A Stochastic Inference-Dual-Based Decomposition Algorithm for TSO–DSO-Retailer Coordination

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***Abstract*—The flexibility services available from embedded resources, being attractive to both the network operators and retailers, pose a problem of co-ordination and market design at the local level. This research considers how a Flexibility Market Operator (FMO) at the local level, analogous to market operators at the wholesale level, can improve the real-time operation of the power systems and efficiently manage the interests of the TSO, DSO, and Retailers. Using generalized disjunctive programming, a stochastic bilevel representation of the task is reformulated as a stochastic mixed-logical linear program (MLLP) with indicator constraints. An Inference-Dual-Based Decomposition (IDBD) Algorithm is developed with sub-problem relaxation to reduce the iterations. Using expected Shapley values, a new payoff mechanism is introduced to allocate the cost of service activations in a fair way. Finally, the performance and benefits of the proposed method are assessed via a case study application.**

***Index Terms*—Stochastic Inference-Dual-Based Decomposition Algorithm, Disjunctive programming, TSO-DSO-Retailer coordination.**

I. Introduction

The effective coordination among Transmission System Operators (TSOs), Distribution System Operators (DSOs), and retailers is crucial for maintaining grid stability and efficiency in real-time power system operations [1]. TSOs manage high-voltage transmission networks, monitor power flows and voltage levels, and facilitate cross-border power exchange. DSOs oversee distribution networks, integrate distributed energy resources, and ensure power quality. Retailers interact with end consumers to efficiently distribute and sell electricity, ensuring reliable supply and managing pricing fluctuations. Optimal resource utilization, renewable energy integration, and demand management are enabled through the effective coordination of all of these participants, thereby enhancing grid reliability and supporting the transition to a sustainable energy system.

A. Motivation

The proliferation of ”Embedded Energy Resources” (EERs) at the local level not only increases the real-time uncertainties faced by the TSO, DSO, and Retailers, but also provides opportunities for these parties to contract with the operators of the EERs to manage their operating risks.

From an optimization viewpoint, the increasing presence of uncertain EERs at the local level poses significant challenges for TSOs, DSOs, and Retailers. The integration of EERs introduces real-time uncertainties and operational risks that need to be effectively managed to ensure reliable and efficient power system operations. Moreover, the traditional approach of independent bilateral contracting between these parties and EER operators leads to suboptimal outcomes, as it fails to consider the interdependencies and cross-impacts among the stakeholders [2].

In addition, since EERs often consist of renewable resources, the use of stochastic coordination, e.g. [3], appears to be most appropriate. To address these optimization challenges, there is a need for coordination schemes that maximize the utilization of EERs while ensuring operational constraints are met. This involves finding optimal strategies for TSOs, DSOs, and Retailers to activate flexibility services in response to real-time uncertainties, while considering their own objectives and the impact on other parties. Additionally, the optimization framework must account for the integration of stochastic renewable energy sources, the behavior of EERs ”behind the meter,” and the impact of retailers’ power demand forecasts.

In the existing liberalized market setting, each party possesses sensitive data that they are unwilling to share openly. This reluctance to share crucial information hampers effective coordination efforts between TSOs and DSOs, hindering the optimization of system operations and overall efficiency. The absence of trusted intermediary results in a sequential flexibility activation procedure that fails to fully address the operational constraints and risks faced by both TSOs and DSOs. The concept of an independent agency needed to be introduced to overcome this limitation and enhance system operations. This agency acts as an intermediary, collecting necessary information from TSOs and DSOs without disclosing it to other market participants. By safeguarding privacy concerns, it is possible to facilitate efficient coordination among TSOs, DSOs, and Retailers, ultimately leading to improved operational efficiency and a fair allocation of resources.

By developing an advanced optimization model, coordination framework, and organizational setup this paper aims to enhance the efficiency and effectiveness of EER activations from an optimization perspective. The proposed coordination framework enables TSOs, DSOs, and Retailers to jointly optimize their decision-making processes, taking into account the uncertain nature of renewable energy sources and the dynamic interactions between stakeholders. The ultimate goal is to achieve optimal resource allocation, reduce operational costs, and improve the overall performance of the power system.

By addressing the optimization challenges associated with EER activations, this research will contribute to the advancement of optimization techniques in the field of energy systems.

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The findings of this study can provide valuable insights for decision-makers, policymakers, and industry practitioners involved in the planning, operation, and management of modern power systems with a high penetration of renewable energy resources.

B. Literature Review

There is extensive background research on the coordination process between TSOs and their interconnected DSOs. In [4], a decentralized algorithm was utilized to implement collaborative TSO-DSO optimal power flow (OPF). Three TSO-DSO coordination schemes were proposed in [5], including two sequential decentralized models and one centrally co-optimized model. In [6], a hierarchical mechanism for TSO-DSO coordination was proposed that considers the dispatch decisions of both TSOs and DSOs. Four TSO-DSO coordination schemes were proposed in the European SmartNet project [7], all of which considered the sequential coordination process. In articles [4]–[7], the retailers' objectives, the stochastic programming, and the cross-dependency between service buyers' objectives have not been modeled in the ancillary flexibility service market. In [8], the authors discuss a deterministic operational coordination problem between TSO and DSOs. The problem is decomposed into TSO and DSO subproblems, and the coordination is achieved through the adjustment of Lagrangian multipliers. To handle the nonlinearities resulting from AC power flow constraints at the TSO level, dynamic linearization techniques are employed. Another framework, presented in [9], focuses on the optimal day-ahead scheduling of power distribution systems. This framework considers the dynamic interaction between TSOs and DSOs, utilizing a mixed-integer linear programming model. However, the approaches discussed in [8] and [9] overlook the integration of stochastic renewable energy sources within the coordination between TSOs and DSOs and do not account for the impact of retailers operating in the electricity market.

In [10], a complementarity model for the TSO-DSO coordination problem under uncertainty is proposed in which two trading markets including day-ahead and real-time have been considered. Although it enhances the TSO-DSO coordination problem from the stochastic programming viewpoint, other research gaps still exist. Three coordination schemes have been proposed in [11] in which the retailer has been included. However, it co-optimizes the TSO-DSO-Retailer coordination problem in a deterministic setting which cannot model the uncertain behavior of renewable energies in the flexibility ancillary service markets properly. Moreover, service buyers require to look beyond the immediate dispatching period to cope with uncertainties over various time intervals in the future [12]. Consequently, to extend this theme of research in those two important dimensions, we develop a stochastic formulation for TSO-DSO-Retailer operational coordination problem with a look-ahead multi-interval (LA-MI) framework.

Three TSO-DSO coordination frameworks have been proposed in the existing literature [13]: the TSO-managed model, the TSO-DSO hybrid-managed model, and the DSO-managed model. The hybrid model involves both the TSO and DSO, with the DSO responsible for bid validation. However, the limitations related to the distribution system's lifespan lead to an underutilization of its maximum capacity, resulting in inefficient resource allocation. On the other hand, the DSO-managed model operates sequentially, prioritizing the DSO's needs, but this approach often fails to achieve overall efficiency in service allocation, leading to suboptimal outcomes. It can also create cross impacts between buyers and facilitate the free-rider strategy, where participants exploit the system without contributing proportionately [13].

In addition, one of the crucial considerations in a liberalized market setting is that each party, TSO, DSO, and Retailer, has proprietary information that they would be unwilling to share. Therefore, to enhance system operations and address the drawbacks of a sequential flexibility activation procedure outlined in [11], as well as mitigate free-rider strategies, we consider the role of an independent agency. This agency would be entrusted with the task of collecting the necessary information from TSO, DSOs, and Retailers. Importantly, this agency would not share the collected data but would focus on facilitating efficient coordination among the stakeholders involved, ultimately leading to improved operational efficiency. We refer to this agency as the Flexibility Market Operator (FMO) and there is already evidence of their emergence [2], [14]. In our formulation, the FMO is an independent, regulated organization through which all market participants who want to buy or sell local flexibility services must trade under their license conditions.

To achieve this from a modeling perspective, therefore, the FMO embeds the stochastic optimization of DSOs and retailers within the stochastic look ahead multi-interval real-time dispatch modeling of the TSO, and as a consequence solves a two-stage stochastic bilevel mixed-logical linear programming problem (MLLP). Further, using the Karush-Kuhn-Tucker (KKT) conditions to reformulate the bilevel stochastic programming problem leads to a large-scale stochastic MLLP problem. Consequently, the utilization of decomposition methods is indispensable, as in [15], [16], and [17]. In these three examples, all binary variables are considered as complicating variables in a Mixed-Integer Linear Programming (MILP) master problem and the sub-problems are linear programs. However, as in [18], in several formulations such as our formulation, due to the existence of numerous binary variables and complementary slackness conditions, it is impossible to have convex sub-problems with zero duality gap since not all binary variables can be considered as complicating variables in the master problem. Consequently, standard duality theory is not applicable [19], and a new modification of the decomposition technique is needed.

Thus, integer sub-problems [20], non-linear constraints [21], and logical propositions [19] are considered in several modifications of Benders decomposition. However, these methods use continuous relaxation to calculate the Benders cuts. Moreover, the execution time of these methods is high, especially with an extensive number of binary variables [20]. Alternatively, since any form of the optimization problem can be considered as a sub-problem in the Inference-Dual-Based Decomposition (IDBD) algorithm [18], the IDBD is very attractive. The proposed decomposition algorithm in our paper combines the concepts of "inference dual" theory and the Benders-like decomposition algorithm. In this sense, it is a major extension of both concepts. Unlike traditional approaches, it eliminates the requirement of a linear optimization problem with a zero duality gap as the resulting subproblem. Therefore, the proposed IDBD algorithm offers the flexibility to address a broad spectrum of convex and non-convex optimization problems as subproblems. The IDBD algorithm does not use continuous relaxation and is computationally superior to the existing methods [22]. In practice, the IDBD cuts are determined using the concept of "inference dual" which depends on a structural investigation of the sub-problem and does not require any dual multiplier. This enables the algorithm to efficiently handle optimization problems with binary/logical variables in the subproblem. However, the IDBD algorithm has not hitherto been developed for two-stage stochastic bilevel programming problems, and no general method to determine the IDBD cuts exists as each problem requires a customized approach [18], [22]. Furthermore, the IDBD algorithm convergence and execution time strongly depend on the inclusion of "sub-problem relaxation" constraints in the master problem [23], which are not similar to common relaxation methods and relate to the specific nature of the complicating variables. Therefore, for any specific problem, a custom formulation is needed.

Moreover, the collaborative activation of flexibility services by TSO, DSOs, and retailers entails a mutual reliance on the value functions of service buyers. Consequently, it becomes imperative to establish a fair compensation mechanism that allocates payoffs commensurate with the actual impacts of each service buyer. The Shapley value method is a widely recognized and respected approach in cooperative game theory for allocating costs. It ensures fairness by considering the marginal contributions of each participant and assessing their influence on various coalitions [24]. This method aims to distribute costs equitably among participants by accounting for all possible combinations. Additionally, the Shapley value method promotes efficiency by incentivizing participants to contribute in a way that maximizes overall benefits, leading to efficient resource utilization and productive collaboration. It also guarantees a balanced allocation of costs by thoroughly evaluating contributions across different coalitions. Another appealing aspect of the Shapley value method is its adherence to the principle of "independence of irrelevant alternatives", meaning that changes in the cooperative game's composition do not affect cost allocation [25]. This method's wide acceptance and extensive research in cooperative game theory have further enhanced its credibility and robustness. In [11], a settlement mechanism was proposed based on the calculation of Shapley values. However, the application of the Shapley value mechanism in the context of a two-stage stochastic optimization problem has not been thoroughly investigated, thereby leaving uncertainties unaddressed. Thus, in light of this research gap, we propose a novel compensation mechanism for service buyers founded on the "expected Shapley value" concept.

C. Contributions

This paper is a major extension of the previously published paper [2] on the TSO-DSO operational coordination framework in a deterministic setting. Accordingly, as compared to [2], the contributions of this paper are as follows:

1) We add the role of retailers into the TSO-DSO operational coordination and develop the IDBD algorithm for a two-stage stochastic TSO-DSO-Retailer operational coordination problem.
2) We propose a novel decomposition technique based on the inference duality theorem to find the optimal solution to our proposed stochastic bilevel Mixed Logical Linear Programming (MLLP) model for the TSO-DSO-Retailer coordination problem.
3) We create a novel formulation to determine the appropriate sub-problem relaxation constraints and Stochastic Inference-Dual-Based Decomposition (SIDBD) cuts.
4) To improve the system operations in a stochastic setting and mitigate the drawbacks of sequential flexibility activation procedures, we propose the establishment of a new agency (FMO) that will collect, but not share, the required information from the TSO, DSO, and Retailers, aiming to facilitate efficient coordination and diminish free-rider strategies.
5) We consider a new approach to determining a fair pricing mechanism amongst the TSO, DSOs, and Retailers, pro-rata to the actual impacts of each of their activations. Extending the deterministic Shapley value approach in [11] to the stochastic optimization context effectively introduces a market allocation mechanism based upon expected Shapley values.

The rest of the paper is organized as follows. Section II explains our market mechanism and problem formulation. The optimization procedures are presented in Section III. Section IV and Section V describe the case studies and the conclusions, respectively.

II. Market Mechanism and Problem Formulation

We assume Demand Response (DR), Photovoltaic (PV) panels, and Wind Turbines (WT) are available as Embedded Energy Resources (EERs) to provide flexibility services to TSO, DSOs, and Retailers. As with [11], we assume that the required flexibility is cleared in a one-shot auction. At the TSO level, the EERs compete with the other transmission-connected operating reserves from the generators to provide flexibility services but they are the only flexibility service providers at the DSO level. Batteries and other storage technologies are included in EERs. For our purposes, they do not need to be modeled separately, as they exhibit a deliverable functionality similar to other EERs, insofar as they cannot be charged and discharged simultaneously. This implies that energy storage devices can participate in the market just like other providers of turn-up and turn-down services, based on bid/offer functions. Nevertheless, energy storage systems introduce distinct inter-temporal and orthogonal constraints into the overall coordination process involving TSO-DSO-Retailer interactions. In a previous paper [2], we extensively discussed an optimization technique for the effective handling of energy storage systems. Hence, for the sake of simplicity here, as in [10], we have excluded the optimization of energy storage systems, given our model's capability to effectively address a large number of intertemporal and orthogonal constraints, as necessary.

A. Coordination Mechanism and Modeling Setup

In our market design, service providers are settled on a "pay-as-bid" basis. We consider 15-minute delivery periods. In Fig.1, we illustrate the LA-MI framework model for the system operating with firm contracts from the forward markets. The LA-MI framework effectively tackles the evolution of uncertainties through the progressive forward market clearing processes. Additionally, we develop a market mechanism based on the market properties discussed in [11].

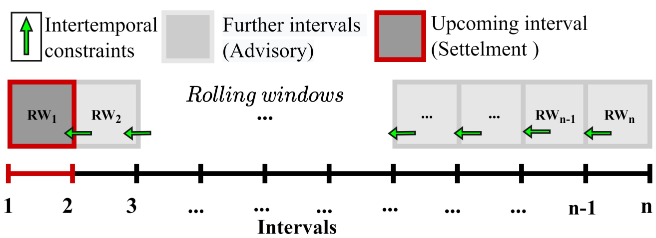

Fig. 1. The proposed LA-MI framework for TSO-DSO-Retailer coordination.

One period ahead, the FMO receives all available information to operate the coordination mechanism, generates scenarios to model different uncertainty sources, embeds DSO-Retailers interest functions within the TSO dispatch model, and solves a two-stage stochastic bilevel TSO-DSO-Retailer operational coordination problem. Then, the flexibility market participants are informed about the market clearing outputs and receive dispatch instructions.

The coordination mechanism proposed in our paper involves an independent organization managing the activations of the EERs in a fair and nondiscriminatory manner. It would effectively be a licensed market operator at the local level. The FMO would represent a development consistent with the unbundling of independent DSOs from DNOs, the licensing of flexibility market platforms and the regulatory directives to ensure the EERs compete in a non-discriminatory way with the larger, incumbent generators. It would need to interact with the independent market agency responsible for the settlement of contracts (eg Elexon is licensed for this role in Britain). This approach is currently the direction of market arrangements in several European designs with the support of the Agency for the Cooperation of Energy Regulators (ACER), which coordinates National Regulators, and the European Association for the Cooperation of Transmission System Operators (ENTSO-E), which coordinates national or regional TSOs in Europe [26]–[28]. Our coordination mechanism is inspired by and aligned with these European designs.

B. Flexibility Interest Functions

Although the impacts of the activated flexibility services are shared, the cost allocation to the respective parties, under the Shapley value criterion, should reflect the relative value to each party if they were to contract singularly from the full availability of services. Thus we have:

1) Expected Shapley Value: When service buyers aim to optimize the expected value of their flexibility interest functions in a stochastic programming problem, we use the expected value criterion and define the expected Shapley value as follows [29].

Definition 1. *The expected Shapley value ϕ is defined as* $\phi_k(M, \vartheta) = \frac{1}{|M!|} \sum_{S \subseteq M} (|s|-1)!(|M|-|s|)!(\mathbb{E}_\omega(\vartheta[S]) - \mathbb{E}_\omega(\vartheta[S]/\{k\}))$

where, $|s|$ is the number of participants in coalition S, $\vartheta[S]$ is the flexibility interest of coalition S, and M is the total number of participants including retailers, DSOs, and TSO. The proof of the validity of the expected Shapley value can be found in [29] which is outside the scope of this paper.

Now, in order to calculate the expected Shapley value, we explain the independent flexibility interests of TSO, DSOs, and retailers as follows. The interest functions of TSO and DSOs are developed based on the same TSO and DSO models in [2]. The differences are the utilization of the two-stage stochastic programming and consideration of the scenarios index which can be seen in the objective functions (1a) and (2a). We use the Nomenclature as summarized in Table IX in the Appendix. Binary variables α_{db}^{DW} and α_{ub}^{UP} are "here-and-now" variables that are independent of different scenarios. These variables represent the state of service activation in our two-stage stochastic programming problem. The rest of the optimization variables are considered "wait-and-see" variables which are related to the second stage and determine the real-time operation of the systems. The variables related to the second stage are scenario-dependent variables. The scenario tree and the first and second stage variables are shown in Fig.2.

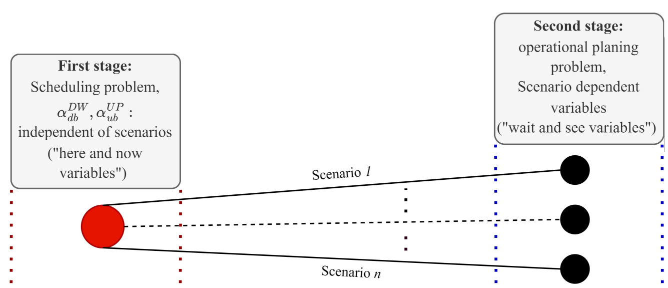

Fig. 2. The two-stage stochastic bilevel programming for TSO-DSO-Retailer operational coordination problem.

2) Expected Flexibility Interest of the TSO: Despite its widespread adoption among system operators for its computational robustness, the DC OPF method exhibits inherent limitations. It neglects essential factors such as reactive power, voltage magnitude, and the consequential influence of reactive power flow on line currents, potentially resulting in suboptimal or insecure solutions. In this study, we employ a novel OPF method proposed in [30] to model the TSO and DSO networks. This OPF method achieves a delicate balance between linearity, convergence guarantee, and accuracy. By incorporating the consideration of reactive power and voltage magnitude while retaining the advantageous features of the linear DC OPF model, our approach aims to mitigate the compromises in accuracy that are typically encountered during network modeling. The utilized AC OPF model incorporates apparent branch flow limits, enabling a more accurate representation of system congestion compared to the DC OPF method, which only enforces surrogate active power flow limits. Furthermore, by considering the impact of voltage/VAR limits on the Locational Marginal Prices (LMP), the proposed method provides a more precise economic signal compared to the DC OPF method. The inclusion of reactive power balance equations facilitates the practical implementation of the reactive power market. In summary, the utilized AC OPF model enhances solution accuracy and economic efficiency, and expands the range of applications compared to the widely used DC OPF model, without requiring any additional information.

In AC power flow calculations, the DC OPF model is commonly used as a basis. However, certain applications require improved accuracy in the linearized network model. To address this, the DC OPF model is re-optimized if any violations occur. During the DC-AC iteration, updating the initial point and formulating a warm-start model can enhance accuracy. The warm-start OPF solution provides a method to improve performance, especially during the iterative process that ensures AC feasibility. The proposed OPF method exploits the quasi-linear relationship between active power and voltage angle, resulting in desirable voltage angle estimates. As losses primarily depend on voltage angle, the OPF model's solution establishes a more favorable "base case system operating condition" for loss linearization. Additionally, a better initial point for nonlinear term expansions further enhances the network model's accuracy.

The combined features of the quasi-linear active power and voltage angle relationship, along with the efficient linearization method employed in the proposed network model, ensure that the warm-start OPF model's solution closely approximates the AC OPF optimum with just one additional iteration. Moreover, compared to the DC OPF method, the AC OPF method effectively handles constraint violations during the iterative process to ensure AC feasibility. In traditional DC-AC iteration, addressing branch flow violations requires modifying branch flow limits in the DC OPF model, which alters the feasible region of system operation. Consequently, the modified DC OPF model may yield sub-optimal or infeasible solutions. Additionally, violations related to reactive power and voltage magnitude cannot be directly addressed. However, by replacing the DC OPF method with the proposed OPF method, constraint violations are likely to be eliminated due to the improved accuracy of the warm-start network model, without the need for modifying operational limits. This approach allows direct constraint of violations in reactive power and voltage. These features significantly reduce the required iterations for ensuring AC feasibility when using the proposed OPF method instead of the DC OPF method.

As a consequence, we formulate the following two-stage stochastic programming problem to specify the TSO's objective based on the proposed AC-OPF model in [30].

$$
\begin{aligned}
&\underset{\Omega_1}{\text{Minimize}} \Big\{ \mathbb{E}_\omega \Big[\sum_{t\in T} \sum_{b\in N_{UP}^{TS}} f_b(P_{g_{bt\omega}}^{TS} - \hat{P}_{g_{bt}}^{TS}) + \\
&\sum_{t\in T} \sum_{b\in N_{DW}^{TS}} \pi_{bt\omega}^{RD} f_b(\hat{P}_{g_{bt}}^{TS} - P_{g_{bt\omega}}^{TS}) + \\
&\sum_{t\in T} \sum_{b\in I^{TS}} \sum_{d\in N_{db}} F_{dbt\omega}^{DWT} \pi_{dbt\omega}^{DW} + \\
&\sum_{t\in T} \sum_{b\in I^{TS}} \sum_{u\in N_{ub}} F_{ubt\omega}^{UPT} \pi_{ubt\omega}^{UP} \Big] + \\
&\sum_{b\in I^{TS}} \Big(\sum_{u\in N_{ub}} \alpha_{ub}^{UP} + \sum_{d\in N_{db}} \alpha_{db}^{DW} \Big) \Big\}
\end{aligned} \tag{1a}
$$

subject to :

$$P_{bct\omega}^{TS} \approx -B_{bc}^{TS} \theta_{bct\omega}^{TS} \ , \forall \ (bc) \in K^{TS}, \ t \in T, \ \omega \in \Theta, \tag{1b}$$

$$Q_{bct\omega}^{TS} \approx -B_{bc}^{TS} \Big(\frac{v_{bt\omega}^{s^{TS}} - v_{ct\omega}^{s^{TS}}}{2} \Big) \tag{1c}$$

$$Q_{bt\omega}^{TS} = \sum_{bc\in K^{TS}} Q_{bct\omega}^{TS} - v_{bt\omega}^{s^{TS}} \sum_{c\in I^{TS}} B_{bc}^{TS}, \tag{1d}$$

$$
\begin{aligned}
&P_{g_{bt\omega}}^{TS} - P_{L_{bt\omega}}^{TS} - \sum_{u\in N_{ub}} F_{ubt\omega}^{UPT} + \sum_{d\in N_{db}} F_{dbt\omega}^{DWT} = \\
&\sum_{(bc)\in K^{TS}} P_{bct\omega}^{TS} + v_{bt\omega}^{s^{TS}} \sum_{c\in I^{TS}} G_{bc}^{TS}, \\
&\forall \ b \in I^{TS} \ , \ t \in T, \ \omega \in \Theta
\end{aligned} \tag{1e}
$$

$$
\begin{aligned}
&P_{L_{pt\omega}}^{TS} = \tilde{P}_{g_{pt\omega}}^{DS} + \sum_{u\in N_{up}} F_{upt\omega}^{UPT} - \sum_{d\in N_{dp}} F_{dpt\omega}^{DWT}, \\
&\quad \forall \ p \in PCC \ , t \in T, \ \omega \in \Theta
\end{aligned} \tag{1f}
$$

$$Q_{L_{pt\omega}}^{TS} = \tilde{Q}_{g_{pt\omega}}^{DS} \ , \ \forall \ p \in PCC \ , \ t \in T, \ \omega \in \Theta \tag{1g}$$

$$
\begin{aligned}
&P_{g_{bt\omega}}^{TS} - P_{g_{b(t-1)\omega}}^{TS} \le \beta_b^{UP} RU_b \ , \\
&\quad \forall \ b \in N_{UP}^{TS}, t \in T, \ \omega \in \Theta
\end{aligned} \tag{1h}
$$

$$
\begin{aligned}
&P_{g_{b(t-1)\omega}}^{TS} - P_{g_{bt\omega}}^{TS} \le \beta_b^{DW} RD_b \ , \\
&\quad \forall \ b \in N_{DW}^{TS}, t \in T, \ \omega \in \Theta
\end{aligned} \tag{1i}
$$

$$
\begin{aligned}
&f_b(P_{g_{bt\omega}}^{TS}) = \tilde{a}_{1,b} P_{g_{bt\omega}}^{TS} + \tilde{a}_{0,b} \ , \\
&\quad \forall \ b \in I^{TS}, \ t \in T, \ \omega \in \Theta
\end{aligned} \tag{1j}
$$

$$
\begin{aligned}
&(F_{ubt}^{UPT} + \tilde{F}_{ubt}^{UPD}) \le \alpha_{ub}^{UP} (\Phi_{upt\omega}^{PV} + \Phi_{upt\omega}^{WT}), \\
&(F_{dbt}^{DWT} + \tilde{F}_{dbt}^{DWD}) \le \alpha_{db}^{DW} \Phi_{dpt\omega}^{DR}, \\
&\forall \ p \in PCC, u \in N_{ub}, d \in N_{db}, t \in T, \omega \in \Theta
\end{aligned} \tag{1k}
$$

$$\underline{P}_b^{TS} \le P_{bt\omega}^{TS} \le \overline{P}_b^{TS} \ , \ \underline{Q}_b^{TS} \le Q_{bt\omega}^{TS} \le \overline{Q}_b^{TS} \ , \tag{1l}$$

$$\underline{v}_b^{s^{TS}} \le v_{bt\omega}^{s^{TS}} \le \overline{v}_b^{s^{TS}} \ , \ \forall \ b \in I^{TS}, \ t \in T, \ \omega \in \Theta \tag{1m}$$

$$
\begin{aligned}
&\underline{P}_{bc}^{TS} \le P_{bct\omega}^{TS} \le \overline{P}_{bc}^{TS} \ , \underline{Q}_{bc}^{TS} \le Q_{bct\omega}^{TS} \le \overline{Q}_{bc}^{TS} \ , \\
&\forall \ (bc) \in K^{TS}, \ t \in T, \ \omega \in \Theta
\end{aligned} \tag{1n}
$$

$$\{F_{ubt\omega}^{UPT}, F_{dbt\omega}^{DWT}\} \in \mathbb{R}_{\ge 0}, \{\alpha, \beta\} \in \{0, 1\} \tag{1o}$$

$$\{v_{bt\omega}^{s^{TS}}, \theta_{bct\omega}^{TS}, P_{bct\omega}^{TS}, Q_{bct\omega}^{TS}, P_{bt\omega}^{TS}, Q_{bt\omega}^{TS}\} \in \mathbb{R} \, , \tag{1p}$$

where, $\theta_{bct\omega}^{TS} = \theta_{bt\omega}^{TS} - \theta_{ct\omega}^{TS} \ , \forall (bc) \in K^{TS}, t \in T, \omega \in \Theta$

Set of variables $\Omega_1 = \{v_{bt\omega}^{s^{TS}}, \theta_{bt\omega}^{TS}, P_{g_{bt\omega}}^{TS}, Q_{g_{bt\omega}}^{TS}, P_{bct\omega}^{TS}, Q_{bct\omega}^{TS}, P_{L_{pt\omega}}^{TS}, P_{bt\omega}^{TS}, Q_{bt\omega}^{TS}, F_{dbt\omega}^{DWT}, F_{ubt\omega}^{UPT}, \beta_b^{UP}, \beta_b^{DW}, \alpha_{ub}^{UP}, \alpha_{db}^{DW}\}$. The objective function (1a) minimizes the reserve and the flexibility service activation costs. Constraints (1b) and (1c) represent the active and reactive power flow on each branch, respectively. Constraints (1d) and (1e) represent the reactive and active power balance at each node, respectively. Constraints (1f) and (1g) represent the linking constraints between the TSO and DSO related to the active and reactive load consumption at the point of common coupling. The activation of reserve capacities considering the ramp rates of operating reserve units are described in (1h) and (1i). In order to adhere to the mathematical principle of implication within the inference dual theorem employed in the IDBD algorithm (Section III-C), it becomes imperative to reassign binary variables, namely α and β, to the flexibility services. The utilization of binary variables is indispensable as the inference dual theorem, rooted in Boolean logic, does not permit the application of the concept of implication to continuous variables. Constraint (1j) shows the offer function of generators. Constraints (1k) represents the maximum available turn-up and turn-down services at each point of common coupling.

3) Expected Flexibility Interest of the DSOs: The following two-stage stochastic programming problem expresses the purpose of DSOs to activate flexibility services based on the proposed AC-OPF model in [2], [30].

$$\underset{\Omega_2}{\text{Minimize}} \left\{ \left(\sum\nolimits_{u \in N_{ub}} \alpha_{ub}^{UP} + \sum\nolimits_{d \in N_{db}} \alpha_{db}^{DW} \right) \right.$$
$$\mathbb{E}_\omega \left[\sum\nolimits_{t \in T} \sum\nolimits_{u \in N_{ub}} F_{ut\omega}^{UPD} \pi_{ut\omega}^{UP} + \right.$$
$$\left. \left. + \sum\nolimits_{t \in T} \sum\nolimits_{d \in N_{db}} F_{dt\omega}^{DWD} \pi_{dt\omega}^{DW} \right] \right\} \tag{2a}$$

subject to :

$$P_{ijt\omega}^{DS} \approx P_{ij\omega,0}^{DS} + (\nabla P_{ij\omega}^{DS}|_0)^T \begin{pmatrix} v_{it\omega}^{s^{DS}} - v_{i\omega,0}^{s^{DS}} \\ v_{jt\omega}^{s^{DS}} - v_{j\omega,0}^{s^{DS}} \\ \theta_{it\omega}^{DS} - \theta_{i\omega,0}^{DS} \\ \theta_{jt\omega}^{DS} - \theta_{j\omega,0}^{DS} \end{pmatrix}, \tag{2b}$$

$$Q_{ijt\omega}^{DS} \approx Q_{ij\omega,0}^{DS} + (\nabla Q_{ij\omega}^{DS}|_0)^T \begin{pmatrix} v_{it\omega}^{s^{DS}} - v_{i\omega,0}^{s^{DS}} \\ v_{jt\omega}^{s^{DS}} - v_{j\omega,0}^{s^{DS}} \\ \theta_{it\omega}^{DS} - \theta_{i\omega,0}^{DS} \\ \theta_{jt\omega}^{DS} - \theta_{j\omega,0}^{DS} \end{pmatrix},$$
$$\forall\, (ij) \in K^{DS},\ t \in T,\ \omega \in \Theta \tag{2c}$$

$$Q_{it\omega}^{DS} = \sum_{(ij) \in K^{DS}} Q_{ijt\omega}^{DS} + v_{it\omega}^{s^{DS}} \sum_{j \in I^{DS}} -B_{ij}^{DS}, \tag{2d}$$

$$P_{g_{it\omega}}^{DS} - P_{L_{it\omega}}^{DS} + PV_{it\omega} + WT_{it\omega} + F_{it\omega}^{DWD} - F_{it\omega}^{UPD} =$$
$$\sum\nolimits_{(ij) \in K_b^{DS}} P_{ijt\omega}^{DS} + v_{it\omega}^{s^{DS}} \sum\nolimits_{j \in I_b^{DS}} G_{ij}^{DS},$$
$$\forall\, i \in I_b^{DS},\ t \in T,\ \omega \in \Theta \tag{2e}$$

$$F_{ut\omega}^{UPD} \leq \alpha_{ub}^{UP} (\Phi_{ut\omega}^{PV} + \Phi_{ut\omega}^{WT}),\ F_{dt\omega}^{DWD} \leq \alpha_{db}^{DW} \Phi_{dt\omega}^{DR},$$
$$\forall\, u \in N_{ub},\ d \in N_{db},\ t \in T,\ \omega \in \Theta \tag{2f}$$

$$\underline{P}_i^{DS} \leq P_{it\omega}^{DS} \leq \overline{P}_i^{DS}, \underline{Q}_i^{DS} \leq Q_{it\omega}^{DS} \leq \overline{Q}_i^{DS},$$
$$\underline{v}_i^{s^{DS}} \leq v_{it\omega}^{s^{DS}} \leq \overline{v}_i^{s^{DS}}, \forall\, i \in I^{DS},\ t \in T,\ \omega \in \Theta \tag{2g}$$

$$\underline{P}_{ij}^{DS} \leq P_{ijt\omega}^{DS} \leq \overline{P}_{ij}^{DS}, \underline{Q}_{ij}^{DS} \leq Q_{ijt\omega}^{DS} \leq \overline{Q}_{ij}^{DS},$$
$$\forall\, (ij) \in K^{DS},\ t \in T,\ \omega \in \Theta \tag{2h}$$

$$\{F_{ut\omega}^{UPD}, F_{dt\omega}^{DWD}\} \in \mathbb{R}_{\geq 0}, \{\alpha_{ub}^{UP}, \alpha_{db}^{DW}\} \in \{0,1\} \tag{2i}$$

$$\{v_{i\omega}^{s^{DS}}, \theta_{i\omega}^{DS}, P_{ij\omega}^{DS}, Q_{ij\omega}^{DS}, P_{i\omega}^{DS}, Q_{i\omega}^{DS}\} \in \mathbb{R} \tag{2j}$$

where,

$$\nabla P_{ij\omega}^{DS}|_0 = \begin{pmatrix} (1 - \frac{v_{j\omega,0}^{DS} cos\theta_{ij\omega,0}^{DS}}{2v_{i\omega,0}^{DS}}) G_{ij}^{DS} - \frac{v_{j\omega,0}^{DS} B_{ij}^{DS} sin\theta_{ij\omega,0}^{DS}}{2v_{i\omega,0}^{DS}} \\ \frac{-v_{i\omega,0}^{DS} G_{ij}^{DS} cos\theta_{ij\omega,0}^{DS}}{2v_{j\omega,0}^{DS}} - \frac{v_{i\omega,0}^{DS} B_{ij}^{DS} sin\theta_{ij\omega,0}^{DS}}{2v_{j\omega,0}^{DS}} \\ v_{i\omega,0}^{DS} v_{j\omega,0}^{DS} G_{ij}^{DS} sin\theta_{ij\omega,0}^{DS} - v_{i\omega,0}^{DS} v_{j\omega,0}^{DS} B_{ij}^{DS} cos\theta_{ij\omega,0}^{DS} \\ -v_{i\omega,0}^{DS} v_{j\omega,0}^{DS} G_{ij}^{DS} sin\theta_{ij\omega,0}^{DS} + v_{i\omega,0}^{DS} v_{j\omega,0}^{DS} B_{ij}^{DS} cos\theta_{ij\omega,0}^{DS} \end{pmatrix}$$

$$\nabla Q_{ij\omega}^{DS}|_0 = \begin{pmatrix} -(1 - \frac{v_{j\omega,0}^{DS} cos\theta_{ij\omega,0}^{DS}}{2v_{i\omega,0}^{DS}}) B_{ij}^{DS} - \frac{v_{j\omega,0}^{DS} G_{ij}^{DS} sin\theta_{ij\omega,0}^{DS}}{2v_{i\omega,0}^{DS}} \\ \frac{v_{i\omega,0}^{DS} B_{ij}^{DS} cos\theta_{ij\omega,0}^{DS}}{2v_{j\omega,0}^{DS}} - \frac{v_{i\omega,0}^{DS} G_{ij}^{DS} sin\theta_{ij\omega,0}^{DS}}{2v_{j\omega,0}^{DS}} \\ -v_{i\omega,0}^{DS} v_{j\omega,0}^{DS} B_{ij}^{DS} sin\theta_{ij\omega,0}^{DS} - v_{i\omega,0}^{DS} v_{j\omega,0}^{DS} G_{ij}^{DS} cos\theta_{ij\omega,0}^{DS} \\ v_{i\omega,0}^{DS} v_{j\omega,0}^{DS} B_{ij}^{DS} sin\theta_{ij\omega,0}^{DS} + v_{i\omega,0}^{DS} v_{j\omega,0}^{DS} G_{ij}^{DS} cos\theta_{ij\omega,0}^{DS} \end{pmatrix}$$

where, Ω_2=$\{v_{it\omega}^{s^{DS}}$, $\theta_{ijt\omega}^{DS}$, $P_{ijt\omega}^{DS}$, $Q_{ijt\omega}^{DS}$, $P_{it\omega}^{DS}$, $Q_{it\omega}^{DS}$, $F_{it\omega}^{UPD}$, $F_{it\omega}^{DWD}$, α_{ub}^{UP}, $\alpha_{db}^{DW}\}$. The objective function (2a) minimizes the activation costs of the flexibility services. The first-order Taylor series expansions of the active and reactive power flow on each branch are explained by equations (2b) and (2c). Constraint (2f) for DSO is similar to (1k) for the TSO. The active and reactive power balance at each bus of the distribution system are expressed by equations (2d) and (2e), respectively.

4) Expected Flexibility Interest of the Retailers: The Retailer activates flexibility services for the peak price periods if the submitted bids by the service providers are lower than the retailer's anticipation of the electricity price ($\pi_{ut\omega}^{EX}$). The following two-stage stochastic programming problem describes the purpose of retailers to activate flexibility services.

$$\underset{\Omega_3}{\text{Minimize}} \left\{ \left(\sum\nolimits_{u \in N_{ub}} \alpha_{ub}^{UP} + \sum\nolimits_{d \in N_{db}} \alpha_{db}^{DW} \right) \right.$$
$$\mathbb{E}_\omega \left[\sum\nolimits_{t \in T} \sum\nolimits_{u \in N_{ub}} F_{ut\omega}^{UPR} (\pi_{ut\omega}^{EX} - \pi_{ut\omega}^{UP}) \right.$$
$$\left. \left. + \sum\nolimits_{t \in T} \sum\nolimits_{d \in N_{db}} F_{dt\omega}^{DWR} (\pi_{dt\omega}^{DW} - \pi_{dt\omega}^{EX}) \right] \right\} \tag{3a}$$

subject to:

$$F_{ut\omega}^{UPR} \leq \alpha_{ub}^{UP} (\Phi_{ut\omega}^{PV} + \Phi_{ut\omega}^{WT}), F_{dt\omega}^{DWR} \leq \alpha_{db}^{DW} \Phi_{dt\omega}^{DR},$$
$$\forall u \in N_{ub},\ d \in N_{db}, t \in T, \omega \in \Theta \tag{3b}$$

$$\{F_{ut\omega}^{UPR}, F_{dt\omega}^{DWR}\} \in \mathbb{R}_{\geq 0}, \{\alpha_{ub}^{UP}, \alpha_{db}^{DW}\} \in \{0,1\} \tag{3c}$$

where, Ω_3=$\{F_{dbt\omega}^{DWR}$, $F_{ubt\omega}^{UPR}$, α_{ub}^{UP}, $\alpha_{db}^{DW}\}$. Constraint (3b) for the retailer is the same as (1k) for the TSO.

III. Optimization Procedure

It is assumed that the EER providers embed all equality and inequality constraints and possible cost functions of their services into the submitted offers. All offer functions are assumed convex.

The proposed coordination process, as illustrated in Fig.3, involves the participation of the FMO, an independent regulated non-profit organization that maintains communication with the TSO, DSOs, Retailers, and EER providers. The FMO obtains the necessary data to formulate a Two-Stage Stochastic Bilevel Programming problem for coordinating the TSO-DSO-Retailer interactions within the flexibility market. Subsequently, the FMO disseminates the dispatch commands to the market participants following the completion of the market clearing process.

The process and various steps of our proposed modeling setup and problem formulation are depicted in Fig. 4, providing a visual representation. In the first step, we reframe the expected flexibility interests of the TSO, DSOs, and Retailers to create our proposed Two-Stage Stochastic Bilevel Programming with Embedded DSO-Retailer Activations. The upper level of our bilevel programming problem addresses the TSO's expected flexibility interest, which is formulated as a Mixed-Integer Linear Programming (MILP) problem. The

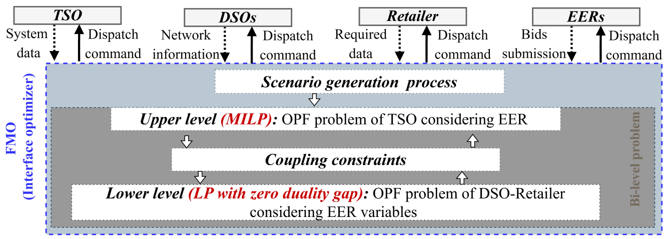

Fig. 3. The proposed coordination process for TSO-DSO-Retailer coordination.

lower level problem involves the merging optimization problem that considers the flexibility interest functions of DSOs and retailers, which can be modeled as a linear programming problem. In the second step, we utilize the Karush-Kuhn-Tucker (KKT) conditions to substitute the lower-level linear programming problem with its optimality conditions. This step transforms the Bilevel problem into a single-level Stochastic Mathematical Program with Complementarity Constraints. In the third step, we reformulate our single-level Stochastic Mathematical Program as a Generalized Disjunctive Programming (GDP) Model. Finally, in the fourth step, we employ our proposed SIDBD algorithm to decompose the GDP model into two Mixed-Logical Linear Programming (MLLP) problems, namely the master and subproblems, which are subsequently solved to address the main problem.

Based on our proposed coordination mechanism and modeling setup, the FMO aims to find a solution to the following two-stage bilevel stochastic programming problem.

A. Two-Stage Stochastic Bilevel Programming with Embedded DSO-Retailer Activations

Here, we assume that the FMO receives all the required data from TSO, DSOs, and retailers to form the optimization problem. Some monitoring devices like PMUs and micro PMUs are required to achieve this goal in both transmission and distribution networks. Then, the FMO considers the stochastic flexibility interest of the DSOs, retailers, and TSO in one programming problem. Therefore, we have a two-stage stochastic bilevel mixed-integer programming problem as follows.

1) Upper Level: The upper level is as follows.

$$\underset{\Omega_4}{\text{Minimize}} \text{ Objective Function of (1a)} \tag{4a}$$

subject to (1b)-(1d), (1h)-(1j), (1l)-(1p), (2i), (3c), and:

$$P^{TS}_{g_{bt\omega}} - P^{TS}_{L_{bt\omega}} - \sum_{u\in N_{ub}} (F^{UPT}_{ubt\omega} + F^{UPD}_{ubt\omega} + F^{UPR}_{ubt\omega}) + \sum_{d\in N_{db}} (F^{DWT}_{dbt\omega} + F^{DWD}_{dbt\omega} + F^{DWR}_{dbt\omega}) = \sum_{(bc)\in K^{TS}} P^{TS}_{bct} + v^{s^{TS}}_{bt\omega} \sum_{c\in I^{TS}} G^{TS}_{bc} \ , \quad \forall\, b\in I^{TS}, t\in T,\ \omega\in\Theta \tag{4b}$$

$$P^{TS}_{L_{pt\omega}} = P^{DS}_{g_{pt\omega}},\ Q^{TS}_{L_{pt\omega}} = Q^{DS}_{g_{pt\omega}},\ v^{s^{TS}}_{pt\omega} = v^{s^{DS}}_{pt\omega}, \theta^{TS}_{pt\omega} = \theta^{DS}_{pt\omega} \quad \forall\, p\in PCC,\ t\in T,\ \omega\in\Theta \tag{4c}$$

$$F^{UPT}_{upt\omega} + F^{UPD}_{upt\omega} + F^{UPR}_{upt\omega} \le \alpha^{UP}_{up}(\Phi^{PV}_{upt\omega} + \Phi^{WT}_{upt\omega}) \ , \quad F^{DWT}_{dpt\omega} + F^{DWD}_{dpt\omega} + F^{DWR}_{dpt\omega} \le \alpha^{DW}_{dp}\Phi^{DR}_{dpt\omega} \ , \quad \forall\, p\in PCC,\ u\in N_{ub},\ d\in N_{db},\ t\in T,\ \omega\in\Theta \tag{4d}$$

$$\{\alpha^{UP}_{ub}, \alpha^{DW}_{db}\} \in \{0,1\} \tag{4e}$$

where, Ω_4={Ω_1, Ω_2, Ω_3, $P^{TS}_{L_{pt\omega}}$, $Q^{TS}_{L_{pt\omega}}$}. Constraints (4c) and (4d) illustrate the common constraints in the TSO-DSO-Retailer coordination problem. Here, variables $P^{DS}_{g_{pt\omega}}$, $Q^{DS}_{g_{pt\omega}}$, $\theta^{DS}_{pt\omega}$, $v^{s^{DS}}_{pt\omega}$, $F^{DWD}_{dbt\omega}$, $F^{UPD}_{ubt\omega}$, $F^{DWR}_{dbt\omega}$, and $F^{UPR}_{ubt\omega}$ are determined through following lower-level optimization problem.

2) Lower Level: We formulate the lower-level problem as follows, which is the merged flexibility interest functions of DSOs and retailers.

$$\underset{\Omega_5}{\text{Minimize}} \sum_{t\in T}\sum_{b\in I^{TS}}\sum_{u\in N_{ub}} F^{UPD}_{ubt\omega}\pi^{UP}_{ubt\omega} + \sum_{t\in T}\sum_{b\in I^{TS}}\sum_{d\in N_{db}} F^{DWD}_{dbt\omega}\pi^{DW}_{dbt\omega} + \sum_{t\in T}\sum_{b\in I^{TS}}\sum_{u\in N_{ub}} F^{UPR}_{ut\omega}(\pi^{EX}_{ut\omega} - \pi^{UP}_{ut\omega}) + \sum_{t\in T}\sum_{b\in I^{TS}}\sum_{d\in N_{db}} F^{DWR}_{dt\omega}(\pi^{DW}_{dt\omega} - \pi^{EX}_{dt\omega}) \tag{5a}$$

subject to (2b)-(2d), (2g)-(2j), and:

$$P^{DS}_{g_{ibt\omega}} - P^{DS}_{L_{ibt\omega}} + PV_{ibt\omega} + WT_{ibt\omega} + F^{DWT}_{ibt\omega} - F^{UPT}_{ibt\omega} + F^{DWD}_{ibt\omega} - F^{UPD}_{ibt\omega} + F^{DWR}_{ibt\omega} - F^{UPR}_{ibt\omega} = \sum_{(ij)\in K^{DS}_b} P^{DS}_{ijbt} + v^{s^{DS}}_{ibt}\sum_{j\in I^{DS}_b} G^{DS}_{ijb} \ , \quad \forall\, b\in I^{TS}\ ,\ i\in I^{DS}_b\ ,\ t\in T,\ \omega\in\Theta \tag{5b}$$

where, Ω_5={$v^{s^{DS}}_{ibt\omega}$, $\theta^{DS}_{ibt\omega}$, $P^{DS}_{ijbt\omega}$, $Q^{DS}_{ijbt\omega}$, $P^{DS}_{ibt\omega}$, $Q^{DS}_{ibt\omega}$, $F^{UPD}_{ibt\omega}$, $F^{DWD}_{ibt\omega}$, $F^{UPR}_{ibt\omega}$, $F^{DWR}_{ibt\omega}$}. The objective function (5a) minimizes the total cost of flexibility service activation. Constraint (5b) is the new power balance with the impact of flexibility service activations. Here, since the lower-level problem is a linear programming problem with a zero duality gap, we replace it with its Karush-Kuhn-Tucker (KKT) optimality conditions, which are presented in the following section. The KKT conditions offer necessary and sufficient conditions for optimality in constrained optimization problems.

3) Reformulated Stochastic Mathematical Program with Complementarity Constraints: Now, we have the single-level reformulated form of our two-stage stochastic bilevel mixed-integer programming problem:

$$\underset{\Omega_4}{\text{Minimize}} \text{ Objective Function of (1a)} \tag{6a}$$

subject to (1b)-(1d), (1h)-(1j), (1l)-(1p), (2b)-(2d), (2i), (2j), (3c), (4b)-(4e), (5b), and:

$$\{\lambda_{ibt\omega}, \Pi_{ibt\omega}, \mu_{ijbt\omega}, \delta_{ijbt\omega}\} \in \mathbb{R},\ \{D\} \in \mathbb{R}_{\ge 0} \tag{6b}$$

$$\lambda_{ibt\omega} + D_{3,ibt\omega} - D_{4,ibt\omega} = 0\ , \quad \Pi_{ibt\omega} + D_{5,ibt\omega} - D_{6,ibt\omega} = 0\ , \quad \forall\, b\in I^{TS}\ ,\ i\in I^{DS}_b\ ,\ t\in T,\ \omega\in\Theta \tag{6c}$$

$$\Pi_{ibt\omega}\sum_{j\in I^{DS}_b} B^{DS}_{ijb} - \lambda_{ibt\omega}\sum_{j\in I^{DS}_b} G^{DS}_{ijb} + D_{1,ibt\omega} - D_{2,ibt\omega} + \sum_{(ij)\in K^{DS}_b}\mu_{ijbt\omega}\left(-\frac{\partial P^{DS}_{ijbt\omega}}{\partial v^{DS}_{ibt\omega}}\right) + \sum_{(ij)\in K^{DS}_b}\delta_{ijbt\omega}\left(-\frac{\partial Q^{DS}_{ijbt\omega}}{\partial v^{DS}_{ibt\omega}}\right) = 0\ , \forall\, b\in I^{TS},\ i\in I^{DS}_b\ , t\in T,\ \omega\in\Theta \tag{6d}$$

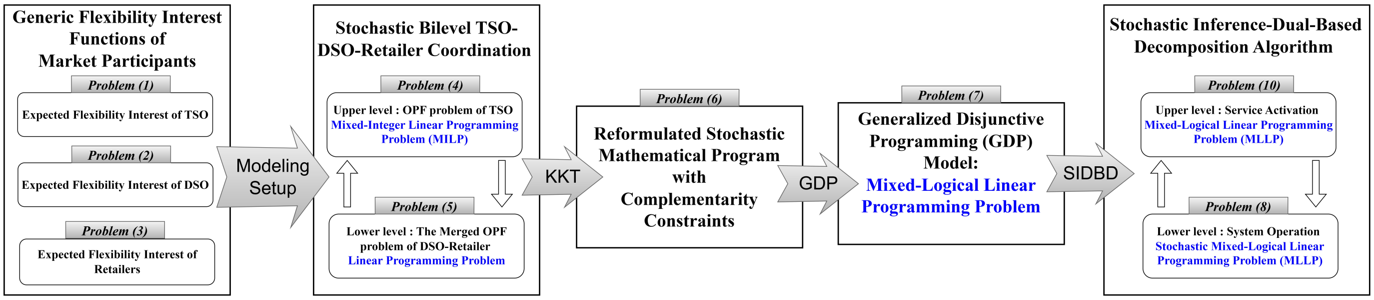

Fig. 4. The proposed problem formulation steps for TSO-DSO-Retailer coordination.

$$\sum\nolimits_{(ij)\in K_b^{DS}} \mu_{ijbt\omega}\left(\frac{\partial P_{ijbt\omega}^{DS}}{\partial \theta_{ibt\omega}^{DS}}\right) + \delta_{ijbt\omega}\left(\frac{\partial Q_{ijbt\omega}^{DS}}{\partial \theta_{ibt\omega}^{DS}}\right) = 0, \quad \forall i \in I_b^{DS}, t \in T,\ \omega \in \Theta \tag{6e}$$

$$S_B \pi_{pt\omega}^{UP} - \lambda_{upt\omega} + D_{11,ibt\omega} = 0, \tag{6f}$$

$$S_B \pi_{pt\omega}^{DW} + \lambda_{dpt\omega} + D_{12,ibt\omega} = 0, \tag{6g}$$

$$S_B(\pi_{ut\omega}^{EX} - \pi_{ut\omega}^{UP}) - \lambda_{upt\omega} + D_{13,ibt\omega} = 0, \tag{6h}$$

$$S_B(\pi_{dt\omega}^{DW} - \pi_{dt\omega}^{EX}) + \lambda_{dpt\omega} + D_{14,ibt\omega} = 0, \quad \forall p \in PCC, u \in N_{up}, d \in N_{dp}, t \in T, \omega \in \Theta \tag{6i}$$

$$0 \le \sum\nolimits_{u\in N_{ub}} F_{upt\omega}^{UPT} \perp \sum\nolimits_{d\in N_{db}} F_{dpt\omega}^{DWT} \ge 0, \quad \forall p \in PCC,\ t \in T, \omega \in \Theta \tag{6j}$$

$$0 \le D_{3,ibt\omega} \perp (\overline{P}_{g_{ib}}^{DS} - P_{g_{ibt\omega}^{DS}}) \ge 0, \tag{6k}$$

$$0 \le D_{4,ibt\omega} \perp (P_{g_{ibt\omega}^{DS}} - \underline{P}_{g_{ib}}^{DS}) \ge 0, \quad \forall\, b \in I^{TS},\ i \in I_b^{DS},\ t \in T,\ \omega \in \Theta \tag{6l}$$

and same complementary slackness conditions as (6k)-(6l) for constraints (2g) and (2h) in problem (5). (6m)

where, Ω_6={Ω_4, Ω_5, D, λ, Π, μ, δ}. The derivative of the Lagrangian function with respect to the optimization variables of the DSO and retailer stochastic programming problems is presented by constraints (6c)-(6i). Constraint (6j) ensures that turn-up and turn-down service activation will not occur at one node of the transmission system, simultaneously. Constraints (6k)-(6m) express the complementary slackness conditions for each inequality constraint in the lower-level optimization problem.

B. Generalized Disjunctive Programming (GDP) Model

In contrast to the reformulated stochastic mathematical program with complementarity constraints, Generalized Disjunctive Programming (GDP) offers several advantages. The GDP surpasses existing optimization models by accommodating flexible disjunctive constraints, thereby rendering it exceptionally suitable for addressing intricate decision-making scenarios amidst uncertainty. Its capability to handle nonlinear and nonconvex problems commonly encountered in real-world optimization challenges enables more precise and realistic modeling.

The realms of real-time operation of power systems and the electricity market are replete with inherent complexities, rendering many real-world optimization problems highly intricate. In order to effectively tackle these challenges, GDP emerges as a powerful tool that facilitates the formulation and solution of such intricate problems. By incorporating GDP into the modeling process, it becomes possible to achieve a more accurate and realistic representation of these complex problems.

One of the key advantages of GDP lies in its ability to incorporate logical relationships between decision variables and constraints. This allows for the explicit specification of logical conditions, utilizing operators such as "or" and "and," thereby capturing the dependencies and interconnections among variables. This capability proves to be particularly valuable when dealing with problems involving binary decision variables or when specific logical conditions must be satisfied. As a result, the modeling process gains enhanced expressiveness, enabling a more comprehensive representation of the problem at hand. Additionally, GDP offers robust support for global optimization, enabling the exploration of the entire feasible region to find the best solutions. The framework employs efficient solution techniques, including decomposition methods, further enhancing its effectiveness in tackling diverse optimization problems.

Therefore, here, we reformulate the optimization problem (6) into a generalized disjunctive programming model as follows.[1]

$$\underset{\Omega_7}{\text{Minimize}} \text{ Objective Function of (1a)} \tag{7a}$$

subject to (1b)-(1d), (1h)-(1j), (1l)-(1p), (2b)-(2d), (2i), (2j), (3c), (4b)-(4e), (5b), (6b)-(6i), and the equivalent disjunctive forms of constraints (6k) and (6l) as follows:

$$\begin{bmatrix} Y_{1,ibt\omega} \\ D_{3,ibt\omega} = 0 \\ D_{4,ibt\omega} = 0 \end{bmatrix} \vee \begin{bmatrix} Y_{2,ibt\omega} \\ D_{3,ibt\omega} = 0 \\ P_{g_{ibt\omega}^{DS}} = \underline{P}_{g_{ib}}^{DS} \end{bmatrix} \vee \begin{bmatrix} Y_{3,ibt\omega} \\ D_{4,ibt\omega} = 0 \\ \overline{P}_{g_{ib}}^{DS} = P_{g_{ibt\omega}^{DS}} \end{bmatrix},$$

$$\begin{aligned} &Y_{1,ibt\omega} \implies \neg Y_{2,ibt\omega} \wedge \neg Y_{3,ibt\omega}, \\ &Y_{2,ibt\omega} \implies \neg Y_{1,ibt\omega} \wedge \neg Y_{3,ibt\omega}, \\ &Y_{3,ibt\omega} \implies \neg Y_{1,ibt\omega} \wedge \neg Y_{2,ibt\omega}, \\ &\quad \forall\, b \in I^{TS},\ i \in I_b^{DS},\ t \in T,\ \omega \in \Theta \end{aligned} \tag{7b}$$

[1] $\vee$:Logical disjunction (OR), $\implies$:Mathematical implication, $\wedge$:Logical conjunction (AND)

$$\text{Similar to (7b) for complementary slackness condition of constraints (2g) and (2h) presented in (6m),} \tag{7c}$$

and the equivalent disjunctive form of constraint (6j) as follows:

$$\begin{gathered}\begin{bmatrix} Y_{21,ibt\omega} \\ 0 \leq \sum_{u\in N_{ub}} F^{UPT}_{upt\omega} \end{bmatrix} \vee \begin{bmatrix} Y_{22,ibt\omega} \\ \sum_{d\in N_{db}} F^{DWT}_{dpt\omega} \geq 0 \end{bmatrix}, \\ Y_{21,ibt\omega} \implies \neg Y_{22,ibt\omega}\ , Y_{22,ibt\omega} \implies \neg Y_{21,ibt\omega}\ , \\ \forall\, b \in I^{TS}\ ,\ i \in I^{DS}_{b},\ t \in T,\ \omega \in \Theta \end{gathered} \tag{7d}$$

where, Ω_7={Ω_6, Y}. Logical disjunctions (7b)-(7d) ensure that all equality and inequality constraints related to each true Boolean variable are added to the main optimization problem. For instance, utilization of the logical propositions in (7b) ensure that only one of the Boolean variables $Y_{1,ibt\omega}$, $Y_{2,ibt\omega}$, and $Y_{3,ibt\omega}$ is true at the same time.

Generally, the Big-M method is of the most common methods to find the solution to GDP problems which reformulates the problem as a mixed-integer linear/nonlinear programming model. In the process of reformulating GDP by the Big-M method, the connections between the binary variables and the corresponding constraints are relatively obfuscated [31]. Moreover, since using the Big-M method for a GDP problem with a large number of variables and complementarity constraints forms an NP-hard problem [32], utilization of the Big-M method leads to weak continuous relaxations and hence it is unsolvable in practice [33]. In our study, we utilized the indicator constraint method in the optimization process of mixed-integer programming (MIP), which has the potential to yield weaker relaxations. Although this characteristic could potentially result in longer computation time, we observed a significant improvement in both performance time and problem complexity in our specific case. This improvement was achieved by implementing our proposed IDBD algorithm, which has a separable structure. Importantly, all IDBD cut calculation problems associated with our method are independent and can be solved simultaneously in a parallel computing mode. Therefore, in this paper, we employ the indicator constraints method and MIP solver CPLEX in the GAMS platform to handle our GDP problem.

Due to a large number of binary variables, the plausible sub-problems will be non-convex. Therefore, obtaining the optimality cuts using the Standard Benders Decomposition (SBD) method is challenging. Since it is not straightforward to solve optimization problems with the non-convexity in the sub-problem with the SBD method, we propose a new Stochastic Inference-Dual-Based Decomposition (SIDBD) algorithm in the current paper as follows.

C. Stochastic Inference-Dual-Based Decomposition (SIDBD) Algorithm

The IDBD's incorporation of logical constraints directly into the decomposition framework enhances modeling flexibility, and it enables the accurate representation of complex problems with logical relationships between decision variables. This leads to improved problem decomposition by grouping variables and constraints based on their logical dependencies, resulting in smaller and more manageable subproblems. Moreover, the IDBD algorithm reduces the computational effort by exploiting logical conditions, minimizing the number of required IDBD cuts. The inclusion of logical relationships also accelerates convergence by guiding the solution process toward feasible regions of the problem space. Furthermore, the IDBD algorithm effectively handles discrete decisions, making it suitable for mixed-integer programming problems. Ultimately, these advantages culminate in higher solution quality, affirming the superiority of the IDBD algorithm in solving optimization problems efficiently and effectively.

Accordingly, in this section, we propose a new modification of the decomposition method for the stochastic TSO-DSO-Retailer coordination problem. In the context of our two-stage stochastic programming problem, we introduce binary variables representing service activation as ”here-and-now” variables that remain constant across scenarios, while other variables are classified as ”wait-and-see” variables, contingent upon specific scenarios. The optimization process follows a sequential approach, where the ”here-and-now” variables are optimized in the first stage, independent of future uncertainty, and the ”wait-and-see” variables are optimized in the second stage, considering real-time operational aspects influenced by encountered scenarios. These steps align with the principles of stochastic programming, effectively addressing uncertainties.

Accordingly, to form the master problem, we consider the binary variables corresponding to the turn-up and turn-down service activations, α^{UP}_{ub} and α^{DW}_{db}, as complicating variables (sub-set N^{mv}_{b}). We also include the sub-problem relaxation constraints that are in the form of the complicating variables in the master problem. Whilst the master problem variables are considered exogenous parameters, we optimize the rest of the variables through the sub-problem. At each iteration of SIDBD, since there is a specific IDBD cut for each scenario, we introduce the concept of ”expected IDBD cuts”. The process of the SIDBD algorithm using the proposed FMO organization and scenario tree generation is shown in Fig. 5.

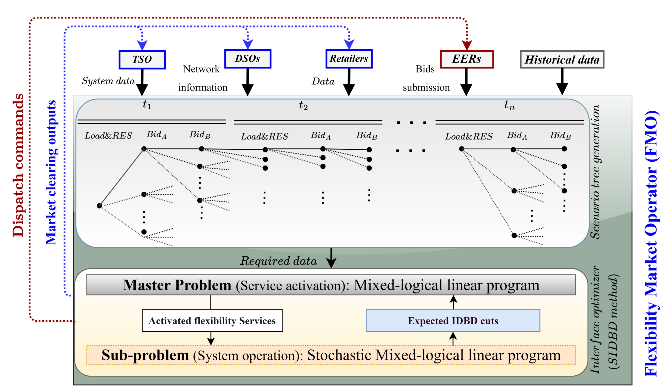

Fig. 5. The stochastic Inference-Dual-Based Decomposition (IDBD) algorithm for solving the embedded TSO-DSO-retailer coordination problem using the proposed FMO organization.

Accordingly, we formulate the sub-problem, master problem, cut calculation process, and sub-problem relaxation as follows.

1) sub-problem: Considering the complicating variables as exogenous parameters $\alpha_{ub}^{UP^*}$ and $\alpha_{db}^{DW^*}$, forms the SIDBD sub-problem corresponding to the GDP problem (7) as follows.

$$\underset{\Omega_7}{\text{Minimize}} \text{ Objective Function of (1a)} \tag{8a}$$

$$\begin{aligned} &\text{subject to (1b)-(1d), (1h)-(1j), (1l)-(1p), (2b)-(2d), (2i),} \\ &\quad \text{(2j), (3c), (4b)-(4e), (5b), (6b)-(6i), (7b)-(7d), and:} \\ &\alpha_{ib}^{UP} = \alpha_{ib}^{UP^*},\ \alpha_{ib}^{DW} = \alpha_{ib}^{DW^*}, \\ &\forall\ b \in I^{TS},\ i \in I_b^{DS},\ t \in T \end{aligned} \tag{8b}$$

At each iteration, the expected upper bound of the original optimization problem (7) is determined through solving the sub-problem (8) for all scenarios. The process is finished provided that the difference between the expected lower and the expected upper bounds is less than a pre-defined tolerance.

2) sub-problem relaxation: The proposed sub-problem relaxation is not a typical sort of relaxation since, rather than the sub-problem variables, it is defined in terms of the master problem variables. The number of SIDBD iterations depends on the process of evaluating all enumeration of complicating variables (i.e. the generated IDBD cuts). Each enumeration of complicating variables forms a specific sub-problem that needs to have a feasible solution to determine the corresponding SIDBD cut. Infeasible sub-problems result in no-good cuts that significantly increase the computational complexity of the SIDBD algorithm [20]. Therefore, removing all enumerations of complicating variables with a resulting infeasible sub-problem is crucial. To achieve this goal, we propose new constraints in the master problem based on the structure of the sub-problem to eliminate the enumerations of complicating variables corresponding to the infeasible sub-problems. These constraints are called "sub-problem relaxation" since they relax the master problem based on the structure of the sub-problem.

Nonetheless, determining appropriate relaxation constraints depends on the structure of the sub-problem of different optimization problems. After utilizing a sub-problem relaxation, the remaining IDBD cuts in the master problem should satisfy the following two properties [31].

Property 1. *The SIDBD cuts related to the infeasible solutions should be excluded from the master problem.*

Property 2. *Any SIDBD cuts related to the feasible solutions must not be excluded from the master problem.*

Property 1 ensures the finite convergence of the SIDBD algorithm when the variables in the master problem have finite domains. In the SIDBD algorithm, each cut generated corresponds to a specific combination of flexibility service activation. The flexibility service is related to the ability to adjust certain parameters or resources in the system. In order to effectively utilize these flexibility services, the FMO needs to determine all acceptable combinations of flexibility service activations in advance. By doing so, the FMO can remove any combinations of flexibility service activation that lead to infeasible subproblems, thereby reducing the computational complexity of the TSO-DSO-retailer coordination problem. Essentially, Property 1 indicates that the FMO should filter out infeasible combinations and focus on feasible ones, thus simplifying the problem.

Property 2 guarantees optimality in the SIDBD algorithm by ensuring that the cuts considered in the master problem do not remove any feasible solutions. Property 2 specifies that the FMO, while utilizing the subproblem relaxation, should not eliminate any combination of flexibility service activation that corresponds to a feasible solution for the subproblem. In other words, the FMO should not discard any feasible options during the optimization process. By considering both Property 1 and Property 2, the SIDBD algorithm can achieve convergence within a small number of iterations, making the overall optimization process more efficient.

Accordingly, we propose sub-problem relaxation constraints in the master problem of the TSO-DSO-Retailer coordination problem as follows.

$$\begin{aligned} &\sum\nolimits_{u\in N_{ub}} \tilde{F}_{upt\omega}^{UP,Min} \le \sum\nolimits_{u\in N_{ub}} \alpha_{up}^{UP} (\Phi_{upt\omega}^{PV} + \Phi_{upt\omega}^{WT})\,, \\ &\sum\nolimits_{d\in N_{db}} \tilde{F}_{dpt\omega}^{DW,Min} \le \sum\nolimits_{d\in N_{db}} \alpha_{dp}^{DW} \Phi_{dpt\omega}^{DR}\,, \\ &\forall\ p \in PCC,\ t \in T,\ \omega \in \Theta \end{aligned} \tag{9}$$

where, $\tilde{F}_{upt\omega}^{UP,Min}$ and $\tilde{F}_{dpt\omega}^{DW,Min}$ are the minimum level of the required flexibility services, calculated by the interface optimizer of FMO, which is needed to be activated to cope with uncertainties in the real-time operation of the system. These constraints ensure that the summation of the maximum possible service activation is greater than the summation of the minimum level of required services.

Considering the proposed sub-problem relaxation (9), Property 1, and Property 2, we have the following Theorem.

Theorem 1. *The proposed sub-problem relaxation* (9) *to determine the SIDBD cuts is valid (i.e. it meets properties* 1 *and* 2 *above) for our stochastic TSO-DSO-Retailer coordination problem.*

Proof. We need to show that Property 1 and Property 2 are valid for our proposed sub-problem relaxation to prove this theorem. Recall that each cut is related to a specific combination of complicating variables that are associated with the activation of different flexibility services. Evidently, due to the power balance equality constraints (4b) and (5b), the sub-problem (8) is infeasible if the summation of the maximum available potential flexibility is lower than the summation of the required services. Consequently, adding constraint (9) to the master problem only excludes the cuts related to the combination of the complicating variable with an infeasible solution and it does not exclude any cut related to the feasible solution of the sub-problem (8). □

The proposed sub-problem relaxation in **Theorem 1** is one of the key contributions of this paper. In our proposed framework, the complicating variables in the master problem are considered "here-and-now" variables at the first stage of the SIDBD algorithm. Each combination of complicating variables in the master problem results in a different sub-problem which forms a specific SIDBD cut. The concept of inference in the inference duality theory (see **Definition 3** and **Theorem 3** in Appendix) is applicable only if the resulting sub-problem has a feasible solution. Since the convergence rate of the SIDBD algorithm depends on finding the SIDBD cuts related to the feasible sub-problems, proposing a method that identifies all SIDBD cuts corresponding to the feasible sub-problems is crucial. The impact of the proposed sub-problem relaxation in the optimization procedure of the SIDBD algorithm is shown in the following illustrative example.

a) Illustrative Example: If the number of combinations of complicating variables with infeasible sub-problems increases, without considering the proposed sub-problem relaxation constraints, the SIDBD algorithm should solve all resulting sub-problems to find the required feasible ones to converge to the final solution. Let consider we have ten binary variables. All enumerations of the complicating variables form 2^{10} combinations. Let assume only 17 out of 1024 combinations result in feasible sub-problems. To avoid any combination with an infeasible sub-problem, we represent the relationship between the infeasibility of the sub-problem and the different enumeration of complicating variables in the master problem, using our sub-problem relaxations constraints. Therefore, we can remove the no-good cuts without solving the sub-problem which highly affects the convergence rate of the algorithm. Consequently, instead of 1024 combinations of complicating variables in the master problem, the number of enumerations decreases to 17.

3) Master Problem: Using the sub-problem relaxation and the concept of strong inference duality (see **Definition 3** and **Theorem 3** in Appendix), our proposed master problem optimizes the complicating variables and the expected values of the lower bound. We formulate the master problem as follows.

$$\underset{g_{rn},\alpha_{ubr}^{UP},\alpha_{dbr}^{DW},Z_r}{\text{Minimize}} \quad Z_r \tag{10a}$$

subject to (4e), (9), and:

$$Z_r \geq \Gamma_{rn} g_{rn}, \forall\, n \in \{1,..,2^{N_b^{mv}}\}, r \in IT \tag{10b}$$

$$g_{rn} \implies \alpha_{ubr}^{UP},\; g_{rn} \implies \alpha_{dbr}^{DW}, \quad \forall\{n \in \{1,..,2^{N_b^{mv}}\}, r \in IT\} \tag{10c}$$

$$\sum\nolimits_{n} g_{rn} = 1,\; \forall\; r \in IT,\; g_{rn} \in \{0,1\} \tag{10d}$$

The objective function (10a) finds the lower bound of our SIDBD algorithm. Constraint (10b) is related to the IDBD cuts. The EER activation modes are modeled through constraint (10c). Constraint (10d) illustrates that, at each iteration, only one combination of service activation is selected.

4) IDBD Cut Calculation: In our proposed SIDBD algorithm, the IDBD cuts constitute the master problem. Here, before proposing our cut calculation process, we need to define expected SIDBD cuts.

Definition 2. *The expected IDBD cut is defined by calculating the expected values of slope and intercepts of different cuts over all scenarios.*

Considering the expected IDBD cuts, the sub-problem (8), and the master problem (10), the optimality theorem for our TSO-DSO-Retailer coordination problem is as follows:

Theorem 2. *our SIDBD algorithm converges to an approximate optimal solution in a finite number of steps.*

Proof. Consider Theorem 1 and recall that the sub-problem (8) returns the expected upper bound. Therefore, the lower bound which is the solution to the master problem should be in the form of the expected value to guarantee the convergence of our proposed SIDBD algorithm. Since the master problem is based on the SIDBD cuts, considering expected SIDBD cuts assures the convergence of the model. □

The following shows the process of calculating the SIDBD cuts. Based on the set of complicating variables ($N_b^{mv} \subseteq N_{ub}$), we have $2^{N_b^{mv}}$ SIDBD cuts. At each iteration, the value of Γ_{rn}, which is the tightest bound to the master problem, is determined based on the optimal values of complicating variables α_{ubr}^{UP*} and α_{dbr}^{DW*} in the sub-problem solutions using the concept of strong inference duality in **Theorem 3**.

$$\begin{aligned}\Gamma_{rn} = &\sum\nolimits_{b\in I^{TS}}\Big(\sum\nolimits_{u\in N_{ub}}\bar{\bar{\alpha}}_{ubrn}^{UP} + \sum\nolimits_{d\in N_{db}}\bar{\bar{\alpha}}_{dbrn}^{DW}\Big)+\\ &\mathbb{E}_\omega\Big(\sum\nolimits_{b\in N_{UP}^{TS}} f_b(\bar{\bar{P}}_{g_{brnt\omega}}^{TS} - \hat{P}_{g_{bt}}^{TS}) +\\ &\sum\nolimits_{b\in N_{DW}^{TS}}\pi_{bt\omega}^{RD} f_b(\hat{P}_{g_{bt}}^{TS} - \bar{\bar{P}}_{g_{brnt\omega}}^{TS})+\\ &\sum\nolimits_{b\in I^{TS}}\sum\nolimits_{d\in N_{db}}\bar{\bar{F}}_{dbrnt\omega}^{DWT}\pi_{dbt\omega}^{DW}+\\ &\sum\nolimits_{b\in I^{TS}}\sum\nolimits_{u\in N_{ub}}\bar{\bar{F}}_{ubrnt\omega}^{UPT}\pi_{ubt\omega}^{UP}\Big)\end{aligned} \tag{11a}$$

where, $\bar{\bar{\alpha}}_{ubrn}^{UP}$ and $\bar{\bar{\alpha}}_{dbrn}^{DW}$ represents the enumeration of the complicating variables, and $\bar{\bar{F}}_{ubrnt\omega}^{UPT}$, $\bar{\bar{F}}_{dbrnt\omega}^{DWT}$, and $\bar{\bar{P}}_{g_{brnt\omega}}^{TS}$ are calculated through following equations.

$$\begin{aligned}&\bar{\bar{F}}_{ubrnt\omega}^{UPT} = \{F_{ubrt\omega}^{UPT*} | \bar{\bar{\alpha}}_{ubrn}^{UP} \xrightarrow{\Omega_7^*} Y_{21,ubrt\omega}{}^*\},\\ &\forall\; b \in I^{TS}, u \in N_{ub}, t \in T, n \in \{1,..,2^{N_b^{mv}}\}, r \in IT\end{aligned} \tag{11b}$$

$$\begin{aligned}&\bar{\bar{F}}_{dbrnt\omega}^{DW} = \{F_{dbrt\omega}^{DWT*} | \bar{\bar{\alpha}}_{dbrn}^{DW} \xrightarrow{\Omega_7^*} Y_{22,dbrt\omega}{}^*\},\\ &\forall\; b \in I^{TS}, d \in N_{db}, t \in T, n \in \{1,..,2^{N_b^{mv}}\}, r \in IT\end{aligned} \tag{11c}$$

$$\begin{aligned}&\bar{\bar{P}}_{g_{brnt\omega}}^{TS} = \{P_{g_{brt\omega}}^{TS}{}^* | \bar{\bar{\alpha}}_{ubrn}^{UP} \xrightarrow{\Omega_7^*} \beta_b^{UP*}\},\\ &\forall b \in N_{UP}^{TS}, i \in I_b^{DS}, t \in T, n \in \{1,..,2^{N_b^{mv}}\}, r \in IT\end{aligned} \tag{11d}$$

$$\begin{aligned}&\bar{\bar{P}}_{g_{brnt\omega}}^{TS} = \{P_{g_{brt\omega}}^{TS}{}^* | \bar{\bar{\alpha}}_{dbrn}^{DW} \xrightarrow{\Omega_7^*} \beta_b^{DW*}\},\\ &\forall b \in N_{DW}^{TS}, i \in I_b^{DS}, t \in T, n \in \{1,..,2^{N_b^{mv}}\}, r \in IT\end{aligned} \tag{11e}$$

Superscript * shows the optimal solution of the sub-problem. Symbol $\longrightarrow$ represents the implication with respect to $\Omega_7{}^*$. **Algorithm 1** explains the process of our proposed SIDBD algorithm.

In summary, using the theorem of strong inference duality (see Appendix VII), our proposed SIDBD algorithm can handle situations where the sub-problem has binary variables. Our proposed sub-problem relaxation technique reduces the number of SIDBD iterations significantly. Furthermore, the indicator constraints method effectively handles the inclusion of constraints in mathematical optimization models by directly incorporating logical conditions. This approach ensures that the constraints are satisfied without introducing large

Algorithm 1 Inference-Dual-Based Decomposition Algorithm

complicating variables: $\alpha_{ubr}^{UP}\ and\ \alpha_{dbr}^{DW}$
initial guess: $\alpha_{ubr}^{UP} \leftarrow \alpha_{ubr}^{UP_{(0)}}$, $LB \leftarrow -\infty$, $UB \leftarrow \infty,\ \ r \leftarrow 1$
while $(UB - LB) \geq \epsilon$ **do**
  if $(r \neq 1)$ **then**
    solve: $MLLP\ master\ problem$ (10)
    $g_{rn}^{*} \leftarrow g_{rn}$, $LB \leftarrow new\ LB$, $g_{rn}^{*} \implies (\alpha_{ubr}^{UP*} \vee \alpha_{dbr}^{DW*})$
  end if
  solve: $Stochastic\ linear\ logic-based\ sub-problem$ (8)
  keep optimal values: $\Omega_7{}^{*} \leftarrow \Omega_7,\ UB \leftarrow new\ UB$,
  for $n \leftarrow 1$ to $2^{N_b^{mv}}$ **do**
    $(\bar{\bar{g}}_{rn} \leftarrow 1) \implies (\bar{\bar{\alpha}}_{ubrn}^{UP} \vee \bar{\bar{\alpha}}_{dbrn}^{DW})$
    $(\bar{\bar{\alpha}}_{ubrn}^{UP} \vee \bar{\bar{\alpha}}_{dbrn}^{DW}) \xrightarrow{\Omega_7^{*}} (Y_{21,ubrt\omega}{}^{*} \vee Y_{22,dbrt\omega}{}^{*} \vee \beta_b^{UP*} \vee \beta_b^{DW*})$
    calculate (11a): $\Gamma_{(r+1)n} \leftarrow \Gamma_{rn},\ \ \bar{\bar{g}}_{rn} \leftarrow 0$
  end for , $r \leftarrow r+1$
end while

parameters (big-Ms) and the associated challenges of finding appropriate values for them.

IV. Case Studies

A. Test System

Here, we evaluate the performance of the proposed SIDBD algorithm in finding the solution to a two-stage stochastic TSO-DSO-Retailer operational coordination problem using a modified IEEE 118-bus test system which is connected to two modified IEEE 33-bus test systems at its bus No.102 and bus No.109. A case study with conflicts among the objectives of service buyers is considered to evaluate the proposed schemes and methods. In order to assess the effectiveness of the proposed SIDBD algorithm in managing the coordination between TSOs, DSOs, and retailers on a larger scale involving more than two DSOs, we conducted additional case studies encompassing three, five, and ten DSOs.

Using the proposed algorithm in [34], a proper number of scenarios are generated considering uncertainty sources including the aggregated load consumption and RES generation, and the submitted offer functions by the service providers. Table I depicts the expected amount of available flexibility services in two considered DSOs. We used the MIP solver CPLEX in the GAMS platform. Our computer had Intel Core i7-8650U (2.11 GHz), and 16GB of RAM.

TABLE I
Expected Available Flexibility Services in the Case Study

Distribution network	*Node*	*Turn-Down*	*Turn-up*
DSO_A	No.07	$DR_1(0.2MW)$	$WT_1(11.3MW)$
	No.08	$DR_2(0.6MW)$	$PV_1(4.5MW)$
	No.24	$DR_3(0.3MW)$	$PV_2(8.3MW)$
	No.30	$DR_4(0.7MW)$	$WT_2(7.8MW)$
	No.32	$DR_5(0.4MW)$	$PV_3(5.5MW)$
DSO_B	No.07	$DR_2(0.4MW)$	$PV_1(7MW)$
	No.24	$DR_2(1.4MW)$	$PV_2(7.1MW)$
	No.25	$DR_3(1.3MW)$	$PV_3(9.7MW)$
	No.32	$DR_4(0.8MW)$	$WT(12.5MW)$

We consider a case study that deals with a situation in DSO_A that the load consumption is unpredictably decreased at buses No.23 − 25 and buses No.30 − 32, and it is increased at buses No.20 − 22 and No.12 − 15. As a result of the considered case study, there are load increments and surplus renewable energy generation, simultaneously. Accordingly, TSO, DSO, and retailers procure flexibility services to cope with possible problems including congestion, energy imbalance, voltage stability, and unexpected increments of cost in their regions.

B. Results and Discussions

We consider eight hours of system operations to evaluate the performance of the proposed SIDBD algorithm and coordination frameworks.

1) Performance of the Proposed SIDBD Algorithm: Table II presents the performance of our SIDBD algorithm, showcasing its results in terms of the number of DSOs, variables, equations, convergence time, and IDBD's iterations. The table includes comparisons with and without incorporating our proposed sub-problem relaxation technique.

TABLE II
The Performance of Our Proposed SIDBD Algorithm For Different Numbers of DSOs

Quantitative Criteria	*Number of DSOs*			
	2	*3*	*5*	*10*
Total Number of Variables	110,276	147,430	208,761	375,984
Total Number of Discrete Variables	403	588	929	2175
Number of Complicating Variables	10	10	15	20
Number of Equations	20,905	28,145	44,532	87,341
Iterations without SP Relaxation	490	521	677	1108
Iterations with SP Relaxation	7	7	8	9
Execution time without SP Relaxation	462 (s)	641 (s)	912 (s)	1486 (s)
Execution time with SP Relaxation	193 (s)	230 (s)	310 (s)	491 (s)

The results unequivocally establish the effectiveness of the proposed SIDBD algorithm in effectively addressing the intricate problem of large-scale coordination among TSO, DSOs, and retailers within a reasonable execution time. Importantly, the SIDBD algorithm exhibits remarkable performance regardless of whether subproblem relaxation is considered. Remarkably, the inclusion of subproblem relaxation significantly enhances the SIDBD algorithm by substantially reducing the number of iterations and execution time.

The complexity of our optimization problem becomes apparent when considering the substantial increase in the total number of variables, which escalates from 110,276 in the scenario involving two DSOs to 375,984 variables in the case involving ten DSOs. Furthermore, the number of binary variables and equations experiences a significant surge. Consequently, it is evident that our optimization problem comprises an abundance of variables and an extensive range of equality and inequality constraints.

Table 1 illustrates the varying execution times associated with different case studies involving interconnected DSOs. It is evident from the perspective of CPU execution time that the utilization of subproblem relaxation reduces the execution time across all case studies. It is crucial to emphasize that the results presented herein are predicated on arbitrary values employed solely for the purpose of evaluating the performance of the proposed method. The ability of TSO and DSOs to accommodate EERs in their respective systems and the efficacy of these services in providing flexibility will invariably depend on the specific circumstances and conditions.

Furthermore, it is noteworthy that our model incorporates an AC-OPF model, which encompasses the inclusion of constraints pertaining to the voltage at the DSO level. This essential inclusion not only renders the problem more realistic, but also introduces a heightened level of complexity to the optimization process.

In the ensuing sections of this paper, we meticulously investigate and analyze the efficacy of the SIDBD algorithm in addressing the intricacies of large-scale coordination among TSO, DSOs, and retailers. Through an in-depth case study involving two DSOs, we comprehensively evaluate the outcomes derived from the implementation of the SIDBD algorithm.

Table III shows the performance of our SIDBD algorithm for different numbers of complicating variables from convergence time and the number of IDBD's iterations viewpoints with and without considering our proposed sub-problem relaxation technique.

TABLE III
EVALUATING THE PERFORMANCE OF OUR PROPOSED SIDBD ALGORITHM ACROSS VARYING NUMBERS OF COMPLICATING VARIABLES: A CASE STUDY WITH TWO DSOS

Quantitative Criteria	*Number of Complicating Variables*			
	3	*4*	*5*	*10*
Number of SIDBD cuts	8	16	32	1024
Iterations without SP Relaxation	4	7	14	490
Iterations with SP Relaxation	2	2	3	7
Execution time without SP Relaxation	945 (s)	807 (s)	726 (s)	462 (s)
Execution time with SP Relaxation	519 (s)	411 (s)	322 (s)	193 (s)

As can be seen, the number of cuts increases by increasing the number of complicating variables. The cuts related to the combination of complicating variables with an infeasible solution are defined as "no-good" cuts. The proposed sub-problem relaxation can find the no-good cuts without solving the sub-problem corresponding to each combination of complicating variables which in turn reduces the execution time and the number of SIDBD iterations significantly, especially when the number of complicating variables increases. For instance, in the case of taking into account five complicating variables, our proposed SIDBD algorithm finds the solution in 3 and 14 iterations with and without considering sub-problem relaxation, respectively. Without sub-problem relaxation, the SIDBD algorithm evaluates all cuts including the cuts related to the infeasible solutions. As a comparator, considering sub-problem relaxation in the master problem removes the cuts related to infeasible conditions of the power balance equality constraints. Furthermore, using the proposed sub-problem relaxation technique improves the performance of the IDBD algorithm from the execution time viewpoint.

Table IV shows the 32 SIDBD cuts associated with all enumerations of five complicating binary variables in DSO_A. As can be seen, at least four services should be activated in order to satisfy the considered power balance constraint in the proposed sub-problem relaxation technique. It means that all combinations of the complicating variables with less than four activated services will end up in an infeasible solution. Consequently, the corresponding cuts to these combinations are labeled as no-good cuts and excluded from the algorithm.

TABLE IV
RESULTS OF SIDBD ALGORITHM WITH FIVE COMPLICATING VARIABLES

IDBD Cuts	*State of flexibility service activation in DSO_A*					*Expected lower bound at each iteration of SIDBD*		
	No.07	*No.08*	*No.24*	*No.30*	*No.32*	*First*	*Second*	*Third*
Cut 1	0	0	0	0	0	no-good	no-good	no-good
Cut 2	0	0	0	0	1	no-good	no-good	no-good
Cut 3	0	0	0	1	0	no-good	no-good	no-good
Cut 4	0	0	1	0	0	no-good	no-good	no-good
Cut 5	0	1	0	0	0	no-good	no-good	no-good
Cut 6	1	0	0	0	0	no-good	no-good	no-good
Cut 7	0	0	0	1	1	no-good	no-good	no-good
Cut 8	0	0	1	0	1	no-good	no-good	no-good
Cut 9	0	1	0	0	1	no-good	no-good	no-good
Cut 10	1	0	0	0	1	no-good	no-good	no-good
Cut 11	0	0	1	1	0	no-good	no-good	no-good
Cut 12	0	1	0	1	0	no-good	no-good	no-good
Cut 13	1	0	0	1	0	no-good	no-good	no-good
Cut 14	0	1	1	0	0	no-good	no-good	no-good
Cut 15	1	0	1	0	0	no-good	no-good	no-good
Cut 16	1	1	0	0	0	no-good	no-good	no-good
Cut 17	0	0	1	1	1	no-good	no-good	no-good
Cut 18	0	1	0	1	1	no-good	no-good	no-good
Cut 19	1	0	0	1	1	no-good	no-good	no-good
Cut 20	0	1	1	0	1	no-good	no-good	no-good
Cut 21	1	0	1	0	1	no-good	no-good	no-good
Cut 22	1	1	0	0	1	no-good	no-good	no-good
Cut 23	0	1	1	1	0	no-good	no-good	no-good
Cut 24	1	0	1	1	0	no-good	no-good	no-good
Cut 25	1	1	0	1	0	no-good	no-good	no-good
Cut 26	1	1	1	0	0	no-good	no-good	no-good
Cut 27	0	1	1	1	1	331139	332938	331128
Cut 28	1	0	1	1	1	331128	**332858**	331140
Cut 29	1	1	0	1	1	331139	332991	331140
Cut 30	1	1	1	0	1	331139	333111	331140
Cut 31	1	1	1	1	0	**330917**	333111	330919
Cut 32	1	1	1	1	1	331139	333111	331140

In the first iteration, based on the objective function of the master problem, cut 31 is selected which has the minimum expected cost. For the further iterations, first, each cut is updated with the maximum expected value for all iterations so far. Second, the master problem selects a cut with the minimum expected value. Consequently, in the second and third iterations, cut number 28 is selected which has the minimum expected value. The summary of this process is shown in Table V.

TABLE V
SELECTED CUTS IN SIDBD ALGORITHM

SIDBD Iteration	*Selected Cut*	*Expected Lower Bound ($)*
First	Cut 31	330917
Second	Cut 28	332858
Third	Cut 28	332858

Fig. 6 depicts the convergence of the proposed SIDBD algorithm with sub-problem relaxation for the case with four advisory intervals and five complicating variables. The proposed SIDBD algorithm has been converged into three iterations.

Since using our proposed sub-problem relaxation technique only removes the infeasible conditions related to the power balance equality constraints, the number of iterations in the case of ten complicating variables equals seven. In fact, our sub-problem relaxation technique cannot find all no-good cuts. The proposed sub-problem relaxation technique can be

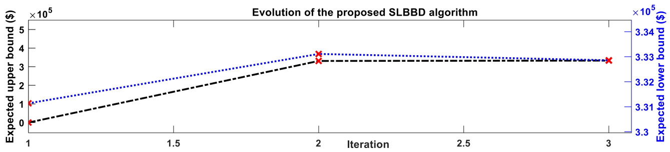

Fig. 6. The convergence of the SIDBD algorithm with five complicating variables using our proposed sub-problem relaxation technique.

enhanced to find the infeasible conditions related to all equality and inequality constraints which is the scope of our future work.

From the stochastic optimization viewpoint, we can employ parallel computing techniques since each scenario is optimized individually and independently. Consequently, the utilization of parallel computing, which is one of the main advantages of our developed SIDBD algorithm, decreases the computation time for large-scale optimization problems, significantly.

2) System Operation Cost: This section presents the findings of a comparative analysis that demonstrates the effectiveness of our proposed framework, namely the embedded DSO-Retailer activation mode with bilevel programming, in contrast to the exogenous DSO-Retailer activation mode utilizing single-level programming in the current electricity market. In the exogenous model, the activation of DSO flexibility is treated as a stochastic exogenous parameter, resulting in the deduction of the anticipated DSO flexibility activation from the total available services for the TSO. This deduction relies on prior knowledge of net energy transactions between the TSO and DSO, encompassing renewable energy output and the requested energy exchange between the TSO and DSO.

TABLE VI
EXPECTED SYSTEM OPERATION COST

Operation Time	*Exogenous DSO-Retailer Activation Model*		*Embedded DSO-Retailer Activation Model*	
	Single-Interval	*Four-Interval*	*Single-Interval*	*Four-Interval*
t_1	82393.8 ($/h)	82430.8 ($/h)	83837 ($/h)	83936 ($/h)
t_2	83107.2 ($/h)	83136.0 ($/h)	83766 ($/h)	83732 ($/h)
t_3	80898.6 ($/h)	80936.4 ($/h)	82928 ($/h)	82934 ($/h)
t_4	Infeasible	88288.6 ($/h)	82931 ($/h)	83164 ($/h)
t_5	Infeasible	83103.9 ($/h)	83603 ($/h)	83047 ($/h)
t_6	Infeasible	85966.8 ($/h)	83512 ($/h)	83044 ($/h)
t_7	Infeasible	85117.0 ($/h)	83315 ($/h)	83061 ($/h)
t_8	Infeasible	82323.5 ($/h)	83636 ($/h)	83673 ($/h)
Total	**-**	**671303** ($)	**667529**($)	**666593**($)

The results presented in Table VI demonstrate that the proposed embedded DSO-Retailer activation model consistently obtains feasible solutions across all generated scenarios. The look-ahead multi-interval framework improves the system operation cost and helps the system operator find feasible solutions compared to the single-interval framework. Additionally, the embedded model reduces system operation costs compared to the exogenous model. Therefore, hereafter, all the results are related to the embedded DSO-Retailer activation model with four intervals.

3) Flexibility Service Activation: As shown in Table VII, there are conflicts between TSO, DSO, and retailers to procure their flexibility services of need. The TSO intends to activate turn-up services due to the surplus energy in the transmission system which makes congestion and voltage problems. On the other side, unlike TSO, DSO tries to activate turn-down services due to the load consumption increment in some parts of the distribution network which makes congestion and voltage problems. At the same time, the retailers compete with DSO to activate turn-down services to decrease their costs.

TABLE VII
EXPECTED FLEXIBILITY SERVICE ACTIVATION AT DSO_A

Time	*TSO service activation (MW)*		*DSO_A service activation (MW)*		*Retailers service activation (MW)*	
	Up	*Down*	*Up*	*Down*	*Up*	*Down*
t_1	8.5	-	-	0.6	-	0.2
t_2	4.4	-	-	0.2	-	0.6
t_3	4.7	-	-	0.2	-	0.6
t_4	12.4	-	-	0.6	-	0.2
t_5	10.5	-	-	0.2	-	0.6
t_6	12.1	-	-	0.2	-	0.6
t_7	10.4	-	-	0.2	-	0.6
t_8	13.4	-	-	0.2	-	0.6

DSO and Retailers activate the available turn-down services at bus No.7 and bus No.8 in the distribution network. The summation of the expected value of available turn-down service at buses No.7 and No.8 equals $0.8\ MW$ at each time of operation in our case study. The results show that our proposed framework, market scheme, and SIDBD algorithm can properly manage the situation with conflict of interest between the service buyers.

4) Payoff Mechanism: Our proposed operational coordination scheme mitigates the conflict between different buyers. However, gaming strategies may occur since the valuations of the service buyers are interdependent. For instance, if TSO activates a turn-up or turn-down service, it can be indirectly beneficial for the other service buyers without any cost. Accordingly, this condition leads to a free-rider strategy where any of the service buyers can anticipate the actions of others for their own benefit. Consequently, we proposed the expected Shapley value calculation to allocate the cost of service activation among the beneficiaries in the coalition in a fair way.

Table VIII compares the results of the proposed expected Shapley value method and the pay-as-bid mechanism in the TSO-DSO-Retailer operational coordination problem.

TABLE VIII
PAYOFF MECHANISM FOR FLEXIBILITY SERVICE ACTIVATION

Time	*Total Cost ($)*	*Expected Cost Without Shapley value($)*			*Expected With Shapley value($)*		
		TSO	*DSO*	*Retailers*	*TSO*	*DSO*	*Retailers*
t_1	114.78	105.82	7.04	1.92	30.95	41.91	41.91
t_2	103.91	88.55	3.68	11.68	26.12	38.90	38.90
t_3	107.46	94.34	3.52	9.6	59.34	24.06	24.06
t_4	245.47	232.35	9.92	3.20	68.69	88.39	88.39
t_5	231.53	216.18	3.84	10.88	65.37	82.76	82.76
t_6	249.38	234.66	3.84	10.88	71.18	89.1	89.1
t_7	266.47	246.75	3.84	10.88	79.95	93.26	93.26
t_8	277.66	263.42	3.52	10.72	82.69	97.49	97.49

As mentioned above in Table VII, TSO activates the turn-up services, and DSO and retailers activate the turn-down services. Aligned with Table VII, the results in Table VIII represent that without considering the expected Shapley value, TSO should pay a higher portion of the costs. However, the proposed payoff mechanism based on the expected Shapley value method demonstrates that DSO and retailers are beneficiaries of the activated services by TSO. For instance, without considering the proposed expected Shapley value, TSO should pay 263.42 \$ for the turn-up service activation at time t_8 which is 94.87 percent of the total service activation costs. As a comparator, since our proposed payoff mechanism allocates the cost pro-rata to the actual impacts of participants in a coalition in a fair way, TSO should pay 82.69 \$ which is corresponding to its actual impacts on the system operation. We have also obtained almost the same results for times t_1 to t_7. Furthermore, utilization of the proposed payoff mechanism shows that DSO and the aggregated retailers have the same impacts on the system operation from the flexibility service activation viewpoint. This results in the fact that the costs of service activation are the same for both.

Therefore, our payoff mechanism properly allocates the cost of service activation among the beneficiaries in a fair way, according to their actual impacts on the system operation.

V. Conclusion

In this paper, the Inference-Dual-Based Decomposition (IDBD) algorithm is developed for a two-stage stochastic TSO-DSO-Retailer operational coordination problem considering a new organizational setup based on the introduced concept of the FMO. A new sub-problem relaxation technique is presented for decreasing the number of iterations and the execution time of the SIDBD algorithm. A new payoff mechanism is proposed based on the expected Shapley value method to allocate the cost of service activation in a fair way. The results show that the proposed SIDBD algorithm manages the coordination problem especially when there are conflicts of interest between the service buyers. The proposed sub-problem relaxation reduces the execution time for finding a solution to a large-scale MLLP problem. Finally, the payoff mechanism allocates the cost of service activation among the service buyers based on their actual impacts on the system operation which, as a result, eliminates the free-rider strategy.

VI. Future work

In this paper, we focused upon solving the TSO-DSO-Retailer coordination for market arrangements typical of the European context. However, we conjecture that with suitable adaptations, the coordination mechanisms can, in principle, be applied to other competitive and regulatory situations, as in North America and elsewhere. This is because the proposed solution algorithm is quite general and it can be applied to solving any bilevel program with binary variables in both upper and lower levels. We suggested the inference-dual theory instead of the value-function theory to handle the bilevel optimization problems which have binary variables at the lower level. Although, we have explained our Inference-Dual-Based Decomposition (IDBD) algorithm in the context of the TSO-DSO-Retailer application, however, the underlying solution algorithm is quite general and it can be tested for other applications beyond this coordination issue.

The authors are presently engaged in developing a fuzzy inference-dual-based decomposition algorithm, aiming to circumvent the necessity of assigning binary variables to service activations.

VII. Appendix

Using the definition of implication in [35], we define the concept of inference dual as follows:

Definition 3. *Consider the optimization problem* $P_1 = \{Minimize\ f(x):\ x \in S,\ x \in D\}$ *with feasibility set* S *and domain set* D*. The inference dual is defined as the problem of inferring the possible tightest lower bound on the optimal value of the objective function* $f(x)$ *from the constraints (*$x \in S$*). The inference dual of* P_1 *is as follows:* $P_2 = \{Maximize\ \psi:\ x \in S \xrightarrow{D} f(x) \geq \psi\}$.

Consequently, with regards to the above definition, the strong duality theorem is introduced as follows.

Theorem 3. *Strong inference duality: The optimization problem* P_1 *and its inference dual problem* P_2 *have the same optimal values.*

Proof. See [2], page 12, Proof of Theorem 1. □

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TABLE IX
NOMENCLATURE

Indices	
b, c	Buses of the transmission system.
i, j	Buses of the distribution system.
d/u	Buses of the distribution system with down/up services.
p	Buses of the transmission and distribution systems with TSO-DSO connection.
t/r	Rolling interval/Iteration of the SIDBD algorithm.
ω	Scenarios.
n	Complicating variables in the SIDBD algorithm
Sets	
I^{TS}/K^{TS}	Buses/Lines of transmission system.
I_b^{DS}/K_b^{DS}	Buses/Lines of a distribution system located at transmission bus b.
N_{db}/N_{ub}	Buses of a distribution system with down/up services located at transmission system bus b.
PCC	Points of common coupling.
N_b^{mv}	Sub-sets of complicating variables in the SIDBD algorithm ($N_b^{mv} \subseteq N_{ub}$).
Θ	Scenarios.
T/IT	Rolling windows/Iteration of the SIDBD algorithm.
Parameters	
$\pi_{ubt\omega}^{UP}/\pi_{dbt\omega}^{DW}$	Flexibility bid of turn-up/down unit u/d at bus b at time t and scenario ω ($\$/MWh$).
$\pi_{bt\omega}^{RD}$	Bid for turn-down generation units at transmission bus b at time t and scenario ω ($\$/MWh$).
$G_{ijb}^{DS}/B_{ijb}^{DS}$	Conductance/Susceptance between buses i and j in distribution system at transmission bus b.
G_{bc}^{TS}/B_{bc}^{TS}	Conductance/Susceptance between buses b and c in transmission system.
RU_b/RD_b	Ramp up/down capability of reserves at bus b (MW/h).
$PV_{ibt\omega}/WT_{ibt\omega}$	Scheduled photovoltaic/wind turbine generation in the forward market at distribution bus i and transmission bus b at time t and scenario ω (MW).
$\Phi_{ubt\omega}^{PV}/\Phi_{ubt\omega}^{WT}$	Offered amount of turn-up service by PV/wind turbine owner at distribution bus u and transmission bus b at time t and scenario ω (MW).
$\Phi_{dbt\omega}^{DR}$	The offered amount of turn-down services by DR aggregators at distribution bus d and transmission bus b at time t and scenario ω (MW).
$\tilde{P}_{g_{pt\omega}}^{DS}/\tilde{Q}_{g_{pt\omega}}^{DS}$	Anticipated active/reactive consumption of the exogenous DSO model by TSO at point of common coupling at time t and scenario ω ($MW/MVar$).
$\tilde{F}_{ipt}^{UPD/DWD}$	Anticipated turn-up/-down service activation by the DSO at the point of common coupling for the exogenous model at time t (MW).
$\overline{P}_{bc}^{TS}/\overline{Q}_{bc}^{TS}$	Upper limit of active/reactive power flow of branch bc in transmission system ($MW/MVar$).
$\underline{P}_{bc}^{TS}/\underline{Q}_{bc}^{TS}$	Lower limit of active/reactive power flow of branch bc in transmission system ($MW/MVar$).
$\overline{P}_{ijb}^{DS}/\overline{Q}_{ijb}^{DS}$	Upper limit of active/reactive power flow of branch ij in distribution system at transmission bus b ($MW/MVar$).
$\underline{P}_{ijb}^{DS}/\underline{Q}_{ijb}^{DS}$	Lower limit of active/reactive power flow of branch ij in distribution system at transmission bus b ($MW/MVar$).
$(\underline{v}^s/\overline{v}^s)_{i/b}$	Limitation of square of the voltage magnitude at bus i/b in distribution/transmission system.
$S_B/\tilde{a}_{1,b}, \tilde{a}_{0,b}$	Base value in per unit system/Anticipated cost coefficients of generator units in transmission bus b.
Variables	
$F_{dbt\omega}^{DWT/D/R}$	Activated turn-down service by TSO/DSO/retailer from distribution bus ($d \in N_{db}$) and transmission bus b at time t and scenario ω (MW).
$F_{ubt\omega}^{UPT/D/R}$	Activated turn-up service by TSO/DSO/retailer from distribution bus ($u \in N_{ub}$) and transmission bus b at time t and scenario ω (MW).
$P_{g_{bt\omega}}^{TS}/P_{g_{ibt\omega}}^{DS}$	Active power of generator b^{th}/i^{th} in TSO/DSO region at time t and scenario ω (MW).
$Q_{g_{bt\omega}}^{TS}/Q_{g_{ibt\omega}}^{DS}$	Reactive power of generator b^{th}/i^{th} in TSO/DSO region at time t and scenario ω ($MVar$).
$P_{bt\omega}^{TS}/Q_{bt\omega}^{TS}$	Active/Reactive power injection at transmission bus b at time t and scenario ω ($MW/MVar$).
$P_{ibt\omega}^{DS}/Q_{ibt\omega}^{DS}$	Active/Reactive power injection at distribution bus i and transmission bus b at time t and scenario ω ($MW/MVar$).
$P_{L_{bt\omega}}^{TS}/Q_{L_{bt\omega}}^{TS}$	Active/Reactive load connected to transmission bus b at time t and scenario ω ($MW/MVar$).
$P_{L_{ibt\omega}}^{DS}/Q_{L_{ibt\omega}}^{DS}$	Active/Reactive load connected to the distribution bus i and transmission bus b at time t and scenario ω ($MW/MVar$).
$P_{bct\omega}^{TS}/Q_{bct\omega}^{TS}$	Active/Reactive power flow from bus b to bus c in transmission system at time t and scenario ω ($MW/MVar$).
$P_{ijt\omega}^{DS}/Q_{ijt\omega}^{DS}$	Active/Reactive power flow from bus i to bus j in distribution system at time t and scenario ω ($MW/MVar$).
$P_{g_{pt\omega}}^{DS}/Q_{g_{pt\omega}}^{DS}$	Active/reactive consumption for the embedded DSO model at point of common coupling at time t and scenario ω ($MW/MVar$).
$(\theta/v^s)_{(b/i)t\omega}$	Phase angle/Square of voltage magnitude at bus b/i in transmission/distribution systems at time t and scenario ω.
$\beta_b^{UP}/\beta_b^{DW}$	Binary variables for up/down reserve activation at bus b.
$\alpha_{ub}^{UP}/\alpha_{db}^{DW}$	Binary variables for up/down service activation at distribution bus $(u/d) \in \{N_{ub}/N_{db}\}$ and transmission bus b.
$\lambda/\Pi/\mu/\delta$	Dual variables related to the equality constraints (1b*), (1c*), (1e*) of our previous article [2], and (2e).
D/Y	Dual variables related to the complementary slackness conditions/Logical indicators.
Γ	Boundary for the determined IDBD cuts in the IDBD algorithm.
g	Binary variables to show the combination of complicating variables.
Abbreviations	
DSO/TSO	Distribution/Transmission system operator.
EER/DR	Embedded energy resources/Demand response.
GDP/RES	Generalized disjunctive programming/Renewable energy sources.
$SBD/LA{-}MI$	Standard Benders decomposition/Look-ahead multi-interval.
$IDBD/SIDBD$	Inference-dual /Stochastic inference-dual-based decomposition algorithm.

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