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Information Use and Acquisition in Price-Setting Oligopolies

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Abstract. Asymmetric price-setting multi-product suppliers have access to multiple sources of information about demand conditions, where the publicity of each source corresponds to the cross-industry correlation of signals received from it. A signal's influence on suppliers' prices is increasing in its publicity as well as in its precision. The emphasis on relatively public information is stronger for smaller suppliers who control narrower product portfolios. When information is endogenously acquired, suppliers listen to only a subset of information sources. This subset is smaller when products are less differentiated and when the industry is less concentrated. Smaller suppliers focus attention on fewer information sources. The inefficiencies arising from information acquisition and use are identified. The associated externalities depend upon the extent of product differentiation, the concentration of the industry, and the degree of decreasing returns to scale.

JEL Classification. C72, D43, D83. **Keywords.** Price competition, uncertainty, Bayesian equilibrium, information acquisition, public and private information.

This paper studies how asymmetrically sized multi-product suppliers facing uncertain demand conditions should acquire and use different sources of information.

The context is a price-setting oligopoly in which each supplier controls a portfolio of horizontally differentiated products. Suppliers learn about industry-wide demand conditions by acquiring and observing signals drawn from a range of information sources. A source is relatively public if the signals observed by different suppliers are relatively correlated. Uncertain demand conditions mean that a supplier's information may be used to infer the information available to others, and so to form (higher order) beliefs about competitors' choices. As a consequence, the publicity of an information source plays an important role in determining its acquisition and use.

The results describe how the imperfectly competitive suppliers acquire and use information about uncertain demand conditions; how acquisition and use respond to characteristics of the industry; and how suppliers' behaviour affects consumer surplus, total profits, and social welfare. They show how the relative acquisition and use of different

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information sources respond to various industry characteristics, and further reveal how acquisition and use vary across the set of differently sized suppliers.

In a symmetric industry, suppliers use relatively public information more intensively. This effect is stronger when products are less differentiated and when the industry is less concentrated. If information is acquired endogenously then suppliers listen to only a subset of information sources. In an asymmetric industry, in which larger suppliers control a broader portfolio of product varieties, the pattern of information use depends upon a supplier's size. Smaller suppliers use relatively public information more intensively than larger suppliers; they use new information less intensively than their larger competitors. If information is acquired endogenously then the incentive to acquire new information is smaller for smaller suppliers. Moreover, smaller suppliers focus their attention on fewer (relatively public) information sources than do their larger rivals. Finally, and from a welfare perspective, suppliers make too much use of new information and place too much emphasis on private information sources.

The interaction of the price-setting oligopolists is equivalent to a coordination game in which players (suppliers) wish their actions (prices) to be close to a fundamental (demand conditions) and to the actions of others (a price index). Such games have received much attention following the work of Morris and Shin (2002). However, the typical model in this literature has (i) a continuum of identical and symmetric players, (ii) just two information sources (one independent-across-players private signal and one commonly observed public signal), and (iii) this information is costless to obtain.

This paper is a first attempt to model asymmetries between the players in such coordination games whilst also generalizing the information structure and acquisition process. The oligopoly setting provides a micro-foundation for this coordination game: its underlying structure endogenously generates asymmetries between the players. These asymmetries manifest in players whose preferences for coordination differ, whose fundamental targets differ, and whose influence on the aggregate (average) action differs.

A model (described in Section 1) is specified in which a set of oligopolists choose prices for portfolios of differentiated products. Industry characteristics include the extent of differentiation, the concentration of the industry, and asymmetries in the sizes of suppliers' product portfolios; these characteristics influence the extent of market power. Consumers' utility is quadratic in the profile of products purchased, and so each product's demand is linearly related to prices and a demand shock.

Each supplier acquires and uses a set of informative signals about the demand shock. Within a normal specification, each information source is characterized by two elements: its precision as a signal of the demand shock, and the conditional (on the demand shock) correlation of signal observations seen by different suppliers. A higher correlation coefficient yields an information source that is more public. For example, a perfectly public signal is perfectly correlated (the same signal is seen by everyone) whereas a perfectly private information source generates (conditional on the true value of the demand shock) independent signal realizations for different suppliers. This linear-quadratic-normal specification generates an equilibrium (characterized in Section 2) in which prices react linearly to signal realizations.

A first question (addressed in Section 3) is this: how do prices respond to the different information sources? In a symmetric industry greater use is made of relatively public information: for signals with the same precision, a supplier's price responds more strongly to the signal with the higher correlation coefficient. This is because prices are strategic complements: suppliers wish to correlate their prices with those of their opponents, and do so by placing greater emphasis on relatively public information. This emphasis is stronger as the industry becomes less concentrated or as product differentiation weakens. Such reductions in market power not only increase the use of relatively public information, but also reduce the correlation between prices and true underlying demand conditions.

Asymmetry is captured via differing sizes of the suppliers' product portfolios. Larger suppliers have greater influence on the aggregate price index, therefore smaller suppliers will care more about learning the prices of others, and so are more heavily influenced by higher-order beliefs about demand conditions. This leads to greater intensity in the use of relatively public information by a small supplier, but less intensive use of new information overall. Any increase in concentration (in the sense of moving products from smaller to larger suppliers) induces an overall increase in new information use.

So far the characteristics of the information available to the suppliers have been exogenously determined. However, if information must be acquired before use, so that each supplier decides how to allocate attention across the different possible information sources, then the precision and publicity of each signal will be determined endogenously. In particular, the more attention is devoted to a particular information source, the more precise and the more correlated the associated signal will become.

With this observation in mind, a second question (answered in Section 4) is this: how does the pattern of information acquisition choices vary with the industry's characteristics? The context is a situation in which each information source is characterized by its underlying quality (the precision with which the underlying information source identifies the true demand shock) and by its clarity (the strength of the relationship between the precision of a supplier's noisy observation of that information source and the attention devoted to it). In equilibrium, suppliers restrict attention to a subset of information sources: those that have the highest clarity. That set shrinks (hence focusing attention on the very clearest information sources, even if they are weaker in underlying quality) as market power wanes. Moreover, in asymmetric industries the focus on very clear (and, endogenously very public) signals is strongest for the smallest suppliers. Again, because smaller suppliers care more for coordination and have less influence on the aggregate price index than do their larger and more price-influential competitors, they find public information relatively valuable. As a result smaller suppliers focus their attention on a still smaller set of relatively clear signals.

A final question (Section 5) follows naturally: is information used efficiently? Of course, prices are inefficiently high (owing to the market power of the differentiated suppliers) and indeed the average prices charged in equilibrium equal those (inefficiently high) prices that would be set in a full-information world. However, a supplier's choice of how its price reacts to the signal realizations also involves externalities.

Firstly, competitors benefit most from a rise in another supplier's price when their own prices are already high. Hence a supplier exerts a positive externality on competitors when making more use of relatively correlated signals: the use of relatively public information is too low from the perspective of the industry's suppliers. Those suppliers would also benefit from greater information use overall.

Secondly, and in contrast, consumers prefer prices to be heterogeneous (consumers exploit bargains amongst different prices) and to react negatively to demand conditions (so bargains arise when the products are most valuable). These concerns mean that, from the perspective of consumers, suppliers make too much use of relatively public information, and too much use of information overall. Welfare results are also available: when returns to scale are constant, Marshallian welfare is maximized with relatively greater use of public information, but ideally would involve no information use at all.

This paper contributes to a literature (described further in Section 6) which has studied information use in oligopolies (Palfrey, 1985; Vives, 1988) and under supply-function competition (Vives, 2011, 2013) with uncertain demand conditions. It joins other recent work (Myatt and Wallace, 2015) in considering the efficiency of information use and acquisition when there are many (differently correlated) information sources. Relative to Myatt and Wallace (2015), this paper considers a price-setting rather than quantity-setting environment (so generating the Bertrand versions of Cournot results) and also extends to admit the analysis of differently sized suppliers. The informational environment is related to that used in various assessments of the social value of information (Morris and Shin, 2002; Angeletos and Pavan, 2004, 2007, 2009; Angeletos, Iovino, and La'O, 2016; Amador and Weill, 2010; Llosa and Venkateswaran, 2013; Colombo, Femminis, and Pavan, 2014; Amador and Weill, 2012; Myatt and Wallace, 2012, 2014).

The paper makes two key contributions. Firstly, it incorporates asymmetries in player sizes into a quadratic-payoff coordination game for the first time, hence admitting a study of the effects of the presence of dominant suppliers (say) or influential players more broadly. Secondly, it does this by endogenously generating the (asymmetric) coordination game from an underlying price-setting oligopoly model using deep parameters (such as the extent of product differentiation and industry concentration), thereby providing a framework for the study of information acquisition and use in such industries.

1. Price Competition with Uncertain Demand

1.1. **Demand and Supply.** A unit interval of differentiated product varieties is indexed by $\ell \in [0,1]$. A representative consumer's consumption profile $q \in \mathcal{R}^{[0,1]}$ yields gross utility $U(q) = \int_0^1 u(q_\ell, Q)\, d\ell$ where total consumption is $Q = \int_0^1 q_{\ell'}\, d\ell'$ and where

$$u(q_\ell, Q) = q_\ell \left(\theta - \frac{\beta q_\ell + (1-\beta)Q}{2} \right). \tag{1}$$

$\beta \in [0,1]$ indexes the degree of product differentiation: the products are completely undifferentiated if $\beta = 0$, but are independent if $\beta = 1$.[1] The demand shifter θ (which is uncertain for suppliers) determines the state of demand conditions.[2]

Facing a profile of prices $p \in \mathcal{R}^{[0,1]}$, the representative consumer maximizes consumer surplus $U(q) - \int_0^1 p_\ell q_\ell\, d\ell$. Writing $P = \int_0^1 p_\ell\, d\ell$ for the aggregate price index, solving this problem yields aggregate demand $Q = \theta - P$ and individual demands

$$q_\ell = \frac{\beta(\theta - p_\ell) + (1-\beta)(P - p_\ell)}{\beta}. \tag{2}$$

The set of product varieties is partitioned into M disjoint sub-intervals offered by M suppliers. The product portfolio $L_m \subseteq [0,1]$ of supplier $m \in \{1, \ldots, M\}$ has size $s_m = \int_{\ell \in L_m} d\ell$. Asymmetric industry specifications are obtained via different portfolio sizes.

The cost of producing each product variety is quadratic in its output. Any linear term is (without loss of generality) absorbed into the demand side, and so only the quadratic term is retained: supplier m's total manufacturing cost is $C_m = \frac{c}{2} \int_{\ell \in L_m} q_\ell^2\, d\ell$. The parameter c indexes the severity of any decreasing returns to scale.

The profit-seeking suppliers simultaneously choose prices. A supplier optimally charges the same price for every variety within its product portfolio. Write p_m for this price and q_m for the corresponding demand (so that $p_\ell = p_m$ and $q_\ell = q_m$ for all $\ell \in L_m$).

1.2. **Profits and Consumer Surplus.** Using the demand function from (2), the per-unit profit earned by supplier m and aggregate consumer surplus are

$$\frac{\text{Profit}_m}{s_m} = p_m \left[\theta - p_m + \frac{(1-\beta)(P - p_m)}{\beta} \right] - \frac{c}{2} \left[\theta - p_m + \frac{(1-\beta)(P - p_m)}{\beta} \right]^2; \tag{3}$$

$$\text{Cons. Surplus} = \frac{1}{2} \sum_{m=1}^{M} s_m \left[(\theta - p_m)^2 + \frac{(1-\beta)(P - p_m)(\theta - p_m)}{\beta} \right]. \tag{4}$$

The profit of a supplier can be re-written to fall within the class of quadratic-payoff coordination games. There are parameters $\pi_m \in (0,1)$ and $\gamma_m \in (0,1)$ such that

$$\text{Profit}_m \propto \text{other terms} - \pi_m (p_m - \gamma_m \theta)^2 - (1 - \pi_m)(p_m - P_{-m})^2, \tag{5}$$

[1]This linear demand specification has been used by Dixit (1979), Singh and Vives (1984), and many others.

[2]The specifications of product varieties, consumer demand, and the production technology described here are identical to those used in the Cournot model of Myatt and Wallace (2015). This model differs by considering Bertrand (price setting) behaviour and by allowing for asymmetrically sized suppliers.

where "other terms" do not depend on p_m, and where $P_{-m} \equiv \sum_{m' \neq m} s_{m'} p_{m'}/(1 - s_m)$ is the average price charged by m's competitors.[3] A supplier's payoff is determined by a weighted average of two quadratic-loss components. $(p_m - \gamma_m \theta)^2$ is a fundamental motive: the supplier would like to price close to the target $\gamma_m \theta$. $(p_m - P_{-m})^2$ is a coordination motive, determined by the distance of a supplier's price from others.

The fundamental target $\gamma_m \theta$ and the relative importance of the fundamental and coordination motives, measured by π_m, both vary with the size s_m of the supplier. The expressions are relatively clean when there are constant returns to scale:

$$c = 0 \quad \Rightarrow \quad \gamma_m = \frac{\beta}{\beta + (1 - (1 - \beta)s_m)} \quad \text{and} \quad \pi_m = \frac{1}{2}\left(1 + \frac{\beta}{1 - (1 - \beta)s_m}\right).$$

Both terms increase in s_m: smaller suppliers prefer to set lower prices ($\gamma_m \theta$ is lower) and place more emphasis on coordination (π_m is lower). These claims also hold for $c > 0$.

As suppliers become small (increasing M and letting $s_m \to 0$), the payoff specification fits within the scope of Morris and Shin (2002), Angeletos and Pavan (2007), Dewan and Myatt (2008, 2012), Myatt and Wallace (2012), and others. The key difference here is the asymmetric setting: the coordination motive $(1 - \pi_m)$, the target fundamental $(\gamma_m \theta)$, and the influence each supplier has on aggregates (via P) all vary with supplier size.

1.3. **Information.** Market conditions are determined by the demand shifter θ. The suppliers share a common prior $\theta \sim N(\bar{\theta}, \sigma^2)$. Supplier m has access to n sources of information about θ. The signal received from the ith source is

$$x_{im} = \theta + \eta_i + \varepsilon_{im}, \tag{7}$$

where $\eta_i \sim \mathrm{N}(0, \kappa_i^2)$ and $\varepsilon_{im} \sim \mathrm{N}(0, \xi_{im}^2)$, and where all the noise terms are uncorrelated.[4]

The common (to all suppliers) shock η_i is noise attributable to the sender of the information. The supplier-specific shock ε_{im} is observation noise attributable to receiver m.[5]

This specification induces a correlation structure for the signal observations. Signals differ in their precision and in their correlation. For instance, if $\xi_{im}^2 = \xi_i^2$ for all m (which is a leading case of interest) then the precision of signal i (as an informative signal of θ) is $\psi_i = 1/(\kappa_i^2 + \xi_i^2)$. Conditional on θ, the correlation coefficient between the observations of two different suppliers is $\rho_i = \kappa_i^2/(\kappa_i^2 + \xi_i^2)$. If observations are more correlated then an information source is more public; that is, in such instances, it makes sense to associate the publicity of a signal directly with its correlation coefficient ρ_i.

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[3]Explicitly, the coefficients γ_m and π_m in this expression are

$$\gamma_m = \frac{\beta + c(1 - (1 - \beta)s_m)}{\beta + (1 + c)(1 - (1 - \beta)s_m)} \quad \text{and} \quad \pi_m = \frac{\beta(\beta + (1 + c)(1 - (1 - \beta)s_m))}{(1 - (1 - \beta)s_m)(2\beta + c(1 - (1 - \beta)s_m))}. \tag{6}$$

The details of (5)–(6) along with some other calculations may be found in the supplementary Appendix B.

[4]The information structure is taken from Dewan and Myatt (2008, 2012) and Myatt and Wallace (2012, 2014, 2015). This structure has also been adopted recently by others: see, for example, Pavan (2014).

[5]Section 4 extends to cases where the variance of ε_{im} is endogenously determined. The "sender" and "receiver" terminology is from Myatt and Wallace (2012), where the information structure is the same, albeit in a context where the focus is the endogenous acquisition of information in the context of a symmetric "beauty contest" quadratic-payoff coordination game with a continuum of players.

2. Equilibrium Characterization

2.1. **Optimal Pricing.** The expected profit of supplier m is a concave quadratic function of the supplier's price p_m. Taking expectations of the expression (3) and differentiating, the first-order condition for a supplier's price choice yields

$$p_m = \pi_m \gamma_m \operatorname{E}[\theta \mid x_m] + (1 - \pi_m) \operatorname{E}[P_{-m} \mid x_m], \tag{8}$$

where the γ_m and π_m are functions of the parameters β, c, and s_m from (6), and where P_{-m} is the (appropriately weighted) average of the prices set by other suppliers.

The solution for p_m applies generally. Note that, if others use aggressive pricing strategies and if demand is expected to be weak (for example, when a signal indicates a negative value for θ) then (8) may yield a negative solution. Relatedly, the pricing choices of the suppliers may (given the realization of true demand conditions) result in negative demands from (2). If negative outputs and prices were disallowed then the strategies considered here would sometimes specify infeasible actions. As noted by Vives (1984, p. 77, fn. 2), "the probability of such [negative price or quantity events] can be made arbitrarily small by appropriately choosing the variances of the model." There are other resolutions. A fuller discussion, within the context of a Cournot model, was offered by Myatt and Wallace (2015).[6]

2.2. **The Full-Information Benchmark.** If demand conditions (via θ) are known then equilibrium prices are readily characterized. The optimal price of supplier m satisfies $p_m = \pi_m \gamma_m \theta + (1 - \pi_m) P_{-m}$. In terms of the average industry price,

$$p_m = \delta_m \left[\frac{\beta\theta}{1-\beta} + P \right] \quad \text{where} \quad \delta_m \equiv \frac{(1-\beta)(\beta + c(1-(1-\beta)s_m))}{\beta + (\beta + c)(1-(1-\beta)s_m)}. \tag{9}$$

δ_m is increasing in s_m, and so in the full-information case larger suppliers charge higher prices: a large supplier internalizes (at least partially) the cannibalizing effect of a price cut on the demand for substitute products. Taking the weighted sum of p_m over all suppliers and re-arranging yields a solution for the average industry price: this is

$$P = \frac{\beta\theta}{1-\beta} \times \frac{\sum_{m=1}^{M} s_m \delta_m}{1 - \sum_{m=1}^{M} s_m \delta_m}. \tag{10}$$

Combining (9) and (10), the equilibrium prices charged are readily obtained.

[6]As discussed by Myatt and Wallace (2015), this ignore-the-problem approach is not entirely satisfactory. They documented other resolutions. For example, the normal specification can be dropped. Its advantage is the linearity of conditional expectations (Li, 1985; Li, McKelvey, and Page, 1987). However, a different specification with the linear regression property could be used. Another approach is to impose linearity directly by insisting that suppliers choose linear strategies. It is acknowledged that non-negativity constraints are sometimes important: some have found that results concerning information sharing in oligopolies (Vives, 1984, for example) can change if non-negativity constraints are respected (Malueg and Tsutsui, 1998; Lagerlöf, 2007). Here, however, the focus is not on information sharing. The shortcut pays dividends by allowing clear results on the relative use of information sources with different correlations.

Proposition 1 (Benchmark Case). *In the complete-information case, where demand conditions are known to suppliers, the equilibrium price of supplier m is*

$$p_m = \frac{\beta\theta}{1-\beta} \times \frac{\delta_m}{1-\sum_{m'=1}^{M} s_{m'}\delta_{m'}}. \tag{11}$$

Smaller suppliers charge lower prices than their larger competitors. Prices are increasing in the degree of product differentiation and in the strength of decreasing returns to scale.

The price index P is a convex function of the suppliers' portfolio sizes. Hence, increasing the concentration of the industry, by shifting product varieties from a smaller to a larger supplier, raises the average price charged and lowers aggregate industry output.

2.3. **Equilibrium.** Strategies are linear if each supplier's price responds linearly to signal realizations. For some intercept term $\bar{p}_m \in \mathcal{R}$ and vector of n weights $w_m \in \mathcal{R}^n$, such a linear strategy takes the form

$$p_m = \bar{p}_m + \sum_{i=1}^{n} w_{im}(x_{im} - \bar{\theta}).$$

$\bar{p}_m$ is the expected price charged by supplier m, and w_{im} measures the response of the price charged to the ith signal of demand conditions.

The properties of the normal imply that the regression $\mathrm{E}[\theta \mid x_m]$ is linear, as is $\mathrm{E}[x_{m'} \mid x_m]$. Hence, if others use linear strategies then $\mathrm{E}[P_{-m} \mid x_m]$ is linear in x_m. Applying (8), the best reply to the linear strategies of others is itself linear in x_m.

Given the use of linear strategies, the model reduces to a simultaneous-move game in which each supplier m chooses $\bar{p}_m$ and $w_m \in \mathcal{R}^n$ to maximize expected profit.[7]

Without fully characterizing equilibrium strategies, the expected prices of suppliers are easily found. Taking expectations of both sides of (8),

$$\begin{aligned}
\bar{p}_m &= \pi_m\gamma_m \,\mathrm{E}[\mathrm{E}[\theta \mid x_m]] + \frac{1-\pi_m}{1-s_m}\sum_{m'\neq m} s_{m'}\left[\bar{p}_{m'} + \sum_{i=1}^{n} w_{im'}\,\mathrm{E}\left[\mathrm{E}[(x_{im'} - \bar{\theta}) \mid x_m]\right]\right] \\
&= \pi_m\gamma_m\bar{\theta} + \frac{1-\pi_m}{1-s_m}\sum_{m'\neq m} s_{m'}\bar{p}_{m'} \\
\Rightarrow \quad \bar{p}_m &= \delta_m\left[\frac{\beta\bar{\theta}}{1-\beta} + \bar{P}\right] \quad \text{where} \quad \bar{P} = \sum_{m'=1}^{M} s_{m'}\bar{p}_{m'}.
\end{aligned}$$

This corresponds to the associated condition from the full-information benchmark case. It implies that the expected price $\bar{p}_m$ charged by supplier m is equal to the price that it would charge in a full-information world when the demand shifter is known to equal $\bar{\theta}$.

[7]If the normal specification were discarded, then this game remains amenable to analysis. It is equivalent to a game in which suppliers are restricted to react linearly to signals of changing demand conditions.

Proposition 2 (Expected Equilibrium Prices and Outputs)**.** *In expectation, suppliers' prices and outputs equal their full-information counterparts. That is,*

$$\bar{p}_m = \mathrm{E}[p_m] = \frac{\beta\bar{\theta}}{1-\beta} \times \frac{\delta_m}{1 - \sum_{m'=1}^{M} s_{m'}\delta_{m'}}. \tag{12}$$

The properties of the full-information benchmark are inherited: smaller suppliers charge lower prices on average; the expected average price is increasing in product differentiation and in the strength of decreasing returns; and increased industry concentration raises the expected average industry price while lowering expected output.

Hence, the presence of demand uncertainty (or the arrival of a zero-mean demand shock) has no effect on average. Nevertheless, second moments of prices (and hence outputs) matter for profits and consumer surplus. As the proof of Lemma 1 will confirm,

$$\begin{aligned}
\frac{\mathrm{E}[\text{Profit}_m]}{s_m} &= \frac{\text{full-information profit}}{s_m} + \text{other terms} && (13)\\
&+ \left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] \mathrm{cov}[\theta, p_m] \\
&- \frac{1-(1-\beta)s_m}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{2\beta}\right] \mathrm{var}[p_m] && (14)\\
&+ \frac{1-\beta}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] \sum_{m' \neq m} s_{m'}\,\mathrm{cov}[p_m, p_{m'}]. && (15)
\end{aligned}$$

The "full information profit" (the profit enjoyed in the absence of any demand shock) depends only on the expected prices of the various suppliers. The "other terms" are those that are outside the control of supplier m. They depend on the signal use of other suppliers, but not on the strategy of supplier m. Moreover, those other terms disappear completely when the production technology exhibits constant returns to scale ($c = 0$).

Examining the remaining terms, supplier m gains from co-movement of its price with the demand shock (via $\mathrm{cov}[\theta, p_m]$), loses from any volatility in its price (via $\mathrm{var}[p_m]$; profit is concave in price), and gains from co-movement of its price with its competitors (via $\mathrm{cov}[p_m, p_{m'}]$; because prices are strategic complements). The first component depends only on the total reaction of a supplier to new information about demand. To measure this define $\bar{w}_m \equiv \sum_{i=1}^{n} w_{im}$ for each supplier m. Using this notation,

$$\mathrm{cov}[\theta, p_m] = \bar{w}_m\sigma^2, \quad \mathrm{var}[p_m] = \bar{w}_m^2\sigma^2 + \sum_{i=1}^{n} w_{im}^2(\kappa_i^2 + \xi_{im}^2),$$

$$\text{and} \quad \mathrm{cov}[p_m, p_{m'}] = \bar{w}_m\bar{w}_{m'}\sigma^2 + \sum_{i=1}^{n} w_{im}w_{im'}\kappa_i^2. \tag{16}$$

The relative importance of each (co)variance component depends upon the size of a supplier's product portfolio. To illustrate the forces at work, consider a special case in which the quadratic component of suppliers' costs is eliminated, so that there are constant returns to scale in production, and where suppliers choose the same weights on their signal realizations. $\operatorname{cov}[p_m, p_{m'}]$ is the same for all pairs, so that

$$\frac{\mathrm{E}[\text{Profit}_m]}{s_m} = \frac{\text{full-information profit}}{s_m} + \text{other terms}$$
$$+ \operatorname{cov}[\theta, p_m] + \frac{(1-\beta)(1-s_m)\operatorname{cov}[p_m, p_{m'}] - [1-(1-\beta)s_m]\operatorname{var}[p_m]}{\beta}.$$

The value of the alignment of a supplier's price with demand conditions (via $\operatorname{cov}[\theta, p_m]$) does not depend on the supplier's size. However, the importance of price volatility ($\operatorname{var}[p_m]$) relative to price co-movement ($\operatorname{cov}[p_m, p_{m'}]$) does depend on s_m. Specifically, the ratio of the coefficient on volatility to the coefficient on co-movement is increasing in s_m. This means when comparing the division of influence between two different information sources, a larger supplier is primarily concerned with the overall noise in signals, whereas a smaller supplier cares more about the covariance of signals.

The general expression for supplier profitability is concave in a supplier's choice of weights $w_m \in \mathcal{R}^n$ and so first-order conditions determine optimality. Those conditions generate the following equilibrium characterization. For this, $w_i = \sum_{m=1}^{M} s_m w_{im}$ is the average weight placed on the ith signal realization (where suppliers are weighted by portfolio sizes) and $\bar{w} = \sum_{i=1}^{n} w_i$ is the average influence of new information on prices.

Lemma 1 (Equilibrium Characterization)**.** *The equilibrium weights satisfy*

$$\left[\kappa_i^2 + (1-\delta_m s_m)\xi_{im}^2\right] w_{im} - \delta_m w_i \kappa_i^2 = \sigma^2 \left[\delta_m \left(\frac{\beta}{1-\beta} + \bar{w}\right) - \bar{w}_m\right]. \tag{17}$$

The first-order condition (17) holds for all information sources and all suppliers. It generates a linear system of nM equations in the nM weights. This system is readily solved using linear algebraic methods. The solution can be found for any parameter constellation, and (for various cases) explicit solutions are reported in later propositions.

For now, however, notice that the coefficient attached to w_{im} is $\kappa_i^2 + (1-\delta_m s_m)\xi_{im}^2$. Hence, the importance of the idiosyncratic noise associated with an information source relative to the common noise depends upon the breadth of a supplier's product portfolio.

3. Information Use

Lemma 1 provides, at least implicitly, a general characterization of the equilibrium weights. To investigate further the properties of these weights it is convenient to focus on three leading cases of interest. Firstly, a general information structure is analysed when suppliers have equally sized product portfolios. The objective is to explore the relative use of different kinds of information (relatively private versus relatively public). Secondly, and maintaining the general information structure, this case is extended to admit the presence of a monopolistically competitive fringe of suppliers. This enables a comparison of the information used by suppliers in the fringe and that used by their larger competitors. Finally, a fully general industry structure (where each supplier has a differently sized portfolio) is studied when a single new information source is available. The aim here is to understand how the reaction of prices to new information varies with the size of suppliers.

3.1. **Symmetric Industries.** A first insight into information use is obtained in a symmetric industry: $s \equiv s_m = 1/M$ and $\xi_{im}^2 = \xi_i^2$ for all m. Applying Proposition 2,

$$\bar{p}_m = \frac{\beta\bar{\theta}}{1-\beta} \times \frac{\delta}{1-\delta} = \gamma\bar{\theta} \quad \forall m, \tag{18}$$

where the subscripts from δ_m and γ_m have been dropped. The solutions to (17) are symmetric across suppliers. The equilibrium is obtained by solving the n equations

$$\left[\frac{(1-\delta)\kappa_i^2 + (1-\delta s)\xi_i^2}{\sigma^2}\right] w_i = \frac{\delta\beta}{1-\beta} - (1-\delta)\bar{w}. \tag{19}$$

Recall that ψ_i and ρ_i are the precision and correlation coefficient of signal i.

Proposition 3 (Symmetric Equilibrium). *If suppliers are symmetric, then the unique linear equilibrium satisfies* $p_m = \bar{p}_m + \sum_{i=1}^n w_i(x_{im} - \bar{\theta})$, *where*

$$w_i \propto \frac{1}{\pi\kappa_i^2 + \xi_i^2} = \frac{\psi_i}{1-(1-\pi)\rho_i} \quad \textit{and where} \quad \pi = \frac{1-\delta}{1-s\delta}. \tag{20}$$

Fixing the correlation coefficients, relatively precise signals have relatively greater influence. Fixing the precisions, relatively correlated signals have relatively greater influence.

The total influence of new information on the prices set by suppliers is

$$\bar{w} = \frac{\gamma\varphi}{1+\varphi}, \quad \textit{where} \quad \varphi \equiv \sum_{i=1}^n \frac{\pi\sigma^2}{\pi\kappa_i^2 + \xi_i^2}. \tag{21}$$

This is increasing in both the precisions and the correlations of suppliers' signals.

The industry makes relatively greater use of relatively public information. Consider the ratio of the weights placed on two different informative signals:

$$\frac{w_i}{w_j} = \frac{\psi_i}{\psi_j}\frac{1-(1-\pi)\rho_j}{1-(1-\pi)\rho_i}.$$

This is increasing in ρ_i and decreasing in ρ_j. The relative use of information also depends upon the characteristics of the industry. Specifically, note that if $\rho_i > \rho_j$ (the ith information source is more public than the jth) then the ratio above is decreasing in π. The parameter π represents a supplier's concern with a fundamental motive (to track demand conditions) relative to a coordination motive (to follow the prices charged by competitors). In terms of the industry's characteristics,

$$\pi = \frac{1-\delta}{1-s\delta} = \frac{\beta(\beta + (1+c)(1-(1-\beta)s))}{(1-(1-\beta)s)(2\beta + c(1-(1-\beta)s))} \quad \text{where} \quad s = \frac{1}{M}.$$

This is increasing in β and s. Greater product differentiation (a rise in β) and an increased share of product varieties (equivalently, a fall in M) give a supplier more market power, and so that supplier focuses more on the fundamental motive.

Proposition 4 (Comparative Statics)**.** *The equilibrium weight placed on a more public signal relative to a more private signal is decreasing in product differentiation, but increasing in the number of competitors and the strength of decreasing returns to scale.*

Formally: if $\rho_i > \rho_j$ then the ratio w_i/w_j is decreasing in β but increasing in M and c.

Proposition 4 reveals how the characteristics of the industry determine the use of different kinds of information. Those characteristics also determine the reaction of prices to shifts in the demand shock. An increase in market power raises prices or, equivalently, induces stronger reactions to perceived improvements in demand conditions. This effect is present in the movement of the baseline expected price in response to changes in expected conditions: recall that $\bar{p}_m = \gamma\bar{\theta}$ from (18). Industry characteristics also influence information use, and this in turn changes how prices respond to new information. One way to measure the response to shocks relative to the level of a supplier's price is the

$$\text{Relative Impact of New Information} \equiv \frac{\partial p_m/\partial \theta}{\partial \bar{p}_m/\partial \bar{\theta}} = \frac{\varphi}{1+\varphi},$$

where φ is defined in (21) and is increasing in π. Factors that raise the suppliers' focus on the fundamental objective (such as greater product differentiation or increased industry concentration) result in a greater (relative) impact of new information on prices.

This property is a reflection of the implicit use by suppliers of the prior mean $\bar{\theta}$ as a perfectly public signal of demand conditions. That is,

$$\rho_i = 1 \quad \Rightarrow \quad \frac{\partial p_m/\partial \bar{\theta}}{\partial p_m/\partial x_{im}} = \frac{\kappa_i^2}{\sigma^2}.$$

Hence an increase in the relative impact of new information (following a change that results in an increase in π) is a consequence of the fact that new information is relatively private compared to the (common, and so perfectly public) prior.

The relationship between market conditions and prices can also be evaluated via the correlation coefficient between a supplier's price and the demand shifter θ. Note that

$$\operatorname{corr}[p_m, \theta] = \frac{\operatorname{cov}[\theta, p_m]}{\sqrt{\sigma^2 \operatorname{var}[p_m]}} = \frac{\bar{w}\sigma^2}{\sqrt{\sigma^2 \operatorname{var}[p_m]}} = \left[1 + \sum_{i=1}^{n} \left(\frac{w_i}{\bar{w}}\right)^2 \frac{\kappa_i^2 + \xi_i^2}{\sigma^2}\right]^{-\frac{1}{2}},$$

where the final equality follows from the substitution of the expression for $\operatorname{var}[p_m]$. Straightforward but tedious derivations confirm that this expression is increasing in π. Hence, if suppliers become more concerned with the fundamental objective then (as expected) their prices correlate more strongly with any shock to demand.

Proposition 5 (Industry Characteristics and Price Responses)**.** *The relative impact of new information on prices and the correlation between prices and the underlying demand conditions are (i) increasing in product differentiation, β, (ii) increasing in the concentration of the industry, s, and (iii) decreasing in the strength of decreasing returns, c.*

The results of Propositions 3–5 are readily compared to those obtained in a Cournot industry. The analysis of Myatt and Wallace (2015) shows that quantity-setting suppliers choose outputs that respond to signals with coefficients $w_i \propto 1/(\pi\kappa_i^2 + \xi_i^2)$. However, in a Cournot world $\pi > 1$, which implies that suppliers place greater emphasis on private information. The key comparative-static results are also reversed. Proposition 2 of Myatt and Wallace (2015) shows that an increase in product differentiation or reduction in the number of competitors (equivalently, an increase in the share of varieties controlled by each supplier) results in a shift in emphasis away from relatively private signals. Here (Proposition 4) it results in a shift toward relatively private signals.

Corollary. *In a price-setting industry, the relative use of new and more private information strengthens as the market power of suppliers rises via heightened product differentiation and industry concentration. In a Cournot industry, the opposite claims hold.*

Quantities are strategic substitutes whereas prices are strategic complements. Cournot suppliers dislike co-movement of their outputs, and so shy away from relatively public information. In contrast, price-setting suppliers gain from co-movement of their prices and so place more emphasis on highly correlated signals.

A key message from the symmetric analysis is a comparative static claim: there is greater use of public information as suppliers become smaller. However, given only a single cross-sectional observation of an industry, it would be interesting to know whether the same claim applies. Necessarily this involves the study of an asymmetric industry. Moreover, comparative static exercises conducted in a symmetric setting unavoidably conflate two issues. As the sizes of the suppliers change, information use (and acquisition) changes for two reasons. The supplier's objective function is affected through an "own-size" effect but also by the size of their competitors. To disentangle these effects, attention now turns to asymmetric industries, in which the sizes of the product portfolios differ between different suppliers. Two issues are investigated: the relationship between supplier size and the relative importance of the different kinds (more or less public) of information; and the impact of the industry's concentration on the reaction of prices to new information about changing demand conditions.

3.2. **A Monopolistically Competitive Fringe.** Consider an industry structure comprising an oligopoly of M leading suppliers with equal shares of the product space, and a monopolistically competitive fringe of suppliers with negligibly sized product portfolios. This is a limiting case of an industry structure in which there are two different types (sizes) of supplier, and where the width of the smaller portfolio is allowed to shrink. This specification, via the introduction of negligibly sized suppliers who have no influence on their larger competitors, usefully isolates the own-size effect mentioned just above.

The pricing strategies of the monopolistically competitive fringe have a negligible effect on the average industry-wide use of each information source, and so w_i is (for each signal) fixed from the perspective of fringe members. For the symmetric oligopolists, the use of information satisfies $w_{im} = w_i$, the solution to which is characterized in (20) of Proposition 3. For any fringe supplier (identified with the subscript F throughout) note that $s_F = 0$ and so (17) in Lemma 1 reduces to

$$\left[\kappa_i^2 + \xi_{iF}^2\right] w_{iF} - \delta_F w_i \kappa_i^2 = \sigma^2 \left[\delta_F \left(\frac{\beta}{1-\beta} + \bar{w}\right) - \bar{w}_F\right]. \tag{22}$$

Continuing with the case $\xi_{iF}^2 = \xi_i^2$ for all i, the solution to these equations along with the characterization of w_i in Proposition 3 combine to give the following result.

Proposition 6 (Information Use in the Competitive Fringe)**.** *The monopolistically competitive fringe uses relatively public information relatively intensively. That is,*

$$\frac{w_{iF}}{w_{jF}} > \frac{w_i}{w_j} \quad \Leftrightarrow \quad \rho_i > \rho_j. \tag{23}$$

Smaller suppliers care more about the coordination motive and less about the fundamental motive than do their larger competitors (π_m is increasing in s_m). More highly correlated (public) information is therefore relatively useful for them. In contrast, larger suppliers have greater influence on the aggregate price index and so care less about others' prices. This reduces the emphasis they place on relatively public information.

3.3. **The Use of New Information.** Propositions 3 and 5 offer results on the use of new information in the context of a symmetric industry. Straightforwardly, such information use increases as that information becomes more precise and as the demand shock itself becomes more uncertain. More subtly, the total influence of new information is also increasing in the publicity of each information source. Turning to industry characteristics, the relative impact (and, indeed, the absolute impact) of new information on prices is increasing in the concentration of the industry.

Here, attention turns to an asymmetric industry where suppliers offer differently sized portfolios of product varieties. To focus on the characterization of information use, however, a situation is considered in which there is only a single source of new information.[8] Specifically, $n = 1$ and so the subscript i can be dropped. Equivalently: $w_{im} = \bar{w}_m$ and $w_i = \bar{w}$. An equilibrium is characterized by M first-order conditions of the form

$$\left[\kappa^2 + (1 - \delta_m s_m)\xi_m^2\right] \bar{w}_m - \delta_m \bar{w} \kappa^2 = \sigma^2 \left[\delta_m \left(\frac{\beta}{1-\beta} + \bar{w}\right) - \bar{w}_m\right]. \tag{24}$$

These equations describe the way in which new information influences prices.

Lemma 2 (Equilibrium with a Single Signal)**.** *In an asymmetric industry where suppliers have access to a single information source, the use of new information satisfies*

$$\bar{w}_m \propto \frac{\delta_m}{\sigma^2 + \kappa^2 + (1 - \delta_m s_m)\xi_m^2} \equiv \mu_m. \tag{25}$$

The influence of new information on the average price is

$$\bar{w} = \frac{\beta\sigma^2}{1-\beta} \times \frac{\sum_{m=1}^M s_m \mu_m}{1 - (\sigma^2 + \kappa^2) \sum_{m=1}^M s_m \mu_m}. \tag{26}$$

[8] A second source of purely public information can be accommodated by absorbing it into the prior.

Inspecting the solution for $\bar{w}_m$, and recalling that δ_m is an increasing function of s_m (and hence so is μ_m), larger suppliers place greater absolute weight on new information relative to smaller suppliers. However, and as noted previously, larger suppliers charge higher prices on average owing to their greater market power. Hence, a more appropriate measure of the use of new information in relative terms is the

$$\text{Relative Impact of New Information} \equiv \frac{\partial p_m/\partial\theta}{\partial\bar{p}_m/\partial\bar{\theta}} \propto \frac{1}{\sigma^2+\kappa^2+(1-\delta_m s_m)\xi_m^2}.$$

This is increasing in s_m: larger suppliers use new information more intensively. Noting that new information is always relatively private compared with a common prior, the intuition is precisely as before: larger suppliers influence the aggregate price index more and are therefore less concerned with the coordination motive (and consequently relatively correlated information) than are their smaller competitors.

The influence of new information on the average price charged can also be evaluated. The solution for $\bar{w}$ is a convex function of the suppliers' product-portfolio, and so increases when products are shifted from a smaller supplier to a larger competitor.

Proposition 7 (The Use of New Information)**.** *Larger suppliers use new information more intensively than smaller suppliers. The total use of new information is increasing in industry concentration, as is the average price.*

The weakening of market power associated with increased concentration lessens the industry's reliance on public information. The common prior over demand conditions (equivalently, the state of demand $\bar{\theta}$ before any shock) acts as a purely public signal: new information is relatively private. Reduced market power lowers the importance of coordination, so raising the emphasis placed on new (relatively private) information and thereby increasing the average price charged by the suppliers.

Summary. In the context of a symmetric price-setting industry, information use exhibits a bias towards relatively public signals. This bias is more pronounced and the co-movement of prices with demand conditions is weaker when suppliers have less market power. These results are the mirror images of those obtained in the Cournot industry of Myatt and Wallace (2015). In an asymmetric price-setting industry, information use differs between suppliers of different sizes. Smaller suppliers (specifically, those in a competitive fringe) exhibit a relatively stronger bias toward relatively public information sources when compared to their larger competitors. Within a more general industrial structure, larger suppliers use new information more intensively. Moreover, an increase in concentration increases the overall use of new information.[9]

[9]These latter results do not have analogues in the quantity-setting Cournot industry of Myatt and Wallace (2015), simply because that earlier paper restricts to a symmetric specification.

4. Information Acquisition

In this section, the model is extended to allow suppliers to acquire information endogenously. The approach follows that of Myatt and Wallace (2012) which is developed below to admit the asymmetries present in the price-setting oligopoly framework.

The characteristics of the suppliers' signals have so far been fixed. The variance κ_i^2 measures sender noise, interpreted as a common error in observing θ made by the information provider, and, as such, is exogenous from the perspective of the suppliers. Receiver noise, on the other hand, measured by ξ_{im}^2, is arguably endogenous. This is noise associated with how much attention supplier m pays to an information source. Increased attention raises the precision with which the signal is observed, and so reduces receiver noise. To capture this idea, let

$$\varepsilon_{im} \sim \mathrm{N}\left(0, \frac{\xi_i^2}{z_{im}}\right),$$

where $z_{im} \geq 0$ is the (private) attention paid by supplier m to information source i. $z_{im} = 0$ is interpreted as m paying no attention to source i: in such a case m will receive a completely uninformative (infinite variance) signal from source i. It is important to note that the attention decision (choice of z_{im}) is made prior to the pricing decision, but that this choice is not observed by a supplier's competitors.

Note that z_{im} is proportional to the precision of the signal which supplier m receives about $\theta + \eta_i$. Within a standard sampling procedure, the precision of a signal is proportional to the sample size. Hence, z_{im} can be interpreted as the expenditure on a sample size used to generate a signal from the ith information source, or as the time spent watching that source. Similarly, $1/\xi_i^2$ can be interpreted as the clarity of that sampling procedure. Source i is clearer than j if $\xi_i < \xi_j$, and a supplier m devotes more attention to i than j if $z_{im} > z_{jm}$. Setting $z_{im} = 0$ is equivalent to ignoring a signal altogether.

Suppose that suppliers have a limited stock of attention, so that $\sum_{i=1}^n z_{im} \leq 1$ for all m. The idea is that suppliers must choose how to allocate their limited attention (or sampling capacity) between the various information sources.[10] An alternative specification would be to assign an (increasing) cost to acquisition. The qualitative features of many of the results below would be unaffected at the cost of some algebraic complexity. In particular, convexity of such a cost function would leave the results largely intact.[11]

4.1. Symmetric Industries.

To identify the optimal acquisition policy $z_m \in \mathcal{R}_+^n$ for supplier m, note that expected per-unit profit given in (13)-(15) depends on $\xi_{im}^2 = \xi_i^2/z_{im}$

[10] Indeed, see Han and Sangiorgi (2015) for a possible micro-foundation of this specification.

[11] Myatt and Wallace (2012, 2015) analyse the case of a convex increasing cost function for information acquisition, the latter paper does so in a (symmetric) Cournot context. The approach taken there could be applied straightforwardly to the current symmetric price-setting industry; given the focus here on asymmetric industries, the stock-of-attention interpretation is retained for expositional convenience.

only through the variance term $\text{var}[p_m]$ of (14). Using (16) this term is

$$\text{var}[p_m] = \bar{w}_m^2\sigma^2 + \sum_{i=1}^{n} w_{im}^2 \left(\kappa_i^2 + \frac{\xi_i^2}{z_{im}}\right),$$

and it enters negatively into the expected profit expression. Given the (endogenously chosen) weights placed on the various signal realizations, supplier m chooses $z_m \in \mathcal{R}_+^n$ to minimize this variance subject to the attention-span constraint $\sum_{i=1}^n z_{im} \leq 1$.

The incentive to give attention to the ith information source is proportional to the weight placed on the signal received from that source: the solution satisfies $z_{im} \propto \xi_i w_{im}$. Hence, in the context of a symmetric equilibrium (with symmetric suppliers) the influence of and the attention paid to the ith information source jointly satisfy

$$w_i \propto \frac{1}{\pi\kappa_i^2 + \xi_i^2/z_i} \quad \text{and} \quad z_i \propto \xi_i w_i.$$

If a signal has no influence ($w_i = 0$) then it receives no attention ($z_i = 0$). Focusing on the set of signals that do receive attention (and so exert influence) the two equations above can be combined. Doing so, there is some constant K (to be determined below) such that the attention paid to the ith information source (if it is used) satisfies

$$z_i = \frac{\xi_i(K - \xi_i)}{\pi\kappa_i^2}.$$

Fixing the clarity of a signal i, attention is inversely proportional to the sender noise (the variance κ_i^2) or, equivalently, is proportional to the underlying quality of the information source. However, attention is non-monotonic in the clarity of the information source. This is a general property that does not depend upon any functional-form assumptions. This is because when ξ_i^2 is small the signal is easy to understand (relatively little attention is needed to comprehend what the sender is trying to say) and so the optimal z_i can be low. Likewise, if ξ_i^2 is sufficiently large then the information source is too expensive, and is ignored: the solution reported above only holds if $\xi_i < K$, if ξ_i^2 is too large, then the solution does not apply and $z_i = w_i = 0$ in equilibrium. Hence attention is maximized for intermediate levels of clarity.

This analysis suggests that only the clearest information sources (where ξ_i is sufficiently small) are candidates to receive attention and so to exert influence. The proof of Proposition 8 goes further by establishing that if an information source receives attention then any clearer information source does so too. Briefly: if an information source i currently receives no attention at all, then a supplier can raise its expected profits by increasing the attention it pays (z_{im}) up from zero while reducing z_{jm} for some j with $\xi_j^2 > \xi_i^2$.

Proposition 8 describes equilibrium information acquisition in a symmetric industry (the m notation is dropped as in Section 3.1). It translates analogous results in Dewan and Myatt (2008) and Myatt and Wallace (2012) to the price-setting oligopoly context.

Proposition 8 (Equilibrium Information Acquisition)**.** *Label the information sources in order of decreasing clarity: $\xi_1 < \cdots < \xi_n$. In a symmetric industry, where $s_m = 1/M$ for all m, there is a unique symmetric equilibrium. In that equilibrium:*

(i) an information source influences prices if and only if it is acquired: $w_i > 0 \Leftrightarrow z_i > 0$*;*

(ii) it is acquired if and only if it is sufficiently clear: $z_i > 0 \Leftrightarrow \xi_i < K$ *for some* K*;*

(iii) the attention paid to an acquired information source satisfies

$$z_i = \max\left\{\frac{\xi_i(K-\xi_i)}{\pi\kappa_i^2}, 0\right\} \quad \textit{where} \quad K = \min_{i\in\{1,\dots,n\}}\left\{\frac{\pi + \sum_{j=1}^{i}\xi_j^2/\kappa_j^2}{\sum_{j=1}^{i}\xi_j/\kappa_j^2}\right\}; \tag{27}$$

(iv) the number of information sources that receive attention falls as the market power of suppliers weakens, via either reduced product differentiation or increased competition, or with a strengthening of any decreasing returns to scale;

(v) amongst those information sources that receive attention, the clearest are endogenously the most public: $\rho_i > \rho_j$ *if and only if* $\xi_i < \xi_j$*; and*

(vi) a fall in market power (via, for example, reduced product differentiation or market entry) shifts attention toward clearer (and more public) information sources.

The key to the fourth claim (and others) is that K is increasing in π. Anything that increases the fundamental motive will result in a higher value for K, and as a result (weakly) more information sources will be acquired. Turning this around, the stronger the coordination motive, the smaller (and clearer) is the set of signals acquired by the suppliers. This means that industries operating under relatively competitive conditions (low differentiation, large number of suppliers) acquire a relatively small number of the very clearest signals. Coordination is paramount in such industries, and these are precisely the information sources most useful for that exercise.

4.2. **A Monopolistically Competitive Fringe.** When there is a symmetric set of M oligopolists and a monopolistically competitive fringe, as in Section 3.2, more can be said about the mix of information that the differently sized suppliers acquire.

For the M oligopolists, the solution reported in Proposition 8 applies: they focus attention on the clearest information sources. Note also that a reduction in concentration (fewer competitors) emphasizes the focus on those clearest sources.

Each member of the competitive fringe also seeks to minimize its price variance subject to the attention-span constraint, and so $z_{iF} \propto \xi_i w_{iF}$. Also, the weights placed on the various signals satisfy the first-order conditions of (22). Combining these, the attention paid by fringe members to the ith information source (if it is used) takes the form

$$z_{iF} = \frac{\xi_i(K_{iF}-\xi_i)}{\kappa_i^2}. \tag{28}$$

The key difference is that whilst K in (27) does not depend directly upon i, K_{iF} in (28) does. In particular it depends linearly upon $w_i\kappa_i^2 \propto K - \xi_i$. This induces a greater bias (relative to the case where K_{iF} is constant with respect to i) toward clearer information sources. The next proposition confirms this formally.

Proposition 9 (Acquisition and a Competitive Fringe)**.** *If the M symmetric oligopolists and the suppliers in the monopolistically competitive fringe acquire the same set of signals, then the fringe acquire (and use) more intensively information which is relatively clear, equivalently, this is information which is relatively public within the oligopoly.*

$$\frac{z_{iF}}{z_{jF}} > \frac{z_i}{z_j} \quad \Leftrightarrow \quad \frac{w_{iF}}{w_{jF}} > \frac{w_i}{w_j} \quad \Leftrightarrow \quad \xi_i^2 < \xi_j^2 \quad \Leftrightarrow \quad \rho_i > \rho_j.$$

Otherwise, suppliers in the competitive fringe acquire a strict subset (consisting of the most clear) of those signals acquired by the M symmetric oligopolists.

A message of Proposition 8 is that the members of a symmetric industry shift attention (and influence) toward clearer (and, endogenously, more public) information sources as the size of each supplier falls. Fixing the industry configuration, Proposition 9 reports a related result from looking across the set of suppliers: a fringe supplier focuses more on clearer information sources than do its larger oligopolist competitors. The driving force here is the one that is common to many results of the paper: smaller suppliers recognize that their competitors have a greater influence over the aggregate price and so care relatively more about coordination. This shifts their information use (and so acquisition) towards information that is (now endogenously) more public: so much so, indeed, that sufficiently private information acquired by their competitors may be ignored altogether.

4.3. **The Acquisition of New Information.** Returning to the specification of Section 3.3, with $n = 1$ and M arbitrarily sized suppliers, suppliers' information acquisition solutions are simple: they choose $z_m = 1$ for all m (the subscript i has been dropped). The derivative of expected profit with respect to z_m reveals the value of new information:

$$\left.\frac{\partial\, \mathrm{E}[\text{Profit}_m]}{\partial z_m}\right|_{z_m=1} = s_m \times \frac{1-(1-\beta)s_m}{\beta} \times \left[1 + \frac{c(1-(1-\beta)s_m)}{2\beta}\right] \times \bar{w}_m^2 \xi^2.$$

The very first term (s_m) simply reflects directly the size of the supplier in question. The second and third terms constitute the (negative) weight that price variance is given in per-unit profit: this is decreasing in s_m as larger suppliers care less about own-price variability. Finally $\bar{w}_m^2$ affects the own-price variance $\mathrm{var}[p_m]$ positively. Given that, from Lemma 2, $\bar{w}_m$ is increasing in s_m (because larger suppliers pay more attention to new information), s_m has conflicting effects on the incentive to acquire new information.

Using the equilibrium value of $\bar{w}_m \propto \mu_m$ from (25), the objective is to check whether

$$F(s_m) = \frac{s_m(1-(1-\beta)s_m)}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{2\beta}\right]\left(\frac{\delta_m}{\sigma^2+\kappa^2+(1-\delta_m s_m)\xi^2}\right)^2 \qquad (29)$$

is increasing or decreasing in s_m. Some straightforward algebra confirms that, so long as c is not too large, if $s_m > s_{m'}$ then $F(s_m) > F(s_{m'})$. The next proposition summarizes.

Proposition 10 (Acquiring New Information)**.** *For c sufficiently small, larger suppliers have a greater incentive to acquire new information than smaller suppliers.*

Recall that Proposition 7 reports that larger suppliers make greater use of new information. Those suppliers (as confirmed in Proposition 10) also have a heightened incentive to acquire information. This incentive is present because they make less use of very public information. As noted earlier, the prior is a kind of perfectly public signal, and so less reliance on it implies greater use (and so acquisition) of new information.

Summary. In a symmetric industry, Section 3.1 reports a bias towards relatively public signals that is more pronounced as market power weakens. If information is acquired endogenously, then the publicity of a signal is determined in equilibrium. The key finding here is that information acquisition exhibits a bias towards clear sources (where little attention is needed to understand what a sender says) which become (endogenously) the most public. Similarly, Section 3.2 reports that smaller suppliers (fringe members) exhibit a relatively stronger bias toward relatively public information. Here, that message carries across to the acquisition of information from relatively clearer sources.

5. The Externalities of Information Use and Acquisition

Attention now turns to welfare considerations: what impact do suppliers' information choices have upon consumer surplus, industry profits, and social welfare? Myatt and Wallace (2015) answer these questions in the context of a fully micro-founded Cournot model. Here the questions are posed in a price-setting industry. Given that prices are strategic complements and quantities strategic substitutes, it is natural to conjecture that many results will be reversed. This turns out indeed to be the case. Primarily for reasons of expositional simplicity and to facilitate a direct comparison between the price and quantity setting worlds, this section restricts to a symmetric specification.

5.1. **Profits, Consumer Surplus, and Welfare.**

Putting aside the use of new information, the core equilibrium price (that is, the intercept $\bar{p}_m$ from a supplier's pricing rule) is too low from the perspective of the industry's suppliers, and too high from the perspective of consumers. The reasons for this are entirely standard. Here, then, the average price charged by supplier is fixed at its equilibrium level and the focus turns to the externalities involved in a supplier's use of informative signals.

For now, the quadratic component of each supplier's cost is eliminated, so that $c = 0$. For this constant-marginal-cost case, the concern for the fundamental motive is

$$\pi = \frac{1}{2}\left[1 + \frac{\beta}{1-(1-\beta)s}\right].$$

If $c = 0$ then the "other terms" of the profit expression (13) disappear. Any externalities from signal use come from the $M-1$ terms in the final line (15) which depend positively on the covariance of a supplier's price with the prices of others. This covariance is increasing in the weight that a competitor places on informative signals. Hence, a supplier (and so the industry) would benefit if all suppliers used their information more intensively. Moreover, a supplier is also affected when a competitor shifts weight between one signal and another. Note that

$$\frac{\partial \operatorname{cov}[p_m, p_{m'}]}{\partial w_{im'}} - \frac{\partial \operatorname{cov}[p_m, p_{m'}]}{\partial w_{jm'}} \propto \frac{\kappa_i^2}{\pi\kappa_i^2 + \xi_i^2} - \frac{\kappa_j^2}{\pi\kappa_j^2 + \xi_j^2} > 0 \quad \Leftrightarrow \quad \rho_i > \rho_j.$$

Hence, from an industry profit perspective, suppliers make too little use of their information sources, and place insufficient emphasis on relatively public information.

The optimal collusive use of information is readily characterized. The proof of Proposition 11 confirms that in a symmetric industry with the play of symmetric strategies,

$$\mathrm{E}[\text{Industry Profit}] = \text{full-information profit} + \operatorname{cov}[\theta, p_m] - \frac{\operatorname{var}[p_m]}{\pi^\dagger} + \frac{(1-\pi^\dagger)\operatorname{cov}[p_m, p_{m'}]}{\pi^\dagger}$$
$$\text{where} \quad \pi^\dagger \equiv \frac{\beta}{1-(1-\beta)s}. \quad (30)$$

$\pi^\dagger$ is the strength of the fundamental motive relative to the coordination motive desired by a collusive regime. Substituting in for the variance and covariances,

$$\mathrm{E}[\text{Industry Profit}] = \text{full-information profit} + (\bar{w} - \bar{w}^2)\sigma^2 - \frac{1}{\pi^\dagger}\sum_{i=1}^{n} w_i^2\left(\pi^\dagger\kappa_i^2 + \xi_i^2\right).$$

$\pi^\dagger < \pi$ and so (from a profit perspective) non-cooperative suppliers insufficiently emphasize coordination. Prices are strategic complements, and so a supplier would prefer to use a higher price when competitors are also offering high prices. The desire to correlate prices results in a collusive desire for public information sources.

In a symmetric industry,

$$\mathrm{E}[\text{Consumer Surplus}] = \text{other terms} - \operatorname{cov}[\theta, p_m] + \frac{\operatorname{var}[p_m]}{2\pi^\dagger} - \frac{(1-\pi^\dagger)\operatorname{cov}[p_m, p_{m'}]}{2\pi^\dagger}.$$

The coefficients on the variance and covariance terms are opposite in sign to those on both individual and industry-wide profit. Consumers prefer prices to be negatively correlated with the state of demand. That is, they prefer to take advantage of low prices when the products are more valuable. The positive coefficient on the variance stems from the usual property that consumers benefit from price variation. For the same reason, positive covariance of different suppliers' prices is undesirable, since it prevents consumers from taking advantage of relative price differences between varieties.

A welfare perspective is obtained by summing industry profit and consumer surplus to obtain the usual Marshallian welfare measure. Industry profit increases one-for-one with the covariance of demand conditions and supplier prices (that is, $\operatorname{cov}[\theta, p_m]$), whereas consumer surplus decreases one-for-one with the same term. These two cancel out when welfare is considered: correlation of prices with demand conditions is irrelevant from a welfare perspective. Thus, information use only matters because it influences the variance and covariance of suppliers' prices. In fact,

$$\begin{aligned}\mathrm{E}[\text{Welfare}] &= \text{full-information welfare} + \frac{\sigma^2}{2} - \frac{\operatorname{var}[p_m]}{2\pi^\dagger} + \frac{(1-\pi^\dagger)\operatorname{cov}[p_m, p_{m'}]}{2\pi^\dagger} \\ &= \text{other terms} - \frac{\bar{w}^2\sigma^2}{2} - \frac{1}{2\pi^\dagger}\sum_{i=1}^{n} w_i^2\left(\pi^\dagger\kappa_i^2 + \xi_i^2\right),\end{aligned}$$

and so welfare is maximized by removing the link between prices and signals.

Proposition 11 summarizes the various externalities associated with information use, and also characterizes both the collusive and socially optimal weights.

Proposition 11 (The Externalities of Information Use)**.** *Consider the equilibrium of a symmetric industry in which there are constant returns to scale in production.*

The effects on industry profit, consumer surplus, and social welfare of (i) a local increase in the use of any signal i, and (ii) a local shift in use away from a relatively private signal j and towards a relatively public signal i, where $\rho_i > \rho_j$, are

		Profit	Surplus	Welfare
(i)	*an increase in any w_i*	$+$	$-$	$-$
(ii)	*a shift from w_j to w_i with $\rho_i > \rho_j$*	$+$	$-$	$+$

The information use that collusively maximizes expected profit satisfies

$$w_i^\dagger \propto \frac{1}{\pi^\dagger\kappa_i^2 + \xi_i^2} \quad \textit{and} \quad \bar{w}^\dagger = \frac{\varphi^\dagger}{2(1+\varphi^\dagger)} \quad \textit{where} \quad \varphi^\dagger = \sum_{i=1}^{n}\frac{\pi^\dagger\sigma^2}{\pi^\dagger\kappa_i^2 + \xi_i^2},$$

and where $\pi^\dagger$ is defined in (30) *and satisfies $\pi^\dagger < \pi$.*

Fixing a level of information use $\bar{w}$, a social planner also prefers to set $w_i \propto 1/(\pi^\dagger\kappa_i^2 + \xi_i^2)$. However, it is socially optimal to ignore new information and set $w_i = 0$ for all i.

In summary, suppliers would prefer to see greater information use and greater relative use made of relatively public information; consumers would prefer to see less information use and greater relative use made of relatively private information. Despite the bias towards relatively public signals, social welfare would increase with a rise in this bias (as would industry profit); whereas welfare would decrease with a rise in the use of new information (as would consumer surplus).

A message is that the use of any information about changing demand conditions is socially undesirable. In a Cournot world, there is a good (welfare) reason for tracking demand conditions: when demand is strong (a rise in θ) it is optimal to produce more, and so outputs should (ideally) react positively to signals of demand.[12] Here, however, production automatically tracks demand conditions because (fixing prices) consumers choose to buy more as θ rises. Linking demand to prices offsets this (useful) effect.

[12]The Cournot analysis of Myatt and Wallace (2015) confirms that new information is socially useful.

5.2. **Decreasing Returns to Scale.** The analysis above restricts to industries in which the production technology exhibits constant returns to scale, so that $c = 0$.

If there are decreasing returns, so that $c > 0$, then the "other terms" from the profit expression for supplier m become relevant. These other terms are independent of m's choice of pricing strategy, but do depend on the information use of others.

For the purposes of compact exposition, it is useful to focus on an industry with monopolistically competitive suppliers so that $M \to \infty$ and so $s \to 0$. In this case,

$$\text{other terms} = -\frac{c\sigma^2}{2} - \frac{c(1-\beta)}{\beta}\left(\frac{(1-\beta)\operatorname{cov}[p_{m'}, p_{m''}]}{2\beta} + \operatorname{cov}[\theta, p_{m'}]\right).$$

These negative terms all carry the coefficient c. This is because they are all determined by the variation in supplier m's output. If $c > 0$ then such variation is costly for a supplier, owing to the convexity of the cost function in output. If competitors make greater use of their information, then their prices move together. This increases the variability of the demand faced by supplier m.

The terms are all increasing in the weight placed on any signal. It follows that the presence of decreasing returns to scale reinforces a social planner's preference to eliminate the use of new information. Furthermore, those other terms are independent of the receiver noise in each information source. Hence, the incorporation of such terms into either an industry profit or welfare objective provides a rationale for a shift away from relatively high sender noise (and so relatively public) information sources.

Nevertheless, once the presence of decreasing returns is incorporated,

$$\begin{aligned}\mathrm{E}[\text{Industry Profit}] = \text{full-information profit} - \frac{c\sigma^2}{2} + [1+c]\,\bar{w}\sigma^2 \\ - \frac{(2+c)\bar{w}^2\sigma^2}{2} - \frac{2+c}{2\pi^\ddagger}\sum_{i=1}^{n} w_i^2(\pi^\ddagger\kappa_i^2 + \xi_i^2) \quad \text{where} \quad \pi^\ddagger = \frac{(2+c)\beta^2}{2\beta + c};\end{aligned}$$

it is straightforward to check that $\pi^\ddagger < \pi$, and so (from an industry perspective) suppliers continue to place too little emphasis on the coordination motive.

Note that the coefficient on the term $\bar{w}\sigma^2$ is $1 + c > 1$. When this is combined with the corresponding term in consumer surplus, $c\bar{w}\sigma^2$ remains. This means there is a local gain to the initial use of new information (the remaining terms of welfare are second order around $w_i = 0$) which means that the socially optimal level of information use is positive.

Proposition 12 (Externalities with Decreasing Returns). *If $c > 0$ then the socially optimal use of new information is positive. Moreover, from both an industry and welfare perspective, there is too little emphasis on relatively public information sources.*

5.3. **Welfare and Information Acquisition.** In an industry with constant returns to scale, there are externalities associated with information use (Proposition 11): suppliers would prefer greater information use and greater relative use made of relatively public information, whereas consumers would prefer less information use and greater relative use of relatively private information. Overall, given constant returns, there is no social value from conditioning prices on demand conditions and so welfare is maximized when all signals are ignored. With decreasing returns, however, it is valuable to reduce the variation in suppliers' demands and so new information is useful (Proposition 12).

If information is acquired endogenously, then (very naturally) under constant returns there is no social value to the acquisition of information; after all, there is no socially beneficial use for it. However, there can be externalities from the information acquisition decisions while fixing the extent to which informative signals are used.

Fix for now the weights placed on signal realizations in an industry. Reducing the information acquired generates noisier prices, which naturally benefits consumers: they may use price variation to exploit relative bargains, which is of course reflected in the usual property that consumer surplus is a convex function of prices.

Greater noise in a supplier's price increases the variance of the demand for competitors' products. If there are constant returns then competitors' profits are linear in such demands and so linear in the price of a supplier. Thus, adding noise does not influence the expected profits of competitors. From an industry perspective, then, there are no externalities from information acquisition. However, if there are decreasing returns ($c > 0$) then competitors' profits are concave in their demands and so concave in a supplier's price: a supplier exerts a negative externality on competitors by reducing information acquisition. In this case, the industry's suppliers would (fixing information use) prefer to see more information acquisition, whereas the consumers would rather see less.

6. Related Literature

This paper relates to studies of information use in quadratic-payoff coordination games, with a focus on how oligopolists' strategies depend upon their available information.

Morris and Shin (2002) prompted others to study information use in quadratic-payoff coordination games where payoffs depend upon the distance of actions from some fundamental and from the average action of others. In a world with public (perfectly correlated) and private (conditionally uncorrelated) signals about the fundamental, actions react more strongly to public information: such information is more important for higher-order beliefs. Frameworks like this have been applied extensively: to investment games, to business cycles, to large oligopoly games, to monopolistically competitive suppliers, to political leadership, to financial markets, and to networked communication (Angeletos and Pavan, 2004, 2007; Hellwig, 2005; Dewan and Myatt, 2008, 2012; Allen, Morris, and Shin, 2006; Calvó-Armengol and de Martí, 2007, 2009; Calvó-Armengol, de Martí, and Prat, 2015; Currarini and Feri, 2015; Fainmesser and Galeotti, 2016).

This paper joins others that allow for multiple information sources and for endogenous information acquisition. The information structure was introduced (in a model of political leadership) by Dewan and Myatt (2008, 2012), was extended (within a "beauty contest" setting) and applied (to Lucas-Phelps island and Cournot oligopoly models) by

Myatt and Wallace (2012, 2014, 2015). Partial correlation of observations (a central feature) has also been incorporated into the work of Angeletos and Pavan (2009), Baeriswyl (2011), and Baeriswyl and Cornand (2007, 2011). Endogenous signal acquisition was also studied by Hellwig and Veldkamp (2009), while the recent models of Colombo, Femminis, and Pavan (2014), Llosa and Venkateswaran (2013), and Pavan (2014) also allow players to choose the precision of a private signal. There are other, also recent, papers in which a public signal emerges endogenously as a noisy aggregate of players' actions (Angeletos, Lorenzoni, and Pavan, 2012; Bayona, 2015).

This paper determines, in the context of a micro-founded differentiated-product price-setting oligopoly, the properties and (in)efficiency of information use and acquisition in a framework which admits a general correlation structure. The use and acquisition of different kinds (more public or more private) of information depend on the players' relative concerns for fundamental and coordination motives. Here, the desire to coordinate is determined endogenously by market characteristics such as product differentiation and the industry's concentration. Moreover, micro-founding the model in this way admits a more meaningful welfare analysis.

A more notable contribution is to allow for asymmetric players. Here, larger players care relatively little about coordination, target a higher fundamental, and exert greater influence on the aggregate action; these differences between larger and smaller players are derived from differently sized product portfolios. The specification allows results about the relative use of information (small suppliers emphasize more public information), the use of new information (larger suppliers and more concentrated industries use new information more intensively), and relative information acquisition (small suppliers acquire information from fewer sources).

This paper also relates more distantly to work that has examined the incentives of oligopolists to share information. In a Cournot industry, information sharing about common demand conditions hurts industry profits (Novshek and Sonnenschein, 1982; Clarke, 1983; Vives, 1984, 1990; Li, 1985; Gal-Or, 1985). However, sharing information about private costs or supplier-specific demand components helps Cournot suppliers to reduce the (undesirable, from a supplier's perspective) correlation between their output choices (Fried, 1984; Li, 1985; Gal-Or, 1986; Shapiro, 1986). These results are reversed in price-setting models (Vives, 1984; Gal-Or, 1986). For a unification of this, and other work (including Sakai, 1986; Kirby, 1988; Sakai and Yamato, 1989), see Raith (1996).

This paper also relates to work that has investigated the efficiency of industries with dispersed private information about demand conditions (Palfrey, 1985; Vives, 1988; Li, McKelvey, and Page, 1987). For example, Vives (1988) used a Cournot model and noted that (private) information is acquired and used efficiently in the competitive limit. The present paper differs by considering a differentiated-product model, by using a price-setting game, by allowing access to multiple signals with different correlation structures, and by offering results away from the monopolistically competitive limit. Notably, the information structure allows results on the inefficient balance of information use across relatively public and private sources. Others (Hwang, 1993, 1995; Hauk and Hurkens, 2001; Vives, 2002) have developed the Vives (1988) model in several directions, while more recent papers (Vives, 2011, 2013) model supply-function competition.

7. Concluding Remarks

The relationship between player-level asymmetries and information acquisition and use explored here has not received much attention in the aforementioned literature. Nonetheless, several of the applications described above may warrant such an exploration: industries have dominant suppliers; political parties and financial markets have big players; and networks have influential agents.

In each case it might be expected that the actions of larger players would have greater impact on aggregates, whereas smaller players would have less influence. The intuition that this leads smaller players to seek relatively public information in order to improve the accuracy of their higher-order beliefs should apply equally to these applications.

The three consequences of asymmetries in the price-setting model described here (larger suppliers influence aggregate prices more; their target fundamental is higher; and their coordination motive lower) may play out in subtly different ways in different applications. However, the intuition that smaller players are relatively biased towards coordination and hence have preferences for relatively public information ought to be robust.

Appendix A. Omitted Proofs

Proof of Proposition 1. Separating out various components,

$$\begin{aligned}
\frac{\text{Profit}_m}{s_m} = {} & \left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right]\theta p_m \\
& - \frac{1-(1-\beta)s_m}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{2\beta}\right] p_m^2 \\
& - \frac{c\theta^2}{2} \\
& - \frac{c(1-\beta)^2(1-s_m)^2}{2\beta^2} P_{-m}^2 \\
& + \frac{(1-\beta)(1-s_m)}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] P_{-m} p_m \\
& - \frac{c(1-\beta)(1-s_m)}{\beta}\theta P_{-m}.
\end{aligned}$$

This is concave in p_m. Taking the first-order condition:

$$\begin{aligned}
\frac{\partial}{\partial p_m}\left[\frac{\text{Profit}_m}{s_m}\right] = {} & \left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right]\theta \\
& - \frac{1-(1-\beta)s_m}{\beta}\left[2 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] p_m \\
& + \frac{(1-\beta)(1-s_m)}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] P_{-m} = 0.
\end{aligned}$$

Note that $(1-s_m)P_{-m} = \sum_{m' \neq m} s_{m'} p_{m'}$. Adding and subtracting the mth term:

$$\begin{aligned}
\left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right]\theta &= \frac{1-(1-\beta)s_m}{\beta}\left[2 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] p_m \\
&\quad + \frac{1-\beta}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] s_m p_m - \frac{1-\beta}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] P \\
&= \frac{1}{\beta}\left[2 - (1-\beta)s_m + \frac{c(1-(1-\beta)s_m)}{\beta}\right] p_m - \frac{1-\beta}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] P.
\end{aligned}$$

This solves to give p_m in terms of the average price P:

$$p_m = \frac{(1-\beta)(\beta + c(1-(1-\beta)s_m))}{\beta + (\beta + c)(1-(1-\beta)s_m)}\left[\frac{\beta\theta}{1-\beta} + P\right] = \delta_m \left[\frac{\beta\theta}{1-\beta} + P\right],$$

where δ_m is as defined in the text. This is increasing in s_m. Hence, looking across the set of industry participants, larger suppliers set higher prices.

Multiplying this by s_m and summing over all suppliers yields an equation in P which solves to yield (10). Substituting back in gives the final solution for p_m reported in (11).

For the final claim, differentiating twice confirms that $s_m\delta_m$ is a convex function of s_m, and hence $\sum_{m=1}^{M} s_m\delta_m$ is (as the sum of convex functions) convex in the sizes of the suppliers. P is a convex function of this summation, and so is convex. □

Proof of Proposition 2. Follows from arguments in the text. □

Proof of Lemma 1. Using the expression from the proof of Proposition 1,

$$\begin{aligned}
\frac{\mathrm{E}[\text{Profit}_m]}{s_m} &= \left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] \mathrm{E}[\theta p_m] \\
&\quad - \frac{1-(1-\beta)s_m}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{2\beta}\right] \mathrm{E}[p_m^2] \\
&\quad - \frac{c\,\mathrm{E}[\theta^2]}{2} \\
&\quad - \frac{c(1-\beta)^2}{2\beta^2} \sum_{m' \neq m} \sum_{m'' \neq m} s_{m'} s_{m''} \,\mathrm{E}[p_{m'} p_{m''}] \\
&\quad + \frac{1-\beta}{\beta}\left[1 + \frac{c(1-(1-\beta)s_m)}{\beta}\right] \sum_{m' \neq m} s_{m'} \,\mathrm{E}[p_m p_{m'}] \\
&\quad - \frac{c(1-\beta)}{\beta} \sum_{m' \neq m} s_{m'} \,\mathrm{E}[\theta p_{m'}].
\end{aligned}$$

The various moments satisfy $\mathrm{E}[\theta^2] = \bar{\theta}^2 + \mathrm{var}[\theta]$, $\mathrm{E}[p_m^2] = \bar{p}_m^2 + \mathrm{var}[p_m]$, and so on. Substitution leads to the claimed expression prior to the statement of the lemma.

$\mathrm{cov}[\theta, p_m]$, $\mathrm{var}[p_m]$, and $\mathrm{cov}[p_m, p_{m'}]$ are obtained straightforwardly. Differentiating,

$$\begin{aligned}
\frac{\partial\, \mathrm{cov}[\theta, p_m]}{\partial w_{im}} &= \sigma^2, \\
\frac{\partial\, \mathrm{var}[p_m]}{\partial w_{im}} &= 2\bar{w}_m\sigma^2 + 2w_{im}(\kappa_i^2 + \xi_{im}^2), \\
\text{and} \quad \frac{\partial\, \mathrm{cov}[p_m, p_{m'}]}{\partial w_{im}} &= \bar{w}_{m'}\sigma^2 + w_{im'}\kappa_i^2.
\end{aligned}$$

Hence the first-order condition for w_{im} is

$$\begin{aligned}0 = &\left[1+\frac{c(1-(1-\beta)s_m)}{\beta}\right]\sigma^2 \\ &-\frac{1-(1-\beta)s_m}{\beta}\left[1+\frac{c(1-(1-\beta)s_m)}{2\beta}\right][2\bar{w}_m\sigma^2+2w_{im}(\kappa_i^2+\xi_{im}^2)] \\ &+\frac{1-\beta}{\beta}\left[1+\frac{c(1-(1-\beta)s_m)}{\beta}\right]\sum_{m'\neq m} s_{m'}[\bar{w}_{m'}\sigma^2+w_{im'}\kappa_i^2].\end{aligned}$$

Re-arranging by adding and subtracting the mth term to the summation, collecting terms, and simplifying, this condition becomes

$$\begin{aligned}0 = &\,\sigma^2\left[\frac{\beta}{1-\beta}+\bar{w}-\frac{\beta+(\beta+c)(1-(1-\beta)s_m)}{(1-\beta)[\beta+c(1-(1-\beta)s_m)]}\bar{w}_m\right] \\ &-\left[\frac{\beta+(\beta+c)(1-(1-\beta)s_m)}{(1-\beta)[\beta+c(1-(1-\beta)s_m)]}-s_m\right][w_{im}\xi_{im}^2] \\ &-\frac{\beta+(\beta+c)(1-(1-\beta)s_m)}{(1-\beta)[\beta+c(1-(1-\beta)s_m)]}w_{im}\kappa_i^2 \\ &+w_i\kappa_i^2.\end{aligned}$$

The large fraction appearing three times in this equation is equal to $1/\delta_m$. Hence:

$$\frac{1}{\delta_m}w_{im}\kappa_i^2+\left[\frac{1}{\delta_m}-s_m\right]w_{im}\xi_{im}^2-w_i\kappa_i^2=\sigma^2\left[\frac{\beta}{1-\beta}+\bar{w}-\frac{\bar{w}_m}{\delta_m}\right],$$

which (on multiplying through) yields the condition reported in the lemma. □

Proof of Proposition 3. Given the definition of π, the first-order condition of (19) is

$$w_i=\frac{\sigma^2/(1-\delta s)}{\pi\kappa_i^2+\xi_i^2}\left[\frac{\delta\beta}{1-\beta}-(1-\delta)\bar{w}\right],$$

which yields the claim regarding the proportionality of w_i. Summing over i,

$$\bar{w}\equiv\sum_{i=1}^{n}w_i=\frac{\sigma^2}{1-\delta s}\left[\frac{\delta\beta}{1-\beta}-(1-\delta)\bar{w}\right]\sum_{i=1}^{n}\frac{1}{\pi\kappa_i^2+\xi_i^2},$$

which is straightforwardly re-arranged to yield the statement of $\bar{w}$ in the proposition. □

Proof of Proposition 4. Follows from arguments in the text. □

Proof of Proposition 5. Follows from arguments in the text. □

Proof of Proposition 6. The first-order condition for a fringe member is

$$\left[\kappa_i^2+\xi_i^2\right]w_{iF}=\delta_F w_i\kappa_i^2+\sigma^2\left[\delta_F\left(\frac{\beta}{1-\beta}+\bar{w}\right)-\bar{w}_F\right].$$

Fix two information sources where $\rho_i>\rho_j$, and take the ratio of their FOCs:

$$\frac{w_{iF}}{w_{jF}}=\frac{\left[\kappa_j^2+\xi_j^2\right]}{\left[\kappa_i^2+\xi_i^2\right]}\frac{w_i\kappa_i^2+\sigma^2\left[\left(\frac{\beta}{1-\beta}+\bar{w}\right)-\frac{\bar{w}_F}{\delta_F}\right]}{w_j\kappa_j^2+\sigma^2\left[\left(\frac{\beta}{1-\beta}+\bar{w}\right)-\frac{\bar{w}_F}{\delta_F}\right]}.$$

Suppose that supplier m is a member of the competitive fringe, so that $w_{im} = w_{iF}$ and $s_m = 0$. The use of signal i relative to j is stronger in comparison to the industry if

$$\frac{\left[\kappa_j^2 + \xi_j^2\right]}{\left[\kappa_i^2 + \xi_i^2\right]} \frac{w_i\kappa_i^2 + \sigma^2\left[\left(\frac{\beta}{1-\beta} + \bar{w}\right) - \frac{\bar{w}_F}{\delta_F}\right]}{w_j\kappa_j^2 + \sigma^2\left[\left(\frac{\beta}{1-\beta} + \bar{w}\right) - \frac{\bar{w}_F}{\delta_F}\right]} > \frac{w_i}{w_j}.$$

Noting that $w_i \propto \psi_i/[1-(1-\pi)\rho_i]$ where $\rho_i = 1/(\kappa_i^2 + \xi_i^2)$, this is equivalently

$$\frac{w_i\kappa_i^2 + \sigma^2\left[\left(\frac{\beta}{1-\beta} + \bar{w}\right) - \frac{\bar{w}_F}{\delta_F}\right]}{w_j\kappa_j^2 + \sigma^2\left[\left(\frac{\beta}{1-\beta} + \bar{w}\right) - \frac{\bar{w}_F}{\delta_F}\right]} > \frac{1-(1-\pi)\rho_j}{1-(1-\pi)\rho_i}.$$

Multiplying up by the denominators on both sides (an inspection of the first-order conditions confirms that the denominator of the left-hand side is positive, and so the direction of inequality is maintained) and re-arranging, the required inequality is

$$[1-(1-\pi)\rho_i]w_i\kappa_i^2 - [1-(1-\pi)\rho_j]w_j\kappa_j^2 > (1-\pi)(\rho_i - \rho_j)\sigma^2\left[\left(\frac{\beta}{1-\beta} + \bar{w}\right) - \frac{\bar{w}_F}{\delta_F}\right].$$

From Proposition 3, the solution for w_i is

$$w_i = \frac{\pi\sigma^2\bar{w}}{\varphi(\pi\kappa_i^2 + \xi_i^2)} = \frac{\pi\sigma^2\bar{w}}{\varphi(\kappa_i^2 + \xi_i^2)(1-(1-\pi)\rho_i)} \quad \text{where} \quad \varphi = \sum_{j=1}^{n} \frac{\pi\sigma^2}{\pi\kappa_j^2 + \xi_j^2}. \tag{31}$$

Substituting in, the required inequality becomes

$$\frac{\pi\sigma^2\bar{w}(\rho_i - \rho_j)}{\varphi} > (1-\pi)(\rho_i - \rho_j)\sigma^2\left[\left(\frac{\beta}{1-\beta} + \bar{w}\right) - \frac{\bar{w}_F}{\delta_F}\right].$$

Given that $\rho_i > \rho_j$, cancel terms on both sides to obtain

$$\frac{\pi\bar{w}}{\varphi} > (1-\pi)\left[\left(\frac{\beta}{1-\beta} + \bar{w}\right) - \frac{\bar{w}_F}{\delta_F}\right].$$

To verify this inequality, return to the first-order condition for a fringe member:

$$w_{iF} = \frac{\delta_F w_i \kappa_i^2}{\kappa_i^2 + \xi_i^2} + \sigma^2\left[\delta_F\left(\frac{\beta}{1-\beta} + \bar{w}\right) - \bar{w}_F\right]\frac{1}{\kappa_i^2 + \xi_i^2}.$$

Sum this over the n information sources

$$\frac{\bar{w}_F}{\delta_F} = \frac{\sum_{i=1}^n w_i\rho_i + \left(\frac{\beta}{1-\beta} + \bar{w}\right)\sigma^2\sum_{i=1}^n \psi_i}{1 + \sigma^2\sum_{i=1}^n \psi_i}.$$

Substitute this back in to the required inequality, to obtain

$$\frac{\pi\bar{w}}{\varphi}\left[1 + \sigma^2\sum_{i=1}^n \psi_i\right] > (1-\pi)\left[\left(\frac{\beta}{1-\beta} + \bar{w}\right) - \sum_{i=1}^n w_i\rho_i\right].$$

Straightforward but tedious re-arrangement shows that this is equivalent to

$$\pi\bar{w}\left(\frac{1+\varphi}{\varphi}\right) > \frac{\beta(1-\pi)}{1-\beta}.$$

Next recall that

$$\bar{w} = \frac{\varphi}{1+\varphi}\frac{\delta\beta}{(1-\beta)(1-\delta)}.$$

Substitution leads to the inequality $1-\delta < \pi$, which holds since $\pi = (1-\delta)/(1-s\delta)$. $\square$

Proof of Lemma 2. The first-order condition (24) is equivalently

$$\left[\sigma^2+\kappa^2+(1-\delta_m s_m)\xi_m^2\right]\bar{w}_m=\delta_m\left[\frac{\beta\sigma^2}{1-\beta}+\bar{w}(\sigma^2+\kappa^2)\right],$$

which immediately yields (25). Solving for $\bar{w}_m$ and summing over m,

$$\bar{w}=\sum_{m=1}^{M}s_m\bar{w}_m=\left[\frac{\beta\sigma^2}{1-\beta}+\bar{w}(\sigma^2+\kappa^2)\right]\sum_{m=1}^{M}\frac{\delta_m s_m}{\sigma^2+\kappa^2+(1-\delta_m s_m)\xi_m^2},$$

which solves straightforwardly to yield (26). □

Proof of Proposition 7. The first claim (larger suppliers use new information more intensively) follows from the arguments in the text. The claims regarding the industry's concentration follows because (for example) $\bar{w}$ is a convex function of the supplier's sizes. By inspection, $\bar{w}$ is an increasing and convex function $\sum_{m=1}^{M}s_m\mu_m$. Hence it is sufficient to show that each element of this summation is convex (since the sum of convex functions is itself convex). Now note that

$$s_m\mu_m=\frac{\delta_m s_m}{\sigma^2+\kappa^2+(1-\delta_m s_m)\xi_m^2}$$

is (again by inspection) an increasing convex function of $\delta_m s_m$. It has already been noted (in the proof of Proposition 1) that $\delta_m s_m$ is a convex function of s_m. □

Proof of Proposition 8. Recall that the expected profit of m satisfies

$$\begin{aligned}\frac{\mathrm{E}[\text{Profit}_m]}{s_m}=\text{other terms}&+\left[1+\frac{c(1-(1-\beta)s_m)}{\beta}\right]\operatorname{cov}[\theta,p_m]\\&-\frac{1-(1-\beta)s_m}{\beta}\left[1+\frac{c(1-(1-\beta)s_m)}{2\beta}\right]\operatorname{var}[p_m]\\&+\frac{1-\beta}{\beta}\left[1+\frac{c(1-(1-\beta)s_m)}{\beta}\right]\sum_{m'\neq m}s_{m'}\operatorname{cov}[p_m,p_{m'}]\end{aligned}$$

where the "other terms" include the full-information profit (if $\theta=\bar{\theta}$) which depends only on the expected prices of the suppliers, and other terms that are outside the control of supplier m.

The covariance terms $\operatorname{cov}[\theta,p_m]=\bar{w}_m\sigma^2$ and $\operatorname{cov}[p_m,p_{m'}]=\bar{w}_m\bar{w}_{m'}\sigma^2+\sum_{i=1}^{n}w_{im}w_{im'}\kappa_i^2$ are linear (and hence concave) in m's choice of weights. The variance term is

$$\operatorname{var}[p_m]=\bar{w}_m^2\sigma^2+\sum_{i=1}^{n}w_{im}^2\left(\kappa_i^2+\frac{\xi_i^2}{z_{im}}\right).$$

The ith element of the summation is convex in w_{im} and z_{im}, and hence $\operatorname{var}[p_m]$ (as the sum of convex functions) is convex in m's choices. It follows that m's profit is concave, and so any interior solution to the expected profit maximization is characterized by first-order conditions.

Consider a symmetric equilibrium, and write $N\subseteq\{1,\dots,n\}$ for the set of information sources that receive attention and are used (so that $w_i=0$ and so $z_i=0$ for $i\notin N$). For these (used) information sources, the minimization of $\operatorname{var}[p_m]$ implies $z_i\propto\xi_i w_i$. The influence w_i of the ith signal satisfies the solution (20) from Proposition 3. Hence:

$$z_i\propto\xi_i w_i\propto\frac{\xi_i}{\pi\kappa_i^2+\xi_i^2/z_i}\quad\Rightarrow\quad z_i=\frac{\xi_i(K-\xi_i)}{\pi\kappa_i^2}$$

for some constant K. Summing over $i \in N$ yields $\sum_{i\in N} z_i = 1$, which upon re-arrangement yields

$$K \equiv \frac{\pi + \sum_{j\in N} \xi_j^2/\kappa_j^2}{\sum_{j\in N} \xi_j/\kappa_j^2}.$$

Note that this is a solution only if $z_i > 0$, which requires $\xi_i < K$. Equivalently, if $\xi_i > K$ (which is true for all higher indices) then an information source must be ignored.

The next step is to show that N comprises the clearest information sources. To do this, first note that the optimal choice of attention implies

$$z_{im} = \frac{\xi_i w_{im}}{\sum_{j=1}^n \xi_j w_{jm}} \quad \Rightarrow \quad \operatorname{var}[p_m] = \bar{w}_m^2 \sigma^2 + \sum_{i=1}^n w_{im}^2 \left(\kappa_i^2 + \frac{\xi_i \sum_{j=1}^n \xi_j w_{jm}}{w_{im}} \right).$$

Using this expression, m's expected profit is determined solely by the weights placed on the various signals. Differentiating with respect to the weight placed on the ith signal,

$$\frac{\partial \operatorname{var}[p_m]}{\partial w_{im}} = 2\bar{w}_m \sigma^2 + 2 w_{im} \kappa_i^2 + 2\xi_i \sum_{j=1}^n \xi_j w_{jm}.$$

Hence, evaluated at a symmetric strategy profile in a symmetric industry,

$$\begin{aligned}
&\frac{\partial}{\partial w_{im}} \left[\frac{\mathrm{E}[\text{Profit}_m]}{s} \right] - \frac{\partial}{\partial w_{(i+1)m}} \left[\frac{\mathrm{E}[\text{Profit}_m]}{s} \right] = \\
&\qquad - \frac{1-(1-\beta)s}{\beta} \left[2 + \frac{c(1-(1-\beta)s)}{\beta} \right] \left(w_i \kappa_i^2 - w_{i+1}\kappa_{i+1}^2 + (\xi_i - \xi_{i+1}) \sum_{j=1}^n \xi_j w_j \right) \\
&\qquad\qquad + \frac{1-\beta}{\beta} \left[1 + \frac{c(1-(1-\beta)s)}{\beta} \right] (1-s)[w_i\kappa_i^2 - w_{i+1}\kappa_{i+1}^2] \\
&\qquad\qquad\qquad > \left(\frac{(1-\beta)(1-s)}{\beta} + \left[2 + \frac{c(1-(1-\beta)s)}{\beta} \right] \right) (w_{i+1}\kappa_{i+1}^2 - w_i \kappa_i^2)
\end{aligned}$$

where the strict inequality is obtained by eliminating the term involving $\xi_{i+1} - \xi_i$. Now suppose that information source $i+1$ is used ($w_{i+1} > 0$) but source i is not ($w_i = 0$). Then

$$\begin{aligned}
&\frac{\partial}{\partial w_{im}} \left[\frac{\mathrm{E}[\text{Profit}_m]}{s} \right] - \frac{\partial}{\partial w_{(i+1)m}} \left[\frac{\mathrm{E}[\text{Profit}_m]}{s} \right] > \\
&\qquad\qquad \left(\frac{(1-\beta)(1-s)}{\beta} + \left[2 + \frac{c(1-(1-\beta)s)}{\beta} \right] \right) w_{i+1}\kappa_{i+1}^2 > 0.
\end{aligned}$$

In other words, a supplier would strictly benefit by shifting attention and influence toward the ith information source. Hence, if an information source has influence, then any clearer source also has influence. Equivalently, there is some i such that $N = \{1, \ldots, i\}$.

Evaluated at a symmetric profile where $w_i = 0$,

$$\begin{aligned}
\frac{\partial}{\partial w_{im}} \left[\frac{\mathrm{E}[\text{Profit}_m]}{s} \right] &= \left[1 + \frac{c(1-(1-\beta)s)}{\beta} \right] \sigma^2 - \frac{1-(1-\beta)s}{\beta} \left[2 + \frac{c(1-(1-\beta)s)}{\beta} \right] \xi_i \sum_{j=1}^n \xi_j w_j \\
&\qquad - \left(\frac{(1-\beta)(1-s)}{\beta} + \left[2 + \frac{c(1-(1-\beta)s)}{\beta} \right] \right) \bar{w}\sigma^2.
\end{aligned}$$

Hence, there is an incentive to introduce the use of the ith signal if

$$\xi_i < \frac{\beta}{\sum_{j=1}^n \xi_j w_j} \frac{[\beta + c(1-(1-\beta)s)]\,\sigma^2 - [\beta + (1+c)(1-(1-\beta)s)]\,\bar{w}\sigma^2}{[2\beta + c(1-(1-\beta)s)][1-(1-\beta)s]}. \tag{32}$$

Simplifying this expression by using the definitions of π and γ, this is equivalent to

$$\xi_i < \frac{\sigma^2\pi[\gamma - \bar{w}]}{\sum_{j=1}^n \xi_j w_j}.$$

From earlier calculations (and inserting $\xi_{im}^2 = \xi_i^2/z_i$ as appropriate), $\bar{w} = \gamma\varphi/(1+\varphi) \Rightarrow \gamma - \bar{w} = \gamma/(1+\varphi) = \bar{w}/\varphi$. Note $w_i = \sigma^2\pi\bar{w}/\varphi(\pi\kappa_i^2 + \xi_i^2/z_i)$ from (31). Substituting the solutions for z_i found above into the w_i terms in $\sum_{j=1}^n \xi_j w_j$ and rearranging, the inequality in (32) becomes $\xi_i < K$. This confirms the equilibrium takes the declared form where $K = K_i$ for some i and

$$K_i \equiv \frac{\pi + \sum_{j=1}^i \xi_j^2/\kappa_j^2}{\sum_{j=1}^i \xi_j/\kappa_j^2},$$

which satisfies $\xi_i \leq K_i < \xi_{i+1}$. There is a unique K_i which satisfies this. Specifically, Lemma 4 of Dewan and Myatt (2008) establishes that the corresponding i also minimizes K_i.

The arguments so far establish claims (i) to (iii). For the comparative static claim of (iv) note that an increase in π (which is determined by β, M, and c) increases K_i for all i and so increases K, which raises the number of information sources that receive attention.

For claim (v), note that

$$\rho_i > \rho_j \quad \Leftrightarrow \quad \frac{\kappa_i^2}{\xi_i^2/z_i} > \frac{\kappa_j^2}{\xi_j^2/z_j} \quad \Leftrightarrow \quad \frac{K - \xi_i}{\pi\xi_i} > \frac{K - \xi_j}{\pi\xi_j} \quad \Leftrightarrow \quad \xi_i < \xi_j.$$

Finally, to establish claim (vi), for $\xi_i < \xi_j$ note that

$$\frac{z_i}{z_j} = \frac{\xi_i\kappa_j^2}{\xi_j\kappa_i^2} \times \frac{K - \xi_i}{K - \xi_j},$$

which is decreasing in K and hence decreasing in π. □

Proof of Proposition 9. Write $N = \{i \mid z_i > 0\}$ and $N_F = \{i \mid z_{iF} > 0\}$ and let $N = N_F$. Define

$$k_0 = \sum_{i \in N} \frac{1}{\kappa_i^2}, \quad k_1 = \sum_{i \in N} \frac{\xi_i}{\kappa_i^2}, \quad \text{and} \quad k_2 = \sum_{i \in N} \frac{\xi_i^2}{\kappa_i^2}. \tag{33}$$

z_i is found in Proposition 8, and given these definitions, for $i \in N$, may be written

$$z_i = \frac{\xi_i}{\pi\kappa_i^2}\left\{\frac{\pi + k_2}{k_1} - \xi_i\right\}.$$

The first-order condition for the M symmetric oligopolists may be written

$$w_i = \frac{\nu z_i}{\xi_i} = \frac{\nu}{\pi\kappa_i^2}\left\{\frac{\pi + k_2}{k_1} - \xi_i\right\} \quad \Rightarrow \quad \bar{w} = \frac{\nu}{\pi}\left\{\frac{(\pi + k_2)k_0 - k_1^2}{k_1}\right\} \tag{34}$$

for some constant ν. Returning to the first-order condition for w_i in (17) for the symmetric case, inserting $\xi_{im}^2 = \xi_i^2/z_i = \nu\xi_i/w_i$, for those signals in use ($i \in N$),

$$\begin{aligned}
(1-\delta)w_i\kappa_i^2 + (1 - s\delta)\nu\xi_i &= \sigma^2\left[\delta\frac{\beta}{1-\beta} - (1-\delta)\bar{w}\right] \\
\Rightarrow \quad w_i + \frac{\nu}{\pi}\frac{\xi_i}{\kappa_i^2} &= \frac{\sigma^2}{\kappa_i^2}\left[\frac{\delta}{1-\delta}\frac{\beta}{1-\beta} - \bar{w}\right] \\
\Rightarrow \quad \bar{w} + \frac{\nu}{\pi}k_1 &= \sigma^2 k_0(\gamma - \bar{w}) \\
\Rightarrow \quad \bar{w}(1 + \sigma^2 k_0) + \frac{\nu}{\pi}k_1 &= \gamma\sigma^2 k_0.
\end{aligned}$$

Now, inserting the equation for $\bar{w}$ in (34) into this expression gives

$$\frac{\nu}{\pi}\left\{\frac{(\pi+k_2)k_0-k_1^2}{k_1}\right\}+\frac{\nu}{\pi}\frac{k_1}{1+\sigma^2 k_0}=\gamma\frac{\sigma^2 k_0}{1+\sigma^2 k_0} \quad\Rightarrow\quad \nu=\frac{\gamma\pi k_1}{(\pi+k_2)(\sigma^{-2}+k_0)-k_1^2}. \tag{35}$$

Note that this final expression for ν is explicit. Substituting back in to earlier expressions for $\bar{w}$ and w_i provides a complete solution to the symmetric oligopoly information-acquisition problem.

Now consider the first-order equations for the competitive fringe. Recall that $N = N_F$, so the same set of signals have positive use for both the oligopoly and the fringe. The first-order condition in z_{iF} may be written, for some constant ν_F,

$$\frac{w_{iF}\xi_i}{z_{iF}}=\nu_F \quad\Rightarrow\quad \xi_{iF}^2\equiv\frac{\xi_i^2}{z_{iF}}=\nu_F\frac{\xi_i}{w_{iF}}.$$

Substituting in this expression for ξ_{iF}^2 in (22), and dividing by κ_i^2, gives, for all $i\in N$,

$$w_{iF}+\nu_F\frac{\xi_i}{\kappa_i^2}-\delta_F w_i=\frac{\sigma^2}{\kappa_i^2}\left[\delta_F\frac{\beta}{1-\beta}-(\bar{w}_F-\delta_F\bar{w})\right]. \tag{36}$$

Summing over $i\in N$ (and here is where it is important that $N=N_F$),

$$\bar{w}_F-\delta_F\bar{w}+\nu_F k_1=\sigma^2 k_0\left[\delta_F\frac{\beta}{1-\beta}-(\bar{w}_F-\delta_F\bar{w})\right].$$

An explicit form for ν_F is required for the later calculations. To obtain this, first solve for $\bar{w}_F-\delta_F\bar{w}$ in the above expression:

$$\bar{w}_F-\delta_F\bar{w}=\delta_F\left\{\frac{\sigma^2 k_0}{1+\sigma^2 k_0}\frac{\beta}{1-\beta}-\frac{\nu_F}{\delta_F}\frac{k_1}{1+\sigma^2 k_0}\right\}$$
$$\Rightarrow\quad \sigma^2\left[\delta_F\frac{\beta}{1-\beta}-(\bar{w}_F-\delta_F\bar{w})\right]=\frac{\sigma^2}{1+\sigma^2 k_0}\left[\delta_F\frac{\beta}{1-\beta}+k_1\nu_F\right]. \tag{37}$$

Returning now to the rewritten first-order condition in (36), pre-multiply by ξ_i, then sum over $i\in N$ (again, note the importance of $N=N_F$ for this step),

$$\nu_F+k_2\nu_F-\delta_F\nu=\sigma^2 k_1\left[\delta_F\frac{\beta}{1-\beta}-(\bar{w}_F-\delta_F\bar{w})\right]=\frac{\sigma^2 k_1}{1+\sigma^2 k_0}\left[\delta_F\frac{\beta}{1-\beta}+k_1\nu_F\right],$$

since by definition $\sum_{i\in N}\xi_i w_i=\nu$ and $\sum_{i\in N_F}\xi_i w_{iF}=\nu_F$. Now this provides an expression for ν_F in terms of ν, for which an explicit solution has been provided in (35): it is convenient to write

$$\left[(1+k_2)(\sigma^{-2}+k_0)-k_1^2\right]\frac{\nu_F}{\delta_F}=\frac{\beta}{1-\beta}k_1+(\sigma^{-2}+k_0)\nu. \tag{38}$$

Returning once again to the rewritten first-order condition in (36), and using the form of the right-hand side given in (37),

$$w_{iF}=\delta_F\left\{w_i+\frac{1}{\kappa_i^2}\left[\frac{1}{\sigma^{-2}+k_0}\frac{\beta}{1-\beta}+\left(\frac{k_1}{\sigma^{-2}+k_0}-\xi_i\right)\frac{\nu_F}{\delta_F}\right]\right\}.$$

The objective is to compare the ratio w_{iF}/w_{jF} with w_i/w_j. Suppose, as in the statement of the proposition that $\xi_i^2<\xi_j^2$ or $\xi_i<\xi_j$, then

$$\frac{w_{iF}}{w_{jF}}>\frac{w_i}{w_j} \quad\Leftrightarrow\quad \frac{w_i+\frac{1}{\kappa_i^2}\left[\frac{1}{\sigma^{-2}+k_0}\frac{\beta}{1-\beta}+\left(\frac{k_1}{\sigma^{-2}+k_0}-\xi_i\right)\frac{\nu_F}{\delta_F}\right]}{w_j+\frac{1}{\kappa_j^2}\left[\frac{1}{\sigma^{-2}+k_0}\frac{\beta}{1-\beta}+\left(\frac{k_1}{\sigma^{-2}+k_0}-\xi_j\right)\frac{\nu_F}{\delta_F}\right]}>\frac{w_i}{w_j}.$$

Now cross-multiplying and cancelling the $w_i w_j$ terms, this is true if and only if

$$\frac{\frac{1}{\sigma^{-2}+k_0}\frac{\beta}{1-\beta}+\left(\frac{k_1}{\sigma^{-2}+k_0}-\xi_i\right)\frac{\nu_F}{\delta_F}}{\frac{1}{\sigma^{-2}+k_0}\frac{\beta}{1-\beta}+\left(\frac{k_1}{\sigma^{-2}+k_0}-\xi_j\right)\frac{\nu_F}{\delta_F}}>\frac{\kappa_i^2 w_i}{\kappa_j^2 w_j}=\frac{\pi+k_2-\xi_i k_1}{\pi+k_2-\xi_j k_1},$$

where the final equality follows from the value of w_i stated in (34). Multiplying across, cancelling common terms, and collecting together what remains gives

$$\begin{aligned}
\frac{w_{iF}}{w_{jF}}>\frac{w_i}{w_j} \quad &\Leftrightarrow \quad (\xi_j-\xi_i)(\pi+k_2)\frac{\nu_F}{\delta_F}>(\xi_j-\xi_i)k_1\frac{1}{\sigma^{-2}+k_0}\left[\frac{\beta}{1-\beta}+k_1\frac{\nu_F}{\delta_F}\right] \\
&\Leftrightarrow \quad (\pi+k_2)(\sigma^{-2}+k_0)\frac{\nu_F}{\delta_F}>k_1\left[\frac{\beta}{1-\beta}+k_1\frac{\nu_F}{\delta_F}\right] \\
&\Leftrightarrow \quad \left[(\pi+k_2)(\sigma^{-2}+k_0)-k_1^2\right]\frac{\nu_F}{\delta_F}>k_1\frac{\beta}{1-\beta} \\
&\Leftrightarrow \quad \frac{\nu_F}{\delta_F}>\frac{\beta}{1-\beta}\left[\frac{k_1}{(\pi+k_2)(\sigma^{-2}+k_0)-k_1^2}\right] \\
&\Leftrightarrow \quad \frac{\beta}{1-\beta}k_1+(\sigma^{-2}+k_0)\nu>\frac{\beta}{1-\beta}k_1\left[\frac{(1+k_2)(\sigma^{-2}+k_0)-k_1^2}{(\pi+k_2)(\sigma^{-2}+k_0)-k_1^2}\right],
\end{aligned}$$

where this last step follows from substituting with (38). Continuing:

$$\begin{aligned}
\frac{w_{iF}}{w_{jF}}>\frac{w_i}{w_j} \quad &\Leftrightarrow \quad (\sigma^{-2}+k_0)\nu>\frac{\beta}{1-\beta}k_1\left[\frac{(1+k_2)(\sigma^{-2}+k_0)-k_1^2}{(\pi+k_2)(\sigma^{-2}+k_0)-k_1^2}-1\right] \\
&\Leftrightarrow \quad (\sigma^{-2}+k_0)\nu>\frac{\beta}{1-\beta}k_1\left[\frac{(1-\pi)(\sigma^{-2}+k_0)}{(\pi+k_2)(\sigma^{-2}+k_0)-k_1^2}\right] \\
&\Leftrightarrow \quad \nu>\frac{\beta}{1-\beta}\left[\frac{(1-\pi)k_1}{(\pi+k_2)(\sigma^{-2}+k_0)-k_1^2}\right] \\
&\Leftrightarrow \quad \frac{\gamma\pi k_1}{(\pi+k_2)(\sigma^{-2}+k_0)-k_1^2}>\frac{\beta}{1-\beta}\left[\frac{(1-\pi)k_1}{(\pi+k_2)(\sigma^{-2}+k_0)-k_1^2}\right],
\end{aligned}$$

from the explicit form of ν given in (35). Cancelling terms, finally,

$$\frac{w_{iF}}{w_{jF}}>\frac{w_i}{w_j} \quad \Leftrightarrow \quad \gamma\pi>\frac{\beta}{1-\beta}(1-\pi). \tag{39}$$

Finally, recalling that $\gamma=\beta/(1-\beta)\times\delta/(1-\delta)$ and $\pi=(1-\delta)/(1-s\delta)$, and so $1-\pi=\delta(1-s)/(1-s\delta)$, the inequality reduces to $1>1-s$, which of course is true. Thus, as required,

$$\frac{w_{iF}}{w_{jF}}>\frac{w_i}{w_j} \quad \Leftrightarrow \quad \xi_i<\xi_j.$$

Moreover, using the proportionality between (i) $\xi_i w_i$ and z_i and (ii) $\xi_i w_{iF}$ and z_{iF},

$$\frac{w_{iF}}{w_{jF}}>\frac{w_i}{w_j} \quad \Leftrightarrow \quad \frac{z_{iF}}{z_{jF}}>\frac{z_i}{z_j}.$$

The oligopoly's optimal information acquisition policy ensures that $\xi_i<\xi_j \Leftrightarrow \rho_i>\rho_j$ (as argued after Proposition 8). This observation completes the proof of the first part of Proposition 9.

The second part of the proposition states two things. First: the competitive fringe also places positive weight only on a set comprising the clearest signals. Second: if the fringe does not use the same set of signals as the oligopoly then it uses strictly fewer.

For the first fact it will be shown that for two signals with $\xi_j^2>\xi_i^2$ (equivalently $j>i$ if the sources are ordered in terms of clarity), if the fringe uses j it must also use i: $z_{jF}>0 \Rightarrow z_{iF}>0$. For the second fact, it is then sufficient to show that any unused signal i by the oligopoly is also unused by the fringe: $z_i=0 \Rightarrow z_{iF}=0$.

First Fact. Consider two signals with $\xi_j^2 > \xi_i^2$. Suppose supplier m places weights w_{jm} and w_{im} on these two signals respectively. The first step is to calculate the sign of the change in expected profits experienced by m if the supplier raised w_{im} and lowered w_{jm} by the same amount (so keeping $\bar{w}_m$ fixed). To do this, recall the expression for expected profits in (13)-(15). Using the expressions for $\mathrm{var}[p_m]$ and $\mathrm{cov}[p_m, p_{m'}]$ in (16), and evaluating at a symmetric equilibrium (so that $\mathrm{cov}[p_m, p_{m'}]$ is the same for all $m' \neq m$),

$$\begin{aligned}\mathrm{E}[\textbf{Profit}_m] &\propto \text{other terms} + \mathrm{cov}[p_m, p_{m'}] - \chi_m \,\mathrm{var}[p_m] \\ &= \text{other terms} + \sum_{i=1}^{n}\Big[w_{im}w_{im'}\kappa_i^2 - \chi_m w_{im}^2(\kappa_i^2 + \xi_{im}^2)\Big],\end{aligned}$$

$$\text{where} \quad \chi_m = \frac{(1-(1-\beta)s_m)(2\beta + c(1-(1-\beta)s_m))}{2(1-\beta)(1-s_m)(\beta + c(1-(1-\beta)s_m))} = \frac{1}{2(1-\pi_m)},$$

and the "other terms" depend on the weights only through $\bar{w}_m$. Now consider the optimal information acquisition decision of supplier m. This satisfies:

$$z_{im} \propto w_{im}\xi_i \quad \Rightarrow \quad z_{im} = \frac{w_{im}\xi_i}{\sum_{j=1}^n w_{jm}\xi_j} \quad \Rightarrow \quad \xi_{im}^2 = \frac{\xi_i^2}{z_{im}} = \frac{\xi_i \sum_{j=1}^n w_{jm}\xi_j}{w_{im}}.$$

Hence:

$$\mathrm{E}[\textbf{Profit}_m] \propto \text{other terms} + \sum_{i=1}^{n}\left[w_{im}w_{im'}\kappa_i^2 - \chi_m\left(w_{im}^2\kappa_i^2 + w_{im}\xi_i \sum_{j=1}^{n} w_{jm}\xi_j\right)\right].$$

Information acquisition terms are eliminated, and so m chooses only weights. Next, differentiating with respect to w_{im} gives

$$\frac{\partial\,\mathrm{E}[\textbf{Profit}_m]}{\partial w_{im}} \propto \frac{\partial[\text{other terms}]}{\partial w_{im}} + w_{im'}\kappa_i^2 - 2\chi_m\left(w_{im}\kappa_i^2 + \xi_i\sum_{j=1}^{n} w_{jm}\xi_j\right),$$

where the derivative of "other terms" is the same for all i, since the effect of a change in w_{im} is felt only via $\bar{w}_m$. This means that, for $i \neq j$,

$$\frac{\partial\,\mathrm{E}[\textbf{Profit}_m]}{\partial w_{im}} - \frac{\partial\,\mathrm{E}[\textbf{Profit}_m]}{\partial w_{jm}} \propto w_{im'}\kappa_i^2 - w_{jm'}\kappa_j^2 - 2\chi_m\left(w_{im}\kappa_i^2 - w_{jm}\kappa_j^2 + (\xi_i - \xi_j)\sum_{k=1}^{n} w_{km}\xi_k\right).$$

Now consider a symmetric oligopoly with a competitive fringe. Here, $w_{im'} = w_i$: the symmetric equilibrium weight placed on the ith signal. Moreover, $\chi_F = (2\beta + c)/2(1-\beta)(\beta + c)$. Hence:

$$\frac{\partial\,\mathrm{E}[\textbf{Profit}_F]}{\partial w_{iF}} - \frac{\partial\,\mathrm{E}[\textbf{Profit}_F]}{\partial w_{jF}} \propto \frac{[w_i\kappa_i^2 - w_j\kappa_j^2]}{2\chi_F} + w_{jF}\kappa_j^2 - w_{iF}\kappa_i^2 + (\xi_j - \xi_i)\sum_{k=1}^{n} w_{km}\xi_k.$$

Since $\xi_j^2 > \xi_i^2$ the oligopoly either acquires both signals, neither, or just i (from Proposition 8). If both are acquired by the oligopoly then, using the equilibrium values of z_i and z_j from (27) and the first-order condition $\xi_i w_i = \nu z_i$ for all i, $w_i\kappa_i^2 > w_j\kappa_j^2$. Else, j is not acquired, and $w_i\kappa_i^2 \geq w_j\kappa_j^2$ since $w_j = 0$. Given that $\xi_j - \xi_i > 0$, this means that, for some positive constant A,

$$\frac{\partial\,\mathrm{E}[\textbf{Profit}_F]}{\partial w_{iF}} - \frac{\partial\,\mathrm{E}[\textbf{Profit}_F]}{\partial w_{jF}} > A(w_{jF}\kappa_j^2 - w_{iF}\kappa_i^2).$$

Now suppose that $w_{jF} > 0$ but $w_{iF} = 0$. Hence supplier m is free to simultaneously raise w_{iF} and lower w_{jF}. However

$$\frac{\partial\,\mathrm{E}[\textbf{Profit}_F]}{\partial w_{iF}} - \frac{\partial\,\mathrm{E}[\textbf{Profit}_F]}{\partial w_{jF}} > A(w_{jF}\kappa_j^2 - w_{iF}\kappa_i^2) = Aw_{jF}\kappa_j^2 > 0,$$

which means that supplier m would indeed wish to do this. This is a contradiction. So, if $w_{jF} > 0$ and $\xi_j^2 > \xi_i^2$ then $w_{iF} > 0$. In other words, the set of information sources that receive attention from (and have influence on) the competitive fringe are the clearest information sources.

Second Fact. It will be shown that $z_i = 0 \Rightarrow z_{iF} = 0$. Equivalently, $w_i = 0 \Rightarrow w_{iF} = 0$. By way of a contradiction, suppose $w_i = 0$ but $w_{iF} > 0$ for some i. The fact that both the oligopoly and the competitive fringe use (potentially different) sets comprising the clearest (lowest ξ_i^2) signals means that $N \subset N_F$, and, ordering the signals $\xi_1^2 < \xi_2^2 < \ldots < \xi_n^2$, if $j > i$ then $w_j = 0$ and for $j < i$, $w_{jF} > 0$. Define an analogous set of quantities to those in (33):

$$\tilde{k_0} = \sum_{j \in N_F} \frac{1}{\kappa_j^2}, \quad \tilde{k_1} = \sum_{j \in N_F} \frac{\xi_j}{\kappa_j^2}, \quad \text{and} \quad \tilde{k_2} = \sum_{j \in N_F} \frac{\xi_j^2}{\kappa_j^2},$$

and let $\hat{k}_x = \tilde{k}_x - k_x = \sum_{j \in N_F \setminus N} \xi_j^x / \kappa_j^2 > 0$. Choose i so that $i = \max\{j \mid w_{jF} > 0\} = \max_{j \in N_F} j$.

Now ν is as found in (35). On the other hand, ν_F/δ_F, implicitly found in (38) derives from the first-order condition for the fringe in (36), which applies for all signals receiving positive weight (those in N_F). Since $N \subset N_F$, the manipulations following (36) can be repeated, noting that for some j, $w_j = 0$, and that $\tilde{k}_x$ must replace k_x everywhere. In particular (38) becomes

$$\left[(1 + \tilde{k}_2)(\sigma^{-2} + \tilde{k}_0) - \tilde{k}_1^2\right] \frac{\nu_F}{\delta_F} = \frac{\beta}{1-\beta} \tilde{k}_1 + (\sigma^{-2} + \tilde{k}_0)\nu. \tag{40}$$

Recall the maintained hypothesis is that $w_i = 0$ but $w_{iF} > 0$ for some $i = \max_{j \in N_F} j$. Take the first-order condition for the weight the fringe places on i from (36). Then, with $w_i = 0$,

$$w_{iF} + \nu_F \frac{\xi_i}{\kappa_i^2} = \frac{\sigma^2}{\kappa_i^2} \left[\delta_F \frac{\beta}{1-\beta} - (\bar{w}_F - \delta_F \bar{w})\right].$$

Substituting for $\bar{w}_F - \delta_F \bar{w}$, which may be calculated analogously to the steps following (36) but replacing k_x with $\tilde{k}_x$ everywhere,

$$w_{iF} = \frac{1}{\kappa_i^2} \left\{ \frac{1}{\sigma^{-2} + \tilde{k}_0} \left[\delta_F \frac{\beta}{1-\beta} + \tilde{k}_1 \nu_F\right] - \nu_F \xi_i \right\}.$$

For this to be positive, as required, the term within the brackets must be positive, or

$$\begin{aligned}
w_{iF} > 0 \quad &\Leftrightarrow \quad \frac{1}{\sigma^{-2} + \tilde{k}_0} \left[\delta_F \frac{\beta}{1-\beta} + \tilde{k}_1 \nu_F\right] > \nu_F \xi_i \\
&\Leftrightarrow \quad \frac{\beta}{1-\beta} > \frac{\nu_F}{\delta_F} \left[\xi_i(\sigma^{-2} + \tilde{k}_0) - \tilde{k}_1\right] \\
&\Leftrightarrow \quad \left[(1 + \tilde{k}_2)(\sigma^{-2} + \tilde{k}_0) - \tilde{k}_1^2\right] \frac{\beta}{1-\beta} > \left[\frac{\beta}{1-\beta} \tilde{k}_1 + (\sigma^{-2} + \tilde{k}_0)\nu\right] \left(\xi_i(\sigma^{-2} + \tilde{k}_0) - \tilde{k}_1\right),
\end{aligned}$$

where the last line follows from the expression for ν_F/δ_F in (40). Now, multiplying out, cancelling common terms, and dividing through by $(\sigma^{-2} + \tilde{k}_0)$, this means that

$$\begin{aligned}
w_{iF} > 0 \quad &\Leftrightarrow \quad (1 + \tilde{k}_2) \frac{\beta}{1-\beta} > \nu \left[\xi_i(\sigma^{-2} + \tilde{k}_0) - \tilde{k}_1\right] + \xi_i \tilde{k}_1 \frac{\beta}{1-\beta} \\
&\Leftrightarrow \quad \frac{\beta}{1-\beta} \left[(1 + \tilde{k}_2) - \xi_i \tilde{k}_1\right] > \gamma\pi \frac{\xi_i(\sigma^{-2} + \tilde{k}_0) - \tilde{k}_1}{\frac{\pi + k_2}{k_1}(\sigma^{-2} + k_0) - k_1},
\end{aligned} \tag{41}$$

where the last line follows from the value of ν found in (35). Consider the fraction on the right-hand side of this last inequality. Note that, using the $\hat{k}_x$ notation,

$$\begin{aligned} & \xi_i(\sigma^{-2}+\tilde{k}_0)-\tilde{k}_1 > \frac{\pi+k_2}{k_1}(\sigma^{-2}+k_0)-k_1 \\ \Leftrightarrow\quad & \xi_i(\sigma^{-2}+k_0+\hat{k}_0)-(k_1-\hat{k}_1) > \frac{\pi+k_2}{k_1}(\sigma^{-2}+k_0)-k_1 \\ \Leftrightarrow\quad & \xi_i(\sigma^{-2}+k_0)+(\xi_i\hat{k}_0-\hat{k}_1) > \frac{\pi+k_2}{k_1}(\sigma^{-2}+k_0). \end{aligned} \tag{42}$$

Now $\xi_i \geq \xi_j$ for all $j \in N_F$, so $\xi_i\hat{k}_0 = \xi_i \sum_{j\in N_F\setminus N} 1/\kappa_j^2 \geq \sum_{j\in N_F\setminus N} \xi_j/\kappa_j^2 = \hat{k}_1$: the second term on the left-hand side is at least zero. The first term on the left-hand side, however, is strictly larger than the term on the right-hand side (since $i \notin N$, from Proposition 8, $\xi_i > K \equiv (\pi+k_2)/k_1$). This confirms the inequality in (42). Now consider the left-hand side of the inequality in (41). Again, using the $\hat{k}_x$ notation, the second term of the inequality is

$$1+\tilde{k}_2-\xi_i\tilde{k}_1 = 1+k_2+\hat{k}_2-\xi_i(k_1+\hat{k}_1) = 1+(k_2-\xi_ik_1)+(\hat{k}_2-\xi_i\hat{k}_1).$$

Once again, $\xi_i > (\pi+k_2)/k_1 \Rightarrow \xi_ik_1-k_2 > \pi$, and $\xi_i\hat{k}_1 = \xi_i\sum_{j\in N_F\setminus N}\xi_j/\kappa_j^2 \geq \sum_{j\in N_F\setminus N}\xi_j^2/\kappa_j^2 = \hat{k}_2$. This means that $1+\tilde{k}_2-\xi_i\tilde{k}_1 < 1-\pi$. In other words, the left-hand side of (41) is less than $(1-\pi)\beta/(1-\beta)$. On the other hand, from the inequality established in (42), the right-hand side of (41) is greater than $\gamma\pi$. But $\gamma\pi > (1-\pi)\beta/(1-\beta)$, as established in (39). The inequality in (41) fails, yielding a contradiction. So $N_F \subseteq N$.

Given the premise of the second part of the proposition, $N_F \subset N$, as required. □

Proof of Proposition 10. Let $n = 1$ and $c = 0$. Now, from (29),

$$F(s_m) = \frac{s_m(1-(1-\beta)s_m)}{\beta}\left(\frac{\delta_m}{\sigma^2+\kappa^2+(1-\delta_ms_m)\xi^2}\right)^2. \tag{43}$$

Take two suppliers, m and m', and suppose the former is larger: $s_m > s_{m'}$. Define $\varsigma = (1-\beta)s_m$ and $\varsigma' = (1-\beta)s_{m'}$, so that $\varsigma > \varsigma'$. Also note that $1 > (1-\beta) \geq (1-\beta)(s_m+s_{m'}) = \varsigma+\varsigma'$. Define

$$\alpha = \frac{\xi^2}{\sigma^2+\kappa^2+\xi^2} \in [0,1].$$

With this notation in place, (43) becomes

$$F(s_m) = \frac{\varsigma(1-\varsigma)}{\beta(1-\beta)}\left(\frac{\delta_m/\xi^2}{\alpha^{-1}-\delta_ms_m}\right)^2 = \frac{\varsigma(1-\varsigma)}{\beta(1-\beta)}\left(\frac{(1-\beta)/(2-\varsigma)\xi^2}{\alpha^{-1}-\varsigma/(2-\varsigma)}\right)^2,$$

since, from (9) with $c = 0$, $\delta_m = (1-\beta)\beta/(\beta+\beta(1-\delta_ms_m)) = (1-\beta)/(2-\varsigma)$ and hence $s_m\delta_m = \varsigma/(2-\varsigma)$. Rearranging this expression for $F(s_m)$ gives

$$F(s_m) = \frac{(1-\beta)}{\beta}\left(\frac{\alpha}{\xi^2}\right)^2 \times \frac{\varsigma(1-\varsigma)}{(2-(1+\alpha)\varsigma)^2},$$

where the first term does not depend upon s_m. Therefore $F(s_m) > F(s_{m'})$, so that the larger supplier (m) has a greater incentive to acquire new information, if and only if

$$\frac{\varsigma(1-\varsigma)}{(2-(1+\alpha)\varsigma)^2} > \frac{\varsigma'(1-\varsigma')}{(2-(1+\alpha)\varsigma')^2}.$$

Multiplying across the (positive) denominators, multiplying out the resultant expressions, and then simplifying, this inequality will hold if and only if

$$4\underbrace{(\varsigma-\varsigma')}_{>0}\times\underbrace{(1-(\varsigma+\varsigma'))}_{>0}>\underbrace{(1+\alpha)(3-\alpha)}_{>0}\times\varsigma\varsigma'\times\underbrace{(\varsigma'-\varsigma)}_{<0}.$$

For $c=0$, larger suppliers have a higher incentive to acquire new information. Moreover, the last expression is strictly negative for $s_m>s_{m'}$. Thus, by continuity, the result will also hold for all positive c sufficiently small, as stated in the proposition. □

Proof of Proposition 11. Evaluated at a symmetric equilibrium,

$$\begin{aligned}\mathrm{E}[\text{Industry Profit}]=\frac{\mathrm{E}[\text{Profit}_m]}{s}&=\left[1+\frac{c(1-(1-\beta)s)}{\beta}\right]\mathrm{E}[\theta p_m]\\&-\frac{1-(1-\beta)s}{\beta}\left[1+\frac{c(1-(1-\beta)s)}{2\beta}\right]\mathrm{E}[p_m^2]\\&-\frac{c\,\mathrm{E}[\theta^2]}{2}\\&-\frac{c(1-\beta)^2s^2}{2\beta^2}\sum_{m'\neq m}\sum_{m''\neq m}\mathrm{E}[p_{m'}p_{m''}]\\&+\frac{(1-\beta)s}{\beta}\left[1+\frac{c(1-(1-\beta)s)}{\beta}\right]\sum_{m'\neq m}\mathrm{E}[p_mp_{m'}]\\&-\frac{c(1-\beta)s}{\beta}\sum_{m'\neq m}\mathrm{E}[\theta p_{m'}].\end{aligned}$$

In terms of variance and covariance terms,

$$\begin{aligned}\mathrm{E}[\text{Industry Profit}]&=\text{full-information profit}-\frac{c\sigma^2}{2}\\&+\left[1+\frac{c(1-(1-\beta)s)}{\beta}\right]\mathrm{cov}[\theta,p_m]\\&-\frac{1-(1-\beta)s}{\beta}\left[1+\frac{c(1-(1-\beta)s)}{2\beta}\right]\mathrm{var}[p_m]\\&-\frac{c(1-\beta)^2s^2}{2\beta^2}\sum_{m'\neq m}\sum_{m''\neq m}\mathrm{cov}[p_{m'},p_{m''}]\\&+\frac{(1-\beta)s}{\beta}\left[1+\frac{c(1-(1-\beta)s)}{\beta}\right]\sum_{m'\neq m}\mathrm{cov}[p_m,p_{m'}]\\&-\frac{c(1-\beta)s}{\beta}\sum_{m'\neq m}\mathrm{cov}[\theta,p_{m'}].\end{aligned}$$

Now set $c=0$, so that there are constant returns to scale. In this case,

$$\begin{aligned}\mathrm{E}[\text{Industry Profit}]&=\text{full-information profit}\\&+\mathrm{cov}[\theta,p_m]-\frac{1-(1-\beta)s}{\beta}\mathrm{var}[p_m]+\frac{(1-\beta)s}{\beta}\sum_{m'\neq m}\mathrm{cov}[p_m,p_{m'}].\end{aligned}$$

Evaluating at a symmetric strategy profile, so that $w_{im}=w_i$ and $\bar{w}_m=\bar{w}$ for all m,

$$\mathrm{cov}[\theta,p_m]=\bar{w}\sigma^2,\ \ \mathrm{var}[p_m]=\bar{w}^2\sigma^2+\sum_{i=1}^n w_i^2(\kappa_i^2+\xi_i^2)\ \text{and}\ \mathrm{cov}[p_m,p_{m'}]=\bar{w}^2\sigma^2+\sum_{i=1}^n w_i^2\kappa_i^2. \quad (44)$$

Substituting in these terms and re-arranging appropriately,

$$\begin{aligned}\mathrm{E}[\text{Industry Profit}] &= \text{full-information profit} \\ &\quad + (\bar{w} - \bar{w}^2)\sigma^2 - \frac{1}{\pi^\dagger}\sum_{i=1}^{n} w_i^2 \left(\pi^\dagger \kappa_i^2 + \xi_i^2\right) \quad \text{where} \quad \pi^\dagger \equiv \frac{\beta}{1-(1-\beta)s}.\end{aligned}$$

Maximization of this and solution of the n linear first-order conditions straightforwardly generates the claims concerning $w_i^\dagger$ and $\bar{w}^\dagger$.

Using the expression from (4), consumer surplus is $\sum_{m=1}^{M} s_m \mathrm{CS}_m$ where

$$\mathrm{CS}_m = \frac{1}{2}\left[(\theta - p_m)^2 + \frac{(1-\beta)(1-s_m)(P_{-m} - p_m)(\theta - p_m)}{\beta}\right].$$

Hence, evaluated at a symmetric equilibrium

$$\begin{aligned}\mathrm{E}[\mathrm{CS}] &= \text{full-information CS} + \frac{\sigma^2}{2} \\ &\quad - \mathrm{cov}[\theta, p_m] + \frac{1}{2}\left(1 + \frac{(1-\beta)(1-s)}{\beta}\right)\mathrm{var}[p_m] - \frac{(1-\beta)(1-s)}{2\beta}\mathrm{cov}[p_m, p_{m'}]\end{aligned}$$

Substituting in the variance and covariances from (44),

$$\mathrm{E}[\mathrm{CS}] = \text{full-information CS} + \frac{\sigma^2}{2} - \bar{w}\sigma^2 + \frac{\bar{w}^2\sigma^2}{2} + \frac{1}{2\pi^\dagger}\sum_{i=1}^{n} w_i^2 \left(\pi^\dagger \kappa_i^2 + \xi_i^2\right),$$

and where $\pi^\dagger$ is the expression used previously. Notice that the terms that depend on relative information use are opposite in sign and half the size of those that are present within the expression for industry profit.

Combining (in a symmetric industry, with $c = 0$) consumer surplus and profit,

$$\begin{aligned}\mathrm{E}[\text{Welfare}] &= \text{full-information welfare} + \frac{\sigma^2}{2} \\ &\quad - \frac{1-(1-\beta)s}{2\beta}\mathrm{var}[p_m] + \frac{(1-\beta)(1-s)}{2\beta}\mathrm{cov}[p_m, p_{m'}].\end{aligned}$$

Notice that the covariance of a supplier's price with the demand shifter ($\mathrm{cov}[\theta, p_m]$) disappears. Substituting in for the remaining variance and covariance terms,

$$\mathrm{E}[\text{Welfare}] = \text{full-information welfare} + \frac{\sigma^2}{2} - \frac{\bar{w}^2\sigma^2}{2} - \frac{1}{2\pi^\dagger}\sum_{i=1}^{n} w_i^2 \left(\pi^\dagger \kappa_i^2 + \xi_i^2\right).$$

This is maximized by setting $w_i = 0$ for each i. However, if positive information is used is considered, so that $\bar{w} > 0$, then it is optimal to set $w_i \propto 1/(\pi^\dagger \kappa_i^2 + \xi_i^2)$, as claimed. □

Proof of Proposition 12. This follows from arguments in the text. Specifically, setting $c > 0$ influences only expected industry profit. Note that

$$\begin{aligned}\mathrm{E}[\text{Industry Profit}] &= \mathrm{E}[\text{Industry Profit} \,|\, c = 0] \\ &\quad + c\left[\bar{w}\sigma^2 - \frac{\sigma^2}{2} - \frac{\bar{w}^2\sigma^2}{2} - \frac{1}{2\pi^\ddagger}\sum_{i=1}^{n} w_i^2(\pi^\ddagger \kappa_i^2 + \xi_i^2)\right].\end{aligned}$$

The last term is increasing in w_i when evaluated at $w_i = 0$ for each i. □

Appendix B. Supplementary Calculations

Profit and Consumer Surplus. The expression for the profit of supplier m in (3) is obtained by substituting in the demand function from (2).

For consumer surplus, note that

$$\text{Consumer Surplus} = \int_0^1 \left(u(q_\ell, Q) - p_\ell q_\ell\right) d\ell = \sum_{m=1}^{M} \left(u(q_m, Q) - p_m q_m\right).$$

Next, using (1), substituting in demand and $Q = \theta - P$,

$$\begin{aligned} u(q_m, Q) - p_m q_m &= q_m \left(\theta - \frac{\beta q_m + (1-\beta)Q}{2} - p_m\right) \\ &= q_m \left(\theta - \frac{\beta(\theta - p_m) + (1-\beta)(P - p_m) + (1-\beta)(\theta - P)}{2} - p_m\right) \\ &= \frac{q_m(\theta - p_m)}{2} \\ &= \frac{1}{2}\left((\theta - p_m)^2 + \frac{(1-\beta)(P - p_m)(\theta - p_m)}{\beta}\right) \end{aligned}$$

Payoffs in a Quadratic-Loss Coordination Game. (5) says that maximization of a supplier's profit objective is equivalent to minimization of the quadratic-loss function $\pi_m(p_m - \gamma_m\theta)^2 + (1 - \pi_m)(p_m - P_{-m})^2$. Equivalently,

$$\text{Profit}_m \propto \text{other terms} - \frac{p_m^2}{2} + \pi_m\gamma_m\theta p_m + (1-\pi_m)p_m P_{-m},$$

where the "other terms" do not depend on p_m. From (6),

$$\gamma_m = \frac{\beta + c(1 - (1-\beta)s_m)}{\beta + (1+c)(1 - (1-\beta)s_m)} \text{ and } \pi_m = \frac{\beta(\beta + (1+c)(1 - (1-\beta)s_m))}{(1 - (1-\beta)s_m)(2\beta + c(1 - (1-\beta)s_m))}.$$

Hence (following some re-arrangement) the coefficients on θp_m and $p_m P_{-m}$ are

$$\begin{aligned} \gamma_m\pi_m &= \frac{\beta(\beta + c(1 - (1-\beta)s_m))}{(1 - (1-\beta)s_m)(2\beta + c(1 - (1-\beta)s_m))} \quad \text{and} \\ 1 - \pi_m &= \frac{(\beta + c(1 - (1-\beta)s_m))\,(1-\beta)(1-s_m)}{(1 - (1-\beta)s_m)(2\beta + c(1 - (1-\beta)s_m))} \end{aligned}$$

From the proof of Proposition 1,

$$\begin{aligned} \frac{\text{Profit}_m}{s_m} &= \text{other terms} + \left[1 + \frac{c[1 - (1-\beta)s_m]}{\beta}\right]\theta p_m \\ &\quad - \left[\frac{1 - (1-\beta)s_m}{\beta} + \frac{c[1 - (1-\beta)s_m]^2}{2\beta^2}\right]p_m^2 \\ &\quad + \left[\frac{c[(1-\beta)^2(1-s_m)^2 + \beta(1-\beta)(1-s_m)]}{\beta^2} + \frac{(1-\beta)(1-s_m)}{\beta}\right]P_{-m}p_m, \end{aligned}$$

where as before "other terms" do not involve p_m. Re-arranging slightly,

$$\begin{aligned} \text{Profit}_m \propto \text{other terms} &+ \left[\frac{\beta^2 + \beta c(1 - (1-\beta)s_m)}{(1 - (1-\beta)s_m)(2\beta + c(1 - (1-\beta)s_m))}\right]\theta p_m \\ &- \frac{p_m^2}{2} + \left[\frac{(\beta + c(1 - (1-\beta)s_m))\,(1-\beta)(1-s_m)}{(1 - (1-\beta)s_m)(2\beta + c(1 - (1-\beta)s_m))}\right]P_{-m}p_m. \end{aligned}$$

By inspection the coefficients on the terms p_m^2, θp_m, and $P_{-m}p_m$ match those from earlier.

The quadratic-payoff-coordination representation can also be used to derive the full-information equilibrium prices reported in Proposition 1. Note that

$$p_m = \pi_m\gamma_m\theta + (1-\pi_m)P_{-m} = \pi_m\gamma_m\theta + \frac{(1-\pi_m)(P - s_m p_m)}{1-s_m}$$
$$\Rightarrow \quad p_m = \frac{(1-s_m)\pi_m\gamma_m\theta + (1-\pi_m)P}{(1-s_m)+s_m(1-\pi_m)} = \frac{1-\pi_m}{1-s_m\pi_m}\left(\frac{(1-s_m)\pi_m\gamma_m\theta}{(1-\pi_m)} + P\right)$$

Substituting in for π_m, note that

$$\frac{1-\pi_m}{1-s_m\pi_m} = \frac{(\beta + c(1-(1-\beta)s_m))\,(1-\beta)(1-s_m)}{(1-(1-\beta)s_m)(2\beta + c(1-(1-\beta)s_m)) - \beta s_m(\beta + (1+c)(1-(1-\beta)s_m))}$$
$$= \frac{(1-\beta)\,(\beta + c(1-(1-\beta)s_m))}{\beta + (\beta+c)(1-(1-\beta)s_m)} = \delta_m,$$

where δ_m is from (9). It is also straightforward to confirm that

$$\frac{(1-s_m)\pi_m\gamma_m}{(1-\pi_m)} = \frac{\beta}{1-\beta}.$$

This verifies the claim in (9). Taking the weighted sum generates P as claimed.

References

ALLEN, F., S. MORRIS, AND H. S. SHIN (2006): "Beauty Contests and Iterated Expectations in Asset Markets," *The Review of Financial Studies*, 19(3), 720–752.

AMADOR, M., AND P.-O. WEILL (2010): "Learning from Prices: Public Communication and Welfare," *Journal of Political Economy*, 118(5), 866–907.

——— (2012): "Learning from Private and Public Observations of Others' Actions," *Journal of Economic Theory*, 147(3), 910–940.

ANGELETOS, G.-M., L. IOVINO, AND J. LA'O (2016): "Real Rigidity, Nominal Rigidity, and the Social Value of Information," *American Economic Review*, 106(1), 200–227.

ANGELETOS, G.-M., G. LORENZONI, AND A. PAVAN (2012): "Wall Street and Silicon Valley: A Delicate Interaction," *MIT* and *Northwestern University*, unpublished manuscript.

ANGELETOS, G.-M., AND A. PAVAN (2004): "Transparency of Information and Coordination in Economies with Investment Complementarities," *American Economic Review: AEA Papers and Proceedings*, 94(2), 91–98.

——— (2007): "Efficient Use of Information and Social Value of Information," *Econometrica*, 75(4), 1103–1142.

——— (2009): "Policy with Dispersed Information," *Journal of the European Economic Association*, 7(1), 11–60.

BAERISWYL, R. (2011): "Endogenous Central Bank Information and the Optimal Degree of Transparency," *International Journal of Central Banking*, 7(2), 85–111.

BAERISWYL, R., AND C. CORNAND (2007): "Monetary Policy and its Informative Value," *University of Munich*, unpublished manuscript.

——— (2011): "Transparency and Monetary Policy Effectiveness," *Annales of Economics and Statistics*, 103/104, 175–194.

BAYONA, A. (2015): “The Social Value of Information with an Endogenous Public Signal,” *ESADE Business School*, unpublished manuscript.

CALVÓ-ARMENGOL, A., AND J. DE MARTÍ (2007): “Communication Networks: Knowledge and Decision,” *American Economic Review: AEA Papers and Proceedings*, 97(2), 86–91.

——— (2009): “Information Gathering in Organizations: Equilibrium, Welfare, and Optimal Network Structure,” *Journal of the European Economic Association*, 7(1), 116–161.

CALVÓ-ARMENGOL, A., J. DE MARTÍ, AND A. PRAT (2015): “Communication and Influence,” *Theoretical Economics*, 10(2), 649–690.

CLARKE, R. N. (1983): “Collusion and the Incentives for Information Sharing,” *Bell Journal of Economics*, 14(2), 383–394.

COLOMBO, L., G. FEMMINIS, AND A. PAVAN (2014): “Information Acquisition and Welfare,” *Review of Economic Studies*, 81(4), 1438–1483.

CURRARINI, S., AND F. FERI (2015): “Information Sharing Networks in Linear Quadratic Games,” *International Journal of Game Theory*, 44(3), 701–732.

DEWAN, T., AND D. P. MYATT (2008): “The Qualities of Leadership: Direction, Communication, and Obfuscation,” *American Political Science Review*, 102(3), 351–368.

——— (2012): “On the Rhetorical Strategies of Leaders: Speaking Clearly, Standing Back, and Stepping Down,” *Journal of Theoretical Politics*, 24(4), 431–460.

DIXIT, A. S. (1979): “A Model of Duopoly Suggesting a Theory of Entry Barriers,” *Bell Journal of Economics*, 10(1), 20–32.

FAINMESSER, I. P., AND A. GALEOTTI (2016): “Pricing Network Effects,” *Review of Economic Studies*, 83(1), 165–198.

FRIED, D. (1984): “Incentives for Information Production and Disclosure in a Duopolistic Environment,” *Quarterly Journal of Economics*, 99(2), 367–381.

GAL-OR, E. (1985): “Information Sharing in Oligopoly,” *Econometrica*, 53(2), 329–343.

——— (1986): “Information Cournot and Transmission-Cournot and Bertrand Equilibria,” *Review of Economic Studies*, 53(1), 85–92.

HAN, J., AND F. SANGIORGI (2015): “Searching for Information,” *Stockholm School of Economics*, unpublished manuscript.

HAUK, E., AND S. HURKENS (2001): “Secret Information Acquisition in Cournot Markets,” *Economic Theory*, 18(3), 661–681.

HELLWIG, C. (2005): “Heterogeneous Information and the Welfare Effects of Public Information Disclosure,” *UCLA*, unpublished manuscript.

HELLWIG, C., AND L. VELDKAMP (2009): “Knowing What Others Know: Coordination Motives in Information Acquisition,” *Review of Economic Studies*, 76(1), 223–251.

HWANG, H.-S. (1993): “Optimal Information Acquisition for Heterogeneous Duopoly Firms,” *Journal of Economic Theory*, 59(2), 385–402.

——— (1995): “Information Acquisition and Relative Efficiency of Competitive, Oligopoly, and Monopoly Markets,” *International Economic Review*, 36(2), 325–340.

KIRBY, A. J. (1988): “Trade Associations as Information Exchange Mechanisms,” *RAND Journal of Economics*, 19(1), 138–146.

LAGERLÖF, J. N. M. (2007): “Insisting on a Non-Negative Price: Oligopoly, Uncertainty, Welfare, and Multiple Equilibria,” *International Journal of Industrial Organization*, 25(4), 861–875.

LI, L. (1985): “Cournot Oligopoly with Information Sharing,” *RAND Journal of Economics*, 16(4), 521–536.

LI, L., R. D. MCKELVEY, AND T. PAGE (1987): “Optimal Research for Cournot Oligopolists,” *Journal of Economic Theory*, 42(1), 140–166.

LLOSA, L. G., AND V. VENKATESWARAN (2013): “Efficiency with Endogenous Information Choice,” *UCLA* and *Pennsylvania State University*, unpublished manuscript.

MALUEG, D. A., AND S. O. TSUTSUI (1998): “Oligopoly Information Exchange when Non-Negative Price and Output Constraints may Bind,” *Australian Economic Papers*, 37(4), 363–371.

MORRIS, S., AND H. S. SHIN (2002): “Social Value of Public Information,” *American Economic Review*, 92(5), 1521–1534.

MYATT, D. P., AND C. WALLACE (2012): “Endogenous Information Acquisition in Coordination Games,” *Review of Economic Studies*, 79(1), 340–374.

——— (2014): “Central Bank Communication Design in a Lucas-Phelps Economy,” *Journal of Monetary Economics*, 63, 64–79.

——— (2015): “Cournot Competition and the Social Value of Information,” *Journal of Economic Theory*, 158, 466–506.

NOVSHEK, W., AND H. F. SONNENSCHEIN (1982): “Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release,” *Bell Journal of Economics*, 13(1), 214–218.

PALFREY, T. R. (1985): “Uncertainty Resolution, Private Information Aggregation, and the Cournot Competitive Limit,” *Review of Economic Studies*, 52(1), 69–83.

PAVAN, A. (2014): “Attention, Coordination, and Bounded Recall,” *Northwestern University*, unpublished manuscript.

RAITH, M. (1996): “A General Model of Information Sharing in Oligopoly,” *Journal of Economic Theory*, 71(1), 260–288.

SAKAI, Y. (1986): “Cournot and Bertrand Equilibria Under Imperfect Information,” *Journal of Economics*, 46(3), 213–232.

SAKAI, Y., AND T. YAMATO (1989): “Oligopoly, Information and Welfare,” *Journal of Economics*, 49(1), 3–24.

SHAPIRO, C. (1986): “Exchange in of Cost Information Oligopoly,” *Review of Economic Studies*, 53(3), 433–446.

SINGH, N., AND X. VIVES (1984): “Price and Quantity Competition in a Differentiated Duopoly,” *RAND Journal of Economics*, 15(4), 546–554.

VIVES, X. (1984): “Duopoly Information Equilibrium: Cournot and Bertrand,” *Journal of Economic Theory*, 34(1), 71–94.

——— (1988): "Aggregation of Information in Large Cournot Markets," *Econometrica*, 56(4), 851–876.

——— (1990): "Trade Association Disclosure Rules, Incentives to Share Information, and Welfare," *RAND Journal of Economics*, 21(3), 409–430.

——— (2002): "Private Information, Strategic Behavior, and Efficiency in Cournot Markets," *RAND Journal of Economics*, 33(3), 361–376.

——— (2011): "Strategic Supply Function Competition with Private Information," *Econometrica*, 79(6), 1919–1966.

——— (2013): "Endogenous Public Information and Welfare in Market Games," *CEPR Discussion Paper*, 8437.