incentives for providers to engage in cherry picking. There is, however, one difference worth outlining. In contrast to the case where cherry picking and upcoding were costless, where welfare was best served by not increasing the number of DRGs, in this case increasing the number of DRGs may confer welfare benefits. This is particularly the case if upcoding is relatively expensive compared to dropping patients. In this case, providers will not engage in too much upcoding and, as a result, the amount of heterogeneity in the high-cost DRG will be limited, thus reducing the financial benefit of dropping high-complexity patients. As a result, providers will only drop a limited number of high-complexity patients, and welfare under two DRGs may be higher than welfare under one DRG.

Turning to the solution with input statistics, the following proposition characterizes the equilibrium outcome.

Proposition 6c *Under the two-DRG payment scheme with input statistics described in §6, if Conditions A are satisfied and if* $\kappa_i \geq 0$ *there exists a unique symmetric Nash equilibrium where all providers choose first-best actions (i.e.,* $(0,0,c^*,\delta^*)$*).*

Importantly for the purposes of this research, yardstick competition based on input statistics as described in the main paper would eliminate incentives to cherry pick and upcode patients and restore first-best investments. To see why this is the case, consider that payments based on input statistics remove incentives to upcode, even when doing so is costless. When it is costly to do so, the incentives to upcode cannot be any greater. Once upcoding has been eliminated so has the heterogeneity in costs within DRG, leading to a drastic reduction in incentives to cherry pick. In fact, as Proposition 6c suggests, if the cost of cherry picking is rising sufficiently fast (as per Conditions A), then the cherry picking is eliminated too for any $\kappa_i \geq 0$.

A2.4. No transfer payments

In the main paper we had assumed that the HO was able to pay a fee per patient episode and a transfer payment. The role of the former was to reimburse providers for the variable cost of treating patients and the role of the latter was to reimburse for the investment in cost reduction. In several healthcare systems, for example, Medicare in the US, providers are paid only a fee per patient episode which has to cover both the variable cost of treatment and the fixed investment cost.

We note that in classic yardstick competition literature (Shleifer 1985), where demand was endogenous to prices, the lack of a transfer payments was inherently problematic. Since patients would have to pay inflated prices (i.e., higher than the marginal cost of treatment) some patients whom the HO would have found optimal to treat will decide not to seek treatment. Furthermore, as a result of this inefficient reduction in demand, providers would find it optimal to invest less in cost reduction. In our case, since demand is exogenous, the impact of the absence of a transfer payment, if any, will be through the way it affects upcoding and cherry-picking incentives.

In the absence of transfer payments, if the HO wanted to implement yardstick competition on the basis of a single DRG, then the payment per patient to hospital i will be given by the average total cost of providing care in all other hospitals:

$$p = \frac{1}{N-1}\sum_{j\neq i}\left[c_j + \delta_j \frac{h(1-\gamma_j)+(1-h)\alpha_j\beta}{1-\gamma_j h} + \frac{R_c(c_j)+R_\delta(\delta_j)}{(1-\gamma_j h)\lambda q}\right]. \tag{27}$$

The first term in the sum represents the average cost for the minor treatment (offered to all patients who are not dropped). The second term represents the average cost of the major treatment (offered to high-complexity patients who are not dropped and to low-complexity patients who are upcoded). The third term represents the average investment cost amortized proportionally on all patients receiving treatment. Clearly, since in this case there is only one DRG, the provider has no incentive to upcode. Furthermore, because the per-patient-episode payment is now inflated by the investment cost, the providers' incentives to lemon drop are diluted in the sense that the additional profit associated with lemon dropping a patient is less in the absence of transfer payments compared to the case where transfer payments are allowed. Nevertheless, the results of Proposition 3 remain largely unchanged.

More specifically, define

$$v_0' := v_0 + \frac{\bar{\gamma}h}{1-\bar{\gamma}h}(R_c(c^{e1}) + R_\delta(\delta^{e1})),$$
$$u_0' := u_0 - \bar{\gamma}h(R_c(c^*) + R_\delta(\delta^*)).$$

The quantity v_0' is the profit of a provider who is paid according to yardstick competition and chooses $(0, c^*, \delta^*)$ when all other providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$ when transfer payments are not allowed, and vice versa for u_0. We will call the cherry-picking-best costs as comparable to first-best costs if either of the following conditions hold:

- $c^* + \delta^* \geq c^{e1} + \delta^{e1} h \frac{1-\bar{\gamma}}{1-h\bar{\gamma}} + \frac{R_c(c^{e1})+R_\delta(\delta^{e1})}{\lambda q(1-\bar{\gamma}h)}$
- $v_0' \leq 0$ or $u_0' \leq 0$.

The following proposition is the equivalent to Propositions 3 and 8 in the absence of transfer payments.

Proposition 2n *In the absence of upcoding ($\bar{\alpha} = 0$), if the HO implements yardstick competition based on a single DRG, then there exists a unique Nash equilibrium:*

- *If cherry-picking-best costs are 'comparable' to first-best costs (as defined above), the equilibrium is symmetric. Providers drop as many patients as possible and invest in cherry-picking-best costs (i.e., all providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$, and $c^{e1} > c^*$, $\delta^{e1} > \delta^*$).*
- *Otherwise, the equilibrium is asymmetric, where $N - \theta'_0$ providers drop as many patients as possible and choose cherry-picking-best costs (i.e., these providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$, and $c^{e1} > c^*$, $\delta^{e1} > \delta^*$) and θ'_0 providers do not engage in cherry picking and invest in cost reduction as much as in first best (i.e., these providers choose $(0, c^*, \delta^*)$). The number of efficient providers θ'_0 is the only integer in the interval $\left[\frac{(N-1)v'_0}{u'_0+v'_0}, \frac{Nv'_0+u'_0}{u'_0+v'_0}\right]$. Furthermore, $\theta'_0 \geq \theta_0$.*

The proposition suggests that, in the absence of transfer payments, then i) the inefficient symmetric equilibrium (where all providers choose to drop the maximum number of patients and underinvest in cost reduction) is the outcome for a smaller set of parameter values; and ii) the asymmetric equilibrium is more efficient in the sense that more providers choose not to engage in cherry picking and invest optimally in cost reduction.

Turning to the case where the HO splits the condition into two DRGs for reimbursement purposes, the payments for the major and the minor condition, set at the average (treatment and investment) cost of all other providers, are given by

$$\bar{p}_{Mi} = \frac{1}{N-1}\sum_{j\neq i}\left[c_j + \delta_j \frac{h(1-\gamma_j)+(1-h)\alpha_j\beta}{h(1-\gamma_j)+(1-h)\alpha_j} + \frac{R_\delta(\delta_j)+R_c(c_j)}{\lambda q(1-h\gamma_j)}\right], \tag{28}$$

$$\bar{p}_{mi} = \frac{1}{N-1}\sum_{j\neq i}\left[c_j + \frac{R_\delta(\delta_j)+R_c(c_j)}{\lambda q(1-h\gamma_j)}\right], \tag{29}$$

respectively.

Before we characterize the equilibrium outcome of this competition, we define the following two quantities:

$$v'_1 := v_1 + \frac{h\bar{\gamma}}{1-h\bar{\gamma}}(R_\delta(\delta^{e1}+R_c(c^{e1})),$$
$$u'_1 := u_1 - h\bar{\gamma}(R_\delta(\delta^*)+R_c(c^*)).$$

As in the case with transfer payments, the quantity v'_1 is the profit of a provider who is paid according to yardstick competition based on two DRGs in the absence of transfer payments and who chooses $(0, c^*, \delta^*)$ when all other providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$, and vice versa for the quantity u'_1. Note that in this case $v'_2 > 0$ while the sign of u'_1 depends on model parameters.

In the absence of transfer payments under two DRGs, we will call the cherry-picking-best costs comparable to first-best costs if either of the following two conditions hold:

- $\lambda q(c^{e1}+\delta^{e1}-c^*-\delta^*) \leq R_\delta(\delta^*)+R_c(c^*)$.
- $u_1' \leq 0$.

Proposition 3n *In the absence of upcoding ($\bar{\alpha}=0$), if the HO implements yardstick competition based on two DRGs without transfer payments, then there exists a unique Nash equilibrium:*

- *If cherry-picking-best costs are 'comparable' to first-best costs (as defined above), the equilibrium is symmetric. Providers will not engage in cherry picking and invest in cost reduction as much as first best (i.e., all providers choose $(0,c^*,\delta^*)$).*
- *Otherwise, the equilibrium is asymmetric, where $N-\theta_1'$ providers drop as many patients as possible and choose cherry-picking-best costs (i.e., these providers choose $(\bar{\gamma},c^{e1},\delta^{e1})$, and $c^{e1}>c^*$, $\delta^{e1}>\delta^*$) and θ_1' providers do not engage in cherry picking and invest in cost reduction as much as in first best (i.e., these providers choose $(0,c^*,\delta^*)$). The number of efficient providers θ_1 is the only integer in the interval $\left[\frac{(N-1)v_0'}{u_0'+v_0'}, \frac{Nv_0'+u_0'}{u_0'+v_0'}\right]$. Furthermore, $\theta_1' \geq \theta_1$.*

Again, the proposition demonstrates that the absence of transfer payments reduces providers' incentives to lemon drop. In contrast to the case with transfer payments, there may exist a symmetric equilibrium where all providers engage in first-best actions. Even if such an equilibrium does not exist, the number of providers that will choose to be inefficient is smaller in the absence of transfer payments compared to the case with transfers.

Turning to the case where the HO uses two DRGs and upcoding is possible, note that the absence of a transfer payment does not affect the value of upcoding – the difference in payment between a low-complexity patient who is upcoded and that of a low-complexity patient who is not remains the same whether transfer payments are used or not. Therefore, condition (7)

$$\beta < \frac{h\delta^{e3}}{h\delta^{e1}+(\delta^{e1}-\delta^{e3})(1-h)\bar{\alpha}}$$

continues to ensure that providers will find upcoding profitable and will engage in it fully. Define

$$v_2' := v_2 + \frac{h\bar{\gamma}}{1-h\bar{\gamma}}(R_\delta(\delta^{e2})+R_c(c^{e1})),$$
$$u_2' := u_2 - h\bar{\gamma}(R_\delta(\delta^{e3})+R_c(c^*)).$$

The quantity v_2' is the profit of a provider who is paid according to yardstick competition and chooses $(\bar{\alpha},0,c^*,\delta^{e3})$ when all other providers choose $(\bar{\alpha},\bar{\gamma},c^{e1},\delta^{e2})$ when transfer payments are not allowed, and vice versa for u_2'. We will call the cherry-picking-upcoding-best (CPU-best) costs comparable to upcoding-best costs if any of the following conditions hold:

- $\lambda q(c^*+\delta^{e3}-c^{e1}-\delta^{e2}\frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}) \geq \frac{R_\delta(\delta^{e2})+R_c(c^{e1})}{\lambda q(1-h\bar{\gamma})}$,
- $v_2' \leq 0$ or $u_2' \leq 0$.

Proposition 4n *If both patient cherry picking and upcoding are possible, if the HO implements yardstick competition based on two DRGs without transfer payments, then there exists a unique Nash equilibrium:*

- *If CPU-best costs are 'comparable' to upcoding-best costs (as defined above), then the equilibrium is symmetric. Providers upcode and drop as many patients as possible and invest in CPU-best costs (i.e., all providers choose* $(\bar{\alpha}, \bar{\gamma}, c^{e1}, \delta^{e2})$*). Furthermore, there is underinvestment in cost reduction compared to first best for the minor condition* $(c^{e1} > c^*)$ *and if* $\beta < \frac{h}{1-h}\bar{\gamma}\bar{\alpha}$ *then there is also underinvestment for the major condition* $(\delta^{e2} > \delta^*)$*, otherwise there is overinvestment.*
- *Otherwise, the equilibrium is asymmetric.* $N - \theta_2'$ *providers upcode and drop as many patients as possible and choose CPU-best costs (i.e., these providers choose* $(\bar{\alpha}, \bar{\gamma}, c^{e1}, \delta^{e2})$*) and* θ_2' *providers upcode as many patients as possible but do not engage in cherry picking and invest in upcoding-best costs (i.e., these providers choose* $(\bar{\alpha}, 0, c^*, \delta^{e3})$*). The number of low-cost providers* θ_2 *is the only integer in the interval* $\left[\frac{(N-1)v_2'}{u_2'+v_2'}, \frac{Nv_2'+u_2'}{u_2'+v_2'}\right]$*. Furthermore,* $\theta_2' \geq \theta_2$*.*

The proposition demonstrates that the presence of upcoding strengthens incentives to drop high-complexity patients, even in the absence of a transfer payment.

Turning to yardstick competition with input statistics, we find it convenient to define

$$\bar{\kappa}' := \min\{c^{e1} + \delta^{e1} - c^* - \delta^* - \frac{R_\delta(\delta^*) + R_c(c^*)}{\lambda q}, \frac{u_1'}{\lambda q h \bar{\gamma}}\}$$

Proposition 6n *Under the two-DRG payment scheme with input statistics described in §6, with* $\kappa_i = \kappa$ *for all* i *then there exists a unique Nash equilibrium:*

- *If* $\kappa > \bar{\kappa}'$*, the equilibrium is symmetric in which all providers choose first-best actions (i.e.,* $(0, 0, c^*, \delta^*)$*).*
- *If* $0 \leq \kappa \leq \bar{\kappa}'$*, the equilibrium is asymmetric. No provider engages in upcoding and* $N - \theta_3$ *providers drop as many patients as possible and choose cherry-picking-best costs (i.e., these providers choose* $(0, \bar{\gamma}, c^{e1}, \delta^{e1})$ *and* $c^{e1} > c^*$*,* $\delta^{e1} > \delta^*$*) and* θ_3' *providers do not engage in cherry picking and invest in cost reduction as much as in first best (i.e., these providers choose* $(0, 0, c^*, \delta^*)$*). The number of efficient providers* $\theta_3' \geq \theta_1'$ *and is non-decreasing in* κ'*. Furthermore,* $\bar{\kappa}' \leq \bar{\kappa}$*.*

The proposition suggests that, compared to the case where transfer payments are allowed, in their absence yardstick competition based on input statistics continues to be effective in eliminating the problem of upcoding, and is even more effective in alleviating the problem of cherry picking.

Collectively, the results of this appendix suggest that the absence of transfer payments helps reduce the problem of cherry picking but does nothing to alleviate the problem of upcoding. In a setting where investment costs are relatively small compared to variable costs, the results of the main paper persist in the absence of transfer fees. According to CMS, for the fiscal year 2022, "operating base payments," which are meant to reimburse "labor and supply costs," were estimated to \$6,122 and "capital base payments," which are meant to reimburse capital expenditures such as "costs for depreciation, interest, rent, and property related insurance and taxes," were estimated to \$473.[8] Therefore, upcoding and patient dropping are likely to remain an issue in the hospital reimbursement setting.

Proofs of results appearing in Appendix 2

Proof of Proposition 7: Under cost-of-service regulation, and irrespective of the number of DRGs used, the provider's payoff is equal to zero. Therefore, the provider resorts to selecting the status quo – zero upcoding and cherry picking and zero investment in cost reduction (i.e., the provider will choose $\alpha=\gamma=0$, $c=c_0$, $\delta=\delta_0$). □

Proof of Proposition 7d: Under cost-of-service regulation the provider's profit is zero, therefore the provider has no incentive to downcode. The proof proceeds as that of Proposition 7.□

Proof of Proposition 4d: The derivative of a provider's profit with respect to ζ_i is given by

$$\frac{\partial}{\partial \zeta_i}\pi_i = \lambda q h(1-\gamma_i)(\bar{c}_{Mi}-\bar{c}_{mi}).$$

This implies that $\frac{\partial}{\partial\zeta_i}\pi_i > 0$ if $\bar{c}_{Mi}-\bar{c}_{mi} > 0$ which suggests that providers will find it optimal not to downcode ($\zeta_i = 1$). Note that

$$\begin{aligned}\bar{c}_{Mi}-\bar{c}_{mi} &= \frac{1}{N-1}\sum_{j\neq i}\delta_j\left[\frac{h(1-\gamma_j)\zeta_j+(1-h)\alpha_j\beta}{h(1-\gamma_j)\zeta_j+(1-h)\alpha_j} - \frac{h(1-\gamma_j)(1-\zeta_j)}{h(1-\gamma_j)(1-\zeta_j)+(1-h)(1-\alpha_j)}\right]\\ &= \frac{1}{N-1}\sum_{j\neq i}\delta_j\left[h(1-\gamma_j)\left(\frac{\zeta_j}{h(1-\gamma_j)\zeta_j+(1-h)\alpha_j} - \frac{1-\zeta_j}{h(1-\gamma_j)(1-\zeta_j)+(1-h)(1-\alpha_j)}\right)\right.\\ &\quad + \left.\frac{(1-h)\alpha_j\beta}{h(1-\gamma_j)\zeta_j+(1-h)\alpha_j}\right].\end{aligned}$$

This is positive if $\frac{\zeta_j}{h(1-\gamma_j)\zeta_j+(1-h)\alpha_j} - \frac{1-\zeta_j}{h(1-\gamma_j)(1-\zeta_j)+(1-h)(1-\alpha_j)}$, which is always the case if $\zeta_i > \alpha_i$. Since $\zeta_i \geq \bar{\zeta}$ and $\alpha_i \leq \bar{\alpha}$, then $\bar{c}_{Mi}-\bar{c}_{mi} > 0$ if $\bar{\zeta} > \bar{\alpha}$. The rest of the proof proceeds as Proposition 4. □

Proof of Proposition 6d: The derivative of a provider's profit with respect to ζ_i is given by

$$\frac{\partial}{\partial \zeta_i}\pi_i = \lambda q h(1-\gamma_i)(\bar{c}_{Mi}-\bar{c}_{mi}+\kappa-\phi_i) > 0.$$

Since $\phi_i-\kappa < \bar{c}_{Mi}-\bar{c}_{mi}$, this implies that if $\bar{c}_{Mi}-\bar{c}_{mi} > 0$ then $\frac{\partial}{\partial\zeta_i}\pi_i > 0$ which suggests that providers will find it optimal not to downcode ($\zeta_i = 1$). Sufficient conditions for $\bar{c}_{Mi}-\bar{c}_{mi} > 0$ are derived in the proof of Proposition 4d and the rest of the proof proceeds as that of Proposition 6.□

[8] See `https://www.medpac.gov/wp-content/uploads/2021/11/medpac_payment_basics_21_hospital_final_sec.pdf`

Proof of Proposition 8: This proof continues from the proof of Proposition 2, where we have shown that if $c^* + \delta^* > c^{e1} + \delta^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$ then there exists a symmetric equilibrium where all providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$. In contrast, if this condition is not satisfied, then a provider that knows all providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$ may find it profitable to deviate to $(0, c^*, \delta^*)$. For this to be the case, the profit of the provider who deviates to $(0, c^*, \delta^*)$ when all other providers choose $(\bar{\gamma}, c^{e1}, \delta^{e1})$ must be positive (as the profit associated with not deviating is zero). After some algebra, this condition can be written as $v_0 \geq 0$. If this is not satisfied (i.e. if $v_0 < 0$), the equilibrium is symmetric and is as given in Proposition 2. Otherwise (i.e. if $v_0 \geq 0$) then no symmetric equilibrium exists.

Turning to the case of asymmetric equilibria, in the Proof of Proposition 2 we have ruled out their existence if $c^* + \delta^* > c^{e1} + \delta^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$. If this condition is not satisfied then an asymmetric equilibrium will exist if the profit of the θ_0 low-cost providers and the profit of the $N - \theta_0$ high-cost providers are non-negative. The profit of a low-cost provider will be $\frac{N-\theta_0}{N-1}v_0$. Similarly, the profit of a high-cost provider will be given by $\frac{\theta_0}{N-1}u_0$. Therefore, for the asymmetric equilibrium to exist it must be the case that $v_0 \geq 0$ and $u_0 \geq 0$. Note that $u_0 > 0$. To see this, note that u_0 can be written as

$$\begin{aligned} u_0 = & \left[-\lambda q\left((1-\bar{\gamma}h)c^{e1} + (1-\bar{\gamma})h\delta^{e1}\right) - R_c(c^{e1}) - R_\delta(\delta^{e1})\right] \\ & + \left[\lambda q\left((1-\bar{\gamma}h)c^* + (1-\bar{\gamma})h\delta^*\right) + R_c(c^*) + R_\delta(\delta^*)\right] \\ & + \bar{\gamma}h(1-h)\delta^*. \end{aligned}$$

Note that the expression $[-\lambda q[(1-\bar{\gamma}h)c + (1-\bar{\gamma})h\delta] - R_c(c) - R_\delta(\delta)]$ is maximized at c^{e1}, δ^{e1}, therefore the sum of the terms in the first two brackets is positive. The third term is also positive, which implies that $u_0 > 0$. For $v_0 \geq 0$ it must be the case that

$$\lambda q(c^{e1} - c^* + h[\frac{1-\bar{\gamma}}{1-\bar{\gamma}h}\delta^{e1} - \delta^*]) \geq R_c(c^*) + R_\delta(\delta^*) - R_c(c^{e1}) - R_\delta(\delta^{e1}). \tag{30}$$

We will next determine the value of θ_0. If one of the θ_0 low-cost providers was to deviate and become a high-cost provider, then their profit would be given by $\frac{\theta_0-1}{N-1}u_0$. If one of the $N-\theta_0$ high-cost providers was to deviate and become a low-cost provider it would be $\frac{N-\theta_0-1}{N-1}v_0$. In the asymmetric equilibrium it must be the case that these deviations are not profitable. Therefore, $\frac{N-\theta_0}{N-1}v_0 > \frac{\theta_0-1}{N-1}u_0$ and $\frac{\theta_0}{N-1}u_0 \geq \frac{N-\theta_0-1}{N-1}v_0$. These conditions imply that θ_0 satisfies $\frac{(N-1)v_0}{u_0+v_0} \leq \theta_0 \leq \frac{Nv_0+u_0}{u_0+v_0}$. If condition (30) is satisfied, this interval contains exactly 1 integer as the difference $\frac{Nv_0+u_0}{u_0+v_0} - \frac{(N-1)v_0}{u_0+v_0} = 1$. Furthermore, this integer is always less than N.

We will then compare θ_1 and θ_0. Note that $v_1 - v_0 = \lambda qh(1-h)\frac{\bar{\gamma}}{1-\bar{\gamma}h}\delta^{e1} > 0$, therefore $v_1 > v_0$. Also, $u_1 - u_0 = -\lambda q\bar{\gamma}h(1-h)\delta^* < 0$, therefore $u_1 < u_0$. It follows that $\frac{v_1}{u_1} > \frac{v_0}{u_0}$, which implies that

$\frac{v_1}{v_1+u_1} > \frac{v_0}{v_0+u_0}$. Since θ_1 is the first integer higher than $(N-1)\frac{v_1}{v_1+u_1}$ and θ_0 is the first integer higher than $(N-1)\frac{v_0}{v_0+u_0}$, it follows that $\theta_1 \geq \theta_0$. $\square$

Proof of Proposition 2c: In the absence of upcoding ($\alpha_i = 0$), under the yardstick competition scheme with a single DRG the profit of provider i is given by

$$\pi_i(c_i, \delta_i, 0, \gamma_i) = \lambda q\left[(1-h\gamma_i)\bar{c}_i - \delta_i h(1-\gamma_i) - c(1-h\gamma_i)\right] - R_c(c_i) - R_\delta(\delta_i) - E_\gamma(\gamma_i) + \bar{R}_i,$$

where $\bar{c}_i = \frac{1}{N-1}\sum_{j\neq i}\left[c_j + \delta_j \frac{h(1-\gamma_j)}{1-h\gamma_j}\right]$, and $\bar{R}_i := \frac{1}{N-1}\sum_{j\neq i}\left[R_c(c_j) + R_\delta(\delta_j)\right]$ as defined in §4. The derivatives of the profit function of provider i are given by:

$$\frac{\partial}{\partial \gamma_i}\pi_i = \lambda q h(c_i + \delta_i - \bar{c}_i) - E'_\gamma(\gamma_i), \tag{31}$$

$$\frac{\partial}{\partial c_i}\pi_i = -\frac{d}{dc}R_c(c_i) - \lambda q(1-h\gamma_i), \tag{32}$$

$$\frac{\partial}{\partial \delta_i}\pi_i = -\frac{d}{d\delta}R_\delta(\delta_i) - \lambda q h(1-\gamma_i). \tag{33}$$

In any equilibrium outcome, the last two conditions, which do not depend on the actions of other providers, will have to be equal to zero for all providers. Otherwise the provider for whom one of these conditions is not zero could increase their profit by changing c_i or δ_i.

Consider a symmetric equilibrium where all providers choose $(\gamma^c, c^c, \delta^c)$. From (31)–(33), these values should satisfy $\lambda q h \delta^c \frac{1-h}{1-\gamma^c h} - E'_\gamma(\gamma^c) = 0$, $-R'_c(c^c) - \lambda q(1-h\gamma^c) = 0$, and $R'_\delta(\delta^c) - \lambda q h(1-\gamma^c) = 0$. Consider a provider that chooses $(\gamma_i, c_i, \delta_i)$ when all other providers choose $(\gamma^c, c^c, \delta^c)$. We will show that this provider will also choose $(\gamma^c, c^c, \delta^c)$. The derivative of the provider's profit is given by $\frac{\partial}{\partial \gamma_i}\pi_i = \lambda q h(c_i + \delta_i - c^c - h\delta^c \frac{1-\gamma^c}{1-\gamma^c h}) - E'_\gamma(\gamma_i)$. Clearly, the derivative is zero if $\gamma_i = \gamma^c$. A sufficient condition for this to be the unique maximizer is $\frac{\partial^2}{\partial \gamma_i^2}\pi_i = (\lambda q h)^2\left(\frac{1}{R''_\delta(\delta_c)} + \frac{1}{R''_c(\delta^c)}\right) - E''_\gamma(\gamma^c) < 0$. A necessary condition for the solution to be in the interval $(0,1]$ is $\lambda q h(c^* + \delta^* - c^c - h\delta^c \frac{1-\gamma^c}{1-\gamma^c h}) - E'_\gamma(0) > 0$. If these conditions hold, $(\gamma^c, c^c, \delta^c)$ is the unique symmetric equilibrium. Note that these conditions are a subset of Conditions A.

The proof above does not rule out asymmetric equilibria, which may exist.$\square$

Proof of Proposition 3c: In the absence of upcoding ($\alpha_i = 0$), under the yardstick competition scheme with two DRGs the profit of provider i is given by

$$\pi_i(c_i, \delta_i, 0, \gamma_i) = \lambda q\left[h(1-\gamma_i)(\bar{c}_{Mi} - \delta_i - c_i) + (1-h)(\bar{c}_{mi} - c_i)\right] - R_c(c_i) - R_\delta(\delta_i) - E_\gamma(\gamma_i) + \bar{R}_i,$$

where $\bar{c}_{Mi} := \frac{1}{N-1}\sum_{j\neq i}[c_j + \delta_j]$, $\bar{c}_{mi} := \frac{1}{N-1}\sum_{j\neq i} c_j$, and $\bar{R}_i := \frac{1}{N-1}\sum_{j\neq i}\left[R_c(c_j) + R_\delta(\delta_j)\right]$ as defined in §4. The derivatives of the profit function of provider i are given by:

$$\frac{\partial}{\partial \gamma_i}\pi_i = \lambda q h(c_i + \delta_i - \bar{c}_{Mi}) - E'_\gamma(\gamma_i), \tag{34}$$

$$\frac{\partial}{\partial c_i}\pi_i = -\frac{d}{dc}R_c(c_i) - \lambda q(1-h\gamma_i), \tag{35}$$

$$\frac{\partial}{\partial \delta_i}\pi_i = -\frac{d}{d\delta}R_\delta(\delta_i) - \lambda qh(1-\gamma_i). \tag{36}$$

In any equilibrium outcome, the last two conditions have to be equal to zero for all providers. Otherwise the provider for whom one of these conditions is not zero could increase their profit by changing c_i or δ_i.

Consider a symmetric equilibrium where all providers choose $(\gamma^c, c^c, \delta^c)$. Equation (34) implies that $\frac{\partial}{\partial \gamma_i}\pi_i = -\frac{d}{d\gamma_i}E_\gamma(\gamma_i) < 0$, therefore $\gamma^c = 0$, which also implies $c^c = c^*, \delta^c = \delta^*$. Consider a provider that chooses $(\gamma_i, c_i, \delta_i)$ when all other providers choose $(0, c^*, \delta^*)$. We will show that this provider will also choose $(0, c^*, \delta^*)$. The derivative of the provider's profit is given by $\frac{\partial}{\partial \gamma_i}\pi_i = \lambda qh(c_i + \delta_i - c^* - \delta^*) - \frac{d}{d\gamma_i}E_\gamma(\gamma_i)$. At $\gamma_i = 0$ this derivative is negative. Furthermore, if $(\lambda qh)^2\left(\frac{1}{R''_\delta(\delta^c)} + \frac{1}{R''_c(\delta^c)}\right) - E''_\gamma(\gamma^c) < 0$ for all all γ_c such that $-R'_c(c^c) - \lambda q(1-h\gamma^c) = 0$, $R'_\delta(\delta^c) - \lambda qh(1-\gamma^c) = 0$, then $\frac{\partial}{\partial \gamma_i}\pi_i < 0$ for all γ_c. This implies that no profitable deviation exists and $(0, c^*, \delta^*)$ is the unique symmetric equilibrium. Note that these conditions are a subset of Conditions A.

The proof above does not rule out asymmetric equilibria, which may exist. □

Proof of Proposition 4c: Under the yardstick competition scheme with two DRGs the profit of provider i is given by

$$\begin{aligned}\pi_i(c_i, \delta_i, \alpha_i, \gamma_i) = {} & \lambda q\{[h(1-\gamma_i) + (1-h)\alpha_i]\bar{c}_{Mi} + (1-h)(1-\alpha_i)\bar{c}_{mi} \\ & - [(h(1-\gamma_i) + (1-h)\alpha_i\beta)\delta_i + (1-h\gamma_i)c_i]\} \\ & - R_c(c) - R_\delta(\delta) - E_\gamma(\gamma_i) - E_\alpha(\alpha_i) + \bar{R}_i,\end{aligned}$$

where $\bar{c}_{Mi} := \frac{1}{N-1}\sum_{j\neq i}\left[c_j + \delta_j \frac{h(1-\gamma_j)+(1-h)\alpha_j\beta}{h(1-\gamma_j)+(1-h)\alpha_j}\right]$, $\bar{c}_{mi} := \frac{1}{N-1}\sum_{j\neq i} c_j$, and $\bar{R}_i := \frac{1}{N-1}\sum_{j\neq i}[R_c(c_j) + R_\delta(\delta_j)]$ as defined in §4. The derivatives of the profit function of provider i are given by:

$$\frac{\partial}{\partial \alpha_i}\pi_i = \lambda q(1-h)(\bar{c}_{Mi} - \bar{c}_{mi} - \beta\delta_i) - E'_\alpha(\alpha_i), \tag{37}$$

$$\frac{\partial}{\partial \gamma_i}\pi_i = \lambda qh(c_i + \delta_i - \bar{c}_{Mi}) - E'_\gamma(\gamma_i), \tag{38}$$

$$\frac{\partial}{\partial c_i}\pi_i = -\frac{d}{dc}R_c(c_i) - \lambda q(1-h\gamma_i), \tag{39}$$

$$\frac{\partial}{\partial \delta_i}\pi_i = -\frac{d}{d\delta}R_\delta(\delta_i) - \lambda q[h(1-\gamma_i) + (1-h)\alpha_i\beta]. \tag{40}$$

In any equilibrium outcome, the last two conditions have to be equal to zero for all providers. Otherwise the provider for whom one of these conditions is not zero could increase their profit by changing c_i or δ_i.

Consider a symmetric equilibrium where all providers choose $(\alpha^d, \gamma^d, c^d, \delta^d)$. From (37)–(40), these values should satisfy $\lambda q h \delta^d \frac{h(1-\beta)(1-\gamma^d)}{h(1-\gamma^d)+(1-h)\alpha^d} - E'_\alpha(\alpha^d) = 0$, $\lambda q h \delta^d \frac{(1-h)(1-\beta)\alpha^d}{h(1-\gamma^d)+(1-h)\alpha^d} - E'_\gamma(\gamma^d) = 0$, $-R'_d(c^d) - \lambda q(1 - h\gamma^d) = 0$, and $R'_\delta(\delta^d) - \lambda q(h(1-\gamma^d) + (1-h)\beta\alpha^d) = 0$. Consider the payoff of a provider that chooses $(\alpha_i, \gamma_i, c_i, \delta_i)$ when all other providers choose $(\alpha^d, \gamma^d, c^d, \delta^d)$. We will show that this provider will also choose $(\alpha^d, \gamma^d, c^d, \delta^d)$. The derivative of the provider's profit is given by $\frac{\partial}{\partial \alpha_i}\pi_i = \lambda q h(\delta^d \frac{h(1-\gamma^d)+(1-h)\alpha^d\beta}{h(1-\gamma^d)+(1-h)\alpha^d} - \beta\delta_i) - \frac{d}{d\alpha_i}E_\alpha(\alpha_i)$ and $\frac{\partial}{\partial \gamma_i}\pi_i = \lambda q h(c_i + \delta_i - c^d - \delta^d \frac{h(1-\gamma^d)+(1-h)\alpha^d\beta}{h(1-\gamma^d)+(1-h)\alpha^d}) - \frac{d}{d\gamma_i}E_\gamma(\gamma_i)$. Clearly, both derivatives are zero if $\alpha_i = \alpha^d, \gamma_i = \gamma^d$. A sufficient condition for this to be the unique maximizer is the Hessian of the profit with respect to α_i and γ_i (when $-R'_d(c_i) - \lambda q(1 - h\gamma_i) = 0$, and $R'_\delta(\delta_i) - \lambda q(h(1-\gamma_i) + (1-h)\beta\alpha_i) = 0$) to be negative definite. Necessary conditions for the solution to be in the interval $(0,1]$ are $\lambda q h(\delta^d \frac{h(1-\gamma^d)+(1-h)\alpha^d\beta}{h(1-\gamma^d)+(1-h)\alpha^d} - \beta\delta^*) - E'_\alpha(0) > 0$ and $\lambda q h(c^* + \delta^* - c^d - \delta^d \frac{h(1-\gamma^d)+(1-h)\alpha^d\beta}{h(1-\gamma^d)+(1-h)\alpha^d}) - E'_\gamma(0) > 0$. If these conditions hold, $(\alpha^d, \gamma^d, c^d, \delta^d)$ is the unique symmetric equilibrium. These conditions are summarized by Conditions B. Furthermore, from (39) and since $\gamma^d > 0$ it follows that $c^d > c^*$. From (40), if $h(1-\gamma^d) + (1-h)\alpha^d\beta > h$ there will be overinvestment in the major treatment $\delta^d > \delta^*$, otherwise underinvestment.

The proof above does not rule out asymmetric equilibria, which may exist.□

Proof of Proposition 6c: Under the yardstick competition scheme with two DRGs with input statistics the profit of provider i is given by

$$\begin{aligned}\pi_i(c_i, \delta_i, \alpha_i, \gamma_i) &= \lambda q\{[h(1-\gamma_i) + (1-h)\alpha_i]\bar{c}_{Mi} + (1-h)(1-\alpha_i)\bar{c}_{mi} \\ &- [(h(1-\gamma_i) + (1-h)\alpha_i\beta)\delta_i + (1-h\gamma_i)c_i]\} \\ &- R_c(c) - R_\delta(\delta) - E_\gamma(\gamma_i) - E_\alpha(\alpha_i) + \bar{R}_i + \kappa_i(M_i - \bar{M}_i) + \phi_i(m_i - \bar{m}_i),\end{aligned}$$

where $M_i = \lambda q(h(1-\gamma_i) + (1-h)\alpha_i)$, $m_i = \lambda q(1-h)(1-\alpha_i)$, $\kappa_i \geq 0$, $\phi_i - \kappa_i > \bar{\delta}_i - \beta\delta^{e3}$, $\bar{\delta}_i := \bar{c}_{Mi} - \bar{c}_{mi}$, $\bar{c}_{Mi} = \frac{1}{N-1}\sum_{j\neq i}\left[c_j + \delta_j \frac{h(1-\gamma_j)+(1-h)\alpha_j\beta}{h(1-\gamma_j)+(1-h)\alpha_j}\right]$, $\bar{c}_{mi} = \frac{1}{N-1}\sum_{j\neq i} c_j$, and $\bar{R}_i = \frac{1}{N-1}\sum_{j\neq i}[R_c(c_j) + R_\delta(\delta_j)]$.

The derivatives of the profit function of provider i are given by:

$$\frac{\partial}{\partial \alpha_i}\pi_i = \lambda q(1-h)(-\beta\delta_i + \bar{\delta}_i + \kappa_i - \phi_i) - E'_\alpha(\alpha_i), \tag{41}$$

$$\frac{\partial}{\partial \gamma_i}\pi_i = \lambda q h(c_i + \delta_i - \bar{c}_{Mi} - \kappa_i) - E'_\gamma(\gamma_i), \tag{42}$$

$$\frac{\partial}{\partial c_i}\pi_i = -\frac{d}{dc}R_c(c_i) - \lambda q(1 - h\gamma_i), \tag{43}$$

$$\frac{\partial}{\partial \delta_i}\pi_i = -\frac{d}{d\delta}R_\delta(\delta_i) - \lambda q[h(1-\gamma_i) + (1-h)\alpha_i\beta]. \tag{44}$$

In any equilibrium, $\frac{\partial}{\partial \alpha_i}\pi_i < 0$, which implies that $\alpha_i = 0$ for all i. In any symmetric equilibrium, $\frac{\partial}{\partial \gamma_i}\pi_i < 0$, therefore a symmetric equilibrium candidate is $(0, 0, c^*, \delta^*)$. The rest of the proof proceeds identically to the Proof of Proposition 4c. □

Proof of Proposition 2n: The proof proceeds similarly to the proofs of Propositions 2 and 8, with the updated notation for cost being comparable and v_0'. The result that $\theta_0' \geq \theta_0$ follows from the observation that $v_0' > v_0$ and $u_0' < u_0$. $\square$

Proof of Proposition 3n: The proof proceeds similarly to the proof of Proposition 3, with the updated notation for cost being comparable. The result that $\theta_1' \geq \theta_1$ follows from the observation that $v_1' > v_1$ and $u_1' < u_1$. $\square$

Proof of Proposition 4n: The proof proceeds similarly to the proof of Proposition 4, with the updated notation for cost being comparable. The result that $\theta_2' \geq \theta_2$ follows from the observation that $v_2' > v_2$ and $u_2' < u_2$. $\square$

Proof of Proposition 6n: The proof proceeds similarly to the proof of Proposition 6. The result that $\bar{\kappa}' \leq \bar{\kappa}$ follows from the observation that $u_1' < u_1$ and $R_\delta(\delta^*) + R_c(c^*) > 0$.$\square$

Appendix 3: The case of two asymmetric providers

In the main analysis we assume that providers are symmetric and have argued that any scheme that achieves first best in the case of symmetric providers can be modified to account for provider heterogeneity based on factors that are observable by the HO and exogenous to the provider. In this section we extent the analysis to examine the case where there is heterogeneity that the HO does not account for in the reimbursement scheme (e.g., because it is not observable or because the HO is not sophisticated enough to make the necessary adjustments). Naturally, the results in this case will depend on the exact way in which providers are asymmetric, but to gain insights on the impact of asymmetry on equilibrium outcomes we will focus on the case where there are two providers ($N = 2$) that differ in the number of patients and in the proportion of high-complexity patients they treat. More specifically we assume that $\lambda_1 q_1 = \lambda q$ and $\lambda_2 q_2 = \eta\lambda q$, where $\eta > 1$ and $h_1 = h$, $h_2 = \xi h$ where $\xi > 1$. This specification ensures that Provider 2 treats more patients in total and more high-complexity patients than Provider 1. Naturally, we require that $h\xi < 1$ (i.e., the proportion of high complexity patients of Provider 2 is less than 1). We continue to assume that providers have access to the same cost reduction technology (i.e., $R_c(.)$ and $R_\delta(.)$ are identical) but, given their different size, they may decide to invest differently in cost reduction. Providers may choose to lemon drop a proportion $\gamma_i \leq \bar{\gamma}$ of their high-complexity patients and upcode a proportion $\alpha_i \leq \bar{\alpha}$ of their low-complexity patients.

In this case, the first best actions of Provider 1 are given by the same conditions as the main text:

$$\gamma_1^* = \alpha_1^* = 0, -\frac{d}{dc} R_c(c_1^*) = \lambda q, -\frac{d}{d\delta} R_\delta(\delta_1^*) = \lambda q h,$$

while the first best actions of Provider 2 become

$$\gamma_2^* = \alpha_2^* = 0, -\frac{d}{dc}R_c(c_2^*) = \lambda q\eta, -\frac{d}{d\delta}R_\delta(\delta_2^*) = \lambda qh\eta\xi.$$

Given the properties of $R_c(.)$ and $R_\delta(.)$ and since $\eta > 1, \xi > 1$, it follows that $c_1^* > c_2^*$ and $\delta_1^* > \delta_2^*$ (i.e., since the larger Provider 2 treats more patients, they find it optimal to invest more heavily in cost reduction and operate at lower cost levels compared to the smaller Provider 1).

Under cost-based yardstick competition, if the HO uses one DRG the fees per patient for each provider are given by $p_{M1} = p_{m1} = \bar{c}_1$ and $p_{M2} = p_{m2} = \bar{c}_2$, where $\bar{c}_i$ is the average treatment cost of the other provider:

$$\begin{aligned} \bar{c}_1 &:= c_2 + \delta_2 \frac{h\xi(1-\gamma_2) + (1-h\xi)\alpha_2\beta}{1-h\xi\gamma_2}, \\ \bar{c}_2 &:= c_1 + \delta_1 \frac{h(1-\gamma_1) + (1-h)\alpha_1\beta}{1-h\gamma_1}. \end{aligned} \tag{45}$$

If the HO breaks the condition into two distinct DRGs the fees per patient episode for the major and the minor condition for each provider are given by $p_{Mi} = \bar{c}_{Mi}$ and $p_{mi} = \bar{c}_{mi}$, respectively, where

$$\begin{aligned} \bar{c}_{M1} &:= c_2 + \delta_2 \frac{h\xi(1-\gamma_2) + (1-h\xi)\alpha_2\beta}{h\xi(1-\gamma_2) + (1-h\xi)\alpha_2} \text{ and } \bar{c}_{m1} := c_2, \\ \bar{c}_{M2} &:= c_1 + \delta_1 \frac{h(1-\gamma_1) + (1-h)\alpha_1\beta}{h(1-\gamma_1) + (1-h)\alpha_1} \text{ and } \bar{c}_{m2} := c_1. \end{aligned} \tag{46}$$

In both cases, the transfer payment for each provider is given by

$$\begin{aligned} \bar{R}_1 &:= R_c(c_2) + R_\delta(\delta_2), \\ \bar{R}_2 &:= R_c(c_1) + R_\delta(\delta_1). \end{aligned} \tag{47}$$

We will analyse the equilibrium outcome for different cases below. In all the analysis we focus on the provider's actions and how they compare to first-best actions. In doing so, we will not impose participation constraints, i.e., we will allow equilibria where one of the two providers may be receiving a negative rent. (If participation constraints apply then in those cases where at least one of the providers receives a negative rent no equilibrium would exist.)

We start by looking at the case were there is no upcoding or cherry picking. The following proposition summarizes the equilibrium actions of the providers.

Proposition 1A: *Under yardstick competition, and irrespective of the number of DRGs used (one or two), in the absence of patient lemon dropping and upcoding ($\bar{\gamma} = \bar{\alpha} = 0$) providers invest in cost reduction optimally.*

Proposition 1A shows that yardstick competition scheme is effective in providing first-best incentives for cost reduction even if the providers are asymmetric. This is the case even if the HO does not adjust for this asymmetry.

We will next examine the case were providers may lemon drop a proportion $\gamma_i \leq \bar{\gamma}$ patients but cannot upcode. We find it convenient to define cherry-picking-best costs $(c_1^{e1}, \delta_1^{e1})$ for Provider 1 as the solutions to

$$-\frac{d}{dc}R_c(c_1^{e1}) = \lambda q(1-\bar{\gamma}h), -\frac{d}{d\delta}R_\delta(\delta_1^{e2}) = \lambda qh(1-\bar{\gamma}),$$

respectively, and $(c_2^{e1}, \delta_2^{e1})$ for Provider 2 as the solutions to

$$-\frac{d}{dc}R_c(c_2^{e2}) = \lambda q\eta(1-\bar{\gamma}\xi h), -\frac{d}{d\delta}R_\delta(\delta_2^{e2}) = \lambda q\eta\xi h(1-\bar{\gamma}),$$

respectively. Due to economies of scale (i.e., $R_c''(.) > 0$, $R_\delta''(.) > 0$), cherry-picking-best costs are higher than first best costs for both providers (i.e., $c_i^{e1} > c_i^*$ and $\delta_i^{e1} > \delta_i^*$).

Proposition 2A: *In the absence of upcoding ($\bar{\alpha} = 0$), if the HO implements yardstick competition based on a single DRG, then*

- *There exists a Nash equilibrium where Provider 1 drops as many patients as possible ($\gamma_1 = \bar{\gamma}$), chooses cherry-picking-best costs $(c_1^{e1}, \delta_1^{e1})$. If $c_1^{e1} + \delta_1^{e1} h \frac{1-\bar{\gamma}}{1-h\bar{\gamma}} < c_2^* + \delta_2^*$, then Provider 2 also drops as many patients as possible ($\gamma_2 = \bar{\gamma}$) and chooses cherry-picking-best costs $(c_2^{e1}, \delta_2^{e1})$. If $c_2^* + \delta_2^* < c_1^{e1} + \delta_1^{e1} \frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$ then Provider 2 does not drop any patients ($\gamma_2 = 0$) and invest optimally in cost reduction (i.e. chooses costs c_2^*, δ_2^*). Otherwise, Provider 2 drops an intermediate proportion γ_2^v of patients, where $0 < \gamma_2^v < \bar{\gamma}$ and chooses intermediate costs (c_2^v, δ_2^v), where $c_2^{e1} > c_2^v > c_2^*$, $\delta_2^{e1} > \delta_2^v > \delta_2^*$.*
- *If $c_1^* + \delta_1^* < c_2^{e1} + \delta_2^{e1} \frac{h\xi(1-\bar{\gamma})}{1-h\xi\bar{\gamma}}$ then there exists a second Nash equilibrium in which Provider 2 drops as many patients as possible ($\gamma_2 = \bar{\gamma}$), and chooses cherry-picking-best costs $(c_2^{e1}, \delta_2^{e1})$. If $c_1^* + \delta_1^* h < c_2^{e1} + \delta_2^{e1}$ then Provider 1 does not drop any patients ($\gamma_1 = 0$) and invests optimally in cost reduction (i.e. chooses costs c_1^*, δ_1^*), otherwise Provider 1 drops an intermediate proportion γ_1^m of patients, where $0 < \gamma_1^m < \bar{\gamma}$ and chooses intermediate costs (c_1^m, δ_1^m), where $c_1^{e1} > c_1^m > c_1^*$, $\delta_1^{e1} > \delta_1^m > \delta^*$.*

This proposition is equivalent to Propositions 2 and 8 of the symmetric case. It shows that if lemon dropping is possible then at least one or both providers will engage in it. As a result of lemon dropping, providers will also underinvest in cost reduction. More specifically, the symmetric equilibrium of Proposition 2, where both providers exercised maximum lemon dropping, still exists and is "symmetric" in the sense that providers still exercise maximum lemon dropping but it is no longer symmetric in the sense that providers choose to operate at different cost levels. Similarly, the asymmetric equilibrium of Proposition 8, where only one provider engaged in lemon dropping, still exists but in this case the provider who engages in lemon dropping could be the larger or, under some conditions, the smaller provider. It is notable that there are no model parameters where the equilibrium does not involve at least some lemon dropping.

We turn to the case where the HO uses two DRGs next.

Proposition 3A: *In the absence of upcoding ($\bar{\alpha}=0$), if the HO implements yardstick competition based on two DRGs, there exists a Nash equilibrium in which Provider 1 drops as many patients as possible and underinvests in cost reduction compared to first best, Provider 2 does not drop any patients and invests optimally in cost reduction. In addition, if $c_2^{e2}+\delta_2^{e2}-c_1^*-\delta_1^*\geq 0$, then there exists a second Nash equilibrium in which Provider 2 drops as many patients as possible and underinvests in cost reduction compared to first best, Provider 1 does not drop any patients and invests optimally in cost reduction.*

This result echoes that of Proposition 3 of the symmetric case. Expanding the number of DRGs is helpful in the sense that now one of the two providers acts optimally. If the providers are sufficiently asymmetric (e.g., even if the larger Provider 2 drops as many patients as possible it still treats more patients that Provider 1 treats if they do not drop any patient, or, to be more precise, so that the first-best cost of the smaller provider are larger than the cherry-picking-best costs of the larger provider) then the equilibrium is unique and the larger Provider 2 is the one that acts optimally. If however the providers are relatively similar an additional equilibrium will emerge the smaller Provider 1 is the one that acts optimally. Therefore, just like in the case of symmetric providers, DRG expansion is not a panacea in this case either.

We turn to the case were in addition to lemon dropping providers can also upcode. Before we present results, we first define the costs $(\delta_1^{e2},\delta_1^{e3})$ as the unique solutions to the equations

$$-\frac{d}{d\delta}R_\delta(\delta)=\lambda q(h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta),-\frac{d}{d\delta}R_\delta(\delta)=\lambda q(h+(1-h)\bar{\alpha}\beta),$$

respectively, and the costs $(\delta_2^{e2},\delta_2^{e3})$ as the unique solutions to the equations

$$-\frac{d}{d\delta}R_\delta(\delta)=\lambda q\eta(\xi h(1-\bar{\gamma})+(1-\xi h)\bar{\alpha}\beta),-\frac{d}{d\delta}R_\delta(\delta)=\lambda q\eta(\xi h+(1-\xi h)\bar{\alpha}\beta),$$

respectively. Note that since $\eta>1,\xi>1,h\xi<1$ it follows that $\delta_2^{e2}<\delta_1^{e2}$, $\delta_2^{e3}<\delta_1^{e3}$, $\delta_i^*>\delta_i^{e3}$ and $\delta_i^{e2}>\delta_i^{e3}$. To focus on cases where upcoding is profitable for both providers, we make the following assumptions that are equivalent to condition (7) of the main text (see discussion in the main text for an explanation as to why these conditions ensure that upcoding is profitable):

$$\delta_2^{e3}\frac{\xi h+(1-h\xi)\bar{\alpha}\beta}{\xi h+(1-h\xi)\bar{\alpha}}>\beta\delta_1^{e1},\delta_1^{e3}\frac{h+(1-h)\bar{\alpha}\beta}{h+(1-h)\bar{\alpha}}>\beta\delta_2^{e1}. \tag{48}$$

For the rest of the analysis we will assume that these two conditions hold.

Proposition 4A: *If both lemon dropping and upcoding are possible, if the HO implements yardstick competition based on two DRG,*

- *There exists a Nash equilibrium where Provider 1 upcodes and drops as many patients as possible ($\alpha_1 = \bar{\alpha}$, $\gamma_1 = \bar{\gamma}$) and chooses costs $(c_1^{e1}, \delta_1^{e2})$. If $c_2^{e1} + \delta_2^{e2} > c_1^{e1} + \delta_1^{e2} \frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$, then Provider 2 also upcodes and drops as many patients as possible ($\alpha_2 = \bar{\alpha}$, $\gamma_2 = \bar{\gamma}$) and chooses costs $(c_2^{e1}, \delta_2^{e1})$. If $c_2^* + \delta_2^{e3} < c_1^{e1} + \delta_1^{e2} \frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$ then Provider 2 upcodes as much as possible does not drop any patients ($\alpha_2 = \bar{\alpha}$, $\gamma_2 = 0$) and chooses costs (c_2^*, δ_2^{e1}). Otherwise, Provider 2 upcodes as much as possible ($\alpha_2 = \bar{\alpha}$), drops an intermediate proportion (γ_2^v) of patients, where $0 < \gamma_2^v < \bar{\gamma}$ and chooses intermediate costs (c_2^v, δ_2^v), where $c_2^{e1} > c_2^v > c_2^*$, $\delta_2^{e3} > \delta_2^v > \delta_2^*$.*
- *If $c_1^* + \delta_1^{e3} < c_2^{e1} + \delta_2^{e2} \frac{\xi h(1-\bar{\gamma})+(1-h\xi)\bar{\alpha}\beta}{\xi h(1-\bar{\gamma})+(1-h\xi)\bar{\alpha}}$ then there exists a second Nash equilibrium in which Provider 2 upcodes as many patients as possible ($\alpha_2 = \bar{\alpha}$) drops as many patients as possible ($\gamma_2 = \bar{\gamma}$), and chooses costs $(c_2^{e1}, \delta_2^{e2})$. If $c_2^{e2} + \delta_2^{e2} > c_1^* + \delta_1^{e3} \frac{h+(1-h)\bar{\alpha}\beta}{h+(1-h)\bar{\alpha}}$ then Provider 1 upcodes as many patients as possible ($\alpha_1 = \bar{\alpha}$) does not drop any patients ($\gamma_1 = 0$) and chooses costs (c_1^*, δ_1^{e3}), otherwise Provider 1 drops an intermediate proportion (γ_1^m) of patients $0 < \gamma_1^m < \bar{\gamma}$ and chooses intermediate costs (c_1^m, δ_1^m), where $c_1^{e1} > c_1^m > c_1^*$, $\delta_1^{e1} > \delta_1^m > \delta_1^{e3}$.*

The results presented in Proposition 4A minor that of Proposition 4 in the symmetric case. If upcoding is possible providers will engage fully in it and, even worse, upcoding will make it optimal for one or both providers to also engage in lemon dropping.

Collectively, this section demonstrates that in the case of two providers that are asymmetric in the number of patients they treat and/or in the proportion of high complexity patients, the results of the symmetric case continue to apply. Namely, in the absence of upcoding, expanding the number of DRGs eliminates lemon dropping incentives only for one of the two providers (usually the larger). The other provider will continue to lemon drop patients. If upcoding is possible then this will, in most cases, reinstate lemon dropping incentives for both provider as well.

Proofs of results appearing in Appendix 3

Proof of Proposition 1A: If there is no upcoding or cherry picking the profit of each provider is given by

$$\pi_1(c_1, \delta_1, 0, 0) = \lambda q \left[h(p_{M1} - \delta_1 - c_1) + (1-h)(p_{m1} - c_1)\right] - R_c(c_1) - R_\delta(\delta_1) + R_c(c_2) + R_\delta(\delta_2)$$

$$\pi_2(c_2, \delta_2, 0, 0) = \lambda q \eta \left[h\xi(p_{M2} - \delta_2 - c_2) + (1-h\xi)(p_{m2} - c_2)\right] - R_c(c_2) - R_\delta(\delta_2) + R_c(c_1) + R_\delta(\delta_1)$$

Since the provider's choice of c_i and δ_i does not affect the reimbursement received, and this is true irrespective of the number of DRGs used by the HO, the profit-maximizing choice of the provider is given by

$$-\frac{d}{dc}R_c(c_1) = \lambda q,\ -\frac{d}{d\delta}R_\delta(\delta_1) = \lambda q h,\ -\frac{d}{dc}R_c(c_2) = \lambda q \eta,\ -\frac{d}{d\delta}R_\delta(\delta_2) = \lambda q h \eta \xi.$$

These conditions are identical to the first order conditions of the welfare-maximization problem. Therefore, the first-best investment decisions constitute a Nash equilibrium. Furthermore, since $R_c'' > 0$ and $R_\delta'' > 0$ these values are unique. □

Proof of Proposition 2A: In the absence of upcoding and if the HO uses a single DRG, the profit of each provider is given by

$$\pi_1 = \lambda q\left[(1-h\gamma_1)\bar{c}_1 - \delta_1 h(1-\gamma_1) - c_1(1-h\gamma_1)\right] - R_c(c_1) - R_\delta(\delta_1) + R_c(c_2) + R_\delta(\delta_2)$$

$$\pi_2 = \lambda q\eta\left[(1-h\xi\gamma_2)\bar{c}_2 - \delta_2 h\xi(1-\gamma_2) - c_2(1-h\xi\gamma_2)\right] - R_c(c_2) - R_\delta(\delta_2) + R_c(c_1) + R_\delta(\delta_1),$$

with $\bar{c}_1 = c_2 + \delta_2\frac{h\xi(1-\gamma_2)}{1-h\xi\gamma_2}$, $\bar{c}_2 = c_1 + \delta_1\frac{h(1-\gamma_1)}{1-h\gamma_1}$. The derivatives of the profit function of Provider 1 are given by:

$$\frac{\partial}{\partial\gamma_1}\pi_1 = \lambda qh(c_1+\delta_1-\bar{c}_1), \tag{49}$$

$$\frac{\partial}{\partial c_1}\pi_1 = -\frac{d}{dc}R_c(c_1) - \lambda q(1-h\gamma_1), \tag{50}$$

$$\frac{\partial}{\partial\delta_1}\pi_1 = -\frac{d}{d\delta}R_\delta(\delta_1) - \lambda qh(1-\gamma_1), \tag{51}$$

and for Provider 2

$$\frac{\partial}{\partial\gamma_2}\pi_2 = \lambda q\eta h\xi(c_2+\delta_2-\bar{c}_2), \tag{52}$$

$$\frac{\partial}{\partial c_2}\pi_2 = -\frac{d}{dc}R_c(c_2) - \lambda q\eta(1-h\xi\gamma_2), \tag{53}$$

$$\frac{\partial}{\partial\delta_2}\pi_2 = -\frac{d}{d\delta}R_\delta(\delta_2) - \lambda q\eta h\xi(1-\gamma_2). \tag{54}$$

In equilibrium the conditions (50), (51), (53), (54) must all be equal to zero, otherwise the provider for whom one of these conditions is not zero could improve their profits.

Turning to (49), we need to check three cases.

- First, if at the optimal solution (49) is positive, i.e., $c_1+\delta_1-c_2-\delta_2 h\xi\frac{1-\gamma_2}{1-h\xi\gamma_2} > 0$, then $\frac{\partial}{\partial\gamma_1}\pi_1 > 0$, therefore Provider 1 must find it optimal to choose $\gamma_1 = \bar{\gamma}$ and $c_1 = c_1^{e1}$, $\delta_1 = \delta_1^{e1}$. Turning to (52) we will examine three subcases:

—First, if at the optimal solution $c_2+\delta_2 > c_1^{e1}+\delta_1^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$ then $\frac{\partial}{\partial\gamma_2}\pi_2 > 0$, therefore Provider 2 must find it optimal to choose $\gamma_2 = \bar{\gamma}$ and $c_2 = c_2^{e1}$, $\delta_2 = \delta_2^{e1}$. At these choices, the condition on (49) is indeed satisfied. However, for the condition on (52) to be satisfied the following condition must hold $c_2^{e1}+\delta_2^{e1} > c_1^{e1}+\delta_1^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$.

—Second, if at the optimal solution $c_2+\delta_2 < c_1^{e1}+\delta_1^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$ then $\frac{\partial}{\partial\gamma_2}\pi_2 < 0$ therefore Provider 2 finds it optimal to choose $\gamma_2 = 0$ and $c_2 = c_2^*$, $\delta_2 = \delta_2^*$. At these choices, the condition on (49) is indeed satisfied. However, for the condition on (52) to be satisfied the following condition must hold $c_2^*+\delta_2^* < c_1^{e1}+\delta_1^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$.

—Third, if at the optimal solution $c_2+\delta_2=c_1^{e1}+\delta_1^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$ then $\frac{\partial}{\partial\gamma_2}\pi_2=0$. Therefore, Provider 2 will find it optimal to choose $\gamma_2=\gamma_2^v$, where $0<\gamma_2^v<\bar{\gamma}$ with c_2^v and δ_2^v such that (53) and (54) are equal to zero. γ_2^v is such that $c_2^v+\delta_2^v=c_1^{e1}+\delta_1^{e1}\frac{h(1-\bar{\gamma})}{1-h\bar{\gamma}}$. At these choices the condition on (49) is satisfied.

- Second, if at the optimal solution (49) is negative, i.e., $c_1+\delta_1-c_2-\delta_2 h\xi\frac{1-\gamma_2}{1-h\xi\gamma_2}<0$ then $\frac{\partial}{\partial\gamma_1}\pi_1\le 0$, suggesting that Provider 1 would find it optimal to choose $\gamma_1=0$ and $c_1=c_1^*$, $\delta_1=\delta_1^*$. Turning to (52) we will examine three subcases:

—First, if $c_2+\delta_2<c_1^*+\delta_1^*h$ then $\frac{\partial}{\partial\gamma_2}\pi_2<0$ therefore Provider 2 finds it optimal to choose $\gamma_2=0$ and $c_2=c_2^*$, $\delta_2=\delta_2^*$. At these choices, the condition on (49) cannot be satisfied, leading to a contradiction. Therefore, this cannot be an equilibrium outcome.

—Second, if $c_2+\delta_2>c_1^*+\delta_1^*h$ then $\frac{\partial}{\partial\gamma_2}\pi_2>0$ therefore Provider 2 finds it optimal to choose $\gamma_2=\bar{\gamma}$ and $c_2=c_2^{e1}$, $\delta_2=\delta_2^{e1}$. At these choices, the conditions on (49) and (52) are both satisfied if $c_1^*+\delta_1^*<c_2^{e1}+\delta_2^{e1}\frac{h\xi(1-\bar{\gamma})}{1-h\xi\bar{\gamma}}$ and $c_2^{e1}+\delta_2^{e1}>c_1^*+\delta_1^*h$.

—Third, if $c_2+\delta_2=c_1^*+\delta_1^*h$ then $\frac{\partial}{\partial\gamma_2}\pi_2=0$. Therefore, Provider 2 will find it optimal to choose $\gamma_2=\gamma^{vv}$, where $0<\gamma_2^{vv}<\bar{\gamma}$ with c_2^{vv} and δ_2^{vv} such that (53) and (54) are equal to zero. γ_2^{vv} is such that $c_1^*+\delta_1^*h=c_2^{vv}+\delta_2^{vv}$. At these choices condition (49) becomes $c_2^{vv}+\delta_2^{vv}\frac{h\xi(1-\gamma_2^{vv})}{1-h\xi\gamma_2^{vv}}>c_1^*+\delta_1^*$. At these choices, adding up (49) and (52) gives $\delta_2^{vv}\frac{1-h\xi}{1-h\xi\gamma_2^{vv}}+\delta_1^*(1-h)<0$, leading to a contradiction. Therefore, this cannot be an equilibrium outcome.

- Third, if at the optimal solution (49) is equal to zero, i.e., $c_1+\delta_1-c_2-\delta_2 h\xi\frac{1-\gamma_2}{1-h\xi\gamma_2}=0$ then $\frac{\partial}{\partial\gamma_1}\pi_1=0$, suggesting that Provider 1 would find it optimal to choose $\gamma_1=\gamma_1^m$ and $c_1=c_1^m$, $\delta_1=\delta_1^m$ such that conditions (50), (51) are equal to zero. Turning to (52) we will examine three subcases:

—First, if $c_2+\delta_2>c_1^m+\delta_1^m h\frac{1-\gamma_1^m}{1-h\gamma_1^m}$ then $\frac{\partial}{\partial\gamma_2}\pi_2>0$ therefore Provider 2 finds it optimal to choose $\gamma_2=\bar{\gamma}$ and $c_2=c_2^{e1}$, $\delta_2=\delta_2^{e2}$. At these choices, the condition on (52) implies that $c_2^{e1}+\delta_2^{e1}>c_1^m+\delta_1^m h\frac{1-\gamma_1^m}{1-h\gamma_1^m}$. Given the condition on (49), this can be written as $\delta_1^m\frac{1-h}{1-h\gamma_1^m}+\delta_2^{e1}\frac{1-h\xi}{1-h\xi\bar{\gamma}}>0$, which is always satisfied. Turning to the condition on (49), there will exist a $\gamma_1^m<\bar{\gamma}$ to satisfy this condition only if $c_1^*+\delta_1^*<c_2^{e1}+\delta_2^{e1}\frac{h\xi(1-\bar{\gamma})}{1-h\xi\bar{\gamma}}$.

—Second, if $c_2+\delta_2<c_1^m+\delta_1^m h\frac{1-\gamma_1^m}{1-h\gamma_1^m}$ then $\frac{\partial}{\partial\gamma_2}\pi_2<0$ therefore Provider 2 finds it optimal to choose $\gamma_2=0$ and $c_2=c_2^*$, $\delta_2=\delta_2^*$. At these choices, the condition on (49) becomes $c_1^m+\delta_1^m=c_2^*+\delta_2^*h\xi$. This cannot be true as $c_1^m>c_2^*$, $\delta_1^m>\delta_2^*$ and $h\xi<1$, leading to a contradiction. Therefore, this cannot be an equilibrium outcome.

—Third, if $c_2+\delta_2=c_1^m+\delta_1^m h\frac{1-\gamma_1^m}{1-h\gamma_1^m}$ then $\frac{\partial}{\partial\gamma_2}\pi_2=0$. Therefore, Provider 2 will find it optimal to choose $\gamma_2=\gamma_2^{mm}$, where $0<\gamma_2^{mm}<\bar{\gamma}$ with c_2^{mm} and δ_2^{mm} such that (53) and (54) are equal to zero. γ_2^{mm} is such that $c_2^{mm}+\delta_2^{mm}=c_1^m+\delta_1^m\frac{h(1-\gamma_1^m)}{1-h\gamma_1^m}$. At these choices, adding up (49) and (52) gives $\delta_2^{mm}\frac{1-h\xi}{1-h\xi\gamma_2^{mm}}+\delta_1^m\frac{1-h}{1-h\gamma_1^{mm}}=0$, leading to a contradiction. Therefore, this cannot be an equilibrium outcome.

Collectively, these cases describe the two equilibria presented in the Proposition.□

Proof of Proposition 3A: In the absence of upcoding ($\bar{\alpha} = 0$), under the yardstick competition scheme with two DRGs, the profit of each provider is given by

$$\pi_1 = \lambda q \left[h(1-\gamma_1)(\bar{c}_{M1} - \delta_1 - c_1) + (1-h)(\bar{c}_{m1} - c_1)\right] - R_c(c_1) - R_\delta(\delta_1) + R_c(c_2) + R_\delta(\delta_2),$$

$$\pi_2 = \lambda q \eta \left[h\xi(1-\gamma_2)(\bar{c}_{M2} - \delta_2 - c_2) + (1-h\xi)(\bar{c}_{m2} - c_2)\right] - R_c(c_2) - R_\delta(\delta_2) + R_c(c_1) + R_\delta(\delta_1),$$

where $\bar{c}_{M1} := c_2 + \delta_2$, $\bar{c}_{m1} := c_2$, $\bar{c}_{M2} := c_1 + \delta_1$, $\bar{c}_{m2} := c_1$. The derivatives of the profit function of Provider 1 are given by:

$$\frac{\partial}{\partial \gamma_1}\pi_1 = \lambda q h(c_1 + \delta_1 - \bar{c}_{M1}), \tag{55}$$

$$\frac{\partial}{\partial c_1}\pi_1 = -\frac{d}{dc}R_c(c_1) - \lambda q(1 - h\gamma_1), \tag{56}$$

$$\frac{\partial}{\partial \delta_1}\pi_1 = -\frac{d}{d\delta}R_\delta(\delta_1) - \lambda q h(1-\gamma_1), \tag{57}$$

and for Provider 2 by

$$\frac{\partial}{\partial \gamma_2}\pi_2 = \lambda q h \eta \xi(c_2 + \delta_2 - \bar{c}_{M2}), \tag{58}$$

$$\frac{\partial}{\partial c_2}\pi_2 = -\frac{d}{dc}R_c(c_2) - \lambda q \eta(1 - h\xi\gamma_2), \tag{59}$$

$$\frac{\partial}{\partial \delta_2}\pi_2 = -\frac{d}{d\delta}R_\delta(\delta_2) - \lambda q h \eta \xi(1-\gamma_2). \tag{60}$$

In equilibrium the conditions (56), (57), (59), (60) must all be equal to zero, otherwise the provider for whom one of these conditions is not zero could improve their profits.

Turning to (55), if $c_1 + \delta_1 - c_2 - \delta_2 > 0$ then $\frac{\partial}{\partial \gamma_1}\pi_1 > 0$, which suggests that Provider 1 would find it optimal to choose $\gamma_1 = \bar{\gamma}$ and $c_1 = c_1^{e1}$, $\delta_1 = \delta_1^{e1}$ and from (58) then $\frac{\partial}{\partial \gamma_2}\pi_2 < 0$, which suggests that Provider 2 would find it optimal to choose $\gamma_2 = 0$, $c_2 = c_2^*$, $\delta_2 = \delta^*$. Given the optimal choices, the condition $c_1 + \delta_1 - c_2 - \delta_2 > 0$ is indeed satisfied.

If $c_1 + \delta_1 - c_2 - \delta_2 \le 0$ then $\frac{\partial}{\partial \gamma_1}\pi_1 \le 0$, which suggests that Provider 1 would find it optimal to choose $\gamma_1 = 0$ and $c_1 = c_1^*$, $\delta_1 = \delta_1^*$ and (58) would imply that $\frac{\partial}{\partial \gamma_2}\pi_2 \ge 0$ which in turn suggests that Provider 2 would find it optimal to choose $\gamma_2 = \bar{\gamma}$, $c_2 = c_2^{e1}$, $\delta_2 = \delta^{e1}$. Given the optimal choices, the condition $c_1 + \delta_1 - c_2 - \delta_2 < 0$ can only be satisfied if $c_1^* + \delta_1^* - c_2^{e1} - \delta_2^1 < 0$. □

Proof of Proposition 4A: In the presence of upcoding, under the yardstick competition scheme with two DRGs, the profit of each provider is given by

$$\begin{aligned}\pi_1 &= \lambda q \left[(h(1-\gamma_1) + (1-h)\alpha_1)\bar{c}_{M1} + (1-h)(1-\alpha_1)\bar{c}_{m1} - h(1-\gamma_1)\delta_1 - (1-\gamma_1 h)c_1\right] \\ &- R_c(c_1) - R_\delta(\delta_1) + R_c(c_2) + R_\delta(\delta_2),\end{aligned}$$

$$\pi_2 = \lambda q\eta\left[(h\xi(1-\gamma_2)+(1-h\xi)\alpha_2)\bar{c}_{M2}+(1-h\xi)(1-\alpha_2)\bar{c}_{m2}-h\xi(1-\gamma_2)\delta_2-(1-\gamma_2 h\xi)c_2\right]$$
$$- R_c(c_2)-R_\delta(\delta_2)+R_c(c_1)+R_\delta(\delta_1),$$

where $\bar{c}_{M1} := c_2+\delta_2\frac{\xi h(1-\gamma_2)+(1-h\xi)\alpha_2\beta}{\xi h(1-\gamma_2)+(1-h\xi)\alpha_2}$, $\bar{c}_{m1} := c_2$, $\bar{c}_{M2} := c_1+\delta_1\frac{h(1-\gamma_1)+(1-h)\alpha_1\beta}{h(1-\gamma_1)+(1-h)\alpha_1}$, $\bar{c}_{m2} := c_1$. The derivatives of the profit function of Provider 1 are given by:

$$\frac{\partial}{\partial \alpha_1}\pi_1 = \lambda q h(\bar{c}_{M1}-\bar{c}_{m1}-\beta\delta_1), \tag{61}$$
$$\frac{\partial}{\partial \gamma_1}\pi_1 = \lambda q h(c_1+\delta_1-\bar{c}_{M1}), \tag{62}$$
$$\frac{\partial}{\partial c_1}\pi_1 = -\frac{d}{dc}R_c(c_1)-\lambda q(1-h\gamma_1), \tag{63}$$
$$\frac{\partial}{\partial \delta_1}\pi_1 = -\frac{d}{d\delta}R_\delta(\delta_1)-\lambda q[h(1-\gamma_1)+(1-h)\alpha_1\beta], \tag{64}$$

and for Provider 2 by

$$\frac{\partial}{\partial \gamma_2}\pi_2 = \lambda q h\eta\xi(\bar{c}_{M2}-\bar{c}_{m2}-\beta\delta_2), \tag{65}$$
$$\frac{\partial}{\partial \gamma_2}\pi_2 = \lambda q h\eta\xi(c_2+\delta_2-\bar{c}_{M2}), \tag{66}$$
$$\frac{\partial}{\partial c_2}\pi_2 = -\frac{d}{dc}R_c(c_2)-\lambda q\eta(1-h\xi\gamma_2), \tag{67}$$
$$\frac{\partial}{\partial \delta_2}\pi_2 = -\frac{d}{d\delta}R_\delta(\delta_2)-\lambda q\eta[h\xi(1-\gamma_2)+(1-h\xi)\alpha_2\beta]. \tag{68}$$

In equilibrium the conditions (63), (64), (67), (68) must all be equal to zero, otherwise the provider for whom one of these conditions is not zero could improve their profits. Condition (61) can be written as $\frac{\partial}{\partial\alpha_1}\pi_1 = \delta_2\frac{\xi h(1-\gamma_2)+(1-h\xi)\alpha_2\beta}{\xi h(1-\gamma_2)+(1-h\xi)\alpha_2} - \beta\delta_1$. Note that the first term is minimized at $\gamma_2 = 0$ and the second term is maximized at δ_1^{e2}. Therefore, (61) is positive if (48) holds and this implies that $\alpha_1 = \bar{\alpha}$. A similar argument shows that if (48) holds then $\alpha_2 = \bar{\alpha}$.

Turning to condition (62), this can be written as $\frac{\partial}{\partial\gamma_1}\pi_1 = c_1+\delta_1-c_2-\delta_2\frac{\xi h(1-\gamma_2)+(1-h\xi)\bar{\alpha}\beta}{\xi h(1-\gamma_2)+(1-h\xi)\bar{\alpha}}$. We need to check 3 cases.

- First, if at the optimal solution (62) is positive, i.e., $c_1+\delta_1-c_2-\delta_2\frac{\xi h(1-\gamma_2)+(1-h\xi)\bar{\alpha}\beta}{\xi h(1-\gamma_2)+(1-h\xi)\bar{\alpha}} > 0$, then $\frac{\partial}{\partial\gamma_1}\pi_1 > 0$, therefore Provider 1 must find it optimal to choose $\gamma_1 = \bar{\gamma}$ and $c_1 = c_1^{e1}$, $\delta_1 = \delta_1^{e2}$. Turning to (66) we will examine three subcases:

—First, if at the optimal solution $c_2+\delta_2 > c_1^{e1}+\delta_1^{e2}\frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$ then $\frac{\partial}{\partial\gamma_2}\pi_2 > 0$, therefore Provider 2 must find it optimal to choose $\gamma_2 = \bar{\gamma}$ and $c_2 = c_2^{e1}$, $\delta_2 = \delta_2^{e2}$. Since $\beta < 1$, at these choices the condition on (62) is indeed satisfied. However, for the condition on (66) to be satisfied the following condition must hold $c_2^{e1}+\delta_2^{e2} > c_1^{e1}+\delta_1^{e2}\frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$.

—Second, if at the optimal solution $c_2+\delta_2 < c_1^{e1}+\delta_1^{e2}\frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$ then $\frac{\partial}{\partial\gamma_2}\pi_2 < 0$ therefore Provider 2 finds it optimal to choose $\gamma_2 = 0$ and $c_2 = c_2^{*}$, $\delta_2 = \delta_2^{e3}$. At these choices, the condition on

(62) is indeed satisfied. However, for the condition on (66) to be satisfied the following condition must hold $c_2^* + \delta_2^{e3} < c_1^{e1} + \delta_1^{e2} \frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$.

—Third, if at the optimal solution $c_2 + \delta_2 = c_1^{e1} + \delta_1^{e2} \frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$ then $\frac{\partial}{\partial \gamma_2}\pi_2 = 0$. Therefore, Provider 2 will find it optimal to choose $\gamma_2 = \gamma_2^v$, where $0 < \gamma_2^v < \bar{\gamma}$ with c_2^v and δ_2^v such that (67) and (68) are equal to zero. γ_2^v is such that $c_2^v + \delta_2^v = c_1^{e1} + \delta_1^{e2} \frac{h(1-\bar{\gamma})+(1-h)\bar{\alpha}\beta}{h(1-\bar{\gamma})+(1-h)\bar{\alpha}}$. At these choices the condition on (62) is satisfied.

- Second, if at the optimal solution (62) is negative, i.e., $c_1 + \delta_1 - c_2 - \delta_2 \frac{\xi h(1-\gamma_2)+(1-h\xi)\bar{\alpha}\beta}{\xi h(1-\gamma_2)+(1-h\xi)\bar{\alpha}} < 0$ then $\frac{\partial}{\partial \gamma_1}\pi_1 \le 0$, suggesting that Provider 1 would find it optimal to choose $\gamma_1 = 0$ and $c_1 = c_1^*$, $\delta_1 = \delta_1^{e3}$. Turning to (66) we will examine three subcases:

—First, if $c_2 + \delta_2 < c_1^* + \delta_1^{e3} \frac{h+(1-h)\bar{\alpha}\beta}{h+(1-h)\bar{\alpha}}$ then $\frac{\partial}{\partial \gamma_2}\pi_2 < 0$ therefore Provider 2 finds it optimal to choose $\gamma_2 = 0$ and $c_2 = c_2^*$, $\delta_2 = \delta_2^{e3}$. At these choices, the condition on (62) cannot be satisfied, leading to a contradiction. Therefore, this cannot be an equilibrium outcome.

—Second, if $c_2 + \delta_2 > c_1^* + \delta_1^{e3} \frac{h+(1-h)\bar{\alpha}\beta}{h+(1-h)\bar{\alpha}}$ then $\frac{\partial}{\partial \gamma_2}\pi_2 > 0$ therefore Provider 2 finds it optimal to choose $\gamma_2 = \bar{\gamma}$ and $c_2 = c_2^{e1}$, $\delta_2 = \delta_2^{e2}$. At these choices, the conditions on (62) and (66) are both satisfied if $c_1^* + \delta_1^{e3} < c_2^{e1} + \delta_2^{e2} \frac{\xi h(1-\bar{\gamma})+(1-h\xi)\bar{\alpha}\beta}{\xi h(1-\bar{\gamma})+(1-h\xi)\bar{\alpha}}$ and $c_2^{e2} + \delta_2^{e2} > c_1^* + \delta_1^{e3} \frac{h+(1-h)\bar{\alpha}\beta}{h+(1-h)\bar{\alpha}}$.

—Third, if $c_2 + \delta_2 = c_1^* + \delta_1^{e3} \frac{h+(1-h)\bar{\alpha}\beta}{h+(1-h)\bar{\alpha}}$ then $\frac{\partial}{\partial \gamma_2}\pi_2 = 0$. Therefore, Provider 2 will find it optimal to choose $\gamma_2 = \gamma^{vv}$, where $0 < \gamma_2^{vv} < \bar{\gamma}$ with c_2^{vv} and δ_2^{vv} such that (67) and (68) are equal to zero. γ_2^{vv} is such that $c_1^* + \delta_1^{e3} \frac{h+(1-h)\bar{\alpha}\beta}{h+(1-h)\bar{\alpha}} = c_2^{vv} + \delta_2^{vv}$. At these choices condition (62) becomes $c_2^{vv} + \delta_2^{vv} \frac{h\xi(1-\gamma^{vv})+(1-h\xi)\bar{\alpha}\beta}{h\xi(1-\gamma^{vv})+(1-h\xi)\bar{\alpha}} > c_1^* + \delta_1^{e3}$. At these choices, adding up (62) and (66) gives $\delta_2^{vv}(1 - \frac{h\xi(1-\gamma^{vv})+(1-h\xi)\bar{\alpha}\beta}{h\xi(1-\gamma^{vv})+(1-h\xi)\bar{\alpha}}) + \delta_1^{e3}(1 - \frac{h+(1-h)\bar{\alpha}\beta}{h+(1-h)\bar{\alpha}}) < 0$, leading to a contradiction. Therefore, this cannot be an equilibrium outcome.

- Third, if at the optimal solution (62) is equal to zero, i.e., $c_1 + \delta_1 - c_2 - \delta_2 \frac{\xi h(1-\gamma_2)+(1-h\xi)\bar{\alpha}\beta}{\xi h(1-\gamma_2)+(1-h\xi)\bar{\alpha}} = 0$ then $\frac{\partial}{\partial \gamma_1}\pi_1 = 0$, suggesting that Provider 1 would find it optimal to choose $\gamma_1 = \gamma_1^m$ and $c_1 = c_1^m$, $\delta_1 = \delta_1^m$ such that conditions (63), (64) are equal to zero. Turning to (66) we will examine three subcases:

—First, if $c_2 + \delta_2 > c_1^m + \delta_1^m \frac{h(1-\gamma_1^m)+(1-h)\bar{\alpha}\beta}{h(1-\gamma_1^m)+(1-h)\bar{\alpha}}$ then $\frac{\partial}{\partial \gamma_2}\pi_2 > 0$ therefore Provider 2 finds it optimal to choose $\gamma_2 = \bar{\gamma}$ and $c_2 = c_2^{e1}$, $\delta_2 = \delta_2^{e2}$. At these choices, the condition on (66) implies that $c_2^{e1} + \delta_2^{e2} > c_1^m + \delta_1^m \frac{h(1-\gamma_1^m)+(1-h)\bar{\alpha}\beta}{h(1-\gamma_1^m)+(1-h)\bar{\alpha}}$. Given the condition on (62), this can be written as $\delta_1^m(1 - \frac{h(1-\gamma_1^m)+(1-h)\bar{\alpha}\beta}{h(1-\gamma_1^m)+(1-h)\bar{\alpha}}) + \delta_2^{e2}(1 - \frac{\xi h(1-\bar{\gamma})+(1-h\xi)\bar{\alpha}\beta}{\xi h(1-\bar{\gamma})+(1-h\xi)\bar{\alpha}}) > 0$, which is always satisfied. Turning to the condition on (62), there will exist a $\gamma_1^m < \bar{\gamma}$ to satisfy this condition only if $c_1^* + \delta_1^{e3} < c_2^{e1} + \delta_2^{e2} \frac{\xi h(1-\bar{\gamma})+(1-h\xi)\bar{\alpha}\beta}{\xi h(1-\bar{\gamma})+(1-h\xi)\bar{\alpha}}$.

—Second, if $c_2 + \delta_2 < c_1^m + \delta_1^m \frac{h(1-\gamma_1^m)+(1-h)\bar{\alpha}\beta}{h(1-\gamma_1^m)+(1-h)\bar{\alpha}}$ then $\frac{\partial}{\partial \gamma_2}\pi_2 < 0$ therefore Provider 2 finds it optimal to choose $\gamma_2 = 0$ and $c_2 = c_2^*$, $\delta_2 = \delta_2^*$. At these choices, the condition on (62) becomes $c_1^m + \delta_1^m = c_2^* + \delta_2^{e3} \frac{\xi h+(1-h\xi)\bar{\alpha}\beta}{\xi h+(1-h\xi)\bar{\alpha}}$. This cannot be true as $c_1^m > c_2^*$, $\delta_1^m > \delta_2^{e3}$, leading to a contradiction. Therefore, this cannot be an equilibrium outcome.

— Third, if $c_2+\delta_2 = c_1^m + \delta_1^m \frac{h(1-\gamma_1^m)+(1-h)\bar{\alpha}\beta}{h(1-\gamma_1^m)+(1-h)\bar{\alpha}}$ then $\frac{\partial}{\partial\gamma_2}\pi_2 = 0$. Therefore, Provider 2 will find it optimal to choose $\gamma_2 = \gamma_2^{mm}$, where $0 < \gamma_2^{mm} < \bar{\gamma}$ with c_2^{mm} and δ_2^{mm} such that (67) and (68) are equal to zero. γ_2^{mm} is such that $c_2^{mm} + \delta_2^{mm} = c_1^m + \delta_1^m \frac{h(1-\gamma_1^m)+(1-h)\bar{\alpha}\beta}{h(1-\gamma_1^m)+(1-h)\bar{\alpha}}$. At these choices, adding up (62) and (66) gives $\delta_2^{mm}(1 - \frac{h\xi(1-\gamma_2^m)+(1-h\xi)\bar{\alpha}\beta}{h\xi(1-\gamma_2^m)+(1-h\xi)\bar{\alpha}}) + \delta_1^m(1 - \frac{h(1-\gamma_1^m)+(1-h)\bar{\alpha}\beta}{h(1-\gamma_1^m)+(1-h)\bar{\alpha}}) = 0$, leading to a contradiction. Therefore, this cannot be an equilibrium outcome.

Collectively, these cases describe the two equilibria presented in the Proposition.□