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Cooper, I A and Nyborg, K G

(2018)

*Consistent valuation of project finance and LBO's using the flow-to-equity method.*

European Financial Management, 24 (1). pp. 34-52. ISSN 1354-7798

DOI: <https://doi.org/10.1111/eufm.12136>

Wiley

<http://onlinelibrary.wiley.com/doi/10.1111/eufm.12...>

This is the peer reviewed version of the following article: 'Consistent valuation of project finance and LBO's using the flow-to-equity method', European Financial Management (2018) 24(1):34-52, which has been published in final form at <http://dx.doi.org/10.1111/eufm.12136>. This article may be used for non-commercial purposes in accordance with Wiley Terms and Conditions for Self-Archiving.

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# Consistent valuation of project finance and LBO's using the flows-to-equity method

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This version: March 2017

## Abstract

The flows-to-equity method is often used to value highly leveraged projects, or transactions, where debt typically amortises over time according to a fixed schedule. This requires a formula that links the changing leverage over time with a time-varying equity discount rate. We show that the extant formulas in the literature and in textbooks yield incorrect discount rates and valuations because they are inconsistent with fixed debt plans. They result in values that are at odds with the Miller and Modigliani result that levered value equals unlevered value plus financing side effects (adjusted present value). The error from using the wrong formula can be large at the currently low levels of interest rates. We derive an equity discount rate formula that captures the effects of a fixed debt plan, potentially expensive debt, and costs of financial distress that, when applied in the flows-to-equity method, yield values that are consistent with adjusted present value. In short, our formula allows for the correct implementation of the flows-to-equity method under fixed debt plans. In the formula, the cost of debt is the promised yield rather than the expected rate of return of debt.

JEL Codes: G12, G24, G31, G32, G33, G34.

Keywords: Valuation, flows-to-equity, equity cash flow, cost of equity, project finance, LBO, cost of equity.

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<sup>1</sup> This research has benefited from a grant from the Research Council of Norway, grant number 1798866/S20. We also thank NCCR-FINRISK for financial support. Cooper: London Business School, Regent's Park, London NW1 4SA, UK. Email: [icooper@london.edu](mailto:icooper@london.edu). Nyborg: Department of Banking and Finance, University of Zurich, Plattenstrasse 14, 8032 Zurich, Switzerland. Email: [nyborg@isb.uzh.ch](mailto:nyborg@isb.uzh.ch).

## **1. Introduction**

The general topic of this paper is the valuation of investments that have fixed debt plans. In other words, at the time the valuation is made the future amount of debt is expected to be a function of time alone. The amount of debt is not expected to fluctuate with the future value of the investment. This type of situation arises in leveraged buyouts (LBO's) (Baldwin 2001a), project finance (Esty 1999), and other highly leveraged transactions (HLT's) where the future amortisation of the debt has been agreed at the time of the investment. Our focus is especially on valuing the equity in such investments directly through the “flows-to-equity” method, whereby the project’s equity free cash flows are discounted at a levered equity rate.

The topic is important because the flows-to-equity method is often used in practice in cases where debt plans are fixed. However, as we show, standard formulas to calculate the equity discount rate result in equity values that are incorrect when debt levels evolve according to a predetermined schedule. They differ from the values one obtains from applying the fundamental idea of adjusted present value that levered value equals unlevered value plus the present value of financing side effects. The main contribution of this paper is to derive a formula for the equity discount rate that, when applied in the flows-to-equity method under fixed debt plans, yields correct equity values. In short, the paper can be viewed as reconciling the flows-to-equity method with adjusted present value for projects with fixed debt plans. Our approach builds on the no-arbitrage valuation approach to valuing interest tax shields in Cooper and Nyborg (2008). We also expand on the basic analysis by incorporating the possibilities of mispriced debt and costs of financial distress into the equity discount rate formula.

A key challenge with using the flows-to-equity method to value projects with fixed debt plans is that the equity discount rate will be time-varying, since leverage, and thus also the risk of equity, changes over time as the debt plan unfolds. Calculating leverage and equity discount rates at different points in time, therefore, requires estimates of equity and debt values each year, or date, of the project’s life. As shown by Esty (1999), the

apparent circularity in this is dealt with through iteration, until what one puts in, in terms of initial values, is what one gets out. Thus, the final estimates of values and time-varying discount rates must satisfy a simple consistency condition. Valuation using iteration is common in other applications in finance, for example, to value new issues of corporate securities with options features such as warrants. The formula we derive in this paper links the equity discount rate to leverage and generates correct valuations using Esty's (1999) iterative implementation of the flows-to-equity method.

The flows-to-equity method has several features that may help explain its popularity in practice, despite the relative complexity of an iterative procedure. For example, as emphasised by Esty (1999) and Baldwin (2001a), the flows-to-equity method

- focuses directly on the cash flows that accrue to equity-holders;
- can allow for time-varying leverage, which is inconsistent with using a constant WACC;
- can allow for a time-varying cost of equity;
- can allow for time-varying effective tax rates;
- can allow for several rounds of financing.

These benefits of the approach are particularly relevant in highly leveraged transactions such as LBO's and project finance.

However, the flows-to-equity approach also has some potential difficulties that may be especially pertinent in the context of highly leveraged transactions. In particular, these transactions tend to use high-yield structured debt, which raises three important issues:

- Should one use the debt's promised yield or expected rate of return as the "cost of debt" when calculating the equity discount rate for use in the flows-to-equity method?

- The cost of debt may contain an element that reflects factors other than credit risk, such as illiquidity. How should these non-risk elements of the cost of debt be incorporated into the valuation?
- Highly leveraged transactions bring a significant chance of financial distress. Is there a simple way of including the effect of this in the valuation?

In this study, we address these questions. We show that it is appropriate to use the debt's promised yield rather than the expected rate of return as the "cost of debt" for the purpose of calculating the equity discount rate in flows-to-equity method. The intuition relates to the fact that the promised yield, rather than the expected rate of return, is used to calculate free cash flows to equity in this method. Consistency, therefore, requires the promised yield to be used also when calculating the equity discount rate. This is an advantage of the flows-to-equity method since it is easier in practice to estimate yields as compared with expected rates of return. We also derive an equity discount rate for use in the flows-to-equity method that incorporates any non-risk elements of the cost of debt and expected cost of financial distress.

The issue as to the correct "cost of debt" has become relatively greater in importance in recent years because of the decrease in the general level of interest rates as well as estimates of the equity market risk premium. In contrast, the discount rate errors from using the incorrect cost of debt are not affected by these developments. So this has become an increasingly important issue in valuation. For highly leveraged structures, the error from using the wrong cost of debt is now as great as more commonly discussed aspects of valuation such as how to estimate the equity market risk premium and beta or which riskless interest rate to use.

The remainder of the paper is structured as follows. Section 2 reviews standard formulas for calculating the equity discount rate for use in the flows-to-equity method and discusses some problems with these. Section 3 derives the appropriate measure for the "cost of debt" in the simple case of perpetuities. Section 4 uses a numerical example to illustrate that the standard formulas found in the literature and in textbooks for the equity

discount rate do not yield correct valuations in the flows-to-equity method under fixed debt plans, even if the correct cost of debt is used. Section 5 derives a formula that works, assuming that the only financing side effect arises from the tax deductibility of interest payments. Section 6 expands on this by allowing for the possibility of mispriced debt and costs of financial distress. Section 7 uses a realistic numerical example to examine which errors matter most. Section 8 gives corresponding formulas for releveraging the overall cost of capital, and Section 9 concludes.

## 2. Standard equity discount rate formulas

This section reviews commonly used formulas for the equity discount rate and clarify the different assumptions that underlie them. The starting point is the standard “Miller and Modigliani with taxes” adjusted present value expression  $V_L = D + E = V_U + PVTS$ , where  $V_L$  is the levered value of the project,  $V_U$  is the unlevered value,  $D$  is the value of the debt,  $E$  is the value of the (levered) equity, and  $PVTS$  is the present value of the tax shields arising from the tax deductibility of interest payments. At this stage, there are no other financing side effects.

Throughout the paper, we consider corporate taxes only, and the corporate tax rate is denoted by  $T$ . Let the expected rate of return (or cost) of the levered equity, unlevered equity, debt, and tax shield component, be denoted by  $R_E$ ,  $R_U$ ,  $R_D$ , and  $R_{TS}$ , respectively.

It follows from the basic adjusted present value identity that

$$\frac{E}{V_L} R_E + \frac{D}{V_L} R_D = \frac{V_L - PVTS}{V_L} R_U + \frac{PVTS}{V_L} R_{TS}.$$

This can be rewritten to give the following expression for the cost of equity in terms of the other parameter:<sup>2</sup>

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<sup>2</sup> See, for example, Cooper and Nyborg (2006) or Dempsey (2013). Time indicators are suppressed for notational simplicity.

$$R_E = \frac{E+D-PVTS}{E}R_U - \frac{D}{E}R_D + \frac{PVTS}{E}R_{TS}. \quad (1)$$

Implementing equation (1) requires a value for  $PVTS$  and an assumption about the tax shield discount rate,  $R_{TS}$ . As is well understood in the literature, these depend on the debt policies pursued by the firm (see, e.g., Cooper and Nyborg 2006 or 2007). The alternative assumptions that are commonly made in the literature and in textbooks are either (a) the debt plan is fixed and  $R_{TS}$  is equal to  $R_D$ , or (b) the amount of debt is a constant fraction of firm value and  $R_{TS}$  is equal to  $R_U$ . These two cases lead to two families of equity discount rate (releveraging) formulas, as shown in Table 1.

**Table 1: Families of equity discount rate formulas**

The table shows formulas for calculating the cost of equity and the beta of equity under different assumptions about the leverage plan.

<i>Panel A: General formula</i>	
General formula for equity discount rate	$R_E = \frac{E+D-PVTS}{E}R_U - \frac{D}{E}R_D + \frac{PVTS}{E}R_{TS} \quad (1)$
<i>Panel B: Perpetual fixed level of debt (Modigliani Miller)</i>	
Cost of equity formula	$R_E = R_U + (D/E)(1-T)(R_U - R_D) \quad (2)$
Equity beta formula	$\beta_E = \beta_U + (D/E)(1-T)(\beta_U - \beta_D) \quad (3)$
Equity beta formula, debt beta zero	$\beta_E = \beta_U [1 + (D/E)(1-T)] \quad (4)$
<i>Panel C: Constant leverage ratio (Miles and Ezzell)*</i>	
General formula for cost of equity	$R_E = R_U + (D/E)(R_U - R_D) \quad (5)$
Equity beta formula	$\beta_E = \beta_U + (D/E)(\beta_U - \beta_D) \quad (6)$
Equity beta formula, debt beta zero	$\beta_E = \beta_U ((E + D) / E) \quad (7)$

\* The formulas in Panel C assume continuous rebalancing (see Cooper and Nyborg 2007 or 2008).

Panel A of Table 1 repeats the general releveraging formula, (1). Panel B provides the formulas derived from case (a) above. These are based on Modigliani and Miller (1963)

and assume perpetual debt at a fixed level. Panel C gives the formulas derived from case (b). These are based on Miles and Ezzell (1980) and assume a constant debt to value ratio.

There are main two concerns with the formulas in Table 1. First, none of these explicitly covers the scenario we wish to focus on, namely, that the debt level changes over time according to a fixed, pre-determined schedule.<sup>3</sup> We will come back to this below.

Second, the formulas in the table, as well as equation (1), are stated in terms of expected rates of return (or betas if the CAPM is assumed to hold). This is a problem with respect to the flows-to-equity method since the cash flows that are discounted in this method are not the expected flows to equity. Rather, they are hybrid flows that mix expected operating cash flows with promised debt repayments. In particular, for each date  $t$ , the free cash flows to equity are defined as (Esty 1999, Berk and DeMarzo 2007):

$$FCFE_t = C_t - D_{t-1}Y(1-T) - (D_{t-1} - D_t), \quad (8)$$

where  $C_t$  is operating free cash flow at time  $t$  (commonly denoted by  $FCFF$ ),  $Y$  is the promised yield (and coupon) on the debt, and  $D_t$  is the level of the debt (principal) at time  $t$ . Thus, it is not clear that Equation (1) and the formulas in Table 1 are appropriate to use in a flows-to-equity valuation.

Heuristically, one might think that replacing the expected rate of return of debt,  $R_D$ , with the promised yield,  $Y$ , in the above equations might result in a set of equity discount rates that work. In this paper, we formally show that this does indeed work when the debt plan is fixed. This can be viewed as an advantage of the flows-to-equity method since the expected rate of return of debt is difficult to estimate (see, e.g., Schaefer and Strebulaev 2008). The yield on debt is often used to proxy its cost (Damodaran 2002, Berk and DeMarzo 2007). For high yield debt, the yield can be significantly different from the expected rate of return (Cooper and Davydenko 2007). The size of this effect can be large. For example, Cooper and Davydenko (2007) provide examples where the promised yield

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<sup>3</sup> All these formulas can be found in standard textbooks. For example, equation (2) is used in Ross, Westerfield, and Jaffe (1996), equation (4) in Damodaran (2002), and equations (5) and (6) in Brealey, Myers, and Allen (2017). Both Esty (1999) and Baldwin (2001b) use equation (7) [which is equivalent to (5) with  $R_D = R_F$  and that the CAPM holds].



spread (over the riskfree rate) is three percent, but the risk premium in the cost of debt is one percent. At current levels, this is of the same order of magnitude as the riskless rate itself. Thus, using the correct “cost of debt” in the releveraging formula is an important concern with respect to the correct implementation of the flows-to-equity method.

To summarize, there are two key, basic issues with respect to the implementation of the flows-to-equity method. First, what is the correct formula to use for the equity discount rate? Clearly, this is a function of debt policy. None of the standard formulas are derived under a debt plan with pre-scheduled debt levels that vary over time. Second, in the appropriate formula, what is the right value to use for the “cost of debt”?

### 3. Implementing the flows-to-equity method with perpetuities

With fixed debt plans, it turns out that the correct “cost of debt” in the equity discount rate formula for use in the flows-to-equity method is the debt’s yield. Here, by way of example, we provide a derivation and intuition of this result in the simple context of perpetuities. Section 5 contains the general and more substantial analysis with amortising debt plans.

In the case of a level perpetuity,  $C$ , and fixed perpetual debt of  $D$ , the APV formula for the value of equity is:<sup>4</sup>

$$E = \frac{C}{R_U} + TD - D. \quad (9)$$

Free cash flows to equity are given by

$$FCFE = C - DY(1 - T) \quad (10)$$

each period, and the value of equity can also be written as

$$E = \frac{FCFE}{R_E}, \quad (11)$$

where  $R_E$  is implicitly defined as the discount rate that equalises the left hand side of (11) with that of (9). In short,  $R_E$  is the appropriate equity discount rate in the flows-to-equity method. Setting (9) and (11) equal to each other and using the expression for  $FCFE$ , we

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<sup>4</sup> Equation (9) assumes that debt is issued at its fair price and that there are no bankruptcy costs.

find that  $R_E$  is given by (2) with the cost of debt set equal to the promised yield on debt,  $R_D = Y$ .

This shows that the appropriate “cost of debt” here is the debt’s yield. The intuition derives from the fact that the flows-to-equity method deducts the full after-tax promised yield from the expected operating cash flows to get the free cash flows to equity, as seen in (8). In other words, the definition of *FCFE* mixes the *expected* cash flow from operations with a *promised* debt payment. As a result, it is correct to use the debt’s yield as the “cost of debt” when calculating the levered equity discount rate. When doing this,  $R_E$  is not the expected rate of return of the equity, rather it is a hybrid equity rate of return that is appropriate to use in the flows-to-equity method.

To get a sense of the error from using the expected rate of return on debt rather than its yield, consider a level perpetuity, a fixed level of debt, and parameter values as follows: riskless interest rate, 2.5%; corporate tax rate, 40%; and equity market risk premium, 5%.<sup>5</sup> These roughly correspond to US capital markets at the current time. Assume also that the CAPM is the appropriate pricing model, the project’s unlevered equity (or asset) beta is 0.6, implying  $R_U = 5.5\%$ , and the initial leverage ratio (D/V) is 0.5.<sup>6</sup> Finally, assume that the yield spread on the debt is 200 basis points, consistent with the high degree of leverage. This spread is the same as that used in the example in Esty (1999), and is consistent with a Baa/BBB rating (Huang and Huang 2012).

With these parameter values, using equation (2) with the cost of debt set equal to its yield gives the equity discount rate for use in the flows-to-equity method as 6.1%. If, instead, the cost of debt is set equal to its expected return one needs to estimate the expected return on that debt. One way of doing this is to use the CAPM applied to the debt. This requires an estimate of the debt beta. Schaefer and Strebulaev (2008, Table 5) give estimated hedge ratios between debt and equity between 0 and 0.25. Using the middle of this range, 0.125, would give a beta for the debt in our example of 0.15. This would

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<sup>5</sup> This is the median level of the US equity market risk premium in the survey by Fernandez et al (2014).

<sup>6</sup> The asset beta, leverage, and debt spread are based on the example from Esty (1999) which we use below.

imply an expected return on the debt of 3.25%. Using equation (2) with the cost of debt set equal to this rather than its yield gives an equity discount rate of 6.85%. Such an error can lead to large mistakes in valuation. For example, given a level perpetuity, a discount rate of 6.1% yields a price to cash flow multiple of 16.4. A discount rate of 6.85% yields a multiple of 14.6. Thus, using the wrong releveraging formula would give a pricing error of more than ten percent. This is as important as many of the other sources of error commonly discussed in valuation.

#### **4. Numerical example of incorrect valuations using standard releveraging formulas in the flows-to-equity method**

In this section, we show, by way of an example, that none of the standard formulas in Table 1 yields the correct value when applied in a flows-to-equity valuation in a setup with an amortising debt plan, even if the debt's yield is used for the cost of debt. In the example, the only financing side effect is the tax deductibility of interest payments so that the correct value can be calculated using standard APV.

Parameter values in the example are: corporate tax rate,  $T = 35\%$ ; yield on debt,  $Y = 5.00\%$ ; risk-free rate,  $R_F = 3.00\%$ ; unlevered cost of equity,  $R_U = 9.00\%$ .

**Insert Table 2 here.**

Table 2, Panel A, sets out the after-tax operating free cash flows (*FCFF*), debt plan, and equity free tax cash flows (*FCFE*). The project has an investment of 100 at time zero and gives rise to after-tax operating free cash flow of 20, 60, 45, 20 in the following years. The debt plan is to borrow 90 and pay it down according to the amortisation schedule shown. The equity free cash flows are the operating cash flows plus the tax saving from interest minus the change in debt, as in (8).

The net equity value at date 0 is given by the APV, which, as seen in the table, is 20.99. The present value of the tax shield is calculated by discounting projected interest

payments at the yield of the debt. Cooper and Nyborg (2008) show that this is consistent with no arbitrage, given certain assumptions about the default process for the debt, which are adopted here.<sup>7</sup>

Panel B of Table 2 computes the value of the investment using the iterative implementation of the flows-to-equity method as laid out by Esty (1999). This is done as follows. From Panel A one first inputs the equity cash flows and the debt plan. In the  $R_E$  column, one enters the releveraging formula to be used, in this case (2), with  $R_D = Y$ . In the “PV equity” column, one enters the equity value (ex cash flow) calculated assuming last period’s equity value grows at  $R_E$ . For example, PV equity at date 1 is  $27.3335 \times 1.2876 - 7.075 = 28.1185$ . The value of the equity is solved for iteratively by choosing an initial end of period equity value (the first row in the fourth column) so that the sum of the discounted equity cash flows equals that equity value less the initial equity outflow.

As seen, the solution when using (2) as the releveraging formula, with the cost of debt set equal to the promised yield of 5.0%, is 23.22, which is 10.6% above the APV calculated in Panel A. Hence, (2) does not give the correct equity discount rate when the debt level varies over time.

If (7) is used to calculate the equity discount rate instead, the procedure yields an equity value of 17.33, or 17.42% below the correct value. Using (5), with  $R_D = Y$ , gives an equity value of 20.79. This is only 0.9% below the correct valuation. While this is a relatively small error, in other examples the error from using (5) may be substantially larger.

This example illustrates that the standard formulas for calculating equity discount rates for use in the flows-to-equity method result in values that are inconsistent with the fundamental adjusted present value identity. This leaves us with the question as to what the correct formula might be?

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<sup>7</sup> See also Molnar and Nyborg (2013).

## 5. The releveraging formula for the cost of equity in the flows-to-equity method with time-varying debt

In this section, we expand on the result from Section 3 to show that the correct releveraging formula using the flows-to-equity method with time-varying debt under a fixed debt plan is a generalization of equation (2), with the promised yield on debt used as its “cost.” We initially assume fairly priced debt and no costs of financial distress so that the debt tax shield continues to be the only financing side effect. This is relaxed in Section 6.

Throughout, we consider a project funded with debt whose level may change over time according to a fixed schedule. The debt face value at time  $t$  is  $D_t$ .<sup>8</sup> The promised yield on the debt is fixed at  $Y$  and the corporate tax rate is  $T$ .<sup>9</sup> The project has expected after-tax unlevered cash flows of  $C_t$ .

We assume that the discount rate for the unlevered flows is constant and equal to  $R_U$ . The unlevered value is calculated by discounting the unlevered free cash flows (after corporate taxes) at the unlevered discount rate:

$$V_{U,t} = \sum_{i=1}^{\infty} \frac{C_{t+i}}{(1+R_U)^i}. \quad (12)$$

The fundamental APV relationship always gives the correct total levered value:

$$V_{L,t} = V_{U,t} + PVTS_t, \quad (13)$$

All leverage-adjusted discount rates are derived from (12). The reason that particular formulas differ is because they make different assumptions about debt policy and, therefore, the size and risk of  $PVTS$ , as discussed in Section 2.

The value of equity can be calculated from the APV formula as:

$$E_t = V_{L,t} - D_t = V_{U,t} + PVTS_t - D_t. \quad (14)$$

<sup>8</sup> Time indicators,  $t$ , are in the subscripts.

<sup>9</sup> Although we treat the interest rates as fixed, the same approach can be used with variable rate debt.

However, the point of the flows-to-equity method is to obtain the equity value by discounting the equity free cash flows, (8).

The equity discount rate,  $R_{E,t}$ , is defined implicitly as the rate required to give the correct value of the equity by discounting equity flows and values period-by-period:

$$E_t = \frac{FCFE_{t+1} + E_{t+1}}{1 + R_{E,t}} \quad (15)$$

where the equity values and equity free cash flow are given by (14) and (8), respectively. A consistent flows-to-equity valuation procedure is the one that delivers an equity value using equation (15) which is the same as that calculated using equation (14).

The final ingredient is an assumption about the risk of  $PVTS$ . With a fixed debt plan and simplifying assumptions regarding the treatment of tax losses, Cooper and Nyborg (2008) show that the value of the debt tax shield is given by:

$$PVTS_t = \sum_{i=0}^{\infty} \frac{D_{t+i} Y^i}{(1+Y)^{i+1}}, \quad (16)$$

where  $Y$  is the promised yield on the debt.

Appendix 1 now shows that  $R_{E,t}$  is given by the following expression

$$R_{E,t} = R_U + \frac{D_t - PVTS_t}{E_t} (R_U - Y). \quad (17)$$

This is seen to be Equation (1) with the “cost of debt” and the tax shield discount rate both being equal to the debt’s yield. This is a consequence of the debt policy and, specifically, (16).

Next, we show that the explicit reference to  $PVTS_t$  in (17) can be eliminated with a bit of additional work. Towards that end, define<sup>10</sup>

$$\alpha_t = \frac{PVTS_t}{TD_t}. \quad (18)$$

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<sup>10</sup> If  $D_t = 0$ , define  $\alpha_t = 0$ .

Hence,  $\alpha_t$  measures the present value of the tax shield resulting from the fixed debt plan as a proportion of what it would be under permanent debt at the current level,  $D_t$ . With constant perpetual debt,  $\alpha_t = 1$ . In general, for HLT's the value of  $\alpha_t$  is less than one, because the level of debt will be expected to reduce over time. However, our approach also allows for debt levels to wax and wane. We can now restate (17) as:

$$R_{E,t} = R_U + \frac{D_t}{E_t} (1 - \alpha_t T) (R_U - Y) \quad (19)$$

If  $\alpha_t = 1$ , this collapses to the MM formula, (2), with the debt yield used as the “cost of debt.” In other words, (2) is the special version of (19) where the debt stays at a constant level in perpetuity.

A final step will eliminate the need to know  $PVTS_t$  in order to estimate the equity discount rate. The trick is that tax shields can be related to the duration of the debt. The modified duration of the aggregate cash flows (interest and repayments) in the fixed debt is

$$MDUR_t = \frac{\sum_{i=1}^{\infty} i B_{t+i} / (1+Y)^i}{D_t (1+Y)}, \quad (20)$$

where  $B_{t+i}$  is the total cash flow going to the debt holders at time  $t + i$ , that is,

$$B_{t+i} = D_{t+i-1} (1+Y) - D_{t+i}. \quad (21)$$

Appendix 2 shows that  $\alpha_t$  is equal to the modified duration of the debt plan divided by the modified duration of a perpetuity, which is equal to  $1/Y$ , that is,

$$\alpha_t = \frac{MDUR_t}{(1/Y)} = MDUR_t Y. \quad (22)$$

Hence, the factor  $\alpha_t$  simply adjusts the releveraging formula for the duration of the debt plan relative to the duration of perpetual debt.

In conclusion, the correct equity discount rate to use in the flows-to-equity method with fixed debt plans is given by (19), with  $\alpha_t$  given by (22). Our formula is the extension of

the basic MM formula, (2), when the level of debt changes according to a predefined schedule over the life of the project. Our analysis also establishes that the “cost of debt” that should be used in the equity discount rate formula for use in the flows-to-equity method is the debt’s promised yield.

## **6. Generalization**

In the previous section, we assumed that there are no costs of financial distress and that debt is priced to have zero NPV to the shareholders of the borrowing firm. However, Almeida and Philippon (2007), among others, have shown that distress costs can have a substantial effect on the net benefit of debt. In addition, Huang and Huang (2012) have shown that a large portion of debt spread arises from sources which do not appear to reflect standard risk factors. As much as three-quarters of the spread on Baa/BBB is not explained by standard risk factors and, therefore, potentially reflects an excessive cost to that type of debt relative to the equilibrium cost of debt. Collin-Dufresne, Goldstein, and Martin (2001) also confirm that there is a component of the risk of debt which does not appear to reflect the risk factors captured in the cost of equity and, therefore, potentially represents an additional component of the cost of debt.

These effects (distress costs and excessive debt yield) are likely to be especially important for highly leveraged transactions. Therefore, in this section, we incorporate them into our valuation procedure by using a simplified version of Almeida and Philippon’s (2007) model. Essentially, we extend their analysis to derive its implications for the flows-to-equity valuation method. We assume that part of the debt spread exceeds fair compensation for default risk and, therefore, represents a loss of NPV to equity-holders. We define a fair interest rate as the rate which would have a zero NPV to shareholders of the borrowing firm, excluding the financing side-effects and incorporate this into our valuation formula. The marginal probability of default per period is assumed to be constant. This is based on the idea that the debt in HLT's is structured to match the maturity structure of debt to the profile of the underlying cash flows. One way of doing this is to make the debt structure generate a constant marginal probability of default.



We wish to value the firm from the perspective of the equity-holders. The side-effects of financing now include the tax shield from debt, distress costs, and the effect of expensive debt. We assume that if default occurs distress costs are a fixed proportion of the face value of debt prior to default. The logic is that the firm value at default is proportional to the amount of debt which has triggered default and the distress costs will be a proportion of the firm value. When expensive debt is issued we allow for its effect in the following way. The impact of the expensive debt on the equity-holders is the amount by which the promised yield exceeds the fair yield that would be required to compensate debt-holders for default risk. This loss of value occurs when the firm is solvent, but is zero in the default state.

We introduce some additional notation:

- Fair promised yield on debt from point of view of equity-holders:  $y$
- Financial distress cost per dollar face value of debt:  $\phi$
- Recovery rate in default per dollar face value of debt:  $\rho$

**Insert Table 3 and Figure 1 here.**

Table 3 shows these financing side-effects in a single-period version of the model. In order to calculate the APV value of the firm, these are the components we need to value. Figure 1 shows the evolution of the components of the adjusted present value in a multiperiod model. At the end of the first period, there is a gain of  $TYD_0$  from the interest tax shield in the solvent state. This is offset by an excess cost of  $(Y - y)D_0$  if the debt is expensive. In the default state, there is a cost of  $\phi D_0$ .

To derive the equity discount rate using these assumptions, we start from the APV formula as before:

$$V_{L,t} = V_{U,t} + PVFS_t \tag{23}$$

where  $PVFS_t$  is the present value at time  $t$  in the solvent state of all future financing side-effects shown in Figure 1 (including the probability of distress costs at future dates). To

determine *PVFS* we need a risk-adjusted probability to use in the valuation tree. As Cooper and Nyborg (2008), we derive the risk-adjusted probability from the condition for fairly-priced debt. Under the risk-neutral probability of default,  $q$ , this must have an expected return equal to the riskless rate. Fairly priced debt pays  $(1 + y)$  per dollar of face value if it does not default and  $\rho(1+y)$  if it does. Thus,

$$(1-q)(1+y) + q(1+y)\rho = (1+R_F). \quad (24)$$

Solving for  $q$  gives the risk-neutral probability of default as:

$$q = \frac{y - R_F}{(1+y)(1-\rho)}. \quad (25)$$

The components of the adjusted present value can be valued using this probability in conjunction with riskless discounting at  $R_F$ . A claim that pays \$1 in the solvent state and 0 in the default state is worth  $(1-q)/(1+R_F)$  at the beginning of the period. \$1 in the default state is worth  $q/(1+R_F)$ . Thus, the loss from expensive debt of  $D(Y-y)$  in the solvent state and 0 in the default state is worth  $D(Y-y)(1-q)/(1+R_F)$  at the beginning of the period.

Using the risk-neutral valuation procedure, we can value all the APV components at time  $t$ :

$$\begin{aligned} PVFS_t &= \sum_{i=0}^{\infty} \frac{D_{t+i}(1-q)^{i+1}YT}{(1+R_F)^{i+1}} - \sum_{i=0}^{\infty} \frac{D_{t+i}(1-q)^{i+1}(Y-y)}{(1+R_F)^{i+1}} - \sum_{i=0}^{\infty} \frac{\phi D_{t+i}(1-q)^i q}{(1+R_F)^{i+1}} \\ &= \sum_{i=0}^{\infty} D_{t+i}YT^* \times \left( \frac{1-q}{1+R_F} \right)^{i+1}, \end{aligned} \quad (26)$$

where

$$T^* = T - \frac{(Y-y)}{Y} - \frac{q\phi}{(1-q)Y}. \quad (27)$$

Define

$$c = \frac{\rho(y - R_F)}{(1 - \rho)(1 + R_F)}, \quad (28)$$

and

$$1 + \gamma = \frac{1 + y}{1 - c}. \quad (29)$$

Substituting these expressions into (26), we obtain

$$PVFS_t = \sum_{i=0}^{\infty} \frac{D_{t+i} Y T^*}{(1 + \gamma)^{i+1}}. \quad (30)$$

This differs from the simple case in two ways. First, it uses an adjusted tax rate,  $T^*$ , which includes the effects of nonzero NPV debt and costs of financial distress. Second, it uses an adjusted yield that allows for the effect of the recovery rate. Note that when  $\rho = 0$  then  $\gamma = y$ , so that the adjusted yield is equal to the fair yield and we “discount” the APV components at the fair yield,  $y$ .

Using the same basic procedure as for the simple case, but with  $PVFS$  given by (30) instead of  $PVTS$  given by (16), we obtain (Appendix 3):

$$R_{E,t} = R_U + \frac{D_t}{E_t} \left[ (1 - \alpha_t^* T^*) (R_U - Y) + \alpha_t^* T^* (\gamma - Y) + (T - T^*) Y \right], \quad (31)$$

where<sup>11</sup>

$$\alpha_t^* = \frac{PVFS_t}{T^* D_t}. \quad (32)$$

Equation (31) parallels (19), but uses  $T^*$  and  $\alpha_t^*$  rather than  $T$  and  $\alpha_t$ . There are also two extra terms. The first involves the difference between  $\gamma$  and  $Y$ , thus reflecting the possibility of mispriced debt (in the sense that the interest rate differs from the fair rate). The second extra term involves the difference between  $T$  and  $T^*$  and captures both mispriced debt and costs of financial distress, as seen in (27).

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<sup>11</sup> If  $D_t = 0$ , define  $\alpha_t^* = 0$ .

The releveraging formula (31) is much more complicated than (19) because of the effect of distress costs, the excess yield, and the recovery rate. While it may be easier to simply use APV, for someone who wishes to use the flows-to-equity method, equation (31) provides a way of doing this that accounts for not only tax shields, but also mispriced debt and costs of financial distress. Our formula may also be useful for someone who wishes to estimate the rate of return to equity as the project unfolds and debt is paid down (with the caveat that our formula is not the expected rate of return, but a hybrid discount rate suitable for use in the flows-to-equity method).

### **7. The size of the effects: What matters most?**

In this section we examine the relative importance of the different factors affecting the cost of equity in an HLT. We first investigate the impact of using an incorrect releveraging formula, then we investigate the impact of distress costs and an excess debt spread. We set the recovery rate to zero in this example.

**Insert Table 4 here.**

We illustrate our results using an updated version of a realistic example studied by Esty (1999). Esty's example has the key features of project finance and LBO's: a relatively large amount of debt, relatively high margins on the debt, and a fixed debt plan. We maintain the basic structure of his example, but update the levels of interest rates and cash flows to be consistent with the current market environment. Table 4 shows the cash flows and leverage of the project. These are based on those in Esty's example, but with the operating cash inflows rescaled to give an IRR for the equity free cash flow of 7%, in line with current levels of capital market variables.

The parameters we use are the same we used in the perpetuity example in Section 3: Corporate tax rate,  $T = 40\%$ ; yield on debt,  $Y = 4.50\%$ ; risk-free rate,  $R_F = 2.50\%$ ; equity market risk premium,  $5\%$ ; and unlevered asset beta,  $\beta_U = 0.6$ . Thus, the unlevered cost

of equity,  $R_U$ , is 5.5% (the CAPM is assumed to hold). The leverage in the project is roughly 50% during the period until the debt is retired and varies over time.

**Insert Table 5 here.**

Table 5 shows the effect of using the wrong releveraging formula. Column (A) gives the correct net equity value, +45,871. Column (B) uses the releveraging formula for beta, Equation (7), which assumes a proportional (Miles-Ezzell) debt policy and zero debt beta. This creates a major valuation error. The net equity value is now estimated as -152,111. The magnitude of the error is confirmed by comparing the estimated value of the cost of equity during the period for which the project is leveraged. On average this is 6.48% for the correct formula, but 9.65% for Equation (7). This error has two sources: using the riskless rate for the cost of debt rather than the debt yield, and using the Miles-Ezzell leverage policy rather than a fixed debt policy. Column (C) shows the effect of using the Miles-Ezzell formula with the debt yield as the cost of debt. The net equity value of +33,897 is now much closer to its correct value and the average cost of equity, 6.64%, is almost correct. Thus the main issue in the choice of releveraging formula is to use the promised yield on debt as the “cost of debt.” As discussed above, this is consistent with the way the flows to equity are calculated.

**Insert Table 6 here.**

Table 6 shows the effect of distress costs and an excess debt spread. Column (A) is the value assuming that both are zero. Column (B) shows the effect of including distress costs of 16.5% ( $\phi = 0.165$ ). This value is consistent with parameter values from Almeida and Philippon (2007). Column (C) shows the effect of assuming that three quarters of the debt spread is excess. That is consistent with the analysis in Huang and Huang (2012). It implies a fair yield,  $y$ , of 3.0%, relative to the riskless rate of 2.5% and the full promised yield of 4.5%. The effect of distress costs is to reduce the net equity value from 45,871 to 13,786. In contrast, the effect of the excess spread reduces the net equity value to -

100,226. Thus, here again, the treatment of the debt spread is the central issue in implementing this approach.

In summary, the analysis of the impact of using the incorrect releveraging formula and the analysis of the impact of distress costs and excess debt spread both indicate the importance of the correct treatment of the debt spread in valuing HLT's. As discussed in the introductory section, the low levels of riskless interest rates have made this a relatively more important issue in the current capital market environment.

### **8. Releveraging the cost of capital directly**

Although our focus in this paper is on the flows to equity method, our approach can also be used to derive adjusted discount rates that apply to the unlevered flows. That is, the rates  $R_{L,t}$  so that when unlevered flows are discounted at these rates we obtain  $V_L$ . The general formula (which can be derived in a way similar to the cost of equity) is:

$$R_{L,t} = R_U [1 - T^* L_t] + T^* L_t (1 - \alpha_t^*) (R_U - Y) + T^* L_t (\gamma - Y), \quad (33)$$

where  $L_t = D_t / V_{L,t}$ . As with the releveraged cost of equity, this is rather complicated. With no distress costs and debt that is issued at fair terms ( $Y = y$ ), this reduces to the simpler expression:

$$R_{L,t} = R_U [1 - TL_t] + TL_t [1 - \alpha_t] (R_U - Y). \quad (34)$$

### **9. Concluding remarks**

We have developed formulas for tax adjusted discount rates in highly levered transactions. Our formulas are best interpreted as being suitable for project finance or other structures where the amount of debt follows a predictable pattern. Our analysis is concerned with developing a consistent method for using the flows-to-equity method. We have shown that the way the equity free cash flow is conventionally calculated implies a specific way of releveraging the cost of equity, which treats the full promised debt yield as the cost of debt. We emphasize that this does not give the cost of equity as it is conventionally

defined as an expected rate of return. Rather, it is a cost of equity that should be used only in the flows-to-equity method.

We have extended the basic framework to allow for debt which has a higher than fair interest rate and distress costs. The formulas in this general scenario parallel those in the simpler case, but involve modified tax and interest rates. These modifications depend on the extent to which the yield spread on the debt is unfair, and the level of distress costs. They are more complex than conventional formulas for releveraging the cost of equity.

Although we focus on the flows-to-equity method, there are alternatives which can be used to value highly leveraged transactions. The WACC and capital cash flow approaches can be used to incorporate the tax benefit of debt directly in the DCF calculation (see Cooper and Nyborg, 2007, for a review). Alternatively, adjusted present value (APV) can be used to separately calculate the tax benefit of the debt (Arzac 1996) and can also include other financing side-effects. All the features that the flows-to-equity method is designed to capture can also be included in the APV approach. In practice, implementing the flows-to-equity approach correctly is, arguably, more complicated than using APV, since iteration is required. Since the consistent version of the flows-to-equity approach is derived from the APV formula, it is an open question as to whether the flows-to-equity method can achieve anything that APV cannot. Still, for someone who wishes to use the flows-to-equity method, it is important to use the correct equity discount rate. Our paper provides just that when debt levels evolve according to a predefined schedule.

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**Appendix 1: Proof of the relationship between  $R_{E,t}$  and  $R_U$**

From equations (12) - (14) in the main text:

$$(E_t + D_t - PVTS_t)(1 + R_U) = E_{t+1} + D_{t+1} - PVTS_{t+1} + C_{t+1}. \quad (A1.1)$$

From equations (14) and (15):

$$E_t(1 + R_{E,t}) = E_{t+1} + C_{t+1} + D_{t+1} - D_t[1 + Y(1 - T)]. \quad (A1.2)$$

From equation (16):

$$PVTS_t(1 + Y) = D_tTY + PVTS_{t+1}. \quad (A1.3)$$

Taking equation (A1.1) plus (A1.3) minus (A1.2) gives:

$$E_tR_U + D_tR_U - PVTS_tR_U - E_tR_{E,t} + PVTS_tY = D_tY. \quad (A1.4)$$

Rearranging (A1.4) gives equation (17) of the main text.

**Appendix 2: The relationship between  $\alpha_t$  and modified duration.**

From equations (20) and (21) of the main text:

$$\begin{aligned} D_t(1 + Y)MDUR_t &= \sum_{i=1}^{\infty} \frac{iD_{t+i-1}}{(1 + Y)^{i-1}} - \sum_{i=1}^{\infty} \frac{iD_{t+i}}{(1 + Y)^i} \\ &= \sum_{i=0}^{\infty} \frac{(i+1)D_{t+i}}{(1 + Y)^i} - \sum_{i=1}^{\infty} \frac{iD_{t+i}}{(1 + Y)^i} \\ &= D_t + \sum_{i=1}^{\infty} \frac{D_{t+i}}{(1 + Y)^i} \\ &= \sum_{i=0}^{\infty} \frac{D_{t+i}}{(1 + Y)^i} \\ &= \frac{PVTS_t(1 + Y)}{YT}, \end{aligned} \quad (A2.1)$$

where the last equality follows from (16). Hence,

$$MDUR_t = \left[ \frac{PVTS_t}{TD_t} \right] \frac{1}{Y} = \frac{\alpha_t}{Y}. \quad (A2.2)$$

**Appendix 3: Proof of the relationship between  $R_{E,t}$  and  $R_U$  in the general case.**

The proof follows along the same lines as in Appendix 1 with  $PVTS$  replaced by  $PVFS$ . Similarly to (A1.1-A1.3):

$$(E_t + D_t - PVFS_t)(1 + R_U) = E_{t+1} + D_{t+1} - PVFS_{t+1} + C_{t+1}. \quad (\text{A3.1})$$

$$E_t(1 + R_{E,t}) = E_{t+1} + C_{t+1} + D_{t+1} - D_t[1 + Y(1 - T)]. \quad (\text{A3.2})$$

$$PVFS_t(1 + \gamma) = D_t T^* Y + PVFS_{t+1}. \quad (\text{A3.3})$$

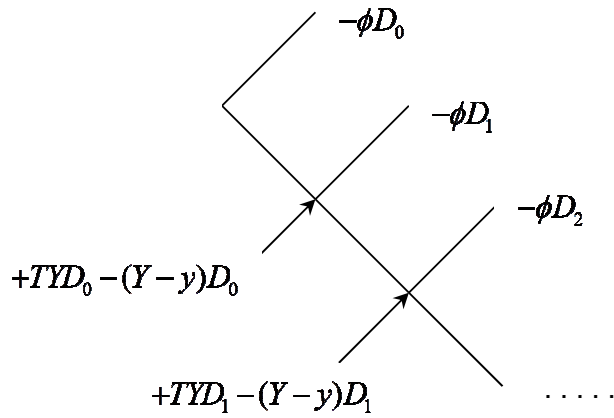
Taking equation (A3.1) plus (A3.3) minus (A3.2) gives:

$$E_t R_U + D_t R_U - PVFS_t R_U - E_t R_{E,t} + PVFS_t \gamma = D_t Y - D_t Y T + D_t Y T^*. \quad (\text{A3.4})$$

Rearranging (A3.4) gives equation (31) of the main text, with  $\alpha_t^*$  given by (32).

### Figure 1: Evolution of the APV components in the multiperiod model

The figure shows the assumptions about the side-effects of financing arising when the firm is either solvent or in default on its debt.  $D_t$  is the value of debt at time  $t$ ,  $Y$  is the promised yield on debt,  $T$  is the corporate tax rate,  $y$  is the fair level of the promised yield, and  $\phi$  is the financial distress cost per dollar of face value of debt.



**Table 2: Example of valuation error in the standard implementation of the flows to equity method**

The table shows an example where the value of a project with a fixed debt plan is calculated using APV in Panel A and the flows-to-equity method in Panel B. In Panel B, the equity discount rate is calculated using Equation (2) in Table 1 with  $R_D = Y$ , namely,

$$R_E = R_U + (D / E)(R_U - Y)(1 - T),$$

where  $R_E$  is the equity discount rate,  $D$  is the value of debt,  $E$  is the value of equity,  $R_U$  is the unlevered cost of capital, and  $Y$  is the debt's promised yield. Parameter values are  $T = 35\%$ ,  $Y = 5\%$ ,  $R_U = 9\%$ . The APV value of the equity is the correct value. So Panel B illustrates the size of error resulting from using Equation (2).

**Panel A: Free cash flows, debt plan, and benchmark adjusted present value**

Year	Operating Cash Flow (FCFF)	Debt	Net Principal Repayment	Interest	Tax saving	Equity Cash Flow (FCFE)	Unlevered discount factor	Discount factor tax shield
0	-100	90	-90	0.00	0.000	-10.000	1.000	1.000
1	20	80	10	4.50	1.575	7.075	0.917	0.952
2	60	30	50	4.00	1.400	7.400	0.842	0.907
3	45	0	30	1.50	0.525	14.025	0.772	0.864
4	20	0	0	0.00	0.000	20.000	0.708	0.823
<b>NPV: 17.766</b>						<b>PVTS: 3.223</b>	<b>APV: 20.99</b>	

**Panel B: Flows-to-equity valuation using Esty's (1999) iterative method with Equation (2) as the releveraging formula, with  $R_D = Y$**

Year	Equity Cash Flow (FCFE)	Debt	PV equity end period	Debt plus equity	Leverage (D/E)	RU (%)	RE (%)	Discount Factor	Present Value of FCFE
0	-10.000	90	33.218	123.218	2.709	9.00	16.04	1.000	-10.000
1	7.075	80	31.472	111.472	2.542	9.00	15.61	0.862	6.097
2	7.400	30	28.985	58.985	1.035	9.00	11.69	0.745	5.516
3	14.025	0	18.349	18.349	0.000	9.00	9.00	0.667	9.360
4	20.000	0	0.000	0.000	0.000	9.00	9.00	0.612	12.245
<b>Sum (PV equity): 23.22</b>									

**Table 3: Financing side-effects in a single period version of the model**

The table lists the assumptions about the side-effects of financing arising when the firm is either solvent or in default on its debt.  $D$  is the value of debt,  $Y$  is the promised yield on debt,  $T$  is the corporate tax rate,  $y$  is the fair level of the promised yield, and  $\phi$  is the financial distress cost per dollar of face value of debt.

Component	State	
	<i>Solvent</i>	<i>Default</i>
Tax saving from debt	$+DYT$	
Distress cost		$-\phi D$
Loss to equity from overpriced debt	$-D(Y - y)$	
Total financing side-effects	$+DYT - D(Y - y)$	$-\phi D$

**Table 4: Operating free cash flow, leverage, and equity free cash flow for the project**

The table shows the operating free cash flow (*FCFF*), leverage, and equity free cash flow (*FCFE*) for the project. The debt yield is 4.5% and the tax rate is 40%.

Year	Operating Free Cash Flow (FCFF)	Debt	Net Principal Repayment	Interest	Tax saving	Equity Free Cash Flow (FCFE)
0	-300,000	0	0	0	0	-300,000
1	-870,000	700,000	-700,000	0	0	-170,000
2	-812,349	1,300,000	-600,000	31,500	12,600	-231,249
3	140,411	1,275,000	25,000	58,500	23,400	80,311
4	141,712	1,250,000	25,000	57,375	22,950	82,287
5	136,631	1,225,000	25,000	56,250	22,500	77,881
6	144,992	1,175,000	50,000	55,125	22,050	61,917
7	140,000	1,125,000	50,000	52,875	21,150	58,275
8	148,451	1,050,000	75,000	50,625	20,250	43,076
9	149,944	975,000	75,000	47,250	18,900	46,594
10	145,055	900,000	75,000	43,875	17,550	43,730
11	153,609	800,000	100,000	40,500	16,200	29,309
12	148,811	700,000	100,000	36,000	14,400	27,211
13	157,455	575,000	125,000	31,500	12,600	13,555
14	152,748	450,000	125,000	25,875	10,350	12,223
15	161,484	300,000	150,000	20,250	8,100	-666
16	159,682	150,000	150,000	13,500	5,400	1,582
17	190,297	0	150,000	6,750	2,700	36,247
18	142,734	0	0	0	0	142,734
19	144,172	0	0	0	0	144,172
20	145,625	0	0	0	0	145,625
21	147,093	0	0	0	0	147,093
22	148,575	0	0	0	0	148,575
23	150,072	0	0	0	0	150,072
24	151,585	0	0	0	0	151,585
25	153,114	0	0	0	0	153,114



**Table 5: The effects of using the wrong releveraging formula**

The table shows the value resulting from estimating the value of the project shown in Table 4 using different releveraging formulas in the flows-to-equity method. The basic parameters are:  $T = 40\%$ ,  $Y = 4.5\%$ ,  $R_F = 2.5\%$ . Equation (19) is the correct releveraging formula. Equation (7) releverages the equity beta using a Miles-Ezzell debt policy based formula and assumes the debt beta is zero. Equation (5) releverages the equity discount rate using a Miles-Ezzell debt policy based formula with  $R_D = Y$ . The calculated net equity value is shown, along with the average estimated cost of equity during the leveraged period.

	(A)	(B)	(C)
$R_E$ formula	Equation (17)	Equation (7)	Equation (5)
Cost of debt in releveraging formula	Promised yield	Riskless rate	Promised yield
Debt plan	Fixed (time-varying)	Proportion of value	Proportion of value
Net equity value	45,871	-152,111	33,897
Average $R_E$ during leveraged period	6.48%	9.65%	6.64%

**Table 6: The effects of distress costs and excess debt yield**

The table shows the effect of distress cost and excess yield on the value of the project shown in Table 4. The basic parameters are:  $T = 40%$ ,  $Y = 4.5%$ ,  $R_F = 2.5%$ . Column (A) has zero distress costs and zero excess yield. Column (B) has distress costs of 0.165. Column (C) has an excess debt spread of 1.5%. The calculated net equity value is shown, along with the average estimated cost of equity during the leveraged period. The recovery rate,  $\rho$ , is set to zero in all cases.

	(A)	(B)	(C)
Distress cost	0	0.165	0
Excess debt spread	0	0	1.5%
Net equity value	45,871	13,786	-100,226
Average $R_E$ during leveraged period	6.48%	6.91%	8.67%