20130514-4

A Model of Financialization of Commodities

SULEYMAN BASAK and ANNA PAVLOVA*

ABSTRACT

We analyze how institutional investors entering commodity futures markets, referred to as the financialization of commodities, affect commodity prices. Institutional investors care about their performance relative to a commodity index. We find that all commodity futures prices, volatilities, and correlations go up with financialization, but more so for index futures than for nonindex futures. The equity-commodity correlations also increase. We demonstrate how financial markets transmit shocks not only to futures prices but also to commodity spot prices and inventories. Spot prices go up with financialization, and shocks to any index commodity spill over to all storable commodity prices.

JEL Classifications: G12, G18, G29

Keywords: Asset pricing, indexing, commodities, futures, spot prices, institutions, money

*Suleyman Basak and Anna Pavlova are at the London Business School and CEPR. We thank participants at the Advances in Commodity Markets, AFA, Arne Ryde, Cowles GE, EFA, MFS, NBER Asset Pricing, NBER Commodities, Paul Woolley Centre, Rothschild Caesarea, and SFI conferences, as well as seminar participants at Bank Gutmann, Berkeley, BIS, BU, Chicago, Geneva, IDC, Imperial, Frankfurt, INSEAD, LBS, LSE, Luxembourg, Melbourne, Monash, Nova, Oxford, St. Gallen, Stanford, Stockholm, and Zurich, as well as Viral Acharya, Steven Baker, Francisco Gomes, Christian Heyerdahl-Larsen, Lutz Kilian, Andrei Kirilenko, Kjell Nyborg, Michel Robe, Sylvia Sarantopoulou-Chiourea, Olivier Scaillet, Dimitri Vayanos, and Wei Xiong for helpful comments. We have also benefitted from the valuable suggestions of Ken Singleton and two anonymous referees. Adem Atmaz provided excellent research assistance. Pavlova gratefully acknowledges financial support from the European Research Council (grant StG263312).

management, asset class.

A sharp increase in the popularity of commodity investing over the past decade has triggered an unprecedented inflow of institutional funds into commodity futures markets, referred to as the financialization of commodities. At the same time, the behavior of commodity prices has become highly unusual. Commodity prices have experienced significant run-ups, and the nature of their fluctuations has changed considerably. An emerging literature on the financialization of commodities attributes this behavior to the emergence of commodities as an asset class, which has become widely held by institutional investors seeking diversification benefits (Buyuksahin and Robe (2014), Singleton (2014)). Starting in 2004, institutional investors have been rapidly building their positions in commodity futures.[1] A Commodity Futures Trading Commission (CFTC) (2008) staff report estimates institutional holdings to have increased from $15 billion in 2003 to over $200 billion in 2008. Many institutional investors hold commodities through a commodity futures index, such as the Goldman Sachs Commodity Index (GSCI), the Dow Jones UBS Commodity Index, or the S&P Commodity Index (SPCI). Tang and Xiong (2012) document that after 2004 the behavior of index commodities has become increasingly different from that of nonindex commodities, with the former becoming more correlated with oil, an important index constituent, and more correlated with the equity market. These intriguing facts could be attributed to the entry of institutional investors into commodity futures markets. The financialization theory has far-reaching implications for regulation: the 2004 to 2008 run-up in commodity prices has prompted many calls for restrictions on the positions of institutions who may have generated the run-up (see Master's (2008) testimony).

While the empirical literature on the financialization of commodities has contributed to the policy debate, theoretical literature on the subject remains scarce. Our goal in this paper is to model the financialization of commodities and to disentangle the effects of institutional flows from the traditional demand and supply effects on commodity futures prices. We particularly focus on identifying the economic mechanisms through which institutions may influence commodity futures prices, volatilities, and their comovement, as well as on how their presence may affect commodity spot prices and inventories.

We develop a multi-good, multi-asset dynamic model with institutional investors and standard futures markets participants. Institutional investors care about their performance relative to a commodity index. They do so because their investment mandate specifies a benchmark index for performance evaluation or because their mandate includes hedging against commodity price inflation. We capture such benchmarking through the institutional objective function. Consistent with extant literature on benchmarking (originating from Brennan (1993)), we posit that the marginal utility of institutional investors increases with the index. In particular, institutional investors do not like to perform poorly when their benchmark index does well and so have an additional incentive to do well when their benchmark does well.[2] All investors in our model invest in the commodity futures markets and the stock market. Prices in these markets fluctuate in response to three possible sources of shocks: (i) commodity supply shocks, (ii) commodity demand shocks, and (iii) (endogenous) changes in wealth shares of the two investor classes. We include in the index only a subset of the traded futures contracts. It is then possible to compare a pair of otherwise identical commodities, one of which belongs to the index and the other does not. We capture the effects of financialization by comparing our economy with institutional investors to an otherwise identical benchmark economy with no institutions. The model is solved in closed form, and all our results below are derived analytically.

We first find that the prices of all commodity futures go up with financialization. However, the price increase is higher for futures belonging to the index than for nonindex futures. This pattern obtains because institutions strive to not fall behind when the index does well, and thus they value assets that pay off more in high-index states. As a result, relative to the benchmark economy without institutions, futures whose returns are positively correlated with those of the index are valued higher. In our model all futures are positively correlated because they are valued using the same discount factor, and so all futures prices go up with financialization. The comovement with the index, however, is higher for futures included in the index. Therefore, prices of index futures rise more than those of nonindex futures. The larger the institutions, the more they affect pricing—or, more formally, the discount factor—making the above effects stronger.

We next find that the volatilities of both index and nonindex futures returns go up with financialization. This is because, absent institutions, there are only two sources of risk: supply risk and demand risk. With institutions present, some agents in the economy (institutional investors) face an additional risk of falling behind the index. This risk is reflected in futures prices and raises the volatilities of futures returns. While the volatilities of all futures rise, those of index futures rise more. Index futures are especially attractive to institutional investors because of their high comovement with the index. Hence, their volatilities rise enough to make them unattractive to normal investors (standard market participants), so that they are willing to sell index futures holdings to institutions.

We also find that the correlations among commodity futures as well as the equity-commodity correlations increase with financialization. The often-quoted intuition for this increase is that commodity futures markets were largely segmented before the inflow of institutional investors in the mid-2000s, and that institutions entering these markets have linked them together, as well as with the stock market, through the cross-holdings in their portfolios. We show that this argument does not need to rely on market segmentation, as the increase in correlations may occur even under complete markets. Benchmarking institutional investors to a commodity index leads to the emergence of this index as a new (common) factor in commodity futures and stock returns. In equilibrium, all assets load positively on this factor, which increases their covariances and correlations. We show that index commodity futures are more sensitive to this new factor, and so their covariances and correlations with each other increase to a greater extent than those for otherwise identical nonindex commodities. Furthermore, we explicitly model demand shocks, which allows us to disentangle the effects of institutional investors from the effects of demand and supply (fundamentals) and conclude that the effects of financialization are sizeable.

To address the question of how commodity spot prices and inventories are affected by financialization in our model, we follow the classical theory of storage (Deaton and Laroque (1992, 1996)) and introduce intermediate consumption and storage decisions. Our main departure from the extant storage literature is that cash flows from storing a commodity are discounted with a (stochastic) discount factor, which is influenced by all investors, including institutions, and not at a constant riskless rate. We show that only *storable* commodity prices are affected by financialization. In the presence of institutions, storable commodity inventories and prices are higher than in the benchmark economy, and again this effect is stronger for commodities included in the index. Storing a commodity is akin to buying an asset whose payoff is the commodity price in the future net of storage costs. In our model this payoff is positively related to the payoff of the commodity index and hence the same intuition developed in the context of futures prices also applies to spot prices of storable commodities. Because the discount factor is affected by institutional investors (and depends on the index), outside shocks to index commodities spill over to prices and inventories of seemingly unrelated commodities. In contrast, there are no spillovers of shocks to nonindex commodities. Outside shocks here are broad and include shocks to a specific index commodity (related to its supply, supply volatility, or demand), to the stock market (stock volatility and return), as well as to the inflow of institutions. These results challenge commonly held views that such shocks do not matter for commodity spot prices.

This paper is related to several strands of literature. The two papers that motivate this work are Tang and Xiong (2012) and Singleton (2014). Singleton examines the 2008 boom/bust in oil prices and argues that flows from institutional investors contribute significantly to that boom/bust. Tang and Xiong document that the comovement between oil and other commodities has risen dramatically following the inflow of institutional investors starting in 2004, and that the commodities belonging to popular indices have been affected disproportionately more. There is no difference in the comovement patterns of index and nonindex commodities pre-2004. Using a proprietary data set from the CFTC, Buyuksahin and Robe (2014) investigate the recent increase in the correlation between equity indices and commodities and argue that this phenomenon is due to the presence of hedge funds that are active in both equity and commodity futures markets. Recently, Henderson, Pearson and Wang (2015)present new evidence on the financialization of commodity futures markets based on commodity-linked notes.

The impact of financialization on commodity futures and spot prices is the subject of much ongoing debate. Surveys by Irwin and Sanders (2011) and Fattouh, Kilian, and Mahadeva (2013) challenge the view that increased speculation in oil futures markets in the post-financialization period is an important determinant of oil prices. Kilian and Murphy (2014) attribute the 2003 to 2008 oil price surge to global demand shocks rather than speculative demand shifts. Hamilton and Wu (2015) examine whether commodity index-fund investing has had a measurable effect on commodity futures prices and find little evidence to support this hypothesis.

There still remains a lack of agreement as to whether institutional investors' trades affect commodity futures prices. Our view is that, given the size of commodity index traders' (a proxy for institutional investors) futures holdings in the data, it is natural to expect that such traders affect prices. Furthermore, similar effects are reasonably well established in other markets, especially equity markets. Starting with Harris and Gurel (1986) and Shleifer (1986), a large body of work documents that prices of stocks that are added to the S&P 500 and other indices increase following the announcement, while prices of stocks that are deleted drop—a phenomenon widely attributed to the price pressure from institutional investors. Similarly, a variety of studies document so-called "asset class" effects, that is "excessive" comovement of assets belonging to the same index or some other visible category of stocks (e.g., Barberis, Shleifer, and Wurgler (2005) for S&P500 versus non-S&P500 stocks, and Boyer (2011) for BARRA value and growth indices). These effects are attributed to the presence of institutional investors.

The closest theoretical work on the effects of institutions on asset prices is that on the Lucas-tree economy by Basak and Pavlova (2013). Basak and Pavlova focus on index and asset class effects in equity markets. Their model does not feature multiple commodities, nor is it designed to address some of the main issues in the debate on financialization, namely, whether institutional investors impact commodity futures and spot prices, as well as inventories. Another related theoretical study of an asset class effect is that by Barberis and Shleifer (2003), whose explanation for this phenomenon is behavioral. However, they do not explicitly model commodities and so cannot address some questions specific to the debate on the financialization of commodities.

Finally, a large and diverse literature going back to Keynes (1923) studies the determination of commodity spot prices in production economies with storage and links the physical markets for commodities with the commodity futures markets.[3] In this literature, Baker (2013) studies the financialization of a storable commodity. His interpretation of financialization is a reduction in transaction costs of households for trading futures. He focuses on a single commodity, while we consider multiple commodities, distinguishing between index and nonindex commodities. More generally, we contribute to this literature by modeling shareholders of storage firms as risk-averse investors (some of which could be institutions), and highlight the influence of our discount factor channel and its role in generating cross-commodity spillovers.

Methodologically, this paper contributes to the asset pricing literature by providing a tractable multi-asset general equilibrium model with heterogeneous investors that is solved in closed form. Pavlova and Rigobon (2007) and Cochrane, Longstaff, and Santa-Clara (2008) highlight the complexities of multi-asset models and provide analytical solutions for the two-asset case. As Martin (2013) demonstrates, the general multi-asset case presents a formidable challenge. In contrast, our multi-asset model is surprisingly simple to solve. Our innovation is to replace the Lucas trees considered in the above literature by zero-net-supply assets (futures) and model only the aggregate stock market as a Lucas tree. The model then becomes just as simple and tractable as a single-tree model.

The remainder of the paper is organized as follows. Sections I and II present our model and demonstrate how institutional investors affect commodity futures prices, volatilities, and their comovement. Section III examines the effect of institutions on commodity inventories and spot prices. Section IV concludes. The Appendix provides all proofs and the Internet Appendix presents the economy with demand shocks.[4]

I. The Model

Our goal in this section is to develop a simple and tractable model of commodity futures markets. We consider a pure-exchange multi-good, multi-asset economy with a finite horizon T. Uncertainty is resolved continuously, driven by a $K+1$-dimensional standard Brownian motion $\omega \equiv (\omega_0, \ldots, \omega_K)^\top$. All consumption in the model occurs at the terminal date T, while trading takes place at all times $t \in [0, T]$.

Commodities. There are K commodities (goods), indexed by $k = 1, \ldots K$. The date-T supply of commodity k, D_{kT}, is the terminal value of the process D_{kt}, with dynamics

$$dD_{kt} = D_{kt}[\mu_k dt + \sigma_k d\omega_{kt}], \tag{1}$$

where μ_k and $\sigma_k > 0$ are constant. The process D_{kt} represents the arrival of news about D_{kT}. We refer to it as the commodity-k *supply news.* The price of good k at time t is denoted by p_{tk}. There is one further good in the economy, commodity 0, which we refer to as the generic good. This good subsumes all remaining goods consumed in the economy apart from the K commodities that we explicitly specify above and serves as the numeraire. The date-T supply of the generic good is D_T, which is the terminal value of the supply news process

$$dD_t = D_t[\mu dt + \sigma d\omega_{0t}], \tag{2}$$

where μ and $\sigma > 0$ are constant. Our specification implies that the supply news processes are uncorrelated across commodities ($dD_{kt}\, dD_{it} = 0$, $dD_{kt}\, dD_t = 0$, $\forall k$, $k \neq i$). This assumption is for expositional simplicity; it can be relaxed in future work. We comment on the case of correlated supply news in footnote 10 (Section II.A) and in Section III.C.

Financial Markets. Available for trading are K standard futures contracts written on commodities $k = 1, \ldots, K$. A futures contract on commodity k matures at time $\tau < T$ and, upon maturity, gets rolled over to the next contract with maturity τ. This process is repeated untill time T, at which point consumption takes place. The payoff of the contract is one unit of commodity k. Each contract is continuously resettled at the futures price f_{kt} and is in zero net supply. Gains/losses on each contract are posited to follow

$$df_{kt} = f_{kt}[\mu_{f_k t} dt + \sigma_{f_k t} d\omega_t], \tag{3}$$

where $\mu_{f_k t}$ and the $K + 1$ vector of volatility components $\sigma_{f_k t}$ are determined endogenously in equilibrium (Section II).

Our model makes a distinction between index and nonindex commodities because we seek to examine theoretically the asset class effect in commodity futures documented by Tang and

Xiong (2012). A *commodity index* includes the first L commodities, $L \leq K$, and is defined as

$$I_t = \prod_{i=1}^{L} f_{it}^{1/L}. \tag{4}$$

This index represents a geometrically weighted commodity index such as, for example, the S&P Commodity Index. For expositional simplicity, our index weighs all commodities equally; this assumption is easy to relax.[5]

In addition to the futures markets, investors can trade in the stock market, S, and an instantaneously riskless bond. The stock market is a claim to the entire output of the economy at time T: $D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$. It is in positive supply of one share and is posited to have price dynamics given by

$$dS_t = S_t[\mu_{St}dt + \sigma_{St}d\omega_t], \tag{5}$$

with μ_{St} and $\sigma_{St} > 0$ endogenously determined in equilibrium. The bond is in zero net supply. It pays a riskless interest rate r, which we set to zero without loss of generality.[6]

We note that our formulation of asset cash flows is standard in the asset pricing literature. The main distinguishing characteristic of our model is that it avoids the complexities of multi-tree economies. This is because only the stock market is in positive net supply, while all other assets (futures) are in zero net supply. As we demonstrate in the ensuing analysis, this model is just as simple and tractable as a single-tree model.

Investors. The economy is populated by two types of market participants: normal investors, $\mathcal{N}$, and institutional investors, $\mathcal{I}$. The (representative) normal investor is a standard market participant, with logarithmic preferences over the terminal value of her portfolio:

$$u_{\mathcal{N}}(W_{\mathcal{N}T}) = \log(W_{\mathcal{N}T}), \tag{6}$$

where $W_{\mathcal{N}T}$ is wealth or consumption.

The institutional investor's objective function, defined over his terminal portfolio value (consumption) $W_{\mathcal{I}T}$, is given by

$$u_{\mathcal{I}}(W_{\mathcal{I}T}) = (a + bI_T)\log(W_{\mathcal{I}T}), \tag{7}$$

where a, $b > 0$. The institutional investor is modeled along the lines of Basak and Pavlova (2013), who study institutional investors in the stock market and also provide microfoundations for such an objective function, as well as a status-based interpretation.[7] There is also a broader interpretation: recently consumers have become more sensitive to commodity prices, and one way to capture this is via a formulation along the lines of (7). The objective function has two key properties: (i) it depends on the index level I_T, and (ii) the marginal utility of wealth is increasing in the benchmark index level I_T. This captures the notion of benchmarking: the institutional investor is evaluated relative to his benchmark index and as a result he cares about the performance of the index. When the benchmark index is relatively high, the investor strives to catch up and so he values his marginal unit of performance highly (his marginal utility of wealth is high). When the index is relatively low, the investor is less concerned about his performance (his marginal utility of wealth is low). We use the commodity market index as the benchmark index because in this work we attempt to capture institutional investors with a mandate to invest in commodities, most of whom are evaluated relative to a commodity index.

An alternative interpretation of the objective function is that the institutional investor has a mandate to hedge commodity price inflation, that is, to deliver higher returns in states in which the commodity price index is high.[8] We think of the terminal date T as the performance evaluation horizon of institutional asset managers, usually three to five years (Bank for International Settlements (2003)).

In this multi-good world, terminal wealth is defined as an aggregate over all goods, a consumption index (or consumption). We take the index to be Cobb-Douglas, that is,

$$W_n = C_{n_0}^{\alpha_0} C_{n_1}^{\alpha_1} \cdot \ldots \cdot C_{n_K}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\}, \tag{8}$$

where $\alpha_k > 0$ for all k. For the case of $\sum_{k=0}^{K} \alpha_k = 1$, the parameter α_k represents the expenditure share on good k. Here we are considering a general Cobb-Douglas aggregator in which the weights do not necessarily add up to one, and hence we label α_k as the "commodity demand parameter."[9] We take the commodity demand parameters to be the same for all investors in the economy. Heterogeneity in demand for specific commodities is not the dimension we would like to focus on in this paper.

A change in α_k represents a demand shift towards commodity k. A change in the demand parameter α_k is the simplest and most direct way of modeling a demand shift, that is, an outward movement in the entire demand schedule, as typical in classical demand theory (Varian (1992)).[10] In Section II.C, we allow the demand parameters α_k to be stochastic in order to capture a more realistic environment with demand shocks. Until then, we keep the demand parameters constant so as to isolate the effects of supply shocks and the effects of financialization (fluctuations in institutional wealth invested in the market) on commodity futures prices.

Institutional and normal investors are initially endowed with fractions $\lambda \in [0, 1]$ and $(1-\lambda)$ of the stock market, providing them with initial assets worth $W_{\mathcal{I}0} = \lambda S_0$ and $W_{\mathcal{R}0} = (1-\lambda)S_0$, respectively. We often refer to the parameter λ as the size of institutions.

Starting with initial wealth W_{n0}, each type of investor $n = \mathcal{N}, \mathcal{I}$, dynamically chooses a portfolio process $\phi_n = (\phi_{n_1}, \ldots, \phi_{n_K})^\top$, where ϕ_n and ϕ_{n_S} denote the fractions of the portfolio invested in futures contracts 1 through K and in the stock market, respectively. The wealth process of investor n, W_n, then follows the dynamics

$$dW_{nt} = W_{nt} \sum_{k=1}^{K} \phi_{n_k t}[\mu_{f_k t} dt + \sigma_{f_k t} d\omega_t] + W_{nt}\phi_{n_S t}[\mu_{St} dt + \sigma_{St} d\omega_t]. \tag{9}$$

II. Equilibrium Effects of Financialization of Commodities

We are now ready to explore how the financialization of commodities affects equilibrium prices, volatilities, and correlations. Since our model is dynamic, our results on the effects of financialization are statements about changes in the entire distribution of commodity prices. To understand the effects of financialization, we often make comparisons with equilibrium in a *benchmark economy*, in which there are no institutional investors. We can specify such an economy by setting $b = 0$ in (7), in which case the institution in our model no longer resembles a commodity index trader but instead behaves like a normal investor. Another way to capture the benchmark economy within our model is to set the fraction of institutions, λ, to zero.

Equilibrium in our economy is defined in a standard way: equilibrium portfolios, asset and time-T commodity prices are such that (i) both normal and institutional investors choose their optimal portfolios, and (ii) futures, stock, bond, and time-T commodity markets clear. Letting $M_{t,T}$ denote the (stochastic) discount factor or the pricing kernel in our model, by no-arbitrage futures prices are given by

$$f_{kt} = E_t[M_{t,T}\, p_{kT}]. \tag{10}$$

The discount factor $M_{t,T}$ is the marginal rate of substitution of any investor, for example the normal investor, in equilibrium.

To develop intuition for our results, it is useful to examine the time-T prices prevailing in our equilibrium. These are reported in the following lemma.

Lemma 1 (Time-T Equilibrium Quantities): *In equilibrium with institutional investors, we obtain the following characterizations for the terminal date quantities:*

$$\textit{Commodity prices:} \quad p_{kT} = \frac{\alpha_k}{\alpha_0}\frac{D_T}{D_{kT}}, \quad p_{kT} = \overline{p}_{kT}; \tag{11}$$

$$\textit{Commodity index:} \quad I_T = \frac{D_T}{\alpha_0}\prod_{i=1}^{L}\left(\frac{\alpha_i}{D_{iT}}\right)^{1/L}, \quad I_T = \overline{I}_T; \tag{12}$$

$$\textit{Stock market value:} \quad S_T = D_T\sum_{k=0}^{K}\frac{\alpha_k}{\alpha_0}, \quad S_T = \overline{S}_T; \tag{13}$$

$$\textit{Discount factor:} \quad M_{0,T} = \overline{M}_{0,T}\left(1 + \frac{b\,\lambda(I_T - E[I_T])}{a + b\,E[I_T]}\right), \tag{14}$$

where $\overline{M}_{0,T} = \frac{e^{(\mu-\sigma^2)T}D_0}{D_T}$, *and the expectation of the time-T index value, $E[I_T]$, is provided in the Appendix, and the quantities with an upper bar denote the corresponding equilibrium quantities prevailing in the economy with no institutions.*

Lemma 1 reveals that the price of good k decreases with the supply of that good, D_{kT}. As supply D_{kT} increases, good k becomes relatively more abundant, and hence its price falls. An increase in the supply of the generic good D_T has the opposite effect. Now good k becomes more scarce relative to the generic good, and hence its price increases. These are classical supply-side effects. A positive shift in α_k represents an increase in demand for good k. As a consequence, the price of good k goes up. This is a classical demand-side effect. Since the index is given by $I_T = \prod_{i=1}^{L} p_{iT}^{1/L}$, the terminal index value inherits the properties of the individual commodity prices. In particular, it declines when the supply of any index commodity i D_{iT} goes up, while it rises when the supply of the generic good D_T increases.

We note that the time-T prices of commodities, and hence the commodity index, coincide with their values in the benchmark economy with no institutions. We have intentionally set up our model in this way. By effectively abstracting away from the effects of financialization on underlying cash flows in (10), we are able to elucidate the effects of institutions in the futures markets coming via the discount factor channel. Financialization, however, can potentially affect time-t, $t < T$, commodity prices in our model. We explore this in Section III.

The stock market is a claim against the aggregate output of all goods in the economy, $D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$, which in this model turns out to be proportional to the aggregate supply of the generic good D_T, due to investors' Cobb-Douglas consumption aggregators. So the aggregate wealth in the economy, the stock market value S_T, in equilibrium is simply a scaled supply of the generic good D_T. The quantity D is an important state variable in our model. In what follows, we refer to it as aggregate wealth, or equivalently, aggregate output.

[INSERT FIGURE 1 HERE]

In the benchmark economy, the discount factor depends only on aggregate output D_T. It bears the familiar inverse relationship with aggregate output (dotted line in Figure 1, Panel A), implying that assets with high payoffs in low-D_T (bad) states get valued higher. In the presence of institutions, the discount factor is also decreasing in aggregate output D_T, albeit at a slower rate. That is, the presence of institutions makes the discount factor less sensitive to news about aggregate output. Additionally, now the discount factor becomes dependent on the supply of each index commodity D_{iT} (Figure 1, Panel B). The channel through which institutions affect the discount factor is apparent from equation (14): the discount factor now becomes dependent on the performance of the index, pricing high-index states higher. This is the channel through which financialization affects asset prices in our model.

The new financialization channel works as follows. Institutional investors have an additional incentive to do well when the index does well. So relative to normal investors, they strive to align their performance with that of the index, performing better when the index does well in exchange for performing poorer when the index does poorly. As highlighted in our discussion of the equilibrium index value in (12), the index does well when aggregate output D_T is high and the supply of index commodity D_{iT} is low. Because of the additional demand from institutions, these states become more "expensive" relative to the benchmark economy (higher Arrow-Debreu state prices or higher discount factor $M_{0,T}$). The financialization channel thus counteracts the benchmark economy inverse relation between the discount factor $M_{0,T}$ and aggregate output, making the discount factor less sensitive to aggregate output D_T (as evident from Figure 1, Panel A). Additionally, it makes the discount factor dependent on and decreasing in each index commodity supply D_{iT}.

The graphs in Figure 1 are important because they underscore the mechanism for the valuation of assets in the presence of institutions. In particular, assets that pay off high in states in which the index does well (high D_T and low D_{iT}) are valued higher than in the benchmark economy with no institutions.

A. Equilibrium Commodity Futures Prices

Proposition 1 (Futures Prices): *In the economy with institutions, the equilibrium futures price of commodity $k = 1, \ldots, K$ is given by*

$$f_{kt} = \overline{f}_{kt} \frac{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{1_{\{k \le L\}}\sigma_k^2 (T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}, \quad (15)$$

where the equilibrium futures price in the benchmark economy with no institutions $\overline{f}_{kt}$ and the quantity $g_i(t)$ are given by

$$\overline{f}_{kt} = \frac{\alpha_k}{\alpha_0} e^{(\mu - \mu_k - \sigma^2 + \sigma_k^2)(T-t)} \frac{D_t}{D_{kt}}, \quad g_i(t) = \frac{\alpha_i}{\alpha_0} e^{(\mu - \mu_i + (1/L+1)\sigma_i^2/2)(T-t)}. \quad (16)$$

Consequently, in the presence of institutions,

(i) Futures prices are higher than in the benchmark economy, $f_{kt} > \overline{f}_{kt}$, $k = 1, \ldots, K$.

(ii) Index futures prices rise more than nonindex future prices for otherwise identical commodities, that is, for commodities i and k with $D_{it} = D_{kt}$, $\forall t$, $\alpha_i = \alpha_k$, $i \le L$, $L < k \le K$.

Proposition 1 reveals that the commodity futures prices in the benchmark economy with no institutions $\overline{f}_{kt}$ inherit the features of time-T futures prices highlighted in Lemma 1. The benchmark economy futures prices rise in response to positive news about aggregate output D_t and fall in response to positive news about the supply of commodity k D_{kt}. In contrast, in the economy with institutions the commodity futures prices f_{kt} depend not only on own supply news D_{kt} but also on supply news for all *index* commodities D_{it}. Other characteristics of index commodities such as the expected growth in their supply μ_i, their volatility σ_i and their demand parameters α_i now also affect the prices of all futures traded in the market. Note that just like in the benchmark economy, supply news D_k and other characteristics of *nonindex* commodities have no spillover effects on other commodity futures.[11] Since there is one consumption date, however, we do not have a meaningful term structure of futures prices. All contracts get rolled over until time T, and so the maturity of a specific futures contract τ does not enter (15).

To understand why all futures prices go up (property (i) of Proposition 1), recall that institutional investors desire high payoffs in states when the index does well. They therefore value assets that pay off highly in those states. All futures in the model are positively correlated with the index even in the benchmark economy because they are all priced using the common discount factor. Hence, all futures prices rise. However, the prices of index futures rise by more (property (ii)). Institutions specifically desire the futures that are included in the index because, naturally, the best way to achieve high payoffs in states when the index does well is to hold index futures. Therefore, index futures have higher prices than otherwise identical nonindex futures.[11]

Corollary 1: *Equilibrium commodity futures prices have the following additional properties.*

(i) All commodity futures prices f_{kt} are increasing in the size of institutions λ, $k = 1, \ldots, K$.

(ii) All commodity futures prices are more sensitive to aggregate output D_t than in the benchmark economy with no institutions, that is, f_{kt} is increasing in D_t at a faster rate than $\overline{f}_{kt}$, $k = 1, \ldots, K$; moreover, index commodity futures are more sensitive to aggregate

output than nonindex commodity futures for otherwise identical commodities.

(iii) *All commodity futures prices f_{kt}, $k = 1, \ldots, K$, react negatively to positive supply news of index commodities D_{it}, $i = 1, \ldots, L$, $k \neq i$, while in the benchmark economy such a price $\overline{f}_{kt}$ is independent of D_{it}; all prices f_{kt}, $k = 1, \ldots, K$, remain independent of nonindex commodities supply news $D_{\ell t}$, unless $k = \ell$.*

(iv) *All commodity futures prices f_{kt}, $k = 1, \ldots, K$, react positively to a positive demand shift towards any index commodity α_i, $i = 1, \ldots, L$, $k \neq i$, while $\overline{f}_{kt}$ is independent of α_i; all prices f_{kt}, $k = 1, \ldots, K$, remain independent of nonindex commodity supply shifts α_ℓ, $\ell \neq k$.*

Figure 2 illustrates the results of the corollary. To develop intuition, we start from properties (iii) and (iv) of the corollary. Panel A shows that, unlike in the benchmark economy, futures prices decrease in response to positive index commodities' supply news D_{it}. Institutional investors strive to align their performance with the index, and as a result affect prices most when the index is high. The index is high when D_{it} is low (supply of index commodity i is scarce) and low when D_{it} is high (supply is abundant). So the effects of institutions on commodity futures prices f_{kt} are most pronounced for low D_{it} realizations and decline monotonically with D_{it}. These effects are absent in the benchmark economy in which agents are not directly concerned about the index. In contrast, futures prices f_{kt} do not react to news about the supply of nonindex commodities (apart from that of own commodity k) because this news does not affect the performance of the index (Panel B and Proposition 1). The demand-side effects on commodity futures prices are presented in Panel C. In contrast to the benchmark economy in which futures prices depend only on own commodity demand parameter α_k, in Panel C we find that futures prices increase in demand parameters α_i for all commodities that are members of the index. An upward shift in demand for any index commodity leads to an increase in that commodity price (Lemma 1) and therefore leads to an increase in the index value. This is favorable for the institutions, and hence their impact on prices becomes increasingly more pronounced as α_k increases. In contrast, these effects are not present for nonindex commodities since a shift in demand for those commodities leaves the index unaffected (Proposition 1). A caveat to this discussion is that we are not formally modeling demand shifts in this section, but rather presenting comparative statics with respect to demand parameters α_k. In an economy with demand uncertainty, investors take this uncertainty into account in their optimization (Section II.C).

To illustrate property (ii), Panel D demonstrates that aggregate output news D_t has stronger effects on futures prices f_{kt} than in the benchmark economy with no institutions. This is because good news about aggregate output increases not only the cashflows of all futures contracts (increases p_{kT}) but also the value of the index. This latter effect is responsible for the amplification of the effect of aggregate output news depicted in Panel D. The higher the aggregate output, the higher the index and hence the stronger the amplification effect. Property (i) shows that commodity futures prices rise when there are more institutions in the market. The more institutions there are, the stronger their effect on the discount factor and hence on all commodity futures prices. Finally, all panels in Figure 2 illustrate that in the presence of institutions, index futures rise more than nonindex futures.

[INSERT FIGURE 2 HERE]

B. Futures Volatilities and Correlations

The past decade in commodity futures markets has been characterized by an increase in volatility, and hence has attracted the attention of policymakers and commentators. In this section we explore commodity futures volatilities in order to highlight the sources of this increased volatility. Our objective is to demonstrate how standard demand and supply risks can be amplified in the presence of institutions. Proposition 2 reports the futures return volatilities in closed form.[12]

PROPOSITION 2 (Volatilities of Commodity Futures): *In the economy with institutions, the volatility vector of loadings of index commodity futures k returns on the Brownian motions are given by*

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + h_{kt}\,\sigma_{It}, \quad h_{kt} > 0, \quad k = 1, \ldots, L, \tag{17}$$

and the analogue for nonindex commodity futures are given by

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + h_t\,\sigma_{It}, \quad h_t > 0, \quad k = L+1, \ldots, K, \tag{18}$$

where $\overline{\sigma}_{f_k}$ is the corresponding volatility vector in the benchmark economy with no institutions and σ_{It} is the volatility vector for the conditional expectation of the index $E_t[I_T]$, given by

$$\overline{\sigma}_{f_k} = (\sigma,\, 0, \ldots, -\sigma_k, 0, \ldots, 0),\ \sigma_{It} = (\sigma,\, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L,\, 0, \ldots, 0), \tag{19}$$

and where h_t and h_{kt} are strictly positive stochastic processes provided in the Appendix with the *property $h_{kt} > h_t$.*

Consequently, in the presence of institutions,

(i) The volatilities of all futures prices, $\|\sigma_{f_k t}\|$, are higher than in the benchmark economy, $k = 1, \ldots K$.

(ii) The volatilities of index futures rise more than those of nonindex futures for otherwise identical commodities, that is, for commodities i and k with $D_{it} = D_{kt}$, $\forall t$, $\alpha_i = \alpha_k$, $i \leq L$, $L < k \leq K$.

The general formulae presented in Proposition 2 can be decomposed into individual loadings of futures returns on the primitive sources of risk in our model, the Brownian motions ω_0, $\omega_1, \ldots, \omega_K$. Figure 3 presents this decomposition and illustrates the role of each individual source of risk. Recall that in our model individual commodities' supply news D_{kt} is independent of each other and of the supply news for the generic good D_t. Each of these processes is driven by own Brownian motion. Since in the benchmark economy a futures price depends only on own D_{kt} and aggregate output D_t, it is exposed to only two primitive sources of risk, namely, Brownian motions ω_k and ω_0. In the presence of institutions, futures prices become additionally dependent on the supply news of all index commodities and therefore exposed to sources of uncertainty $\omega_1, \ldots \omega_L$. (The dependence is negative, as revealed by Corollary 1.) Additionally, as argued in Corollary1, shocks to D_t are amplified in the presence of institutions. Proposition 2 formalizes the intuition by explicitly reporting the loadings on ω_0, $\omega_1, \ldots, \omega_K$, the driving forces behind D, $D_1, \ldots, D_K$, respectively. Hence, commodity futures become more volatile for two reasons: (i) their volatilities are amplified because futures prices react more strongly to news about aggregate output D_t and (ii) their prices now depend on additional shocks driving index commodity supply news $D_1, \ldots, D_L$. Our model delivers increased sensitivity to the underlying shocks in the presence of institutions because the shocks affect the value of the index.

[INSERT FIGURE 3 HERE]

Figure 4 provides an illustration. All plots in the figure are against an index commodity's supply news D_i, which is a new state variable identified by our model, but the plots against aggregate output D look similar. Figure 4 Panel A also reveals that the volatilities of index and nonindex futures are differentially affected by the presence of institutions. Tang and Xiong (2012) document that since 2004, and especially during 2008, index commodities have exhibited higher volatility increases than nonindex commodities. Our results are consistent with these findings.[13] The volatilities of index futures are higher than those of nonindex futures because, by construction, index futures pay off more when the index does well. The volatilities of index futures become high enough to make them unattractive to normal investors (standard market participants), so that they are willing to sell the index futures to institutions.

[INSERT FIGURE 4 HERE]

We next turn to examining the (instantaneous) correlations of futures returns, defined as $corr_t(i,k) = \sigma_{f_i t} \cdot \sigma_{f_k t} / (\|\sigma_{f_i t}\| \, \|\sigma_{f_k t}\|)$. Recent evidence indicates that the financialization of commodities markets has coincided with a sharp increase in the correlations across a wide range of commodity futures returns (Tang and Xiong (2012)). The increase in correlations is especially pronounced for index futures returns. Tang and Xiong hypothesize that while the commodity markets were largely segmented before 2000, the inflow of institutional investors who hold multiple commodities in the same portfolio has linked the commodity futures markets and increased the correlations among commodities, especially index commodities. Our model shows that one does not need to rely on market segmentation to produce these effects. Arguably, commodity market speculators investing across commodity markets were present before 2004. Our model produces both the increase in the correlations among commodities and the higher increase in the correlations among index commodities under complete markets.[14] Our key mechanism is that in the presence of institutional investors benchmarked to a commodity index, this index (more precisely, $E_t[I_T] = D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}$) emerges as a common factor in the returns of all commodities, raising their correlations. However, the sensitivity to this new factor is higher for index commodity futures (Proposition 2), making their returns more correlated than those of nonindex futures. The above intuition is precise for covariances, but it also carries through to the correlations because the effect of rising volatilities is smaller than the effect of rising covariances. Figure 4 Panel B plots the correlations occurring in our model.

Moreover, the new factor $E_t[I_T]$ can be mapped in our model into the performance to date of institutional investors relative to normal investors or the time-t wealth distribution in the economy.[15] Intuitively, because institutional investors have an additional reason to hold index futures, they end up with a long position in these futures, while normal investors take the other side. So the higher the expected level of the index, the higher the portfolio return of institutions relative to that of normal investors. As institutional investors get wealthier relative to normal investors, they comprise a higher proportion of the market. This effect is similar to that of fund inflows, with inflows following good relative performance to date. The resulting volatilities and correlations become time-varying and depend, in principle, on all state variables and parameters of the model (see Proposition 2). We have attempted to identify the variables and parameters that the volatilities and correlations react to the most. We find that volatilities and correlations go up significantly in the presence of a common demand shock (Section II.C, and the Internet Appendix). Additionally, they are quite sensitive to the underlying supply news and aggregate output volatilities (σ_i and σ). The latter can be mapped within our model into the VIX (see Proposition A1): the higher the VIX, the higher are the commodity return volatilities and correlations. Finally, they are sensitive to the size of institutional investors λ and their relative performance to date (as captured by $E_t[I_T]$), although the sensitivity to both of these is lower.

Since investors in our model invest in both the futures and stock markets, one may expect that the effects we find are also present in equity-commodity correlations. Our main focus is on commodity markets, however, and so we do not incorporate all driving forces pertinent in stock markets.[16] The quantities $corr_t(S,k) = \sigma_{f_S t} \cdot \sigma_{f_k t}/(||\sigma_{St}||\, ||\sigma_{f_k t}||)$, for all k, are the (instantaneous) equity-futures correlations in our model, where the stock market level and volatility vector are presented in the Appendix (Proposition A1). These correlations always rise in the presence of institutions. In other words, we do get theoretical confirmation within our model supporting the assertion that the recent rise in the equity-commodity correlations can be attributed to financialization.[17] Figure 4, Panel C depicts the equity-commodity correlations in our model. The correlations of the stock market and commodity futures returns go up because both the stock market and the commodities returns depend positively on the new common factor- the commodity index. The correlations of the stock market and index commodities is higher than those with nonindex commodities because index commodity futures have a higher loading on the new factor.

C. Economy with Demand Shocks

Our setting so far has been missing demand shocks, and such shocks have been argued to be critical in understanding the behavior of oil prices and the prices of other commodities (Fattouh, Kilian, and Mahadeva (2013)). Commentators often link the increase in commodity prices and cross-commodity correlations to China, whose high growth has led to an increase in demand for a number of key commodities. In the Internet Appendix, we introduce common demand shocks, which affect a group of commodities. In particular, in the consumption index (8) of investors,

$$W_n = C_{n_0}^{\alpha_0} C_{n_1}^{\alpha_1} \cdot \ldots \cdot C_{n_K}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\},$$

we allow two demand parameters, α_1 and α_2, to be strictly positive stochastic processes. These processes are modeled so that the demand for a commodity is increasing with aggregate output (as in the model of oil prices of Dvir and Rogoff (2009)). We may think of commodities 1 and 2 as representing energy commodities, both in the commodity index.

Within this richer setting, we demonstrate the validity of earlier results that all futures prices and their volatilities are higher in the presence of institutions, with those of index futures exceeding those of nonindex futures (Proposition IA in the Internet Appendix). However, these effects are stronger than those in the baseline model. In the presence of demand shocks, the index becomes more volatile and so institutional investors' incentive to not fall behind the index is even stronger, amplifying our earlier results. Furthermore, the cross-commodity futures return correlations go up sizably, reaching the levels documented in the post-financialization period in the data (Tang and Xiong (2012)). Within this setup we can disentangle the effects of financialization from the effects of fundamentals (demand and supply) by comparing economies with and without institutional investors. Within a plausible numerical illustration, presented in the Internet Appendix, we quantify the fraction of the futures price increase that is due to financialization and find it to be sizeable. Our results support the view advocated in Kilian and Murphy (2014) that fundamentals, especially demand shocks, are important in explaining commodity prices, but we also find that there is a contribution of financialization, with the presence of institutions amplifying the effects of rising demand.[18]

III. Commodity Prices and Inventories

Commodity spot prices are important determinants of the cost of living worldwide. Spiralling food and energy prices observed in recent years have sparked intense debate as to whether the inflow of institutional investors into futures markets may be hurting millions of households. In his congressional testimony, Masters (2008) argues that the price spiral is unequivocally due to the inflow of institutional commodity investors. In a formal study, Singleton (2014) presents evidence in favor of this view.

The framework we have developed so far does not carry direct implications for intermediate commodity spot prices p_{kt}, $t < T$. To determine prices p_{kt} one needs to extend our model and incorporate additional features. In this section we introduce intermediate consumption and storage. Towards that end we adopt the classical optimal storage framework for commodity pricing following Deaton and Laroque (1992,1996). The main departure from Deaton and Laroque is that the cash flows of storage firms are priced by a discount factor that reflects the risk aversion of their shareholders and that is influenced by institutional investors (because institutional investors can hold shares of storage firms). This departure highlights how financial markets transmit outside shocks not only to futures prices (Section II) but also to commodity spot prices and inventories, via the discount factor channel.

A. *Incorporating Storage*

We introduce additional economic agents, namely, consumers and firms operating within a competitive storage sector. These new agents exist alongside our normal and institutional investors. The new agents are present over two dates, t and $t+1$. The firms make a one-time decision at time t to store a commodity until time $t+1$. To close the model, we incorporate consumers who consume at times t and $t+1$.[19] The choice of a one-time storage decision is for tractability; the complexity and intractability of the dynamic storage framework is well acknowledged in the literature (Deaton and Laroque (1992), Dvir and Rogoff (2009)). Our setting has the added complexities of having risk-averse investors and elaborate financial markets. Our formulation is the simplest possible setup sufficient to illustrate our main economic mechanism.[20]

Commodities. As before we have $K+1$ commodities, but now the economy is additionally endowed with output D_t, D_{kt} units of commodities $k = 1, \ldots, K$ at time t and $D_{t+1}, D_{k\,t+1}$ at time $t+1$. One of these commodities, x, is storable in the sense that putting aside X_t units of the commodity at time t yields $(1-\delta)X_t$ units of this commodity at time $t+1$, where δ is the storage cost. Inventory X_t is optimally chosen by the storage firms. The remaining commodities are non storable. Here we make a distinction between storable and non storable commodities because we intend to demonstrate that the effects of financialization on storable versus non storable commodities are very different.

The total amount of the storable commodity at times t and $t+1$ is, respectively, $D_{xt} - X_t$

and $D_{x\,t+1} + (1-\delta)X_t$. A governing entity distributes these quantities of commodity x and all of the output of the other commodities to consumers in the form of the endowment (or labor income).

Competitive Storage Sector. The firms in the competitive storage sector at time t buy X_t units of good x at the equilibrium price p_{xt} and carry an inventory over to the next period, liquidating it at time $t+1$. The shares in these firms are traded by investors in our economy, both the normal and institutional investors. Firm shares are in zero net supply. A firm's objective is to choose an optimal inventory level X_t so as to maximize its value given by

$$-p_{xt}X_t + E_t\left[M_{t,t+1}\,p_{x\,t+1}(1-\delta)X_t\right]. \tag{20}$$

For tractability, we abstract away from inventory stockouts and do not impose an explicit non-negativity constraint on inventories.[21]

Firms in the storage sector are perfectly competitive, which ensures that the equilibrium prices p_{xt}, $p_{x\,t+1}$ must satisfy

$$p_{xt} = (1-\delta)E_t\left[M_{t,t+1}\,p_{x\,t+1}\right]. \tag{21}$$

Otherwise, if for example p_{xt} in equation (21) were less than the quantity on the right-hand side, the firms would have an incentive to store more at time t and sell the commodity at time $t+1$. The relationship (21) underpins the classical theory of storage (see Deaton and Laroque (1992) and others). Our main departure from this literature is that we do not assume that the shareholders of the storage firms are risk-neutral, whereby the discounting of future cash flows is at a constant riskless interest rate. Instead, our firm shareholders discount future cash flows using a stochastic discount factor. Since the shareholders are the normal and institutional investors in our economy, the relevant stochastic discount factor is that determined from the equilibrium conditions in the financial markets in which our investors trade (Section II), which reflects their attitudes towards risk. The importance of replacing the riskless interest rate by a stochastic discount factor in the pricing of storable commodities has recently been emphasized by Singleton (2014). Casassus and Collin-Dufresne (2005) also employ such discounting, and it implicitly appears in earlier works in finance such as Gibson and Schwartz (1990). Note that equation (21) holds with equality because we have abstracted away from the non negativity constraints on inventories. We further note that, as evident from equations (20) and (21), the value of the firm's profits is zero. Firm shares can be viewed as redundant assets in our economy, and so they are priced as redundant securities under complete markets.

Investors. The investors are the same as in Section I, normal and institutional investors.

Consumers. The storage literature typically specifies the demand functions for commodities in reduced form. Here we opt to microfound these functions by explicitly modeling the end consumers of commodities. We model the consumers $\mathcal{C}$ as consumer-workers who live hand-to-mouth from time t until $t+1$. At the two dates t and $t+1$ they receive an endowment (labor income), which they fully consume. These consumers neither save nor invest and are distinct from the investors in our model. Such hand-to-mouth consumers were introduced by Campbell and Mankiw (1989) and first applied in an asset pricing context in Weil (1992).[22] The value of the representative consumer's endowment at times t and $t+1$ is respectively,

$$W_{\mathcal{C}t} = D_t + p_{1t}D_{1t} + \ldots + p_{xt}(D_{xt} - X_t) + \ldots + p_{Kt}D_{Kt},$$

$$W_{\mathcal{C}t+1} = D_{t+1} + p_{1t+1}D_{1t+1} + \ldots + p_{xt+1}(D_{xt+1} + (1-\delta)X_t) + \ldots + p_{Kt+1}D_{Kt+1}.$$

Since the consumer consumes all his endowment every period, at each $s = t,\ t+1$ the consumer maximizes his utility given by

$$\log C_{0s}^{\alpha_0} C_{1s}^{\alpha_1} \cdot \ldots \cdot C_{Ks}^{\alpha_K}$$

subject to the budget constraint

$$C_{0s} + p_{1s}C_{1s} + \ldots + p_{ks}C_{ks} = W_{\mathcal{C}s}.$$

The demand for commodities resulting from this optimization is closely related to reduced-form demand functions adopted in the storage literature.

Equilibrium now involves the following additional market-clearing conditions for each good:

$$C_{0s} = D_s,\ C_{ks} = D_{ks},\ k \neq x,\ s = t, t+1, \tag{22}$$

$$C_{xt} = D_{xt} - X_t,\ C_{xt+1} = D_{xt+1} + (1-\delta)X_t. \tag{23}$$

B. Equilibrium Commodity Prices and Inventories

Proposition 3 reveals how the discount factor is affected by institutional investors and summarizes the equilibrium commodity prices and inventories in our economy with storage.

Proposition 3 (Commodity Prices and Inventories): *In the economy with institutions, the equilibrium inventories X_t of the storable commodity satisfy*

$$\frac{D_t}{D_{xt} - X_t} = (1-\delta)E_t\left[M_{t,t+1}\frac{D_{t+1}}{D_{xt+1} + (1-\delta)X_t}\right], \tag{24}$$

where the stochastic discount factor $M_{t,t+1}$ is given by

$$M_{t,t+1} = e^{\left(\mu-\sigma^2\right)}\frac{D_t}{D_{t+1}}\frac{A + b\lambda D_{t+1}\prod_{i=1}^{L}\left(g_i\left(t+1\right)/D_{it+1}\right)^{1/L}}{A + b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}}, \tag{25}$$

the deterministic function $g_i\left(t\right)$ is as in (16), and the constant A is reported in the Appendix.

The equilibrium commodity prices are given by

$$p_{xt} = \frac{\alpha_x}{\alpha_0}\frac{D_t}{D_{xt} - X_t} \qquad \textit{(storable commodity)}, \tag{26}$$

$$p_{x\,t+1} = \frac{\alpha_k}{\alpha_0}\frac{D_{t+1}}{D_{x\,t+1} + (1-\delta)X_t} \qquad \textit{(storable commodity)}, \tag{27}$$

$$p_{ks} = \frac{\alpha_k}{\alpha_0}\frac{D_s}{D_{ks}},\ s = t, t+1, k \neq x \qquad \textit{(non storable commodities)}. \tag{28}$$

In the benchmark economy with no institutions, the equilibrium inventories $\overline{X}_t$ satisfy (24) with $M_{t,t+1}$ replaced by its corresponding benchmark value $\overline{M}_{t,t+1} = e^{(\mu-\sigma^2)}D_t/D_{t+1}$. The benchmark economy commodity prices are given by (26) to (28) with $\overline{X}_t$ replacing X_t.

Consequently, in the presence of institutions,

(i) The storable commodity time-t price is higher than in the benchmark economy, $p_{xt} > \overline{p}_{xt}$.

(ii) The inventory of the storable commodity is higher than in the benchmark economy, $X_t >$

$\overline{X}_t$.

(iii) An index commodity's price and inventory rise more than those of a nonindex commodity for an otherwise identical storable commodity.

Proposition 3 reveals that financialization affects commodity inventories and prices, but only for those commodities that can be stored. The prices of non storable commodities at each time are determined by the supply and demand at that time. Such commodities cannot be viewed as investable assets, and hence relationship (21) does not apply to them. Since most of the commodities for which futures are traded are storable, albeit at some cost, we now focus on the storable commodity in our model, following Deaton and Laroque (1992) and subsequent works.

Proposition 3 reports, in closed form, the discount factor prevailing in our economy. Even in the benchmark economy with no institutions, this discount factor is stochastic because it depends on the aggregate output. Therefore, in the benchmark economy, the discounting of storage firms' cash flows is not at a constant (riskless) rate, as in much of the extant storage literature. This implies that the expected return on storing a commodity in our model, just like that on any other asset, depends on the covariance of the return from holding that commodity with the discount factor. Since the discount factor is determined in financial markets, the financial markets and commodity spot markets are therefore intertwined. In particular, outside shocks affecting financial markets may also affect commodity spot prices.

[INSERT FIGURE 5 HERE]

The discount factor in the economy with institutions, presented in Proposition 3, depends additionally on the characteristics of index commodities (e.g., D_i, α_i, σ_i, $i = 1, \ldots L$). This is due to the presence of institutional investors, whose marginal utilities are represented in the discount factor. As a consequence, the prices of storable commodities are affected by the presence of institutions. In particular, as Proposition 3 demonstrates, commodity prices and inventories are higher than in the benchmark economy with no institutions. The intuition is as follows. As the demand for commodities and hence commodity prices in our model are increasing with aggregate output (as also discussed in Dvir and Rogoff (2009)), all commodity prices comove positively with each other and with the commodity index. Since institutional investors strive to not fall behind when the commodity index does well, they particularly value such assets, that is, assets that pay off more in the states when the index is expected to do well.[23] Storing a unit of a commodity from time t until $t + 1$ can be viewed as investing in an asset whose future (time-$t + 1$) return is positively correlated with that of the commodity index. Hence, the price of such assets, p_{xt} in this case, is higher in the presence of institutions. Because there is a one-to-one mapping in our model between the inventory of commodity x and its time-t price p_{xt} (equation (26)), the inventory of commodity x has to increase at the same time. Moreover, if commodity x is included in the index, its price naturally comoves more with the index. Therefore, storing such a commodity is especially attractive to institutional investors and the effects we have described are stronger, in particular, the commodity price and inventories increase by more. Finally, at the end of the storage period $t + 1$, inventories have to be liquidated and all of the available commodities consumed. Since inventories are higher in the presence of institutions, mechanically the time-$t+1$ commodity price has to be lower in the presence of institutions. This effect of the last storage date holds even in an extension of our framework to multiple storage decisions, provided that firms live for a finite number of periods. This suggests that an infinite horizon model with storage is perhaps more appropriate, but it is beyond the scope of the current work. Figure 5 illustrates the results of Proposition 3.

C. *Cross-Commodity Spillovers and the Impact of Income Shocks*

We now turn to describing cross-commodity and other spillovers in our model. These spillovers act through the discount factor channel and are novel to our model.

PROPOSITION 4 (Spillovers): *Consider an index commodity i.*

(i) In the economy with institutions, the following spillovers from commodity i to the storable commodity x occur.

	Increase in		
Spillover to	*Supply of index commodity D_{it}*	*Demand of index commodity α_i*	*Mean growth μ_i or volatility σ_i of supply*
Price of storable commodity x, p_{xt}	$-$	$+$	$\neq 0$
Inventory of storable commodity x, X_t	$-$	$+$	$\neq 0$

(ii) From a nonindex commodity ℓ, there are no spillovers to any commodity, that is, all entries in the table above are zero. Furthermore, there are no such spillovers in the benchmark economy with no institutions.

(iii) In the economy with institutions, an inflow of institutions (an increase in λ) increases the storable commodity's price p_{xt} and inventory X_t.

Proposition 4(i) shows how outside shocks affecting an index commodity spill over to all other storable commodities' spot prices and inventories. The main driver behind such spillovers is that the discount factor in (21) is affected by institutions. In particular, it becomes dependent on the characteristics of index commodities. The intuition is as follows. Consider a negative supply shock to an index commodity. Recall that such a shock implies that the commodity price, and hence the index value, is expected to be higher. This benefits institutions, which hold more of the index than normal investors. As institutions become relatively wealthier in the economy, their weight in the discount factor rises. Consequently, their effects on the assets they trade become more pronounced. In particular, prices of all assets that are positively correlated with the index go up. One such asset is the storable commodity. Institutions increase their demand for storage of the commodity (provided by the storage firms), boosting the commodity's spot price and inventories. When there are multiple storable commodities, this mechanism induces comovement among seemingly unrelated storable commodities, in the spirit of Kyle and Xiong (2001). Now consider a positive demand shift for an index commodity. Following such a shift, the affected commodity's price is higher and therefore the index value is expected to be higher. This increases institutions' demand for all assets that are positively correlated with the index and in particular their demand for commodity storage. Hence, prices and inventories of all storable commodities rise. Finally, consider a shift in the mean growth rate or volatility of any index commodity's supply. Such a shift affects the discount factor in the economy with institutions (see equation (25)) and hence has an impact on every storable commodity's price and inventory. We do not report the sign of the effect, as one can show analytically that it can be positive or negative depending on parameter values. In contrast, a shock to a nonindex commodity does not affect the discount factor and hence does not spill over to other commodities (provided, of course, that the shock does not have a common component with shocks to index commodities) because such shocks do not alter the discount factor (Proposition 4(ii)). There are also no such spillovers in the benchmark economy with no institutions because in that economy the discount factor depends only on the aggregate output and is not directly affected by index commodities. Of course, if shocks to nonindex commodities had a common component with shocks to index commodities, a shock to a nonindex commodity would affect other commodities. This would give rise to a natural interdependence, present also in the benchmark economy, and such comovement would not be amplified with institutions present.

As evident from Proposition 4(iii), an inflow of institutional investors, which in our model is captured by an increase in λ, increases prices and inventories of storable commodities. This occurs because the incentives of institutions to do well relative to the commodity index are impounded into the discount factor, and the more institutions there are, the bigger their influence on the discount factor. The resulting increase in the storable commodity's price and inventory, revealed in Proposition 3, therefore becomes more pronounced.

We note that spillovers via financial markets may present challenges for identification strategies commonly used in empirical work. Since supply and demand shocks are not directly observable, they have to be inferred from commodity prices, inventories, and other observable variables. An econometrician may then interpret an increase in the prices and inventories of a commodity as, for example, a result of a shift in the demand for that commodity. Proposition 4 shows that such an increase may have nothing to do with demand or supply for that particular commodity but may come from a shock to another commodity that is transmitted via financial markets.

There are also spillovers from the stock market to the spot price of the storable commodity. Identifying spillovers due to institutions, however, requires a more nuanced approach because stock and commodity prices comove even in the benchmark economy with no institutions as they load on a common factor, aggregate output D. So in the economy with institutions one needs to separate the interdependence from genuine spillovers occurring only in the presence of institutions. We can do so within our model by comparing the economies with and without institutions and focusing on the difference, which we call a spillover. Figure 6 plots the spillovers from the stock market to commodity prices. In particular, in the presence of institutions, higher stock market returns and volatility spill over to commodity prices, pushing them up. We are not able to analytically sign the spillover, but for reasonable parameter values it is positive, as in Figure 6. The spillovers occur via our discount factor channel. Recent empirical evidence documents spillovers akin to those occurring in Figure6. Diebold and Yilmaz (2012) document volatility spillovers from the U.S. stock market to commodity markets in recent data.

[INSERT FIGURE 6 HERE]

Our final goal in this section is to examine the effects of income shocks on commodity inventories and prices.[24] In classical storage models (e.g., Deaton and Laroque (1996)), storage always has stabilizing effects on prices. This is because a positive income shock (temporarily) increases a commodity price, but at the same time firms have a reduced incentive to store the commodity because they expect lower prices in the future. This makes the commodity more abundant today. So storage "leans against the wind" and mitigates the effects of the shock on commodity prices. Dvir and Rogoff (2009) challenge this conclusion on empirical grounds because the above mechanism is unable to deliver sufficient persistence of commodity prices. They point out that income in Deaton and Laroque (1996) and related literature is assumed to follow an AR(1) process in levels, and this assumption leads to a drop in storage following a positive income shock. Dvir and Rogoff (2009) propose to consider instead *permanent* income (demand) shocks, which end up implying that firms actually store more following such shocks (because prices are expected to remain high in the future).

Our setting naturally lends itself to an exploration of permanent income shocks because all our driving processes are persistent (geometric Brownian motions). Figure 7 depicts the effects of an income shock on prices and inventories in our model adopted to the Deaton and Laroque (1996) setting (dashed line), in our benchmark economy without institutional investors (dotted line), and in the economy with institutions (solid line). The Deaton-Laroque line corresponds to an economy with discounting at the constant riskless rate and output D_t following an AR(1) process ($\log D_{t+1} = \rho \log D_t + \varepsilon_{t+1}$, $\rho \in (0,1)$, $\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2)$).

Consistent with the classical storage literature, in the Deaton-Laroque case inventories indeed decrease in response to a positive income shock (a positive change in D_t). In contrast, in our benchmark economy with no institutions we find that inventories *do not respond* to income shocks. There are now two counteracting forces. As before, a positive income shock increases the price p_{xt} (see, for example, equation (26)). In the Deaton-Laroque case, this reduces incentives to store. But now the shock is permanent, and income and prices are expected to remain high in the future, which increases storage. In Dvir and Rogoff (2009) the latter force dominates, but in our case the two forces exactly offset each other. This suggests that simply replacing an AR(1) output processes by any I(1) process is not sufficient to make inventories increasing in an income shock, one needs a more nuanced specification, as for example the one suggested by Dvir and Rogoff (2009). On the the other hand, in the presence of institutions we see that inventories go up with a shock, magnifying the effects of an income shock on commodity prices. Firms have an incentive to store more because higher output today implies higher levels of the commodity price index in the future. Anticipating that, institutional investors increase their demand for all assets whose payoffs are correlated with the index, and in particular increase their demand for storage. This result complements the findings of Dvir and Rogoff (2009) but here we offer an alternative channel and a more nuanced view on the connection between permanent income shocks and commodity inventories.

[INSERT FIGURE 7 HERE]

IV. Conclusion and Discussion

In this paper we explore theoretically how the presence of institutional investors may affect commodity prices and their dynamics. We find that in the presence of institutions, futures prices of all commodities rise, with futures prices of index commodities increasing to a greater extent. We also find that in the presence of institutional investors, shocks to the fundamentals (demand and supply) of index commodities get transmitted to the prices of all other commodities. Furthermore, the volatilities and correlations of all commodity futures returns rise in the presence of institutions, with those of index commodities increasing by more. Finally, we find that storable commodity spot prices and inventories go up in the presence of institutions. The financial markets serve as a conduit in transmitting outside shocks to commodity spot prices.

To keep the paper focused, we have not explored our model's implications for commodity futures risk premia. The risk premium is defined as the difference between the expected spot price of a commodity and its futures price. This quantity should be positive according to hedging pressure theory (Keynes (1930), Hicks (1939), Hirshleifer (1988)). If producers of a commodity want to hedge their price risk by selling futures contracts, then arbitrageurs who take the other side of the contract should receive the risk premium in compensation for taking that risk. According to our model, the buying pressure from institutional investors exerts a similar effect in the opposite direction, which should reduce the risk premium. Consistent with this prediction, Hamilton and Wu (2014) document that on average the risk premium in crude oil futures has decreased and become more volatile since 2005. Moreover, it would be interesting to explore the effects of institutions on the futures curve. In our model, the (very simplistic) futures curve for any commodity exhibits a contango in the classical sense (as defined by Keynes) in that the futures price exceeds its corresponding expected spot price, but only in the presence of institutions. Owing to its one-consumption-date nature, however, our model is not immediately suitable for a proper analysis of futures curves, but its extended version could be.

Our model also has implications for the open interest in futures markets. Cheng, Kirilenko, and Xiong (2014) show that the positions of commodity index traders fall in response to an increase in overall economic uncertainty, as captured by the VIX Volatility Index. We anticipate that, qualitatively, our model delivers this implication. In a recent paper, Hong and Yogo (2012) document that open interest predicts asset prices and macroeconomic variables. It would be interesting to examine whether our model delivers this intriguing finding.

In our model information is symmetric and investors have the same beliefs. Sockin and Xiong (2015) develop a model with asymmetric information in which producers learn about the state of the economy from futures prices, a channel absent in our framework. In our model, trade between investors occurs because their interim relative performance fluctuates. Another realistic motive for trade is expectations-based speculation, driven by investors' differences of opinion. It would be desirable if not straightforward to extend our model to include asymmetric information, expectations-based speculation, and inefficient risk sharing. Finally, our analysis of financialization is based on comparing economies with and without institutional investors. It would be desirable to address the deeper issue of why institutions entered the commodity futures markets in the first place. We leave these important extensions to future research.

REFERENCES

Acharya, Viral V., Lars A. Lochstoer, and Tarun Ramadorai, 2013, Limits to arbitrage and hedging: Evidence from commodity markets, *Journal of Financial Economics* 109, 441-465.

Baker, Steven D., 2013, The financialization of storable commodities, Working paper, University of Virginia.

Bank for International Settlements, 2003, Incentive structures in institutional asset management and their implications for financial markets, Committee on the Global Financial System, Bank for International Settlements, Basel.

Barberis, Nicholas, and Andrei Shleifer, 2003, Style investing, *Journal of Financial Economics* 68, 161-199.

Barberis, Nicholas, Andrei Shleifer, and Jeffrey Wurgler, 2005, Comovement, *Journal of Financial Economics* 75, 283-317.

Basak, Suleyman, and Anna Pavlova, 2013, Asset prices and institutional investors, *American Economic Review* 103, 1728-1758.

Boons, Martijn, Frans De Roon, and Marta Szymanowska, 2014, The price of commodity risk in stock and futures markets, Working paper, Tilburg University.

Boyer, Brian H., 2011, Style-related comovement: fundamentals or labels? *Journal of Finance* 66, 307-332.

Brennan, Michael J., 1993, Agency and asset pricing, Working paper, UCLA.

Buyuksahin, Bahattin, Michael S. Haigh, Jeffrey H. Harris, James A. Overdahl, and Michel A. Robe, 2008, Fundamentals, trader activity and derivative pricing, Working paper.

Buyuksahin, Bahattin, and Michel A. Robe, 2014, Speculators, commodities and cross-market linkages, *Journal of International Money and Finance* 42, 38-70.

Campbell, John Y., and Gregory N. Mankiw, 1989, Consumption, income and interest rates: Reinterpreting the time series evidence, in *NBER Macroeconomics Annual* (MIT Press), Olivier J. Blanchard and Stanley Fischer, eds.

Casassus, Jaime, and Pierre Collin-Dufresne, 2005, Stochastic convenience yield implied from commodity futures and interest rates, *Journal of Finance* 60, 2283-2331.

Cheng, Ing-Haw, Andrei Kirilenko, and Wei Xiong, 2015, Convective risk ows in commodity futures markets, *Review of Finance* 19, 1733-1781.

Cochrane, John H., Francis A. Longstaff, and Pedro Santa-Clara, 2008, Two trees, *Review of Financial Studies* 21, 347-385.

Commodity Futures Trading Commission, 2008, Staff report on commodity swap dealers and index traders with Commission recommendations.

Deaton, Angus, and Guy Laroque, 1992, On the behaviour of commodity prices, *Review of Economic Studies* 59, 1-23.

Deaton, Angus, and Guy Laroque, 1996, Competitive storage and commodity price dynamics, *Journal of Political Economy* 104, 896-923.

Diebold, Francis X., and Kamil Yilmaz, 2012, Better to give than to receive: Predictive directional measurement of volatility spillovers, *International Journal of Forecasting* 28, 57-66.

Dvir, Eyal, and Kenneth S. Rogoff, 2009, Three epochs of oil, NBER Working paper 14927.

Fattouh, Bassam, Lutz Kilian, and Lavan Mahadeva, 2013, The role of speculation in oil markets: What have we learned so far? *Energy Journal* 34, 7-33.

Gibson, Rajna, and Eduardo S. Schwartz, 1990, Stochastic convenience yield and the pricing of oil contingent claims, *Journal of Finance* 45, 959-976.

Gorton, Gary B., Fumio Hayashi, and K. Geert Rouwenhorst, 2013, The fundamentals of commodity futures returns, *Review of Finance* 17, 35-105.

Hamilton, James D., and Jing C. Wu, 2014, Risk premia in crude oil futures prices, *Journal of International Money and Finance* 42, 9-37.

Hamilton, James D., and Jing C. Wu, 2015, Effects of index-fund investing on commodity futures prices, *International Economic Review* 56, 187-205.

Harris, Lawrence, and Eitan Gurel, 1986, Price and volume effects associated with changes in the S&P 500 list: New evidence for the existence of price pressures, *Journal of Finance* 41, 815-829.

Henderson, Brian J., Neil D. Pearson, and Li Wang, 2015, New evidence on the financialization of commodity markets, *Review of Financial Studies* 28, 1285-1311.

Hicks, John R., 1939, Value and capital, *Oxford University Press*, Cambridge.

Hirshleifer, David, 1988, Residual risk, trading costs, and commodity futures risk premia, *Review of Financial Studies* 1, 173-193.

Hong, Harrison, Wenxi Jiang, Na Wang, and Bin Zhao, 2014, Trading for status, *Review of Financial Studies* 27, 3171-3212.

Hong, Harrison, and Motohiro Yogo, 2012, What does futures market interest tell us about the macroeconomy and asset prices? *Journal of Financial Economics* 105, 473-490.

Irwin, Scott H., and Dwight R. Sanders, 2011, Index funds, financialization, and commodity futures markets, *Applied Economic Perspectives and Policy* 33, 1-31.

Kaplan, Greg, and Giovanni L. Violante, 2014, A model of the consumption response to fiscal stimulus payments, *Econometrica* 82, 1199-1239.

Keynes, John M., 1923, Some aspects of commodity markets, *Manchester Guardian Commercial*, European Reconstruction Series, 13, 784-786.

Keynes, John M., 1930, *A Treatise on Money* (Macmillan, London).

Kilian, Lutz, and Daniel P. Murphy, 2014, The role of inventories and speculative trading in the global market for crude oil, *Journal of Applied Econometrics* 29, 454-478.

Koijen, Ralph S.J., 2014, The cross-section of managerial ability, incentives, and risk preferences, *Journal of Finance* 69, 1051-1098.

Kyle, Albert S., and Wei Xiong, 2001, Contagion as a wealth effect, *Journal of Finance* 56, 1401-1440.

Martin, Ian, 2013, The Lucas orchard, *Econometrica* 81, 55-111.

Masters, Michael W., 2008, Testimony before the Committee on Homeland Security and Governmental Affairs, U.S. Senate.

Pastor, Lubos, and Pietro Veronesi, 2012, Uncertainty about government policy and stock prices, *Journal of Finance* 67, 1219-1264.

Pavlova, Anna, and Roberto Rigobon, 2007, Asset prices and exchange rates, *Review of Financial Studies* 20, 1139-1180.

Routledge, Bryan R., Duane J. Seppi, and Chester S. Spatt, 2000, Equilibrium forward curves for commodities, *Journal of Finance* 55, 1297-1338.

Shleifer, Andrei, 1986, Do demand curves for stocks slope downward? *Journal of Finance* 41, 579-590.

Singleton, Kenneth J., 2014, Investor ows and the 2008 boom/bust in oil prices,*Management Science* 60, 300-318.

Sockin, Michael, and Wei Xiong, 2015, Informational frictions and commodity markets, *Journal of Finance* 70, 2063-2098.

Tang, Ke, and Wei Xiong, 2012, Index investment and the financialization of commodities, *Financial Analysts Journal* 68, 54-74.

Varian, Hal R., 1992, *Microeconomic Analysis* (Norton, New York).

Weil, Philippe, 1992, Hand-to-mouth consumers and asset prices, *European Economic Review* 36, 575-583.

Notes

[1]Related empirical literature dates the start of the financialization of commodity futures around 2004 (Buyuksahin et al. (2008), Irwin and Sanders(2011), Tang and Xiong (2012), Hamilton and Wu (2014), Boons, De Roon and Szymanowska (2014), among others), and some of these works explicitly test for and confirm a structural break around 2004.

[2]One may reasonably argue that there is also a category of institutional investors who want to perform well when the index does poorly (e.g., hedge funds).

[3]In this strand of literature, a recent paper by Sockin and Xiong (2015) shows that price pressure from investors operating in futures markets (even if driven by nonfundamental factors) can be transmitted to spot prices of underlying commodities. Acharya, Lochstoer, and Ramadorai (2013) stress the importance of capital constraints on futures' markets speculators and argue that frictions in financial (futures) markets can feed back into production decisions in the physical market. In a similar framework, Gorton, Hayashi, and Rouwenhorst (2013) derive endogenously the futures basis and the risk premium and relate them to inventory levels. Routledge, Seppi, and Spatt (2000) derive the term structure of forward prices for storable commodities, highlighting the importance of nonnegativity constraints on inventories.

[4]The Internet Appendix is available in the online version of the article on the *Journal of Finance* website.

[5]To model other major commodity indices such as the Goldman Sachs Commodity Index and the Dow Jones UBS Commodity Index, it is more appropriate to define the index as $I_t = \sum_{i=1}^{L} w_i f_{it}$, where the weights w_i add up to one. Although one could still obtain a closed-form characterization of futures prices along the lines of that in Proposition 1 and its stated properties, further properties are less analytically tractable.

[6]This is a standard feature of models that do not have intermediate consumption. In other words, there is no intertemporal choice that would pin down the interest rate. Our normalization is commonly employed in models with no intermediate consumption (see, for example, Pastor and Veronesi (2012) for a recent reference).

[7]Direct empirical support for the status-based interpretation of our model is provided in Hong et al. (2014), who adopt the formulation in (7) in their analysis. Empirical work estimating objectives of institutional investors remains scarce, with the notable exception of Koijen (2014).

[8]One could reasonably argue that there is also a category of institutional investors whose marginal utility is decreasing in the index level, for example, hedge funds, which may prefer higher payoffs when the index does poorly.

[9]In what follows, we are interested in comparative statics with respect to α_k. The expenditure share on commodity k, $\alpha_k/\sum_{k=0}^{K}\alpha_k$, is monotonically increasing in α_k. Hence, all our comparative statics for α_k are equally valid for expenditure shares $\alpha_k/\sum_{k=0}^{K}\alpha_k$.

[10]For example, an increase in demand for soya beans due to the invention of biofuels and concerns about the environment.

[11]Recall that in our specification supply news is uncorrelated across commodities. Otherwise, nonindex commodity news would affect index commodity futures, but this effect is normal interdependence, also present in the benchmark economy without institutions.

[11]One major difference between this model and the one-good stock market economy of Basak and Pavlova (2013) is that in their analysis nonindex security prices are unaffected by the presence of institutions, although institutions are modeled similarly. Consequently, in contrast to our findings, their nonindex assets have zero correlation among themselves and with index assets, and the nonindex asset prices and volatilities are not affected by institutional investors. The key reason for these differences is that in Basak and Pavlova (2013), the cashflows of nonindex securities are exogenous and are uncorrelated with the index. Here, nonindex cashflows, which are endogenously determined commodity prices, end up being correlated with the index.

[12]The notation $||z||$ denotes the square root of the dot product $z \cdot z$.

[13]In Figure 4 we do not attempt to generate realistic magnitudes of volatility increases, rather, we simply illustrate our comparative statics results in Proposition 2. For more realistic magnitudes of the volatilities, see our richer model in Section II.C. See also Internet Appendix Figure IA2.

[14]This result can be shown analytically when the volatilities of commodity supply news are the same, that is, $\sigma_k = \sigma_j$, $\forall k, j = 1, \ldots, K$. For different volatility supply news parameters, all cross-correlations (including the stock) can be analytically shown to increase for $L = 1$.

[15]Indeed, it can be shown that the time-t wealth distribution is $W_{\mathcal{I}t}/W_{\mathcal{N}t} = (a + bE_t[I_T])/(a + bE[I_T])$.

[16]For example, investors who are benchmarked to a stock market index (e.g., S&P 500) would have a confounding effect on the stock market's valuation. Their index would also appear in the equilibrium stock market level. Our model can be extended to incorporate such investors.

[17]Tang and Xiong (2012) document that the correlation between the GSCI commodity index and the S&P500 rose after 2004, and was especially high in 2008. Similarly, Buyuksahin and Robe (2014) find that the GSCI-S&P500 correlation has risen since the 2008 financial crisis, but not before.

[18]This amplification effect suggests that the specifications used in structural econometric models of commodity prices, such as those in Kilian and Murphy (2014), may not be time-invariant, and in particular the sensitivity of commodity prices to structural shocks may have changed since the inflow of institutional investors starting in 2004. This is a testable implication that we leave for future empirical work.

[19]We note that we can easily recast our baseline model (Section I) in discrete time so that, like the storage decision, asset allocation decisions are made over discrete intervals. The only changes needed to discretize the model would be to take the supply news to be discrete-time analogues of the processes in equations (1) and (2) (each $D_t, D_{kt}, k = 1, \ldots, K$ is conditionally lognormal) and to complete the financial markets with enough zero-net-supply securities to compensate for the loss of spanning owing to the removal of continuous re-trading. With these two changes, our key insights, and in particular all our expressions and results in Proposition 1 and Corollary 1, remain. We can therefore frame our model with storage in discrete time.

[20]Two related recent works in finance that employ commodity storage models in the presence of financial markets, Acharya, Lochstoer, and Ramadorai (2013) and Gorton, Hayashi, and Rouwenhorst (2013), also resort to one-time storage decisions. Notably, Baker (2013) considers dynamic storage decisions.

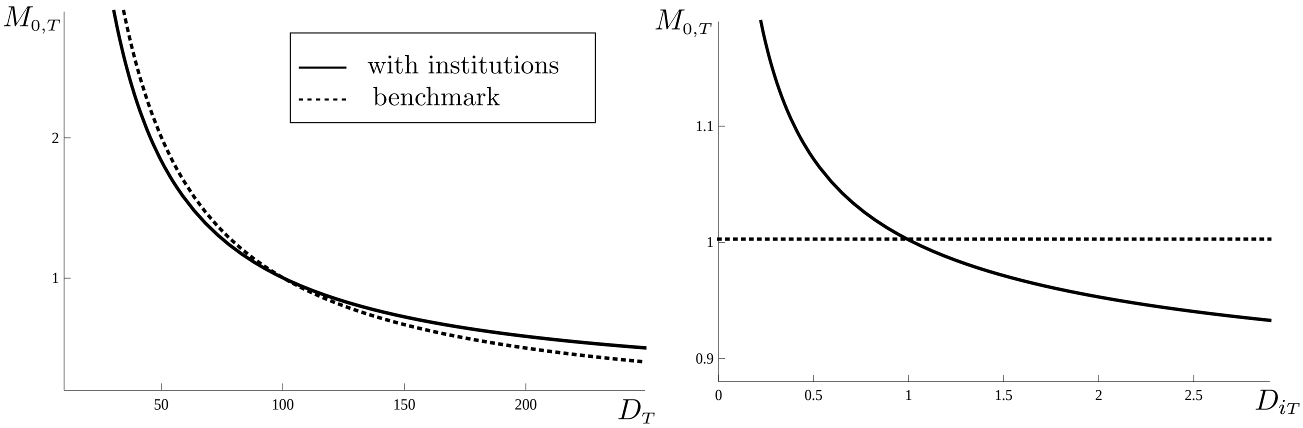
[21]Our focus here is on the interaction between financial markets and commodity prices and inventories, and we chose to highlight these interactions in the simplest possible way. It is possible to impose non negativity constraints on inventories, as in the storage literature, but our analysis here does not have much to add regarding the effects of inventory stockouts beyond what is already reported in the literature.

[22]This way of modeling consumers has been employed extensively in macroeconomics– it has been argued that accounting for hand-to-mouth consumers helps rationalize aggregate consumption data and is important for policy experiments (e.g., impact of a fiscal stimulus). In a recent paper, Kaplan and Violante (2014) document

that in the U.S. between 18% and 37% of households live hand-to-mouth (consume all of their paycheck).

[23]If a commodity is scarce at time $t + 1$ and hence its price is high, it is expected to remain scarce over the horizon of the investors because our driving processes are persistent.

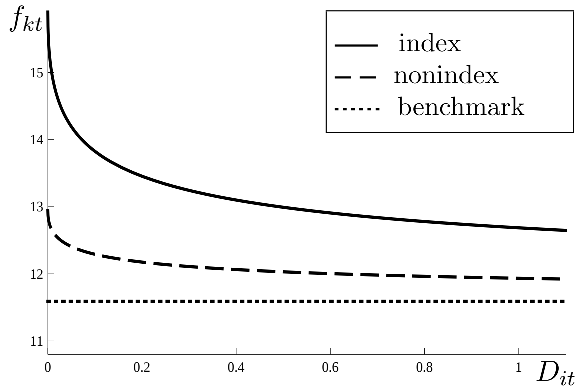
[24]The shocks we are considering here are shocks to the income of consumers. Such shocks induce shifts in the demand schedule for each commodity, and therefore are commonly referred to as *demand* shocks in models that specify consumer demand exogenously.



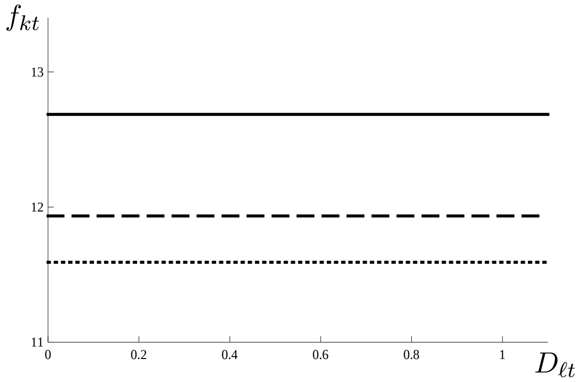
Panel A. Effect of aggregate output D_T

Panel B. Effect of index commodity supply D_{iT}

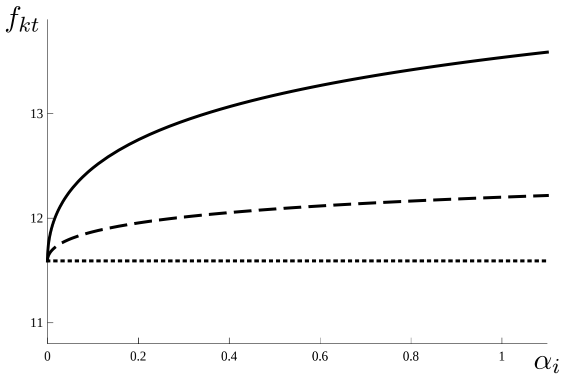
Figure 1. Discount factor. This figure plots the discount factor in the presence of institutions against aggregate output D_T and against an index commodity's supply D_{iT}. The dotted lines correspond to the discount factor in the benchmark economy with no institutions. The plots are typical. The parameter values, when fixed, are: $L = 2$, $K = 5$, $a = 1$, $b = 1$, $T = 5$, $\lambda = 0.2$, $\alpha_0 = 0.7$, $D_T = D_0 = 100$, $D_{kT} = D_{k0} = 1$, $\mu = \mu_k = 0.05$, $\sigma = 0.15$, $\sigma_k = 0.25$, and $\alpha_k = 0.06$, $k = 1, \ldots K$. We discuss the parameters and their choices in the Internet Appendix.



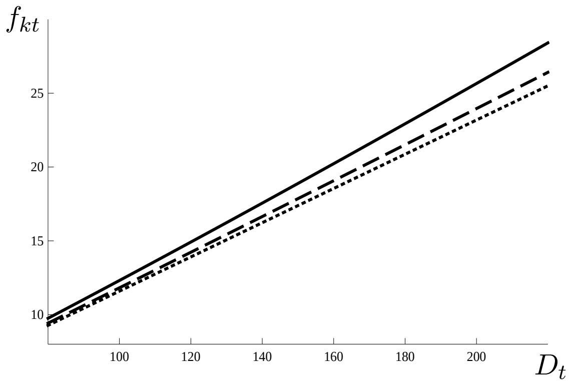
Panel A. Effect of index commodity supply news D_i



Panel B. Effect of nonindex commodity supply news D_ℓ



Panel C. Effect of index commodity demand parameter α_i



Panel D. Effect of aggregate output D_t

Figure 2. Futures prices. This figure plots the equilibrium futures prices against several key quantities. The plots are typical. We set $t = 0.1$, $D_t = 100$, and $D_{kt} = 1$, $k = 1, \dots K$. The solid line is for index futures, the dashed line is for nonindex futures, and the dotted line is for the benchmark economy. The remaining parameter values (when fixed) are as in Figure 1.

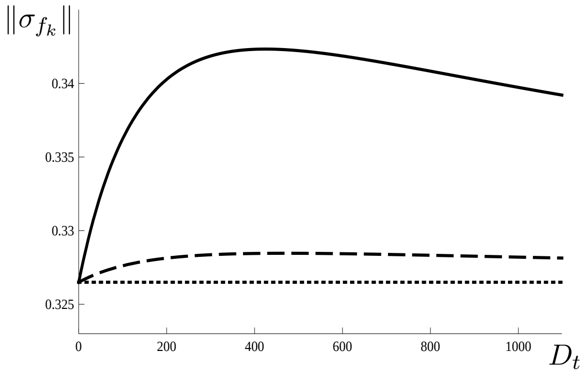
	Sources of risk associated with								
	Generic	Index commodities					Nonindex commodities		
	ω_0	ω_1	$\dots$	ω_k	$\dots$	ω_L	ω_{L+1}	$\dots$	ω_K
Loadings									
Benchmark $\overline{\sigma}_{f_k}$	σ	0	$\dots$	$-\sigma_k$	$\dots$	0	0	$\dots$	0
Index σ_{f_k}	$\sigma(1+h_{kt})$	$-\sigma_1 \frac{1}{L} h_{kt}$	$\dots$	$-\sigma_k(1+\frac{1}{L} h_{kt})$	$\dots$	$-\frac{1}{L}\sigma_L h_{kt}$	0	$\dots$	0

Panel A. Index commodity futures $k = 1, \dots, L$

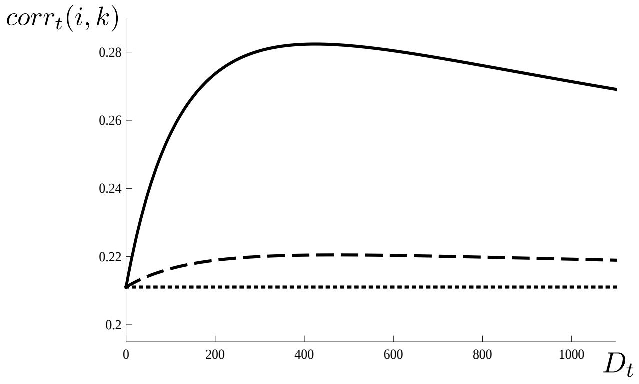
	Sources of risk associated with									
	Generic	Index commodities			Nonindex commodities					
	ω_0	ω_1	$\dots$	ω_L	ω_{L+1}	$\dots$	ω_k	$\dots$	ω_K
Loadings									
Benchmark $\overline{\sigma}_{f_k}$	σ	0	$\dots$	0	0	$\dots$	$-\sigma_k$	$\dots$	0
Nonindex σ_{f_k}	$\sigma(1+h_t)$	$-\sigma_1 \frac{1}{L} h_t$	$\dots$	$-\frac{1}{L}\sigma_L h_t$	0	$\dots$	$-\sigma_k$	$\dots$	0

Panel B. Nonindex commodity futures $k = L+1, \dots, K$

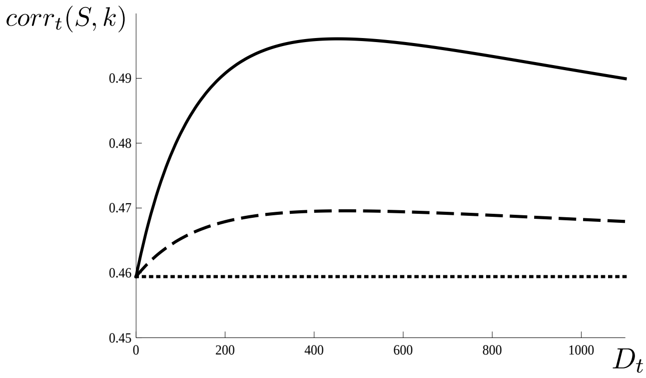
Figure 3. Individual volatility components of Futures Prices.



Panel A. Volatilities

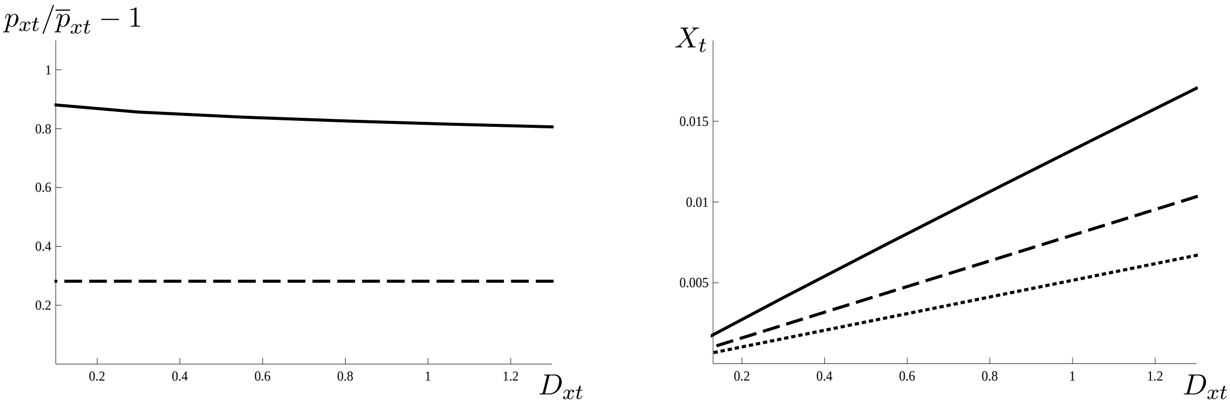


Panel B. Cross-commodity correlations



Panel C. Equity-commodity correlations

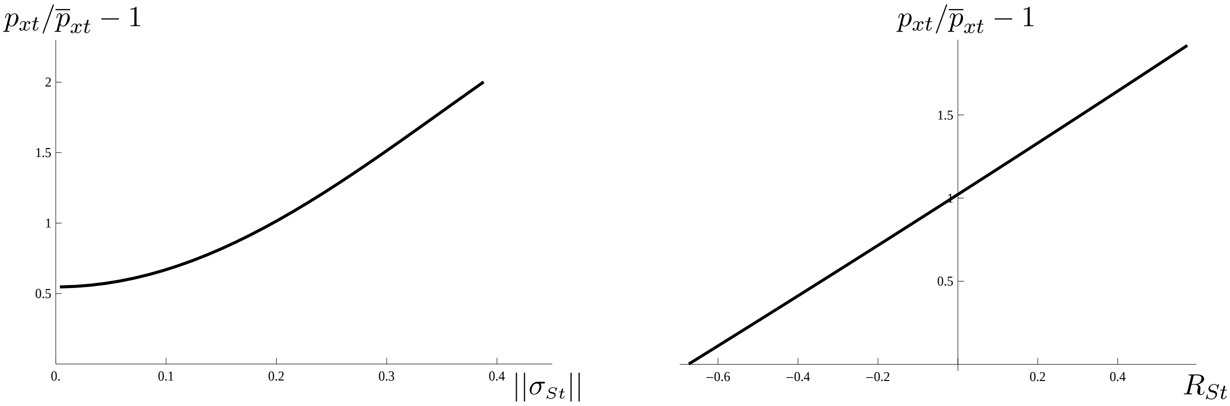
Figure 4. Commodity futures volatilities, cross-commodity correlations, and equity-commodity correlations. This figure plots the commodity futures volatility $||\sigma_{f_k t}||$, cross-commodity correlations $\sigma_{f_i t} \cdot \sigma_{f_k t}/(\|\sigma_{f_i t}\| \, \|\sigma_{f_k t}\|)$, and equity-commodity correlations $\sigma_{f_S t} \cdot \sigma_{f_k t}/(\|\sigma_{St}|| \, ||\sigma_{f_k t}\|)$ in the presence of institutions against aggregate output D_t. The solid line is for index futures, the dashed line is for nonindex futures, and the dotted line is for the benchmark economy. The parameter values are as in Figure 2.



Panel A. Price difference relative to benchmark (%)

Panel B. Commodity inventory

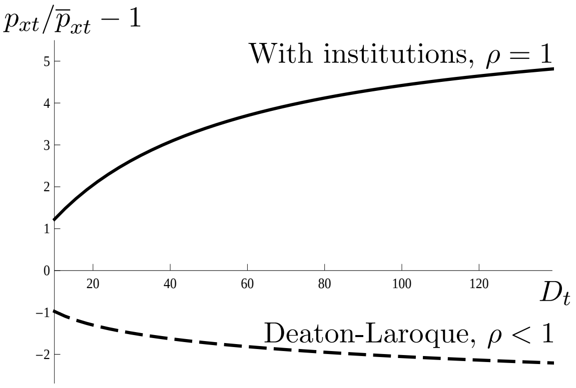
Figure 5. Commodity prices and inventories. Panel A plots the difference between the storable commodity price in the economy with institutions relative to that in the benchmark economy (in %). Panel B plots the inventory of the storable commodity x. The plots are typical. The solid line is for index commodities, the dashed line is for nonindex commodities, and the dotted line is for the benchmark economy. We set the storage cost to $\delta = 0.02$. The remaining parameters are from Figure 2.



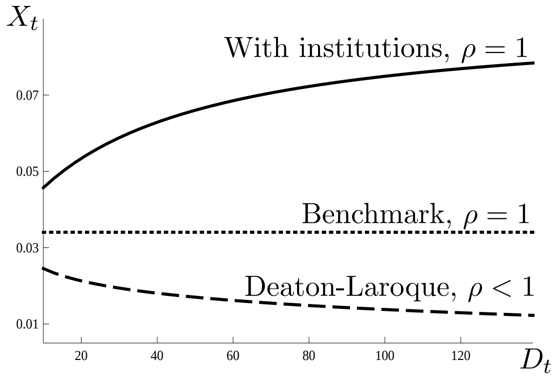
Panel A. Spillover of stock volatility

Panel B. Spillover of stock return

Figure 6. Spillovers from stock market. This figure plots commodity prices (difference relative to benchmark (%)) against stock volatility $||\sigma_S||$ and stock return (%), where $R_{St} \equiv S_t/S_0 - 1$. We set $\delta = 0.02$, $\rho = 0.99$,and $\mu_x = 0$. The remaining parameters are as in Figure 2.



Panel A. On commodity price (difference relative to benchmark with $\rho = 1$ (%))



Panel B. On commodity inventory

Figure 7. Effects of an income shock. This figure plots commodity prices and inventories against consumers' income (D_t). We set the gross interest rate for the Deaton-Laroque (1996) case to $R = 1.02$. The remaining parameters are as in Figure 6.

Appendix. Proofs

Proof of Lemma 1: We first determine institutional and normal investors' optimal demands in each commodity. Since the securities market is dynamically complete in our setup with $K+1$ risky securities and $K+1$ sources of risk ω, there exists a state price density process, ξ, such that the time-t value of a payoff Q_T at time T is given by $E_t\big[\xi_T Q_T\big]/\xi_t$. In our setting, the state price density is a martingale. Accordingly, investor n's, $n = \mathcal{N}, \mathcal{I}$, dynamic budget constraint (9) can be restated as

$$E_t\Big[\xi_T \sum_{k=0}^{K} p_{kT} C_{n_k T}\Big] = \xi_t W_{nt}. \tag{A1}$$

Maximizing the institutional investor's expected objective function (7), with the Cobb-Douglas aggregator (8) substituted in, subject to (A1) evaluated at time $t=0$ leads to the institution's optimal demand for commodity $k = 1, \ldots, K$ and the generic good of

$$C_{\mathcal{I}_k T} = \frac{\alpha_k \left(a + bI_T\right)}{y_{\scriptscriptstyle I} p_{kT} \xi_T}, \tag{A2}$$

$$C_{\mathcal{I}_0 T} = \frac{\alpha_0 \left(a + bI_T\right)}{y_{\scriptscriptstyle I} \xi_T}, \tag{A3}$$

where $1/y_{\scriptscriptstyle I}$ solves A1 evaluated at $t = 0$. Substituting (A2) and (A3) into (A1) at $t = 0$, we obtain

$$\frac{1}{y_{\scriptscriptstyle I}} = \frac{\lambda \xi_0 S_0}{\sum_{j=0}^{K} \alpha_j \left(a + bE\left[I_T\right]\right)}.$$

Consequently, the institution's optimal commodity demands are given by

$$C_{\mathcal{I}_k T} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{\lambda \xi_0 S_0}{p_{kT} \xi_T} \frac{a + b I_T}{a + bE\left[I_T\right]}, \qquad k = 1, \ldots, K, \tag{A4}$$

$$C_{\mathcal{I}_0 T} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\lambda \xi_0 S_0}{\xi_T} \frac{a + b I_T}{a + bE\left[I_T\right]}. \tag{A5}$$

Similarly, we obtain the normal investor's optimal commodity demands at time T as

$$C_{\mathcal{N}_k T} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{\left(1 - \lambda\right) \xi_0 S_0}{p_{kT} \xi_T}, \qquad k = 1, \ldots, K, \tag{A6}$$

$$C_{\mathcal{N}_0 T} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\left(1 - \lambda\right) \xi_0 S_0}{\xi_T}. \tag{A7}$$

We now determine the equilibrium prices at time T. To obtain the equilibrium state price density, we impose the market clearing condition for the generic good, $C_{\mathcal{N}_0 T} + C_{\mathcal{I}_0 T} = D_T$, and substitute (A5) and (A7) to obtain

$$\frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{\xi_T} \left(1 - \lambda + \lambda \frac{a + b I_T}{a + bE\left[I_T\right]}\right) = D_T,$$

which after rearranging leads to the equilibrium terminal state price density

$$\xi_T = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{D_T} \left(1 + \frac{\lambda b \left(I_T - E\left[I_T\right]\right)}{a + bE\left[I_T\right]}\right). \tag{A8}$$

The equilibrium state price density in the benchmark economy with no institutions is obtained by considering the special case of $b = 0$ in (A8). The time-T discount factor is defined as

$M_{0,T} = \xi_T/\xi_0$, which after substituting (A8) leads to expression (14) reported in Lemma 1.

To determine the equilibrium commodity prices at T, we impose the market clearing condition $C_{\mathcal{N}_k T} + C_{\mathcal{I}_k T} = D_{kT}$ for each commodity $k = 1, \ldots, K$, and substitute (A4) and (A6) to obtain

$$\frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{\xi_0 S_0}{p_{kT} \xi_T} \left(1 - \lambda + \lambda \frac{a + bI_T}{a + bE\left[I_T\right]} \right) = D_{kT},$$

which after substituting the equilibrium state price density (A8) and rearranging leads to the equilibrium commodity price expressions (11) in Lemma 1. Substituting the equilibrium commodity prices (11) that are in the commodity into the definition of the index (4) leads to the equilibrium commodity index value (12). Moreover, substituting the equilibrium commodity prices (11) into the stock market terminal value $S_T = D_T + \sum_{k=1}^{K} p_{kT} D_{kT}$ leads to expression (13) in Lemma 1. To determine the unconditional expectation of the index, we make use of the fact that D_T, D_{iT}, $i = 1, \ldots, L$, are lognormally distributed and hence obtain

$$E\left[I_T\right] = E\left[\frac{D_T}{\alpha_0} \prod_{i=1}^{L} \left(\frac{\alpha_i}{D_{iT}} \right)^{1/L} \right] = e^{\left(\mu - \frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)T} \frac{D_0}{\alpha_0} \prod_{i=1}^{L} \left(\frac{\alpha_i}{D_{i0}} \right)^{1/L}. \tag{A9}$$

Finally, we note that the equilibrium commodity and stock prices at time T are as in the benchmark economy with no institutions (the special case of $b = 0$, $a = 1$).

Q.E.D.

Proof of Proposition 1: By no arbitrage, the time-t futures price of a futures contract with maturity τ on commodity $k = 1, \ldots, K$ is given by $f_{kt} = E_t\left[\xi_{t+\tau} p_{k\,t+\tau}\right]/\xi_t$. Iteratively substituting the next rollover price upon maturity, $f_{k\,t+\tau}$, untill T, we obtain

$$f_{kt} = \frac{E_t\big[\xi_T p_{kT}\big]}{\xi_t}. \tag{A10}$$

We proceed by first determining the equilibrium state price density process ξ. Since the state price density process is a martingale, its time-t value is given by

$$\begin{aligned} \xi_t &= E_t\big[\xi_T\big] \\ &= \bar{\xi} E_t\left[1/D_T\right]\left(a + b\left(1-\lambda\right)E\left[I_T\right] + \lambda b \frac{E_t\left[I_T/D_T\right]}{E_t\left[1/D_T\right]}\right), \end{aligned} \tag{A11}$$

where the second equality follows by substituting ξ_T from (A8) and rearranging, and

$$\bar{\xi} = \frac{\alpha_0}{\sum_{j=0}^{K}\alpha_j}\frac{\xi_0 S_0}{a + bE\left[I_T\right]}. \tag{A12}$$

Substituting (12) and using the fact that D_T, D_{iT}, $i = 1,\ldots,L$, are lognormally distributed, we obtain

$$\begin{aligned} E_t\left[I_T/D_T\right] &= \frac{1}{\alpha_0}E_t\left[\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0}e^{-\frac{1}{L}\sum_{i=1}^{L}\left(\mu_i - \frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)(T-t)}\prod_{i=1}^{L}\left(\alpha_i/D_{it}\right)^{1/L}. \end{aligned} \tag{A13}$$

Substituting (A9), (A13), and $E_t\left[1/D_T\right] = e^{\left(\sigma^2-\mu\right)(T-t)}/D_t$ into (A11), we obtain

$$\xi_t = \bar{\xi}\frac{e^{\left(\sigma^2-\mu\right)(T-t)}}{D_t}\left(a + b\left(1-\lambda\right)D_0\prod_{i=1}^{L}\left(g_i\left(0\right)/D_{i0}\right)^{1/L} + b\lambda e^{-\sigma^2(T-t)}D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}\right), \tag{A14}$$

where $g_i\left(t\right)$ is as given in (16).

To compute the expected deflated futures payoff of commodity $k = 1, \ldots, K$, we substitute (A8) and (11), and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \bar{\xi}\frac{\alpha_k}{\alpha_0}E_t\left[1/D_{kT}\right]\left(a + b\left(1-\lambda\right)E\left[I_T\right] + b\,\lambda\,\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]}\right), \tag{A15}$$

where $\bar{\xi}$ is as in (A12).

For nonindex futures contracts $k = L+1, \ldots, K$, we proceed by considering

$$\begin{aligned} E_t\left[I_T/D_{kT}\right] &= \frac{1}{\alpha_0}E_t\left[D_T/D_{kT}\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}\right] \\ &= \frac{1}{\alpha_0}E_t\left[D_T\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}\right]E_t\left[1/D_{kT}\right], \end{aligned}$$

where in the first equality we have substituted (12) and in the second we have made use of the fact that D_{kT} is independent of D_T, D_{iT}, $i = 1, \ldots, L$. Consequently, using the fact that D_T, D_{iT}, $i = 1, \ldots, L$, are lognormally distributed, we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = D_t\prod_{i=1}^{L}\left(g_i\left(t\right)/D_{it}\right)^{1/L}, \tag{A16}$$

where $g_i(t)$ is as in (16). Substituting (A14) to (A16), (A9) and $E_t[1/D_{kT}] = e^{\left(\sigma_k^2-\mu_k\right)(T-t)}/D_{kt}$ into (A10), and rearranging, we arrive at the equilibrium nonindex futures price expression reported in (15) for $k = L+1, \ldots, K$. The equilibrium futures price $\bar{f}_k$ in the benchmark economy with no institutions (16) follows by considering the special case of $a = 1$, $b = 0$ in (15).

For index futures contracts $k = 1, \ldots, L$, we substitute (12) and again compute

$$E_t[I_T/D_{kT}] = \frac{1}{\alpha_0} E_t\left[D_T/D_{kT} \prod_{i=1}^{L} (\alpha_i/D_{iT})^{1/L}\right]$$

$$= \frac{1}{\alpha_0} e^{\left(-\mu+\mu_k+\left(\frac{1}{L}+1\right)\sigma_k^2-\frac{1}{L}\sum_{i=1}^{L}\left(\mu_i-\frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)(T-t)} \frac{D_t}{D_{kt}} \prod_{i=1}^{L} (\alpha_i/D_{it})^{1/L}.$$

Using $E_t[1/D_{kT}] = e^{\left(\sigma_k^2-\mu_k\right)(T-t)}/D_{kt}$ we obtain

$$\frac{E_t[I_T/D_{kT}]}{E_t[1/D_{kT}]} = e^{\frac{1}{L}\sigma_k^2(T-t)} D_t \prod_{i=1}^{L} (g_i(t)/D_{it})^{1/L}, \tag{A17}$$

where $g_i(t)$ is as in (16). Substituting (A14),(A15),(A17), and (A9) into (A10) and rearranging leads to the equilibrium index futures price expression reported in (15) for $k = 1, \ldots, L$. The property (i) that the futures prices are higher than in the benchmark economy follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (15) is strictly greater than one. Similarly, the property (ii) that the index futures price increase is higher than that of nonindex futures follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (15) is higher for an otherwise identical index futures.

Q.E.D.

Proof of Corollary 1. The stated properties follow by taking appropriate partial derivatives of the expressions (15) and (16), and comparing the relevant magnitudes of the partial derivatives of interest.

Q.E.D.

Proof of Proposition 2: We write the equilibrium index futures price in (15) for $k = 1, \ldots, L$ as

$$f_{kt} = \bar{f}_{kt} \frac{Z_t}{Y_t}, \tag{A18}$$

where

$$\overline{f}_{kt} = \frac{\alpha_k}{\alpha_0} e^{(\mu - \mu_k - \sigma^2 + \sigma_k^2)(T-t)} \frac{D_t}{D_{kt}},$$

$$Z_t = a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2 (T-t)/L} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L},$$

$$Y_t = a + b(1-\lambda) D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2 (T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L},$$

with $g_i\,(t)$ as in (16).

Applying Itô's Lemma to both sides of (A14), we obtain

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + \sigma_{Zt} - \sigma_{Yt}, \tag{A19}$$

where

$$\overline{\sigma}_{f_k} = (\sigma,\, 0, \ldots, -\sigma_k, 0, \ldots, 0)\,,$$

$$\sigma_{Zt} = \frac{b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2(T-t)/L} D_t \prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}\sigma_{It},$$

$$\sigma_{Yt} = \frac{b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}\sigma_{It},$$

and σ_{It} is the volatility vector of $D_t \prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L} = E_t\left[I_T\right]$ given by

$$\sigma_{It} = \left(\sigma,\, -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L,\, 0, \ldots, 0\right).$$

We note that $Y_t\sigma_{Yt} = Z_t\sigma_{Zt}e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)}$. Hence, we have

$$\begin{aligned} Z_t\sigma_{Zt}Y_t - Y_t\sigma_{Yt}Z_t &= Z_t\sigma_{Zt}\left(Y_t - e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)}Z_t\right) \\ &= Z_t\sigma_{Zt}\left(1 - e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)}Z_t\right)\left(a + b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}\right), \end{aligned} \tag{A20}$$

where the second equality follows by substituting Z_t and Y_t and manipulating terms. Substituting (A20) into the expression $\sigma_{Zt} - \sigma_{Yt} = \left(Z_t\sigma_{Zt}Y_t - Y_t\sigma_{Yt}Z_t\right)/Y_tZ_t$ and then into (A19) leads to the equilibrium volatility vector of loadings of index commodity futures in (17), where

$$h_{kt} = \frac{b\,\lambda\, e^{\sigma_k^2(T-t)/L}\left(1 - e^{-\left(\sigma^2+\sigma_k^2/L\right)(T-t)}\right)\left(a + b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}\right)}{a + b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{\sigma_k^2(T-t)/L}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}$$

$$\times \frac{D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}} > 0, \quad \text{(A21)}$$

with $g_i\left(t\right)$ as in (16).

To determine the volatility vector of loadings of nonindex futures $k = L+1,\ldots,K$, as reported in (18), we follow the same steps as above for index futures and obtain the stochastic process h_t as

$$h_t = \frac{b\,\lambda\left(1 - e^{-\sigma^2(T-t)}\right)\left(a + b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L}\right)}{a + b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}$$

$$\times \frac{D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0\prod_{i=1}^{L}\left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}} > 0, \quad \text{(A22)}$$

where $g_i\left(t\right)$ is as in (16).

The property that the volatilities of all futures prices are higher than in the benchmark economy follows immediately from (17) and (18). To prove property (ii), we note that for commodities i and k with $D_{it} = D_{kt}$, $\alpha_i = \alpha_k$, we have $h_{kt} > h_t$ from (A21) and (A22), and hence the volatility increase for index futures is higher than that for otherwise identical nonindex futures.

Q.E.D.

Proposition A1 (Stock Market Level and Volatility): In the economy with institutions, the equilibrium stock market level and volatility vector are given by

$$S_t = \overline{S}_t \frac{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, e^{-\sigma^2(T-t)} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}, \tag{A23}$$

$$\sigma_{St} = \overline{\sigma}_S + h_{St}\sigma_{It}, \quad h_{St} > 0, \tag{A24}$$

where $\overline{S}_t$ and $\overline{\sigma}_S$ are the corresponding quantities in the benchmark economy with no institutions, given by

$$\overline{S}_t = \frac{\sum_{k=0}^{K} \alpha_k}{\alpha_0} e^{(\mu-\sigma^2)(T-t)} D_t, \quad \overline{\sigma}_S = \sigma, \tag{A25}$$

h_{St} is a strictly positive stochastic process given by (A22), and σ_{It} is as in Proposition 2. Consequently, in equilibrium, the stock market level and its volatility $\|\sigma_{St}\|$ are higher in the presence of institutions.

Proof of Proposition A1: By no arbitrage, the stock market level is given by

$$S_t = \frac{E_t\left[\xi_T D_T\right]}{\xi_t}. \tag{A26}$$

To compute the expected deflated stock market payoff, we substitute (A8) and (12) to obtain

$$E_t\left[\xi_T D_T\right] = \bar{\xi} \sum_{k=0}^{K} \frac{\alpha_k}{\alpha_0} \left(a + b(1-\lambda)D_0 \prod_{i=1}^{L} \left(g_i(0)/D_{i0}\right)^{1/L} + b\,\lambda\, D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L} \right), \tag{A27}$$

where we have used the fact that D_T, D_{iT}, $i = 1, \ldots, L$ are lognormally distributed, $\bar{\xi}$ is as in (A12) and $g_i(t)$ is as in (16). Substituting (A27) and (A14) into (A26) and manipulating leads to the reported equilibrium stock market level in (A23). The equilibrium stock market level $\bar{S}_t$ in the benchmark economy (A25) follows by considering the special case of $a = 1$, $b = 0$ in (13).

To derive the stock market volatility vector (A24), we follow the same steps for the index futures in the proof of Proposition 2 and obtain the stochastic process h_{St} as in (A22). The property that the stock market level and its volatility are higher than those in the benchmark follow straightforwardly from the expressions (A23) to (A25).

Q.E.D.

Proof of Proposition 3: Maximizing the consumer's objective function leads to optimal commodity demands

$$C_{0s} = \frac{\alpha_0}{\sum_{j=0}^{K} \alpha_j} W_{Cs}, \; C_{ks} = \frac{\alpha_k}{\sum_{j=0}^{K} \alpha_j} \frac{W_{Cs}}{p_{ks}}, \quad s = t, \, t+1, \tag{A28}$$

implying

$$p_{ks} = \frac{\alpha_k}{\alpha_0} \frac{C_{0s}}{C_{ks}}, \quad s = t, \, t+1. \tag{A29}$$

Substituting the market clearing conditions (22) and (23) for each good into (A29) leads to the equilibrium storable and non storable commodity prices reported in (26) to (28). Substituting the storable commodity equilibrium prices (26) and (27) into (21) leads to the equilibrium inventories satisfying (24). The stochastic discount factor in (24) is given by $M_{t,t+1} = \xi_{t+1}/\xi_t$ and is determined by substituting the equilibrium state price density ξ_t in (A14), leading to the expression in (25), where

$$A = a + b\,(1-\lambda)\,D_0 \prod_{i=1}^{L} \left(g_i\,(0)\,/D_{i0}\right)^{1/L}. \tag{A30}$$

To prove the stated properties, we first note that the equilibrium inventories in the benchmark economy with no institutions $\overline{X}_t$ satisfy

$$\begin{aligned}
\frac{D_t}{D_{xt}-\overline{X}_t} &= (1-\delta)E_t\left[\frac{\overline{\xi}_{t+1}}{\overline{\xi}_t}\frac{D_{t+1}}{D_{xt+1}+(1-\delta)\overline{X}_t}\right] \\
&= (1-\delta)E_t\left[\frac{\overline{\xi}_{t+1}}{\overline{\xi}_t}D_{t+1}\right]E_t\left[\frac{1}{D_{xt+1}+(1-\delta)\overline{X}_t}\right] \\
&< (1-\delta)E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right]E_t\left[\frac{1}{D_{xt+1}+(1-\delta)\overline{X}_t}\right],
\end{aligned} \tag{A31}$$

where the second equality uses the independence of D_{t+1} and D_{xt+1}, $\overline{\xi}_t$ is given by (A14) with $b=0$, and the inequality follows from the stock market implication in Proposition A1 and the law of iterated expectations applied to (A31). On the other hand, the equilibrium inventories with institutions satisfy

$$\frac{D_t}{D_{xt}-X_t} = (1-\delta)E_t\left[\frac{\xi_{t+1}}{\xi_t}\frac{D_{t+1}}{D_{xt+1}+(1-\delta)X_t}\right] \tag{A32}$$

$$\begin{aligned}
&= (1-\delta)E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right]E_t\left[\frac{1}{D_{xt+1}+(1-\delta)X_t}\right] \\
&\quad + \frac{1-\delta}{\xi_t}\mathrm{Cov}_t\left(\xi_{t+1}D_{t+1}, \frac{1}{D_{xt+1}+(1-\delta)X_t}\right).
\end{aligned} \tag{A33}$$

Substituting ξ_t from (A14) and using the independence of D_t, D_{xt}, D_{it}, we see that both arguments of the covariance term are decreasing in D_{xt+1}, implying that the covariance term is positive if commodity x is in the index and is zero if commodity x is not in the index, and hence

$$\frac{D_t}{D_{xt} - X_t} \geq (1-\delta) E_t \left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} \right] E_t \left[\frac{1}{D_{xt+1} + (1-\delta) X_t} \right]. \tag{A34}$$

The inequalities (A31) and (A34) imply the inventory property result (ii), which then implies the commodity price property (i). The last property (iii) follows from the fact that the weak inequality in (A34) holds with a strict inequality for an index commodity x and holds with equality for a nonindex commodity. *Q.E.D.*

Proof of Proposition 4: We first prove the properties in (i). Rearranging (A32) gives

$$\frac{D_t}{(1-\delta)} = E_t \left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{D_{xt} - X_t}{D_{xt+1} + (1-\delta) X_t} \right]. \tag{A35}$$

Taking the derivative with respect to the supply of index commodity D_{it} of both sides of (A35) yields

$$0 = E_t \left[\frac{\partial}{\partial D_{it}} \left(\frac{\xi_{t+1}}{\xi_t} D_{t+1} \right) \frac{D_{xt} - X_t}{D_{xt+1} + (1-\delta) X_t} + \frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{\partial}{\partial D_{it}} \left(\frac{D_{xt} - X_t}{D_{xt+1} + (1-\delta) X_t} \right) \right]. \tag{A36}$$

Substituting ξ_t from (A14), we have

$$\frac{\xi_{t+1}}{\xi_t} D_{t+1} = D_t e^{(\mu - \sigma^2)} \frac{A + b\lambda D_{t+1} \prod_{i=1}^{L} \left(g_i\left(t+1\right) / D_{it+1} \right)^{1/L}}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i\left(t\right) / D_{it} \right)^{1/L}}, \tag{A37}$$

where $g_i(t)$ is as in (16) and A as in (A30). Hence, we have

$$\frac{\partial}{\partial D_{it}} \frac{\xi_{t+1}}{\xi_t} D_{t+1} = -\frac{A}{LD_{it}} \frac{1}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} \left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} - D_t e^{\left(\mu-\sigma^2\right)}\right], \quad \text{(A38)}$$

and

$$E_t\left[\frac{A + b\lambda D_{t+1} \prod_{i=1}^{L} \left(g_i(t+1)/D_{it+1}\right)^{1/L}}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}\right] = \frac{A + b\lambda D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} > 1, \quad \text{(A39)}$$

implying from (A37) that

$$E_t\left[\frac{\xi_{t+1}}{\xi_t} D_{t+1}\right] > D_t e^{\left(\mu-\sigma^2\right)}. \quad \text{(A40)}$$

Substituting (A38) into the first expectation of (A36) gives

$$E_t\left[\frac{\partial}{\partial D_{it}}\left(\frac{\xi_{t+1}}{\xi_t} D_{t+1}\right) \frac{D_{xt} - X_t}{D_{xt+1} + (1-\delta) X_t}\right]$$

$$= -\frac{A}{LD_{it}} \frac{D_{xt} - X_t}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}} E_t\left[\left(\frac{\xi_{t+1}}{\xi_t} D_{t+1} - D_t e^{\left(\mu-\sigma^2\right)}\right) \frac{1}{D_{xt+1} + (1-\delta) X_t}\right]$$

$$= -\frac{A}{LD_{it}} \frac{D_{xt} - X_t}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i(t)/D_{it}\right)^{1/L}}$$

$$\times\left(E_t\left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} - D_t e^{\left(\mu-\sigma^2\right)}\right] E_t\left[\frac{1}{D_{xt+1} + (1-\delta) X_t}\right] + \mathrm{Cov}_t\left[\frac{\xi_{t+1}}{\xi_t} D_{t+1}, \frac{1}{D_{xt+1} + (1-\delta) X_t}\right]\right) < 0,$$

where the inequality follows from (A40) and the fact that the covariance term is positive as in the proof of Proposition 3 (see equation (A33)). Since the first expectation of (A36) is negative, the second expectation term must be positive, that is,

$$0 < E_t \left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{\partial}{\partial D_{it}} \left(\frac{D_{xt} - X_t}{D_{xt+1} + (1-\delta) X_t} \right) \right]$$

$$= -E_t \left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{D_{xt+1} + (1-\delta) D_{xt}}{\left(D_{xt+1} + (1-\delta) X_t\right)^2} \right] \frac{\partial X_t}{\partial D_{it}},$$

implying $\partial X_t / \partial D_{it} < 0$. Therefore, from (26), we deduce that

$$\frac{\partial p_{xt}}{\partial D_{it}} = \frac{\alpha_x}{\alpha_0} \frac{D_t}{\left(D_{xt} - X_t\right)^2} \frac{\partial X_t}{\partial D_{it}} < 0.$$

To prove the spillover property from the demand of the index commodity, we take the derivative of both sides of (A35) with respect to α_i:

$$0 = E_t \left[\frac{\partial}{\partial \alpha_i} \left(\frac{\xi_{t+1}}{\xi_t} D_{t+1} \right) \frac{D_{xt} - X_t}{D_{xt+1} + (1-\delta) X_t} + \frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{\partial}{\partial \alpha_i} \left(\frac{D_{xt} - X_t}{D_{xt+1} + (1-\delta) X_t} \right) \right]. \quad \text{(A41)}$$

Taking the derivative of (A37), substituting $\partial g_i(t) / \partial \alpha_i = g_i(t) / \alpha_i$, $\partial A / \partial \alpha_i = (A - a) / L\alpha_i$, and manipulating, we obtain

$$\frac{\partial}{\partial \alpha_i} \frac{\xi_{t+1}}{\xi_t} D_{t+1} = \frac{a}{\alpha_i L} \frac{1}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i(t) / D_{it}\right)^{1/L}} \left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} - D_t e^{\left(\mu - \sigma^2\right)} \right]. \quad \text{(A42)}$$

Substituting (A42) into the first expectation of (A41) gives

$$E_t\left[\frac{\partial}{\partial\alpha_i}\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}\right)\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)X_t}\right]$$

$$=\frac{a}{\alpha_i L}\frac{D_{xt}-X_t}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}D_{t+1}-D_t e^{\left(\mu-\sigma^2\right)}\right)\frac{1}{D_{xt+1}+(1-\delta)X_t}\right]$$

$$=\frac{a}{\alpha_i L}\frac{D_{xt}-X_t}{A+b\lambda e^{-\sigma^2}D_t\prod_{i=1}^{L}\left(g_i(t)/D_{it}\right)^{1/L}}$$

$$\times\left(E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}-D_t e^{\left(\mu-\sigma^2\right)}\right]E_t\left[\frac{1}{D_{xt+1}+(1-\delta)X_t}\right]+\mathrm{Cov}_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1},\frac{1}{D_{xt+1}+(1-\delta)X_t}\right]\right)>0,$$

where the inequality follows from (A40) and the covariance term is positive as in equation (A33) of the proof of Proposition 3. Since the first expectation of (A41) is positive, the second expectation term must be negative, that is,

$$0>E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{\partial}{\partial\alpha_i}\left(\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta)X_t}\right)\right]$$

$$=-E_t\left[\frac{\xi_{t+1}}{\xi_t}D_{t+1}\frac{D_{xt+1}+(1-\delta)D_{xt}}{\left(D_{xt+1}+(1-\delta)X_t\right)^2}\right]\frac{\partial X_t}{\partial\alpha_i},$$

implying $\partial X_t/\partial\alpha_i>0$. Therefore, from (26), we deduce that

$$\frac{\partial p_{xt}}{\partial\alpha_i}=\frac{\alpha_x}{\alpha_0}\frac{D_t}{\left(D_{xt}-X_t\right)^2}\frac{\partial X_t}{\partial\alpha_i}>0.$$

To demonstrate the spillover from the mean growth μ_i and volatility σ_i of the index commodity supply, we note that the equilibrium inventories X_t must satisfy (A32), which is driven by the state price density ξ_t. The equilibrium ξ_t as given in (A14) is itself driven by both μ_i

and σ_i, leading to the stated dependence.

To prove property (ii), we again note that the equilibrium inventories X_t must satisfy (A32). Consider a decrease in D_{it} while keeping all other state variables and the inventory X_{kt} fixed. If commodity i is not in the index, ξ_s, $s = t, t+1$, and D_{kt} are independent of D_{it} (see (1) and (2) and (A14)), and D_{it} does not directly enter (A33). Hence, X_{kt} is independent of D_{it}. From (26), the price p_{kt} is also unchanged. The same goes for α_i. Furthermore, in the benchmark economy with no institutions, the state price density $\bar{\xi}_t$ is given by (A14) with $b = 0$, which is not dependent on D_{it} or α_i. Hence, in the benchmark economy the storable good's inventory and price do not depend on D_{it} or α_i.

To prove property (iii), the spillover from an inflow of institutions, we take the partial derivative of both sides of (A35) with respect to λ:

$$0 = E_t \left[\frac{\partial}{\partial \lambda} \left(\frac{\xi_{t+1}}{\xi_t} D_{t+1} \right) \frac{D_{xt} - X_t}{D_{xt+1} + (1-\delta) X_t} + \frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{\partial}{\partial \lambda} \left(\frac{D_{xt} - X_t}{D_{xt+1} + (1-\delta) X_t} \right) \right]. \tag{A43}$$

Taking the derivative of (A37), we obtain

$$\begin{aligned} \frac{\partial}{\partial \lambda} \frac{\xi_{t+1}}{\xi_t} D_{t+1} = {} & D_t e^{\left(\mu - \sigma^2\right)} \frac{\partial A/\partial \lambda + b D_{t+1} \prod_{i=1}^{L} \left(g_i\left(t+1\right)/D_{it+1}\right)^{1/L}}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i\left(t\right)/D_{it}\right)^{1/L}} \\ & - \frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{\partial A/\partial \lambda + b e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i\left(t\right)/D_{it}\right)^{1/L}}{A + b\lambda e^{-\sigma^2} D_t \prod_{i=1}^{L} \left(g_i\left(t\right)/D_{it}\right)^{1/L}}, \end{aligned} \tag{A44}$$

where $\partial A/\partial \lambda = -b D_0 \prod_{i=1}^{L} \left(g_i\left(0\right)/D_{i0}\right)^{1/L}$. Substituting (A44) into the first expectation of

(A43) yields

$$
\begin{aligned}
& E_t\left[\frac{\partial}{\partial \lambda}\left(\frac{\xi_{t+1}}{\xi_t} D_{t+1}\right) \frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta) X_t}\right] \\
= & E_t\left[D_t e^{\left(\mu-\sigma^2\right)} \frac{\partial A / \partial \lambda+b D_{t+1} \prod_{i=1}^L\left(g_i(t+1) / D_{it+1}\right)^{1 / L}}{A+b \lambda e^{-\sigma^2} D_t \prod_{i=1}^L\left(g_i(t) / D_{it}\right)^{1 / L}} \frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta) X_t}\right] \\
- & E_t\left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{\partial A / \partial \lambda+b e^{-\sigma^2} D_t \prod_{i=1}^L\left(g_i(t) / D_{it}\right)^{1 / L}}{A+b \lambda e^{-\sigma^2} D_t \prod_{i=1}^L\left(g_i(t) / D_{it}\right)^{1 / L}} \frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta) X_t}\right] .
\end{aligned}
$$

Following similar steps as those in the proof of property (i), after some algebra we deduce that the above expression is positive. Since the first expectation of (A43) is positive, the second expectation term must be negative, that is,

$$
\begin{aligned}
0 & > E_t\left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{\partial}{\partial \lambda}\left(\frac{D_{xt}-X_t}{D_{xt+1}+(1-\delta) X_t}\right)\right] \\
& =-E_t\left[\frac{\xi_{t+1}}{\xi_t} D_{t+1} \frac{D_{xt+1}+(1-\delta) D_{xt}}{\left(D_{xt+1}+(1-\delta) X_t\right)^2}\right] \frac{\partial X_t}{\partial \lambda},
\end{aligned}
$$

implying $\partial X_t / \partial \lambda > 0$. Hence, from (26) we obtain

$$
\frac{\partial p_{xt}}{\partial \lambda}=\frac{\alpha_x}{\alpha_0} \frac{D_t}{\left(D_{xt}-X_t\right)^2} \frac{\partial X_t}{\partial \lambda}>0 .
$$

Q.E.D.