S Basak and A Pavlova
A model of financialization of commodities
Article



Basak, S and Pavlova, A

(2016)

*A model of financialization of commodities.*

Journal of Finance, 71 (4). pp. 1511-1556. ISSN 0022-1082

DOI: https://doi.org/10.1111/jofi.12408

Wiley
http://onlinelibrary.wiley.com/doi/10.1111/jofi.12...

# Internet Appendix for "A Model of Financialization of Commodities"

SULEYMAN BASAK and ANNA PAVLOVA*

## I. Economy with Demand Shocks

In this appendix we introduce commodity demand shocks to our baseline model. While our setup with supply-side-only uncertainty is capable of delivering our main insights, we need a richer model to explore the quantitative effects of financialization. It has been argued in the literature that demand shocks are very important in explaining the behavior of oil prices and the prices of other commodities (see Fattouh, Kilian, and Mahadeva (2013) for a survey). For example, Kilian and Murphy (2014) argue that the 2004 to 2008 surge in oil prices can be attributed to demand shocks. Furthermore, commentators frequently attribute commodity price rises to increased demand from China, arguing that as its industry grows and population becomes wealthier its consumption basket becomes more commodity-intensive. They also link

*Citation format: Basak, Suleyman, and Anna Pavlova, Internet Appendix for "A Model of Financialization of Commodities", *Journal of Finance* [DOI STRING]. Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

the increase in cross-commodity correlations to China, whose high growth has led to a simultaneous increase in demand for a number of key commodities. Here, we consider correlated demand shocks and show that such shocks alone can generate a sizeable increase in futures prices and their comovement. Importantly, demand shocks also magnify the effects of financialization. Using our model we can disentangle how much of an increase in futures prices and their comovement can be attributed to positive demand shocks alone and how much to financialization.

To incorporate demand shocks, we make the following modification to our model. In consumption index (8) of investors, repeated here for expositional clarity,

$$W_n = C_{n_0}^{\alpha_0} C_{n_1}^{\alpha_1} \cdot \ldots \cdot C_{n_K}^{\alpha_K}, \qquad n \in \{\mathcal{N}, \mathcal{I}\}, \tag{IA1}$$

we allow the two demand parameters $\alpha_1$ and $\alpha_2$ to be stochastic. Shocks to $\alpha_1$ and $\alpha_2$ then represent shifts in demand for goods 1 and 2 in the commodity index; we hereafter refer to them as demand shocks. We do not consider shocks to demand for other goods, but our model can be extended to incorporate such shocks. We assume that $\alpha_1$ and $\alpha_2$ are strictly positive processes with dynamics

$$d\alpha_{1t} = \alpha_{1t}\sigma_{\alpha_1} dw_{0t}, \; d\alpha_{2t} = \alpha_{2t}\sigma_{\alpha_2} dw_{0t}, \tag{IA2}$$

where $\sigma_{\alpha_1}, \sigma_{\alpha_2} > 0$ are constants. Implicit in this assumption is that $\alpha_1$ and $\alpha_2$ are driven by the same source of risk. This source of risk, Brownian motion $w_0$, is the one driving aggregate output $D$, that is, $\alpha_1$ and $\alpha_2$ have one-to-one mappings with aggregate output.[1] Now an investor's time-$T$ demand for good 1 is not simply a (decreasing) function of its price $p_{1T}$, but also an (increasing) function of aggregate output $D_T$ (through $\alpha_{1T}$). (The demand for good 2 is determined analogously). The latter assumption has been advocated by Dvir and Rogoff (2009) in their model of oil prices. In the numerical illustration that follows, we associate commodities 1 and 2 with energy and refer to them as energy 1 and energy 2. We include futures on both energy 1 and energy 2 in the index, consistent with energy futures being included in all popular commodity indices.

Proposition IA1 reports the equilibrium futures prices and their return volatilities in the economy with demand shocks in closed form.

Proposition IA1 (Futures Prices and Volatilities with Demand Shocks): *In the economy with institutions and demand shocks, the equilibrium futures price of commodity $k = 1, \ldots, K$ and its associated volatility vector of loadings are given by*

$$f_{kt} = \overline{f}_{kt} \frac{B + b\,\lambda e^{\left(1_{\{k \le L\}}\sigma_k^2/L + 1_{\{k \le 2\}}(L\sigma + \sigma_{\alpha 1} + \sigma_{\alpha 2})\sigma_{\alpha k}/L\right)(T-t)} \alpha_{1t}^{1/L} \alpha_{2t}^{1/L} D_t \prod_{i=1}^{L} \left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}}{B + b\lambda e^{-(\sigma^2 + (\sigma\sigma_{\alpha 1} + \sigma\sigma_{\alpha 2})/L)(T-t)} \alpha_{1t}^{1/L} \alpha_{2t}^{1/L} D_t \prod_{i=1}^{L} \left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}}, \tag{IA3}$$

$$\sigma_{f_k t} = \overline{\sigma}_{f_k} + \hat{h}_{kt}\sigma_{It}, \quad \hat{h}_{kt} > 0, \tag{IA4}$$

*where* $B, \hat{g}_i(t) > 0$, $\overline{f}_{kt}$ *is the equilibrium futures price in the benchmark economy with no institutions,* $\overline{\sigma}_{f_k}$ *is its corresponding volatility vector,* $\sigma_{It}$ *is the volatility vector of the conditional expected index* $E_t[I_T]$*, given by*

$$\overline{f}_{kt} = \frac{\left(\alpha_k 1_{\{k>2\}} + \alpha_{kt} 1_{\{k\leq 2\}}\right) e^{(\mu-\mu_k-\sigma^2+\sigma_k^2)(T-t)}}{\alpha_0} \frac{D_t}{D_{kt}}, \tag{IA5}$$

$$\overline{\sigma}_{f_k} = (\sigma + \sigma_{\alpha 1} 1_{\{k=1\}} + \sigma_{\alpha 2} 1_{\{k=2\}}, 0, \ldots, -\sigma_k, 0, \ldots, 0), \tag{IA6}$$

$$\sigma_{It} = (\sigma + \tfrac{1}{L}(\sigma_{\alpha 1} + \sigma_{\alpha 2}), -\tfrac{1}{L}\sigma_1, \ldots, -\tfrac{1}{L}\sigma_L, 0, \ldots, 0), \tag{IA7}$$

*and the constant* $B$*, the deterministic quantity* $\hat{g}_i(t)$*, and the stochastic process* $\hat{h}_{kt}$ *are explicitly provided in the proof located at the end of this appendix.*

*Consequently, in equilibrium, all futures prices and their volatilities* $\|\sigma_{f_k}\|$ *are higher than in the benchmark economy.*

Proposition IA1 confirms our earlier result that all futures prices are higher in the presence of institutions, with prices of index futures exceeding those of nonindex futures. The distinguishing feature of our economy with demand shocks is that these effects become stronger than in the economy without demand shocks and the cross-commodity futures return correlations increase sizably, reaching the levels documented in the post-financialization period in the data (Tang and Xiong (2012)). Below we identify plausible parameter values to assess the quantitative importance of our results.

Table IAI summarizes the parameter values for our numerical illustration. Since we have taken commodities 1 and 2 to represent energy, we infer the demand parameters $\alpha_{1t}$ and $\alpha_{2t}$ from the energy expenditure share in total consumption. The total expenditure share in our model is given by $(\alpha_{1t} + \alpha_{2t})/(\sum_{k=0,k\neq 1,2}^{K} \alpha_k + \alpha_{1t} + \alpha_{2t})$. For convenience, we set our baseline parameters such that $\sum_{k=0,k\neq 1,2}^{K} \alpha_k + \alpha_{1t} + \alpha_{2t} = 1$ at time $t$. We obtain data on the energy expenditure share in the U.S. from Bureau of Economic Analysis Table 2.3.5U from 1959:M1 through 2012:M12.[2] The average expenditure share in the sample is about 6%, and so we set $\alpha_{1t} = \alpha_{2t} = 0.03$. As also noted by Hamilton (2013), the energy expenditure share series is very volatile. We set $\sigma_{\alpha_1} = \sigma_{\alpha_2} = 9.8\%$ to match the volatility estimate obtained from the series. Finally, the series does not have a deterministic trend, and we cannot reject the null that it has a unit root, which supports our specification in (IA2). The expenditure share of the generic good is taken to be 70%, and the remaining expenditure is spread equally across the remaining commodities (other than energy). We set the volatility of the process for the generic good's supply news $D_t$ to be consistent with the stock market volatility expression (IA25) in Section III, Supplement to Table IAI, below. Using the value of 16% for aggregate U.S. stock market volatility in the data,[3] the parameter value for the volatility of the generic good's supply news, $\sigma$, is then around 15%, which is consistent with the aggregate dividend volatility in the data. The model-implied volatility parameters of the processes for $D_1$, $D_2$, and $D_k$, $k = 3, \ldots, K$ are obtained from equation (IA7) using data on the average volatilities of the energy-sector and

**Table IAI. Parameter Values and State Variables**

| Parameter or State Variable | Symbol | Value |
|---|---|---|
| Mean growth rate of generic good's supply news | $\mu$ | 0.05 |
| Volatility of generic good's supply news | $\sigma$ | 0.15 |
| Mean growth rate of commodity $k$ supply news, $k = 1, \ldots, K$ | $\mu_k$ | 0.05 |
| Volatility of commodities 1 and 2 (energy) supply news | $\sigma_1, \sigma_2$ | 0.29 |
| Volatility of commodity $k \neq 1, 2$ (non-energy) supply news | $\sigma_k$ | 0.24 |
| Volatility of commodities 1 and 2 demand shocks | $\sigma_{\alpha_1}, \sigma_{\alpha_2}$ | 0.098 |
| Demand parameter, generic good | $\alpha_0$ | 0.7 |
| Time-0 and time-$t$ demand parameter for energy | $\alpha_{10}$, $\alpha_{20}$, $\alpha_{1t}$, $\alpha_{2t}$ | 0.03 |
| Demand parameter, commodity $k = 1, \ldots K$ | $\alpha_k$ | 0.08 |
| Number of commodities | $K$ | 5 |
| Number of commodities in the index | $L$ | 2 |
| Terminal date | $T$ | 5 years |
| Current date | $t$ | 0.1 years |
| Size of institutions | $\lambda$ | 0.2 |
| Objective function parameters | $a$, $b$ | 1 |
| Time-0 and time-$t$ supply of generic good | $D_0$, $D_t$ | 100 |
| Time-0 and time-$t$ supply of commodity $k$, $k = 1, \ldots, K$ | $D_{k0}$, $D_{kt}$ | 1 |

non-energy sector futures from Gorton, Hayashi, and Rouwenhorst (2013). We set the mean growth rates to $\mu = \mu_k = 0.05$ for all $k$; our results do not vary much for a wide range of alternative values for these parameters.

The horizon $T$ is set in line with the typical performance evaluation horizons of fund managers, usually three to five years (BIS (2003)). We interpret our parameter $\lambda$ as the fraction of investors in the commodity futures market that are benchmarked to a commodity index. The closest measure for this in the data is the fraction of commodity index traders in this market. Using proprietary data, Aulerich, Irwin, and Garcia (2010) and Cheng, Kirilenko, and Xiong (2015) provide suggestive evidence of what the value of $\lambda$ could be, based on the percent of total open interest held by commodity index traders (net long positions). The magnitudes vary across commodities, averaging around 0.32 post-2004 (Aulerich et al.'s data). Since this measure is just a rough proxy, we set $\lambda = 0.2$ and report our results for a range of values around $\lambda = 0.2$. Another reason to be conservative in our choice of $\lambda$ is that our model is general equilibrium, and hence our institutional investors could be interpreted as the percentage of commodity-oriented institutional investors in *all* capital markets, suggesting that $\lambda$ should be smaller.[4]

Figure IA1 illustrates the results of Proposition IA1 and disentangles the contribution of financialization over and above that of fundamentals (demand and supply). We vary the demand parameter $\alpha_{1t}$ for energy commodity 1 to highlight the contribution of increasing/decreasing demand. As one can see from the figure, increasing demand for energy pushes up its futures price even in the benchmark economy with no institutions (the dashed lines).[5] But in the presence of institutions, the futures price increases even more, especially in the presence of demand shocks (solid blue line). This is because there is now an additional risk in the economy—shifts in demand for energy—that affects the value of the index. Therefore, assets whose payoffs are positively correlated with these demand shocks—the energy futures—become even more valuable than in the economy without demand shocks. Proposition IA1 also uncovers that demand shocks to energy spill over into other futures prices. The mechanism for these spillovers echoes that in Section II of the main article– the spillovers to other futures occur because demand shocks affect the value of the index.

The effects above are quantitatively important. As revealed by Table IAII, for our baseline

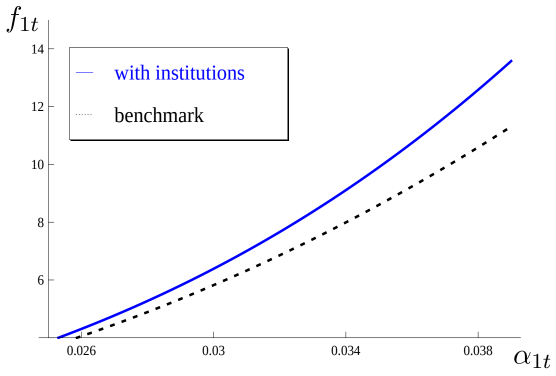



**Figure IA1. Futures prices.** This figure plots index futures 1's price in the economy with demand shocks (solid blue line) against the demand parameter $\alpha_{1t}$ for energy commodity 1. The dotted black line is for the corresponding prices in the benchmark economy with no institutions. The parameter values are as in Table IAI.

parameterization, we find that the increase in energy futures price with financialization is 9.71%. This result is sensitive to the parameter $\lambda$, with our estimates for the price increase ranging from 4.71% for $\lambda = 0.1$ to 14.80% for $\lambda = 0.3$. We also perform sensitivity analysis around our parameter values for energy supply news volatility and report the resulting values in Table IAII. Our results are not out of line with the findings of Kilian and Murphy (2014) that fluctuations in fundamentals are important in explaining the fluctuations in commodity prices, but we also stress a significant contribution of financialization. However, we should add the caveat that our model abstracts from several important influences on commodity futures prices and so our quantitative results should be taken with caution.

Table IAII also highlights that the magnitudes of the impact of financialization on futures prices are rather sensitive to the volatility of supply news; the more volatile the energy commodity supply news is, the greater the increase in commodity futures prices that is attributable

**Table IAII. Increase in Energy Futures Prices with Financialization,** $(f_{1t} - \overline{f}_{1t})/\overline{f}_{1t}$**.** This table reports the increases in energy futures prices that are attributable to financialization. The parameter values are as in Table IAI, unless specified otherwise. The figure in bold is for our baseline parameter values (in Table IAI).

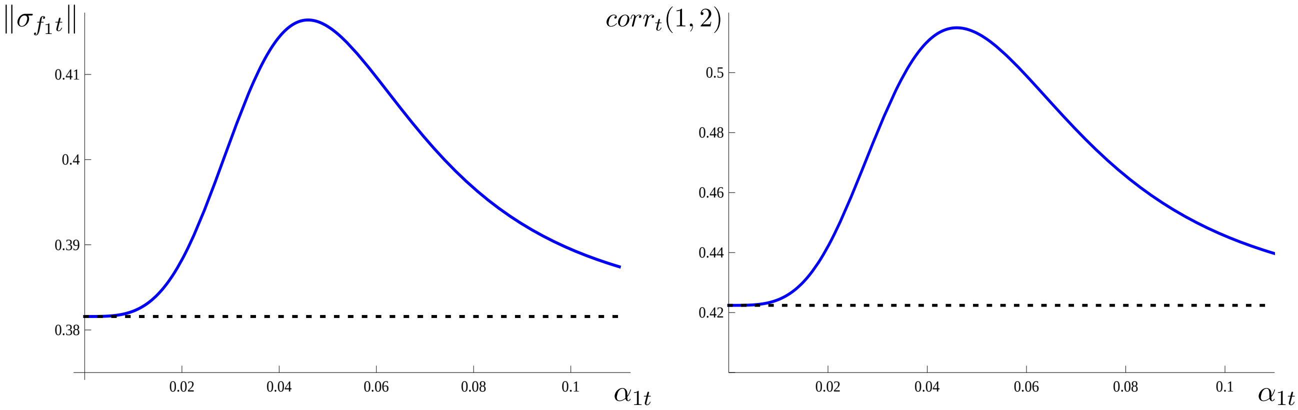
| | | $\lambda = 0.1$, $\alpha_{1t} = 0.03$ | $\lambda = 0.2$, $\alpha_{1t} = 0.03$ | $\lambda = 0.3$, $\alpha_{1t} = 0.03$ | $\lambda = 0.1$, $\alpha_{1t} = 0.04$ | $\lambda = 0.2$, $\alpha_{1t} = 0.04$ | $\lambda = 0.3$, $\alpha_{1t} = 0.04$ |
|---|---|---|---|---|---|---|---|
| | 0.24 | 4.00% | 8.13% | 12.38% | 9.81% | 17.60% | 24.56% |
| $\sigma_1$ | 0.29 | 4.78% | **9.71%** | 14.80% | 11.71% | 21.31% | 29.31% |
| | 0.34 | 5.79% | 11.76% | 17.91% | 14.16% | 25.74% | 35.40% |

to financialization. In unreported analysis, we find that the effects of financialization are also stronger the larger the aggregate output news volatility $\sigma$ and the larger the demand uncertainly $\sigma_{\alpha_i}$. These comparative statics may shed light on the time-variation in futures prices. During periods of high uncertainty the effects of financialization are amplified, pushing prices much higher than what could have been justified by fundamentals (supply and demand) alone.

Kilian and Murthy further argue that most of the 2003 to 2008 increase in energy prices (specifically, oil prices) was due to global demand shocks. Our model delivers this result, but also uncovers an important interaction: the effects of financialization become stronger with higher global demand. To illustrate this implication, we explore the effects of an upward demand shift for energy from the baseline value of $\alpha_{1t} = 0.03$ to 0.04—a 33% increase. The last three columns of Table IAII present the (recomputed) increases in futures prices that are attributable to financialization. As one can see clearly, financialization becomes significantly more important. For our baseline parameter values, the energy futures price increase attributable to financialization rises from 9.71% to 21.31%.[6]

Proposition IA1 also confirms that our remaining results in Section II of the main article continue to hold in the presence of demand shocks. In particular, futures return volatilities are higher in the presence of institutions. Moreover, we find in our numerical illustration that they become much higher in the presence of demand shocks. To understand the intuition behind this new result, it is useful to note that energy futures are exposed to an additional source of risk, demand shocks, and so become more volatile even in the benchmark economy with no institutions. Additionally, the membership of energy futures in the index makes the index riskier than in the economy without demand shocks. Since falling behind the index is a source of risk for institutional investors, all futures prices depend on the expected index, as we have highlighted before. The (expected) index appears as a new risk factor in the futures prices, and this factor is now more volatile (higher $||\sigma_I||$). Consequently, all futures prices are more volatile as well. In terms of magnitudes, for our baseline parameter values, the volatility of energy futures prices in our model rises from around 0.33 in the economy without demand shocks (Section II, Figure 4) to 0.41 in the economy with demand shocks. Figure IA2, Panel A presents the sensitivity of these magnitudes to the energy demand parameter $\alpha_{1t}$.

The commodity futures return correlations also rise with financialization. As before, this is because the expected index emerges as a common factor affecting all assets in the economy, and hence the covariances of all assets with each other increase more than in the economy without demand shocks. The same ends up being true for the corresponding correlations. To illustrate




Panel A. Volatility of energy futures

Panel B. Energy futures return correlation

**Figure IA2. Volatilities and correlations.** Panel A plots return volatility of energy futures (solid blue line). Panel B plots return correlations of energy futures, $corr_t(1, 2)$ (solid blue line). Both plots are against the energy demand parameter $\alpha_{1t}$. The dotted lines are for the corresponding quantities in the benchmark economy with no institutions. The parameters are from Table IAI.

quantitatively the effects of financialization on the correlations , in Figure IA2, Panel B we plot energy futures return correlations in the economies with and without institutions. Our model with demand shocks delivers large correlation increases, part of which are due to fundamentals (the common demand shock) and part of which are due to the presence of institutions. For example, in the baseline model in the benchmark economy (Section II, Figure 4 of the main article), the correlation between the two index futures returns is 0.21, while in the economy with the common demand shock and institutions it rises to 0.5 (of this increase, the rise from 0.21 to 0.42 is attributed to fundamentals). These magnitudes are roughly consistent with the evidence in Tang and Xiong (2012), who document that the average correlation of index commodities with oil rose from 0.1 pre-2004 to about 0.5 in 2009. Furthermore, the correlation of nonindex commodities in this numerical exercise is only 0.28—much lower than that of index (energy) commodities, which is roughly in line with the data (Tan and Xiong (2012)). This is because in our numerical illustration nonindex commodities are not affected by the common demand shock, which we believe is a reasonable assumption. That could be part of the explanation behind the large difference in the index versus nonindex commodities returns correlations in the data.

# II. Proof of Proposition IA1

We first consider investors' optimal demands in each commodity. Maximizing the institutional investor's expected objective function (7), subject to (A1) evaluated at $t = 0$, leads to the institution's optimal demand for commodity $k = 3, \ldots, K$ and the generic good as in (A2) of Section II, and demand for commodities 1 and 2 as

$$C_{\mathcal{I}_1 T} = \frac{\alpha_{1T}\left(a + bI_T\right)}{y_I p_{1T} \xi_T}, \quad C_{\mathcal{I}_2 T} = \frac{\alpha_{2T}\left(a + bI_T\right)}{y_I p_{2T} \xi_T}. \tag{IA8}$$

Here, $1/y_I$ solves (A1) evaluated at $t = 0$, and using the lognormal distribution property of $D_T, D_{kT}, \alpha_{1T}, \alpha_{2T}$, is given by

$$\frac{1}{y_I} = \frac{\lambda\, \xi_0 S_0}{a\left(\sum_j \alpha_j + \alpha_1 + \alpha_2\right) + b\left(\sum_j \alpha_j E\left[I_T\right] + E\left[\alpha_{1T} I_T\right] + E\left[\alpha_{2T} I_T\right]\right)}, \tag{IA9}$$

where henceforth the summation $\sum_j$ denotes the summation over all commodities but the first two, that is, $j = 0, 3, \ldots, K$, and $\alpha_1 \equiv \alpha_{10}$, $\alpha_2 \equiv \alpha_{20}$ denote the initial values of the processes $\alpha_{1t}$ and $\alpha_{2t}$, respectively. Similarly, we obtain the normal investor's optimal commodity demands at time $T$ for $k = 0, 3, \ldots, K$ as in (A6) and (A7), and for commodities 1 and 2 as

$$C_{\mathcal{N}_1 T} = \frac{\alpha_{1T}\,(1-\lambda)\,\xi_0 S_0}{(\sum_j \alpha_j + \alpha_1 + \alpha_2) p_{1T} \xi_T}, \quad C_{\mathcal{N}_2 T} = \frac{\alpha_{2T}\,(1-\lambda)\,\xi_0 S_0}{(\sum_j \alpha_j + \alpha_1 + \alpha_2) p_{2T} \xi_T}. \tag{IA10}$$

To determine the equilibrium state price density, we impose market-clearing for the generic good, $C_{\mathcal{N}_0 T} + C_{\mathcal{I}_0 T}$, substitute (A5) and (A7), and rearrange to obtain at $T$

$$\xi_T = \bar{\xi}\frac{1}{D_T}\left(B + b\lambda I_T\right), \tag{IA11}$$

where

$$\bar{\xi} = \frac{\alpha_0 \xi_0 S_0}{a\left(\sum_j \alpha_j + \alpha_1 + \alpha_2\right) + b\left(\sum_j \alpha_j E\left[I_T\right] + E\left[\alpha_{1T} I_T\right] + E\left[\alpha_{2T} I_T\right]\right)}, \tag{IA12}$$

$$B = a + b\,(1-\lambda)\,\frac{\sum_j \alpha_j E\left[I_T\right] + E\left[\alpha_{1T} I_T\right] + E\left[\alpha_{2T} I_T\right]}{\sum_j \alpha_j + \alpha_1 + \alpha_2}. \tag{IA13}$$

To determine the equilibrium commodity prices at $T$, we impose the market clearing condition $C_{\mathcal{N}_k T} + C_{\mathcal{I}_k T} = D_{kT}$ for each commodity $k = 1, \ldots, K$, and substitute (A4),(A6), (IA8) and (IA10) and the equilibrium state price density (IA11) to obtain the same commodity prices

(11) as in Lemma 1 for $k = 3, \ldots, K$, and for commodities 1 and 2 we obtain

$$p_{1T} = \frac{\alpha_{1T}}{\alpha_0}\frac{D_T}{D_{1T}}, \quad p_{2T} = \frac{\alpha_{2T}}{\alpha_0}\frac{D_T}{D_{2T}}. \tag{IA14}$$

Substituting the equilibrium commodity prices (11), (IA14), into the definition of the index (4) leads to the time-$T$ equilibrium commodity index value

$$I_T = \frac{\alpha_{1T}^{1/L}\alpha_{2T}^{1/L}}{\alpha_1^{1/L}\alpha_2^{1/L}}\frac{D_T}{\alpha_0}\prod_{i=1}^{L}\left(\alpha_i/D_{iT}\right)^{1/L}. \tag{IA15}$$

Hence, making use of the lognormal property of $D_T$, $D_{iT}$, $\alpha_{1T}$, $\alpha_{2T}$, we deduce the unconditional expected index value to be

$$E\left[I_T\right] = e^{\left(\mu+\frac{1}{L}\left(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2}+\frac{1}{2}\left(\frac{1}{L}-1\right)\left(\sigma_{\alpha 1}^2+\sigma_{\alpha 2}^2\right)+\frac{1}{L}\sigma_{\alpha 1}\sigma_{\alpha 2}\right)-\frac{1}{L}\sum_{i=1}^{L}\left(\mu_i-\frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)\right)T}\frac{D_0}{\alpha_0}\prod_{i=1}^{L}\left(\frac{\alpha_i}{D_{i0}}\right)^{1/L}, \tag{IA16}$$

and the unconditional expectations in(IA12) and (IA13) as

$$E\left[\alpha_{1T}I_T\right] = \alpha_1 e^{\left(\sigma\sigma_{\alpha 1}+\left(\sigma_{\alpha 1}+\sigma_{\alpha 2}\right)\sigma_{\alpha 1}/L\right)T}E\left[I_T\right],$$

$$E\left[\alpha_{2T}I_T\right] = \alpha_2 e^{\left(\sigma\sigma_{\alpha 2}+\left(\sigma_{\alpha 1}+\sigma_{\alpha 2}\right)\sigma_{\alpha 2}/L\right)T}E\left[I_T\right].$$

We now determine the equilibrium futures prices. First, the equilibrium time-$t$ state price density follows by taking the conditional expectation of (IA11), substituting (IA15), and using the lognormality of $D_T$, $D_{iT}$, $\alpha_{1T}$, $\alpha_{2T}$. After some algebra we then get

$$\xi_t = \bar{\xi} e^{\left(\sigma^2-\mu\right)(T-t)} \frac{1}{D_t} \left( B + b\lambda e^{-\left(\sigma^2+(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2})/L\right)(T-t)} \alpha_{1t}^{1/L} \alpha_{2t}^{1/L} D_t \prod_{i=1}^{L} \left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L} \right), \quad \text{(IA17)}$$

where $\bar{\xi}$ and $B$ are as in (IA12) and (IA13), and and

$$\hat{g}_i\left(t\right) = \frac{\alpha_i}{\alpha_0 \alpha_1^{1/L} \alpha_2^{1/L}} e^{\left(\mu+\frac{1}{L}\left(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2}+\frac{1}{2}\left(\frac{1}{L}-1\right)\left(\sigma_{\alpha 1}^2+\sigma_{\alpha 2}^2\right)+\frac{1}{L}\sigma_{\alpha 1}\sigma_{\alpha 2}\right)-\mu_i+\frac{1}{2}\left(\frac{1}{L}+1\right)\sigma_i^2\right)(T-t)}. \quad \text{(IA18)}$$

To compute the expected deflated futures payoff of commodity $k = 3, \ldots, K$, we substitute (IA11) and (11), and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \frac{\bar{\xi}}{\alpha_0} \alpha_k \frac{e^{\left(\sigma_k^2-\mu_k\right)(T-t)}}{D_{kt}} \left( B + b\lambda \frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} \right). \quad \text{(IA19)}$$

For nonindex futures contracts $k = L+1, \ldots, K$, using the lognormality of $D_T$, $D_{iT}$, $\alpha_{1T}$, $\alpha_{2T}$, and substituting (IA15) we obtain

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = \alpha_{1t}^{1/L} \alpha_{2t}^{1/L} D_t \prod_{i=1}^{L} \left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}, \quad \text{(IA20)}$$

where $\hat{g}_i\left(t\right)$ is as in (IA18). For index futures contracts except for the first two commodity futures, $k = 3, \ldots, L$, we get

$$\frac{E_t\left[I_T/D_{kT}\right]}{E_t\left[1/D_{kT}\right]} = e^{\frac{1}{L}\sigma_k^2(T-t)} \alpha_{1t}^{1/L} \alpha_{2t}^{1/L} D_t \prod_{i=1}^{L} \left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}. \quad \text{(IA21)}$$

Finally, for the first two index futures contracts $k = 1, 2$, we substitute (IA11) and (IA14) and rearrange to obtain

$$E_t\left[\xi_T p_{kT}\right] = \frac{\bar{\xi}}{\alpha_0}\alpha_{kt}\frac{e^{\left(\sigma_k^2-\mu_k\right)(T-t)}}{D_{kt}}\left(B + b\lambda\frac{E_t\left[\alpha_{kT}I_T/D_{kT}\right]}{E_t\left[\alpha_{kT}/D_{kT}\right]}\right).$$

Using the lognormality of $D_T$, $D_{iT}$, $\alpha_{1T}$ $\alpha_{2T}$, and substituting (IA15) we then obtain

$$\frac{E_t\left[\alpha_{kT}I_T/D_{kT}\right]}{E_t\left[\alpha_{kT}/D_{kT}\right]} = e^{\left(\frac{1}{L}(L\sigma+\sigma_{\alpha 1}+\sigma_{\alpha 2})\sigma_{\alpha k}+\frac{1}{L}\sigma_k^2\right)(T-t)}\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}. \tag{IA22}$$

Substituting (IA19) to (IA22) and (IA17) into (A10) and rearranging leads to the equilibrium index futures price expression reported in (IA3). The equilibrium futures price $\bar{f}_k$ in the benchmark economy with no institutions (IA5) follows by considering the special case of $a = 1$, $b = 0$ in (IA3). The property that the futures prices are higher than in the benchmark economy follows by observing that the factor multiplying $\bar{f}_{kt}$ in expression (IA3) is strictly greater than one.

To derive the equilibrium volatility vector of loadings, we apply Itô's Lemma to the futures price expression (IA3) and follow similar steps as in the proof of Proposition 2 to deduce (IA4) in Proposition IA1, with

$$\hat{h}_{kt} = \frac{b\,\lambda\,B\left(e^{\left(1_{\{k\leq L\}}\sigma_k^2/L+1_{\{k\leq 2\}}(L\sigma+\sigma_{\alpha 1}+\sigma_{\alpha 2})\sigma_{\alpha k}/L\right)(T-t)}-e^{-\left(\sigma^2+(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2})/L\right)(T-t)}\right)}{B+b\,\lambda e^{\left(1_{\{k\leq L\}}\sigma_k^2/L+1_{\{k\leq 2\}}(L\sigma+\sigma_{\alpha 1}+\sigma_{\alpha 2})\sigma_{\alpha k}/L\right)(T-t)}\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}}$$
$$\times\frac{\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}}{B+b\,\lambda e^{-(\sigma^2+(\sigma\sigma_{\alpha 1}+\sigma\sigma_{\alpha 2})/L)(T-t)}\alpha_{1t}^{1/L}\alpha_{2t}^{1/L}D_t\prod_{i=1}^{L}\left(\hat{g}_i\left(t\right)/D_{it}\right)^{1/L}}>0, \qquad \text{(IA23)}$$

where $B$ and $\hat{g}_i\left(t\right)$ as in (IA13) and (IA18),, respectively. The property that volatilities of all futures price returns are higher than in the benchmark economy follows immediately from (IA4) since $h_{kt} > 0$.

*Q.E.D.*

# III. Supplement to Table IAI: Stock Volatility

To determine the equilibrium stock market volatility for our numerical illustration, we note that in the benchmark economy with no institutions the stock market's terminal value is given by

$$\overline{S}_T = \frac{\sum_j \alpha_j + \alpha_{1T} + \alpha_{2T}}{\alpha_0} D_T,$$

where we have substituted (11) and (IA8). Following similar steps as in the determination of equilibrium futures prices above, we arrive at the following equilibrium stock market level and its associated vector of loadings in the benchmark economy with demand shocks:

$$\bar{S}_t = \frac{\sum_j \alpha_j + \alpha_{1t} + \alpha_{2t}}{\alpha_0} e^{\left(\mu - \sigma^2\right)(T-t)} D_t, \tag{IA24}$$

$$\bar{\sigma}_{St} = \left(\sigma + \frac{\alpha_{1t}\sigma_{\alpha_1} + \alpha_{2t}\sigma_{\alpha_2}}{\sum_j \alpha_j + \alpha_{1t} + \alpha_{2t}}, 0, \ldots, 0\right). \tag{IA25}$$

# REFERENCES

Aulerich, Nicole M., Scott H. Irwin, and Philip Garcia, 2010, The price impact of index funds in commodity futures markets: Evidence from the CFTC's daily large trader reporting system, Working paper, University of Illinois at Urbana-Champaign.

Bank for International Settlements, 2003, Incentive structures in institutional asset management and their implications for financial markets, Committee on the Global Financial System, Bank for International Settlements, Basel.

Cheng, Ing-Haw, Andrei Kirilenko, and Wei Xiong, 2015, Convective risk ows in commodity futures markets, *Review of Finance* 19, 1733-1781.

Dvir, Eyal, and Kenneth S. Rogoff, 2009, Three epochs of oil, NBER Working paper 14927.

Fattouh, Bassam, Lutz Kilian, and Lavan Mahadeva, 2013, The role of speculation in oil markets: What have we learned so far? *Energy Journal* 34, 7-33.

Gorton, Gary B., Fumio Hayashi, and K. Geert Rouwenhorst, 2013, The fundamentals of commodity futures returns, *Review of Finance* 17, 35-105.

Hamilton, James D., 2013, Oil prices, exhaustible resources, and economic growth, *Handbook of Energy and Climate Change*, (Edward Elgar Publishing, Cheltenham).

Kilian, Lutz, and Daniel P. Murphy, 2014, The role of inventories and speculative trading in the global market for crude oil, *Journal of Applied Econometrics* 29, 454-478.

Tang, Ke, and Wei Xiong, 2012, Index investment and the financialization of commodities, *Financial Analysts Journal* 68, 54-74.

Vassilev, Georgi D., 2010, Essays on commodity futures and volatility, Ph.D. Dissertation, University of Southern California.

# Notes

[1]Using the dynamics in (2) and (IA2), one can establish the following mapping between $\alpha_{it}$ and $D_t$: $\alpha_{it} = \alpha_{i0}\, e^{-\sigma^2_{\alpha_i}/2\, t+\sigma_{\alpha_i}/\sigma[\log Y_t/Y_0-(\mu-\sigma^2/2)t]}$, $i = 1, 2$.

[2]Hamilton (2013) uses the same data source in his detailed analysis of the energy expenditure share.

[3]We recognize that equilibrium models with logarithmic preferences, such as ours, are unable to deliver a realistic equity premium if one requires that aggregate consumption equals aggregate dividend and uses consumption data to infer model parameters. To match the equity premium, one would need to employ more sophisticated preferences or break the relationship that aggregate consumption equals aggregate dividend (e.g., by assuming that the stock market is a levered claim on aggregate consumption).

[4]In our baseline model we assume for simplicity that these investors are benchmarked only against a commodity index. However, we can generalize our model and benchmark our institutions against a composite index that includes both equities and commodities, that is, redefine our index in (4) to also include the stock market. Thus, instead of interpreting our institutional investor as an individual commodity manager, one could interpret it as, for example, a pension fund as a whole, which invests in equities as well as commodities. The assets under management of pension funds and other related institutional asset managers are a sizeable fraction of global capital markets. Therefore, the associated $\lambda$ should be high. One can show that our results continue to hold in this extension. That said, for the purposes of this paper, our leading interpretation of $\lambda$ is the fraction of (long-only) institutional investors in the commodity futures market. Much of the financialization literature has been motivated by the fact that there has been a significant inflow of such investors into this market.

[5]Equation (IA5) reveals that the benchmark price of energy futures is linear in the energy 1 demand parameter $\alpha_{1t}$. However, there is also an indirect dependence of benchmark futures prices of all commodities on $\alpha_{1t}$ through aggregate output $D_t$. This is because $\alpha_{1t}$ and $D_t$ are driven by the same source of risk, the Brownian motion $\omega_0$, and an increase in $\alpha_{1t}$ always coincides with an increase in $D_t$. That is why the plot of the futures price against $\alpha_{1t}$ is convex, even in the benchmark economy without institutions.

[6]Table IAII demonstrates that our results are quite sensitive to energy supply news volatility $\sigma_1$. For robustness, we re examine our results using parameter values based on Vassilev (2010). Vassilev includes fewer

commodities, and his data imply parameter values $\sigma_1 = 0.33$ and $\sigma_k = 0.25$, $k > 2$. (The remaining parameter values remain as in Table IAI). In this new exercise, we find that the increase in energy futures prices with financialization rises from 9.71% to 11.37%. For an upward demand shift for energy from $\alpha_{1t} = 0.03$ to 0.04, this value becomes 24.88%.