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Pricing and Capacity Allocation for Shared Services

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We study the pricing and capacity allocation problem of a service provider who serves two distinct customer classes. Customers within each class are inherently heterogeneous in their willingness to pay for service, but their utilities are also affected by the presence of other customers in the system. Specifically, customer utilities depend on how many customers are in the system at the time of service as well as who these other customers are. We find that if the service provider can price discriminate between customer classes, pricing out a class, i.e., operating an exclusive system, can sometimes be optimal and that depends only on classes' perceptions of each other. If the provider must charge a single price, an exclusive system is even more likely. We extend our analysis to a service provider who can prevent class interaction by allocating separate capacity segments to the two customer classes. Under price discrimination, allocating capacity is optimal if the "net appreciation" between classes, as defined in the paper, is negative. However, under a single–price policy, allocating capacity can be optimal even if this net appreciation is positive. We describe in detail how the nature of asymmetry in classes' perception of each other determines the optimal strategy.

Key words: customer mix, crowding, pricing, capacity allocation

1. Introduction

In many service systems, service is simultaneously delivered to many customers who share the same physical environment. For example, members of a gym workout in the same space and share the equipment, passengers on a cruise ship share the common areas on the ship, and customers of a nightclub enjoy the dancefloor together. In such facilities, an individual customer's perception of the service quality is highly influenced by the composition of the clientele. For example, some female gym members do not enjoy sharing the same facility with males, and in nightclubs and bars, males typically have strong preferences for other customers being female (Skinner et al. 2005, Kubacki et al. 2007). Other service settings where customer satisfaction is influenced by the others' characteristics (such as age, socioeconomic status, intellectual capabilities, etc.) include social clubs, health clubs, schools (Buchanan 1965, Basu 1989, Sandler and Tschirhart 1997), beauty salons (Moore et al. 2005), recreational parks, adventure sports (Thakor et al. 2008), restaurants (Huang 2008), and professional conferences (Gruen et al. 2007).

Demand management for such service establishments, where each customer's satisfaction depends on who the fellow customers are, can be particularly challenging. The service provider who is facing this challenge has two powerful tools: pricing and capacity allocation, which, in its more extreme form, might even mean choosing to serve only certain segments of the population. Restricting access to certain customer segments may seem like a radical solution, but in practice it is more prevalent than expected. Such restriction could be direct or a result of a "forced" self-selection. Gyms and health clubs employ direct restriction when they choose to become women–only establishments or allocate certain times of the week for the exclusive use of families with kids. On the other hand, some firms design the service experience so as to appeal to a particular segment and let the customers self–select. This is the idea behind theme cruises and nightclubs catering to different types of clientele on different floors of the venue or at different nights of the week by carefully choosing the music and decoration. If such capacity allocation or restriction options are not available, or as a complementary tool, firms also use pricing as a means to manage their capacity and composition of their clientele, and maximize their profits. For example, nightclubs use various pricing promotions (e.g., "ladies' nights") to attract the "right" mix of customers.

Such practices are prevalent but that does not mean that they are devoid of controversy. "Ladies' nights" have long been criticized by some as being discriminatory against men and this led to a number of lawsuits being filed over the years (Rank 2011). Recently some gyms have been the center of attention due to similar policies. In 2007, a complaint against the Las Vegas Athletic Club (Friess (2007), Friess (2008)) led Nevada to pass a law in 2011 making gender-based pricing legal when used for promotional purposes (Schoenmann 2011). A more recent controversy was caused by Fitness USA, which abruptly decided to make two of its locations in Michigan women-only. The company preferred to offer its services exclusively to females and charge them a higher price, even if that meant angering several, male and female, customers (Komer 2013). In general, even though women-only health clubs have occasionally drawn ire, and some argue about their legality, they are popular and common in and outside the U.S. It is also important to note that the revenues associated with the leisure industry, where customer mix effects are prominent, are quite high. In the UK, it generates over £200bn of revenue every year, provides 2.6m jobs and represents 9% of the workforce (Wyman 2012). Similarly, in the US, the health club industry has annual revenues of $27bn (*IBISWorld* 2014b) and the nightlife industry of $24bn (*IBISWorld* 2014a). All these figures point to the importance of investigating the optimal pricing and capacity allocation strategies in these contexts.

In the establishments described above, the two fundamental questions the service provider needs to answer are: given the available capacity, what is the "optimal" customer mix and how should this mix be achieved? The objective of this paper is to provide insights into these two questions, which are inextricably linked. The optimal mix could be so that the system is an exclusive one, where service is offered to one segment of the population or an inclusive one where customers from different segments interact. Alternatively, the provider may also choose to allocate capacity for the exclusive use of each segment. Another interesting dimension is whether or not and how firm's pricing policy affects such decisions. Our analysis sheds light on these questions, helps identify conditions that would lead firms to choose one strategy over the other, and explains some of the existing practices we observe in the service industry.

The main challenge in investigating these questions is that no prior work can serve as the foundation for our modeling effort. Despite the fact that the operations management literature is rich in articles that deal with pricing, demand management, and capacity control in the context of service operations, the focus is not on the service process itself. Specifically, the "service" experience in these articles is typically not influenced by the characteristics of the others with whom they share the service experience (or service is simply not a shared experience), whereas delays in access to service is the important dimension of the problem. As a consequence, most papers consider queueing-based formulations. In contrast, for the service settings we are interested in, capturing the delivery of the service process (specifically, who the other customers are and how many of them there are)—as opposed to delays in access to service—is far more important. Thus, one of the main contributions of this paper is the development of a novel stylized formulation that permits detailed analysis of pricing and capacity allocation decisions for such settings.

Our model assumes that the service provider serves two classes of customers. Customers of one class have stochastically larger intrinsic valuations for the received service. Each customer knows the distribution of service valuations for both customer classes and uses this information along with the price to decide whether or not to purchase service. We focus first on the pricing decision and assume that the provider does not have the option to allocate different capacity segments to different customer classes, but can deny service to one of the two classes altogether. We consider two different settings; in §4.1, the firm has the flexibility to charge different prices to different classes, and in §4.2, the firm has to charge everyone the same price. When price discrimination is allowed, the firm might choose to exclude a particular class from service only due to the classes' perceptions of each other, i.e. the customer mix effects. Additionally, increasing the capacity might increase utilization. This surprising phenomenon is observed when customers are symmetric in their inherent willingness to pay for service and the customer mix effects are mild but disappears as the asymmetry increases. When the firm is forced to choose a single price, a strong asymmetry in the feelings of the two classes about each other urges the provider to restrict access to a single class in order to be profitable. (Interestingly, this is not true when there is mutual dislike.) This suggests that attempts to achieve price "fairness" by disallowing price discrimination might lead the service provider to deny service to one class.

In §4.3 and §4.4 of the paper, assuming that the firm can allocate capacity to different customer classes, we study the optimal allocation and pricing policy. In Section 5, we compare the different strategies to shed light into the design of such a service system and we find that if the firm can price discriminate, whether or not the firm chooses to allocate capacity depends purely on classes' perceptions of each other, not on any potential willingness–to–pay asymmetry between classes. However, this choice is more complicated if the firm has to charge the same price to both classes. In most cases, a firm that cannot price discriminate is more likely to prefer capacity allocation; however, this is not always true if customer classes are asymmetric in their inherent willingness to pay for service. In Section 6, we gain further insights via numerical examples and discuss the robustness of our results through a sensitivity analysis.

2. Literature Review

Prior work in the operations management literature has mostly investigated questions related to pricing and capacity control in service establishments where queueing prior to service is a critical aspect of the service experience. Thus, this body of work typically considers models that capture congestion effects and delay-sensitive customers (e.g., Naor 1969, Mendelson 1985, Mendelson and Whang 1990, Afèche 2013, Afèche and Pavlin 2015) and/or queue lengths provide signals of the service quality (e.g., see Debo and Veeraraghavan 2009, Veeraraghavan and Debo 2009, 2011). This is unlike our formulation which captures the service process during delivery but not the delays in access to service. Specifically, we focus on the consumption of a service good where class heterogeneity and the total number of customers has an impact on the customers' utility. To the best of our knowledge, the effects of this customer-to-customer interactions and their influence on the firms' pricing and capacity allocation decisions, has not been analytically studied before.

One paper that is relatively closer to our work is Johari and Kumar (2010), which considers positive-only network effects together with congestion effects. It is motivated by online services and these two effects are formulated in a way that is more general than our approach in that the effects not only depend on the number of active users in the system but also on the load these users generate. However, unlike the case in our model, Johari and Kumar ignore possible asymmetry in how customers from different segments feel about each other. Furthermore, their focus is completely different from ours. The authors are not interested in pricing and capacity allocation decisions for a profit–maximizing firm, but rather focus on the optimal number of users from the users' and the manager's perspectives. The gap between the two optima is discussed along with its implications.

In the economics literature, there are some articles related to our paper. A significant portion of these articles belong to a stream of work on "club theory," which originated from the seminal papers by Tiebout (1956) and Buchanan (1965). (For an extensive review of this literature, see Cornes and Sandler 1996 and Sandler and Tschirhart 1997.) However, this literature typically investigates questions that are completely different from ours. Specifically, except for a few papers (Hearne 1988, Basu 1989) that we discuss further below, the traditional club theory has not focused on pricing and/or capacity allocation considerations of a profit–maximizing firm. Moreover, again except for a few papers (e.g., Basu 1989, Brueckner and Lee 1989, Scotchmer 1997, Becker and Murphy 2000), the club theory literature has typically assumed that customers are homogeneous and their utilities do not depend on the characteristics of the other individuals in the facility.

Hearne (1988) focuses on the optimal pricing mechanisms of a monopolistic club and apart from the focus, the paper is different from ours in that the customers are assumed to be homogeneous. Basu (1989) is generally interested in contexts where recipients of a service are automatically associated with a certain status. In the schools context, for instance, rich students are willing to pay more than poor students and (rich or poor) students' willingness to pay depends on what fraction of the school population is clever. This work is purely interested in whether the schools should be allowed to charge different prices. Similarly, Brueckner and Lee (1989) are motivated by schools with two groups in the population. The paper characterizes the Pareto–efficient club configurations and carries out an equilibrium analysis for a competition model. Scotchmer (1997) defines a new notion of approximate competitive equilibrium in a setting where the utility of each customer type depends on the number of customers from each type. She shows that there exists such an equilibrium when the economy is sufficiently large. Note that none of Basu (1989), Brueckner and Lee (1989) and Scotchmer (1997) develop insights into the optimal pricing and capacity allocation decisions from an individual club's perspective. The model of Chapter 5 in Becker and Murphy (2000) is the most relevant to our work because it also assumes that the utility of a customer depends on the ratio of customers from one class. Despite this similarity, however, they assume that prices are determined through a competitive bidding process and there exists no service provider who sets prices to maximize profits.

Outside the club theory literature, another stream of articles within the economics literature deals with systems where customers experience positive network effects. Armstrong (2006) and Rochet and Tirole (2003) study two-sided markets where the two groups of agents interact via a, not necessarily physical, platform and the focus is on pricing mechanisms to attract the right mix of agents from both groups and achieve a "good" balance. Since the focus is not restricted to physical platforms, there is no consideration of capacity allocation nor crowding effects and the main attention is driven to mechanisms to gain market share in a competitive environment.

There is also some literature that refers to the "network effect" as the effect that other users in the network have on the utility of an individual user. For example, see Oren and Smith (1981), and more recently Candogan et al. (2012). These articles ignore the possibility that network effects across different groups within the population could be different. To the best of our knowledge, the only exception to this is Katz and Spiegel (1996) that uses a similar demand formulation but with no capacity considerations. There is also a large body of work that focuses on congestion effects leaving out positive network externalities. For examples of such work, we refer the reader to MacKie-Mason and Varian (1995), Wang and Schulzrinne (2006), and references therein.

Finally, there are many articles in the marketing literature that investigate customer-to-customer interactions (CCI) in services (see Nicholls 2010 for an extensive review). A number of articles empirically study CCI in various service environments including nightclubs (Skinner et al. 2005, Kubacki et al. 2007), professional conferences (Gruen et al. 2007), adventure sports (Thakor et al. 2008), beauty salons (Moore et al. 2005), cruise ships (Huang and Hsu 2010), and organized tours (Wu 2007), and find that customers can have strong preferences regarding who they share their service experience with. Moreover, some articles discuss the importance of the management of CCI in the service industry and point to various strategies the providers might employ. Among these, Martin (1996) and Grove and Fisk (1997) discuss operational issues including the effective use of capacity, which we also address in this paper. In particular, Martin (1996) investigates customers' perceptions of and reactions to the others' behavior. He suggests improving service experience through capacity allocation via physical separation or time allocation for the use of different segments who might not enjoy the interaction. This is a practice widely used and we also investigate it. In the same spirit, Grove and Fisk (1997) establish conditions under which the system's capacity is fully utilized or underutilized due to the presence or behavior of others and call for more research into identifying the optimal capacity for systems that serve many customers simultaneously.

3. Model

We consider a service system associated with a leisure facility with capacity $K > 0$ and serves two distinct customer classes, each one with the same finite size $\Lambda > 0$. We later consider different class sizes in a numerical study. Class membership of a customer is observable to the service provider and to all the other customers. Customers enjoy the leisure facility and their utilities consist of three different components. In the absence of other customers, the service value of class–1 customers is uniformly distributed on the line segment $[0, 1]$. Likewise, the service value of class–2 customers is uniformly distributed on the line segment $[a, 1+a]$, $a \geq 0$. Thus, on average, class–2 customers have the same or larger inherent willingness to pay for service than class–1 customers. In the presence of other customers, however, there are two components that may affect customer utility and depend on λ_1 and λ_2, the number of customers in the system belonging to class 1 and 2, respectively. To facilitate game theoretic treatment we treat customers as non-atomic (infinitesimal) and therefore, λ_1 and λ_2 as continuous parameters. Customers of a particular class might like or dislike sharing the same service environment with the other class. Moreover, their satisfaction can be dependent on the overall crowd size. In mathematical terms, the gross utilities U_1 and U_2 of a customer x in class 1 and 2, respectively, are given by

$$\begin{aligned} U_1(x,\lambda_1,\lambda_2) &= x + b_1\lambda_2/(\lambda_1+\lambda_2) + c\left((\lambda_1+\lambda_2)/K\right), \quad 0 \le x \le 1, \\ U_2(x,\lambda_2,\lambda_1) &= x + b_2\lambda_1/(\lambda_1+\lambda_2) + c\left((\lambda_1+\lambda_2)/K\right), \quad a \le x \le 1+a. \end{aligned} \tag{1}$$

The terms $b_1\lambda_2/(\lambda_1+\lambda_2)$ and $b_2\lambda_1/(\lambda_1+\lambda_2)$ in (1) captures the customer-mix effect on class–1 and class–2 customer utilities, respectively. We assume that customers of each class are homogenous in their perception of the customer mix and this is represented by the parameters b_1 and b_2. If $b_1 > 0$ ($b_1 < 0$), customers of class 1 prefer a customer mix with more (fewer) class–2 customers and if $b_2 > 0$ ($b_2 < 0$), customers of class 2 prefer a customer mix with more (fewer) class–2 customers. We also define $b \equiv b_1 + b_2$ as the "net appreciation" between the two customer classes and will be useful in presenting our results. The reader should note that this net appreciation term has a very specific meaning in our stylized formulation and one should be careful when interpreting the practical implications of our results particularly in regards to how customers' perceptions of each other affect the optimal policy decisions.

Customers' experience might also be affected by the *crowding level*, which is defined as $(\lambda_1 + \lambda_2)/K$. Depending on the leisure activity, an undercrowded system or/and an overcrowded system might not be desirable for an enjoyable experience, which in turn reduces customer utility. The continuously differentiable function $c : [0,1] \to \mathbb{R}$ in (1) captures these effects on customer utilities. We assume that $c''(\cdot) < 0$, thereby guaranteeing a uniquely optimal crowding level for an arbitrary customer. To avoid an empty system in equilibrium, we also assume that $c(0) > -1$. It is important to note that we impose no further restrictions on $c(\cdot)$; it can take positive or negative values, it can be monotone or unimodal. In fact, there are some service experiences where the overall *crowding level* in the system may not influence customers' utilities, i.e. $c \equiv 0$, or service experiences where the customers' utilities are affected only for certain crowding levels. In both cases, our results still hold. However, we assume that whatever the crowding effects are, they are symmetric across classes.

We consider a game in which the leisure facility first chooses the prices (p_1, p_2) simultaneously and commits to them. The customers arrive to the service facility, observe the price p_i, if from the class i, and decide whether to join the system or not. A customer with service value x_i has a strategy space $s_i(x_i) = 1$ (customer joins the system) or $s_i(x_i) = 0$ (customer does not join the system). A Nash Equilibrium $(s_1(x_1), s_2(x_2))$ of this game will be such that

$s_1(x_1)=1$ if and only if $U_1(x_1,\Lambda\int_0^1 s_1(y)dy,\Lambda\int_a^{1+a} s_2(y)dy)\geq p_1$ and $s_2(x_2)=1$ if and only if $U_2(x_2,\Lambda\int_a^{1+a} s_2(y)dy,\Lambda\int_0^1 s_1(y)dy)\geq p_2$ using (1).

Proposition 1. *For a continuously differentiable function $c:[0,1]\to\mathbb{R}$ such that $c''(\cdot)<0$, there exists a unique Nash Equilibrium (NE) such that a customer x_i from class i will pay p_i and join the system, if $x_i\geq x_i^*$, $i=1,2$ where x_1^*,x_2^* satisfy*

$$x_1^*+b_1\frac{a+1-x_2^*}{a+2-x_1^*-x_2^*}+c\left(\frac{\Lambda(a+2-x_1^*-x_2^*)}{K}\right)=p_1,$$
$$x_2^*+b_2\frac{1-x_1^*}{a+2-x_1^*-x_2^*}+c\left(\frac{\Lambda(a+2-x_1^*-x_2^*)}{K}\right)=p_2.$$

Since there is a unique mapping between the NE (x_1^*,x_2^*) and (λ_1,λ_2), with $\lambda_1=\Lambda\int_{x_1^*}^1 s_1(x_1)dx_1=\Lambda(1-x_1^*)$, $\lambda_2=\Lambda\int_{x_2^*}^{a+1} s_2(x_2)dx_2=\Lambda(a+1-x_2^*)$, the NE can be equivalently expressed in terms of (λ_1,λ_2) and the equilibrium prices will be derived as follows

$$p_1(\lambda_1,\lambda_2)=1-\lambda_1/\Lambda+b_1\lambda_2/(\lambda_1+\lambda_2)+c\left((\lambda_1+\lambda_2)/K\right), \tag{2}$$
$$p_2(\lambda_2,\lambda_1)=1+a-\lambda_2/\Lambda+b_2\lambda_1/(\lambda_1+\lambda_2)+c\left((\lambda_1+\lambda_2)/K\right). \tag{3}$$

The structure of the solution is provided separately for the different cases in the next sections. Because customer utilities depend on λ_1 and λ_2, which are equilibrium quantities, a potential customer must construct beliefs about their equilibrium values when deciding to join the system. In turn, these beliefs must be confirmed in equilibrium, that is, customers should act rationally with respect to information and be able to correctly predict the equilibrium values, as a result. As in all definitions of equilibrium, customers' choices and beliefs are determined simultaneously.

Before moving on to the analysis, we briefly comment on the case in which classes are identical and customer-mix effects do not exist or are ignored, i.e., $a=0$ and $b_1=b_2=0$. In that case, it is easy to show that the service provider 1) always prefers to have both classes in the system to sustain higher prices; 2) charges both classes the same price even when price discrimination is allowed. Therefore, if classes are identical and the customer mix does not affect customer utilities, neither capacity allocation nor price discrimination are of any value to a service provider. As we demonstrate in this paper, asymmetry in the willingness to pay for service and/or customer-mix effects make both price discrimination and capacity allocation effective tools to the service providers, and explain to a great extent what is observed in practice.

To help with the exposition in the rest of the paper, we introduce the following terminology; we call a system *exclusive*, if no interaction between the two classes is allowed and *inclusive* otherwise. Exclusivity can be a result of restricting access to a single class or allocating capacity for the exclusive use of each class. We call a system *full* if its crowding level is equal to one; we call a system *not full* if its crowding level is strictly less than one. Also, we refer to the case $a = 0$ as *symmetric* and to the case $a > 0$ as *asymmetric* with classes described as being symmetric and asymmetric respectively. Notice in equation (1) that the two customer classes are possibly different in two dimensions: their feelings about each other and their (inherent) willingness to pay for service and therefore, our definition of symmetry is with a slight abuse of the terminology[1].

4. Optimal pricing and capacity allocation decisions

We start our analysis in §4.1 with a leisure facility where the two classes share the whole capacity and the service provider is allowed to charge them differently. We call this scenario *price discrimination without capacity allocation* (CS-DP). We continue in §4.2 with the more restrictive pricing policy, where the provider must charge the same price to all customers and we will call this scenario *single price without capacity allocation* (CS-SP). In both CS-DP and CS-SP, however, the provider can choose to restrict access to one class only, i.e., run an exclusive system.

If the service provider is better off running an exclusive system, she might, in fact, choose to allocate separate capacity segments for the exclusive use of each customer class. The service provider might be able to divert customers to the "right" location depending on their class identities or she can design the service and the service environment for different segments to induce customers to self–select. In a nightclub, this usually happens by hosting theme nights on different days of the week so as to appeal to customers with particular tastes. Nightclubs with adequate space might also provide a private area for members who are willing to pay a premium so as not to socialize with the rest of the clientele. In §4.3, we study the service provider's problem under the assumption that she exercises her option to allocate capacity to each customer class and price discriminate and we call this scenario *price discrimination with capacity allocation* (CA-DP). We then restrict the problem to the single price case in §4.4 and we call this scenario *single price with capacity allocation* (CA-SP). We use (P1), (P2), (P3), and (P4) to represent the mathematical formulations of the optimization problems that correspond to CS-DP, CS-SP, CA-DP and CA-SP, respectively.

4.1. Price discrimination without capacity allocation

We start our analysis with a leisure facility, a nightclub for instance, where the two classes, the male and the female customers, share the whole capacity and the service provider is allowed to charge them differently. In that setting, typically male customers are willing to pay more, not for the service per se, but because they are considered to gain more from the interaction with female customers, than female customers do (Armstrong 2006). The service provider's objective is to charge prices so as to maximize the total profit. Hence, an individually rational provider who can charge a different price to each class maximizes revenue by solving the following problem:

$$\begin{array}{ll} \max_{\lambda_1,\lambda_2} & R(\lambda_1,\lambda_2) = \lambda_1 p_1(\lambda_1,\lambda_2) + \lambda_2 p_2(\lambda_2,\lambda_1) \\ \text{s.t.} & \lambda_1+\lambda_2 \le K,\ 0 \le \lambda_1 \le \Lambda,\ 0 \le \lambda_2 \le \Lambda. \end{array} \tag{P1}$$

[1] Classes are truly symmetric only if $a = 0$ and $b_1 = b_2$.

We first establish some basic properties of the optimal solution $(\lambda_1^*, \lambda_2^*)$ to problem (P1).

LEMMA 1. *(i) If $a = 0$, then $\lambda_1^* \lambda_2^* > 0$ if and only if $\lambda_1^* = \lambda_2^*$.*

(ii) If $a = 0$, a feasible solution to $(P1)$ at which $\lambda_1 = \lambda > 0$, $\lambda_2 = 0$, is revenue–equivalent to a feasible solution to $(P1)$ at which $\lambda_1 = 0$, $\lambda_2 = \lambda > 0$.

(iii) If $a > 0$, then $\lambda_2^* > \lambda_1^*$.

The properties described in Lemma 1 suggest that the customer mix effects play no role. According to the lemma, even if class-1 customers are very fond of class-2 customers but the latter despise the former, the provider will admit the same number of customers from each class if $a = 0$, and will admit more customers from class 2 if $a > 0$. This may seem to suggest that when it comes to the customer mix in equilibrium, the customer-mix effects are irrelevant. However, the customer mix effects implicitly play a role when the provider has to determine if she will operate an exclusive or an inclusive leisure facility ($\lambda_1^* \lambda_2^* > 0$ or $\lambda_1^* \lambda_2^* = 0$), as we will see in Proposition 2 below. Nevertheless, it is true that *if* it is optimal for the provider to admit both classes, most customers will be from the class that has the higher willingness to pay for service regardless of any asymmetry in how classes feel about being around each other. This result is due to the provider's ability to internalize any asymmetry in the *linear* customer-mix effects by charging different prices. For example, the male customers of a nightclub might end up paying a much higher price than the female customers; in fact, the price differential will be so large that the same number of customers from both classes will eventually choose to join the system.

The next proposition characterizes the general structure of the NE, i.e., the structure of the optimal solution to (P1).

PROPOSITION 2. *If customers from different classes are allowed to share the same space and the service provider can price discriminate, the optimal solution to the revenue maximization problem has the following properties:*

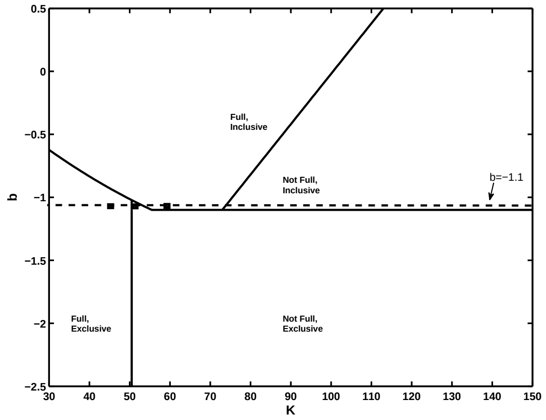
(i) There exists threshold $b^*(K)$ such that $\lambda_1^* = 0$, $\lambda_2^* > 0$ if $b \leq b^*(K)$, and $\lambda_2^* \geq \lambda_1^* > 0$ if $b > b^*(K)$.

(ii) If $K \leq \min\{\Lambda(1 + a + c(1))/2, 2(1 + c(1))\Lambda/3\}$, then $\lambda_1^* + \lambda_2^* = K$.

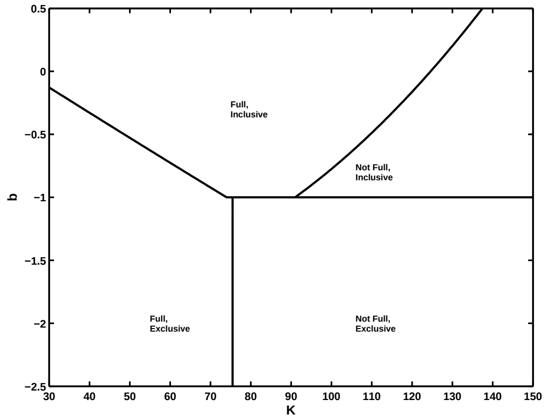
(iii) If K is sufficiently large, then $\lambda_1^* + \lambda_2^* < K$.

(iv) If b is sufficiently positive so that $\lambda_1^* > 0$ $\forall K$, or if b is sufficiently negative so that $\lambda_1^* = 0$ $\forall K$, then there exists $K^*(b)$ such that $\lambda_1^* + \lambda_2^* = K$ if $K \leq K^*(b)$, and $\lambda_1^* + \lambda_2^* < K$ if $K > K^*(b)$.

Before we discuss the implications of Proposition 2 in detail, we should highlight an important point. The reader might note that the structural properties as stated in the proposition depend on b_1 and b_2 only through the term $b = b_1 + b_2$. This is in fact not surprising as one can show that the



(a) $a = 0$



(b) $a = 0.5$

Figure 1 Structure of the optimal policy under price discrimination without capacity allocation when $\Lambda = 100$.

optimization problem (P1) can equivalently be expressed in terms of b alone. As we will see later in the paper, however, this is not the case when the service provider cannot price discriminate and thus the optimal solution has a more complex relationship with the interaction terms b_1 and b_2.

Proposition 2 characterizes the basic structure of the NE and provides insights into the two key decisions the service provider needs to make. First, she needs to decide whether to admit customers from both classes (inclusive system) or to restrict access to the customers with higher willingness-to-pay (exclusive system). Second, she needs to decide whether the existing capacity should be fully utilized or intentionally kept underutilized at profit-maximizing prices. Figure 1 illustrates the different system types that arise in equilibrium if classes are symmetric (a) or asymmetric (b). In what follows, we first discuss the most important insights in the case of symmetric classes and then we highlight the differences that arise if classes are asymmetric. When following this discussion, the reader would likely find it helpful to refer to the graphs in Figure 1. In *symmetric classes*, Proposition 2-(i) states that although the system capacity is a factor in deciding whether or not the system should be inclusive or exclusive, only the net appreciation term b is relevant. More specifically, if the net appreciation between classes is sufficiently negative, the provider is better off leaving one class out of the system. Although it is possible that one class likes the other (e.g., $b_1 > 0$), if the feelings of the other class are opposite and much more intense (i.e., $b_2 << -b_1$), then an exclusive system helps prevent customer-mix effects from hurting revenues. For example, some female customers of gyms and health clubs are not willing to share the same workout space with male customers. If this disutility of female customers is strong, the service provider might find it more profitable to run an exclusive system. This might be the motivation behind Fitness USA's decision to go women-only.

Parts (ii), (iii), and (iv) of Proposition 2 characterize how the choice regarding the number of customers in the system should be made to fill in the capacity. Not surprisingly, if the capacity is sufficiently small, there are enough customers who would be willing to pay a high price and the system will be fully utilized regardless of an exclusive or inclusive system. On the other hand, if capacity is very large, running a full system is suboptimal as it would necessitate charging unjustifiably low prices or would be outright impossible.

Part (iv) of Proposition 2 strengthens these structural properties further. When customer-mix effects are so powerful that a system is either always inclusive or always exclusive regardless of its capacity, then progressively larger capacities can only imply transitions from full to not full systems. However, if the customer-mix effects are relatively weak, which is possibly the most common scenario in a leisure facility, then we have some interesting and unexpected changes in the preference for exclusivity and crowding level (Figure 1(a)). We use a numerical example to illustrate this. Consider a small absolute value of the net appreciation effect, $b = -1.1$, and $\Lambda = 100$, $a = 0$. We will consider three different capacity levels of $K = 45, 50, 60$. (See the dashed line and squares on Figure 1(a) to follow the rest of the paragraph.) If $K = 45$, the system is in the regime of part (ii) of Proposition 2, i.e., a full exclusive system is optimal and the corresponding revenue is $R(0, 45) = 24.75$. On the other hand, the highest revenue an inclusive system could yield is $R(22.5, 22.5) = 22.5$. In this case, the limited capacity does not allow the provider to adequately counter the negative customer-mix effects by admitting more customers from both classes. Suppose that capacity increases to $K = 51$. Now, the most profitable system is still exclusive but not full, with 50 customers and revenue $R(0, 50) = 25$, whereas the highest revenue an inclusive system could yield is $R(25.5, 25.5) = 23.97$. In this case, again, capacity is not sufficient to result in enough revenue for an inclusive system to be optimal. Finally, suppose that capacity increases even further to $K = 60$. The optimal system now is full and inclusive, with 30 customers from each class and revenue $R(30, 30) = 25.5$. On the other hand, the highest revenue an exclusive system could yield remains at $R(0, 50) = 25$. At this capacity level, the provider can admit enough customers from both classes to make up for the revenue she loses due to negative customer-mix effects. It is the negative customer interaction effects that hurt revenues of inclusive systems, thus making it difficult to make a general statement about the effect of capacity changes based on intuition alone. In the absence of such effects, admitting customers from both classes would raise the average price customers pay compared to an exclusive system with the same number of customers.

In *asymmetric classes*, the asymmetry in the willingness to pay for service does not change substantially the structure of the equilibrium (Figure 1(b)). However, there are two noteworthy differences. First, the net appreciation between classes needs to be higher for inclusivity to be the optimal choice because the provider can simply find more customers in class–2 than in class–1 to pay a good price for service. Second, when the two classes are strongly asymmetric, as in the case of Figure 1(b), an increase in the system capacity can never result in the optimal crowding level changing from "not full" to "full." This is in contrast to the symmetric and weakly asymmetric cases (Figure 1(a)), where a capacity increase can switch the optimal policy from "exclusive, not full" to "inclusive, full." This difference is due to the fact that class-1 customers' low willingness-to-pay combined with sufficiently strong negative customer effects between the two classes does not justify admitting class-1 customers in the more asymmetric cases. Thus, the system remains exclusive as capacity increases and operating a full system does not become a better alternative.

We conclude this section with a comparison of the prices that classes pay when they coexist. The revenue achieved by the service provider depends on the overall asymmetry of the classes (b, a) that determines the proportion of the customers that will join the facility (λ_1, λ_2). The extra flexibility that she possesses, she uses it by charging prices that reflect the classes' feelings; higher b_i implies higher p_i. Equations (8) and (9) imply that $p_2^* - p_1^* = a + (\lambda_1^* - \lambda_2^*)/\Lambda + (b_2\lambda_1^* - b_1\lambda_2^*)/(\lambda_1^* + \lambda_2^*)$. Wherefore, if classes are symmetric and the provider runs an inclusive system, the class that likes (dislikes) the other the most (the least) pays a higher price for service and in particular, $p_2^* - p_1^* = (b_2 - b_1)/2$. This might explain why "ladies" are offered discounts to compensate for their weaker utility of having "gentlemen" around in in nightclubs or why some colleges offer reduced tuition to students of high caliber.

With asymmetric classes ($a > 0$), the price comparison is not straightforward. In this case, $\lambda_1^* < \lambda_2^*$ and class–2 customers might end up paying *less* than class–1 customers, although they can afford a higher price for service. The reason is that if class–1 customers value the presence of class 2 much more than class–2 customers value them in return ($b_1 >> b_2$), the former will end up paying more than the latter although they are not as wealthy on average. This result partially explains why famous and wealthy individuals enjoy a free ride at certain social events; the strong desire of less wealthy and less famous people to be around them might give rise to this phenomenon.

4.2. Single price without capacity allocation

As discussed in Section 1, price discrimination is a sensitive issue and can be illegal, or not ethical, when it is based on a demographic factor. Whether or not it is implemented depends on a combination of factors including what the law says about the practice, whether the law is enforced, customers' attitude, and the provider's ability to manage customer perceptions. When the manager is constrained to charging a single price, she has to either charge the optimal unique price to both classes or she may offer the service to only one of the two classes.

Using (8) and (9), the constraint $p_1 = p_2$ implies

$$[b_1/(\lambda_1 + \lambda_2) + 1/\Lambda]\lambda_2 = [b_2/(\lambda_1 + \lambda_2) + 1/\Lambda]\lambda_1 + a. \tag{4}$$

Because $a \geq 0$, a NE in which the provider charges a single price and $\lambda_1^* \lambda_2^* > 0$ is possible only if $b_1 < 0$ and $b_2 < -a$, or if $b_2 > -K/\Lambda$ and $b_1 > a - K/\Lambda$. Hence, without proof, the following lemma.

LEMMA 2. *The service provider can charge a single price in a NE in which $\lambda_1^* \lambda_2^* > 0$ only if $b_1 < 0$ and $b_2 < -a$, or if $b_2 > -K/\Lambda$ and $b_1 > a - K/\Lambda$.*

The necessary conditions of Lemma 2 essentially say that there is a limit to how differently the two customer classes can feel about each other and still allow a profitable single–price policy with both classes being admitted. Interestingly, if the dislike between customer classes is mutual, this is not sufficient for the provider to deny service to one of the classes. In that case, there are always customers who are willing to pay the asking price and bear with the customers from the other class due to the inherent heterogeneity within customer classes. The intensity of the customer feelings determines the ratio of the two classes in the facility and as a result, when there is strong asymmetry in the two classes' mutual appreciation, it is not profitable to maintain an inclusive facility using a single price. Although Lemma 2 identifies conditions under which an inclusive system with single price might be profitable, the provider might be better off running an exclusive system (Figure 2).

To solve the optimization problem (P2), the service provider first solves the following problem (P2′), which enforces the single–price constraint and ignores the possibility that the service can be limited to only one class. Problem (P2′) is essentially problem (P1) with the addition of the single–price constraint (4).

$$\begin{array}{ll} \max_{\lambda_1,\lambda_2} & R(\lambda_1,\lambda_2) = \lambda_1 p_1(\lambda_1,\lambda_2) + \lambda_2 p_2(\lambda_2,\lambda_1) \\ \text{s.t.} & \lambda_1 + \lambda_2 \leq K \\ & [b_1/(\lambda_1+\lambda_2) + 1/\Lambda]\lambda_2 = [b_2/(\lambda_1+\lambda_2) + 1/\Lambda]\lambda_1 + a \\ & 0 \leq \lambda_1 \leq \Lambda, 0 \leq \lambda_2 \leq \Lambda. \end{array} \tag{P2$'$}$$

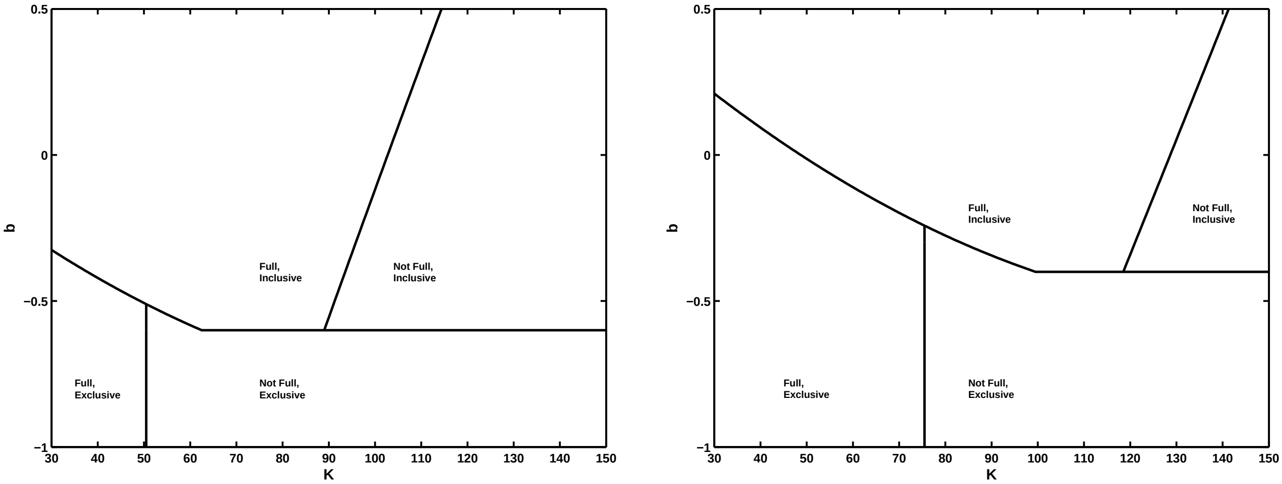
The solution to (P2) is then obtained by comparing the optimal solution to (P2′) with the optimal solution under which the service is restricted to class–2 customers. (There is no need to consider the case where service is restricted to class–1 customers because such a solution is guaranteed to not lead to higher revenue. Restriction to either class leads to the same revenue only if $a = 0$.)

We first establish some basic properties of the optimal solution $(\lambda_1^*, \lambda_2^*)$ to problem (P2).

LEMMA 3. *(i) If $a = 0$ and $b_1 \geq b_2$, then either $\lambda_1^* \lambda_2^* = 0$ or $\lambda_1^* \geq \lambda_2^*$. Similarly, if $a = 0$ and $b_2 \geq b_1$, then either $\lambda_1^* \lambda_2^* = 0$ or $\lambda_2 * \geq \lambda_1^*$*

(ii) If $a = 0$, a feasible solution to $(P2)$ at which $\lambda_1 = \lambda > 0$, $\lambda_2 = 0$, is revenue–equivalent to a feasible solution to $(P2)$ at which $\lambda_1 = 0$, $\lambda_2 = \lambda > 0$.

According to Lemma 3, if classes are symmetric, the provider either admits only one customer class, or she runs an inclusive system with more customers from the class that likes (dislikes) the other more (less). Since the classes are not truly symmetric, when $a = 0$ but $b_1 \neq b_2$, the single-price constraint does not permit a customer mix with an equal number of customers from both classes.



(a) $a = 0$ (b) $a = 0.5$

Figure 2 Structure of the optimal policy under single-price policy without capacity allocation when $\Lambda = 100$, $b_2 = 0$.

For example, if $b_1 > b_2$ and the provider charges a single price, there will be more class–1 than class-2 customers who are willing to pay that price and the optimal customer mix will have more customers from class–1.

The next proposition characterizes the overall structure of the NE in the case of single price, i.e., the structure of the optimal solution to (P2). (We slightly abuse notation by using the same symbols, $b^*(K)$ and $K^*(b)$, in both propositions, although they might correspond to different values.)

Proposition 3. *If customers from both classes are allowed to share the same space and the service provider cannot price discriminate, the optimal solution has the following properties, where we let $\Delta b \equiv |b_1 - b_2|$.*

(i) There exists a threshold $b^*(K)$ such that $\lambda_1^* = 0$ if $b \le b^*(K)$, and $\lambda_1^* > 0$ if $b > b^*(K)$. Furthermore, if $a = 0$ and there exists a net appreciation value $\tilde{b}$ such that $b_1 + b_2 = \tilde{b}$ and $\lambda_1^* \lambda_2^* > 0$ if $\Delta\tilde{b} = 0$, then there exists threshold $\Delta b^*(K) > 0$ such that $\lambda_1^*\lambda_2^* > 0$ if $\Delta b \le \Delta b^*(K)$, and $\lambda_1^*\lambda_2^* = 0$ if $\Delta b > \Delta b^*(K)$.

(ii) If $K \le \min\{\Lambda(1 + a + c(1))/2, \Lambda(1 + c(1))\}$, then $\lambda_1^* + \lambda_2^* = K$.

(iii) If K is sufficiently large, then $\lambda_1^* + \lambda_2^* < K$.

(iv) If b is sufficiently positive so that $\lambda_1^* > 0$ $\forall K$, or if b is sufficiently negative so that $\lambda_1^* = 0$ $\forall K$, then there exists $K^*(b)$ such that $\lambda_1^* + \lambda_2^* = K$ if $K \le K^*(b)$, and $\lambda_1^* + \lambda_2^* < K$ if $K > K^*(b)$.

A quick read of Proposition 3 reveals that each of its statements corresponds to an analogous statement in Proposition 2, which is also evident by comparing Figures 1 and 2. There is, however, one important difference. The second part of Proposition 3-(i) states that, for a given net appreciation term, unless the two individual terms b_1 and b_2 are sufficiently close to each other, service will be restricted to one class. In other words, a single-price policy leads to an exclusive system when this asymmetry is sufficiently large unlike in the price discrimination case where the asymmetric customer-mix effects are absorbed by the differential pricing (Figures 1 and 2). The revenue is tightly constrained by the single–price condition. As we explained in the discussion of Lemma 2, this condition critically depends on how different terms b_1 and b_2 are from each other. Thus, when the provider charges a single price, not only the net appreciation term, but also the individual terms b_1 and b_2 are important. In other words, the customer mix effects on revenue, which are symmetric across classes under price discrimination, become asymmetric under the single–price clause. This result practically implies that when regulators attempt to achieve "price fairness" by disallowing price discrimination, they might inadvertently be forcing the service provider to exclude an entire class of customers from service if that is practically feasible. Although there is no evidence to conclude that this is the reason why gyms like Fitness USA, convert some of their locations to women-only establishments, they are very likely to be affected by similar underlying dynamics. By restricting access to females, these gyms not only become more appealing to women and increase their willingness to pay for the experience, but also bypass any possible restriction (legal or otherwise) to charge the same price to both men and women. It is also interesting that, due to this phenomenon, the optimal price may have a non-monotonic relationship with the capacity. Specifically, one might expect that the optimal price would decrease with an increase in system capacity but as it turns out, a larger capacity might mean the optimality of an inclusive system with a higher price.

4.3. Price discrimination with capacity allocation

Capacity allocation with or without price discrimination is a prevalent practice. For example, theme cruises typically occupy part of a cruise ship while the rest is filled with passengers on a regular tour. Similarly, some health clubs, or public swimming pools allocate their capacity to different customer classes through space separation or time allocation.

If the service provider can allocate capacity, she needs to decide the capacity to allocate to each class as well as the optimal number of customers to admit. In the subsequent analysis, $(1-x)K$ denotes the fraction of capacity allocated to class–1 customers and xK denotes the fraction of capacity allocated to class–2 customers. The equilibrium prices are modified as follows

$$p_1(\lambda_1) = 1 - \lambda_1/\Lambda + c\left(\lambda_1/((1-x)K)\right), \tag{5}$$

$$p_2(\lambda_2) = 1 - \lambda_2/\Lambda + a + c\left(\lambda_2/(xK)\right), \tag{6}$$

where the customer mix effects disappear since the two classes do not coexist. As previously, we study first the price discrimination policy (CA-DP) and in §4.4, we focus on the single–price policy (CA-SP).

Given the choices of capacity allocation and price discrimination, the service provider is faced with the following revenue maximization problem:

$$\begin{array}{ll} \max_{\lambda_1,\lambda_2,x} & R(\lambda_1,\lambda_2,x)=\lambda_1 p_1(\lambda_1)+\lambda_2 p_2(\lambda_2) \\ \text{s.t.} & 0\le\lambda_1\le(1-x)K,\ 0\le\lambda_2\le xK \\ & 0\le x\le 1. \end{array} \tag{P3}$$

We first establish the uniqueness of the optimal solution to problem (P3), as well as some important properties of x^*, the optimal fraction of capacity allocated to class 2.

LEMMA 4. *(i) There exists a unique optimal solution to* $(P3)$.

(ii) If $\lambda_1^*\lambda_2^*>0$*, crowding levels are the same in both capacity segments, i.e.,* $\frac{\lambda_1^*}{(1-x^*)K}=\frac{\lambda_2^*}{x^*K}$.

(iii) The optimal allocation fraction for class 2, x^**, equals* $\lambda_2^*/(\lambda_1^*+\lambda_2^*)$.

(iv) If $a=0$, $x^*=1/2$. *In addition,* x^* *is increasing in* a.

Lemma 4 is a key result for the remainder of our analysis. The fact that the classes are identical in their sensitivity towards crowding and that the crowding disutility function c is (strictly) concave explains the identical crowding levels in both segments. Furthermore, they are inextricably linked to each other because the two capacity segments share the same total capacity. Hence, there is a unique capacity allocation that results in equal crowding levels in the two segments. In the absence of customer mix effects, the capacity will be equally split when the two classes are symmetric but in the asymmetric case, more capacity will be allocated to the class that values the service more.

4.4. Single price with capacity allocation

In this section, we describe the optimization problem of the service provider when capacity allocation is an option but prices need to be the same for both classes. First, note that the single–price constraint is relevant only when $0<x<1$. In that case, enforcing $p_1=p_2$ in (5) and (6) yields,

$$\lambda_2/\Lambda-c\,(\lambda_2/(xK))=\lambda_1/\Lambda-c\,(\lambda_1/((1-x)K))+a. \tag{7}$$

As in the case of capacity sharing with single–price restriction, the single–price constraint disappears when $x=0$ or $x=1$, and the problem is solved in two stages. First, the service provider solves the following optimization problem:

$$\begin{array}{ll} \max_{\lambda_1,\lambda_2,x} & R(\lambda_1,\lambda_2,x)=\lambda_1 p_1(\lambda_1)+\lambda_2 p_2(\lambda_2) \\ \text{s.t.} & 0\le\lambda_1\le(1-x)K,\ 0\le\lambda_2\le xK \\ & 0<x<1 \\ & \lambda_2/\Lambda-c(\lambda_2/(xK))=\lambda_1/\Lambda-c(\lambda_1/[(1-x)K])+a. \end{array} \tag{P4$'$}$$

The solution to optimization problem (P4) is then obtained by comparing the optimal solution to (P4′) with the optimal solution under which the whole capacity is reserved for class–2 customers. In section 5, we use the optimization problem (P4) to prove a number of results on how the policies compare with each other with respect to their optimal revenues.

5. Policy Comparison

In this section, we focus on the most important aspect of the service provider's decision; the policy to adopt depending on the attributes of the customer base. We compare the revenues under the different scenarios and provide valuable analytical results for the design of such a service system.

We start with the case where the manager has the flexibility to charge different prices to the two customer classes and we establish a useful link between the optimal solution to problem (P1) and the optimal solution to problem (P3) in the next corollary.

COROLLARY 1. *If the service provider can price discriminate and $b = 0$, the optimal revenue and customer mix are the same with or without capacity allocation.*

Corollary 1 essentially says that if the net appreciation term is zero, the ability to allocate capacity does not change anything: the provider makes the same revenue with or without capacity allocation, and the resulting customer mix is the same. The linear customer mix effects allow the provider to absorb any significant asymmetry in how the two classes feel about each other (e.g., $b_1 >> 0$ and $b_2 << 0$) through price discrimination. If these asymmetric customer-mix effects are roughly the same in absolute value, then there is not much to gain from separation. Corollary 1 might leave one with the impression that prices with and without capacity allocation are the same. That is not true in general. Unless $b_1 = b_2 = 0$, a simple pairwise comparison of equations (8)–(9) and (5)–(6) reveals that the provider charges different prices when she allocates capacity and when she does not. For example, if $b_1 > 0, b_2 < 0, b_1 + b_2 = 0$, class–1 customers pay a lower price when the provider allocates capacity than when she does not because they lose the benefit of interacting with class–2 customers that they like. The opposite is true for class–2 customers. This price difference leaves the net customer utility unaffected but points to an important implication of an operational decision: depending on whether or not the service provider uses capacity allocation, customers from both classes can end up enjoying different service values and paying significantly different prices without affecting the service provider's revenue. We illustrate this in detail in Table 1 later.

In general, when $b \neq 0$, the service provider has to choose between capacity allocation and sharing. The next theorem provides sufficient conditions for the optimality of each strategy.[2]

THEOREM 1. *If the service provider can allocate capacity and can price discriminate, the capacity allocation decision is as follows:*

(i) If $b \leq 0$, it is optimal to allocate capacity.

(ii) If $b \geq 0$, it is optimal to not allocate capacity.

[2] In the statements of Theorems 1-2, note that the optimality of not allocating capacity does not necessarily imply an inclusive system; it implies that the provider cannot achieve strictly higher revenue by allocating capacity.

Theorem 1 confirms the reality of many service systems, in which providers allocate capacity to mitigate negative interactions between different customer classes. An interesting observation in Theorem 1 is that any asymmetry in the classes' willingness to pay for service (i.e., the value of a) does not affect the sufficient conditions. Although one might expect larger asymmetry to favor capacity allocation, the capacity allocation only aims to prevent customer interactions that hurt the overall customer experience and has nothing to do with customers' willingness to pay. The provider takes into account any asymmetry in the willingness to pay for service by letting more class–2 customers in the optimal customer mix through pricing or by allocating more capacity to them (when classes use different capacity segments). The flexibility of charging different prices allows the service provider to deal with the asymmetry in the willingness-to-pay through pricing. As a result, parameter a plays no role in the service provider's decision on capacity allocation. This is not the case when both classes have to be charged the same price and as a result the asymmetry parameter a becomes a significant factor, as the following theorem indicates.

THEOREM 2. *Suppose that the service provider cannot discriminate but has the flexibility to allocate capacity for the exclusive use of each class. Then, there exists* $b^*(a)$ *such that*

(i) If $b_1 \leq 0$, $b_2 \leq 0$, *it is optimal to allocate capacity.*

(ii) If $b_2 > 0 > b_1$, *then*

(a) *If* $b_1 \leq a - K/\Lambda$, *it is optimal to allocate capacity.*

(b) *If* $b_1 > a - K/\Lambda$ *and* $b \leq b^*(a)$ *(with* $b^*(a) \geq 0$*), it is optimal to allocate capacity.*

(c) *If* $b_1 > a - K/\Lambda$ *and* $b \geq b^*(a)$ *(with* $b^*(a) \geq 0$*), it is optimal to not allocate capacity.*

(iii) If $b_1 > 0 > b_2$, *then*

(a) *If* $b_2 \leq -K/\Lambda$, *or* $b_1 \leq a - K/\Lambda$ *and* $b_2 > -K/\Lambda$, *it is optimal to allocate capacity.*

(b) *If* $b_1 > a - K/\Lambda$, $b_2 > -K/\Lambda$ *and* $\lambda_1^* = 0$ *in* $(P2)$, *it is optimal to allocate capacity.*

(c) *If* $b = 0$, $\frac{a}{2} \geq b_1 > a - \frac{K}{\Lambda}$, $b_2 \geq -\frac{a}{2}$ *and* $\lambda_1^* > 0$ *in* $(P2)$, *it is optimal to not allocate capacity.*

(iv) If $b_1 \geq 0$, $b_2 \geq 0$, *it is optimal to not allocate capacity.*

Furthermore, if $b = b^*(a)$, *allocating and not allocating capacity yield the same revenue to the provider.*

As Theorem 2 shows, the provider's decision regarding capacity allocation is more complicated if she cannot price discriminate. There are two important observations we can make by comparing Theorems 1 and 2. First, a single–price policy leads to capacity allocation in more cases than price discrimination does. Second, when deciding on the capacity allocation, the ability to price discriminate allows the service provider to determine the optimal choice with less information on customer-mix effects. Nonetheless, notice that parts (i) and (iv) of Theorem 2 are analogous to parts (i) and (ii) of Theorem 1, respectively. If classes dislike each other, it is better to separate

	S1: $b_1 = b_2 = 0$		S2: $b_1 = 0.3, b_2 = -0.3$		S3: $b_1 = 0.6, b_2 = -0.6$	
	Capacity Allocation	Capacity Sharing	Capacity Allocation	Capacity Sharing	Capacity Allocation	Capacity Sharing
Double Price	$p_1^* = 0.73$ $p_2^* = 0.98$		$p_1^* = 0.73$ $p_2^* = 0.98$	$p_1^* = 0.92$ $p_2^* = 0.87$	$p_1^* = 0.73$ $p_2^* = 0.98$	$p_1^* = 1.12$ $p_2^* = 0.77$
	$R(27.5, 52.5) = 71.125$					
Single Price	$p^* = 0.85$		$p^* = 0.85$	$p^* = 0.89$	$p^* = 0.85$	$p^* = 0.81$
	$R(15, 65) = 68$		$R(15, 65) = 68$	$R(30, 50) = 71$	$R(15, 65) = 68$	$R(45, 35) = 65$

Table 1 Optimal Revenue (evaluated at the optimal arrival rates) and Prices under Capacity Allocation and Capacity Sharing under the price discrimination and the single price policy for three different scenarios (S1-S3) with zero net appreciation when $\Lambda = 100, K = 80, a = 0.5$

them. If there is mutual appeal, it is more profitable to refrain from capacity allocation. Thus, if class feelings are mutual, neither the pricing policy nor the asymmetry in the willingness to pay for service have an impact on the capacity allocation decision.

The provider's choice is less straightforward if class perceptions go in opposite directions. When $a = 0$, the two cases are completely symmetric and parts (ii) and (iii) of Theorem 2 are identical. If $b_2 > 0 > b_1$, the sufficient conditions of Theorem 2-(ii) confirm that the single–price constraint results in capacity allocation in more cases than price discrimination does. If $b_1 > 0 > b_2$, and classes' feelings toward each other are so different that the provider's best choice without capacity allocation is an exclusive system as outlined in Lemma 2, then she is better off allocating capacity when there is such a flexibility (parts (iii)-a and (iii)-b of Theorem 2). What is more interesting is the case when conditions are such that the provider's optimal choice is to run an inclusive system, i.e., not allocating capacity, and price discrimination is not an option (Theorem 2(ii)-c,(iii)-c and (iv)) and we investigate that in more detail in Section 6 using some numerical examples.

6. Numerical Examples and Sensitivity Analysis

In this section, we first expand on our discussion of Theorems 1 and 2 via a numerical study. Then, we investigate the importance of the customer mix effects in service systems and the sensitivity of the different policies to various parameters. Finally, we study the validity of our results when we relax the same class size assumption for the two customer classes.

Comparison of the different policies for asymmetric classes

We use three different sets of parameters with zero net appreciation ($b = 0$) to gain insights into how the provider's revenues change depending on the capacity allocation decision and the pricing policy followed. We set $\Lambda = 100$, $K = 80$ and $a = 0.5$. The optimal solutions are provided in Table

1. As discussed earlier, when the manager can price discriminate, she will attract the same mix of customers, independently of the capacity decision, by charging different prices, and yield the same revenue ($\lambda_1^* = 27.5, \lambda_2^* = 52.5$ and $R(27.5, 52.5) = 71.125$). However, in the single price policy, mixing the customers or allocating capacity yields the same revenue only when $b_1 = b_2 = 0$ (S1). The revenues might be the same, but prices can be different for the two capacity allocation decisions. Suppose now that $b_1 = 0.3$ and $b_2 = -0.3$ so that $b_1 > 0 > b_2$. These changes do not affect the revenue under capacity allocation (because different classes do not interact), but they change the revenue of an inclusive system. Specifically, the new solution yields higher revenue than before, $R(30, 50) = 71$, and thus operating an inclusive system with both classes sharing the whole capacity is strictly better than allocating capacity for the exclusive use of each class. There are two interesting points to highlight using this example. First, although asymmetry in customer-mix effects hurts revenue when classes are ex ante symmetric ($a = 0$), that may not be the case when classes are ex ante asymmetric ($a > 0$). Second, when customers are no longer indifferent about the presence of customers from the other class, it is strictly preferable to have a system where both classes share the facility. As we explain below, both are consequences of the same price constraint.

If classes are ex ante asymmetric and $b_1 = b_2 = 0$, class 2 would pay more for service than class 1 if the provider could price discriminate. However, the single–price constraint requires that class–1(–2) customers pay more(less) than what they would have had under price discrimination, thereby resulting in inefficient pricing. Suppose now that $b = 0$ but $b_1 > 0 > b_2$. In that case, all else being equal, class–1 customers are willing to pay more than class–2 customers to be around customers of the other type; in other words, the effect of the asymmetry in class feelings is in line with the single–price mandate. What does this mean for the revenue of an inclusive system? Compared to the case where each class is indifferent about the other's presence ($b_1 = b_2 = 0$), it is better to have a small asymmetry in perceptions, with class–1 customers having slight preference for having class–2 customers around while class–2 customers having slight preference for not having class–1 customers around. As a result, in S2, price discrimination has little benefit. However, if this asymmetry in perceptions is strong (S3), it becomes critical in implementing a single–price policy and will force the provider to separate the classes or admit one class only. Also, if the asymmetry is in the opposite direction, with class–2 customers enjoying the presence of class–1 customers, class feelings are no longer in line with the single–price mandate and an inclusive system is not the preferred choice of the service provider.

Part (iii)-c of Theorem 2 states some particularly interesting conditions that guarantee the optimality of capacity sharing; as long as the net appreciation term is zero, a small asymmetry in classes' feelings about each other increases the revenue of a system when classes are not separated, and this can also be observed in Table 1 (S2). Because it is strictly better to not separate classes if b_1

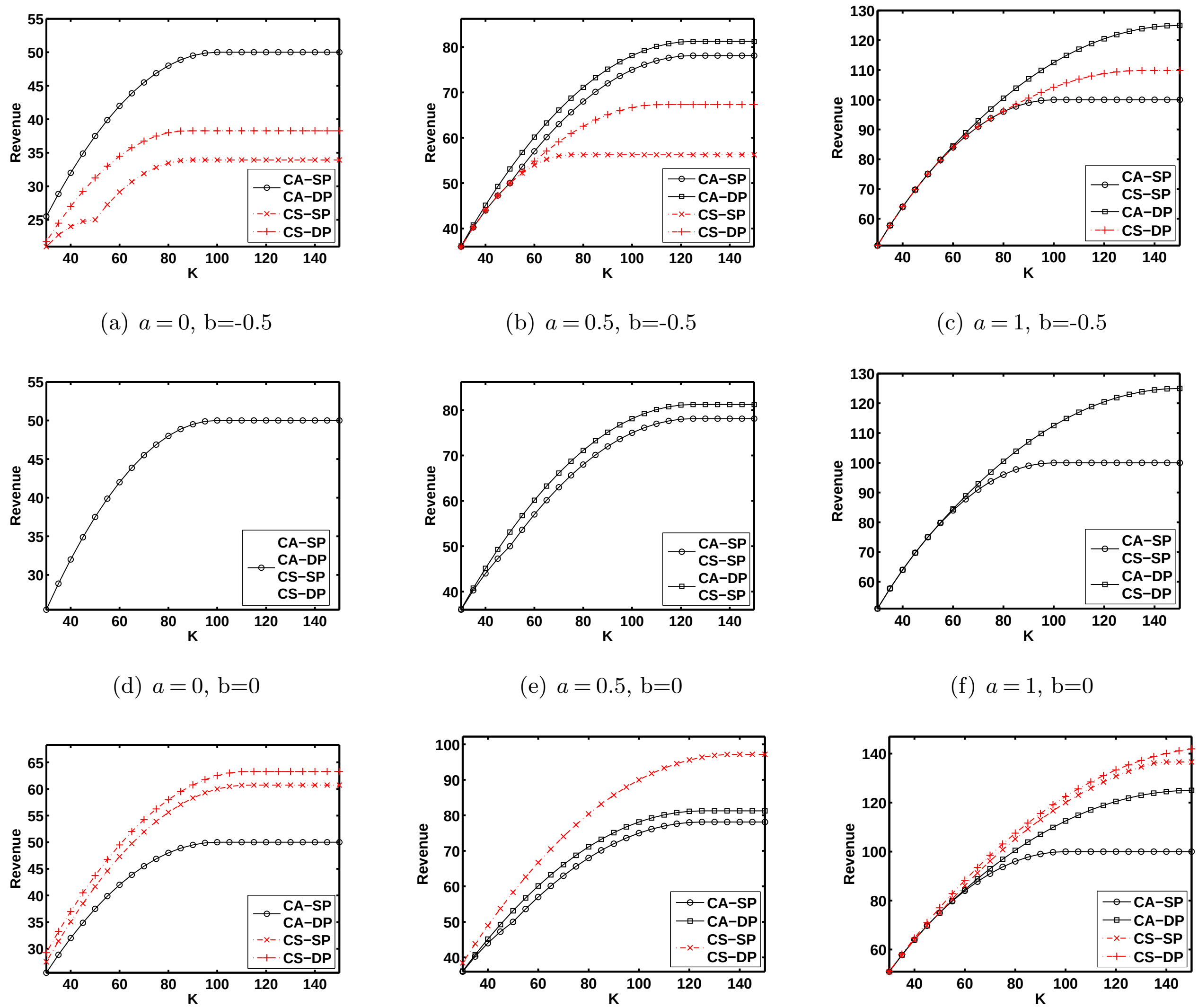
and b_2 are sufficiently small in absolute value and $b = 0$, the provider would also be better off doing so for small yet negative values of b. This means that in some cases, a single–price policy makes capacity allocation less likely than price discrimination does. This might appear to be contradicting one of the insights we have obtained so far, i.e., that single–price policies lead to more exclusivity. It is true that if the provider's choice is only between an inclusive but sharing system and an exclusive system with only one class admitted, then a single–price mandate always leads to more exclusivity because that mandate disappears in an exclusive system. However, exclusivity as a result of separating the two customer classes by capacity allocation does not make the single–price mandate disappear. In that case, there might be some benefit from keeping ex ante asymmetric classes together and mitigating the pricing inefficiency that a single price causes, even if these classes feel differently about being around each other and their net appreciation is negative.

Value of capturing customer mix effects

To further highlight the value of capturing the customer mix effects, we compare the optimal revenues with the revenues we would have achieved had we ignored the parameters b_1 and b_2 by assuming $b_1 = b_2 = 0$. We will follow the examples in Table 1 to make this comparison and use S1 as a benchmark. In S2, when customers can share the service facility and there is price discrimination (CS-DP), the revenue would be $R'(41.5, 36.5) = 67.75$ instead of $R(27.5, 52.5) = 71.125$. When both classes pay the same price (CS-SP), that is $p = 0.85$, then the revenue would be $R'(32.7, 47.3) = 68$ instead of the optimal $R(30, 50) = 71$. Similarly, for the set of parameters in S3, under price discrimination (CS-DP), the revenue would drop to $R'(0, 52.5) = 51.19$ compared to $R(27.5, 52.5) = 71.125$. In this case, ignoring the customer mix effects forces the system to become an exclusive one due to the high price charged to class–1 customers. For the single price policy (CS-SP), the revenue would be $R'(40.4, 31.4) = 60.95$ instead of $R(45, 35) = 65$. These examples are indicative of how high the losses can be and also confirm the fact that a suboptimal capacity allocation strategy might be followed. Taking into consideration that these losses become higher and more discernible when $b \neq 0$ further supports the operational importance of an appropriate capacity allocation decision and pricing strategy.

Sensitivity analysis with respect to system capacity and the strength of customer asymmetry and interaction effects

We have also conducted numerical studies to understand the impact of the parameters a, b, K on the revenue under different policies. Some of the interesting examples are shown in Figure 3. When the net appreciation is negative, capacity allocation is superior to mixing the customers and with higher a, chances are higher to operate an exclusive system at least under low capacity (a small facility can be filled up with high-paying customers). As b increases, mixing the customers becomes



(a) $a = 0$, b=-0.5 (b) $a = 0.5$, b=-0.5 (c) $a = 1$, b=-0.5

(d) $a = 0$, b=0 (e) $a = 0.5$, b=0 (f) $a = 1$, b=0

(g) $a = 0$, b=0.5 (h) $a = 0.5$, b=0.5 (i) $a = 1$, b=0.5

Figure 3 **Comparison of the revenues for the four different policies: capacity allocation (CA) with single price (SP) or price discrimination (DP) and shared capacity (CS) as the capacity increases for different values of the net appreciation and a when $\Lambda = 100$.**

more profitable with $b = 0$ making the policies equivalent and $b > 0$ making capacity sharing the preferred choice. Not surprisingly, price discrimination is at least as good as single price policy and thus, the service provider has the incentive to price discriminate, even when illegal, and incur a penalty up to a certain level. Using an example from the figure with $a = 1$ (when $a = 0$, the pricing strategy does not matter) and $b = -0.5$, $K = 150$, the manager can achieve 25% more revenue if she charges the two classes differently (Figure 3(c)). Finally, as one can observe from Figure 3, investing in capacity can benefit the facility but only up to a certain point.

Λ_1	$\Lambda_2 = 100$			
	CA-SP	CA-DP	CS-DP	CS-SP
50	96	99	96	99
100	96	100.5	96	100.5
150	96	101.4	96	101.4

Λ_2	$\Lambda_1 = 100$			
	CA-SP	CA-DP	CS-DP	CS-SP
50	50	72.3	50	72.33
100	96	100.5	96	100.5
150	117.3	117.4	117.3	117.4

Table 2 Optimal Revenue for the different policies as the class sizes change ($K = 80$, $b = 0$, $a = 1$).

Different class sizes

Heretofore, we focused on leisure facilities that attract two customer classes of the same size but this might not be always the case. One interesting fact we observed from our numerical experiments is that for high values of a the changes in the size of class 2 have a higher impact on the revenue than changes in Λ_1. This is due to the asymmetry of the two classes in terms of their willingness to pay for the service (Table 2). Moreover, we observed that under price discrimination, as the size of one class increases, while the other is constant or decreasing, the system tends to operate in an exclusive manner more often than before ($b^*(K)$ is higher). But if a is higher and class 2 is small, i.e. the high value customers are few, then the facility is better off admitting a mix of customers. In other words, the manager has to exhaust her options of attracting the high value customers but might be limited by their class size. It is also important to note that our numerical analysis suggests that the results of Theorem 1 continue to hold. Not surprisingly, however, the conditions of Theorem 2 have to be modified to account for different class sizes.

7. Conclusions

This paper deals with a particular type of service setting, where service takes an extended period of time and is shared by others so that what happens during service or more specifically who else is there during service is a very important determinant of the customers' utility. Despite the prevalence of such services in practice, these features are sometimes ignored by the service managers and they have received limited attention in the operations literature. One of the important contributions of this paper is the development of a stylized framework that can be helpful in building new models to investigate various research questions (e.g., effects of competition) regarding shared service systems.

We developed a framework to provide insights into the use of pricing and capacity allocation as leverages to control the customer mix and crowding. Some of our findings conform to what we observe in practice and our intuition (for example, the use of discounts if there is asymmetry between how different classes feel about each other), whereas others are either counter–intuitive or help us gain a deeper understanding of some of the issues for which intuition is nonexistent. For example, we find that if the service provider is restricted to charge the same price to two highly asymmetric (either with respect to mutual appreciation or willingness-to-pay) customer classes, the service provider can be profitable only by offering the service to only one class. Interestingly, however, when there is mutual dislike between the two classes, the facility can profitably serve both classes. In short, when faced with sufficiently asymmetric customer classes, the best action for the service provider is to restrict access to a particular class of customers or to allocate different portions of its capacity for the exclusive use of different customer classes. Thus, strong asymmetry requires some sort of discrimination or capacity allocation for the survival of the firm.

For a service provider who can use price discrimination, the choice between allocating capacity for the exclusive use of different classes and making the whole capacity available to all its customers depends purely on the customer mix effects, not on crowding effects, capacity, or the degree of asymmetry in the two customer classes' willingness to pay. Specifically, capacity allocation is desirable if the net appreciation is negative. If price discrimination is not an option, capacity allocation could be desirable even if the net appreciation is positive. Thus, in *many* cases, disallowing price discrimination makes it more likely for firms to serve the different customer classes separately. It is, however, possible that restriction to a single-price policy might lead the provider to switch to an inclusive system with the whole capacity available to both classes. This can only happen if the class with the lower willingness to pay for service likes the other class because only in this case, inclusivity helps reduce the gap between the willingness-to-pay of the two classes.

Our results highlight the importance of having a deeper understanding of customer-mix effects on the utilities of different customer classes, because they are highly relevant in choosing the pricing and capacity allocation policies to be employed. Many articles in the marketing literature have established the presence and importance of these effects, but we are not aware of any work that has aimed to quantify them. To take advantage of the insights, a rough estimate of the parameters might sometimes be sufficient to determine the right strategy. However, some quantification of the customer mix effects, i.e. the sign of b and/or which effect is dominant could be critical in maximizing profit. Thus, one avenue for future research is to develop a framework that can be utilized in measuring customer-mix effects empirically in different service settings. Capturing the valuation for the service is also challenging, yet necessary, to determine the optimal pricing policy. In this direction, economists and marketing researchers have used surveys, experiments, transactions data to infer the willingness to pay of the customers (Wertenbroch and Skiera 2002). Most of these methods can be put into use when estimating customer mix effects.

In some of the service settings we have discussed, the service establishment can gain some pooling benefit if it allows the two customer classes to share the facility (or possibly incur a cost to separate the physical space). This is something we ignored in our formulation. If this benefit were to be considered, our results would change accordingly; the threshold on the customer mix effects would be negative for the capacity allocation to be optimal accounting for the pooling loss. As expected, the new threshold would depend on the actual cost savings from pooling resources; the higher the saving, the lower the threshold would be. In other words, when the savings from pooling are higher, the classes' appreciation of each other would need to be stronger in the negative direction for capacity allocation to be optimal. In some cases, changing the capacity allocation strategy might be costly as it may require rebuilding the whole facility. In that case, the problem is more complex and its analysis would require a formulation that is different from the one we considered in this paper. If rebuilding the facility is an option to the provider, i.e. she is not restricted by the actual size of the facility, then at the beginning of the time horizon, she has to take into consideration several factors including the size of the investment, the competition, the market targeted etc., and investigate how much profit the firm would make at different levels of capacity investment in order to make an optimal decision.

Our model assumed that there is no demand uncertainty and customers make their joining decision simultaneously knowing the behavior of all the other customers. However, it would also be interesting to consider a formulation with stochastic demand and sequential arrivals, so that the manager can dynamically adjust the admission price to control demand. Another interesting direction would be to study multiple competing facilities, each offering different capacity arrangements to their customers.

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