Does improved information improve incentives?*

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Abstract

This paper studies the value of more precise signals on agent performance in an optimal contracting model with endogenous effort. With limited liability, the agent's wage is increasing in output only if output exceeds a threshold, else it is zero regardless of output. If the threshold is sufficiently high, the agent only beats it, and is rewarded for increasing output through greater effort, if there is a high noise realization. Thus, a fall in output volatility reduces effort incentives – information and effort are substitutes – offsetting the standard effect that improved information lowers the cost of compensation. We derive conditions relating the incentive effect to the underlying parameters of the agency problem.

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1 Introduction

Since the seminal contributions of Holmstrom (1979) and Shavell (1979), the moral hazard literature has shown that superior information on agent performance can improve the principal's payoff. This result has implications for many contracting applications, such as compensation, financing, insurance, and regulation. However, information is also costly. Thus, to determine whether the principal should invest in information, we must analyze what its benefits depend on – in particular, relate them to the underlying parameters of the agency problem. Doing so will identify the situations in which information is most valuable, and thus its acquisition most justified.

Analyzing the benefits of information requires an optimal contracting approach. Assuming a particular contracting form may lead to misleading results – for example, when contracts are suboptimal, adding noise to the contract may be desirable, suggesting that precision has strictly negative value. As is well-known, solving for the optimal contract in a general setting is highly complex (e.g., Grossman and Hart, 1983). We consider the standard framework of risk neutrality and limited liability, originally analyzed by Innes (1990) and widely used in a number of settings (e.g., Biais, Mariotti, Rochet, and Villeneuve, 2010; Clementi and Hopenhayn, 2006; DeMarzo and Fishman, 2007a, 2007b; DeMarzo and Sannikov, 2006). This framework allows for an optimal contracting approach and involves the agent receiving a call option, as observed in practice.[1]

We model the option contract as based on output and information as affecting output volatility, but the model is virtually identical if the contract is instead based on a separate performance signal and information affects the volatility of this signal. We start by assuming a general output distribution and an endogenous implemented effort level. An increase in information precision (fall in volatility), in the sense of a mean-preserving spread of output, has two effects, each with a clear economic interpretation. First, a fall in volatility reduces the value of the option and thus the expected wage: the direct effect. Second, it changes the agent's effort incentives: the incentive effect. While the direction of the direct effect is unambiguous – the standard intuition that information reduces the cost of contracting – the incentive effect can either be positive or negative, i.e., effort and precision can be complements or substitutes. The heart of this paper analyzes this incentive effect. Our most general result is to derive a

[1]While options are not the only instruments used in practice, Dittmann and Maug (2007) find that the payoff structure provided by a CEO's overall compensation package resembles an option.

condition that determines the sign of this effect, and thus whether effort and precision are complements (precision increases incentives) or substitutes (precision decreases incentives). The condition is simple and easy to check, as we show with a simple example.

We then consider the common case of output distributions with a location parameter, i.e., effort shifts the location of the distribution without affecting its shape, as in the normal and uniform distributions. This allows us to relate the sign of the incentive effect to the strike price of the option. We show that the incentive effect is positive if and only if the strike price is below a threshold, and negative otherwise. The intuition is as follows. Since the wage is positive only if output exceeds the strike price, increasing effort increases the wage only if output ends up higher than the strike price – if output still ends up below, the agent receives zero, regardless of output and thus his effort. Increasing precision moves probability mass from the tails towards the center of the distribution. If the strike price is low, this change moves mass from below to above the strike price, increasing the probability that output exceeds it. In simple language, the agent thinks "If I work harder, I'll get paid more unless I get so unlucky that output falls below the target. Now that precision is higher, I'm unlikely to suffer bad enough luck, so it's worth it for me to work harder." Thus, effort and precision are complements, and so the principal's benefit from increasing precision is even higher than when focusing on the direct effect alone.

On the other hand, if the strike price is high, increasing precision shifts mass from above to below the strike price, reducing the probability that output exceeds it. In simple language, the agent thinks "The target is so high that, even if I did work, I wouldn't meet it unless I also got lucky. Now that precision is higher, I'm unlikely to get lucky enough to meet the target, so there's no point in working." Thus, effort and precision are substitutes, and so the principal's benefit from increasing precision is lower than when focusing on the direct effect alone. As a result, the net benefit of precision may be insufficient to justify its cost.

Since the strike price is endogenous, we next relate the incentive effect to the underlying parameters of the agency problem – specifically the cost function. In any contracting setting, the effect of the cost function on the strength of incentives (here, captured by the option's delta and thus the strike price) depends on whether the implemented effort level is fixed or endogenous. With endogenous effort, a less convex cost of effort typically leads to the principal implementing a higher effort level and thus offering stronger incentives; with fixed effort, a lower cost of effort means that weaker incentives are needed to implement the given effort level. We therefore analyze both cases. Where effort is endogenous, the sign of the incentive effect depends on the convexity of the cost function. If the cost function is sufficiently convex, inducing effort is costly and so the principal implements a low effort level. As a result, the strike price is high relative to the effort level, and so precision reduces incentives. This result contrasts intuition that information should be more valuable when the agency problem is strong. In addition, an analysis focusing only on the direct effect of precision, and ignoring the incentive effect, would suggest that the value is highest when the option is at-the-money – i.e., a moderate initial strike price and a moderate agency problem.

A fixed effort level arises with a binary effort space, which is often used for tractability. In addition to being tractable, a fixed effort level is also realistic if the benefits of effort are large relative to the costs, as with CEOs of large firms (Edmans and Gabaix, 2011), because the principal always wishes to implement full productive efficiency. The sign of the incentive effect depends on the cost of effort: where the cost is high, the principal must give the agent strong incentives to induce effort. These incentives are provided by a low strike price, and so precision increases incentives.

It is limited liability, and not risk neutrality, that is key to our results. We show that the incentive effect similarly depends on the strike price when the agent is risk-averse. Regardless of whether the agent is risk-neutral or risk-averse, when there is limited liability, the optimal contract pays the agent zero when output is below a threshold. Thus, he is only rewarded for marginal increases in effort if output exceeds the threshold, the probability of which depends on output volatility.

We also analyze the case in which the principal can renege on any pre-announced level of precision after the agent has exerted effort. Then, there is no incentive effect (the agent ignores the initial announcement when selecting effort) and so the principal only considers the direct effect when choosing the final level of precision. Thus, her benefit from increasing precision is higher ex ante than ex post if and only if the incentive effect is positive. In this case, she may wish to commit to a high level of precision ex ante. We also show that the level of precision chosen by the principal generally differs from the socially optimal level, and may exceed it. This result suggests that regulations to increase information disclosure may move us further from the social optimum. In addition, if the agent had bargaining power and chose precision, he would typically select a different level from the principal, so control rights matter for efficiency.

Our results have a number of implications for compensation contracts. Most importantly, they highlight that information may not improve incentives, contrary to conventional wisdom that more precise signals make incentive provision easier. They also identify the settings in which investing in information is optimal for the principal. Using the fixed effort model as an example, when incentives are strong (weak) to begin with, e.g., for CEOs (rank-and-file managers), an increase in precision increases (reduces) incentives. One way in which the principal can invest in information is to engage in relative performance evaluation ("RPE"). There is very little evidence that RPE is used for rank-and-file managers, and only modest evidence of its usage for CEOs.[2] Bebchuk and Fried (2004) interpret this rarity as evidence that CEO contracts are inefficient. However, to evaluate this argument, we need to identify the settings in which the value of information is smallest, and compare them to the cases in which RPE is particularly absent in reality. That RPE is more common for CEOs than managers is consistent with the above prediction.

In addition to the gains from precision, our analysis also studies the impact of exogenous changes in precision, such as changes in stock market efficiency. For example, an increase in volatility raises (lowers) the incentives of agents with out-of-the-money (in-the-money) options. If firms recontract in response, CEOs with in-the-money options should receive the highest increase in incentives.

As Innes (1990) showed, in addition to compensation, the model can also be applied to a financing setting where the agent (entrepreneur) raises financing from the principal (investor), in which case the contract is risky debt, and the strike price represents its face value. Our model sheds light on the settings in which the investor's incentive to reduce output volatility is highest. As with investing in information, doing so is potentially costly – implementing a risk management system is expensive, and imposing covenants can stifle investment. An analysis based on the direct effect would suggest that risk management is most valuable for firms at the bankruptcy threshold, as then the value of debt is most sensitive to volatility. This is consistent with standard intuition that risk management incentives are increasing in loan size (up to the bankruptcy threshold), because the lender has more at stake. This intuition is incomplete because it ignores the incentive effect. When the face value of debt is low, equity is in-the-money and the incentive effect is positive. As a result, risk management raises

[2]While Aggarwal and Samwick (1999) and Murphy (1999) document almost no use, more recent evidence by Albuquerque (2009), Gong, Li, and Shin (2011), and De Angelis and Grinstein (2017) find evidence of RPE. See Edmans, Gabaix, and Jenter (2017) for a review of the evidence on RPE.

effort incentives, further increasing its value over and above the direct effect. Thus, surprisingly, risk management may be more valuable for firms that are some distance from bankruptcy, and when the investor has little debt at stake. Separately, the model suggests that the entrepreneur's initial wealth affects the value of precision. Where wealth is low, the entrepreneur needs to raise debt with a high face value which, as is known, reduces effort incentives. Our results suggest that increases in precision may further exacerbate the incentive problems originating from low initial wealth, since the incentive effect is then negative.

Dittmann, Maug, and Spalt (2013) also consider the effect on effort when analyzing a specific form of increased precision – indexing stock and options – and similarly show that indexation may weaken incentives. They use a quite different setting, reflecting the different aims of each paper. Their goal is to calibrate real-life contracts, and so they fix the implemented effort level, restrict the contract to comprising salary, stock, and options, and hold stock constant when changing the contract to restore the agent's incentives upon indexation. They acknowledge that the actual savings from indexation will be different if the principal recontracts optimally. Our primary goal is theoretical. We take an optimal contracting approach, where the contract adjusts optimally to changes in precision, and the implemented effort level is endogenous.

The interaction between effort and (mean-preserving) risk has been studied by other papers. In Holmstrom and Milgrom (1987), risk has no direct effect on effort (unlike in our paper), but greater risk reduces the optimal pay-performance sensitivity and thus incentives. In this sense, precision always increases effort, unlike in our paper. Gjesdal (1982) shows that incentives can increase with a different type of risk – giving the agent a stochastic contract where, for a given output level, the agent receives a lottery rather than a wage. If the agent's utility is non-separable in the wage and his action, higher effort may directly reduce the agent's risk aversion; giving him a lottery incentivizes him to exert effort to reduce his aversion to the lottery. In our paper, a deterministic contract is optimal (as is common in reality) and risk instead involves changing the precision of the performance measure, allowing our model to apply to risk management and monitoring. The channel through which risk affects effort is also very different.

This paper proceeds as follows. Section 2 presents the model. The main results are presented in Section 3, where we study the effects from increased signal precision and relate the incentive effect to the underlying parameters of the agency problem. Section 4 discusses applications and alternative modeling assumptions, and Section 5 concludes. The proofs are presented in Appendix A.

2 The model

We consider a standard principal-agent model with risk neutrality and limited liability, as in Innes (1990). At time $t = -1$, the principal offers a contract to the agent. At $t = 0$, the agent chooses effort e from a non-empty, compact subset of the real line $\mathcal{E}$. The agent's cost of exerting effort e is $C(e)$, where $C(\cdot)$ is continuous and increasing. As is standard, effort can refer not only to working rather than shirking, but also choosing projects to maximize firm value rather than private benefits, or not diverting cash flows. We normalize the lowest effort level to 0 and its cost to $C(0) = 0$. At $t = 1$, output q is realized.

The principal does not observe the agent's effort, but observes the realization of output. Output is continuously distributed according to a cumulative distribution function ("CDF") $F_\theta(q|e)$ with a continuous probability density function ("PDF") $f_\theta(q|e)$ that satisfies the monotone likelihood ratio property ("MLRP"): for any $e_H > e_L$ and any θ, $\frac{f_\theta(q|e_H)}{f_\theta(q|e_L)}$ is strictly increasing in q.[3] Intuitively, MLRP means that higher outputs indicate higher effort.

To ensure existence of an optimal contract, we assume that output has a finite mean $\mathbb{E}_\theta[q|e] < \infty$ and the integral $\int_X^\infty q f_\theta(q|e)dq$ is a continuous function of e for each $X \in \mathbb{R}$. Moreover, to simplify notation, we assume that the CDF $F_\theta(q|e)$ is differentiable with respect to θ.

The parameter θ orders the precision of the output distribution in the sense of a mean-preserving spread ("MPS"). Formally, for any $\theta \geq \theta'$, $\mathbb{E}_\theta\left[q|e\right] = \mathbb{E}_{\theta'}\left[q|e\right]$, and

$$F_\theta(q|e) \left\{ \begin{array}{c} \leq \\ \geq \end{array} \right\} F_{\theta'}(q|e) \text{ for } q \left\{ \begin{array}{c} < \\ > \end{array} \right\} q_e. \quad (1)$$

for some q_e.[4] Since a higher θ removes noise from the distribution of output without without affecting its mean, θ represents how informative output q is about effort e.

[3]Note that MLRP implies first-order stochastic dominance ("FOSD"): $F_\theta(q|e)$ is strictly decreasing in e for each fixed (θ, q). Thus, throughout the paper, we use "effort increases output" as a shorthand for "effort improves the distribution of output in the sense of FOSD."

[4]This definition follows Machina and Rothschild (2008) and states that the distributions differ by a single MPS. This notion implies second-order stochastic dominance ("SOSD"), which allows distributions to differ by a sequence of MPSs.

The precision parameter θ may be chosen by the principal, or result from exogenous changes such as a reduction in economic uncertainty. Our goal is to analyze the value of information and its effect on incentives, which applies to either setting.[5]

The agent is paid a "wage" $W_\theta(q)$ and the principal receives a "profit" $R_\theta(q) = q - W_\theta(q)$. The agent is risk-neutral and so maximizes his expected wage

$$\mathbb{E}_\theta\left[W_\theta(q)\,|e\right] = \int_{-\infty}^{\infty} W_\theta(q)\, f_\theta(q|e)\, dq,$$

less the cost of effort. His reservation utility is zero and there is no discounting.

Following Innes (1990), we make two assumptions on the set of feasible contracts. First, there is a limited liability constraint on the agent:

$$W_\theta(q) \geq 0\ \forall\, q. \tag{2}$$

Second, a monotonicity constraint ensures the principal's payoff is non-decreasing in output:

$$\eta \geq W_\theta(q+\eta) - W_\theta(q) \tag{3}$$

for all $\eta > 0$. Innes (1990) justifies this constraint on two grounds. First, if it did not hold, the principal would have incentives to sabotage output. Second, if it did not hold, the agent would gain more than one-for-one for increases in output. Thus, he would have incentives to borrow on his own account to increase output.

The principal wishes the agent to exert effort level e^*, and so the contract must satisfy the following incentive compatibility constraint:

$$e^* \in \arg\max_e \mathbb{E}_\theta\left[W_\theta(q)\,|e\right] - C(e). \tag{4}$$

Following standard arguments, the participation constraint will be slack and can be ignored in the analysis that follows. The principal is also risk-neutral and thus chooses a contract $W_\theta(q)$ and an effort level e^* to maximize her expected profit $R_\theta(q)$ subject to the limited liability (2), monotonicity (3), and incentive (4) constraints.

We discuss three features of our modeling setup. First, following Innes (1990), the principal contracts on output q and so changes in the precision of the performance

[5] While we consider the effect of changing the volatility of output, Chaigneau, Edmans, and Gottlieb (2017) derive necessary and sufficient conditions for whether the addition of a new signal has strictly positive value under contracting constraints.

measure change the volatility of output, as with risk management. We have also analyzed a model in which q is non-contractible, as in Baker (1992). For example, the principal may not have a contractible objective (e.g., a non-profit firm or government agency, or a private firm with no traded stock); the agent may be only one employee in a team and only the team's output is observable (e.g., a public firm where the employee has very little effect on the stock price); effort may contribute to a long-term project such as R&D where, by the time verifiable output is fully realized, the agent may no longer be with the firm; or the effort may have non-quantifiable outcomes, such as corporate social responsibility initiatives or actions with externalities on other divisions (see also Malenko (2016)). In this model, there is a separate signal $s = q + \eta$ on which contracts could be written. Thus, the precision of the signal s could be affected without changing output volatility, and so this model applies to improving monitoring technology or filtering out noise.[6] All results continue to hold (because of risk neutrality, changing the volatility of output has no effect), but the notation is more complex due to the introduction of an additional variable. Appendix B analyzes an alternative framework where both output q and a separate signal s are contractible, and changes in precision change the volatility of s but not q. In the core model, since the "signal" equals "output", we will use these terms interchangeably.

Second, in Innes (1990), the agent offers the contract and maximizes his utility subject to the principal's participation constraint. Since it is the principal who will typically invest in information, we model her as offering the contract so that she captures the surplus and thus the benefits from precision. Appendix F considers the agent offering the contract and choosing precision. Third, while Innes (1990) assumes continuous effort and the first-order approach ("FOA"), we do not impose any such structure on effort for our most general results.

2.1 The optimal contract

This section solves for the optimal contract holding precision θ fixed. The analysis is similar to Innes (1990). Our main results will come in Section 3, which analyzes the gains from increasing θ.

[6] In that model, there is a similar justification for the monotonicity constraint (3). Rather than borrowing on his own account to increase the signal, the agent could exert effort to manipulate the signal. If the marginal cost of increasing the signal by 1 is μ, the contract slope will equal μ (any greater slope will induce manipulation). Thus, all the results will hold except that the contract slope is now μ rather than 1.

Lemma 1 below establishes that the optimal payment to the agent $W_\theta(\cdot)$ is a call option on q with strike price X_θ. Alternatively, the optimal payment to the principal $R_\theta(\cdot)$ is risky debt with face value X_θ.

Lemma 1 *(Optimal contract) For each θ, there exists an optimal contract with*

$$W_\theta(q) = \max\{0, q - X_\theta\}, \quad (5)$$

$$R_\theta(q) = \min\{q, X_\theta\}, \quad (6)$$

for some X_θ.

As in Innes (1990), the intuition is as follows. The absolute value of the likelihood ratio is highest in the tails of the distribution of q, so output is most informative about effort in the tails. The principal cannot incentivize the agent in the left tail by giving negative wages (due to limited liability), and so she incentivizes him in the right tail by giving high wages. With an upper bound on the slope, the optimal contract involves call options on q with the maximum feasible slope, i.e., $\frac{\partial W_\theta}{\partial q}(q) = 1$. Since the agent's positive wage for high output cannot be offset by a negative wage for low output, his expected wage $\mathbb{E}_\theta[W_\theta(q)|e]$ strictly exceeds his reservation utility of zero, and so he enjoys rents.

Substituting the contract (5) into the incentive constraint (4), the effort that the agent chooses when offered a contract with strike price X_θ is given by

$$e_\theta(X_\theta) \in \arg\max_e \int_{X_\theta}^{\infty} (q - X_\theta) f_\theta(q|e)dq - C(e). \quad (7)$$

In general, $e_\theta(X_\theta)$ may not be single-valued. Whenever this is the case, we follow the standard approach of choosing the effort level preferred by the principal.

3 The value of information

This section calculates the value of information to the principal, by studying the effect of changes in signal precision on the principal's profits. Section 3.1 shows that the total effect of precision contains both a direct and incentive effect, and provides a condition that determines the sign of the incentive effect that holds for all output distributions. Section 3.2 shows that, for distributions with a location parameter, the sign of the incentive effect depends on the strike price of the option. Section 3.3 relates the initial strike price of the option – and thus the incentive effect and the value of information – to the underlying parameters of the agency problem.

3.1 The incentive effect

We initially assume that the effort function $e_\theta(X_\theta)$ is differentiable with respect to θ. This is for simplicity of exposition and transparency of intuition; we drop this assumption later.

Differentiating the principal's expected profits with respect to precision yields:

$$\overbrace{\frac{d}{d\theta}\mathbb{E}_\theta\left[R_\theta(q)|e_\theta(X_\theta)\right]}^{\text{Total Effect}} = \overbrace{\frac{\partial}{\partial\theta}\mathbb{E}_\theta\left[R_\theta\left(q\right)|e_\theta(X_\theta)\right]}^{\text{Direct Effect}} + \underbrace{\frac{d}{dX_\theta}\mathbb{E}_\theta\left[R_\theta\left(q\right)|e_\theta(X_\theta)\right]}_{\text{Zero by Envelope Theorem}}\tfrac{\partial X_\theta}{\partial\theta} + \underbrace{\frac{\partial}{\partial e}\mathbb{E}_\theta\left[R_\theta\left(q\right)|e_\theta(X_\theta)\right]\frac{\partial e_\theta(X_\theta)}{\partial\theta}}_{\text{Incentive Effect}} \tag{8}$$

The first term is the *direct effect.* Holding constant the strike price and effort level, an increase in output precision reduces the value of the agent's option W and increases the principal's expected profit R. Due to limited liability, the principal's profit, $\min\{q, X_\theta\}$, is concave in q; the agent's option, $\max\{0, q - X_\theta\}$, is convex in q. Since θ orders the distribution of q in terms of SOSD, the expected profit (wage) is increasing (decreasing) in θ. This reduction in pay is the benefit of precision highlighted by Bebchuk and Fried (2004) in their argument that the lack of RPE is inefficient. In the Holmstrom (1979) setting of a risk-averse agent, an increase in precision reduces the risk borne by the agent and thus allows the principal to lower the expected wage. In our setting of risk neutrality and limited liability, precision directly reduces the expected wage by lowering the value of the option.

The second term is the effect of re-optimizing the strike price on profits, which is zero by the envelope theorem – the principal had already optimized it for the initial level of precision. The third term is the *incentive effect*, which arises because the increase in precision affects the agent's incentive to exert effort. An increase in effort raises profits ($\frac{\partial}{\partial e}\mathbb{E}\left[R_\theta\left(q\right)|e_\theta(X_\theta)\right] > 0$), but whether precision increases or decreases effort is ambiguous. Our goal is to analyze the determinants of this effect and relate them to the underlying parameters of the agency problem.

Even when $\frac{\partial e_\theta(X_\theta)}{\partial\theta} < 0$ and the incentive effect counteracts the direct effect, it can never outweigh it. The total effect is always weakly negative from revealed preference: if reducing precision reduced the principal's profit, she would have added in randomness to the contract, and so the initial contract would not have been optimal.

For a given contract, if the agent's effort level increases (decreases) with precision, we say that the incentive effect is positive (negative) and that effort and precision are complements (substitutes). Differentiating the incentive constraint (4) with respect to e and θ, the incentive effect is positive if and only if

$$\frac{\partial^2}{\partial e \partial \theta} \mathbb{E}_\theta \left[W_\theta \left(q \right) | e \right] \geq 0. \tag{9}$$

and negative if the reverse inequality holds.

Integrating by parts, we can rewrite the agent's expected wage as follows:

$$\mathbb{E}[W_\theta \left(q \right) | e, \theta] = \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e) dq. \tag{10}$$

Precision does not affect the mean output $\mathbb{E}[q|e]$, and effort does not affect the strike price X_θ. Thus, only the cross-partial of the third term on the right-hand side ("RHS") of (10) is non-zero. Therefore, precision increases (decreases) effort, i.e., effort and precision are complements (substitutes), if

$$\frac{\partial^2}{\partial \theta \partial e} \int_{-\infty}^{X} F_\theta(q|e) dq \geq (\leq)\, 0 \ \forall e, X. \tag{11}$$

We now formalize the above argument and generalize it to cases where the effort function is not differentiable with respect to precision.[7] To allow for this case, and thus to accommodate situations where the FOA fails or the effort space is discrete, Definition 1 replaces the derivatives that determine the total and direct effects by their discrete counterparts. Let $\Pi(\theta) \equiv \mathbb{E}_\theta[R_\theta(q)|e_\theta(X_\theta)]$ denote the principal's expected profit when precision equals θ.

Definition 1 *The incentive effect of precision is positive at θ if*

$$\frac{\Pi(\theta') - \Pi(\theta)}{\theta' - \theta} \geq \frac{\mathbb{E}_{\theta'}[R_\theta(q)|e_\theta(X_\theta)] - \mathbb{E}_\theta[R_\theta(q)|e_\theta(X_\theta)]}{\theta' - \theta} \tag{12}$$

[7] Since the envelope theorem only holds at differentiability points of the value function, the decomposition (8) may not be well defined if the agent's effort $e_\theta \left(X_\theta \right)$ is not differentiable in θ.

for all $\theta' > \theta$ *in an open neighborhood of* θ*, and negative at* θ *if*

$$\frac{\Pi(\theta') - \Pi(\theta)}{\theta' - \theta} \leq \frac{\mathbb{E}_{\theta'}[R_\theta(q)|e_\theta(X_\theta)] - \mathbb{E}_\theta[R_\theta(q)|e_\theta(X_\theta)]}{\theta' - \theta} \tag{13}$$

for all $\theta' < \theta$ *in an open neighborhood of* θ*.*

The left-hand side ("LHS") of (12) and (13) corresponds to the total effect: the total change in profits when precision changes from θ to θ'. The RHS is the direct effect: the change in profits when when precision changes from θ to θ' but we hold the strike price and effort fixed. The incentive effect is the difference. Definition 1 is analogous to the decomposition in (8), except that it considers sub- and super-gradients to incorporate situations where $e_\theta(X_\theta)$ is not differentiable.

Proposition 1 generalizes the conditions in (11), for the incentive effect to be positive and negative, without assuming a differentiable effort function:

Proposition 1 *(Effect of precision on incentives) The incentive effect of precision is positive if, for all* $e_H > e_L$ *and all* X*,*

$$\frac{\partial}{\partial\theta}\int_{-\infty}^{X} [F_\theta(q|e_H) - F_\theta(q|e_L)]\, dq \geq 0. \tag{14}$$

The incentive effect of precision is negative if, for all $e_H > e_L$ *and all* X*,*

$$\frac{\partial}{\partial\theta}\int_{-\infty}^{X} [F_\theta(q|e_H) - F_\theta(q|e_L)]\, dq \leq 0. \tag{15}$$

Proposition 1 provides a sufficient condition for the incentive effect to be positive that is both general and simple.[8] The condition is general: it can be applied under any output distribution and any assumption about the effort space. In particular, we do not require the first-order approach to hold or the effort space to be an interval. The condition is also simple and thus easy to check: while the expected wage contains several terms (equation (10)), Proposition 1 requires us to check only one term. Appendix B provides an analogous condition for the case in which both output and a separate signal are contractible, and changes in precision affect the distribution of the signal rather than output.

[8]Formally, condition (14) states that $\int_{-\infty}^{X} F_\theta(q|e)$ is supermodular in (e, θ), whereas (15) states that it is submodular in (e, θ). As usual, supermodularity (submodularity) can be interpreted as e and θ being complements (substitutes).

Example 1 illustrates the results from Proposition 1 in a simple case where effort has a closed-form solution:

Example 1 *Suppose the cost of effort is quadratic $C(e) = \frac{e^2}{2}$ and that output belongs to a linear distribution family (Hart and Holmstrom, 1987):*

$$F_\theta(q|e) = eF_\theta^H(q) + (1-e)\,F_\theta^L(q), e \in [0,1],$$

where $F_\theta^H(q)$ and $F_\theta^L(q)$ are CDFs. In this case, we can write the agent's optimal effort in closed form,

$$e_\theta\,(X) = \mathbb{E}[q|e=1] - \mathbb{E}[q|e=0] + \int_{-\infty}^{X} \left[F_\theta^H(q) - F_\theta^L(q)\right] dq,$$

which allows us to obtain the conditions from Proposition 1 directly:

$$\frac{\partial e_\theta\,(X_\theta)}{\partial \theta} = \int_{-\infty}^{X_\theta} \frac{\partial}{\partial \theta} \left[F_\theta^H(q) - F_\theta^L(q)\right] dq.$$

3.2 The incentive effect and the strike price

Proposition 1 gives sufficient conditions for the incentive effect to be positive or negative regardless of the strike price of the option. An important limitation, however, is that for most distributions, $\frac{\partial}{\partial\theta}\int_{-\infty}^{X}\left[F_\theta(q|e_H) - F_\theta(q|e_L)\right] dq$ is positive for some values of X and negative for other values of X, and so neither (14) nor (15) holds. In these cases, the sign of the incentive effect will depend on the strike price. This section relates the incentive effect to the strike price for output distributions with a location parameter.

We henceforth decompose output as follows:

$$q = e + \varepsilon, \tag{16}$$

where ε is continuously distributed according to a PDF g_θ and CDF G_θ with full support on an interval of the real line. Equation (16) is without loss of generality, since we can always define "noise" ε as the difference between effort and output. In practice, noise can result from a market or industry shock, the contribution of other agents, or measurement error.

When output has a location parameter, the noise distribution g_θ is not a function of e: exerting effort shifts the distribution of output rightward without affecting its shape.

As a result, distributions with a location parameter are commonly used in agency settings; examples include the normal, uniform, logistic, and Laplace distributions. Then, $f_\theta(q|e) \equiv g_\theta(q-e)$ and $F_\theta(q|e) \equiv G_\theta(q-e)$ denote the PDF and CDF of output conditional on effort e. As before, we assume that effort increases output in the sense of MLRP and that θ orders the precision of output in terms of a MPS. For distributions with a location parameter, the former entails that, for any $e_H > e_L$ and any θ, $\frac{g_\theta(q-e_H)}{g_\theta(q-e_L)}$ is strictly increasing in q. The latter entails that the mean of ε is the same for all θ and that there exists $\widehat{\varepsilon}$ such that

$$\frac{\partial G_\theta}{\partial \theta}(\varepsilon)\begin{Bmatrix} \leq \\ \geq \end{Bmatrix} 0 \text{ for } \varepsilon \begin{Bmatrix} < \\ > \end{Bmatrix} \widehat{\varepsilon}. \tag{17}$$

For tractability, we assume that the FOA is valid: the effort space is an interval and that the agent's incentive constraint (7) can be replaced by its first-order condition:[9]

$$\frac{d}{de}\mathbb{E}_\theta\left[W_\theta\left(q\right)|e\right] = C'(e) \tag{18}$$

With the FOA, the agent's effort, implicitly defined by (18), is a differentiable function of θ, which simplifies the analysis of the impact of θ on effort.

Proposition 2 shows that the sign of the incentive effect depends on whether the strike price of the option is above or below a fixed threshold.

Proposition 2 *(Effect of information on the strike price) Suppose the FOA is valid. There exists $\widehat{X}$, independent of θ, such that:*

(i) if $X_\theta < \widehat{X}$, effort and precision are complements and the incentive effect of precision is positive;

(ii) if $X_\theta > \widehat{X}$, effort and precision are substitutes and the incentive effect of precision is negative.

The intuition is as follows. The agent's marginal benefit of effort is its positive effect on the value of his option. When output has a location parameter, an increase in effort by one unit shifts the distribution of output by $1. Since the agent receives a positive wage if and only if $q \geq X_\theta$, increasing effort only increases the wage if output

[9]To verify the validity of the FOA, it suffices to check that, at the strike price chosen by the principal under this relaxed program, the agent's payoff is a single-peaked function of effort. A sufficient condition for the validity of FOA in our setting is $\sup_\varepsilon\{g_\theta(\varepsilon)\} < \inf_e\{C''(e)\}$, which we will assume throughout this section.

ends up higher than the strike price – if output still ends up below the strike price, then he receives no return for his effort. Recall that increasing precision moves probability mass from the tails towards the center of the distribution. If X_θ is low, raising θ shifts mass from below to above X_θ, increasing the probability that output exceeds the strike price. On the other hand, if X_θ is high, raising θ shifts mass from above to below X_θ, reducing the probability that output exceeds the strike price.

Importantly, the threshold is independent of θ. The threshold will depend on the effort level that the principal wishes to implement; hypothetically, this effort level might depend on θ. However, effort is a smooth function of θ. Since the threshold separates the areas where effort is increasing and decreasing in θ, effort is constant in θ at the threshold, and so the threshold is independent of θ. This independence is important to allow us to sign the incentive effect as precision changes.[10]

Appendix C shows that similar results hold when the agent is risk-averse, and so the driver of our results is limited liability, rather than risk aversion. With limited liability, regardless of whether the agent is risk-neutral or risk-averse, the optimal contract pays zero when output is below a threshold (regardless of the level of output) and a positive amount, increasing in output, when output exceeds the threshold. (The only difference is that, with risk aversion, the contract may not be linear above the threshold, and so the contract may not be an option). Thus, regardless of the agent's utility function, he is only rewarded for marginal increases in output if output ends up greater than the threshold, as only then is his wage strictly increasing in output. As a result, precision affects incentives by affecting the likelihood that output exceeds the threshold.

3.3 The incentive effect and the cost of effort

Proposition 2 has shown that the sign of the incentive effect depends on the strike price X_θ. This result is particularly useful since it is often difficult to solve agency models, i.e., obtain the optimal contract and effort level as functions of the exogenous parameters. Proposition 2 allows us to understand the incentive effect – and in particular the intuition for why it depends on the strike price – without fully solving for these quantities.

One limitation is that, because the strike price X_θ is endogenous, Proposition 2 does

[10] For example, suppose precision increased and we observed that the strike price rose. If the original incentive effect were negative, we can conclude that it remains negative, but we would be unable to do so if the threshold depended on θ.

not relate the incentive effect to the underlying parameters of the agency problem. We now relate the incentive effect to the cost of effort. Before doing so, we note that the strike price X_θ reflects the strength of incentives: a lower X_θ corresponds to a higher option delta and thus stronger incentives. Generally, the link between the strength of incentives and the cost of effort depends on whether the implemented effort level is endogenous or fixed (see the survey of Edmans and Gabaix (2016), Section 3.3). When the implemented effort level is endogenous, a less convex cost of effort means that it is cheaper to provide incentives; as a result, the principal typically induces higher effort and raises incentives to do so (see, e.g., Holmstrom and Milgrom, 1987). When the implemented effort level is fixed, a lower cost of effort means that fewer incentives are needed to implement this effort level.

Both the endogenous and fixed effort level may be appropriate in different circumstances. For example, Edmans and Gabaix (2011) show that, if the agent is a CEO who has a multiplicative impact on firm value and the firm is large, the benefits of effort (increased firm value) swamp the costs (increased incentives) since firm value is much larger than the CEO's salary. In this case, the fixed effort level corresponds to full productive efficiency (taking all value-adding projects) and the principal wishes to implement it regardless of the cost of effort. Moreover, a fixed effort level arises in binary effort models, which are often used for tractability. We thus study both frameworks, starting with the endogenous-effort model that we have analyzed thus far.

3.3.1 Endogenous effort level

When effort is endogenous, the key feature of the cost of effort function is its convexity. This convexity parametrizes the severity of the agency problem: The higher the convexity, the more rapidly the marginal cost of effort rises with the level of effort, and so the greater the incentives the principal must provide to increase effort.

Let κ parametrize the convexity of the cost of effort, $\frac{\partial^3 C}{\partial e^2 \partial \kappa}(e;\kappa) > 0$. A typical example is the case of quadratic costs $C(e;\kappa) = \frac{\kappa e^2}{2}$, where $\frac{\partial^2 C}{\partial e^2}(e;\kappa) = \kappa$. We now index the strike price by $X_{\theta,\kappa}$ and the effort level by $e_{\theta,\kappa}$.

Proposition 3 shows that the incentive effect is positive if and only if convexity κ is below a threshold.

Proposition 3 *(Effect of cost function on incentive effect, endogenous effort) Suppose the FOA is valid. There exists $\bar{\kappa}$ such that the incentive effect is positive (negative) if*

$\kappa < (>) \bar{\kappa}$.

The intuition for Proposition 3 is as follows. Recall that $\widehat{\varepsilon}$ is the inflection point of $\frac{\partial G_\theta}{\partial \theta}(\varepsilon)$ defined in (17). If $X_{\theta,\kappa} - e_{\theta,\kappa} \leq \widehat{\varepsilon}$, precision increases incentives by moving mass from below the strike price to above it; if $X_{\theta,\kappa} - e_{\theta,\kappa} \geq \widehat{\varepsilon}$, precision reduces incentives by moving mass from above the strike price to below it. Thus, to determine the effect of convexity κ on effort incentives, we must determine how it affects the difference between the initially optimal strike price and the initially optimal effort: $X_{\theta,\kappa} - e_{\theta,\kappa}$.

An increase in convexity κ directly reduces the effort chosen by the agent $e_{\theta,\kappa}$. The effect on the optimal strike price $X_{\theta,\kappa}$ is ambiguous: since it is more costly to provide incentives when effort costs are more convex, the principal may reduce incentives by increasing the strike price, or she may compensate for the decrease in effort by providing higher incentives through a lower strike price. Even if the principal chooses to reduce the strike price, the increase in incentives is always outweighed by the direct reduction in effort from an increase in convexity – if it were not, the principal should have chosen a lower strike price before, when the incentive effect of doing so was higher since the cost of effort was less convex. The proof also shows that the fall in effort exceeds any fall in the strike price. Thus, when convexity is high, $X_{\theta,\kappa} - e_{\theta,\kappa} \geq \widehat{\varepsilon}$ and so precision reduces incentives.

Example 2 below illustrates this intuition in a simple model that admits a closed-form solution:

Example 2 *Suppose the cost function is quadratic* $C(e) = \frac{\kappa e^2}{2}$ *and that noise* ε *is uniformly distributed in* $[-\frac{1}{\theta}, \frac{1}{\theta}]$*, so that* $\widehat{\varepsilon} = 0$*. The first-order condition that determines the agent's effort choice yields:*

$$e = \frac{1 - \theta X}{2\kappa - \theta}, \tag{19}$$

and the second-order condition is satisfied as long as $\theta < 2\kappa$*. Suppose this inequality holds so that the FOA is valid. Substituting (19) into the principal's profit function and maximizing with respect to the strike price, we obtain the optimal strike price and effort:*

$$X^*_{\theta,\kappa} = \frac{1}{\theta} - \frac{1}{\kappa} + \frac{\theta}{2\kappa^2}, \quad e^*_{\theta,\kappa} = \frac{\theta}{2\kappa^2}. \tag{20}$$

Note that, while effort is always decreasing in κ*, the strike price is not monotonic: it decreases (increases) in* κ *if* $\kappa \leq (\geq) \theta$*. Nevertheless,* $X^*_{\theta,\kappa} - e^*_{\theta,\kappa} = \frac{1}{\theta} - \frac{1}{\kappa}$ *is increasing* *in κ. Setting $X^*_{\theta,\kappa} - e^*_{\theta,\kappa}$ equal to $\widehat{\varepsilon} = 0$, we find that, consistent with Proposition 3, the incentive effect is positive if $\kappa \leq \theta$ and negative if $\kappa \geq \theta$.*

3.3.2 Fixed effort level

We now relate the incentive effect to the cost of effort in the case of a fixed effort level. Formally, suppose the effort space is $\mathcal{E} = \{\underline{e}, \overline{e}\}$, where $e = \underline{e}$ ("shirking") costs zero and $e = \overline{e}$ ("working") costs $C > 0$. The principal wishes to implement $e = \overline{e}$.[11]

The incentive constraint is now

$$\mathbb{E}\left[W_\theta\left(q\right)|\overline{e}\right] - \mathbb{E}\left[W_\theta\left(q\right)|\underline{e}\right] \geq C, \tag{21}$$

and we refer to the LHS of (21) as effort incentives. Recall that, from Lemma 1, the optimal contract is an option.

Differentiating the principal's expected profits with respect to precision yields:

$$\frac{d}{d\theta}\mathbb{E}\left[R_\theta\left(q\right)|\overline{e}\right] = \underbrace{\frac{\partial}{\partial\theta}\mathbb{E}\left[R_\theta\left(q\right)|\overline{e}\right]}_{\textit{direct effect}} + \underbrace{\frac{d}{dX_\theta}\mathbb{E}\left[R_\theta\left(q\right)|\overline{e}\right]\frac{\partial X_\theta}{\partial\theta}}_{\textit{incentive effect}}.$$

The direct effect is unchanged, but the incentive effect is different from the case where effort is endogenous. Since effort is now fixed at $\bar{e}$, the principal instead responds to a change in precision by changing the strike price. Intuitively, when precision reduces effort incentives (i.e., it tightens the incentive constraint (21)), the principal must lower the strike price to increase the delta and restore incentives ($\frac{dX_\theta}{d\theta} \leq 0$). This lower strike price increases the expected wage and lowers the principal's profit ($\frac{d}{dX_\theta}\mathbb{E}\left[R_\theta\left(q\right)|\overline{e}\right] > 0$).[12] Conversely, when precision increases incentives (i.e., it relaxes the incentive constraint), the principal can increase the strike price without violating incentive compatibility ($\frac{dX_\theta}{d\theta} \geq 0$), thus reducing the expected wage. The incentive effect is thus now the effect of precision on profits through changes in the strike price, rather than changes in the effort level.

As in the general model of Section 3.1, the direct effect is always positive but the

[11] The principal wishes to implement high effort if $X_\theta - \int_{-\infty}^{X_\theta} F_\theta(q|\overline{e})dq \geq \mathbb{E}[q|\underline{e}]$, where the strike price X_θ is implicitly determined by the incentive constraint: $\mathbb{E}[q|\overline{e}] - \mathbb{E}[q|\underline{e}] + \int_{-\infty}^{X_\theta}\left[F_\theta(q|\overline{e}) - F_\theta(q|\underline{e})\right]dq = C$.

[12] With endogenous effort, $\frac{d}{dX_\theta}\mathbb{E}\left[R_\theta\left(q\right)|\overline{e}\right] = 0$: a higher strike price reduces the value of the option and increases the expected profit, but also reduces effort and reduces the expected profit. By the envelope theorem, these two effects cancel out. With fixed effort, the second effect does not exist.

incentive effect is ambiguous. We again relate it to the severity of the agency problem. With continuous effort, this severity was parametrized by the convexity of the cost function. With binary effort, this severity is captured by the cost of effort C. Lemma 2 below shows that the strike price falls with C. Intuitively, when effort is costly, a high option delta is needed to induce effort, which corresponds to a low strike price.

Lemma 2 *(Effect of effort cost on strike price) The strike price X_θ is strictly decreasing in the cost of effort C.*

Recall that Proposition 2 established that the incentive effect is positive if $X_\theta < \widehat{X}$ and negative if $X_\theta > \widehat{X}$ (while derived for the case of endogenous efforts, it is straightforward to verify the result for fixed efforts). Thus, the incentive effect is positive when the strike price is low, i.e., incentives are strong. In turn, Proposition 3 showed that, for an endogenous effort level, incentives are strong when the agency problem is mild (the cost of effort is less convex). Lemma 2 now shows that, for a fixed effort level, incentives are strong when the agency problem is severe (the cost of effort C is high). Thus, the incentive effect is positive when the agency problem is severe. Proposition 4 states this result formally.

Proposition 4 *(Effect of cost function on incentive effect, fixed effort) There exists a constant $\widehat{C}$ such that, if $C > (<) \widehat{C}$, the incentive effect of precision is positive (negative) and $\frac{dX_\theta}{d\theta} > (<)\, 0$.*

While Proposition 2 guarantees that a single cutoff $\widehat{X}$ separates the regions where the incentive effect is positive and negative (for continuous effort), we do not know in general where this cutoff lies. A benefit of the fixed effort model is that, since we know the implemented effort level, we can relate the cutoff to it. Indeed, Corollary 1 shows that, when the distribution is symmetric (as with the normal, uniform, logistic, and Laplace distributions), $\widehat{X}$ lies half-way between the expected output when the agent works $\overline{e}$ and the expected output when he shirks $\underline{e}$, i.e., $\widehat{X} = \frac{\overline{e}+\underline{e}}{2}$. Thus, we do not need to solve for the optimal strike price as a function of the exogenous parameters of the model and take derivatives with respect to θ to sign the incentive effect. It is sufficient to observe whether the strike price is above or below the threshold $\frac{\overline{e}+\underline{e}}{2}$: the strike price is a "sufficient statistic" for the direction of the incentive effect.

Corollary 1 *(Symmetric distributions) Suppose that G_θ is symmetric around zero. Then, the incentive effect is positive (negative) if $X_\theta < (>) \frac{\overline{e}+\underline{e}}{2}$.*

We now discuss the intuition using symmetric distributions in the location-scale family, that is, output distributions with a CDF that can be written as $F_\sigma(q|e) = G\left(\frac{q-e}{\sigma}\right)$. Examples include the normal, uniform, and logistic distributions. Such distributions clarify the intuition since precision θ can be parametrized by volatility $\sigma = \frac{1}{\theta}$, which allows us to discuss the intuition using the familiar concept of the option "vega": the sensitivity of its value to σ.

The agent's effort incentives stem from the difference in the value of two options. If he works, he receives an option-when-working worth $\mathbb{E}\left[W_\sigma(q)\,|\overline{e}\right]$; if he shirks, he receives an option-when-shirking worth $\mathbb{E}\left[W_\sigma(q)\,|\underline{e}\right]$. His effort incentives are given by the difference, i.e.,

$$\mathbb{E}\left[W_\sigma(q)\,|\overline{e}\right] - \mathbb{E}\left[W_\sigma(q)\,|\underline{e}\right]. \tag{22}$$

Thus, the incentive effect of increasing volatility σ (reducing precision θ) is given by

$$\frac{\partial}{\partial\sigma}\left\{\mathbb{E}_\sigma\left[W(q)\,|\overline{e}\right] - \mathbb{E}_\sigma\left[W(q)\,|\underline{e}\right]\right\}\Big|_{W(q)=W_\sigma(q)}. \tag{23}$$

An increase in volatility raises the value of each option, because the vega of an option is positive, but does so to different degrees since the options have different vegas. The incentive effect of increasing volatility is thus the vega of the option-when-working, $\frac{\partial}{\partial\sigma}\mathbb{E}_\sigma\left[W(q)\,|\overline{e}\right]$, minus that of the option-when-shirking, $\frac{\partial}{\partial\sigma}\mathbb{E}_\sigma\left[W(q)\,|\underline{e}\right]$.

The vega of an option is highest for an at-the-money option (see Claim 1 in Appendix D[13]), and declines when the option moves either in- or out-of-the-money. Thus, the vega of the option-when-working is highest at $X = \overline{e}$ and the vega of the option-when-shirking is highest at $X = \underline{e}$. If the initial strike price is $X_\sigma = \widehat{X} = \frac{\overline{e}+\underline{e}}{2}$, both options are out-of-the-money by $\frac{\overline{e}-\underline{e}}{2}$. They thus have the same vega (see Claim 2 in Appendix D), and so effort incentives are independent of σ. We thus have $\frac{dX_\sigma}{d\sigma} = 0$.

When $X_\sigma < \widehat{X}$, the option-when-shirking is closer to at-the-money, and so it has a higher vega. An increase in σ raises the value of the option-when-shirking more than the option-when-working and thus reduces effort incentives. Thus, the strike price must be lowered to restore effort incentives, and so $\frac{dX_\sigma}{d\sigma} < 0$. When $X_\sigma > \widehat{X}$, the option-when-working is closer to at-the-money than the option-when-shirking. An increase in σ raises effort incentives, and so $\frac{dX_\sigma}{d\sigma} > 0$.

[13] It is well-known that for lognormal distributions, the vega is highest for at-the-money options (as maturity approaches zero). Claim 1 extends this result to all distributions with a location and scale parameter.

Note that the LHS of (23) is related to the option's vanna.[14] The vanna of an option is given by $\frac{\partial^2 \mathbb{E}[W]}{\partial q \partial \sigma}$ – the cross-partial of its value with respect to both output q and volatility σ, or alternatively the derivative of its delta ($\frac{\partial \mathbb{E}[W]}{\partial q}$) with respect to volatility σ. When an option is in-the-money, its vanna is negative. Its delta – and thus the agent's effort incentives – decreases with volatility, and thus increase with precision: effort and precision are complements. In contrast, when an option is out-of-the-money, its vanna is positive, and so effort and precision are substitutes.

Graphical illustration We now demonstrate graphically the direct and incentive effects, to illustrate the importance of taking the incentive effect into account when calculating the value of information. We consider the common case of a normal distribution, which is symmetric. Figure 1 illustrates how the direct and incentive effects change with the severity of the moral hazard problem (parametrized by C). As is standard for graphs of option values, the figure contains the strike price X on the x-axis; since X is strictly decreasing in C (Lemma 2), there is a one-to-one mapping between X and C.

To understand Figure 1, the direct effect, $\frac{\partial \mathbb{E}[W_\sigma(q)|\bar{e}]}{\partial \sigma}$, is the vega of the option-when-working. It tends to zero as the strike price approaches either $-\infty$ or ∞, and is greatest when the option is at-the-money, i.e., $X = 1$. The incentive effect, $\frac{\partial \mathbb{E}[W_\sigma(q)|\bar{e}]}{\partial X_\sigma} \frac{dX_\sigma}{d\sigma}$, is positive for $X < \widehat{X}$ and thereafter negative; when X crosses above $\widehat{X}$ it becomes increasingly negative but then returns to zero. The total effect $\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}$ combines these effects. While the direct effect is initially increasing in X, this is outweighed by the fact that the incentive effect is initially decreasing in X. Thus, in Figure 1, the total gains from precision are monotonically decreasing in X.

An analysis focusing purely on the direct effect would suggest that the value of information is greatest when the initial option is at-the-money, which corresponds to a moderate strike price and a moderate cost of effort. In contrast, considering the total effect shows that, for a fixed effort level, the value of information is monotonically increasing in the severity of the agency problem. Appendix E shows analytically that this monotonic effect is general to the normal distribution, rather than applying only to the specific parameter values chosen in Figure 1.

[14]The difference is that the vanna is defined for local changes in output q, but equation (23) concerns potentially non-local changes in effort and thus output.

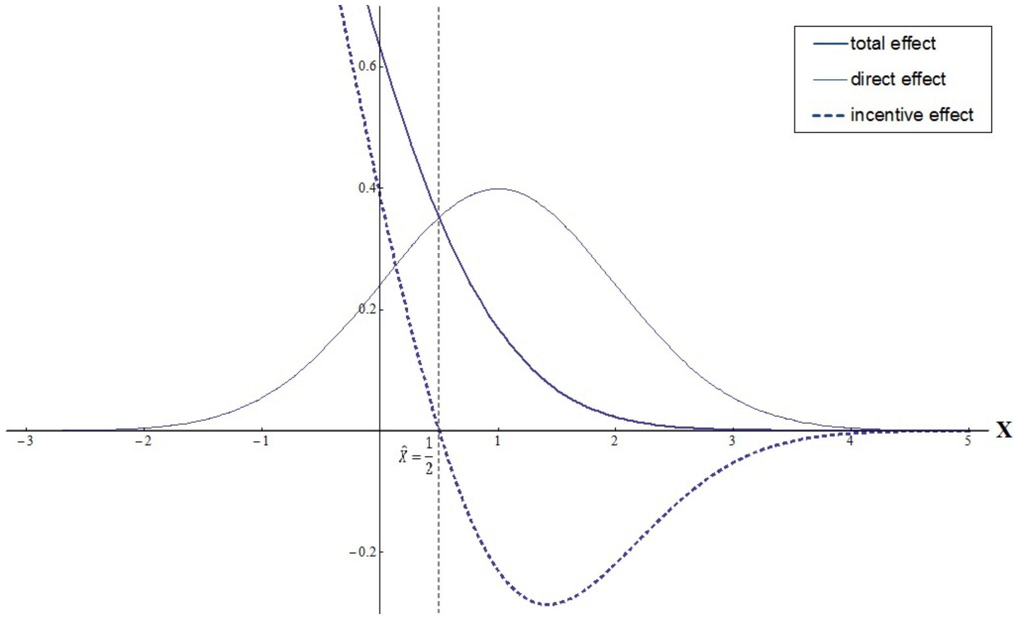

Figure 1: Total and partial derivative of expected pay with respect to σ for a range of values of X, for $\underline{e} = 0$, $\bar{e} = 1$, and $\sigma = 1$.

Ex ante and ex post incentives We end this section by contrasting the principal's ex ante and ex post incentives to invest in precision. Our analysis thus far assumes that the principal chooses precision ex ante, i.e., before the agent chooses effort, and can commit to this choice. She thus takes into account the effect of precision on both the value of the agent's option (the direct effect) and his effort (the incentive effect). We now consider the case in which the principal cannot commit to an initial level of precision, but instead chooses precision ex post, i.e., after the agent has exerted effort but before output is realized. Any level of precision announced before the agent has exerted effort is not credible as the agent will rationally anticipate that the principal will change precision to the level that maximizes her payoff ex post, and so any pre-announced level is irrelevant.

When precision is chosen ex post, the principal's marginal benefit of precision corresponds to the direct effect only, since the strike price and the agent's effort have already been chosen.[15] Thus, the difference between the principal's ex ante and ex post benefits from precision is given entirely by the incentive effect. If the incentive

[15]By backwards induction, the principal will select a strike price that induces the agent to work given the precision level that he will choose ex post.

effect is positive, the principal would choose a higher level of precision when the choice is made ex ante than ex post. In this case, the principal would like to commit to a high level of precision if such commitment is possible. Conversely, if the incentive effect is negative, the principal would like to commit to a low level of precision.

4 Discussion

4.1 Applications

This section discusses applications of our results, starting with compensation contracts. Most importantly, our results show that, when employers decide whether to increase the precision with which they monitor agent performance, they should consider the effect on the agent's incentives. This effect can be positive or negative, and so the total benefits of precision can be markedly higher or lower than an analysis focused on the direct effect alone. For all output distributions with a location parameter, and regardless of whether the effort level is fixed or endogenous, the incentive effect is positive when the strike price is low. In turn, this threshold is low when the effort level is fixed and agency problems are severe, or when the effort level is endogenous and agency problems are mild.

Thus, the model provides guidance on where the principal should invest in precision, within a firm. For example, if the fixed effort model applies, agents with high-powered incentives (such as CEOs) should be evaluated more precisely than those with low-powered incentives (such as rank-and-file managers). Relatedly, the model has implications for the optimality of RPE, which is costly as it involves forgoing the benefits of pay-for-luck documented by prior research (e.g., Oyer, 2004; Axelson and Baliga, 2009; Gopalan, Milbourn, and Song, 2010; Hoffman and Pfeil, 2010; Hartman-Glaser and Hébert, 2016). The results suggest that RPE need not be optimal, as it can reduce effort incentives. This effect is particularly likely where incentives are low-powered to begin with, consistent with RPE being even rarer for rank-and-file managers than for executives.

In addition, Proposition 2 suggests that exogenous changes in volatility (see Gormley, Matsa, and Milbourn (2013) and De Angelis, Grullon, and Michenaud (2017) for natural experiments) or stock market efficiency will have different effects on the incentives of agents depending on the moneyness of their outstanding options. This result applies both within a firm and across firms. In particular, increases in precision will lower (raise) the incentives of CEOs with out-of-the-money (in-the-money) options. Thus, where CEOs have out-of-the-money options, firms may wish to reduce the strike prices to restore incentives. Option repricing is documented empirically by Brenner, Sundaram, and Yermack (2000), although they do not study if it is prompted by falls in volatility.[16] Relatedly, Bebchuk and Fried (2004) advocate out-of-the-money options because they only reward a manager upon good performance. It is already known that a disadvantage of such options is that they provide weaker incentives compared to in-the-money options, due to their lower deltas; our model shows that this disadvantage is increasing in the level of precision.

A second application is to financing contracts, where the principal (investor) receives debt with face value of X_θ, and the entrepreneur (agent) holds equity – a call option on firm value with a strike price equal to the face value of debt. This application is relevant for both mature firms, and also young firms since they frequently raise debt and the entrepreneur holds levered equity, as shown by Robb and Robinson (2014).[17] Our results shed light on the investor's incentives to reduce output volatility via risk management. Such risk management has several interpretations: the investor can implement risk management herself since she retains control rights on output; she stipulates in the contract that the entrepreneur implement the above measures; or she has a menu of projects that she can finance and thus can choose project risk. Standard intuition would suggest that these incentives are increasing in the size of her debt claim, and thus her value-at-risk, but this intuition ignores the effect of risk management on effort. If the initial face value is low ($X_\theta < \widehat{X}$), a fall in output volatility raises effort incentives. This reinforces the direct effect of risk management, that it increases the value of the investor's risky debt due to its concave payoff. Thus, surprisingly, risk management may be more valuable for firms that are some distance from bankruptcy, and when the investor has little debt at stake. In contrast, if the initial face value is high ($X_\theta > \widehat{X}$), risk management reduces effort incentives, offsetting the direct effect.

[16] Acharya, John, and Sundaram (2000) also study the repricing of options theoretically, although in responses to changes in the mean rather than volatility of the signal.

[17] This is the original interpretation of the contract in Innes (1990). In his model, the agent has the bargaining power; under the financing interpretation of our model, the principal continues to hold the bargaining power so that she captures the surplus and so has incentives to invest in precision. (Appendix F analyzes the case in which the agent has bargaining power and chooses precision).

4.2 Binding participation constraint

In the core model, the agent's participation constraint is slack and he earns rents. The direct effect arises because information reduces the value of the option and thus the agent's rents. This section considers the case in which the agent's participation constraint binds and so the principal must restore the value of the option (by reducing the strike price) to maintain the agent's participation following an increase in precision. Even though the direct effect is fully offset by the reduction in the strike price, we will show that the total effect of information is typically non-zero.

Now assume that the agent's reservation utility is given by $\overline{U}$ rather than zero. As in the core model, the optimal contract is an option,[18] and so the agent's participation constraint is now given by

$$\int_{X_\theta}^{\infty} (q - X_\theta) f_\theta(q|e^*) dq \geq \overline{U} + C(e^*). \tag{24}$$

An increase in precision reduces the value of the agent's option on the LHS of the participation constraint (24). When the participation constraint is binding, the principal must reduce X_θ to maintain the agent's participation.[19] This reduction must fully offset the direct effect so that the participation constraint continues to hold with equality, i.e., $\frac{\partial X_\theta}{\partial \theta}$ must satisfy

$$\frac{d}{dX_\theta} \mathbb{E}_\theta \left[W_\theta \left(q \right) | e_\theta(X_\theta) \right] \frac{\partial X_\theta}{\partial \theta} = -\frac{\partial}{\partial \theta} \mathbb{E}_\theta \left[W_\theta \left(q \right) | e_\theta(X_\theta) \right],$$

where the RHS is the direct effect in (8). Indeed, if the effort level were fixed ($e^* = \overline{e}$), the value of information to the principal would be exactly zero. Total surplus is constant at $\mathbb{E}[q|\overline{e}]$, since effort is constant. With a binding participation constraint, the agent's

[18]The proof of Lemma 1 can be adapted by letting the initial non-option contract also satisfy the agent's participation constraint. Then the corresponding option contract constructed in the proof has the same expected value as the initial non-option contract, and it leaves both the principal's and agent's objective functions unchanged, since the effort level is the same. Because the agent chooses effort optimally and can achieve the same expected utility by choosing the same effort level under the new option contract as under the initial contract, he is better off under the option contract with the higher effort level than under the initial contract with the initial effort level. Thus, with a binding participation constraint, the initial non-option contract remains dominated by an option contract.

[19]Since the agent chooses effort optimally, we know from the envelope theorem that the total effect of a change in X_θ on the agent's expected utility net of effort cost is simply equal to the partial effect, holding effort constant.

utility is constant at $\overline{U} + C(\bar{e})$. Since the principal's profit equals total surplus minus the agent's utility, it would also be constant. Intuitively, when the agent's participation constraint binds, precision cannot be used to reduce the cost of compensation, and so has no value to the principal.

However, when the implemented effort level is endogenous, the value of information is typically non-zero. The fall in the strike price required to maintain the agent's participation increases the option's delta and thus effort incentives. We call this the "participation effect". The total value of information to the principal in (8) is now given by:

$$\underbrace{\frac{d}{d\theta}\mathbb{E}_\theta\left[R_\theta(q)|e_\theta(X_\theta)\right]}_{\text{Total Effect}} = \underbrace{\frac{\partial}{\partial e}\mathbb{E}_\theta\left[R_\theta\left(q\right)|e_\theta(X_\theta)\right]\frac{\partial e_\theta(X_\theta)}{\partial\theta}}_{\text{Incentive Effect}} + \underbrace{\frac{\partial}{\partial e}\mathbb{E}_\theta[R_\theta(q)|e_\theta(X_\theta)]\frac{\partial e_\theta(X_\theta)}{\partial X_\theta}\frac{\partial X_\theta}{\partial\theta}}_{\text{Participation Effect}}. \tag{25}$$

There is no longer a direct effect. When the agent's participation constraint is binding, the value of information to the principal stems entirely from its effect on effort – it cannot be used to reduce the agent's rents.

The "effort effect" $\frac{de}{d\theta}$ now has two components. These can be more clearly seen as follows:

$$\frac{de}{d\theta} = \frac{\partial e}{\partial\theta} + \frac{\partial e}{\partial X}\frac{\partial X}{\partial\theta}. \tag{26}$$

First, as in the core model, there is the "incentive effect" $\frac{\partial e}{\partial\theta}$, which can be positive or negative depending on whether effort and precision are complements or substitutes. Second, there is an additional "participation effect", which is strictly positive: since $\frac{\partial X}{\partial\theta} < 0$ and $\frac{\partial e}{\partial X} < 0$, $\frac{\partial e}{\partial X}\frac{\partial X}{\partial\theta} > 0$. Thus, if effort and precision are not substitutes, the incentive effect is non-negative and so precision increases effort due to the positive participation effect. Even if effort and precision are substitutes, effort can still rise, if the negativity of the incentive effect is outweighed by the positivity of the participation effect.

Note that a similar result holds if we instead assume Nash bargaining, where the agent captures a fraction α of the total surplus. The participation effect is replaced by a "bargaining effect": following an increase in precision, the strike price falls to maintain the agent's share of total surplus, which in turn increases effort incentives.

Appendix F studies the opposite case in which the principal's participation constraint binds. Specifically, the agent (entrepreneur) has full bargaining power and chooses precision. He raises an amount $I > 0$ from the principal (investor) to fund a project which produces output q, and so the principal's expected payoff must equal I. There is a similar participation effect to this section, and the value of information again stems entirely from its effect on effort, which remains (26).

In addition to demonstrating robustness, the analysis delivers two additional results. First, the value of precision depends on the entrepreneur's initial wealth. The amount I can be thought of as the amount of new financing that the entrepreneur must raise, net of his initial wealth. Thus, if wealth is low, I is high, which leads to a high face value of debt. It is already known that this reduces effort incentives, since part of the gains from effort go to the investor. Appendix F shows that increases in precision may further exacerbate the incentive problems originating from low initial wealth, since the incentive effect is then negative. In contrast, firms with abundant internal wealth have high incentives to increase precision. Second, the level of precision will be different than when it is chosen by the principal. The party that chooses precision internalizes the change in effort triggered by an increase in precision only to the extent that it affects him/her, depending on the initial contract. For example, with a low face value of debt X_θ, the principal has close-to-safe debt and so benefits little from changes in effort, while the agent is close to the residual claimant. Thus, the agent is affected more by the effort effect (26) than the principal. As a result, if the effort effect is positive, he will choose a higher level of precision. Due to these differential benefits from precision, control rights matter – the chosen level of information depends on which party has bargaining power and chooses precision.

4.3 Social welfare and regulation

Appendix G analyzes the effect of precision on social welfare and, in particular, whether the principal overinvests or underinvests relative to the social optimum. Total surplus depends only on effort; the wage is a pure transfer from the principal to the agent with no effect on total surplus. Since the principal does not take the agent's utility into account when choosing the level of precision, she typically does not choose the socially optimal level. This has implications for the optimality of regulation.

For example, when effort is fixed, total surplus is independent of precision. Nevertheless, the principal has an incentive to increase precision to reduce the agent's wage, and so she always overinvests in precision compared to the social optimum. This result is interesting since most regulation increases the precision of performance metrics, such as reporting and disclosure requirements or mandatory audits. When effort is endogenous, the principal overinvests in precision relative to the social optimum whenever precision reduces the agent's utility and underinvests in precision whenever it increases the agent's utility.[20]

5 Conclusion

This paper uses an optimal contracting model to study the value of information – a more precise signal of agent performance – to the principal. We show that increasing signal precision has two effects on the principal's profit. The first is the direct effect: reducing signal volatility lowers the value of the agent's option, and unambiguously increases profit. This is the standard effect of precision considered by arguments that information is valuable to the principal. Our paper focuses on a second, indirect effect: reducing signal volatility changes the agent's effort incentives. Crucially, this effect may be negative and harm the principal – effort and precision may be substitutes – offsetting the benefits of the direct effect.

We derive a general condition that determines whether effort and precision are complements or substitutes, and thus the sign of the incentive effect, that holds for any output distribution and does not assume the first-order approach. When the output distribution has a location parameter, and the first-order approach is valid, we show that this condition is satisfied – precision increases incentives – if and only if the strike price of the option is low. We then relate the strike price of the option – and thus the incentive effect of effort – to the underlying parameters of the agency problem, to determine the conditions under which information is most valuable to the principal. When the effort level is endogenous, the principal will choose a low effort level and thus a high strike price (relative to this effort level) when the cost of effort is sufficiently convex – i.e., the agency problem is severe – as then incentive provision is difficult. When the effort level is fixed, as may be the case for CEOs of large firms, the strike price is high when the cost of effort is low – i.e., the agency problem is mild – as then weak incentives are needed to implement the fixed effort level.

In a compensation setting, our results have implications for the situations in which

[20] Unlike in the fixed effort case, it is possible for an increase in precision to augment the agent's utility. This may occur if it optimal for the principal to significantly reduce the strike price to implement a higher effort level.

information on agent performance is most valuable, and how firms should recontract in response to changes in the signal precision. In a financing setting, they have implications for the value of risk management. In particular, the incentives to manage risk may, surprisingly, be high when the face value of debt is low. Even though the investor has little skin in the game, risk management is especially valuable due to its positive effect on the agent's incentives.

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A Proofs

Proof of Lemma 1

The proof adopts Lemma 1 from Matthews (2001) to a setting with a continuum of signals and general supports. Let (W^*, e^*) be a feasible contract and consider the option contract $W^O = \max\{0, q - X\}$ where the strike price X is chosen so that both contracts have the same expected payment under effort e^*:

$$\int_{-\infty}^{\infty} W^*(q) f(q|e^*) dq = \int_{-\infty}^{\infty} W^O(q) f(q|e^*) dq. \tag{27}$$

It is straightforward to show that the contract W^O exists and is unique. We will show that replacing W^* by W^O increases effort and raises the principal's expected profit.

Let e^O be an optimal effort for the agent when he is offered the option contract:

$$e^O \in \arg\max_{e \in E} \int_{-\infty}^{\infty} W^O(q) f(q|e) dq - C(e).$$

Since the agent chooses e^* when offered W^* and e^O when offered W^O, we must have:

$$\int_{-\infty}^{\infty} W^O(q) f(q|e^O) dq - C(e^O) \geq \int_{-\infty}^{\infty} W^O(q) f(q|e^*) dq - C(e^*),$$

$$\int_{-\infty}^{\infty} W^*(q) f(q|e^*) dq - C(e^*) \geq \int_{-\infty}^{\infty} W^*(q) f(q|e^O) dq - C(e^O).$$

Combining these two inequalities, we obtain

$$\int_{-\infty}^{\infty} \left[W^O(q) - W^*(q)\right] \left[f(q|e^O) - f(q|e^*)\right] dq \geq 0. \tag{28}$$

Since both contracts have the same expected value under effort e^* and the option contract pays the lowest feasible amount for $q < X$ and has the highest possible slope for $q > X$, there exists $\bar{q} \geq X$ such that

$$W^O(q) \left\{ \begin{array}{c} \leq \\ \geq \end{array} \right\} W^*(q) \text{ for all } q \left\{ \begin{array}{c} \leq \\ \geq \end{array} \right\} \bar{q}. \tag{29}$$

We will now show by contradiction that $e^* \leq e^O$. Suppose that $e^* > e^O$. Then:

$$\begin{aligned}
0 \; & \leq \int_{-\infty}^{\infty} \left[W^O(q) - W^*(q)\right] \left[\frac{f(q|e^O)}{f(q|e^*)} - 1\right] f(q|e^*)dq \\
& = \int_{-\infty}^{\infty} \left[W^O(q) - W^*(q)\right] \frac{f(q|e^O)}{f(q|e^*)} f(q|e^*)dq - \underbrace{\int_{-\infty}^{\infty} \left[W^O(q) - W^*(q)\right] f(q|e^*)dq}_{0} \\
& = \int_{-\infty}^{\bar{q}} \left[W^O(q) - W^*(q)\right] \frac{f(q|e^O)}{f(q|e^*)} f(q|e^*)dq + \int_{\bar{q}}^{\infty} \left[W^O(q) - W^*(q)\right] \frac{f(q|e^O)}{f(q|e^*)} f(q|e^*)dq \\
& < \int_{-\infty}^{\bar{q}} \left[W^O(q) - W^*(q)\right] \frac{f(\bar{q}|e^O)}{f(\bar{q}|e^*)} f(q|e^*)dq + \int_{\bar{q}}^{\infty} \left[W^O(q) - W^*(q)\right] \frac{f(\bar{q}|e^O)}{f(\bar{q}|e^*)} f(q|e^*)dq \\
& = \frac{f(\bar{q}|e^O)}{f(\bar{q}|e^*)} \int_{-\infty}^{\infty} \left[W^O(q) - W^*(q)\right] f(q|e^*)dq = 0
\end{aligned}$$

where the first line divides and multiplies the expression inside the integral in (28) by $f(q|e^*)$, the second line adds a term that equals zero (from (27)), the third line splits the integral between outputs lower and higher than $\bar{q}$, and the fourth line uses MLRP and (29). These inequalities give us a contradiction ($0 < 0$), showing that $e^* \leq e^O$.

To conclude the proof, we need to show that the principal's profits from offering the option contract (W^O, e^O) are higher than with the original contract (W^*, e^*):

$$\int \left[q - W^O(q)\right] f\left(q|e^O\right) dq \geq \int \left[q - W^*(q)\right] f(q|e^*)\, dq.$$

Subtracting $\int \left[q - W^O(q)\right] f(q|e^*)\, dq$ from both sides, gives:

$$\int \left[q - W^O(q)\right] \left[f\left(q|e^O\right) - f(q|e^*)\right] dq \geq \int \left[W^O(q) - W^*(q)\right] f(q|e^*)\, dq = 0,$$

where the expression on the RHS equals zero by (27). Rearranging this expression, it follows that the principal profits from the replacement if

$$\int \left[q - W^O(q)\right] f\left(q|e^O\right) dq \geq \int \left[q - W^O(q)\right] f(q|e^*)\, dq,$$

which is true because $q - W^O(q) = \min\{q, X\}$ is an increasing function of q and $f(q|e^O)$ first-order stochastically dominates $f(q|e^*)$ ($e^O \geq e^*$ and FOSD is implied by MLRP).

Proof of Proposition 1

We start with two auxiliary lemmas that will be useful for the main proof. Lemmas 3 and 4 show that the incentive effect is positive if $e_\theta(X_\theta)$ is increasing in θ and negative if $e_\theta(X_\theta)$ is decreasing in θ. The principal's program is:

$$V(\theta) \equiv \max_{X_\theta} R(X_\theta, \theta), \tag{30}$$

where $R(X_\theta, \theta) \equiv X_\theta - \int_{-\infty}^{X_\theta} F_\theta(q|e_\theta(X_\theta))dq$. Let $X_\theta^* \in \arg\max_{X_\theta} R(X_\theta, \theta)$.

Lemma 3 *Suppose $e_\theta(X)$ is a non-decreasing function of θ at $X = X_\theta^*$ in an interval $[\theta, \theta + \Delta)$. Then,*

$$\frac{V(\theta') - V(\theta)}{\theta' - \theta} \geq -\frac{\int_{-\infty}^{X_\theta^*} \left[F_{\theta'}(q|e_{\theta'}(X_\theta^*)) - F_\theta(q|e_{\theta'}(X_\theta^*))\right] dq}{\theta' - \theta} \tag{31}$$

for all $\theta' \in [\theta, \theta + \Delta)$. Moreover, if $V(\theta)$ is right-hand differentiable, then $V'(\theta+) \geq -\int_{-\infty}^{X_\theta^*} \frac{\partial F_\theta}{\partial \theta}(q|e_\theta(X_\theta^*))dq$.

Proof. Since X_θ^* is solution of program (30), $V(\theta') \geq R(X_\theta^*, \theta')$ and $V(\theta) = R(X_\theta^*, \theta)$. Therefore, for any $\theta' > \theta$,

$$\frac{V(\theta') - V(\theta)}{\theta' - \theta} \geq \frac{R(X_\theta^*, \theta') - R(X_\theta^*, \theta)}{\theta' - \theta}. \tag{32}$$

Substituting the expression for R, we have

$$\begin{aligned} R(X_\theta^*, \theta') - R(X_\theta^*, \theta) &= \int_{-\infty}^{X_\theta^*} \left[F_\theta(q|e_\theta(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*))\right] dq \\ &= \int_{-\infty}^{X_\theta^*} \left[\underbrace{F_\theta(q|e_\theta(X_\theta^*)) - F_\theta(q|e_{\theta'}(X_\theta^*))}_{+} + F_\theta(q|e_{\theta'}(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*)) \right] dq \\ &\geq -\int_{-\infty}^{X_\theta^*} \left[F_{\theta'}(q|e_{\theta'}(X_\theta^*)) - F_\theta(q|e_{\theta'}(X_\theta^*))\right] dq, \end{aligned}$$

where the inequality used the fact that $F_\theta(q|e)$ is decreasing in e (FOSD) and $e_\theta(X_\theta^*) \leq e_{\theta'}(X_\theta^*)$. Substituting back in (32), establishes (31). For the second claim, take the limit as $\theta' \searrow \theta$. ■

Lemma 4 *Suppose $e_\theta(X)$ is a non-increasing function of θ at $X = X_\theta^*$ in an interval $(\theta - \Delta, \theta]$. Then,*

$$\frac{V(\theta') - V(\theta)}{\theta' - \theta} \leq -\frac{\int_{-\infty}^{X_\theta^*} \left[F_\theta(q|e_{\theta'}(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*))\right] dq}{\theta' - \theta} \tag{33}$$

for all $\theta' \in (\theta - \Delta, \theta]$. *Moreover, if* $V(\theta)$ *is left-hand differentiable, then* $V'(\theta-) \leq -\int_{-\infty}^{X_\theta^*} \frac{\partial F_\theta}{\partial \theta}(q|e_\theta(X_\theta^*))dq$.

Proof. Since X_θ^* solves the maximization program in (30), $V(\theta') \geq R(X_\theta^*, \theta')$ and $V(\theta) = R(X_\theta^*, \theta)$. Therefore, for any $\theta' < \theta$,

$$\frac{V(\theta') - V(\theta)}{\theta' - \theta} \leq \frac{R(X_\theta^*, \theta') - R(X_\theta^*, \theta)}{\theta' - \theta}. \tag{34}$$

From the definition of R, we have

$$\begin{aligned} R(X_\theta^*, \theta') - R(X_\theta^*, \theta) &= \int_{-\infty}^{X_\theta^*} \left[F_\theta(q|e_\theta(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*))\right] dq \\ &= \int_{-\infty}^{X_\theta^*} \left[\underbrace{F_\theta(q|e_\theta(X_\theta^*)) - F_\theta(q|e_{\theta'}(X_\theta^*))}_{+} + F_\theta(q|e_{\theta'}(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*))\right] dq \ , \\ &\geq -\int_{-\infty}^{X_\theta^*} \left[F_\theta(q|e_{\theta'}(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*))\right] dq \end{aligned}$$

where we used FOSD and $e_{\theta'}(X) \geq e_\theta(X_\theta)$ to establish that $F_\theta(q|e_\theta(X_\theta^*)) \geq F_\theta(q|e_{\theta'}(X_\theta^*))$. Substituting in (34), establishes (33). The second claim takes the limit as $\theta' \nearrow \theta$. ∎

The full proof of Proposition 1 now follows. The agent's best-response correspondence is

$$e_\theta(X_\theta) \equiv \arg\max_e \int_{X_\theta}^{\infty} (q - X_\theta) f_\theta(q|e)dq - C(e). \tag{35}$$

By the maximum theorem, $e_\theta(X_\theta)$ is non-empty and compact-valued. Recall that the agent chooses the effort preferred by the principal whenever $e_\theta(X_\theta)$ is not single-valued. We now show that the principal will strictly prefer the highest one. To see this, recall from (10) that expected wage can be written:

$$\mathbb{E}[W_\theta(q)|e, \theta] = \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e)dq. \tag{36}$$

Suppose the agent is indifferent between efforts $e_H > e_L$. Then,

$$\mathbb{E}[q|e_H] - \mathbb{E}[q|e_L] = \int_{-\infty}^{X_\theta} \left[F_\theta(q|e_L) - F_\theta(q|e_H)\right] dq + C(e_H) - C(e_L) > 0,$$

where the inequality follows from $e_H > e_L$, $C' > 0$, and FOSD. Using this inequality, we obtain

$$\mathbb{E}_\theta[R_\theta(q)|e_H] = X_\theta - \int_{-\infty}^{X_\theta} F_\theta(q|e_H)dq > X_\theta - \int_{-\infty}^{X_\theta} F_\theta(q|e_L)dq = \mathbb{E}_\theta[R_\theta(q)|e_L],$$

showing that the principal has a higher profit under e_H than under e_L.

Using the expression for the agent's wage from (36), the agent's effort is the largest element of

$$\arg\max_e \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e)dq - C(e).$$

The result then follows from Topkis's theorem, since the objective function satisfies increasing (decreasing) differences if $\frac{\partial^2}{\partial\theta\partial e}\left[\int_{-\infty}^{X} F_\theta(q|e)dq\right] \geq (\leq) 0$ for all e, X.

Proof of Proposition 2

With a location parameter, $F_\theta(q|e) \equiv G_\theta(\varepsilon)$. Plugging this into (10) yields

$$\mathbb{E}_\theta[W_\theta(q)|e] = \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta - e} G_\theta(\varepsilon)d\varepsilon. \tag{37}$$

Maximizing with respect to effort, we can rewrite the first- and second-order conditions as:

$$1 - G_\theta(X - e_\theta(X)) = C'(e_\theta(X)), \tag{38}$$

$$g_\theta(X - e_\theta(X)) - C''(e_\theta(X)) < 0. \tag{39}$$

Applying the implicit function theorem to (38), it follows that $\frac{\partial}{\partial X}e_\theta(X) \leq 0$, and, by MPS ($\frac{\partial G_\theta}{\partial\theta}(\varepsilon) \leq (\geq) 0$ for $\varepsilon < (>) \hat{\varepsilon}$), $\frac{\partial}{\partial\theta}e_\theta(X) \geq (\leq) 0$ if $X - e_\theta(X) < (>) \widehat{\varepsilon}$. Since $\lim_{X\searrow-\infty} X - e_\theta(X) = -\infty$, $\lim_{X\nearrow+\infty} X - e_\theta(X) = +\infty$, and $X - e_\theta(X)$ is a strictly increasing and continuous function of X, if follows that there exists a unique $\widehat{X}_\theta$ that solves

$$\widehat{X}_\theta - e_\theta(\widehat{X}_\theta) = \widehat{\varepsilon}. \tag{40}$$

Totally differentiating (40), gives:

$$\frac{d\widehat{X}_\theta}{d\theta}\left[1 - \frac{\partial e_\theta}{\partial X}(\widehat{X}_\theta)\right] = \frac{\partial e_\theta}{\partial\theta}(\widehat{X}_\theta).$$

Since $\frac{\partial e_\theta}{\partial X}(\widehat{X}_\theta) \leq 0$ and $\frac{\partial e_\theta}{\partial\theta}(\widehat{X}_\theta) = 0$, it follows that $\frac{d\widehat{X}_\theta}{d\theta} = 0$, and so $\widehat{X}_\theta$ is constant in θ.

Proof of Proposition 3

The agent's expected payoff equals:

$$\mathbb{E}[W(q)|e,\theta] - C(e;\kappa) = e - X + \int_{-\infty}^{X-e} G_\theta(\varepsilon)d\varepsilon - C(e;\kappa),$$

so the agent's optimal effort is given by the following first-order condition:

$$1 - G_\theta(X-e) - \frac{\partial C}{\partial e}(e;\kappa) = 0.$$

The principal's profits equal

$$X - \int_{-\infty}^{X-e_{\theta,\kappa}(X)} G_\theta(\epsilon)d\epsilon.$$

We introduce the "distance from $\widehat{\varepsilon}$" variable $Z \equiv X - e$. As seen in the proof of Proposition 2, for the location family, $\frac{\partial e_\theta}{\partial \theta} \geq (\leq) 0$ if $Z \leq (\geq) \widehat{\varepsilon}$. It suffices to show that the distance from $\widehat{\varepsilon}$ chosen by the principal, $Z^*_{\kappa,\theta} = X^*_{\kappa,\theta} - e^*_{\kappa,\theta}$, is increasing in κ.

We can rewrite the principal's profits in terms of (Z, e) instead of (X, e), i.e.,

$$\Pi(Z;\kappa) \equiv Z + e^*_{\theta,\kappa}(Z) - \int_{-\infty}^{Z} G_\theta(\epsilon)d\epsilon$$

where $e^*_{\theta,\kappa}(Z)$ is implicitly determined by:

$$1 - G_\theta(Z) - \frac{\partial C}{\partial e}(e;\kappa) = 0.$$

Total differentiation of the previous equality establishes that:

$$\frac{\partial e^*_{\theta,\kappa}(Z)}{\partial Z} = -\frac{g_\theta(Z)}{\frac{\partial^2 C}{\partial e^2}(e;\kappa)} < 0, \quad \frac{\partial e^*_{\theta,\kappa}(Z)}{\partial \kappa} = \frac{-\frac{\partial^2 C}{\partial e \partial \kappa}(e;\kappa)}{\frac{\partial^2 C}{\partial e^2}(e;\kappa)} > 0.$$

Note that the principal's profit $\Pi(Z,\kappa)$ has increasing differences, since

$$\frac{\partial^2 \Pi}{\partial Z \partial \kappa}(Z;\kappa) = \frac{g_\theta(Z)}{\left[\frac{\partial^2 C}{\partial e^2}(e;\kappa)\right]^2} \frac{\partial^3 C}{\partial e^2 \partial \kappa}(e;\kappa) > 0.$$

By Topkis's theorem, the solution $Z^*_{\kappa,\theta} = X^*_{\kappa,\theta} - e^*_{\kappa,\theta}$ is increasing in κ.

Proof of Lemma 2

The agent works if the incentive constraint holds:

$$\mathbb{E}[W_\theta(q)\,|\overline{e}] - \mathbb{E}[W_\theta(q)\,|\underline{e}] \geq C. \tag{41}$$

We define X_θ implicitly by the binding incentive constraint:

$$\int_{X_\theta}^{\infty} (q - X_\theta)\left[f_\theta(q|\overline{e}) - f_\theta(q|\underline{e})\right] dq = C. \tag{42}$$

It is straightforward to show that X_θ exists and is unique. By the implicit function theorem,

$$\frac{dX_\theta}{dC} = \frac{1}{F_\theta(X_\theta|\overline{e}) - F_\theta(X_\theta|\underline{e})} < 0.$$

Proof of Proposition 4

As discussed in the main text, the only term in the decomposition of the expected wage (10) with a non-zero cross-partial is $\int_{-\infty}^{X_\theta} F_\theta(q|e)dq$. Using the incentive constraint in (21), precision and incentives are complements (substitutes) if and only if

$$\frac{\partial}{\partial\theta}\int_{-\infty}^{X_\theta}\left[F_\theta(q|\overline{e}) - F_\theta(q|\underline{e})\right] dq > (<) 0. \tag{43}$$

Since $F_\theta(q|e) = G_\theta(q-e)$, we have

$$\begin{aligned}\frac{\partial}{\partial\theta}\int_{-\infty}^{X_\theta}\left[F_\theta(q|\overline{e}) - F_\theta(q|\underline{e})\right] dq &= \frac{\partial}{\partial\theta}\left\{\int_{-\infty}^{X_\theta-\overline{e}} G_\theta(\varepsilon)\,d\varepsilon - \int_{-\infty}^{X_\theta-\underline{e}} G_\theta(\varepsilon)\,d\varepsilon\right\} \\ &= -\int_{X_\theta-\overline{e}}^{X_\theta-\underline{e}} \frac{\partial G_\theta}{\partial\theta}(\varepsilon)\,d\varepsilon. \end{aligned} \tag{44}$$

Therefore, precision and incentives are complements (substitutes) if and only if the expression in (44) is positive (negative). In addition, using (10) we get

$$\frac{\partial}{\partial X_\theta}\left\{\mathbb{E}\left[W_\theta(q)|\overline{e}\right] - \mathbb{E}\left[W_\theta(q)|\underline{e}\right]\right\} = \frac{\partial}{\partial X_\theta}\int_{-\infty}^{X_\theta}\left[F_\theta(q|\overline{e}) - F_\theta(q|\underline{e})\right] dq = F_\theta(X_\theta|\overline{e}) - F_\theta(X_\theta|\underline{e}),$$

which is negative for any X_θ by FOSD, so that incentives are decreasing in the strike price. In sum, if precision and incentives are complements (substitutes), then X_θ as defined in equation (42) must be increased (decreased) following a rise in θ, i.e., $\frac{dX_\theta}{d\theta} \geq (\leq) 0$.

From the definitions of a MPS in (1) and of G_θ, we know that $\frac{\partial G_\theta}{\partial \theta}$ alternates signs only once, and $\frac{\partial G_\theta}{\partial \theta} \leq (\geq) \ 0$ for q small (large) enough. Therefore, there exists $\widehat{X}$ such that $-\int_{X_\theta-\overline{e}}^{X_\theta-\underline{e}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon)\, d\varepsilon$ is nonnegative for $X_\theta < \widehat{X}$, and nonpositive for $X_\theta > \widehat{X}$. In sum, precision increases incentives and $\frac{dX_\theta}{d\theta} \geq 0$ if $X_\theta < \widehat{X}$, while precision decreases incentives and $\frac{dX_\theta}{d\theta} \leq 0$ if $X_\theta > \widehat{X}$. Finally, we know from Lemma 2 that the initial strike price X_θ is decreasing in C, which completes the proof.

Proof of Corollary 1

Recall that the strike price X_θ is implicitly defined as the solution to

$$\overline{e} - \underline{e} - \int_{X-\overline{e}}^{X-\underline{e}} G_\theta(\varepsilon) d\varepsilon = C.$$

By the implicit function theorem,

$$\frac{dX_\theta}{d\theta} = \frac{\int_{X_\theta-\overline{e}}^{X_\theta-\underline{e}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon) d\varepsilon}{G_\theta(X_\theta - \overline{e}) - G_\theta(X_\theta - \underline{e})}. \tag{45}$$

The denominator is negative; so $\frac{dX_\theta}{d\theta} \geq (\leq) 0$ if $\int_{X_\theta-\overline{e}}^{X_\theta-\underline{e}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon)\, d\varepsilon \leq (\geq)\ 0$.

Since G_θ is symmetric for any θ, $G_\theta(\varepsilon) = 1 - G_\theta(-\varepsilon)$ for any θ, ε. In particular,

$$\frac{\partial G_\theta}{\partial \theta}(\varepsilon) = -\frac{\partial G_\theta}{\partial \theta}(-\varepsilon).$$

so, by MPS, $\frac{\partial G_\theta}{\partial \theta}(x) \geq 0 \ \Leftrightarrow \ x \geq 0$.

Evaluating at $X_\theta = \frac{\overline{e}+\underline{e}}{2}$, gives

$$\int_{-\frac{\overline{e}-\underline{e}}{2}}^{\frac{\overline{e}-\underline{e}}{2}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon) d\varepsilon = 0.$$

Note that $\frac{dX_\theta}{d\theta} \leq 0$ whenever $X > \overline{e}$, since, in that case, $\frac{\partial G_\theta}{\partial \theta}(\varepsilon) \geq 0$ for all $\varepsilon \in (X - \overline{e},\ X - \underline{e})$. Similarly, $\frac{\partial G_\theta}{\partial \theta}(\varepsilon) \leq 0$ whenever $X < \underline{e}$. Finally, for $\underline{e} \leq X \leq \overline{e}$, we have

$$\frac{d}{dX}\left[\int_{X-\overline{e}}^{X-\underline{e}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon) d\varepsilon\right] = \frac{\partial G_\theta}{\partial \theta}(X - \underline{e}) - \frac{\partial G_\theta}{\partial \theta}(X - \overline{e}) \geq 0,$$

where the inequality follows from the fact that $X-\underline{e} \geq 0 \geq X-\overline{e}$ implies $\frac{\partial G_\theta}{\partial \theta}(X-\underline{e}) \geq 0$ and $\frac{\partial G_\theta}{\partial \theta}(X - \overline{e}) \leq 0$. Therefore, the numerator in (45) is increasing in X.

Supplementary Appendix for "Does Improved Information Improve Incentives?"

B Separate signal and output

In the core model, output q is the only contractible variable. As discussed in Section 2, the model remains essentially unchanged if output were unobservable and contracts were instead based on a separate signal s. This section considers an alternative model in which both output q and a separate signal s are contractible, and changes in precision affect the distribution of s rather than output q. This setting allows the model to apply to improvements in monitoring technology when the agent's output is also contractible.

In our core model, there was a single contractible signal (output) and so we could consider general signal distributions. With two contractible signals, tractability requires us to specialize to a binary effort and signal distribution.[21]

Let (q, s) denote a state, which consists of an output $q \in \mathbb{R}$ and a signal $s \in \{L, H\}$. Let $f_\theta(q, s|e)$ denote the PDF of state (q, s) conditional on effort e and precision θ. As in Section 3.3.2, we assume binary effort ($\mathcal{E} = \{\underline{e}, \bar{e}\}$) where the principal wishes to implement $\bar{e}$. Let $p_\theta(e) \equiv \Pr(s = H|e, \theta)$ denote the probability of a high signal, which is good news about effort: $p_\theta(\bar{e}) \geq p_\theta(\underline{e})$.[22] Define the likelihood ratio $I(\theta) \equiv \frac{p_\theta(\bar{e})}{p_\theta(\underline{e})} \geq 1$. Now, the precision parameter θ orders the likelihood ratio, i.e., $I'(\theta) \geq 0$. This condition can be rewritten:

$$\frac{\frac{\partial p_\theta}{\partial \theta}(\bar{e})}{p_\theta(\bar{e})} \geq \frac{\frac{\partial p_\theta}{\partial \theta}(\underline{e})}{p_\theta(\underline{e})}. \tag{46}$$

We first show that the optimal contract is an option, due to the same intuition as before, but now the strike price $X_{s,\theta}$ decreases in the signal realization (as in Chaigneau, Edmans, and Gottlieb (2016)). Since a high signal is good news about effort, rewarding the agent with a lower strike price improves incentives.

[21] This is not possible in the core model where signal is output, since we need more than two outputs to study the structure of the optimal contract (with only two outputs, the contract would involve only two payments, and so options would be indistinguishable from stock or bonuses). Innes (1993) considers two contractible signals, both with general signal distributions. However, the second signal is not informative about effort, which is why a general distribution is feasible without a loss of tractability.

[22] This assumption is without loss of generality, since we can always relabel states.

Lemma 5 *There exists an optimal contract with* $W_\theta(q,s) = \max\{0, q - X_{s,\theta}\}$*, where* $X_{H,\theta} < X_{L,\theta}$*.*

In the core model, where precision affects output, it affects incentives by changing the likelihood that output exceeds the strike price, and thus that the agent benefits from marginal increases in output. In this extension, precision affects the signal but not output, and so this effect disappears. Instead, precision affects incentives by having differential effects on the likelihood that working and shirking lead to a high signal and thus low strike price. This effect is given in Proposition 5, which is the analogy of Proposition 1 in the main model:

Proposition 5 *(Effect of precision on incentives, separate signal) The incentive effect of precision is positive (negative) if*

$$\frac{\partial p_\theta(\bar{e})}{\partial \theta}\left[X_{L,\theta} - X_{H,\theta} - \int_{X_{H,\theta}}^{X_{L,\theta}} F(q|\bar{e})dq\right] - \frac{\partial p_\theta(\underline{e})}{\partial \theta}\left[X_{L,\theta} - X_{H,\theta} - \int_{X_{H,\theta}}^{X_{L,\theta}} F(q|\underline{e})dq\right] \geq (\leq) 0. \tag{47}$$

The incentive effect comprises two components. The first is how precision increases the probability that working leads to a high signal (and thus low strike price), $\frac{\partial p_\theta(\bar{e})}{\partial \theta}$, multiplied by the increase in the agent's expected wage upon a low strike price. If the agent always exercised his call option, he would always pay the strike price, and so he would benefit fully by the lower strike price $X_{L,\theta} - X_{H,\theta}$. However, since he does not always exercise the option and pay the strike price, his benefit is reduced by $\int_{X_{H,\theta}}^{X_{L,\theta}} F(q|\bar{e})dq$.[23] The overall effect is positive: since the value of the agent's call option is decreasing in the strike price, (10) yields $\frac{d}{dX}\left[X - \int_{-\infty}^{X} F(q|e)dq\right] \geq 0$. The second is how precision increases the probability that shirking leads to a high signal, multiplied by the increase in the agent's expected wage upon a high signal.

[23] To further understand the origin of this term, note that the value of a put option is given by

$$\Pr(q < X|e)\,\mathbb{E}\left[(X-q)\,|q < X, e\right] = \int_{-\infty}^{X} -(q-X)\,f(q|e)dq = \int_{-\infty}^{X} F_\theta(q|e)\,dq,$$

where the final equality follows from integration by parts. Thus, $\int_{X_{H,\theta}}^{X_{L,\theta}} F(q|\bar{e})dq = \int_{-\infty}^{X_{L,\theta}} F(q|\bar{e})dq - \int_{-\infty}^{X_{H,\theta}} F(q|\bar{e})dq$ is the difference in the value of two put options. Indeed, equation (10) can be interpreted as the familar put-call parity equation. The agent's call option contains an implicit put option – the option not to exercise the call, and thus not to pay the strike price. Thus, the agent's gain from a lower strike price is reduced by the fall in the value of his implicit put option.

From (46), the effect of precision on the probabilities is very general: the proportional increase in the probability of a high signal upon working exceeds the proportional increase in the probability of a high signal upon shirking, but it could be that both probabilities increase, both decrease, or the former increases and the latter decreases. Due to this generality, the incentive effect of precision can be positive or negative. The examples below respectively give situations in which θ increases incentives, decrease incentives, and has no effect on incentives.

Example 3 *Suppose* $\frac{\partial p_\theta}{\partial \theta}(\underline{e}) \leq 0$ *and* $\frac{\partial p_\theta}{\partial \theta}(\bar{e}) \geq 0$ *for all* θ*. Then, from (47),* θ *increases incentives.*

If precision increases the probability that working leads to a high signal and reduces the probability that shirking leads to a high signal, incentives automatically rise.

Example 4 *Let* $p_\theta(\bar{e}) = 1 - \theta$ *and let* $p_\theta(\underline{e}) = \frac{1}{2} - \theta$ *for* $\theta \in (0, \frac{1}{2})$*. Then, the LHS of (47) is equal to*

$$\int_{X_{H,\theta}}^{X_{L,\theta}} \left[F(q|\bar{e}) - F(q|\underline{e})\right] dq < 0,$$

where the inequality follows from FOSD, and so precision decreases incentives.

Here, a rise in θ *reduces* the probability that both working and shirking lead to a high signal by the same absolute amount. However, because θ involves a lower proportional reduction in $p_\theta(\bar{e})$, the likelihood ratio still rises and so a rise in θ still corresponds to greater precision. Consider the limit case where $\theta \to \frac{1}{2}$. Then, $p_\theta(\bar{e}) \to \frac{1}{2}$ and $p_\theta(\underline{e}) \to 0$, so the likelihood ratio tends to infinity: a high signal is perfectly informative. A fall in θ reduces precision because the high signal could now be generated by shirking. However, it *increases* the agent's incentives. The probabilities that both working and shirking generate the high signal – and thus the low strike price – increase by the same amount. Under working, the agent benefits more from the lower strike price, because he is more likely to exercise the call option and pay the lower strike price.

Example 5 *Suppose that signals are entirely uninformative if precision is sufficiently low. That is, there exists* $0 \in \Theta$ *under which* $p_0(\bar{e}) = p_0(\underline{e})$*. Then, we must have* $X_{H,0} = X_{L,0}$ *and*

$$\left.\frac{\partial \Psi}{\partial \theta}\right|_{\theta=0} = \frac{\partial p_\theta}{\partial \theta}(\bar{e}) \times 0 - \frac{\partial p_\theta}{\partial \theta}(\underline{e}) \times 0 = 0.$$

If the initial signal is entirely uninformative, an increase in precision has no incentive effect.

When the signal is entirely uninformative, the strike price is optimally independent of the signal realization. Then, improvements in signal precision do not directly affect the value of the option, so the incentive effect is zero.

B.1 Proofs

Proof of Lemma 5

Let $(W^*(q,s), e^*)$ be a feasible contract and consider an option contract $W^O_s = \max\{0, q - X_s\}$ where the strike price X_s is chosen so that, for each realization of the signal s, both contracts have the same expected payment under effort e^*:

$$\int_{-\infty}^{\infty} W^*(q,s) f(q|e^*) dq = \int_{-\infty}^{\infty} W^O(q,s) f(q|e^*) dq.$$

It is straightforward to show that the option contract W^O exists and is unique. As in the proof of Lemma 1, we will verify that replacing W^* by W^O increases effort, which, in turn, raises the principal's expected profit.

Let

$$e^O \in \arg\max_{e\in\mathcal{E}} \sum_{s=H,L} \int_{-\infty}^{\infty} W^O(q,s) f_\theta(s,q|e) dq - C(e).$$

Since the agent chooses effort e^* when offered W^* and e^O when offered W^O, we must have:

$$\begin{aligned} \sum_{s=H,L} \int_{-\infty}^{\infty} W^O(q,s) f_\theta(q,s|e^O) dq - C(e^O) &\geq \sum_{s=H,L} \int_{-\infty}^{\infty} W^O(q,s) f_\theta(q,s|e^*) dq - C(e^*), \\ \sum_{s=H,L} \int_{-\infty}^{\infty} W^*(q,s) f_\theta(q,s|e^*) dq - C(e^*) &\geq \sum_{s=H,L} \int_{-\infty}^{\infty} W^*(q,s) f_\theta(q,s|e^O) dq - C(e^O). \end{aligned}$$

Combining these two inequalities, we obtain

$$\sum_{s=H,L} \int_{-\infty}^{\infty} \left[W^O(q,s) - W^*(q,s)\right] \left[f_\theta(q,s|e^O) - f_\theta(q,s|e^*)\right] dq \geq 0 \tag{48}$$

Since, conditional on each s, both contracts have the same expected value under effort

e^* and the option contract pays the lowest feasible amount for $q < X_s$ and has the highest possible slope for $q > X_s$, there exists $\bar{q}_s \geq X_s$ such that

$$W^O(q,s)\left\{\begin{array}{c}\leq\\ \geq\end{array}\right\}W^*(q,s) \text{ for all } q\left\{\begin{array}{c}\leq\\ \geq\end{array}\right\}\bar{q}_s. \tag{49}$$

In order to obtain a contradiction, suppose that $e^* > e^O$. By MLRP, $\frac{f_\theta(q_H,s|e^O)}{f_\theta(q_H,s|e^*)} \leq \frac{f_\theta(q_L,s|e^O)}{f_\theta(q_L,s|e^*)}$ for any $q_H \geq q_L$ and any s. Rewrite (48) as

$$\begin{aligned}
0 &\leq \sum_{s=H,L}\int_{-\infty}^{\infty}\left[W^O(q,s)-W^*(q,s)\right]\left[\frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)}-1\right]f_\theta(q,s|e^*)dq \\
&= \textstyle\sum_{s=H,L}\int_{-\infty}^{\infty}\left[W^O(q,s)-W^*(q,s)\right]\left[\frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)}-1\right]f_\theta(q,s|e^*)dq \\
&\quad +\textstyle\sum_{s=H,L}\underbrace{\displaystyle\int_{-\infty}^{\infty}\left[W^O(q,s)-W^*(q,s)\right]f_\theta(q,s|e^*)dq}_{0} \\
&= \sum_{s=H,L}\int_{-\infty}^{\infty}\left[W^O(q,s)-W^*(q,s)\right]\frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)}f_\theta(q,s|e^*)dq \\
&= \sum_{s=H,L}\left\{\begin{array}{c}\int_{-\infty}^{\bar{q}_s}\left[W^O(q,s)-W^*(q,s)\right]\frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)}f_\theta(q,s|e^*)dq\\ +\int_{\bar{q}_s}^{\infty}\left[W^O(q,s)-W^*(q,s)\right]\frac{f_\theta(q,s|e^O)}{f_\theta(q,s|e^*)}f_\theta(q,s|e^*)dq\end{array}\right\} \\
&< \sum_{s=H,L}\frac{f_\theta(\bar{q}_s,s|e^O)}{f_\theta(\bar{q}_s,s|e^*)}\left\{\begin{array}{c}\int_{-\infty}^{\bar{q}_s}\left[W^O(q,s)-W^*(q,s)\right]f_\theta(q,s|e^*)dq\\ +\int_{\bar{q}_s}^{\infty}\left[W^O(q,s)-W^*(q,s)\right]f_\theta(q,s|e^*)dq\end{array}\right\} \\
&= \sum_{s=H,L}\frac{f_\theta(\bar{q}_s,s|e^O)}{f_\theta(\bar{q}_s,s|e^*)}\left\{\int_{-\infty}^{\infty}\left[W^O(q,s)-W^*(q,s)\right]f_\theta(q,s|e^*)dq\right\}=0
\end{aligned}$$

where the last inequality follows by the definition of W^O. But this is a contradiction $(0 < 0)$, so we must have $e^* \leq e^O$.

Next, we show that the principal's profit is higher with the option contract:

$$\sum_{s=H,L}\int_{-\infty}^{\infty}\left[q-W^O(q,s)\right]f_\theta\left(q,s|e^O\right)dq \geq \sum_{s=H,L}\int_{-\infty}^{\infty}\left[q-W^*(q,s)\right]f_\theta(q,s|e^*)\,dq$$

Subtracting $\sum_{s=H,L}\int_{-\infty}^{\infty}\left[q-W^O(q,s)\right]f_\theta(q,s|e^*)\,dq$ from both sides and rearranging, gives:

$$\sum_{s=H,L}\int_{-\infty}^{\infty}\left[q-W^{O}\left(q,s\right)\right]f_{\theta}\left(q,s|e^{O}\right)dq\geq\sum_{s=H,L}\int_{-\infty}^{\infty}\left[q-W^{O}\left(q,s\right)\right]f_{\theta}\left(q,s|e^{*}\right)dq,$$

showing that the principal indeed profits from the substitution.

It remains to be shown that $X_H < X_L$. Since the optimal contract is an option, we can write the principal's program as

$$\min_{X_L,X_H}\ p_\theta(\bar{e})\int_{X_H}^{\infty}(q-X_H)f(q|\bar{e})dq+[1-p_\theta(\bar{e})]\int_{X_L}^{\infty}(q-X_L)f(q|\bar{e})dq$$

subject to

$$\int_{X_H}^{\infty}(q-X_H)\left[f(q|\bar{e})p_\theta(\bar{e})-f(q|\underline{e})p_\theta(\underline{e})\right]dq+\int_{X_L}^{\infty}(q-X_L)\left\{f(q|\bar{e})\left[1-p_\theta(\bar{e})\right]-f(q|\underline{e})\left[1-p_\theta(\underline{e})\right]\right\}dq\geq C$$

The necessary first-order conditions give:

$$-p_\theta(\bar{e})\left[1-F(X_H|\bar{e})\right]+\lambda\left\{p_\theta(\bar{e})\left[1-F(X_H|\bar{e})\right]-\left[1-F(X_H|\underline{e})\right]p_\theta(\underline{e})\right\}=0$$

$$\therefore\frac{1}{\lambda}=1-\frac{p_\theta(\underline{e})\left[1-F(X_H|\underline{e})\right]}{p_\theta(\bar{e})\left[1-F(X_H|\bar{e})\right]}.$$

$$-\left[1-p_\theta(\bar{e})\right]\left[1-F(X_L|\bar{e})\right]+\lambda\left\{\left[1-p_\theta(\bar{e})\right]\left[1-F(X_L|\bar{e})\right]-\left[1-p_\theta(\underline{e})\right]\left[1-F(X_L|\underline{e})\right]\right\}=0$$

$$\therefore\frac{1}{\lambda}=1-\frac{\left[1-p_\theta(\underline{e})\right]\left[1-F(X_L|\underline{e})\right]}{\left[1-p_\theta(\bar{e})\right]\left[1-F(X_L|\bar{e})\right]}.$$

Thus, the optimality conditions are

$$\frac{1-p_\theta(\bar{e})}{p_\theta(\bar{e})}\cdot\frac{1-F(X_L|\bar{e})}{1-F(X_H|\bar{e})}=\frac{1-p_\theta(\underline{e})}{p_\theta(\underline{e})}\cdot\frac{1-F(X_L|\underline{e})}{1-F(X_H|\underline{e})}.$$

Note that $p_\theta(\bar{e}) > p_\theta(\underline{e})$ implies $\frac{1-p_\theta(\bar{e})}{p_\theta(\bar{e})} < \frac{1-p_\theta(\underline{e})}{p_\theta(\underline{e})}$. Thus, the optimality condition implies

$$\frac{1-F(X_L|\bar{e})}{1-F(X_L|\underline{e})}>\frac{1-F(X_H|\bar{e})}{1-F(X_H|\underline{e})}.$$

We claim that this implies that $X_H < X_L$. To see this, note that differentiation gives

$$\frac{d}{dX}\left(\frac{1-F(X|\bar{e})}{1-F(X|\underline{e})}\right)=\frac{1-F(X|\bar{e})}{1-F(X|\underline{e})}\left[\frac{f(X|\underline{e})}{1-F(X|\underline{e})}-\frac{f(X|\bar{e})}{1-F(X|\bar{e})}\right]>0,$$

which is negative because strict MLRP implies:

$$\frac{f(X|\bar{e})}{1-F(X|\bar{e})}<\frac{f(X|\underline{e})}{1-F(X|\underline{e})}.$$

Indeed, since $\frac{f(q|\bar{e})}{f(q|\underline{e})}$ is strictly increasing in q by MLRP, $\frac{f(q+\delta|\bar{e})}{f(q+\delta|\underline{e})} > \frac{f(q|\bar{e})}{f(q|\underline{e})}$ for all $\delta > 0$. Rearrange this inequality as $f(q|\underline{e})f(q+\delta|\bar{e}) > f(q+\delta|\underline{e})f(q|\bar{e})$. Integrate both sides with respect to δ:

$$f(q|\underline{e})\underbrace{\int_0^\infty f(q+\delta|\bar{e})d\delta}_{1-F(q|\bar{e})}>f(q|\bar{e})\underbrace{\int_0^\infty f(q+\delta|\underline{e})d\delta}_{1-F(q|\underline{e})}\;\Leftrightarrow\;\frac{f(q|\underline{e})}{1-F(q|\underline{e})}>\frac{f(q|\bar{e})}{1-F(q|\bar{e})}.$$

Proof of Proposition 5

Let Ψ denote the agent's marginal benefit from effort. Using the decomposition of the expected wage in equation (10), this can be rewritten

$$\begin{aligned}\Psi(\theta;X_H,X_L)\equiv\;&\mathbb{E}[q|\bar{e}]-\mathbb{E}[q|\underline{e}]\\&+p_\theta(\underline{e})\left[X_H-\int_{-\infty}^{X_H}F(q|\underline{e})dq\right]+(1-p_\theta(\underline{e}))\left[X_L-\int_{-\infty}^{X_L}F(q|\underline{e})dq\right]\\&-p_\theta(\bar{e})\left[X_H-\int_{-\infty}^{X_H}F(q|\bar{e})dq\right]-(1-p_\theta(\bar{e}))\left[X_L-\int_{-\infty}^{X_L}F(q|\bar{e})dq\right].\end{aligned}$$

The agent's incentive constraint requires his marginal benefit from effort to exceed its cost, i.e.,

$$\Psi(\theta;X_H,X_L)\geq C. \tag{50}$$

As in Section 3.3.2, precision increases (decreases) incentives if it relaxes (tightens) the incentive constraint (50), i.e., $\frac{\partial\Psi}{\partial\theta}(\theta;X_{H,\theta},X_{L,\theta})\geq(\leq)0$. Differentiating Ψ yields Proposition 5.

C Risk aversion

We now extend the core model to incorporate risk aversion. We make the following small changes to the model to allow us to use the framework of Jewitt, Kadan, and Swinkels (2008) which solves for optimal contracts with contracting constraints (but do not study effect of changes in precision); all other assumptions are unchanged.

Output is now given by $q \in [\underline{q}, \overline{q}]$ and the agent chooses effort in $e \in \left(0, \overline{E}\right)$. The main change is that the agent's utility of wealth is now given by $u(W)$, which is twice continuously differentiable with $u' > 0$, $u'' < 0$, and $\lim_{W\to\infty} u(W) = \infty$. His objective function is $\mathbb{E}[u(W_\theta\left(q\right))|e] - C(e)$, with $C > 0, C' > 0, C'' > 0$.

We continue to assume limited liability and that the participation constraint is slack. However, we no longer need to assume the monotonicity constraint. With risk neutrality, Innes (1990) showed that, without monotonicity, the optimal contract is highly discontinuous: the agent receives 0 for $q < \widetilde{X}$, and the entire output q (rather than the residual $q - X$) for $q \geq \widetilde{X}$. Thus, the agent's wage jumps from 0 to q at $q = \widetilde{X}$. Thus, monotonicity is required to rule out discontinuous contracts. With risk aversion, monotonicity is not necessary for the contract to be continuous.

The principal wishes to implement effort level e^*. As in Section 3.2, we assume that the FOA is valid and that output has a location parameter, and we also assume that an optimal contract exists. Thus, for a given θ, the principal's problem is to choose a function $W_\theta(\cdot)$ to minimize $\mathbb{E}[W_\theta\left(q\right)|e^*]$, subject to limited liability and the following incentive constraint (for simplicity, we drop the subscript θ from the PDF):

$$\frac{d}{de}\int_{-\infty}^{\infty} u(W_\theta\left(q\right))f(q|e^*)dq = C'(e^*). \tag{51}$$

We refer to the LHS of (51) as "effort incentives". Proposition 1 in Jewitt, Kadan, and Swinkels (2008) implies that the optimal contract is defined implicitly by:

$$\frac{1}{u'(W_\theta\left(q\right))} = \begin{cases} \mu\frac{f_e(q|e^*)}{f(q|e^*)} & \text{if } \mu\frac{f_e(q|e^*)}{f(q|e^*)} \geq \frac{1}{u'(0)}, \\ \frac{1}{u'(0)} & \text{if } \mu\frac{f_e(q|e^*)}{f(q|e^*)} < \frac{1}{u'(0)}, \end{cases} \tag{52}$$

where $\mu > 0$, the shadow price of the incentive constraint, is unique, and where $f_e(q|e)$ denotes the first derivative of the PDF with respect to e. Let q^* be implicitly defined by $\mu\frac{f_e(q^*|e^*)}{f(q^*|e^*)} = \frac{1}{u'(0)}$ which is unique due to MLRP. Intuitively, q^* is the highest value of q such that the wage is zero under the optimal contract. It is analogous to the threshold X_θ in Lemma 1, except that it is not called a strike price, since the optimal contract need not be an option.

With MLRP, (52) can be rewritten as

$$W_\theta(q) = \begin{cases} u'^{-1}\left(1/\left(\mu \frac{f_e(q|e^*)}{f(q|e^*)}\right)\right) & \text{if } q \geq q^*, \\ 0 & \text{if } q < q^*. \end{cases} \tag{53}$$

Let $g_\theta(x) \equiv f(x|0)$.[24] We can rewrite:

$$\int_{-\infty}^{\infty} u(W_\theta(q)) f(q|e^*) dq = \int_{-\infty}^{\infty} u(W_\theta(e^* + \varepsilon)) g_\theta(\varepsilon) d\varepsilon,$$

and so the incentive constraint in (51) becomes

$$\int_{-\infty}^{\infty} W'_\theta(e^* + \varepsilon)\, u'(W_\theta(e^* + \varepsilon)) g_\theta(\varepsilon) d\varepsilon = C'(e^*).$$

When precision increases in an MPS sense, the distribution of ε can be divided into three regions: the left tail, the right tail, and the centre. Thus, there exist ε_a and ε_b such that

$$\frac{dg_\theta(\varepsilon)}{d\theta} \begin{cases} > 0 & \text{if } \varepsilon \in (\varepsilon_a, \varepsilon_b), \\ \leq 0 & \text{if } \varepsilon \notin (\varepsilon_a, \varepsilon_b), \end{cases} \tag{54}$$

An increase in θ shifts mass away from the left tail ($\varepsilon \leq \varepsilon_a$) and right tail ($\varepsilon \geq \varepsilon_b$) and towards the centre ($\varepsilon_a < \varepsilon < \varepsilon_b$). Similar to equation (9) in the core model, the incentive effect is positive if and only if

$$\frac{\partial^2}{\partial e \partial \theta} \mathbb{E}_\theta \left[u\left(W_\theta(q)\right)|e\right] \geq 0.$$

Proposition 6 gives the effect of precision on incentives when we allow for risk aversion, although it also holds for the case of risk neutrality.

Proposition 6 *(Effect of precision on incentives, risk aversion) (i) When the limited liability constraint is binding for all $q < e^* + \varepsilon_b$, then effort and precision are substitutes.*

(ii) When the limited liability constraint is binding for all $q < e^* + \varepsilon_a$, let $q^* \geq e^* + \varepsilon_a$ be the level of output such that the limited liability constraint is binding if and only if

[24] With a location parameter, output q is equal to effort plus white noise. By construction, g_θ is the PDF of the white noise.

$q < q^*$. *Then, effort and precision are complements if and only if* q^* *is such that* $q^* < \widehat{q}$.

The intuition is as follows. In general, the effect of precision on effort is complex, because there are up to three relevant regions. Part (i) studies the case in which limited liability binds for all $q < e^* + \varepsilon_b$, and so the wage is positive only in the right tail. An increase in precision reduces the right tail and thus lowers incentives, regardless of whether the agent is risk-neutral or risk-averse. The intuition is as in the core model. An increase in precision reduces the likelihood that output ends up in the right tail, and thus the agent is rewarded for increases in effort.

Part (ii) studies the case in which limited liability binds for all $q < e^* + \varepsilon_a$, and so the wage is positive only in the centre and right tail. We define q^* as the threshold output above which the wage is positive, analogous to the threshold X_θ in the core model. Effort and precision are complements if and only if this threshold is below a cutoff $\widehat{q}$ – analogous to the condition $X_\theta < \widehat{X}$ in Proposition 2. Again, the intuition is as in the core model. If the threshold q^* is low, then the agent will be paid for any increases in effort unless he suffers a sufficiently negative shock to push output below the threshold. An increase in precision reduces the risk of such shocks, and so raises incentives. If the threshold q^* is high, the agent will be paid for any increases in effort only if he enjoys a sufficiently positive shock to push output above the threshold. An increase in precision reduces the likelihood of such shocks, and so reduces incentives.[25]

C.1 Log utility and linear likelihood ratio

With risk aversion, in general it is not possible to pin down the shape of the optimal contract – it may be convex, concave, or have both convex and concave regions. However, as shown by Chaigneau, Edmans, and Gottlieb (2016), if the agent has log

[25]If limited liability does not bind for all $q < e^* + \varepsilon_a$, i.e. the wage is positive in part of the left tail (as well as the centre and right tails), one might think that precision and effort are always complements, similar to the core model for low X_θ – the wage is positive except for very low output realizations, and increases in precision reduce the risk of such output realizations. However, this need not be the case. When the wage is positive in the left tail, we must consider how precision affects incentives in the left tail (not just the center and right tail). As discussed at the end of Section C.1, a risk-averse agent's incentives are given by utility-adjusted pay-performance sensitivity. Due to diminishing marginal utility, the agent places higher weight on (dollar) pay-performance sensitivity in the left tail than at the center or in the right tail. While an increase in precision redistributes probability mass from the right tail towards the center, it also redistributes mass from the left tail to the center, and so may reduce incentives. Thus, the effect of precision on incentives is ambiguous when limited liability does not bind for all $q < e^* + \varepsilon_a$.

utility and the likelihood ratio is linear (as with the (truncated) normal and gamma distributions), the optimal contract is an option. This result is stated in Proposition 7.

Proposition 7 *(Log utility, linear likelihood ratio) With log utility and a likelihood ratio linear in q, the optimal contract gives the agent $\hat{b}_\theta$ options with strike price q^*:*

$$W_\theta(q) = \hat{b}_\theta \max\{q - q^*, 0\}. \tag{55}$$

Moreover, if the output distribution is symmetric, these options are at-the-money: $q^* = \mathbb{E}[q|e^*] = e^*$. Then, an increase in precision does not change the effort incentives of a risk neutral agent, but it increases the effort incentives of a risk averse agent.

The optimal contract gives the agent $\hat{b}_\theta$ options on q with strike price q^*, which is such that the likelihood ratio is equal to zero at $q = q^*$: the agent gets a positive wage whenever the likelihood ratio is positive. Expected firm value is always $\mathbb{E}[q|e^*] = e^*$, so that an option with strike price e^* is at-the-money. Moreover, if we assume that the output distribution is symmetric (as with the normal distribution), then the level of output such the likelihood ratio turns positive is $q^* = e^*$. In this case, the optimal contract involves at-the-money options.

With an at-the-money option and a symmetric distribution, an increase in precision does not change the likelihood that the option ends up in-the-money (which remains at 1/2). Thus, it does not change the expected pay-performance sensitivity of the option, and therefore effort incentives – regardless of precision, if the agent increases effort by 1 unit, there is a 1/2 probability that the option ends up in-the-money, in which case the incremental effort increases his pay by $\hat{b}_\theta$ units. With risk aversion, effort incentives depend on the *utility-adjusted* pay-performance sensitivity. An at-the-money option contract gives the agent a constant reward of $\hat{b}_\theta$ for every one unit increase in output above expected output (e^*). Due to diminishing marginal utility, a reward of $\hat{b}_\theta$ has less effect on the agent's utility at high levels of wealth (i.e., in the right tail) than at low levels of wealth (i.e., in the center). An increase in precision redistributes probability mass from the right tail to the centre, i.e., from low to high utility-adjusted pay-performance sensitivity regions, and thus raises effort incentives.

C.2 Proofs

Proof of Proposition 6

We start with part (i). For a given contract, the effect of a marginal increase in precision on the LHS of the incentive constraint is given by:

$$\int_{-\infty}^{\infty} W'_\theta\left(e^*+\varepsilon\right)u'(W_\theta\left(e^*+\varepsilon\right))\frac{dg_\theta(\varepsilon)}{d\theta}d\varepsilon. \tag{56}$$

When limited liability is binding for $q<\varepsilon_b+e^*$, $W_\theta\left(e^*+\varepsilon\right)=0$ for $\varepsilon<\varepsilon_b$. Thus, (56) becomes:

$$\int_{\varepsilon_b}^{\infty} W'_\theta\left(e^*+\varepsilon\right)u'(W_\theta\left(e^*+\varepsilon\right))\frac{dg_\theta(\varepsilon)}{d\theta}d\varepsilon. \tag{57}$$

Whether the agent is risk-neutral or risk-averse, we have $u'>0$, and MLRP ensures $W'_\theta>0$. In addition, from the definition of ε_b in (54), $\frac{dg_\theta(\varepsilon)}{d\theta}\leq 0$ for $\varepsilon>\varepsilon_b$, so (57) is negative.

We now turn to part (ii). For a given contract, when limited liability is binding for $q<q^*$, where $q^*\geq e^*+\varepsilon_a$, the effect of a marginal increase in precision on the LHS of the incentive constraint is given by:

$$\int_{q^*-e^*}^{\varepsilon_b} W'_\theta\left(e^*+\varepsilon\right)u'(W_\theta\left(e^*+\varepsilon\right))\frac{dg_\theta(\varepsilon)}{d\theta}d\varepsilon+\int_{\varepsilon_b}^{\infty} W'_\theta\left(e^*+\varepsilon\right)u'(W_\theta\left(e^*+\varepsilon\right))\frac{dg_\theta(\varepsilon)}{d\theta}d\varepsilon. \tag{58}$$

As above, $u'>0$ and $W'_\theta>0$. Moreover, $\frac{dg_\theta(\varepsilon)}{d\theta}$ is positive under the first integral and negative under the second integral. The expression in (58) is therefore strictly decreasing in q^*, and strictly negative for $q^*\geq e^*+\varepsilon_b$. If there exists $q^*\in[e^*+\varepsilon_a,e^*+\varepsilon_b]$ such that the expression in (58) is equal to zero, then let $\widehat{q}$ be equal to this value of q^*. If such a value does not exist, i.e., the expression in (58) is strictly negative for $q^*\geq e^*+\varepsilon_a$, then let $\widehat{q}=-\infty$.

Proof of Proposition 7

A linear likelihood ratio can be written as $\frac{f_e(q|e^*)}{f(q|e^*)}\equiv a+bq$ with $b>0$ due to MLRP. In addition, log utility yields $u'^{-1}(W)=\frac{1}{W}$. Thus, (53) can be written as

$$W_\theta\left(q\right)=\begin{cases}\hat{b}_\theta(q-q^*) & \text{if } q\geq q^*,\\ 0 & \text{if } q<q^*,\end{cases} \tag{59}$$

where $\hat{b}_\theta=\mu b>0$ and $q^*=-\frac{a}{b}$ (which implies that $\hat{b}_\theta(q-q^*)=0$ at $q=q^*$). Equation

(59) can thus be rewritten as in (55).

With MLRP, there is only one value q^* such that $\frac{f_e(q^*|e^*)}{f(q^*|e^*)} = 0$. For a distribution with a location parameter, this equality implies that the derivative of the PDF with respect to q is equal to zero at q^*, and at this point only. If the distribution is symmetric, and if the derivative of the PDF is equal to zero at a single point, this point must be the point around which distribution is centered, i.e., it must be the mean of the distribution e^*. That is, for a symmetric distribution with a location parameter that satisfies MLRP, the likelihood ratio $\frac{f_e(q|e^*)}{f(q|e^*)}$ is only equal to zero at $q = e^*$. With a linear likelihood ratio, this implies $a + be^* = 0$, so that $q^* = e^*$.

In this case, the expression in (58) can be rewritten as

$$\int_0^\infty \hat{b}_\theta u'(\hat{b}_\theta(e^* + \varepsilon - e^*)) \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon. \tag{60}$$

With a risk-neutral agent, u' is a positive constant, and this expression has the same sign as $\int_0^\infty \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon$, which is equal to zero because a change in θ is a MPS and the distribution of ε is symmetric around zero for any θ. With a risk-averse agent, the expression in (60) can be rewritten as

$$\int_0^{\varepsilon_b} \hat{b}_\theta u'(\hat{b}_\theta(e^* + \varepsilon - e^*)) \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon + \int_{\varepsilon_b}^\infty \hat{b}_\theta u'(\hat{b}_\theta(e^* + \varepsilon - e^*)) \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon. \tag{61}$$

As above, $\int_0^\infty \frac{dg_\theta(\varepsilon)}{d\theta} d\varepsilon = 0$, and the first integral in (61) is positive while the second is negative because of (54). With a risk-averse agent, $u'' < 0$ and so the first integral outweighs the second, so that the expression in (61) is positive.

D Location-scale distributions

Claim 1 *For distributions parametrized with e and σ such that $F_\sigma(q|e) = G\left(\frac{q-e}{\sigma}\right)$, the option vega is highest when $X_\sigma = e$.*

Proof. The agent's expected pay under volatility σ and effort e is given by

$$\mathbb{E}[W_\sigma(q)|e] = \int_{X_\sigma}^\infty (q - X_\sigma) f_\sigma(q|e) dq. \tag{62}$$

Using the same integration by parts as in (10) yields

$$\mathbb{E}\left[W_{\theta}\left(q\right)|e\right]=\mathbb{E}[q|e]-X_{\sigma}+\int_{-\infty}^{X_{\sigma}}F_{\sigma}\left(q|e\right)dq. \tag{63}$$

Thus, the vega of the option is

$$\nu=\frac{\partial}{\partial\sigma}\mathbb{E}\left[W_{\theta}\left(q\right)|e\right]=\frac{\partial}{\partial\sigma}\left\{\mathbb{E}[q|e]-X_{\sigma}+\int_{-\infty}^{X_{\sigma}}F_{\sigma}\left(q|e\right)dq\right\}.$$

Since $F_{\sigma}\left(q|e\right)=G\left(\frac{q-e}{\sigma}\right)$, we have

$$\nu=\frac{\partial}{\partial\sigma}\left\{\int_{-\infty}^{X_{\sigma}}G\left(\frac{q-e}{\sigma}\right)dq\right\}=-\frac{1}{\sigma}\int_{-\infty}^{X_{\sigma}}\frac{q-e}{\sigma}g\left(\frac{q-e}{\sigma}\right)dq.$$

Using the change of variables $y=\frac{q-e}{\sigma}$ gives

$$\nu=\int_{-\infty}^{\frac{X_{\sigma}-e}{\sigma}}(-y)g(y)dq. \tag{64}$$

Since $g(y)>0$, this expression is maximized for $X_{\sigma}=e$, i.e., for an at-the-money option.[26] ■

Claim 2 shows that, for symmetric distributions, the vegas of the option-when-working and option-when-shirking are equal for $X_{\sigma}=\frac{\underline{e}+\overline{e}}{2}$.

Claim 2 *For symmetric distributions parametrized by e and σ such that $F_{\sigma}\left(q|e\right)=G\left(\frac{q-e}{\sigma}\right)$, the vegas of the option-when working and the option-when-shirking are equal for $X_{\sigma}=\frac{\underline{e}+\overline{e}}{2}$.*

Proof. Using (64), for $X_{\sigma}=\frac{\underline{e}+\overline{e}}{2}$, the vega $\nu_{\overline{e}}$ of the option-when-working ($e=\overline{e}$) is

$$\nu_{\overline{e}}=\int_{-\infty}^{\frac{X_{\sigma}-\overline{e}}{\sigma}}(-y)g(y)dq=\int_{-\infty}^{\frac{\underline{e}-\overline{e}}{2\sigma}}(-y)g(y)dq.$$

[26] With high effort, $e=\overline{e}$, so the option-when-working is ATM for $X_{\sigma}=\overline{e}$. With low effort, $e=\underline{e}$, so the option-when-shirking is ATM for $X_{\sigma}=\underline{e}$.

Likewise, for $X_\sigma = \frac{\underline{e}+\overline{e}}{2}$, the vega $\nu_{\underline{e}}$ of the option-when-shirking ($e = \underline{e}$) is

$$\nu_{\underline{e}} = \int_{-\infty}^{\frac{X_\sigma - \underline{e}}{\sigma}} (-y)g(y)dq = \int_{-\infty}^{\frac{\overline{e}-\underline{e}}{2\sigma}} (-y)g(y)dq.$$

In addition,

$$\int_{-\infty}^{\frac{\overline{e}-\underline{e}}{2\sigma}} (-y)g(y)dq = \int_{-\infty}^{\frac{\underline{e}-\overline{e}}{2\sigma}} (-y)g(y)dq + \int_{\frac{\underline{e}-\overline{e}}{2\sigma}}^{\frac{\overline{e}-\underline{e}}{2\sigma}} (-y)g(y)dq. \tag{65}$$

For a symmetric distribution, we have $\int_{-\frac{\overline{e}-\underline{e}}{2\sigma}}^{\frac{\overline{e}-\underline{e}}{2\sigma}} (-y)g(y)dq = 0$. Equation (65) then implies that $\nu_{\overline{e}} = \nu_{\underline{e}}$. ■

E Normal distribution

This section provides analytical results to support the graph in Figure 1, which illustrates the direct and incentive effects for the Normal distribution. The proofs are in Section E.1.

Let φ and Φ denote the PDF and CDF of the standard normal distribution, respectively. As shown in Appendix E.1, the total and direct effects are respectively given by:

$$\frac{d\mathbb{E}\left[W_\sigma\left(q\right)|\overline{e}\right]}{d\sigma} = \varphi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right) - \left[1 - \Phi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right)\right] \frac{\varphi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right)}, \text{ and} \tag{66}$$

$$\frac{\partial\mathbb{E}\left[W_\sigma\left(q\right)|\overline{e}\right]}{\partial\sigma} = \varphi\left(\frac{X_\sigma - \overline{e}}{\sigma}\right). \tag{67}$$

The direct effect is discussed in the main text. The incentive effect, $\frac{\partial\mathbb{E}[W_\sigma(q)|\overline{e}]}{\partial X_\sigma}\frac{dX_\sigma}{d\sigma}$, consists of two components. The first is the change in strike price required to maintain incentive compatibility, $\frac{dX_\sigma}{d\sigma}$. From Corollary 1 and using $\sigma = \frac{1}{\theta}$, $\frac{dX_\sigma}{d\sigma} > 0$ if and only if $X_\sigma > \widehat{X} = \frac{\overline{e}+\underline{e}}{2}$. Indeed, for the normal distribution, not only does $\frac{dX_\sigma}{d\sigma}$ turn from negative to positive as X_σ crosses above $\widehat{X}$, but it is also monotonically increasing in X_σ, i.e., monotonically decreasing in the cost of effort. This result is stated in Lemma 6 below:

Lemma 6 *(Normal distribution, change in strike price): Suppose ε is normally distributed. Then, the effect of volatility on the strike price is decreasing in the cost of effort, i.e.,*

$$\frac{d^2 X_\sigma}{d\sigma dC} < 0. \tag{68}$$

The second component is the change in the value of the option when the strike price increases, $\frac{\partial \mathbb{E}[W_\sigma(q)|\bar{e}]}{\partial X_\sigma}$. This change is always negative, and its negativity is increasing in the moneyness of the option. Overall, as X falls below $\widehat{X}$ and the option becomes increasingly in the money, both $\frac{dX_\sigma}{d\sigma}$ and $\frac{\partial \mathbb{E}[W_\sigma(q)|\bar{e}]}{\partial X_\sigma}$ become increasingly negative, and so the overall incentive effect $\frac{\partial \mathbb{E}[W_\sigma(q)|\bar{e}]}{\partial X_\sigma}\frac{dX_\sigma}{d\sigma}$ becomes monotonically more positive. However, as X rises above $\widehat{X}$, the two components of the incentive effect move in opposite directions. On the one hand, greater precision increasingly worsens incentives ($\frac{dX_\sigma}{d\sigma}$ becomes more positive). On the other hand, $\frac{\partial \mathbb{E}[W_\sigma(q)|\bar{e}]}{\partial X_\sigma}$ rises towards zero: when the option is deeply out-of-the-money, its value is small to begin with and thus little affected by the strike price. Overall, the impact of X on incentives is non-monotonic. As X initially rises above $\widehat{X}$, the incentive effect becomes increasingly negative but subsequently rises to zero.

Proposition 8 proves for the normal distribution that the value of information is monotonically increasing in C (the exogenous parameter that drives X).

Proposition 8 *(Normal distribution, effect of cost of effort on value of information) Suppose ε is normally distributed. Then,* $\frac{d}{dC}\left\{\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}\right\} > 0$.

E.1 Proofs

Proof of Equations (66) and (67)

First, with σ instead of θ, the decomposition in (8) can be rewritten as

$$\frac{d}{d\sigma}\mathbb{E}\left[W_\sigma(q)\,|\bar{e}\right] = \underbrace{\frac{\partial}{\partial\sigma}\mathbb{E}\left[W_\sigma(q)\,|\bar{e}\right]}_{\textit{direct effect}} + \underbrace{\frac{\partial}{\partial X_\sigma}\mathbb{E}\left[W_\sigma(q)\,|\bar{e}\right]\frac{dX_\sigma}{d\sigma}}_{\textit{incentive effect}}. \tag{69}$$

Second,

$$\begin{aligned}
\frac{\partial \mathbb{E}[W_\sigma(q)|e]}{\partial \sigma} &= \frac{\partial}{\partial \sigma}\int_{X_\sigma}^{\infty}(q-X_\sigma)\frac{1}{\sigma}\varphi\left(\frac{q-e}{\sigma}\right)dq = \frac{\partial}{\partial \sigma}\int_{X_\sigma - e}^{\infty}\frac{q+e-X_\sigma}{\sigma}\varphi\left(\frac{q}{\sigma}\right)dq \\
&= \frac{\partial}{\partial \sigma}\int_{X_\sigma - e}^{\infty}\frac{q}{\sigma}\varphi\left(\frac{q}{\sigma}\right)dq - (X_\sigma - e)\frac{\partial}{\partial \sigma}\int_{X_\sigma - e}^{\infty}\frac{1}{\sigma}\varphi\left(\frac{q}{\sigma}\right)dq \\
&= \frac{\partial}{\partial \sigma}\left\{\left[-\sigma\varphi\left(\frac{q}{\sigma}\right)\right]_{X_\sigma - e}^{\infty}\right\} - (X_\sigma - e)\frac{\partial}{\partial \sigma}\left\{1-\Phi\left(\frac{X_\sigma - e}{\sigma}\right)\right\} \\
&= \varphi\left(\frac{X_\sigma - e}{\sigma}\right) - \sigma\frac{X_\sigma - e}{\sigma^2}\varphi'\left(\frac{X_\sigma - e}{\sigma}\right) + (X_\sigma - e)\left(-\frac{X_\sigma - e}{\sigma^2}\right)\varphi\left(\frac{X_\sigma - e}{\sigma}\right) \\
&= \varphi\left(\frac{X_\sigma - e}{\sigma}\right) - \frac{X_\sigma - e}{\sigma}\varphi'\left(\frac{X_\sigma - e}{\sigma}\right) + \frac{X_\sigma - e}{\sigma}\varphi'\left(\frac{X_\sigma - e}{\sigma}\right) = \varphi\left(\frac{X_\sigma - e}{\sigma}\right).
\end{aligned}$$

where the fourth and sixth equalities use $\varphi'(x) = -x\varphi(x)$, and the fifth equality uses $\varphi(x) \to_{x\to\infty} 0$. This establishes (67). In addition, it follows that

$$\frac{\partial}{\partial \sigma}\left\{\mathbb{E}\left[W_\sigma(q)|\bar{e}\right] - \mathbb{E}\left[W_\sigma(q)|\underline{e}\right]\right\} = \varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right). \tag{70}$$

Third,

$$\begin{aligned}
\frac{\partial \mathbb{E}[W_\sigma(q)|e]}{\partial X_\sigma} &= \frac{\partial}{\partial X_\sigma}\int_{X_\sigma}^{\infty}(q-X_\sigma)\frac{1}{\sigma}\varphi\left(\frac{q-e}{\sigma}\right)dq \\
&= \int_{X_\sigma}^{\infty}-\frac{1}{\sigma}\varphi\left(\frac{q-e}{\sigma}\right)dq = -\left(1-\Phi\left(\frac{X_\sigma - e}{\sigma}\right)\right).
\end{aligned} \tag{71}$$

It follows that

$$\begin{aligned}
\frac{\partial}{\partial X_\sigma}\left\{\mathbb{E}\left[W_\sigma(q)|\bar{e}\right] - \mathbb{E}\left[W_\sigma(q)|\underline{e}\right]\right\} &= -\left(1-\Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)\right) + \left(1-\Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)\right) \\
&= \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right).
\end{aligned} \tag{72}$$

which is strictly negative because of MLRP, which implies FOSD.

Fourth, according to Lemma 1, following a change in σ the strike price X_σ adjusts so that the incentive constraint remains satisfied as an equality, so:

$$\frac{\partial\left\{\mathbb{E}[W_\sigma(q)|\bar{e}] - \mathbb{E}[W_\sigma(q)|\underline{e}]\right\}}{\partial \sigma} + \frac{\partial\left\{\mathbb{E}[W_\sigma(q)|\bar{e}] - \mathbb{E}[W_\sigma(q)|\underline{e}]\right\}}{\partial X_\sigma}\frac{dX_\sigma}{d\sigma} = 0.$$

Rearranging and using the results in equations (70) and (72):

$$\frac{dX_\sigma}{d\sigma} = -\frac{\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)}. \tag{73}$$

Using the results above, we can rewrite (69) as

$$\frac{d\mathbb{E}[W_\sigma(q)\,|\bar{e}]}{d\sigma} = \varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) + \left[1 - \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)\right] \frac{\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)}. \tag{74}$$

This establishes (66).

Proof of Lemma 6

As X_σ is strictly decreasing in C (see Proposition 4), inequality (68) holds if and only if $\frac{d\frac{dX_\sigma}{d\sigma}}{dX} > 0$. As established in the proof of equations (66) and (67) above,

$$\frac{dX_\sigma}{d\sigma} = -\frac{\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)}.$$

To simplify notation, define

$$x \equiv \frac{X_\sigma - \underline{e}}{\sigma}, t \equiv \frac{\bar{e} - \underline{e}}{\sigma}.$$

We wish to show that $\forall t > 0$,

$$f(x,t) \equiv [\varphi(x) - \varphi(x-t)]^2 - [\Phi(x) - \Phi(x-t)][\varphi'(x) - \varphi'(x-t)] > 0, \quad \forall x, \tag{75}$$

where

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\Phi(x) = \int_{-\infty}^{x} \varphi(y)\, dy.$$

For $t = 0$, $f(x,0)$ is trivially 0. Since $\varphi(x) = \varphi(-x)$, we have $\Phi(x) - \Phi(x-t) = \Phi(-x+t) - \Phi(-x)$ and $\varphi'(x) - \varphi'(x-t) = \varphi'(-x+t) - \varphi'(-x)$. As a consequence, $f(x,t) = f(-x+t,t)$. We thus only have to study $x \geq \frac{t}{2} > 0$.

We first analyze the term $\varphi'(x) - \varphi'(x-t)$. Since

$$\varphi'(x) = -\frac{x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}},$$

$$\varphi'(x) - \varphi'(x-t) = \varphi(x-t)(-xe^{-t(x-t/2)} + x - t).$$

When $x \geq t/2$, the function $e^{-t(x-t/2)} - 1 + \frac{t}{x}$ is only equal to zero at one point, since it monotonically decreases from 2 to -1. Let that point be x_0. Then

$$\varphi'(x) - \varphi'(x-t) \begin{cases} < 0 & \frac{t}{2} \leq x < x_0 \\ = 0 & x = x_0 \\ > 0 & x > x_0 \end{cases}.$$

We know that when $x \in [\frac{t}{2}, x_0]$, $f(x,t) > 0$ since $[\varphi(x) - \varphi(x-t)]^2 > 0$ and $\Phi(x) - \Phi(x-t) > 0 \; \forall x$, so that (75) is proven for $x \in [\frac{t}{2}, x_0]$

We now prove (75) for $x > x_0$. In this interval (omitting the argument t):

$$f(x,t) > 0 \iff g(x) \equiv \frac{f(x,t)}{\varphi'(x) - \varphi'(x-t)} > 0.$$

To prove the latter, we first calculate

$$\begin{aligned} g'(x) &= \frac{2[\varphi(x) - \varphi(x-t)][\varphi'(x) - \varphi'(x-t)]^2 - [\varphi(x) - \varphi(x-t)]^2[\varphi''(x) - \varphi''(x-t)]}{[\varphi'(x) - \varphi'(x-t)]^2} \\ &\quad -[\varphi(x) - \varphi(x-t)] \\ &= \frac{[\varphi(x) - \varphi(x-t)]\varphi(x-t)^2}{[\varphi'(x) - \varphi'(x-t)]^2} \left[\left(e^{-t(x-t/2)} - 1\right)^2 + t^2 e^{-t(x-t/2)} \right] \\ &< 0, \quad x \in (x_0, \infty), \end{aligned}$$

where in the last step we used the fact that $\varphi(x) < \varphi(x-t)$ when $x > t/2$. Therefore,

$$g(x) > 0 \quad \forall x \in (x_0, \infty) \iff \lim_{x \to \infty} g(x) \geq 0.$$

Since

$$g(x) = \frac{[\varphi(x) - \varphi(x-t)]^2}{\varphi'(x) - \varphi'(x-t)} - \Phi(x) + \Phi(x-t)$$
$$= \frac{1}{\sqrt{2\pi}} e^{-(x-t)^2/2} \frac{\left(e^{-t(x-t/2)} - 1\right)^2}{-xe^{-t(x-t/2)} + x - t} - \Phi(x) + \Phi(x-t),$$

it is clear that

$$\lim_{x \to \infty} g(x) = 0.$$

Proof of Proposition 8

Using the chain rule,

$$\frac{d}{dC}\left\{\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}\right\} = \frac{d}{dX_\sigma}\left\{\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}\right\}\frac{dX_\sigma}{dC}$$

Since $\frac{dX_\sigma}{dC} < 0$ (Lemma 2), we have $\frac{d}{dC}\left\{\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}\right\} > 0$ if and only if $\frac{d}{dX_\sigma}\left\{\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}\right\} <$ 0.

Using (66) and $\varphi'(x) = -x\varphi(x)$ for the normal distribution, we have

$$\frac{d}{dX_\sigma}\left\{\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}\right\} = \frac{d}{dX_\sigma}\left\{\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \left[1 - \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)\right]\frac{\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)}\right\}$$

$$= \frac{1}{\sigma}\Bigg(-\frac{X_\sigma - \bar{e}}{\sigma}\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) + \left[\frac{X_\sigma - \bar{e}}{\sigma}\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \frac{X_\sigma - \underline{e}}{\sigma}\varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)\right]\frac{1 - \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)}$$
$$+ \left[\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)\right]\frac{\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)}$$
$$- \frac{1 - \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)}{\left(\Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)\right)^2}\left(\varphi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right) - \varphi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right)\right)^2\Bigg) \quad (76)$$

Multiplying all terms by $\sigma\left(\Phi\left(\frac{X_\sigma - \underline{e}}{\sigma}\right) - \Phi\left(\frac{X_\sigma - \bar{e}}{\sigma}\right)\right) > 0$, the expression in (76) has the same sign as

$$\left[\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}-\frac{X_\sigma-\underline{e}}{\sigma}\right]\left[\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\right]\right.$$
$$\left.-\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right]\right]-\frac{\overline{e}-\underline{e}}{\sigma}\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right]. \quad (77)$$

Since the last term in (77) is always negative, the expression in (77) is negative if the first line in (77) is negative. We now prove the latter.

The hazard rate $\varphi(x)/(1-\Phi(x))$ of the normal distribution is increasing, which implies that

$$\frac{\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{1-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}>\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}{1-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}.$$

Rearranging, we have

$$\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\right]-\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)\left[1-\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\right]>0 \quad (78)$$

Define

$$d(X_\sigma,\overline{e})\equiv\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}.$$

If $d(X_\sigma,\overline{e})<\frac{X_\sigma-\underline{e}}{\sigma}$, then combining with (78) establishes that (77) is negative, as desired. We now show that $d(X_\sigma,\overline{e})<\frac{X_\sigma-\underline{e}}{\sigma}$, by proving first that $d(X_\sigma,\overline{e})\longrightarrow_{\overline{e}\to 0}\frac{X_\sigma-\underline{e}}{\sigma}$ and second that $d(X_\sigma,\overline{e})$ is decreasing in $\overline{e}$.

First,

$$\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)=-\varphi'\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\frac{\overline{e}-\underline{e}}{\sigma}+O(\overline{e}^2)$$
$$\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)=\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\frac{\overline{e}-\underline{e}}{\sigma}+O(\overline{e}^2).$$

Using $\varphi'(x)=-x\varphi(x)$ for the normal distribution, we have

$$\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}\longrightarrow_{\overline{e}\to\underline{e}}\frac{\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)\frac{(\overline{e}-\underline{e})(X_\sigma-\underline{e})}{\sigma^2}}{\frac{\overline{e}-\underline{e}}{\sigma}\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}=\frac{X_\sigma-\underline{e}}{\sigma}.$$

Second,

$$\frac{d}{d\overline{e}}\left\{\frac{\varphi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)-\varphi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)}{\Phi\left(\frac{X_\sigma-\underline{e}}{\sigma}\right)-\Phi\left(\frac{X_\sigma-\overline{e}}{\sigma}\right)}\right\}=\frac{d}{d\overline{e}}\left\{\frac{\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma} q\exp\left\{-\frac{q^2}{2}\right\}dq}{\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\exp\left\{-\frac{q^2}{2}\right\}dq}\right\}$$

$$=\frac{1}{\sigma}\frac{\frac{X_\sigma-\overline{e}}{\sigma}\exp\left\{-\frac{(X_\sigma-\overline{e})^2}{2\sigma^2}\right\}\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\exp\left\{-\frac{q^2}{2}\right\}dq-\exp\left\{-\frac{(X_\sigma-\overline{e})^2}{2\sigma^2}\right\}\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}q\exp\left\{-\frac{q^2}{2}\right\}dq}{\left(\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\exp\left\{-\frac{q^2}{2}\right\}dq\right)^2}.$$

This expression has the same sign as

$$\frac{X_\sigma-\overline{e}}{\sigma}\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\exp\left\{-\frac{q^2}{2}\right\}dq-\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}q\exp\left\{-\frac{q^2}{2}\right\}dq$$

$$=\int_{(X_\sigma-\overline{e})/\sigma}^{(X_\sigma-\underline{e})/\sigma}\left[\frac{X_\sigma-\overline{e}}{\sigma}-q\right]\exp\left\{-\frac{q^2}{2}\right\}dq<0.$$

This establishes that $d(X_\sigma,\overline{e})$ is decreasing in $\overline{e}$, which completes the proof.

F Principal's participation constraint binds

In the core model, the principal has full bargaining power and offers the contract to the agent. The opposite assumption is for the agent to have full bargaining power and offer the contract to the principal, as in Innes (1990). Under this assumption, the agent (entrepreneur) raises an amount $I > 0$ from the principal (investor) to fund a project which produces output q. Now, it is the principal's, rather than the agent's, participation constraint that binds. We also assume that it is the agent who now chooses precision, since it is he that captures the surplus and thus bears any gains or losses from the incentive effect.

As discussed in Section 4.1, the optimal contract involves the principal receiving risky debt with face value X_θ. The entrepreneur, who holds equity in a levered firm, chooses X_θ to maximize his payoff, subject to his incentive constraint (79), the investor's participation constraint (80), and the contracting constraints (2) and (3):

$$\max_{X_\theta} \int_{X_\theta}^{\infty} (q - X_\theta) f_\theta(q|e^*)dq - C(e^*)$$

$$\text{subject to} \quad e^* \in \arg\max_e \int_{-\infty}^{\infty} W_\theta(q) f_\theta(q|e)dq - C(e), \tag{79}$$

$$\int_{-\infty}^{X_\theta} q f_\theta(q|e^*)dq + \int_{X_\theta}^{\infty} X_\theta f_\theta(q|e^*)dq = I. \tag{80}$$

We assume that there exists a face value X_θ and associated effort e^* such that the LHS of (80) is at least I (i.e., an optimal contract exists). Since $I > 0$, we have $X_\theta > 0$.

For a given debt contract, higher precision increases the investor's expected payoff on the LHS of (80) – lower risk increases the value of risky debt. This is the analog of the "direct effect" in the core model. This in turn allows the entrepreneur to reduce X_θ while still satisfying the investor's participation constraint, fully offsetting the direct effect. In turn, a lower X_θ increases the entrepreneur's effort incentives. This is a similar participation effect to Section 4.2 and means that the value of information again stems entirely from its effect on effort, which remains (26).

There are two differences when it is the agent rather than the principal who offers the contract and chooses precision. The first is that the level of precision chosen will be different. From (25), the value of information to the principal is given by

$$\frac{\partial}{\partial e}\mathbb{E}_\theta \left[R_\theta\left(q\right)|e_\theta(X_\theta)\right]\frac{de}{d\theta}, \tag{81}$$

where $\frac{de}{d\theta}$, in turn, is given by (26). The value of information to the agent when he offers the contract and chooses precision is given by

$$\frac{\partial}{\partial e}\mathbb{E}_\theta \left[W_\theta\left(q\right)|e_\theta(X_\theta)\right]\frac{de}{d\theta}. \tag{82}$$

The quantities given by (81) and (82) will typically be different, for the reasons given in the main text. The second difference is that the distinction between ex ante and ex post incentives is not the same as in the core model. If the effort effect is positive, the agent would like to commit to the highest possible level of precision ex ante, as this would maximize the value of the investor's debt claim, allowing him to minimize the face value of debt X_θ. This in turn maximizes his effort incentives, increasing total surplus and his profits (since he has full bargaining power, he captures the full surplus). However, if commitment is not possible and precision is chosen once the contract has been written and effort has been exerted, the agent will select the lowest possible level of θ to maximize the value of his levered equity. As a result, the agent may wish to commit to a high level of precision, for example by accepting covenants in the debt contract. The agent's benefits from precision are always higher ex ante than ex post, whereas the principal's benefits are higher ex ante if and only if the incentive effect is positive, and lower otherwise.[27]

G Social welfare

The core model studies the effect of precision on the principal's profits and, in particular, the agent's effort incentives. This section analyzes the effect of precision on social welfare and, in particular, whether the principal overinvests or underinvests in information relative to the social optimum.

We start with the fixed effort model of Section 3.3.2 as the results are clearest. Total surplus is determined entirely by expected output, $\mathbb{E}_\theta [q|e]$ minus the cost of effort; it does not depend on the wage as this is a mere transfer from the principal to the agent. In turn, expected output depends only on effort and not precision (since changes in precision represent a MPS, $\mathbb{E}_\theta [q|e]$ is independent of θ.) Thus, when effort is fixed, it is independent of precision, and so total surplus is independent of precision. In contrast, the principal has strict incentives to increase precision, because doing so reduces the expected wage: the total effect of precision is positive.[28] As a result, the principal always overinvests in precision: increases in precision have no social benefit but the principal has incentives to undertake them.

We now move to the endogenous effort model. Now, precision may change total surplus by affecting the implemented effort level. Let the principal choose precision at

[27]Note that the analysis of ex ante and ex post incentives when the principal chooses precision was with the fixed effort model. With endogenous effort, the analysis becomes substantially more complex and a pure strategy equilibrium may not even exist. This is because the agent will anticipate the future level of precision, and thus may change his effort choice.

[28]Indeed, if increasing precision were costless, the principal would choose infinite precision. Then, effort of $\overline{e}$ would lead to output of $\overline{e}$ with certainty, and the optimal contract would pay the agent C if $q = \overline{e}$ and zero otherwise. Thus, the agent is paid his cost of effort and does not receive any rents.

a cost $\kappa(\theta)$, where $\kappa(\cdot)$ is an increasing function. Her profit is given by

$$\Pi(\theta) \equiv \max_X \left\{ X - \int_{-\infty}^{X} F_\theta(q|e_\theta(X))dq - \kappa(\theta) \right\}.$$

The agent's utility is given by

$$\mathcal{A}(\theta) \equiv \max_e \left\{ \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e)dq - C(e) \right\}$$

and chooses effort to maximize $\mathcal{A}(\theta)$. Total surplus is the sum of profit and utility, i.e.,

$$\mathcal{U}(\theta) \equiv \Pi(\theta) + \mathcal{A}(\theta).$$

Let θ^S (θ^P) be the level of precision that maximizes social welfare (profit). We wish to study whether $\theta^S \gtrless \theta^P$, i.e., whether the principal underinvests or overinvests in precision relative to the social optimum. Define Z as the weighted average between social welfare and profit:

$$Z(\theta; \alpha) \equiv \alpha \mathcal{U}(\theta) + (1-\alpha)\Pi(\theta),$$

so that Z equals social welfare when $\alpha = 1$ and profit when $\alpha = 0$. The principal overinvests (underinvests) in precision if and only if the level of precision that maximizes Z is decreasing (increasing) in α, i.e., $\theta^S \leq (\geq) \theta^P$ if

$$\frac{\partial^2 Z}{\partial\theta\partial\alpha} \leq (\geq) 0.$$

We have

$$\frac{\partial^2 Z}{\partial\alpha\partial\theta} = \frac{d}{d\theta}[\mathcal{U}(\theta) - \Pi(\theta)] = \mathcal{A}'(\theta).$$

Thus, the principal overinvests (underinvests) in precision if and only if the agent's utility decreases (increases) with precision, i.e., $A'(\theta) \leq (\geq) 0$.

The intuition is that the principal has two incentives to increase precision. The first is that precision may increase effort. This incentive is fully aligned with the social welfare function. The second is that precision may reduce the agent's wage, redistributing surplus from the agent to herself. This effect has no impact on total surplus. Thus, the principal overinvests in precision if and only if precision reduces the agent's utility.

To study the impact on the agent's utility, note that, by the envelope theorem,

$$\begin{aligned}\mathcal{A}'(\theta) &= \frac{d}{d\theta}\left\{\mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e)dq - C(e)\right\}\Bigg|_{e=e_\theta(X)} \\ &= -\left[1 - F_\theta(X_\theta|e_\theta(X))\right]\frac{dX_\theta}{d\theta} + \int_{-\infty}^{X_\theta} \frac{\partial F_\theta}{\partial \theta}(q|e_\theta(X))dq\end{aligned}$$

Since precision orders F in terms of SOSD, the second term is negative. Holding constant the strike price, a decrease in precision reduces the value of an option – the well-known effect of volatility on option value. If the strike price does not fall with precision ($\frac{dX_\theta}{d\theta} \geq 0$), the first term is non-positive and so the agent's utility falls with precision overall. The agent's utility can only rise with precision if the strike price falls sufficiently to outweigh the impact of reduced volatility.

We apply this analysis to the model from Example 2. Recall from (20) that

$$\begin{aligned} & X^*_{\theta,\kappa} = \frac{1}{\theta} - \frac{1}{\kappa} + \frac{\theta}{2\kappa^2} \\ \implies \quad & \frac{dX^*_{\theta,\kappa}}{d\theta} = -\frac{1}{\theta^2} + \frac{1}{2\kappa^2},\end{aligned}$$

where we needed $\theta \leq 2\kappa$ for the FOA to be valid. Therefore,

$$\frac{dX^*_{\theta,\kappa}}{d\theta} \geq 0 \iff \theta \geq \sqrt{2}\kappa.$$

Thus, when $\theta \leq \sqrt{2}\kappa$, the principal underinvests in precision and when $\sqrt{2}\kappa \leq \theta \leq 2\kappa$, the principal overinvests in precision.

Overall, the force common to both the fixed and endogenous effort models is that the principal overinvests in precision if and only if the agent's utility falls with precision, since this fall is a transfer to the principal with no social benefit. With fixed effort, the agent's utility unambiguously falls with precision and so the principal unambiguously overinvests – intuitively, if agent utility rose with precision, the principal would have added randomness to the initial contract. With endogenous effort, agent utility can rise if the change in precision makes it optimal for the principal to significantly reduce the strike price to implement a higher effort level.

Finally, note that, if the agent's participation constraint is binding, as in Section 4.2, the principal's incentives to invest in precision are fully aligned with social welfare.

This is because, with a binding participation constraint, the agent's rents are zero regardless of the level of precision. Thus, the principal cannot use precision to reduce the agent's rents. Her only motive to increase precision is to increase incentives, which is fully aligned with social welfare.

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