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A Multifactor Perspective on Volatility-Managed Portfolios

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ABSTRACT

Moreira and Muir question the existence of a strong risk-return trade-off by showing that investors can improve performance by reducing exposure to risk factors when their volatility is high. However, Cederburg et al. show that these strategies fail out-of-sample, and Barroso and Detzel show they do not survive transaction costs. We propose a conditional multifactor portfolio that outperforms its unconditional counterpart even out-of-sample and net of costs. Moreover, we show that factor risk prices generally decrease with market volatility. Our results demonstrate that the breakdown of the risk-return trade-off is more puzzling than previously thought.

A FUNDAMENTAL PREMISE IN FINANCE is that there is a strong risk-return trade-off. Moreira and Muir (2017) challenge this premise by showing that in-

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vestors can increase Sharpe ratios by reducing exposure to risk factors when their volatility is high. The intuition underlying their findings is that, in the absence of a strong risk-return trade-off for factor returns, factor exposure can be scaled back during times of high volatility without a proportional reduction in returns. Their work is a challenge to structural models of time-varying expected returns, which typically predict that the market risk-return trade-off strengthens during periods of high volatility. However, Cederburg et al. (2020) show that the performance gains from volatility management are not achievable out-of-sample because of estimation error, and Barroso and Detzel (2021) show that transaction costs erode any such gains.

While the papers above focus on volatility-managed *individual-factor* portfolios, we provide a *multifactor* perspective by proposing a novel conditional mean-variance multifactor portfolio whose weights on each factor decrease with market volatility. We show that this strategy outperforms the unconditional multifactor portfolio even out-of-sample and net of costs. Our findings show that estimation error and transaction costs do not explain the gains from volatility management, and hence, the breakdown of the risk-return trade-off is more puzzling than previously thought.

The key distinguishing feature of our approach to volatility management is that we focus on multifactor portfolios. Although it is informative to study the effect of volatility management for each individual factor, the stochastic discount factor (SDF) that prices all assets is determined by the conditional mean-variance *multifactor* portfolio. Thus, to evaluate the asset-pricing implications of volatility management, one needs to test whether the volatility-managed multifactor portfolio outperforms its unconditional counterpart. Moreover, as explained by Chernov, Lochstoer, and Lundeby (2022), this test provides information about the joint dynamics of factor returns. In particular, if our conditional multifactor portfolio, which reduces its weights on the factors when market volatility increases, outperforms its unconditional counterpart, then we must have the counterintuitive result that the relation between the conditional mean vector and covariance matrix of factor returns weakens with market volatility.[1]

Our approach to volatility management differs in three other ways from that in the existing literature. First, our conditional multifactor portfolios allow the relative weights on the different factors to vary with market volatility. In contrast, Moreira and Muir (2017, section I.E) consider a conditional fixed-weight multifactor portfolio whose relative weight on each factor does not vary with volatility and Barroso and Detzel (2021, ftn. 12) consider a portfolio that assigns an equal relative weight to each factor. Second, we evaluate conditional multifactor portfolios that are optimized accounting for transaction

[1] To see this, note that the conditional mean-variance multifactor portfolio of a single-period investor with constant relative risk-aversion γ is $w_t = \Sigma_t^{-1}\mu_t/\gamma$, where μ_t and Σ_t are the conditional mean vector and covariance matrix of factor excess returns. Thus, the conditional mean vector and covariance matrix satisfy the relation $\mu_t = \gamma\Sigma_t w_t$. Therefore, if the conditional multifactor portfolio weights w_t decrease with market volatility, then the relation between the conditional mean and covariance matrix weakens with market volatility.

costs. Third, we account for the reduction in transaction costs associated with the netting of trades across the different factors combined in the multifactor portfolio, an effect termed *trading diversification* by DeMiguel et al. (2020).[2]

Our conditional mean-variance multifactor portfolio achieves an out-of-sample and net-of-costs Sharpe ratio that is 13% higher than that of the unconditional mean-variance multifactor portfolio, with the difference statistically significant at the 1% level.[3] We identify three main drivers of the favorable performance of our conditional multifactor portfolio. The first is trading diversification. In particular, although both the unconditional and the conditional multifactor portfolios benefit from the netting of trades across multiple factors, the benefits are larger for the conditional portfolio because the transaction costs of the volatility-managed factors are much larger than those of the unmanaged factors. For instance, ignoring trading diversification, we find that while the net-of-costs mean return of all nine unmanaged factors is positive, that of four of the nine managed factors is negative. However, accounting for trading diversification, the net mean return of all nine managed factors becomes positive.

The second driver of the performance of our conditional multifactor portfolio is that it is optimized accounting for transaction costs, which significantly improves its performance relative to the unconditional multifactor portfolio. Again, even though the performance of both the conditional and the unconditional portfolios improves when they are optimized accounting for transaction costs, the benefits are larger for the conditional portfolios because the transaction costs of trading the managed factors are larger.

The third driver of the performance of our conditional portfolios is that they allow the *relative* weight on each factor to vary with market volatility. Indeed, our conditional multifactor portfolio optimally times some of the factors aggressively while assigning an almost-constant weight to others. As a result, the average exposure of our conditional portfolio to the various factors can differ substantially from that of the unconditional and fixed-weight portfolios. For instance, our conditional portfolio has a larger average exposure to the value, momentum, and betting-against-beta factors compared with the unconditional and fixed-weight portfolios but a smaller average exposure to the investment factors.

To explain the economic mechanism underlying the performance of the conditional multifactor portfolios, in Figure 1 we illustrate how the risk-return

[2] Other papers that have also documented that combining characteristics reduces transaction costs include Barroso and Santa-Clara (2015a), Frazzini, Israel, and Moskowitz (2015), and Novy-Marx and Velikov (2016).

[3] Barroso and Detzel (2021) show that the volatility-managed *market* portfolio outperforms the market factor during high-sentiment periods, but underperforms the market during low-sentiment periods. However, Section VIII of the Internet Appendix shows that the out-of-sample performance of the conditional multifactor portfolio is significantly better than that of the unconditional portfolio during *both* high- and low-sentiment periods. We conclude that sentiment does not explain the out-of-sample and net-of-costs performance of our proposed multifactor strategy. The Internet Appendix is available in the online version of this article on *The Journal of Finance* website.

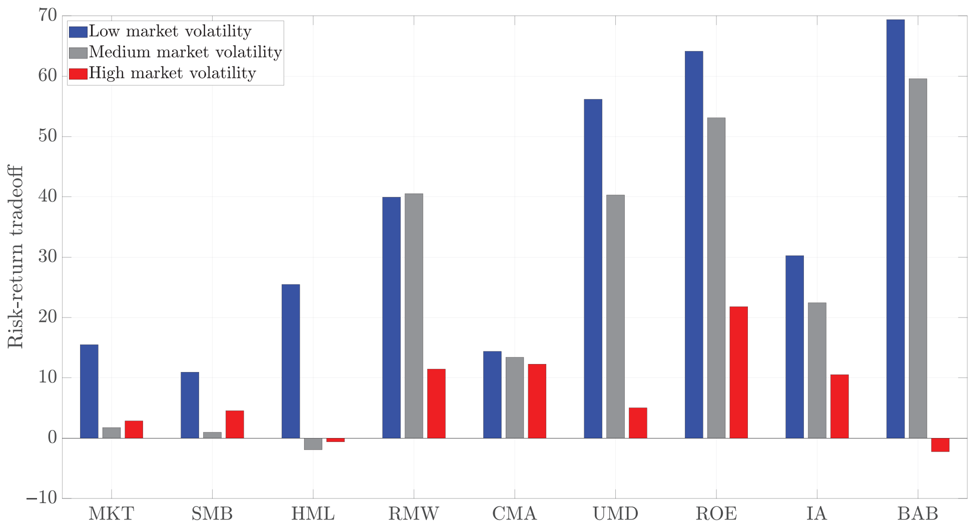


Figure 1. Factor risk-return trade-off and market volatility. This barplot illustrates how the risk-return trade-off for the nine factors in our data set varies with realized market volatility. We first use the monthly time series of realized market volatility to sort the months in our sample into terciles. For each factor, we then estimate the risk-return trade-off for month t as the realized factor return for month $t + 1$ divided by the monthly realized factor variance estimated as the sample variance of daily returns for month t. Finally, we report the risk-return trade-off averaged across the months in each tercile. Blue bars correspond to the tercile containing low-market-volatility months, gray bars to medium-market-volatility months, and red bars to high-market-volatility months. The sample spans January 1977 to December 2020. (Color figure can be viewed at wileyonlinelibrary.com)

trade-off for the nine factors varies with market volatility. In the figure, we use the monthly time series of realized market volatility to sort the months in our sample into volatility terciles and report the risk-return trade-off for each factor averaged across the months in each tercile. Our key finding is that for all nine individual factors the risk-return trade-off *weakens* with market volatility. This explains why our conditional multifactor portfolio, which reduces exposure to the risk factors when realized market volatility is high, outperforms its unconditional counterpart. Moreover, the weakening of the risk-return trade-off is substantial for some of the factors (UMD, ROE, and BAB) but less striking for others (SMB and CMA). This motivates our choice to consider a conditional multifactor portfolio that allows the relative weights on the different factors to vary with market volatility.

To understand the asset pricing implications of our findings, we also estimate a conditional SDF whose price of risk for each of the nine factors is an affine function of inverse realized market volatility. Consistent with the results in Figure 1, we find that the price of risk for individual factors generally *decreases* with realized market volatility. This is a counterintuitive result because one expects the price of risk for systematic factors to remain constant or increase with market volatility. We also observe that the reduction in the price of risk with market volatility is more significant for some factors than others. Our analysis therefore shows that conditioning on volatility helps construct an SDF that better spans the investment opportunity set, but the importance of volatility management varies across factors.

Our work is related to the literature on factor timing. Early contributions include Fleming, Kirby, and Ostdiek (2001, 2003), who evaluate the gains from volatility timing across multiple asset classes, and Marquering and Verbeek (2004), who study volatility and return timing of a market index. More recently, Ehsani and Linnainmaa (2022) and Gupta and Kelly (2019) study the performance of factor-momentum strategies. Gómez-Cram (2021) shows that the market can be timed using a business-cycle predictor derived from macroeconomic data. There are also papers that, like ours, study the timing of *combinations* of factors. For instance, Miller et al. (2015) develop a dynamic portfolio approach using classification tree analysis. Bass, Gladstone, and Ang (2017), Hodges et al. (2017), Amenc et al. (2019), and Bender, Sun, and Thomas (2018) study multifactor portfolios conditional on macroeconomic state variables. De Franco, Guidolin, and Monnier (2017) consider a multivariate Markov regime-switching model for the three Fama-French factors. Haddad, Kozak, and Santosh (2020) time the market and the first five principal components of a large set of equity factors using the value spread of the principal components as the timing variable. In contrast to these papers, our focus is on multifactor portfolios whose relative weights change with market volatility.

Our work is also related to the literature on the relation between market risk and return. Although some papers find a positive relation between market risk and return (French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992)), others find a negative relation (Breen, Glosten, and Jagannathan (1989), Nelson (1991), Glosten, Jagannathan, and Runkle (1993)). In addition, Lochstoer and Muir (2022) show that slow-moving beliefs about stock market volatility could explain a weak, or even negative, market risk-return trade-off. We contribute to this literature by showing that the risk-return trade-off for the nine factors we consider weakens with realized market volatility.

The rest of the paper is organized as follows. Section I describes our data and methodology for constructing conditional multifactor portfolios. Section II reports the performance gains of our conditional multifactor portfolios. Section III investigates the sources of these gains. Section IV studies the broader economic implications of our work by estimating a conditional SDF whose price of risk for each factor varies with market volatility. Section V concludes. The Appendix provides a description of the construction of the nine factors we consider. The Internet Appendix contains a large number of robustness tests and additional results.

I. Data and Methodology

In this section, we first describe the data used for our empirical analysis. We then explain how we construct conditional multifactor portfolios and account for transaction costs.

A. Data

We compile data for the same nine factors considered by Moreira and Muir (2017) and Barroso and Detzel (2021).[4] To do so, we first download from the authors' websites excess returns for the market (MKT), small-minus-big (SMB), high-minus-low (HML), robust-minus-weak (RMW), conservative-minus-aggressive (CMA), and momentum (UMD) factors of Fama and French (2018), the profitability (ROE) and investment (IA) factors of Hou, Xue, and Zhang (2015), and the betting-against-beta (BAB) factor of Frazzini and Pedersen (2014). Every factor (other than MKT and BAB) is the return of a long-short portfolio of stocks with one dollar in the long leg and one dollar in the short leg. The MKT and BAB factors are also zero-cost portfolios because their investment in the long leg is equal to that in the short leg once we account for their negative position in the risk-free asset.

We also construct these nine value-weighted factor portfolios independently in order to calculate the transaction costs required to trade the stocks comprising the factor portfolios.[5] To do this, we combine data from CRSP and Compustat for every stock traded on the NYSE, AMEX, and NASDAQ exchanges from January 1967 to December 2020.[6] We then drop stocks for firms with a negative book-to-market ratio.

For the out-of-sample analysis, we use an expanding-window approach, with the first estimation window consisting of 120 months starting from January 1967. The out-of-sample results therefore correspond to the period January 1977 to December 2020. To ensure a fair comparison with the out-of-sample results, the in-sample results are evaluated for the same period, January 1977 to December 2020.

B. Conditional Mean-Variance Multifactor Portfolios

We start by defining *individual* volatility-managed factors, as in Moreira and Muir (2017). Specifically, the return of the k^{th} volatility-managed factor is given by

$$r^{\sigma}_{k,t+1} = \frac{c}{\sigma^2_{k,t}} r_{k,t+1}, \tag{1}$$

[4] In the main body of the manuscript, we consider the same set of factors as Moreira and Muir (2017) and Barroso and Detzel (2021) so that we can compare our results to theirs. However, Section IV in the Internet Appendix shows that our findings are robust to considering a larger set of 66 factors that includes the nine factors considered by Moreira and Muir (2017) plus 57 factors from Green, Hand, and Zhang (2017).

[5] We find that the correlation of each of our factor returns with that of the original factor is above 90%.

[6] Moreira and Muir (2017) use data from 1926 to 2015 for MKT, SMB, HML, and UMD, from 1963 to 2015 for RMW and CMA, and from 1967 to 2015 for ROE and IA. Our multifactor analysis exploits all nine factors, so to ensure that we have data for all the factors over the entire sample period, our sample spans 1967 to 2020. Section VII of the Internet Appendix shows that our main findings are robust to evaluating the performance of the conditional *multifactor* portfolios over the first and second halves of our sample.

where $r_{k,t+1}$ is the k^{th} unmanaged factor return for month $t+1$, $\sigma^2_{k,t}$ is the realized variance of the k^{th} factor for month t estimated as the sample variance of daily factor returns, and c is a scaling parameter that ensures the volatility of the managed factor coincides with that of the unmanaged factor. The volatility-managed individual-factor *portfolio* is then the mean-variance combination of the unmanaged factor and its managed counterpart.[7]

Although the bulk of their analysis focuses on individual factors, Moreira and Muir (2017) also consider timing the unconditional mean-variance multifactor portfolio. In particular, they construct the optimal combination of the unconditional mean-variance multifactor portfolio and its managed counterpart, obtained by scaling the unconditional portfolio by the inverse of its past-month return variance. The resulting portfolio assigns the same relative weight to each factor as the unconditional multifactor portfolio, and thus we refer to it as the "conditional *fixed-weight* multifactor portfolio."

In contrast to timing individual factors or a conditional fixed-weight multifactor portfolio, we consider a conditional mean-variance multifactor portfolio that allows the relative weights of the different factors to vary with market volatility. For simplicity, we employ market volatility to time all nine factors, and Section X of the Internet Appendix shows that this is a conservative choice because the performance is even stronger when we time each factor using its own volatility or the average volatility of factors other than the market.[8] Note also that we use market volatility instead of market variance as our conditioning variable because Moreira and Muir (2017, section II.B) and Barroso and Detzel (2021, section 3.3) point out that using volatility can help reduce the transaction costs of volatility-managed factor portfolios.[9]

A conditional multifactor portfolio at time t can be expressed as

$$w_t(\theta_t) = \sum_{k=1}^{K} x_{k,t}\theta_{k,t}, \tag{2}$$

[7] Liu, Tang, and Zhou (2019) show that the out-of-sample Sharpe ratio of the managed factor is smaller when they estimate the scaling parameter c using a rolling window of past data. However, the scaling parameter c does not affect the out-of-sample performance of the volatility-managed individual-factor portfolio because it is obtained by selecting the weight on the unmanaged and managed factors that maximizes the mean-variance utility in the estimation window. In particular, for the k^{th} factor we compute the values of a_k and b_k that maximize the mean-variance utility of the returns of the portfolio of the k^{th} unmanaged and managed factors $(a_k r_{k,t+1} + b_k r^{\sigma}_{k,t+1})$ over each estimation window. Using equation (1), the mean-variance portfolio return can be rewritten as $a_k r_{k,t+1} + (b_k c) r_{k,t+1}/\sigma^2_{k,t}$. From this expression it may appear that the mean-variance portfolio depends on c. However, using the change of variables $\hat{b}_k = b_k c$, we can rewrite the return of the mean-variance portfolio as $a_k r_{k,t+1} + \hat{b}_k r_{k,t+1}/\sigma^2_{k,t}$, which does not depend on c. Thus, the values of a_k and $\hat{b}_k$ that maximize the mean-variance utility of the volatility-managed individual-factor portfolio are independent of c, and hence the volatility-managed individual-factor portfolio is also independent of c.

[8] In Section XV of the Internet Appendix, we also consider seven alternative conditioning variables and find that exploiting another conditioning variable, in addition to market volatility, does not help improve performance significantly.

[9] Moreover, Cejnek and Mair (2021) show that using volatility also reduces leverage.

where K is the number of factors, $x_{k,t} \in \mathbb{R}^{N_t}$ is the stock portfolio associated with the k^{th} factor at time t, in which N_t is the number of stocks at time t, $\theta_{k,t}$ is the portfolio weight on the k^{th} factor at time t, and $\theta_t = (\theta_{1,t}, \theta_{2,t}, \ldots, \theta_{K,t})$ is the factor-weight vector at time t. We parameterize each factor weight, $\theta_{k,t}$, as an affine function of the inverse of market volatility,

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t}, \tag{3}$$

where σ_t is the realized market volatility estimated as the sample volatility of the daily market returns in month t. Note that a positive b_k implies that the portfolio reduces exposure to the k^{th} factor when realized market volatility is high. Also, this parameterization allows for the weight of each factor to vary differently with market volatility because, in general, $b_i \neq b_j$ for $i \neq j$.

Let $r_{t+1} \in \mathbb{R}^{N_t}$ be the vector of stock returns for month $t+1$ and $r_{k,t+1} \equiv x_{k,t}^\top (r_{t+1} - r_{f,t+1} e_t) \in \mathbb{R}$ be the k^{th} factor return for month $t+1$, where $r_{f,t+1}$ is the return of the risk-free asset at time $t+1$ and e_t is the N_t-dimensional vector of ones. The return of a conditional multifactor portfolio can then be written as

$$r_{p,t+1}(\theta_t) = \sum_{k=1}^{K} r_{k,t+1}\theta_{k,t} = \sum_{k=1}^{K} r_{k,t+1}\left(a_k + b_k \frac{1}{\sigma_t}\right), \tag{4}$$

where the second equality follows from substituting (3) into (2).

For convenience, we also define the "extended" factor portfolio-weight matrix $X_{\text{ext},t}$, factor-return vector $r_{\text{ext},t+1}$, and factor-weight vector η as

$$X_{\text{ext},t} \equiv \begin{bmatrix} x_{1t}^\top \\ x_{2t}^\top \\ \vdots \\ x_{Kt}^\top \\ x_{1t}^\top \times \frac{1}{\sigma_t} \\ x_{2t}^\top \times \frac{1}{\sigma_t} \\ \vdots \\ x_{Kt}^\top \times \frac{1}{\sigma_t} \end{bmatrix}^\top, \quad r_{\text{ext},t+1} \equiv \begin{bmatrix} r_{1,t+1} \\ r_{2,t+1} \\ \vdots \\ r_{K,t+1} \\ r_{1,t+1} \times \frac{1}{\sigma_t} \\ r_{2,t+1} \times \frac{1}{\sigma_t} \\ \vdots \\ r_{K,t+1} \times \frac{1}{\sigma_t} \end{bmatrix}, \quad \text{and} \quad \eta \equiv \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \\ b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix}, \tag{5}$$

respectively. The conditional *mean-variance* multifactor portfolio is then given by the extended factor-weight vector, η, that optimizes the net-of-transaction-costs mean-variance utility of an investor with risk-aversion parameter γ,

$$\max_{\eta \geq 0} \quad \widehat{\mu}_{\text{ext}}^\top \eta - \widehat{\text{TC}}(\eta) - \frac{\gamma}{2} \eta^\top \widehat{\Sigma}_{\text{ext}} \eta, \tag{6}$$

where $\widehat{\mu}_{\text{ext}}$ and $\widehat{\Sigma}_{\text{ext}}$ are the sample mean and covariance matrix of the extended factor-return vector, $\widehat{\mu}_{\text{ext}}^\top \eta$ and $\eta^\top \widehat{\Sigma}_{\text{ext}} \eta$ are the sample mean and variance of the conditional multifactor portfolio return, and $\widehat{\mathrm{TC}}(\eta)$ is its sample transaction cost.[10] Note that because all of our factors are zero-cost portfolios, we do not need to impose a constraint that the weights of the conditional multifactor portfolio add up to one.

To alleviate the impact of estimation error, we discipline the conditional multifactor portfolios by assigning a nonnegative weight to each unmanaged factor $a_k \geq 0$ and a higher weight to each factor when volatility is low $b_k \geq 0$; that is, we impose the constraint that $\eta \geq 0$. These nonnegativity constraints are also economically meaningful in the sense of Campbell and Thompson (2008) because one would expect the optimal portfolio to load positively on the unmanaged factors and reduce exposure when volatility is high. However, in Section XII of the Internet Appendix, we show that our findings are robust to both relaxing and dropping entirely the nonnegativity constraints on the factor weights of the multifactor portfolios. Moreover, in Section XIII of the Internet Appendix, we show that our findings are also robust to constraining the leverage of the conditional multifactor portfolio to be at most 20% higher than that of the unconditional multifactor portfolio and to dropping low-institutional-ownership stocks from the sample.

C. Modeling Transaction Costs

We now explain how we compute the sample transaction cost of a conditional multifactor portfolio. First, note that the vector of stock trades required to rebalance the conditional multifactor portfolio at time $t+1$ is

$$\Delta w_{t+1}(\eta) = w_{t+1}(\eta) - w_t(\eta)^+, \tag{7}$$

where

$$w_{t+1}(\eta) = X_{\text{ext},t+1}\eta \quad \text{and} \tag{8}$$

$$w_t(\eta)^+ = w_t(\eta) \circ (e_t + r_{t+1}) \tag{9}$$

are the conditional multifactor portfolio at time $t+1$ and the conditional multifactor portfolio before rebalancing at time $t+1$, respectively, in which $x \circ y$ is the Hadamard or componentwise product of vectors x and y.

Given an estimation window with T historical observations of stock returns and factor portfolios, the average transaction cost incurred for rebalancing the conditional multifactor portfolio can be estimated as

[10] Following Moreira and Muir (2017), Barroso and Detzel (2021), and Cederburg et al. (2020), in the main body of the manuscript we study volatility management in the context of a short-term mean-variance investor with a one-month horizon, but Section IX in the Internet Appendix shows that our main findings are robust to evaluating performance over investment horizons up to 18 months. Note also that Moreira and Muir (2019) solve the optimal portfolio for a long-term investor with Epstein-Zin utility and find that the intertemporal hedging demands are small, and thus, their findings about the volatility-managed market portfolio based on mean-variance utility are robust to considering more general utility functions.

$$\widehat{\mathrm{TC}}(\eta) = \frac{1}{T-1} \sum_{t=1}^{T-1} \| \Lambda_t \Delta w_{t+1}(\eta) \|_1, \tag{10}$$

where $\|a\|_1 = \sum_{i=1}^N |a_i|$ denotes the 1-norm of the N-dimensional vector a, and the transaction-cost matrix at time t, Λ_t, is the diagonal matrix whose i^{th} diagonal element contains the transaction-cost parameter $\kappa_{i,t}$ of stock i at time t. Note that the transaction-cost term in equation (10) accounts for the netting of the rebalancing trades across multiple factors. That is, the transaction cost is computed by first netting the rebalancing trades across the K factor portfolios and then charging the transaction cost at the individual-stock level.

To isolate the benefit of trading diversification, we also compute transaction costs ignoring the netting of trades across factors. In this case, in contrast to (10), we estimate the transaction cost of the conditional multifactor portfolio by charging for the transaction cost *before* aggregating the rebalancing trades across the K factors,

$$\widehat{\mathrm{TC}}(\eta) = \frac{1}{T-1} \sum_{t=1}^{T-1} \sum_{k=1}^{K} \| \Lambda_t \left(x_{k,t+1}\theta_{k,t+1} - x_{k,t}^{+}\theta_{k,t} \right) \|_1, \tag{11}$$

where $x_{k,t}^{+} = x_{k,t} \circ (e_t + r_{t+1})$.

To compute the stock-level transaction-cost parameter $\kappa_{i,t}$, we use the two-day corrected method proposed by Abdi and Ranaldo (2017) to estimate the monthly bid-ask spread of the i^{th} stock as

$$\widehat{s}_{i,t} = \frac{1}{D} \sum_{d=1}^{D} \widehat{s}_{i,d}, \quad \widehat{s}_{i,d} = \sqrt{\max\{4(\mathrm{cls}_{i,d} - \mathrm{mid}_{i,d})(\mathrm{cls}_{i,d} - \mathrm{mid}_{i,d+1}), 0\}}, \tag{12}$$

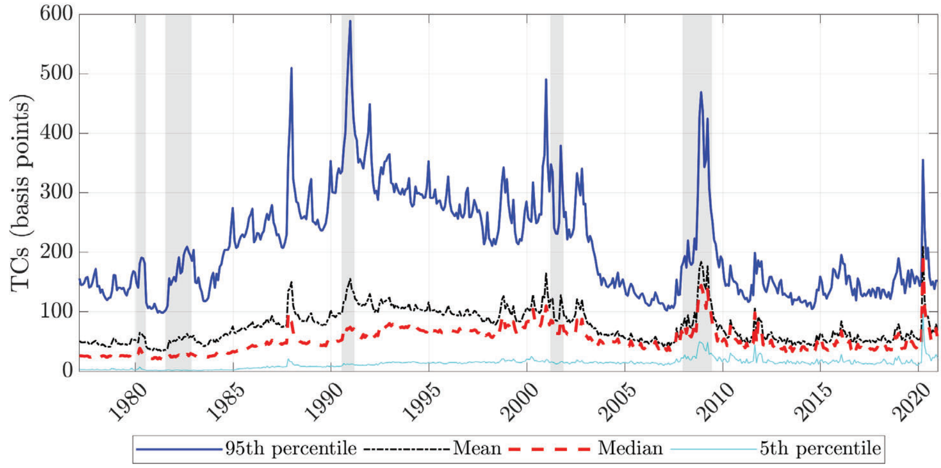
where D is the number of days in month t, $\widehat{s}_{i,d}$ is the two-day bid-ask spread estimate, $\mathrm{cls}_{i,d}$ is the closing log-price on day d, and $\mathrm{mid}_{i,d}$ is the mid-range log-price on day d; that is, the mean of daily high and low log-prices.[11] Finally, because the effective trading cost is half the bid-ask spread, the transaction-cost parameter for the i^{th} stock is $\kappa_{i,t} = \widehat{s}_{i,t}/2$.[12]

Figure 2 depicts the time series of transaction costs for January 1977 to December 2020 at the individual stock level (Panel A) and the average transaction

[11] Following Novy-Marx and Velikov (2016) and Barroso and Detzel (2021), we replace missing observations of transaction-cost parameters in month t with the transaction cost for the stock that is the closest match in terms of market capitalization and idiosyncratic volatility, or the closest match in terms of only one of these two characteristics if the other is missing, or the cross-sectional mean transaction-cost parameter if both characteristics are missing. We estimate idiosyncratic volatility as the standard deviation of residuals from a CAPM regression over the three months of daily data ending in month t.

[12] Sections XVI and XVII of the Internet Appendix, respectively, show that our findings are robust to considering proportional transaction costs estimated using the low-frequency average bid-ask spread of Chen and Velikov (2023) and quadratic price-impact costs estimated using the results of Novy-Marx and Velikov (2016).

Panel (A) *Individual-stock level*



Panel (B) *Factor level*

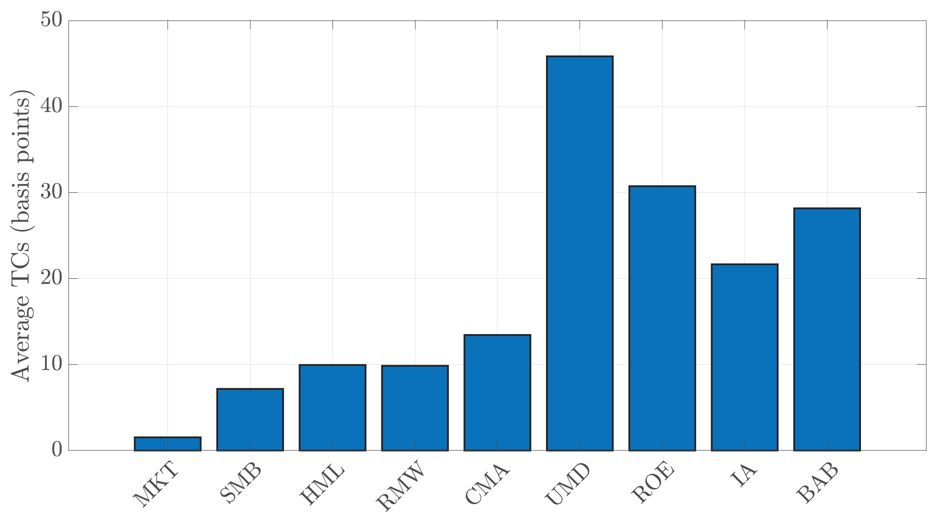


Figure 2. Transaction costs at the stock and factor levels. This figure depicts several descriptive statistics of the transaction costs estimated using the method of Abdi and Ranaldo (2017) described in equation (12). Panel A depicts how the 95^{th} percentile, mean, median, and 5^{th} percentile of individual stock transaction-cost parameters vary over time for the out-of-sample period from January 1977 to December 2020, with NBER recessions shaded in gray. Panel B depicts the average monthly transaction cost of trading the nine factors in our data set. (Color figure can be viewed at wileyonlinelibrary.com)

cost at the factor level (Panel B). Panel A shows that the transaction costs of individual stocks are highly time-varying, with the variation being stronger for less liquid stocks. The transaction costs of individual stocks are particularly large during NBER recessions, which are shaded in gray.

Panel B of Figure 2 shows that the factor with the highest average transaction cost is momentum (UMD), while the factor with the lowest average cost is the market (MKT).[13] Panel B also shows that our transaction-cost estimates are very similar to those in Barroso and Detzel (2021). For instance, our estimate of the transaction cost to rebalance the unmanaged SMB factor is around 7.5 basis points (bps) and Barroso and Detzel (2021, Figure 2, Panel D) estimate 7 bps; for HML, we estimate 10 bps and they 8; for RMW, we estimate 10 bps and they 10; for CMA, we estimate 13 bps and they 14; for UMD, we estimate 46 bps and they 50; for ROE, we estimate 31 bps and they 29; for IA, we estimate 22 bps and they 21; and for BAB, we estimate 28 bps and they 33.

II. Performance Gains from Volatility Management

In this section, we study the economic gains from volatility management. Section II.A evaluates the performance of the volatility-managed *individual*-factor portfolios, which are the focus of the existing literature. Section II.B evaluates the out-of-sample and net-of-costs performance of our proposed conditional *multifactor* portfolio.

A. Volatility-Managed Individual-Factor Portfolios

To set the stage for analyzing our multifactor portfolios, we first evaluate the in-sample performance of the volatility-managed *individual-factor* portfolios, which are the focus of Moreira and Muir (2017). We then assess the performance of these strategies net of transaction costs and out-of-sample, which allows us to confirm the findings of Barroso and Detzel (2021) and Cederburg et al. (2020), respectively.[14]

For each of the nine factors we consider, Table I reports the annualized Sharpe ratio of the unmanaged factor, $\text{SR}(r_k)$, the volatility-managed individual-factor portfolio, $\text{SR}(r_k, r_k^{\sigma})$, which is the mean-variance combination of the unmanaged factor and its managed counterpart given in (1), and the p-value for the difference between these two Sharpe ratios.[15] We consider an

[13] Section XVIII of the Internet Appendix shows that the transaction costs that we estimate for trading the market factor are very small and do not drive any of the results in our manuscript.

[14] To facilitate comparison with the existing literature, the individual-factor portfolios considered in this section are obtained by using inverse factor variance to time each of the factors. Section III of the Internet Appendix shows that the results are similar if, instead of using inverse factor variance to time the individual factors, we use inverse market volatility, which is the conditioning variable for our multifactor portfolios.

[15] We use bootstrap to construct one-sided p-values for the difference in Sharpe ratios. First, we generate 10,000 bootstrap samples of the returns of the volatility-managed individual-factor portfolio and the unmanaged factor using the stationary block-bootstrap method of Politis and

Table I

Performance of Volatility-Managed Individual-Factor Portfolios

For each of the nine factors we consider, this table reports the annualized Sharpe ratios of the unmanaged factor, $SR(r_k)$, the volatility-managed individual-factor portfolio, $SR(r_k, r_k^\sigma)$, which is the mean-variance combination of the unmanaged factor and its managed counterpart given in (1), and the p-value for the difference between these two Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample but ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs considering trading diversification. To facilitate comparison, both the in-sample and the out-of-sample performance are evaluated over the January 1977 to December 2020 period.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
	Panel A: In-Sample without Transaction Costs								
$SR(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$SR(r_k, r_k^\sigma)$	0.585	0.246	0.215	0.739	0.419	1.088	1.153	0.621	1.397
p-value$(SR(r_k, r_k^\sigma) - SR(r_k))$	0.242	0.366	0.337	0.033	0.311	0.000	0.001	0.094	0.000
	Panel B: In-Sample and Net of Transaction Costs but without Trading Diversification								
$SR(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$SR(r_k, r_k^\sigma)$	0.521	0.125	0.053	0.356	0.159	0.251	0.331	0.107	0.703
p-value$(SR(r_k, r_k^\sigma) - SR(r_k))$	0.464	0.500	0.500	0.500	0.500	0.249	0.407	0.500	0.161
	Panel C: Out-of-Sample but Ignoring Transaction Costs								
$SR(r_k)$	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
$SR(r_k, r_k^\sigma)$	0.408	0.068	0.194	0.527	0.355	1.035	1.094	0.605	1.321
p-value$(SR(r_k, r_k^\sigma) - SR(r_k))$	0.899	0.929	0.390	0.467	0.897	0.000	0.000	0.062	0.000
	Panel D: Out-of-Sample and Net of Transaction Costs but Ignoring Trading Diversification								
$SR(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$SR(r_k, r_k^\sigma)$	0.324	−0.295	−0.041	−0.453	−0.047	0.194	0.269	−0.127	0.690
p-value$(SR(r_k, r_k^\sigma) - SR(r_k))$	0.976	1.000	0.879	1.000	1.000	0.342	0.672	1.000	0.281
	Panel E: Out-of-Sample and Net of Transaction Costs with Trading Diversification								
$SR(r_k)$	0.519	0.125	0.053	0.356	0.159	0.114	0.311	0.107	0.606
$SR(r_k, r_k^\sigma)$	0.433	0.035	0.089	0.226	0.153	0.209	0.324	0.193	0.746
p-value$(SR(r_k, r_k^\sigma) - SR(r_k))$	0.917	0.858	0.243	0.965	0.547	0.097	0.405	0.029	0.064

Romano (1994) with an average block size of five. Second, we construct the empirical distribution of the difference between the Sharpe ratios of the returns of the volatility-managed individual-factor portfolio and the unmanaged factor, $SR(r_k, r_k^\sigma) - SR(r_k)$, across the 10,000 bootstrap samples. Third, we compute the p-value as the frequency with which this difference is smaller than investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample and ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs with trading diversification. Our sample spans January 1967 to December 2020 and, similar to the base-case analysis in Cederburg et al. (2020), we evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data.[16] Thus, the out-of-sample results are for January 1977 to December 2020. The out-of-sample return of each volatility-managed individual-factor portfolio is evaluated for the month following the last month of each estimation window. To ensure a fair comparison with the out-of-sample results, the in-sample results are computed for the same period, January 1977 to December 2020.[17]

Panel A of Table I confirms the main finding of Moreira and Muir (2017): in-sample and ignoring transaction costs, the Sharpe ratio of the volatility-managed individual-factor portfolio, $\text{SR}(r_k, r_k^\sigma)$, is greater than that of the unmanaged factor, $\text{SR}(r_k)$, for all nine factors, with the difference being statistically significant at the 10% level for five of the factors (RMW, UMD, ROE, IA, and BAB).[18]

Panel B reports the performance in-sample and net of transaction costs but, as in Barroso and Detzel (2021), ignoring the trading diversification benefits from combining the unmanaged and managed factors. Comparing Panels A and B, we observe that transaction costs greatly diminish the performance of the volatility-managed individual-factor portfolios. In fact, the transaction cost of the managed factor is so large for five of the nine factors—SMB, HML, RMW, CMA, and IA—that when considering the optimal combination of the

zero across the bootstrap samples. Section XXIV of the Internet Appendix shows that the inference is robust to using three other approaches to compute p-values: (i) the approach of Jobson and Korkie (1981), (ii) the approach of Ledoit and Wolf (2008), and (iii) an alternative bootstrap approach to compare in-sample Sharpe ratios that accounts for how the conditional portfolios are constructed.

[16] Cederburg et al. (2020) report that their results are not sensitive to the length of the estimation window: "We therefore consider specifications with 20-year ($K = 240$) and 30-year ($K = 360$) initial estimation periods. These designs produce roughly the same number of positive Sharpe ratio and CER differences that the base case does." They also report that their results are not sensitive to the value chosen for the risk aversion parameter: "Using a lower ($\gamma = 2$) or higher ($\gamma = 10$) risk aversion parameter leads to almost identical results to the base case with $\gamma = 5$."

[17] As mentioned in footnote 6, we consider a sample spanning the period 1977 to 2020, for which there are data to construct all nine factors we consider, so that we can evaluate the performance of the conditional multifactor portfolios that exploit all nine factors. It is possible, however, to evaluate the performance of each of the volatility-managed *individual-factor* portfolios for longer samples. Section I of the Internet Appendix shows that our main findings are robust to considering such longer samples.

[18] In Section XXV of the Internet Appendix, we show that the alphas of the nine volatility-managed individual-factor portfolios with respect to their unmanaged counterparts are positive, and that they are statistically significant for the same five factors (RMW, UMD, ROE, IA, and BAB).

unmanaged and the volatility-managed factors, the investor assigns zero weight to the managed factor, which explains why the Sharpe ratio of the individual-factor portfolio is the same as that of the unmanaged factor.[19] For the other four factors, the improvement in Sharpe ratio from volatility management is not statistically significant. We therefore conclude that even in-sample, the gains from volatility management are completely eroded by transaction costs, confirming the result in Barroso and Detzel (2021).

Panel C shows that the *out-of-sample* Sharpe ratio of the volatility-managed individual-factor portfolios in the absence of transaction costs is lower than the in-sample Sharpe ratio in Panel A for all nine factors. We also observe that the optimal combination of the unmanaged and volatility-managed factors delivers an out-of-sample Sharpe ratio, $\mathrm{SR}(r_k, r_k^\sigma)$, that can be smaller than that of even the unmanaged factor, $\mathrm{SR}(r_k)$; this is the case for the MKT, SMB, and CMA factors.[20] The out-of-sample gains from volatility management are statistically significant at the 10% level for only four of the nine factors (UMD, ROE, IA, BAB). Overall, our results show that, consistent with Cederburg et al. (2020), estimation error diminishes the gains from volatility management.

Panel D shows that transaction costs erode the out-of-sample performance further. In particular, once we account for both estimation error and transaction costs ignoring trading diversification, the Sharpe ratio for five of the nine volatility-managed individual-factor portfolios becomes negative. Moreover, the Sharpe ratio of the optimal combination of the unmanaged and volatility-managed factors is lower than that of the corresponding unmanaged factor for all factors except UMD and BAB, with neither being statistically significant, which drives home the point that estimation error and transaction costs offset entirely the gains from volatility-managing individual factors.

Comparing the Sharpe ratios of the volatility-managed individual-factor portfolios, $\mathrm{SR}(r_k, r_k^\sigma)$, in Panels D and E, we find that accounting for the netting of trades across the unmanaged and managed factors improves the performance of all nine portfolios. Moreover, with trading diversification, volatility management improves performance for five of the nine factors, with the

[19] Note that the *p*-values are not well defined in Panel B of Table I for the SMB, HML, RMW, CMA, and IA factors because the difference in Sharpe ratios is zero for every bootstrap sample, and thus, the entire empirical distribution for the difference in Sharpe ratios is concentrated at zero. For these cases, we set the *p*-value equal to 0.5, which is the level one would expect for a one-sided test when the Sharpe ratios of two portfolios are statistically indistinguishable.

[20] The first row of each panel in Table I reports the performance of each of the *unmanaged individual factors*, that is, assuming the investor assigns a constant weight to the factor. Note that the Sharpe ratio of the unmanaged factor does not depend on the value of the constant weight because both the average and standard deviation of the factor returns are linear in the weight. Accordingly, we simply report the Sharpe ratio of the *unscaled* unmanaged factor return, which does not require any estimation, and thus the in-sample and out-of-sample Sharpe ratios of the unmanaged factor reported in the first row of Panels A and C coincide. Moreover, there are no trading diversification benefits from trading an unmanaged factor in isolation, and thus its Sharpe ratio with costs is the same whether one accounts for trading diversification or not. Consequently, the Sharpe ratios reported in the first row of Panels B, D, and E also coincide.

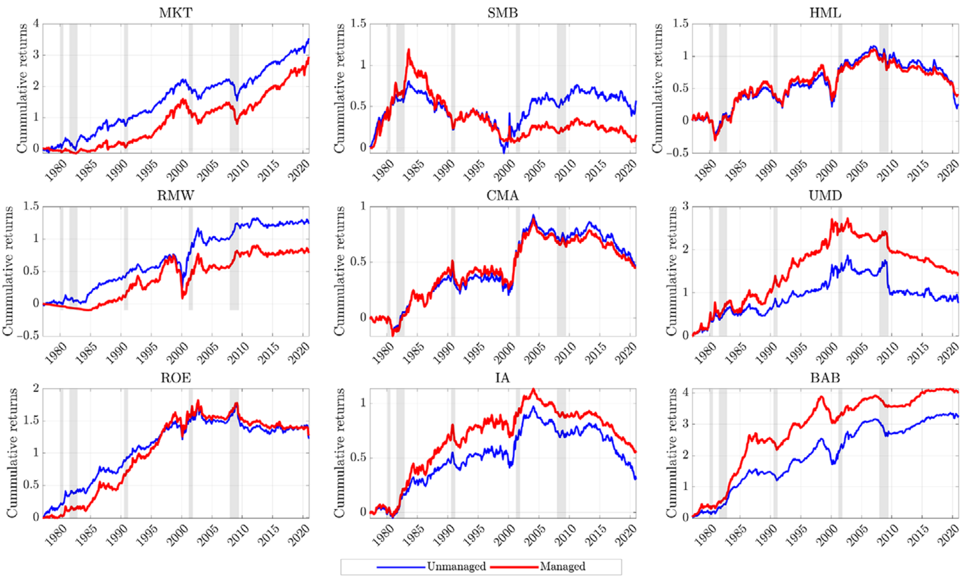


Figure 3. Cumulative returns of individual-factor portfolios. The nine graphs in this figure depict the out-of-sample cumulative returns net of transaction costs with trading diversification of each unmanaged factor (blue line) and its associated volatility-managed individual factor portfolio (red line) over the out-of-sample period from January 1977 to December 2020. The cumulative returns are reported in dollars, and the volatility-managed individual-factor portfolio is scaled to have the same volatility as the unmanaged factor. (Color figure can be viewed at wileyonlinelibrary.com)

improvement being statistically significant at the 10% level for three factors (UMD, IA, BAB).[21] Thus, trading diversification partially alleviates the concerns raised by Barroso and Detzel (2021) and Cederburg et al. (2020), but it does not fully resurrect the gains from volatility managing individual factors.[22]

To illustrate these results, Figure 3 depicts the out-of-sample cumulative returns net of transaction costs with trading diversification of each unmanaged factor (blue line) and its associated volatility-managed individual-factor portfolio (red line) over the out-of-sample period from January 1977 to December 2020. The cumulative returns are reported in dollars and the volatility-managed individual factor portfolio is scaled to have the same volatility as the

[21] This finding is consistent with Barroso and Santa-Clara (2015b), Cederburg and O'Doherty (2016), and Barroso, Detzel, and Maio (2021), who show that timing the volatility of the momentum and betting-against-beta factors produces substantial gains.

[22] Section II of the Internet Appendix shows that it is possible to improve the out-of-sample and net-of-costs performance of the volatility-managed individual-factor portfolios by assigning equal weights to the unmanaged and managed factors and estimating realized volatility using a six-month window of daily returns instead of a one-month window. After implementing these strategies to alleviate the impact of estimation error and transaction costs, the number of volatility-managed individual-factor portfolios that significantly outperform their unconditional counterpart at the 10% level increases from three to five.

unmanaged factor.[23] These plots show again that, with trading diversification, volatility management improves the performance for five of the nine factors, although (as shown in Table I) the difference is statistically significant for only three factors.

We conclude from the evidence presented above that, consistent with the findings of Barroso and Detzel (2021) and Cederburg et al. (2020), a volatility-managed portfolio based on an individual factor typically fails to significantly outperform its unmanaged counterpart when performance is measured out-of-sample and net of transaction costs.

B. Conditional Mean-Variance Multifactor Portfolio

In the previous section, we evaluate the performance of the volatility-managed individual-factor portfolios, which have been the focus of the existing literature. In this section, we provide a multifactor perspective by evaluating the benefits of volatility management for an investor who has access to multiple factors. To do so, we compare the out-of-sample and net-of-costs performance of two portfolios: the conditional mean-variance multifactor portfolio (CMV) obtained by solving Problem (6) and the unconditional mean-variance multifactor portfolio (UMV) obtained by solving Problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$, that is, under the constraint that its weights on the K factors do not vary with market volatility.

For each multifactor portfolio, Table II reports the out-of-sample annualized mean, standard deviation, Sharpe ratio of returns net of transaction costs, and p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios.[24] For completeness, the table also reports in percentage the annualized alpha of the time-series regression of the conditional portfolio out-of-sample returns net of transaction costs on those of the unconditional portfolio, the Newey-West t-statistic for the alpha, and the out-of-sample transaction costs accounting for trading diversification of the unconditional and conditional portfolios.[25] The portfolios are constructed exploiting all nine factors in our data set. We use an expanding-window approach and the out-of-sample period spans January 1977 to December 2020.

[23] Note that the unmanaged factor and the volatility-managed individual factor portfolio are both self-financing portfolios, and therefore earn payoffs rather than returns, but for simplicity we refer to the payoffs as returns. We calculate the cumulative return of each self-financing portfolio by adding the dollar payoffs over the entire out-of-sample period.

[24] Consistent with the conditional multifactor portfolio Problem (6), we compute the annualized net mean return as the difference between the out-of-sample gross mean return and the transaction cost, $E[r_{p,t+1}] - \text{TC}$, the standard deviation as $\text{stdev}(r_{p,t+1})$, and the Sharpe ratio as the ratio of these two quantities. We use the procedure described in footnote 15 to construct one-sided p-values for the difference in Sharpe ratios.

[25] For the Newey-West alpha t-statistic, we use a one-month lag throughout the manuscript. We have computed all Newey-West alpha t-statistics using alternative lags of five and 10 months and the inference (i.e., whether they are larger than two in absolute value) does not change for any of the t-statistics.

Table II

Performance of Conditional Mean-Variance Multifactor Portfolio

This table reports the out-of-sample and net-of-costs performance of two multifactor portfolios: the conditional mean-variance multifactor portfolio (CMV) obtained by solving Problem (6) and the unconditional mean-variance multifactor portfolio (UMV) obtained by solving Problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$, that is, under the constraint that its weights on the K factors are constant over time. For each multifactor portfolio, the table reports the out-of-sample annualized mean, standard deviation, Sharpe ratio of returns net of transaction costs accounting for trading diversification, and p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios. The table also reports the annualized alpha of the time-series regression of the conditional portfolio out-of-sample returns net of transaction costs on those of the unconditional portfolio, alpha Newey-West t-statistic, and out-of-sample transaction costs of the unconditional and conditional portfolios. The portfolios are constructed using all nine factors in our data set. We report out-of-sample performance from January 1977 to December 2020.

	UMV	CMV
Mean	0.430	0.477
Standard deviation	0.458	0.449
Sharpe ratio	0.940	1.062
p-value($\text{SR}_{\text{CMV}} - \text{SR}_{\text{UMV}}$)		0.006
α		0.066
$t(\alpha)$		3.637
TC	0.163	0.213

Table II shows that the conditional multifactor portfolio delivers an out-of-sample Sharpe ratio of net returns that is significantly larger than that of the unconditional portfolio. In particular, the conditional portfolio achieves a Sharpe ratio of 1.062, which is 13% higher than that of the unconditional portfolio, with the difference being statistically significant at the 1% level. The conditional portfolio also has a significantly positive annualized alpha with a t-statistic above three.[26] The table shows that the conditional portfolio has slightly lower volatility than its unconditional counterpart, and although the conditional portfolio incurs larger transaction costs, its gross mean return more than compensates for the additional trading costs associated with factor timing.[27]

Figure 4 plots the cumulative out-of-sample net returns of the unconditional (UMV) and conditional (CMV) multifactor portfolios. The returns are reported

[26] Note that the magnitude of the alpha of the conditional multifactor portfolios is not comparable to that of standard asset pricing factors. This is because while standard asset pricing factors assign a weight of one dollar to each of their long and short legs, the multifactor portfolio assigns a weight to its long and short legs that varies over time and is not generally equal to one dollar on average. Moreover, multiplying the conditional multifactor portfolio by a scalar larger than one will proportionally inflate its alpha. However, the alpha t-statistic of the conditional multifactor portfolio is invariant to scaling and therefore can be compared to the alpha t-statistics of standard asset pricing factors.

[27] Section XIX of the Internet Appendix shows that the conditional multifactor portfolio is also less risky than its unconditional counterpart in terms of alternative risk measures such as value-at-risk, maximum drawdown, skewness, and kurtosis.

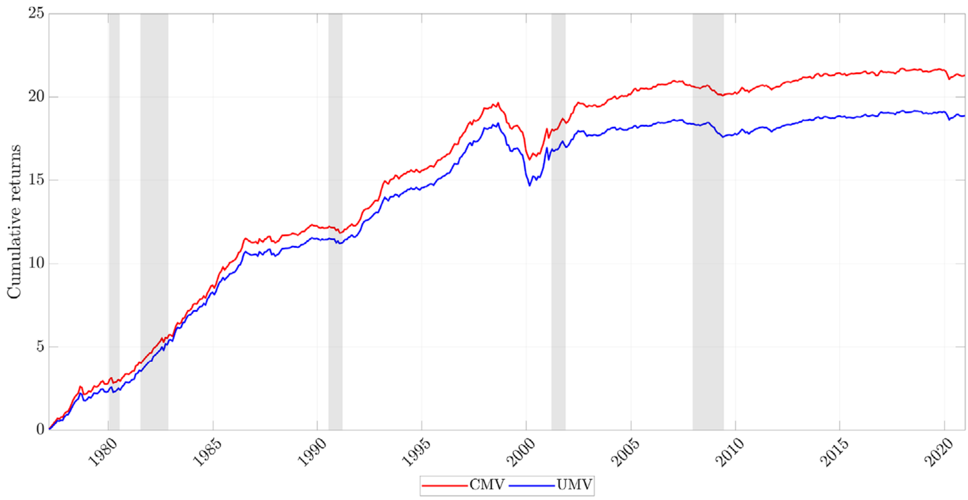


Figure 4. Cumulative returns of multifactor portfolios. This figure depicts the out-of-sample cumulative returns net of transaction costs of the unconditional (UMV) and conditional (CMV) mean-variance multifactor portfolios over the out-of-sample period from January 1977 to December 2020. Returns are reported in dollars and the conditional multifactor portfolio is standardized to have the same volatility as its unconditional counterpart. (Color figure can be viewed at wileyonlinelibrary.com)

in dollars and the conditional multifactor portfolio is standardized to have the same volatility as its unconditional counterpart. Figure 4 shows that the conditional multifactor portfolio outperforms the unconditional portfolio steadily over the entire sample.[28]

Finally, comparing the performance of the conditional *multifactor* portfolio in Table II with the performance of the volatility-managed *individual*-factor portfolios in Panel E of Table I, we see that, not surprisingly, the conditional multifactor portfolio also outperforms substantially the volatility-managed individual-factor portfolios out-of-sample and net of transaction costs.

[28] Section V of the Internet Appendix shows that our findings are not driven by the performance of a particular factor because the conditional multifactor portfolio outperforms its unconditional counterpart even after excluding each of the nine factors one at a time or replacing the BAB factor with a more conventional value-weighted BAB factor. In addition, Section VI of the Internet Appendix shows that the out-of-sample and net-of-costs performance of the conditional multifactor portfolio is significantly better than that of its unconditional counterpart during high-volatility and crises periods. Finally, Section XI of the Internet Appendix shows that the performance of the conditional multifactor portfolio can be further improved by estimating realized market volatility using daily market returns over three-, six-, or 12-month estimation windows. However, to facilitate comparison with existing literature, we focus our analysis on the conditional multifactor portfolios obtained using one-month realized volatility.

III. Understanding the Conditional Multifactor Portfolio

The results in the previous section demonstrate that the conditional multifactor portfolio significantly outperforms its unconditional counterpart out-of-sample and net of transaction costs. In this section, we conduct various experiments to understand the sources of the favorable performance of the conditional multifactor portfolio.

A. Disentangling the Source of the Gains

Table III reports the performance of three multifactor portfolios optimized either ignoring or accounting for transaction costs. The three multifactor portfolios are: (i) the unconditional multifactor portfolio (UMV), (ii) the conditional fixed-weight multifactor portfolio (CFW) of Moreira and Muir (2017),[29] and (iii) the conditional multifactor portfolio (CMV). Columns (1) to (3) report the performance of the three multifactor portfolios optimized ignoring transaction costs, while columns (4) to (6) account for transaction costs. The four panels in the table report portfolio performance evaluated in a different way: Panel A in-sample without transaction costs, Panel B out-of-sample but without transaction costs, Panel C out-of-sample with transaction costs but ignoring trading diversification, and Panel D out-of-sample with transaction costs and trading diversification.

We first discuss the performance of the three multifactor portfolios optimized ignoring transaction costs, which is reported in columns (1) to (3) of Table III. Comparing columns (1) and (2) in Panel A, we confirm the finding in Moreira and Muir (2017) that, in-sample and ignoring transaction costs, timing the unconditional multifactor portfolio leads to a statistically significant increase in the Sharpe ratio from 1.441 to 1.735. Column (3) shows that allowing the relative weight of each factor to vary with market volatility, one can obtain a slightly higher Sharpe ratio of 1.844. Panel B shows that the gains from volatility managing multiple factors are significant even out-of-sample, if one ignores transaction costs, although they are smaller than those in-sample. This is in contrast to the result in Table I that the volatility-managed *individual-factor* portfolios typically fail to significantly outperform the unmanaged factor out-of-sample. This result suggests that combining multiple factors can help to alleviate the impact of the estimation error associated with some of the noisier factors. However, columns (1) to (3) of Panel C show that accounting for transaction costs while ignoring trading diversification eliminates the out-of-sample gains from volatility-managing multiple factors. Finally, Panel D shows that if one accounts for trading diversification by netting out the rebalancing trades across the multiple factors, then both conditional multifactor portfolios outperform their unconditional counterpart even out-of-sample and net of transaction costs. Thus, the main

[29] Specifically, the CFW portfolio is the optimal combination of the unconditional mean-variance multifactor portfolio and its managed counterpart, obtained by scaling the unconditional portfolio by the inverse of its past-month return variance.

Table III

Understanding the Performance of the Multifactor Portfolios

This table reports the performance of three multifactor portfolios optimized either ignoring or accounting for transaction costs (TC). The three multifactor portfolios are: (i) the unconditional multifactor portfolio (UMV), (ii) the conditional fixed-weight multifactor portfolio (CFW) of Moreira and Muir (2017), and (iii) our conditional multifactor portfolio (CMV). Columns (1) to (3) report the performance of the three multifactor portfolios optimized ignoring transaction costs, while columns (4) to (6) account for transaction costs. The four panels report portfolio performance evaluated in a different way: Panel A in-sample without transaction costs, Panel B out-of-sample without transaction costs, Panel C out-of-sample with transaction costs without trading diversification, and Panel D out-of-sample with transaction costs with trading diversification. The sample period and quantities reported for each portfolio are the same as in Table II.

	Optimized Ignoring TC			Optimized Accounting for TC		
	UMV (1)	CFW (2)	CMV (3)	UMV (4)	CFW (5)	CMV (6)
Panel A: In-Sample without Transaction Costs						
Mean	0.415	0.602	0.680	0.301	0.314	0.432
Standard deviation	0.288	0.347	0.369	0.218	0.222	0.250
Sharpe ratio	1.441	1.735	1.844	1.378	1.415	1.726
p-value($SR_{CMV} - SR_{UMV}$)		0.000	0.000		0.005	0.000
α		0.187	0.213		0.009	0.107
$t(\alpha)$		5.738	6.684		3.073	7.638
TC	0.000	0.000	0.000	0.000	0.000	0.000
Panel B: Out-of-Sample without Transaction Costs						
Mean	0.753	0.783	0.925	0.593	0.626	0.690
Standard deviation	0.580	0.520	0.569	0.458	0.445	0.449
Sharpe ratio	1.299	1.506	1.625	1.295	1.407	1.537
p-value($SR_{CMV} - SR_{UMV}$)		0.000	0.000		0.002	0.000
α		0.140	0.239		0.059	0.126
$t(\alpha)$		5.012	5.797		3.808	6.414
TC	0.000	0.000	0.000	0.000	0.000	0.000
Panel C: Out-of-Sample with Transaction Costs without Trading Diversification						
Mean	0.412	0.313	0.349	0.347	0.343	0.332
Standard deviation	0.580	0.520	0.569	0.458	0.445	0.449
Sharpe ratio	0.710	0.601	0.613	0.758	0.772	0.739
p-value($SR_{CMV} - SR_{UMV}$)		0.986	0.930		0.329	0.721
α		−0.035	−0.027		0.012	0.001
$t(\alpha)$		−1.610	−0.727		0.967	0.031
TC	0.341	0.470	0.576	0.246	0.283	0.358

(Continued)

Table III—*Continued*

Panel D: Out-of-Sample with Transaction Costs and Trading Diversification						
Mean	0.517	0.511	0.575	0.430	0.457	0.477
Standard deviation	0.580	0.520	0.569	0.458	0.445	0.449
Sharpe ratio	0.891	0.984	1.010	0.940	1.026	1.062
p-value($SR_{CMV} - SR_{UMV}$)		0.017	0.072		0.002	0.006
α		0.072	0.103		0.046	0.066
$t(\alpha)$		3.094	2.759		3.407	3.637
TC	0.236	0.272	0.350	0.163	0.169	0.213

finding from columns (1) to (3) is that it is crucial to net out trades when accounting for transaction costs of volatility-managed multifactor portfolios.

We now discuss the performance of the three multifactor portfolios optimized accounting for transaction costs. Comparing columns (2) and (5) in Panel D, we observe that optimizing the conditional fixed-weight portfolio for transaction costs increases its out-of-sample and net-of-costs Sharpe ratio by around 4%. More importantly, a key takeaway from comparing columns (4) and (5) of Table III is that the conditional fixed-weight portfolio significantly outperforms its unconditional counterpart even in the presence of transaction costs and estimation error, once we optimize the portfolio for transaction costs and account for trading diversification. Moreover, column (6) of Panel D shows that allowing the relative weights on the different factors to vary with market volatility leads to a further moderate increase in the out-of-sample and net-of-costs Sharpe ratio. The conditional multifactor portfolios optimized accounting for transaction costs in columns (5) and (6) also incur lower transaction costs than the corresponding strategies in columns (2) and (3), where the factor weights are not optimized for transaction costs.

Summarizing, Table III shows that the favorable performance of the conditional multifactor portfolios has three drivers: (i) taking trading diversification into account when evaluating performance, (ii) accounting for transaction costs and trading diversification when optimizing portfolio weights, and (iii) allowing the relative weights on different factors to vary with market volatility. In the rest of this section, we examine these determinants more closely.

B. Trading Diversification of Multifactor Portfolios

To investigate the source of the trading diversification benefits that are one of the critical drivers of the favorable performance of the conditional multifactor portfolios, Figure 5 compares three quantities for each factor: (i) its mean gross return, (ii) its mean return net of transaction costs ignoring trading diversification, and (iii) its mean return net of transaction costs accounting for trading diversification.[30] For the case in which we account for trading diversification, we use the factor weights that solve Problem (6) in-sample.

[30] We use equations 12 and 14 of DeMiguel et al. (2020) to compute the transaction cost of each factor accounting for and ignoring trading diversification, respectively. We then report the

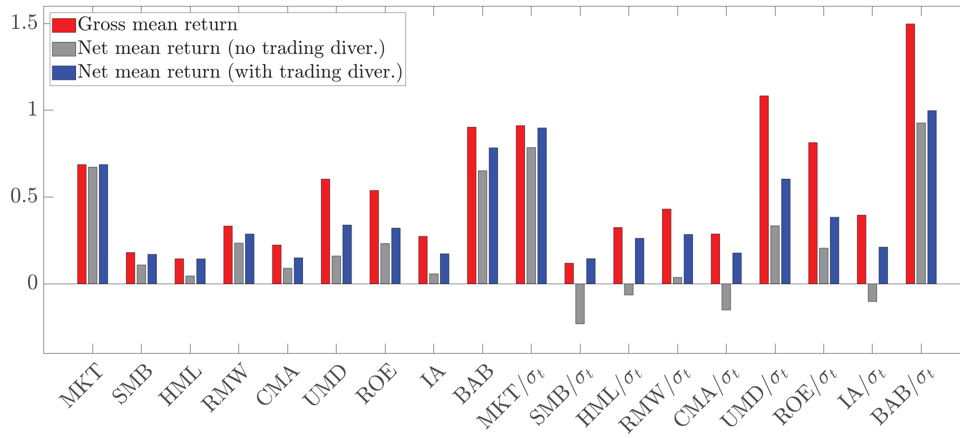


Figure 5. Gross and net-of-costs mean factor returns. This barplot depicts the monthly average factor returns (in percentage) of the nine unmanaged and volatility-managed factors. The figure compares three quantities for each factor: (i) its mean gross return, (ii) its mean return net of transaction costs ignoring trading diversification, and (iii) its mean return net of transaction costs accounting for trading diversification. For the case in which we account for trading diversification, we use the factor weights that solve Problem (6) in-sample. The sample spans January 1977 to December 2020. (Color figure can be viewed at wileyonlinelibrary.com)

We highlight four findings from Figure 5. First, comparing the mean gross return of each factor (red bar) with its mean net return when ignoring trading diversification (gray bar), we observe that transaction costs substantially reduce mean returns. For instance, when ignoring trading diversification, the mean net returns of three of the unmanaged factors (HML, UMD, and IA) are less than half their mean gross returns.

Second, transaction costs are even more critical for the profitability of the managed factors, with four of them (SMB/σ_t, HML/σ_t, CMA/σ_t, and IA/σ_t) having *negative* mean net returns when we ignore trading diversification.

Third, trading diversification helps explain why the conditional multifactor portfolio outperforms the unconditional multifactor portfolio even in the presence of transaction costs. In particular, although both multifactor portfolios benefit from the netting of trades across factors, the benefits are relatively larger for the conditional portfolios because they exploit managed factors, which are more expensive to trade.[31]

Fourth, most of the benefits from trading diversification arise from the netting of trades across different factors rather than across just the managed and

difference between the gross mean return of each factor and its transaction cost without or with trading diversification.

[31] Note that the managed SMB factor achieves a mean net return accounting for trading diversification that is larger than its mean gross return. This is because the rebalancing trades of the conditional mean-variance multifactor portfolio are negatively correlated with those of the managed SMB factor. Thus, one can effectively exploit the managed SMB factor at a negative transaction cost.

Table IV

Sources of Trading Diversification Benefits

This table reports the out-of-sample and net-of-costs performance of the unconditional (UMV) and conditional (CMV) multifactor portfolios. We evaluate the performance of the conditional multifactor portfolio for three cases: (i) taking trading diversification fully into account (that is, netting trades across all unmanaged and managed factors), (ii) taking trading diversification into account only partially (netting trades only across the unmanaged and managed versions of each individual factor, but not across different factors), and (iii) ignoring trading diversification altogether. The sample period and quantities reported for each portfolio are the same as in Table II.

	Case (i) with Full Trading Div. within and Across Factors		Case (ii) with Trading Div. Only within Factors	Case (iii) without Any Trading Div.
	UMV	CMV	CMV	CMV
Mean	0.430	0.477	0.351	0.332
Standard deviation	0.458	0.449	0.449	0.449
Sharpe ratio	0.940	1.062	0.783	0.739
p-value($SR_{CMV} - SR_{UMV}$)		0.006	1.000	1.000
α		0.066	−0.060	−0.079
$t(\alpha)$		3.637	−3.228	−4.223
TC	0.163	0.213	0.339	0.358

unmanaged versions of each individual factor. To demonstrate this, Table IV reports the out-of-sample performance of the conditional multifactor portfolio evaluated for three cases: (i) taking trading diversification fully into account (that is, netting trades across all unmanaged and managed factors), (ii) taking trading diversification into account only partially (netting trades only across the unmanaged and managed versions of each individual factor, but not across different factors), and (iii) ignoring trading diversification altogether.

Case (iii) in Table IV shows that the out-of-sample Sharpe ratio of the conditional multifactor portfolio when we ignore trading diversification is 0.739, which is smaller than that of its unconditional counterpart when we ignore trading diversification, 0.758, as shown earlier in Table III (column (4) of Panel C). Case (ii) in Table IV shows that allowing for trading diversification across just the unmanaged and managed versions of each individual factor increases the Sharpe ratio of the conditional multifactor portfolio only marginally from 0.739 to 0.783. However, Case (i) shows that allowing for trading diversification across all unmanaged and managed factors substantially increases the Sharpe ratio of the conditional multifactor portfolio from 0.783 to 1.062, making it significantly higher than that of the unconditional portfolio, 0.940.[32]

In summary, most of the trading diversification benefits enjoyed by the conditional multifactor portfolio arise from the netting of trades across different factors. Thus, the favorable performance of the conditional *multifactor*

[32] One can make the same inference by comparing the alpha t-statistics or transaction costs of these three portfolios instead of their Sharpe ratios.

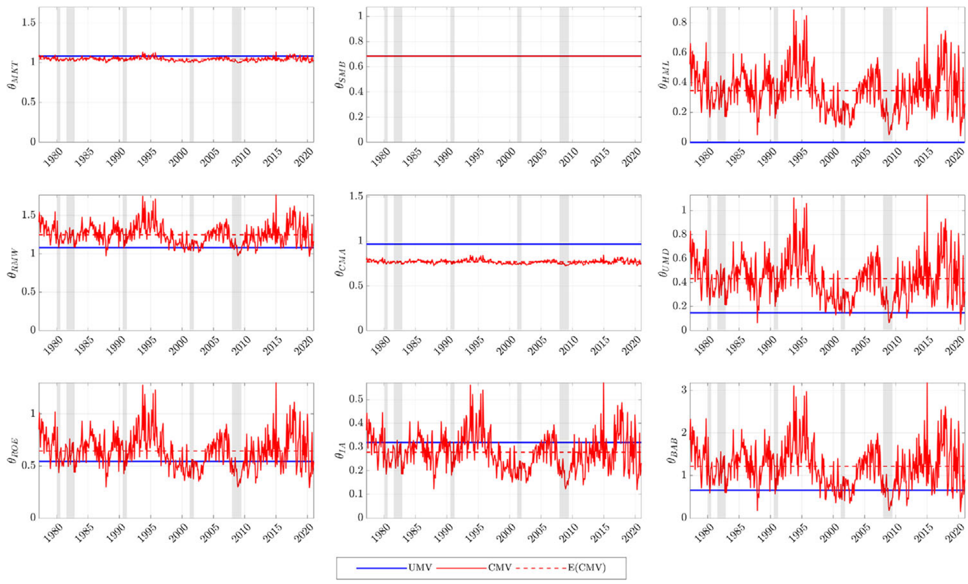


Figure 6. Weights of unconditional and conditional multifactor portfolios. This figure depicts the in-sample weights of the unconditional multifactor portfolio (UMV, blue line) and the conditional multifactor portfolio (CMV, solid red line) from January 1977 to December 2020. The figure also depicts the average weights of the conditional multifactor portfolio (E[CMV], dashed red line). Each of the nine graphs depicts the weights for a particular factor. (Color figure can be viewed at wileyonlinelibrary.com)

portfolio compared to the volatility-managed *individual-factor* portfolios is explained in part by the benefits of trading diversification across multiple factors.[33]

C. Time Variation in Multifactor Portfolio Weights

We now study how the conditional multifactor portfolio benefits from the ability to time the various factors *differentially*, which is ruled out for the conditional fixed-weight portfolios. Figure 6 plots the in-sample weights over the January 1977 to December 2020 period of the unconditional multifactor portfolio (UMV, blue line) and the conditional multifactor portfolio (CMV, solid red line) that account for transaction costs and trading diversification.[34] The

[33] Another reason that the multifactor portfolio outperforms the individual-factor portfolios is that it takes advantage of the risk diversification benefits from combining multiple factors. Section XX of the Internet Appendix shows that the market and size factors are moderately negatively correlated with the seven other factors, and thus multifactor portfolios benefit substantially from risk diversification across factors.

[34] We consider in-sample weights in this section so that the weights of the unconditional mean-variance portfolio are constant over time, which allows us to interpret the time variation of the conditional multifactor portfolio weights. However, we show in Section XXI of the Internet Appendix that our insights are robust to considering the out-of-sample weights of the conditional and unconditional multifactor portfolios.

figure also depicts the average weights of the conditional multifactor portfolio (E[CMV], dashed red line).

Figure 6 shows that the unconditional multifactor portfolio assigns a strictly positive weight to every factor except value (HML), to which it assigns zero weight. This is not surprising given that Panel B of Table I shows that the Sharpe ratio of net returns of the HML factor is only 5.3%, the smallest across the nine factors. The figure also shows that the conditional multifactor portfolio assigns an almost-constant weight to three factors (MKT, SMB, CMA), while aggressively timing the other six (HML, RMW, UMD, ROE, IA, BAB). For instance, the weights of the conditional multifactor portfolio on these six factors drop dramatically during the Great Recession and after the early 2000s recession, but they increase during periods of low market volatility, such as 1992 to 1997. Thus, our conditional portfolio takes advantage of the opportunity to time factors differentially.

Because our conditional multifactor portfolio times the factors differentially, it optimally assigns an average weight to each factor that differs substantially also from that of the unconditional portfolios, as shown in Figure 6. For example, the conditional portfolio assigns a much higher average weight to the value (HML), momentum (UMD), and betting-against-beta (BAB) factors than the unconditional portfolio. Interestingly, allowing the relative weight of each factor to vary with market volatility "resurrects" the value (HML) factor, to which the conditional multifactor portfolio assigns a substantial average weight of about 0.35. In contrast, the conditional portfolio assigns a substantially lower average weight to the investment factors (CMA and IA) than the unconditional portfolio.[35]

IV. Economic Mechanism and Implications

In this section, we first characterize the economic mechanism driving the performance of the conditional multifactor portfolio by studying how the risk-return trade-off for individual factors varies with market volatility. We then study the broader economic implications of our work by estimating a conditional SDF whose price of risk for each factor varies with market volatility.

A. Factor Risk-Return Trade-Off and Market Volatility

Moreira and Muir (2017, Figure 1) show that the risk-return trade-off for the market weakens with market volatility and that this explains the outperformance of the volatility-managed market portfolio. In particular, they find

[35] In contrast to our conditional multifactor portfolio, the conditional *fixed-weight* portfolio is obtained by timing the unconditional multifactor portfolio in its entirety, and thus its relative weight on each factor coincides with that of the unconditional portfolio. Consequently, the conditional fixed-weight portfolio has zero weight on HML, just like the unconditional portfolio. In Section XXVI of the Internet Appendix, we show that also for the other factors, the average weight assigned by the conditional fixed-weight portfolio is similar to that assigned by the unconditional portfolio.

that "there is little relation between lagged volatility and average returns, but there is a strong relation between lagged volatility and current volatility. This means that the mean-variance trade-off weakens in periods of high volatility." Thus, a mean-variance investor should decrease exposure to the market when realized market volatility is high.

We extend the analysis in Moreira and Muir (2017) to the other factors in our data set, besides the market. Our key finding is that for all nine individual factors, the risk-return trade-off *weakens* with realized market volatility. This explains why our conditional multifactor portfolio, which reduces exposure to the risk factors when realized market volatility is high, outperforms its unconditional counterpart.

Figure 1 in the introduction depicts how the risk-return trade-off for the nine factors varies with realized market volatility. In the figure, we first use the monthly time series of realized market volatility to sort the months in our sample into terciles. For each factor, we then estimate the risk-return trade-off for month t as the realized factor return for month $t+1$ divided by the monthly realized factor variance estimated as the sample variance of daily returns for month t. Finally, we report the risk-return trade-off averaged across the months in each tercile.[36]

Figure 1 shows that the risk-return trade-off for all nine factors *weakens* with realized market volatility. To see this, note that the risk-return trade-off for the low-market-volatility tercile (blue bars) is higher than that for the high-market-volatility tercile (red bars) for every factor. Moreover, the weakening of the risk-return trade-off is substantial for some of the factors (UMD, ROE, and BAB) but less striking for others (MKT, SMB, and CMA).[37] This explains why the conditional multifactor portfolio assigns an almost-constant weight to the MKT, SMB, and CMA factors but a time-varying weight to the rest of the factors, as shown in Figure 6.

B. Conditional Stochastic Discount Factor

To understand the broader economic implications of our work, we also estimate a conditional SDF whose price of risk for each factor can vary with inverse market volatility. To do so, we extend the unconditional approach of Barroso and Maio (2021) to study how the prices of risk for the nine factors vary with market volatility.

[36] Section XXII of the Internet Appendix shows that the findings from Figure 1 are robust to estimating the factor risk-return trade-off for each volatility tercile, instead of estimating it for each month and then computing the average risk-return trade-off across the months in that volatility tercile.

[37] Note that the mean-variance trade-off for some of the factors (UMD, ROE, and BAB) is above 50 for the low-market-volatility tercile, which may seem unreasonably large. These large trade-offs stem from a combination of high mean returns and low variance. However, we have also computed the corresponding Sharpe ratios and find that they are below one for every factor across all three terciles. For instance, for the low-market-volatility tercile, the Sharpe ratio for the BAB factor is 0.83 and for the ROE factor is 0.59.

To set the stage for our empirical analysis, we assume that, conditional on realized market volatility σ_t, there is an SDF that prices tradable assets, that is, $0 = E_t(M^{\sigma}_{t+1} r^e_{t+1})$, where M^{σ}_{t+1} is the conditional SDF at time $t+1$ and r^e_{t+1} is the vector of excess asset returns at time $t+1$. Furthermore, we assume that the price of risk for the k^{th} factor is an affine function of inverse realized market volatility at time t so that

$$M^{\sigma}_{t+1} = 1 - \sum_{k=1}^{K} (\alpha_k + \beta_k \tilde{\sigma}_t^{-1}) \tilde{r}_{k,t+1}, \tag{13}$$

where $\tilde{\sigma}_t^{-1}$ is the demeaned inverse realized market volatility at time t, $\tilde{\sigma}_t^{-1} = 1/\sigma_t - E(1/\sigma_t)$, and $\tilde{r}_{k,t+1}$ is the conditionally demeaned k^{th} factor return at time $t+1$, $\tilde{r}_{k,t+1} = r_{k,t+1} - E_t(r_{k,t+1})$. The SDF prices every traded factor return, and thus[38]

$$E_t(r_{i,t+1}) = \sum_{k=1}^{K} (\alpha_k + \beta_k \tilde{\sigma}_t^{-1}) \text{cov}_t(r_{i,t+1}, r_{k,t+1}), \quad \text{for } i = 1, 2, \ldots, K, \tag{14}$$

where $\text{cov}_t(r_{i,t+1}, r_{k,t+1})$ is the conditional covariance between the i^{th} and k^{th} factor returns.

One could estimate the coefficients α_k and β_k by running a pooled conditional regression for the K equations in (14), but this would require estimating a large number of $K(K+1)/2$ realized factor variances and covariances each month. We employ two approaches to address the challenge of estimating a large covariance matrix. First, one could ignore the correlations between the different factors, which would lead to a more parsimonious (robust) approach. Second, one could estimate the pooled conditional regression in (14) but use a shrinkage estimator of the covariance matrix of factor returns. We pursue the first approach below; in Section XXIII of the Internet Appendix, we show that our findings are similar when using the second approach. Ignoring the correlations between different factors, (14) simplifies to

$$E_t(r_{k,t+1}) = (\alpha_k + \beta_k \tilde{\sigma}_t^{-1}) \text{var}_t(r_{k,t+1}), \quad \text{for } k = 1, 2, \ldots, K, \tag{15}$$

which requires estimating only K realized factor-return variances each month. Moreover, equation (15) implies that, when factor returns are uncorrelated, the conditional risk-return trade-off for the k^{th} factor, given by $E_t(r_{k,t+1})/\text{var}_t(r_{k,t+1})$, is equal to the price of risk for the k^{th} factor, $\alpha_k + \beta_k \tilde{\sigma}_t^{-1}$, that is,

$$\frac{E_t(r_{k,t+1})}{\text{var}_t(r_{k,t+1})} = \alpha_k + \beta_k \tilde{\sigma}_t^{-1}, \quad \text{for } k = 1, 2, \ldots, K. \tag{16}$$

[38] To see this, note that $0 = E_t(M^{\sigma}_{t+1} r_{i,t+1}) = E_t(M^{\sigma}_{t+1}) E_t(r_{i,t+1}) + \text{cov}_t(M^{\sigma}_{t+1}, r_{i,t+1})$. Because $E_t(M^{\sigma}_{t+1}) = 1$, we have that $E_t(r_{i,t+1}) = -\text{cov}_t(M^{\sigma}_{t+1}, r_{i,t+1}) = \sum_{k=1}^{K} (\alpha_k + \beta_k \tilde{\sigma}_t^{-1}) \text{cov}_t(r_{i,t+1}, r_{k,t+1})$.

Table V

Factor Risk Prices and Market Volatility

This table reports the coefficients α_k and β_k for the time-series regression defined in equation (17) for the nine factors in our data set. The numbers in square brackets are Newey-West t-statistics. The sample spans January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
α_k	6.668	5.479	7.573	30.467	13.341	33.565	46.118	20.979	41.824
	[3.790]	[1.494]	[1.258]	[4.499]	[2.240]	[6.676]	[7.101]	[3.820]	[8.678]
β_k	0.512	0.155	1.002	1.155	0.329	1.561	1.361	0.770	2.311
	[2.358]	[0.420]	[2.270]	[2.236]	[0.749]	[3.840]	[2.621]	[1.982]	[7.204]

To test how the price of risk for the k^{th} factor varies with market volatility, we estimate the time-series regression

$$\frac{r_{k,t+1}}{\sigma^2_{k,t}} = \alpha_k + \beta_k \tilde{\sigma}_t^{-1} + \epsilon_{k,t+1}, \tag{17}$$

where $\sigma^2_{k,t}$ is the monthly realized variance of the k^{th} factor estimated as the sample variance of daily returns over month t, and $\epsilon_{k,t+1}$ is the residual at time $t+1$. Note that one can estimate the unconditional price of risk for the k^{th} factor as $E(r_{k,t+1}/\sigma^2_{k,t}) = E(\alpha_k + \beta_k \tilde{\sigma}_t^{-1} + \epsilon_{k,t+1}) = \alpha_k$ because $E(\tilde{\sigma}_t^{-1}) = 0$ and $E(\epsilon_{k,t+1}) = 0$. Thus, to test whether the *unconditional* price of risk for the k^{th} factor is positive, we can use the t-statistic for the estimated coefficient α_k. More importantly, to test whether the *conditional* price of risk for the k^{th} factor weakens with market volatility, we can use the t-statistic for the estimated coefficient β_k.

Table V reports the results for the time-series regressions in (17) for the nine factors. Our first observation is that the estimated coefficient α_k is positive for all nine factors. This indicates that, as one would expect, the unconditional price of risk for the nine factors is positive. Moreover, the unconditional price of risk is significant for MKT, RMW, CMA, UMD, ROE, IA, and BAB.

More importantly, our second observation from Table V is that, consistent with the results in Figure 1, the estimated coefficient β_k is positive for every individual factor, which indicates that the conditional price of risk for all nine factors decreases with realized market volatility. This is a counterintuitive result because one expects that the price of risk of systematic risk factors should *not* decrease with market volatility. We also observe that the reduction in the price of risk is significant for some of the factors (MKT, HML, RMW, UMD, ROE, and BAB) but not for others (SMB, CMA, and IA).[39] Thus, although conditioning on volatility helps construct an SDF that better spans the investment

[39] The statistical significance of β_k for the UMD and BAB factors is consistent with the findings by Barroso and Santa-Clara (2015b), Cederburg and O'Doherty (2016), and Barroso, Detzel, and Maio (2021) that timing the volatility of the UMD and BAB factors produces substantial gains.

opportunity set, the importance of conditioning on volatility to achieve this goal varies across factors.[40]

V. Conclusion

We develop a new strategy that exploits market volatility to time investment in popular asset pricing factors. Instead of timing an individual equity factor conditional on its variance or timing a fixed combination of factors conditional on the variance of that combination, we consider a conditional multifactor portfolio whose relative weight on each factor can vary with market volatility. We show that the conditional multifactor portfolio outperforms its unconditional counterpart even out-of-sample and net of transaction costs. To study the economic mechanism driving the performance of the conditional multifactor portfolio, we estimate the factor risk-return trade-off and prices of risk and find that they generally decrease with market volatility. This is counterintuitive because one would expect the price of risk of systematic factors to remain constant or increase with market volatility. Thus, the breakdown of the most fundamental premise in finance, that between risk and returns, is more puzzling than previously thought.

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Appendix

Factor Definitions

We consider the same nine factors as Moreira and Muir (2017). This includes the six factors—MKT, SMB, HML, RMW, CMA, and UMD—constructed as in Fama and French (2018), the Hou, Xue, and Zhang (2015) profitability and investment factors (ROE and IA), and the Frazzini and Pedersen (2014) BAB factor. The SMB and HML Fama-French factors are constructed using six value-weighted portfolios formed as the intersection of stocks sorted independently on two size buckets (big and small) and three book-to-market buckets (value, neutral, and growth). The RMW, CMA, and UMD factors are constructed using six value-weighted portfolios formed as the intersection of stocks sorted independently on two size buckets (big and small) and three buckets using operating profitability, asset growth, and prior returns from month −12 to −2, respectively. Similarly, the ROE and IA factors of Hou, Xue,

[40] Note that $\tilde{\sigma}_t^{-1}$ can be negative, and thus, the expected risk-return trade-off predicted by the conditional regression in (17), $E_t(r_{k,t+1})/\sigma_{k,t}^2 = \alpha_k + \beta_k \tilde{\sigma}_t^{-1}$, could be negative. However, we find empirically that the expected risk-return trade-off predicted by the regression is positive for every month for SMB, RMW, CMA, ROE, and IA, for more than 95% of the months for UMD and BAB, for more than 85% of the months for MKT, and for around 70% of the months for HML. This is reassuring because, for the factors we consider, the expected risk-return trade-off should be positive.

and Zhang (2015) are constructed using 18 value-weighted portfolios formed as the intersection of stocks sorted independently on two size buckets (big and small), three profitability (return on equity) buckets, and three investment (asset growth) buckets. The Fama-French and Hou, Xue, and Zhang (2015) factors use NYSE breakpoints to define the value-weighted portfolios for the construction of the factors. Below, we summarize how each factor is constructed.

(i) Market (MKT): Excess return on the value-weighted portfolio of all NYSE, AMEX, and NASDAQ firms with a CRSP share code of 10 or 11.
(ii) Size (SMB): Average return on the three value-weighted small portfolios minus the average return on the three value-weighted big portfolios.
(iii) Value (HML): Average return on the two high-book-to-market value-weighted portfolios minus the average return on the two low-book-to-market value-weighted portfolios.
(iv) Robust-minus-weak (RMW): Average return on the two robust (i.e., high) operating-profitability value-weighted portfolios minus the average return on the two weak (i.e., low) operating-profitability value-weighted portfolios.
(v) Conservative minus aggressive (CMA): Average return on the two conservative-investment (i.e., low asset growth) value-weighted portfolios minus the average return on the two aggressive-investment (i.e., high asset growth) value-weighted portfolios.
(vi) Momentum (UMD): Average return on the two high-prior-return value-weighted portfolios minus the average return on the two low-prior-return value-weighted portfolios.
(vii) Profitability (ROE): Average return on the six high-return-on-equity value-weighted portfolios minus average return on six low-return-on-equity value-weighted portfolios.
(viii) Investment (IA): Average return on the six low-asset-growth value-weighted portfolios minus the average return on the six high-asset-growth value-weighted portfolios.
(ix) Betting against beta (BAB): Excess return of market neutral portfolio that buys a rank-weighted portfolio of low-beta stocks and shorts a rank-weighted portfolio of high-beta stocks.[41]

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.
Replication Code.