LBS Research Online

Trade Credit in Competition: A Horizontal Benefit

Heikki Peura

Imperial College Business School, South Kensington Campus, London SW7 2AZ, United Kingdom, h.peura@imperial.ac.uk

S. Alex Yang

London Business School, Regent's Park, London NW1 4SA, United Kingdom, sayang@london.edu

Guoming Lai

McCombs School of Business, University of Texas at Austin, Austin TX 78712, guoming.lai@mccombs.utexas.edu

Trade credit is a widely adopted industry practice. Prior research has focused on how trade credit benefits firms by improving *vertical* supply chain relationships. This paper offers a novel perspective by examining whether trade credit benefits suppliers through a *horizontal* channel. Under the classic Bertrand competition framework, we analyze two competing firms' price decisions with and without trade credit. We find that when the firms are financially constrained, trade credit softens horizontal price competition. Specifically, with trade credit, the firms will behave less aggressively in setting their prices for fear of incurring additional financing costs, resulting in equilibrium prices above the marginal cost, even if the products are perfect substitutes. Equilibrium profits under trade credit may thus be strictly higher than those under cash contracts. Furthermore, we find that with trade credit, a financially stronger firm may be able to exclude its weaker competitor from the market. We also investigate the relationship between the firms' financial strength and their physical capacity in the competition with trade credit. We find that the horizontal benefit of trade credit over cash contracts increases as either the firms' physical capacities increase or their financial status weakens. Therefore, with trade credit, firms' financial constraints are a partial substitute for the role that physical capacity plays in price competition. Finally, we study the firms' choice between offering trade credit and cash contracts. We find that trade credit is the equilibrium contract form if customers value trade credit, suggesting that the horizontal benefit of trade credit may complement its vertical roles.

Key words: trade credit, Bertrand competition, financial capacity, physical capacity, operations–finance interface

History: First draft: February 2015. Current draft: March 2016.

1. Introduction

Trade credit (i.e. financing extended by suppliers to their buyers for the purchase of goods or services) is a common component of supply chain contracts. According to the *Financial Times*, in 2007, 90% of world merchandise trade was financed by trade credit to the value of $25 trillion (Williams 2008). In the U.S., the account of trade payables at the aggregate level is the second-largest liability in the balance sheets of non-financial firms, and its amount has been increasing at an annual pace of 2–6% in recent years (Federal Reserve Board 2014). Across industries, trade

credit is offered not only by large firms with easy access to capital markets, but also by small suppliers who are often financially constrained (Giannetti et al. 2011). In fact, dominant retailers in the developed world regularly carry enormous amounts of accounts payable extended by their much smaller suppliers in emerging markets.

Despite the common notion that trade credit facilitates transactions in supply chains, it is puzzling why small and financially constrained suppliers widely extend trade credit as it ties up a significant proportion of their already limited capital, which may lead to financial difficulties (Loten 2012). Some argue that this is simply a result of the buyer's bargaining power (Ng et al. 1999, Klapper et al. 2012). However, when the buyer's financing cost is lower than the supplier's, asking for lower prices instead of longer terms would benefit the buyer more: The buyer must eventually compensate the supplier for the more expensive financing costs borne by the supplier to extend trade credit. Other theories rationalize the prevalence of trade credit by exploring its operational role. For example, trade credit can be either voluntarily used by suppliers to signal their product quality, as they are not afraid of possible returns (Long et al. 1993), or demanded by buyers to incentivize suppliers to improve quality in the face of no-payment risk in case the products fail (Babich and Tang 2012). This quality assurance role of trade credit is particularly important when the suppliers are small and have little trading history.

While most academic research, including the above literature, focuses on the vertical roles of trade credit (how trade credit influences the relationship between suppliers and buyers), practitioners often emphasize trade credit as a necessity to compete horizontally. For example, in manufacturing and wholesale industries, which are often characterized by intense competition due to low technology barriers in making commoditized products (e.g., apparel and household consumables), offering trade credit to downstream buyers has become the business standard — not offering favorable payment terms may simply exclude a firm from business (Pike et al. 1998, Paul and Boden 2012). In such circumstances, except for some general perception that trade credit has become a necessary strategy for suppliers to compete for business, there is a lack of a thorough understanding of its role in horizontal competition. Specifically, can the wide adoption of trade credit be further rationalized by how it influences horizontal competition between financially constrained firms? From the suppliers' perspective, could there be a silver lining to tying up capital by extending trade credit?

To explore the possible horizontal benefit of trade credit in competition, we analyze a model with two financially constrained firms competing under the classic Bertrand framework. To focus on the role of trade credit in competition, we assume away other factors such as information asymmetry that may also influence the firms' decisions under trade credit. In the model, the firms' products are perfect substitutes and their production costs are identical, and thus the buyers will purchase

from whoever offers the lowest price. In the case that the same price is offered, the demand is split evenly between the firms. Clearly, without physical capacity limits or trade credit, competition will result in equilibrium prices approaching marginal production cost — the well-known Bertrand paradox.

We contrast this classical result with the case where the firms engage in price competition when offering trade credit. In the face of potential liquidity shocks, trade credit exposes the firms to higher financing costs, which actually alleviate their incentives to undercut each other. As a consequence, we find that in equilibrium, the two firms set prices significantly higher than their marginal cost so that both obtain higher profits than under cash transactions. In this sense, trade credit creates a horizontal benefit to suppliers by alleviating aggressive competition. Thus, even though it has long been seen as a cost of doing business, trade credit as an industry standard can in fact increase firms' profitability. We further find that the lower the firms' cash positions, the less incentive they have to reduce their prices. Their profit margins will therefore be higher if they become more financially constrained. For the same reason, higher demand ties up more capital in trade credit and hence may also increase profits. Complementing the findings of prior literature, this analysis shows how using trade credit can benefit not only downstream buyers but also upstream suppliers in supply chains.

To deepen our understanding of this horizontal role of trade credit, we extend the model in three directions. First, as the above mechanism relies on the suppliers' exposure to financial risk, we investigate how it is affected by their potentially different financial constraints. As long as the two firms have sufficiently similar financial status, both achieve positive profits through markups. But when one of the firms has a significantly lower financial capacity, we show that the firm with a stronger financial position may be able to use trade credit to exclude the weaker competitor from business in certain situations (even though they have the same operational efficiency). This result is in line with evidence that trade credit may be applied as a competitive strategy in practice (Pike et al. 1998, Paul and Boden 2012, Barrot 2016).

Second, we study how the horizontal benefit of trade credit is affected by the presence of physical capacity constraints, which are also known to be able to soften price competition. We find that under trade credit, in addition to maintaining positive margins as in the basic model, firms' profits are higher when their physical capacities increase. This is in contrast with the conventional finding that larger capacities hurt firms' profits under cash-on-delivery (COD). This result also implies that offering trade credit can be especially beneficial when the industry is characterized by excess capacity, tight financial constraints, and intense competition (which, for instance, is often a common scenario for the small suppliers of the aforementioned large retailers). Our paper thus sheds light on

the connection between firms' financial and physical capacities: Through trade credit, constraints in the former play a similar role in softening competition as in the latter.

Finally, we extend the model to examine the firms' equilibrium choice between trade credit and COD contracts. Using a two-stage game where the firms choose contract terms (trade credit or COD) before the price game, we find that both firms offer trade credit in equilibrium as long as the trade credit premium from its vertical roles such as quality assurance is sufficiently high. In such a situation, the suppliers are able to secure a profit margin higher than this vertical premium. However, when this premium is low, a prisoner's dilemma arises — both firms may offer COD contracts although they would be strictly better off if both offered trade credit. These findings suggest that the horizontal benefit of trade credit complements the vertical ones identified in prior literature. They are also consistent with the observation that despite its prevalence, trade credit is not used in all circumstances (Giannetti et al. 2011).

The remainder of this paper is organized as follows. We review the related literature in Section 2. Section 3 describes the basic model and analyzes the symmetric competition case with trade credit. Sections 4, 5 and 6 extend the basic model by examining the implications of asymmetric financial status, constraints of physical capacity, and the choice between trade credit and COD, respectively. We conclude the paper and summarize testable hypotheses for future empirical work in Section 7. All proofs are in the Appendix.

2.   Related Literature

Our paper is closely related to the literature on theories of trade credit, which can be largely classified into two categories. The first category focuses on the financing advantages that the suppliers might have over the banks to extend credit. Earlier works such as Meltzer (1960) and Schwartz (1974) argue that trade credit may substitute for access to traditional capital markets, allowing suppliers to share their low-cost financing with their buyers who are more financially constrained. Other financing advantages of trade credit include lower transaction costs (Emery 1984), tax savings (Brick and Fung 1984), private information about the buyers' creditworthiness (Smith 1987, Biais and Gollier 1997), liquidation advantage (Longhofer and Santos 2003, Frank and Maksimovic 2004, Fabbri and Menichini 2010), mitigating buyers' opportunistic behavior (Burkart and Ellingsen 2004, Chod 2015), and incentive to grant concession/provide insurance under buyers' financial distress (Wilner 2000, Cuñat 2007). In the second category of theories, trade credit serves an operational role in supply chain relationships, such as signaling product quality (Lee and Stowe 1993, Long et al. 1993), deterring suppliers' moral hazard (Kim and Shin 2012, Babich and Tang 2012, Rui and Lai 2015), sharing demand risk (Kouvelis and Zhao 2012, Yang and Birge 2009, 2011), and price discrimination (Brennan et al. 1988). Similar to the studies in the second category, our

paper also points out an operational role of trade credit. However, different from the above research, which focuses on vertical supply chain relationships, our research takes a horizontal perspective.

While the horizontal role of trade credit has received little attention from a theoretical standpoint, several empirical studies have investigated the impact of competition on trade credit usage. The findings are mixed. For instance, Petersen and Rajan (1997) and McMillan and Woodruff (1999) argue that competition among suppliers may reduce the use of trade credit because suppliers may lose the power to enforce payments. By contrast, Fisman and Raturi (2004) find that competition may in fact increase trade credit usage for both demand- and supply-side reasons (e.g. by reducing the hold-up effect and increasing the switching cost). Hyndman and Serio (2010) observe that monopolies are less likely to offer trade credit than suppliers in competitive settings. Fabbri and Klapper (2013) also document that suppliers in more competitive industries tend to extend more trade credit. Barrot (2016) finds that longer trade credit terms allow financially stronger firms to deter entry. Lee and Zhou (2015) show that trade credit may play either a (horizontally) competing or a (vertically) collaborative role in supply chains. Our paper complements this literature by offering theoretical support to existing empirical evidence. In addition, our results also lead to several testable hypotheses.

By adopting the classic Bertrand competition framework, our work is closely related to the body of literature that offers solutions to the Bertrand paradox of marginal cost pricing (Bertrand 1883). One of the earliest remedies to the Bertrand paradox is capacity constraints (Edgeworth 1897). Subsequently, Levitan and Shubik (1972), Dasgupta and Maskin (1986a,b), and Allen and Hellwig (1986) have shown the existence of mixed strategy equilibria when price-setting firms are capacity constrained. Kreps and Scheinkman (1983) further show that price competition can be softened if firms can commit to capacity/quantity levels ex ante. Our paper complements this literature in two respects. First, we find that under trade credit, a firm's financial capacity plays a role (partly) substitutable to its physical capacity in softening price competition. Second, similar to quantity commitment in Kreps and Scheinkman (1983), our results suggest that in the presence of financing costs, trade credit can be seen as a commitment device that leads to positive margins. Other solutions include product differentiation (Shaked and Sutton 1983, Porteus et al. 2010), repeated interaction (Tirole 1988), and, more closely related to our work, production cost convexity. In this literature, Dixon (1984) first establishes the existence of mixed strategy equilibria when the firms have convex production costs. The result is subsequently extended to more general cost functions (Maskin 1986), pure strategy equilibria (Dastidar 1995), cost uncertainty (Wambach 1999), and repeated interactions (Weibull 2006). From a technical perspective, the trade credit scheme in our

model has a role similar to that of convex production costs.[1] However, focusing on the explanations of the wide adoption of trade credit in practice, our work differs from theirs in several aspects. First, our main results concern characterizing the horizontal benefit of trade credit and how it is influenced by various factors, instead of exploring the theoretical possibility of resolving the Bertrand paradox. Second, our focus on trade credit leads to results that are not studied previously, such as the interaction between financial constraint and physical capacity (Section 5) and the choice of adopting a convex cost structure (trade credit) or not (Section 6). Last, instead of convex production costs, our insights are built upon a convex cost structure from the financial side, which does not depend on production technologies.

Finally, our paper fits in the operations-finance interface literature, which examines firms' operational decisions in the presence of financial constraints. Representative papers in this literature include Babich and Sobel (2004), Buzacott and Zhang (2004), Xu and Birge (2004), Dada and Hu (2008), Lai et al. (2009), Aydin et al. (2011), Boyabatlı and Toktay (2011), Li et al. (2013), and Dong et al. (2015). Particularly related to our paper, several studies have analyzed optimal inventory policies in a dynamic setting with the provision of trade credit (Gupta and Wang 2009, Federgruen and Wang 2010, Huh et al. 2011, Luo and Shang 2013). In addition, by linking financial risks to price competition, our paper is also similar to Babich et al. (2007), who examine how the correlation between competing suppliers' disruption risks influences their pricing decisions, and to Peura and Bunn (2015), who show how financial targets may result in dynamic pricing patterns in price competition.

3. Price Competition with Trade Credit

Consider two identical firms $i = 1, 2$ selling a homogeneous product, with installed production (physical) capacity K and marginal cost $c > 0$. After observing market demand, the firms engage in Bertrand (price) competition under one of two payment schemes commonly observed in practice: cash on delivery (COD) and trade credit. We denote $\delta \geq 0$ as the *trade credit premium* that buyers are willing to pay for trade credit contracts over COD. The premium captures the vertical value of trade credit as identified in prior literature (e.g., customers are willing to pay more for goods delivered under trade credit contracts because the delayed payment allows them to check the quality of the products, or better manage their own cash flow). Buyers' willingness to pay (WTP) is thus v_c with COD and $v = v_c + \delta$ with trade credit, with $v \geq v_c > c$. In the basic model, to isolate the customer preference effect from our focus on competition, we assume that $\delta = 0$ so that

---

[1] Some results in our paper, e.g., Propositions 1 and 3 (equilibrium pricing under trade credit with symmetric and asymmetric financial capacities, respectively), share some technical similarities with the above literature (Dastidar 1995).

$v = v_c$. Intuitively, allowing $\delta > 0$ would make trade credit more attractive compared to COD, which only strengthens our results. To focus on identifying the horizontal benefit of trade credit in price competition, we treat the payment terms (trade credit or COD) as given and focus on the pricing stage. Section 6 extends the basic model by allowing the two firms to choose payment schemes (trade credit or COD) before the pricing stage and $\delta > 0$.
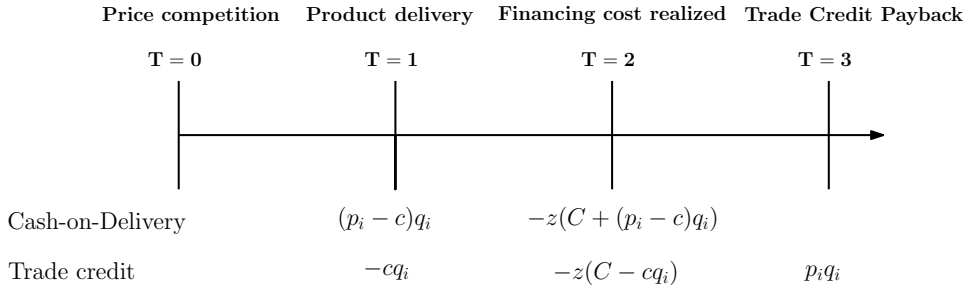
The sequence of events and cash flows corresponding to each event under both contract types are illustrated in Figure 1. At $T = 0$, observing demand D, the two firms simultaneously offer prices p_i $(i = 1, 2)$. In this section, we focus on the case that demand is less than a single firm's physical capacity $(D < K)$. Section 5 extends the basic model to study the impact of physical capacity constraints $(D \geq K)$. As long as prices are below buyers' WTP v, the market is split as follows:

$$q_i(p_i, p_j) = \begin{cases} D & \text{if } p_i < p_j, \\ D/2 & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j. \end{cases} \tag{1}$$

Each firm satisfies all of the demand it faces (e.g., Dastidar 1995).[2]

At $T = 1$, production costs (cq_i) are incurred and goods are delivered to buyers. If the firms compete under COD, revenue $(p_i q_i)$ is also realized.

Figure 1 **Sequence of events and cash flows with cash on delivery and trade credit.**

	Price competition	Product delivery	Financing cost realized	Trade Credit Payback
	T = 0	T = 1	T = 2	T = 3
Cash-on-Delivery		$(p_i - c)q_i$	$-z(C + (p_i - c)q_i)$	
Trade credit		$-cq_i$	$-z(C - cq_i)$	$p_i q_i$

In a deviation from the classic Bertrand game, we assume that each firm may incur a liquidity shock at $T = 2$. This liquidity shock captures various cash flow uncertainties faced by the firms, such as losses in other lines of business, an unexpected order that requires a large amount of upfront investment (e.g., procuring raw material), natural disasters, legal and product liabilities, etc. Each firm can manage its liquidity using its financial capacity C, which can be seen as the firm's cash on hand or secured line of credit (Sufi 2009). We first assume that the two firms are financially

---

[2] For the ease of exposition, we assume the market demand D is inelastic and the two firms split the demand equally when they offer the same price $(p_1 = p_2)$. As shown in Online Appendix D.2 and D.3, our qualitative results remain unchanged when demand follows a general form or is unequally allocated to the two firms under equal prices.

symmetric. Section 4 relaxes this assumption and discusses the implications of asymmetric financial status.

Financial capacity C determines each firm's expected financing costs when the liquidity shock is realized. Let the cost be $z(x)$, where x is each firm's financial capacity at $T = 2$.

ASSUMPTION 1. *Each firm's expected financing cost $z(\cdot) > 0$ is decreasing and convex.*

The property of convexity is assumed in many related papers such as Froot et al. (1993), Babich (2010), and Dong and Tomlin (2012) and is satisfied by many commonly used forms of financing costs. Specifically, consider the following three forms of financing costs.

1. The liquidity shock ξ_i follows the cumulative distribution function (CDF) $G(\cdot)$ and probability density function (PDF) $g(\cdot)$. If a firm's financial capacity (x) is not sufficient to cover this liquidity shock ξ_i $(\xi_i > x)$, the firm needs to refinance, incurring a fixed re-financing cost $c_R > 0$. Therefore, the *expected* financing cost is $z(x) = c_R[1 - G(x)]$; this is convex when $G(\cdot)$ is concave at the tail, which is a reasonable assumption as a random variable unbounded above has to have a concave CDF as it increases. This specification will be used in numerical examples later in this paper.

2. When the refinancing cost is proportional, $z(x) = s \int_x^{+\infty} (\xi - x) dG(\xi)$, where s is the unit transaction cost, for example, the loss proportion due to a fire sale, and $\xi - x$ is the amount to be financed. It is easy to verify that $\frac{d^2 z(x)}{dx^2} = sg(x) > 0$, where $g(\cdot)$ is the PDF. Obviously, a linear combination of fixed and proportional costs also satisfies this assumption.

3. Consider a continuous-time model where the aggregated liquidity shock is $\xi_t = \sigma B_t$, in which B_t is a standard Brownian motion. Therefore, with financial capacity $x \geq 0$, the probability that the firm will need financing within the trade credit duration τ is equivalent to the probability that the first passage time that the Brownian motion reaches x is less than τ $G(x; \tau) = 2\Pr\left(B_t > \frac{x}{\sigma}\right) = 2\Phi\left(-\frac{x}{\sigma\sqrt{\tau}}\right)$, where $\Phi(\cdot)$ is the CDF of the standard normal distribution. Therefore, under fixed financing cost c_R, the financing cost $z(x) = 2c_R\Phi\left(-\frac{x}{\sigma\sqrt{\tau}}\right)$, which is obviously convex and decreasing in x for sufficiently large x.

Combining the expected financing cost with firms' operational profit $(p_i - c)q_i$, the firm's total expected profit with COD is:

$$\pi_i = (p_i - c)q_i - z(C + (p_i - c)q_i). \tag{2}$$

In contrast to COD, with trade credit, the firm's operational revenue $(p_i - c)q_i$ is received only at $T = 3$. Therefore, firm i's financial capacity at $T = 2$, i.e. when the liquidity shock is realized, is

$C - cq_i$.[3] The difference in the timing of receiving payments from the buyers with COD and with trade credit captures the fact that as trade credit ties up a firm's working capital, the firm will be more vulnerable to liquidity shocks with trade credit. Indeed, according to our model, with trade credit, the firms lower their financial capacity and will therefore incur higher expected financing costs. Therefore, firm i's total expected profit under trade credit is:

$$\pi_i = (p_i - c)q_i - z(C - cq_i). \tag{3}$$

To focus on the role of trade credit in competition, we assume away other factors that may influence the use of trade credit. Specifically, we assume that both firms are risk neutral, that there is no information asymmetry, and, without loss of generality, that the interest rates faced by the firms and their potential buyers are zero. Finally, buyers always pay the full amount and hence the firms have no risk related to payment default.

3.1. Equilibrium with COD

With COD, the two firms have already collected revenues when the liquidity shock is realized, and firm i's financing cost $z(C + (p_i - c)q_i)$ decreases in its operational profit. Therefore, the firms' objective to maximize expected total profit is aligned with maximizing operational profit only, and hence the only difference between this setting and the standard Bertrand competition is that our model takes the firms' financing costs into consideration.

LEMMA 1. *With COD, the two firms set their prices at the marginal cost in equilibrium, that is, $p_{u,i}^c = c$, and their corresponding profits are $\pi_{u,i}^c = -z(C)$, $i = 1, 2$.*

Since the firms collect their revenues before the liquidity shock, their financial capacities do not influence price competition, and it is therefore always optimal to undercut the competitor to gain more market share. The equilibrium price then equals the marginal cost, as in standard Bertrand competition.[4]

3.2. Equilibrium with trade credit

As shown above, with COD, the firm's operational and financial motives are aligned, and the incentive to improve operational profit leads to classic cut-throat price competition. With trade credit, the driving forces behind price competition become more complicated than with COD. Consider the following illustrative example.

---

[3] In the main body of the paper, for expositional brevity, we assume that after extending credit, the firms can only receive payment when trade credit is due ($T = 3$). But, in practice, firms may use factoring to obtain (part of) the cash (at a discount) before trade credit is due. Factoring, however, is not without cost (Klapper 2006). As shown in Appendix D.1, the main insights of the paper remain unchanged when the cost associated with factoring is non-negligible.

[4] The financing cost can be seen as a cost for the firm to stay in business, which allows the firm to earn profit in the future. Thus, an immediate negative profit does not prevent the firm from competing in the current transaction.

EXAMPLE 1. Two identical firms face fixed demand $D = 2$. Before production, each firm has financial capacity $C = 3$. The production cost is $c = 1$. After production, each firm faces a liquidity shock with distribution $\mathcal{N}(0, 1)$. If its financial capacity is less than the realized liquidity shock, the firm incurs a refinancing cost $c_R = 0.5$. Therefore, the firm's expected financing cost is $z(x) = c_R \Phi(-x)$, where $\Phi(\cdot)$ is the CDF of a standard normal distribution. With COD, the firms set $p = c = 1$ in the Bertrand equilibrium. With trade credit, suppose both firms set prices at $p = 1.065$ and split the market evenly, each selling one unit. At this price, the probability of refinancing is 2.3%. Hence, the overall profit earned by each firm is $p - c - c_R \Phi\left(-\left(C - \frac{cD}{2}\right)\right) = 1.065 - 1 - 0.5 \cdot (0.023) \approx 0.054$. If a firm deviates and lowers the price to $1.065 - \epsilon$, it captures the entire market, but the probability of refinancing jumps to 15.9%. Then, the profit earned is $2(p - c) - c_R \Phi(-(C - cD)) = 2(1.065 - \epsilon - 1) - 0.5 \cdot (0.159) \approx 0.051 < 0.054$. Therefore, there is no incentive to undercut the competitor, even with a strictly positive operational margin, as the additional financing cost outweighs the gains.

The intuition behind the above example is best illustrated by comparing the profit functions under trade credit (equation 3) and that under COD (equation 2). On the one hand, with trade credit, a firm can still increase its operational profit by undercutting the competitor, similar to the case under COD. But on the other hand, unlike COD, under which the firm's financing cost is also lowered with a larger market share, the gain in market share under trade credit adds pressures to the firm's financing situation. As shown in (3), by increasing its market share from $\frac{D}{2}$ to D, the firm's financing capacity drops from $C - \frac{cD}{2}$ to $C - cD$, and hence its expected financing cost jumps from $z\left(C - \frac{cD}{2}\right)$ to $z(C - cD)$. Balancing these forces, the two firms' incentive to undercut each other in order to gain more market share is weakened, alluding to the possibility that their equilibrium prices may strictly exceed the marginal cost. This intuition is formalized in the following proposition.[5]

PROPOSITION 1. *With trade credit, a symmetric pure strategy equilibrium exists if and only if* $v \geq c + M_{u,p}\left(C + \frac{cD}{2}\right)$. *The equilibrium price is* $p_{u,i}^s = \min\left\{v, c + M_{u,p}(C)\right\}$ *for* $i = 1, 2$, *and the resulting profit is* $\pi_{u,i}^s = \left(p_{u,i}^s - c\right)\frac{D}{2} - z\left(C - \frac{cD}{2}\right)$, *where* $M_{u,p}(x) = \frac{2}{D}\left[z\left(x - cD\right) - z\left(x - \frac{cD}{2}\right)\right]$.

When the buyers' WTP is sufficiently high, or equivalently, when the firms' financial capacity is relatively low, trade credit enables the firms to maintain a strictly positive margin. As the firms' financial capacity C tightens, the margin gradually increases to the buyers' WTP v. However, as C continues to decrease, the financial burden associated with trade credit keeps increasing. At the extreme, the financial cost could become so large that the firms may prefer not to serve any demand. Otherwise, by creating tension between the firms' operational profit and financial costs, trade credit allows the firms to maintain a positive margin.

---

[5] In addition to the pure strategy equilibrium, there may also exist a mixed strategy equilibrium with trade credit, which only exists if the pure strategy equilibrium exists, and is also Pareto-dominated by the pure strategy equilibrium when both exist. We leave the details to Lemma B.1 in the Appendix.

COROLLARY 1. *Under the pure strategy equilibrium, the profit margin with trade credit* $(M_{u,p}(C))$ *decreases in industry-level financial capacity C and increases in demand D.*

Intuitively, higher demand will increase the margin, and greater financial capacity will reduce it. In the extreme case, if the firms are not financially constrained at all, we have the standard Bertrand equilibrium.

An analogous intuition holds on how demand influences the firms' profits. While the profit margin increases in D, the financial burden also increases as D gets larger. We can show that for low values of demand, the margin effect dominates and the profits increase in D, but for higher demand, the risk becomes disproportionately high and the financial burden effect dominates.

COROLLARY 2. *The pure strategy equilibrium profit $(\pi_{u,i}^s)$ first increases and then decreases in D.*

3.3. The horizontal benefit of trade credit

While trade credit allows the firms to maintain a positive profit margin, the associated financing cost is also higher than that with COD. Between the operational margin and financial risk, which will be the dominant force? We now compare these effects through the value of trade credit, defined as the difference between the profits with trade credit and those with COD.

COROLLARY 3. *The equilibrium with trade credit (weakly) Pareto dominates that with COD if the former exists.*

The value of trade credit is positive as long as a pure strategy equilibrium exists or, equivalently, when the buyers' WTP is sufficiently high. The logic behind Corollary 3 is as follows: With trade credit, when a firm offers a price higher than its competitor's, and hence is completely squeezed out of the market, its payoff is exactly the same as that with COD. The existence of a trade credit equilibrium dictates that such a deviation is profitable. As a result, the value of trade credit is positive as long as the equilibrium exists.

How is the horizontal benefit of trade credit influenced by the firms' characteristics? As the firms' profits with COD are independent of demand D, according to Corollary 2, the benefit of trade credit first increases and then decreases with demand. On the other hand, while it is obvious that with COD the firms' profits always increase as the firms' financial capacities increase, it is less clear whether financial strength always translates into higher profits with trade credit: Greater financial capacity reduces the financing cost, but it also lowers the operational margin as deviating becomes less costly. Combining these different effects, how does the benefit of trade credit change as the two firms become financially stronger?

COROLLARY 4. *Suppose $z'''(x) \leq 0$. The horizontal benefit of trade credit is first increasing and then decreasing in C.*

Relating the condition in Corollary 4 to the different forms of financing costs discussed previously, we can see that $z'''(x) \leq 0$ is equivalent to the PDF of the liquidity shock ξ is decreasing under proportional refinancing cost, or is convexly decreasing under fixed refinancing cost, which in general holds for any unbounded tail distribution. Under such conditions, Corollary 4 reveals that if the firms' financial capacities are very low, the effect of C on reducing the financing cost dominates, and hence the horizontal benefit of trade credit will increase as the firms become financially stronger. However, as C continues to increase, while both the effect of limiting operational margin and the marginal benefit of reducing the financing cost become smaller, the former remains relatively more pronounced under $z'''(x) \leq 0$. As a result, an increase of C will reduce the value of trade credit, echoing the intuition that as C becomes very large, the equilibrium outcome reverts to the classic Bertrand result and the horizontal benefit of trade credit disappears.

Figure 2 depicts how the horizontal benefit of trade credit is affected by the model parameters. This value is presented as a fraction of the corresponding value of monopoly pricing (i.e. the difference between the maximal possible profit from serving the demand and COD). First, Figure 2(a), complementing Corollary 4, shows that the value of offering trade credit is first increasing and then decreasing in the firms' financial capacity. With low levels of financial capacity, the buyers' WTP binds and increasing C can significantly reduce financing costs. With high financial capacities, the operational margin is reduced more than the risk as C increases, and thus the benefit of offering trade credit decreases. Similar non-monotone effects are also apparent for the production and refinancing marginal costs, as shown in Figures 2(b) and 2(c). This is because increasing costs have a similar effect to decreasing financial capacity. Specifically, the relative value of trade credit increases as long as the buyers' WTP does not bind: Higher marginal costs can lead to greater profit margins—which outweigh the increase in the expected financing costs—by softening the competition. However, when the costs continue to increase, the buyers' WTP will be binding and thus the benefits of trade credit will stop increasing; yet, the expected financing costs will continue to increase as the costs increase. Lastly, Figure 2(d) illustrates an increase in demand will similarly increase the value of trade credit at first by improving profit margins and then reduce it as the financing risk starts to dominate.

4. Predating a Financially Weaker Competitor under Trade Credit

We have seen the fundamental mechanism through which trade credit softens price competition with financially constrained firms, assuming symmetric financial capacities. But what happens

Figure 2   The relative value of trade credit compared to monopoly pricing.

(a) Financial capacity (C)

(b) Production cost (c)

(c) Refinancing cost (c_R)
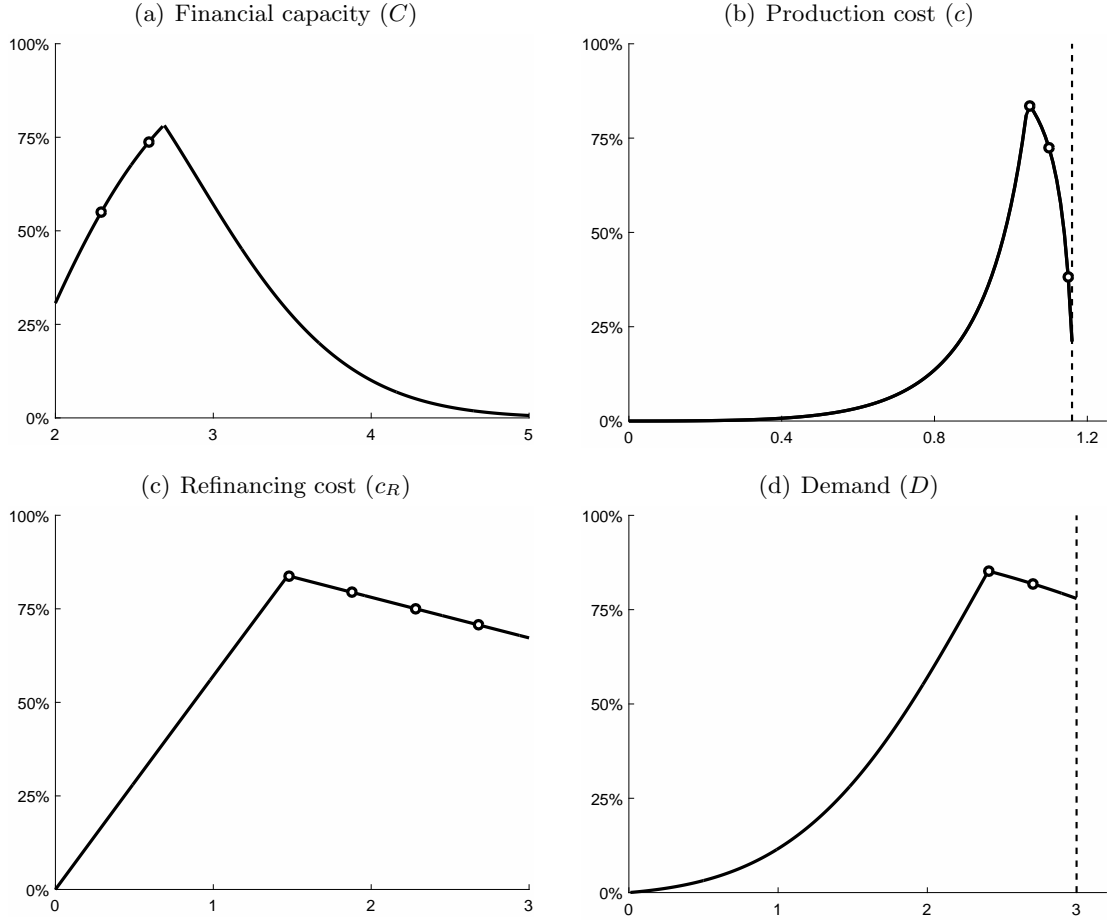
(d) Demand (D)

Note: The line segments with markers represent that WTP v is binding in this region. The dashed vertical lines represent the existence condition. Baseline parameters: demand $D = 2$, production cost $c = 1$, WTP $v = 1.2$, financial capacity $C = 3$, cost of refinancing $c_R = 1$, liquidity shock follows $\mathcal{N}(0,1)$, (expected) financing cost $z(x) = c_R \Phi(-x)$.

when one firm is financially stronger than its competitor? How will this financial asymmetry influence the effectiveness of trade credit on limiting competition? We now extend the above analysis to the case where $C_1 > C_2$, that is, Firm 1 is financially stronger. Evidently, with COD, the imbalance in financial capacities is irrelevant and both firms still set prices at marginal cost. Therefore, we only need to focus on the results with trade credit.

PROPOSITION 2. *With trade credit, a symmetric pure strategy equilibrium exists when* $C_2 \geq C_1 - \frac{cD}{2}$ *and* $v \geq c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)$. *The equilibrium price* $p_{u,i}^s = \min\left\{v, c + M_{u,p}(C_1)\right\}$ *and the resulting profits are* $\pi_{u,i}^s = \left(p_{u,i}^s - c\right)\frac{D}{2} - z\left(C_i - \frac{cD}{2}\right)$ *for* $i = 1, 2$.

Proposition 2 shows that there exists a symmetric pure strategy equilibrium with trade credit when two conditions are satisfied. First, it requires that the difference between C_1 and C_2 is no

greater than the "inventory" investment ($\frac{cD}{2}$). When the two firms' financial capacities differ by more than this amount, the price regions where the two firms have no incentive to deviate do not overlap. The second condition for the existence of the equilibrium is simply the participation condition from Proposition 1, adapted to the financially weaker firm: The buyers' WTP needs to be greater than the lowest price that prevents the financially weaker firm from not serving any demand. When the equilibrium exists, as the two firms make the same operational profit, the financially weaker firm earns a lower profit due to its higher expected financing cost.

Furthermore, note that the equilibrium price is determined by the markup supported by the financially stronger firm, as its financial strength means that it can afford to offer products on credit at a lower price. Intuitively, as its relative financial strength continues to grow, it may further reduce price to such a level that the financially weaker firm finds it unprofitable to sell anything in the market. The following example illustrates this intuition.

Example 2. Consider the same setting as in Example 1, with the only difference that $C_2 = 1.5 < C_1$. Suppose Firm 1 still sets the price at $p = 1.065$; in Example 1, this allowed the two firms to split the market. However, as Firm 2 now has less financial capacity, serving half of the demand at $p = 1.065$ leads to a probability of refinancing at $\Phi(-1.5 + 1) = 31\%$ and a profit at $(1.065 - 1) - 0.5 \cdot 0.31 = -0.089$. On the other hand, if Firm 2 chooses to stay away from the market by setting a price higher than 1.065, its refinancing probability falls to 7%, with the corresponding profit at -0.033. Therefore, financially weaker Firm 2 will in fact prefer to stay out of the market. As a result, Firm 1 serves the entire demand $D = 2$ with a refinancing probability of 16% and a profit of $2 \cdot (1.065 - 1) - 0.5 \cdot 0.16 = 0.051$. In fact, in equilibrium, Firm 1 can achieve an even higher price of $p = 1.13$, with Firm 2 staying out of the market.

Due to its higher tolerance, the financially stronger firm is able to obtain a positive profit while excluding the more constrained firm from the market. We formalize this intuition in the following proposition.

PROPOSITION 3. *(a) An asymmetric pure strategy equilibrium exists if $C_2 \leq C_1 - \frac{cD}{2}$ and $v > c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)$, or $v \in \left[c + M_{u,m}(C_1), c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)\right]$. The firms set prices $p_{u,1}^a = \min\left\{v, c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)\right\}$ and $p_{u,2}^a = p_{u,1}^a + \epsilon$, where $\epsilon > 0$ is sufficiently small, and their profits are $\pi_{u,1}^a = (p_{u,1}^a - c)D - z(C_1 - cD)$ and $\pi_{u,2}^a = -z(C_2)$.*

(b) A mixed strategy equilibrium exists if $v > c + M_{u,m}(C_2)$. The firms set prices in the range $[c + M_{u,m}(C_2), v]$ and their profits are $\pi_{u,1}^m = (v - c)D - z(C_1 - cD)$ and $\pi_{u,2}^m = -z(C_2)$, where $M_{u,m}(x) = \frac{1}{D}\left[z(x - cD) - z(x)\right]$.

Proposition 3 characterizes two equilibria in which the two firms split the market asymmetrically. The asymmetric pure strategy equilibrium arises when one firm has significantly larger financial

capacity and hence finds it profitable to drive the price to a level that is too low for the financially weaker firm to sell its products at. Similarly, in the mixed strategy equilibrium, the financially stronger firm offers a lower price than its financially weaker competitor, and hence gains a larger market share. Under both equilibria, the financially weaker firm's expected profit equals its profit under COD, while the financially stronger firm is strictly better off under trade credit. As such, in the presence of financial asymmetry, trade credit exhibits a predation effect: A firm may leverage its financial advantage to the operational dimension through trade credit, and (effectively) deter a financially weaker competitor.

Figure 3   Existence of different equilibria with asymmetric financial capacities.
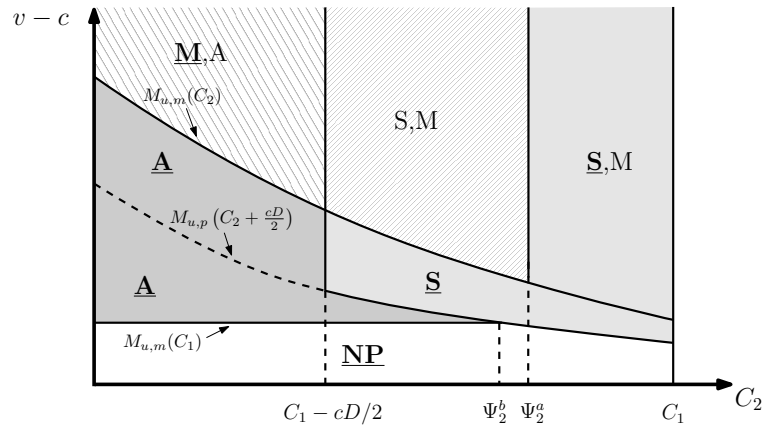
Note. The (weakly) Pareto dominant equilibria is marked as bold and underlined. "S" represents the symmetric pure strategy equilibrium, "A" represents the asymmetric pure strategy equilibrium, "M" represents the (asymmetric) mixed strategy equilibrium, and "NP" represents that no firm participates with trade credit. Ψ_2^a satisfies $M_{u,m}(\Psi_2^a) = M_{u,p}(C_1)$, and Ψ_2^b satisfies $M_{u,p}\left(\Psi_2^b + \frac{cD}{2}\right) = M_{u,m}(C_1)$.

Figure 3 summarizes the existence conditions of different equilibria in Propositions 2 and 3, as well as the Pareto-dominating equilibrium by comparing firms' payoffs.[6] As shown, when the buyers' WTP is very low, i.e., $v < \min\left(M_{u,p}\left(C_2 + \frac{cD}{2}\right), M_{u,m}(C_1)\right)$, there exists no equilibrium under trade credit (Region NP). This is because the firms' highest possible profit margin is $v - c$: When v is very low, the gain from the operational margin would not be able to justify the extra financing cost incurred due to trade credit. For a reasonably high v, the Pareto-dominating equilibrium depends on the difference between the two firms' financial capacities. When the two firms' financial situations are similar, the symmetric pure strategy equilibrium (S) Pareto dominates, and hence the two firms split the market evenly. To the other extreme, when the difference between C_1 and C_2 is very large, only asymmetric equilibria (in pure A or mixed strategies M) exist, and the

---

[6] For expositional brevity, the technical results are summarized in Proposition B.1 in the Appendix.

financially weaker firm is effectively pushed out of the market. This is consistent with the finding in Klapper et al. (2012) that when facing larger and financially stronger buyers, those suppliers that are financially stronger than their peers offer more trade credit, and that in Barrot (2016) that restricting trade credit duration allows more entry into an industry. Finally, we note that when $C_1 - C_2$ is in the middle region, there is a scenario that the financially weaker firm prefers the symmetric equilibrium (S), while the stronger one favors the mixed strategy equilibrium (M).

Furthermore, the impact of financial capacities on the firms' profits in different equilibria $(S, A,$ and $M)$ is summarized in Corollary 5. Two observations can be made. First, intuitively, a firm's profit decreases in its competitor's financial capacity. Second, while the financially weaker firm can always benefit if its own financial capacity increases, the financially stronger firm may actually prefer its own financial capacity to be maintained at a moderate level, which allows some exposure to financial risk.

> COROLLARY 5. *(a) The financially weaker firm's profit π_2 is increasing in its own financial capacity C_2 in all equilibria, and it is (weakly) decreasing in C_1 in (S).*
>
> *(b) The financially stronger firm's profit π_1 is increasing in its own financial capacity C_1 in (A) and (M), and first increasing and then decreasing in it in (S), assuming $z'''(x) \leq 0$. It is decreasing in C_2 in (A) and (M) and independent of it in (S).*

Finally, as with the results in Section 3, the equilibrium profits with trade credit (weakly) dominate those with COD in all trade credit equilibria, confirming that offering trade credit allows firms to maintain positive profit margins at the expense of higher financial costs.

5. Trade Credit and Physical Capacity Constraints

We have seen that when demand is low enough that firms' physical production capacities are irrelevant, trade credit improves their profitability. Does this result continue to hold when production capacity becomes tight? In this section, we address this question by focusing on the case where the demand cannot be completely served by one firm. For ease of exposition, we focus on the case of identical financial and physical capacities, that is, $C_1 = C_2 = C$ and $K_1 = K_2 = K$, with $K \in \left(\frac{D}{2}, D\right)$. The qualitative insights remain valid when the firms have constrained physical capacity and *asymmetric* financial capacities.

Given the physical capacity constraints, the lower-pricing firm serves demand up to its capacity, while the higher-pricing firm serves the residual demand:

$$q_i(p_i, p_j) = \begin{cases} K & \text{if } p_i < p_j \\ \frac{D}{2} & \text{if } p_i = p_j \\ D - K & \text{if } p_i > p_j. \end{cases} \tag{4}$$

5.1. Equilibrium with COD

Without physical capacity constraint, the equilibrium under COD was simply marginal cost pricing. However, under physical capacity constraints, no such pure strategy equilibrium exists: If both firms set prices equal to their marginal costs, it would be beneficial to deviate to a higher price to serve the residual demand. On the other hand, if both firms set prices higher than the marginal costs, then it would be profitable to undercut the competitor. Consequently, only a mixed strategy equilibrium exists.

LEMMA 2. *Given $K \in \left(\frac{D}{2}, D\right)$, under COD, there exists only a mixed strategy equilibrium in which the expected profits are: $\pi_{c,i}^c = (v-c)(D-K) - z(C + (v-c)(D-K))$ for $i = 1, 2$.*

In the above equilibrium, the firms' expected profits are equal to a residual monopolist's profit with the price at the buyers' WTP. The expected profits increase with demand and decrease with physical capacity K, approaching the profits under perfect competition when D decreases to K and the shared monopoly profits when D increases to $2K$.

5.2. Equilibrium with trade credit

Given that physical capacity constraints allow firms to maintain positive profit margins, it is not clear whether they can derive additional benefits from trade credit. To examine this question, we next establish possible pure and mixed strategy pricing equilibria with trade credit and then quantify the value of trade credit.

PROPOSITION 4. *Given $K \in \left(\frac{D}{2}, D\right)$, under trade credit, there exists a symmetric pure strategy equilibrium when $v - c \in \left[M_{u,p}\left(C + \frac{cD}{2}\right), \overline{m}_{c,p}\right)$, with $p_{c,i}^s = \min\{v, c + m_{c,p}\}$ and profits $\pi_{c,i}^s = (p_{c,i}^s - c)\frac{D}{2} - z\left(C - \frac{cD}{2}\right)$, where $m_{c,p} = \frac{z(C-cK) - z\left(C - \frac{cD}{2}\right)}{K - \frac{D}{2}}$ and $\overline{m}_{c,p} = m_{c,p} + \frac{z(C-cK) + z(C-c(D-K)) - 2z\left(C - \frac{cD}{2}\right)}{D - K}$.*

The existence of the pure strategy equilibrium again requires the buyers' WTP v to be above a certain level so that the potential benefits of a higher margin dominate the associated financial costs. But unlike the uncapacitated case, the buyers' WTP also needs to be lower than a particular threshold. The intuition is as follows. Without physical capacity constraints, deviating up, that is, offering a price higher than that of the competitor, is very costly for a firm as it will simply be excluded from the market. With capacity constraints, however, the firm's cost of deviating up decreases since its competitor cannot serve the whole market by itself, and the firm can therefore serve the residual demand $(D - K)$ at the buyers' WTP v. Deviating up hence becomes more lucrative when v increases. However, the pure strategy profit margin is constrained by the incentive to undercut and capture a larger market share, which is independent of v when v exceeds a threshold. Thus, the pure strategy equilibrium does not exist for large v.

When the equilibrium exists, the markup $m_{c,p}$ corresponds to the highest equilibrium price with trade credit that the firm will offer without being tempted to undercut its competitor. We can show that $m_{c,p}$ decreases in C, and the price $p_{c,i}^s$ hence (weakly) decreases in C. That is, the equilibrium price is v when C is sufficiently small. As C becomes larger, competition becomes more intense and the equilibrium price is lowered to $c + m_{c,p}$.

COROLLARY 6. *With trade credit, if the symmetric pure strategy equilibrium exists, $\pi_{c,i}^s$ increases in K, strictly so if $v > c + m_{c,p}$.*

Under COD, greater physical capacity intuitively hurts firms' profitability because the market becomes more competitive, with the equilibrium profits in Lemma 2 approaching the uncapacitated Bertrand outcome when K goes to D. By contrast, in the pure strategy equilibrium under trade credit, Corollary 6 reveals that firms are actually better off with more physical capacity as undercutting becomes less profitable and they can therefore hold higher markups.

With trade credit, there also exists a mixed strategy equilibrium if the buyers' WTP v is sufficiently large, as in the previous sections. The expected equilibrium profits are equal to the profit from setting the price at v and serving the residual demand $D - K$.

PROPOSITION 5. *Given $K \in \left(\frac{D}{2}, D \right)$, with trade credit, there exists a mixed strategy equilibrium when $v > c + m_{c,m}$, with expected profits $\pi_{c,i}^m = (v - c)(D - K) - z\left(C - c(D - K)\right)$, where $m_{c,m} = \frac{z(C - cK) - z[C - c(D-K)]}{2K - D}$.*

Figure 4 illustrates the existence conditions of the pure and mixed strategy equilibria as summarized in Propositions 4 and 5. As shown, the two equilibria exist simultaneously for $v \in (m_{c,m}, \overline{m}_{c,p})$. Corollary 7 confirms that, in this region, the pure strategy equilibrium is Pareto dominant. However, when customers' WTP v is high or demand D is large, the pure strategy equilibrium may not exist as deviating becomes more lucrative. Therefore, the mixed strategy one becomes the unique equilibrium.

COROLLARY 7. *Given $K \in \left(\frac{D}{2}, D \right)$, with trade credit, the pure strategy equilibrium, if exists, Pareto dominates the mixed strategy equilibrium.*

5.3. The horizontal benefit of trade credit

Next, we compare the firms' profits in the Pareto-dominant equilibrium under trade credit with those under COD. The following result shows that the profits with trade credit are higher than those with COD if the physical capacities for both firms are sufficiently high.

PROPOSITION 6. *If $v - c \geq M_{u,p}\left(C + \frac{cD}{2}\right)$, there exists a threshold physical capacity $K^* \in \left(\frac{D}{2}, D \right]$ such that the firms' profits with trade credit are higher than those with COD if and only if $K \geq K^*$.*

Figure 4   Existence of different equilibria with physical capacity constraints.
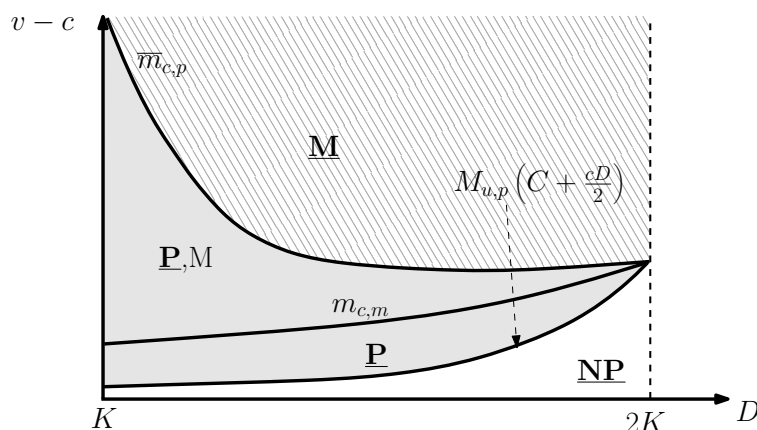
Note. The Pareto dominant equilibrium is marked as bold and underlined. "P" represents the pure strategy equilibrium, "M" represents the mixed strategy equilibrium, and "NP" represents that no firm participates with trade credit.

Figure 5   Firm's profit with trade credit and COD under physical constraints relative to monopoly pricing.
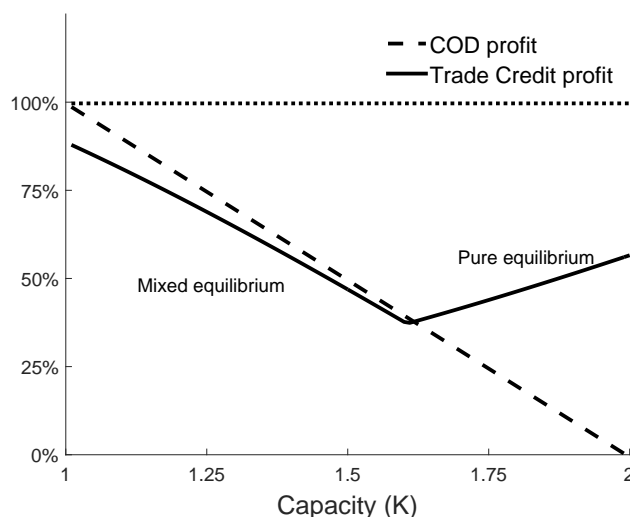
Note. In the figure, the firm's profit under monopoly pricing is normalized to 100% (the horizontal dotted line), and the firm's profit under COD without capacity constraint is normalized to 0. Parameters: demand $D = 2$, production cost $c = 1$; WTP $v = 1.2$; financial capacity $C = 3$; cost of refinancing $c_R = 1$, liquidity shock distributed $\mathcal{N}(0,1)$, (expected) financing cost $z(x) = c_R \Phi(-x)$.

As we have seen, with high K, COD pricing becomes more competitive, whereas trade credit allows firms to sustain higher markups. Trade credit profits therefore dominate the ones under COD with high enough K. This result is illustrated in Figure 5. With COD, the firms' profits decrease in capacity as the mixed strategy equilibrium approaches the standard Bertrand equilibrium. With trade credit, the symmetric pure strategy pricing equilibrium exists when the physical capacity is high enough relative to the demand. In contrast to the mixed strategy equilibria, the prices in the

pure strategy equilibrium increase in capacity. This is because undercutting becomes less lucrative with higher physical capacity, which allows higher markups. Therefore, this result confirms that even with physical capacity constraints, which allow firms to extract positive profits, trade credit can still serve as a mechanism to soften competition as long as the capacity constraints are not too tight. In this sense, our analysis connects the effects of constrained physical and financial capacities. Through trade credit, constraining financial capacity effectively acts as a substitute for constraining physical capacity by allowing positive markups.

6.   Choice between Trade Credit and COD

In the previous sections, we have assumed that both firms offer trade credit to quantify the benefit of doing so compared to the case without trade credit. Such an assumption is compatible with the practice that as a business norm, trade credit has been adopted by the majority of the firms in most industries. In this section, we relax this assumption and allow the two firms to choose freely whether to offer trade credit or COD in the first place. Such an extension allows us to explore whether offering trade credit is indeed an equilibrium choice. For expositional brevity, we focus on the uncapacitated case ($D \leq K$) with equal financial capacities $C_1 = C_2 = C$. In addition, we allow a positive trade credit premium $\delta \geq 0$, which captures the other (vertical) roles trade credit may play in buyers' willingness to pay, e.g., quality assurance (Long et al. 1993, Babich and Tang 2012), lower transaction cost (Ferris 1981), and hence $v \geq v_c$.

To endogenize the choice between trade credit and COD, we model the firms' interaction in a two-stage game. In the first stage (the contract stage), each firm decides whether to offer trade credit; and then in the second stage (the pricing stage), observing its competitor's first-stage decision, each firm decides its price. We assume all information is public as in standard Bertrand models. For expositional brevity, for the first stage game, we only consider pure strategy equilibria, and to avoid trivial cases, we focus on the scenario where an equilibrium with trade credit exists in Proposition 1, i.e., $\delta \geq M_{u,p}\left(C + \frac{cD}{2}\right) - (v_c - c)$.

Clearly, depending on whether each firm chooses trade credit or not, there are four possible price subgames in the second stage: one with both offering trade credit, one with both offering COD, and two asymmetric ones where one firm offers trade credit while the other offers COD. The former two have already been studied in Section 3. By further investigating the two asymmetric subgames and comparing the firms' payoffs under each subgame, which we detail in Appendix B.3, we obtain the following result on their equilibrium choice between trade credit and COD.

PROPOSITION 7. *When $\delta \geq M_{u,m}(C)$, the unique equilibrium is that both firms choose to offer trade credit, and then set the price at $c + \min\{\delta + (v - c), M_{u,p}(C)\}$.*

Proposition 7 confirms that when trade credit is sufficiently valuable to customers, both firms offer trade credit. To verify that this is indeed an equilibrium outcome, consider Firm 1's incentive to deviate from trade credit to COD when it expects its competitor (Firm 2) to offer trade credit and subsequently set a price p. Clearly, as trade credit is valuable to customers, to win over demand from Firm 2 in the price subgame, Firm 1 will have to lower its price to at least $p - \delta$ so that customers' surplus when purchasing from Firm 1 is no less than that from Firm 2. Therefore, when δ is sufficiently large, to unilaterally deviate to COD would only be harmful, which leads to the outcome that both of them offer trade credit in equilibrium and split the market with a positive margin in the second stage. Note that while the vertical benefit of trade credit sustains the equilibrium outcome that both firms offer trade credit in the first stage, the result that the firms can maintain a positive markup in the price competition is driven by the horizontal benefit of trade credit as identified earlier. In fact, when $\delta < M_{u,p}(C)$, the profit margin that the two firms can obtain by offering trade credit is greater than the trade credit premium, suggesting that the horizontal effect of trade credit may actually allow the firms to extract a markup higher than how much the customers value the trade credit.

However, as δ decreases, even though the firms would still be better if both offered trade credit compared to if both offered COD, they may have the incentive to deviate unilaterally with the hope to win over the entire market by offering COD and then pricing out its competitor. In fact, when $\delta < M_{u,m}(C)$, a prisoner's dilemma can arise where both firms offer COD. For ease of exposition, we refer the reader to Appendix B.3 for related technical details.

The above results reveal an intriguing insight about the complementarity between the horizontal and vertical benefits of trade credit from the suppliers' perspective. On the one hand, when the suppliers can freely choose whether to offer trade credit, the horizontal benefit of trade credit is achieved only if there is also a considerably large vertical benefit. In such a scenario, the horizontal effect can allow the suppliers to gain a margin higher than the premium that the vertical benefit contributes. On the other hand, in the absence of the horizontal effect (e.g., with unlimited financial capacity), the vertical benefit of trade credit, if it is offered, would simply be enjoyed by the customers rather than the suppliers as they would engage in a fierce competition and set their prices to their marginal costs. Moreover, the above results also reveal that in a perfectly competitive setting, trade credit may not necessarily be offered despite its vertical value for the customers. When this value is not large enough, the incentive to undercut each other may simply drive the suppliers to offer COD contract only.

7. Conclusion

Trade credit is a common business practice in many industries. However, it is puzzling why even small or financially disadvantaged suppliers extend trade credit to financially stronger buyers.

Complementing the prior literature that rationalizes this puzzle through vertical supply chain interactions, this paper examines trade credit from a competition aspect and identifies a horizontal benefit of trade credit. We find that it is the financial risk faced by suppliers that allows them to use trade credit to soften competition. When offering trade credit, firms face a fundamental tradeoff between financial risk and operational margin. Unlike cash on delivery, with trade credit, a firm facing financial risk is less willing to undercut its competitor, since a gain in market share leads to a larger amount of capital sunk in the goods sold and hence a (disproportionately) larger financing cost. Due to this role, trade credit allows firms to maintain positive margins under the classic Bertrand competition framework. Furthermore, we find that when one firm is financially stronger than its competitor, trade credit may allow the stronger firm to predate the weaker competitor. We also show how firms' financial and physical capacities are connected through trade credit, and in particular how constrained financial capacity may have a similar pricing impact to constrained physical capacity. Finally, we identify the complementarity between the horizontal and vertical benefits of trade credit, and we find that trade credit arises in equilibrium when the vertical benefit is sufficiently large.

Note that to isolate the impact of trade credit on horizontal competition, we have weakened the role of buyers and assumed that suppliers' financing costs are due to liquidity shocks not directly related to buyers. However, it is possible that when offering trade credit, suppliers are exposed to buyers' credit risk. This credit risk could potentially serve a similar role to the suppliers' internal liquidity shocks that we modeled in this paper, with the slight difference that a supplier's exposure to buyers' credit risk is related to its market share. Therefore, we hypothesize that it may become even more costly for a supplier to undercut its competitor as a larger market share not only ties up more capital, as shown in this paper, but also increases credit risk, and hence, trade credit could actually be more effective in softening competition. In addition, for tractability, we focus on firms with installed capacity. However, as shown in Section 5, relative to COD, trade credit is more valuable as firms' physical capacities increase, leading to the conjecture that trade credit also allows firms to support larger physical capacities.

Our results complement some of the existing empirical findings on the use of trade credit under competition. For example, Fisman and Raturi (2004) document that trade credit is widely used in competitive industries. Our results provide a possible theoretical explanation for this phenomenon. Moreover, we also find that the benefit of trade credit in softening competition is greater if buyers' WTP for a product is higher. This is consistent with the finding in Giannetti et al. (2011) that suppliers that offer products with higher added value tend to extend more trade credit. Finally, our results also lead to some new testable hypotheses. For example, our model suggests that trade credit serves as a substitute for constraints in physical capacity in softening competition, indicating

that trade credit may be used more heavily in industries with overcapacity. We would also expect that with dispersion in financial capacities, trade credit may allow financially stronger firms to obtain larger market shares. More generally, our results suggest that through the effect of trade credit on horizontal competition, financial shocks (either industry-wide or firm-specific) may yield significantly stronger pricing impact with trade credit than without it. The empirical investigation of these questions is left for future research.

References

Allen, B., M. Hellwig. 1986. Bertrand-Edgeworth oligopoly in large markets. *The Review of Economic Studies* **53**(2) 175–204.

Aydin, G., V. Babich, D. Beil, Z. Yang. 2011. Decentralized supply risk management. *The Handbook of Integrated Risk Management in Global Supply Chains* 387–424.

Babich, V. 2010. Independence of capacity ordering and financial subsidies to risky suppliers. *Manufacturing & Service Operations Management* **12**(4) 583–607.

Babich, V., A.N. Burnetas, P.H. Ritchken. 2007. Competition and diversification effects in supply chains with supplier default risk. *Manufacturing & Service Operations Management* **9**(2) 123–146.

Babich, V., M.J. Sobel. 2004. Pre-IPO operational and financial decisions. *Management Science* **50**(7) 935–948.

Babich, V., C. S Tang. 2012. Managing opportunistic supplier product adulteration: Deferred payments, inspection, and combined mechanisms. *Manufacturing & Service Operations Management* **14**(2) 301–314.

Barrot, J.-N. 2016. Trade credit and industry dynamics: Evidence from trucking firms. *The Journal of Finance* **71**(5) 1975–2016.

Bertrand, J. 1883. *Review of Walrass théorie mathématique de la richesse sociale and Cournots recherches sur les principes mathematiques de la theorie des richesses in Cournot oligopoly: Characterization and applications. edited by A. F. Daughety*. Cambridge University Press.(1988).

Biais, B., C. Gollier. 1997. Trade credit and credit rationing. *Review of Financial Studies* **10**(4) 903–937.

Boyabatlı, O., L.B. Toktay. 2011. Stochastic capacity investment and flexible vs. dedicated technology choice in imperfect capital markets. *Management Science* **57**(12) 2163–2179.

Brennan, M.J., V. Miksimovic, J. Zechner. 1988. Vendor financing. *Journal of Finance* **43**(5) 1127–1141.

Brick, I.E., W.K.H. Fung. 1984. Taxes and the theory of trade debt. *Journal of Finance* **39**(4) 1169–1176.

Burkart, M., T. Ellingsen. 2004. In-kind finance: A theory of trade credit. *American Economic Review* **94**(3) 569–590.

Buzacott, J.A., R.Q. Zhang. 2004. Inventory management with asset-based financing. *Management Science* **50**(9) 1274–1292.

Chod, J. 2015. Inventory, risk shifting, and trade credit. Working paper, Boston College.

Cuñat, V. 2007. Trade credit: suppliers as debt collectors and insurance providers. *Review of Financial Studies* **20**(2) 491–527.

Dada, M., Q. Hu. 2008. Financing newsvendor inventory. *Operations Research Letters* **36**(5) 569–573.

Dasgupta, P., E. Maskin. 1986a. The existence of equilibrium in discontinuous economic games, I: Theory. *The Review of economic studies* **53**(1) 1–26.

Dasgupta, P., E. Maskin. 1986b. The existence of equilibrium in discontinuous economic games, II: Application. *The Review of Economic Studies* **53**(1) 27–41.

Dastidar, K. G. 1995. On the existence of pure strategy bertrand equilibrium. *Economic Theory* **5**(1) 19–32.

Dixon, H. 1984. The existence of mixed-strategy equilibria in a price-setting oligopoly with convex costs. *Economics Letters* **16**(3) 205–212.

Dong, L., S. Y. Tang, B. Tomlin. 2015. Production chain disruptions: Inventory, preparedness, and insurance. Working Paper.

Dong, L., B. Tomlin. 2012. Managing disruption risk: The interplay between operations and insurance. *Management Science* **58**(10) 1898–1915.

Edgeworth, F. Y. 1897. La teoria pura del monopolio. *Giornale degli economisti* 13–31.

Emery, G.W. 1984. A pure financial explanation for trade credit. *Journal of Financial and Quantitative Analysis* **19**(3) 271–285.

Fabbri, D., L Klapper. 2013. Bargaining power and trade credit. *Journal of Corporate Finance* Forthcoming.

Fabbri, D., A. M. C Menichini. 2010. Trade credit, collateral liquidation, and borrowing constraints. *Journal of Financial Economics* **96**(3) 413–432.

Fabra, N., N. H. von der Fehr, D. Harbord. 2006. Designing electricity auctions. *The RAND Journal of Economics* **37**(1) 23–46.

Federal Reserve Board. 2014. Financial accounts of the united states (third quater 2014). URL `http://www.federalreserve.gov/releases/z1/current/z1.pdf`. Accessed at February 26, 2015.

Federgruen, A., M. Wang. 2010. Inventory subsidy versus supplier trade credit in decentralized supply chains. Working Paper, Columbia Business School.

Ferris, J.S. 1981. A transactions theory of trade credit use. *Quarterly Journal of Economics* **96**(2) 243–270.

Fisman, R., M. Raturi. 2004. Does competition encourage credit provision? evidence from african trade credit relationships. *Review of Economics and Statistics* **86**(1) 345–352.

Frank, M., V. Maksimovic. 2004. Trade credit, collateral, and adverse selection. Unpublished manuscript, University of Maryland.

Froot, K. A, D. S Scharfstein, J. C Stein. 1993. Risk management: Coordinating corporate investment and financing policies. *The Journal of Finance* **48**(5) 1629–1658.

Giannetti, M., M. Burkart, T. Ellingsen. 2011. What you sell is what you lend? explaining trade credit contracts. *Review of Financial Studies* **24**(4) 1261–1298.

Gupta, D., L. Wang. 2009. A stochastic inventory model with trade credit. *Manufacturing & Service Operations Management* **11**(1) 4–18.

Huh, W. T., G. Janakiraman, A. Muharremoglu, A. Sheopuri. 2011. Inventory systems with a generalized cost model. *Operations research* **59**(4) 1040–1047.

Hyndman, K., G. Serio. 2010. Competition and inter-firm credit: Theory and evidence from firm-level data in indonesia. *Journal of Development Economics* **93**(1) 88–108.

Kim, S., H. Shin. 2012. Sustaining production chains through financial linkages. *The American Economic Review* **102**(3) 402–406.

Klapper, L. 2006. The role of factoring for financing small and medium enterprises. *Journal of Banking and Finance* **30**(11) 3111–3130.

Klapper, L., L. Laeven, R. Rajan. 2012. Trade credit contracts. *Review of Financial Studies* **25**(3) 838–867.

Kouvelis, P., W. Zhao. 2012. Financing the newsvendor: supplier vs. bank, and the structure of optimal trade credit contracts. *Operations Research* **60**(3) 566–580.

Kreps, D. M, J. A Scheinkman. 1983. Quantity precommitment and Bertrand competition yield Cournot outcomes. *The Bell Journal of Economics* **14**(2) 326–337.

Lai, G., L.G. Debo, K. Sycara. 2009. Sharing inventory risk in supply chain: The implication of financial constraint. *Omega* **37**(4) 811–825.

Lee, H-H., J. Zhou. 2015. Collaboration or competition? the role of trade credit in supply chains. Working Paper.

Lee, Y.W., J.D. Stowe. 1993. Product risk, asymmetric information, and trade credit. *Journal of Financial and Quantitative Analysis* **28**(2) 285–300.

Levitan, R., M. Shubik. 1972. Price duopoly and capacity constraints. *International Economic Review* 111–122.

Li, L., M. Shubik, M. J Sobel. 2013. Control of dividends, capital subscriptions, and physical inventories. *Management Science* **59**(5) 1107–1124.

Long, M.S., I.B. Malitz, S.A. Ravid. 1993. Trade credit, quality guarantees, and product marketability. *Financial Management* 117–127.

Longhofer, S.D., J.A.C. Santos. 2003. The paradox of priority. *Financial Management* **32**(1) 69–81.

Loten, A. 2012. Small Firms' Big Customers Are Slow to Pay. *Wall Street Journal* .

Luo, W., K. Shang. 2013. Managing inventory for entrepreneurial firms with trade credit and payment defaults. *Available at SSRN* .

Maskin, E. 1986. The existence of equilibrium with price-setting firms. *The American Economic Review* 382–386.

McMillan, J., C. Woodruff. 1999. Interfirm relationships and informal Credit in Vietnam*. *Quarterly Journal of Economics* **114**(4) 1285–1320.

Meltzer, A.H. 1960. Mercantile credit, monetary policy, and size of firms. *The Review of Economics and Statistics* **42**(4) 429–437.

Ng, C.K., J.K. Smith, R.L. Smith. 1999. Evidence on the determinants of credit terms used in interfirm trade. *Journal of Finance* **54**(3) 1109–1129.

Paul, S. Y, R. Boden. 2012. *Getting paid: lessons for and from SMEs*. ACCA (Association of Chartered Ceterified Accountants).

Petersen, M.A., R.G. Rajan. 1997. Trade credit: theories and evidence. *Review of Financial Studies* **10**(3) 661–691.

Peura, Heikki, Derek W Bunn. 2015. Dynamic pricing of peak production. *Operations Research* **63**(6) 1262–1279.

Pike, R., N. S. Cheng, L. Chadwick. 1998. *Managing Trade Credit for Competitive Advantage: A Study of Large UK Companies*. Chartered Institute of Management Accountants.

Porteus, E. L, H. Shin, T. I Tunca. 2010. Feasting on leftovers: Strategic use of shortages in price competition among differentiated products. *Manufacturing & Service Operations Management* **12**(1) 140–161.

Rui, H., G. Lai. 2015. Sourcing with deferred payment and inspection under supplier product adulteration risk. *Production and Operations Management* **24**(6) 934–946.

Schwartz, R.A. 1974. An economic model of trade credit. *Journal of Financial and Quantitative Analysis* **9**(4) 643–657.

Selten, Reinhard. 1975. Reexamination of the perfectness concept for equilibrium points in extensive games. *International journal of game theory* **4**(1) 25–55.

Shaked, A., J. Sutton. 1983. Natural oligopolies. *Econometrica* 1469–1483.

Smith, J.K. 1987. Trade credit and informational asymmetry. *Journal of Finance* **42**(4) 863–872.

Sufi, A. 2009. Bank lines of credit in corporate finance: An empirical analysis. *Review of Financial Studies* **22**(3) 1057–1088.

Tirole, J. 1988. *The Theory of Industrial Organization*. MIT press.

Wambach, A. 1999. Bertrand competition under cost uncertainty. *International Journal of Industrial Organization* **17**(7) 941–951.

Weibull, J. W. 2006. Price competition and convex costs. Tech. rep., SSE/EFI Working Paper Series in Economics and Finance.

Williams, F. 2008. World Bank urged to lift trade credit finance. *Financial Times* Accessed at December 20, 2014.

Wilner, B. S. 2000. The exploitation of relationships in financial distress: The case of trade credit. *The Journal of Finance* **55**(1) 153–178.

Xu, X., J.R. Birge. 2004. Joint production and financing decisions: modeling and analysis. Available at SSRN: http://ssrn.com/paper=652562.

Yang, S. A., J. R. Birge. 2009. How Inventory is (Should Be) Financed: Trade Credit in Supply Chains with Demand Uncertainty and Costs of Financial Distress. *SSRN eLibrary* doi:10.2139/ssrn.1734682.

Yang, S. A., J. R Birge. 2011. Trade credit in supply chains: multiple creditors and priority rules. *Available at SSRN 1840663* .

Appendix A: List of Notation

Table 1 summarizes a list of notation.

Appendix B: Supplemental Results

B.1. Supplemental Results for Section 3

Lemma B.1 *With trade credit, there exists a mixed strategy equilibrium when $v > c + M_{u,m}(C)$, in which the prices set by the firms follow the cumulative distribution function in the range $[c + M_{u,m}(C), v]$ as follows:*

$$H_i(p) = \frac{[p - c - M_{u,m}(C)]D}{(p-c)D + z(C) - z(C - cD)}, \quad i = 1, 2, \tag{5}$$

The firms' corresponding expected profits are: $\pi_{u,i}^m = -z(C)$ for $i = 1, 2$.

B.2. Supplemental Results for Section 4

Proposition B.1 *With trade credit and asymmetric financial capacities,*

1. *for $C_2 \leq C_1 - \frac{cD}{2}$ and $v - c \in [M_{u,m}(C_1), M_{u,m}(C_2)]$, or $C_1 - \frac{cD}{2} < C_2 \leq \Psi_2^b$ and $v - c \in [M_{u,m}(C_1), M_{u,p}(C_2 + \frac{cD}{2})]$, only the asymmetric pure strategy equilibrium exists;*

2. *for $C_2 \leq C_1 - \frac{cD}{2}$ and $v - c \geq M_{u,m}(C_2)$, both firms (weakly) prefer the mixed equilibrium to the asymmetric pure equilibrium;*

3. *for $C_2 \in \left(C_1 - \frac{cD}{2}, C_1\right)$ and $v - c \in [M_{u,p}\left(C_2 + \frac{cD}{2}\right), M_{u,m}(C_2)]$, only the symmetric pure strategy equilibrium exists;*

4. *for $C_2 \in \left(C_1 - \frac{cD}{2}, \Psi_2^a\right)$ and $v - c \geq M_{u,m}(C_2)$, Firm 1 prefers the mixed strategy equilibrium while Firm 2 prefers the symmetric pure strategy equilibrium;*

5. *for $C_2 \in (\Psi_2^a, C_1)$ and $v - c \geq M_{u,m}(C_2)$, both firms prefer the symmetric pure strategy equilibrium.*

Table 1   Notation

D	demand realization
c	unit production cost
K	firm's physical capacity, $i = 1, 2$
C_i	firm i's financial capacity, $i = 1, 2$
v_c	buyers' willingness to pay (WTP) with cash on delivery
v	buyers' WTP with trade credit
δ	trade credit premium, $\delta = v - v_c$
$z(x)$	cost of financing with financial capacity x
$\pi_{j,i}^k$	firm i's equilibrium profit under scenarios j and k, with $j = u, c$ representing whether the physical capacity is binding (u represents $K > D$ and c represents $D \in (K, 2K)$), and $k = c, s, a, m$ represents the contract form and type of equilibrium. Specifically, $k = c$ represents COD, $k = s, a, m$ represents the symmetric pure strategy, asymmetric pure strategy, and mixed strategy equilibria with trade credit
$p_{i,j}^k$	firm i's equilibrium price under scenarios j and k
$M_{u,p}(x)$	$M_{u,p}(x) = \frac{2}{D}\left[z(x - cD) - z\left(x - \frac{cD}{2}\right)\right]$
$M_{u,m}(x)$	$M_{u,m}(x) = \frac{1}{D}\left[z(x - cD) - z(x)\right]$
Ψ_2^a	$\Psi_2^a \in \left(C_1 - \frac{cD}{2}, C_1\right)$ satisfies $M_{u,m}(\Psi_2^a) = M_{u,p}(C_1)$
Ψ_2^b	$\Psi_2^b \in \left(C_1 - \frac{cD}{2}, C_1\right)$ satisfies $M_{u,m}(C_1) = M_{u,p}\left(\Psi_2^b + \frac{cD}{2}\right)$
$m_{c,p}$	$m_{c,p} = \frac{\left[z(C - cK) - z\left(C - \frac{cD}{2}\right)\right]}{K - \frac{D}{2}}$
$\overline{m}_{c,p}$	$\overline{m}_{c,p} = m_{c,p} + \frac{z(C - cK) + z(C - c(D - K)) - 2z\left(C - \frac{cD}{2}\right)}{D - K}$
$m_{c,m}$	$m_{c,m} = \frac{z(C - cK) - z[C - c(D - K)]}{2K - D}$

B.3.   Supplemental Results for Section 6

To analyze the two-stage game as laid out in Section 6, we first derive equilibrium prices for when just one firm offers trade credit (Appendix B.3.1), and then endogenize the choice of trade credit (Appendix B.3.2). Proofs of related results are in Appendices C.4. In this section, we use superscript (i, j), where $i, j = T, C$, to represent the two firms' choice of contract forms. For example, (C, T) represents the case that Firm 1 offers COD and Firm 2 offers TC.

B.3.1.   Price Game Equilibrium with Asymmetric Trade Credit Choice

Without loss of generality, we focus on the price subgame that Firm 2 offers trade credit and Firm 1 offers a COD contract. Under such contract choices, firms set prices p_i for $i = 1, 2$. Then firm 2 serves demand

$$q_2 = \begin{cases} D & p_2 - p_1 < \delta \\ D/2 & p_2 - p_1 = \delta \\ 0 & p_2 - p_1 > \delta. \end{cases} \tag{6}$$

while Firm 1 serves $q_1 = D - q_2$. Lemma B.2. establishes the possible equilibria, which can be in pure strategies (symmetric or asymmetric) or mixed strategies depending on the magnitude of δ.

Lemma B.2 *In the price game where only Firm 2 offers trade credit, there exist the following equilibria.*

(A1) *When $\delta \leq M_{u,p}\left(C + \frac{cD}{2}\right)$, there exists an asymmetric equilibrium where Firm 1 serves the entire demand, with profits*

$$\Pi_{1,A_1}^{C,T} = (\min\{v_c, c - \delta + M_{u,p}\left(C + \frac{cD}{2}\right)\} - c)D - z(C + (\min\{v_c, c - \delta + M_{u,p}\left(C + \frac{cD}{2}\right)\} - c)D)$$
(7)

$$\Pi_{2,A_1}^{C,T} = -z(C).$$
(8)

(A2) *When $\delta \geq M_{u,p}(C)$, there exists an asymmetric equilibrium where Firm 2 serves the entire demand, with profits*

$$\Pi_{1,A_2}^{C,T} = -z(C)$$
(9)

$$\Pi_{2,A_2}^{C,T} = \delta D - z(C - cD).$$
(10)

(S) *When $\delta \in \left[M_{u,p}\left(C + \frac{cD}{2}\right), M_{u,p}(C)\right]$, a "symmetric" equilibrium exists where the firms split the market equally, with $p_1 = c$, $p_2 = c + \delta$, and profits*

$$\Pi_{1,S}^{C,T} = -z(C)$$
(11)

$$\Pi_{2,S}^{C,T} = \frac{\delta D}{2} - z\left(C - \frac{cD}{2}\right).$$
(12)

(M1) *When $\delta \in [M_{u,m}(C), x_{B_1}]$ and $v \geq c + M_{u,m}(C)$, there exists a mixed strategy equilibrium with profits*

$$\Pi_{1,M_1}^{C,T} = -\delta D + z(C - cD) - z(C) - z(C + (\underline{p} - c)D)$$
(13)

$$\Pi_{2,M_1}^{C,T} = -z(C).$$
(14)

where $\underline{p} = c - \delta + \frac{1}{D}(z(C - cD) - z(C))$, and x_{B_1} is solved from $x = \frac{1}{D}\left(z(C - cD) - z(C + (v - x - c)D)\right)$.

**(M2)** *When $\delta \geq x_{B_1}$, there exists a mixed strategy equilibrium with profits*

$$\Pi_{1,M_2}^{C,T} = -z(C)$$
(15)

$$\Pi_{2,M_2}^{C,T} = \delta D - z(C - cD).$$
(16)

As shown in Lemma B.2, pure and mixed strategy equilibria may coexist. The following proposition identifies the Pareto-dominating equilibrium (when there is one) by comparing firms' profits under different equilibria.

Proposition B.2 *When only Firm 2 offers trade credit, the (weakly) Pareto-dominating price equilibrium is as follows.*

1. *For $\delta \leq M_{u,p}\left(C + \frac{cD}{2}\right)$, the mixed strategy equilibrium (M1) weakly Pareto dominates.*

2. *For $\delta \in [M_{u,p}\left(C + \frac{cD}{2}\right), M_{u,m}(C))$, there is no Pareto-dominating equilibrium: firm 1 prefers the mixed strategy equilibrium (M1) while firm 2 prefers the symmetric pure strategy equilibrium (S).*

3. *For $\delta \in [M_{u,m}(C), M_{u,p}(C)]$. The symmetric pure strategy equilibrium (S) weakly dominates.*

Figure 6   Existence of different equilibria when only Firm 2 offers trade credit.
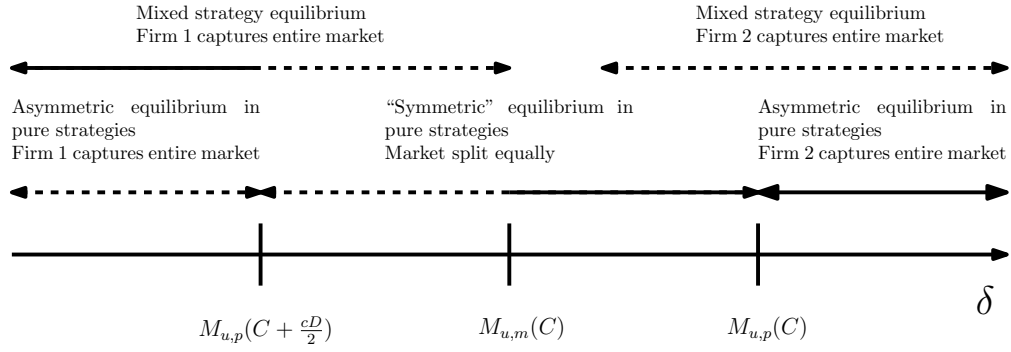
Note. Bold stroke: Pareto-dominant equilibrium.

4. *For $\delta > M_{u,p}(C)$, the asymmetric pure strategy equilibrium (A2) with weakly dominates.*

The existence of different equilibria, as well as the Pareto dominating one, is illustrated in Figure 6. As shown, there is generally a Pareto-dominating equilibrium, apart from when $\delta \in \left[M_{u,p}\left(C + \frac{cD}{2} \right), M_{u,m}(C) \right)$. In this range, the trade credit firm prefers the symmetric pure strategy equilibrium while the cash contract firm prefers the mixed strategy equilibrium.

B.3.2.   The choice between trade credit and COD   The two firms' choice between trade credit and COD depends on their profits under different price subgames as specified in the earlier sections of the paper (for (T,T) and (C,C)) and the previous section (for (C,T)). We have already shown in Proposition 7 that (T,T) is the unique equilibrium for $\delta \geq M_{u,M}(C)$. In the rest of this section, we examine the equilibrium contract choice for $\delta > M_{u,M}(C)$.

We focus on the scenario where $\delta < M_{u,p}\left(C + \frac{cD}{2} \right)$. In this case, under (C,T), the Pareto-dominating equilibrium is (M1), and Firm 1's profit is $\Pi_{1,A_1}^{C,T}$, which is greater than his profit under (T,T) as specified in Proposition 1. Therefore, if Firm 2 offers trade credit, Firm 1 has the incentive to deviate to COD, and hence (T,T) is not an equilibrium. Intuitively, this is because that when δ is small, Firm 2 can win over the whole market by only lowering the price a little bit. The resulting profit is higher than when he also offers trade credit and split the market with Firm 1. Symmetrically, we can see that Firm 2 has no incentive to deviate from (C,T) to (T,T).

Next, note that under (C,T), Firm 2's payoff is $-z(C)$, which is the same as his payoff under (C,C). Therefore, Firm 1 has no (strict) incentive to deviate to COD if his competitor offers COD. Therefore, (C,T) is an equilibrium. Symmetrically, Firm 2 has no incentive to deviate from (C,C) to (C,T) either, and hence (C,C) is also an equilibrium.

In summary, for $\delta < M_{u,p}\left(C + \frac{cD}{2} \right)$, there exists three Nash equilibria in the contract form stage: (C,C), (T,C), and (C,T). Note that while both firms are better off under (T,T) than (C,C), as shown in Proposition 1, (C,C) is an equilibrium while (T,T) is not. In other words, when the vertical value of trade credit is low, a Prisoner's Dilemma may arise where both firms offer COD. Further, we observe multiple equilibria in this case. To choose which one is more preferable/stable, one often apply different equilibrium refinements, such

as trembling-hand perfect equilibrium Selten (1975). However, such refinement is beyond the scope of the current paper and we leave it to future research.

Finally, note that when $\delta \in \left(M_{u,p}\left(C + \frac{cD}{2}\right), M_{u,m}(C)\right)$, there is no Pareto-dominating equilibrium in the price subgame. If both firms "believe" (M1) will be the equilibrium in the second stage, we can again show that (C, C) is a Nash equilibrium for the first stage. The details are similar to the previous case and are omitted here.

Appendix C: Proofs

C.1. Proofs for Section 3 and Appendix B.1

Proof of Lemma 1. The price competition follows the classic Bertrand competition result, and the total profit is the operational profit, zero in this case, minus the financing cost. $\square$

Proof of Proposition 1. When both firms offer contracts with trade credit, the firms' expected total profits by choosing the unit trade credit price p_i are

$$\pi_i = (p_i - c)q_i - z(C - cq_i). \tag{17}$$

In a symmetric pure strategy equilibrium, both firms would set $p_i = p \in [c, v]$ and each serves $q_i = \frac{D}{2}$. The resulting expected profits are:

$$\pi_E = \frac{(p - c)D}{2} - z\left(C - \frac{cD}{2}\right). \tag{18}$$

For this to be an equilibrium, both undercutting and increasing prices will be unprofitable. When deviating down from this (undercutting the competitor and hence serving the entire demand) would give profit: $\pi_D = (p - c)D - z(C - cD)$. Deviating up, on the other hand, would simply yield $\pi_U = -z(C)$. For this equilibrium to exist, the trade credit price p and a firm's financial conditions need to satisfy the following conditions:

$$\frac{(p - c)D}{2} - z\left(C - \frac{cD}{2}\right) \geq (p - c)D - z(C - cD), \tag{19}$$

$$\frac{(p - c)D}{2} - z\left(C - \frac{cD}{2}\right) \geq -z(C). \tag{20}$$

Or equivalently, for both firms, $p \in [\underline{p}, \overline{p}]$, where $\underline{p} = c + M_{u,p}\left(C + \frac{cD}{2}\right)$ and $\overline{p} = c + M_{u,p}(C)$.

In addition, note that the equilibrium price also has to satisfy $p \in [c, v]$. Therefore, the symmetric equilibrium exists if and only if $[\underline{p}, \overline{p}] \cap [c, v] \neq \emptyset$. As $z(\cdot)$ is convex, $z(C) - 2z\left(C - \frac{cD}{2}\right) + z(C - cD) \geq 0$, and hence, $[\underline{p}, \overline{p}] \neq \emptyset$. Noting that $\underline{p} > c$, $[\underline{p}, \overline{p}] \cap [c, v] \neq \emptyset$ if and only if $\underline{p} \leq v$, that is, $v \geq c + M_{u,p}\left(C + \frac{cD}{2}\right)$. When this condition is satisfied, a symmetric pure strategy equilibrium exists with equilibrium price $p \in \left[c + M_{u,p}\left(C + \frac{cD}{2}\right), \min\{v, c + M_{u,p}(C)\}\right]$. Among them, $p = \min\{v, c + M_{u,p}(C)\}$ is Pareto dominating. $\square$

Proof of Corollary 1. Looking first at $M_{u,p}$, for D, we have

$$\frac{dM_{u,p}}{dD} = \frac{2}{D^2}\left[-cDz'(C - cD) + \frac{cD}{2}z'\left(C - \frac{cD}{2}\right) - z(C - cD) + z\left(C - \frac{cD}{2}\right)\right], \tag{21}$$

which is positive as z is convex decreasing. For C, we have

$$\frac{dM_{u,p}}{dC} = \frac{2}{D}\left[z'(C - cD) - z'\left(C - \frac{cD}{2}\right)\right], \tag{22}$$

which is negative as z is convex decreasing.

With $z(C) = 2\gamma\Phi\left(-\frac{C}{\sigma\sqrt{\tau}}\right)$, we have

$$\frac{dM_{u,p}}{d\tau} = \frac{4\gamma}{D\sigma\tau\sqrt{\tau}}\left[C_a\phi\left(-\frac{C_a}{\sigma\sqrt{\tau}}\right) - C_b\phi\left(-\frac{C_b}{\sigma\sqrt{\tau}}\right)\right], \tag{23}$$

where $C_a := C - cD < C - \frac{cD}{2} =: C_b$ and $\phi := \Phi'$ represents the PDF of the standard normal distribution. The sign of the expressions is determined by the terms in the square brackets; this is positive when

$$\frac{C_a}{C_b} > \frac{\phi\left(-\frac{C_b}{\sigma\sqrt{\tau}}\right)}{\phi\left(-\frac{C_a}{\sigma\sqrt{\tau}}\right)}. \tag{24}$$

The LHS is independent of τ, while the RHS is increasing in τ and we have $\lim_{\tau\downarrow 0} = 0$ and $\lim_{\tau\uparrow\infty} = 1$. Since $\frac{C_a}{C_b} \in (0,1)$, the above expression holds if and only if τ is low enough. This implies that $M_{u,p}$ is first increasing and then decreasing in τ. $\square$

Proof of Corollary 2. Consider two cases. First, when $v > c + M_{u,p}(C)$,

$$\frac{d\pi_{u,i}^s}{dD} = -cz'(C - cD) + cz'\left(C - \frac{cD}{2}\right), \tag{25}$$

which is clearly positive. When $v \le c + M_{u,p}(C)$,

$$\frac{d\pi_{u,i}^s}{dD} = \frac{v-c}{2} + \left(\frac{c}{2}\right)z'\left(C - \frac{cD}{2}\right), \tag{26}$$

which is positive if and only if $v > c - cz'\left(C - \frac{cD}{2}\right)$. Note that due to the convexity of $z(\cdot)$, $M_{u,p}\left(C + \frac{cD}{2}\right) < -cz'\left(C - \frac{cD}{2}\right) < M_{u,p}(C)$. Combining the two scenarios, we have that $\pi_{u,i}^s$ increases in D as long as $v > c - cz'\left(C - \frac{cD}{2}\right)$. Due to the convexity of $z(\cdot)$, $-z'\left(C - \frac{cD}{2}\right)$ increases in D. Therefore, the above condition is equivalent to the condition that D is sufficiently small. That is, $\pi_{u,i}^s$ increases in D when D is small and then decreases in D. $\square$

Proof of Lemma B.1. This lemma is a special case of Proposition 3 with $C_1 = C_2 = C$. $\square$

Proof of Corollary 3. This result follows directly from the proof of Proposition 1. $\square$

Proof of Corollary 4. When v does not bind for the symmetric equilibrium ($v > c + M_{u,p}(C)$ or equivalently C sufficiently high), we have

$$\frac{d(\pi_{u,i}^s - \pi_{u,i}^c)}{dC} = z'(C - cD) - 2z'\left(C - \frac{cD}{2}\right) + z'(C), \tag{27}$$

which is negative if $z'(\cdot)$ is concave, or equivalently $z'''(\cdot) \le 0$.

When v does bind, we have

$$\frac{d(\pi_{u,i}^s - \pi_{u,i}^c)}{dC} = -z'\left(C - \frac{cD}{2}\right) + z'(C), \tag{28}$$

which is positive. $\square$

C.2. Proofs for Section 4 and Appendix B.2

Proof of Proposition 2. Similarly to the proof in Proposition 1, let $\underline{p}_i = c + M_{u,p}\left(C_i + \frac{cD}{2}\right)$ and $\bar{p}_i = c + M_{u,p}(C_i)$. While the convexity of $z(\cdot)$ can guarantee that $[\underline{p}_i, \bar{p}_i] \neq \emptyset$, it is not sufficient to guarantee the existence of the symmetrical pure equilibrium. Instead, the symmetrical pure equilibrium exists if and only if $[\underline{p}_1, \bar{p}_1] \cap [\underline{p}_2, \bar{p}_2] \cap [c, v] \neq \emptyset$. As $C_1 \geq C_2$, $[\underline{p}_1, \bar{p}_1]$ and $[\underline{p}_2, \bar{p}_2]$ overlap if and only if $\bar{p}_1 \geq \underline{p}_2$, that is,

$$z(C_1 - cD) - z\left(C_1 - \frac{cD}{2}\right) \geq z\left(C_2 - \frac{cD}{2}\right) - z(C_2). \tag{29}$$

Or equivalently, $C_2 \geq C_1 - \frac{cD}{2}$. Furthermore, $[\underline{p}_2, \bar{p}_1] \cap [c, v] \neq \emptyset$ if and only if $v \geq \underline{p}_2$, as desired. The equilibrium price and profit follow directly. $\square$

Proof of Proposition 3. We first consider the existence condition for the asymmetric pure strategy equilibrium. Assume that $p_1 < p_2$ so that Firm 1 would serve the entire demand. The corresponding profits are:

$$\pi_1 = (p_1 - c)D - z(C_1 - cD). \tag{30}$$

$$\pi_2 = -z(C_2). \tag{31}$$

For (p_1, p_2) to be an equilibrium, neither firm should have an incentive to deviate. First, Firm 2 would not deviate to a lower price if

$$-z(C_2) \geq (p_1 - c)D - z(C_2 - cD) \tag{32}$$

and would not deviate to an equal price if:

$$-z(C_2) \geq (p_1 - c)\frac{D}{2} - z\left(C_2 - \frac{cD}{2}\right). \tag{33}$$

Equivalently, $p_1 \leq \min\left(\bar{p}_1^L, \bar{p}_1^E\right)$, where

$$\bar{p}_1^L = c + \frac{z(C_2 - cD) - z(C_2)}{D} \tag{34}$$

$$\bar{p}_1^E = c + \frac{2[z\left(C_2 - \frac{cD}{2}\right) - z(C_2)]}{D}. \tag{35}$$

As $z(\cdot)$ is convex, $\bar{p}_1^L \geq \bar{p}_1^E$. Therefore, Firm 2 has no incentive to enter the market when $p_1 \leq \bar{p}_1^E$.

Second, for the equilibrium to exist, we also need to confirm that Firm 1 would not deviate. Setting a price $p_1 < p_2$, Firm 1 would always benefit from increasing its price up to $p_2 - \epsilon$, for a sufficiently small $\epsilon > 0$. The prices would thus need to be essentially equal in any equilibrium. In addition, we need to check for Firm 1's incentives to deviate to prices higher than or equal to p_2 and lose some of its market share. However, unlike for Firm 2, Firm 1's option to deviate depends on p_1. Specifically, we consider the following two scenarios.

1. $p_1 < v$. In this scenario, the problem faced by Firm 1 is symmetric to that of Firm 2. For Firm 1 not to deviate to a higher price and lose the entire market, we need:

$$(p_1 - c)D - z(C_1 - cD) \geq -z(C_1). \tag{36}$$

Similarly, for Firm 1 not to deviate to an equal price and split the market with Firm 2, p_1 satisfies:

$$(p_1 - c)D - z(C_1 - cD) \geq (p_1 - c)\frac{D}{2} - z\left(C_1 - \frac{cD}{2}\right). \tag{37}$$

Note that the price on the right-hand side should be the price set by Firm 2, which is (virtually) equal to p_1.[7] Combining the above two conditions leads to: $p_1 \geq \max\left(\underline{p}_1^H, \underline{p}_1^E\right)$, where

$$\underline{p}_1^H = c + \frac{z(C_1 - cD) - z(C_1)}{D}; \tag{38}$$

$$\underline{p}_1^E = c + \frac{2[z(C_1 - cD) - z\left(C_1 - \frac{cD}{2}\right)]}{D}. \tag{39}$$

As $z(\cdot)$ is convex, we have $\underline{p}_1^H < \underline{p}_1^E$. Therefore, Firm 1 does not deviate if and only if $p_1 \geq \underline{p}_1^E$. Finally, we also need $p_1 \in [c, v)$. Combining the three conditions, the asymmetric equilibrium exists if $[c, v) \cap [\underline{p}_1^E, \overline{p}_1^E] \neq \emptyset$. First, note that $\overline{p}_1^E \geq \underline{p}_1^E$ if and only if

$$z(C_1 - cD) - z\left(C_1 - \frac{cD}{2}\right) \leq z\left(C_2 - \frac{cD}{2}\right) - z(C_2). \tag{40}$$

That is, $C_2 \leq C_1 - \frac{cD}{2}$. Second, we also need $v > \underline{p}_1^E$, that is, $v > c + M_{u,p}(C_1)$. When these conditions are satisfied, asymmetric equilibria exist. Furthermore, when $v > \overline{p}_1^E = c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)$, $p_1 = c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)$ is Pareto dominant. Otherwise, $p_1 = v - \epsilon$ is Pareto dominant.

2. $p_1 = v$. In this scenario, the deviation of equal price and splitting the market do not exist; therefore, we only need to make sure that deviating up and losing the entire market is not profitable, that is:

$$(v - c)D - z(C_1 - cD) \geq -z(C_1), \tag{41}$$

or equivalently, $v \geq c + M_{u,m}(C_1)$. Combining this with the condition that Firm 2 does not deviate, that is, $v \leq c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)$, we have that $p_1 = v$ is an equilibrium for $v \in \left[c + M_{u,m}(C_1), c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)\right]$.

Combining the two scenarios, we notice that the region where $p_1 = v - \epsilon$ is Pareto dominating is included in the region where $p_1 = v$ is Pareto dominating. Therefore, we have:

1. when $C_2 \leq C_1 - \frac{cD}{2}$ and $v > c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)$, the Parato-dominating asymmetric equilibrium is $p_1 = c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)$.

2. when $C_2 \leq \Psi_2^b$ and $v \in \left[c + M_{u,m}(C_1), c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)\right]$, the Pareto-dominating asymmetric equilibrium is $p_1 = v$.

The equilibrium profits follow immediately.

Next, we consider the mixed strategy equilibrium. Let $H_i(p)$ denote the CDF of the equilibrium mixed strategy of firm i. Mixed strategies are set by making the competitor's expected profit equal in all cases:

$$\pi_i(p) = H_j(p)[-z(C_i)] + (1 - H_j(p))[(p - c)D - z(C_i - cD)]. \tag{42}$$

Re-arranging the terms, we have:

$$H_j(p) = \frac{(p - c)D - \pi_i(p) - z(C_i - cD)}{(p - c)D + z(C_i) - z(C_i - cD)}. \tag{43}$$

This can be solved by noting that there will be a continuous support of prices[8] and requiring that the distribution reaches zero and one at the ends. Let $\underline{p}_i$ and $\overline{p}_i$ be the infimum and the supremum of the prices

---

[7] Firm 2 is in fact indifferent in setting any price up to v; however, in equilibrium the prices must be virtually equal.

[8] To show this, we need some technical arguments, which can be found in, for example, Fabra et al. (2006).

of firm i. We need to have $\underline{p}_1 = \underline{p}_2 = \underline{p}$ (to guarantee continuity of $H_j(p)$). Since the expected profits will be equal for any price, we can write $\pi_i(p) = \pi_i$. Setting $H_j(\underline{p}) = 0$ yields

$$\pi_i = (\underline{p} - c)D - z(C_i - cD). \tag{44}$$

The support of prices is bounded from above by v. Continuing to assume that $C_1 \geq C_2$, we have

$$\lim_{p \uparrow v} (H_1(p) - H_2(p)) = (v - \underline{p}) \left(\frac{1}{v - c - M_{u,m}(C_2)} - \frac{1}{v - c - M_{u,m}(C_1)} \right). \tag{45}$$

The limit is positive if $M_{u,m}(C_2) \geq M_{u,m}(C_1)$, which is true for convex $z(\cdot)$. Therefore, we must have that $H_2(v) \leq H_1(v) = 1$. We can use this to get

$$H_1(v) = \frac{(v - c)D - \pi_{2,u}^m - z(C_2 - cD)}{(v - c)D + z(C_2) - z(C_2 - cD)} = 1 \tag{46}$$

$$\pi_{u,2}^m = -z(C_2) \tag{47}$$

$$\pi_{u,1}^m = z(C_2 - cD) - z(C_2) - z(C_1 - cD) \tag{48}$$

as desired, with the corresponding price mixing CDFs: $H_i(p) = \frac{(p - c + M_{u,m}(C_2))D}{(p-c)D + z(C_j) - z(C_j - cD)}$, for $p < v$, and $H_i(p) = 1$ for $p = v$. $\square$

Proof of Proposition B.1. Let us compare the existence conditions of the different equilibria. If $C_2 \geq C_1 - \frac{cD}{2}$ and $v \geq c + M_{u,p}\left(C_2 + \frac{cD}{2}\right)$, the symmetric equilibrium exists. The asymmetric equilibrium exists when $C_2 \leq C_1 - \frac{cD}{2}$ and $v \geq c + M_{u,m}(C_1)$, or $C_2 \leq \Psi_2^b$ and $v - c \in [M_{u,m}(C_1), M_{u,p}\left(C_2 + \frac{cD}{2}\right)]$. Furthermore, the mixed strategy equilibrium exists when $v \geq c + M_{u,m}(C_2)$. We can see that $M_{u,m}(C_2)$ is greater than both $M_{u,m}(C_1)$ and $M_{u,p}\left(C_2 + \frac{cD}{2}\right)$; hence, the mixed equilibrium only exists when a pure strategy equilibrium also exists.

Let us next compare expected profits with the asymmetric and mixed equilibria. Recall that the profits in the asymmetric equilibrium are $\pi_{2,u}^a = -z(C_2)$ and

$$\pi_{1,u}^a = \begin{cases} (v - c)D - z(C_1 - cD) & \text{if } c + M_{u,m}(C_1) < v < c + M_{u,p}\left(C_2 + \frac{cD}{2}\right) \\ 2M_{u,p}\left(C_2 + \frac{cD}{2}\right)D - z(C_1 - cD) & \text{if } v > c + M_{u,p}\left(C_2 + \frac{cD}{2}\right) \end{cases} \tag{49}$$

while the profits in the mixed equilibrium are

$$\pi_{2,u}^m = z(C_2 - cD) - z(C_2) - z(C_1 - cD) \tag{50}$$

$$\pi_{2,u}^m = -z(C_2). \tag{51}$$

Clearly, for Firm 2 the profits are equal. It is easy to see that whenever the two equilibria exist together, the price cap does not bind for the asymmetric equilibrium.

Then, for Firm 1, the mixed strategy profits are higher if

$$z(C_2 - cD) + z(C_2) \geq 2z\left(C_2 - \frac{cD}{2}\right), \tag{52}$$

which is always true for convex z. The mixed strategy equilibrium is hence better when the two equilibria exist together.

Now let us compare the mixed equilibrium with the symmetric pure strategy equilibrium, with profits

$$\pi_{u,i}^s = \begin{cases} (v-c)\frac{D}{2} - z\left(C_i - \frac{cD}{2}\right) & \text{for } p_{u,i}^s = v \\ z(C_1 - cD) - z\left(C_1 - \frac{cD}{2}\right) - z\left(C_i - \frac{cD}{2}\right) & \text{for } p_{u,i}^s = M_{u,p}(C_1). \end{cases} \tag{53}$$

For Firm 2, if the price cap does not bind, the mixed strategy equilibrium is better if

$$-z(C_2) \geq z(C_1 - cD) - z\left(C_1 - \frac{cD}{2}\right) - z\left(C_2 - \frac{cD}{2}\right) \tag{54}$$

$$z\left(C_2 - \frac{cD}{2}\right) - z(C_2) \geq z(C_1 - cD) - z\left(C_1 - \frac{cD}{2}\right), \tag{55}$$

which is never true when the symmetric equilibrium exists. If the price cap does bind, the mixed equilibrium is better when

$$-z(C_2) \geq (v-c)\frac{D}{2} - z\left(C_2 - \frac{cD}{2}\right). \tag{56}$$

Or equivalently,

$$v \leq c + M_{u,p}(C_2) \leq c + M_{u,m}(C_2), \tag{57}$$

which is never true when the mixed equilibrium exists. Therefore, the symmetric equilibrium is always better for Firm 2.

For Firm 1, when the price cap does not bind, we similarly have

$$z(C_2 - cD) - z(C_2) \geq 2z(C_1 - cD) - 2z\left(C_1 - \frac{cD}{2}\right), \tag{58}$$

which is equivalent to $C_2 \leq \Psi_2^a$. When the price cap does bind, that is, $v < c + M_{u,p}(C_1)$, we have

$$z(C_2 - cD) - z(C_2) - z(C_1 - cD) \geq (v-c)\frac{D}{2} - z\left(C_1 - \frac{cD}{2}\right), \tag{59}$$

or equivalently, $v + [c + M_{u,p}(C_1)] \leq 2[c + M_{u,m}(C_2)]$. However, since we know that $c + M_{u,m}(C_2) \leq v \leq c + M_{u,p}(C_1)$ in this case, this is never true. Therefore, for Firm 1, the mixed strategy equilibrium is only better when the price cap does not bind and C_2 is sufficiently low. □

Proof of Corollary 5.   This result follows directly from the proofs of Propositions 2 and 3.

C.3.   Proofs for Section 5

Proof of Lemma 2.   It is well known that in capacity-constrained price competition without trade credit, there is no pure strategy equilibrium (see Tirole 1988). Unlike the traditional Bertrand competition, in this model the firm's financing cost also needs to be taken into consideration, as when $p_i > c$, the firm makes a strictly positive profit and hence the cash on hand will be strictly greater than C, which differs from the unconstrained capacity case.

Setting the expected profit equal for all $p < v$:

$$\pi_i(p) = H_j(p)[(p-c)(D-K) - z(C + (p-c)(D-K))]$$
$$+ (1 - H_j(p))[(p-c)K - z(C + (p-c)K)]. \tag{60}$$

Solving this, we have for $i = 1, 2$,

$$H_i(p) = \frac{(p-c)K - z(C + (p-c)K) - \pi_j}{(p-c)(2K-D) + z(C + (p-c)(D-K)) - z(C + (p-c)K)}. \tag{61}$$

It is obvious that both firms should have the same lower bound of price range. Otherwise, the one with the lower price range should deviate by increasing the lower end of the range to make more profit. Let the lower bound be $\underline{p}$. Setting $H_i(\underline{p}) = 0$, we have for $i = 1, 2$:

$$\pi_i = (\underline{p} - c)K - z(C + (\underline{p} - c)K). \tag{62}$$

On the other hand, we need to have $H_i(v) = 1$ for $i = 1, 2$. Therefore,

$$\pi_i = (v-c)(D-K) - z(C + (v-c)(D-K)). \tag{63}$$

Substitute this back to $\underline{p}$ and we have $\underline{p} = c + \frac{D-K}{K}(v-c)$. $\square$

Proof of Proposition 4. Under the symmetric pure strategy, with equal prices p, the firms' expected profits will be

$$\pi_{E,i} = (p-c)q_i - z(C - cq_i) = (p-c)\frac{D}{2} - z\left(C - \frac{cD}{2}\right). \tag{64}$$

If a firm deviates below, it will serve its capacity K; if it deviates above, the residual demand will be $D - K$. It is clear that deviating down should be done by undercutting and deviating up, by setting the price equal to the price cap. The profits of deviating down and up are

$$\pi_{D,i} = (p-c)K - z(C - cK); \tag{65}$$

$$\pi_{U,i} = (v-c)(D-K) - z(C - c(D-K)), \tag{66}$$

respectively. The pure strategy equilibrium exists if both deviations are unprofitable, that is,

$$(p-c)\frac{D}{2} - z\left(C - \frac{cD}{2}\right) \geq (p-c)K - z(C - cK); \tag{67}$$

$$(p-c)\frac{D}{2} - z\left(C - \frac{cD}{2}\right) \geq (v-c)(D-K) - z(C - c(D-K)). \tag{68}$$

The above conditions can be re-written as $p \in [\underline{p}, \overline{p}]$, where $\overline{p} = c + m_{c,p}$, and

$$\underline{p} = c + \frac{2(D-K)}{D}(v-c) + \frac{2[z\left(C - \frac{cD}{2}\right) - z(C - c(D-K))]}{D}. \tag{69}$$

In addition, note that similar to the case with $D < K$, the firm can also deviate up by setting $p > v$ and not serving any demand. For the equilibrium to exist, this deviation also needs to be unprofitable, that is,

$$(p-c)\frac{D}{2} - z\left(C - \frac{cD}{2}\right) \geq -z(C). \tag{70}$$

That is, $p \geq c + M_{u,p}\left(C + \frac{cD}{2}\right)$.

Combining the above two regions for p with that $p \in [c, v]$, the equilibrium exists if and only if $[\underline{p}, \overline{p}] \cap \left[c + M_{u,p}\left(C + \frac{cD}{2}\right), v\right] \neq \emptyset$. Note that due to the convexity of $z(\cdot)$, $m_{c,p} > M_{u,p}\left(C + \frac{cD}{2}\right)$. Therefore, the sufficient and necessary condition for the equilibrium to exist becomes $v \geq \max\left\{\underline{p}, c + M_{u,p}\left(C + \frac{cD}{2}\right)\right\}$, or equivalently, $v \in \left[c + M_{u,p}\left(C + \frac{cD}{2}\right), c + \overline{m}_{c,p}\right]$.

When this condition is satisfied, the Pareto-dominating equilibrium is $p = \min(v, c + m_{c,p})$. The profits follow immediately. $\square$

Proof of Corollary 6.   When $v \leq c + m_{c,p}$, $\pi_{c,i}^s$ is independent of K. When $v > c + m_{c,p}$, $\frac{d\pi_{c,i}^s}{dK} > 0$ if and only if $\frac{dm_{c,p}}{dK} > 0$, or equivalently,

$$\frac{c}{(2K - D)} \left\{ -z'(C - cK) - \frac{\left[z(C - cK) - z\left(C - \frac{cD}{2}\right)\right]}{c\left(K - \frac{D}{2}\right)} \right\} > 0, \tag{71}$$

which always holds as $z(\cdot)$ is convex decreasing. $\square$

Proof of Proposition 5.   Each firm will choose its mixing distribution so that the expected profit of the competitor is equal for all price p:

$$\pi_i(p) = (p - c)[H_j(p)(D - 2K) + K] - H_j(p)z(C - c(D - K)) - [1 - H_j(p)]z(C - cK). \tag{72}$$

Solving this, we get

$$H_i(p) = \frac{(p - c)K - z(C - cK) - \pi_j}{(p - c)(2K - D) - z(C - cK) + z(C - c(D - K))}. \tag{73}$$

As we need $H_i(v) = 1$, and hence $\pi_i = (v - c)(D - K) - z(C - c(D - K))$. On the other hand, we need $H_i(\underline{p}) = 0$, where $\underline{p}$ is the lowest end of the price range, that is,

$$\pi_i = (\underline{p} - c)K - z(C - cK), \quad i = 1, 2, \tag{74}$$

or equivalently,

$$\underline{p} = c + \frac{D - K}{K}(v - c) + \frac{z(C - cK) - z(C - c(D - K))}{K}. \tag{75}$$

Finally, note that we need the mixing distribution to be positive. The numerator is clearly positive. Therefore, the denominator should also be positive, that is,

$$(\underline{p} - c)(2K - D) - z(C - cK) + z(C - c(D - K)) > 0, \tag{76}$$

or equivalently,

$$\frac{D - K}{K}(v - c) + \frac{z(C - cK) - z(C - c(D - K))}{K} > \frac{z(C - cK) - z(C - c(D - K))}{2K - D}. \tag{77}$$

That is, $v > c + m_{c,m}$ as desired. $\square$

Proof of Corollary 7.   We compare the symmetric pure equilibrium and mixed equilibrium profits according to Propositions 4 and 5. Consider two scenarios based on the symmetric pure equilibrium. For $v < c + m_{c,p}$, we have

$$\pi_{c,i}^s = (v - c)\frac{D}{2} - z\left(C - \frac{cD}{2}\right), \quad i = 1, 2. \tag{78}$$

Comparing with the mixed equilibrium (with trade credit), the pure strategy profit is higher if:

$$(v - c)\frac{D}{2} - z\left(C - \frac{cD}{2}\right) \geq (v - c)(D - K) - z(C - c(D - K)), \tag{79}$$

which holds when the symmetric equilibrium exists.

Second, when $v \geq c + m_{c,p}$, recall the profit under pure equilibrium is:

$$\pi_{c,i}^s = \left(\frac{D}{2K - D}\right)\left[z(C - cK) - z\left(C - \frac{cD}{2}\right)\right] - z\left(C - \frac{cD}{2}\right). \tag{80}$$

Compared with the mixed equilibrium, it is easy to see that the symmetric equilibrium is more appealing if and only if $v > c + m_{c,p}$, which again is guaranteed when the symmetric equilibrium exists. $\square$

Proof of Proposition 6. We will first show that trade credit is more likely to be preferred with higher capacity, that is, $\frac{d(\pi_{c,i}^s - \pi_{c,i}^c)}{dK} \geq 0$. When $v \leq c + m_{c,p}$, this is true when

$$\frac{d}{dK}\left[(v-c)\left(K - \frac{D}{2}\right) - \left(z\left(C - \frac{cD}{2}\right) - z(C + (v-c)(D-K))\right)\right] \geq 0 \tag{81}$$

$$\iff (v-c) - (v-c)z'(C + (v-c)(D-K)) \geq 0, \tag{82}$$

which is clearly true with $z'(\cdot) \leq 0$.

Similarly, when $v > c + m_{c,p}$, we consider whether

$$\frac{d}{dK}\left[\left(\frac{D}{2K-D}\right)\left[z(C - cK) - z\left(C - \frac{cD}{2}\right)\right] - z\left(C - \frac{cD}{2}\right)\right.$$
$$\left. - \left((v-c)(D-K) - z(C + (v-c)(D-K))\right)\right] \geq 0. \tag{83}$$

Here, the derivative of the last two terms is again

$$v - c - (v-c)z'(C + (v-c)(D-K)) \geq 0, \tag{84}$$

while for the first terms, we have

$$\left(\frac{D}{(2K-D)^2}\right)\left(-c(2K-D)z'(C-cK) - 2\left[z(C-cK) - z\left(C - \frac{cD}{2}\right)\right]\right), \tag{85}$$

which is positive as long as $z(\cdot)$ is convex.

By combining the above two scenarios we can show that trade credit becomes more attractive with higher K. Next, we show that trade credit is indeed preferred for some values of $K \in \left(\frac{D}{2}, D\right)$. We can show, as with the above derivations, that $\frac{d\overline{m}_{c,p}}{dK} \geq 0$, and furthermore, $\overline{m}_{c,p} \uparrow \infty$ when $K \uparrow D$. Therefore, with high enough K we will have the pure strategy equilibrium as long as $v - c \geq M_{u,p}\left(C + \frac{cD}{2}\right)$.

Suppose first that $v > c + m_{c,p}$, i.e. the WTP does not bind. Then, when $K = D$, the trade credit and cash profits are $z(C - cD) - 2z\left(C - \frac{cD}{2}\right)$ and $-z(C)$, respectively. With convexity of $z(\cdot)$, trade credit is then preferred. Due to the monotonocity shown above, there exists a K^* such that trade credit is preferred with $K \geq K^*$.

On the other hand, when $v \leq c + m_{c,p}$ (WTP binds), the equilibrium with trade credit is preferred at $K = D$ when

$$(v-c)\left(K - \frac{D}{2}\right) \geq z\left(C - \frac{cD}{2}\right) - z(C + (v-c)(D-K)) \tag{86}$$

$$(v-c)\frac{D}{2} \geq z\left(C - \frac{cD}{2}\right) - z(C), \tag{87}$$

which is exactly the existence constraint for the equilibrium. Hence, $K^* \leq D$ in this case, with equality when the above condition holds with equality. $\square$

C.4. Proofs for Sections 6 and Appendix B.3

Proof of Lemma B.2. We consider three types of equilibria: asymmetric pure strategy ones, symmetric pure strategy ones, and mixed strategy ones.

Asymmetric pure strategy equilibria. Consider first the asymmetric equilibrium where $q_1 = D$ (the firm offering COD captures the entire market). The profits are then

$$\pi_1 = (p_1 - c)D - z(C + (p_1 - c)D) \tag{88}$$

$$\pi_2 = -z(C). \tag{89}$$

As long as $p_1 \geq c$, firm 1 would never deviate as it would both lose profits and add risk. Firm 2 could deviate either to equal price (for market share $D/2$) or to a slightly lower price (for D). Due to convexity of z, the former is more profitable. Deviation does not happen if

$$p_1 \leq c - \delta + \frac{2}{D} \left[z \left(C - \frac{cD}{2} \right) - z(C) \right]. \tag{90}$$

The RHS needs to exceed c, which is true for

$$\delta \leq \frac{2}{D} \left[z \left(C - \frac{cD}{2} \right) - z(C) \right] = M_{u,p} \left(C + \frac{cD}{2} \right), \tag{91}$$

which is the existence condition of this equilibrium. If it exists, the price is set just low enough to discourage deviation. Furthermore, the price cannot exceed v_c. Hence, profits are

$$\pi_{1,A_1}^{0,1} = (\min\{v_c, c - \delta + M_{u,p} \left(C + \frac{cD}{2} \right)\} - c)D - z(C + (\min\{v_c, c - \delta + M_{u,p} \left(C + \frac{cD}{2} \right)\} - c)D) \tag{92}$$

$$\pi_{2,A_1}^{0,1} = -z(C). \tag{93}$$

The other asymmetric equilibrium has $q_2 = D$, with profits

$$\pi_1 = -z(C) \tag{94}$$

$$\pi_2 = (p_2 - c)D - z(C - cD). \tag{95}$$

Firm 1 would always deviate to capture market share if $p_2 \geq c + \delta$, so firm 2 will set this price (just lower) in the equilibrium. Firm 2 could again deviate to either equal price or a higher one, with equal price more attractive. The resulting constraint is

$$\delta \geq \frac{2}{D} \left[z(C - cD) - z \left(C - \frac{cD}{2} \right) \right] = M_{u,p}(C), \tag{96}$$

and we note that there is a gap between this and the previous constraint. Again, the price cannot exceed v (which is ruled out by $v \geq c + \delta$), so that profits are

$$\pi_{1,A_2}^{C,T} = -z(C) \tag{97}$$

$$\pi_{2,A_2}^{C,T} = \delta D - z(C - cD). \tag{98}$$

Symmetric pure strategy equilibrium. In the middle range for δ, there exists a "symmetric" pure strategy equilibrium. That is, prices are set so that $p_2 = p_1 + \delta$, and demand is split equally. Profits are then

$$\pi_1 = (p_1 - c)\frac{D}{2} - z \left(C + \frac{(p_1 - c)D}{2} \right) \tag{99}$$

$$\pi_2 = (p_2 - c)\frac{D}{2} - z \left(C - \frac{cD}{2} \right). \tag{100}$$

The symmetric equilibrium requires $p_1 = c$, as otherwise firm 1 will deviate. For firm 2, this results in the exact constraints employed in the above asymmetric pure strategy equilibrium. Therefore, a symmetric equilibrium will exist when $\delta \in [M_{u,p}(C), M_{u,p}(C + \frac{cD}{2})]$, and hence the profits are:

$$\pi_{1,S}^{C,T} = -z(C) \tag{101}$$

$$\pi_{2,S}^{C,T} = \frac{\delta D}{2} - z\left(C - \frac{cD}{2}\right). \tag{102}$$

Mixed strategy equilibrium. Finally, there can also exist a mixed strategy equilibrium. Following the derivation of mixed strategies in earlier sections, let us write the profits of the firms given their competitors' mixing strategies $H_i(p)$.

$$\pi_1 = H_2(p)[-z(C)] + (1 - H_2(p))[(p-c)D - z(C + (p-c)D)] \tag{103}$$

$$\pi_2 = H_1(p)[-z(C)] + (1 - H_1(p))[(p-c)D - z(C - cD)] \tag{104}$$

Solving for the distributions, we have

$$H_2(p) = \frac{(p-c)D - z(C + (p-c)D) - \pi_1}{(p-c)D - z(C + (p-c)D) + z(C)} \tag{105}$$

$$H_1(p) = \frac{(p-c)D - z(C - cD) - \pi_2}{(p-c)D - z(C - cD) + z(C)}. \tag{106}$$

Note, however, the different ranges of prices set by the firms. Firm 2 can set prices up to v, and should not set prices below $c + \delta$. In essence, this amounts to a shift in the price comparisons. To accommodate this, let us rewrite

$$H_1(p) = \frac{(p + \delta - c)D - z(C - cD) - \pi_2}{(p + \delta - c)D - z(C - cD) + z(C)}. \tag{107}$$

Now, we can solve for the equilibrium as before. We have

$$\pi_1 = (\underline{p} - c)D - z(C + (\underline{p} - c)D) \tag{108}$$

$$\pi_2 = (\underline{p} - c)D - z(C - cD), \tag{109}$$

with $\underline{p}$ defined with respect to c.

The next step is to find which distribution has a higher limit at $p \uparrow v_c = v - \delta$. Calculating the limits, it is $H_1()$ if and only if $\delta \leq \frac{1}{D}(z(C - cD) - z(C + (v - \delta - c)D))$ (which can be written as $\delta \leq x_{B_1}$). This results in two different mixed strategy profit combinations.

First, if $H_1(v)$ is equal to one, we have profits

$$\pi_{1,M_1}^{C,T} = -\delta D + z(C - cD) - z(C) - z(C + (\underline{p} - c)D) \tag{110}$$

$$\pi_{2,M_1}^{C,T} = -z(C). \tag{111}$$

where $\underline{p} = c - \delta + \frac{1}{D}(z(C - cD) - z(C))$. Here, we require $\underline{p} \geq c$, that is, $\delta \leq \frac{1}{D}(z(C - cD) - z(C)) = M_{u,m}(C) \in [M_{u,p}(C), M_{u,p}(C + \frac{cD}{2})]$, which is stricter than the previous condition x_{B_1}. Additionally, we require that $v_c \geq \underline{p}$, that is,

$$v_c \geq c - \delta + M_{u,m}(C) \tag{112}$$

$$v \geq c + M_{u,m}(C). \tag{113}$$

Second, if $H_2(v)$ is equal to one, we have profits

$$\pi_{1,M_2}^{C,T} = -z(C) \tag{114}$$

$$\pi_{2,M_2}^{C,T} = \delta D - z(C - cD) \tag{115}$$

which are equal to the profits in equilibrium (A2). $\quad\square$

Proof of Proposition B.2. The results follow directly from Lemma B.2. $\square$

Proof of Proposition 7.   To see that for $\delta \geq M_{u,m}(C)$, (T,T) is the unique equilibrium, we first show that for Firm 1, his profit under (T,T) is higher than that under (C,T). To see this, note that under (C,T), according to Proposition B.2, under the Pareto-dominating equilibrium, $\pi_1^{C,T} = -z(C)$, which is lower than $\pi_1^{T,T}$ according to Proposition 1. Therefore, (T,T) is an equilibrium, and (C,T) is not. By symmetry, (T,C) is not an equilibrium either.

Next, we show that for Firm 2, his profit under (C,C) is lower than that under (C,T). To see this, note that under (C,C), $\pi_2^{C,C} = -z(C)$. Now consider Firm 2's profit under the Pareto-dominating equilibrium in (C,T). For $\delta \in (M_{u,m}(C), M_{u,p}(C))$, the Pareto-dominating equilibrium is (S), and hence $\pi_2^{C,T} = \frac{\delta D}{2} - z\left(C - \frac{cD}{2}\right)$, which is greater than $-z(C)$. For $\delta > (M_{u,m}(C), M_{u,p}(C))$, the Pareto-dominating equilibrium is (A2), and hence $\pi_2^{C,T} = \delta D - z(C - cD)$, which is also greater than $-z(C)$. Therefore, (C,C) is not an equilibrium.

Combining the above two steps, we can see that (T,T) is the only equilibrium. $\square$

Online Appendix: Supplemental Analysis

D.1.   The impact of factoring

As shown previously, the horizontal benefit of trade credit hinges on the fact that the financing cost the firms incur is more pronounced under trade credit, reducing their incentive to undercut each other. In practice, firms may employ some tools (e.g., insurance) to lower the financing cost. One such tool that is particularly related to trade credit is factoring, which allows the firm to sell (part of) its accounts receivable for immediate cash (at a discount) (Klapper 2006). In this appendix, we study how the existence of a factoring market influences the horizontal benefit of trade credit.

While factoring helps lowering the firm's financing cost, it is not costless for a firm to adopt this instrument. According to Klapper (2006), due to various practical considerations, there are two caveats related to factoring. First, apart from interest, the factor charges a *service fee*, which reflects the costs expended by the factor in completing the transaction, such as evaluating buyers' creditworthiness and verifying the authenticity of the invoices. Second, when factoring their invoices, the firms typically only receive a fraction of the value of invoice, known as the *advance rate*. The remaining is paid to firms after invoices are paid by the firms' buyers. In addition, factoring also imposes indirect costs to the selling firms, such as additional administrative costs and loss of customer goodwill. Such costs are often non-negligible, as indirectly supported by the fact that only a small fraction of trade credit is factored (Klapper 2006).

To incorporate the presence of the factoring market in the current model, we assume that both competing firms have the *option* to sell their account receivables to a factor after making their sales but prior to the financial shock. That is, after the price competition, each firm can choose whether to factor its (entire) accounts receivable. In addition, when applying factoring, the firm faces an advance rate $R_a \in (0,1)$ and a discount factor (interest and fees) $d \in (0,1)$. Therefore, with a selling price p_i and quantity q_i, the firm receives $dR_a p_i q_i$ before the liquidity shock, and $d(1-R_a)p_i q_i$ after. As a result, the total firm's profit is:

$$\pi_i^F = (dp_i - c)q_i - z(C + (dR_a p_i - c)q_i), \tag{116}$$

To understand how the the discount factor d and the advance rate R_a influence the benefit of trade credit, we first consider the impact of d. Intuitively, a lower advance rate makes factoring less attractive. Thus, we set $R_a = 1$, which allows us to show that the horizontal benefit of trade credit is still present when the only friction in the factoring market is the discount factor $d \in (0,1)$.

In this case, note that since the factoring decision is only taken after p_i and q_i are known, the firm factors its accounts receivable if and only if

$$dp_i q_i - z(C + (dp_i - c)q_i) > p_i q_i - z(C - cq_i). \tag{117}$$

By examining the above equation, clearly, if $d = 1$, the firm will factor, as the profits are then simply COD profits. On the other hand, it will not factor with $d = 0$ as it would receive none of the sales revenue. This suggests that the firms will only use factoring above some discount threshold—although the factoring decision will also depend on the endogenously determined price and quantity. This intuition is indeed confirmed by the following proposition, which summarizes the price game equilibrium in the presence of a factoring market.

Proposition D.1 *Under the base model in the paper (Section 3, $C_i = C_j = C$ and $D > K$), suppose that the firms can engage in factoring at advance rate $R_a = 1$ and discount factor $d \in (0, 1)$. Then, under trade credit, there exist thresholds $\underline{d}$ and $\overline{d}$ such that*

1. *if $d < \underline{d}$, the firms do not employ factoring and the equilibrium price is equal to the one without the factoring option.*

2. *if $d \in [\underline{d}, \overline{d}]$, the firms do not employ factoring and the equilibrium price is lower than that without the factoring option.*

3. *if $d > \overline{d}$, the firms employ factoring and the price game equilibrium is $p = c/d$.*

The results in Proposition D.1 are illustrated in Figure 7. In the presence of a factoring market, price competition can be divided into three segments depending on the cost associated with factoring. First, if the factoring market is efficient ($d > \overline{d}$, Region III in Figure 7), the price game intuitively resembles the COD equilibrium of marginal cost pricing, but with a higher $p = \frac{c}{d}$. In other words, while the two firms set their prices above marginal cost, the corresponding profits are absorbed by the factoring market. In the other extreme, when the cost associated with factoring is significant ($d < \underline{d}$, Region I), the firms do not engage in factoring, and Proposition 1 continues to hold. Between these two extremes (Region II), while the firms still do not engage in factoring, the availability of factoring forces them to adjust their prices down to preempt the option, allowing them to partially recover the horizontal benefit of trade credit. In sum, Proposition D.1 suggests that the horizontal benefit of trade credit is present as long as factoring costs are non-negligible.

Figure 7   Equilibrium price under trade credit in the presence of a factoring market with discount factor d.
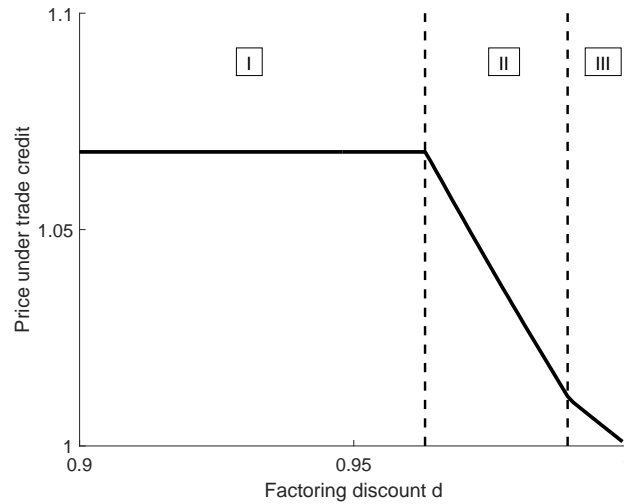
Note. Parameters: production cost $c = 1$; cost of refinancing $c_R = 0.5$, liquidity shock distributed $\mathcal{N}(0, 1)$, (expected) financing cost $z(x) = c_R \Phi(-x)$. Factor advance rate $R_a = 1$.

Regarding the impact of the advance rate R_a, mirroring the case above, let us set $d = 1$, i.e., the firms can factor their accounts receivable without any discount. Consider two extreme cases: on the one hand, when $R_a = 1$, factoring is frictionless, and hence the firm's profit is the same as that under COD. On the other

hand, if $R_a = 0$, the firm's profit is the same as that in Section 3, where trade credit is shown to be beneficial to the firm. Therefore, although the firms always factor their invoices, we can show that, similarly to the impact of d, the horizontal benefit of trade credit is present as long as R_a is not very close to 1, and the benefit is more significant for a lower R_a.

D.2. The horizontal benefit of trade credit under general price competition

In the following section, we show that our results (trade credit softens competition and enhances firms' profitability) remain unchanged under Bertrand competition with elastic demand and general price competition (under mild technical conditions). All proofs are in Appendix D.4.

D.2.1. Bertrand competition with elastic demand. Suppose that instead of the inelastic demand considered above, demand reacts to the price set by the firms. Observing the demand, the two firms simultaneously offer prices p_i ($i = 1, 2$). As long as these prices are below buyers' WTP, the market is split as follows:

$$q_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j, \\ D(p_i)/2 & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j. \end{cases} \tag{118}$$

As in literature (Dastidar 1995), we assume that each firm satisfies all of the demand it faces.

Proposition D.2 *When firms face demand function in (118), there exists a symmetric equilibrium with price $p > c$. The firms' profits with trade credit are higher than those under COD.*

Proposition D.2 confirms that our main result continues to hold under a Bentrand competition with an elastic demand curve. Intuitively, this is because the horizontal benefit of trade credit lays on the fact under trade credit that when one firm under-cuts its competitor, the financing cost associated to a larger production quantity dominates the gain on operational profit when the firm's profit margin is already low. This mechanism is independent of price elasticity.

D.2.2. General (non-Bertrand) price competition. In this section, we examine the case that the two firms compete under a general, non-Bertrand, price competition. In this competition, each firm faces a general demand function $q_i(p_i, p_j)$ that satisfies the following conditions:

$$\frac{\partial q_i}{\partial p_i} < 0; \quad \text{and} \quad \frac{\partial q_i}{\partial p_j} > 0. \tag{119}$$

i.e., the demand faced by firm i decreases in its own price, and increases in its competitor's price. A specific demand function that satisfies this condition is the linear demand curve:

$$q_i(p_i, p_j) = a - p_i + p_j. \tag{120}$$

Proposition D.3 *Suppose each firm faces demand $q_i(p_i, p_j)$ satisfies (119), and there exists unique equilibria to the resulting price games (both under COD and under trade credit). Then the equilibrium price under trade credit is higher than that under COD.*

Further, when the firm's demand function follows (120),

1. *the equilibrium prices under COD is $p = a + c$, and that under trade credit is $p = a + c - cz'(C - ca)$.*

2. *the firm's equilibrium profit under TC is higher than those under COD if and only if*

$$-z'(C - ca) \geq \frac{z(C - ca) - z(C + a^2)}{ca} \qquad (121)$$

The above proposition has two implications. First, even under a general (non-Bertrand) competition, trade credit still results in higher equilibrium prices than COD. This confirms that the principle behind higher prices under trade credit is robust. Simply put, the higher equilibrium price under trade credit is because the risk associated with selling under trade credit reduces the incentive to undercut the competitor's price. Take the linear demand case (120) as an example: Unlike in the Bertrand model, the firms now earns positive profit margin under both COD and trade credit. Since $z(x)$ decreases in x, the markup under trade credit (relative to COD), $-cz'(C - ca)$, is positive. Note that the markup increases in demand intercept a, echoing the result in Corollary 1 that the profit margin under trade credit increases in demand D under Bertrand competition. In addition, the magnitude of $z'(x)$ approximately increases in $z''(x)$. Therefore, the more convex the financing cost function is in this region, the higher markup the firms can support under trade credit.

Second, the proposition characterizes conditions under which trade credit improves firms' profits in equilibrium. Note that in general, between trade credit with COD, a higher operational margin $(p - c)$ under trade credit (than under COD) does not necessarily translate to a higher profit because trade credit intrinsically involves a higher financing cost. In fact, trade credit leads to a higher overall profit if the markup it can support is sufficiently high, or equivalently, when the financing cost function $z(x)$ is sufficiently convex, as depicted in (121). Note that any convex $z(x)$ will automatically satisfy $-z'(C - ca) \geq \frac{z(C - ca) - z(C)}{ca}$. Comparing this expression with (121), we can observe for trade credit to improve the firms' profit, $-z'(x)$ need not only to increases in x, but increases sufficiently fast. We illustrate the condition through the following numerical example and Figure 8.

EXAMPLE 3. Suppose that each firm faces the linear demand (120), and the production costs $c = 1$. The liquidity shock distributed $\mathcal{N}(0, 1)$; The firm's expected financing cost given cash position x is then $z(x) = c_R \Phi(-x)$, i.e., if a firm's financial capacity is less than the liquidity shock, it incurs a refinancing cost $c_R = 0.5$. For simplicity, we assume the willingness-to-pay v is high enough not to bind prices.

Under these parameters, Figure 8 illustrates how the horizontal benefit of trade credit changes according to the demand intercept a (the x-axis) and the financial capacity C (different lines). As shown, when financial capacities are reasonably high, the horizontal benefit of trade credit is positive and increases with demand. Intuitively, this is because when C is large, the reduction in financing cost by shifting from trade credit to COD, $z(C - ca) - z(C + a^2)$, is dominated by the increase in markup under trade credit $-cz'(C - ca)$. However, when the firms become financially weaker (low C), the above tradeoff is flipped. Thus, COD becomes the preferred term, and more so as the market size increases.

In summary, relating the above results to the ones in the main body of the paper, we have seen that our main results — that trade credit results in higher equilibrium prices and profits than COD — are relatively robust under different demand functions.

Figure 8 **The impact of demand parameter a and financial capacities C on the horizontal benefit of trade credit**
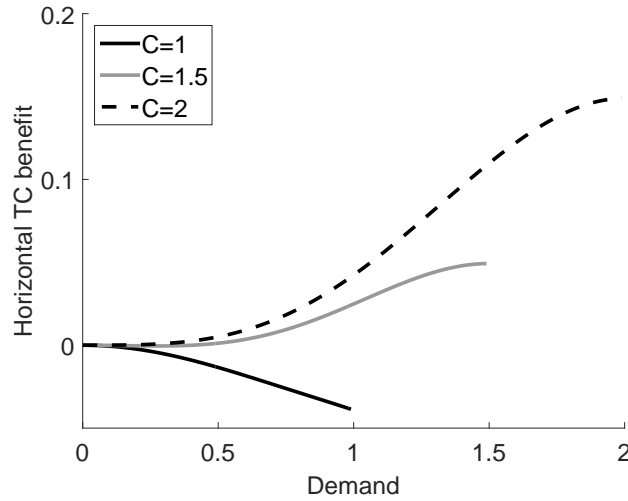
Note. Parameters: production cost $c = 1$; cost of refinancing $c_R = 0.5$, liquidity shock distributed $\mathcal{N}(0,1)$, (expected) financing cost $z(x) = c_R \Phi(-x)$. Note that the horizontal benefit is only well defined in the range of a where the financing cost $z(x)$ is convex at $x = C - ca$. The profit functions for different C are therefore cut off in different points to ensure this convexity in the normal distribution.

D.3. The impact of general unequal sharing rules

In the main body of the paper, we have assumed that firms setting equal prices will split the market equally. In this appendix, following Maskin (1986), we generalize this sharing rule by assuming that the demand is split as follows:

$$q_i(p_i, p_j) = \begin{cases} D & \text{if } p_i < p_j, \\ \alpha_i D & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j, \end{cases} \tag{122}$$

where $\alpha_i \geq 0$ and $\alpha_1 + \alpha_2 = 1$. That is, the demand is split at an arbitrary fraction when the firms set equal prices. The following result generalizes Proposition 1 under this sharing rule.

Proposition D.4 *Under the base model in the paper (Section 3, $C_i = C_j = C$ and $D > K$), suppose that demand is split according to (122). Then, under trade credit, a symmetric pricing equilibrium with $p > c$ exists as long as the willingness-to-pay v is sufficiently high.*

As shown, even under general unequal sharing rules, trade credit still allows the firms to price at a level strictly higher than marginal cost, consistent with Proposition 1. In other words, trade credit still softens horizontal competition. While the proof of the Proposition is fairly technical, intuitively, this is because the horizontal benefit of trade credit is mainly driven by the convexity of the financing cost $z(\cdot)$. While unequal sharing rules influence the magnitude of the effect as the firm's undercutting incentive is influenced by the share of demand allowed to him when the two firms offer equal prices, qualitatively, the effect is robust. Under such equilibrium prices, the firms are also able to earn an equilibrium profit higher than that under COD.

Regarding the quantitative impact of unequal sharing rule, we can show that the markups achieved by the firms (weakly) decrease as α deviates farther away from $1/2$ (either higher or lower due to the symmetry of the two firms). The result is intuitive: the less demand a firm serves according to the sharing rule, the higher its incentive to try to capture more market share by undercutting. Therefore, the equilibrium price has to adjust down accordingly to discourage such undercutting.

D.4. Proofs

Proof of Proposition D.1. Let us extend the symmetric price game equilibrium of Proposition 1. For simplicity, we shall focus on the case where the willingness-to-pay v does not bind. For ease of exposition, denote the profits when factoring and selling quantity q at price p

$$\pi_q^F(p) = (dp - c)q - z(C + (dp - c)q), \tag{123}$$

where we note that a firm expecting to factor would not set a price below c/d. When not factoring, the profits are, as before,

$$\pi_q(p) = (p - c)q - z(C - cq). \tag{124}$$

First, we note that there are thresholds $d_q^*(p)$ such that a firm serving demand $q \in \{D, D/2\}$ at price p will factor after the price game if and only if $d \geq d_q^*(p)$. We can see this from

$$\pi_q^F(p) > \pi_q(p) \iff dp_i q_i - z(C + (dp_i - c)q_i) > p_i q_i - z(C - cq_i), \tag{125}$$

where the left-hand side is continuous and increasing in d and the inequality holds for $d = 1$ but not $d = 0$; the thresholds thus exist. Furthermore, $d_D^*(p) \leq d_{D/2}^*(p)$, that is, a firm is more likely to factor when it serves the entire demand.

The firms will never factor for $d < d_D^*(p)$. Let p^* be the equilibrium price without the factoring option. Then, for $d < \underline{d} = d_D^*(p^*)$, the equilibrium will be exactly the one without the factoring option, and we hence have item 1 in the proposition.

For the remainder of the proposition, we need to consider how a change in price p affects the firms' subsequent decision to factor. We shall show that given a discount d, either the firms never factor or there exists a price threshold such that the firms will factor if and only if the price is below this threshold.

The firms will factor at price p when serving demand D if

$$\pi_D^F(p) > \pi_D(p) \iff \eta_D(p) \triangleq dpD - z(C + (dp - c)D) - pD + z(C - cD) > 0. \tag{126}$$

Differentiating, we can see that factoring is more likely happen with a lower p if and only if

$$\frac{d\eta_D(p)}{dp} > 0 \iff D(-1 + d - dz'(C + (dp - c)D)) > 0. \tag{127}$$

Furthermore, the second derivative

$$\frac{d\eta_D^2(p)}{dp^2} = D^2 d^2 z''(C + (dp - c)D)) > 0. \tag{128}$$

Noting that $\eta_D(0) = 0$, it is continuous and $\lim_{p \to \infty} \eta_D(p) \to -\infty$, the positive second derivative implies that $\eta_D(p)$ is either always decreasing or first increasing and then decreasing in p. In the first case, the firms

would never factor, and in the second case they would factor if and only if the price is below a threshold. A similar argument holds for demand $D/2$.

With these auxiliary results, we are ready to move on to parts 2 and 3 of the proposition. Suppose $d > \underline{d}$, and further that d and p^* are such that the firms only factor at D. We shall first show that there then exists a threshold $\hat{p}$ such that the firms will undercut and factor if and only if $p > \hat{p}$. Denote by p^* the Pareto-optimal price trade-credit equilibrium price in the absence of factoring. This price is such that

$$\pi_{D/2}(p^*) = \pi_D(p^*) \iff \frac{p^* D}{2} - z\left(C - \frac{cD}{2}\right) = p^* D - z(C - cD), \tag{129}$$

it cannot now be supported in equilibrium given inequality (126), as undercutting and then factoring is too lucrative. At price $p \leq p^*$, either firm will therefore undercut if

$$\pi_D^F(p) > \pi_{D/2}(p) \iff (dp - c)D - z(C + (dp - c)D) - \frac{(p - c)D}{2} + z\left(C - \frac{cD}{2}\right) > 0. \tag{130}$$

Notice that when the firms are indifferent between undercutting and sharing the demand at price p, i.e., (130) holds with equality, this implies through inequality (126) that the firms will factor at this price as well (for $p \leq p^*$).

To see that there exists a threshold $\hat{p}$ such that the firms will undercut and factor if and only if $p > \hat{p}$ let us differentiate from (130):

$$\frac{d(\pi_D^F(p) - \pi_{D/2}(p))}{dp} = D\left(d - dz'(C + (dp - c)D) - \frac{1}{2}\right) \tag{131}$$

$$\frac{d^2(\pi_D^F(p) - \pi_{D/2}(p))}{dp^2} = -D^2 d^2 z'(C + (dp - c)D) \leq 0. \tag{132}$$

We can readily check that the difference is positive for high values and negative for low values. It must be either decreasing or first increasing and then decreasing in p; either way there exists a threshold such that the firm will undercut and factor if and only if $p > \hat{p}$.

However, $\hat{p}$ is assuming that the firms would only factor after serving the entire demand D — but not after serving $D/2$. From above, there exists a threshold $\check{p}$ such that the firms will factor for demand $D/2$ iff $p < \check{p}$. Now if $\check{p} \leq \hat{p}$, the equilibrium price will be $p = \hat{p}$, and neither firm will factor. If, on the other hand, $\check{p} > \hat{p}$, the firms will factor when serving either demand D or $D/2$ at $\hat{p}$, the maximum possible price that precludes undercutting. Then the equilibrium price is equal to $p = c/d$ as both firms expect to factor and the price game reduces to the Bertrand equilibrium with marginal cost adjusted by d.

To see when $\check{p} > \hat{p}$, note that $\pi_q^F(c/d) = -z(C)$ both when selling D and $D/2$. We also have that for $p > c/d$, $\pi_D^F(p) > \pi_{D/2}^F(p)$. Recall that $\pi_D^F(p) \geq \pi_{D/2}(p)$ iff p sufficiently high, and $\pi_{D/2}^F(p) \geq \pi_{D/2}(p)$ iff p sufficiently low. Therefore if $\pi_{D/2}(c/d) \leq \pi_q^F(c/d)$, we have $\check{p} \geq c/d \geq \hat{p}$ and the equilibrium is $p = c/d$ and if $\pi_{D/2}(c/d) > \pi_q^F(c/d) = -z(C)$, $\check{p} > \hat{p}$ and $p > c/d$. Letting $p = c/d$ in the non-factoring profits, we have

$$\pi_{D/2}(c/d) = \frac{cD}{2}\left(\frac{1}{d} - 1\right) - z\left(C - \frac{cD}{2}\right), \tag{133}$$

which is decreasing in d. Therefore $\check{p} > \hat{p}$ with a sufficiently high d and vice versa, and we have parts 2 and 3 of the proposition. $\square$

Proof of Proposition D.2. The firms' total expected profit with COD and TC is:

$$\pi_i^{COD} = (p_i - c)q_i - z(C + (p_i - c)q_i) \tag{134}$$

$$\pi_i^{TC} = (p_i - c)q_i - z(C - cq_i). \tag{135}$$

The equilibrium under COD is evidently still pricing at marginal cost $p_i = c$. Under TC, a symmetric pure-strategy equilibrium exists when deviating by undercutting or setting a higher price is not profitable:

$$(p_i - c)D(p_i)/2 - z(C - cD(p_i)/2) \geq (p_i - c)D(p_i) - z(C - cD(p_i)) \tag{136}$$

$$(p_i - c)D(p_i)/2 - z(C - cD(p_i)/2) \geq -z(C). \tag{137}$$

This simplifies to:

$$(p_i - c)D(p_i)/2 \leq z(C - cD(p_i)) - z(C - cD(p_i)/2) \tag{138}$$

$$(p_i - c)D(p_i)/2 \geq z(C - cD(p_i)/2) - z(C). \tag{139}$$

Consider the second inequality. Let us show that there exists a unique solution (so that with a higher price, the inequality holds). For existence, note first that the inequality does not hold at $p_i = 0$. Assuming that sharing monopoly profits is profitable for the firms, the inequality holds at the monopoly price, and hence there exists a p_i^* at which it holds with equality. For uniqueness, moving everything to the LHS and differentiating, we have:

$$\pi_i^{TC}(p_i) = D(p_i)/2 + D'(p_i)/2[p_i - c + z(C - cD(p_i)/2)]. \tag{140}$$

We want to show that this is positive. The first term is positive, and so is the second if the term in square brackets is negative. Suppose the inequality does not hold:

$$(p_i - c)D(p_i)/2 \leq z(C - cD(p_i)/2) - z(C). \tag{141}$$

By the convexity of z, we have

$$z(C - cD(p_i)/2) - z(C) \leq cz'(C - cD(p_i)/2)D(p_i)/2, \tag{142}$$

and therefore

$$p_i - c + z(C - cD(p_i)/2) \leq 0. \tag{143}$$

Thus the solution is unique. We can use a similar argument for the first inequality.

The range exists when

$$z(C - cD(p_i)) - 2z(C - cD(p_i)/2) + z(C) \geq 0, \tag{144}$$

which is true by the convexity of $z(\cdot)$. The Pareto-dominating equilibrium price is hence the p_i that solves

$$(p_i - c)D(p_i)/2 = z(C - cD(p_i)) - z(C - cD(p_i)/2). \tag{145}$$

The firms are evidently better off than in the COD equilibrium, which yields profits equal to an upward deviation under trade credit. $\square$

Proof of Proposition D.3. Under COD, solving the FOCs we have

$$q_i(p_i, p_j) + (p_i - c)q_i'(p_i, p_j) = 0. \tag{146}$$

Suppose specifically that each firm faces demand $q_i(p_i, p_j) = a - p_i + p_j$. Then we have the (symmetric) equilibrium price $p = a + c$.

Under TC, the FOC for firm i is:

$$q_i(p_i, p_j) + q_i'(p_i, p_j)(p_i - c) + cq_i'(p_i, p_j)z'(C - cq_i(p_i, p_j)) = 0. \tag{147}$$

Noticing that the first two terms are equal to the COD FOC, they must be zero at the COD price. Assuming concavity and hence a unique COD equilibrium, and noting that the last term of the TC FOC is positive, the TC price must be higher than the COD price.

For the specific demand function, substituting the derivative, for the (symmetric) equilibrium, we have:

$$a - (p_i - c) - cz'(C - ca) = 0 \tag{148}$$

$$p = a + c - cz'(C - ca), \tag{149}$$

which is higher than the COD price.

The difference in profits is:

$$\pi^{TC} - \pi^{COD} = (a + c - cz'(C - ca) - c)a - z(C - ca) - [a^2 - z(C + a^2)] \tag{150}$$

$$= -caz'(C - ca) - z(C - ca) + z(C + a^2). \tag{151}$$

From the convexity of $z(\cdot)$, we know that $-acz'(C - ca) - z(C - ca) + z(C) \geq 0$. Here the condition for TC to yield higher profits than COD is stricter, as we need:

$$-caz'(C - ca) - z(C - ca) + z(C) \geq \Delta z \geq 0, \tag{152}$$

where $\Delta z = z(C) - z(C + a^2)$, as desired. $\square$

Proof of Proposition D.4. Let $\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha$. We shall show that there exists a price at which the firms will not deviate from equal pricing given this sharing rule.

The firms' expected total profits by choosing the price p_i are

$$\pi_i = (p_i - c)q_i - z(C - cq_i). \tag{153}$$

In a symmetric pure strategy equilibrium, both firms set $p_i = p \in [c, v]$ and serve demand $q_1 = \alpha D$, $q_2 = (1 - \alpha)D$. The resulting expected profits are:

$$\pi_{E1} = (p - c)\alpha D - z(C - c\alpha D) \tag{154}$$

$$\pi_{E2} = (p - c)(1 - \alpha)D - z(C - c(1 - \alpha)D). \tag{155}$$

For this to be an equilibrium, both undercutting and increasing prices should be unprofitable. When deviating down from this (undercutting the competitor and hence serving the entire demand) would give profit:

$$\pi_D = (p - c)D - z(C - cD). \tag{156}$$

Deviating up, on the other hand, would simply yield $\pi_U = -z(C)$. For this equilibrium to exist, the trade credit price p and a firm's financial conditions need to satisfy the following conditions for firm 1:

$$(U1) \quad (p-c)\alpha D - z(C - c\alpha D) \geq (p-c)D - z(C - cD), \tag{157}$$

$$(L1) \quad (p-c)\alpha D - z(C - c\alpha D) \geq -z(C). \tag{158}$$

This is equivalent to $p \in [\underline{p}_1, \overline{p}_1]$, where

$$\overline{p}_1 = c + \frac{1}{(1-\alpha)D}[z(C - cD) - z(C - c\alpha D)], \tag{159}$$

$$\underline{p}_1 = c + \frac{1}{\alpha D}[z(C - c\alpha D) - z(C)]. \tag{160}$$

For firm 2, the corresponding conditions are:

$$(U2) \quad (p-c)(1-\alpha)D - z(C - c(1-\alpha)D) \geq (p-c)D - z(C - cD), \tag{161}$$

$$(L2) \quad (p-c)(1-\alpha)D - z(C - c(1-\alpha)D) \geq -z(C), \tag{162}$$

that is, $p \in [\underline{p}_2, \overline{p}_2]$, where

$$\overline{p}_2 = c + \frac{1}{\alpha D}[z(C - cD) - z(C - c(1-\alpha)D)], \tag{163}$$

$$\underline{p}_2 = c + \frac{1}{(1-\alpha)D}[z(C - c(1-\alpha)D) - z(C)]. \tag{164}$$

As $z(\cdot)$ is convex, we can verify that $[\underline{p}_i, \overline{p}_i] \neq \emptyset \ \forall i$. In addition, for the equilibrium to exist, we also need $\min\{\overline{p}_1, \overline{p}_2\} \geq \max\{\underline{p}_1, \underline{p}_2\}$. To identify the condition, consider two steps.

1. $\overline{p}_1 \geq \overline{p}_2$ if and only if

$$\frac{1}{\alpha(1-\alpha)D}[(2\alpha - 1)z(C - cD) - \alpha z(C - c\alpha D) + (1-\alpha)z(C - c(1-\alpha)D)] \geq 0. \tag{165}$$

Suppose $\alpha < 1/2$, which implies $C - cD \leq C - (1-\alpha)cD \leq C - \alpha cD$. Let $t = \frac{\alpha}{1-\alpha}$. The left hand side of the above equation can be re-written as:

$$\frac{1}{\alpha D}[z(C - c(1-\alpha)D) - (1-t)z(C - cD) - tz(C - c\alpha D)]. \tag{166}$$

Note that $C - (1-\alpha)cD = (1-t)(C - cD) + t(C - \alpha cD)$. Therefore, by the convexity of $z(\cdot)$, the above expression is negative, as desired, and hence $\overline{p}_1 \leq \overline{p}_2$ for $\alpha < 1/2$. Similar steps show the opposite result for $\alpha > 1/2$.

2. Repeating this analysis for $\max\{\underline{p}_1, \underline{p}_2\}$, we get the same result: $\underline{p}_1 \leq \underline{p}_2$ iff $\alpha < 1/2$.

Combining the above two cases, for $\min\{\overline{p}_1, \overline{p}_2\} \geq \max\{\underline{p}_1, \underline{p}_2\}$, we need:

$$\underline{p}_1 < \overline{p}_2, \quad \text{if } \alpha > 1/2, \tag{167}$$

$$\underline{p}_2 < \overline{p}_1, \quad \text{if } \alpha < 1/2. \tag{168}$$

Suppose $\alpha \geq 1/2$. Then $\overline{p}_2 - \underline{p}_1 = \frac{1}{\alpha D}\Delta U$, where

$$\Delta U = z(C - cD) - z(C - c(1-\alpha)D - z(C - c\alpha D) - z(C). \tag{169}$$

Note that for $\alpha = 1$, $\Delta U = 0$; for $\alpha = 1/2$, by the convexity of $z(\cdot)$,

$$\Delta U = z(C) - 2z\left(C - \frac{cD}{2}\right) + z(C - cD) \geq 0. \tag{170}$$

Further, taking the derivative with respect to α, we have

$$\frac{d\Delta U}{d\alpha} = cD(z'(C - c\alpha D) - z'(C - c(1-\alpha)D) \leq 0, \tag{171}$$

where the inequality follows from z being decreasing and $\alpha \geq 1/2$. Therefore $\underline{p}_1 < \overline{p}_2$ for $\alpha \geq 1/2$.

A similar argument can be made for the other case $\alpha < 1/2$. Therefore, $\min\{\overline{p}_1, \overline{p}_2\} \geq \max\{\underline{p}_1, \underline{p}_2\}$ and there exists a range of prices $[\underline{p}, \overline{p}]$ within which neither firm will deviate.

Finally, for the equilibrium to exist, we also need $p \in [c, v]$. Therefore, the symmetric equilibrium exists if and only if $[\underline{p}, \overline{p}] \cap [c, v] \neq \emptyset$, that is, when v is sufficiently high. The corresponding equilibrium price will then be set at $p = \min\{\overline{p}, v\}$.

We can also show that the markup achieved by the firms decreases as we move away from $\alpha = 1/2$. To see this, consider without loss of generality $\alpha \leq 1/2$, so that the markup is $\overline{p}$ (assuming that v does not bind). Then we have

$$\frac{d\overline{p}}{d\alpha} = \frac{1}{(1-\alpha)^2 D}[z(C - cD) - z(C - c\alpha D) + (1-\alpha)cDz'(1 - c\alpha D)], \tag{172}$$

which is positive due to the convexity of $z(\cdot)$. Hence the further we move away from $\alpha = 1/2$, the lower the markup. $\quad\square$