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Government as Borrower of First Resort

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Abstract

We examine optimal provision of riskless government bonds under asymmetric information and safe asset scarcity. Paradoxically, corporations have incentives to issue junk debt precisely when intrinsic demand for safe debt is high since uninformed investors then migrate to risky overheated debt markets. Uninformed demand stimulates informed speculation which drives junk debt prices closer to fundamentals, encouraging pooling at high leverage. Acting as borrower of first resort, the government can issue safe bonds which siphon off uninformed demand for risky corporate debt and reduce socially wasteful informed speculation. Thus, government bonds either eliminate pooling at high leverage or improve risk sharing in such equilibria. The optimal quantity of government bonds is increasing in intrinsic demand for safe assets and non-monotonic in marginal Q.

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In recent years the set of safe stores of value has contracted. For example, the credit crisis of 2007/8 revealed the exposure of senior tranches of securitizations to correlated defaults. The Eurozone crisis called into question the safety of some sovereign debts. Finally, fiscal weakness undermined confidence in deposit insurance in some jurisdictions. At the same time, these crises stimulated investor demand for safe stores of value. The perceived combination of diminished supply and increased demand for safe assets has led some to argue there is a scarcity of safe assets. In this vein, a recent Global Financial Stability Report of the IMF (2012) states, “In the future, there will be rising demand for safe assets, but fewer of them will be available...”

Of course, one might reasonably expect any safe asset scarcity problem to be self-correcting. Gourinchas and Jeanne (2012) argue “the economy as a system will strive to compensate for any shortage.” For example, if pension funds and insurers have a strong demand for long-duration safe stores of value, one might expect an issuing corporation with long-lived capital assets to adopt a low leverage ratio, supplying long maturity debt with low default risk. However, Stein (2013) points out that it is the junk bond market that has grown in recent years, with record volumes of high-yield debt issuance, leveraged loans, and dividend recapitalization transactions, as well as high debt-to-EBITDA multiples in leveraged buyouts. In light of these trends, Stein, and other policymakers have called for tools to identify and respond to debt market “overheating.”

In this paper we put forward a positive framework for understanding the conjunction of safe asset scarcity and an overheated corporate debt market. We then develop a normative framework to analyze whether and how variation in the supply of safe government bonds can be used as a policy tool to influence equilibrium in corporate debt markets and increase social welfare. The model is predicated on a canonical corporate finance friction: asymmetric information between the corporation and investors regarding cash flows. Following Ross (1977), the privately-informed corporation chooses a debt face value and then uses the proceeds to fund a scalable investment delivering a private benefit $Q > 1$ per unit invested. The terminal period payoff on the asset-in-place backing the debt is either low ($L$) or high ($H$). This payoff is verifiable ex post, but
unobservable to investors ex ante.

We depart from the extant debt signaling literature by allowing investors to purchase corporate debt in a securities market modeled à la Kyle (1985). There are perfectly competitive market-makers who clear the market and a speculator who can exert costly effort to acquire a noisy signal regarding the asset payoff. In addition, there is a continuum of uninformed investors who would prefer to carry funds across periods using a riskless store of value. There is safe asset scarcity, which the corporation can remedy by issuing riskless debt. The positive question addressed is whether the corporation can be expected to eliminate the problem of safe asset scarcity by providing uninformed investors with an information-insensitive store of value. The normative question addressed is whether the government can increase social welfare by acting as a borrower of first resort with an eye toward influencing the corporate debt market equilibrium.

As we show, regardless of the severity of the safe asset scarcity problem, the equilibrium set always includes pooling at riskless debt. Further, the equilibrium set always includes a separating equilibrium where the corporation with positive private information signals this by issuing riskless debt with low face value, leaving the shareholder to bear the residual cash flow risk. If either of these two equilibria is implemented, the initial problem of safe asset scarcity is self-correcting, with uninformed investors being insulated from adverse selection.

Critically, the problem of safe asset scarcity need not self-correct. This is because, as we show, if there is sufficiently high intrinsic demand for safe storage, the private sector may actually pool at risky rather than safe debt. That is, risky debt can be imposed on investors precisely when doing so generates a large negative externality. The argument is as follows. If there is safe asset scarcity, a portion of uninformed investor demand migrates to the risky corporate debt market. The prospect of trading against uninformed investors encourages speculator information production and informed trading. In turn, informed trading brings the risky debt price closer to fundamentals. And with less severe underpricing, a corporation with positive private information is more willing to pool at risky debt.
The preceding argument suggests a role for the treasury or central bank to increase welfare by varying the quantity of safe government bonds made available to investors, with the goal of inducing the corporate sector to implement the socially preferred equilibrium. To see this formally, suppose as we do that the government acts as a Stackelberg leader, having the ability to offer the public access to safe storage (bonds), with the public storage being insufficient to fill the intrinsic uninformed demand for safe assets. Despite this constraint, the government can potentially ensure an adequate aggregate supply of safe assets by increasing the amount of safe government bonds offered. After all, uninformed investors will substitute any available riskless government bonds for risky corporate debt in their portfolios. The anticipation of less uninformed trading in the corporate debt market deters speculator information production. This widens the gap between debt prices and fundamentals. With sufficient underpricing anticipated, high debt pooling equilibria unravel. Notice, the role of government debt here is to crowd-out risky debt and crowd-in riskless corporate debt.

Although it may be feasible for the government to eliminate the high debt pooling equilibrium in the manner just described, doing so does not necessarily increase social welfare. On one hand, anticipation of adverse selection in the risky debt market will induce distortions in the portfolios of uninformed investors. Against these costs, government must weigh the NPV of the incremental investment financed by higher debt issuance. As we show, crowding out junk debt (and investment), increases social welfare only if marginal $Q$ is sufficiently low relative to intrinsic storage demand.

The arguments made thus far suggest that the optimal government debt policy is binary: Offer the maximum feasible amount of government bonds if the goal is to eliminate the high debt pooling equilibrium, and zero otherwise. However, we show it may also be optimal for the government to offer an intermediate amount of bonds. If marginal $Q$ is high, social welfare will be higher if there is pooling at risky corporate debt. But that does not imply optimality of zero government debt. Rather, offering a limited supply of government bonds serves to increase social welfare given pooling at risky debt. After all, government bonds can still siphon uninformed demand, decrease speculator
effort, and mitigate distortions in the portfolios of uninformed investors. Here government policy must be fine-tuned since too much siphoning can put in jeopardy the high debt pooling equilibrium.

We turn now to related literature. Perhaps most similar to our paper is that of Stein (2012) who also argues that central bank operations can serve to mitigate negative externalities associated with private sector debt decisions, increasing social welfare. In his model, banks can choose between long and short-term debt, with households placing higher value on the latter due to its relative safety. The privately optimal amount of short-term debt (“money”) creation can be socially excessive. This is because banks fully capture the social value of safe assets, enjoying lower financing costs when they rely on short-term debt. However, they do not always internalize the fire-sale externality that emerges should a common shock force them to liquidate assets in order to deliver on their short-term debts. Here the central bank can cool the short-term bank debt market by withdrawing reserves.

Gorton and Pennacchi (1990) also analyze the equilibrium supply of riskless debt in a setting where uninformed investors prefer safe storage. In their model, uninformed investors carve out a safe debt claim for themselves. In contrast, we show a privately informed issuer can have the opposite incentive, switching from riskless to risky debt when uninformed investors have high demand for safe storage.

In the model of Woodford (1990), government bonds directly increase welfare by providing agents with an intertemporal store of value. Holmström and Tirole (1998) analyze the social welfare benefits of government bonds when limited verifiability constrains the private supply of stores of value. They consider a setting with hidden action and potential production inefficiencies. In their model the role of government debt is a direct one, in that it increases aggregate storage dollar for dollar. In our model, government bonds can have a disproportionate multiplier effect on the aggregate supply of safe assets by crowding out corporate junk debt and crowding in riskless corporate debt.

In the model of Gorton and Ordonez (2013), projects are positive NPV but loans must be backed by collateral. Some producers have low quality collateral and will be cut off from credit if investors acquire information. The government can increase producer collateral by making them a gift of its
bonds backed by future tax collections. In their model, safe government debt serves to crowd-in risky borrowing while in our model government debt serves to crowd-out risky borrowing.

Krishnamurthy and Vissing-Jorgensen (2012) emphasize that investors value liquidity and safety, with variation in government debt leading to variation in equilibrium compensation for illiquidity and risk. Greenwood, Hanson and Stein (2010) develop and test a theory of corporate debt maturity predicated upon a related mechanism, arguing corporations can lower their cost of debt capital by filling the gaps left when the government changes its debt maturity structure.

The most important difference between our proposed theory and existing gap filling theories is that we argue corporations may not fill government debt gaps like-for-like. In particular, we predict that a sufficiently large reduction in government debt will indeed increase the supply of corporate debt, but this debt will be risky not safe. Consistent with our argument, Greenwood and Hanson (2013) find that the high-yield share of corporate debt flotations is actually inversely related to Treasury yields. This suggests that corporations respond to safe asset scarcity by supplying more risky debt.

Graham, Leary and Roberts (2014A) shed additional light on this empirical question. They document a negative cross-sectional relationship between firm-specific debt-to-asset ratios and the ratio of government debt to corporate assets. Importantly, this negative relationship is statistically indistinguishable for corporations with high and low default risk. Instead of focusing on cross-sectional leverage relationships, Graham, Leary and Roberts (2014B) analyze the time series relationship between aggregate corporate and government leverage ratios. They find that the ratio of aggregate corporate debt to aggregate corporate assets is inversely related to the government debt to GDP ratio. As they show, this relationship is particularly pronounced over the period 1950 thru 1970, when the aggregate corporate leverage ratio increased from 10% to 35% while the ratio of government debt to GDP decreased from 100% to 10%. Consistent with the cross-sectional evidence, the propensity of corporations to increase leverage over this time period held across high and low leverage categories. It is hard to reconcile these stylized facts with existing gap filling theories.
After all, if the goal is to increase the supply of safe assets in response to a decrease in government debt, riskier firms should decrease not increase leverage. Moreover, it is not clear that the increased leverage of safer firms is consistent with an increased supply of safe assets. After all, increases in leverage imply reductions in credit quality.

The remainder of the paper is organized as follows. Section I describes the economic setting. Section II describes bond pricing. Section III describes the equilibrium corporate leverage decision. Sections IV and V analyze the optimal provision of safe government bonds. Section VI considers model extensions.

I. The Economic Setting

There are two periods (1 and 2) and three categories of agents: government, corporation(s), and investors. The government and corporation offer bonds and investors buy them. All investors enter the model with sufficiently large endowments in period 1 to purchase their desired portfolios. The period 2 endowments of investors are not verifiable so they cannot issue securities backed by them. Holmström and Tirole (1998) also rule out unsecured credit based on limits on verifiability.

Investors cannot privately store their period 1 endowments, e.g. privately stored goods will be stolen or decay. The absence of such private storage creates the possibility of a scarcity of stores of value, in the spirit of Holmström and Tirole (1998). In fact, each investor could be endowed with limited safe storage without changing results. The critical assumption is that private storage, here normalized to zero, is smaller than intrinsic storage demand.

The government has the unique ability to store goods from period 1 to period 2 without any risk of theft or decay. To illustrate clearly the ability of the government to raise social welfare through its borrower-of-first-resort capability, it is assumed to have no other capabilities. In particular, the government can neither verify endowments in order to collect taxes nor redistribute resources.\footnote{Endowing government with ability to tax and redistribute creates a trivial rationale for government to transfer funds to positive NPV investments.}
our parable economy, the government simply has the ability to collect goods from investors in period 1, place them in public storage, and return them in period 2. Essentially, government bond investors receive risk-free inflation-protected debt claims. The government’s ability to provide such risk free stores of value is assumed to be limited, however. In particular, the maximum capacity of the public storage facility is $G \in [0, \infty)$. 

The objective of the government is to maximize a utilitarian social welfare function placing equal weight on each agent. The government acts as a Stackelberg leader in debt markets, with a Cournot structure discussed as an extension. In particular, the government moves first by specifying the amount of storage $G \in [0, G]$ that it will make available to investors. If requested storage exceeds $G$, it will be allocated on a pro rata basis.

The corporate sector acts as follower in the Stackelberg game. Specifically, just after the government specifys $G$, a private corporation chooses its debt level. We initially focus on the leverage decision of a single corporation, with interdependence between corporate capital structure decisions analyzed as an extension. The corporation is controlled by a manager-owner ("the manager" below) who cannot raise outside equity funding due to his ability to costlessly divert discretionary cash flow. The manager has vNM utility function over consumption $QC_1 + C_2$, where $Q > 1$. The corporation has an asset-in-place but no internal funds, and the manager has no other funds. The asset-in-place will generate an observable and verifiable cash flow in period 2. The cash flow is either $L$ or $H$, where $H > L > 0$. The asset type, denoted $T \in \{L, H\}$, is equivalent to the cash flow the asset-in-place will generate. Each asset type is equally likely, and investors do not observe asset type. In contrast, the manager privately observes the asset type in period 1.

The privately informed manager chooses his corporation’s leverage by specifying a debt face value $D$ due in period 2. The proceeds raised by the debt flotation are used to finance a dividend in period 1. The manager enjoys limited liability so the period 2 payoff on the debt is the minimum of $T$ and $D$. As captured by the manager’s utility function, each unit of funding the corporation receives in

\footnote{With outside equity, the qualitative welfare tradeoffs remain. Low corporate leverage leads to efficient risk sharing and high leverage leads to high investment.}
period 1 provides the manager with $Q$ units of utility. There are two alternative interpretations for why the manager utility parameter $Q$ is greater than one. First, one may think of the manager as being impatient. Second, one may think of the manager as using the funds received from investors to finance a new investment providing him with a private benefit. In this, our chosen interpretation, $Q$ represents marginal $Q$.

There are three categories of investors who invest in government and corporate bonds. There is a measure one continuum of uninformed investors (UI). The UI are analogous to the liquidity traders of Gorton and Pennacchi (1990). They are akin to pension funds and insurance companies in that they are risk-averse and have a preference for safe storage. The UI have identical stochastic period 2 endowments $Y_2 \in \{Z - N, Z\}$ with the parametric assumption $0 < N \leq Z$. Each realization of $Y_2$ is equiprobable. The negative endowment shock satisfies the following two inequalities:

$$N > \bar{G}$$

$$N \leq \frac{L}{2}.$$  

The random endowment shock generates noisy bond demand, as one observes in reality, and is essential if informed speculation is to be profitable. The first inequality above implies the government cannot fill all intrinsic demand for safe assets. The second inequality implies the corporation has the ability to meet the intrinsic demand for safe assets by issuing debt with face value $L$. The specific form of the second inequality plays an additional technical role ensuring market-makers never face a call to take short positions, which is infeasible for them given limits on verifiability.

Each UI has linear utility over period 1 consumption and concave utility over period 2 consumption. We follow the tractable specification of risk-aversion of Dow (1998) in that the period 2 utility of each UI is piecewise linear, with a concave kink at a critical consumption level which is just equal to $Z$.\footnote{Assuming perfect correlation only serves to simplify the algebra.} The UI differ in the intensity of their aversion to consumption shortfalls. An uninformed

\footnote{Other smooth utility functions could be assumed, with more complex aggregate UI demands.}
investor with preference parameter \( \theta \) has vNM utility function:

\[
U(C_1, C_2; \theta) \equiv C_1 + \theta \min\{0, C_2 - Z\}.
\]  

(1)

Each UI is averse to period 2 consumption falling below the critical level \( Z \), creating an intrinsic demand for safe storage when confronted with a low terminal endowment. The intensity of aversion to low terminal consumption is captured by \( \theta \). The \( \theta \) parameters have support \( \Theta \equiv [1, \infty) \) with density \( f \) and cumulative density \( F \). The distribution is atomless, with \( f \) strictly positive and continuously differentiable. It is apparent that when faced with the prospect of a low future endowment, each UI would like to invest in a riskless security delivering \( N \) units in period 2, bringing \( C_2 \) up to the critical level \( Z \).

There is a risk-neutral speculator \( S \) with vNM utility function \( C_1 + C_2 \). Her period 1 endowment is \( Y_1^s \) and her period 2 endowment is normalized at zero without loss of generality. The speculator is unique amongst investors in that she observes a private signal \( s \in \{ s_L, s_H \} \) of the true asset type. The speculator chooses the precision of her signal, \( \sigma \), from a feasible set \( \Sigma = [\frac{1}{2}, 1] \). Signal precision is defined as follows:

\[
\sigma \equiv \Pr[T = H | s = s_H] = \Pr[T = L | s = s_L].
\]

The speculator must exert costly effort in order to generate an informative signal. The speculator’s effort cost function \( e \) is twice continuously differentiable, strictly increasing, and strictly convex, with

\[
\lim_{\sigma \downarrow \frac{1}{2}} e(\sigma) = 0 \quad \lim_{\sigma \uparrow \frac{1}{2}} e'(\sigma) = 0 \quad \lim_{\sigma \uparrow 1} e'(\sigma) = \infty.
\]

Since \( e' \) is strictly increasing, it has a well-defined inverse

\[
\Psi \equiv [e']^{-1}.
\]
In addition to the uninformed investors and speculator, there are a large number of risk-neutral market-makers (MM below). Each MM has vNM utility function $C_1 + C_2$. Their aggregate period 1 endowment is $Y_{1mm}$ and their period 2 endowment is normalized at zero without loss of generality.

Investors form beliefs regarding the true asset type based upon the manager’s choice of $D$. The speculator and UI anonymously submit simultaneous orders for government safe storage and corporate debt. Prior to submitting her orders, the speculator pays the effort cost $e(\sigma)$ and observes her signal $s$ regarding the asset type $T$. Prior to placing orders, the UI privately observes their period 2 endowment. The corporate debt price is set as in Kyle (1985): the MM observe aggregate order flows and bid up the corporate debt price until it reaches its conditional expected payoff.

We solve for pure strategy perfect Bayesian equilibria (PBE). For each $D \in D$ that may be chosen by the manager, each investor must have an assessment $a : D \rightarrow [0, 1]$ regarding the probability that the true asset type is $H$. In response to debt face values chosen on the equilibrium path, investor beliefs must be consistent with Bayes’ rule. Actions of all agents must be sequentially optimal given their beliefs and the actions of the other agents.

Our primary interest is in pinning down the socially optimal amount of government bonds in light of the effect on corporate leverage. The central mechanism is the interplay between government borrowing and asymmetric information in the corporate debt market. To see this, note that if the asset type were common knowledge, the manager would sell debt with face value equal to the true cash flow ($D = T$). The MM would then set the debt price $P = T$. Since the corporate debt would be priced at its true payoff, the speculator would have no incentive to exert costly effort. On the other hand, the UI with low terminal endowments would purchase $N/T$ units of corporate debt, ensuring they achieve their critical consumption level $Z$. That is, under symmetric information regarding firm type, the manager would raise $T$ units of outside funding and first-best sharing of risks would be achieved across investors. Thus, safe government bonds would be superfluous under common knowledge of the asset type.
II. The Corporate Debt Market

We solve via backward induction. Consider first the corporate debt price \((P)\). We know:

\[ D \leq L \Rightarrow P = D. \tag{2} \]

If the corporation issues riskless debt, the speculator has no incentive to acquire a costly signal. If say \(D = L\), UI hit with a negative endowment shock can submit orders for \(N/L\) units of the debt, just enough to achieve the target terminal period consumption level \(Z\). This corresponds to a perfect sharing of risks across investors.

Consider next the pricing of debt for higher face values. Here we must distinguish between two types of equilibria. In a separating equilibrium the choice of debt face value varies with the true type \(T\), fully revealing the manager’s private information. In such cases, the MM will set the debt price equal to its true type-contingent payoff. We have:

\[ \text{Separating Equilibrium} \Rightarrow P = \min\{D, T\}. \tag{3} \]

If \(D\) reveals \(T\), the speculator has no incentive to exert costly effort. Further, if the asset type is revealed, each UI hit with a negative income shock can submit an order for \(N = \min\{D, T\}\) units of the debt, just enough to achieve the target consumption \(Z\). This corresponds to perfect risk sharing across investors.

Consider next price determination in a pooling equilibrium in which \(D \in (L, H]\) is invariant to \(T\). Here the debt price set by the MM will depend upon order flow. Consider then the aggregate demand of the UI. The UI enter the debt market holding their prior belief. If the UI have a high period 2 endowment, they have no motive to buy any debt. Conversely, if the UI anticipate the low period 2 endowment, they may be willing to buy debt. Let \(x^*(\theta, D, G, \sigma)\) denote the optimal \(\theta\)-contingent demand for an UI in the event of a low period 2 endowment. Aggregate UI demand in the event of a negative period 2 endowment shock is:

\[ X_U(D, G, \sigma) = \int_{\theta}^{\infty} x^*(\theta, D, G, \sigma) f(\theta) d\theta. \tag{4} \]
We will return to the determination of the UI demand function \( x^* \) below.

The speculator relies on the trading of the UI to provide camouflage. In fact, as shown below, her trading gain is increasing in UI demand for corporate debt. Since the UI prefer safe stores of value, they would put all their savings in the government storage if this were feasible. To limit their access to such safe assets, the speculator will submit an infinite order for government bonds, causing them to be allocated on a pro rata basis.\(^5\) It follows that in the event of a low period 2 endowment, the UI will have a residual demand for safe storage equal to \( N - G \). The critical role played by the government bond offering is to alter the amount of residual UI storage demand migrating to the corporate debt market.

Consider next the speculator’s optimal order in the corporate debt market. Since she cannot short-sell, her optimal strategy is to place a buy order for the debt if and only if she receives the positive signal \( s_H \). As in Maug (1998), if the speculator is to make positive expected trading gains, she must choose her order size such that the MM cannot infer her signal. This can only be achieved by choosing an order size such that MM cannot distinguish between no UI endowment shock combined with speculator buying versus UI endowment shock combined with speculator not buying. Thus, the speculator will submit an order for \( X_U \) units of corporate debt upon observing a positive signal and place zero order otherwise. Critically, the size of the speculator’s order size, and hence her effort incentive, is constrained by the equilibrium volume UI demand.

Table 1 depicts the order flow possibilities.

\(^5\) Alternatively, the speculator could submit a random order no less than \( G \). Both suffice to mask the UI endowment state as required to confound the MM.
Table 1: Aggregate Demand Outcomes

<table>
<thead>
<tr>
<th>Type</th>
<th>Speculator Signal</th>
<th>UI Period 2</th>
<th>Speculator Order</th>
<th>UI Order</th>
<th>Aggregate Order</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>$s_H$</td>
<td>Z - N</td>
<td>$X_U$</td>
<td>$X_U$</td>
<td>$2X_U$</td>
<td>$\frac{\sigma}{4}$</td>
</tr>
<tr>
<td>H</td>
<td>$s_H$</td>
<td>Z</td>
<td>$X_U$</td>
<td>0</td>
<td>$X_U$</td>
<td>$\frac{\sigma}{4}$</td>
</tr>
<tr>
<td>H</td>
<td>$s_L$</td>
<td>Z - N</td>
<td>0</td>
<td>$X_U$</td>
<td>$X_U$</td>
<td>$\frac{1-\sigma}{4}$</td>
</tr>
<tr>
<td>H</td>
<td>$s_L$</td>
<td>Z</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{1-\sigma}{4}$</td>
</tr>
<tr>
<td>L</td>
<td>$s_L$</td>
<td>Z - N</td>
<td>0</td>
<td>$X_U$</td>
<td>$X_U$</td>
<td>$\frac{\sigma}{4}$</td>
</tr>
<tr>
<td>L</td>
<td>$s_L$</td>
<td>Z</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{\sigma}{4}$</td>
</tr>
<tr>
<td>L</td>
<td>$s_H$</td>
<td>Z - N</td>
<td>$X_U$</td>
<td>$X_U$</td>
<td>$2X_U$</td>
<td>$\frac{1-\sigma}{4}$</td>
</tr>
<tr>
<td>L</td>
<td>$s_H$</td>
<td>Z</td>
<td>$X_U$</td>
<td>0</td>
<td>$X_U$</td>
<td>$\frac{1-\sigma}{4}$</td>
</tr>
</tbody>
</table>

The MM set the debt price based upon aggregate demand ($X_A$) as follows:

$$P(X_A) = D \Pr(T = H|X_A) + L \Pr(T = L|X_A) \quad \forall \quad X_A \in \{0, X_U, 2X_U\}. \quad (5)$$

As shown in Table 1, the MM will face one of three order flows. The highest and lowest order flows fully reveal the speculator signal, while the intermediate order flow leaves the MM confounded as to the signal and the true asset type. Using Bayes’ rule the MM form beliefs as follows:

$$\Pr[T = H|X_A = 2X_U] = \sigma \quad (6)$$
$$\Pr[T = H|X_A = X_U] = \frac{1}{2}$$
$$\Pr[T = H|X_A = 0] = 1 - \sigma.$$ 

It follows from equations (5) and (6) that the debt price is increasing in aggregate demand. Further, the responsiveness of the debt price to aggregate demand is increasing in the speculator’s signal precision. Intuitively, the MM revise beliefs more aggressively in response to order flow if the speculator has more precise information.

Continuing the backward induction, we must pin down Nash configurations for the pair $(\sigma, X_U)$, the speculator signal precision and UI demand. Consider first the speculator’s signal precision. The
speculator chooses some $\tilde{\sigma} \in \Sigma$ taking as given the UI demand factor $X_U$ and the pricing rule used by the MM, with the pricing rule itself being predicated upon the speculator signal precision postulated by the MM, call it $\sigma$. Using Table 1, the speculator’s expected trading gain is:

$$\Pi(\tilde{\sigma}, \sigma, X_U) = \left[ \frac{\tilde{\sigma}}{4} [D - P(2X_U)] + \frac{\tilde{\sigma}}{4} [D - P(X_U)] \right] \times X_U. \tag{7}$$

An incentive compatible speculator signal precision, call it $\sigma_{ic}$, equates the marginal change in expected trading gains resulting from a change in $\tilde{\sigma}$ with the marginal effort cost. This implies:

$$e'(\sigma_{ic}) = \Pi_1(\tilde{\sigma}_{ic}, \sigma, X_U) = \frac{1}{2} (D - L)X_U. \tag{8}$$

Thus:

$$\sigma_{ic}(X_U) = \Psi \left[ (D - L)X_U/2 \right]. \tag{9}$$

Since $\Psi$ is increasing, it follows from the preceding equation that, holding all else constant, speculator effort is increasing in the uninformed corporate debt demand $X_U$. Intuitively, higher UI demand allows the speculator to place larger orders and to make higher trading gains, strengthening her effort incentive.

We now return to determining the debt demands of the individual UI in the event of a low future endowment. In order to formulate their optimal debt demand, the UI must form an expectation of the equilibrium debt price, conditional on being hit with a negative endowment shock. From Table 1 it follows:

$$E[P|Y_2 = Z - N] = \frac{1}{2} \left[ D + L + (D - L) \left( \sigma - \frac{1}{2} \right) \right]. \tag{10}$$

Equation (10) shows the UI face adverse selection in that they expect to pay a price in excess of the unconditional expected debt payoff of $(D + L)/2$. Intuitively, a negative endowment shock implies higher expected order flow. And in the presence of an informed speculator, the MM will respond to high order flow by setting a higher debt price. In fact, the price set by the MM is more sensitive to order flow the higher the speculator’s signal precision. Thus, the intensity of the adverse selection problem, as perceived by the UI, is increasing in the speculator’s signal precision.
With their conditional expectation of the debt price determined, we can now pin down the optimal debt demand for those UI with low period 2 endowments. The optimal corporate debt demand maximizes expected period 2 utility less the expected debt price. The program is:

\[
\max_{x \geq 0} \quad \frac{1}{2} \theta \min\{0, -N + G + xL\} + \frac{1}{2} \theta \min\{0, -N + G + xD\} - xE[P|Y_2 = Z - N].
\] (11)

Solving the preceding program, we obtain the following characterization of the optimal UI portfolios:

\[
\theta \in [1, \theta_1) \Rightarrow x^*(\theta) = 0
\]
\[
\theta \in [\theta_1, \theta_2) \Rightarrow x^*(\theta) = \frac{N - G}{D}
\]
\[
\theta \geq \theta_2 \Rightarrow x^*(\theta) = \frac{N - G}{L}
\]

where

\[
\theta_1(\sigma, D) \equiv 1 + \left(\sigma - \frac{1}{2}\right) \left(\frac{D - L}{D + L}\right)
\]
\[
\theta_2(\sigma, D) \equiv 1 + \frac{D}{L} + \left(\sigma - \frac{1}{2}\right) \left(\frac{D - L}{L}\right).
\] (13)

From the preceding equations we see that if \(\theta\) is sufficiently low, adverse selection dominates the storage motive and the respective investor boycotts the corporate debt market. Those UI with intermediate values of \(\theta\) partially insure in the sense of buying just enough units of corporate debt to ensure they will achieve the target consumption \(Z\) if \(T = H\), implying consumption falls short of \(Z\) if \(T = L\). Finally, if \(\theta\) is sufficiently high, the investor completely insures in the sense of purchasing enough units of corporate debt to ensure he achieves his target consumption level even if \(T = L\), implying his consumption actually overshoots \(Z\) if \(T = H\).

Integrating over the individual debt demands, we obtain the following expression for aggregate UI demand:

\[
X_U(D, G, N, \sigma) = (N - G) \left(\frac{1}{L} \left[1 - F(\theta_2(\sigma, D))\right] + \frac{1}{D} \left[F(\theta_2(\sigma, D)) - F(\theta_1(\sigma, D))\right]\right).
\] (14)

There are two points worth noting regarding the aggregate UI demand schedule. First, the UI demand cutoffs \(\theta_1\) and \(\theta_2\) are both increasing in \(\sigma\), implying aggregate UI demand is decreasing in
the speculator signal precision posited by the UI. Second, aggregate UI demand increases linearly with the size of the residual safe storage demand $N - G$. That is, higher government bond supply decreases uninformed demand for corporate debt. This is the central lever at the government’s disposal in our model.

For each given debt face value $D \in (L, H]$ at which we wish to consider the possibility of a pooling equilibrium of the full game, we can now determine the continuation equilibrium values for uninformed demand and speculator signal precision. Such continuation equilibrium pairs will be denoted $(X_{eq}^U, \sigma_{eq})$, and are found as solutions to equations (9) and (14). Substituting the uninformed demand equation (14) into the speculator’s incentive compatibility condition (9), equilibrium is defined implicitly by the following equation:

$$
\Psi \left[ \frac{1}{2} (D - L) X_U (\sigma_{eq}) \right] - \sigma_{eq} = 0.
$$

(15)

The appendix shows the continuation equilibrium defined by the equation (15) is unique.

Figure 1 depicts the UI demand schedule and speculator signal precision in the event of pooling at risky debt, with equilibrium found at their intersection. The upward sloping line depicts the schedule $\sigma_{uc}$. From equation (9) it follows that this schedule is increasing in $X_U$. Intuitively, the speculator’s effort incentive is higher when there is a larger volume of uninformed trading providing camouflage. The downward sloping line depicts the schedule $X_U$. From equation (14) it follows this schedule is strictly decreasing in $\sigma$. Intuitively, the UI face a more severe adverse selection problem when the speculator has more precise information. They respond by cutting their corporate debt demand.

Applying the Implicit Function Theorem to equation (15) we find:

$$
\frac{\partial \sigma_{eq}}{\partial G} = \frac{\frac{1}{2} \Psi'(\cdot)(D - L) \frac{\partial X_U}{\partial G}}{1 - \frac{1}{2} \Psi'(\cdot)(D - L) \frac{\partial X_U}{\partial \sigma}} < 0
$$

(16)

$$
\frac{\partial X_U}{\partial G} = -\frac{1}{L} \left[ 1 - F(\theta_2(\sigma, D)) \right] - \frac{1}{D} \left[ F(\theta_2(\sigma, D)) - F(\theta_1(\sigma, D)) \right] < 0.
$$

The preceding equations show that the equilibrium level of speculator effort is decreasing in $G$. Intuitively, an increase in $G$ reduces UI demand for corporate debt. And it is this demand that
provides the camouflage and subsidy to informed speculation. As shown in Figure 1, an increase in \( G \) manifests itself as a shift downward of the schedule \( X_U \), implying lower equilibrium speculator signal precision.

Again applying the Implicit Function Theorem to equation (15) it can be verified that an increase in the size of the negative endowment shock \( N \) would increase speculator signal precision. In particular:

\[
\frac{\partial \sigma_{eq}}{\partial N} = -\frac{\partial \sigma_{eq}}{\partial G} > 0. \tag{17}
\]

Similarly, a first-order stochastic dominant shift in the \( \theta \) parameters would also increase speculator effort, since this too increases uninformed demand at each given level of speculator effort. Both effects arise from the fact that UI trading provides the subsidy to informed speculation.

The following lemma summarizes the key result from this section, showing the role of government debt in determining incentives for speculative information production.

**Lemma 1** If there is pooling at risky corporate debt, speculator effort in the continuation equilibrium is increasing in the intrinsic demand for safe assets (\( N \)) and decreasing in the quantity of riskless government debt (\( G \)).

### III. The Corporate Leverage Choice

This section analyzes equilibrium leverage choice in light of the debt pricing rules described in the previous section. Again, the equilibrium concept is perfect Bayesian equilibrium. We consider the Intuitive Criterion refinement as an extension.

The following lemma is useful in characterizing potential equilibria.

**Lemma 2** The set of equilibria includes all debt face value configurations such that a manager owning a low value asset-in-place attains at least \( V_L^{\text{min}} \equiv QL \), while a manager owning a high value asset-in-place attains at least \( V_H^{\text{min}} \equiv QL + H - L \).
Proof. To see the necessity of each type attaining the posited minimum utility, note that regardless of what beliefs investors might form in response, the manager can always attain the stated minimum by issuing debt with face value $L$. For sufficiency, consider a posited equilibrium in which each type makes at least the posited minimum. This equilibrium can be sustained if investors impute a deviation to the manager holding a low value asset. Given such beliefs, any deviation will yield the deviating manager a payoff no greater than $V_{T}^{\min}$.

Based on the preceding lemma, we obtain the following proposition.

**Proposition 1** There is no equilibrium in which the manager chooses debt with face value less than $L$. There is a pooling equilibrium in which, regardless of the true asset type, the manager chooses riskless debt with face value $L$. The set of separating equilibria are those in which the owner of a high value asset issues debt with face value $L$ while the owner of a low value asset issues debt with face value in $(L, H]$.

**Proof.** The first statement in the proposition follows from Lemma 2 and the fact that debt with face value less than $L$ provides the issuer with less than $V_{T}^{\min}$. The second statement in the proposition follows from Lemma 2 and the fact that debt with face value $L$ generates issuer utility equal to $V_{T}^{\min}$. The last statement in the proposition follows from the fact that the low type cannot make more than $QL$ in a separating equilibrium. So he must make $QL$ in any separating equilibrium. For this reason, the high type cannot sell debt at price exceeding $L$ in a separating equilibrium. So the high type must market debt with face value $L$ in any separating equilibrium. The low type can then issue debt with face value in $(L, H]$ in any separating equilibrium. Each type then attains his respective minimum utility.

Consider finally whether there exist pooling equilibria in which the issuer, regardless of the true type, chooses some $D > L$. From Lemma 2 we know that any viable pooling equilibrium has the property that the issuer attains at least his type-contingent minimum utility $V_{T}^{\min}$. With this in mind, we use Table 1 to compute the type-contingent expected utility of the issuer in the event of pooling at some face value $D \in (L, H]$. The expected utility ($V_{T}$) of the issuer in the event of
pooling is equal to $Q$ times the type-conditional expectation of the debt price plus the terminal period dividend. We have:

$$
V_H(D) = Q[I(\sigma^{eq})D + (1 - I(\sigma^{eq}))L] + H - D = V_H^{\min} + [QI(\sigma^{eq}) - 1][D - L] \quad (18)
$$

$$
V_L(D) = Q[I(\sigma^{eq})L + (1 - I(\sigma^{eq}))D] = V_L^{\min} + Q(D - L)[1 - I(\sigma^{eq})]
$$

$$
I(\sigma) = \frac{3}{4} + \sigma^2 - \sigma.
$$

The endogenous variable $I$ plays an important role in the model, capturing the informational efficiency of prices. For example, if $I = 1/2$ the debt price is completely uninformative, as would be the case in a standard signaling model sans informed trading. In fact, the function $I$ is increasing in $\sigma$ with

$$
I \left( \frac{1}{2} \right) = \frac{1}{2}, \quad I(1) = \frac{3}{4}.
$$

As shown in equation (18), the high (low) type benefits (suffers) from an increase in $I$. Further from this same equation it is apparent that the low type is always better off under pooling at risky debt than in the separating equilibrium (or pooling at riskless debt with face value $L$). In contrast, the high type is only better off if $QI \geq 1$. In fact, from Lemma 2 and equation (18) we have the following proposition characterizing the set of potential pooling equilibria with risky debt.

**Proposition 2** If the funding value $Q \leq 4/3$, there is no pooling equilibrium in which the manager, regardless of the true asset type, chooses a debt face value greater than $L$. For $Q \in (4/3, 2)$, pooling at risky debt can be sustained if and only if the residual storage demand $N - G$ is sufficiently high to ensure:

$$
\sigma^{eq} \geq \frac{1}{2} + \sqrt{1/Q - 1/2}.
$$

If $Q \geq 2$, pooling can be sustained at any face value in $(L, H]$.

The intuition for Proposition 2 is as follows. From Lemma 2 we know a pooling equilibrium can be sustained if and only if, regardless of the true asset type, the issuer is better off pooling than
he would be under the issuance of riskless debt with face value $L$. The owner of a low quality asset is always better off in the event of pooling with $D > L$, since he benefits from overpricing of his debt. Whether the owner of a high value asset is better or worse off depends on the magnitude of competing effects. On one hand, by raising the debt face value, the issuer raises more funding in expectation, which is valuable given $Q > 1$. On the other hand, the owner of the high value asset knows the market will underprice his debt. This latter effect is attenuated by high speculator effort, which serves to drive price closer to fundamentals. Thus, the critical signal precision threshold for sustaining a pooling equilibrium is decreasing in $Q$.

Given our interest in social welfare, it is worthwhile to contrast the risky debt pooling equilibria described in Proposition 2 with the other equilibria. The attractive feature of pooling at risky debt is higher investment. The unattractive feature is that risk-sharing is distorted as the UI distort the portfolios as shown in equation (13).

In light of these welfare tradeoffs, a fundamental question to be addressed is whether one should expect corporations to offer riskless debt to investors in response to a large intrinsic demand ($N$) for safe assets. Here Proposition 2 offers a stark negative result showing that, paradoxically, the corporation may be induced to issue risky rather than riskless debt precisely when investor demand for, and the welfare gains from, safe storage is strongest. Intuitively, as shown in Section II, an increase in the intrinsic demand for safe storage ($N$) has the effect of stimulating UI demand for risky debt. And with greater uninformed demand for risky debt, an informed speculator has more camouflage for her trades in the high-yield debt market. Anticipating this ability to place larger orders, she will exert more effort. Prices will be driven closer to fundamentals (higher $I$) and pooling is more readily sustained ($QI \geq 1$). Further, although we focus on comparative statics with respect to $N$, the same chain of reasoning shows that a first-order stochastic dominant shift in the risk-aversion parameters ($\theta$) would increase uninformed demand for risky debt, encouraging pooling at risky debt, precisely when the social welfare benefits of riskless corporate debt are highest.

Finally, although we do not assume this to be this case, it can be argued that a risky debt
pooling equilibrium is “focal” relative to the separating equilibria or pooling at safe debt. After all, it follows from Lemma 2 that when such an equilibrium exists, the issuer is better off than in the separating equilibria or pooling at riskless debt. And this is true regardless of the issuer’s true type.

IV. Optimal Government Debt in Economy Without Informed Speculation

As a precursor to our analysis of the optimal quantity of government bonds in an economy with speculation, this section contains a brief analysis of the potential role for government bonds to increase social welfare in an economy where there is no possibility of informed speculation. This analysis will help clarify the causal mechanisms operative in the full model, as well as helping to distinguish our theory from existing theories of liquidity supply, e.g. Holmström and Tirole (1998).

To set a benchmark, consider first social welfare if the type of the asset-in-place was common knowledge. Since investment has positive NPV, the manager would raise the maximum funding possible by marketing debt with face value $T$, converting each unit of funds raised into $Q$ units of private benefits. The speculator and market-makers would have total consumption equal to their endowments. Each UI facing a low period 2 endowment would invest $N$ in period 1 in order to receive $N$ in period 2, insuring against any consumption shortfall. Thus, with symmetric information, social welfare is:

$$W^* = \frac{1}{2} (H + L) Q + Y_1^s + Y_{1mm}^s + Y_{1ui}^s - \frac{1}{2} N.$$  \hfill (20)

Consider next an economy in which the speculator can only receive an uninformative signal. The analysis of Sections II and III are still applicable. To cover the present case we can simply set $\sigma$ equal to 1/2. Doing so, consider first the portfolio decisions of the UI as derived in equation (13). With $\sigma = 1/2$ we obtain the following expressions for the optimal UI portfolios:

$$\theta \in [1, \theta_2^{\sigma=1/2}] \Rightarrow x^*(\theta) = \frac{N - G}{D}$$

$$\theta \geq \theta_2^{\sigma=1/2} \Rightarrow x^*(\theta) = \frac{N - G}{L}$$

$$\theta_2^{\sigma=1/2} = 1 + \frac{D}{L}.$$
Notice, even though there is no informed speculation, the UI still have distorted portfolios relative to the case of symmetric information between the firm and investors. Intuitively, even if the corporate debt has a price equal to its expected payoff, as is the case if \( \sigma = 1/2 \), the UI have distorted savings decisions since they do not know the true debt payoff. Consequently, the debt claim is not tailored to their precise need for a payoff equal to \( N \) with probability one.

However, the preceding equation does reveal that relative to the case where the speculator receives an informative signal, portfolio distortions are less severe if \( \sigma = 1/2 \). In particular, here all UI buy some form of insurance, whereas with \( \sigma > 1/2 \) some UI boycott the corporate debt market altogether. In the present case, some UI underinsure and will only cover the negative endowment shock if the firm is a high type. Others overinsure, and will receive a payoff in excess of the negative shock if the firm is a high type. Both distortions create deadweight loss.

Consider next equilibrium corporate leverage in the absence of informed speculation. From Proposition 2 it follows that the equilibrium set then has a simple characterization. If \( Q < \frac{1}{2} \), there is no possibility of pooling at any \( D > L \). If \( Q \geq \frac{1}{2} \), pooling can be sustained at any face value \( D \in (L, H] \). Notice, in contrast to the full model with informed speculation, here equilibrium corporate leverage is independent of the level of uninformed storage demand as captured by the parameter \( N \).

If \( Q < \frac{1}{2} \) there is no risk-sharing distortion as the corporate sector supplies the UI with the riskless debt they need to perfectly cover any negative endowment shock. However, there is a deadweight loss relative to symmetric information in the form of underinvestment. With this in mind consider the optimal quantity of government debt. It is apparent that changes in government debt here have no effect on risk-bearing by the UI, nor any effect on equilibrium investment. Thus, any \( G \in [0, \overline{G}] \) is an optimal government debt supply if \( Q < \frac{1}{2} \) and \( \sigma = 1/2 \).

Consider next the optimal supply of government debt when growth options are more profitable, with \( Q \geq \frac{1}{2} \). As shown in Proposition 1, it is possible that equilibrium will entail pooling at riskless debt with face value \( D = L \) or that a separating equilibrium occurs. In either of these two cases,
government debt supply is irrelevant since the UI achieve perfect insurance against endowment shocks, while corporate investment would be left unchanged.

From Proposition 2 we know that with \( Q \geq 2 \) it is also possible for firms to pool at some \( D \in (L, H] \). The question is whether and how the supply of government bonds can affect social welfare under such an equilibrium. Relative to social welfare under symmetric information (\( W^* \)), we have the following deadweight loss in the event of pooling:

\[
DWL = \frac{1}{2} (H - D)Q + \frac{1}{2} (N - G) \left[ \frac{1}{2} \left( \frac{D - L}{L} \right) [1 - F(\theta_2)] \right. \\
\left. + \frac{1}{2} \left( \frac{D - L}{L} \right) \int_1^{\varrho^* = 1/2} (\theta - 1) f(d\theta) \right].
\]  \( (22) \)

In the preceding equation we see that deadweight loss arises from underinvestment and distorted risk bearing. As shown in equation (21), in the event of pooling at risky debt, some UI underinsure and others overinsure. Here by supplying riskless debt the government directly increases social welfare by allowing the UI to substitute public debt with a known payoff for corporate debt with an uncertain payoff. Effectively, the government debt reduces the residual storage need of the UI from \( N \) to \( N - G \), with the preceding equation revealing a direct welfare benefit. Since social welfare is increasing in \( G \), here it is optimal for the government to offer the maximum feasible amount of safe storage, implying \( G^* = \overline{G} \).

Based on the preceding analysis, we have the following proposition.

**Proposition 3** If informed speculation is impossible then the government can maximize total social welfare by offering \( G^* \in [0, \overline{G}] \) if \( Q < 2 \) and \( G^* = \overline{G} \) if \( Q \geq 2 \).

Before closing this section it is worth emphasizing that in the absence of informed speculation, government debt has no effect on the equilibrium choice of corporate leverage. That is, there is no crowding-out of risky debt, as is the case when informed speculation is possible. Consequently, the role of government debt here is a direct one, allowing the UI to substitute out of corporate debt and into government debt dollar for dollar. There is no riskless debt multiplier effect.
V. Optimal Government Debt in Economy With Informed Speculation

This section analyses the role that variation in the quantity of safe government debt can play in increasing social welfare. Following the same steps as in Section IV, we first consider equilibrium in the absence of government debt. We then consider optimal government debt in light of the anticipated equilibrium sans intervention. Contrasting equilibria in which informed speculation does and does not occur, Pareto improvements are impossible since uninformed investors are worse off when informed speculation occurs while the speculator is better off. Therefore, we take the perspective of a utilitarian social planner placing equal weight on all agents.

A. Equilibrium With Zero Government Debt

This section considers the prospective equilibrium the government anticipates should it not issue debt. Proposition 1 shows it is always possible to sustain pooling at riskless debt with face value $L$ as well as separating equilibria in which the type is revealed. For brevity, we label the riskless debt pooling equilibrium LPOOL, while a separating equilibrium is labeled SEP. If either equilibrium will be implemented with or without government debt, then the choice of $G$ has no effect on total social welfare. After all, risk-sharing is already first-best, and the government debt would have no effect on corporate investment.

In order for government debt to alter social welfare, it must be that the private sector would implement pooling at risky debt in its absence. Recall, Lemma 2 and equation (18) show $QI \geq 1$ is necessary for pooling at risky debt. With this in mind, we consider $G = 0$ and let $N_{RP}$ denote the critical value of the parameter $N$ at which it is possible to sustain pooling at risky debt with face value $D > L$. We call such a risky debt pooling equilibrium RPOOL. By definition:

$$QI[(\sigma(N_{RP})] = 1 \Rightarrow N_{RP}'(Q) = -\frac{Q^{-2}}{\partial Q/ \partial \sigma} < 0. \quad (23)$$

Apparently, the function $N_{RP}$ is decreasing in $Q$. Intuitively, the high type finds pooling at risky debt more attractive if project NPV is increased. Restoring high type indifference between RPOOL and LPOOL (or SEP) requires lowering $N$, with a concomitant reduction in speculator effort. It
follows from equation (19) that:

$$\lim_{Q \to \frac{4}{3}} N_{RP}(Q) = \infty$$  \hspace{1cm} (24)$$
$$N_{RP}(2) = 0.$$  

That is, sustaining RPOOL would require an infinite amount of uninformed trading volume as \( Q \) converges to \( 4/3 \), and requires zero uninformed trading volume if \( Q \geq 2 \). The downward sloping schedule in Figure 2 depicts \( N_{RP} \). It is only possible to sustain RPOOL at points on or to the right of this schedule.

B. Optimal Government Debt

This subsection pins down the optimal government debt supply on the various regions depicted in Figure 2. If LPOOL or SEP will be implemented with or without government debt, then the choice of \( G \) has no effect on total social welfare and no further analysis is necessary. So, we shall confine attention to the interesting case where RPOOL would be implemented sans intervention. This is only possible on Regions II-VI.

We pin down the optimal choice of \( G \) in response to RPOOL in two steps. We first show that offering some government debt, while still sustaining RPOOL increases social welfare. We then consider whether pruning RPOOL is both feasible and optimal. To begin with the first assertion, note that if the government offers \( G \leq N - N_{RP}(Q) \), the condition \( QI \geq 1 \) is satisfied and so RPOOL remains an equilibrium. It is readily verified social welfare is increasing in \( G \) given that RPOOL is still implemented. To see this, note that in RPOOL we have the following deadweight loss relative to symmetric information \((W^*)\):

$$DWL^{RPOOL} = \frac{1}{2} (H - D)(Q - 1) + e(\sigma^{eq}) + \frac{1}{2} (N - G) \left[ \int_{\theta_1}^{\theta_2} (\theta - 1) f(\theta) d\theta + \frac{1}{2} \left( \frac{D-L}{D} \right) \left[ 1 - F(\theta_2) \right] \right] + \frac{1}{2} \left( \frac{D-L}{D} \right) \int_{\theta_1}^{\theta_2} (\theta - 1) f(\theta) d\theta$$  \hspace{1cm} (25)$$

The first term in equation (25) captures underinvestment relative to symmetric information re-
garding type. The second term captures speculator effort costs which are zero under symmetric information. The third term is based upon the UI portfolios derived in equation (12). The first term in the large brackets captures the fact that those UI with \( \theta \in [1, \theta_1] \) bypass the purchase of corporate debt altogether, despite the fact that there would be a social gain of \( \theta - 1 \) per unit of incremental safe assets purchased by these investors. The second term in the large brackets represents the social cost associated with overinsurance by extremely risk-averse UI. The final term in the large brackets reflects the fact that adverse selection induces UI with \( \theta \in (\theta_1, \theta_2) \) to only partially insure against costly consumption shortfalls.

Differentiating equation (25) with respect to \( G \) we obtain:

\[
\frac{dDWL^{RPOOL}}{dG} = e'(\sigma) \frac{\partial \sigma}{\partial G} - \frac{1}{2} \left[ \int_{\theta_1}^{\theta_2} (\theta - 1)f(d\theta) + \frac{1}{2} \left( \frac{D-L}{L^2} \right) [1 - F(\theta_2)] \right] \\
+ \frac{1}{2} \left( \frac{D-L}{D} \right) \int_{\theta_1}^{\theta_2} (\theta - 1)f(d\theta)
\]

From the fact that speculator effort is decreasing in \( G \) (equation (16)) and the fact that the two UI portfolio cutoffs are increasing in \( \sigma \) (equation (13)) it follows that social welfare in RPOOL is increasing in \( G \). The intuition is as follows. First, the provision of safe public debt serves to reduce speculator effort costs. Second, the induced reduction in speculator effort reduces the portfolio distortions of marginal uninformed investors. This effect is captured by the final term in the preceding equation. Finally, with additional safe assets, inframarginal uninformed investors substitute out of corporate bonds which represent an imperfect savings vehicle. This effect is captured by the term with large square brackets in the preceding equation.

We have the following lemma which shows that if RPOOL is to be implemented, the government will find it optimal to issue the maximum debt consistent with RPOOL remaining an equilibrium.

**Lemma 3** If the corporation pools at risky debt with face value \( D > L \), social welfare is increasing
The optimal government debt supply for implementing such an equilibrium is $\tilde{G}$, with

$$\tilde{G} \equiv G \text{ for } Q \geq 2$$

and

$$\tilde{G} \equiv \min\{G, N - N_{RP}(Q)\} \text{ for } Q \in \left(\frac{4}{3}, 2\right).$$

The government has the ability to prune RPOOL if $G > N - N_{RP}(Q)$. We now evaluate whether it is socially optimal to prune RPOOL in order to bring about SEP or LPOOL. To begin, note that we have the following reduction in social welfare relative to the symmetric information case ($W^*$) in SEP and LPOOL:

$$DWL^{SEP} = DWL^{LPOOL} = \frac{1}{2}(Q - 1)(H - L).$$  \hspace{1cm} (27)

As reflected in the preceding equation, the only distortion in SEP and LPOOL is underinvestment. These equilibria are socially attractive in that the corporation effectively supplies the UI with riskless debt, so there are no savings distortions and no costly speculator effort.

To pin down the optimal government debt supply, it will be convenient to compute the critical value of $Q$, call it $Q_{PUB}$, at which the social planner would be indifferent between LPOOL (or SEP) and RPOOL, with the latter evaluated at $G = 0$. Equating the respective deadweight losses we have:

$$Q_{PUB} = 1 + \frac{\epsilon[\sigma(N)] + \frac{1}{2}N \left[ \int_{1}^{\theta_1} (\theta - 1)f(d\theta) + \frac{1}{2} \left( \frac{D-L}{L} \right) [1 - F(\theta_2)] + \frac{1}{2} \left( \frac{D-L}{D} \right) \int_{\theta_1}^{\theta_2} (\theta - 1)f(d\theta) \right]}{\frac{1}{2}(D - L)}. \hspace{1cm} (28)$$

Apparently, $Q_{PUB}$ is increasing in $N$. Intuitively, an increase in the size of the endowment shock raises the speculator effort and risk sharing costs arising from pooling at risky debt. Maintaining social planner indifference across the equilibria then necessitates a compensating increase in $Q$, which raises deadweight costs of underinvestment, with underinvestment higher at LPOOL than RPOOL. The schedule $Q_{PUB}$ is shown in Figure 2, with the planner preferring to LPOOL to RPOOL at points to the right of the schedule.
Turning to Figure 2, a positive amount of government debt is isomorphic to reducing $N$ to $N' = N - G$. With this in mind, consider first starting an arbitrary point inside Region II. Here risk sharing concerns dominate investment given that $N$ is high relative to $Q$. Consequently, the planner prefers LPOOL to RPOOL. Here the optimal government debt supply is $G$. To see this, suppose first $G \leq N - N_{RP}$, so that pruning being infeasible. In this case, supplying maximal debt is still optimal since the welfare loss in RPOOL is decreasing in $G$. If instead $G > N - N_{RP}$, it is optimal to prune RPOOL by offering maximal debt. To see this, note that social welfare is higher under LPOOL than RPOOL as one crosses the pruning boundary from Region II to Region I.

Consider next an arbitrary point inside Region III. Since $Q \geq 2$, RPOOL remains an equilibrium regardless of $G$. Further, here the planner prefers RPOOL to LPOOL given that investment has such high NPV. Consequently, provided that RPOOL would still be played, since it is still in the equilibrium set, $G$ is optimal. Here the goal is to implement RPOOL with minimal deadweight loss. Less formally, one may view this strategy as risky to the government since it makes RPOOL less attractive to the high type, potentially bringing about the socially inferior equilibrium LPOOL. Thus, we may properly think of $G$ as being a socially optimal debt supply on Region III under the assumption that RPOOL will continue to be played given that it remains an element of the equilibrium set.

Consider next starting at an arbitrary point inside Region IV. Here too $G$ is optimal. To see this, note that initially the planner prefers LPOOL to RPOOL. By supplying small amounts of debt the government makes RPOOL more efficient. By supplying still more debt the government increases the efficiency of RPOOL to the point that RPOOL becomes preferred to LPOOL. This is seen in Figure IV as a horizontal move from Region IV into Region III. And once inside Region III, the argument in the preceding paragraph applies.

Consider next an arbitrary point inside Region V. Notice that even with $G = 0$ the planner prefers RPOOL to LPOOL. By supplying positive amounts of government debt the planner increases social welfare in RPOOL still further. Thus, the optimal policy here is to issue issue maximal
debt consistent with RPOOL being played. For example, applying the same reasoning as above
\( G = \min\{\overline{G}, N - N_{RP}(Q)\} \) is optimal under the conjecture that RPOOL will still be implemented
given that it remains an element of the equilibrium set.

Consider finally starting at an arbitrary point inside Region VI. Here too \( G = \min\{\overline{G}, N - N_{RP}(Q)\} \) is an optimal government debt supply. To see this, note that the planner prefers LPOOL
to RPOOL at the initial point. By supplying small amounts of debt the government makes RPOOL
more efficient. By supplying still more debt the government increases the efficiency of RPOOL to
the point that RPOOL becomes preferred to LPOOL. This is seen as a horizontal move from Region
VI into Region V. And once inside Region V, the argument made in the preceding paragraph applies.

Based on the preceding discussion we have the following characterization of the optimal government bond offering.

**Proposition 4** The choice of \( G \) is irrelevant if a separating equilibrium or pooling at riskless debt
will occur with or without government debt. If \( Q \geq 2 \) and pooling at risky debt will occur with or
without government debt, the optimal government bond offering is \( \overline{G} \). If \( Q \) is sufficiently low relative
to \( N \), it is optimal for the government to attempt pruning of risky debt equilibria by offering \( \overline{G} \).
For intermediate values of \( Q \) relative to \( N \), the government should offer the maximal feasible debt
consistent with risk debt pooling, with \( G = \min\{\overline{G}, N - N_{RP}(Q)\} \) being optimal.

Recall, Proposition 3 showed that if informed speculation is impossible, the optimal government
bond supply is invariant to \( N \). In contrast, Proposition 4 shows that if informed speculation is
possible, optimal government bond supply is increasing in the size of intrinsic storage demand, as
captured by the endowment shock parameter \( N \). The intuition is as follows. In cases where the
government would like to prune RPOOL, it must increase its bond offering in response to increases in
\( N \). After all, larger endowment shocks translate into higher uninformed demand for risky corporate
debt, making pooling at risky debt more attractive to the high type. In cases where the government
would like to implement RPOOL, while minimizing risk sharing distortions, if \( N \) increases, the
government can increase its bond offering but still preserve the issuer’s willingness to implement
RPOOL.

Recall, Proposition 3 showed that if informed speculation is impossible, the optimal government bond supply is increasing in $Q$. In contrast, Proposition 4 shows that if informed speculation is possible, the optimal government bond supply is non-monotonic in marginal $Q$. The intuition is as follows. For very low values of $Q$ relative to $N$ (Region II) the government would like to prune RPOOL, so maximal debt is optimal. For intermediate values of $Q$ (Regions V and VI), the government optimally supplies an intermediate amount of debt in order to avoid pruning the risky debt equilibrium. If $Q \geq 2$, the government again wants to avoid pruning the risky debt equilibrium but can do so while offering maximal debt. Here the high value of investment induces pooling at risky debt even with maximal government bond siphoning.

VI. Extensions

This section shows that one may relax various modeling assumptions with the central arguments still being applicable.

A. The Intuitive Criterion

To begin, we return to Section III and consider which of the perfect Bayesian equilibria satisfy the Intuitive Criterion of Cho and Kreps (1987), following with a discussion of implications for optimal government debt supply.

A posited equilibrium fails to satisfy the Intuitive Criterion if one of the issuer types would benefit from choosing a different face value, provided this were sufficient to convince investors of his true type, with the other type being strictly worse off choosing that same face value regardless of the beliefs formed in response. We have the following proposition.

**Proposition 5** All separating equilibria satisfy the Intuitive Criterion, as does pooling at riskless debt with face value $L$. If $Q \leq 3/2$, pooling at risky debt does not satisfy the Intuitive Criterion. For $Q > 3/2$, pooling at risky debt satisfies the Intuitive Criterion if and only if the residual storage
demand $N - G$ is sufficiently high to ensure:

$$\sigma^{eq} \geq \frac{1}{2} \left[ 1 + \sqrt{4Q/(2Q - 1) - 1} \right].$$

(29)

Proof. Given any PBE, the low type never gains from deviating if doing so identifies him, so attention can be confined to the high type’s incentive to deviate. Consider then any separating equilibrium or pooling at $D = L$. Only a deviation to a face value greater than $L$ can make the high type strictly better off, but then the low type would also gain from this deviation under some beliefs. So the Intuitive Criterion admits imputing such a deviation to the low type. Given such beliefs, there is no incentive for either type to deviate. Next, consider that pruning a pooling PBE featuring face value $D > L$ via the Intuitive Criterion demands finding a deviation $D_0$ such that:

$$QD_0 + H - D_0 > V_H(D)$$

$$QD_0 < V_L(D).$$

The final inequality stated in the proposition implies no such $D_0$ exists.

The preceding proposition shows that only a subset of the risky debt pooling equilibria described in Proposition 2 satisfy the Intuitive Criterion. Apparently, satisfaction of the Intuitive Criterion demands a higher level of speculator effort and informational efficiency. It follows that under the Intuitive Criterion, the pruning of RPOOL via the supply of government debt, should that be desired, can be achieved with lower government debt capacity ($\mathcal{G}$) given that a smaller reduction in speculator signal precision suffices. On the other hand, maintaining RPOOL as an equilibrium, should that be desired, is only possible with a smaller government bond offering. This last point again highlights the delicate balancing inherent in a government strategy of siphoning off demand from the corporate debt market while still hoping to implement RPOOL, albeit in a more socially efficient form.

B. Multiple Corporate Debt Issuers

This subsection considers how the equilibrium set described in Section III is affected if there is more than one corporate debt issuer, following with implications for the optimal government debt
supply.

To limit the number of equilibrium permutations, attention is confined to symmetric equilibria with two i.i.d. corporations. We begin by noting that once again it is possible to sustain a given debt configuration as an equilibrium provided that regardless of type, a manager gets at least his payoff from issuing riskless debt with face value $L$ (Lemma 2). Further, the existence of another issuer has no effect on a manager’s equilibrium payoff in any of the separating equilibria described in Proposition 1. So one symmetric equilibrium entails both managers signaling their private information as described in Proposition 1. Similarly, the existence of another issuer has no effect on a manager’s equilibrium payoff in the event that he issues riskless debt with face value $L$ regardless of his type (pooling). Thus, there is another symmetric equilibrium in which both managers issue riskless debt with face value $L$ regardless of their true asset type.

Consider finally equilibria in which the manager chooses $D > L$ regardless of $T$ (Proposition 2). In this class of equilibria, the presence of another issuer does indeed have an effect on the manager’s payoff. To see this, note that, as shown in the appendix, the presence of a rival debt issuer causes an inward shift of the uninformed debt demand curve $X_U$ facing each issuer. We know an inward shift in the uninformed demand curve results in a reduction of speculator effort ($\sigma^{eq}$) in the continuation equilibrium. And with lower speculator effort, pooling at risky debt becomes less viable since the necessary condition (29) is less likely to be satisfied. Intuitively, the presence of another corporate debt issuer siphons off some of the uninformed debt demand, resulting in lower informational efficiency in the event of pooling. And with lower informational efficiency, a high type is less willing to pool given that he will face more severe underpricing of his debt. However, with sufficiently large endowment shocks ($N$), this equilibrium may still be implemented by the private sector even if there are multiple corporate issuers.

It follows that with multiple issuers the pruning of RPOOL via the supply of government debt, should that be desired, can be achieved with lower government debt capacity ($\overline{G}$) given that a smaller reduction in speculator signal precision suffices. On the other hand, maintaining RPOOL
as an equilibrium, should that be desired, is only possible with a smaller government bond offering.

C. Cournot Game

We have assumed that the government acts as the first-mover in debt markets. This can be viewed as approximating reality inasmuch as the future stock of government debt can be anticipated by firms given budget projections. For example, during the 1990s it was widely anticipated that the stock of U.S. government debt on offer would dwindle. Further, central banks often provide some form of forward guidance regarding their intent to increase or decrease the stock of government debt held on their balance sheets. Although not a full commitment, forward guidance can be viewed as offering an approximation of the benefits associated with first-mover status.

The preceding comments notwithstanding, it is worthwhile to consider which of our posited equilibria would survive if the government choice of $G$ was made simultaneously with the corporate choice of $D$. To this end, let $(G^*, D^*_L, D^*_H)$ denote the optimal government bond supply and induced corporate debt face values (across types) in the original Stackelberg game. Since $(D^*_L, D^*_H)$ were optimal in response to $G^*$ in the Stackelberg game, they are a Nash response to $G^*$ in the Cournot game. Thus, we need only assess whether $G^*$ is optimal for the government given that the corporate sector implements $(D^*_L, D^*_H)$.

If $(D^*_L, D^*_H) = (L, L)$, as in LPOOL, or $(D^*_L, D^*_H) = (D > L, L)$, as in SEP, social welfare is invariant to $G$ and so the original $G^*$ is Nash. It follows that the risky debt pruning policy, with $G^* = \overline{G}$, described as optimal on Region II of Figure 2, remains an optimal government policy in the Cournot game.

The final case to be considered are those Stackelberg equilibria where the government chooses $G^*$ with the intent to induce pooling at risky debt, $D^*_L = D^*_H > L$. Here we recall from Lemma 3 that in such an RPOOL, social welfare is increasing in $G$. Thus, the posited vector $(G^*, D^*_L, D^*_H)$ is only a Cournot equilibrium if $G^* = \overline{G}$. This condition is satisfied on Regions II, III and IV of Figure 2, as well as on Regions V and VI if $\overline{G} \leq N - N_{RP}(Q)$.

If instead $(Q, N)$ fall into Regions V or VI, but with $\overline{G} > N - N_{RP}(Q) = G^*$, then the posited
Stackelberg equilibrium is not a Cournot equilibrium. Intuitively, in such a Stackelberg equilibrium the government implements RPOOL by refraining from offering the maximal feasible debt. But in response to RPOOL, maximal debt is optimal. At a deeper level one can view such fine-tuning policies as suffering from a time-consistency problem. The government wants the corporation to choose a high debt face value, to achieve high expected investment. But once the corporation has done so, the government has the incentive to choose $G$ with the goal of minimizing the risk-sharing distortions arising when corporations pool at risky debt. Here one sees a role for reputational concerns.

Conclusions

In recent years there has been increasing concern over a potential scarcity of safe assets. Seemingly paradoxically, corporations have responded by increasing the supply of junk debt, consistent with a more general historical negative correlation between government bond yields and the high-yield share in total corporate debt.

In this paper we present a positive framework for understanding the conjunction of safe asset scarcity and “overheated” corporate debt markets. This provides the foundation for a normative framework for thinking about the welfare consequences of government-supplied safe bonds. We start from a canonical debt signaling framework, adding one additional element: endogenous trading by uninformed investors and an informed speculator. We argue that an overheated debt market, with low social welfare, emerges when safe asset shortages support a speculative high yield debt market. If there is a safe asset shortage, uninformed demand migrates to the junk debt market. The increase in uninformed demand spurs speculator information production. This drives prices of junk debt closer to fundamentals, encouraging firms with positive information to pool at high face values. The social benefit of such an outcome is high corporate investment. One social cost of this outcome is the cost of speculator information production. A resulting social cost of asymmetric information across investors is distorted portfolios. And to the extent that some uninformed investors are biased away
from saving adequately, distress costs may result. Paradoxically, we show the corporate sector is more likely to impose the negative externality associated with risky debt precisely when it is large.

In this economy, the government can increase social welfare by offering to investors even a limited amount of safe bonds. For example, the government can offer safe bonds with an eye toward deterring pooling at risky corporate debt. Safe government bonds siphon off uninformed demand from junk debt markets. This lowers speculative information production, driving prices away from fundamentals. If this effect is sufficiently strong, corporations will opt to issue riskless debt instead of junk debt. That is, riskless government bonds serve to crowd-in safe corporate debt, while crowding out investment financed by risky debt. This increases social welfare if marginal $Q$ is sufficiently low. Alternatively, the government can supply safe government bonds with an eye toward increasing the efficiency with which the private market implements pooling at risky debt. Here the siphoning effect of government bonds serves to induce marginal reductions in socially wasteful speculator effort and mitigates the extent of investor-level portfolio distortions.

One can also think of our framework as allowing one to understand an unconventional channel through which central bank operations may operate, alleviating corporate leverage externalities. For example, contractions in the central bank balance sheet increase the amount of government bonds in circulation, siphoning off demand for risky corporate debt and cooling off speculative debt markets. As we show, such operations increase social welfare when risk-sharing concerns dominate concerns over corporate investment incentives.
Lemma A1: Existence of Unique Continuation Equilibrium

Define the function $\Gamma$ with domain $[1/2, 1]$ based upon the speculator’s incentive condition as follows:

$$
\Gamma(\sigma) \equiv \Psi \left[ \frac{1}{2} (D - L)(N - G) \left( \frac{1}{L} [1 - F(\theta_2(\sigma, D))] + \frac{1}{D} [F(\theta_2(\sigma, D)) - F(\theta_1(\sigma, D))] \right) \right]
$$

The function $\Gamma$ is continuous and strictly decreasing with $\Gamma(1/2) > 1/2$. It follows there exists a unique solution to the equation $\Gamma(\sigma) = \sigma$ in $(1/2, 1)$. \(\blacksquare\)

Lemma A2: Reduction in Uninformed Demand with Two Corporate Debt Issuers

For brevity, let $\phi \equiv N - G$. Consider then the portfolio problem of an individual UI. Let $k$ denote the number of defaults against which the agent wants to insure, with $k \in \{0, 1, 2\}$. In this connection, let $x_k$ denote the number of units of debt of each issuer the investor must hold in order to achieve a payoff of $\phi$ given that there are $k$ defaults. We have:

$$
x_k = [kL + (2 - k)D]^{-1} \phi
$$

We can pin down the optimum portfolio here using perturbation arguments. Consider first an investor anticipating the low future endowment who holds zero units of debt. His gain from increasing his holdings of each issuer’s debt infinitesimally is equal to

$$
\theta \left[ \frac{1}{4} (2L) + \frac{1}{4} (2D) + \frac{1}{2} (L + D) \right] - 2E(P).
$$

Consider next an investor with initial portfolio holding of $x_0$ contemplating an increase in his holdings. His gain from increasing his holdings of each issuer’s debt infinitesimally is equal to

$$
\theta \left[ \frac{1}{4} (2L) + \frac{1}{2} (L + D) \right] - 2E(P).
$$

Finally, consider an investor with initial portfolio holding of $x_1$ contemplating an increase in his holdings. His gain from increasing his holdings of each issuer’s debt infinitesimally is equal to

$$
\theta \left[ \frac{1}{4} (2L) \right] - 2E(P).
$$
From the preceding perturbation gain equations we obtain the following critical cutoffs for a net gain to increasing the portfolio:

\[
\hat{\theta}_1 \equiv \frac{2E(P)}{L+D}; \hat{\theta}_2 \equiv \frac{4E(P)}{2L+D}; \hat{\theta}_3 \equiv \frac{4E(P)}{L}.
\]

And we have the following portfolio rule:

\[
\theta \leq \hat{\theta}_1 \Rightarrow x^*(\theta) = 0
\]
\[
\theta \in (\hat{\theta}_1, \hat{\theta}_2) \Rightarrow x^*(\theta) = \frac{\phi}{2D}
\]
\[
\theta \in (\hat{\theta}_2, \hat{\theta}_3) \Rightarrow x^*(\theta) = \frac{\phi}{L+D}
\]
\[
\theta \geq \hat{\theta}_3 \Rightarrow x^*(\theta) = \frac{\phi}{2L}.
\]

In contrast, with one issuer we had the following thresholds:

\[
\theta_1 = \frac{2E[P]}{L+D}, \theta_2 = \frac{2E[P]}{L}.
\]

And the following portfolio rule.

\[
\theta \leq \theta_1 \Rightarrow x^*(\theta) = 0
\]
\[
\theta \in (\theta_1, \theta_2) \Rightarrow x^*(\theta) = \frac{\phi}{D}
\]
\[
\theta \geq \theta_2 \Rightarrow x^*(\theta) = \frac{\phi}{L}.
\]

And we verify that for all \( \theta \) demand is lower with two issuers than with one issuer:

\[
\theta \in (\theta_1, \hat{\theta}_2) : \frac{\phi}{2D} < \frac{\phi}{D}
\]
\[
\theta \in (\hat{\theta}_2, \theta_2) : \frac{\phi}{L+D} < \frac{\phi}{D}
\]
\[
\theta \in (\theta_2, \hat{\theta}_3) : \frac{\phi}{L+D} < \frac{\phi}{L}
\]
\[
\theta > \hat{\theta}_3 : \frac{\phi}{2L} < \frac{\phi}{L}.
\]
References


Figure 1: Uninformed Demand and Speculator Signal with Risky Debt

Functional forms are: \( e(\sigma) = [1 + \ln(\frac{1}{2})] - \ln(1 - \sigma) - 2\sigma \) and \( F(\theta) = 1 - \frac{1}{\theta} \).

\( L = 50, D = H = 700, N = 12 \). Base \( G = 5 \) on the solid \( X_U \) curve and \( G = 6 \) on the dashed curve.

Figure 2: Private versus Public Preferences

Same functional forms and parameters as in Figure 1. \( N \in [0, 14] \).