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Faccini, R, Mumtaz, H and [Surico, P](#)

(2016)

*International fiscal spillovers.*

Journal of International Economics, 99 (March). pp. 31-45. ISSN 0022-1996

DOI: <https://doi.org/10.1016/j.jinteco.2015.11.009>

Elsevier

<http://www.sciencedirect.com/science/article/pii/S...>

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# International Fiscal Spillovers\*

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November 2015

## Abstract

A two-country business cycle model featuring nominal rigidities, countercyclical mark-ups, rule of thumb consumers and government spending reversals is used to identify inequality predictions that are robust across a range of empirically plausible parameterizations. These robust inequality restrictions are imposed onto a regime-change factor model for the United States and its main trade partners to estimate the international fiscal spillovers. The effects of U.S. government spending on foreign real activity are found to be sizable and significant, operating mainly by lowering real interest rates rather than boosting trade balances. In contrast, there seems to be only limited evidence of state dependence in the international transmission of fiscal policy.

JEL CLASSIFICATION: E3, E6, F4

KEY WORDS: regime-change factor model, fiscal spillovers, international transmission.

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\*We are grateful to Giancarlo Corsetti and two anonymous referees for very useful comments and suggestions. Surico gratefully acknowledges financial support from the European Research Council (Starting Grant 263429 and Consolidator Grant 647049). The graphs in this paper are best viewed in color.

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# 1 Introduction

The great recession of 2007-09 has reignited the discussion in policy and academic circles about the economic circumstances under which fiscal policy (and government spending in particular) can stimulate the economy, both domestically and internationally. On the theoretical side, recent contributions have shown that accommodative monetary policy has the potential to alter the transmission of fiscal policy in closed economy models (Hall, 2009, Woodford, 2011, and Christiano, Eichenbaum and Rebelo, 2011) as well as in multi-country models (Cook and Devereux, 2011 and Coenen et al. 2011).

On the empirical side, Canova and Pappa (2011) report that whenever a fiscal expansion is associated with negative real short-term interest rates, the domestic fiscal multipliers in the United States, United Kingdom and the Euro area tend to be somewhat larger than the estimates based on various identification schemes reported in Blanchard and Perotti (2002), Mountford and Uhlig (2009) and Barro and Redlick (2011). Auerbach and Gorodnichenko (2012) show that the fiscal multipliers are typically larger during recessions whereas, using a longer sample, Ramey and Zubairy (2014) find little evidence for state-dependent government spending multipliers in the United States.

While the dynamic response of the real exchange rate to a U.S. fiscal shock has been the subject of a rapidly growing empirical literature (Monacelli and Perotti, 2011, Ravn, Schmitt-Grohe and Uribe, 2012, and Enders, Müller and Scholl, 2011), little is known on whether international fiscal spillovers –defined as the response of foreign output to a domestic fiscal shock conditional on fiscal policy abroad– are (i) positive, (ii) heterogeneous across trade partners and (iii) varying over time.

In this paper, we address this important gap in the literature by identifying international fiscal spillovers. Our reference framework is a two-country real business cycle model featuring countercyclical markups (in the spirit of Ravn, Schmitt-Grohe and Uribe, 2012), sticky prices and wages, rule of thumb consumers (a la Galí, Lopez-Salido and Valles, 2007), and government spending reversals (following Corsetti, Meier and Müller, 2010 and 2012). The contributions above have shown that each of these channels has the potential to alter the effects of government spending.

The theoretical framework is used to derive a set of sign restrictions in the dynamic

responses to a government spending shock that are robust across a range of empirically plausible parameterizations of these theoretical mechanisms. The robust sign restrictions are then imposed onto a change point factor model for the U.S. economy and its main trade partners over the post-Bretton Woods period. Following Kilian and Murphy (2012), we impose the additional restriction that the size of the domestic fiscal multiplier cannot be implausibly higher than the point estimates available in the literature for the U.S.. Finally, while the empirical model allows fiscal policy in the foreign economy to adjust following a U.S. government spending shock, the analysis in Section 4 reveals that the response of foreign government spending is often insignificant across countries and regimes, implying that our estimates can be interpreted as the international fiscal spillovers holding foreign government spending constant.

The choice of a factor model fulfills our desire to identify government spending shocks using large information, which Forni and Gambetti (2010) and Gambetti (2010) have shown to ameliorate the non-fundamentalness problem raising from fiscal foresight in small-scale VARs. More specifically, as shown conceptually by Leeper, Walker and Yang (2013), whenever government policies are anticipated by the public and the variables used by the econometrician span a smaller information set than available to the agents, identification strategies based on combinations of VAR residuals fail to recover the structural shocks. The reason is that the VAR residuals are still contaminated by the component of government spending that the agents could have predicted using the variables omitted by the econometrician. In contrast, a large information approach, as taken in this paper, is more likely to avoid the distorted inference associated with fiscal foresight.

Time-variation is introduced because our sample is characterized by significant changes in (i) the conduct of fiscal policy (Davig and Leeper, 2006, and Bianchi and Ilut, 2011) and monetary policy (Cogley and Sargent, 2005), (ii) business cycle conditions and (iii) the volatility of structural shocks (Primiceri, 2005, and Sims and Zha, 2006), ranging from the 1970s great inflation to the great moderation and finally the great recession. To avoid taking a stand a-priori on the most relevant source of changes (and its precise timing), our statistical model identifies in the data the most likely break points.

Our main results can be summarized as follows. First, the probability of a positive re-

sponse of foreign output to an unanticipated increase of government spending in the United States is typically larger than fifty percent over the post-Bretton Woods period (especially after 1984), with the largest effects recorded for Canada and the United Kingdom. Second, an expansionary U.S. government spending shock leads to a significant decrease in real rates, both domestically and internationally, but small and insignificant changes in the trade balances. We interpret this as suggestive that the international transmission of fiscal policy might operate through a financial channel rather than a trade channel. Third, we find little support for regime dependence: both the spending multipliers and the international transmission of government spending shocks seem remarkably stable over the statistical different regimes identified by our factor model, and neither the adoption of the Euro nor the state of the business cycle (either internationally or domestically) seem to have led to a significant change in the international transmission of U.S. fiscal policy.

In the rest of the paper, Section 2 introduces the theoretical framework and illustrates the way we nest a number of hypotheses for the international transmission of fiscal policy. It also reports the inequality predictions (for the dynamic effects of a government spending shock on the endogenous variables) that are robust to a wide perturbation of the parameter space. These theory-robust sign restrictions are imposed in Section 3 onto a factor model for the U.S. economy and some of its main trade partners. Results are presented in Section 4 before conclusions. The appendices provide details of the theoretical model, the estimation of the empirical model, data and variance decomposition. We also relegate to the Appendices a discussion of the propagation of the various theoretical transmission channels, and further details on the identification of the sign restrictions.

## 2 Theoretical framework and sign restrictions

The reference framework is a two-country New-Keynesian model augmented with counter-cyclical markups, rule of thumb consumers and government spending reversals. Each of these ingredients is meant to exemplify a specific channel within a broad class of competing models for the international transmission of fiscal policy.

There are two symmetric countries, and in each country two types of firms: final good and intermediate good firms. Final good firms combine home and foreign intermediate products

into a homogeneous consumption good. We assume home bias in the production of the consumption good as a reduced-form device to modeling trade openness. While final good firms operate under perfect competition, intermediate producers set their price under monopolistic competition and Calvo price stickiness, using differentiated labor services as the only factor of production.

On the household side we introduce both asset holders and rule of thumb consumers. These two types of agents differ in that only asset holders can access international capital markets and transfer wealth into the future. We assume that the elasticity of substitution varies procyclically with aggregate output, so as to give rise to countercyclical markups. As for policy, the monetary institution is captured by a Taylor rule, while the government takes the shape of a fiscal rule that allows for spending and taxes to respond to the real level of debt, so as to produce spending reversals.

In the special case where prices and wages are fully flexible, the mark-up is constant, the budget is balanced at all times and there are no rule-of-thumb consumers, the model boils down to the standard neo-classical model. Introducing procyclical elasticity of substitution over this benchmark gives us the counter-cyclical markup model; introducing both price and wage rigidity coupled with either limited asset market participation or fiscal feedback rules will provide a benchmark for the rule of thumb model and spending reversal model, respectively. Because the different specifications allowing for rule of thumb consumers, spending reversals and countercyclical markups are relatively standard in the literature, details of the model and derivation of the log-linearized system of equation is relegated to the web Appendix C. We refer to the web Appendix D for an illustration of differences and similarities in the propagation of the various theoretical channels.

Using the nested framework where rule of thumb consumers, countercyclical markups, government spending reversals as well as stickiness in wages and prices are allowed to interact with each other, we are able to identify sign restrictions for government spending shocks that are common across empirically plausible perturbations of the parameter space. We find that following a positive government spending shock, (i) government spending, (ii) taxes, and (iii) domestic output, increase on impact, while the response of (iv) the government budget surplus, is non-positive. Furthermore, the nesting model generates a positive comovement

(v) between short-term nominal interest rate and inflation, and (vi) between consumption and real exchange rate.<sup>1</sup>

It is worth noting that our sign restrictions on the joint response of output, budget surplus and taxes uniquely identify the government spending shock. Any other shock that is included in the nesting model would generate opposite comovements between primary budget surplus and taxes. For instance, expansionary TFP, monetary, preference, labor supply, and markup shocks would increase surplus and decrease taxes. For more details on the computation and the robustness of these sign restrictions we refer to the web Appendix in Section D.3.

On the basis of this finding, our theory-based strategy to identify a government spending shock in the data is to impose the common sign restrictions (i) to (vi), which are reported in Table 1, onto the factor model of Section 3. The ultimate goal is to let the data inform us about the sign and size of the international fiscal spillovers, namely the dynamic response of foreign output to unanticipated government purchases in the United States.

### 3 The empirical framework

In this section, we present the dynamic factor model with independent break points in the factor loadings and idiosyncratic variances, which will be used to investigate any possible time-variation in the international transmission of government spending shocks. We lay out here the estimation and identification procedures while providing a more detailed description in Appendix B.

The reasoning behind the choice of a model with time-varying parameters is twofold. First, a large empirical literature has documented significant variation in the evolution of the volatility of real activity, inflation and interest rates in a number of advanced economies over the post-WWII period. Second, our sample has been arguably characterized by several monetary and fiscal regimes, both in the U.S. and its main trade partners, featuring different degrees of commitment (and success) to fight inflation and stabilize debt.

As discussed in the introduction, the identification of a structural shock requires that no variable available to the agents (but omitted by the econometrician) could predict the shock.

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<sup>1</sup>The parameter values used in the simulations are drawn from uniform distributions over 10,000 repetitions. Our results indicate that the inequality predictions (i) to (iv) are satisfied in every single draw. The restrictions (v) and (vi) are instead satisfied in 97.7% and 99.4% of the draws, respectively.

To confront this problem, which is referred to in the literature as ‘non-fundamentalness’, we follow Gambetti (2010) and Forni and Gambetti (2010) and use a dynamic factor model as an efficient way to summarize the information in a large dataset of variables for the U.S. and its main trade partners.

### 3.1 A change point factor model

In the empirical model, we assume that a few factors summarize the comovements among a large cross-section of observables  $X_{it}$  according to the following specification:

$$X_{it} = \beta_{i,S}F_t + \sigma_{i,Q}e_{it} \quad (1)$$

$$F_t = c + \sum_{j=1}^L \lambda_j F_{t-j} + \Sigma^{1/2}v_t \quad (2)$$

where the factors  $F_t = \{f_{1t}, \dots, f_{K,t}\}$  and  $L = 2$ . We allow for *independent* structural breaks in the factor loadings  $\beta_i$  and the variances  $\sigma_i$ . As shown below, this specification is preferred to an alternative with fixed coefficients and to another that allows for breaks in the parameters of the transition equation. Following Chib (1998),  $M$  structural breaks are introduced through the unobserved discrete state variable  $S$ . This state variable is assumed to follow a  $M + 1$  state Markov chain process with a restricted transition probability matrix  $\tilde{P}$ .<sup>2</sup> The transition probabilities  $p_{ij} = p(S_t = j | S_{t-1} = i)$  are given by

$$p_{ij} > 0 \text{ if } i = j, p_{ij} > 0 \text{ if } j = i + 1 \quad (3)$$

$$p_{M+1M+1} = 1, p_{ij} = 0 \text{ otherwise.}$$

Analogously, the state variable  $Q$  follows an  $M + 1$  Markov chain process with a similarly restricted transition probability matrix  $\tilde{Q}$ , which is independent of  $\tilde{P}$ .

The process described in (3) is a Markov switching model where transitions are allowed in a sequential manner. For example, to move from regime 1 to regime 3, the process has to visit regime 2. Similarly, and without loss of generality, transitions to past regimes are not allowed. Note, however, that this is not restrictive as two (non-consecutive) regimes that were similar or identical to one another would simply be given two labels.<sup>3</sup>

<sup>2</sup>Kim and Nelson (1999, Chapter 10) provide further examples of factor models with switching parameters. Del Negro and Otrok (2005) were the first to consider a factor model with time-varying factor loadings.

<sup>3</sup>The proposed model has computational advantages over a more ‘conventional’ Markov switching specifi-



## 3.2 Estimation

The model is estimated using a Gibbs sampling algorithm. Details of the priors and conditional posteriors are given in the appendix. Here we sketch the main steps:

1. Given a value for the factors, draw the VAR parameters.
  - The VAR coefficients  $c, \lambda_j$  have a normal conditional posterior, while the conditional posterior of the covariance matrix  $\Sigma$  is Inverse Wishart.
2. Given a value for the factors, draw the factor loadings  $(\beta_{i,S})$ , the variance of the idiosyncratic components  $\sigma_{i,Q}$  and the state variable  $S_t$  and  $Q_t$ .
  - Given data on  $X_{i,t}$  and  $F_t$ , equation (1) is a system of equations with independently switching coefficients and variances. Following Kim and Nelson (1999, Chapter 9), we use the Multi-Move Gibbs sampling to draw  $S_t$  from the joint conditional density  $f(S_t | \beta_{i,S}, \sigma_{i,Q}, \tilde{P}, Q_t)$  and  $Q_t$  from the joint conditional density  $f(Q_t | \beta_{i,S}, \sigma_{i,Q}, \tilde{Q}, Q_t)$ .
  - Conditional on  $S_t$  and  $Q_t$ , standard results for regression models can be used and the coefficients and the variances are simulated from a normal and inverse gamma distribution.
3. Conditional on  $S_t$  and  $Q_t$ , elements of  $\tilde{P}$  and  $\tilde{Q}$  are drawn from the Dirichlet distribution.
4. Simulate the factors conditional on all the other parameters.
  - This is done by employing the methods described in Carter and Kohn (1994).
5. Go to step 1.

The autocorrelations of the retained draws (see Appendix B) show little variation which provides some evidence of convergence of the algorithm.

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caution. In the latter, the labels of the regimes are not identified and researchers typically refer to the properties of a particular time series so that, for instance, a high regime corresponds to a higher unconditional mean for a specific endogenous variable. Given the factor structure as well as the large panel dimension, however, this strategy is not feasible in our context and the choice of a specific variable for labeling the regimes may not be innocuous. Hence, we adopt the simpler structure for regime transitions described in equation (20).

### 3.3 Model comparison

We carry out a model comparison exercise to determine the number and location of breaks and the number of factors. In particular, we estimate three versions of the factor model: (i) a model with fixed parameters, (ii) a model with  $M$  breaks in the vectors  $\beta'_i s$  and parameters  $\sigma'_i s$  of the observation equation 1 and (iii) a model with  $M$  breaks in the vectors  $\sum_{j=1}^L \lambda'_j s$  and the matrix  $\Sigma^{1/2}$  of the transition equation 2. Note that in (ii) and (iii) we allow for independent breaks in the coefficients and variances as described above.

We assume  $M = 1, 2, 3$  and in each case allow for the possibility of up to eight factors. The model comparison is carried out via the Bayesian Deviance Information Criterion ( $DIC$ ). Introduced in Spiegelhalter et al. (2002), the  $DIC$  is a generalisation of the Akaike information criterion – it penalises model complexity while rewarding fit to the data. The  $DIC$  is defined as

$$DIC = \bar{D} + p_D.$$

The first term  $\bar{D} = E(-2 \ln L(\Xi_i)) = \frac{1}{M} \sum_i (-2 \ln L(\Xi_i))$  where  $L(\Xi_i)$  is the likelihood evaluated at the draws of all of the parameters  $\Xi_i$  in the MCMC chain. This term measures goodness of fit. The second term  $p_D$  is defined as a measure of the number of effective parameters in the model (or model complexity). This is defined as  $p_D = E(-2 \ln L(\Xi_i)) - (-2 \ln L(E(\Xi_i)))$  and can be approximated as  $p_D = \frac{1}{M} \sum_i (-2 \ln L(\Xi_i)) - \left(-2 \ln L\left(\frac{1}{M} \sum_i \Xi_i\right)\right)$ .<sup>4</sup> Prior distributions on the parameters in our model and the presence of latent variables implies that the number of parameters (as used in the calculation of the Akaike and Schwarz information criterion) do not necessarily represent model complexity. The definition of the effective number of parameters used in the computation of the  $DIC$  avoids this problem. Note that the model with the lowest estimated  $DIC$  is preferred. Calculation of the  $DIC$  requires the calculation of the likelihood of the change point factor model. In our application this is done via the approximate filter discussed in Kim and Nelson (1999).<sup>5</sup>

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<sup>4</sup>The first term in this expression is an average of  $-2$  times the likelihood function evaluated at each MCMC iteration. The second term is  $(-2)$  times the likelihood function evaluated at the posterior mean.

<sup>5</sup>For an accurate approximation of the likelihood function, Kim and Nelson (Chapter 5, 1999) recommend to keep track of the regimes in three periods:  $t$ ,  $t-1$  and  $t-2$ . As our empirical model involves  $(M+1)^2$  regimes (in a combination of factor loading and volatility breaks), their recommendation amounts to keep track of  $[(M+1)^2]^3$  possible trajectories for the evolution of the parameters. To keep the estimation computationally tractable, we therefore consider up to four possible regimes (i.e.  $M=3$ ). This leads to more than four

Table 2 reports the DIC statistics for each estimated model. The fixed parameter specifications is not favoured by the DIC criterion, which in fact provides support for the model with 3 (independent) breaks in the factor loadings and the idiosyncratic variances of the observation equation and seven factors. Note that this model also has a lower DIC than models (not shown in the table) with drifting factor loadings (DIC= 8275) or parameter drift and stochastic volatility in the VAR transition equation (DIC=17042). In other words, for our dataset a model with structural breaks appears favored relative to one that allows for smooth time-variation. In addition, the model with 3 independent breaks in the parameters of the observation equation and seven factors is also preferred to restricted specifications (not reported in Table 2) that only allow for 3 breaks in the factor loadings (DIC=2952) or 3 breaks in idiosyncratic variances (DIC=-1873.64). Finally, our preferred change point factor specification compares favorably with a model in which recessions feature as a possible separate regime as the latter is associated with a DIC statistics of 1837.03.

### 3.4 Computation of the impulse response functions

We calculate the impulse responses  $\Delta_t$  of  $F_t$  to a government spending shock. With these in hand, the regime-specific impulse responses of each underlying variables can be easily obtained using the observation equation of the model. The impulse response  $\Delta_t$  is estimated using a contemporaneous impact matrix  $A_0$  which is calculated to satisfy the sign restrictions in Table 1, which are based on the theoretical framework of Section 2.

In addition to these sign restrictions, we impose ‘plausibility’ restrictions on the short-run response of domestic and foreign real per-capita GDP growth requiring the contemporaneous impact of the government spending shock to be less than 0.6%. Coupled with a government spending-GDP ratio of about 20%, these restrictions map into fiscal multipliers smaller than three, consistently with both the top end of the structural VAR confidence bands and the quantitative analysis in Cook and Deveraux (2001) for the home-bias case when the nominal interest rate responds neither in the domestic nor in the foreign country. As emphasized by Kilian and Murphy (2012), impulse response functions based on sign restrictions only, implicitly assume that all admissible models, including those with economically implausible

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thousands possible regime permutations.

fiscal multipliers, are equally likely. Finally, to explore the possibility that the zero lower bound on the domestic short-term nominal interest rate may have amplified the effects of fiscal policy during the most recent period, in the fourth regime only we add the additional restrictions that the short-term rate does not move by more than ten basis points either way on impact.<sup>6</sup>

The sign restrictions are implemented as follows: let  $\Sigma = PP'$  be the Cholesky decomposition of the transition equation covariance matrix  $\Sigma$ . We draw a  $4 \times 4$  matrix,  $J$ , from the  $N(0, 1)$  distribution. We take the  $QR$  decomposition of  $J$ , which gives us a candidate structural impact matrix as  $A_0 = PQ$ . Then, we compute the impulse response of  $X_{1,t}, \dots, X_{N,t}$ . We check if these impulse responses satisfy our sign and plausibility restrictions. If this is the case, we store  $A_{0,t}$  and move to the next Gibbs iteration.

### 3.5 Data

We fit the change point factor model described above to an international panel of 143 quarterly series over the post-Bretton woods sample 1975Q1-2010Q4. The United States is treated as the domestic economy while the foreign block is made up of Canada, France, Germany, Japan and the United Kingdom. These countries account for the lion share of trade volumes with the United States.<sup>7</sup> As for the domestic variables, we include –among others– real government spending, real net taxes, real GDP, CPI, 3-month Treasury bills rate, 10 year government bond yields, real private consumption expenditure, real wage, investment, terms of trade and CPI real effective exchange rate. The foreign block includes government spending, real GDP, personal consumption expenditures, investment, the trade balance, CPI and the 3-month Treasury bills rate for each country. The inclusion of foreign government spending fulfills our desire to control for fiscal interventions abroad which, if omitted, may distort the inference drawn upon the estimated effects of U.S. government spending on its main trade partners GDP. As shown in Section 4, however, the response of government spending abroad is typically insignificant.

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<sup>6</sup>While results are not sensitive to the specific cut off of ten basis points, they are meant to exemplify a scenario in which the short-term nominal rate is forced to remain close to its sub-sample average, which in the fourth regime is 0.4% on an annual basis.

<sup>7</sup>China is excluded because of the possible currency manipulation over part of our sample period.

All real variables are expressed in per capita terms. All variables but interest rates and terms of trade are in log-difference. A full description of the individual time series and their sources is provided in Appendix C where we also report the forecast error variance decomposition and the contribution of the factors to the total variance of the main variables in our panel (Table 4).

## 4 Results

In this section, we present the main results of the paper, namely the dynamic responses of U.S. and some of its main trade partner variables to a U.S. government spending shock. The impulse responses are obtained using the estimates from the change point factor model of Section 3. We begin by discussing the statistical regimes identified by the empirical model and then, we move to the impulse response function analysis. We start with the reaction of domestic variables before looking at the magnitude of the fiscal spillovers on foreign real activity. Finally, we explore the international transmission mechanism of the U.S. government spending shock as well as assess the role of the business cycle abroad and the introduction of a single currency in the Euro area in confounding our results.

### 4.1 Regimes

Modeling time-variation in both variances and parameters of a large fiscal panel is attractive because both the structure of the economy, the volatility of the shocks and the stance of economic policy may have changed over the post-Bretton Woods period.

Our setup is flexible enough to capture time variation along these dimensions either through a break in the factor loadings, a break in the variances or a break in both. The results are reported in Figure 1 where different color bands represent the regimes identified by the breakpoints in the factor loadings (top panel) and the breakpoints in the idiosyncratic variances (bottom panel).<sup>8</sup>

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<sup>8</sup>The 68% central posterior bands for the distribution of the probability  $p_{ii}$  that the factor loadings remain in the same regime  $i$  are [92.5% 98.4%] for  $p_{11}$ , [91.8% 98.2%] for  $p_{22}$  and [96.6% 99.2%] for  $p_{33}$ , whereas for the distribution of the probability  $Q_{ii}$  associated with the idiosyncratic variances these statistics are [95.9% 99.0%] for  $Q_{11}$ , [91.9% 98.2%] for  $Q_{22}$  and [94.4% 98.6%] for  $Q_{33}$ . By definition of last regime,  $p_{44}=Q_{44}=1$ .

While the empirical model allows for the break points in the two panels to be unsynchronized, the results in Figure 1 suggests that the last regime has been characterized by a virtually simultaneous shift in factor loadings and variances. On the other hand, the break in the variances during the first half of the 1980s seems to have preceded the break in the factor loadings by a couple of years whereas the beginning of the third regime preceded the break in variances by one year. The top panel reveals that the second regime lasted longer than any other regime and largely overlapped with the period of Great moderation in the idiosyncratic variances recorded in the bottom panel. Finally, the third period appears to mark the run-up to the fourth regime, which coincides with the global crisis triggered by the great recession.

## 4.2 The response of domestic variables and fiscal spillovers

This section reports impulse responses for domestic variables and fiscal spillovers in each of the regimes estimated on the basis of the factor loading breakpoints identified in the previous section. In the next section, we will look at the international transmission mechanism and the role played by the foreign business cycle.

Figures 2 and 3 report the responses of domestic variables to a fiscal shock in the United States normalized to have a one percent impact on government spending. Given the tight credible sets for the very high estimates of  $p_{ii}$  (with  $i=1$  to 3), we compute impulse responses conditional on being in a particular regime but we have verified that similar results are obtained when we incorporate regime uncertainty (which however makes the computation more burdensome).

Each row refers to a different variable while the columns report the central 68% credible set of the estimated impulse responses for each of the four regimes. A number of interesting results emerge from Figure 2. There is substantial evidence of a significant increase in US real GDP in all regimes but the first, where the response of real GDP is only significant on impact. Estimates of the contemporaneous effect are centered on 0.2 in regimes 2 and 3 and on 0.25 in regimes 1 and 4.<sup>9</sup> The response of inflation is statistically indistinguishable from

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<sup>9</sup>Using the 20% government spending-GDP ratio as rule of thumb, the credible sets in the first row of Figure 2 would approximately map into short-run multipliers between 0.2 and 2.1 across regimes, with an average point estimate around 1.1.

zero at all horizons in regimes 1 and 3, while in regimes 2 and 4 it is positive and significant. The response of the short-term real rate is flat and insignificant at all horizons in regime 1, while in the other regimes the median responses are strongly negative in the short-term, albeit still insignificant.<sup>10</sup> The median responses of the long-term *real* rate always lies in positive territory in regime one. In any other regime, the probability of the long-term rate being negative in the short-term is larger than 50 percent.

The dynamic effects of a fiscal shock on government spending, net taxes, consumption and the real effective exchange rate are presented in Figure 3. The response of net taxes is positive and significant in all regimes. Furthermore, only in the first regime the median response of government spending always remains in positive territory. The probability of government spending being negative is larger than 50 percent at all times beyond the first twenty quarters in regimes 2 and 3, and beyond fifteen quarters in regime 4. The response of consumption is positive and significant in all regimes, but only on impact in regimes 1 and 4, whereas the real exchange rate significantly depreciates at all times and regimes.

Moving to the response of foreign output, in Figure 4 we note that international spillovers from a U.S. fiscal expansion tend to be positive and significant, except in the first regime, where they are still positive, but insignificant. In regimes 2 and 4, spillovers are significantly positive for all countries in our sample, with the exception of Germany in period 2 where the spillovers are initially negative and then turn positive after about 10 quarters. In regime 3 instead, spillovers are positive and significant, but only for three out of five countries, namely Japan, the U.K. and France. The largest peak effect on foreign GDP across all regimes is found for Canada and the U.K., with the peak impact on Japan, France and Germany being substantially lower. While the point estimates for the peak effects are in line with the size of the fiscal spillovers in Corsetti and Müller (2014), we note that they are significantly larger than the prediction of any of the theoretical model discussed in Section 2 (Figure 13 in the web-Appendix D.2).

Finally, we note that the international fiscal spillovers in regime 4 do not appear necessarily different from the spillovers in the other regimes, suggesting that the last sub-sample is most

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<sup>10</sup>The response of the short-term nominal interest rate in regime 4 is constrained by the restriction meant to exemplify a zero lower bound type of scenario. Without this restriction, the response of both domestic and foreign output would be slightly smaller than shown in figures 2 and 4.

likely capturing a break in macroeconomic volatility, consistent with the evidence in Figure 5. In other words, we find little empirical support for the notion that the international transmission mechanism changed significantly during the most recent global crisis.

### 4.3 The international transmission mechanism

In this section, we shed light on the determinants of the international spillovers from a U.S. fiscal stimulus to foreign economic activity. We do so by discussing the estimated responses of the trade balance, short-term real rates, consumption and investment in foreign countries, which are reported in Figures 5 to 8. Starting with the trade balance, we notice that the impulse responses are typically insignificant across regimes and countries. The only exceptions are concentrated in regime 2, mostly for the U.S./U.K. and only to a lesser extent for Japan and France. Overall, central estimates for the response of the US trade balance are negative and persistent, consistent with the twin deficits hypothesis. However, the effects of government spending shocks on the trade balance are estimated to be quantitatively small, both in the US and abroad, with peak effects below 0.2 across all regimes and countries. Even for Canada, the only country in our sample for which central estimates indicate a trade surplus, the estimates are below 0.1. The results that the trade balance seems unlikely to drive fiscal spillovers is in line with the findings in Corsetti and Müller (2014).

On the other hand, we find evidence that fiscal spillovers operate by lowering the real interest rate abroad (see Figure 6) in all regimes but regime 1. In regime 2, which spans most of the time in our sample, the fall in the short-term real rate is statistically significant for all countries except Canada; in regimes 3 and 4 the probability of a fall in real rates in the aftermath of the fiscal stimulus remains persistently above 50% for all countries. In other words, Figures 5 and 6 suggest that the mechanism by which government spending shocks spill-over to foreign countries might operate through a financial channel, rather than the trade balance.

The decrease in short-term real rates is consistent with the estimated responses of consumption and investment, reported in Figures 7 and 8. In regime 2, the response of consumption is positive and significant for all countries except Germany, where the response remains always insignificant. In regime 4, the increase in consumption is positive and significant for



Canada, Japan and France, but insignificant for Germany. The response of consumption in the U.K. is instead significantly negative after about 10 quarters. In regimes 1 and 3 instead, the point estimate for the response of consumption is always positive, but never significant, with the exception of Japan in regime 3.

The adjustment in investment broadly mimics the adjustment in consumption across countries and regimes, with central estimates of the impulse responses being positive for all countries and regimes a few quarters after the shock, but attaining statistical significance only over a subset of country-regime pairs. In regimes 2 and 4, investment increases significantly in all countries, with the exception of the U.K. in regime 4. In regime 1 instead, the increase in investment is always statistically indistinguishable from zero, while in regime 3 the response is significant only in Canada and France.

Finally, in Figure 9 we report impulse responses for foreign government spending. In almost all countries and regimes we find little evidence of significant responses in foreign spending, with error bands generally being very wide around central estimates (France and Canada represent sporadic exceptions). Arguably, lower real interest rates induced by the US fiscal stimulus could improve fiscal sustainability abroad and induce a delayed increase in spending in foreign countries. But we find little evidence in support of this hypothesis.

In summary, we find little evidence of structural breaks in the transmission of fiscal spillovers after 1985, when U.S. fiscal policy begins to produce significant domestic and cross-border effects. The international transmission of government spending shocks does not appear to operate through the trade balance (with the only possible exception of regime 2) but most likely through a financial channel in the form of a decrease in real rates abroad, which in turn stimulates consumption and investment.

#### **4.4 State-dependence**

The regimes in our factor model are identified statistically. But a possible economic interpretation is that they reflect different states of the domestic or the international business cycle. Indeed, a prominent literature for the U.S., exemplified by the contributions of Auerbach and Gorodichenko (2012) and Ramey and Zubairy (2014), has studied (reaching opposite conclusions) whether the government spending multiplier is larger during periods of slack in

economy activity. In analogy to these studies, we ask whether the international transmission of fiscal policy depends on the presence of a recession abroad or on the introduction of a single currency among some of the U.S. main trade partners.

More specifically, we estimate two Factor Augmented VAR models where the stochastic regime shifts are replaced with a dummy variable. In the first model, the dummy takes the value of one during periods in which at least one of the countries in the panel is in recession and takes the value of zero otherwise.<sup>11</sup> The second model sets the value of the dummy to one after the first quarter of 1999 to account for any possible break associated with the adoption of the Euro.

Figure 10 records the response of foreign output to a U.S. government spending shock conditioned on the two regimes, with the left (right) column corresponding to periods of recessions (no recessions) abroad. The median responses appear remarkably similar across the two regimes, suggesting that international fiscal spillovers tend to vary neither with the state of the foreign economy nor with the state of the domestic economy. The results are similar when the impulse responses are conditioned to the sub-samples before and after adoption of the Euro, thereby providing little evidence in favour of the hypothesis that the transmission of U.S. fiscal policy (in Europe) changed systematically after this date (see Figure 11).

## 5 Conclusions

What are the effects of a fiscal expansion in the United States on foreign real activity? This paper has searched for international fiscal spillovers using theory-robust sign restrictions and a factor model. Our evidence suggests that an increase in U.S. government spending tends to have a positive influence on its main trade partners. The transmission mechanism appears to operate through a financial channel (as exemplified by negative real rates abroad) rather than a trade channel and appears to have been remarkably stable over time.

The ongoing period of policy retrenchment is likely to offer new challenges for modelling the interaction between fiscal and monetary policies. Furthermore, the current reversal of government spending and other policy interventions is suggestive of the possible start of a new regime. Whether the international fiscal spillovers associated with a newly identified

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<sup>11</sup>We use NBER (OECD) recession dates (recession indicators) for the U.S. (for the remaining countries).

fiscal consolidation era may be significantly different from the past is of course an empirical question. But the strategy outlined in this paper appears well placed to evaluate in future research any possible change in the international transmission of fiscal policy.

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Table 1: Theory-robust sign restrictions imposed in the empirical model

Variable	Sign Restriction
US real government spending	$\geq 0$
US real net taxes	$\geq 0$
(US real primary fiscal surplus) $_t$	$\leq 0$ for $t = 1, 2$
US real GDP	$\geq 0$
Corr(US consumption, US real exchange rate)	$\geq 0$
Corr(US inflation, US short-term nominal rate)	$\geq 0$

Note: An increase in the real exchange rate corresponds to a depreciation. Unless otherwise stated, the inequality constraints are imposed only on impact.

Table 2: Model Selection via Deviance Information Criterion

	2 Factors	3 Factors	4 Factors	5 Factors	6 Factors	7 Factors	8 Factors
Fixed Parameter	14880.5	8950.2	6757.3	4521.5	2525.4	1345.7	429.5
1 break (obs.)	7520.9	3944.1	2325.4	308.8	-1068.3	-1075.6	-3807.1
2 breaks (obs.)	6612.8	3100.1	-408.1	-1749.2	-3333.1	-4082.6	-4190.9
3 breaks (obs.)	3591.8	1560.1	-2291.3	-3542.9	-4245.1	-6200.9	-5566.7
1 break (trans.)	10290.4	8348.6	5994.9	4554.5	2433.0	502.5	796.2
2 breaks (trans.)	9421.8	7518.3	5711.1	4127.3	3361.7	1039.8	1014.6
3 breaks (trans.)	9869.9	7541.1	5710.7	3965.1	2249.0	1155.9	297.8



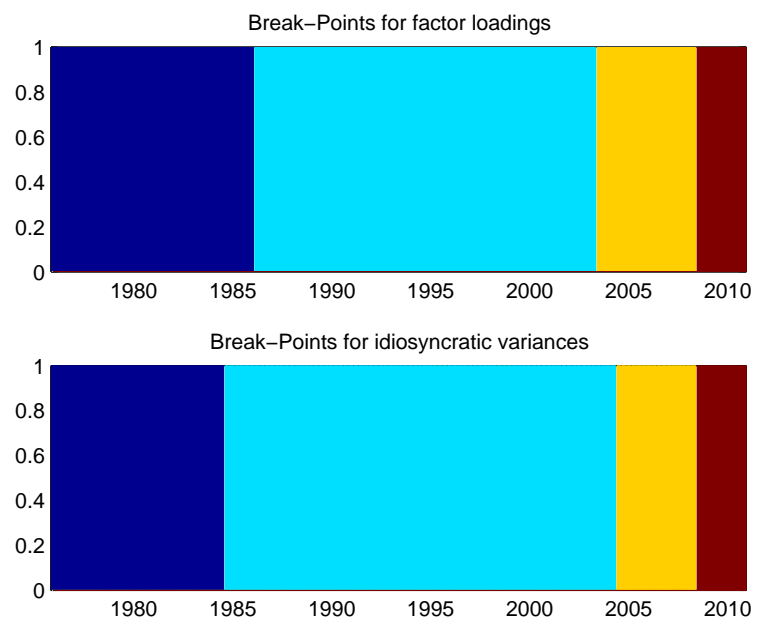


Figure 1: Identified breakpoints and regime probabilities.

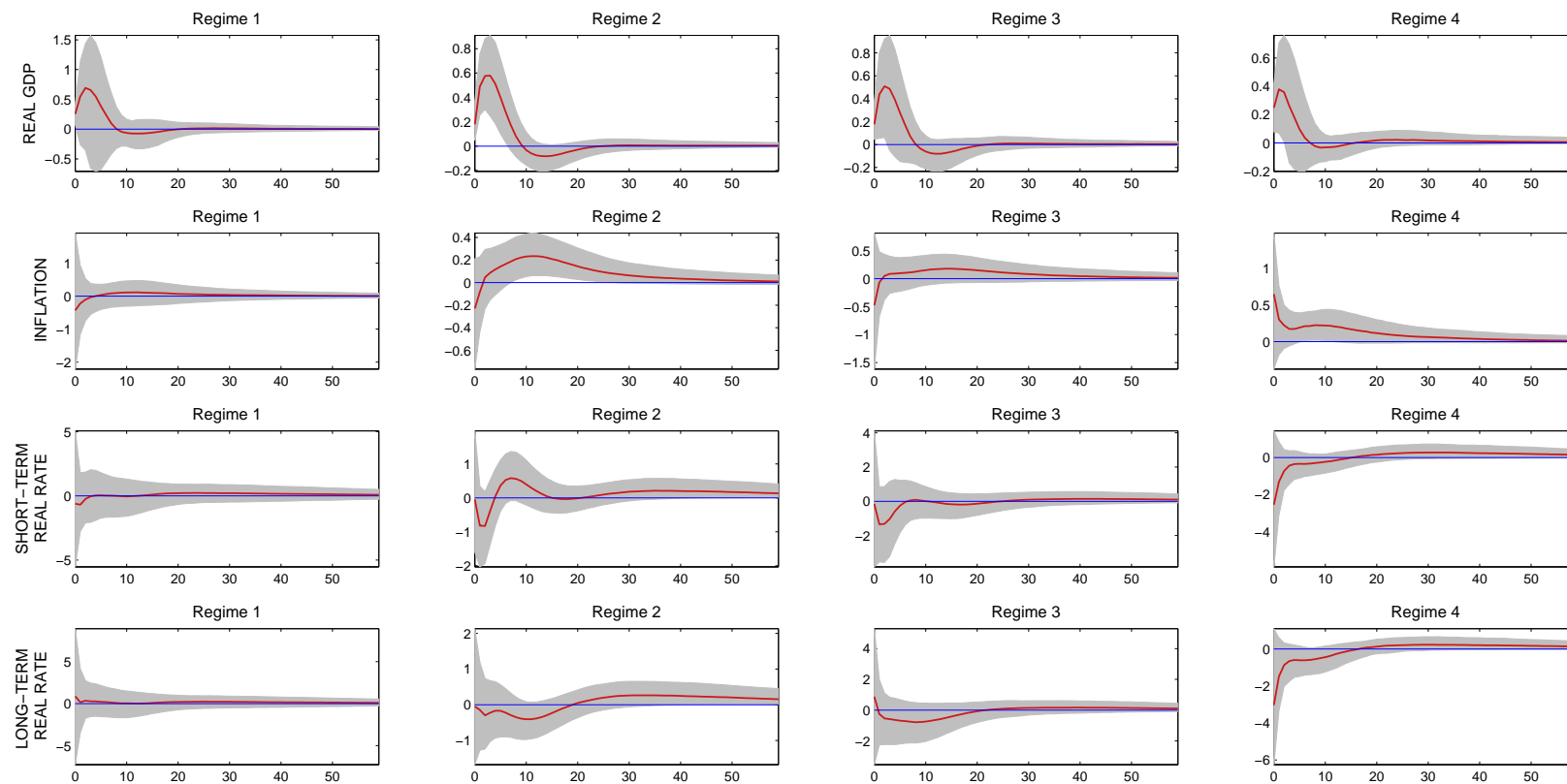


Figure 2: Dynamic responses of U.S. variables to a government spending shock normalized to have a 1% impact on U.S. government spending. Central 68% credible set.

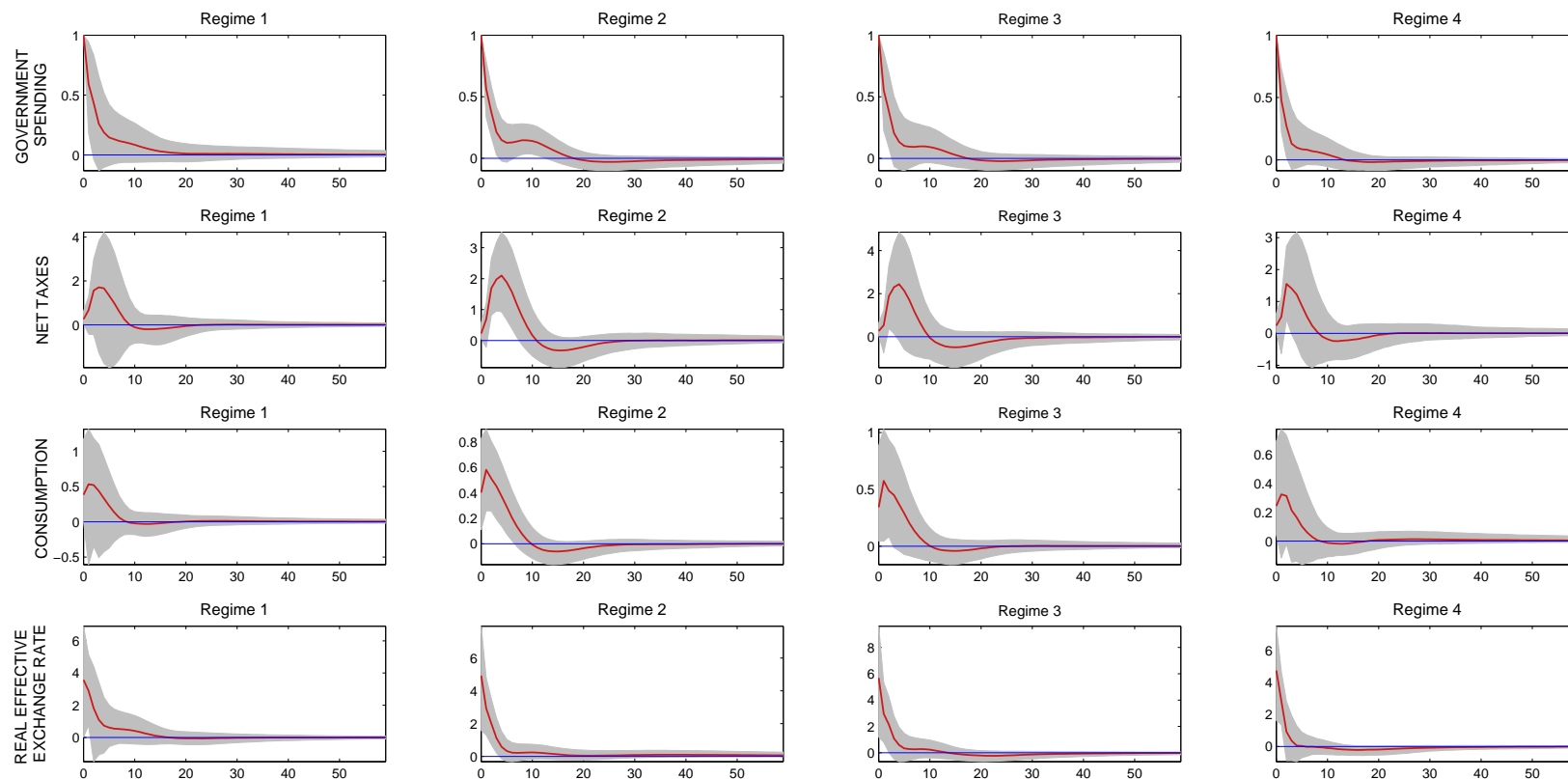


Figure 3: Dynamic responses of U.S. variables to a government spending shock normalized to have a 1% impact on U.S. government spending. An increase in the real exchange rate corresponds to a depreciation. Central 68% credible set.

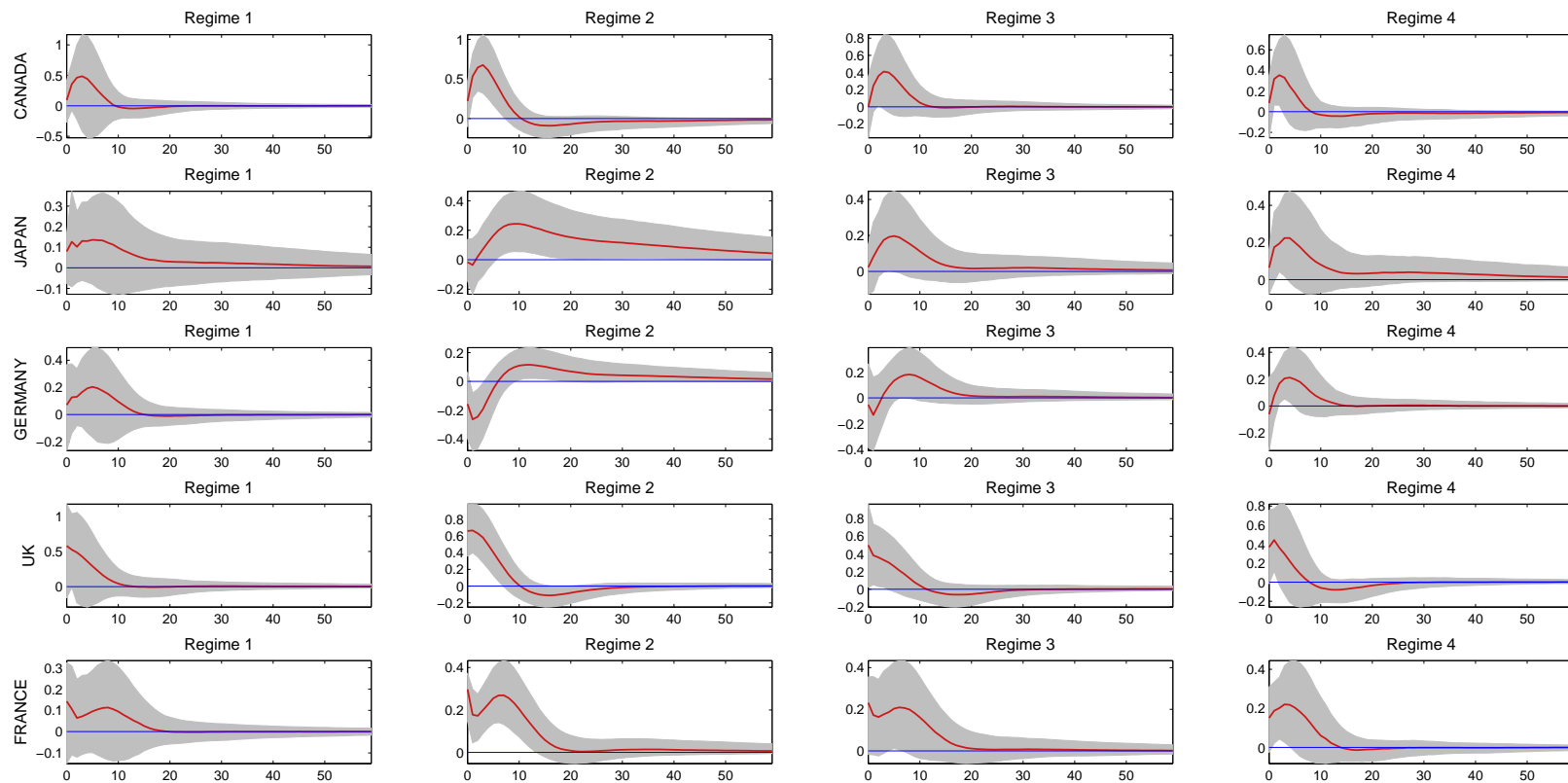


Figure 4: Dynamic responses of foreign real GDP to a government spending shock normalized to have a 1% impact on U.S. government spending. 68% central posterior credible set.

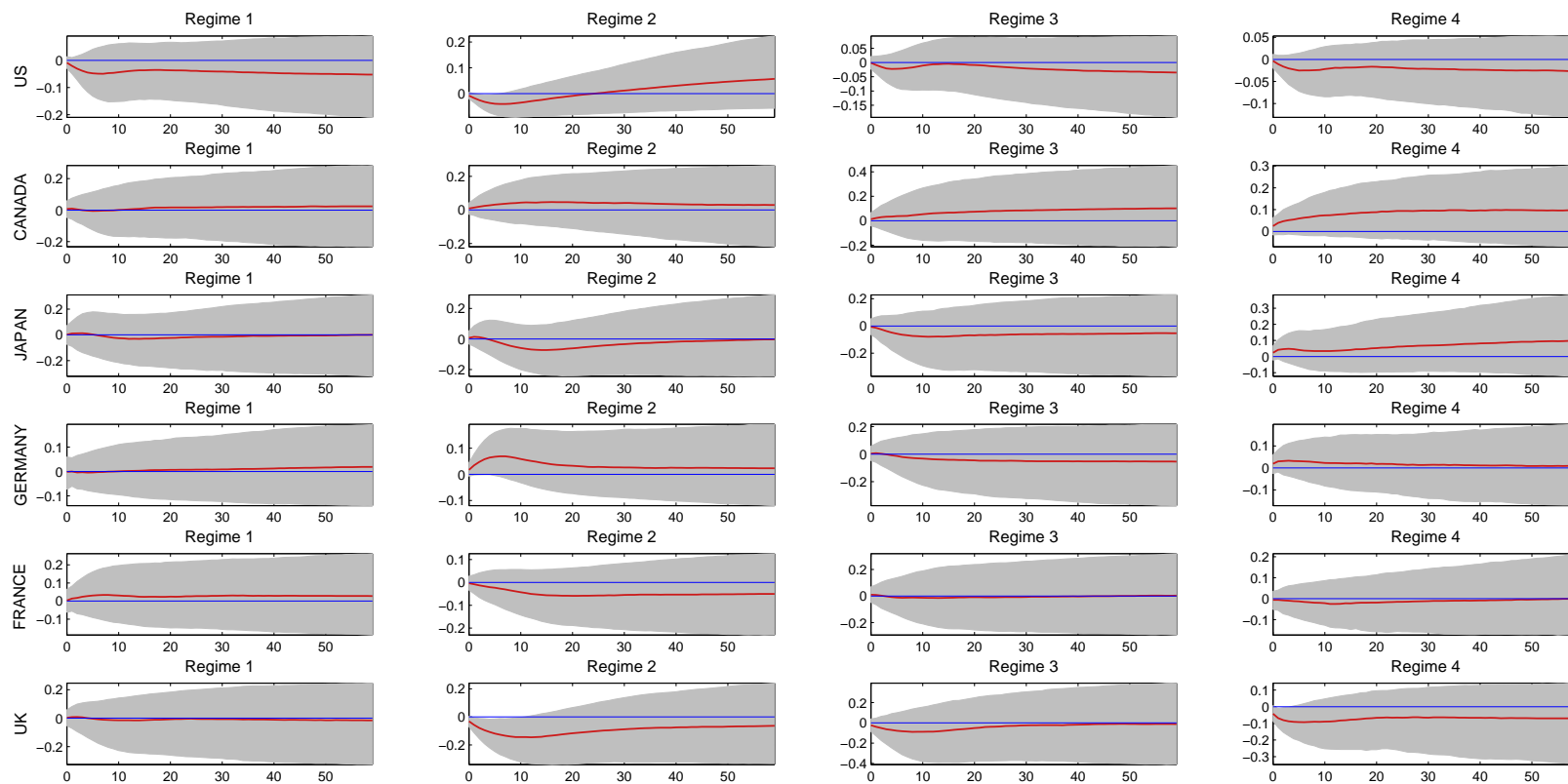


Figure 5: Dynamic responses of the trade balance to a government spending shock normalized to have a 1% impact on U.S. government spending. 68% central posterior credible set.

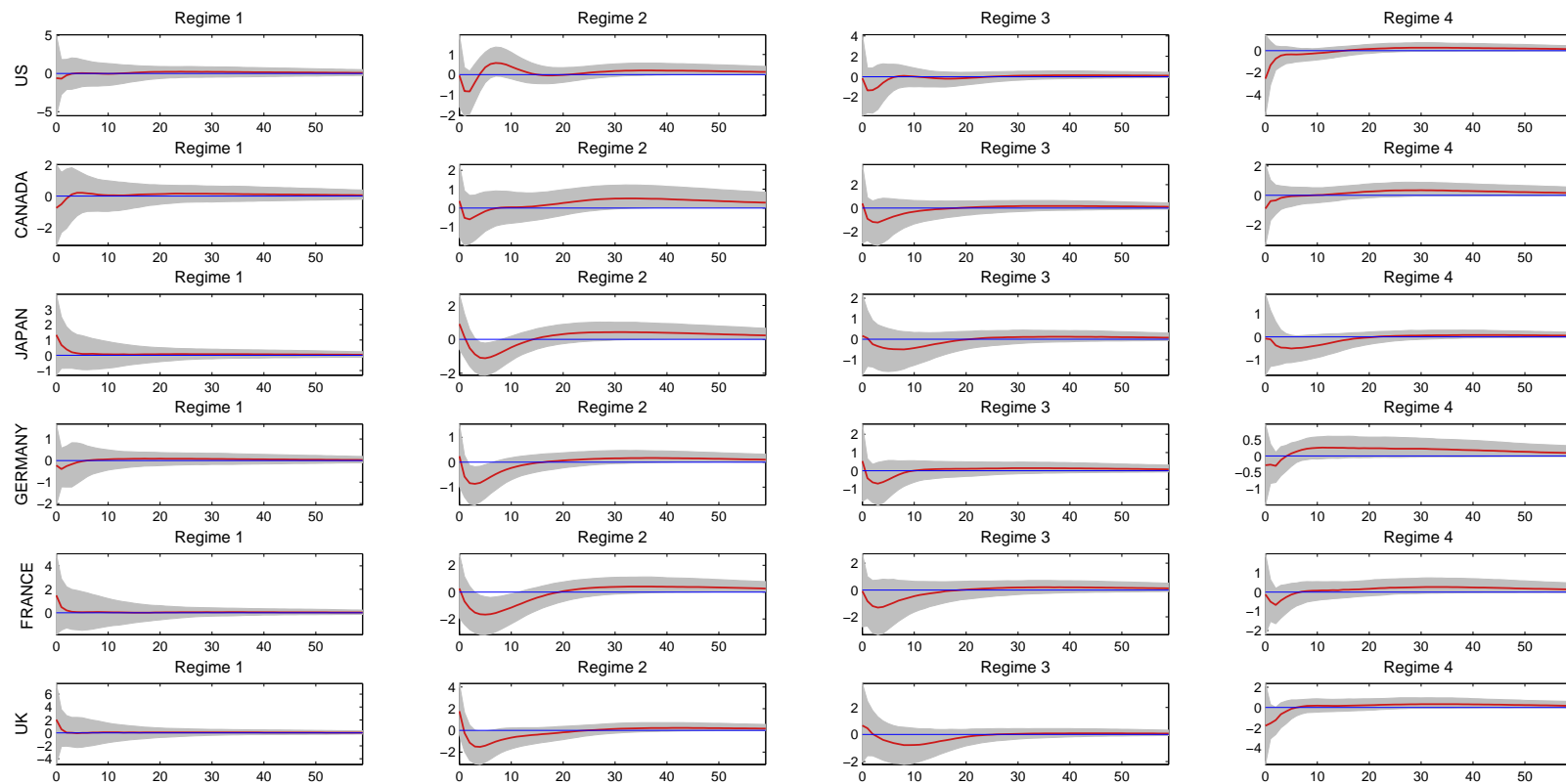


Figure 6: Dynamic responses of short-term real rates to a government spending shock normalized to have a 1% impact on U.S. government spending. 68% central posterior credible set.

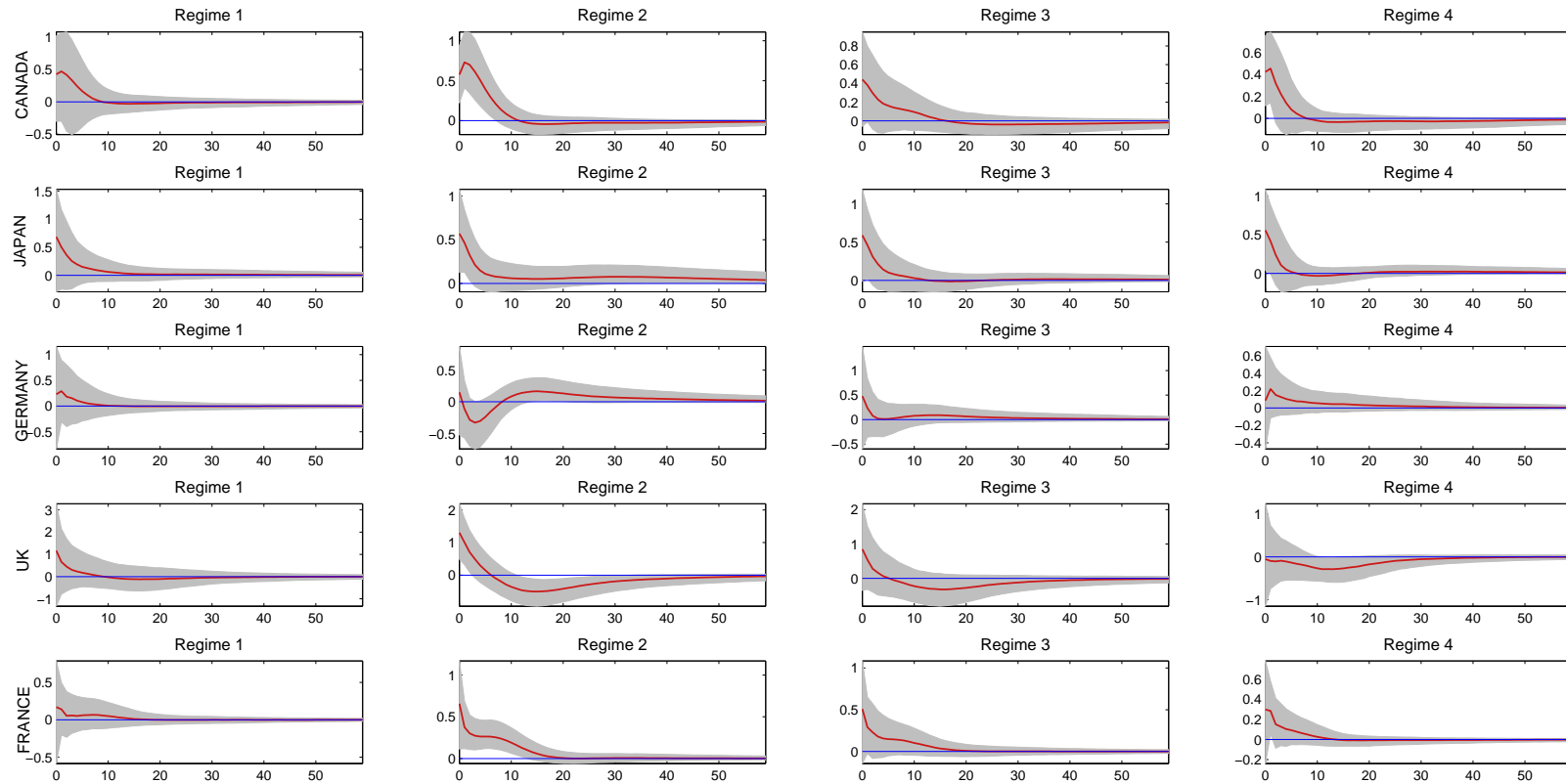


Figure 7: Dynamic responses of foreign consumption to a government spending shock normalized to have a 1% impact on U.S. government spending. 68% central posterior credible set.

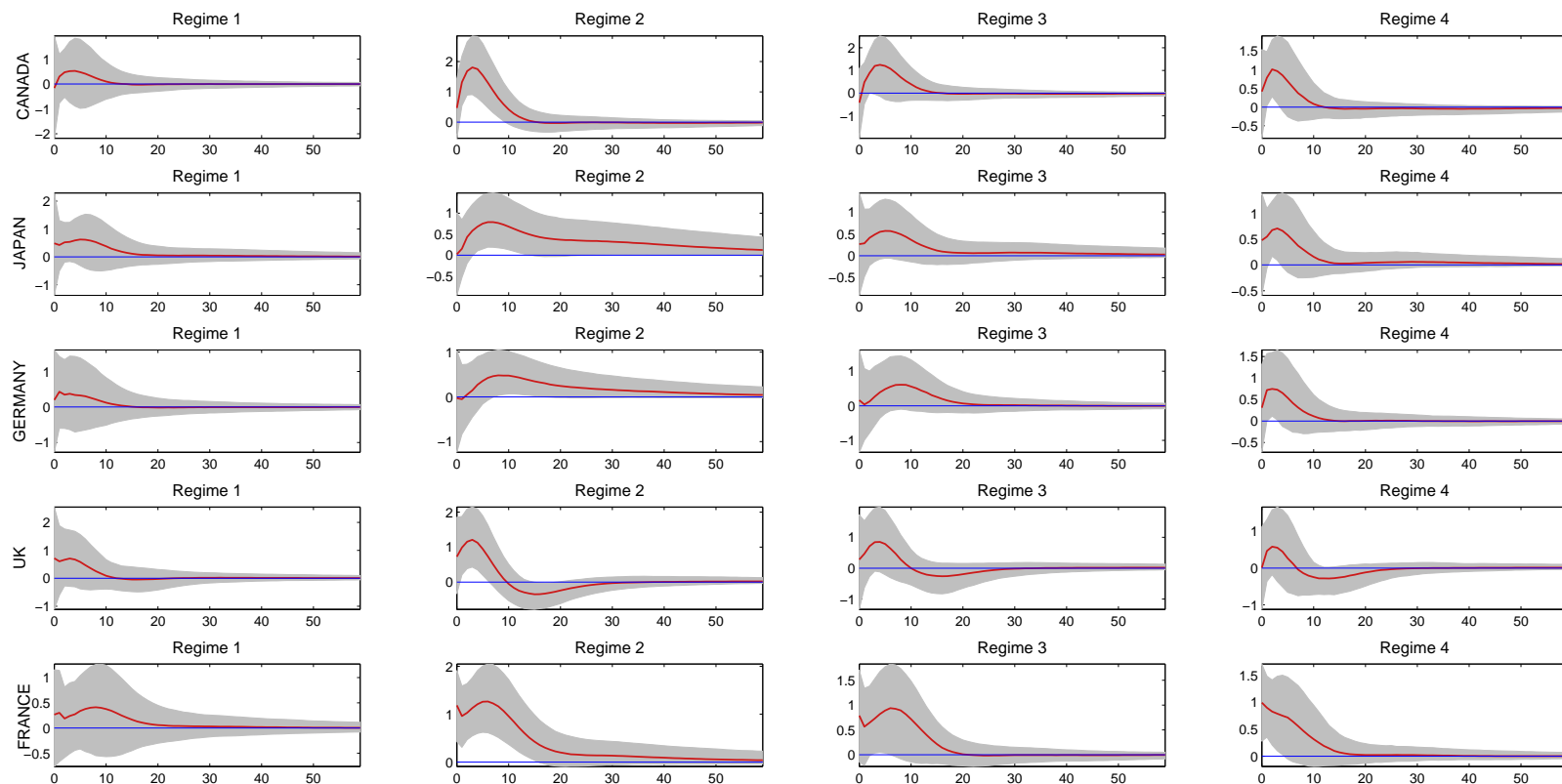


Figure 8: Dynamic responses of foreign investment to a government spending shock normalized to have a 1% impact on U.S. government spending. 68% central posterior credible set.



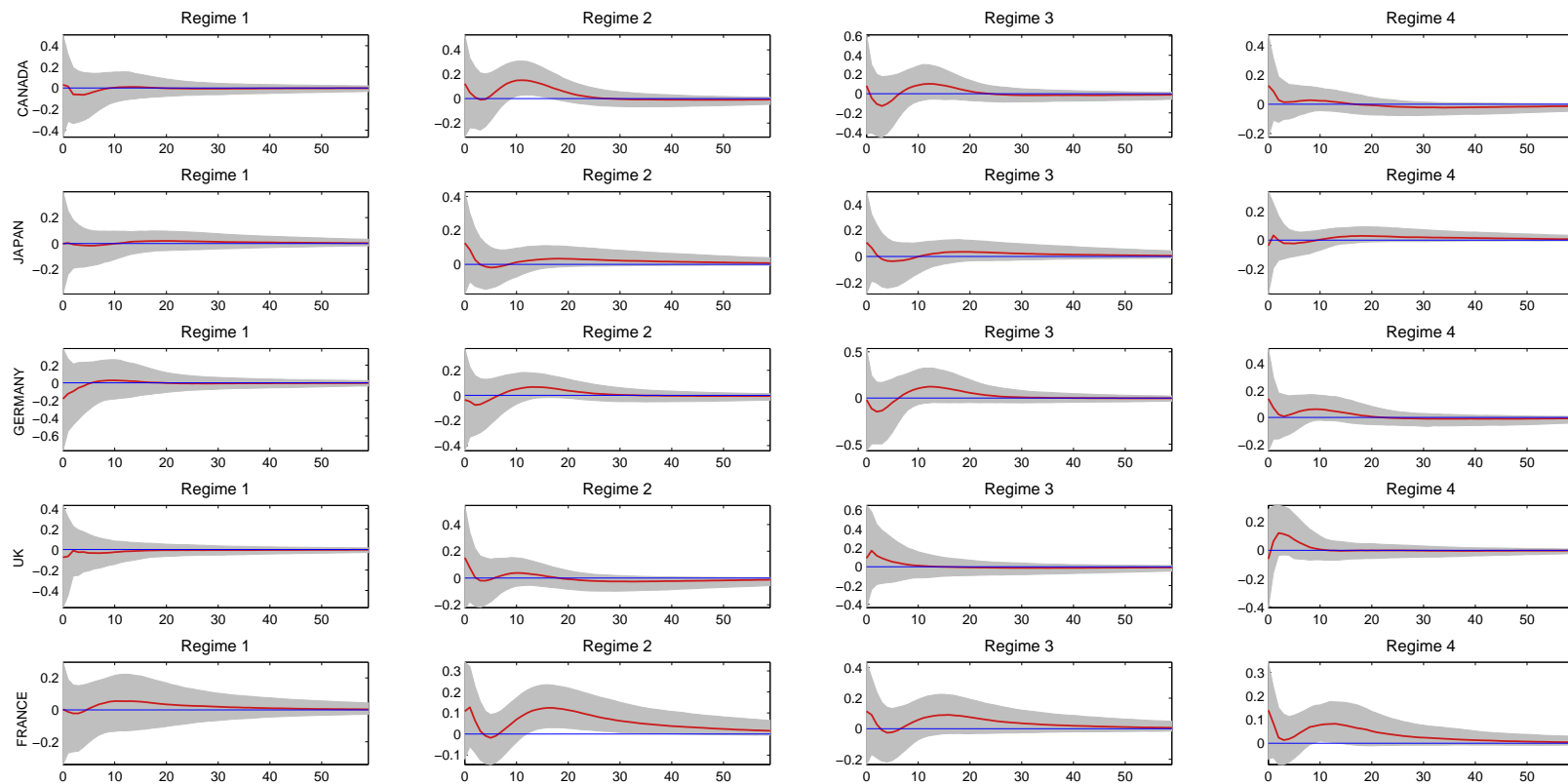


Figure 9: Dynamic responses of foreign government spending to a US government spending shock normalized to have a 1% impact on U.S. government spending. 68% central posterior credible set.

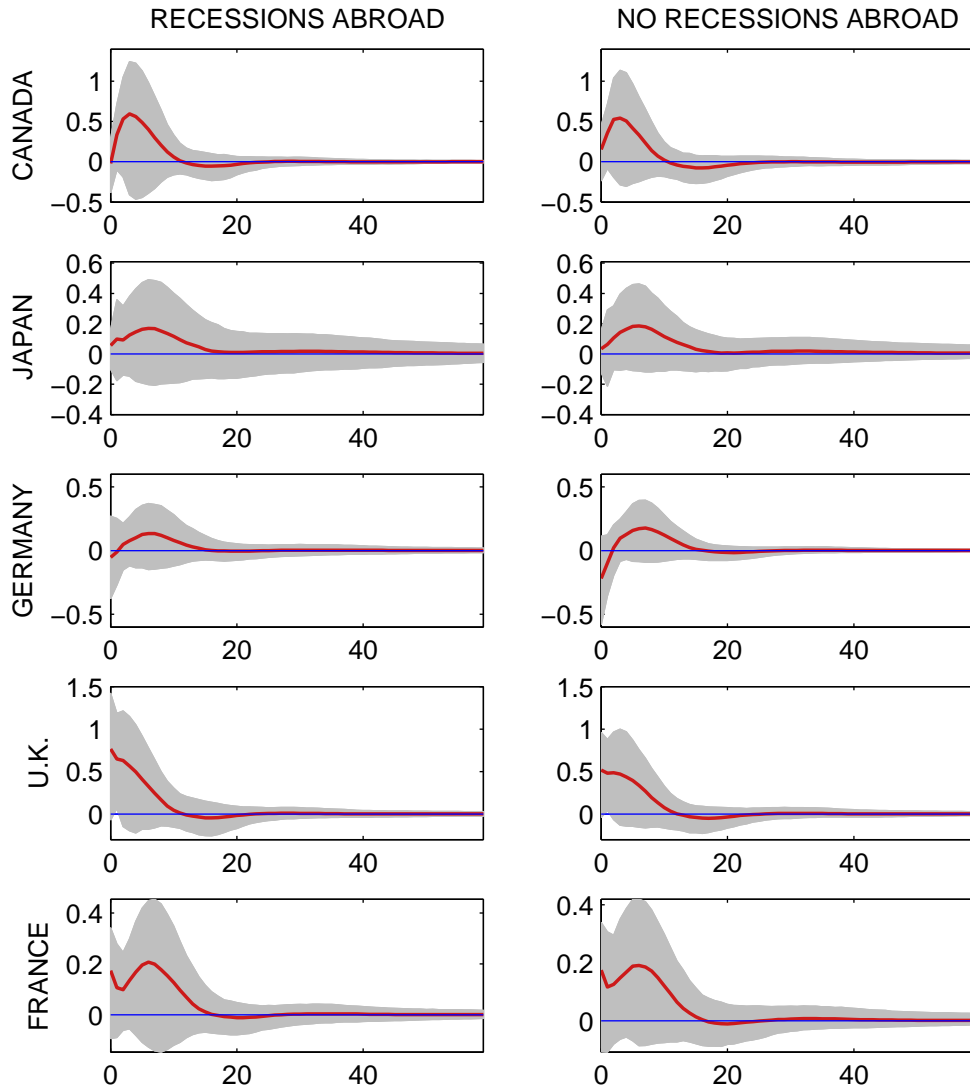


Figure 10: Dynamic responses of foreign GDP to a US government spending shock normalized to have a 1% impact on U.S. government spending. The impulse responses are conditional to recession in at least one country (regime 1) and no recession (regime 2). 68% central posterior credible set.

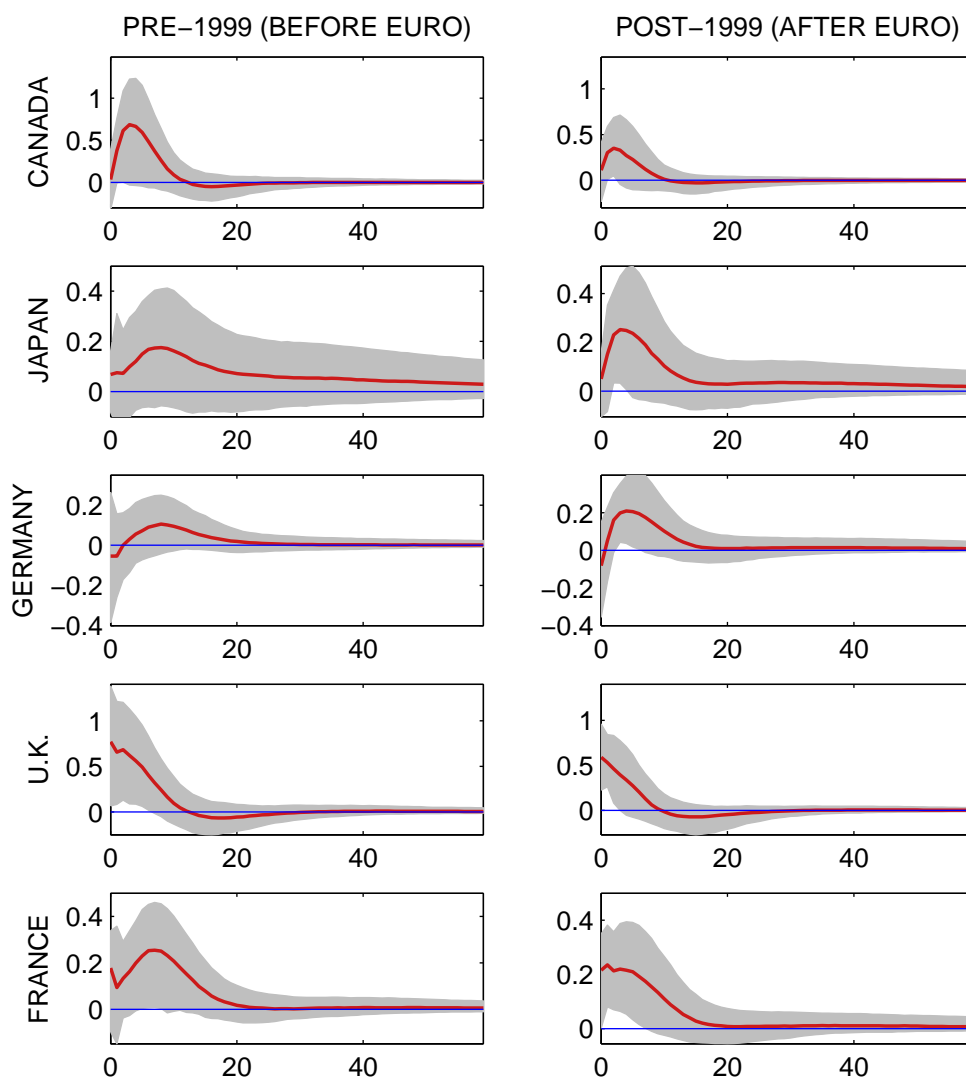


Figure 11: Dynamic responses of foreign DGP to a US government spending shock normalized to have a 1% impact on U.S. government spending. Impulse responses in regime 1 refer to the pre-1999 period, responses in regime 2 are post-1999. 68% central posterior credible set.

## A Gibbs sampler for the switching factor model

Consider the factor model defined by the following equations:

$$X_{it} = \beta_{i,S}F_t + \sigma_{i,Q}e_{it} \quad (4)$$

$$F_t = c + \sum_{j=1}^L \lambda_j F_{t-j} + \Sigma^{1/2}v_t \quad (5)$$

where  $S$  follows a  $M+1$  state Markov chain process with a restricted transition probability matrix  $\tilde{P}$ . Similarly, the state variable  $Q$  follows an  $M+1$  Markov chain process with a similarly restricted transition probability matrix  $\tilde{Q}$ .

The Gibbs sampler proceeds in the following steps:

Step 1. Sampling the parameters of the observation equation:  $c, \lambda_j$  and  $\Sigma$ . Conditional on an initial value for  $F_t$  (obtained via a principal component estimator), equation 5 is a Bayesian VAR model. Collecting the VAR coefficients ( $K \times (K \times L + 1)$ ) vector  $\Upsilon$  the LHS of equation 5 into the matrix  $Y_t$  and the RHS (ie lags and the intercept terms) of equation 5 into the matrix  $x_t$ , the conditional posterior of the VAR coefficients and the covariance matrix is given by

$$\begin{aligned} G(\Upsilon \setminus \Sigma) &\sim N(\text{vec}(\Upsilon^*), \Sigma \otimes (x^*{}'x^*)^{-1}) \\ G(\Sigma \setminus \Upsilon) &\sim IW(S^*, T^*) \end{aligned}$$

where

$$\begin{aligned} \Upsilon_S^* &= (x^*{}'x^*)^{-1} (x^*{}'Y^*) \\ S^* &= (Y^* - x^*\Upsilon^*)'(Y^* - x^*\Upsilon^*) \end{aligned}$$

where  $Y^* = [Y_t; Y_D]$ ,  $x^* = [x_t; X_D]$ .  $Y_D$  and  $X_D$  are dummy observations that implement the normal inverse Wishart prior and are defined as

$$Y_D = \begin{pmatrix} \frac{\text{diag}(\gamma_1\sigma_1 \dots \gamma_N\sigma_N)}{\tau} \\ 0_{N \times (P-1) \times N} \\ \dots \\ \text{diag}(\sigma_1 \dots \sigma_N) \\ \dots \\ 0_{1 \times N} \end{pmatrix}, \text{ and } X_D = \begin{pmatrix} \frac{J_P \otimes \text{diag}(\sigma_1 \dots \sigma_N)}{\tau} & 0_{NP \times 1} \\ 0_{N \times NP} & 0_{N \times 1} \\ \dots \\ 0_{1 \times NP} & c \end{pmatrix}$$

where  $\sigma_1 \dots \sigma_N$  represents standard deviations of the error term of an AR model estimated using the initial principal component estimate of the factors,  $\gamma_1$  to  $\gamma_N$  denotes the prior mean for the coefficients on the first lag,  $\tau$  is the tightness of the prior on the VAR coefficients and  $c$  is the tightness of the prior on the constant terms. We set  $\tau = 0.01$  and  $c = 0.000001$  in our implementation. The results are robust to higher values for  $\tau$  but these results become imprecise in regimes with a few number of observations.

Step 2. Sampling the parameters of the transition equation:  $\beta_{i,S}$ . Given a draw for  $F_t, S_t, Q_t$  and  $\sigma_{i,Q}$  the observation equation 4 is a sequence of linear regressions in each of the  $M$  regimes and heteroscedastic disturbances. The pattern of heteroscedasticity is determined by  $\sigma_{i,Q}$  and the state variable  $Q_t$ . The factor loadings in regime  $S_t = j$  are sampled from

$$\beta_{i,S} \sim N(\beta^*, M^*)$$

where the conditional mean and variance are estimated using the Kalman filter that takes the discrete changes in  $\sigma_{i,Q}$  into account. In other words, for each  $i$  and  $S = s$ , we express the regression model as the following state space system

$$\begin{aligned} X_t &= \beta_{s,t} F_t + \sigma_Q e_t \\ \beta_{s,t} &= \beta_{s,t-1} \end{aligned}$$

The final iteration of the Kalman filter delivers  $\beta^* = \beta_{s,T \setminus T}$  and  $M^* = P_{T \setminus T}$ . The Kalman filter is initialised using the prior mean  $B_{0 \setminus 0} = \hat{B}_i$  where  $\hat{B}_i$  represents the estimated factor loadings using the principal component estimator. The prior variance  $P_{0 \setminus 0}$  equals  $I_k \times 0.2$  where  $I_k$  is a  $k \times k$  identity matrix. Note that this is an application of the Carter and Kohn (2004) algorithm.

Step 3. The variance of the idiosyncratic components  $\sigma_{i,Q}$  for  $Q = j$  is sampled from the inverse Gamma density:

$$\sigma_{i,Q}^2 \sim IG(\sigma_i^*, T + V_0)$$

where  $\sigma_i^* = \tilde{e}'_{it} \tilde{e}_{it} + \sigma_0$ . The residual  $\tilde{e}_t = \iota[Q = j] \left( \sum_s^S I[S = s] (\tilde{X}_{it} - \tilde{F}_t \beta_{i,S}) \right)$  where  $I[S = s]$  is an indicator function while  $\iota[Q = j]$  selects observations when regime  $Q = j$ . We set the prior scale parameter  $\sigma_0 = 0.1$  and the prior degrees of freedom  $V_0 = 1$ .

Step 4. Sampling the markov states:  $S$ . Given a draw of the parameters of the observation equation from step 2 and an initial value for the transition probabilities  $\tilde{P}$  and the unobserved factors, the unobserved state variable  $S$  is drawn using Multi-Move Gibbs sampling to draw from the joint conditional density  $f(S_t | X_t, F_t, \beta_{i,S}, \sigma_{i,Q}, \tilde{P}, Q_t)$ . Kim and Nelson (1999, chapter 9) show that the Markov property of  $S_t$  implies that

$$f(S_t | Z_t) = f(S_T | X_T) \prod_{t=1}^{T-1} f(S_t | S_{t+1}, X_t) \quad (6)$$

where we have suppressed the conditioning arguments. This density can be simulated in two steps:

- Calculating  $f(S_T | X_T)$ : The Hamilton (1989) filter provides  $f(S_t | X_t), t = 1, \dots, T$ . The last iteration of the filter provides  $f(S_T | X_T)$ . Note that conditioning on  $Q_t$  allows us to take into account changes in  $\sigma_i$  across time when running the filter.

- Calculating  $f(S_t|S_{t+1}, X_t)$ : Kim and Nelson (1999) show that

$$f(S_t|S_{t+1}, X_t) \propto f(S_{t+1}|S_t) f(S_t|X_t) \quad (7)$$

where  $f(S_{t+1}|S_t)$  is the transition probability matrix and  $f(S_t|X_t)$  is obtained via Hamilton (1989) filter in step a. Kim and Nelson (1999) (pp. 214) show how to sample  $S_t$  from (7).

Step 5. Sampling the Markov states:  $Q_t$ . Given the parameters of the observation equation, a draw for the transition probabilities  $\tilde{Q}$ , the Markov states  $S_t$  and the factors, the algorithm in Step 4 above is used to simulate  $Q_t$ .

Step 6. Sampling the transition probabilities:  $\tilde{P}$ . The prior for the non zero elements of the transition probability matrix  $p_{ij}$  is of the following form

$$p_{ij}^0 = D(u_{ij})$$

where  $D(\cdot)$  denotes the Dirichlet distribution and  $u_{ij} = 15$  if  $i = j$  and  $u_{ij} = 1$  if  $i \neq j$ . This choice of  $u_{ij}$  implies that the regimes are fairly persistent. The posterior distribution is:

$$p_{ij} = D(u_{ij} + \eta_{ij})$$

where  $\eta_{ij}$  denotes the number of times regime  $i$  is followed by regime  $j$ .

Step 7. Sampling the transition probabilities  $\tilde{Q}$ : The priors and conditional posteriors are in step 5.

Step 8. Sampling the factors:  $F_t$  Given a draw for the parameters of the observation and transition equation and the state variable  $S_t$  the Carter and Kohn (1994) algorithm is used to draw from the conditional posterior of  $F_t$ .

To compute the impulse responses, we use 500,000 replications of the Gibbs sampler, discarding the first 10,000 replications as burn-in and retaining those draws which satisfy the sign restrictions set out above. In Figure 12 below we plot the recursive means of a sample of 1000 retained draws. The stability of these provides evidence for convergence.

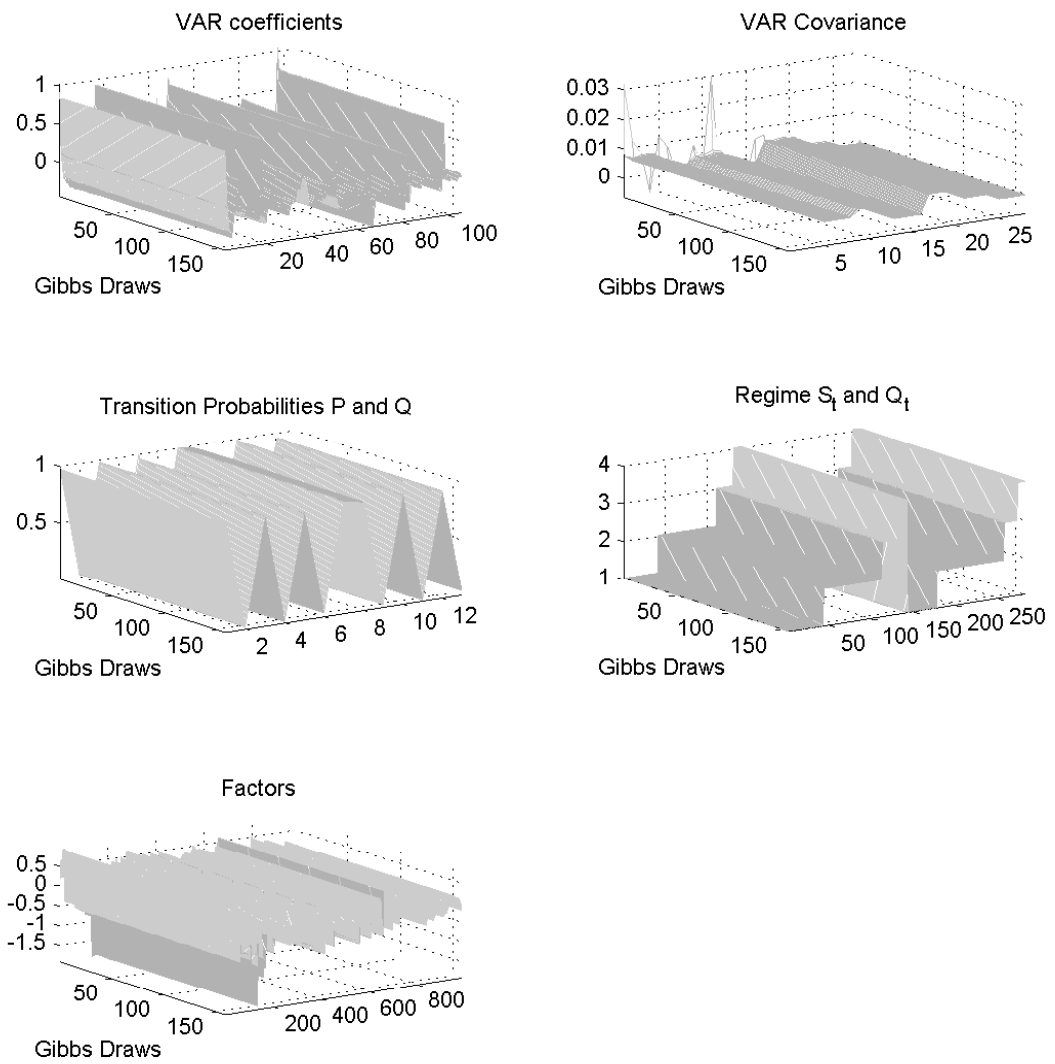


Figure 12: Recursive means of Gibbs draws.

## B Data description and variance decomposition

The variables in the dataset used for the estimation of the factor model are listed in table 3 below. The table also reports the data source and the transformation applied to the variable. BEA refers to Bureau of Economic Analysis (<http://www.bea.gov/>), FRED is Federal Reserve Economic Data (<http://research.stlouisfed.org/fred2/>), IFS is the IMF's International Financial Statistics ([www.imfstatistics.org/](http://www.imfstatistics.org/)) and GFD is the Global Financial Database ([www.globalfinancialdata.com](http://www.globalfinancialdata.com)). LD refers to the log difference transformation.

US fiscal variables are constructed as follows:

- Government spending: government consumption expenditures and gross investment (BEA Table 1.15 Line 21) deflated by GDP deflator (FRED series id GDPDEF) and divided by population ( FRED series id POP).
- Net Taxes: current receipts (BEA Table 3.1 Line 1) minus current transfer payments (BEA Table 3.1 Line 17) and interest payments (BEA Table 3.1 Line 22) deflated by GDP deflator and divided by population



Table 3: Data description, sources and transformations

Variable	Source	Transformation
Industrial Production index	FRED	LD
Industrial Production index Final Product	FRED	LD
Industrial Production index Consumer Goods	FRED	LD
Industrial Production index Durable Consumer Goods	FRED	LD
Industrial Production index Non-Durable Consumer Goods	FRED	LD
Industrial Production: Business Equipment	FRED	LD
Industrial Production: Materials	FRED	LD
Industrial Production: Durable Materials	FRED	LD
Industrial Production: Non-Durable Materials	FRED	LD
Industrial Production: Manufacturing	FRED	LD
Industrial Production: Electric and Gas Utilities	FRED	LD
Personal Consumption Expenditures: Durable Goods	FRED	LD
Personal Consumption Expenditures: Nondurable Goods	FRED	LD
Personal Consumption Expenditures: services	FRED	LD
Fixed Private investment	FRED	LD
Fixed Non-residential Private investment	FRED	LD
Fixed residential private investment	FRED	LD
Real Exports	FRED	LD
Real Imports	FRED	LD
Capacity Utilization: Total Industry	FRED	LD
Industrial Production: Nondurable Manufacturing (NAICS)	FRED	LD
Employment (construction)	FRED	LD
Employment (health and education)	FRED	LD
Employment (financial services)	FRED	LD
Employment (good producing)	FRED	LD
Employment (government)	FRED	LD
Employment (information services)	FRED	LD
Employment (leisure)	FRED	LD
Employment (natural resources mining)	FRED	LD

Employment (other services)	FRED	LD
Employment (professional and business services)	FRED	LD
Employment (retail trade)	FRED	LD
Employment (service providing ind)	FRED	LD
Employment (trade transportation utilities)	FRED	LD
Employment (wholesale trade)	FRED	LD
Civilian employment	FRED	LD
Civilian labor force	FRED	LD
Civilian participation rate	FRED	LD
Nonfarm Business Sector: Average Weekly Hours	FRED	LD
Average Weekly Overtime Hours of Production and Nonsupervisory	FRED	LD
Employees: Manufacturing		
Unemployment Rate	FRED	none
Average (Mean) Duration of Unemployment	FRED	LD
Civilians Unemployed for 5-14 Weeks	FRED	LD
Civilians Unemployed for 15-26 Weeks	FRED	LD
Civilians Unemployed - Less Than 5 Weeks	FRED	LD
Civilians Unemployed - 15 Weeks & Over	FRED	LD
Civilians Unemployed for 27 Weeks and Over	FRED	LD
Housing Starts: Total: New Privately Owned Housing Units Started	FRED	LD
Housing Starts in Northeast Census Region	FRED	LD
Housing Starts in Midwest Census Region	FRED	LD
Housing Starts in South Census Region	FRED	LD
Housing Starts in West Census Region	FRED	LD
ISM Manufacturing: New Orders Index	FRED	none
ISM Manufacturing: Inventories Index	FRED	none
ISM Manufacturing: Supplier Deliveries Index	FRED	none
GDP deflator	FRED	LD
Personal Consumption Expenditures: Chain-type Price Index	FRED	LD
Personal Consumption Expenditures: Chain-type Price Index Less	FRED	LD
Food and Energy		
Consumer Price Index for All Urban Consumers: All Items Less	FRED	LD

Food & Energy		
Gross Private Domestic Investment: Chain-type Price Index	FRED	LD
Consumer Price Index for All Urban Consumers: Energy	FRED	LD
Consumer Price Index for All Urban Consumers: Food	FRED	LD
Consumer Price Index for All Urban Consumers: Housing	FRED	LD
Consumer Price Index for All Urban Consumers: Apparel	FRED	LD
Consumer Price Index for All Urban Consumers: Transportation	FRED	LD
Consumer Price Index for All Urban Consumers: Medical Care	FRED	LD
Consumer Price Index for All Urban Consumers: Other G. and S.	FRED	LD
Export Price	IFS	LD
Import Price	IFS	LD
Description: Economist All-Commodity Dollar Index	GFD	LD
West Texas Intermediate Oil Price (US\$/Barrel)	GFD	LD
Nonfarm Business Sector: Output Per Hour of All Persons	FRED	LD
Nonfarm Business Sector: Real Compensation Per Hour	FRED	LD
Nonfarm Business Sector: Unit Labor Cost	FRED	LD
Federal Funds Rate	FRED	none
6-Month Treasury Bill: Secondary Market Rate	GFD	none
USA 1-year Constant Maturity Note Yield	GFD	none
USA 5-year Note Constant Maturity Yield	GFD	none
6mth-3mth	GFD	none
12mth-3mth	GFD	none
10yr-3mth	GFD	none
AAA-10yr	GFD	none
AAB-10yr	GFD	none
M1 Money Stock	GFD	LD
MZM Money Stock	GFD	LD
M2 Money Stock	GFD	LD
Monetary Base	GFD	LD
Non Borrowed Reserves	GFD	LD
Total Reserves	GFD	LD
Commercial and Industrial Loans at All Commercial Banks	GFD	LD

Total Consumer Credit Outstanding	GFD	LD
S&P 500 Total Return Index (w/GFD extension)	GFD	LD
Dow Jones Industrials Total Return Index	GFD	LD
S&P 500 Monthly Dividend Yield	GFD	none
S&P 500 P/E Ratio (As Reported Earnings)	GFD	none
Canada Real GDP Per Capita	IFS	LD
Canada CPI All items	IFS	LD
Canada 3 month Treasury Bill Rate	IFS	none
Canada Real Consumption	IFS	LD
Canada Gross Fixed Capital Formation	IFS	LD
Japan Real GDP per capita	IFS	LD
Japan CPI	IFS	LD
Japan 3 month Treasury Bill Rate	IFS	none
Japan Real Consumption	IFS	LD
Japan Gross Fixed Capital Formation	IFS	LD
Germany real GDP per capita	IFS	LD
Germany CPI	IFS	LD
Germany 3 month Treasury Bill Rate	IFS	none
Germany Real Consumption per capita	IFS	LD
Germany Gross Fixed Capital Formation	IFS	LD
UK Real GDP Per Capita	IFS	LD
UK CPI	IFS	LD
UK 3 month treasury bill rate	IFS	none
UK Real Consumption per Capita	IFS	LD
UK Gross Fixed Capital Formation	IFS	LD
France Real GDP per capita	IFS	LD
France CPI	IFS	LD
France 3 month treasury bill rate	IFS	none
France Consumption Per Capita	IFS	LD
France Gross fixed Capital formation	IFS	LD
Canadian Dollars to 1 US Dollar Real exchange Rate	IFS	LD
Yen Per Dollar real exchange rate	IFS	none

German Marks Per Dollar real exchange rate	IFS	none
UK Pounds Per Dollar real exchange rate	IFS	none
French Francs Per Dollar real exchange rate	IFS	none
Government Spending per capita	Authors' calculations	none
Net Taxes per capita	Authors' calculations	none
Real GDP Per Capita	FRED	LD
CPI	FRED	LD
3 month treasury bill yield	GFD	none
10 year Govt Bond Yield	GFD	none
Real Consumption expenditure per capita	FRED	LD
Terms of Trade	IFS	LD
Real Effective Exchange rate	IFS	LD
Canada Real Government Final Consumption Expenditure	OECD	LD
Japan Real Government Final Consumption Expenditure	OECD	LD
Germany Real Government Final Consumption Expenditure	OECD	LD
UK Real Government Final Consumption Expenditure	ONS	LD
France Real Government Final Consumption Expenditure	OECD	LD
Canada Real Government Gross Fixed Capital Formation	OECD	LD
Japan Real Government Gross Fixed Capital Formation	OECD	LD
France Real Government Gross Fixed Capital Formation	OECD	LD
UK Real Government Gross Fixed Capital Formation	ONS	LD
Canada Real Exports	GFD	LD
Japan Real Exports	GFD	LD
Germany Real Exports	GFD	LD
UK Real Exports	GFD	LD
France Real Exports	GFD	LD
Canada Real Imports	GFD	LD
Japan Real Imports	GFD	LD
France Real Imports	GFD	LD
Germany Real Imports	GFD	LD
UK Real Imports	GFD	LD

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Table 4: Contribution to the conditional and unconditional variance (%)

<i>variable</i>	<i>Forecast Error Variance Decomposition</i>			<i>Total Variance by the factors</i>
	<i>1 quarter</i>	<i>4 quarters</i>	<i>20 quarters</i>	
	<i>REGIME 1</i>			
Real GDP	0.3	5.3	7.8	59.6
Inflation	12.7	11.1	10.0	85.8
Short-term rate	8.4	8.3	8.5	97.9
Long-term rate	3.3	5.3	6.4	97.5
Govt spending	18.6	19.7	17.9	10.7
Net taxes	0.0	2.3	5.5	60.0
Consumption	3.3	10.8	11.1	49.1
Exchange rate	5.0	10.5	11.3	43.4
	<i>REGIME 2</i>			
Real GDP	0.6	9.3	11.1	63.6
Inflation	5.5	5.1	11.4	94.7
Short-term rate	5.9	4.3	9.7	99.5
Long-term rate	5.8	4.0	6.9	97.2
Govt spending	24.8	21.8	20.0	20.0
Net taxes	0.1	3.8	8.0	33.2
Consumption	4.7	13.8	13.7	54.2
Exchange Rate	17.3	19.5	18.3	41.7
	<i>REGIME 3</i>			
Real GDP	0.5	8.2	9.9	71.0
Inflation	8.3	7.3	10.6	86.9
Short-term rate	11.7	10.9	9.7	98.0
Long-term rate	6.3	7.4	7.8	98.3
Govt spending	20.6	19.8	18.4	33.1
Net taxes	0.0	2.6	6.5	73.0
Consumption	3.1	11.3	12.2	79.5
Exchange Rate	13.2	15.8	15.4	63.9
	<i>REGIME 4</i>			
Real GDP	1.6	6.5	7.4	74.0
Inflation	7.0	7.7	12.0	82.1
Short-term rate	0.0	1.4	5.3	97.7
Long-term rate	2.7	3.5	5.9	97.6
Govt spending	37.9	32.2	25.5	16.9
Net taxes	0.0	2.4	4.7	12.1
Consumption	2.9	8.4	8.3	71.4
Exchange Rate	20.9	22.9	21.5	79.5

Note: the first three columns report the percentage of variance explained by the government expenditure shock at horizons of one, four and twenty quarters. The last column refers to the percentage of unconditional variance explained by the factors.

## C The model -Web-Appendix-

### C.1 Firms

**Final good firms.** The consumption good is produced by final good firms as a bundle of home and foreign intermediate goods, and it is sold to consumers in perfect competition. In turn intermediate goods are produced by a continuum of monopolistically competitive firms at home and abroad under imperfect competition. The subscript  $i \in [0, 1]$  is used to index intermediate good firms together with their products and prices. We denote intermediate home goods by  $Y_{H,t}(i)$  and imported goods by  $Y_{F,t}(i)$ . Final good firms minimize expenditure subject to the aggregation constraint:

$$C_t = \Phi Y_{H,t}^{v/2} Y_{F,t}^{1-v/2}, \quad 1 \leq v \leq 2,$$

where  $\Phi = (v/2)^{v/2} (1 - (v/2))^{v/2}$ ,  $C_t$  denotes the aggregate consumption good, and  $Y_{Ht}$  and  $Y_{Ft}$  are the bundles of goods produced in the home and foreign country, respectively. The parameter  $v$  captures home bias in the production of the consumption good: the higher is  $v$ , the more closed is the economy. In the special case where  $v = 1$  there is no home-bias while for  $v = 2$  the economy is autarkic. Final good firms in the foreign country are symmetric, with the aggregation technology having weight  $v/2$  on the foreign good as in Cook and Devereaux (2011).  $Y_{Ht}$  and  $Y_{Ft}$  are CES aggregates over a continuum of goods:

$$Y_{Ht} = \left[ \int_0^1 Y_{H,t}(i)^{1-1/\varepsilon_{Y,t}} di \right]^{\frac{1}{(1-1/\varepsilon_{Y,t})}}, \quad Y_{Ft} = \left[ \int_0^1 Y_{F,t}(i)^{1-1/\varepsilon_{Y,t}^*} di \right]^{\frac{1}{(1-1/\varepsilon_{Y,t}^*)}},$$

where  $\varepsilon_{Y,t} > 1$  and  $\varepsilon_{Y,t}^* > 1$  are the time varying elasticities of substitution for the home and foreign goods, respectively. We assume that the elasticity of substitution is procyclical and follows the process  $\hat{\varepsilon}_{Y,t} = \eta \hat{y}_{H,t} \xi_t$  for the home country good, and  $\hat{\varepsilon}_{Y,t}^* = \eta \hat{y}_{F,t}$  for the foreign country good, where  $\hat{\varepsilon}_{Y,t}$  and  $\hat{y}_{H,t}$  are used to denote percentage deviations of the home elasticity of substitution and intermediate output from their steady-state values and  $\xi_t$  captures a mark-up shock, which follows a log-linear AR(1) stochastic process. The parameter  $\eta > 0$  governs the procyclical behavior of the elasticity of demand: the higher is  $\eta$ , the more countercyclical is the mark-up.<sup>12</sup>

Expenditure minimization implies the following price indices associated with the home and foreign intermediate good bundles:

$$P_{H,t} = \left[ \int_0^1 P_{H,t}(i)^{1-\varepsilon_{Y,t}} di \right]^{\frac{1}{1-\varepsilon_{Y,t}}}, \quad P_{F,t} = \left[ \int_0^1 P_{F,t}(i)^{1-\varepsilon_{Y,t}^*} di \right]^{\frac{1}{1-\varepsilon_{Y,t}^*}},$$

---

<sup>12</sup>This reduced form process for the elasticity of substitution is intended to capture the dynamics of the mark-up as described by Ravn, Schmitt-Grohe and Uribe (2007).

and the aggregate CPI is  $P_t = P_{H,t}^{v/2} P_{F,t}^{1-v/2}$ . Minimization of expenditure by the domestic final good producers yields the following demand functions for the home and foreign intermediate good bundles:

$$Y_{H,t}^D = \frac{v}{2} \frac{P_t}{P_{H,t}} C_t,$$

$$Y_{F,t}^D = \left(1 - \frac{v}{2}\right) \frac{P_t}{P_{F,t}} C_t.$$

The demand for a generic variety  $Y_{H,t}(i)$  and  $Y_{F,t}(i)$  is given by:

$$Y_{H,t}^D(i) = \left[ \frac{P_{H,t}(i)}{P_{H,t}} \right]^{-\varepsilon_{Y,t}} Y_{H,t}^D,$$

$$Y_{F,t}^D(i) = \left[ \frac{P_{F,t}(i)}{P_{F,t}} \right]^{-\varepsilon_{Y,t}^*} Y_{F,t}^D.$$

Symmetric demand functions can be derived for the foreign final good producer.

**Intermediate good firms.** Home intermediate firms employ labor to produce a differentiated good according to the production function:

$$Y_{H,t}(i) = \Psi_t L_t(i),$$

where  $\Psi_t$  is a standard TFP shock that follows an AR(1) log-linear stochastic process and  $L_t(i)$  is an aggregator of differentiated labor services supplied by the households, which we define below. The unit cost of labor services is denoted by  $W_t$  and is interpreted as the aggregate nominal wage index. Firm  $i$  maximizes profits defined as  $\Pi_t(i) = P_{H,t}(i)Y_{H,t}(i) - W_t L_t(i)$ , and resets the price according to Calvo pricing, where the probability of readjusting prices in each period is  $1 - \delta_p$ . The home firm faces the elasticity of substitution  $\varepsilon_{Y,t}$  when selling to home and foreign customers, as well as to the domestic government. Profit maximization leads to the following optimal price setting condition:

$$\tilde{P}_{H,t}(i) = \frac{E_t \sum_{s=0}^{\infty} \varepsilon_{Y,t+s} \Lambda_{t,t+s} \delta_p^s \frac{W_{t+s}}{A_{t+s}} Y_{t+s}(i)}{E_t \sum_{s=0}^{\infty} (\varepsilon_{Y,t+s} - 1) \Lambda_{t,t+s} \delta_p^s Y_{t+s}(i)}, \quad (8)$$

where  $E_t$  denotes the expectation operator,  $\tilde{P}_{H,t}(i)$  is the new price set by the firms that are allowed to readjust their prices and  $\Lambda_{t,t+j} = \beta^j (P_t / \vartheta_t C_{A,t}^{-\sigma}) (\vartheta_{t+j} C_{A,t+j}^{-\sigma} / P_{t+j})$  is the stochastic nominal discount factor of the households, which we derive in the following section. In the aggregate, the price index for the home good follows the process given by:

$$P_{H,t} = \left[ (1 - \delta_p) \tilde{P}_{H,t}^{1-\varepsilon_{Y,t}} + \delta_p P_{H,t-1}^{1-\varepsilon_{Y,t}} \right]^{1/(1-\varepsilon_{Y,t})}. \quad (9)$$

The behavior of foreign firms and the foreign good price index can be described analogously.



## C.2 Households

The world is populated by a unit measure of monopolistically competitive households indexed by  $h \in [0, 1]$  (Erceg et al., 2000). Each household supplies differentiated labor services  $L_t(h)$  to the production sector and receives a wage payment  $W_t(h)$ . These different types of labor are combined into the following Dixit-Stiglitz aggregator, which is used as an input in the intermediate sector:

$$L_t = \left[ \int_0^1 L_t(h)^{1-1/\varepsilon_L} dh \right]^{\frac{1}{(1-1/\varepsilon_L)}},$$

where  $\varepsilon_L$  denotes the elasticity of substitution between different types of labor services. The unit cost associated with the labor index is:

$$W_t = \left[ \int_0^1 W_t(h)^{1-\varepsilon_L} dh \right]^{\frac{1}{1-\varepsilon_L}},$$

and the aggregate demand for labor services of type  $h$  is:

$$L_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{-\varepsilon_L} L_t. \quad (10)$$

A fraction  $1 - \lambda$  of households are asset holders and they are indexed by subscript  $A$ . These households are the owners of firms and have access to the financial market. The remaining fraction of households  $\lambda$  do not participate to the asset market and are indexed by  $N$ .

**Asset holders.** In each period, the asset holding household derives utility from consumption and disutility from work. This household maximizes lifetime utility defined as:

$$U_{A,t}(h) = E_t \sum_{s=0}^{\infty} \beta^s \vartheta_{t+s} \left[ \frac{1}{1-\sigma} C_{A,t+s}^{1-\sigma}(h) - \frac{\chi_{t+s}}{1+\varphi} L_{A,t+s}^{1+\varphi}(h) \right], \quad (11)$$

where  $\beta$  is the discount factor,  $\sigma$  governs the degree of risk aversion,  $\varphi$  is the inverse Frisch elasticity of labor supply,  $\vartheta_t$  is a preference shock and  $\chi_t$  is a labor supply shock, both following a standard AR(1) log-linear process.

Households supply labor services to a continuum of firms in their own country and receive in return the nominal average wage  $W_t(h)$ . Each period households receive profits  $\Upsilon_t$  and pay lump sum taxes  $T_{A,t}$ . Letting  $\Omega_{t+1}$  denote the payoff in units of domestic currency in period  $t + 1$  of the portfolio held at the end of period  $t$ , the budget constraint of the household is given by:

$$P_t C_{A,t}(h) + E_t \Lambda_{t,t+1} \Omega_{t+1}(h) = W_t(h) L_{A,t}(h) + \Upsilon_t(h) - T_{A,t}(h) + \Omega_t(h),$$

where  $\Lambda_{t,t+1}$  is the stochastic discount factor for the one-period ahead nominal payoffs relevant to the domestic household. Foreign asset holder households have analogous preferences and face an analogous budget constraint.

Combining the first order conditions with respect to  $C_{A,t}(h)$  and  $\Omega_{t+1}(h)$  yields (after invoking symmetry and thus dropping the household-specific index  $h$ ):

$$E_t \beta \left( \frac{C_{A,t+1}}{C_{A,t}} \right)^{-\sigma} \frac{\vartheta_{t+1}}{\vartheta_t} \frac{P_t}{P_{t+1}} = E_t \Lambda_{t,t+1}. \quad (12)$$

If  $E_t \Lambda_{t,t+1}$  is the price of a riskless one-period discount bond paying one unit of domestic currency in  $t + 1$ , and  $R_t = 1/(E_t \Lambda_{t,t+1})$  is its gross return, the equation above can be rearranged to obtain the standard Euler condition:

$$\beta R_t E_t \left( \frac{C_{A,t+1}}{C_{A,t}} \right)^{-\sigma} \frac{\vartheta_{t+1}}{\vartheta_t} \frac{P_t}{P_{t+1}} = 1. \quad (13)$$

Under the assumption of complete asset markets, a first order condition analogous to (12) must hold for the foreign country:

$$E_t \beta \left( \frac{C_{A,t+1}^*}{C_{A,t}^*} \right)^{-\sigma} \left( \frac{S_t P_t^*}{S_{t+1} P_{t+1}^*} \right) = E_t \Lambda_{t,t+1},$$

where  $S_t$  denotes the nominal exchange rate (home price of the foreign currency), and  $P_t^* = P_{F,t}^{*v/2} P_{H,t}^{*1-v/2}$  is the foreign CPI. The real exchange rate is defined as  $\mathcal{Q}_t = S_t P_t^*/P_t$  and home terms of trade as  $\mathcal{T}_t = S_t P_{F,t}^*/P_{H,t}$ . Combining the domestic and the foreign Euler conditions to eliminate  $\Lambda_{t,t+1}$  and assuming that the law of one price holds in individual goods and both home and foreign composite consumption goods (i.e., so that  $P_{F,t} = S P_{F,t}^*$ ), it is possible to obtain:

$$\vartheta_t C_{A,t}^\sigma = C_{A,t}^{*\sigma} \mathcal{Q}_t = C_{A,t}^{*\sigma} \mathcal{T}_t^{v-1}, \quad (14)$$

which implies that state contingent marginal utilities are equalized across countries.

Forward looking households set wages in staggered contracts. Each period, households reset their wage with probability  $1 - \delta_w$ . In any period in which the household is allowed to renegotiate the wage, the household maximizes the utility function in (11) subject to a sequence of demand schedules for their labor type (10). The first order condition reads:

$$\sum_{s=0}^{\infty} (\beta \delta_w)^s E_t \frac{L_{A,t+s|t}}{\chi_{t+s} C_{A,t+s}^\sigma} \left\{ \frac{\tilde{W}_t}{P_{t+s}} - \frac{\varepsilon_L}{\varepsilon_L - 1} MRS_{A,t+s|t} \right\} = 0, \quad (15)$$

where  $L_{A,t+s|t}$  denotes the quantity demanded at time  $t + s$  of a labor type whose wage was renegotiated at time  $t$ , and  $MRS_{A,t+s|t} \equiv \chi_{t+s} C_{A,t+s}^\sigma L_{A,t+s|t}^\varphi$  is the marginal rate of substitution between time consumption and labor at time  $t + s$  conditional on the wage being renegotiated at time  $t$ .

**Non asset holders.** Non asset holders, also known as rule of thumb consumers (ROT) choose consumption  $C_{N,t}$  and supply labor  $L_{N,t}$  to maximize the flow of utility  $U_{N,t}$  on a period-by-period basis:

$$U_{N,t}(h) = \frac{1}{1-\sigma} C_{N,t}^{1-\sigma}(h) - \frac{\lambda_t}{1+\varphi} L_{N,t}^{1+\varphi}(h),$$

subject to the constraint that consumption expenditure equals net income:

$$P_t C_{N,t}(h) = W_t L_{N,t}(h) - T_{N,t}(h). \quad (16)$$

The above budget constraint assumes that non asset holders set their wage to be the average wage of the optimizing households. Since ROT consumers face the same labor demand schedule as the forward looking households, each ROT household works the same number of hours as the average for asset holding households.

### C.3 Monetary and fiscal policy

We assume that the monetary authority sets the gross nominal interest rate  $R_t$  according to the following Taylor rule:

$$\frac{R_t}{R} = \left[ \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \right]^{(1-\rho_R)} \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \varsigma_t, \quad (17)$$

where variables without time subscript indicate steady-state values and  $\pi_t = P_t/P_{t-1}$  denotes the rate of inflation of the consumption bundle. The parameters  $\phi_\pi$  and  $\phi_y$  measure the strength of the response of the short-term nominal interest rate to deviations of the home CPI inflation rate and domestic output from their respective steady-state values. Interest rate smoothing is captured by the parameter  $\rho_R$ , and  $\varsigma_t$  denotes a standard monetary shock, which follows an AR(1) log-linear stochastic process.

Since the expectation hypothesis holds in this framework, nominal and real long term rates are related to the expected path of short term rates. For example, the real yield on a  $n$ -period bond at time  $t$ ,  $R_{t,t+n}^r$ , is related to the sequence of one-period real rates as follows:

$$R_{t,t+n}^r = E_t \prod_{j=0}^{n-1} \left( \frac{R_{t+j}}{\pi_{t+j+1}} \right), \quad (18)$$

For future reference we define the long term real interest rate as the real yield of a bond of infinite duration, that is,  $\lim_{n \rightarrow \infty} R_{t,t+n}^r$ .

Following Corsetti, Meier and Müller (2010), we assume that the government can finance its spending either through lump-sum taxes,  $T_t$ , or through issuance of one-period nominal bonds,  $D_t$ . The budget constraint of the government is:

$$R_t^{-1} D_{t+1} = D_t + P_{H,t} G_t - T_t, \quad (19)$$

where  $T_t = \lambda T_{N,t} + (1 - \lambda) T_{A,t}$ , and  $G_t$  denotes government spending, which is assumed to be directed exclusively on domestic goods. Denoting real debt by  $D_t^r = D_t/P_{t-1}$  and real taxes as  $T_t^r = T_t/P_t$ , we let fiscal policy be described by the following feedback rule:

$$G_t = (1 - \rho)G + \rho G_{t-1} + \psi_{GD} D_t^r + \theta_t, \quad (20)$$

$$T_t^r = G \left( \frac{P_{H,t} G_t}{P_t G} \right)^{\psi_{TG}} + \psi_{TD} D_t^r, \quad (21)$$

where  $\theta_t$  denote exogenous i.i.d. shocks to government spending. The response of government spending to government debt is captured by the parameter  $\psi_{GD}$ , while the parameters  $\psi_{TG}$  and  $\psi_{TD}$  capture the responsiveness of taxes to government spending and debt, respectively. Taxes for asset holders and ROT consumers are assumed to follow the same aggregate rule defined in equation (21).

## C.4 Equilibrium

Market clearing requires that the supply of intermediate goods equals government spending plus total demand from home and foreign final good firms. The demand for a particular variety of the home good is therefore defined by:

$$Y_{H,t}(i) = \left[ \frac{P_{H,t}(i)}{P_{H,t}} \right]^{-\varepsilon_{Y,t}} \left[ \frac{v}{2} \frac{P_t}{P_{H,t}} C_t + \left( 1 - \frac{v}{2} \right) \frac{S_t P_t^*}{P_{F,t}} C_t^* + G_t \right].$$

Since all firms are identical at equilibrium, aggregating across firms yields the market clearing condition for the home good:

$$Y_{H,t} = \frac{v}{2} \frac{P_t}{P_{H,t}} C_t + \left( 1 - \frac{v}{2} \right) \frac{S_t P_t^*}{P_{H,t}} C_t^* + G_t. \quad (22)$$

By symmetry, the aggregate market clearing condition for foreign output can be written:

$$Y_{F,t} = \frac{v}{2} \frac{P_t^*}{P_{F,t}^*} C_t^* + \left( 1 - \frac{v}{2} \right) \frac{P_t}{S_t P_{F,t}^*} C_t + G_t^*. \quad (23)$$

In addition, total consumption in the home country is given by:

$$C_t = \lambda C_{N,t} + (1 - \lambda) C_{A,t}. \quad (24)$$

An analogous condition holds for the foreign country.

## D The international transmission of fiscal policy -Web-Appendix-

In this section we illustrate the transmission mechanism of government spending shocks by comparing impulse responses that are produced under different restrictions of the parameter

space. We begin by illustrating impulse responses in the neo-classical benchmark. Then, we distinguish between three competing (but not necessarily mutually exclusive) specifications, which differ by the nature of the restrictions they impose on either the structure of the economy or the stance of fiscal policy: (i) countercyclical mark-up, (ii) rule of thumb consumers and (iii) spending reversals. In the last part of this section, we show that despite the different set of assumptions, it is possible to identify a number of inequality constraints that are not overturned by empirically plausible perturbations of the parameter space spanning all theoretical specifications.

## D.1 Parameterization

In Table 5, we report the values taken by the parameters of the model under three specifications, in each of which only one of the mechanisms for the transmission of fiscal policy described above is at play. For illustrative purposes, we also report a further restrictive parameterization, which corresponds to the neo-classical benchmark. The top panel reports the set of common values, which are relatively standard. The discount factor is set to 0.99, and government spending is assumed to be 20% of output in steady state, which implies a private consumption-output ratio of 0.8. Following Cook and Devereux (2011), we set the elasticity of intertemporal substitution to 2 and the inverse of the Frisch elasticity to 1. The home bias in consumption is 1.7, which implies an import-output ratio of 0.12 as in Corsetti, Meier and Müller (2010), whereas the elasticity of substitution across labor services is set to 4.5 (Galí, 2011). To develop intuition for the logic behind each mechanism, in the illustrative calculations of this section we restrict monetary policy to react only to inflation, with a parameter of 1.5 as suggested by Taylor (1993). In the nested model used below to derive the theory-robust sign restrictions, we verify robustness to a range of values for the interest rate response to output and the smoothing parameter. Finally, the persistence of government spending is 0.9, consistent with Galí and Perotti (2003).

The neo-classical benchmark in the first column is characterized by monopolistically competitive households and firms, flexible prices ( $\delta_p = 0$ ), flexible wages, ( $\delta_w = 0$ ), and balanced government budget ( $\psi_{TG} = 1$ ). The parameterization in the second column differs from the neoclassical benchmark only insofar as the mark-up is assumed to be countercyclical. The remaining two columns belong to the family of new-keynesian models. Beyond the assumptions of sticky prices and wages, the model in column three (four) differs from the benchmark neoclassical model for the introduction of rule of thumb consumers (policy reversals).

In the second column, we select a value for the parameter governing the countercyclicality of the price markup lying at the lowest range of values that are sufficient to generate an increase in consumption and a real exchange depreciation in response to an unanticipated increase in government spending. For higher values of  $\eta$ , the transmission of fiscal policy would remain qualitatively unchanged. For the new-Keynesian models, we calibrate the degree of price rigidity so that the probability of keeping prices fixed at any given point in time is 50% as in Christiano Eichenbaum and Evans (2005), implying an average frequency of

Table 5: Parameterization

<i>description of the parameters</i>		<i>values</i>			
discount factor	$\beta$	0.99			
consumption-output ratio	$c_y$	0.8			
elasticity of intertemporal substitution	$\sigma$	2			
inverse of Frisch elasticity	$\varphi$	1			
home bias in consumption	$v$	1.7			
elasticity of labor substitution	$\varepsilon_L$	4.5			
interest rate response to inflation	$\phi_\pi$	1.5			
interest rate response to output	$\phi_y$	0			
interest rate smoothing	$\rho_R$	0			
gov. spending autocorrelation	$\rho$	0.9			
		<i>theoretical models</i>			
		NC	CM	ROT	SR
probability of price fixed	$\delta_p$	<b>0</b>	0	0.5	0.5
probability of wage fixed	$\delta_w$	<b>0</b>	0	0.8	0.8
elasticity of markup to demand	$\eta$	0	<b>1.1</b>	0	0
share of rule of thumb consumers	$\lambda$	0	0	<b>0.4</b>	0
gov. spending sensitivity of taxes	$\psi_{TG}$	1	1	1	<b>0</b>
debt sensitivity of spending	$\psi_G$	0	0	0	<b>-0.02</b>
debt sensitivity of taxes	$\psi_{TD}$	0	0	0	<b>0.02</b>

Note: same values apply to the foreign economy. NC, ROT, CM and SR stands for Neo-Classical, Rule Of Thumb, Countercyclical Markup and Spending Reversals.

price adjustment of two quarters. In addition, we assume that in each quarter, the probability that a household experiences a nominal wage change is 20%, implying a coefficient of  $\delta_w = 0.8$ . This number lies within the range of values consistent with estimates by Barattieri, Basu and Gottshalk (2014) and Gertler, Sala and Trigari (2008). The share of rule of thumb consumers in the third model is equal to 0.4, consistent with evidence in Campbell and Mankiw (1989), and Misra and Surico (2014). The last column refers to the spending reversals model, which relaxes the assumption of a balanced government budget at each point in time by allowing spending and taxes to depend on the level of real debt in a way that is consistent with the evidence in Galí and Perotti (2003). The parameters governing the evolution of government spending and taxation are calibrated along the lines of Corsetti, Meier and Müller (2010).<sup>13</sup>

To illustrate differences and similarities across the three specifications and the neo-classical benchmark, Table 5 makes clear that in each column there is only one specific mechanism at play for the transmission of fiscal policy, as the relevant parameters associated with the identifying features of the other models are set so as to shut down those alternative channels.

## D.2 Dynamic responses to a government spending shock

In Figure 13, we present the dynamic responses of domestic and foreign variables to a domestic government spending shock in the neo-classical benchmark and the three specifications augmented according to the parameter values in the last three columns of Table ???. The blue solid line refers to the neo-classical transmission mechanism and the red dots to the model with countercyclical mark-up. As for the new-Keynesian models, the purple circles and green squares denote the impulse responses associated with the existence of rule of thumb consumers and spending reversals, respectively.

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<sup>13</sup>As discussed in Corsetti, Meier and Mueller (2010), a coefficient of  $\psi_{TD} = 0.02$  ensures the stationarity of debt even in the absence of spending reversals, i.e., it ensures that the condition  $(1 - \psi_{TD})/\beta < 1$  holds at the calibrated equilibrium.

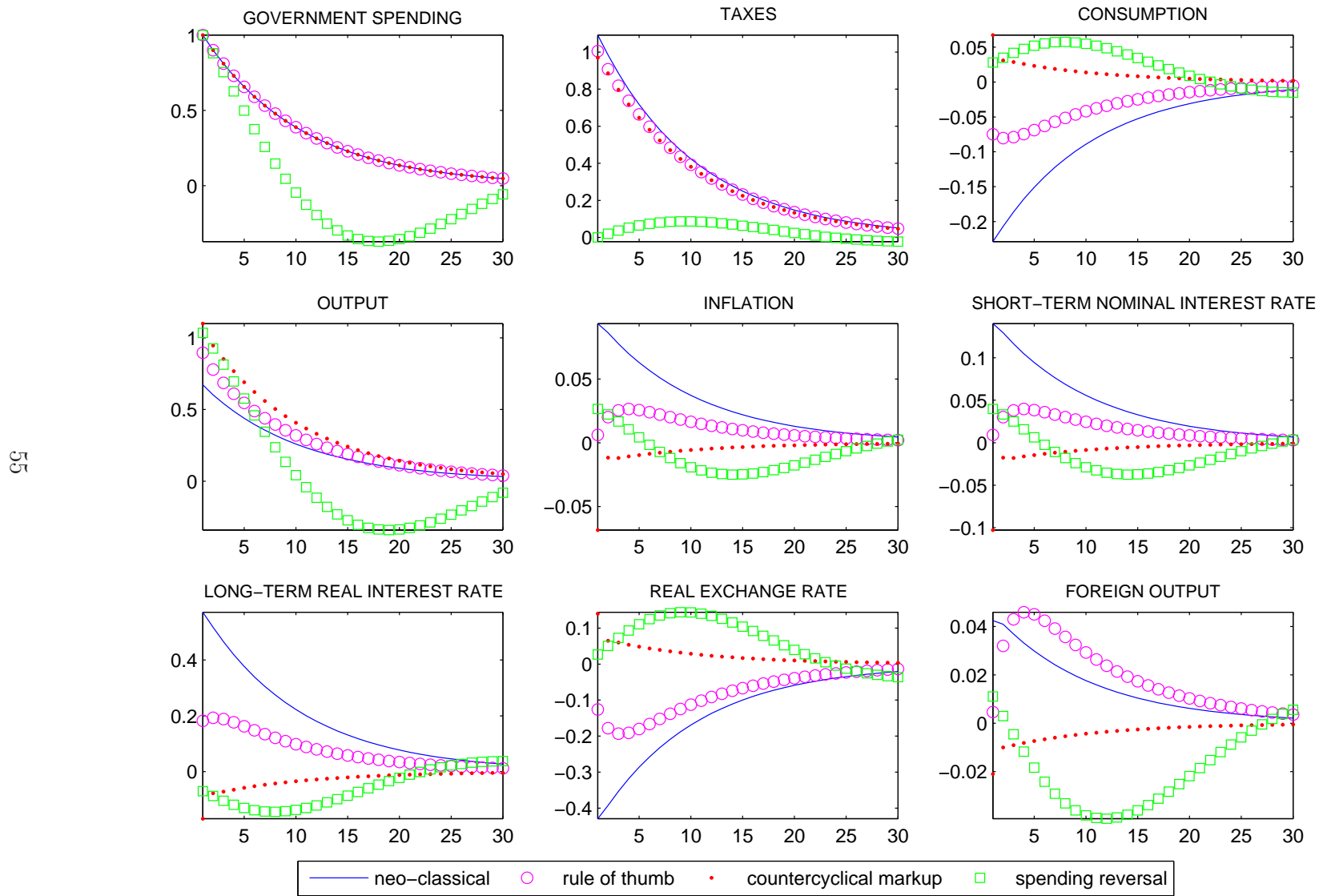


Figure 13: Dynamic responses to unanticipated increase in domestic government spending. An increase in the real exchange rate corresponds to a depreciation. The responses of government spending, taxes and consumption are expressed as a percent of GDP.



The impulse response analysis uncovers a set of inequality predictions that are shared by all specifications of our two-country framework. Following a positive fiscal shock, (i) government spending,  $G_t$ , (ii) taxes,  $T_t/P_t$ , and (iii) domestic output,  $Y_{H,t}$ , increase on impact, while the response of (iv) the government budget surplus,  $(T_t - P_{H,t}G_t)/P_t$ , is non-positive. Furthermore, in all models there is a positive comovement between (v) short-term nominal interest rate and inflation, and (vi) consumption and real exchange rate. As we will show in the next section, these sign restrictions are robust to empirically plausible changes in model assumptions and parameter values, and therefore will form the basis for our identification strategy.

As for the ambiguous predictions, both the countercyclical markup and the spending reversals models generate a fall in the long-term real interest rate. This is crucial for the ability of the spending reversals model to produce a positive response of consumption and boost domestic output. At our calibrated equilibrium, a value of  $\lambda = 0.4$  is not sufficient for the interaction between sticky prices and rule of thumb consumers to generate a rise in private consumption (Galí, Lopez-Salido and Valles, 2007).

Moving to the open economy variables, we note that the neo-classical model yields a real exchange appreciation stemming from international risk-sharing on consumption, which further boosts foreign output. A share of 40% ROT consumers instead does not suffice to reverse the real exchange rate appreciation, but is important to boost the response of foreign output.

The countercyclical markup and the spending reversals are the only models predicting a CPI real exchange rate depreciation. The two mechanisms, however, are quite different. In the countercyclical markup model, an increase in domestic public spending produces a decline in the markup of domestically sold goods. This triggers a real exchange depreciation and a decline of foreign output because of the loss of competitiveness abroad. According to the spending reversals mechanism, in contrast, the domestic fiscal expansion is associated with both an increase in foreign output and a real exchange depreciation. The reason is that the expectations of future tax rises are so strong as to generate negative long-term interest rates at home and abroad, thereby stimulating output globally.

The assumption of wage stickiness allows the spending reversal model to generate a depreciation of the real exchange rate and an increase in domestic consumption without relying on implausibly high values of price stickiness: at our calibrated equilibrium, the signs of the impulse responses in Figure 13 arise independently of the value chosen for price rigidity. Absent wage stickiness, the spending reversal model would have hard times to generate responses of consumption and exchange rate like in Figure 13 for values of  $\delta_p$  below 0.9.

It is important to emphasize that the assumption of wage rigidity has no consequences for the identification of the theory-robust sign restrictions described above. Without wage stickiness, the association between an increase (decrease) in consumption and real exchange depreciation (appreciation) emerges as a robust prediction of both the rule of thumb consumers and spending reversal models, independently of the degree of price stickiness. The reason for this result is that, for values of  $\delta_p$  below 0.9, consumption and real exchange rate

switch sign simultaneously in both specifications, thereby generating an additional robust prediction that can be exploited in the empirical analysis.

### D.3 Theory-robust sign restrictions

The previous section was intended to illustrate differences and similarities in the international transmission of fiscal policy across the various specifications. In this section, we check formally that the sign-restrictions (i) to (vi) are robust to a wide range of perturbations of the parameter space in a nested framework where rule of thumb consumers, countercyclical markups, government spending reversals as well as stickiness in wages and prices are allowed to interact with each other.

The ranges of parameter values used for the simulations of the nesting model are reported in Table 2 and they reflect the ranges of values typically encountered in the empirical literature. An exception is represented by the share of rule of thumb consumers and the intertemporal elasticity of substitution, whose ranges are set to conservative values. The reason for this choice is that higher values lead to indeterminacy whenever all transmission mechanisms are allowed to operate simultaneously. Indeed, a high share of rule of thumb consumers makes the model prone to indeterminacy, particularly in the presence of countercyclical markups. As a sensitivity check, however, we have verified that our results are robust to applying values of  $\lambda \in [0, 0.3]$  and  $\sigma \in [1, 20]$  to the benchmark specification, namely setting  $\eta = 0$  but letting all other parameters to vary as in Table 2.<sup>14</sup>

Table 7 reports the sign restrictions implied by the model for an extended set of shocks that buffet our model economy, namely fiscal, TFP, monetary, preferences, labor supply and markup shocks. In this table we restrict attention to the same variables reported in Table 1: government spending, taxes, primary budget surplus, output and the correlations between consumption and real exchange rate, and inflation and nominal interest rate. For each shock, we have computed impulse responses by drawing parameter values from the uniform distributions in Table 2 over 10,000 repetitions. In addition, we have assumed that the autocorrelation coefficients for all the shocks in Table 7 are drawn from a uniform distribution with support  $[0.7, 0.98]$ , with the exception of monetary shocks, for which the autocorrelation coefficient has support  $[0, 0.9]$ .

Our results for the fiscal shock indicate that the sign restrictions (i) to (iv) are satisfied in every single draw. The restrictions (v) and (vi) are instead satisfied in 97.7% and 99.4% of the draws, respectively.<sup>15</sup> All sign restrictions for technology, preference, monetary and markup shocks were satisfied in more than 99% of the draws, while in the case of labor supply shocks all sign restrictions are satisfied almost 99% of the times. The results indicate that with the only exception of a fiscal shocks, any other shock that increases output would generate an

<sup>14</sup>In this setting values of  $\lambda$  above 0.3 significantly enlarge the indeterminacy region.

<sup>15</sup>The inequality constraints (i) to (vi) are also robust to two significant departures from the model derived in Section 2: (i) non-unitary trade elasticities along the range of values reported by Corsetti, Dedola and Leduc (2008), (ii) drawing the parameters in Table 2 independently for the home and foreign economies so as to break down the assumption of symmetry between the two countries.

Table 6: Parameter values used in the simulation of the nested model

<i>description of the parameters</i>		range
discount factor	$\beta$	0.99
consumption-output ratio	$c_y$	[0.75, 0.85]
elasticity of intertemporal substitution	$\sigma$	[1, 3.5]
inverse of Frisch elasticity	$\varphi$	[0.5, 5]
home bias in consumption	$v$	[1.4, 1.9]
elasticity of labor substitution	$\varepsilon_L$	[4, 12]
interest rate response to inflation	$\phi_\pi$	[1.1, 2.0]
interest rate response to output gap	$\phi_y$	[0, 0.25]
interest rate smoothing	$\rho_R$	[0, 0.8]
gov. spending autocorrelation	$\rho$	[0.7, 0.98]
probability of price fixed	$\delta_p$	[0.3, 0.7]
probability of wage fixed	$\delta_w$	[0.3, 0.9]
elasticity of markup to demand	$\eta$	[1.1, 1.3]
share of rule of thumb consumers	$\lambda$	[0, 0.15]
gov. spending sensitivity of taxes	$\psi_{TG}$	[0, 0.5]
debt sensitivity of spending	$\psi_{GD}$	[-0.02, -0.04]
debt sensitivity of taxes	$\psi_{TD}$	[0.02, 0.04]

Note: Parameter values are randomly drawn from a uniform distribution. Same ranges apply to the foreign economy. Results are based on 10,000 repetitions.

Table 7: Theoretical sign restrictions: all shocks

	Fiscal	TFP	Monetary	Pref.	L. Supply	Mark-up
	$\theta$	$\Psi$	$\varsigma$	$\vartheta$	$\chi$	$\xi$
government spending	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$
taxation	$\geq 0$	$\leq 0$	$\leq 0$	$\leq 0$	$\leq 0$	$\leq 0$
budget surplus (at $t = 2$ )	$\leq 0$	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$
domestic output	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$
correl. $(\hat{\pi}_t, \hat{r}_t)$	$\geq 0$	$\geq 0$	$N/A$	$\geq 0$	$\geq 0$	$\geq 0$
correl. $(C_t, Q_t)$	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$	$\geq 0$

increase in the budget surplus, a decrease in real debt and, through equation (21), a decrease in taxes. Hence, fiscal shocks are uniquely identified, in our model, by the joint response of output, budget surplus and taxes.

On the basis of this finding, our theory-based strategy to identify a fiscal shock in the data is to impose the common sign restrictions (i) to (vi) onto the factor model of Section 3.

## E Log linearized system

**Households.** Let hat lowercase variables denote percent deviations from their steady state values, with the exception of the interest rate and inflation, where  $\hat{r}_t$  and  $\hat{\pi}_t$  denote deviations in percentage points. In the problem of the asset holding households, a log-linearization of the Euler equation (13) yields the following expression:

$$-\sigma \hat{c}_{A,t} + \hat{\vartheta}_t = -\sigma \hat{c}_{A,t+1} + \hat{\vartheta}_{t+1} + (\hat{r}_t - E_t \hat{\pi}_{t+1}).$$

Defining  $\tilde{c}_{A,t} = (C_{A,t} - C_A)/Y_H$  as log deviations in output units, the equation above can be rewritten as follows:

$$-\sigma \tilde{c}_{A,t} + c_y \hat{\vartheta}_t = -\sigma \tilde{c}_{A,t+1} + c_y \hat{\vartheta}_{t+1} + c_y (\hat{r}_t - E_t \hat{\pi}_{t+1}), \quad (25)$$

where  $c_y$  denotes the output share of consumption and we have made use of the assumption that steady state consumption is identical across household types, that is,  $C_A = C_N = C$ . This stationary equilibrium can always be achieved by an appropriate choice of taxes  $T_A$  and  $T_N$ . Similarly, the Euler equation for the foreign country writes:

$$-\sigma \tilde{c}_{A,t}^* = -\sigma \tilde{c}_{A,t+1}^* + c_y (\hat{r}_t^* - E_t^* \hat{\pi}_{t+1}^*). \quad (26)$$

Taking a Taylor expansion of equation (15) around a zero inflation steady state we get the

following wage setting rule:

$$\hat{w}_t^* = (1 - \beta\delta) \sum_{s=0}^{\infty} (\beta\delta)^s E_t \{ \widehat{mrs}_{A,t+s|t} + \hat{p}_{t+s} \},$$

where  $\mu^w = \ln \frac{\varepsilon_L}{\varepsilon_L - 1}$ . Using  $\widehat{mrs}_{A,t} = \hat{\chi}_t + \sigma \hat{c}_{A,t} + \varphi \hat{l}_{A,t}$  and a log-linear approximation of the labor demand equation (10) we can write:

$$\widehat{mrs}_{A,t+s|t} = \widehat{mrs}_{A,t+s} + \varphi (\hat{l}_{A,t+s|t} - \hat{l}_{A,t+s}) = \widehat{mrs}_{A,t+s} - \varepsilon_L \varphi (\hat{w}_t^* - \hat{w}_{t+s}).$$

Combining the two equations above and rearranging we get:

$$\hat{w}_t^* = \frac{1 - \beta\delta}{1 + \varepsilon\varphi} (\widehat{mrs}_{A,t} + \hat{p}_t + \varepsilon\varphi \hat{w}_t) + \beta\delta E_t \hat{w}_{t+1}^*$$

A log linear approximation of the wage index reads as follows:

$$\hat{w}_t = \delta_w \hat{w}_{t-1} + (1 - \delta_w) \hat{w}_t^*.$$

These last two equations can be combined to get the wage Phillips Curve:

$$\hat{\pi}_t^w = \kappa^w \left( \sigma \tilde{c}_{A,t} + c_y \hat{\chi}_t + c_y \varphi \hat{l}_{A,t} - c_y \hat{w}_t^r \right) + \beta E_t \hat{\pi}_{t+1}^w, \quad (27)$$

where  $\kappa^w = \frac{(1-\beta\delta)(1-\delta)}{\delta[1+\varepsilon_L\varphi]}$  and  $\hat{w}_t^r$  denotes deviations of the real wage from its steady-state value. The wage inflation equation for the foreign country reads:

$$\hat{\pi}_t^{w*} = \kappa^w \left( \sigma \tilde{c}_{A,t}^* + c_y \varphi \hat{l}_{A,t}^* - c_y \hat{w}_t^{r*} \right) + \beta E_t \hat{\pi}_{t+1}^{w*}. \quad (28)$$

In addition, the change in the real wage can be expressed as the difference between nominal wage inflation and CPI inflation:

$$\hat{w}_t^r = \hat{w}_{t-1}^r + \hat{\pi}_t^w - \hat{\pi}_t, \quad (29)$$

In the foreign country, the following symmetric equation holds:

$$\hat{w}_t^{r*} = \hat{w}_{t-1}^{r*} + \hat{\pi}_t^{w*} - \hat{\pi}_t^*. \quad (30)$$

The budget constraint for the ROT consumers in equation (16) can be linearized as follows:

$$Y_H \tilde{c}_{N,t} = \frac{W L_N}{P} \left( \hat{w}_t^r + \hat{l}_{N,t} \right) - Y_H \widetilde{tax}_t^r,$$

where  $\widetilde{tax}_t^r = \frac{(T_{N,t}/P_t - T_N/P)}{Y_H}$ . Since  $L^A = L^N = L$ , the above equation can be rewritten as:

$$\tilde{c}_{N,t} = \hat{w}_t^r + \hat{l}_{N,t} - \widetilde{tax}_t^r. \quad (31)$$

An analogous equation will determine consumption for the ROT households in the foreign country:

$$\tilde{c}_{N,t}^* = \hat{w}_t^{r*} + \hat{l}_{N,t}^* - \widetilde{tax}_t^{r*}. \quad (32)$$

Since forward-looking and ROT consumers are paid the same wage and face the same labor demand schedule, their labor supply will be identical, both in the home and in the foreign country:

$$\hat{l}_{N,t} = \hat{l}_{A,t}, \quad (33)$$

$$\hat{l}_{N,t}^* = \hat{l}_{A,t}^*. \quad (34)$$

From the problem of the asset holding households, the interest rate parity condition in equation (14) can be linearized and written:

$$\sigma \tilde{c}_t - c_y \hat{\vartheta}_t = \sigma \tilde{c}_t^* + c_y (v - 1) \hat{\tau}_t. \quad (35)$$

**Firms.** The linearized production function for the home and the foreign countries can be written as follows:

$$\hat{y}_t = \hat{\Psi}_t + \hat{l}_t, \quad (36)$$

$$\hat{y}_t^* = \hat{l}_t^*. \quad (37)$$

Using the first order condition in equation (8) and the law of motion for the price index in equation (9) we can derive the Phillips curve for the home and the foreign economy:

$$\hat{\pi}_{H,t} = \frac{(1 - \beta \delta_p)(1 - \delta_p)}{\delta_p} \left[ \hat{w}_t^r - \eta \hat{y}_t + \left(1 - \frac{v}{2}\right) \hat{\tau}_t \right] + E_t \beta \hat{\pi}_{H,t+1}, \quad (38)$$

$$\hat{\pi}_{F,t} = \frac{(1 - \beta \delta_p)(1 - \delta_p)}{\delta_p} \left[ \hat{w}_t^{r*} - \eta \hat{y}_t^* - \left(1 - \frac{v}{2}\right) \hat{\tau}_t \right] + E_t \beta \hat{\pi}_{F,t+1}. \quad (39)$$

**Term structure.** A log-linearization of equation (18) around the stationary equilibrium can be used to express the real yield on a bond of infinite duration expressed in deviations from the steady state and denoted by  $\bar{r}_t^r$ , as the infinite sum of expected short-term real rates:

$$\bar{r}_t^r = E_t \sum_{j=0}^{\infty} (\hat{r}_{t+j}^r - \hat{\pi}_{t+1}).$$

Iterating forward on equation (25), and using the assumption that the model is stationary and therefore consumption reverts to its steady state value (i.e.,  $\lim_{s \rightarrow \infty} c_{t+s} = 0$ ),  $\bar{r}_t^r$  can be rewritten as a function of consumption in log deviations from output:

$$\bar{r}_t^r = \frac{-\sigma \tilde{C}_{A,t}}{c_y}. \quad (40)$$

A similar relationship holds for the foreign country:

$$\bar{r}_t^{r*} = \frac{-\sigma \tilde{C}_{A,t}^*}{c_y}. \quad (41)$$

**Price indices, terms of trade and real exchange rate.** Having defined the real exchange rate as  $\mathcal{Q}_t = S_t P_t^* / P_t$  and home terms of trade as  $\mathcal{T}_t = S_t P_{F,t}^* / P_{H,t}$ , substituting the price indices into  $\mathcal{Q}_t$  it is possible to derive the following linearized relationship:

$$\hat{q}_t = (v - 1) \hat{\tau}_t \quad (42)$$

Using the definition of the price indices it is possible to derive the following expressions for domestic and foreign CPI inflation:

$$\hat{\pi}_t = \hat{\pi}_{H,t} + \left(1 - \frac{v}{2}\right) (\hat{\tau}_t - \hat{\tau}_{t-1}), \quad (43)$$

$$\hat{\pi}_t^* = \hat{\pi}_{F,t} - \left(1 - \frac{v}{2}\right) (\hat{\tau}_t - \hat{\tau}_{t-1}). \quad (44)$$

**Monetary and fiscal policies.** Linearizing the Taylor rule in equation (17) yields the two following expressions for the home and the foreign country:

$$\hat{r}_t = \rho_R \hat{r}_{t-1} + (1 - \rho_R) \left[ \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_y \right] + \hat{\varsigma}_t, \quad (45)$$

$$\hat{r}_t^* = \rho_R \hat{r}_{t-1}^* + (1 - \rho_R) \left[ \phi_\pi \hat{\pi}_t^* + \phi_y \hat{y}_y^* \right]. \quad (46)$$

Denoting  $\tilde{g}_t = (G_t - G) / Y_H$  and  $\tilde{d}_t^r = D_t / (P_{t-1} Y_H)$ , the government spending feedback rule in (20) becomes for the home and the foreign country:

$$\tilde{g}_t = \rho_G \tilde{g}_{t-1} + \psi_{GD} \tilde{d}_t^r + e_t, \quad (47)$$

$$\tilde{g}_t^* = \rho_G \tilde{g}_{t-1}^* + \psi_{GD} \tilde{d}_t^{r*}. \quad (48)$$

Linearizing the tax feedback rule in equation (21) yields the following expressions for the home and the foreign country:

$$\widetilde{tax}_t^r = \psi_{TG} \left[ \tilde{g}_t - (1 - c_y) \left(1 - \frac{v}{2}\right) \hat{\tau}_t \right] + \psi_{TD} \tilde{d}_t^r, \quad (49)$$

$$\widetilde{tax}_t^{r*} = \psi_{TG} \left[ \tilde{g}_t^* + (1 - c_y) \left(1 - \frac{v}{2}\right) \hat{\tau}_t \right] + \psi_{TD} \tilde{d}_t^{r*}. \quad (50)$$

The log-linearized conditions for the domestic government budget constraint in equation (19) can be written as follows:

$$\beta \tilde{d}_t^r = \tilde{d}_{t-1}^r - (1 - c_y) \left(1 - \frac{v}{2}\right) \hat{\tau}_{t-1} + \tilde{g}_{t-1} - \widetilde{tax}_{t-1}^r. \quad (51)$$

A similar condition holds for the foreign country:

$$\beta \tilde{d}_t^{r*} = \tilde{d}_{t-1}^{r*} + (1 - c_y) \left(1 - \frac{v}{2}\right) \hat{\tau}_{t-1} + \tilde{g}_{t-1}^* - \widetilde{tax}_{t-1}^{r*}. \quad (52)$$

**Equilibrium.** The linearized expression for aggregate consumption in equation (24) reads as follows:

$$\tilde{c}_t = \lambda \tilde{c}_{N,t} + (1 - \lambda) \tilde{c}_{A,t}, \quad (53)$$

while the analogous equation for the foreign country writes:

$$\tilde{c}_t^* = \lambda \tilde{c}_{N,t}^* + (1 - \lambda) \tilde{c}_{A,t}^*. \quad (54)$$

In order to linearize the market clearing condition for the home good in equation (22) it is convenient to express  $P_t/P_{H,t} = \mathcal{T}_t^{1-v/2}$  and  $SP^*/P_H = \mathcal{T}_t^{v/2}$ . Substituting for consumption and government spending in deviations from output yields:

$$\hat{y}_{H,t} = \frac{v}{2} \tilde{c}_t + \left(1 - \frac{v}{2}\right) \tilde{c}_t^* + 2c_y \frac{v}{2} \left(1 - \frac{v}{2}\right) \hat{\tau}_t + \tilde{g}_t, \quad (55)$$

where  $\tilde{g}_t = (G_t - G)/Y_H$ . The market clearing condition for the foreign economy in equation (23) can be rearranged in a similar way to write:

$$\hat{y}_{F,t}^* = \frac{v}{2} \tilde{c}_t^* + \left(1 - \frac{v}{2}\right) \tilde{c}_t - 2c_y \frac{v}{2} \left(1 - \frac{v}{2}\right) \hat{\tau}_t + \tilde{g}_t^*. \quad (56)$$

Equations (25) to (56) provide a complete characterization of the dynamic system used in the simulations.