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When Customers Anticipate Liquidation Sales: Managing Operations under Financial Distress

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The presence of strategic customers may force an already financially distressed firm into a death spiral: Sensing the firm's financial difficulty, customers may wait strategically for deep discounts in liquidation sales. In turn, such waiting lowers the firm's profitability and increases the firm's bankruptcy risk. Using a two-period model to capture these dynamics, this paper identifies customers' strategic waiting behavior as a source of a firm's cost of financial distress. We also find that customers' anticipation of bankruptcy can be self-fulfilling: When customers anticipate a high bankruptcy probability, they prefer to delay their purchases, making the firm more likely to go bankrupt than when customers anticipate a low probability of bankruptcy. Such behavior has important operational and financial implications. First, the firm acts more conservatively when either facing more severe financial distress or a large share of strategic customers. As its financial situation deteriorates, the firm lowers inventory alone when financial distress is mild or only a small share of customers are strategic and lowers both inventory and price in the presence of severe financial distress and a large fraction of strategic customers. Under optimal price and inventory decisions, strategic waiting accounts for a large part of the firm's total cost of financial distress, although a larger proportion of strategic customers may result in a lower probability of bankruptcy. In addition to inventory reduction and (immediate) price discount, we find that a deferred discount, in the form of rebates and/or store credits for future purchases, can act as an effective mechanism to mitigate strategic waiting. As a contingent price reduction, deferred discounts align the interests of customers and the firm and are most effective when the fraction of strategic customers is high and the firm's financial distress is at a medium level.

Key words: financial distress; liquidation sale; strategic customers; inventory; pricing; deferred discount; rebate; store credit

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1. Introduction

Squeezed by disappointing demand and financial pressure, many major US retailers, including Linens 'n Things in May 2008, Circuit City in November 2009, Borders in February 2011, and, most recently, Sports Authority in March 2016, have filed for bankruptcy. Many others, such as Sears and Radio Shack, while operating as going concerns, have closed a large share of their existing stores (Isidore 2014, Wahba 2016a). These are not isolated cases. In fact, according to Gaur et al. (2014), 15% of US public retailers entered bankruptcy in the past 20 years.

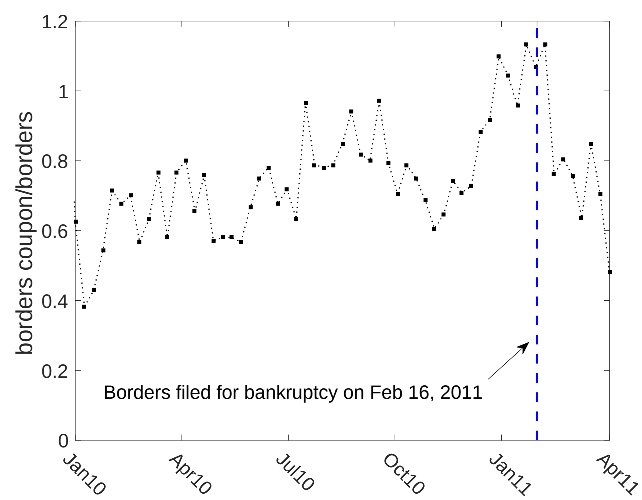
Another challenge retailers face is increasingly sophisticated customers. Due to a confluence of technology, economy, and social norms, it has become increasingly common across all income brackets and a wide variety of goods for customers to wait an extraordinary amount of time to purchase a good at the lowest possible price (Silverstein and Butman 2006, Paragon 2011). Recent academic research has found strong empirical support for such strategic waiting behavior. For example, Li et al. (2014) quantify that between 5.2% and 19.2% of customers purposely delay air ticket purchases in anticipation of possible future price discounts. Similar strategic behaviors are empirically documented for console video games (Nair 2007), textbooks (Chevalier and Goolsbee 2009), and soft drinks (Hendel and Nevo 2013). In a controlled laboratory environment, Osadchiy and Bendoly (2013) find that, facing a future purchase opportunity, up to 79% of customers exhibit forward-looking behavior. Such strategic waiting behavior can have a significant detrimental impact on firms' profitability (Su and Zhang 2008, Cachon and Swinney 2009).

Retailers' financial difficulties can be another reason for customers to postpone their purchases strategically as bankruptcy and large-scale store closures are often followed by liquidation sales. For example, in May 2016, after a failed reorganization, Sports Authority immediately started liquidating all 463 stores (Wahba 2016b). In 2014, Radio Shack liquidated the inventory of the 1,100 stores it closed. The amount of inventories liquidated during these sales is tremendous. According to Craig and Raman (2015), the value of inventory sold during the liquidations of Linens 'n Things, Circuit City, and Borders alone was more than \$3 billion. To add to the pressure faced by retailers, liquidation sales, as regulated by state laws, are limited to short times periods such as 60 or 90 days (Ohio Administrative Code Chapter 109:4-3-17, Massachusetts General Laws Part 1, Chapter 93). To liquidate a large amount of inventory within such a short space of time, retailers inevitably offer deep price discounts; this may entice consumers to postpone purchases when they observe a retailer's weakening financial situation.

In addition to the above empirical evidence on customers' strategic waiting behavior, recent research also finds that customers can reasonably assess a firm's level of financial distress, in particular, the probability of bankruptcy, and thus incorporate such information into their purchasing behavior. For example, Hortaçsu et al. (2013) find that shifting an automaker's probability of default from zero to near-certain bankruptcy reduces the average market value of that producer's used cars by \$1,400 on a \$28,000 car. Such evidence is consistent with previous research on the wisdom of the crowd that, collectively, average people can make accurate forecasts of complicated events, often more so than individual experts (Ho and Chen 2007, Surowiecki 2005).

Anecdotal evidence also supports the possibility that customers may react in anticipation of a firm's financial distress and the subsequent liquidation sale. For example, many websites enable customers to have access to information on a company's financial difficulties, the progress of liquidation, and how to cash in on liquidation sales (Bowsher 2011). Discussions around (the possibility of) liquidation sales are also hot topics on online forums. Interest in possible goods deals around bankruptcy is also reflected in online search volume. As shown in Figure 1, the (relative) search volume for "Borders coupon" rose gradually prior to Borders' bankruptcy filing. Similar patterns are also present around other retailers' bankruptcy. While this phenomenon may be attributed to other factors, one possibility is that customers searched for bargains more actively as they became increasingly aware of the firm's financial difficulty.

Figure 1 **Ratio of (normalized) weekly Google search volumes for "Borders coupon" to those for "Borders" around the bankruptcy of Borders (GoogleTrend 2015)**



Motivated by the above phenomena, this paper focuses on examining the operational and financial implications of strategic customer behavior as a source of financial distress.[1] Specifically, the

[1] In addition to interactions between retailers and individual consumers, the dynamics described above are also present in business-to-business settings where business buyers may strategically time their purchases in response to a seller's financial distress. The goods purchased may also be financial assets or investment projects. For example, prior to its bankruptcy in April 2016, SunEdison, the solar developer, was in the process of selling part of its asset portfolio, which is now likely to be sold in liquidation (Wesoff 2016). To reflect this, in the remainder of the paper, we refer to the seller as the *firm* and the buyers as *customers*.

paper investigates the following three questions. First, how do customers react to a firm's financial distress, and how does this reaction influence the firm's probability of bankruptcy in return? Second, under such behavior, how do consumer characteristics and financial conditions jointly influence the firm's operational decisions, such as inventory and price, as well as its profitability? Third, apart from inventory and price, is there another mechanism that may alleviate the adverse impact of strategic consumer behavior on financially distressed firms?

To answer these questions, we incorporate two salient features into the classic newsvendor model. First, we use the firm's *level of financial distress* (τ) to capture the amount of profit the firm needs to make in order to avoid bankruptcy. Higher τ generally leads to a higher probability of bankruptcy. Second, we capture consumer characteristics using *the fraction of strategic customers* (α), i.e., the share of customers in the market that may time their purchases strategically in anticipation of a liquidation sale.

Using this model, we find that collectively, customers' strategic waiting behavior can have a significant impact on a firm's probability of bankruptcy. More importantly, customers' anticipation of bankruptcy can be self-fulfilling: When strategic customers believe that a firm's probability of bankruptcy is high, they react by waiting in the first period due to the high likelihood of obtaining a bargain in the following period's liquidation sale. Such waiting in turn leads to a higher actual probability of bankruptcy than when customers anticipate a low probability of bankruptcy. Such dynamics may serve as a channel that contributes to the death spiral faced by distressed retailers, as alluded to by industry experts (Sozzi 2016).

The possibility of liquidation sales and strategic waiting leads to several important implications for a firm's operational decisions and performance. First, the threat of financial distress is aggravated as the proportion of strategic customers increases. Second, when inducing customers to purchase, the firm first lowers its inventory and then offers a price discount. As the level of financial distress (τ) increases, the firm lowers its inventory regardless of α, which is consistent with the empirical findings that distressed retailers lower their inventory levels (Chevalier 1995, Matsa 2011). However, the price discount only increases in τ when τ is low or α is high. Third, over a wide range of levels of financial distress, the firm's probability of bankruptcy decreases in the proportion of strategic customers.

In addition, we argue that deferred discounts, such as (non-cash) rebates or store credit for future purchases, can be more effective than immediate price discounts in mitigating strategic waiting. This is because, unlike immediate discounts, whose value is independent of strategic customers' behavior, deferred discounts are more valuable when the firm's probability of bankruptcy is lower. This contingency better aligns the interests of the firm and customers, nudging strategic customers to purchase early. We also find that deferred discounts are most valuable when a firm faces medium financial distress and many strategic customers.

The contribution of our paper is twofold. First, as an initial attempt to link strategic customer behavior to financial distress, the paper examines how strategic customers react to a firm's financial distress and point out that customers' strategic waiting for liquidation sales may serve as an important source of financial distress. Second, by characterizing how firms respond to financial distress and the corresponding customer behavior by adjusting inventory levels and offering (immediate) price discounts and/or deferred discounts, our paper may offer possible explanations for anecdotal evidence and motivate future empirical research.

2. Related Literature

Focusing on the operations of a financially distressed firm in the presence of strategic customers, our paper is closely related to two streams of literature: the operations–finance interface and consumer-driven operations management.

The operations–finance interface literature stresses that a firm's financial situation can have a significant impact on its operational decisions, which in turn influences the firm's financial health. In this stream of literature, Xu and Birge (2004), Babich and Sobel (2004), Dada and Hu (2008), Boyabatlı and Toktay (2011), Alan and Gaur (2011), Dong and Tomlin (2012), Li et al. (2013), Chod and Zhou (2013), and Luo and Shang (2013) study how a firm links its operational decisions, such as inventory and capacity investment, to its financing decisions in the presence of financial market imperfections. Yang and Birge (2009), Kouvelis and Zhao (2011), and Kouvelis and Zhao (2012) examine how to structure different types of supply chain contracts when one party in the supply chain is financially constrained. Papers in this stream mostly use the cost of financial distress in its reduced form as the main source of market imperfection. Our paper complements this literature by focusing on strategic customer behavior as a source of financial distress and examining the corresponding implications for a firm's operational decisions and performance. In a related segment of literature, Babich et al. (2007), Swinney and Netessine (2009), Babich (2010), and Yang et al. (2015) study the externality of one firm's financial distress on other firms in the supply chain. Similarly, we also endogenize the impact of bankruptcy by including customers as an integral part of the supply chain. Finally, Craig and Raman (2015) characterize the detailed operational decisions, such as transshipment, store closures, and dynamic pricing, during a retailer's liquidation sale. Using real-world data, they show that the efficiency of liquidation sales can be significantly improved when various operational levers are optimized jointly. Our paper complements theirs by emphasizing the possibility that liquidation sales have a significant impact on firms' pre-bankruptcy operational decisions and performance when customers anticipate a firm's risk of bankruptcy and the subsequent liquidation sale.

By focusing on strategic customer behavior as an additional source of financial distress, our paper is also related to the expanding literature on consumer-driven operations management and, in particular, to studies that focus on the implications of customers' forward-looking behavior. See Netessine and Tang (2009) for an overview of related works. Research in this literature focuses on identifying the adverse effects of strategic consumer behavior and proposing various forms of operational mitigation, such as quantity commitment (Su and Zhang 2008, Liu and van Ryzin 2008), price commitment (Aviv and Pazgal 2008, Lai et al. 2010), display format (Yin et al. 2009), quick response (Cachon and Swinney 2009), early-purchase reward (Aviv and Wei 2015), and group buying (Surasvadi et al. 2015). Our paper also highlights the adverse impact of strategic consumer behavior but with a focus on how such behavior interacts with a firm's financial distress. In addition, we argue that deferred discounts, such as (mail-in) rebates and store credit, serve as an effective mechanism to mitigate strategic waiting.[2] Furthermore, several recent papers examine the impact of strategic consumer behavior on other common operational decisions such as demand learning (Aviv et al. 2015), product quality (Yu et al. 2014, Papanastasiou and Savva 2015), and new product launches (Lobel et al. 2016). Similarly, our paper studies the impact of strategic consumer behavior on operational decisions under financial distress.

To the best of our knowledge, Hortaçsu et al. (2011) is the only extant paper to model the interaction between bankruptcy and customers' behavior in anticipation of bankruptcy. Our paper differs from theirs in two ways. First, in their paper the channel that lowers customers' first-period valuation is because of a lack of after-sales service; we focus on the possibility of deeper discounts in the future, which is closely related to a firm's operational decisions, such as price and inventory. Second, Hortaçsu et al. (2011) call for public policy, such as a government guarantee, to reduce the impact of bankruptcy anticipation on consumers' product valuations, while our paper focuses on reducing the bankruptcy feedback effect through operational levers that the firm can control.

Finally, by examining deferred discounts, our paper is also related to the literature on rebates, which can be seen as a specific form of deferred discount. As a widely used marketing tool, rebates have been studied in both the marketing (Soman 1998, Lu and Moorthy 2007) and operations management literatures (Chen et al. 2007, Cho et al. 2009, Arya and Mittendorf 2013). We complement the above papers by showing that, as deferred discounts, rebates better align customers' interests with those of the firm when the latter is in financial distress.

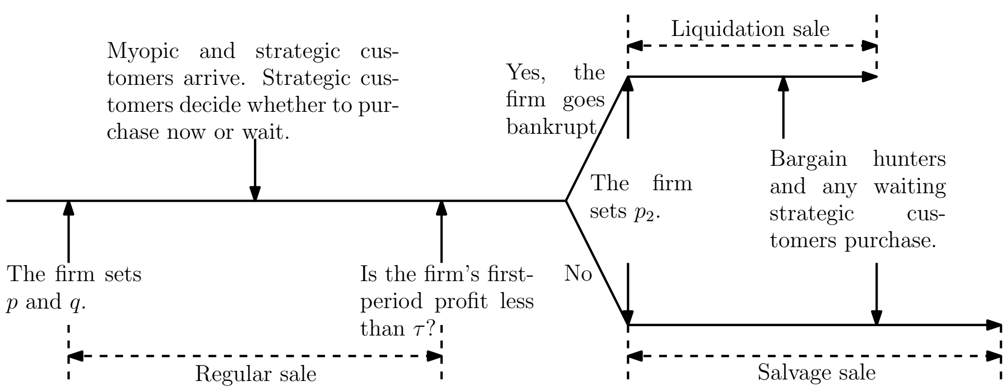
[2] For financially distressed firms, some mechanisms discussed in the literature may be less effective. For example, it is difficult for firms in bankruptcy to honor their price-matching commitments; some may face legal requirements when invalidating prior commitments to protect creditors ex-post. However, as shown later, this lack of commitment power is the exact mechanism that makes deferred discounts effective.

3. The Basic Model

We model a firm selling a single type of product to customers over two periods. The sequence of events is illustrated in Figure 2. In the first period (the *regular sale*), the firm sets both the "full" (or "regular") price p and the inventory level q, which is procured at unit cost c.[3] The total number of customers who may purchase in the first period is a random variable $D \in [d_l,\, d_h)$ with a cumulative distribution function (CDF) $F(\cdot)$, probability density function (PDF) $f(\cdot)$, complementary CDF $\bar{F}(\cdot) = 1 - F(\cdot)$, and failure rate $h(\cdot) = f(\cdot)/\bar{F}(x)$. The demand distribution is assumed to have an increasing failure rate (IFR), a mild condition that is satisfied by most commonly used distributions. Unsatisfied first-period demand is lost. Let $R_1(D; p, q)$ be the realized first-period revenue under the first-period demand D and decisions (p, q). The specific form of $R_1(\cdot)$ depends on customers' purchase decisions and is detailed later.

In the second period, depending on the firm's financial status, as described later, the firm sets its second-period price p_2 to clear its leftover inventory.[4] Let $R_2(D; p, q)$ be the realized second-period revenue under the *optimal* p_2. Following the literature (Jensen 2001, Ayotte and Morrison 2009, Becker and Strömberg 2012), we assume that the firm's objective is to maximize its expected profit over the two periods, i.e. $\pi(p, q) = -cq + \mathbb{E}[R_1(D; p, q) + R_2(D; p, q)]$, which is also equivalent to maximizing the firm's value.[5]

Figure 2 Sequence of events



[3] In Section 6, we extend the basic model by allowing the firm to offer a *deferred discount* in addition to setting price and inventory.

[4] As we show later, if the firm is not bankrupt, the inventory-clearing price p_2 also maximizes the firm's second-period revenue. In bankruptcy, the firm cannot credibly commit not to clear the entire inventory. Therefore, the inventory-clearing price is more appropriate. In addition, as suggested by Lemma C.1, we expect our main qualitative insights to remain unchanged even if the firm chooses p_2 to maximize revenue.

[5] The firm's value is composed of both equity and debt value. Thus, even if the firm goes bankruptcy in the second period, as we will detail in the next section, the revenue from liquidation still belongs to part of the firm's debt value, and hence is included in the firm's objective function.

3.1. Level of financial distress and the possibility of bankruptcy

We capture the firm's level of financial distress using $\tau \in (-\infty, +\infty)$. τ can be seen as the firm's net debt, i.e. debt minus liquid assets. The greater τ, the more financially distressed the firm. At the end of the first period, the firm is forced into bankruptcy if and only if its first-period cashflow $-cq + R_1(D; p, q)$ is lower than τ. To focus on the firm's operational decisions and its interaction with consumers, we take τ as exogenous. This assumption is also supported by the empirical finding that it is very costly, if not impossible, to reduce debt in the short term (Heider and Ljungqvist 2015). Using the first-period cashflow and τ as triggers for bankruptcy, this model is consistent with two commonly observed phenomena relating to corporate bankruptcy and default. First, in practice, many companies enter bankruptcy for liquidity difficulties (Taub 2008, De La Merced 2012). Second, most debt contracts are associated with covenants on performance measures such as profitability and cash flow. If a firm's performance fails to meet the performance target, the corresponding covenant is violated and the lender (e.g. the bank) often seizes control of the firm (Roberts and Sufi 2009). In both cases, firms enter bankruptcy when their performance fails to meet an existing threshold.[6]

The firm's financial status influences its second-period operations. If the firm manages to avoid bankruptcy, it continues normal operations and salvages its remaining inventory over a long period of time (a *salvage sale*). On the other hand, if the firm enters bankruptcy, it needs to liquidate its leftover inventory over a short period of time (a *liquidation sale*), as is often required by law. Intuitively, due to the difference between the salvage and liquidation sales, the second-period price p_2 set by the firm may be lower under a liquidation sale than a salvage sale. As such, the two-period model naturally captures the make-or-break season and the possible subsequent bankruptcy period faced by financially distressed retailers (Mui and Marr 2008, Loeb 2015).

3.2. Customer behavior and its link with bankruptcy

To capture strategic customer behaviors, especially how such behaviors are influenced by a firm's financial status, we assume that the population of consumers is divided into three segments, similar to Cachon and Swinney (2009). All customers are assumed to be risk-neutral. Customer characteristics are summarized in Table 1.

Two of the three segments of customers arrive in the first period. Among them, $(1-\alpha)D$ are *myopic*; they purchase in the first period as long as their surplus is non-negative, that is, $p \leq v$. The rest of the customers (αD) are *strategic*, where α represents the fraction of strategic customers the firm faces in the market. Observing price p and inventory q, strategic customers decide whether to

[6] When determining the firm's bankruptcy threshold, taking into consideration the possible salvage value of leftover inventory should not change our qualitative insights as long as such salvage value is lower when the firm's financial distress is more severe, or refinancing is costly.

purchase in the first period or to wait by comparing the surplus of buying early ($v-p$) with the expected surplus of waiting until the second period under a *rational belief* about other strategic customers' behavior.[7] As all strategic customers are homogenous, we focus on symmetric equilibria. In addition, we confine our analysis to pure strategy equilibria. Therefore, two possible equilibria exist: either all strategic customers decide to purchase in the first period (*the buy-equilibrium*) or all wait (*the wait-equilibrium*).

Table 1 Characteristics of consumer segments

Segment	Number	Period-1 valuation	Period-2 valuation (salvage sale)	Period-2 valuation (liquidation sale)
Myopic	$(1-\alpha)D$	v	n/a	n/a
Strategic	αD	v	s	s
Bargain-hunting	$+\infty$	n/a	s	b

The third customer segment is formed of *bargain hunters*, who only arrive in the second period. To reflect the impact of the firm's financial status and operational modes (salvage vs. liquidation sale) on customers, we assume that the bargain hunters' valuation is s under the salvage sale and b under the liquidation sale, with $b<s<c$. This assumption captures the reality in two ways. First, among bargain hunters, some, with a low valuation b, monitor the firm's liquidation status closely and hence can jump in immediately after the firm announces liquidation. Others, while having a high valuation s, may only visit the store according to their regular shopping schedule and grab a deal if they see one. As a result, when the firm is not bankrupt, it can afford to run the salvage sale for a longer period and wait for the high-valuation bargain hunters to show up. However, a liquidation sale is time limited, and hence, the firm can only sell to those bargain hunters with a lower valuation. Second, an individual bargain hunter's valuation may drop during liquidation as liquidation sales may not offer a satisfactory shopping experience; for example, they face limited payment options, a more restrictive return policy, and fewer staff to assist customers (Strain 2009). Finally, similar to Su (2010), we assume that when both strategic and bargain-hunting customers are present in the second period, the inventory is efficiently rationed, that is, the demand from high-valuation customers is satisfied first.

[7] In the literature, several papers assume customers can observe q (e.g. Liu and van Ryzin 2008), while others assume customers cannot observe q and instead form a rational expectation about it (e.g. Cachon and Swinney 2009). Su and Zhang (2008, 2009) compare both scenarios and quantify the value of a firm's commitment to an inventory level q in mitigating the adverse effect of strategic waiting. With the understanding that the commitment of q improves the firm's profitability, we assume that the firm can reveal q to customers using various mechanisms, as pointed out in Liu and van Ryzin (2008) and Su and Zhang (2008, 2009). However, as the analysis in Online Appendix E reveals, assuming strategic customers *cannot* observe inventory q does not change our qualitative insights.

At a high level, the difference between s and b captures how urgent, or inefficient, the liquidation sale is. As shown later, this difference causes two sources of *indirect cost of financial distress*. First, as bargain hunters' valuation is lowered in bankruptcy, the firm may have to reduce the second-period price during liquidation sales, commonly known in the literature as the cost of "fire sales" (Shleifer and Vishny 2011). Second, strategic customers may wait for the potential lower price in liquidation, hurting the firm's profit in the first period.

The remainder of the paper is organized as follows. We examine strategic customer behavior in §4. Section 5 analyzes the firm's profit and characterizes its optimal inventory and pricing strategies. Sections 6 and 7 study how deferred discounts can mitigate strategic waiting and alleviate financial distress. Section 8 concludes the paper. The appendix includes a list of notations. All proofs are included in the online appendices, which also include additional technical results.

4. Strategic Customers' Purchase Decision and Self-fulfilling Bankruptcy

We analyze the model through backward induction. Observing price (p) and inventory (q), to decide whether to purchase at the regular price p or wait for a possible liquidation sale in the event of bankruptcy, a typical strategic customer weighs the consumer surplus of purchasing ($v-p$) against the expected surplus of waiting, which depends on the price distribution in the second period.

LEMMA 1. *The firm's second-period price $p_2 = b$ if and only if the firm is in bankruptcy and the first-period realized demand is less than q. Otherwise, $p_2 = s$.*

Lemma 1 reveals that without bankruptcy, the firm should always set the second-period price at s. As a result, strategic customers' waiting surplus is zero; hence, these customers do not wait to purchase if they believe the firm will not go bankrupt. In this sense, our model degenerates to the classic newsvendor model with salvage value s when the level of financial distress (τ) is sufficiently low.

The firm may still set p_2 to s in bankruptcy. This happens when the total inventory q is less than the realized first-period demand D or, equivalently, when there are more customers waiting strategically than leftover inventory, in which case the firm clearly has no incentive to set a price lower than s. As such, waiting strategic customers are left with zero surplus whether they purchase or not. This is consistent with the evidence that some customers waiting for deep discounts were disappointed by the high prices of certain items in Circuit City's liquidation sale (Marco 2009).

Finally, when the firm goes bankrupt and leftover inventory exceeds the number of waiting strategic customers ($q > D$), the firm is forced to lower the price to b as it needs to attract bargain hunters to clear its inventory. This leaves strategic customers with a strictly positive surplus $s-b$, and, hence, an incentive to wait if the first-period price p is sufficiently high.

Given the second-period price distribution, it is clear that the maximum waiting surplus for strategic customers is $s-b$. Therefore, strategic customers always purchase early when $p \le v-(s-b)$. In addition, early purchase is also guaranteed when $p \le v$ and $q \le d_l$. To avoid trivial cases, we confine our analysis in this section and the following to the region $(p,q) \in \Omega_0 := \{(p,q) | p \in (v-(s-b), v] \text{ and } q \ge d_l\}$.

Moving to strategic customers' first-period purchase decisions, note that the likelihood that the second-period price will be b depends on the joint probability of $q > D$ and the event of the firm's bankruptcy. While $q > D$ depends only on the demand distribution and the firm's inventory decision q, the probability that the firm will go bankrupt is in fact influenced by strategic customers' behavior, which is in turn affected by their belief about the probability of bankruptcy. For example, if strategic customers anticipate a high probability of bankruptcy, and hence a higher chance of getting a bargain in liquidation, they should find it more appealing to wait, which, in turn, lowers the firm's first-period profit and increases the probability of bankruptcy. Therefore, the firm's bankruptcy probability and strategic customers' purchase or wait decisions need to be determined jointly under a rational expectations framework, i.e. individual customers form a rational belief about other customers' behavior and its impact on the probability of bankruptcy.

Proposition 1. *Let* $Q_s(p) := F^{-1}\left(\frac{v-p}{s-b}\right)$ *and* $Q_b(p) := \max\left[Q_s(p), \frac{(1-\alpha)pQ_s(p)-\tau}{c}\right]$.

1. *An equilibrium where all strategic customers purchase in the first period* (*the* buy-equilibrium) *exists if and only if* $q \le \max\left(\frac{pQ_s(p)-\tau}{c}, Q_s(p)\right)$. *Under the buy-equilibrium, the firm goes bankrupt if and only if* $D < d_\tau^B := \frac{cq+\tau}{p}$.

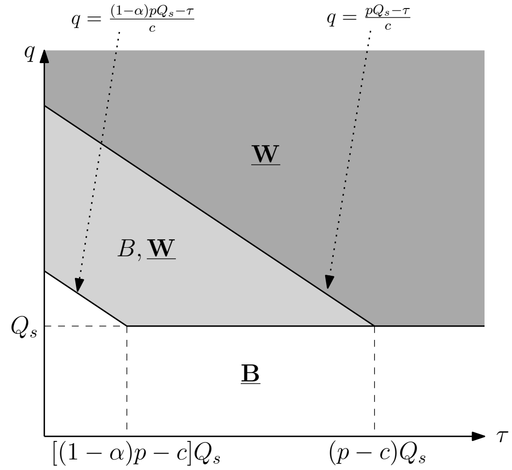
2. *An equilibrium where all strategic customers wait in the first period (the* wait-equilibrium*) exists if and only if* $q > Q_b(p)$. *Under the wait-equilibrium, the firm goes bankrupt if and only if* $D < d_\tau^W := \frac{cq+\tau}{(1-\alpha)p}$.

3. *When both equilibria exist, strategic customers' surplus in the wait-equilibrium is greater than that in the buy-equilibrium, that is, the wait-equilibrium is more appealing to strategic customers.*

Proposition 1 is illustrated in Figure 3. As shown, the buy-equilibrium exists when the firm's inventory is lower than a threshold composed of two pieces, capturing the events governing the second-period price distribution as summarized in Lemma 1. When the firm is in deep distress (large τ), $d_\tau^B > q$ and the customers' waiting surplus is determined solely by inventory availability and is independent of the firm's financial situation. Therefore, customers purchase if and only if $v-p \ge (s-b)F(q)$ or, equivalently, $q \le Q_s(p)$. Similarly, for a smaller τ, strategic customers purchase early if and only if $v-p \ge (s-b)F\left(d_\tau^B\right)$ or, equivalently, $q \le \frac{pQ_s(p)-\tau}{c}$.

Symmetrically, the wait-equilibrium exists when q is sufficiently high. Note that while the existence region for the buy-equilibrium is independent of the proportion of strategic customers (α),

Figure 3 Strategic customers' behavior in equilibrium under p.



Notes. B (W) represents that the buy-equilibrium (the wait-equilibrium) exists in the region. The equilibrium that is more appealing to strategic customers is in bold font and underlined.

the region for the wait-equilibrium expands as α increases, suggesting that strategic customers are collectively more powerful in their capability to nudge the firm into bankruptcy.

In addition, note that when both τ and q are at a medium level, the two equilibria may co-exist, leading correspondingly to two possibilities for the firm's profit and bankruptcy probability. In this region, strategic customers' belief about bankruptcy becomes a self-fulfilling prophecy: When strategic customers expect that the firm's probability of bankruptcy is high, and hence wait, the firm indeed enters bankruptcy with a probability $F\left(\frac{cq+\tau}{(1-\alpha)p}\right)$ that increases with the share of strategic consumers. Similarly, if strategic customers believe the firm is unlikely to go bankrupt, and hence purchase early, the probability of bankruptcy decreases to $F\left(\frac{cq+\tau}{p}\right)$. The existence of multiple equilibria hinges upon two critical features of the model: the distress level τ and fraction of strategic customers α. First, note that the region of multiple equilibria expands as α increases. In other words, a greater proportion of strategic customers amplifies the self-fulfilling prophecy of bankruptcy. Second, multiple equilibria exist when τ is in the medium range. Indeed, if the firm is sufficiently distressed, i.e. $\tau > (p-c)Q_s$, the firm goes bankrupt even if all inventory is sold in the first period ($d_\tau^W > d_\tau^B > q$), and hence, customers' waiting surplus becomes independent of their belief about the probability of bankruptcy.

Interestingly, when both equilibria exist, even though it is intuitive that the firm makes a higher profit under the buy-equilibrium, the wait-equilibrium is always more appealing to strategic customers. The logic is as follows. On the one hand, strategic customers' surplus of purchasing early is independent of their belief about the firm's bankruptcy probability and, hence, is identical under both equilibria. On the other hand, their surplus of waiting is higher under the wait-equilibrium due to the higher probability of bankruptcy. Consequently, when the wait-equilibrium exists, the surplus under the equilibrium with waiting is always higher than that of purchasing, which is also the surplus under the buy-equilibrium.

To distill strategic customers' behavior from these observations, we define the *buy-region* as $\Omega^B = \{(p,q) \in \Omega_0 \mid q \leq Q_b(p)\}$ and the *wait-region* as $\Omega^W = \{(p,q) \in \Omega_0 \mid q > Q_b(p)\}$, each corresponding to the price and inventory pairs that induce strategic customers to buy or wait, respectively, in the first period. Note that the buy-region is influenced by both α and τ through $Q_b(p)$. In other words, both the level of financial distress (τ) and fraction of strategic customers (α) constrain the firm's feasible set of prices and inventory positions to those that induce customers to purchase early.

5. Optimal Operational Response to Financial Distress

Built on the understanding of how strategic customers react to a firm's financial status (τ), in this section, we explore how such interactions shape a firm's optimal operational decisions (p and q) and performance (profitability and probability of bankruptcy). We first lay out the firm's profit function in the presence of both financial distress and strategic customers in Section 5.1, followed by a benchmark without strategic customers ($\alpha = 0$) in Section 5.2. In Sections 5.3 and 5.4, we examine how the fraction of strategic customers (α) influences the firm's decisions and performance under mild (low τ) and severe financial distress (high τ), respectively. Finally, Section 5.5 uses numerical results to complement the above analytical results and offers a complete picture of how τ and α jointly affect the firm's decisions and performance.

5.1. The firm's profit function

To characterize how the strategic customers' behavior established in Section 4 influences the firm's profit function, we first consider the case where $(p,q) \in \Omega_B$. According to Figure 3, for $(p-c)q \geq \tau$, i.e. $d_\tau^B \leq q$, the firm's profit can be discussed under three scenarios depending on the realized demand D. First, when $D \geq q$, the firm sells everything it has in the first period, and hence, its first-period revenue $R_1 = pq$ and its second-period revenue $R_2 = 0$. Second, when $D \in [d_\tau^B, q)$, the firm sells D at regular price p; thus, $R_1 = pD$. As it avoids bankruptcy, the firm salvages the leftover inventory at price s, leading to $R_2 = s(q-D)$. Third, when $D < d_\tau^B$, the firm also sells D at regular price p ($R_1 = pD$). However, as it goes bankrupt, and $D < q$, according to Lemma 1, the firm must liquidate the leftover inventory at price b, and hence, $R_2 = b(q-D)$.

Integrating over D across the three scenarios and rearranging terms, we can see that when $(p,q) \in \Omega^B$ and $(p-c)q > \tau$, the firm's total expected profit $\pi = -cq + \mathbb{E}[R_1 + R_2]$ is:

$$\pi_L^B(p,q) = (p-c)q - (p-s)\int_{d_l}^{q}(q-x)dF(x) - (s-b)\int_{d_l}^{d_\tau^B}(q-x)dF(x), \tag{1}$$

where the subscript B represents that $(p,q)\in\Omega_B$ and the superscript L represents that the level of financial distress is *low*, i.e. $\tau \le (p,c)q$. Observe that the first two terms of (1) are identical to a traditional newsvendor profit function with price p. The unique feature of financial distress is reflected in the last term, i.e. $(s-b)\int_{d_l}^{d_\tau^B}(q-x)dF(x)$, which equals to the additional price discount the firm has to offer in liquidation $(s-b)$ multiplied by the expected leftover inventory conditional on that the firm is bankrupt $D<d_\tau^B$.

On the other hand, if $(p-c)q<\tau$, i.e. $d_\tau^B>q$, the second scenario above ($D\in[d_\tau^B,q]$) disappears, and hence, the firm's profit function is:

$$\pi_H^B(p,q)=(p-c)q-(p-b)\int_{d_l}^{q}(q-x)dF(x). \tag{2}$$

Similarly, the firm's profit under $(p,q)\in\Omega_W$ follows:[8]

$$\pi_W(p,q)=(p-c)q-(p-s)\int_{d_l}^{\frac{q}{1-\alpha}}[q-(1-\alpha)x]dF(x)-(s-b)\int_{d_l}^{\min(q,d_\tau^W)}[q-(1-\alpha)x]dF(x). \tag{3}$$

As shown, in addition to the term associated with liquidation, when customers are not induced to purchase early, the first-period demand faced by the firm is essentially $(1-\alpha)D$.

5.2. The benchmark without strategic customers ($\alpha=0$)

To isolate how financial distress alone influences the firm's operational decisions, we first establish a benchmark in the absence of strategic customers ($\alpha=0$).

LEMMA 2. *Let $q^{NV}:=\bar{F}^{-1}\left(\frac{c-s}{v-s}\right)$, and $q_b^{NV}:=\bar{F}^{-1}\left(\frac{c-b}{v-b}\right)$. In the absence of strategic customers ($\alpha=0$), the firm's optimal price is $p^*=v$. The optimal inventory q^* follows:*

1. *for $\tau\le vd_l-cq^{NV}$, $q^*=q^{NV}$;*
2. *for $\tau\in(vd_l-cq^{NV},\, vd_l-cq_b^{NV})$, $q^*\in(q^{NV},\, q_b^{NV})$ decreases in τ;*
3. *for $\tau>vd_l-cq_b^{NV}$, $q^*=q_b^{NV}$.*

Under the optimal inventory, the firm's profit decreases in τ and its probability of bankruptcy increases in τ.

Two observations are notable. First, without strategic customers, the firm does not offer any price discount, that is, it charges customers' valuation v in the first period regardless of financial distress. This is intuitive as in our model, price discount is used only to induce strategic customers to purchase early. Second, we find that the firm's inventory follows three stages. First, when the firm is financially healthy (τ is very low, Statement 1 in Lemma 2) and bankruptcy is not a concern, it simply orders q^{NV}, the newsvendor quantity with salvage price s. At the other extreme, when τ is extremely high (Statement 3), bankruptcy is unavoidable and the firm orders q_b^{NV}, the newsvendor

[8] See Lemma C.2 in the appendix for detailed derivations.

quantity with the lower liquidation price b. The difference between q^{NV} and q_b^{NV} captures how the probability of bankruptcy (and the corresponding liquidation sale) influences the firm's inventory level. The lower b relative to s, the lower q_b^{NV} is relative to q^{NV}. Finally, between the above two scenarios (Statement 2), as τ increases, the firm gradually lowers its inventory from q^{NV} to q_b^{NV} to cope with the deteriorating financial status and the increasing chance of salvaging its leftover inventory at a lower price. This is consistent with the empirical findings in Chevalier (1995) and Matsa (2011) that distressed retailers often lower their inventory levels significantly.

This benchmark establishes that the presence of financial distress is the fundamental driver of the firm's profit reduction. Such reduction is caused by two inter-dependent effects. First, as τ increases, keeping inventory constant, the firm's probability of bankruptcy, and hence the probability of liquidating its leftover inventory at price b, increases, leading to lower revenue from liquidation. Second, to (partially) alleviate the first effect, the firm lowers its inventory q, reducing the (expected) first-period profit.

5.3. The firm's operational decisions under mild financial distress (low τ)

Apart from the two effects identified in Section 5.2, the presence of strategic customers also influence the firm's decision by interacting with τ. We examine such interaction with low τ in this section and high τ in the next.

PROPOSITION 2. *Let $T^D(\alpha) := (1-\alpha)vd_l - cq^{NV}$. In the presence of strategic customers ($\alpha > 0$), the firm's optimal price p^* and inventory q^* are:*

1. *for $\tau \le T^D(\alpha)$, $p^* = v$ and $q^* = q^{NV}$.*
2. *for $\tau > T^D(\alpha)$, $q^* < q^{NV}$.*
   (a) *$\exists\, T^B(\alpha) > T^D(\alpha)$ such that for $\tau \le T^B(\alpha)$, (p^*, q^*) satisfy $q^* = \frac{(1-\alpha)p^* Q_s(p^*) - \tau}{c}$.*
   (b) *$\exists\, \delta > 0$ such that for $\tau \le T^D(\alpha) + \delta$, $p^* = v$ if $f(d_l) > 0$ and $p^* < v$ if $f(d_l) = 0$.*

Proposition 2 (Statement 1) reveals that the presence of strategic customers aggravates the firm's financial distress significantly. Specifically, the threat of financial distress, as measured by the threshold $T^D(\alpha)$, becomes greater as the firm faces more strategic customers. In fact, when the firm needs to deviate from the *newsvendor benchmark* (v, q^{NV}), i.e. $\tau = T^D(\alpha)$, the minimal profit made by the firm is $vd_l - cq^{NV}$, strictly greater than $T^D(\alpha)$. This is because for (v, q^{NV}) to be optimal, the firm needs to ensure that its probability of bankruptcy under the (hypothetical) wait-equilibrium is zero. Otherwise, according to Proposition 1, the wait-equilibrium exists and becomes the more appealing one for strategic customers. In this sense, the existence of strategic customer behavior induces the firm to adopt a more conservative operational strategy.

As the level of financial distress (τ) increases beyond $T^D(\alpha)$ (Statement 2), we observe two features of the solution. First, when τ is relatively low, it is always optimal for the firm to induce strategic customers to purchase early as the cost of doing so is low. In particular, note that the relationship between p^* and q^* corresponds to the downward-sloping segment of the buy–wait boundary identified in Figure 3. This suggests that when τ is relatively low, the firm eliminates strategic waiting by lowering the probability of bankruptcy under the (hypothetical) wait-equilibrium $F(d_\tau^W)$.

Second, observe that when $\tau > T^D(\alpha)$, while the firm always lowers the inventory level, the firm may only lower the price as financial distress further deepens. This pecking order reflects the different roles played by inventory and price in mitigating distress: Lowering inventory alleviates the adverse impact of financial distress by both reducing the firm's procurement cost and inducing strategic customers to purchase. Price discount, however, is a double-edged sword. On the one hand, it reduces the strategic waiting motive and, hence, mitigates financial distress indirectly. On the other hand, lowering the price increases financial distress directly due to lower margins. Therefore, it is only employed when the cost of deterring strategic waiting through lowering inventory is high.

5.4. The firm's operational decisions under severe financial distress (high τ)

As shown above, when τ is low, the firm always finds it profitable to induce customers to purchase. However, as shown in the following proposition, this result no longer holds when the firm faces severe financial distress (high τ).

Proposition 3. *$\exists\, T_h$ and $A^B > 0$ such that for $\tau \geq T_h$, the firm's optimal price p^* and inventory q^* are:*

1. *for $\alpha \leq A^B$, $(p^*, q^*) = (v, q_H^W)$, where q_H^W is determined by:*

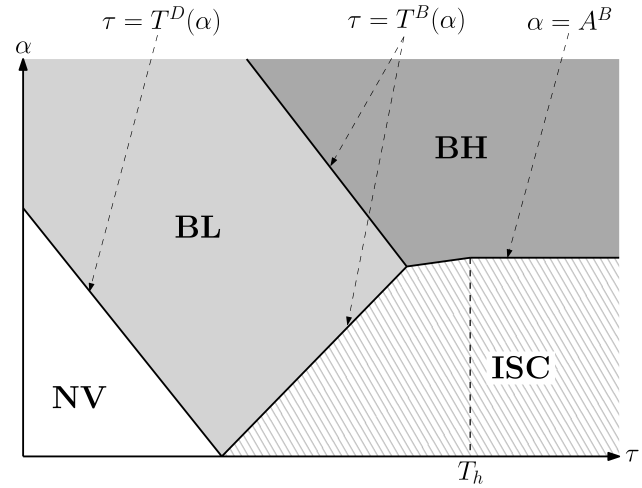
$$q_H^W = (1-\alpha)\bar{F}^{-1}\left(\frac{(c-b)-(s-b)\bar{F}(q_H^W)\left[1-\alpha q_H^W h(q_H^W)\right]}{v-s}\right); \tag{4}$$

2. *for $\alpha > A^B$, $(p^*, q^*) = (p_H^B, q_H^B)$, where $p_H^B = v-(s-b)F(q_H^B)$ and q_H^B is determined by:*

$$q_H^B = \bar{F}^{-1}\left(\frac{c-b}{(v-b)-(s-b)\left[F(q_H^B)+h(q_H^B)\int_{d_l}^{q_H^B}\bar{F}(x)dx\right]}\right). \tag{5}$$

The implications of Proposition 3, together with those of Proposition 2, are illustrated in Figure 4. In particular, the two statements in Proposition 2 are illustrated in Regions NV and BL, respectively. When the firm is in deeper distress, the optimal strategy bifurcates depending on the proportion of strategic customers: When a large share of customers are strategic (Region BH), the firm continues to adjust its price and inventory to induce strategic customers to purchase early. However, differently from Region BL, the optimal price and inventory in this region correspond to the *horizontal* segment of the buy–wait boundary in Figure 3, i.e. $q^* = Q_s(p^*)$. In other words,

Figure 4 Illustration of the firm's optimal strategies.



Notes. **NV** represents the region where the *newsvendor* solution is optimal; **BL** represents inducing customers to *buy* under *low* financial distress by limiting the probability of bankruptcy under the (hypothetical) wait-equilibrium d_τ^W; **BH** represents inducing customers to *buy* under *high* financial distress by limiting inventory q^*; and **ISC** represents the strategy that *ignores strategic consumers*, i.e. $(p^*, q^*) \in \Omega^W$.

the firm eliminates strategic waiting by directly limiting the leftover inventory available in the liquidation sale.

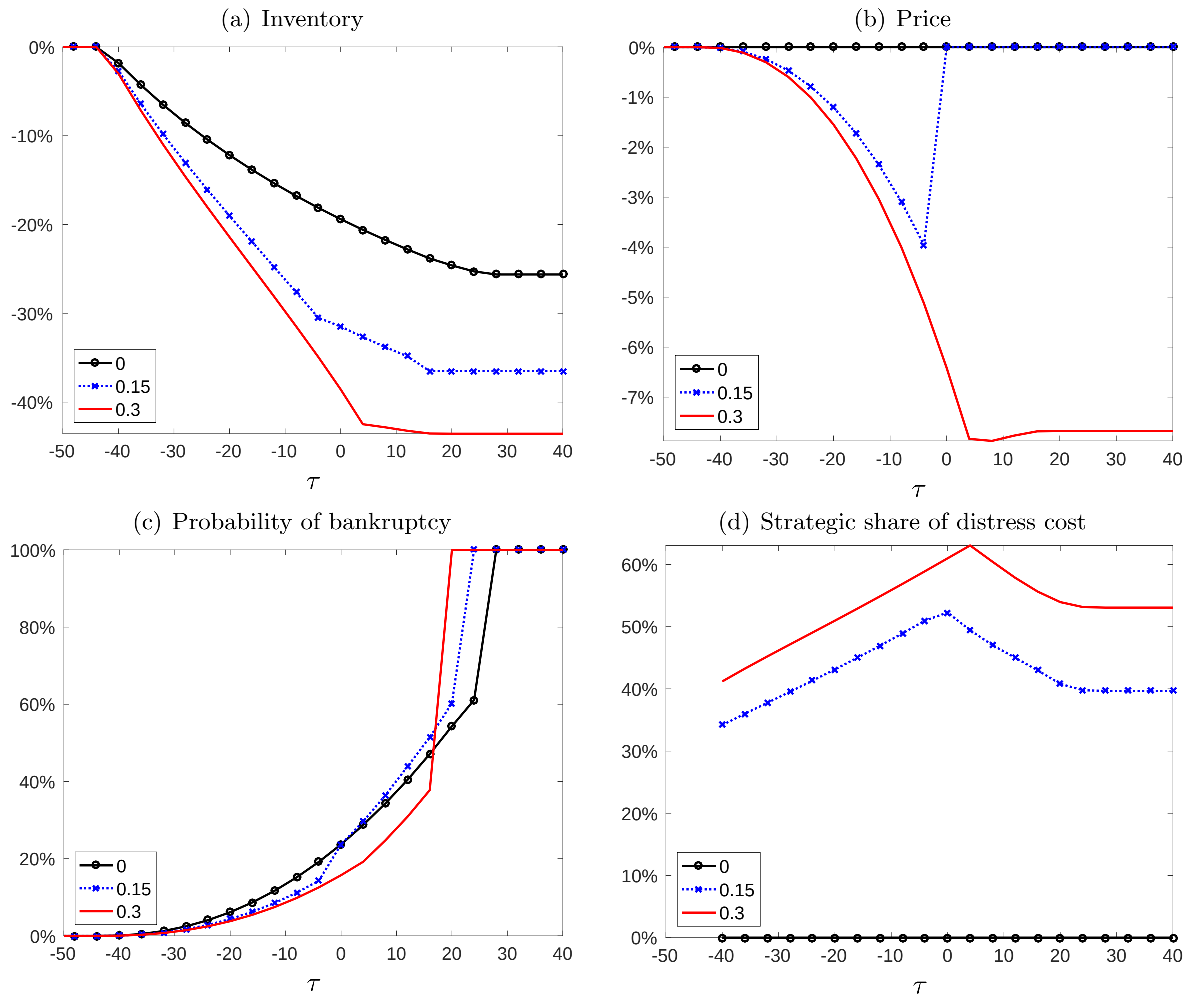
On the other hand, with only a small proportion of strategic consumers (Region *ISC* in Figure 4), it is actually optimal for the firm to *not* induce strategic customers to purchase early. The intuition is as follows. For sufficiently large τ, the firm's profit function is depicted in (2), where $\pi_H^B(p_H^B, q_H^B)$ is independent of α. This is in contrast to the situation with low τ (Proposition 2), where the cost of inducing customers to purchase is lower when α is small. Because of this difference, as financial distress deepens, the relative cost to induce strategic customers to purchase is high when α is low. Therefore, the firm is indeed better off letting the small share of strategic customers wait. The contrast between the firm's pricing strategy in Regions *BL* and that in *BH*, as well as those in the other two regions, highlights that the composition of customers faced by a distressed firm could have not only a quantitative, but also qualitative, impact on the firm's operational decisions.

5.5. Numerical results

To complement the analytical results presented in the previous sections, we conduct comprehensive numerical studies.[9] Figure 5 offers a representative view of these results. As shown in Figure 5(a), as τ increases, the firm's inventory level gradually drops from the newsvendor level and eventually remains at the low level specified in Proposition 3 for sufficiently large τ. Furthermore, the optimal inventory level also declines in the presence of a greater proportion of strategic customers.

[9] We refer the readers to Online Appendix E for other parameters we have used for robustness checks.

Figure 5 **The impact of level of financial distress (τ) and fraction of strategic customers (α) on optimal decisions and performance**



Notes: $D \sim Triangular(0,100,50)$, $v=1$, $c=0.6$, $s=0.5$, $b=0.3$. Different lines represent different fractions of strategic customers (α), as marked in the legend. Figure 5(a) (5(b)) represents the inventory (price) change in percentage relative to the the newsvendor benchmark (v, q^{NV}). Figure 5(d) represents the strategic share of distress cost, defined as the proportion of total distress cost caused by strategic consumers, i.e. $\frac{\Pi(\tau,\alpha)-\Pi(\tau,0)}{\Pi(\tau,\alpha)-\Pi(-\infty,0)}$, where $\Pi(\tau,\alpha)$ is the firm's *optimal* profit under (τ,α). In Figure 5(d), the strategic share of distress cost is not defined when τ is low as the total distress cost is zero.

In contrast to inventory, the pattern for the optimal price discount, as shown in Figure 5(b), varies distinctly for different levels of α, echoing Proposition 3. When $\alpha = 0$, there is no incentive for the firm to lower the price. At the low α level, the firm starts to offer a price discount when moving out of Region NV and increases the price discount as τ increases. However, the optimal price goes back to the full price in the high-distress region when there are few strategic consumers since it becomes too costly to induce them to buy and to let the myopic customers freeride the discounts. When there are more strategic customers in the market (high α) and the financial distress is severe (high τ), strategic customers should not be ignored: The firm still offers moderate price discounts to push all strategic consumers to make purchases.

Combining the patterns exhibited in Figures 5(a) and 5(b), we note that in the presence of financial distress, firms may emphasize on different operational levers depending on their consumer compositions (α), which can be related to product characteristics. For example, related to the existing empirical research on strategic customer behavior, which has identified that a larger fraction of customers is strategic when the product price is high (Li et al. 2014), our results imply that retailers in a market where the average price of the products is low, and hence lower α (e.g., supermarkets), should pre-dominantly apply the inventory lever. However, for sellers focusing on high-valued product categories (e.g., electronics and automobiles), it is crucial to accompany inventory reduction with price discounts.

As shown in Figure 5(c), the firm's probability of bankruptcy under optimal operational decisions increases in τ. However, under the same τ, a greater share of strategic customers does not necessarily lead to a higher probability of bankruptcy. In fact, for lower τ, the firm's bankruptcy probability decreases in α. The reason lies exactly on the firm's operational response to strategic customer behavior: When τ is low, the firm's optimal strategy is to induce early purchase (Region BL in Figure 4) by offering (p, q) that eliminate the wait-equilibrium. As α increases, i.e., more customers may wait strategically, eliminating the wait-equilibrium requires the firm to reduce its inventory and/or price more aggressively, actually reducing the probability of bankruptcy. However, as the cost associated with such mitigation increases in both τ and α, for higher τ, the firm gives up its efforts to lower the bankruptcy probability, causing the probability to jump to 100%, as shown in Figure 5(c).

Finally, Figure 5(d) illustrates the proportion of the total cost of financial distress that is due to the presence of strategic customers, which we call the *strategic share of distress cost.* As shown, fixing τ, the strategic share of distress cost increases with α. On the other hand, with the same proportion of strategic customers, the strategic share of distress cost is highest when τ is at a medium level, when the self-fulfilling nature of customers' anticipation of bankruptcy has the strongest effect.

6. Using Deferred Discounts to Induce Early Purchase

While both inventory reduction and price discounts alleviate the cost of financial distress, neither mitigates a fundamental challenge faced by the firm; that is, the wait-equilibrium is always more appealing to strategic customers when both equilibria exist. This hurts not only the firm's profitability but also social welfare as the customers' valuation of the products declines over time. Is there a mechanism that nudges strategic customers to purchase early when the wait-equilibrium exists? In this section, we argue that *deferred discount* acts as exactly such a mechanism.

As the name suggests, deferred discounts benefit customers not immediately but in a later period, which is often specified by the firm. Many widely used marketing tools can be seen as a form of deferred discount. For example, rebate allows customers to receive a partial refund some time after purchase. Consumer electronics stores such as Ritz Camera and CompUSA are among the firms that frequently offer rebates. Recently, a real estate developer in Qinhuangdao, China that was facing financial pressure offered a 40% price discount through a rebate that would be returned to customers at a rate of 10% per year over four years (Wang 2014). Another form of deferred discount is store credit that can only be applied to a future purchase or service. For example, AT&T offered a $50 discount on the new iPhone 6 upgrade in the form of bill credit applied over three subsequent billing cycles (Siegal 2014).

An important feature of deferred discounts is that their value is contingent on the firm's future financial status as deferred discounts are often not honored when the firm goes bankrupt due to the presence of other claims owed to creditors with higher priority. For example, when the DVD drive maker CenDyne filed for bankruptcy in 2003, it stopped honoring rebates (Shim 2003). Similarly, other forms of deferred discount, such as store credit, coupons, and gift cards, were not accepted after Circuit City filed for bankruptcy (McCraken 2009). As we show later, this contingency allows deferred discounts to better align customers' interests with the firm's in the presence of financial distress.

To incorporate deferred discounts into the base model, we augment the firm's operational decisions to include not only the price p and inventory q but also a deferred discount with face value $t > 0$. Customers receive the value t if and only if they purchase in the first period *and* the firm survives to the second period.[10] The sequence of events is the same as under the base model (Figure 2). In the rest of the section, we characterize customers' purchase behavior under (p, q, t). The impact of deferred discounts on the firm's operational decisions and performance is studied in Section 7.

Under (p, q, t), customers decide whether to purchase in the first period or wait until the second period by comparing their expected payoffs in these two periods. Obviously, their expected payoff in waiting is exactly the same as in Section 4. However, if they decide to purchase in the first period, in addition to the immediate surplus $v - p$, they also obtain the deferred discount with expected value $t\bar{F}(d_\tau)$, where $\bar{F}(d_\tau)$ is the firm's survival probability. Therefore, customers benefit more from early purchase when the firm's probability of bankruptcy is low. In this sense, deferred discounts better align customers' interests with those of the seller. This alignment is absent in immediate discount, under which customers benefit from the firm's financial failure. This intuition

[10] To keep the model tractable, we assume that the redemption of the deferred discounts is guaranteed if the firm survives at the end of the first period. Our main insights should remain unchanged as long as the firm does not go bankrupt with certainty in the future.

is formalized in the following proposition. In preparation, similar to the definition of Ω_0 in Section 4, we confine our analysis to $(p,q,t) \in \Omega_0^{dd} := \{(p,q,t) | p \in (v-(s-b), v] \text{ and } q \geq d_l \text{ and } t > 0\}$.

PROPOSITION 4. *Let $Q_s^{dd}(p,t) := F^{-1}\left(\frac{v-p+t}{s-b+t}\right)$ and $Q_s^{dd,a}(p,t)$ and $Q_s^{dd,b}(p,t)$ satisfy:*

$$(s-b)F\left(Q_s^{dd,a}\right) + tF\left(\frac{cQ_s^{dd,a}+\tau}{p}\right) = v-p+t; \tag{6}$$

$$(s-b)F\left[\frac{cQ_s^{dd,b}+\tau}{(1-\alpha)p}\right] + tF\left(\frac{cQ_s^{dd,b}+\tau}{p}\right) = v-p+t. \tag{7}$$

For any $(p,q,t) \in \Omega_0^{dd}$,

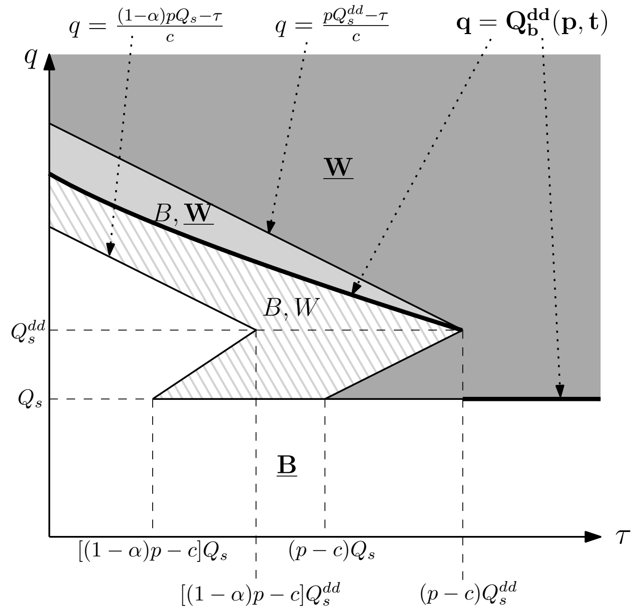
1. *the buy-equilibrium exists if and only if $q \leq \min\left(Q_s, \frac{\tau}{p-c}\right)$ or $q \in \left(\frac{\tau}{p-c}, \frac{pQ_s^{dd}-\tau}{c}\right]$;*
2. *the wait-equilibrium exists if and only if $q \in \left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right)$ or $q > \max\left(\frac{\tau}{(1-\alpha)p-c}, \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}\right)$;*
3. *the buy-equilibrium is more appealing to strategic customers when $q = Q_b^{dd}(p,t)$. The wait-equilibrium is more appealing when $q > Q_b^{dd}(p,t)$, where $Q_b^{dd}(p,t)$ follows:*

$$Q_b^{dd}(p,t) = \begin{cases} Q_s^{dd,b} & \text{if } \tau \leq [(1-\alpha)p-c]Q_s^{dd,a}, \\ Q_s^{dd,a} & \text{if } \tau \in ([(1-\alpha)p-c]Q_s^{dd,a}, (p-c)Q_s^{dd}], \\ Q_s & \text{if } \tau > (p-c)Q_s^{dd}. \end{cases} \tag{8}$$

Proposition 4 is illustrated in Figure 6. By comparing Figures 3 and 6, we note several similarities between Propositions 1 and 4. Indeed, as t moves to zero, $Q_s^{dd}(\cdot)$ and $Q_b^{dd}(\cdot)$ in Proposition 4 degenerate to $Q_s(\cdot)$ and $Q_b(\cdot)$ in Proposition 1, respectively. In general, the buy-equilibrium exists when the inventory level is relatively low, while the wait-equilibrium exists for a higher inventory level. In addition, when τ is not extremely high, both equilibria may co-exist for inventory at the medium level.

Aside from the above similarities, Figure 6 also reveals two distinctions between Propositions 1 and 4 caused by the deferred discount t. First, note that under a given τ, the range of q such that a buy- (wait-)equilibrium exists may not be continuous. This is due to a jump in the value of the deferred discount corresponding to a jump in the bankruptcy probability. Take the buy-equilibrium for $\tau \in [(p-c)Q_s, (p-c)Q_s^{dd}]$ as an example. First, when $q < Q_s$, the low inventory level alone ensures the existence of the buy-equilibrium, as in Section 4. However, for $q \in \left(Q_s, \frac{\tau}{p-c}\right)$, because $\tau < (p-c)q$, the firm's survival probability is zero, deeming deferred discounts valueless. However, the increase in q gives customers a better chance of getting a bargain in liquidation, nudging customers to wait. As such, the buy-equilibrium no longer exists. Finally, when q increases beyond $\frac{\tau}{p-c}$, the firm's upside potential increases; the value of a deferred discount sees an immediate jump from zero to $t[1-F(d_\tau^B)]$. For this reason, the buy-equilibrium arises again. Similar situations happen with the wait-equilibrium for the same reason.

Figure 6 Strategic customers' behavior in equilibrium under p and deferred discount t.



Notes. B (W) represents that the buy-equilibrium (wait-equilibrium) exists in the region. The equilibrium that is more appealing to strategic customers is in bold font and underlined. In the shaded area, both the buy- and wait-equilibria exist and customers may prefer either depending on the specific magnitudes of multiple parameters. We omit the details because $q < Q_b^{dd}(p,t)$ cannot be an optimal solution when the buy-equilibrium is more appealing to customers at $q = Q_b^{dd}(p,t)$.

Second, and more importantly, by introducing deferred discounts t, the buy-equilibrium may become the most appealing even if both equilibria co-exist in the region. Indeed, as shown in Figure 6, while both equilibria co-exist over a wide region when $q \leq Q_b^{dd}(p,t)$, deferred discounts are able to push the maximum inventory level under which the buy-equilibrium is more appealing to $Q_b^{dd}(p,t)$. How is it that the buy-equilibrium can be more appealing than the wait-equilibrium under deferred discount? The reason lies in the contingent nature of deferred discount. Specifically, in the presence of deferred discount, customers' surplus of purchasing is different under the buy- and wait-equilibria: When a strategic customer anticipates that all other customers will purchase in the first period, the customer's own surplus of purchasing, $v - p + t\bar{F}(d_\tau^B)$, is higher than when he anticipates that no peers will purchase $v - p + t\bar{F}(d_\tau^W)$, as $d_\tau^B < d_\tau^W$. By contrast, under immediate discount, customers' surplus from purchasing, $v - p$, is the same under the buy- and wait-equilibrium, while their surplus of waiting, $(s-b)F(\min(q, d_\tau))$ is higher under the wait-equilibrium. In this sense, under immediate discount, the firm and customers have conflicting interests: Under bankruptcy, the firm loses while customers always gain. This conflict of interests is (partially) resolved by introducing deferred discount, under which customers also benefit from the firm's survival.

7. When Are Deferred Discounts Most Valuable to the Firm?

Understanding that the contingency embedded in deferred discounts provides an additional incentive for customers to purchase early, in this section, we examine how this effect translates into higher profits for the firm. Specifically, we focus on the following question: *Under what conditions* is employing deferred discounts most valuable to the firm?

7.1. The firm's profit function with deferred discount

To answer the above question, we first examine the firm's profit function under (p,q,t), which we denote as $\pi^{dd}(p,q,t)$. Note that with $t=0$, $\pi^{dd}(p,q,t)$ degenerates to $\pi(p,q)$, as studied in Section 5. We focus on the case where $q=Q_b^{dd}(p,q,t)$ and $\tau \le (p-c)q$.[11]

As deferred discounts do not add value to the firm in the above two scenarios, for brevity of exposition, we focus on the scenario where $q=Q_b^{dd}(p,q,t)$ and $\tau \le (p-c)q$. Analyzing the firm's payoffs depending on different demand realizations as in Section 5.1, we have:

$$\pi^{dd}(p,q,t) = \pi_L^B(p,q) - t\int_{d_\tau^B}^{q} \min(x,\, q)dF(x), \tag{9}$$

where $\pi_L^B(p,q)$ follows (1) in Section 5. As deferred discounts are only redeemable when the firm survives, i.e. the demand is no less than d_τ^B, the total cost of offering deferred discounts is t multiplied by the expected first-period sales when the firm survives. The benefit of deferred discount, on the other hand, is that it is able to support higher p and q yet still induce customers to purchase early by pushing the buy–wait boundary from $q=Q_b(p)$ in Figure 3 to $q=Q_b^{dd}(p,t)$ in Figure 6. Deferred discounts are valuable if and only if the benefit outweighs the cost.

7.2. When deferred discounts can (or cannot) be valuable

While it is clear that the firm's profit will not be worse off by having t as an additional lever, the next proposition offers some insight into the conditions under which offering deferred discounts can *strictly* improve the firm's profit as well as when it cannot.

Proposition 5. *Let (p^*,q^*) be the firm's optimal decision without deferred discounts and $(p^{dd,*},q^{dd,*},t^{dd,*})$ be the firm's optimal decision with deferred discounts, i.e. $(p^*,q^*)=\arg\max \pi(p,q)$ and $(p^{dd,*},q^{dd,*},t^{dd,*})=\arg\max \pi^{dd}(p,q,t)$.*

1. *When any of the following three conditions holds, offering deferred discounts does not improve the firm's profit, i.e. $\pi^{dd}(p^{dd,*},q^{dd,*},t^{dd,*})=\pi(p^*,q^*)$*
   (a) *$\tau \le T^D(\alpha)$;*
   (b) *$q^{dd,*} > Q_b^{dd}(p^{dd,*},t^{dd,*})$;*

[11] As shown later in Proposition 5, those (p,q,t) that do not satisfy these conditions will be either *only* as good as without deferred discounts $(t=0)$ or dominated by other decisions.

(c) $(p^{dd,*} - c)\, Q_s^{dd}(p^{dd,*}) > \tau$.

2. *When $p^* < v$ and the firm's probability of bankruptcy is sufficiently small under (p^*, q^*), offering deferred discounts strictly improve the firm's profit, i.e.* $\pi^{dd}(p^{dd,*}, q^{dd,*}, t^{dd,*}) > \pi(p^*, q^*)$.

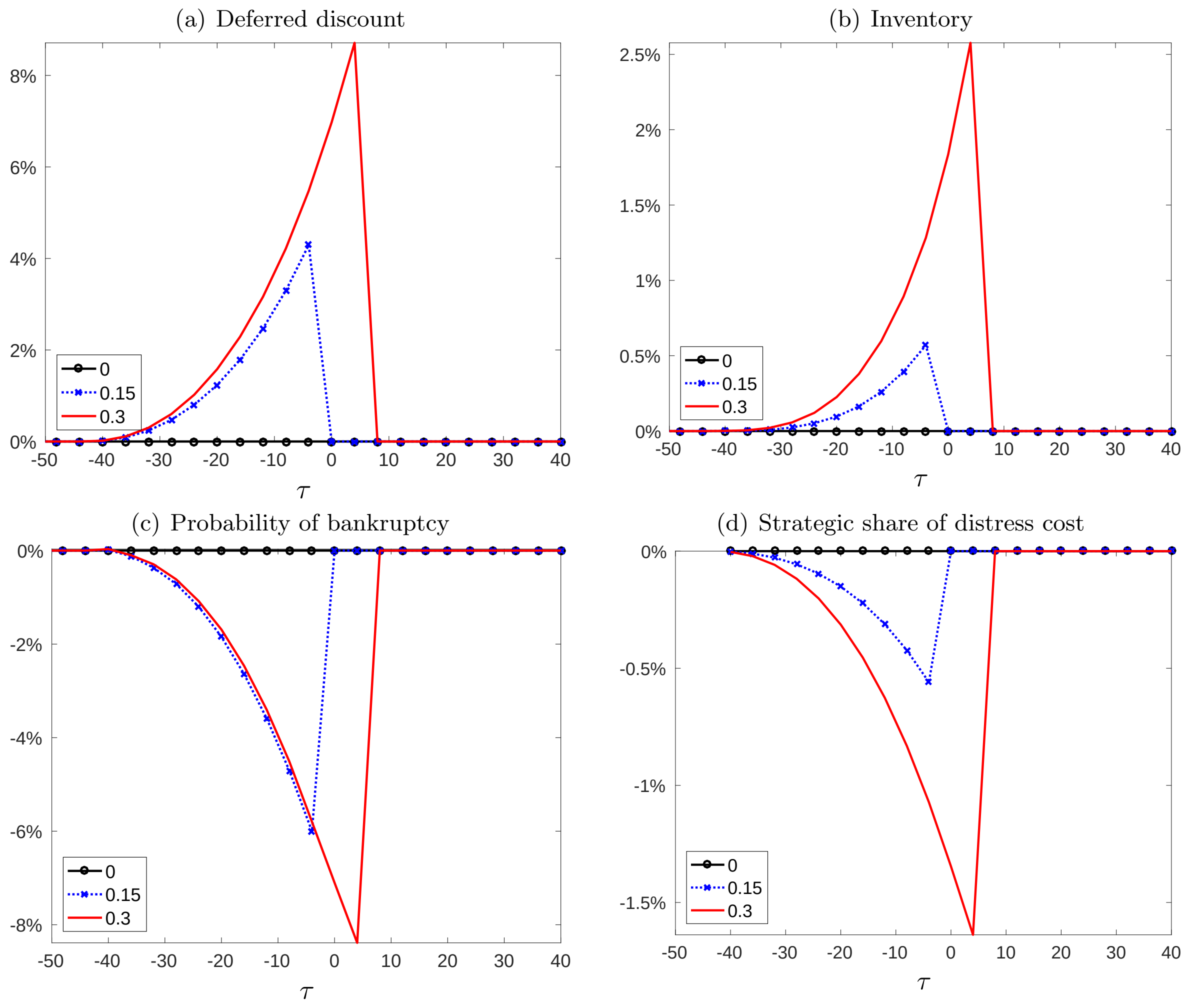
Statement 1 in Proposition 5 identifies several conditions under which deferred discounts are *not* valuable. As a mechanism that induces customers to purchase, offering deferred discounts is clearly not beneficial in the absence of financial distress, i.e. $\tau \le T^D(\alpha)$, or when it cannot induce customers to purchase, i.e. $q^{dd,*} > Q_b^{dd}(p^{dd,*}, t^{dd,*})$. Finally, when the level of financial distress is high, i.e. $\tau > (p^{dd,*} - c)\, Q_s^{dd}(p^{dd,*})$, the probability of redeeming a deferred discount is zero, also rendering deferred discounts valueless. The above three conditions correspond to Region *NV* and, roughly, Regions *ISC* and *BH* in Figure 4, respectively.

Statement 2 in Proposition 5 shows that in the part of Region *BL* that neighbors Region *NV*, offering deferred discounts is strictly beneficial to the firm. The intuition behind this result is as follows. The boundary of Region *NV*, $T^D(\alpha)$, is determined so that the firm will not go bankrupt even if all customers wait, i.e. $F(d_\tau^W) = 0$. This is exactly because, according to Proposition 1, in order to induce customers to purchase, we need to eliminate the wait-equilibrium. For the same reason, according to Proposition 2, as τ increases slightly beyond $T^D(\alpha)$ (and when $f(d_l) > 0$), the firm needs to lower its price and inventory immediately, even though the firm's lowest possible profit $pd_l - cQ_b(p)$ is still greater than $T^D(\alpha)$. In this region, by employing a small deferred discount $t \approx v - p$, the firm is able to stock at $Q_b^{dd}(v, t)$ as characterized in Proposition 4 while still eliminating strategic waiting. Such an increase in inventory level directly translates to an increase in the firm's profit.

7.3. The impact of deferred discounts on operational decisions and performance

To complement Proposition 5, we conduct numerical experiments using the same parameters as in Section 5.5. A representative set of results is illustrated in Figure 7. Specifically, Figure 7(a) shows that deferred discounts are employed when the firm's financial distress is at a medium level. This region corresponds to the low τ region in Figure 6, where both the buy- and wait-equilibria exist and deferred discounts are able to push the buy–wait boundary upward. When τ is extremely high, the two equilibria do not co-exist, rendering deferred discounts valueless. Both phenomenon echo Proposition 5. In addition, the firm offers a larger deferred discount when it faces a greater share of strategic customers. This result is again consistent with the role deferred discounts play in better aligning the interests of the firm and its customers. Due to this effect, the optimal inventory level with deferred discounts is greater than that without (Figure 7(b)). Such changes in operational decisions also lead to performance improvement. As shown in Figures 7(c) and 7(d), employing deferred discounts reduces both the firm's probability of bankruptcy and the strategic share of

Figure 7 **The usage of deferred discounts and its impact on the firm's operational decisions and performance under** (α, τ)



Notes. $D \sim Triangular(0, 100, 50)$,$v = 1,\ c = 0.6,\ s = 0.5,\ b = 0.3$. Different lines represent different α, with the corresponding numbers in the legend. In Figure 7(a), the optimal amount of deferred discount is plotted as a fraction of v. Figures 7(b), 7(c), and 7(d) plot the percentage differences between the corresponding quantities under the optimal decisions *with* deferred discount $(p^{dd,*}, q^{dd,*}, t^{dd,*})$, and those without (p^*, q^*). As such, a positive (negative) number suggests the inventory with deferred discounts is higher (lower) than that without.

distress cost. Indeed, our numerical results suggest that deferred discounts can strictly improve the firm's profits over the entire Region *BL* depicted in Figure 4 and also in the parts of Region *ISC* and Region *BH* that neighbor Region *BL*.

In summary, by better aligning the interests of the firm and its customers in the presence of financial distress, deferred discounts enrich the firm's toolbox for fighting financial difficulties caused by customers' strategic behavior. In addition, we find that deferred discounts are most valuable to the firm when it faces a medium level of financial distress and many strategic customers. This is consistent with anecdotal evidence that rebates have been frequently employed by consumer electronics stores facing financial pressure, such as CompUSA and Ritz Camera.

8. Conclusion

Financial difficulties and strategic customers pose major challenges for firms in terms of operational strategies and financial performance. This paper focuses on the interaction between these two challenges. Specifically, we find that customers' strategic behavior when anticipating a liquidation sale can play an important role in determining a firm's bankruptcy risk.

The dynamics linking customers' strategic behavior and the firm's probability of bankruptcy have important implications for common operational levers such as inventory and price. In particular, we find that as a firm's financial situation worsens, aggressive price discounting may not be the most effective strategy to induce customers to purchase early. Instead, the firm should first lower inventory and then reduce both price and inventory. As the level of financial distress increases further, it may be optimal for the firm to cut back its price discount when there is only a small proportion of strategic customers. In addition to inventory and price discounting, we argue that deferred discounts, such as rebates and store credit, are an effective mechanism for mitigating a firm's financial distress. Deferred discounts create value by better aligning customers' interests with those of the firm. As such, deferred discounts are most valuable to a firm when its level of financial distress is not too high and a large proportion of its customers are strategic.

On the financial side, the paper points out that while a firm's value always decreases as it becomes more financially distressed or faces a larger share of strategic customers, facing a large proportion of strategic consumers may actually lower a firm's probability of bankruptcy as it adopts a more conservative operational strategy, alluding to that firms with different consumer characteristics may adopt different capital structures.

As an initial attempt to link customer behavior explicitly to the cost of financial distress and the corresponding operational decisions, this work can be extended in several directions. First, focusing on operational implications, we treat the firm's financing decisions as exogenous. Extending the model to endogenous financing decisions represents a possible extension. In addition, our model leads to several predictions about the relationships among the fraction of strategic customers, level of financial distress, and operational metrics, some of which may serve as testable hypotheses for future empirical research by combining methods used to identify strategic customers and financial distress.

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Appendix A: List of Notation

Table 2 summarizes the paper's notation. In general, superscript B (W) represents the quantity under the buy- (wait-)equilibrium, while superscript dd represents the quantity with *deferred discount*.

Appendix B: Proofs

B.1. Proofs of results in Section 4.

Proof of Lemma 1. Consider the following two scenarios.

1. When the firm does not go bankrupt, it enters the salvage sale. Facing an infinite number of bargain hunters with valuation s, the firm should set $p_2 = s$.

2. When the firm goes bankrupt, it enters the liquidation sale and faces bargain hunters with valuation b and waiting strategic customers with valuation s. Obviously, when the inventory available for liquidation is less than the number of waiting strategic customers or, equivalently, if the total inventory q is less than the realized demand D in the first period, the firm can liquidate all inventory at price s. Otherwise, the firm needs to set $p_2 = b$ to sell part of the inventory to the bargain hunters. □

Proof of Proposition 1. We prove the three statements in sequence.

Statement 1: The existence condition for the buy-equilibrium. By definition, the sufficient and necessary conditions for a buy-equilibrium to exist are that under the belief that all other customers buy in the first period, a strategic customer does not have the incentive to deviate from buying to waiting. Mathematically, this is equivalent to

$$v - p \geq (s-b)F\left(\min(d_\tau^B, q)\right). \tag{10}$$

Consider two cases.

1. When $d_\tau^B < q$ ($q > \frac{\tau}{p-c}$), equation (10) is equivalent to $F(d_\tau^B) \leq \frac{v-p}{s-b}$. This condition always holds when $d_\tau^B \leq d_l$; that is, $q \leq \frac{pd_l - \tau}{c}$. When $d_\tau^B \geq d_l$, $F(d_\tau^B) \leq \frac{v-p}{s-b}$ is equivalent to $q \leq \frac{pQ_s - \tau}{c}$. Combining the two cases, as $Q_s \geq d_l$, the buy-equilibrium exists if $q \in \left(\frac{\tau}{p-c}, \frac{pQ_s-\tau}{c}\right]$.

2. When $d_\tau^B \geq q$ ($q \leq \frac{\tau}{p-c}$), equation (10) is equivalent to $q \leq Q_s$. Combining the two scenarios, a buy-equilibrium exists if and only if $q \leq \min\left(\frac{\tau}{p-c}, Q_s\right)$ or $q \in \left(\frac{\tau}{p-c}, \frac{pQ_s-\tau}{c}\right]$.

Table 2 Notation

τ	the firm's net financial obligation. The firm goes bankrupt when the first-period profit is less than τ
α	the proportion of first-period customers that are strategic, $\alpha \in [0,1]$
D	the random number of first-period customers, $D \in [d_l, d_h)$, $D \in F(\cdot)$
v	the first-period valuation of myopic and strategic customers
c	the firm's unit procurement cost
s	the second-period valuation of strategic customers and bargain hunters in salvage sale
b	the second-period valuation of bargain hunters in salvage sale
p	the first-period price set by the firm
q	the first-period inventory set by the firm
p_2	the second-period price set by the firm
d_τ^j	$j = B, W$, the minimum first-period demand realization to avoid bankruptcy when strategic customers buy (wait) in the first period.
Ω_0	$\Omega_0 := \{(p,q) \mid p \in (v-(s-b), v] \text{ and } q \geq d_l\}$
$Q_s(p)$	$Q_s = F^{-1}\left(\frac{v-p}{s-b}\right)$.
$Q_b(p)$	the buy-wait boundary, $Q_b = \max\left(Q_s, \frac{(1-\alpha)pQ_s - \tau}{c}\right)$
Ω^B	the buy-region $\Omega^B = \{(p,q) \in \Omega_0 \mid q \leq Q_b(p)\}$
Ω^W	the wait-region $\Omega^W = \{(p,q) \in \Omega_0 \mid q > Q_b(p)\}$
$\pi(p,q)$	the firm's total profit under price p and inventory q
q^{NV}	$q^{NV} = \bar{F}^{-1}\left(\frac{c-s}{v-s}\right)$
q_b^{NV}	$q_b^{NV} = \bar{F}^{-1}\left(\frac{c-b}{v-b}\right)$
$T^D(\alpha)$	$T^D(\alpha) = (1-\alpha)vd_l - cq^{NV}$
t	the amount of deferred discount, $t \geq 0$.
Ω_0^{dd}	$\Omega_0^{dd} := \{(p,q,t) \mid p \in (v-(s-b), v], q \geq d_l, \text{ and } t > 0\}$
$Q_s^{dd}(p,t)$	the buy-wait boundary under (p,t) for high τ, $Q_s^{dd}(p,t) = F^{-1}\left(\frac{v-p+t}{s-b+t}\right)$.
$Q_s^{dd,a}(p,t)$	the buy-wait boundary under (p,t) for low τ, $(s-b)F\left(Q_s^{dd,a}\right) + tF\left(\frac{cQ_s^{dd,a}+\tau}{p}\right) = v-p+t$.
$Q_s^{dd,b}(p,t)$	the buy-wait boundary under (p,t) for medium τ, $(s-b)F\left[\frac{cQ_s^{dd,b}+\tau}{(1-\alpha)p}\right] + tF\left(\frac{cQ_s^{dd,b}+\tau}{p}\right) = v-p+t$.
$\pi^{dd}(p,q,t)$	the firm's total profit under p, q, and deferred discount t

Noticing that $\frac{\tau}{p-c} < \frac{pQ_s-\tau}{c}$ if and only if $Q_s > \frac{\tau}{p-c}$, we can further simplify the above range by discussing two scenarios. First, when $\tau > (p-c)Q_s$, we have $\frac{\tau}{p-c} > Q_s$, that is, $\min\left(\frac{\tau}{p-c}, Q_s\right) = Q_s$. Also, $\frac{\tau}{p-c} > \frac{pQ_s-\tau}{c}$, that is, $\left(\frac{\tau}{p-c}, \frac{pQ_s-\tau}{c}\right] = \emptyset$. Combining the two parts, when $\tau > (p-c)Q_s$, the buy-equilibrium exists if and only if $q \leq Q_s$.

Similarly, when $\tau \le (p-c)Q_s$, we have $\min\left(\frac{\tau}{p-c}, Q_s\right) = \frac{\tau}{p-c}$. We also have $\frac{\tau}{p-c} \le \frac{pQ_s-\tau}{c}$, that is, $\left(\frac{\tau}{p-c}, \frac{pQ_s-\tau}{c}\right] \ne \emptyset$. Combining the two parts, when $\tau \le (p-c)Q_s$, the buy-equilibrium exists if and only if $q \le \frac{pQ_s-\tau}{c}$.

Note that $Q_s > \frac{pQ_s-\tau}{c}$ if and only if $\tau > (p-c)Q_s$. Therefore, combining the two scenarios, i.e. $\tau > (p-c)Q_s$ and $\tau \le (p-c)Q_s$, the buy-equilibrium exists if and only if $q \le \max\left(Q_s, \frac{pQ_s-\tau}{c}\right)$, as desired.

Statement 2: The existence condition for the wait-equilibrium. Symmetrically, the wait-equilibrium exists if and only if a customer's expected surplus of waiting is larger than the surplus of buying in the first period, assuming all customers wait. That is,

$$v - p < (s-b)F(\min(d_\tau^W, q)). \tag{11}$$

Again, consider two scenarios.

1. When $d_\tau^W < q$, (11) is equivalent to $q > \frac{(1-\alpha)pQ_s-\tau}{c}$. Combining the condition $d_\tau^W < q$, that is, $q > \frac{\tau}{(1-\alpha)p-c}$, the wait-equilibrium exists if $q > \max\left[\frac{\tau}{(1-\alpha)p-c}, \frac{(1-\alpha)pQ_s-\tau}{c}\right]$.
2. When $d_\tau^W \ge q$, that is, $q \le \frac{\tau}{(1-\alpha)p-c}$, (11) is simplified to $v-p < (s-b)F(q)$, that is, $q > Q_s$. The condition for the existence of the wait-equilibrium is $q \in \left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right]$.

Note that when $\tau < [(1-\alpha)p-c]Q_s$, we have $\left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right] = \emptyset$. Furthermore, $\frac{(1-\alpha)pQ_s-\tau}{c} > \frac{\tau}{(1-\alpha)p-c}$. Therefore, the wait-equilibrium exists if and only if $q > \frac{(1-\alpha)pQ_s-\tau}{c}$.

Similarly, when $\tau \ge [(1-\alpha)p-c]Q_s$, we have $\left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right] \ne \emptyset$, and $\frac{(1-\alpha)pQ_s-\tau}{c} \le \frac{\tau}{(1-\alpha)p-c}$. Therefore, the wait-equilibrium exists if and only if $q > Q_s$.

Note that $Q_s \ge \frac{(1-\alpha)pQ_s-\tau}{c}$ if and only if $\tau \ge [(1-\alpha)p-c]Q_s$. Therefore, the above two conditions can be combined, and hence, the wait-equilibrium exists if and only if $q > \max\left(Q_s, \frac{(1-\alpha)pQ_s-\tau}{c}\right)$, as desired.

Statement 3: Comparing the two equilibria. Note that customers prefer the wait-equilibrium to the buy-equilibrium if and only if their expected surplus under the wait-equilibrium is higher, that is, $v-p < (s-b)F\left(\min(q, d_\tau^W)\right)$, which is exactly the condition for the wait-equilibrium to exist. Therefore, customers prefer the wait-equilibrium to the buy-equilibrium as long as the wait-equilibrium exists. □

B.2. Proofs of results in Section 5.

Proof of Lemma 2. Using Lemma C.2, we can write out the firm's profit function with $\alpha = 0$ for $(p,q) \in \Omega_0$ as follows:

$$\pi(p,q) = (p-c)q - (p-s)\int_{d_l}^{q}(q-x)dF(x) - (s-b)\int_{d_l}^{\min(d_\tau^B, q)}(q-x)dF(x). \tag{12}$$

It is easy to see that $\pi(p,q)$ increases in p, and hence, $p^* = v$, allowing us to rewrite the above equation as:

$$\pi(v,q) = (v-c)q - (v-s)\int_{d_l}^{q}(q-x)dF(x) - (s-b)\int_{d_l}^{\min(d_\tau^B, q)}(q-x)dF(x). \tag{13}$$

Next, we solve $\pi(v,q)$ for the optimal inventory q. Note that the first statement in Lemma 2 ($p^* = v$, $q^* = q^{NV}$ for $\tau \le vd_l - cq^{NV}$) is a special case of the first statement in Proposition 2. See the corresponding proof for details. For $\tau > vd_l - cq^{NV}$, depending on the relative magnitude of q, d_l, and d_τ^B, we discuss the following three scenarios.

1. When $q > d_l \geq d_\tau^B$, i.e. $q \in \left(d_l, \frac{vd_l-\tau}{c}\right]$, the optimal solution is the boundary one, i.e. $q = \frac{vd_l-\tau}{c}$.
2. When $q > d_\tau^B > d_l$, i.e. $q > \max\left(\frac{vd_l-\tau}{c}, \frac{\tau}{v-c}\right)$. The firm's profit function can be simplified to:

$$\pi_L^0(v,q) = (v-c)q - (v-s)\int_{d_l}^{q}(q-x)dF(x) - (s-b)\int_{d_l}^{d_\tau^B}(q-x)dF(x). \tag{14}$$

Similar to the proof in Lemma C.3, we π_L^0 is pseudo-concave for $\tau < (v-c)q$. Therefore, the optimal solution is either the boundary one, i.e. $\max\left(\frac{vd_l-\tau}{c}, \frac{\tau}{v-c}\right)$, or the unique interior solution that satisfies $\frac{d\pi_L^0}{dq} = 0$. Let the interior solution be q_L^0, where

$$q_L^0 = \bar{F}^{-1}\left\{\left(\frac{c-b}{v-s}\right) - \left(\frac{s-b}{v-s}\right)\bar{F}\left[\frac{cq_L^0+\tau}{v}\right]\left[1 - \left(\frac{c[(v-c)q_L^0-\tau]}{v^2}\right)h\left(\frac{cq_L^0+\tau}{v}\right)\right]\right\}. \tag{15}$$

3. When $d_\tau^B \geq q > d_l$ or, equivalently, $q \in \left(\frac{vd_l-\tau}{c}, \frac{\tau}{v-c}\right]$, the firm's profit function can be simplified to:

$$\pi_H^0(v,q) = (v-c)q - (v-b)\int_{d_l}^{q}(q-x)dF(x). \tag{16}$$

It is easy to check that π_H^0 is concave, and hence, the optimal solution is either the interior optima q_b^{NV} or the boundary solution $\frac{\tau}{v-c}$. However, note that as $\frac{d\pi_H^0}{dq} - \frac{d\pi_L^0}{dq} < 0$, and hence, $\frac{\tau}{v-c}$ cannot be optimal.

Summarizing the above three scenarios, there are three candidates q for the global optima: $\frac{vd_l-\tau}{c}$, q_L^0, and q_b^{NV}. To compare the three solutions, we consider the following two cases depending on the magnitude of τ.

1. When $\tau \in ((vd_l - cq^{NV}), (v-c)d_l]$, only $\frac{vd_l-\tau}{c}$ and q_L^0 are relevant. Define $T^0 \in [vd_l - cq^{NV}, (v-c)d_l]$ as:

$$(v-s)\bar{F}\left(\frac{vd_l - T^0}{c}\right) - (s-b)\left[\frac{(v-c)d_l - T^0}{v}\right]h(d_l) = c-s. \tag{17}$$

We can verify that when $\tau \leq T^0$, $q_L^0 \leq \frac{vd_l-\tau}{c}$, and hence $\frac{vd_l-\tau}{c}$ is optimal; otherwise, q_L^0 is optimal.

2. When $\tau > (v-c)d_l$, only q_L^0 and q_b^{NV} are relevant. To compare the two, it is easy to see that when $\frac{d\pi_H^W}{dq}|_{q=\frac{\tau}{v-c}} > 0$, q_L^0 is optimal; otherwise q_b^{NV} is optimal. Note that $\frac{d\pi_H^W}{dq}|_{q=\frac{\tau}{v-c}} > 0$ is equivalent to $\tau > (v-c)q_b^{NV}$.

Combining the two cases, the optimal q^* can be characterized as:

$$q^* = \begin{cases} q^{NV} & \text{for } \tau \leq vd_l - cq^{NV}; \\ \frac{vd_l-\tau}{c} & \text{for } \tau \leq (vd_l - cq^{NV}, T_0); \\ q_L^0 & \text{for } \tau \leq (T_0, (v-c)q_b^{NV}); \\ q_b^{NV} & \text{for } \tau > (v-c)q_b^{NV}. \end{cases} \tag{18}$$

The monotonicity of q^* follows directly by comparing the four quantities over the relevant range of τ. □

Proof of Proposition 2. We prove this proposition in four steps.

Step 1: (v, q^{NV}) **is optimal if and only if** $\tau \leq T^D(\alpha)$**.** To see this, it is obvious that (v, q^{NV}) is optimal for $\tau \leq T^D(\alpha)$. To show that (v, q^{NV}) is not optimal for $\tau > T^{NV}(\alpha)$, note that for $\tau > T^{NV}(\alpha)$, strategic customers prefer to wait under (v, q^{NV}). Therefore, the relevant profit function at (v, q^{NV}) is either π_L^W or π_H^W, as in Lemma C.2. Assume π_L^W is the relevant profit function. It is easy to check that $\frac{d\pi_L^W}{dq}|_{q=q^{NV}} < 0$. Furthermore, for $\epsilon \in \left(0, q^{NV} - \frac{(1-\alpha)vd_l-\tau}{c}\right)$, $(v, q^{NV} - \epsilon) \in \Omega^W$. As $\pi_L^W(v,q)$ is continuously differentiable at $q = q^{NV}$, $\exists\, \epsilon \in \left(0, q^{NV} - \frac{(1-\alpha)vd_l-\tau}{c}\right)$ such that $\pi(v, q^{NV} - \epsilon) > \pi(v, q^{NV})$. Hence, (v, q^{NV}) cannot be optimal. Similarly, we can also show that (v, q^{NV}) cannot be optimal when π_H^W is the relevant profit function.

Step 2: $q^* < q^{NV}$ for $\tau \geq T^D(\alpha)$. To see this, note that according to Lemma C.3, if $(p^*, q^*) \in \Omega^W$, then $q^* < (1-\alpha)q^{NV}$. Similarly, if $(p^*, q^*) \in \Omega^B$, we can show that for any $p^* \leq v$, at $q^* \geq q^{NV}$, $\frac{d\pi_L^W}{dq}_{|q=q^*} < 0$, so $q^* < q^{NV}$.

Step 3: The existence of $T^B(\alpha)$. First, according to Corollary C.1, for $\tau \leq (1-\alpha)(vd_l - cq^{NV})$, only the buy-region solution can be optimal; second, according to Lemma C.5, for $\tau \leq T^{B,1}(\alpha)$, the optimal buy-region solution satisfies $q^{B,*} = \frac{(1-\alpha)p^* Q_s(p^{B,*}) - \tau}{c}$. Therefore, $\exists\, T^B(\alpha) > \min\left((1-\alpha)(vd_l - cq^{NV}),\, T^{B,1}(\alpha)\right)$ such that $q^* = \frac{(1-\alpha)p^* Q_s(p^*) - \tau}{c}$.

Step 4: The optimal p^* for $\tau \leq T^D(\alpha) + \epsilon$. Following the last step, we know that for sufficiently small ϵ, the optimal solution (p^*, q^*) satisfies $q^* = \frac{(1-\alpha)p^* Q_s(p^*) - \tau}{c}$. Therefore, to prove Statement 2(b), we only need to show that for sufficiently small ϵ, the resulting d_τ^W under (p^*, q^*) satisfies $d_\tau^W = d_l$ for $f(d_l) > 0$, and $d_\tau^W > d_l$ for $f(d_l) = 0$.

To show this, in preparation, define function $\bar{\pi}_L^B(y) = \pi_L^B\left(p(y), q(y)\right)$, with $p(y) = v - (s-b)F(y)$ and $q(y) = \frac{(1-\alpha)p(y)y - \tau}{c}$. Under this definition, y can be seen as the wait-equilibrium default threshold d_τ^W under (p, q). Let $y^* = \arg\max \bar{\pi}_L^B(y)$. Clearly, we have $y^* \geq d_l$. Note that at $y = d_l$, we have $p(y) = v$, $\frac{dp(y)}{dy} = -(s-b)f(d_l)$, $q(y) = \frac{(1-\alpha)vd_l - \tau}{c}$, and $\frac{dq(y)}{dy} = \left(\frac{1-\alpha}{c}\right)[v - (s-b)d_l f(d_l)]$. Finally, $d_\tau^B = \frac{cq(y)+\tau}{p(y)} = (1-\alpha)y < d_l$. Taking the derivative of $\bar{\pi}_L^B$ with respect to y at $y = d_l$, we have:

$$\frac{d\hat{\pi}_L^B}{dy} = \frac{(1-\alpha)v}{c}\left[(v-c) - (v-s)F(q)\right] - (s-b)f(d_l)\left[\frac{(1-\alpha)d_l}{c}\left[(v-c) - (v-s)F(q)\right] + \int_{d_l}^{q} \bar{F}(x)dx\right]. \quad (19)$$

Clearly, for $f(d_l) = 0$, $\frac{d\hat{\pi}_L^B}{dy}\,|_{x=d_l} > 0$, therefore $y^* > d_l$ or, equivalently, $p^* < v$.

On the other hand, for $f(d_l) > 0$, for sufficiently small ϵ, $q \approx q^{NV}$, and hence, $\frac{d\hat{\pi}_L^B}{dx}\,|_{x=d_l} < 0$, i.e. $p^* = v$ is a local optima. Now consider $y > d_l$, taking the derivative of $\bar{\pi}_L^B$ with respect to y:

$$\frac{d\bar{\pi}_L^B}{dy} = \left[(p-c) - (p-s)F(q)\right]\frac{dq}{dy} + \left(\int_{d_l}^{q} \bar{F}(x)dx\right)\frac{dp}{dy}. \quad (20)$$

Assume that the global optima $y^* > d_l$, i.e. $p^* < v$. According to Lemma C.5, we must have $\frac{dq(y^*)}{dy^*} > 0$. In addition, for $y^* > d_l$ to be the global optima, we must also have $\frac{d\bar{\pi}_l^B}{dy} = 0$. As $\frac{dq(y^*)}{dy^*} > 0$ and $\frac{dp(y^*)}{dy^*} < 0$, $\frac{d\bar{\pi}_l^B}{dy} = 0$ only if $(p-c) - (p-s)F(q(y^*)) > 0$, i.e. $q(y^*) < F\left(\frac{p-c}{p-s}\right)$. As $p^* < v$, we have $q(y^*) < q^{NV}$. On the other hand, for sufficiently small ϵ, $q(d_l) = \frac{(1-\alpha)vd_l - \tau}{c}$ is arbitrarily close to q^{NV}, and hence, $q(y^*) < q(d_l)$. As $p(y^*) < v = p(d_l)$, the profit under $(v, q(d_l))$ is greater than $(p(y^*), q(y^*))$ contradicts the assumption that $y^* > d_l$ is the global optima. Therefore, $\left(v, \frac{(1-\alpha)vd_l - \tau}{c}\right)$ is optimal for $\tau < T^D(\alpha) + \epsilon$. $\square$

Proof of Proposition 3. According to Lemma C.6, for sufficiently large τ, $(p_H^B,\, q_H^B)$ is the optimal buy-region solution. Similarly, according to Corollary C.2, (v, q_H^W) is the only possible solution in the wait-region that can be globally optimal. Therefore, $\exists\, T_h$ such that for $\tau > T_h$, $(p^*, q^*) = (p_H^B,\, q_H^B)$ or (v, q_H^W).

Next, note that $\pi_H^W(v, q_H^W)$ decreases while $\pi_H^W(p_H^B, q_H^B)$ remains constant. In addition, as $q_H^B < q_b^{NV}$, (v, q_H^W) dominates (p_H^B, q_H^B) at $\alpha = 0$. On the other hand, when α is sufficiently high, according to Corollary C.1, (v, q_H^W) is dominated by (p_H^B, q_H^B). Therefore, there exists a $A^B > 0$ such that $\pi_H^B(p_H^B, q_H^B) = \pi_H^W(v, q_h^W)$ at $\alpha = A^B$, and $\pi_H^B(p_H^B, q_H^B) > \pi_H^W(v, q_h^W)$ if and only if $\alpha > A^B$. $\square$

B.3. Proofs of results in Section 6.

Proof of Proposition 4. We prove the three statements separately.

Statement 1: The sufficient and necessary conditions for the buy-equilibrium to exist. The buy-equilibrium exists if and only if, assuming all other strategic customers purchase early, the expected surplus for one customer from buying in the first period is greater than the surplus of waiting. We consider two scenarios.

1. When $(p-c)q > \tau$ or, equivalently, $d_\tau^B > q$, the firm's survival probability is zero, and hence deferred discount has no value to customers. Therefore, the buy-equilibrium exists if and only if $v-p \geq (s-b)F(q)$ or, equivalently, $q \leq Q_s$.

2. When $(p-c)q \leq \tau$, that is, $d_\tau^B \leq q$, the firm survives with probability $F(d_\tau^B)$. The buy-equilibrium exists if and only if $v-p+t[1-F(d_\tau^B)] \geq (s-b)F(d_\tau^B)$ or, equivalently, $q \leq \frac{pQ_s^{dd}-\tau}{c}$.

To summarize the above two scenarios, the buy-equilibrium exists if and only if $q \in \left(\frac{\tau}{p-c}, \frac{pQ_s^{dd}-\tau}{c}\right]$ or $q \leq \min\left(Q_s, \frac{\tau}{p-c}\right)$.

Statement 2: The sufficient and necessary conditions for the wait-equilibrium to exist. The wait-equilibrium exists if and only if, assuming all other strategic customers wait, the expected surplus for one customer from waiting is larger than the surplus of buying in the first period. We again consider two scenarios.

1. When $\tau > [(1-\alpha)p-c]q$, that is, $q < d_\tau^W$. In this case, the firm's survival probability is zero and deferred discount therefore has no value. Thus, the wait-equilibrium exists if and only if $v-p < (s-b)F(q)$, i.e. $q > Q_s$.

2. When $\tau \leq [(1-\alpha)p-c]q$, we have $q \geq d_\tau^W$. The firm's survival probability is $F(d_\tau^W)$. Thus, the wait-equilibrium exists if and only if $v-p+t < (s-b+t)F(d_\tau^W)$, i.e. $q > \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}$.

Summarizing the above two scenarios, the wait-equilibrium exists if and only if $q \in \left(Q_s, \frac{\tau}{(1-\alpha)p-c}\right)$ or $q > \max\left(\frac{\tau}{(1-\alpha)p-c}, \frac{(1-\alpha)pQ_s^{dd}-\tau}{c}\right)$.

Statement 3: The region in which the buy-equilibrium is more appealing. According to Lemma C.11, we know that when $q = Q_b^{dd}$, the buy-equilibrium is more appealing to strategic customers.

In addition, the statement that strategic customers prefer the wait-equilibrium when $q > Q_b^{dd}$ follows from the proof of Lemma C.11. Specifically, consider two scenarios.

1. When $\tau \leq (p-c)Q_s^{dd}$, according to the proof of Lemma C.11, both equilibria exist when $q \in \left(Q_b^{dd}, \frac{pQ_s^{dd}-\tau}{c}\right]$, and customers prefer to wait. When $q > \frac{pQ_s^{dd}-\tau}{c}$, only the wait-equilibrium exists. Therefore, customers wait when $q > Q_b^{dd}$.

2. When $\tau > (p-c)Q_s^{dd}$, as shown, the two equilibria are mutually exclusive, and hence, customers wait when $q > Q_b^{dd}$. $\square$

B.4. Proofs of results in Section 7.

Proof of Proposition 5. For Statement 1, we consider the three conditions separately.

1. If $\tau \leq T^D(\alpha)$, it is obvious that (v, q^{NV}) is optimal.

2. If $q > Q_b^{dd}(p,t)$, strategic customers wait and the corresponding profit function is:

$$\pi^{dd}(p,q,t) = \pi^W(p,q) - t\int_{d_\tau^W}^{\frac{q}{1-\alpha}} \min((1-\alpha)x, q)dF(x) < \pi^W(p,q) \tag{21}$$

3. If $(p-c)Q_s^{dd}(p) < \tau$, without loss of generality, assume $q \le Q_b^{dd}(p,t)$, strategic customers purchase early, and the corresponding profit function $\pi^{dd}(p,q,t) = \pi_H^B(p,q)$.

For Statement 2, we consider the case where the bankruptcy risk under (p^*, q^*) is zero. It is clear that when $p^* < v$ and the corresponding $d_\tau^B \le d_l$, i.e. $p^* d_l - cq^* \ge \tau$, the corresponding profit function without deferred discount is $\pi_L^B()$. Following (9), $\pi^{dd}(v, q^*, v-p^*) = \pi_L^B(p^*, q^*)$. In addition, note that $Q_b^{dd}(v, v-p^*) > Q_b(p^*)$ for $p^* d_l - cq^* \ge \tau$. Therefore, there exists $q' > q^*$ such that under $(v, q', v-p^*)$ we have $d_\tau^B \le d_l$, and $q' \le Q_b^{dd}(v, v-p^*)$. Therefore, $\pi^{dd}(v, q', v-p^*) > \pi^{dd}(v, q^*, v-p^*) = \pi_L^B(p^*, q^*)$. Due to continuity, the result continues to hold when the bankruptcy risk is sufficiently small, as desired. □

Appendix C: Supplemental Results

This appendix includes supplemental materials and technical lemmas that support the proofs of the analytical results presented in the paper. The proofs of these results are in Online Appendix D.

C.1. Supplemental results for Section 4.

Lemma C.1. Assume the firm chooses the second-period price p_2 to maximize revenue.

1. If strategic customers purchase in the first period, the distribution of p_2 is the same as in Lemma 1.
2. If strategic customers wait,

$$p_2 = \begin{cases} b & \text{for } \; D < \min\left(\frac{cq+\tau}{(1-\alpha)p}, \left(\frac{b}{\alpha s+(1-\alpha)b}\right) q\right); \\ s & \text{otherwise.} \end{cases} \tag{22}$$

Remark: By comparing the above result with Lemma 1), we can see that the distribution of the second-period price p_2 follows a similar structure under the two objectives (inventory clearance and revenue maximization): The firm sets $p_2 = b$ if and only if the realized demand is lower than the minimum of the bankruptcy threshold d_τ and (an increasing function of) inventory level q. Note that d_τ increases in τ, the firm's level of financial distress. This suggests that the second-period price p_2 is more likely to be b as τ increases (the firm becomes more financially distressed). This is exactly the force that drives our results on customer behavior and optimal decisions. For example, this result directly causes the piecewise linear pattern of the buy–wait boundary in Figure 3: When τ is small, d_τ is smaller than (the function of) q, and the probability that $p_2 = b$ is determined by τ, as is the customers' waiting incentive. Therefore, the $Q_b(p)$ decreases in τ. However, as τ becomes larger, the probability that $p_2 = b$ depends only on q, as does the customers' waiting incentive, leading to $Q_b(p)$ independent of τ. Therefore, we would expect the qualitative insights in this paper to remain unchanged if the firm maximizes its revenue in the second period.

That said, we can also observe that the probability that $p_2 = b$ is lower under the revenue-maximization objective than the inventory-clearance objective. As a result, customers actually have *less* incentive to wait, rendering strategic waiting less harmful for the firm. Therefore, we would expect the effects identified in our results to be smaller under the revenue-maximization objective than that under the inventory-clearance objective.

C.2. Supplemental results for Section 5.

LEMMA C.2. The firm's total expected profit under $(p,q) \in \Omega_0$ $\pi(p,q)$ is:

$$\pi(p,q) = \begin{cases} \pi_L^B(p,q) & \text{for} \quad (p,q) \in \Omega^B \quad \text{and} \quad (p-c)q \geq \tau \\ \pi_H^B(p,q) & \text{for} \quad (p,q) \in \Omega^B \quad \text{and} \quad (p-c)q < \tau \\ \pi_L^W(p,q) & \text{for} \quad (p,q) \in \Omega^W \quad \text{and} \quad [(1-\alpha)p-c]q \geq \tau \\ \pi_H^W(p,q) & \text{for} \quad (p,q) \in \Omega^W \quad \text{and} \quad [(1-\alpha)p-c]q < \tau, \end{cases} \tag{23}$$

where $\pi_L^B(p,q)$ and $\pi_H^B(p,q)$ are defined in (1) and (2), respectively, in the main body of the paper and

$$\pi_L^W(p,q) = (p-c)q - (p-s)\int_{d_l}^{\frac{q}{1-\alpha}} [q-(1-\alpha)x]dF(x) - (s-b)\int_{d_l}^{d_\tau^W} [q-(1-\alpha)x]dF(x); \tag{24}$$

$$\pi_H^W(p,q) = (p-c)q - (p-s)\int_{d_l}^{\frac{q}{1-\alpha}} [q-(1-\alpha)x]dF(x) - (s-b)\int_{d_l}^{q} [q-(1-\alpha)x]dF(x). \tag{25}$$

Remark: The above lemma summarizes the firm's profit function according to different regions for (p,q). In addition, note that (3) is a combination of (24) and (25).

LEMMA C.3. Let $(p^{W,*}, q^{W,*})$ be the optimal price and inventory in the wait-region, i.e. $(p^{W,*}, q^{W,*}) = \arg\max_{(p,q)\in\Omega^W} \pi(p,q)$. For $\tau > T^D(\alpha)$, $(p^{W,*}, q^{W,*}) = \arg\max_{(p,q)\in\Omega_0} \pi(p,q)$ only if:

1. $p^{W,*} = v$, and
2. $q^{W,*} = q_L^W$ or q_H^W, where q_H^W is defined in (4) in Proposition 3, and

$$q_L^W = (1-\alpha)\bar{F}^{-1}\left\{\left(\frac{c-b}{v-s}\right) - \left(\frac{s-b}{v-s}\right)\bar{F}\left[\frac{cq_L^W+\tau}{(1-\alpha)v}\right]\left[1-\left(\frac{c[(v-c)q_L^W-\tau]}{(1-\alpha)v^2}\right)h\left(\frac{cq_L^W+\tau}{(1-\alpha)v}\right)\right]\right\}. \tag{26}$$

Remark: The above lemma identifies the *necessary* conditions for a solution (p,q) in the wait-region to be the global optimal one. As we are only interested in the global optima, we need only focus on the solutions above.

COROLLARY C.1. $(p^*, q^*) \notin \Omega^W$ for $\tau \leq (1-\alpha)(vd_l - cq^{NV})$ or $\alpha \geq 1 - \frac{d_l}{q^{NV}}$.

Remark: These conditions show that the firm is better off inducing customers to purchase when τ is small or α is large.

COROLLARY C.2. $\exists\, T^W(\alpha)$ such that $\pi(v, q_H^W) \geq \pi(v, q_L^W)$ if and only if $\tau \geq T^W(\alpha)$.

Remark: This result shows that when τ is sufficiently high, (v, q_H^W) is the only possible wait-region solution that is globally optimal.

LEMMA C.4. Let $\hat{\pi}_H^B(q) := \pi_H^B(v-(s-b)F(q), q)$. $\hat{\pi}_H^B(q)$ is pseudo-concave in q.

LEMMA C.5. Let $(p^{B,*}, q^{B,*})$ be the optimal price and inventory in the buy-region, i.e. $(p^{B,*}, q^{B,*}) = \arg\max_{(p,q)\in\Omega^B} \pi(p,q)$. $(p^{B,*}, q^{B,*})$ satisfy $q^{B,*} = Q_b(p^B)$ and $\frac{dQ_b(p^{B,*})}{dp^{B,*}} < 0$ if $p^{B,*} < v$.

LEMMA C.6. For $\tau > T^D(\alpha)$, $\exists\, T^{B,1}(\alpha)$ and $T^{B,2}(\alpha)$ where $T^{B,2}(\alpha) \geq T^{B_1}(\alpha) > T^D(\alpha)$ such that

1. for $\tau \leq T^{B,1}(\alpha)$, $q^{B,*} = \frac{(1-\alpha)p^{B,*}Q_s(p^{B,*})-\tau}{c}$;
2. for $\tau \in (T^{B,1}(\alpha), T^{B,2}(\alpha)$, $q^{B,*} = Q_s(p^{B,*})$ and $(p^{B,*}-c)q^{B,*}) < \tau$;
3. for $\tau > T^{B,2}(\alpha)$, $q^{B,*} = q_H^B$ as in (5) and $p^{B,*} = v-(s-b)F(q_H^B)$.

Remark: This lemma has two implications: First, it shows that for low τ ($\tau < T^{B,1}(\alpha)$), the optimal solution induces customers to purchase early by constraining the probability of bankruptcy under the *hypothetical* wait-equilibrium $F(d_\tau^W)$; for high τ, it induces early purchase by directly limiting the inventory level. Second, it gives the exact optimal solution in the buy-region for sufficiently high τ.

C.3. Technical results for Section 6.

Lemma C.7. The following three statements are equivalent.

1. $Q_s^{dd,a} \geq Q_s^{dd}$;
2. $\tau \leq (p-c)Q_s^{dd}$;
3. $\tau \leq (p-c)Q_s^{dd,a}$.

Lemma C.8. $Q_s^{dd,b} \in \left(\frac{(1-\alpha)pQ_s^{dd}-\tau}{c}, \frac{pQ_s^{dd}-\tau}{c}\right)$.

Lemma C.9. The following three statements are equivalent.

1. $Q_s^{dd,a} \leq Q_s^{dd,b}$;
2. $\tau \leq [(1-\alpha)p-c]Q_s^{dd,a}$;
3. $\tau \leq [(1-\alpha)p-c]Q_s^{dd,b}$.

Lemma C.10. With deferred discount t, when multiple equilibria exist,

1. for $q < d_\tau^B$, the wait-equilibrium is more appealing to strategic customers;
2. for $q \in [d_\tau^B, d_\tau^W)$, the buy-equilibrium is more appealing to strategic customers if and only if $q \leq Q_s^{dd,a}$;
3. for $q > d_\tau^W$, the buy-equilibrium is more appealing to strategic customers if and only if $q \leq Q_s^{dd,b}$.

Lemma C.11. When both the buy- and wait-equilibria exist, the buy-equilibrium is more appealing to strategic customers if one of the following two sets of conditions holds:

1. $\tau \leq [(1-\alpha)p-c]Q_s^{dd,a}$ and $q \leq Q_s^{dd,b}$;
2. $\tau \in ([(1-\alpha)p-c]Q_s^{dd,a}, (p-c)Q_s^{dd}]$ and $q \leq Q_s^{dd,a}$.