Journal of Financial Economics 149 (2023) 497–535

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec

Debt dynamics and credit risk[☆]

Peter Feldhütter[a,*], Stephen Schaefer[b]

[a] Department of Finance, Copenhagen Business School, Solbjerg Plads 3, A4.07, Frederiksberg 2000, Denmark
[b] London Business School, Regent's Park, London NW1 4SA, United Kingdom

ARTICLE INFO

Article history:
Received 22 December 2021
Revised 23 June 2023
Accepted 23 June 2023

JEL classification:
C23
G12

Keywords:
Structural models
Debt levels
Default rates
Default boundary
Credit risk

ABSTRACT

We investigate how the dynamics of corporate debt policy affect the pricing of corporate bonds. We find empirically that debt issuance has a significant stochastic component that is imperfectly correlated with shocks to asset value. As a consequence, the volatility of leverage is significantly higher than asset volatility over short horizons. At long horizons, the relation between leverage and asset volatility is reversed due to mean reversion in leverage. We incorporate these stochastic debt dynamics into structural models of credit risk, both standard diffusion models as well as newer models with stochastic volatility and jumps. Including stochastic debt gives more accurate predictions of credit spreads in both the cross-section and the time series.

1. Introduction

In most structural models of credit risk, there is no significant role for the dynamics of a firm's capital structure. Thus, almost none of the extensive theoretical and empirical literature on the dynamics of corporate capital structure has found its way into the literature on pricing corporate debt. Indeed, most structural models of credit risk assume that the amount of debt is constant, either explicitly, or implicitly through the assumption of a constant default boundary.[1] Default in structural models occurs when the value of a firm's assets breaches the default boundary and so default probabilities and bond prices depend on the dynamics of both a firm's asset value and the default boundary. Excluding the dynamics of the default boundary therefore misses one of the two key components of the dynamics of credit risk.

Although most models assume that the amount of debt is constant, there are a few exceptions. For example, the Black and Cox (1976) model allows the amount of debt to grow at a deterministic rate but this implies that changes in the amount of debt are perfectly predictable. Thus, the model proposed by Collin-Dufresne and Goldstein (2001) (CDG) represents a major step forward be-

☆ We thank the editor Nikolai Roussanov, two anonymous referees and Zhan Shi, Jing-Zhi Huang (discussant), Alexey Ivashchenko, Nils Friewald and seminar participants at Copenhagen Business School, Banque de France, Nova SBE, Western Finance Association's Annual Meeting 2020 (online), Michigan State University's Virtual Finance Seminar (online), NHH Norwegian School of Economics, the Tinbergen Institute, Aalto University School of Business, ESSEC Business School (Paris) and Tsinghua University for valuable comments and suggestions received. We are also very grateful to Alex Lipton and Vadim Kaushansky for very kindly providing us with their software for computing the density of first hitting times in Ornstein–Uhlenbeck processes. Peter Feldhütter is grateful for support from the Danish Finance Institute and the Center for Big Data in Finance (Grant no. DNRF167).

* Corresponding author.
E-mail addresses: pf.fi@cbs.dk (P. Feldhütter), sschaefer@london.edu (S. Schaefer).

[1] Examples where the amount of debt is constant include Merton (1974); Longstaff and Schwartz (1995); Leland (1998); Chen et al. (2018); Feldhütter and Schaefer (2018); Du et al. (2019); Bai et al. (2020), and Huang et al. (2020a).

https://doi.org/10.1016/j.jfineco.2023.06.007
0304-405X/

cause it includes dynamic adjustment of leverage. As CDG point out, firms' adjustment of their leverage is not only a prediction of many models of optimal capital structure (e.g., Fischer et al., 1989) but an evident empirical fact. In CDG's model, a firm adjusts the volume of its debt towards a target leverage ratio, so that leverage is mean reverting.

The CDG model thus deals with one major deficiency of structural models but there remains a second. Default occurs when $V_t \leq dK_t$ where V_t is the market value of the firm's assets, K_t is the face value of debt and d is the default boundary, implying that default occurs when

$$l_t = k_t - v_t \geq -\log(d) \tag{1}$$

where l_t is log-leverage and, thus, the probability of default depends on the evolution of the distribution of leverage. In structural models with either constant debt or time-varying but deterministic debt as in the Black–Cox model, the volatility of log-leverage, both instantaneously and at any future date, is simply equal to the volatility of assets. In the CDG model, firms issue or retire debt to adjust their leverage towards their target, but because debt issuance in the model is locally deterministic, shocks to leverage come only from the shocks to asset value. This implies that instantaneously, leverage volatility in the CDG model is also equal to asset volatility while for longer horizons, due to mean reversion in leverage, it is lower than asset volatility. Later structural models of credit risk build on the insight of CDG and add realistic features such as debt adjustment costs and macro-economic uncertainty (Hackbarth et al., 2006; Bhamra et al., 2010; Chen, 2010, and others), but share the same basic mechanism that, given the state of the economy, shocks to leverage come only from shocks to asset value.

We carry out an empirical analysis of the dynamics of debt in the light of two fundamental theories of leverage, the trade-off theory and the pecking order theory of Myers and Majluf (1984). To shed new light on their relative importance for structural models of credit risk, we follow Welch (2004) and investigate the response of the level of debt to future equity returns. The trade-off theory predicts high (low) debt growth in response to high (low) equity returns, while the pecking order theory predicts the opposite. Specifically, we group firms with similar leverage and, within each group, classify them as either 'high-' or 'low equity return' depending on whether their *future* equity returns are above or below the median. We find that, over the period on which equity returns are conditioned, firms with *low* equity returns take on *more* debt than firms with high equity returns. This short-run pattern is consistent with the pecking order theory. In contrast, in the years following the period on which equity returns are conditioned, high return firms increase debt more than low return firms and end up with more debt. This long-run pattern is consistent with a target leverage ratio. Both patterns are robust across time periods, when controlling for differences in firm size, leverage, cash holdings and survivorship bias.

While the long-run response of debt to equity returns is consistent with the predictions of the CDG model, the short-run response is not. Importantly, the differences in response for different horizons shows that shocks to debt issuance are imperfectly correlated with shocks to asset value. We therefore propose a new structural model that is able to capture both the short- and long-run response. Since our focus is on the consequences of the dynamics of the level of debt, we initially maintain the standard assumptions about both the dynamics of firm value (Geometric Brownian Motion) and risk premia (a constant Sharpe ratio). In the model – the 'stochastic debt' (SD) model – leverage is mean reverting, as in the CDG model, but debt issuance is stochastic and locally negatively correlated with changes in the firm's asset value. The stochastic component of debt issuance and its negative correlation with the firm's asset value have a significant effect on both the level and term structure of the volatility of leverage and, therefore, on credit spreads.

To see the importance of adding stochastic debt to structural models, Fig. 1 plots the standard deviation of log-leverage (thick black line) and log-firm value (thin red line) at annual horizons from one to 20 years for a number of cases. Panels A, B and C give model values with parameters estimated later in the paper while Panel D shows the average volatility of firm value and leverage computed from individual firms that are present in all years in our data sample (1988–2017). Panel A shows that for models where the future amount of debt is deterministic, the volatilities of firm value and leverage are identical at all horizons. This is the case for the Merton and Black–Cox models as well as newer models with added realism such as stochastic volatility and jumps investigated later in the paper.[2] Panel B shows the corresponding values for the CDG model. Here, as in the Merton and Black–Cox models, the local volatility of leverage and firm value are equal but mean reversion in leverage attenuates the volatility of leverage at longer horizons and leverage volatility is always lower than asset volatility. As described above, the SD model that we propose differs from existing models in that debt adjustment is stochastic. Panel C shows that, as in the CDG model, mean reversion in leverage results in the volatility of leverage converging to a limiting value as the horizon becomes distant. However, the stochastic component in a firm's debt adjustment (and its negative correlation with firm value) means that in the short run the volatility of leverage is higher than the volatility of firm value. Comparing panels A-C and D, it is clear that the SD model captures the relation between the volatility of leverage and the volatility of firm value better than existing models.[3]

[2] Examples include Cremers et al. (2008); McQuade (2018); Du et al. (2019), and Bai et al. (2020).

[3] Leverage volatility is overstated if leverage changes are predictable. To examine this further, we estimate the volatility of $\epsilon_{i,t}$ in the regression $\log(L_{i,t+T}) - \log(L_{i,t}) = \beta' X_{i,t} + \epsilon_{i,t}$ where $X_{i,t}$ includes a constant, market-to-book assets ratio, tangibility, profits, log of assets, leverage, market-wide expected inflation, Moody's BBB-AAA spread, the last year's US equity market return and the last year's changes in firm leverage and debt. Adjusting for predictable changes only lowers leverage volatility at 1-year horizon slightly, from 40.2% to 39.0%, while the reduction in the leverage volatility at long horizons is more sizeable, for example the volatility at a 20-year horizon is reduced from 93.4% to 69.8%. The result that short horizons leverage volatility is higher than asset volatility while the reverse is true for long maturities is the same. Results are available on request.

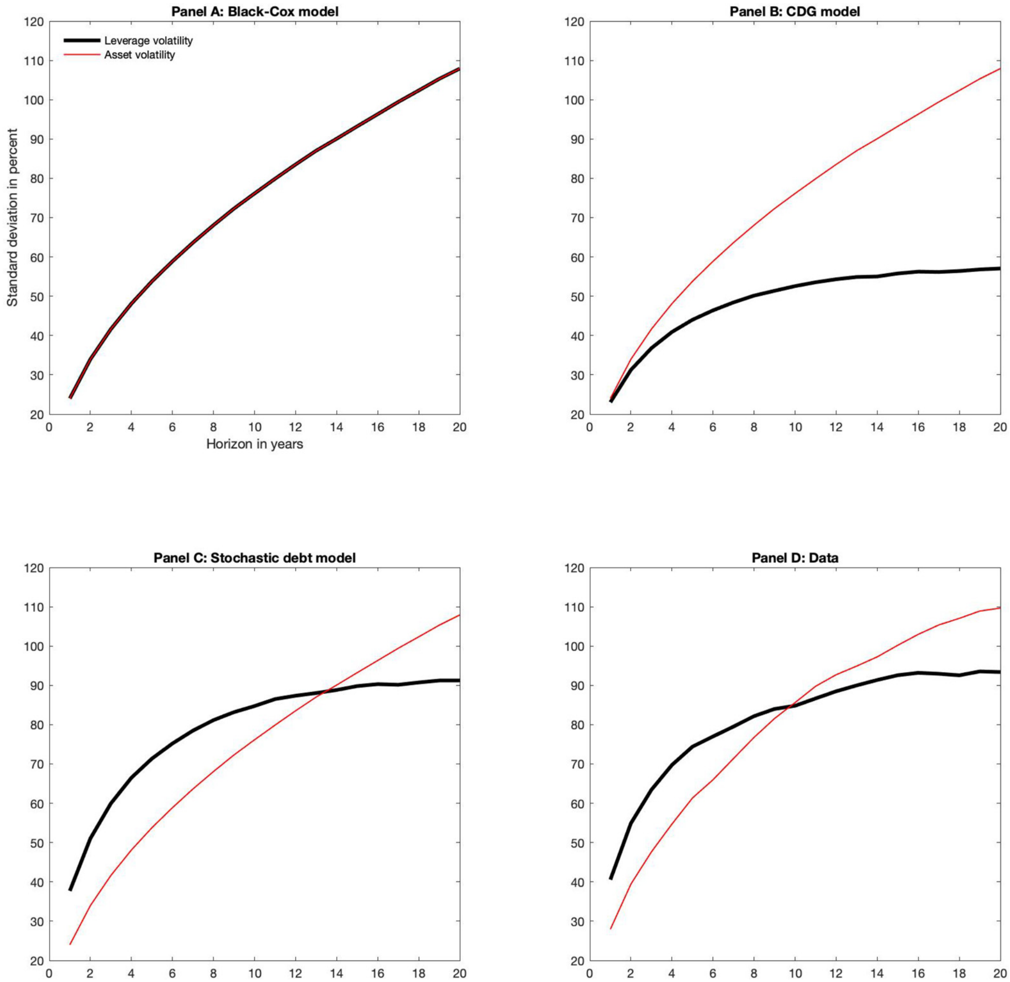

Fig. 1. *Leverage and asset volatility.* This graph shows volatility of leverage - $\sqrt{Var(\log(\frac{L_{t+T}}{L_t}))}$ - and volatility of asset value - $\sqrt{Var(\log(\frac{V_{t+T}}{V_t}))}$ - as a function of the horizon $T-t$ in years. In the Black–Cox model log-firm value is given as $dv_t = (\mu - \delta - \frac{\sigma^2}{2})dt + \sigma dW_t$ and the only source of volatility in leverage. Panel A shows the volatility for different horizons, where we have used average parameters from the empirical section, $\sigma = 0.24$, $\mu = 0.1028$, and $\delta = 0.05$. In the stochastic debt and CDG models, log-debt is given as $dk_t = \lambda(\nu - l_t)dt + \sigma_k dW_{k,t}$, where $l_t = k_t - v_t$ and $\sigma_k = 0$ in the CDG model. The volatilities in the CDG and stochastic debt models in Panel B and C are based on a simulated time series of 1,000,000 years. The parameters in the stochastic debt model are set to the empirically estimated values $\lambda = 0.1814$, $\nu = -1.0046$, $\sigma_k = 0.2706$ and $\rho = -0.1868$ (where ρ is the correlation between W_t and $W_{k,t}$), while they are $\lambda = 0.1732$ and $\nu = -1.0007$ in the CDG model. Panel D shows empirical volatilities, based on firms that have available data in all 30 years of the data sample, 1988–2017, and that have a leverage of at least 0.01 in all years (a total of 238 firms). Leverage is defined as $L_t = \frac{D_t}{V_t}$ where D_t is the book value of debt and V_t is the market value of the firm's assets. As an empirical proxy for the market value of assets, we use the market value of equity plus the book value of debt.

Using US corporate bond yield data for the period 1988–2018, we first calibrate the models to match historical default rates and then investigate their ability to match both spreads and spread volatility. The pricing is out-of-sample in the sense that we do not use bond spreads as part of the model calibration. We find that the SD model has the smallest average pricing errors as a result of its more accurate pricing of short-term bonds. The reason for the better performance of the SD model is that, compared to the other models and particularly for short-term bonds, it produces higher spreads for safe firms and lower spreads for risky firms. This is the case in both the cross-section and the time series: In the cross-section the SD model predicts higher (lower) spreads for firms with low (high) leverage and in the time series higher (lower) monthly spreads in calm (volatile) periods.

The distinct predictions of the SD model for short-term bonds is due the model's higher leverage volatility for short horizons. The default boundary for the different models is estimated by calibrating each model to match historical default rates and this means that, for average levels of leverage, default probabilities – and thus model spreads – are similar across the models. However, higher leverage volatility in the SD model means that the estimated default boundary is lower and the sensitivity of spreads to changes in leverage is lower, and so firms with low (high) leverage have higher (lower) spreads in the SD model compared to the other models. Consistent with this, we find that the volatility of spreads in the SD model match actual volatility better. For example, the volatility of changes in monthly average spreads for bonds with a maturity below three years is 23–24 bps in the SD model, reasonably close to the actual volatility of 23 bps, while it is 41–62 in the CDG model.

To shed further light on the importance of stochastic debt, we also incorporate this feature in the model with stochastic volatility and jumps proposed in Du et al. (2019) and estimate the model parameters for the largest firm in our sample, Walmart, by fitting the model to the firm's CDS spread curves. The SD model outperforms the stochastic volatility-jump (SVJ) model, RMSEs are 9 bps vs. 14 bps. The reason for the better pricing performance is that the stochastic debt model can match the average concave spread curve of Walmart while the SVJ model cannot. The SVJ model has the same leverage volatility as asset volatility as shown in Fig. 1 Panel A and, thus, being able to break the one-to-one link between asset- and leverage-volatility (as shown in Panel C) is important for pricing. Combining the SVJ and stochastic debt models improves the pricing performance further with RMSEs down from 9–14 bps to 3 bps.

We carry out a number of robustness checks. We show that when we use the actual market value of debt (instead of using book value as an empirical proxy) when calculating firm value, our empirical results are similar. Furthermore, we show that the results are also robust to using Treasury yields instead of swap rates as the riskfree rate and using CDS premiums as a measure of credit spreads. In all cases, we find the inclusion of stochastic debt to be important when pricing corporate credit risk.

We are not aware of any papers that analyse the dynamics of the level of debt and then use these dynamics to distinguish between different structural models of credit risk. Some papers test a number of different structural models: for example, Huang and Huang (2012) (HH) investigate a variety of models, each calibrated so as to match exactly the historical default rate at each maturity and rating. Recognising that the historical default rate for a given maturity and rating provides only a very noisy estimate of the expected default rate, we calibrate to a cross-section of historical default rates. Furthermore, we investigate spread predictions in the cross-section and in the time series, while HH focus on average spreads. Eom et al. (2004) [EHH] test structural models with different dynamics for the default boundary and report that most of the models overpredict spreads. Rather than calibrate to historical default rates, they assume that the default point is equal to the face value of debt. Huang et al. (2020b) use GMM to estimate a range of structural models (including the CDG model). They pay particular attention to the first and second moments of CDS spreads and equity returns over a relatively short period (2002–2004), while we focus on debt dynamics and corporate bond spreads over a longer period (1988–2018). Bai et al. (2020) reject the joint assumption of the firm value following a Geometric Brownian Motion and the level of debt being constant. They suggest firm value dynamics that differ from standard models while, here, we suggest different dynamics for the firm's debt. Dorfleitner et al. (2011) propose a general specification of the default boundary and calibrate their model to CDS premia of two firms, but do not conduct a large-scale empirical analysis as we do. Finally, Flannery et al. (2012) investigate the empirical relation between future changes in leverage and current credit spreads, but do not isolate the contribution of the level of debt or incorporate their findings into a structural model.

2. New facts about the dynamics of debt

The amount of debt that a firm has, and the way this changes over time, plays a key role in structural models of credit risk as it determines the firm's default boundary, and so has a major impact on its default probability and credit spread. Moreover, in pricing bonds or estimating default probabilities, it is the amount of debt at the time of default which is critical rather than the amount of debt at the price observation date. Thus, in order to price bonds or estimate default probabilities, we need to characterise the dynamics of a firm's debt level. The empirical evidence on this point is surprisingly limited and in this section we present new facts.

Our empirical analysis is guided by the main predictions of the two leading theories of capital structure: the trade-off theory and the pecking order theory of Myers and Majluf (1984). First, we provide estimates of the growth rate of debt, both the unconditional growth rate – averaging over all firms – and the growth rate conditioned on current leverage. Since asset values grow over time, the stationary leverage theory predicts that, on average, the level of debt should also grow. Furthermore, if firms have a target leverage ratio, there should be a negative relation between current leverage and future growth of debt. Second, we follow Welch (2004) and investigate the growth rate of debt conditioned on future equity returns. The stationary leverage theory predicts that to maintain the same leverage ratio firms with low equity returns have low debt growth rates. The pecking order theory predicts a preference for debt rather than equity, as a result of lower information costs for debt, and to the extent that firms with low returns have a higher need for outside funds, it predicts a higher debt growth rate for these firms. Like Welch (2004), we examine firms' debt choice over the same horizon as the equity shock but also at shorter and longer horizons.

We first investigate how the average level of debt changes over time. For firm i in year t we denote the nominal gross amount of debt by $D_{i,t}$.[4] For a future horizon T (measured in years) we calculate the log-growth in debt

[4] Details about the data used here and later in the paper are given in Appendix A but the following provides some key points and definitions. Firm variables are collected in the CRSP/Compustat Merged Database and computed as in Feldhütter and Schaefer (2018). For a given firm and year the nominal amount of debt is the debt in current liabilities plus long-term debt. We restrict our analysis to industrial firms and to be consistent with the corporate bond data set, we restrict the firm data we use to the period 1988–2017. The leverage ratio is calculated as (nominal amount of debt)/(market value of equity + nominal amount of debt). The number of firm-year observations with both the level of debt and market value of equity available is 131,971 and the number of firms is 14,503.

Table 1
Future debt relative to current debt. For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. The table shows the average log-ratio for different initial leverage ratios and future horizons. The final columns of the table shows for selected horizons the growth rate calculated as exp (log-growth rate) −1. Standard errors clustered at firm level is in parentheses and the number of observations in brackets. The data is from CRSP/Compustat and the sample period is 1988–2017.

	Log-growth										Growth (in percent)		
Horizon (years)	1	2	3	4	5	6	7	8	9	10	1	5	10
Leverage 0–1	0.09	0.18	0.28	0.38	0.47	0.55	0.63	0.70	0.78	0.86	9	60	136
	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)			
	[94,138]	[81,591]	[71,423]	[63,079]	[56,081]	[49,990]	[44,573]	[39,795]	[35,421]	[31,522]			
Leverage 0–0.2	0.21	0.42	0.61	0.78	0.92	1.05	1.16	1.27	1.36	1.44	23	152	324
	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)			
	[47,690]	[41,374]	[36,285]	[32,046]	[28,509]	[25,494]	[22,753]	[20,375]	[18,294]	[16,459]			
Leverage 0.2–0.4	−0.01	0.01	0.04	0.07	0.12	0.17	0.21	0.26	0.30	0.37	−1	12	45
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)			
	[23,453]	[20,624]	[18,259]	[16,214]	[14,478]	[12,889]	[11,522]	[10,281]	[9148]	[8160]			
Leverage 0.4–0.6	−0.05	−0.08	−0.09	−0.08	−0.06	−0.02	0.02	0.04	0.08	0.12	−5	−5	13
	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)			
	[13,055]	[11,410]	[9910]	[8729]	[7699]	[6837]	[6076]	[5450]	[4810]	[4226]			
Leverage 0.6–0.8	−0.09	−0.15	−0.21	−0.22	−0.20	−0.20	−0.19	−0.14	−0.10	−0.06	−8	−18	−6
	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.05)			
	[7238]	[6025]	[5156]	[4516]	[4016]	[3567]	[3165]	[2766]	[2397]	[2037]			
Leverage 0.8–1	−0.18	−0.32	−0.42	−0.45	−0.45	−0.42	−0.35	−0.33	−0.28	−0.29	−17	−36	−25
	(0.01)	(0.02)	(0.03)	(0.04)	(0.04)	(0.05)	(0.06)	(0.07)	(0.09)	(0.10)			
	[2702]	[2158]	[1813]	[1574]	[1379]	[1203]	[1057]	[923]	[772]	[640]			

for firm i between t and T as:

$$R_{it}^T = \log\left(\frac{D_{i,t+T}}{D_{i,t}}\right) \tag{2}$$

and discard the observation if the amount of debt for firm i at time $t+T$ is not reported.[5] We measure the log debt ratio rather than the simple ratio for two reasons. First, log is more robust to outliers and, second, in all the models that we investigate (except those where the amount of debt is constant), the dynamics of debt are defined in logs. Our estimate of the average growth rate for horizon T is

$$R^T = \frac{1}{N^T}\sum_i\sum_t R_{it}^T \tag{3}$$

where N^T is the total number of observations of R_{it}^T (across firms and time).

Table 1 shows the average log debt ratio in our sample period 1988–2017. On average, the amount of debt increases substantially. For all firms, the face value of debt after 10 years is 136% higher and the increase is highly statistically significant. This fact may seem obvious in the sense that, since asset values grow over time, if nominal debt did not also grow at a similar rate, the average level of corporate leverage would tend to zero. Despite this, a zero growth rate in nominal debt is the most common assumption in the credit risk literature.

As mentioned in the introduction we are interested in when leverage $L_t := \frac{K_t}{V_t}$ crosses a threshold where K_t is the book value of debt and V_t is the market value of the firm. As an empirical proxy for the market value of the firm, we use the book value of debt plus the market value of equity and our measure of leverage is thus $\frac{K_t}{E_t+K_t}$ where E_t is the market value of equity.

The relation between a firm's initial leverage and the growth rate of its debt is shown in Fig. 2 (and Table 1). Firms with lower leverage have a higher future growth rate of debt. For example, over a 10-year horizon firms with a leverage of between zero and 20% increase their nominal amount of debt by an average of 324%, while for firms with a leverage between 40% and 60% the increase is only 13%. We also see that highly levered firms reduce their debt. For example, again over a horizon of ten years, firms with a leverage of more than 80% decrease their nominal debt by an average of 25%.[6] Overall, Fig. 2 and Table 1 document both the growth in firm debt over time and a negative relation between current leverage and the future growth rate of nominal debt.

The growth rates in debt documented so far are consistent with leverage being stationary. However, when we condition on a shock to asset value, as in Welch (2004), a different pattern emerges. For each of the leverage groups in Fig. 2 and Table 1, we sort firms according to future equity returns. Specifically, for a given leverage group, we calculate for each firm and year t, the three-year future equity return between t and $t+3$ (taking into account dividends and stock splits), and then sort firms in year t into two groups according to whether their return is below or above the median. For example, if there were 200 firms in 1995 with a leverage between 0.4 and 0.6, we calculate their equity returns between 1995 and 1998, and designate the 100 firms with equity returns above (below) the me-

[5] We restrict our analysis to firm-year observations with a positive amount of debt which is the case for 84.0% of the observations.

[6] One may worry about a simple average across time and firms for the following reason. If, in recessions, firms are more highly leveraged and have lower debt growth rates, the low debt growth rate of highly leveraged firms may partially be due to the low debt growth rates in recessions. To address this concern we have also, for each leverage group, calculated the average debt growth rate for each year in the sample and then calculated the average across years. Results are very similar and available on request.

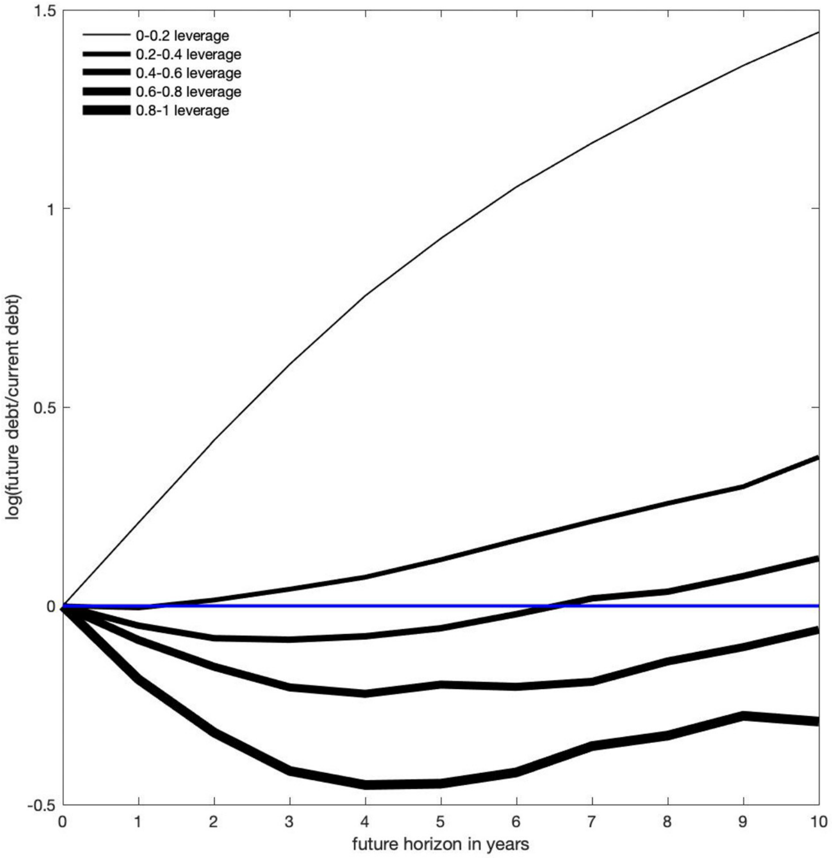

Fig. 2. *Future debt growth as a function of initial leverage.* For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. The figure shows the average ratio for different initial leverage ratios and future horizons. The data is from CRSP/Compustat and the sample period is 1988–2017.

dian return as 'high' ('low') equity return. We repeat this for the remaining years in the sample to arrive at our final sample of high and low equity return firm-years. (Firms may, of course, switch between high and low return over the years).

Fig. 3 shows the average cumulative growth in log-debt over six years: three years either side of the three-year horizon over which equity returns are measured. Beyond the three-year horizon, the growth in debt for high-return firms is higher than for low return firms, consistent with firms having a target leverage ratio. But, at the 1- and 2-year horizon, apart from firms with leverage higher than 80%, the opposite is true and low return firms issue more debt than high return firms. For example, Table 2 reports that, for firms with an initial leverage between 20% and 40% that experience a high three-year equity return, the change in debt level after one year is approximately zero, while firms that experience a low return increase their debt by 4%. After 10 years, i.e., seven years after the equity shock, the pattern is reversed: for the same leverage bracket, high return firms increase their debt by 64% and low return firms by only 6%. The table shows that the differences in debt growth rates are statistically significant except in a few cases. Furthermore, we see that the average leverage for low and high equity return firms is very similar and so the results are unlikely to be due to imperfectly controlling for leverage.[7]

This result is consistent with firms maintaining a stationary leverage ratio in the long run. However, if firms always adjusted their debt levels towards their target leverage ratio, we would find that firms with low equity returns would have lower debt growth rates at all horizons. Thus, while the long-run results in Fig. 3 are consistent with firms having a target leverage ratio, the short-run behaviour is consistent with the pecking order theory, i.e.,

[7] In some of the cases shown in Fig. 3, the value of log future debt shows a 'kink' at the three-year horizon. Kinks are also visible other related figures that we describe below (e.g., Figs. 4 and 6). The reason for the kink is that, when we condition on the 3-year equity return, the expected value of log debt at a future time t, depends on the correlation between the time-t value of debt and the equity return over 3 years. As t moves through 3 years, the period over which equity return is measured no longer includes all of the period over which log debt is measured. We show in Appendix B that, under our model, although the correlation is continuous as t moves through the conditioning horizon, the first derivative of the correlation is not, i.e., there is a kink (see Eqs. (53)–(55)). When we then compute the conditional expectation in Eq. (55), the kink in correlation carries over to the expected values.

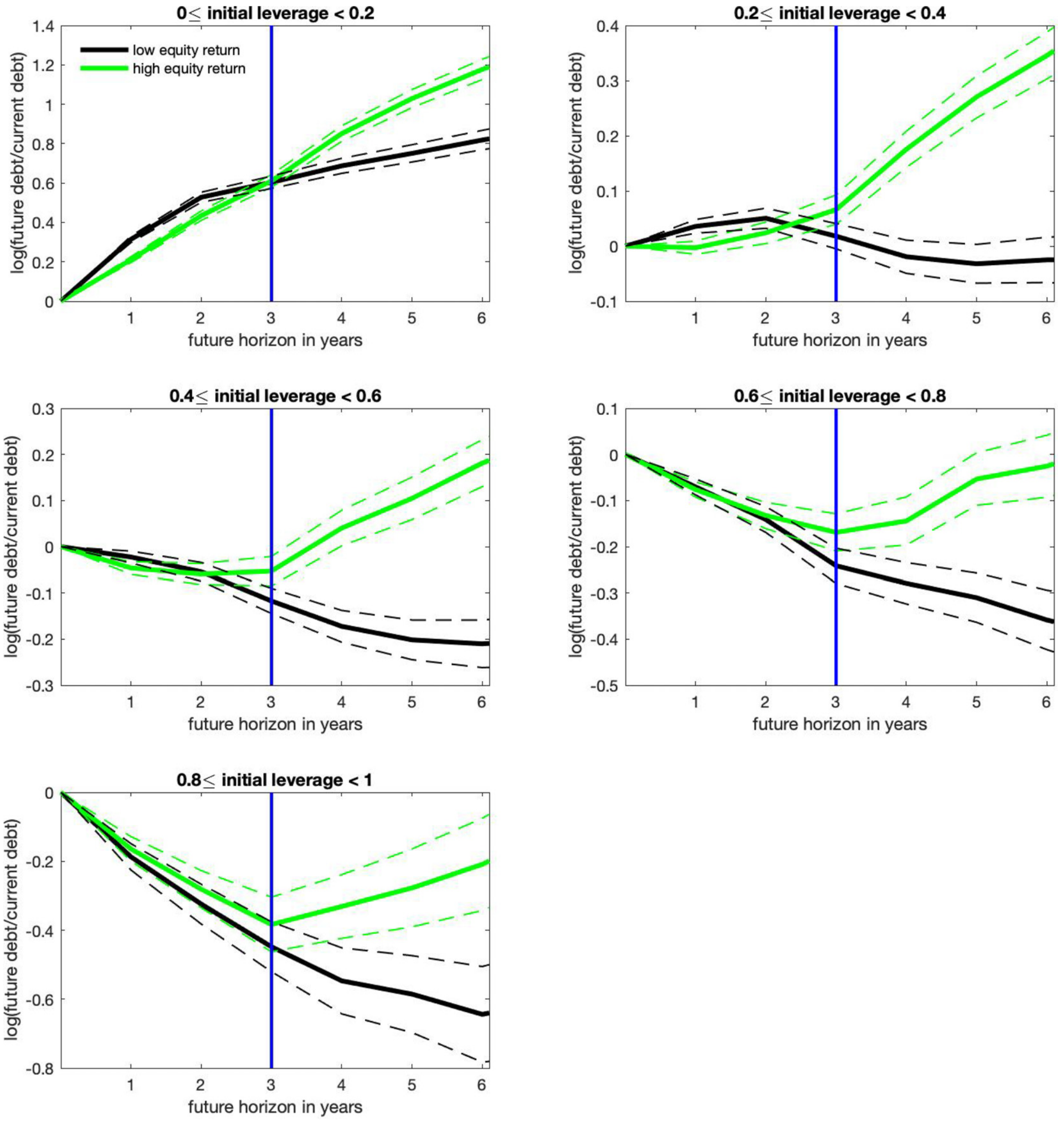

Fig. 3. *Future debt growth conditional on future three-year equity returns.* For firm i, year t, and horizons 1,...,20, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median between t and $t+3$ 'High (Low) future equity return' firms. The figure shows the average log-ratio for high and low future equity return firms. The dashed lines mark 95% confidence levels based on standard errors clustered at the firm level. The data is from CRSP/Compustat and the sample period is 1988–2017.

firms that suffer a negative value shock rely first on debt financing.

Welch (2004) finds a similar result over a one-year horizon: firms respond to poor performance with higher debt levels and to good performance with higher equity issuance. Empirical evidence in Brown et al. (2021) and Norden and Weber (2010) provide more detailed evidence on how firms manage their debt. Brown et al. (2021) find that a temporary negative shock to cash flows leads firms to draw on their credit lines and this applies particularly to high-quality firms. Norden and Weber (2010) document that, 12 months prior to default, firms experience a decline in their cash flows and draw heavily on their credit lines and that this is also the case for surprise defaults in 'high grade' firms. Thus, the existing empirical evidence is consistent with our finding that firms that are subject to a negative shock take on more debt. Further, it appears from the literature that credit lines are an important channel through which this occurs.

Fig. 4 shows the future growth of short- and long-term debt separately. We see that both types of debt show the same pattern, namely that low equity return firms increase debt in the short run and decrease debt in the long-run. The pattern is much stronger in short-term debt than in long-term debt and the figure also shows that low equity return firms accumulate less cash (and so do not hoard

Table 2
Future debt relative to current debt conditional on future three-year equity returns. For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median between t and $t+3$ 'High (Low) future equity return' firms. The table reports the average log-ratio for high and low future equity return firms as well as the difference. The final column shows the average initial leverage ratio. In parentheses are standard errors, clustered at the firm level, of the differences and '*' indicate significance at 95% level and '**' at 99% level. The data is from CRSP/Compustat and the sample period is 1988–2017.

Horizon (years)	1	2	3	4	5	6	7	8	9	10	Average leverage
Leverage 0–0.2											
High equity return	0.21	0.43	0.61	0.85	1.03	1.18	1.31	1.40	1.48	1.56	0.07
Low equity return	0.31	0.53	0.60	0.69	0.75	0.82	0.88	0.96	1.06	1.14	0.08
Difference	-0.10**	-0.09**	0.01	0.16**	0.28**	0.36**	0.43**	0.44**	0.41**	0.42**	-0.01**
	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.05)	(0.05)	(0.00)
Leverage 0.2–0.4											
High equity return	−0.00	0.02	0.07	0.18	0.27	0.35	0.43	0.51	0.55	0.64	0.29
Low equity return	0.04	0.05	0.02	−0.02	−0.03	−0.03	−0.02	−0.01	0.02	0.06	0.29
Difference	-0.04**	-0.03*	0.05**	0.19**	0.30**	0.37**	0.45**	0.52**	0.53**	0.58**	0.00
	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.05)	(0.00)
Leverage 0.4–0.6											
High equity return	−0.05	−0.06	−0.05	0.04	0.10	0.18	0.24	0.29	0.35	0.38	0.49
Low equity return	−0.02	−0.05	−0.12	−0.17	−0.20	−0.21	−0.20	−0.23	−0.21	−0.18	0.49
Difference	-0.02*	-0.00	0.06**	0.21**	0.31**	0.39**	0.44**	0.52**	0.56**	0.57**	-0.00
	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)	(0.04)	(0.05)	(0.05)	(0.06)	(0.00)
Leverage 0.6–0.8											
High equity return	−0.08	−0.13	−0.17	−0.14	−0.05	−0.03	0.02	0.06	0.07	0.12	0.69
Low equity return	−0.07	−0.14	−0.24	−0.28	−0.31	−0.36	−0.40	−0.32	−0.29	−0.25	0.69
Difference	-0.01	0.01	0.07**	0.13**	0.26**	0.33**	0.42**	0.38**	0.36**	0.37**	-0.00
	(0.01)	(0.02)	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.06)	(0.07)	(0.08)	(0.00)
Leverage 0.8–1											
High equity return	−0.16	−0.28	−0.38	−0.33	−0.28	−0.21	−0.11	−0.06	0.02	0.03	0.87
Low equity return	−0.19	−0.32	−0.45	−0.55	−0.59	−0.64	−0.61	−0.64	−0.68	−0.72	0.88
Difference	0.02	0.04	0.06	0.22**	0.31**	0.43**	0.50**	0.58**	0.71**	0.74**	-0.01**
Difference	(0.03)	(0.04)	(0.05)	(0.07)	(0.08)	(0.10)	(0.11)	(0.12)	(0.15)	(0.19)	(0.00)

cash when hit by a negative shock) and that our main result is very similar if we net cash from the firm's total debt. Finally, Fig. 4 shows the future change in leverage. We see that the joint effect of a negative equity shock and an increase in debt leads to a substantial increase in leverage that reverts back after the shock only slowly.

In Section 6.3 we show that these results are robust to controlling for firm size, using a different data period, accounting for survivorship bias, and matching firms exactly on leverage and cash holdings.

With these stylized facts as yardsticks, we now compare the ability of structural models of credit risk to match actual debt dynamics. Since, as we have already noted, many models in the literature assume, counterfactually, that the amount of debt is constant, this is an area in which structural models could potentially be improved and this is what we turn to next. Later, in Section 5.3, we discuss and implement a model with stochastc asset volatility and jumps in asset value.

3. Structural models and their dynamics of debt

In this section we discuss the four structural diffusion models that we implement in the main analysis: three that are well-known from the literature and a new model that we propose. We focus on the dynamics of debt in each case since our assumptions on asset value dynamics and risk premia are standard (and common across the different models). We then describe how the model parameters are estimated and compare their debt level dynamics with those documented in the previous section.

3.1. Structural models

For all four models we assume that the market value of the firm's assets follows a Geometric Brownian Motion under the natural measure,

$$\frac{dV_t}{V_t} = (\mu - \delta)dt + \sigma dW_t \tag{4}$$

where δ is the payout rate to debt and equity holders, μ is the expected return on the firm's assets and σ is the volatility of returns on the assets.

The firm defaults the first time that the value of the firm hits the default boundary (from above). We assume that the default boundary is a constant fraction, d, of the face value of debt at the time of default, K_t, and so τ, the default time is given by:

$$\tau = \inf\{t | V_t \le d \times K_t\}. \tag{5}$$

The models we examine differ only in their assumptions about the dynamics of the firm's nominal debt, K_t. We denote log-firm value as $v_t = \log(V_t)$ and log-debt as $k_t = \log(K_t)$, and so the default time may also be written as:

$$\tau = \inf\{t | l_t \ge -log(d)\} \tag{6}$$

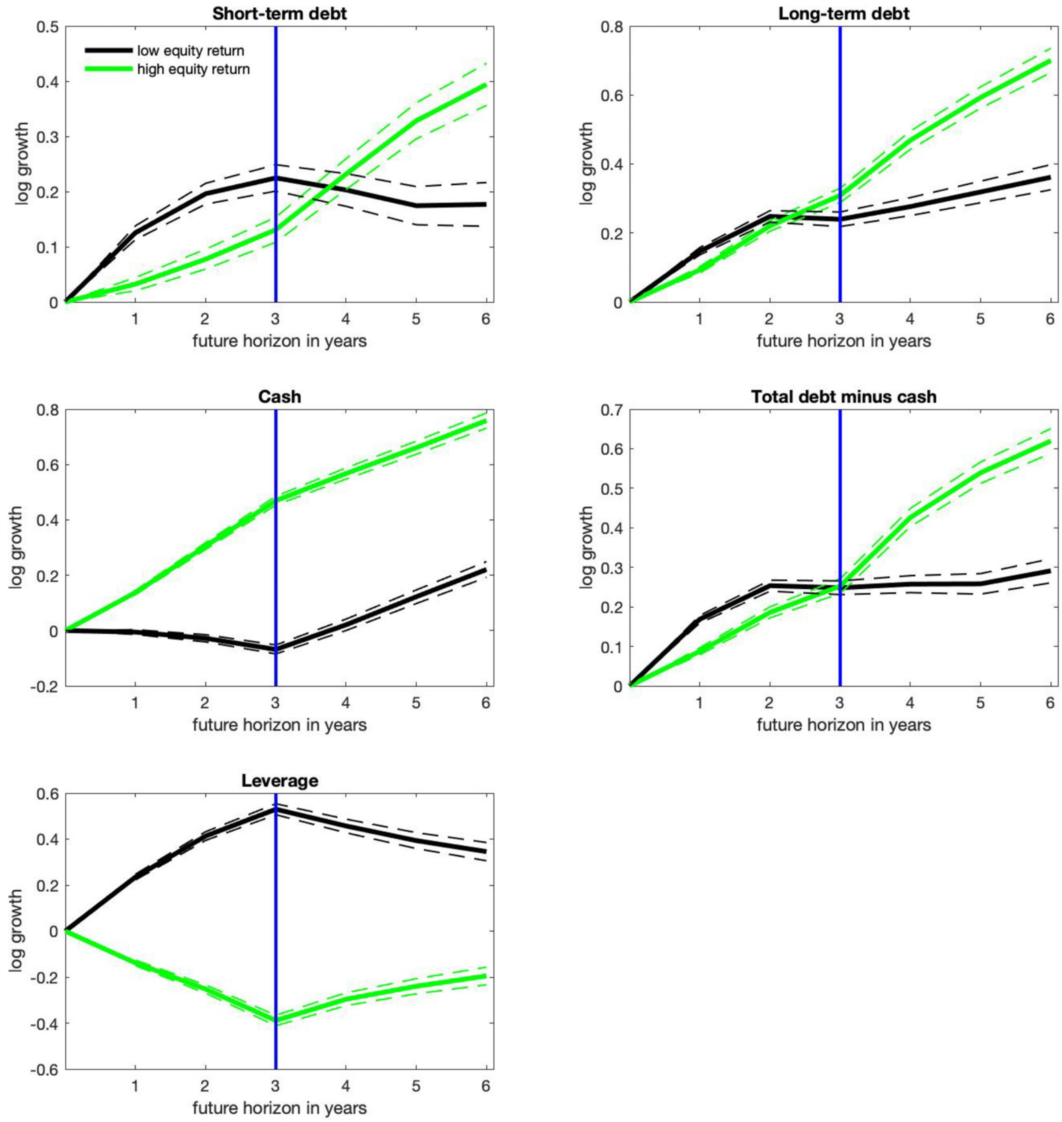

Fig. 4. *Future growth in short-term debt, long-term debt, cash, and leverage conditional on future three-year equity returns.* For firm i, year t, and horizons 1,...,20, we calculate $\log\left(\frac{B_{i,t+T}}{B_{i,t}}\right)$ where $B_{i,t}$ is the variable of interest for firm i in year t and T is the horizon in years. For each firm-year in the sample, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median between t and $t+3$ 'High (Low) future equity return' firms. The figure shows the average log-ratio for high and low future equity return firms. The dashed lines mark 95% confidence levels based on standard errors clustered at the firm level. The data is from CRSP/Compustat and the sample period is 1988–2017.

where log-leverage is $l_t = k_t - v_t$. We next describe the evolution of debt in the different models we implement.

3.1.1. Constant level of debt [BC-0G]

The most common assumption in the literature is that the level of debt remains constant (see footnote 1 for examples), i.e.

$$k_t = k_0 \text{ for all } t > 0. \tag{7}$$

This corresponds to the Black and Cox (1976) model with growth in the default boundary set to zero; we refer to this model as BC-0G.

3.1.2. Constant growth in debt [BC]

In the Black and Cox (1976) model, the level of debt is $K_t = K_0 e^{\gamma t}$, i.e.

$$k_t = k_0 + \gamma t \tag{8}$$

where $\gamma > 0$ is the growth rate of debt. In this case, the level of debt increases deterministically over time. Since firms do, on average, increase their debt over time, it is perhaps surprising that this model is never used in the literature.[8]

[8] Bao (2009) and Feldhütter and Schaefer (2018) implement a model they refer to as the Black–Cox model, but in our terminology this is the BC-0G model.

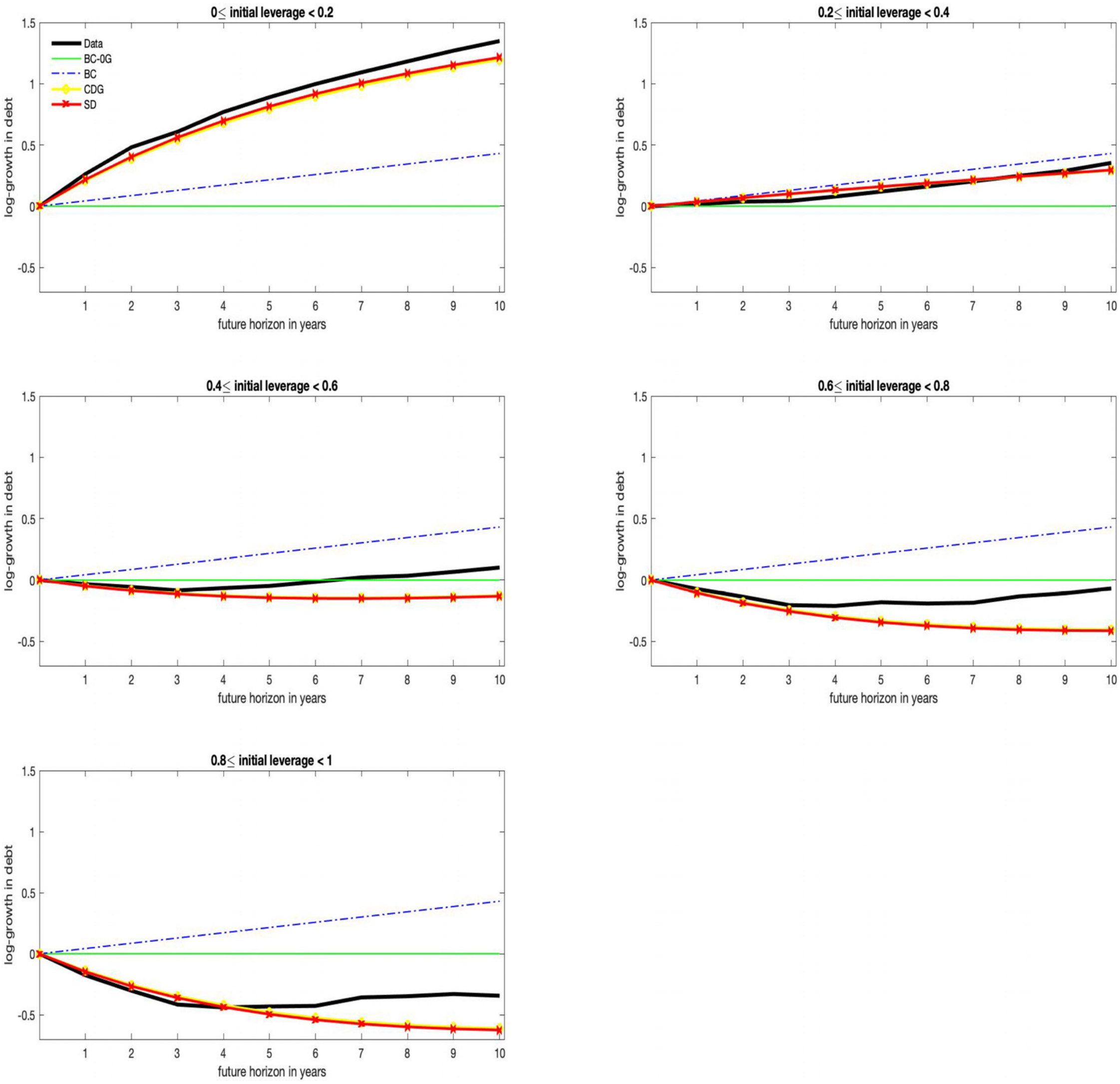

Fig. 5. *Future debt relative to current debt, model-fit.* For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. 'Data' shows the average log-ratio for different initial leverage ratios and future horizons in the data. The figure also shows fitted values from structural models. 'BC-0G' refers to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. The data is from CRSP/Compustat and the sample period is 1988–2017.

3.1.3. Deterministic debt adjustment [CDG]

Collin-Dufresne and Goldstein (2001) propose a structural model in which leverage is stationary and the adjustment to target leverage is locally deterministic. Specifically, the dynamics of the log-debt level, k_t, in the CDG model are given by

$$dk_t = \lambda(\nu - l_t)dt \tag{9}$$

where $\lambda > 0$ and l_t is log-leverage. If log-leverage is above (below) a target ν, the firm reduces (increases) its level of debt. In the long run, the expected level of debt is proportional to firm value. This model has been used in Eom et al. (2004) and Huang and Huang (2012), among others.

3.1.4. Stochastic debt adjustment [SD]

To accommodate the features of debt dynamics that are documented in Section 2, we propose a model with mean reversion, where the dynamics of the debt level are given by:

$$dk_t = \lambda(\nu - l_t)dt + \sigma_k dW_{k,t} \tag{10}$$

where the correlation between the shock to debt, W_k, and the shock to firm value, W, is ρ. Mean reversion, $\lambda(\nu - l_t)dt$, implies that the expected level of debt is proportional to firm value in the long run, as is the case in the

Table 3

Bond summary statistics. The sample consists of noncallable bonds with fixed coupons issued by industrial firms. This table shows summary statistics for the data set. The data sample cover the period 1988Q2-2018Q1. 'Number of bonds' is the number of bonds that appear (in a particular rating and maturity range) at some point in the sample period. For each quote we calculate the bond's time since issuance and 'Age' is the average time since issuance across all quotes. 'Coupon' is the average bond coupon across all quotes. 'Amount outstanding' is the average outstanding amount of a bond issue across all quotes. 'Time-to-maturity' is the average time until the bond matures across all quotes.

	AAA	AA	A	BBB	BB	B	C	all
0–20-year bond maturity								
Number of bonds	66	322	1486	1422	563	410	152	3129
Age	4.99	4.68	6.1	7.24	6.79	7.82	11.1	6.59
Coupon	5.6	6.23	6.99	7.61	7.58	7.46	7.73	7.18
Amount outstanding ($mm)	610	520	296	324	274	282	275	333
Time-to-maturity	6.87	6.45	6.69	7.11	6.78	6.16	7.90	6.82
Number of observations	2617	10,667	36,144	31,172	11,907	5679	2873	101,059
0–3-year bond maturity								
Number of bonds	26	144	765	730	262	238	61	1884
Age	4.72	5.78	6.95	8.25	7.05	7.2	9.58	7.29
Coupon	3.94	4.73	5.82	6.62	6.92	6.63	7.44	6.15
Amount outstanding ($mm)	707	707	342	361	293	297	304	382
Time-to-maturity	1.59	1.43	1.45	1.39	1.51	1.47	1.61	1.44
Number of observations	513	2706	9668	8228	3072	1882	729	26,798
3–10-year bond maturity								
Number of bonds	52	230	1025	864	330	184	80	2219
Age	3.43	3.77	4.73	5.78	5.41	6.78	8.74	5.16
Coupon	5.81	6.47	7.19	7.86	7.72	7.84	7.54	7.37
Amount outstanding ($mm)	667	488	291	326	280	292	258	334
Time-to-maturity	6.17	6.23	6.11	6.34	6.02	5.73	5.74	6.15
Number of observations	1605	6121	18,967	15,385	6263	2608	1097	52,046
10–20-year bond maturity								
Number of bonds	16	80	293	347	129	77	51	745
Age	10.3	6.09	8.48	9.11	9.81	11	14.5	9.12
Coupon	6.65	7.6	8	8.15	8.04	7.93	8.13	8
Amount outstanding ($mm)	329	350	252	281	235	235	274	270
Time-to-maturity	14.55	14.56	14.92	14.91	14.93	14.52	14.54	14.84
Number of observations	499	1840	7509	7559	2572	1189	1047	22,215

CDG model. However, the presence of the stochastic component, $\sigma_k dW_{k,t}$, means that, in the short run, leverage deviates from a deterministic drift towards the firm's target leverage ratio.

3.2. Estimation of debt dynamics parameters

We estimate the parameters for each model by matching the model-implied debt dynamics to the historical estimates documented in Section 2. Specifically, we denote by $\overline{D}^g_{i,T}$ the historical average T-year log-growth in debt for firms with an initial leverage in range L_i. We use initial leverage ranges $L_1 = [0; 0.2], L_2 = [0.2; 0.4], L_3 = [0.4; 0.6], L_4 = [0.6; 0.8]$, and $L_5 = [0.8; 1]$, for which the historical log-growth rates are given in Table 1. We denote the historical average T-year log-growth in debt for firms with an initial leverage that is in leverage group L_i, and experiences a three-year equity shock above (below) the median by $\overline{D}^{g,H}_{i,T}$ ($\overline{D}^{g,L}_{i,T}$) and the difference by $\overline{\Delta D}^g_{i,T} = \overline{D}^{g,H}_{i,T} - \overline{D}^{g,L}_{i,T}$. These conditional historical growth rates are given in Table 2.

All four models share the same dynamics for the value of the firm (equation (4)) and we set the corresponding parameters to average values (estimated later and given in Table 4): $\hat{\Theta}^P_v = (\mu, \delta, \sigma) = (0.0996, 0.044, 0.24)$.[9] The BC-0G model requires no debt-dynamics parameters. For the other models, the debt dynamics parameters are given as $\Theta^{BC} = \gamma$, $\Theta^{CDG} = (\lambda, \nu)$, and $\Theta^{SD}_k = (\lambda, \nu, \sigma_k, \rho)$. We estimate these parameters for each model by minimizing the weighted squared differences between historical debt growth rates and model-implied debt growth rates where the weights are given by the precision with which the historical growth rates are estimated:

$$\min_{\Theta_k} \sum_{i=1}^{5} \sum_{T=1}^{10} \left(\left[\frac{\overline{D}^g_{i,T} - D^g_T(L^m_i, \hat{\Theta}^P_v, \Theta_k)}{SD^{\overline{D}}_{i,T}} \right]^2 + \left[\frac{\overline{\Delta D}^g_{i,T} - \Delta D^g_T(L^m_i, \hat{\Theta}^P_v, \Theta_k)}{SD^{\overline{\Delta D}}_{i,T}} \right]^2 \right). \quad (11)$$

Here $SD^{\overline{D}}_{i,T}$ are the standard errors given in Table 1, $SD^{\overline{\Delta D}}_{i,T}$ are the standard errors given in Table 2, L^m_i is the mid point of leverage interval L_i, $D^g_T(L^m_i, \hat{\Theta}^P_v, \Theta_k)$ is the model-implied growth rate in debt, and $\Delta D^g_T(L^m_i, \hat{\Theta}^P_v, \Theta_k)$ is the

[9] Specifically, the average riskfree rate is $r = 0.04679$ and with a Sharpe ratio of $\theta = 0.22$ we have that $\mu = r + \theta\sigma = 0.04679 + 0.22 * 0.24 = 0.0996$.

Table 4
Firm summary statistics. For each bond yield observation, the leverage ratio, equity volatility, asset volatility, and payout ratio are calculated for the issuing firm on the day of the observation. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility, calculated as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Note that since some firms change rating in our sample period, the sum of firms over rating classes is greater than 'all'. Firm variables are computed using data from CRSP and Compustat.

	#firms	Mean	10th	25th	Median	75th	90th
				Leverage ratio			
AAA	15	0.12	0.03	0.05	0.07	0.14	0.25
AA	68	0.17	0.07	0.11	0.17	0.20	0.24
A	231	0.28	0.11	0.17	0.25	0.36	0.50
BBB	353	0.35	0.15	0.23	0.32	0.44	0.55
BB	208	0.44	0.20	0.32	0.42	0.56	0.68
B	122	0.56	0.26	0.38	0.58	0.71	0.87
C	50	0.69	0.41	0.56	0.72	0.87	0.95
all	571	0.33	0.11	0.18	0.29	0.44	0.60
				Equity volatility			
AAA	15	0.24	0.16	0.19	0.23	0.29	0.36
AA	68	0.25	0.16	0.20	0.24	0.29	0.38
A	231	0.31	0.19	0.24	0.29	0.36	0.41
BBB	353	0.33	0.21	0.25	0.31	0.39	0.47
BB	208	0.44	0.27	0.31	0.40	0.50	0.62
B	122	0.55	0.28	0.35	0.48	0.68	0.85
C	50	0.61	0.36	0.44	0.56	0.74	0.98
all	571	0.34	0.20	0.24	0.31	0.40	0.50
				Asset volatility			
AAA	15	0.22	0.17	0.20	0.20	0.24	0.25
AA	68	0.22	0.20	0.20	0.22	0.24	0.25
A	231	0.24	0.18	0.20	0.23	0.25	0.30
BBB	353	0.24	0.16	0.20	0.22	0.26	0.33
BB	208	0.26	0.17	0.21	0.26	0.31	0.34
B	122	0.27	0.17	0.22	0.25	0.30	0.37
C	50	0.26	0.11	0.19	0.23	0.31	0.40
all	571	0.24	0.17	0.20	0.23	0.26	0.32
				Payout ratio			
AAA	15	0.042	0.012	0.021	0.044	0.059	0.073
AA	68	0.037	0.010	0.018	0.034	0.050	0.065
A	231	0.041	0.017	0.024	0.037	0.052	0.071
BBB	353	0.047	0.018	0.028	0.042	0.059	0.085
BB	208	0.046	0.018	0.027	0.040	0.055	0.076
B	122	0.049	0.023	0.033	0.044	0.060	0.079
C	50	0.051	0.027	0.039	0.049	0.059	0.073
all	571	0.044	0.016	0.025	0.040	0.055	0.076

Table 5
Estimates of the default boundary. For each of the models we estimate the default boundary – the fraction of the total face value of debt at which the firm defaults – by minimizing the distance between average model-implied default probabilities and historical average default rates for a cross-section of horizons 1,...,20 years and ratings AAA, AA, A, BBB, BB, B. 'BC-0G' to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. The historical average default rates are based on defaults of U.S. industrial firms in the period 1970–2017.

Model	Default boundary estimate ($\hat{d}$)
BC-0G	0.8614
BC	0.7315
CDG	0.9978
CDG-FL	0.9103
SD	0.7322
SD-FL	0.6707

model-implied difference in growth rates between low and high 3-year equity shock firms. Appendix B provides formulae for the expected level of debt for each model.[10]

3.3. Model-implied debt dynamics

For each of the models, Fig. 5 shows the expected value of the increase in (log) debt, over horizons from one to ten years along with the corresponding average values from the data. The BC-0G model assumes that the level of debt remains constant which is clearly counterfactual. The growth rate in the BC model is estimated to be $\hat{\gamma} = 0.0430$ and, in this sense, the model is more reasonable than the

[10] The model-implied growth rate in debt is 0 in the BC-0G model, γt in the BC model, and given in Eq. (43) for the CDG and SD models. The model-implied difference in growth rates between low and high equity shock firms is 0 in the BC-0G and BC models and given in Eq. (55) for the CDG and SD models.

Table 6

Actual and model credit spreads. For every model we imply out the default boundary by minimizing the difference between model-implied and historical default rates. This table shows average actual and model-implied corporate bond yield spreads. Spreads are grouped according to remaining bond maturity at the spread observation date. 'Actual spread' is the actual spread to the swap rate. 'BC-0G' to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. 'Inv' includes bonds rated AAA, AA, A, and BBB, while 'Spec' includes bonds rated BB, B, and C. Confidence bands are simulation-based following Feldhütter and Schaefer (2018) and are at the 95% level. * implies significance at the 5% level and ** at the 1% level. The sample period is 1988–2018 for spread of bonds with a maturity more than three years and 2002–2018 for bonds with a maturity less than three years.

		All	Short	Medium	Long
Inv	Actual spread	84	77	72	121
	BC-0G	88 (49;132)	73 (22;162)	82 (46;112)	122 (91;142)
	BC	99 (64;128)	36* (12;66)	93 (55;123)	189** (149;214)
	CDG	73 (25;123)	117 (21;217)	51 (22;84)	72** (38;103)
	CDG-FL	60 (21;109)	81 (18;183)	53 (21;86)	52*** (24;76)
	SD	107 (62;144)	67 (28;110)	107 (63;143)	153 (103;189)
	SD-FL	86 (48;119)	59 (25;94)	89 (50;122)	111 (70;140)
Spec	Actual spread	413	409	387	470
	BC-0G	392 (223;578)	447 (162;873)	401 (250;518)	308*** (240;354)
	BC	363 (238;468)	248 (103;415)	412 (269;514)	398*** (333;435)
	CDG	345 (119;607)	641 (152;1208)	259 (116;426)	170*** (86;270)
	CDG-FL	385 (166;621)	502 (147;1008)	371 (179;528)	275*** (159;353)
	SD	349 (210;466)	423 (208;635)	353 (227;448)	251*** (177;302)
	SD-FL	400 (249;518)	408 (209;594)	428 (277;536)	334*** (239;392)

BC-0G model because it at least implies an increase in the level of debt over time. However, the model does not capture cross-sectional differences in the growth rate of debt. As Fig. 5 again shows, there is a large difference in the average historical growth rate of debt for firms with low and high leverage while the BC model implies no difference between these two cases. (Table A1 gives the values).

The parameters defining the expected path of log-debt in the CDG model are estimated to be $(\hat{\lambda}, \hat{\nu}) = (0.1732, -1.0007)$. The half-life of the leverage adjustment is $\log(2)/0.1732 = 4.0$ years, and thus firms slowly adjust their leverage to changes in firm value. The estimate of ν corresponds to a target leverage ratio of 0.31.[11] Fig. 5 shows that the CDG and SD models fit the cross-section of historical average debt growth rates substantially better than the BC-0G and BC models.

Fig. 6 shows the historical and model-implied differences between the debt levels for firms with an above- and below-median value shock (Table A2 in the Appendix gives the values). In the CDG model, regardless of the initial leverage, firms with a positive value shock increase debt more than firms with a negative equity shock. As we have seen this is inconsistent with actual firm behaviour in the short run where a positive value shock typically leads to a smaller increase in debt than a negative value shock. In this regard, the short-run predictions of the CDG model are less accurate than the BC and BC-0G models, which predict no difference in the level of debt between high and low shock firms. Thus, the CDG model implies that firms always "pull away" from default when there is a negative shock by reducing debt while the actual behaviour of firms is, on average, to increase debt by more for a negative shock than for a positive shock.

In estimating the parameters of the SD model, $(\lambda, \nu, \sigma_k, \rho)$, Eqs. (43) and (55) in the Appendix show that σ_k and ρ are not separately identified; only the product $\sigma_k\rho$ is identified. The estimates of the three parameters are $(\hat{\lambda}, \hat{\nu}, \widehat{\sigma_k\rho}) = (0.1814, -1.0046, -0.0505)$. The estimates of λ and ν are similar to those in the CDG model, and consequently, as Fig. 5 shows, the average future level of debt, conditional on initial leverage, is very similar in the two models.

While the SD and CDG models imply similar unconditional future expected levels of debt, they have very different predictions for firms experiencing a positive or negative value shock. Fig. 6 shows that, in the short run, while the SD model implies more debt issuance in the short run by negative-shock firms relative to positive-shock firms, the CDG model implies the opposite. The key parameter that leads to this behaviour in the SD model is ρ, which is negative (since $\sigma_k\rho$ is negative) and this implies that a negative shock to firm value leads to a positive shock to debt issuance. After the shock, and consistent with the empirical evidence, high return firms increase their debt more than low return firms. The negative value of ρ in the SD model means that firms' debt issuance policy resembles pecking order behaviour at short horizons and trade-off behaviour at long horizons.

The speed of mean reversion of corporate leverage has been investigated extensively in the literature although, as Frank and Goyal (2008) point out, the magnitude of the mean reversion parameter is not a settled issue. Examples of reported estimates include 0.07–0.15 (Fama and French, 2002), 0.17–0.23 (Huang and Ritter, 2009), 0.13–0.39 (Lemmon et al., 2008), and 0.34 (Flannery and Rangan, 2006). Our estimates of 0.1814 for the SD model and 0.1732 for the CDG model are within, but towards the lower end of, the range of estimates found in the literature.[12]

In the next part of the paper we calibrate the models to historical default rates using a panel data set of firms.

[11] Eq. (40) shows that the *log* target leverage is $\bar{l} = \nu - \frac{\mu-\delta-\frac{\sigma^2}{2}}{\lambda} = -1.0007 - \frac{0.0996-0.044-\frac{0.24^2}{2}}{0.1732} = -1.1554$. The corresponding value of target leverage is $\exp(\bar{l}) = \exp(-1.1554) = 0.31$.

[12] When estimating the (common) mean reversion parameter we assume a common target leverage. Flannery and Rangan (2006) and Lemmon et al. (2008) find that including firm-specific heterogeneity in the estimation increases the estimate of speed of mean reversion and one may therefore hypothesise that different values of ν may lead to substantially higher estimates of λ. To examine this we reestimate λ and $\sigma_k\rho$ while holding the target leverage fixed at different values. We find that a target leverage fixed in the range 30–60% (0–100%) produces a mean reversion in the range 0.11–0.17 (0.00–0.17), which suggests that allowing for a firm-specific ν would not materially increase the speed of mean reversion estimate.

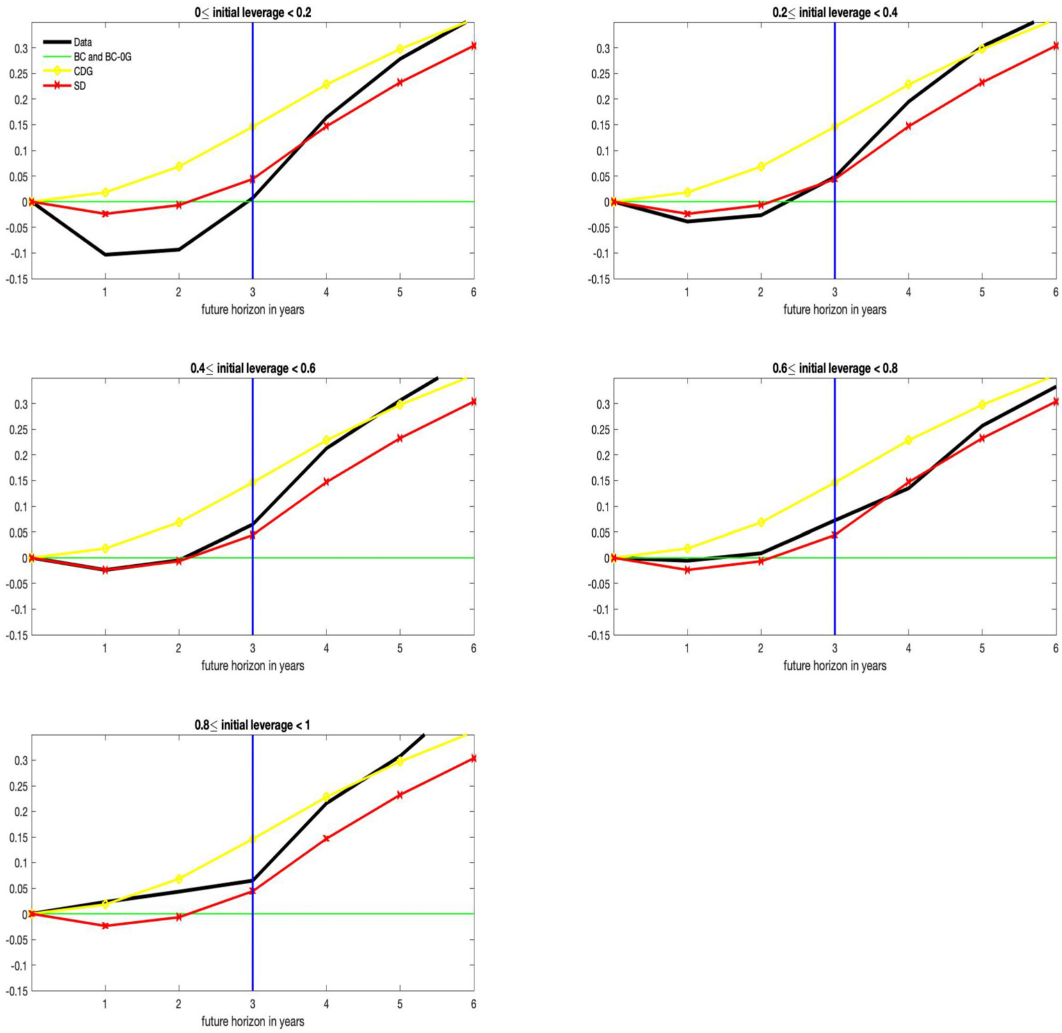

Fig. 6. *Future debt of firms with high future three-year equity returns minus future debt of firms with low three-year equity returns.* For firm i, year t, and horizons 1,...,20, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median 'High (Low) future equity return' firms. The figure shows both the fitted values for the different models and the values from the data of the difference in the average increase in log-debt for high and low future equity return firms. 'BC-0G' refers to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. The data is from CRSP/Compustat and the sample period is 1988–2017.

For each model, the value of corporate bonds depends on the dynamics of leverage – the ratio of debt to firm value – rather than the dynamics of debt and firm value separately. As Appendix B shows, leverage dynamics in the CDG and SD models are isomorphic and, in both models, are given by:

$$dl_t = \lambda\left(\bar{l} - l_t\right)dt + \sigma_l dW_{l,t}. \tag{12}$$

The crucial difference between the models is the size of leverage volatility σ_l. In the CDG model σ_l is equal to asset volatility, σ, while in the SD model:

$$\sigma_l = \sqrt{\sigma_k^2 + \sigma^2 - 2\rho\sigma_k\sigma} \tag{13}$$

where σ_k is the volatility of log-debt and ρ is the correlation between innovations in the firm's asset value and innovations in its debt.

We assume that firms with high asset volatility have high debt volatility and, specifically, that both the correlation, ρ, and the ratio of debt volatility to asset volatility, κ, are constant across firms, i.e., that for firm i, $\sigma_k^i = \kappa\sigma^i$. This implies that, ω, the ratio of leverage volatility to asset volatility, given by:

$$\omega \equiv \frac{\sigma_l}{\sigma} = \sqrt{\kappa^2 + 1 - 2\rho\kappa}, \tag{14}$$

is also constant across firms. To estimate ω we first find all firm-years where the leverage is at least 1% (a total of 99,707 firm-years). For every firm i and

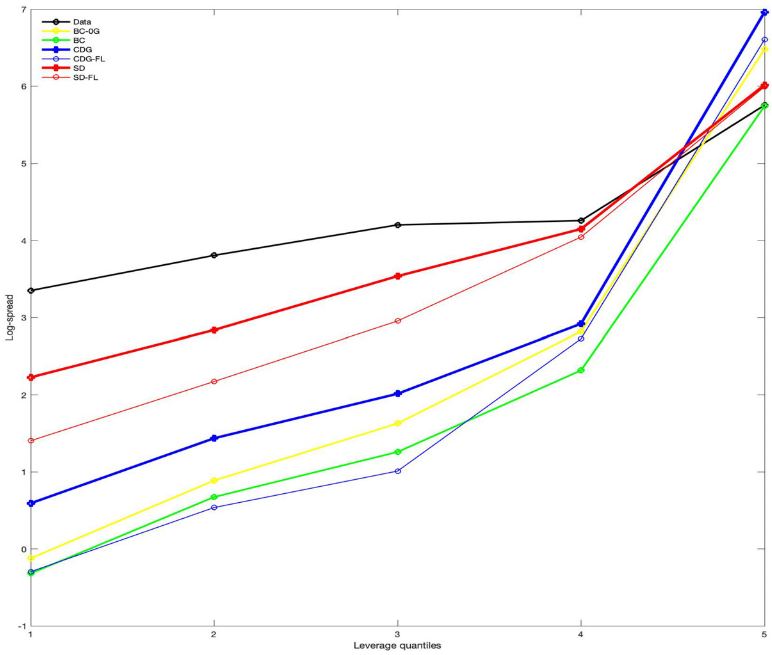

Fig. 7. *Actual and model short-term investment grade credit spreads across leverage quintiles.* For bonds with an investment grade rating and a bond maturity less than three years, we calculate the average actual spread (in bps) of all bond-month observations where the issuing firm has a leverage in the lowest quintile, second-lowest quintile, ..., fifth-lowest quantile and compute the log of the average spread. For each leverage quintile, the figure shows the log of the average spread in the data and for each of the models. The sample period is 2002:07–2018:03.

year t we then calculate the annual log-change in asset value, $\log(\frac{V^i_{t+1}}{V^i_t})$, and leverage, $\log(\frac{L^i_{t+1}}{L^i_t})$, and, for all firms that have available data in all 30 years of the data sample, 1988–2017 (238 firms), calculate the ratio $\frac{\sqrt{Var(\log(\frac{L^i_{t+1}}{L^i_t}))}}{\sqrt{Var(\log(\frac{V^i_{t+1}}{V^i_t}))}}$. The average value of this ratio is 1.5027. Using results in Appendix B.2, over discrete intervals of length t, this ratio corresponds to $\omega\sqrt{\frac{1-e^{-2\lambda t}}{2\lambda t}}$ and therefore our estimate of ω is $1.5027\sqrt{\frac{2\lambda t}{1-e^{-2\lambda t}}}$; with $\lambda = 0.1814$ and $t = 1$ we have that $\hat{\omega} = 1.6409$. Both in estimating the default boundary in Section 4 and in pricing bonds in Section 5, we use this $\hat{\omega} = 1.6409$ to estimate a firm's leverage volatility as $\hat{\omega}$ times its asset volatility.

From the estimate of $\hat{\omega} = 1.6409$ and Eq. (14), together with the value of $\kappa\rho = \frac{\hat{\sigma_\kappa \rho}}{\sigma} = \frac{-0.0505}{0.24} = -0.2106$ obtained above, we find $\kappa = 1.1275$ and $\rho = -0.1868$. It appears, therefore, that the average volatility of log-debt is similar in magnitude to asset volatility and that this, combined with a small (and negative) correlation between shocks to debt and shocks to asset value, results in a level of leverage volatility that is substantially higher than asset volatility.

In Section 6.7, as a robustness check, we estimate κ and ρ directly from the time series of log-debt changes. This gives an estimate of the ratio of leverage volatility to asset volatility that is very similar.

4. Matching historical default rates

We implement the four structural models described in Section 3.1 and, again, relegate details about the data to Appendix A.[13]

For each model, we assume that the parameters for debt adjustment are common to all firms and equal to those estimated in Section 3.3. For the deterministic and stochastic debt adjustment models, we also implement versions where we allow the target leverage ratio to be firm-specific. In this case, we calculate the average historical log-leverage $\hat{l^i}$ for each firm i and use this as the target log-leverage.[14] This is motivated by Lemmon et al. (2008); Huang and Ritter (2009), and others who find that firms have a target leverage which is firm-specific, stable and only to a lesser extent explained by firm characteristics or macroeconomic factors.

A large part of the literature that investigates the ability of structural models to price corporate debt matches models to a single historical default rate at a given ma-

[13] Briefly, we use a data set of monthly corporate bond yield spreads from Merrill Lynch and TRACE for the period 1996–2018 for non-callable fixed-rate bonds issued by industrial firms. The number of bond-month observations is 119,765.

[14] In the models, the parameter of interest ν is different from target leverage. Equation (40) shows that the relation between ν and target leverage $\bar{l}$ is $\bar{l} = \nu - \frac{\mu-\delta-\frac{\sigma^2}{2}}{\lambda}$. We use a common adjustment $\frac{\mu-\delta-\frac{\sigma^2}{2}}{\lambda}$ for all firms given as $\frac{0.0996-0.044-\frac{0.24^2}{2}}{0.1732} = 0.1547$ in the CDG model and $\frac{0.0996-0.044-\frac{0.24^2}{2}}{0.1814} = 0.1477$ in the SD model.

turity and for a specific rating. Feldhütter and Schaefer (2018) show that this results in very noisy estimates of default probabilities and, further, that matching models to default rates across horizons and ratings vastly improves precision. We therefore use their approach and extract a default boundary – d in Eq. (5) – common to all firms but specific to each model, that provides the best fit to the cross-section of historical default rates.

Before we describe the procedure for finding d, it is important to discuss the relation between d and the recovery rate R_{bond} we use when pricing corporate bonds later. In the simplest case, bond holders receive the remaining firm value at default, and d must be equal to the recovery rate R_{bond}, i.e. $d = R_{bond}$. In reality, firms have both bank and bond debt (and potential other kinds of debt). For example, considering only bank and bond debt, we have

$$d = \frac{Ba}{D} R_{bank} + \frac{Bo}{D} R_{bond} \tag{15}$$

where Ba is nominal bank debt, Bo is nominal bond debt and $D = Ba + Bo$. Bank debt is senior to bond debt in default, and if absolute priority is upheld, holders of bank debt are fully repaid before bond holders receive anything. This implies that the recovery rate of bank debt is significantly higher than that of bond debt. Empirically, this is the case: Schwert (2020) reports an average loan recovery rate of 84% and an average bond recovery rate of 32%. While most firms have modest amounts of bank debt, they increase it dramatically when in financial distress as Rauh and Sufi (2010) show and therefore at default Ba/D is substantial. This implies that R_{bond} is significantly lower than d. It is beyond the scope of this paper to specify the dynamics of bank and bond debt separately and instead, we focus on specifying total debt and estimate d such that the models match historical default rates and use the historical bond recovery rate when pricing corporate bonds.

To find d, for each model, we use the following procedure. For each observed spread in the data sample on bond i with a time-to-maturity T issued by firm j and observed on date t, we calculate the firm's T-year default probability $\pi^P(dL_{jt}, \Theta^P_{jt}, T)$ where L_{jt} is the time-t estimate of the firm's leverage ratio and Θ^P_{jt} is a vector containing the relevant parameters for the specific model. Formulae for default probabilities are given in Appendix B. Summary statistics for the bond sample are given in Table 3 and for firm-level quantities in Table 4. We assume a constant asset Sharpe ratio θ and so the drift in firm value is $\theta\sigma_j + r^T_t - \delta_{jt}$, where r^T_t is the T-year riskfree rate. As is common in the literature, we use an asset Sharpe ratio of 0.22 based on Chen et al. (2009).

For a given rating a and maturity T - rounded up to the nearest integer year - we find all bond observations in the sample with the corresponding rating and maturity. For a given calendar year y we calculate the average default probability $\overline{\pi}^P_{y,aT}(d)$ and we then calculate the overall average default probability for rating a and maturity T, $\overline{\pi}^P_{aT}(d)$, by computing the mean across the N years, $\overline{\pi}^P_{aT}(d) = \frac{1}{N}\sum_{y=1}^N \overline{\pi}^P_{y,aT}(d)$. We denote by $\hat{\pi}^P_{aT}$ the corresponding historical default frequency. For rating categories AAA, AA, A, BBB, BB, and B and horizons of 1–20 years we find the value of d that minimizes the sum of absolute differences between the annualized historical and model-implied default rates by solving

$$\min_{\{d\}} \sum_{a=AAA}^{B} \sum_{T=1}^{20} \frac{1}{T} \left| \overline{\pi}^P_{aT}(d) - \hat{\pi}^P_{aT} \right|. \tag{16}$$

Bai et al. (2020) find that estimates of leverage are biased for C-rated firms when using book values of debt as a proxy for market values of debt when calculating firm value and this may bias the estimate of the default boundary. For this reason we exclude the rating category C in the estimation of the default boundary in Eq. (16) for all models. We return to this issue in Section 6.6.

Moody's provide an annual report with historical cumulative default rates and these are extensively used in the academic literature. The default rates are based on a long history of default experience for firms in different industries and different regions of the world. In Appendix C we use Moody's default database to calculate historical default rates for U.S. industrial firms and find them to be economically and statistically significantly different from those published by Moody's for global firms. We therefore use historical default rates for U.S. industrial firms calculated using default data from the period 1970–2017; these are given in Table A3 ('US industrial firms, equal-weight').

Table 5 shows the estimated default boundaries. The estimate of 0.8614 in the BC-0G model is similar to the estimate of 0.8944 in Feldhütter and Schaefer (2018), while the default boundary is lower in the BC model because the future level of debt is higher than in the BC-0G model. The SD model has a lower boundary than other models because the volatility of leverage is higher. In the BC and CDG models the volatility of leverage is equal to the volatility of assets, σ, while in the SD model the volatility of leverage is equal to $1.6409 \times \sigma$ as discussed in the previous section. Thus, the key differences between the SD model and the BC/CDG models are a higher leverage volatility and, largely for this reason, a lower default boundary.

Davydenko (2013) uses the market value of bonds, bank loans, and equity of defaulting firms to estimate the market value of the firm at default and finds an average value of 66% of the face value of debt. The default boundary estimates of 0.67–0.73 in the SD models are thus close to his estimate, providing further support for the SD models.

5. Pricing of corporate bonds

In this section we compare different models in terms of their ability to capture the term structure of average spreads, average spreads across leverage quintiles, and the time series variation of monthly average spreads. Following Eom et al. (2004); Bao (2009); Huang and Huang (2012); Feldhütter and Schaefer (2018); Huang et al. (2020a) and others, we assume that if default occurs, investors receive at maturity a fraction of the originally promised face value, but now with certainty. Assuming the bond is a zero-coupon bond, the credit spread, s, is then calculated as:

$$s = y - r = -\frac{1}{T}\log\left[1 - (1-R)\pi^Q(T)\right] \tag{17}$$

where y is the yield-to-maturity, r is the riskless rate, R is the recovery rate, T is the bond maturity and $\pi^Q(T)$ is the risk-neutral default probability.

Since the Black–Cox and stationary leverage models are one-factor models, all claims have the same Sharpe ratios and so the bond Sharpe ratio is the same as the asset Sharpe ratio. In the stochastic debt model this may not be the case and we assume that the bond Sharpe ratio is the same as the asset Sharpe ratio, 0.22. We discuss this assumption further in Section 6.1. We set the recovery rate to 33.48% which is Moody's (2018a)'s average recovery rate, as measured by post-default trading prices, for senior unsecured bonds for the period 1983–2017.

Table 4 shows summary statistics for the firms that have bonds outstanding in our sample. We see that the most common rating is BBB followed by A, i.e. lower investment grade ratings. We also see that equity volatility increases as rating decreases and the average equity volatility is 24% for AAA and 61% for C. The increase in equity volatility is largely due to increasing leverage while asset volatility changes much less with rating: the average asset volatility is 22% for AAA and 26% for C.

5.1. Average spreads

Table 6 gives average actual and model spreads for different bond maturities. We report spreads for three maturity ranges - 0–3 years (Short), 3–10 years (Medium), and 10–20 years (Long) - and, with some notable exceptions, the table shows that the models capture average spreads for both IG and HY bonds reasonably well. For investment grade bonds, the main exceptions are the BC model, that predicts long-term spreads that are too high, and the CDG models for which long-term spreads are too low. All the models underpredict long-term speculative grade spreads, consistent with the evidence in Feldhütter and Schaefer (2018).

Next, we compare the performance of the models for different levels of leverage and focus on short-term spreads since this is the area that has presented most difficulty to diffusion-based structural models. Fig. 7 gives the log of average spreads on short-term investment grade bonds broken down by leverage quintile, and shows that, except for the highest leverage quintile, all the models underpredict short-term spreads. However, for quintiles 1–4, the predicted spreads in the SD and SD-FL models are always closest to the average actual spreads. For the quintile with the highest leverage, all the models overestimate spreads and – except for the BC model that, as Table 6 shows, provides the worst overall predictions of short-term investment grade spreads – the SD and SD-FL models are again closest to the actual data.

Table 6 and Fig. 7 are informative about the models' ability to capture the term structure of average spreads and the variation of average spreads with leverage. However, they do not help us detect differences in pricing accuracy in time series and this is the issue that we turn to next.

Table 7

Pricing errors of monthly credit spreads. This table shows how well structural models match average monthly credit spreads. For a given rating r and maturity m, we find all bonds at the end of a given month t that have this rating and maturity, calculate the average actual credit spread (in basis points) to the swap rate, s^a_{rmt}, and do this for all months in the sample. For each model, we likewise calculate a time series of monthly average model credit spread (in basis points) $s^M_{rm1},\ldots,s^M_{rmT}$. This table shows the average absolute pricing error $1/T\sum_{t=1}^{T}\left|s^a_{rmt}-s^M_{rmt}\right|$. 'Short' includes bond maturities in the range 0–3 years, 'Medium' 3–10 years, and 'Long' 10–20 years. 'BC-0G' refers to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. 'Inv' includes bonds rated AAA, AA, A, and BBB, while 'Spec' includes bonds rated BB, B, and C. The sample period for 'Short' is 2002:07-2018:03 while it is 1988:03-2018:03 for 'Medium' and 'Long'.

		Average	Short	Medium	Long
All	BC-0G	63	109	40	42
	BC	67	92	46	62
	CDG	119	204	53	99
	CDG-FL	87	127	41	92
	SD	52	65	50	42
	SD-FL	57	67	50	53
Inv	BC-0G	44	72	31	29
	BC	55	53	37	76
	CDG	75	136	34	53
	CDG-FL	62	83	33	72
	SD	45	36	51	48
	SD-FL	39	38	42	37
Spec	BC-0G	194	247	169	166
	BC	181	241	173	130
	CDG	285	357	223	277
	CDG-FL	218	282	177	196
	SD	185	180	165	211
	SD-FL	179	185	185	166

5.2. Monthly average spreads

We next investigate the models' ability to capture the time series variation of spreads by calculating monthly averages of actual and model-implied spreads. Specifically, for each month $t=1,\ldots,T$ we calculate the average actual and model-implied spread (in basis points), s^a_t and s^M_t respectively. Table 7 shows the average absolute pricing error in basis points

$$\frac{1}{T}\sum_{t=1}^{T}\left|s^a_t-s^M_t\right| \tag{18}$$

for a given rating class and range of bond maturity. Across maturities, the SD and SD-FL models have the smallest average errors of 52 and 57 bps while the errors in the other models are in the range 63–119 bps. The differences in model performance is predominantly due to large differences in the models' ability to capture short-term spreads. For example, for investment-grade bonds the SD and SD-FL models have average errors of 36 and 38 bps, respectively, for investment grade while the remaining models have errors in the range 53–136.

Fig. 8 shows why the SD and SD-FL models do better than the other models in capturing short-term spreads (the

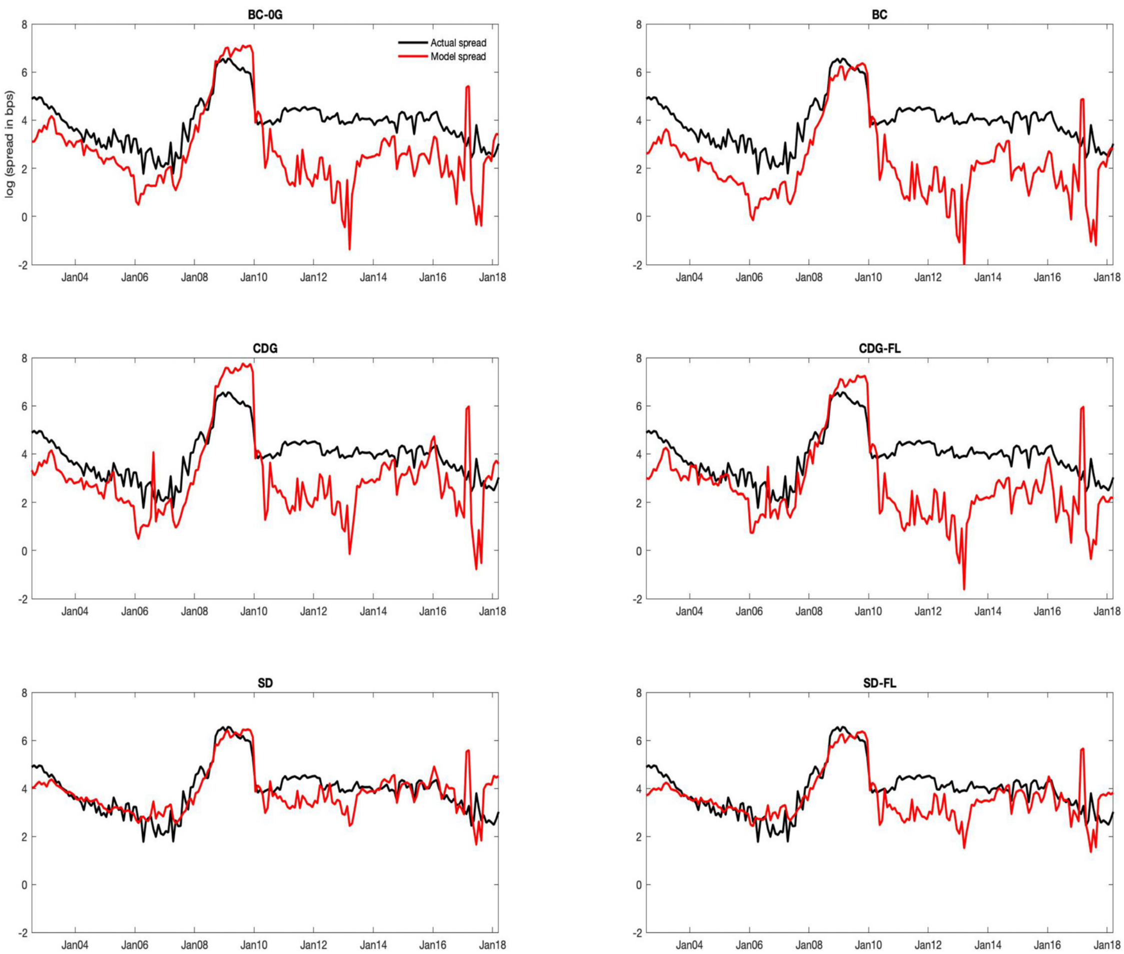

Fig. 8. *Actual and model short term investment grade credit spreads.* For the investment grade rating, bond maturity less than three years, and each month in the sample, we find all bonds at the end of a given month t that have this rating and maturity, and calculate the average actual credit spread s^a_{rmt}. For each model, we similarly calculate a time series of monthly average model credit spread $s^M_{rm1}, \ldots, s^M_{rmT}$. This figure shows the time series of log credit spreads. The sample period is 2002:07–2018:03.

figure shows log-spreads since the high spreads during the crisis would otherwise dominate the graphs). In the figure, the monthly investment grade log spreads for short maturities in the SD and SD-FL models are close to actual spreads (apart from the period 2010–2014 when all the models underpredict spreads), while the predicted spreads in the other models are too low for almost the entire sample period, except during the crisis. While it is not immediately clear from the figure, the predictions of BG-0G and CDG models flip sharply during the 2008–2009 financial crisis and the model spreads become much too high. For example, the average actual spread during (outside) 2008–2009 is 382 bps (48 bps), while it is 674 bps (16 bps) in the BG-0G model and 370 bps (44 bps) in the SD model. Although the average short-term investment grade spread in the BG-0G model of 73 bps matches the average actual spread of 77 bps well, this is achieved by predicting spreads that are too low during normal times and too high during the financial crisis. The predictions in the CDG and CDG-FL models are similar to those in the BC-0G model. In contrast, the average spread in the SD model is slightly lower (67 bps) than in the BC-0G model but closer to the actual spread in both normal times and during the crisis.

The excess sensitivity of short-term spreads to leverage, when leverage is high, in the BC-0G and both CDG models, evident in Fig. 8, reappears in a different form in their predictions of spread volatility. Table 8 shows the standard deviation of changes in the monthly spread for each of the models as well as for the data. Focussing again on short-term investment grade spreads, the standard deviation in the SD models is 20 bps (SD) and 18 bps (SD-FL), which is slightly higher than the value of 16 bps in the data. In contrast, the volatilities in the BC-0G and CDG models are too high: 25 bps in the BC-0G model and 30 bps and 43 bps respectively in the CDG and CDG-FL models. In summary, the SD models provide a better fit to actual short-term spreads in both normal times and in the crisis and they also provide a better fit to the volatility of short-term spreads. To better understand these results, and the relationship between them, we interpret the findings

Table 8
Volatility of monthly credit spreads. This table shows how well structural models match the volatility of monthly credit spreads. For a given rating r and maturity m, we find all bonds at the end of a given month t that have this rating and maturity, calculate the average actual credit spread (in basis points) to the swap rate, s^a_{rmt}, and do this for all months in the sample. For each model, we likewise calculate a time series of monthly average model credit spread (in basis points) $s^M_{rm1}, \ldots, s^M_{rmT}$. This table shows the standard deviation, in basis points, of monthly changes in the average credit spread. 'Short' includes bond maturities in the range 0–3 years, 'Medium' 3–10 years, and 'Long' 10–20 years. 'BC-0G' refers to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. 'Inv' includes bonds rated AAA, AA, A, and BBB, while 'Spec' includes bonds rated BB, B, and C. The sample period for 'Short' is 2002:07-2018:03 while it is 1988:03-2018:03 for 'Medium' and 'Long'.

		Average	Short	Medium	Long
All	Data	20	23	18	19
	BC-0G	22	34	17	14
	BC	18	18	18	17
	CDG	27	61	12	9
	CDG-FL	23	41	16	12
	SD	15	24	12	10
	SD-FL	16	23	14	11
Inv	Data	14	16	13	14
	BC-0G	18	25	15	14
	BC	16	13	16	18
	CDG	21	43	10	9
	CDG-FL	18	30	14	10
	SD	14	20	11	11
	SD-FL	14	18	12	11
Spec	Data	51	55	57	42
	BC-0G	53	83	47	30
	BC	41	47	46	31
	CDG	66	141	36	22
	CDG-FL	60	103	45	31
	SD	41	67	31	23
	SD-FL	41	62	36	25

through the lens of the distance-to-default measure

$$DD_T = \frac{\ln(\frac{V_0}{K_0}) - \ln(d) + \left(\mu_l - \frac{1}{2}\sigma_l^2\right)T}{\sigma_l\sqrt{T}} \tag{19}$$

$$= -\frac{1}{\sigma_l\sqrt{T}} l_0 + \frac{\left(\mu_l - \frac{1}{2}\sigma_l^2 - \frac{\ln(d)}{\sqrt{T}}\right)\sqrt{T}}{\sigma_l} \tag{20}$$

where DD_T is the T-year distance-to-default and μ_l and σ_l the drift and volatility of leverage. Although the formula is derived from the Merton model, Jessen and Lando (2015) show its ranking of firms according to their default risk is strongly robust to deviations in model specification from the Merton model, such as asset dynamics and the default boundary.

On average, across horizons and rating classes, the default boundary d is estimated such that DD_T is consistent with historical average default frequencies and therefore a higher σ_l leads to a lower d. Furthermore, the sensitivity of DD_T to changes in leverage, $-\frac{1}{\sigma_l\sqrt{T}}$, is decreasing in σ_l and T. Leverage, being observable, has the same value in the different models and so, since σ_l is higher in the SD model, when leverage is either unusually high or low, its impact on default probabilities and spreads is lower. This also implies that the impact of higher leverage volatility on the volatility of DD_T, and thus spreads, is also smaller in the SD model and that the difference is most pronounced for short maturities.[15]

5.3. Stochastic volatility and jumps in firm value

So far we have compared the pricing performance of the stochastic debt model with well-known and established diffusion models. These models have parameters that are relatively easy to estimate over time and across firms allowing us to estimate the parameters based on the firm value dynamics. Once the default boundary is estimated using default rates, we then price corporate debt out-of-sample in the sense that spreads are not used as inputs when estimating any of the parameters. An advantage of doing so is that the pricing performance of the models is not given in advance; if model parameters were estimated by fitting to spreads, the SD model would automatically have the best pricing performance since the SD model nests the other models.

In this section we compare the SD model with recent models that include stochastic volatility and jumps in firm value. Since the models are non-nested and it is less straightforward to extract the jump and stochastic volatility parameters from firm value dynamics, we instead estimate the risk-neutral parameters by fitting the models to spreads as in Du et al. (2019). While this approach does not ensure that the models' default rates are consistent with historical default rates, the advantage is that, in principle, the risk-neutral parameters can be estimated with high precision based on a single day of a yield spread curve.

We follow Du et al. (2019) and model firm value V_t, the variance of firm value A_t, and log-debt k_t under the risk-neutral measure as

$$\frac{dV_t}{V_t} = \left(r - \delta - \xi\overline{\eta}^Q\right)dt + \sqrt{A_t}dW_{1,t} + dJ_t^Q \tag{21}$$

$$dA_t = \lambda_A(\nu_A - A_t)dt + \sigma_A dW_{2,t} \tag{22}$$

$$dk_t = \lambda(\nu - l_t)dt + \sigma_k dW_{3,t} \tag{23}$$

where r is the riskfree rate, δ is the payout rate, and J_t^Q is a jump process with constant intensity ξ and a random jump size equal to η^Q. Conditional on a jump, firm value V_t jumps to $V_t e^{\mu^Q}$ with $\mu^Q \sim N(\overline{\mu}^Q, \gamma^2)$. This implies that $\overline{\eta}^Q = E(\eta^Q) = E(e^{\mu^Q}) = e^{\overline{\mu}^Q + \frac{1}{2}\gamma^2}$. Finally, the correlation matrix C of the Brownian motions is

$$C = \begin{bmatrix} 1 & \rho_V & \rho_k \\ \rho_V & 1 & 0 \\ \rho_k & 0 & 1 \end{bmatrix}, \tag{24}$$

[15] The BC model has the same leverage volatility as the BC-0G model and yet has low standard deviations in Table 8. The reason for this is that the model underpredicts short-term default probabilities (as Table 6 shows) and thus firms are considered too "safe" relative to what is consistent with the data. This makes credit spreads less sensitive to changes in leverage.

that is, volatility and debt have correlation ρ_V and ρ_k, respectively, with firm value. We estimate eleven parameters $(\xi, \overline{\eta}^Q, \gamma, \lambda_A, \nu_A, \sigma_V, \rho_A, \lambda, \nu, \sigma_k, \rho_k)$, observe two state variables (V_t and A_t) and set the default boundary d to 0.85 as it is poorly identified.[16]

We estimate three models for the issuer with most bonds in our sample, Walmart, and fit the models at the end of each year 2003–2017 in the sample for which we have CDS premiums at both shortest and longest CDS maturities of 0.5 and 30 years respectively.[17] Walmart is rated AA throughout the sample period, has a low leverage between 0.08 and 0.22 in the period 2001–2018 and an average asset volatility of 0.18, thus an example of a low-risk firm with a high investment grade rating for which, as a result, it might be difficult for a pure diffusion model to fit the credit spread curve. For a given date, we estimate the parameters by using Monte-Carlo and minimize RMSEs of CDS premiums across maturities.[18] The models are the SD model (A_t is constant and there are no jumps in firm value), the stochastic volatility with jumps (SVJ) model (k_t is constant), and the full model with stochastic debt and volatility and jumps in firm value (SVJ-SD).

Table 9 Panel B shows the RMSEs of the estimated spreads for the three models. The average RMSE is 14bps for the SVJ model, 9bps for the SD model and 3bps for the SVJ-SD model showing that stochastic debt helps significantly in matching average CDS spread curves. The improvement is most dramatic for the 30-year maturity where the SVJ model has three times as large average RMSE as the SD model. Panel B also shows that the differences in fit increases when 2008 is excluded: the average RMSE is 14bps for the SVJ model, 4bps for the SD model and 2bps for the SVJ-SD model. Even at a short horizon of one year the SD model has lower average RMSE than the SVJ model, both when including and excluding 2008.

Fig. 9 shows the average model-implied CDS curves of the three models together with the average actual CDS curve. The SVJ-SD model fails to capture the concavity of the yield curve which leads to large pricing errors at short (6 months), intermediate (10-year) and long (30 year) maturities. As the individual spread curves in Fig. 10 show, the poor fit becomes particularly pronounced after the financial crisis in 2008. It may be surprising that the SVJ model cannot match a concave spread curve, but during the post-crisis period Walmart's payout rate exceeded the riskfree rate, making the risk-neutral asset drift negative (before considering jumps that amplify the effect), and this fact is difficult to reconcile with the concave spread curve of a safe issuer.

Fig. 10 shows that while the SVJ model fits CDS curves significantly worse on average than the SD model, it does well in 2008 where the CDS curve was quite flat and high at short maturities. Thus, it suggests that although the SD model outperforms the SVJ model in normal periods, jumps are necessary to match CDS curves in a period with high uncertainty.

Fig. 9 shows that adding jumps and stochastic volatility to the SD model improves the fit particularly at the shortest maturities below two years where the SD model underpredicts spreads. The fit improvement comes in particular from jumps (not shown) and suggests that adding jumps to stochastic debt is important to capture spreads at very low maturities.

Overall, this section shows that recognising the stochastic nature of debt is important in explaining the CDS curves for Walmart, a firm with a high investment grade rating and with the most transactions in our sample.

6. Further topics and robustness

In this section we explore the extent to which the results are robust to alternative assumptions regarding key quantities. In Section 6.1 we discuss the relation between bond and asset Sharpe ratios. The consequences of including jumps in the amount of debt are discussed in Section 6.2. In Section 6.3 we reexamine our main finding on debt dynamics when we (i) restrict the sample of firms to those that exist every year in our data sample, (ii) use data for the period 1965–1987 (rather than 1988–2017), and (iii) match firms exactly on firm size, leverage, and cash. We examine how bond pricing errors are affected by using (i) Treasury yields instead of swap rates as the riskfree rate in Section 6.4, (ii) CDS spreads instead of bond spreads in Section 6.5, and (iii) the actual market value of debt instead of the book value when calculating firm value in Section 6.6. Finally, Section 6.7 discusses an alternative method of estimating the parameters of debt dynamics.

6.1. Bond Sharpe ratio

In the Black–Cox and CDG models, there is one risk factor and so the Sharpe ratio is the same for the firm's equity, debt and all other claims on its assets. In the stochastic debt model, there are two risk factors and the Sharpe ratios are now not necessarily the same for different claims issued by the same firm. Appendix B.4 shows that when we write the dynamics of debt as

$$dk_t = \lambda(\nu - l_t)dt + \sigma_k\left(\rho dW_t^P + \sqrt{1-\rho^2}dW_{2,t}\right) \qquad (25)$$

where W_t^P is the Brownian motion driving the asset value in Eq. (4) and $W_{2,t}$ is a Brownian motion uncorrelated with W_t^P, the asset Sharpe ratio in a CAPM world is $SR_V =$

[16] In the previous sections, consistent with the assumptions of the BC, CDG and SD models, we use a constant asset volatility for each firm by calculating a time series average of unlevered asset volatilities of the firm. Here, we calculate a time-varying asset variance by unlevering equity volatility, calculated as 255 times the variance of the past three months' daily equity returns. Firm value and payout rate are calculated as in the previous sections (see Appendix A) while the riskfree rate is the 10-year swap rate. When estimating the parameters and default boundary jointly, the default boundary typically becomes unrealistically low and leverage volatility unreasonably large and for this reason we fix the boundary.

[17] Most years have maturities $\frac{1}{2}$, 1, 2, 3, 4, 5, 7, 10, 15, 20 and 30 years. Missing maturities are 4-year in 2003, 4- and 15-year in 2005, 30-year in 2006 and 20-year in 2010.

[18] We use 100,000 simulations and discretise the dynamics using steps of one month. When estimating the parameters, we use Matlab's fminsearch and start the optimization with the estimated parameters in Table VII in Du et al. (2019) for the SVJ model parameters and our estimates for the SD model parameters.

Table 9

Stochastic volatility-jump model with stochastic debt. For the issuer with most bonds in our sample, Walmart, we estimate a model with stochastic asset volatility and jumps in asset value (SVJ), the stochastic debt model (SD), and a model with stochastic asset volatility, jumps in asset value and stochastic debt (SVJ-SD) by fitting to the year-end CDS premium curves in the years 2003, 2004, ..., 2017. Panel A reports the average parameter estimates where ξ is jump intensity, $\overline{\eta}^Q$ is average jump size, γ is jump volatility, λ_A is mean-reversion of volatility, ν_A is mean of volatility, σ_A is volatility of volatility, ρ_V is correlation between volatility and asset value, λ is mean reversion of debt, ν is mean of debt, σ_k is volatility of debt, and ρ_k is correlation between dent and asset value. Panel B reports root-mean-squared errors (RMSEs) overall and across CDS maturities $\frac{1}{2}$, 1, 2, 3, 4, 5, 7, 10, 15, 20 and 30 years.

Panel A: Average parameter estimates

	ξ	$\overline{\eta}^Q$	γ	λ_A	ν_A	σ_A	ρ_V	λ	ν	σ_k	ρ_k
SVJ	0.010	−0.827	0.064	3.925	0.022	0.062	−0.230				
SD								0.314	−1.827	0.451	−0.112
SVJ-SD	0.010	−0.722	0.043	4.052	0.023	0.315	−0.179	0.249	−1.557	0.230	−0.116

Panel B: Pricing performance (RMSEs)

2003–2017												
Maturity	All	0.5	1	2	3	4	5	7	10	15	20	30
SVJ	13.81	13.15	12.01	9.19	6.82	4.21	4.25	10.40	13.60	9.55	8.39	34.05
SD	9.19	20.14	11.06	7.55	6.79	7.60	5.35	4.17	2.75	3.85	6.13	11.60
SVJ-SD	2.68	3.12	2.25	1.72	2.15	2.29	2.30	2.54	4.03	1.42	1.82	4.26
2003–2017 excl. 2008												
SVJ	14.24	13.55	12.17	9.33	7.04	4.29	3.84	10.61	14.06	9.76	8.66	35.33
SD	4.28	7.09	8.20	6.17	2.59	2.83	2.71	1.76	2.11	0.98	2.98	2.55
SVJ-SD	2.31	3.20	1.70	1.64	1.91	2.16	1.83	2.61	3.33	1.24	1.45	3.11

$\rho_{V,M}SR_M$ – where $\rho_{V,M}$ is the correlation between asset value and the market and SR_M is the market Sharpe ratio. The corresponding bond Sharpe ratio is

$$SR_b = -\left(\frac{\sigma_k\rho - \sigma}{\sigma_l}\rho_{V,M} + \frac{\sigma_k}{\sigma_l}\sqrt{1-\rho^2}\rho_{W_2,M}\right)SR_M \tag{26}$$

where $\rho_{W_2,M}$ is the correlation between $W_{2,t}$ and the market. Fig. 11 shows the relation between the bond Sharpe ratio and $\rho_{W_2,M}$ as well as the (constant) asset Sharpe ratio. In the figure we use the estimated values $\sigma = 0.24, \sigma_k = 1.1275\sigma$, $\rho = -0.1868$ and assume that $SR_M = 0.44$ and $\rho_{V,M} = 0.5$ such that the asset Sharpe ratio is 0.22. To recap: $dW_{2,M}$ is the component of the shock to a firm's debt that is uncorrelated with the firm's assets. If this component has a significant negative correlation with the market, the bond Sharpe ratio is higher than the asset Sharpe ratio, while the reverse is the case if the correlation is positive. Thus, the model flexibly allows for both a positive and negative wedge between the asset and bond Sharpe ratios.

Previous literature such as Chen et al. (2009); Chen (2010); Feldhütter and Schaefer (2018), and Bai et al. (2020) rely on an equity Sharpe ratio of 0.22 estimated by Chen et al. (2009) when calibrating one-factor structural models. Since all claims in one-factor models have the same Sharpe ratio, there is no need to distinguish between the Sharpe ratios of the different assets. In the stochastic debt model, this is not so and, in this case, the Sharpe ratio requires more attention. When pricing bonds we have used a common bond Sharpe ratio of 0.22 for all the models (and all firms). This implies that we have set $\rho_{W_2,M}$ for each firm such that the asset and bond Sharpe ratios are both equal to 0.22. The reason for this choice is that we want to be sure that any pricing differences between the models are due to differences in debt dynamics and not to differences in bond risk premia. It remains, however, an interesting issue for future research.

6.2. Jumps in debt

Our assumption that a firm's total debt follows a diffusion process, is consistent with a firm using credit lines or short-term commercial paper to make continuous adjustments to its borrowings. However, for firms that raise debt in the corporate bond market, the issuance and retirement amounts are often significant relative to the firm's total debt and these adjustments may be more reasonably viewed as jumps. Furthermore, in theoretical models of dynamic capital structure with costly adjustment, changes in debt typically arise as jumps (see for example Hackbarth et al., 2006; Strebulaev, 2007; Bhamra et al., 2010; Chen, 2010; Geelen, 2017; Geelen, 2019).

This section examines the impact on credit spreads of jumps in a firm's total debt and focuses on short-term spreads because this is where the impact of shocks to debt is greatest. We include a jump component, J_t, in the dynamics of k_t, log total debt and choose the parameters so that (i) jumps constitute a significant fraction of the variability in shocks to debt, and (ii) the risk neutral distribution of leverage has the same mean and variance as in the diffusion case.

Specifically, we assume that under the risk neutral measure:

$$dk_t = \lambda(\nu_{JD}^Q - l_t)dt + \sigma_{k,JD}dW_{k,t} + dJ_t, \tag{27}$$

where the parameters are as previously defined and the "JD" subscript indicates the jump-diffusion model. The jump process, J_t, has intensity η, and jumps that are normally distributed with mean ξ and standard deviation ζ. The risk-neutral dynamics of the firm's assets are unchanged:

$$d\nu_t = (r - \delta - \frac{\sigma}{2})dt + \sigma dW_t, \tag{28}$$

and so, the risk-neutral dynamics of leverage can be written as:

$$dl_t = dk_t - d\nu_t = \lambda(\bar{l}_{JD}^Q - l_t)dt + \sigma_{l,JD}dW_{l,t} + dJ_t. \tag{29}$$

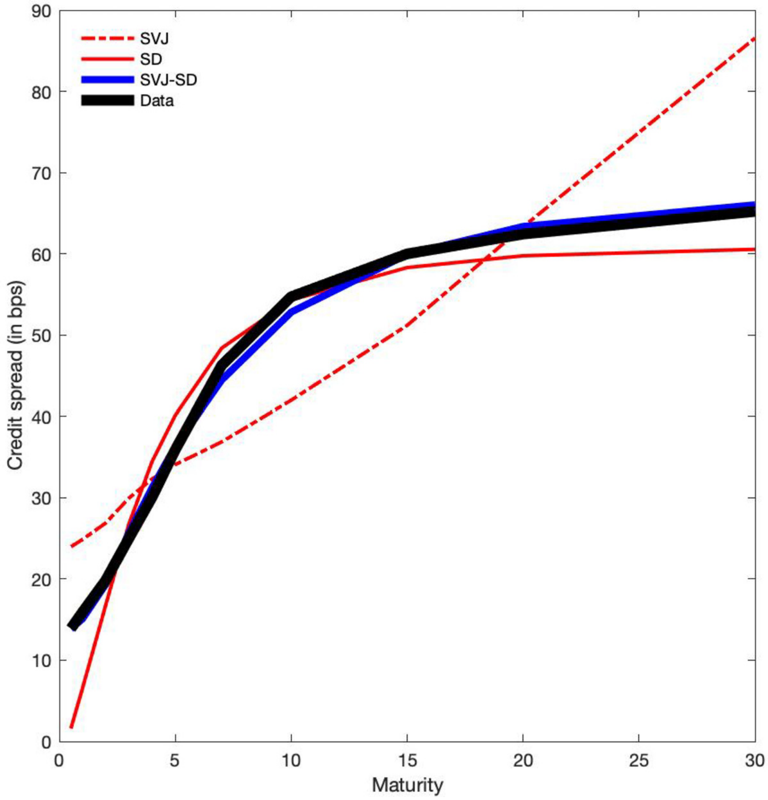

Fig. 9. *Stochastic volatility-jump model with stochastic debt.* For the issuer with most bonds in our sample, Walmart, we estimate a model with stochastic asset volatility and jumps in asset value (SVJ), the stochastic debt model (SD), and a model with stochastic asset volatility, jumps in asset value and stochastic debt (SVJ-SD) by fitting to the year-end CDS premium curves in the years 2003, 2004, ..., 2017. The graph shows the average actual CDS curves as well as the average model-implied yield curves for the years where there are available CDS premiums at all maturities (2005, 2007–2009 and 2011–2017).

where the parameters are as previously defined and the 'JD' subscript indicates the jump-diffusion model. The jump process, J_t, has intensity η, and jumps that are normally distributed with mean ξ and standard deviation ζ.

To provide a significant role for jumps, we set $\sigma_{k,JD}$, the volatility of the diffusion component of shocks to total debt equal to $\sigma_k/2$, i.e., half its value in the diffusion model. We set the jump intensity to one and the mean jump size equal to $0.9\sigma_k$ and so jumps occur once per year on average and have an average size equal to 90% of the annual standard deviation of total debt in the diffusion model. We then set the standard deviation of the jump size, ζ, so that the variance of log-leverage in the diffusion and jump-diffusion models is the same and $\bar{l}_{JD}^{Q}$ so that the mean value of log-leverage under the risk-neutral measure is also equal in two models.

The first two moments of log-leverage in the jump-diffusion model are given by Das (2002)

$$E(l_{t+\tau}|l_t) = \left(\bar{l}_{JD}^{Q} + \frac{\eta\xi}{\lambda}\right)(1-e^{-\lambda\tau}) + l_t e^{-\lambda t}, \tag{30}$$

$$Var(l_{t+\tau}|l_t) = \left(\sigma_{l,JD}^2 + \eta(\xi^2+\zeta^2)\right)\left(\frac{1-e^{-2\lambda\tau}}{2\lambda}\right). \tag{31}$$

Given the mean jump size, ξ, the assumption that $\sigma_{k,JD} = \sigma_k/2$ and the parameters of the diffusion model, it is simple to show that, to equate the variance of log-leverage in the two models, the standard deviation of the jump size, ζ, must be set equal to:

$$\zeta = \sqrt{\frac{\frac{3}{4}\sigma_k^2 - \rho\sigma_k\sigma}{\eta} - \xi^2} = 0.0880.$$

Finally, to equate the mean value of leverage under the risk-neutral measure in the two models, we set $\bar{l}_{JD}^{Q} = \bar{l}^{Q} - \frac{\eta\xi}{\lambda}$.

Fig. 12 compares the 18-month log-credit spread for the (SD) diffusion model and the corresponding jump-diffusion model. For the diffusion model, the default boundary is left unchanged at 0.7322. In the jump-diffusion model, the default boundary is set to 0.675 so that, for a leverage ratio of 0.5 and a maturity of 18 months, the credit spread in the two models is the same.

For safe firms, i.e., those with low leverage, spreads are higher in the jump-diffusion model than in the diffusion model while the reverse is the case for risky firms, i.e., those with high leverage. The relation between the results for the JD model and the SD model thus mirrors that between the results for the SD models and the other models in Fig. 7, i.e., higher spreads for low leverage firms and

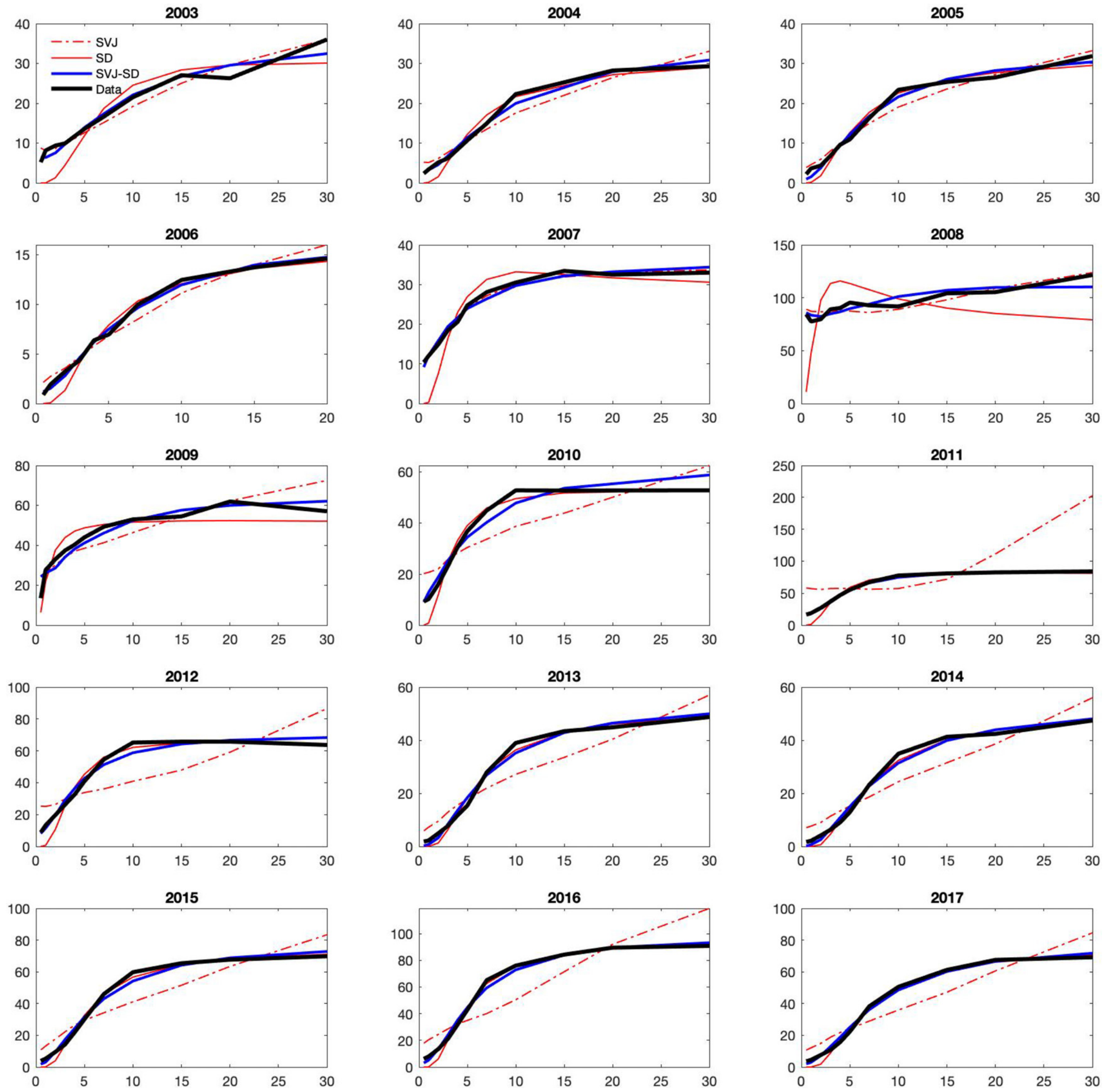

Fig. 10. *Stochastic volatility-jump model with stochastic debt.* For the issuer with most bonds in our sample, Walmart, we estimate a model with stochastic asset volatility and jumps in asset value (SVJ), the stochastic debt model (SD), and a model with stochastic asset volatility, jumps in asset value and stochastic debt (SVJ-SD) by fitting to the year-end CDS premium curves in the years 2003, 2004, ..., 2017. The graph shows the actual CDS curves as well as the model-implied yield curves.

lower spreads for high leverage firms. The main difference between the SD model and the CDG and other diffusion models is the result of a higher leverage volatility and, as Fig. 12, shows, jumps in debt amplify this effect, with still higher spreads for low leverage firms and still lower spreads for high leverage firms.

6.3. Debt dynamics conditional on equity returns

In Section 2 we document that short run (long run) changes in firms' debt levels are negatively (positively) correlated with their short run equity returns. In this section we show that this pattern is robust to using a different sample period, accounting for survivorship bias, and matching firms exactly on leverage, size and cash holdings.

6.3.1. Different sample period

When we examine debt dynamics in Section 2 we use firm data from CRSP/Compustat for the period 1988–2017. In particular, we start our sample period in 1988 to be consistent with the bond data sample. CRSP/Compustat has data available prior to 1988 and to see whether debt dynamics follow a similar pattern in the earlier period, Fig. A1 shows debt growth rates for high and low equity return firms in the period 1965–1987. The figure shows that the pattern of debt dynamics for the 1965–1987 sample is similar to that for 1988–2017. Over a three-year horizon, except in the case of high leverage, firms that experience a low equity return have higher growth rate of debt than firms with a high equity return. As before, after the shock, the pattern is reversed.

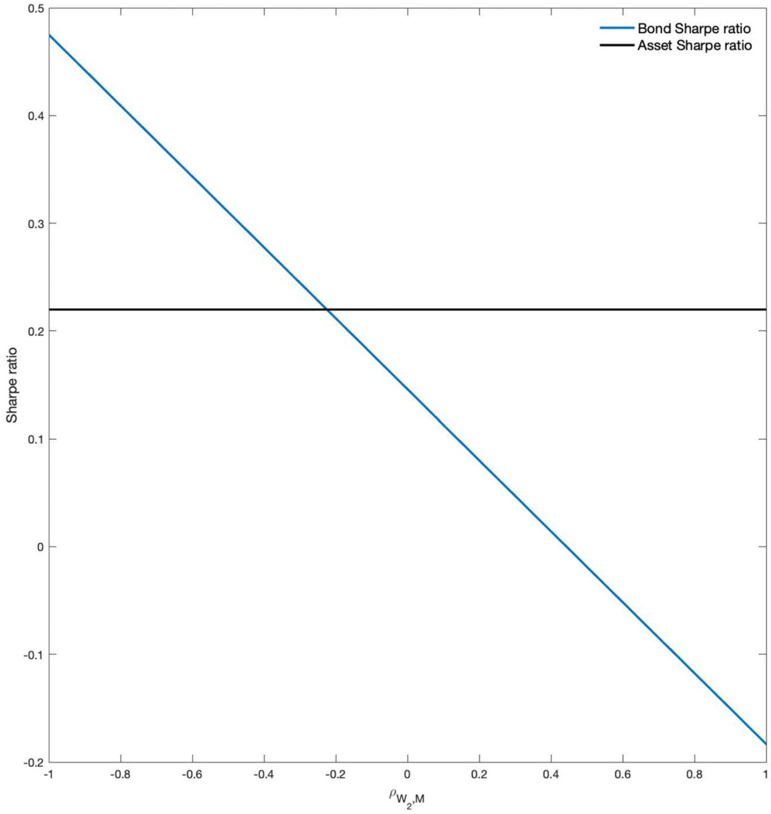

Fig. 11. *Sharpe ratios in the stochastic debt model.* The graph plots the asset and bond Sharpe ratios in the stochastic debt model as a function of $\rho_{W_2,M}$, the correlation between shocks to the market and the component of shocks to debt uncorrelated with shocks to the asset value. The remaining parameters are set to the estimated values, as explained in the text.

6.3.2. Survivorship bias

When we examine debt dynamics, we use all available firm data and firms may not be in the data base for the whole sample period. They may have missing data for several reasons such as default or going private through an LBO. Since the patterns of debt growth rates we document are prevalent for all leverage groups, we think it unlikely that our results are driven by firms exiting the sample. For example, default exits are much more common for highly leveraged firms than for lowly leveraged firms, while LBOs are more frequent for firms with low leverage than firms with high leverage. Nevertheless, to further address this concern, we look at the subset of 238 firms that have data in CRSP/Compustat for every year in the 1988–2017 period, in total 30 years, and leverage of at least 0.01 in all years. Fig. A2 shows that the pattern of debt growth rates of high and low equity shock firms is again – although more noisy due to the smaller sample – similar to those documented in Section 2.

6.3.3. Selection bias

It is possible that there is a selection bias such that on average high- and low-return firms are different in some dimension. For example, Frank and Goyal (2003) find that small high-growth firms are more likely to issue equity in response to a financing deficit. To explore this issue, we control for firm size in the following way. For each year t, we sort the firms that have available data in year t as well as in year $t+3$ according to their firm size in year t. The two largest firms make up the first pair, numbers 3 and 4, in terms of size, the second pair, and so forth (if there is an uneven number of firms, the smallest firm is discarded). In each pair the firm with the higher (lower) equity return between t and $t+3$ is classified as a "high equity return" ("low equity return") firm. The advantage of this approach is that we can precisely match the sizes of high and low return firms. The disadvantage is that the average difference in return between high and low returns will be smaller and the corresponding difference in the debt growth rates also smaller. Fig. A3 ("Same firm size") shows debt growth when firms are sorted according to size. We see that the pattern documented in Section 2 is robust to controlling for firm size.

Lemmon and Zender (2010) show that firms with debt capacity concerns are more likely to use equity financing. It is plausible that firms with lower leverage have both greater debt capacity and also lower expected returns on equity. In this case, low equity return firms could be more likely to have lower leverage and spare debt capacity, leading to the negative correlation between short-run debt growth and short-run equity returns (even though this explanation cannot explain the positive long-run correlation). While we attempt to control for leverage by looking at firms within leverage intervals, there remain small differ-

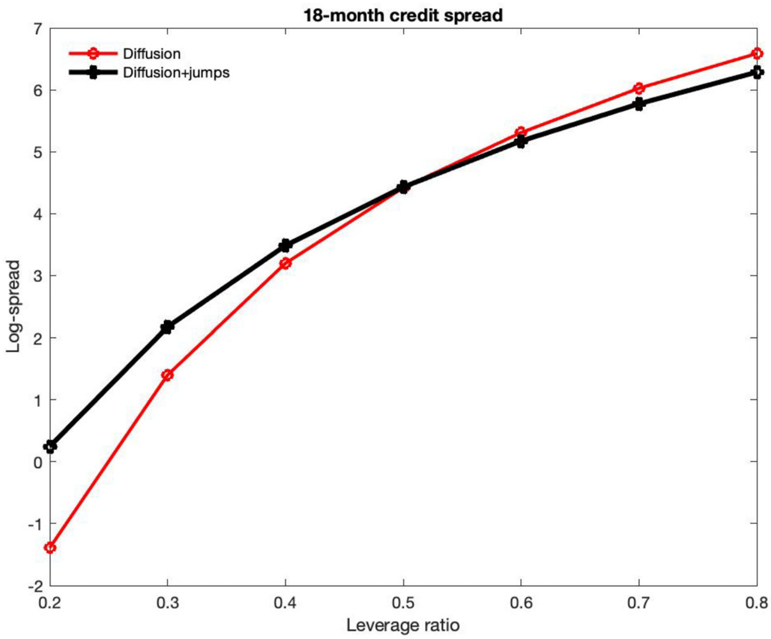

Fig. 12. *Credit spreads with jumps in debt.* In the main analysis log-debt in the SD model is given as $dk_t = \lambda(\nu^Q - l_t)dt + \sigma_k dW_{k,t}$, where l_t is log-leverage. The figure shows log-credit spreads for this benchmark model ('Diffusion') as well as the SD model where debt can also jump, $dk_t = \lambda(\nu^Q_{JD} - l_t)dt + \sigma_{k,JD} dW_{k,t} + dJ_t$, where J_t is a jump process with jump intensity η and jumps are normally distributed with mean ξ and standard deviation ζ ('Diffusion+jumps'). In both cases the dynamics of assets are given by $dv_t = (r - \delta - \frac{\sigma^2}{2})dt + \sigma dW_t$ where $r = 0.05$, $\delta = 0.05$, $\sigma = 0.24$, and the recovery rate is 33.38%. For debt dynamics the mean reversion parameter is $\lambda = 0.1814$ while $\sigma_k = 0.2706$ and $\nu^Q = -0.6593$ in the diffusion case and $\nu^Q = -2.0017$, $\sigma_k = 0.1353$, $\eta = 1$, $\xi = 0.2435$, and $\zeta = 0.0880$ in the jump-diffusion case. The default boundary is 0.7322 in the diffusion case and 0.675 in the jump-diffusion case.

ences in average leverage between high and low equity return firms as Table 2 shows. We therefore pair firms as above according to leverage (instead of on firm size) and the results in Fig. A3 ("Same initial leverage") show that the documented pattern in debt growth is also robust to controlling precisely for leverage.

Similarly, firms with high levels of cash have more debt capacity and lower expected returns and this might explain the negative debt-equity correlation. Fig. A3 ("Same initial cash level") shows that this is not the case and that the pattern in debt growth is robust to sorting on cash (as a percentage of firm value).

6.4. The riskfree rate

In the main analysis we use swap rates as riskfree rates. Traditionally, Treasury yields have been used as riskfree rates, but recent evidence shows that swap rates are a better proxy than Treasury yields. A major reason for this is that Treasury bonds enjoy a convenience yield that pushes their yields below riskfree rates (Feldhütter and Lando, 2008; Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016). The convenience yield is for example due to the ability to post Treasuries as collateral with a significantly lower haircut than other securities. Nevertheless, Treasury yields are used as the riskfree rate in some studies and we therefore also calculate actual bond yield spreads using Treasury yields as riskfree rates. Specifically, we replace swap rates with Treasury par yields from Gürkaynak et al. (2006) when we calculate actual spreads. Table A5 shows the average monthly pricing errors in this case. Pricing errors are substantially larger for all models when using the Treasury yield instead of the swap rate, consistent with the view that Treasury yields contain a convenience yield that is outside the scope of this paper. Most importantly, however, the relative ranking of the models does not change. In particular, the SD models capture short-term spreads better than the other models.

6.5. CDS spreads

There is substantial evidence that corporate bond prices are affected by bond market illiquidity and the size of the illiquidity component has been shown to be significant during the financial crisis 2008–2009 (see for example Bao et al., 2011; Dick-Nielsen et al., 2012; Feldhütter, 2012; Feldhütter and Schaefer, 2018; Huang et al., 2020a). The presence of an illiquidity component in spreads could distort our results. Longstaff et al. (2005) and others argue that CDS spreads are less affected by illiquidity and thus may be a better proxy for 'true' credit spreads than bond spreads. Accordingly, as a robustness check, we recompute estimates of the model pricing errors using CDS premiums.

Specifically, we replace bond yield spreads in our sample with CDS spreads whenever possible and discard the remaining bond yield spreads. We use daily USD CDS spreads from Markit for contracts on senior unsecured debt with the modified restructuring clause. These data are available from 2001. For each bond-month observation in our main bond sample, we find CDS spreads on the last day in the month for which there exists at least one CDS spread on the issuing firm. If available, we linearly interpolate between the two CDS spreads nearest in maturity to compute a synthetic CDS spread with the same maturity as the bond. If the bond maturity is lower (higher) than that of the available CDS with the lowest (highest) maturity, we use the CDS spread with the lowest (highest) maturity. The period covered in the final CDS sample runs from January 2001 (the beginning of the CDS sample) to March 2018 (the end of the bond sample). The CDS sample has 44,175 observations and we winsorize the CDS spreads at the 1% and 99% level. The mean (median) CDS spread is 135bps (56bps) while the mean (median) bond spread is 160bps (69bps) and the correlation between the CDS and bond spread is 84%.

Table A6 shows monthly pricing errors when using CDS spreads to measure the credit spread. Comparing these results to those in Table 7, that contains the corresponding results using bond spreads, we note that the results are similar to those for bonds in that the SD models have the smallest average errors (61–74bps) compared to 88–129bps for the other models and, as before, the improved fit is mainly due to a much better fit to short-term spreads.

6.6. Using market values of debt

In the main analysis we use the book value of debt as a proxy for the market value when calculating firm value. Since firms issue debt close to par this proxy works well in most cases. However, for firms that have recently gone into distress, the proxy may be less accurate, and for this reason we exclude C-rated firms when estimating the default boundary.

To further mitigate the concern that using book value of debt in our calculation of firm value affects our main results, we use actual market values of debt from Bretscher et al. (2021) (BFKS). Using a comprehensive data set on bond and loan valuations from secondary market transactions, BFKS calculate the market value of debt for a cross-section of firms for the period 1998–2018 and we use their market values to calculate the leverage ratio for firm i in month t as

$$\frac{D_{it}^{BV}}{D_{it}^{MV}+E_{it}^{MV}} \tag{32}$$

where D^{BV} is the book value of debt, D^{MV} is the market value of debt and E^{MV} is the market value of equity.[19] For every bond-month observation in our main sample, we replace the leverage ratio calculated using the book value of debt with the market leverage in Eq. (32) and discard the bond-month observation if market data from BFKS is not available. The data set in this sample has 66,438 bond-month observations compared to 101,059 in the main sample. The results are similar when using market values of debt as Table A7 shows:[20] the SD models have the lowest pricing errors (59–62bps compared to 83–101bps for the other models) and the models outperform particularly when pricing short-maturity bonds (57–77bps compared to 114–162bps for the other models).[21]

Table 10

Alternative estimates of debt-asset correlation. The first row (labelled 'Data') shows the 1-, 2-, and 3-year debt-equity correlations using firm-year observations based in firms that have available data in all 30 years of the data sample. The second row ('Benchmark model') shows model-implied correlations between changes in log-level of debt and log-returns of equity in the stochastic debt model, where firm value is given as $dv_t=(\mu-\delta-\frac{\sigma^2}{2})dt+\sigma dW_t$ and log-debt is given as $dk_t=\lambda(\nu-l_t)dt+\sigma_k dW_{k,t}$ where $\sigma=0.24$, $\mu=0.1028$, and $\delta=0.05$, $\lambda=0.1814$, $\nu=-1.0046$, $\sigma_k=0.2706$ and $\rho=-0.1868$ (where ρ is the correlation between W_t and $W_{k,t}$). The number in the second column is the instantaneous correlation while the next three columns show correlations for 1-, 2-, and 3-year changes. The third row shows the correlation ρ in the benchmark model that minimizes the sum of squared errors between model-implied and data 1-, 2-, and 3-year correlations (where empirical debt-equity correlations proxy for debt-asset correlations). The data is from CRSP/Compustat and the sample period is 1988–2017.

	$T=0$	$T=1$	$T=2$	$T=3$
Data		−0.034	−0.024	0.010
Benchmark model	−0.1868	−0.104	−0.016	0.075
$\hat{\rho}$ from data	−0.1878			

6.7. Alternative estimation of debt dynamics parameters

In the main analysis we estimate ρ, the correlation between debt and asset value, from the dynamics of debt and the ratio of leverage volatility and asset volatility. In this section we estimate the correlation directly from data on changes in debt and equity returns.

We focus on firms that have available data in all 30 years of the data sample and for each firm we calculate the correlation between log equity returns (taking into account dividends and stock splits) and log book debt changes at horizons of one, two, and three years. We use equity-debt correlations as a proxy for the correlations between asset values and book debt.[22] The first row 'Data' in Table 10 shows the correlations and we see that the 1- and 2-year correlations are (modestly) negative while the 3-year correlation is positive.

For comparison the second row 'Benchmark model' in the table shows the model-implied correlations as well as the instantaneous correlation ($T=0$). An analytical expression for the correlation over discrete intervals is derived in Eq. (58). We see that the benchmark model shows a similar pattern of correlations from negative at two years or

[19] We are grateful to Bretscher et al. (2021) for sharing the data with us and details of how the market value is calculated is given in their paper.

[20] We include C-rated firms when estimating the default boundary because there is no potential bias when using market values of debt and in the estimation we winsorize leverage ratios at $\frac{0.9}{k}$ where k is the default boundary (the highest leverage in the sample is 2.36 and if we did not winsorize the upper bound on the default boundary would be $1/2.36=0.42$).

[21] We have left out results on average spreads, volatility of spreads and default boundary estimates because they are similar to the those in the main section, but they are available on request.

[22] We do not estimate the correlation between changes in debt and changes in a firm's asset value because extreme observations due to corporate events bias the estimate upwards (for example, a merger between two identical firms doubles debt and asset values). The correlation between changes in debt and equity is not the same as the correlation between debt and a firm's asset value but, since the correlation is small in this case, we ignore this difference.

less to a positive three-year correlation. Furthermore, the instantaneous correlation is substantially lower (i.e., more negative) than the discrete-time correlations and therefore it is important to account for this difference when estimating the model correlation from data counterparts.

To estimate the instantaneous correlation from data correlations, we hold the parameters of the SD model fixed, except ρ, which we estimate by minimizing the sum of squared differences between the empirical and model-implied correlations at one, two and three years. The estimate is $\hat{\rho} = -0.1878$, shown in the last row of the table, which is virtually identical to the estimate obtained earlier.

7. Conclusion

We investigate how the dynamics of corporate debt policy affect the pricing of corporate bonds. We find empirically that debt issuance has a significant stochastic component that has a modest, and in fact often negative, correlation with shocks to the firm's asset value. As a consequence of both these features – the significant size of shocks to debt and their correlation with the firm's assets – the volatility of leverage is significantly higher than the volatility of asset returns over short horizons. At long horizons, the relation between leverage volatility and asset volatility is reversed due to mean reversion in leverage.

We incorporate these debt dynamics into the default boundary within both a standard structural model (where firm value follows a Geometric Brownian Motion and the Sharpe ratio is constant) as well as a newer structural model with stochastic asset volatility and jumps in asset value. In the context of structural models of credit risk, incorporating debt dynamics results in a stochastic default boundary and a volatility of leverage that is substantially higher at short horizons than asset volatility. Compared to existing diffusion models, the model provides more accurate pricing predictions in both the cross-section and time series and in terms of both absolute pricing errors as well as volatility of spread changes. The improvement relative to standard diffusion models comes mainly from more accurate pricing of short-maturity investment grade bonds. This is due to the model's predictions, relative to existing models, of higher spreads for firms with low credit risk and lower spreads for firms with high credit risk. Compared to a model with stochastic volatility and jumps but deterministic debt, including stochastic debt again leads to more accurate predictions of the term structure of credit spreads.

Our analysis combining stochastic debt with jumps and stochastic volatility is exploratory and limited to Walmart. Fixing the default boundary, then for a given maturity, and relative to a model with constant volatility and deterministic debt, both stochastic debt and the inclusion of jumps and stochastic volatility in asset dynamics result in higher leverage volatility and, therefore, higher spreads. However, across maturity the results are different: jumps and stochastic volatility increase credit spreads at all maturities while, as a result of mean reversion in leverage, our model of stochastic debt increases credit spreads at short maturities but decreases them at long maturities. Furthermore, jumps and stochastic volatility imply a one-to-one relation between asset volatility and leverage volatility, while with stochastic debt, this is no longer true. We leave it for future work to investigate in detail the relative contribution of jumps, stochastic volatility and stochastic debt.

Declaration of Competing Interest

None declared.

Data availability

Data will be made available on request.

Appendix A. Data

In our analysis we use firm variables (for example, leverage and equity volatility) along with corporate bond prices and individual bond information and we also calculate historical default rates. We focus on the US market and our main data sources are: firm variables from CRSP/Compustat, corporate bond quotes from the Lehman Brothers and Merrill Lynch databases, corporate bond transaction prices from the Trade Reporting and Compliance Engine (TRACE), bond information from the Mergent Fixed Income Securities Database and default data from Moody's Default and Recovery Database. The data sources are well known and used in a large number of studies. Below we provide a brief description of each.

A1. Firm variables

Firm variables are collected in the CRSP/Compustat Merged Database and computed as in Feldhütter and Schaefer (2018). For a given firm and year the *nominal amount of debt* is the debt in current liabilities (DLCQ) plus long-term debt (DLTTQ) in the fourth quarter of the year. We restrict our analysis to industrial firms and so exclude utilities (SIC codes 4900–4949) and financials (SIC codes 6000–6999). To be consistent with the corporate bond data set, we restrict the firm data we use to the period 1988–2017. The *leverage ratio* is calculated as (nominal amount of debt)/(market value of equity + nominal amount of debt) where the *market value of equity* is calculated as the number of shares outstanding (CSHOQ) times the closing share price in the quarter (PRCC). The number of firm-year observations with both the level of debt and market value of equity available is 131,971 and the number of firms is 14,503.

Equity volatility $\sigma_{E,t}$ is computed as $\sqrt{255}$ times the standard deviation of daily stock returns in the past three years. If there are no return observations on more than half the days in the three-year window, we do not calculate equity volatility. We follow Feldhütter and Schaefer (2018) and calculate *asset volatility* at time t as $(1-L_t)\sigma_{E,t}$ and multiply this by 1 if $L_t < 0.25$, 1.05 if $0.25 < L_t \le 0.35$, 1.10 if $0.35 < L_t \le 0.45$, 1.20 if $0.45 < L_t \le 0.55$, 1.40 if $0.55 < L_t \le 0.75$, and 1.80 if $L_t > 0.75$. For a given firm we then compute the average asset volatility over the sample period and use this constant asset volatility for every day in the sample period.

Table A1

Log ratio of future debt relative to current debt, model-fit. For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. 'Data' shows the average log-ratio for different initial leverage ratios and future horizons in the data. The table also shows fitted values from structural models. 'BC-0G' refers to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks.

Horizon (years)	1	2	3	4	5	6	7	8	9	10
Leverage 0–0.2										
Data	0.21	0.42	0.61	0.78	0.92	1.05	1.16	1.27	1.36	1.44
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.04	0.09	0.13	0.17	0.22	0.26	0.30	0.34	0.39	0.43
CDG	0.21	0.39	0.54	0.68	0.79	0.90	0.98	1.06	1.13	1.20
SD	0.22	0.40	0.56	0.70	0.81	0.92	1.01	1.08	1.15	1.21
Leverage 0.2–0.4										
Data	−0.01	0.01	0.04	0.07	0.12	0.17	0.21	0.26	0.30	0.37
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.04	0.09	0.13	0.17	0.22	0.26	0.30	0.34	0.39	0.43
CDG	0.03	0.07	0.10	0.13	0.16	0.19	0.21	0.24	0.27	0.29
SD	0.04	0.07	0.10	0.13	0.16	0.19	0.22	0.24	0.27	0.30
Leverage 0.4–0.6										
Data	−0.05	−0.08	−0.09	−0.08	−0.06	−0.02	0.02	0.04	0.08	0.12
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.04	0.09	0.13	0.17	0.22	0.26	0.30	0.34	0.39	0.43
CDG	−0.05	−0.08	−0.11	−0.13	−0.14	−0.14	−0.15	−0.14	−0.14	−0.13
SD	−0.05	−0.09	−0.11	−0.13	−0.14	−0.15	−0.15	−0.15	−0.14	−0.13
Leverage 0.6–0.8										
Data	−0.09	−0.15	−0.21	−0.22	−0.20	−0.20	−0.19	−0.14	−0.10	−0.06
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.04	0.09	0.13	0.17	0.22	0.26	0.30	0.34	0.39	0.43
CDG	−0.10	−0.18	−0.25	−0.30	−0.33	−0.36	−0.38	−0.40	−0.40	−0.41
SD	−0.11	−0.19	−0.26	−0.31	−0.35	−0.37	−0.39	−0.41	−0.41	−0.41
Leverage 0.8–1										
Data	−0.18	−0.32	−0.42	−0.45	−0.45	−0.42	−0.35	−0.33	−0.28	−0.29
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0.04	0.09	0.13	0.17	0.22	0.26	0.30	0.34	0.39	0.43
CDG	−0.14	−0.25	−0.35	−0.42	−0.48	−0.52	−0.56	−0.58	−0.60	−0.61
SD	−0.15	−0.27	−0.36	−0.44	−0.50	−0.54	−0.57	−0.60	−0.61	−0.62

The *payout rate* is the total outflow to stake holders divided by firm value. This is computed as the sum of the previous year's interest payments, dividend payments, and net stock repurchases divided by the sum of market value of equity and book value of debt. The payout ratio is winsorized at 0.13 as in Feldhütter and Schaefer (2018).

A2. Corporate bond yield spreads

We use several sources to arrive at our U.S. corporate bond data set for the period April 1988 to March 2018. For the period April 1988 to December 1996 we use monthly quote data from the Lehman Brothers Fixed Income Database and include only actual quotes. For the period January 1997 to June 2002, we use quotes provided by Merrill Lynch (ML) on all corporate bonds included in the ML investment grade and high-yield indices. For each bond-month we use the last quote in the month. Feldhütter and Schaefer (2018) show that there is a significant bid-bias in bond quotes for short-maturity bonds and we therefore follow Feldhütter and Schaefer (2018) and exclude ML and Lehman quotes for bonds with a maturity less than three years. For the period July 2002-June 2017 we use transactions data from Enhanced TRACE and for the period July 2017-March 2018 transactions data from standard TRACE. We filter transactions according to Dick-Nielsen (2009, 2014) and focus on transactions with a volume of $100,000 or more. When using TRACE, we calculate one yield observation for each bond-month by computing the median yield for the bond in the month.[23] When we match yield observations to firm variables, we use firm variables from the day the median is observed.

A3. Bond information

We obtain bond information from the Mergent Fixed Income Securities Database (FISD) and limit the sample to senior unsecured fixed rate or zero coupon bonds. We exclude bonds that are callable, convertible, putable, perpetual, foreign denominated, Yankee, or have sinking fund provisions.[24] We use only bonds issued by industrial firms

[23] If there are N observations in a month where N is even, we sort the observations increasingly and use the $N/2$'th observation.

[24] For bond rating, we use the lower of Moody's rating and S&P's rating. If only one of the two rating agencies have rated the bond, we use that

Table A2
Log ratio of future debt relative to current debt conditional on future equity returns, model-fit. For firm i, year t, and horizons 1,...,10 years, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median 'High (Low) future equity return' firms. The table shows the difference in log-ratio for high and low future equity return firms. The table also shows fitted values from structural models. 'BC-0G' refers to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. The sample period is 1988–2017.

Horizon (years)	1	2	3	4	5	6	7	8	9	10
Leverage 0–0.2										
Data	−0.10	−0.09	0.01	0.16	0.28	0.36	0.43	0.44	0.41	0.42
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.02	0.07	0.15	0.23	0.30	0.36	0.40	0.45	0.48	0.51
SD	−0.02	−0.01	0.04	0.15	0.23	0.30	0.36	0.41	0.45	0.49
Leverage 0.2–0.4										
Data	−0.04	−0.03	0.05	0.19	0.30	0.37	0.45	0.52	0.53	0.58
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.02	0.07	0.15	0.23	0.30	0.36	0.40	0.45	0.48	0.51
SD	−0.02	−0.01	0.04	0.15	0.23	0.30	0.36	0.41	0.45	0.49
Leverage 0.4–0.6										
Data	−0.02	−0.00	0.06	0.21	0.31	0.39	0.44	0.52	0.56	0.57
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.02	0.07	0.15	0.23	0.30	0.36	0.40	0.45	0.48	0.51
SD	−0.02	−0.01	0.04	0.15	0.23	0.30	0.36	0.41	0.45	0.49
Leverage 0.6–0.8										
Data	−0.01	0.01	0.07	0.13	0.26	0.33	0.42	0.38	0.36	0.37
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.02	0.07	0.15	0.23	0.30	0.36	0.40	0.45	0.48	0.51
SD	−0.02	−0.01	0.04	0.15	0.23	0.30	0.36	0.41	0.45	0.49
Leverage 0.8–1										
Data	0.02	0.04	0.06	0.22	0.31	0.43	0.50	0.58	0.71	0.74
BC-0G	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0
CDG	0.02	0.07	0.15	0.23	0.30	0.36	0.40	0.45	0.48	0.51
SD	−0.02	−0.01	0.04	0.15	0.23	0.30	0.36	0.41	0.45	0.49

and restrict our sample to bonds with a maturity of less than 20 years to be consistent with the maturities of the default rates we use as part of the estimation. After merging the bond and firm data, the number of bond-month observations is 119,765. We winsorize spreads at the 1% and 99% level.

A4. Riskfree rates

As in Feldhütter and Schaefer (2018); Bai et al. (2020), and others we calculate corporate bond yield spreads relative to the swap rate and, for a given date, use the available rates among the 1-week, 1-month, 2-month, and 3-month LIBOR and 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20-year swap rates, interpolating linearly to obtain a swap rate at the exact maturity of the bond. Before 1998 the longest swap maturity is 10 years and so, for longer maturities in the early period, we use the (interpolated) Treasury CMT rate plus the swap spread at the longest maturity for which a swap rate is available. LIBOR and swap rates are downloaded from Bloomberg.

A5. Default data

Data on defaults are from Moody's Analytics' Default and Recovery Database (DRD v2.0). In the period from 1919 to 2018, the database contains the rating history for 27,750 unique firms and 11,024 default events. There are four events that constitute a debt default: a missed interest or principal payment, a bankruptcy filing, a distressed exchange, and a change in the payment terms of a credit agreement or indenture that results in a diminished financial obligation. Soft defaults ('dividend omission' and 'BFSR default') appear in the database, but we follow Moody's and exclude these when calculating default rates. The database includes information on the (latest) company industry and domicile.

We set the recovery rate to 33.48% which is Moody's (2018a)'s average recovery rate, as measured

rating. We track rating changes on a bond, so the same bond can appear in several rating categories over time.

Table A3

Average cumulative default rates, 1970–2017. 'Moody's report' refers to default rates for 1970–2017 published in Moody's (2018). 'All firms, cohort-weight' is the calculated default rate when using Moody's methodology and their default database. 'US industrial firms, cohort-weight' is the calculated default rate when using Moody's methodology and restricting the sample to US industrial firms in their default database. 'US industrial firms, equal-weight' is the calculated default rate when using Moody's methodology and restricting the sample to US industrial firms in their default database, with one change applied to their methodology: default rates from different cohorts are weighted equally instead of weighted by the cohort size. Cohorts are formed on a monthly basis and 'obs' is the sum of all cohort sizes.

	Horizon (years)															Obs.
Horizon (years)	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20	
AAA																
Moody's report	0.00	0.01	0.01	0.03	0.08	0.14	0.19	0.25	0.31	0.38	0.52	0.64	0.73	0.80	0.80	
All firms, cohort-weight	0.00	0.01	0.01	0.02	0.06	0.10	0.14	0.18	0.22	0.27	0.37	0.46	0.52	0.57	0.57	85,708
US industrial firms, cohort-weight	0.00	0.00	0.00	0.07	0.24	0.41	0.59	0.77	0.95	1.14	1.55	1.86	2.10	2.26	2.26	17,019
US industrial firms, equal-weight	**0.00**	**0.00**	**0.00**	**0.06**	**0.16**	**0.29**	**0.42**	**0.51**	**0.62**	**0.72**	**0.96**	**1.17**	**1.34**	**1.46**	**1.46**	**17,019**
AA																
Moody's report	0.02	0.06	0.11	0.19	0.29	0.40	0.52	0.63	0.71	0.79	1.02	1.29	1.49	1.78	2.26	
All firms, cohort-weight	0.02	0.08	0.15	0.25	0.38	0.49	0.62	0.74	0.86	0.98	1.29	1.61	1.86	2.19	2.69	262,844
US industrial firms, cohort-weight	0.00	0.01	0.04	0.18	0.31	0.40	0.47	0.55	0.62	0.66	0.83	1.18	1.42	1.71	2.35	43,725
US industrial firms, equal-weight	**0.00**	**0.01**	**0.02**	**0.12**	**0.21**	**0.29**	**0.35**	**0.42**	**0.48**	**0.51**	**0.64**	**0.96**	**1.19**	**1.49**	**2.16**	**43,725**
A																
Moody's report	0.05	0.16	0.33	0.52	0.74	0.99	1.26	1.54	1.85	2.15	2.74	3.36	4.08	4.82	5.53	
All firms, cohort-weight	0.06	0.17	0.35	0.55	0.79	1.06	1.35	1.65	1.95	2.26	2.90	3.57	4.35	5.14	5.87	517,501
US industrial firms, cohort-weight	0.03	0.12	0.27	0.41	0.60	0.84	1.08	1.33	1.58	1.84	2.43	3.07	3.76	4.56	5.33	135,673
US industrial firms, equal-weight	**0.02**	**0.10**	**0.22**	**0.35**	**0.52**	**0.74**	**0.96**	**1.20**	**1.44**	**1.70**	**2.28**	**2.91**	**3.60**	**4.42**	**5.22**	**135,673**
BBB																
Moody's report	0.17	0.44	0.77	1.16	1.55	1.95	2.35	2.77	3.24	3.75	4.88	6.07	7.29	8.56	9.65	
All firms, cohort-weight	0.18	0.48	0.82	1.22	1.62	2.02	2.42	2.85	3.32	3.83	4.93	6.13	7.40	8.64	9.76	494,030
US industrial firms, cohort-weight	0.18	0.52	0.97	1.54	2.11	2.73	3.36	4.09	4.91	5.74	7.42	9.17	11.07	13.05	14.82	163,580
US industrial firms, equal-weight	**0.16**	**0.50**	**0.98**	**1.59**	**2.21**	**2.91**	**3.66**	**4.53**	**5.49**	**6.43**	**8.26**	**10.09**	**11.95**	**13.99**	**15.83**	**163,580**
BB																
Moody's report	0.92	2.52	4.38	6.36	8.20	9.90	11.39	12.85	14.34	15.88	18.81	21.54	24.31	26.65	28.71	
All firms, cohort-weight	0.92	2.43	4.17	6.05	7.79	9.39	10.85	12.26	13.70	15.16	17.91	20.44	23.06	25.19	27.07	282,165
US industrial firms, cohort-weight	1.11	2.99	5.12	7.37	9.52	11.63	13.59	15.50	17.44	19.42	23.49	27.56	31.93	35.60	38.50	135,738
US industrial firms, equal-weight	**1.05**	**2.80**	**4.83**	**7.07**	**9.37**	**11.79**	**14.12**	**16.39**	**18.64**	**20.85**	**25.79**	**30.70**	**35.79**	**39.90**	**42.80**	**135,738**
B																
Moody's report	3.45	8.15	12.96	17.32	21.31	24.92	28.17	30.92	33.42	35.51	38.76	41.71	44.52	46.87	48.71	
All firms, cohort-weight	3.50	8.13	12.83	17.06	20.94	24.42	27.57	30.23	32.59	34.64	37.78	40.64	43.45	45.87	47.69	333,112
US industrial firms, cohort-weight	3.81	8.87	14.04	18.65	22.96	26.85	30.38	33.46	36.22	38.70	42.79	46.76	50.61	53.89	56.93	203,217
US industrial firms, equal-weight	**4.43**	**9.15**	**13.86**	**18.08**	**22.33**	**26.28**	**30.00**	**33.40**	**36.58**	**40.53**	**45.68**	**50.45**	**55.06**	**57.16**	**58.81**	**20,3217**
C																
Moody's report	10.22	18.04	24.64	30.17	34.67	38.09	41.12	44.04	46.76	48.90	51.35	51.84	52.43	52.53	52.53	
All firms, cohort-weight	9.09	16.03	22.04	26.97	31.01	34.16	36.83	39.24	41.33	42.82	45.11	46.11	47.34	47.97	49.34	203,057
US industrial firms, cohort-weight	8.96	16.19	22.60	27.97	32.55	36.04	39.08	42.44	46.23	49.34	54.24	56.39	59.27	61.68	64.32	133,834
US industrial firms, equal-weight	**8.39**	**14.78**	**20.70**	**26.06**	**31.27**	**35.07**	**38.09**	**41.36**	**44.96**	**48.18**	**56.51**	**61.50**	**65.04**	**66.36**	**67.76**	**133,834**

by post-default trading prices, for senior unsecured bonds for the period 1983–2017.

Appendix B. Analytical results

All models assume that firm-value dynamics follows a geometric Brownian Motion (see Eq. (4)):

$$\frac{dV_t}{V_t} = (\mu - \delta)dt + \sigma dW_t, \tag{33}$$

where μ is the expected return on the firm's assets, δ is the payout ratio, and σ is the asset return volatility. The dynamics for the log of firm value, $v_t = \log V_t$, is

$$dv_t = \left(\mu - \delta - \frac{\sigma^2}{2}\right)dt + \sigma dW_t. \tag{34}$$

B1. Default probabilities

In the Black–Cox model the debt level is given as $K(t) = K_0 e^{\gamma t}$ and the cumulative default probability at time t is (see Bao, 2009)

$$\pi^P(dL_0, \Theta^P, t) = N\left[-\left(\frac{(-\log(dL_0) + a_0 t}{\sigma\sqrt{t}}\right)\right] + \exp\left(\frac{2\log(dL_0)a_0}{\sigma^2}\right)N\left[\frac{\log(dL_0) + a_0 t}{\sigma\sqrt{t}}\right]$$
$$a_0 = \mu - \delta - \gamma - \frac{\sigma^2}{2} \tag{35}$$

where $L_0 = \frac{K_0}{V_0}$ is the current leverage and $\Theta^P = (\mu, \sigma, \delta, \gamma)$. The risk-neutral default probability, π^Q, is obtained by replacing μ with r in Eq. (35). The default probability in the constant boundary model is given by setting $\gamma = 0$ in (35).

In the stochastic debt model the dynamics of the log-debt level, k_t, is (see Eq. (10))

$$dk_t = \lambda(\nu - l_t)dt + \sigma_k dW_{k,t} \tag{36}$$

Table A4

Average cumulative default rates of US industrials vs. the rest, 1970–2017. 'US industrial firms' is the calculated default rate when using Moody's methodology and restricting the sample to US industrial firms in their default database, with one change applied to their methodology: default rates from different cohorts are weighted equally instead of weighted by the cohort size. 'Firms that are not US industrials' refers to the calculated default rate using all observations in the Moody's default database that are not industrial firms from the US. Cohorts are formed on a monthly basis and 'obs' is the sum of all cohort sizes. '*', '**', and '***' show when the difference is outside the 95%, 99%, and 99.9% confidence band, respectively, where the confidence bands are calculated according to Appendix C.3.

	Horizon (years)													Obs.
	1	2	3	4	5	6	8	10	12	14	16	18	20	
AAA														
US industrial firms	0.00	0.00	0.00	0.06	0.16	0.29	0.51	0.72	0.96	1.17	1.34	1.46	1.46	16,344
Firms that are not US industrials	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	65,892
Difference	0.00	−0.00	−0.00	0.05	0.15	0.29	0.51*	0.72*	0.95*	1.16	1.33	1.46	1.46	
AA														
US industrial firms	0.00	0.01	0.02	0.12	0.21	0.29	0.42	0.51	0.64	0.96	1.19	1.49	2.16	42,162
Firms that are not US industrials	0.02	0.06	0.12	0.20	0.32	0.47	0.84	1.08	1.37	1.65	1.82	2.10	2.37	207,636
Difference	−0.02	−0.05	−0.10	−0.08	−0.11	−0.18	−0.42	−0.57	−0.72	−0.69	−0.64	−0.60	−0.21	
A														
US industrial firms	0.02	0.10	0.22	0.35	0.52	0.74	1.20	1.70	2.28	2.91	3.60	4.42	5.22	128,700
Firms that are not US industrials	0.04	0.14	0.31	0.50	0.69	0.90	1.38	1.92	2.43	2.92	3.61	4.38	5.13	355,512
Difference	−0.02	−0.04	−0.09	−0.15	−0.17	−0.16	−0.18	−0.22	−0.15	−0.02	−0.01	0.05	0.10	
BBB														
US industrial firms	0.16	0.50	0.98	1.59	2.21	2.91	4.53	6.43	8.26	10.09	11.95	13.99	15.83	151,602
Firms that are not US industrials	0.16	0.41	0.71	1.06	1.41	1.69	2.14	2.61	3.30	4.21	5.09	5.87	6.55	306,102
Difference	0.00	0.08	0.27	0.52	0.80	1.22*	2.38**	3.82**	4.96**	5.88***	6.85***	8.12***	9.27***	
BB														
US industrial firms	1.05	2.80	4.83	7.07	9.37	11.79	16.39	20.85	25.79	30.70	35.79	39.90	42.80	121,470
Firms that are not US industrials	0.93	2.26	3.81	5.44	6.88	8.01	9.98	11.87	13.45	14.60	15.74	16.80	18.58	131,898
Difference	0.11	0.54	1.02	1.63	2.49	3.77*	6.41**	8.98***	12.33***	16.10***	20.05***	23.10***	24.22***	
B														
US industrial firms	4.43	9.15	13.86	18.08	22.33	26.28	33.40	40.53	45.68	50.45	55.06	57.16	58.81	174,288
Firms that are not US industrials	4.22	8.75	12.91	16.46	19.63	22.57	26.88	30.34	32.08	34.69	37.51	39.62	40.68	109,848
Difference	0.21	0.40	0.95	1.62	2.70	3.70	6.52**	10.19***	13.59***	15.76***	17.55***	17.54***	18.12**	
C														
US industrial firms	8.39	14.78	20.70	26.06	31.27	35.07	41.36	48.18	56.51	61.50	65.04	66.36	67.76	98,364
Firms that are not US industrials	7.72	12.46	16.77	20.61	23.71	26.95	30.95	31.81	32.47	32.80	33.10	33.11	33.24	50,310
Difference	0.67	2.33	3.92	5.45	7.56	8.13	10.41	16.36	24.05**	28.70**	31.94**	33.25*	34.52**	

If we assume that the bond Sharpe ratio is θ, then l_t follows the following risk-neutral dynamics[25]

$$dl_t = \lambda(\bar{l}^Q - l_t)dt + \sigma_k dW_{k,t} - \sigma dW_t^Q, \quad (37)$$

where $\bar{l}^Q = \nu - \frac{r+(\sigma-\sigma_l)\theta-\delta-\frac{\sigma^2}{2}}{\lambda}$. It is convenient to write the dynamics as

$$dl_t = \lambda(\bar{l}^Q - l_t)dt + \sigma_l dW_{l,t} \quad (38)$$

where $\sigma_l = \sqrt{\sigma_k^2 + \sigma^2 - 2\rho\sigma_k\sigma}$. Since the default time is $\tau = \inf\{t|0 \le l_t + \log(d)\}$, we define $\tilde{l}_t = l_t + log(d)$ which has the dynamics

$$d\tilde{l}_t = \lambda(\bar{\bar{l}}^Q - \tilde{l}_t)dt + \sigma_l dW_{l,t}, \quad (39)$$

where $\bar{\bar{l}}^Q = \bar{l}^Q + \log(d)$. The default time is the first time $\tilde{l}$ hits 0.

Log-leverage in Eq. ((39)) follows an Ornstein–Uhlenbeck (OU) process and the default time is therefore the first hitting time (FHT) of an OU process. There are no closed-form solutions for the distribution of the FHT of an OU process in the general case and we use numerical methods.

B2. Expected value of debt level

The expected debt level in the Merton model is constant while it is deterministically increasing in the Black–Cox model.

To calculate the expected debt level in the stochastic debt model, we can write the dynamics for leverage under the natural measure as:

$$dl_t = \lambda(\bar{l}^P - l_t)dt + \sigma_l dW_{l,t} \quad (40)$$

where W^P is the Brownian motion driving firm value, W_i is independent of W^P, and $\bar{l}^P = \nu - \frac{\mu-\delta-\frac{\sigma^2}{2}}{\lambda}$.

We have that both ν_t and l_t are normally distributed,

$$\nu_t = \nu_0 + \left(\mu - \delta - \frac{\sigma^2}{2}\right)t + \sigma W_{\nu,t} \quad (41)$$

$$l_t = \bar{l}^P + e^{-\lambda t}(l_0 - \bar{l}^P) + (\sigma_k\rho - \sigma)e^{-\lambda t}\int_0^t e^{\lambda s} dW_{\nu,s} + \sigma_k\sqrt{1-\rho^2}e^{-\lambda t}\int_0^t e^{\lambda s} dW_{i,s} \quad (42)$$

[25] It is straightforward to show that the dynamics in the SD model of l_t under P and Q is the same as the dynamics in the CDG model with σ replaced by σ_l and ν replaced by ν^* where $\nu^* = \nu + \frac{\sigma_V^2}{2\lambda} - \frac{\sigma_l^2}{2\lambda} + \frac{\sigma_l\theta^V}{\lambda} - \frac{\sigma_V\theta^V}{\lambda}$. Assuming a bond Sharpe ratio of θ in the CDG model then gives rise to the risk-neutral dynamics in Eq. (37). Alternatively, one can assume that the CAPM holds and derive the result using the Feynman–Kac theorem.

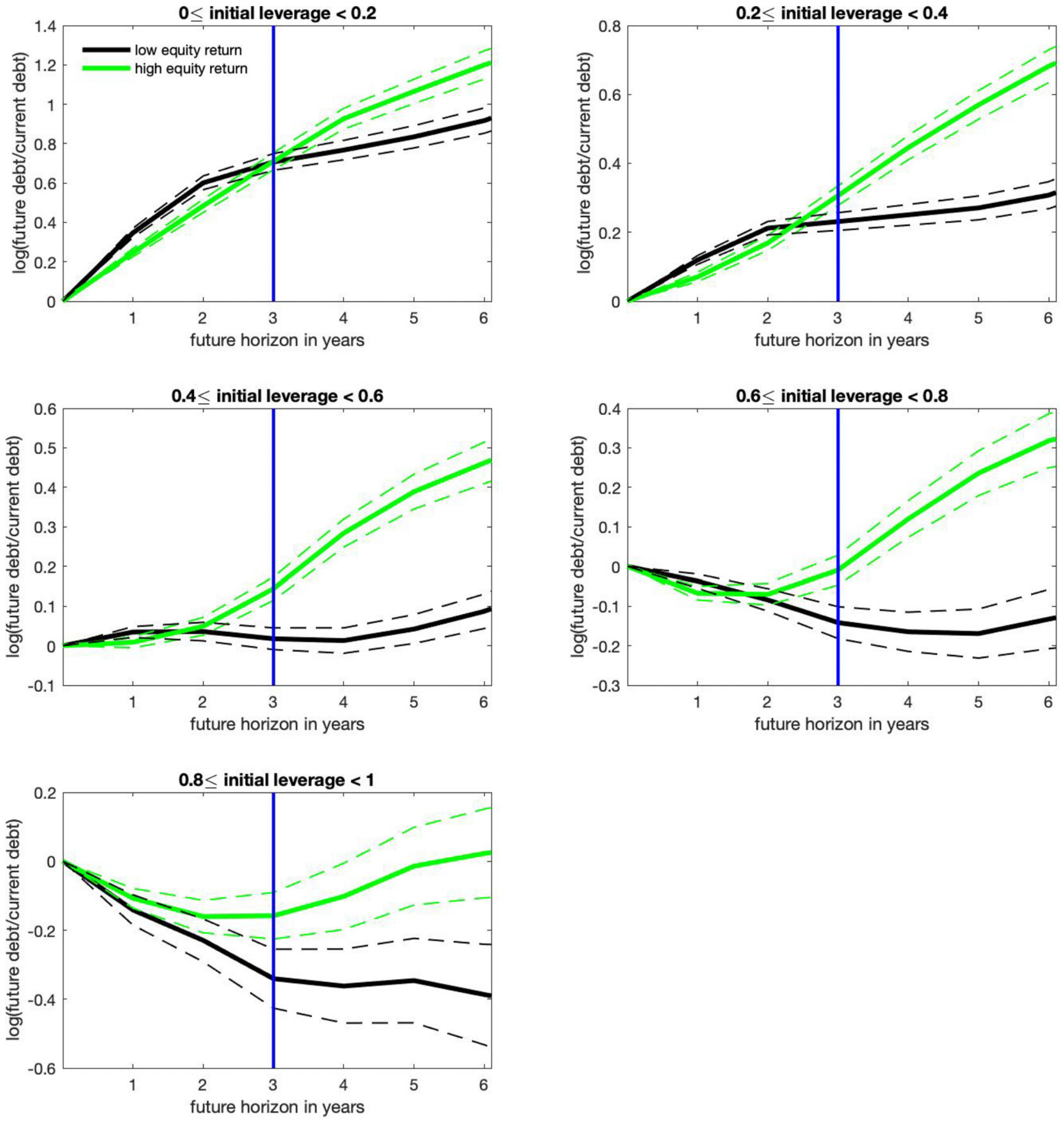

Fig. A1. *Future debt growth conditional on future three-year equity returns for different leverage ratios 1965–1987.* For firm i, year t, and horizons 1,...,20, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median 'High (Low) future equity return' firms. The figure shows the average log-ratio for high and low future equity return firms. The dashed lines mark 95% confidence levels based on standard errors clustered at the firm level. The data is from CRSP/Compustat and the sample period is 1965–1987.

This immediately gives

$$E(k_t) = \nu_0 + \left(\mu - \delta - \frac{\sigma^2}{2}\right)t + \bar{l}^P + e^{-\lambda t}\left(l_0 - \bar{l}^P\right). \tag{43}$$

The covariance is

$$Cov(l_t, \nu_u) = Cov\Bigg((\sigma_k\rho - \sigma)e^{-\lambda t}\int_0^t e^{\lambda s}dW_{\nu,s} + \sigma_k\sqrt{1-\rho^2}e^{-\lambda t}\int_0^t e^{\lambda s}dW_{i,s}, \sigma W_{\nu,u}\Bigg) \tag{44}$$

$$= (\sigma_k\rho - \sigma)e^{-\lambda t}\sigma E\left(\int_0^t e^{\lambda s}dW_{\nu,s} \times W_{\nu,u}\right) \tag{45}$$

$$= (\sigma_k\rho - \sigma)e^{-\lambda t}\sigma E\left(\int_0^t e^{\lambda s}dW_{\nu,s} \times \int_0^u dW_{\nu,s}\right) \tag{46}$$

$$= (\sigma_k\rho - \sigma)e^{-\lambda t}\sigma E\left(\int_0^{\min(t,u)} e^{\lambda s}ds\right) \tag{47}$$

$$= (\sigma_k\rho - \sigma)e^{-\lambda t}\sigma \frac{1}{\lambda}\left(e^{\lambda \min(t,u)} - 1\right) \tag{48}$$

where the Itô isometry is used. Their correlation is

$$\rho_{l_t,\nu_u} = Corr(l_t, \nu_u) = \frac{(\sigma_k\rho - \sigma)e^{-\lambda t}\sigma\frac{1}{\lambda}(e^{\lambda\min(t,u)} - 1)}{\sqrt{\sigma^2 u}\sigma_{l_t}} \tag{49}$$

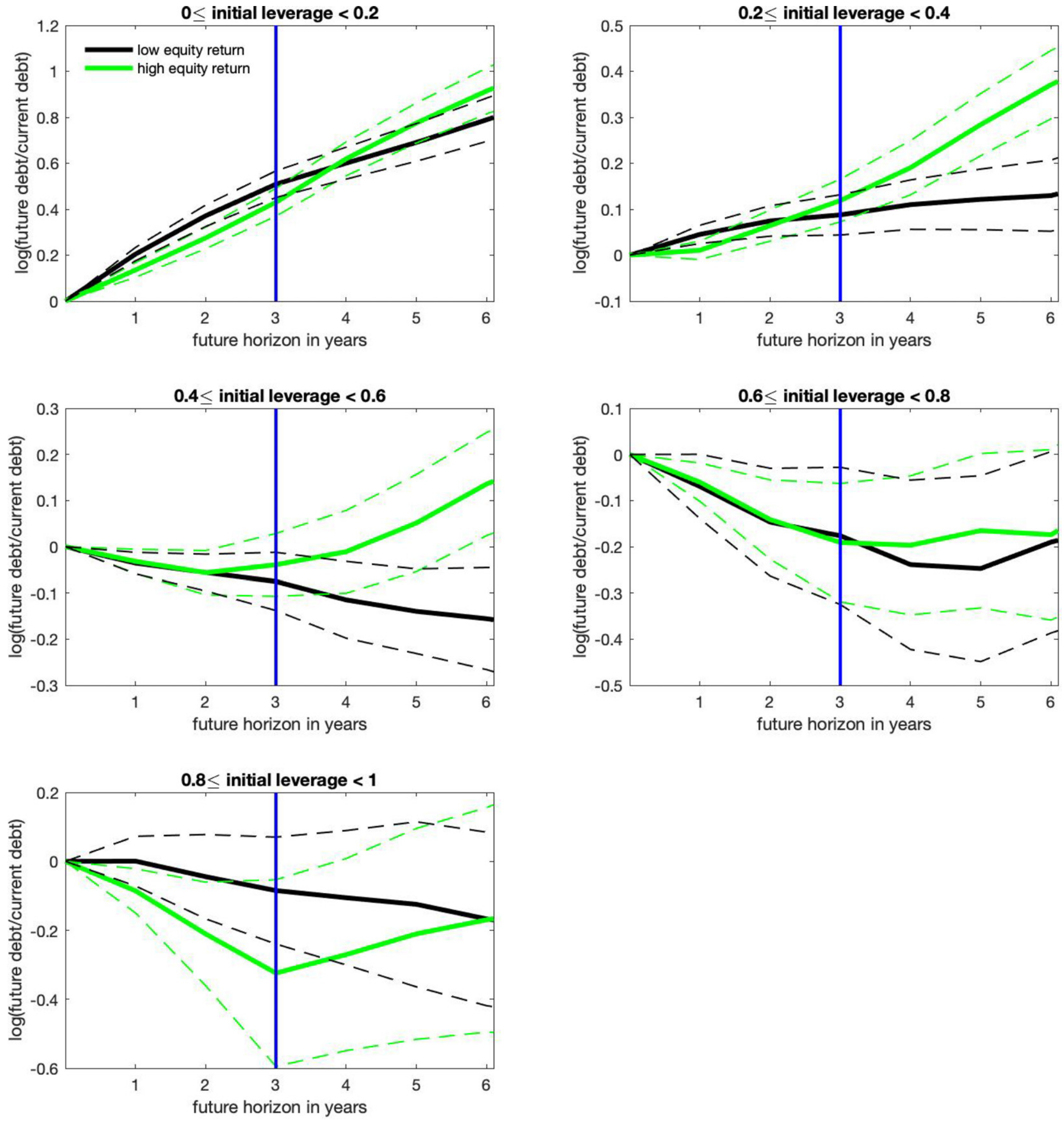

Fig. A2. *Log ratio of future debt relative to current debt conditional on future equity returns for surviving firms.* In this figure we restrict the sample to 238 firms that have data in CRSP/Compustat every year in the sample period 1988–2017 and have a leverage of at least 0.01 in all years. For firm i, year t, and horizons 1,...,10, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. For each firm-year in the sample where the initial leverage ratio at time t of the firm is in a certain interval, we calculate the future three-year equity return between t and $t+3$ and label firms with a return higher (lower) than the (within this leverage group) median 'High (Low) future equity return' firms. The figure shows the average log-ratio for high and low future equity return firms. The dashed lines mark 95% confidence levels based on standard errors clustered at the firm level. The data is from CRSP/Compustat and the sample period is 1988–2017.

$$= \frac{(\sigma_k\rho - \sigma)e^{-\lambda t}(e^{\lambda \min(t,u)} - 1)}{\lambda \sigma_{l_t}\sqrt{u}} \qquad (50)$$

where we have that $\sigma_{l_t} = \sigma_l\sqrt{\frac{1-e^{-2\lambda t}}{2\lambda}}$ which is well-known from the properties of the Ornstein–Uhlenbeck process.

According to Azzalini and Valle (1996, p. 716–717) we have that $\frac{l_t - E_0(l_t)}{\sigma_{l_t}}$ given $\frac{\nu_u - E_0(\nu_u)}{\sigma_{\nu_u}} > 0$ is skew-normal distributed with mean $\sqrt{\frac{2}{\pi}}\rho_{l_t,\nu_u}$ and therefore

$$E[l_t|\nu_u > E_0(\nu_u)] = \sqrt{\frac{2}{\pi}}\frac{(\sigma_k\rho - \sigma)e^{-\lambda t}(e^{\lambda \min(t,u)} - 1)}{\lambda\sqrt{u}}$$

$$+ \bar{l}^P + e^{-\lambda t}(l_0 - \bar{l}^P). \qquad (51)$$

Now

$$E[\nu_t|\nu_u > E_0(\nu_u)] = \nu_0 + \left(\mu - \delta - \frac{\sigma^2}{2}\right)t + \sigma E[W_{\nu,t}|W_{\nu,u} > 0] \qquad (52)$$

and using the results in Azzalini and Valle (1996) we have $E[W_{\nu,t}|W_{\nu,u} > 0] = \sqrt{\frac{2}{u\pi}}\min(t,u)$ so

$$E[k_t|\nu_u > E_0(\nu_u)] = \sqrt{\frac{2}{\pi}}\frac{(\sigma_k\rho - \sigma)e^{-\lambda t}(e^{\lambda \min(t,u)} - 1)}{\lambda\sqrt{u}}$$

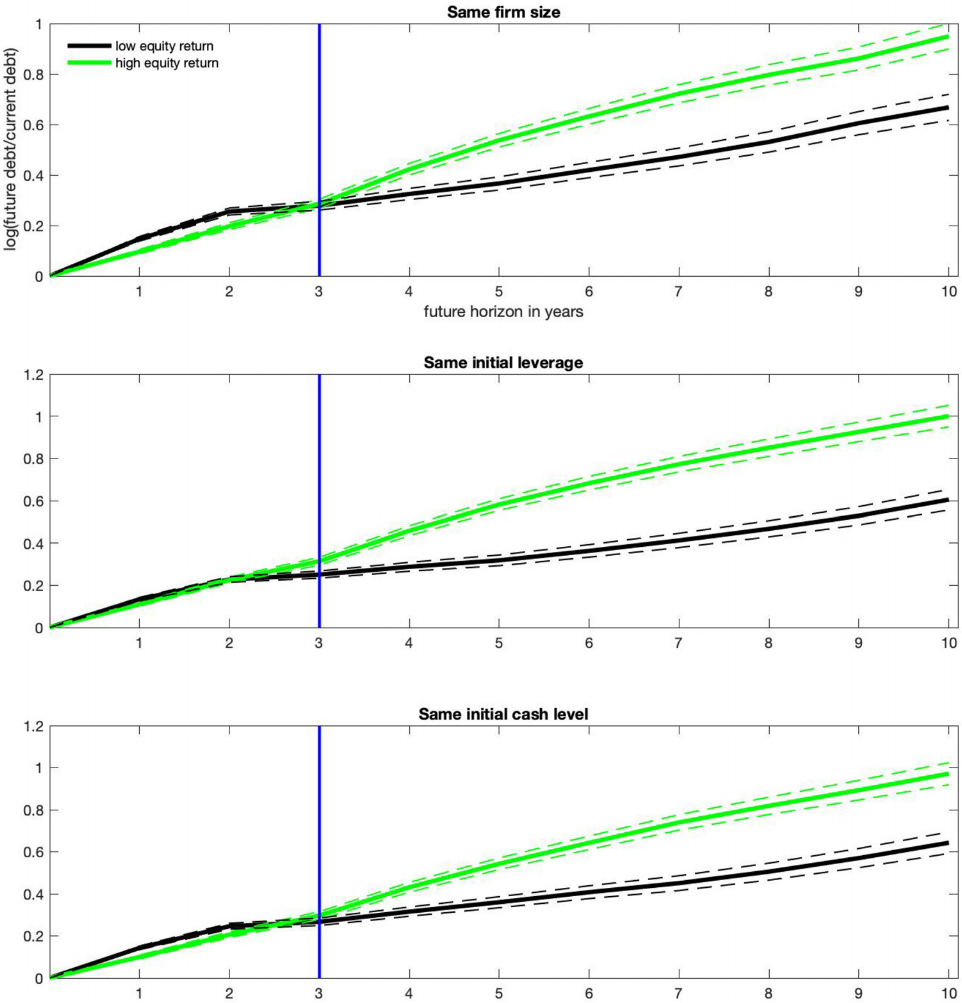

Fig. A3. *Log ratio of future debt relative to current debt conditional on future equity returns for firms with same leverage, firm size, and cash holdings.* For each year t we use the subsample of firms for which data exists in year t and $t+3$. We sort these firms according to their firm size (market value of equity + nominal value of debt) and pair the two largest firms, the third- and fourth-largest, etc. For each pair, we denote the firm with the higher (lower) equity return between t and $t+3$ a 'high equity return' ('low equity return') firm in year t. For firm i, year t, and horizons 1,...,10, we calculate $\log\left(\frac{D_{i,t+T}}{D_{i,t}}\right)$ where $D_{i,t}$ is the nominal level of debt for firm i in year t and T is the horizon in years. The top figure show the average log-ratio for high and low future equity return firms for horizons 1–10 years. In the middle figure firms are sorted into pairs according to their leverage. In the lower figure firms are sorted according to their cash levels, measured as cash divided by firm value. The dashed lines mark 95% confidence levels based on standard errors clustered at the firm level. The data is from CRSP/Compustat and the sample period is 1988–2017.

$$+\bar{l}^P + e^{-\lambda t}(l_0 - \bar{l}^P) + \nu_0 + \left(\mu - \delta - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{\frac{2}{u\pi}}\min(t,u). \tag{53}$$

Likewise

$$E[k_t|\nu_u < E_0(\nu_u)] = -\sqrt{\frac{2}{\pi}}\frac{(\sigma_k\rho - \sigma)e^{-\lambda t}(e^{\lambda\min(t,u)} - 1)}{\lambda\sqrt{u}} + \bar{l}^P + e^{-\lambda t}(l_0 - \bar{l}^P) + \nu_0 + \left(\mu - \delta - \frac{\sigma^2}{2}\right)t - \sigma\sqrt{\frac{2}{u\pi}}\min(t,u). \tag{54}$$

so

$$E[k_t|\nu_u > E_0(\nu_u)] - E[k_t|\nu_u < E_0(\nu_u)] = \sqrt{\frac{8}{\pi u}}\left[\frac{(\sigma_k\rho - \sigma)e^{-\lambda t}(e^{\lambda\min(t,u)} - 1)}{\lambda} + \sigma\min(t,u)\right]. \tag{55}$$

Since ν_u is normally distributed, its mean and the median are the same and so the expected value of k_t in the equation above is unchanged if we condition on ν_u being greater than the median instead of the mean. Also, since there is a monotone relation between the value of equity and the value of the firm, the expected value of k_t is also

Table A5

Pricing errors of monthly credit spreads when using Treasury yields as risk-free rates. For a given rating r and maturity m, we find all bonds at the end of a given month t that have this rating and maturity, calculate the average actual credit spread (in basis points) to the Treasury yield, s^a_{rmt}, and do this for all months in the sample. For each model, we likewise calculate a time series of the monthly average model credit spread (in basis points) $s^M_{rm1}, \dots, s^M_{rmT}$. The table shows the average absolute pricing error $1/T\sum_{t=1}^T \left|s^a_{rmt} - s^M_{rmt}\right|$. 'Short' includes bond maturities in the range 0–3 years, 'Medium' 3–10 years, and 'Long' 10–20 years. 'BC-0G' refers to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. 'Inv' includes bonds rated AAA, AA, A, and BBB, while 'Spec' includes bonds rated BB, B, and C. The sample period for 'Short' is 2002:07-2018:03 while it is 1988:03-2018:03 for 'Medium' and 'Long'.

		Average	Short	Medium	Long
All	BC-0G	117	229	59	62
	BC	117	234	55	61
	CDG	170	285	95	129
	CDG-FL	145	240	72	123
	SD	102	187	61	58
	SD-FL	105	193	57	66
Inv	BC-0G	74	148	37	36
	BC	81	145	33	63
	CDG	111	195	65	73
	CDG-FL	100	156	54	90
	SD	69	120	40	46
	SD-FL	66	127	35	37
Spec	BC-0G	337	532	254	225
	BC	333	558	255	185
	CDG	430	623	325	343
	CDG-FL	358	557	261	256
	SD	328	461	251	273
	SD-FL	315	466	257	222

Table A6

Pricing errors of monthly CDS spreads. For a given rating r and maturity m, we find all CDS spreads in a given month t that have this rating and maturity, calculate the average actual CDS spread (in basis points), s^a_{rmt}, and do this for all months in the sample. For each model, we likewise calculate a time series of the monthly average model credit spread (in basis points) $s^M_{rm1}, \dots, s^M_{rmT}$. This table shows the average absolute pricing error $1/T\sum_{t=1}^T \left|s^a_{rmt} - s^M_{rmt}\right|$. 'Short' includes CDS maturities in the range 0–3 years, 'Medium' 3–10 years, and 'Long' 10–20 years. 'BC-0G' refers to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. 'Inv' includes CDS observations of firm with rating AAA, AA, A, and BBB, while 'Spec' includes CDS observations of firms with rating BB, B, and C. The sample period for 'Short' is 2002:07-2018:03 while it is 2001:01-2018:03 for 'Medium' and 'Long'.

		Average	Short	Medium	Long
All	BC-0G	91	151	55	67
	BC	92	85	59	132
	CDG	129	277	48	63
	CDG-FL	88	168	47	50
	SD	74	72	52	98
	SD-FL	61	65	57	62
Inv	BC-0G	76	113	33	82
	BC	85	65	30	161
	CDG	97	193	42	55
	CDG-FL	62	124	38	25
	SD	73	56	32	131
	SD-FL	46	55	27	56
Spec	BC-0G	192	258	171	147
	BC	185	197	184	173
	CDG	252	391	128	236
	CDG-FL	206	289	157	170
	SD	172	174	145	199
	SD-FL	187	180	199	181

unchanged if we condition on the equity value – rather than the firm value – being greater than the median.

The same formulas hold for the stationary leverage model with $\sigma_k = \rho = 0$.

B3. Debt-firm value correlation at discrete intervals

Since $k_t = \ell_t + v_t$, it follows that $cov(k_t, v_t) = cov(\ell_t, v_t) + var(v_t)$, where (implicitly) covariance and variance are conditional on the value of the variables at time zero, i.e., computed using a differencing interval of t. So, using Eq. (44), we have:

$$cov(k_t, v_t) = (\sigma_k\rho - \sigma)\sigma h(t) + var(v_t) \tag{56}$$

where $h(t) = \frac{1}{\lambda}(1 - e^{-\lambda t})$. Since

$$var(k_t) = \sigma_k^2 \frac{1}{2\lambda}(1 - e^{-2\lambda t}) \equiv \sigma_k^2 g(t) \tag{57}$$

and $var(v_t) = \sigma^2 t$ we have

$$corr(k_t, v_t) = \left(\frac{1}{\sqrt{t}\sqrt{g(t)}}\right)\left[\left(\rho - \frac{\sigma}{\sigma_k}\right)h(t) + \frac{\sigma}{\sigma_k}t\right] \tag{58}$$

B4. Sharpe ratios

In the Black–Cox and stationary leverage models there is one source of risk – the Brownian motion driving asset value in equation (4) – and so the equity, bond, and asset Sharpe ratios are identical. In the stochastic debt model there are two sources of risk: a Brownian motion driving asset value in Eq. (4) and a Brownian motion driving debt changes in (10). This implies that the equity, asset, and debt Sharpe ratios are not necessarily the same even when assuming the CAPM holds. To see this, we assume that the value of the market follows

$$\frac{dM_t}{M_t} = \mu_M dt + \sigma_M dW_{M,t}. \tag{59}$$

Note that in the stochastic debt model the bond price is locally perfectly negatively correlated with log-leverage and the correlation between log-leverage and the return of the market, $\rho_{l,M}$, is

$$\rho_{l,M} = \frac{cov(dl_t, \frac{dM_t}{M_t})}{\sigma_l\sigma_M} = \frac{cov(\sigma_k dW_{k,t} - \sigma dW_t^P, \sigma_M dW_{M,t})}{\sigma_l\sigma_M} = \frac{\sigma_k}{\sigma_l}\rho_{k,M} - \frac{\sigma}{\sigma_l}\rho_{V,M}. \tag{60}$$

Table A7
Pricing errors of monthly credit spreads using market values. This table shows how well structural models match average monthly credit spreads. For a given rating r and maturity m, we find all bonds at the end of a given month t that have this rating and maturity, calculate the average actual credit spread (in basis points) to the swap rate, s^a_{rmt}, and do this for all months in the sample. For each model, we likewise calculate a time series of monthly average model credit spread (in basis points) $s^M_{rm1},\dots,s^M_{rmT}$. This table shows the average absolute pricing error $1/T\sum_{t=1}^T\left|s^a_{rmt}-s^M_{rmt}\right|$. 'Short' includes bond maturities in the range 0–3 years, 'Medium' 3–10 years, and 'Long' 10–20 years. 'BC-0G' refers to the Black–Cox model with zero growth in debt. 'BC' refers to the Black–Cox model. 'CDG' refers to the Collin-Dufresne and Goldstein (2001) model where all firms have a common long-run target leverage. 'SD' refers to a model where the firm adjusts the level of debt such that a long-run leverage, common to all firms, is targeted, and the level of debt is subject to random shocks. 'FL' refers to models where the long-run target leverage is firm specific and calculated as the historical average firm leverage. 'Inv' includes bonds rated AAA, AA, A, and BBB, while 'Spec' includes bonds rated BB, B, and C. The sample period for 'Short' is 2002:07-2018:03 while it is 2000:01-2018:03 for 'Medium' and 'Long'.

		Average	Short	Medium	Long
All	BC-0G	83	151	36	62
	BC	101	162	68	72
	CDG	95	131	49	104
	CDG-FL	99	114	59	123
	SD	59	77	46	52
	SD-FL	62	57	45	83
Inv	BC-0G	67	131	36	33
	BC	96	140	55	93
	CDG	60	91	41	49
	CDG-FL	79	88	50	100
	SD	58	73	47	53
	SD-FL	51	57	37	58
Spec	BC-0G	190	238	146	185
	BC	184	252	189	111
	CDG	242	270	172	285
	CDG-FL	212	249	168	219
	SD	171	145	141	227
	SD-FL	154	139	148	176

We can write the dynamics of debt as

$$dk_t=\lambda(\nu-l_t)dt+\sigma_k\left(\rho dW_t^P+\sqrt{1-\rho^2}dW_{2,t}\right). \tag{61}$$

where $dW_{2,t}$ is the innovation to k that is orthogonal to dW_t^P and it follows that

$$\rho_{k,M}=\rho\times\rho_{V,M}+\sqrt{1-\rho^2}\times\rho_{W_2,M}. \tag{62}$$

Note that since $|\rho_{k,M}|\le 1$, we have that

$$-\left(\frac{1+\rho\times\rho_{V,M}}{\sqrt{1-\rho^2}}\right)\le\rho_{W_2,M}\le\left(\frac{1-\rho\times\rho_{V,M}}{\sqrt{1-\rho^2}}\right). \tag{63}$$

In the CAPM $SR_i=\rho_{i,M}SR_M$, so the bond Sharpe ratio, SR_b, is given as

$$\begin{aligned}SR_b&=-\rho_{l,M}SR_M\\&=-\left(\frac{\sigma_k\rho-\sigma}{\sigma_l}\rho_{V,M}+\frac{\sigma_k}{\sigma_l}\sqrt{1-\rho^2}\rho_{W_2,M}\right)SR_M\end{aligned} \tag{64}$$

while the asset Sharpe ratio is $SR_V=\rho_{V,M}SR_M$.

Appendix C. Default rate calculations

Moody's provide an annual report with historical cumulative default rates and these are extensively used in the academic literature as estimates of default probabilities. The default rates are based on a long history of default experience for firms in different industries and different regions of the world.

A number of studies find that ratings across industries and regions are not comparable: Cornaggia et al. (2017) find that default rates of financial institutions are significantly different from default rates of similarly-rated non-financial institutions, Cantor and Falkenstein (2001) find default rates of speculative-grade utility firms are significantly different from default rates of similarly-rated non-utility firms, while Cantor et al. (2004) find that ratings are more accurate for European firms than for North American firms.

As in most studies of structural models of credit risk we focus on U.S. industrial firms. To compare apples with apples we therefore use Moody's default database to calculate historical default rates for the subset of U.S. industrial firms in the Moody's database. Hamilton and Cantor (2006) discuss how Moody's calculate default rates (Moody's approach is based on the methodology in Altman, 1989; Asquith et al., 1989) and we review their methodology in Appendix C.1.

In Appendix C.2 we detail our calculation of default rates for U.S. industrial firms and compare our results with those published by Moody's for all global firms. We find that default rates for U.S. industrial firms are economically different from Moody's published default rates for all rated firms. In Appendix C.3 we show that the differences in default rates are also statistically significant.

C1. Moody's default rate calculations

Assume that there is a cohort of issuers formed on date y holding rating z. The number of firms in the cohort during a future time period is $n_y^z(t)$ where t is the number of periods from the initial forming date (time periods are measured in months in the main text). In each period there are three possible mutually exclusive end-of-period outcomes for an issuer: default, survival, and rating withdrawal. The number of defaults during period t is $x_y^z(t)$, the number of withdrawals is $w_y^z(t)$, and the number of issuers during period t is defined as

$$n_y^z(t)=n_y^z(0)-\sum_{i=1}^{t-1}x_y^z(i)-\sum_{i=1}^{t-1}w_y^z(i)-\frac{1}{2}w_y^z(t). \tag{65}$$

The marginal default rate during time period t is

$$d_y^z(t)=\frac{x_y^z(t)}{n_y^z(t)} \tag{66}$$

and the cumulative default rate for investment horizons of length T is

$$D_y^z(T)=1-\prod_{t=1}^{T}\left[1-d_y^z(t)\right]. \tag{67}$$

The average cumulative default rate is

$$\overline{D}^z(T)=1-\prod_{t=1}^{T}\left[1-\overline{d}^z(t)\right] \tag{68}$$

where $\overline{d}^z(t)$ is the average marginal default rate.[26]

For a number of cohort dates y in a historical data set Y, Moody's calculate the average marginal default rate as a weighted average, where each period's marginal default rate is weighted by the relative size of the cohort

$$\overline{d}^z(t) = \frac{\sum_{y\in Y} x_y^z(t)}{\sum_{y\in Y} n_y^z(t)}. \tag{69}$$

We label default rates based on Eq. (69) for *cohort-weighted* default rates. In the presence of macroeconomic risk as modelled in Feldhütter and Schaefer (2018) it is more robust to use *equal-weighted* default rates where the average marginal default rate is calculated as

$$\overline{d}^z(t) = \frac{1}{N_Y}\sum_{y\in Y}\frac{x_y^z(t)}{n_y^z(t)} \tag{70}$$

where N_Y is the number of cohorts in the historical dataset Y.

C2. Calculating default rates for U.S. industrial firms

Moody's default database appears to have a more extensive coverage of firms in the last 50 years compared to the previous 50 years. Specifically, there are 9055 firms with a rating at some point in the period 1919–1969 while there are 27,549 in the period 1970–2018. We therefore restrict our calculation of default rates to start from January 1, 1970.

We calculate historical default rates for industrial firms but, as a check on our methodology, we first replicate Moody's default rates for all firms. Table A3 shows Moody's (2018a) reported historical default rates 1970–2017 and, in row 2, default rates for all firms calculated using Moody's methodology and their default database for the same sample period (January 1, 1970 to January 1, 2018) as in Moody's (2018a). The calculated default rates are close, but not identical, to Moody's reported default rates. For example, the 10-year BBB default rate, a focal point in the academic literature, is calculated to be 3.83% while Moody's report 3.75%. We do not expect to replicate Moody's rates exactly because Moody's (2018b) note that "you will not be able to replicate the exhibits exactly, as the researchers have access to non-public information that is not included in the database".

When we restrict the sample to US industrial firms, historical default rates change quite dramatically as 'US industrial firms, cohort weight' (row 3 in Table A3) shows. The BBB 10-year default rate, for example, is estimated to be 5.74% which is 53% higher than Moody's estimate. Thus, there is a substantial effect of restricting the sample to U.S. industrial firms. We show in Appendix C.3 that the difference in default rates when using all firms and when using US industrial firms is statistically significant and therefore it is important to restrict the sample to correspond to firms used in our empirical analysis.[27]

Moody's calculate average default rates by using a cohort-weighted average of default rates. This leads to an uneven weighting across time. For example, default rates for AAA (B) during the decade 1970–1979 are weighted 4.5 times higher (23 times lower) than default rates during the most recent decade 2008–2017. In the presence of macroeconomic risk, it is preferable to have an even weighting across time, and we therefore equally-weight default rates. Table A3 shows that there is a moderate effect on default rates of equal weighting. The 10-year BBB default rate increases from 5.74% to 6.43%. However, there is no clear pattern in the direction that default rates change generally.

C3. Are default rates of US industrial firms different from those of all firms

In this section we calculate the statistical significance of the difference in default rates calculated using US industrial firms and firms that are not US industrials using default data from 1970–2017. We do so by calculating the distribution of the difference under the assumption that both sets of firms have the same ex ante default probability.

Specifically, for a given rating r and horizon h (in years 1,...,20), we record the number of firms in the January 1970, January 1971, ..., January 2017+1-h US industrial cohorts, $n^1_{1970}, n^1_{1971}, \ldots, n^1_{1977+1-h}$ (sample 1) and the corresponding number in cohorts of the remaining firms, $n^2_{1970}, n^2_{1971}, \ldots, n^2_{1977+1-h}$ (sample 2). We calculate the historical equal-weighted default rate, $\hat{p}_{r,h}$ as in the previous Section C.1 for all firms by combining the cohorts n^1_y and n^2_y (combined sample).

In year 1, corresponding to cohort 1970, we have $n^1_{1970} + n^2_{1970}$ firms, where firm i's value under the natural measure follows a GBM,

$$\frac{dV_t^i}{V_t^i} = (\mu - \delta)dt + \sigma dW_{it}^P. \tag{71}$$

We assume every firm has one h-year bond outstanding, and a firm defaults if firm value is below face value at bond maturity, $V_h^i \leq F$. Using the properties of a Geometric Brownian Motion, the default probability is

$$p = P(W_{iT}^P - W_{i0}^P \leq c) \tag{72}$$

where $c = \frac{\log(F/V_0) - (\mu - \delta - \frac{1}{2}\sigma^2)T)}{\sigma}$. This implies that the unconditional default probability is $N(\frac{c}{\sqrt{T}})$ where N is the cumulative normal distribution. For a given default probability $\hat{p}_{r,h}$ we can always find c such that Eq. (72) holds, so in the following we use $\hat{p}_{r,h}$ instead of the underlying Merton parameters that give rise to $\hat{p}_{r,h}$.

We introduce systematic risk by assuming that

$$W_{iT}^P = \sqrt{\rho}W_{sT} + \sqrt{1-\rho}W_{iT} \tag{73}$$

where W_i is a Wiener process specific to firm i, W_s is a Wiener process common to all firms, and ρ is the pairwise correlation between percentage firm value changes, which

[26] Note that this calculation assumes that marginal default rates are independent.

[27] It may be surprising that the difference is significant given that Feldhütter and Schaefer (2018) show that default rates have large confidence bands. However, the difference in default rates for two samples exposed to the same macroeconomic shocks is more precisely estimated than either default rate separately.

we set to $\rho = 0.2002$ following Feldhütter and Schaefer (2018). All the Wiener processes are independent. We simulate the realized default frequencies in the year 1-cohort separately for sample 1 and 2 by simulating one systematic, common for both samples, and $n^1_{1970} + n^2_{1970}$ idiosyncratic processes in Eq. (73).

In year 2 we form a cohort of $n^1_{1971} + n^2_{1971}$ new firms. The firms in year 2 have characteristics that are identical to those of the previous firms at the point they entered the index in year 1. We calculate the realized h-year default frequency of the year 2-cohort as we did for the year 1-cohort. Crucially, the common shock for years 1–9 for the year 2-cohort is the same as the common shock for years 2–10 for firms in the year 1-cohort. We repeat the same process for $48 - h$ years and calculate the overall realized cumulative 10-year default frequency for the two samples, $\tilde{p}^1_{r,h}$ and $\tilde{p}^2_{r,h}$, by taking an average of the default frequencies across the $48 - h$ cohorts, and compute the difference $\tilde{d}^{s_1}_{r,h} = \tilde{p}^1_{r,h} - \tilde{p}^2_{r,h}$. Finally, we repeat this entire simulation 100,000 times, get $\tilde{d}^{s_1}_{r,h}, \tilde{d}^{s_2}_{r,h}, \ldots, \tilde{d}^{s_{100,000}}_{r,h}$. A historical difference is significant at say the 5% level of this historical difference is smaller than the 2.5% quantile or larger than the 97.5% quantile in the simulated distribution of differences.

There are three approximations in this calculation. First, in the main text we use monthly cohorts while we use yearly cohorts in the simulation. Second, we assume all firms are replaced each year, while this is not so in the actual cohorts. These two approximations partially counterweight each other. Third, we do not use marginal default rates as above.

Table A4 shows the statistical significance of the difference in default rates. We see that long-term default rates for risky firms (rated BBB or below) are higher than other firms and the difference is statistically highly significant. For example, the 20-year BBB default rate for US industrial firms is 15.83% which is 242% higher than that of other firms and the difference is significant at the 0.1% level.

References

Altman, E., 1989. Measuring corporate bond mortality and performance. J. Finance 44, 909–922.
Asquith, P., Mullins, D., Wolff, E., 1989. Original issue high yield bonds: aging analyses of defaults, exchanges, and calls. J. Finance 44, 923–952.
Azzalini, A., Valle, A., 1996. The multivariate skew-normal distribution. Biometrika 83, 715–726.
Bai, J., Goldstein, R., Yang, F., 2020. Is the credit spread puzzle a myth? J. Financ. Econ. (137) 297–319.
Bao, J., 2009. Structural Models of Default and the Cross-Section of Corporate Bond Yield Spreads. Working paper.
Bao, J., Pan, J., Wang, J., 2011. The illiquidity of corporate bonds. J. Finance 66, 911–946.
Bhamra, H.S., Kuehn, L.-A., Strebulaev, I.A., 2010. The levered equity risk premium and credit spreads: a unified framework. Rev. Financ. Stud. 23 (2), 645–703.
Black, F., Cox, J., 1976. Valuing corporate securities: some effects of bond indenture provisions. J. Finance 31, 351–367.
Bretscher, L., Feldhütter, P., Kane, A., Schmid, L., 2021. Marking to Market Corporate Debt. Working Paper.
Brown, J., Gustafson, M., Ivanov, I., 2021. Weathering cash flow shocks. J. Finance 76, 1731–1772.
Cantor, R., Falkenstein, E., 2001. Testing for rating consistency in annual default rates. J. Fixed Income 11 (2), 36–51.
Cantor, R., Stumpp, P., Madelain, M., De Bodard, E., 2004. Measuring the quality and consistency of corporate ratings across regions. In: Special Comment. New York: Moody's Investors Services, pp. 1–18.
Chen, H., 2010. Macroeconomic conditions and the puzzles of credit spreads and capital structure. J. Finance 65 (6), 2171–2212.
Chen, H., Cui, R., He, Z., Milbradt, K., 2018. Quantifying liquidity and dewfault risks of corporate bonds over the business cycle. Rev. Financ. Stud. 31 (3), 852–897.
Chen, L., Collin-Dufresne, P., Goldstein, R.S., 2009. On the relation between the credit spread puzzle and the equity premium puzzle. Rev. Financ. Stud. 22, 3367–3409.
Collin-Dufresne, P., Goldstein, R., 2001. Do credit spreads reflect stationary leverage ratios? J. Finance 56, 1929–1957.
Cornaggia, J., Cornaggia, K., Hund, J., 2017. Credit ratings across asset classes: a long-term perspective. Rev. Finance 21, 465–509.
Cremers, M., Driessen, J., Maenhout, P., 2008. Explaining the level of credit spreads: option-implied jump risk premia in a firm value model. Rev. Financ. Stud. 21, 2209–2242.
Das, S.R., 2002. The surprise element: jumps in interest rates. J. Econom. 106, 27–65.
Davydenko, S. A., 2013. When do firms default? A Study of the Default Boundary. Working paper.
Dick-Nielsen, J., 2009. Liquidity biases in TRACE. J. Fixed Income 19 (2), 43–55.
Dick-Nielsen, J., 2014. How to Clean Enhanced TRACE Data. Unpublished Manuscript.
Dick-Nielsen, J., Feldhütter, P., Lando, D., 2012. Corporate bond liquidity before and after the onset of the subprime crisis. J. Financ. Econ. 103, 471–492.
Dorfleitner, G., Schneider, P., Veza, T., 2011. Flexing the default barrier. Quant. Finance 11, 1729–1743.
Du, D., Elkamhi, R., Ericsson, J., 2019. Time-varying asset volatility and the credit spread puzzle. J. Finance 74, 1841–1885.
Eom, Y.H., Helwege, J., Huang, J.-Z., 2004. Structural models of corporate bond pricing: an empirical analysis. Rev. Financ. Stud. 17 (2), 499–544.
Fama, E.F., French, K.R., 2002. Testing trade-off and pecking order prediction about dividends and debt. Rev. Financ. Stud. 15, 1–33.
Feldhütter, P., 2012. The same bond at different prices: identifying search frictions and selling pressures. Rev. Financ. Stud. 25, 1155–1206.
Feldhütter, P., Lando, D., 2008. Decomposing swap spreads. J. Financ. Econ. 88, 375–405.
Feldhütter, P., Schaefer, S., 2018. The myth of the credit spread puzzle. Rev. Financ. Stud. 8, 2897–2942.
Fischer, E., Heinkel, R., Zechner, J., 1989. Dynamic capital structure choice: theory and tests. J. Finance 44, 19–40.
Flannery, M.J., Nikolova, S., Oztekin, O., 2012. Leverage expectations and bond credit spreads. J. Financ. Quant. Anal. 47, 689–714.
Flannery, M.J., Rangan, K., 2006. Partiel adjustment toward target capital structures. J. Financ. Econ. 79, 469–506.
Frank, M., Goyal, V., 2003. Testing the pecking order theory of capital structure. J. Financ. Econ. 67, 217–248.
Frank, M., Goyal, V., 2008. Trade-off and pecking order theories of debt. In: Eckbo, B.E. (Ed.), Handbook of Empirical Corporate Finance, pp. 135–202.
Geelen, T., 2017. Debt maturity and lumpy debt. Working Paper.
Geelen, T., 2019. Information dynamics and debt maturity. Working Paper.
Gürkaynak, R.S., Sack, B., Wright, J.H., 2006. The U.S. Treasury Yield Curve: 1961 to the Present. Working Paper. Divisions of Reasearch and Statistics and Monetary Affairs, Federal Reserve Board, Washington, D.C..
Hackbarth, D., Miao, J., Morellec, E., 2006. Capital structure, credit risk, and macroeconomic conditions. J. Financ. Econ. 82, 519–550.
Hamilton, D. T., Cantor, R., 2006. Measuring corporate default rates. Special Comment. New York: Moody's Investors Services, pp. 1–16.
Huang, J.-z., Huang, M., 2012. How much of the corporate-treasury yield spread is due to credit risk? Rev. Asset Pricing Stud. 2 (2), 153–202.
Huang, J.-z., Nozawa, Y., Shi, Z., 2020a. The Global Credit Spread Puzzle. Working Paper.
Huang, J.-z., Shi, Z., Zhou, H., 2020. Specification analysis of structural credit risk models. Rev. Finance 23, 45–98.
Huang, R., Ritter, J., 2009. Testing theories of capital structure and estimating the speed of adjustment. J. Financ. Quant. Anal. 44, 237–271.
Jessen, C., Lando, D., 2015. Robustness of distance-to-default. J. Bank. Finance 50, 493–505.
Krishnamurthy, A., Vissing-Jorgensen, A., 2012. The aggregate demand for treasury debt. J. Polit. Econ. 120 (2), 233–267.
Leland, H., 1998. Agency costs, risk management, and capital structure. J. Finance 53, 1213–1243.
Lemmon, M., Roberts, M., Zender, J., 2008. Back to the beginning: persistence and the cross-section of corporate capital structure. J. Finance 63, 1575–1608.
Lemmon, M., Zender, J., 2010. Debt capacity and tests of capital structure theories. J. Financ. Quant. Anal. 45, 1161–1187.

Longstaff, F., Mithal, S., Neis, E., 2005. Corporate yield spreads: default risk or liquidity? New evidence from the credit-default swap market. J. Finance 60 (5), 2213–2253.
Longstaff, F., Schwartz, E.S., 1995. A simple approach to valuing risky fixed and floating rate debt. J. Finance 50, 789–819.
McQuade, T.J., 2018. Stochastic Volatility and Asset Pricing Puzzles. Working Paper. Harvard University.
Merton, R., 1974. On the pricing of corporate debt: the risk structure of interest rates. J. Finance 29, 449–470.
Moody's, 2018. Annual Default Study: Corporate Default and Recovery Rates, 1920–2017. Moody's Investors Service, pp. 1–60.
Moody's, 2018. Moody's Analytics Default & Recovery Database (DRD): Frequently Asked Questions and Reference Guide. Moody's Analytics, pp. 1–12.
Myers, S.C., Majluf, N.S., 1984. Corporate financing and investment decisions when firms have information that investors do not have. J. Financ. Econ. 13, 187–221.
Nagel, S., 2016. The liquidity premium of near-money assets. Q. J. Econ. 131, 1927–1971.
Norden, L., Weber, M., 2010. Credit line usage, checking account activity, and default risk of bank borrowers. Rev. Financ. Stud. 23, 3665–3699.
Rauh, J., Sufi, A., 2010. Capital structure and debt structure. Rev. Financ. Stud. 23, 4242–4280.
Schwert, M., 2020. Does borrowing from banks cost more than borrowing from the market? J. Finance 75 (2), 905–947.
Strebulaev, I., 2007. Do tests of capital structure theory mean what they say? J. Finance 62 (4), 1747–1787.
Welch, I., 2004. Capital structure and stock returns. J. Polit. Econ. 112, 106–131.